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Modeling Expected Travel Distances for a Common Warehouse Design with Multiple Docks

Modeling Expected Travel Distances for a Common Warehouse Design with Multiple Docks

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

By

Mahmut Tutam
Gazi University
Bachelor of Science in Industrial Engineering, 2011

May 2015
University of Arkansas

This thesis is approved for recommendation to the Graduate Council.

Dr. John Austin White, Jr.
Thesis Director

Dr. Russell D. Meller
Committee Member

Dr. Letitia M. Pohl
Committee Member

Abstract

This thesis develops expected travel distance expressions for both single- and dual-command operations in a unit-load warehouse design with multiple docks. Storage racks are aligned perpendicular to the wall containing docks. Results are presented for continuous and discrete formulations. Because of the importance of how docks are located on a wall, different dock locations are investigated, including uniformly distributed docks along one wall, specified distances between adjacent docks located symmetrically about the mid-point of a warehouse wall, and any distribution of locations along one wall. Among the results obtained, we find that the width-to-depth ratio of the storage area (commonly called shape factor) that minimizes expected distance traveled is a function of the number of docks and their locations. We find that the spacing between adjacent docks and the distance the first dock is from either the left end of the wall containing the docks or the centerline of the warehouse can significantly affect the optimal shape factor. Two cases are treated for the distance between adjacent docks: a) the distance is a function of the width of the storage area or the width of the storage area is a function of the number of docks and the distance between them and b) the distance is a fixed value. In the former case, our results are consistent with those obtained by others; however, in the latter case, some of our results will be surprising to many who have studied similar design problems.

Acknowledgement

I would like to express my deep gratitude and thanks to my advisor, Professor John Austin White, for being a tremendous mentor through the completion of my master’s thesis. I appreciate his guidance, encouragement and useful critiques of the thesis. His experience and deep knowledge in industrial engineering contributed staggeringly. I can never find enough words to express my gratitude and appreciation to him. I hope that one day I will become as good an advisor to my students as Dr. White has been to me. Also, I am thankful to his wife, Mary Elizabeth Quarles White, for her understanding and allowing us to study during the holidays.

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Finally, my special thanks go to my family for their endless support, concern and love not only during my master’s study, but my whole life in all good and tough days. Without their belief and patience, I would never be where I am today. To all relatives, especially my guarantors; Yakup Tutam and Hacikaya Polat, friends and others who shared their moral and psychological supports, thank you.

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1 Introduction

Unit-load warehouses are commonly used in industry. Such warehouses receive and ship products on pallets or in other unit-load quantities. In a unit-load warehouse, storage racks are frequently aligned perpendicular to the wall containing the dock(s) to form picking aisles which are used to perform storage and retrieval operations. For unit-load storage and retrieval operations, either single- or dual-command operations are used.

In a single-command operation, a worker travels from a dock to a storage location where the unit-load is stored and returns empty to the dock or a worker travels empty from a dock to a storage location, retrieves a unit-load and transports it to the dock. A dual-command operation occurs when a worker travels from a dock to a storage location where the unit-load is stored, then travels empty to another storage location where another unit-load is retrieved before returning to the dock.

The performance of a warehouse is impacted by the number and locations of docks. In the warehouse configuration we consider, a single dock, centrally located along one wall is often assumed, with subsequent calculations based on this assumption. However, warehouses typically have multiple docks for receiving and shipping. Therefore, this study focuses on formulating expected travel distance expressions with multiple docks in a traditional layout with both single- and dual-command storage and retrieval operations. Our results indicate that the expected travel distances increase with an increasing number of docks.

The shape factor, the ratio of warehouse width to warehouse depth, is another important design parameter because the shape of the warehouse directly affects the number and length of the picking aisles. Studies in the literature are inadequate to show the relations between the shape factor and the number and locations of docks, as well as the distance between docks. Based on the formulation developed, we acknowledge that the shape factor is affected by the distance between docks in addition to the number and locations of the docks. Specifically, we found that the optimal shape factor decreases with an increasing number of docks if the distance between docks is a function of the width of the storage area; whereas, the optimal shape factor increases with an increasing number of docks if the distance between docks is a fixed value.

In the thesis, two approaches are used in formulating expected distances: discrete and continuous. Discrete formulations include the locations of aisles and storage locations within the warehouse and account for travel between any two storage locations in the warehouse occurring along an aisle or a combination of aisles; whereas, continuous formulations treat the interior of the warehouse as a continuous region with rectilinear travel occurring between any two storage points in the region.

Although discrete formulations yield more accurate representations of expected travel distances, results obtained from continuous approximations might not be significantly different from those obtained from discrete formulations. Continuous formulations tend to be more easily solved than their counterpart discrete formulations; as such, insights regarding the impact of the number and locations of docks on expected distance traveled are more easily obtained. In our research, the percentage error introduced by using a continuous approximation is obtained by comparing results from continuous formulations with discrete formulations.

Throughout this thesis, a random storage policy and rectilinear travel are assumed. We limit our study to travel on the floor of the warehouse and do not consider travel required to access tiers in the rack above floor-level. Hence, two travel distances are of interest: travel that is parallel to the wall(s) containing the dock(s) and travel that is perpendicular to the wall(s) containing the dock(s). In the study, we concentrate on parallel travel because, with rectilinear travel, neither perpendicular travel nor travel-between distance is affected by the dock(s) location(s).

Throughout, we assume constant travel velocity and ignore pickup and deposit times. Workers use picking aisles to access racks and use cross-aisles at the ends of the picking aisles to move between picking aisles. Picking aisles are considered to have the same width and to be two-way, such that workers performing storages and retrievals can access each side of the aisle.

The remainder of the thesis is organized as follows. First, we review the literature of the warehouse layout we consider for both single- and dual-command operations. In Section 3, we present the notation used to model travel distances for the warehouse configuration we consider. Section 4 addresses continuous space formulations and Section 5 provides discrete formulations for both parallel and perpendicular travel. In Section 6, we provide both a discrete formulation and a formulation that includes continuous approximation components for both single- and dual-command operations. (Multi-dock formulations are provided in Sections 4, 5 and 6.) While presenting the models, we perform sensitivity analyses to illustrate the effect various design decisions can have on expected distance traveled and on the warehouse shape that minimizes expected distance traveled. In Section 7, we examine the impact on shape factor and expected distance traveled by restricting certain docks to be used for receiving and others to be used for shipping. In Section 8, we examine a mixture of single-command and cross-docking travel in the warehouse and its impact on the shape factor. In Section 9, the research findings are summarized, design conclusions are drawn, the significance of the research is addressed, and recommendations are provided for further research. [1]

2 Literature Review

Francis (1967) appears to have been the first to model single-command travel distance for the warehouse layout we consider. He obtained the shape factor for a single dock that is centrally located along one wall. Bassan et al. (1980) provided cost models for the warehouse layout we consider by developing optimal design parameters. They analyzed optimal location of docks and concluded that all docks should be located as near as possible to the center of a single wall. Ratliff and Rosenthal (1983) developed a procedure to calculate the minimum pick tour for an order-picker. They defined this problem as an order-picking problem and recognized it is a travelling salesman problem which is an NP-Hard problem.

Hall (1993) compared some routing strategies for a manual order picker under assumptions of a random storage policy and a centrally located dock. Peterson (1997) and de Koster and van der Poort (1998) used simulation to compare the results for optimal and heuristic routes with a random storage policy for the warehouse layout we consider. Hwang et al. (2004) used simulation results to demonstrate the validity of their formulaic results.

Mayer (1961) appears to have been the first researcher to model dual-command operations for the warehouse layout we consider by permitting an order-picker to move two loads in a cycle. Malmborg and Krishnakumar (1987) assumed an order-picker interleaves a storage and a retrieval in each operation in their formulation of order picking costs in the warehouse layout we

[1] Some parts of this thesis can be found in the paper by Tutam and White (2015), particularly the first three sections of the thesis.

consider. Pohl et al. (2009) were the first to describe optimal dual-command travel distances for a range of traditional layouts. They modeled the expected single- and dual-command travel distances in the warehouse layout we consider under the assumption of a centrally located dock. They considered the layout to be a special case of a multi-aisle automated storage and retrieval system (AS/RS) by ignoring vertical travel in the model derived by Hwang and Ko (1988) and assuming the dock is centrally located on a warehouse wall. They determined the number of aisles which minimizes dual-command travel distances for traditional layouts. Expected distance between two random points in a warehouse is defined as the "travel-between" (*TB*) distance in their model.

As noted, shape factor directly affects the number and length of the picking aisles. The shape factor which minimizes the expected travel distance in a warehouse is defined as the optimal shape factor and denoted as S^*. Francis (1967) investigated the impact of warehouse shape and concluded that the optimal width-to-depth ratio is 2:1 for a centrally located dock. Specifically, for single-command operations, Thomas and Meller (2014) show that S_{SC}^* ranges from 2:1 to 1.5:1 as the number of docks ranges from one to infinity. They considered docks are located over the entire width of the warehouse; hence, if the width increases, either the number of docks or the distance between adjacent docks increases. Our work differs from theirs by treating the number of docks and the spacing between adjacent docks as a design parameter, rather than a decision variable. In addition, we do not require that docks be uniformly distributed about the centerline of the warehouse.

3 Notation

In this section, we define the notation used in the thesis, as illustrated in Figure 1.

a = distance between centerlines of adjacent aisles

n = number of picking aisles

W = width of the storage area

L = length of picking aisles

v = half the width of a cross-aisle

D = depth of the storage area ($D = L + 4$v)

A = total storage area ($A = WD$)

S = shape factor ($S = W / D$)

k = number of docks

ω = the width of a dock

δ = distance between the centerlines of two adjacent docks (i.e. i^{th} and $(i+1)^{th}$ docks) ($\delta > \omega$)

SC = single-command distance

TB = travel-between distance

DC = dual-command distance (DC = SC + TB)

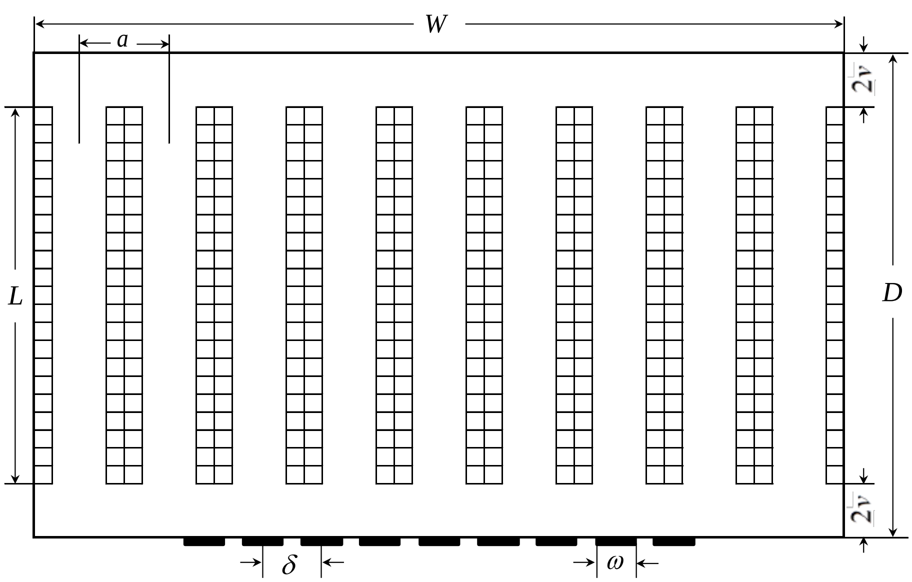

Figure 1: Warehouse notation

4 Continuous Space

In this section, we develop models of expected travel distance for single-command operations in a unit-load warehouse with one or more dock(s). However, in contrast to Figure 1, we ignore the storage racks and aisles and assume that storage and retrieval points are uniformly distributed over a rectangular region of width W and depth D.

We calculate the expected rectilinear distance to perform a single-command operation between a centrally located dock and a random location in the warehouse by treating separately the two dimensions of rectilinear travel: (1) parallel travel, and (2) perpendicular travel. Given a storage width of W and uniformly distributed destination points, the expected parallel distance between a centrally located dock and a random point is $W / 4$; therefore, the roundtrip expected distance is $W / 2$. Given a storage depth of D and uniformly distributed destination points, the expected perpendicular distance between a centrally located dock and a random point is $D / 2$; hence, the roundtrip expected distance is D. Finally, the expected distance to perform a single-command operation is

$$E[SC] = W/2 + D \tag{1}$$

By using the relationship between a given area ($A = WD$) and the shape factor ($S = W / D$), we can obtain the width and length of the warehouse with respect to the given area and the shape factor as follows. Because $W = SD$, $A = SD^2$ and $D = \sqrt{A/S}$. Hence, $W = SD = \sqrt{AS}$. Therefore, replacing W and D with $\sqrt{AS}$ and $\sqrt{A/S}$ respectively, in Equation 1, the expected single-command distance can be expressed as a function of A and S

$$E[SC] = \sqrt{A\,S}/2 + \sqrt{A/S} \tag{2}$$

Taking the first derivative of Equation 2 with respect to *S*, setting it equal to zero, and solving for *S*, we obtain the optimal shape factor, $S^*_{SC} = 2$. The same result is obtained by Francis (1967). So, for a single, centrally located dock, the optimal shape factor (width-to-length) is 2:1.[2]

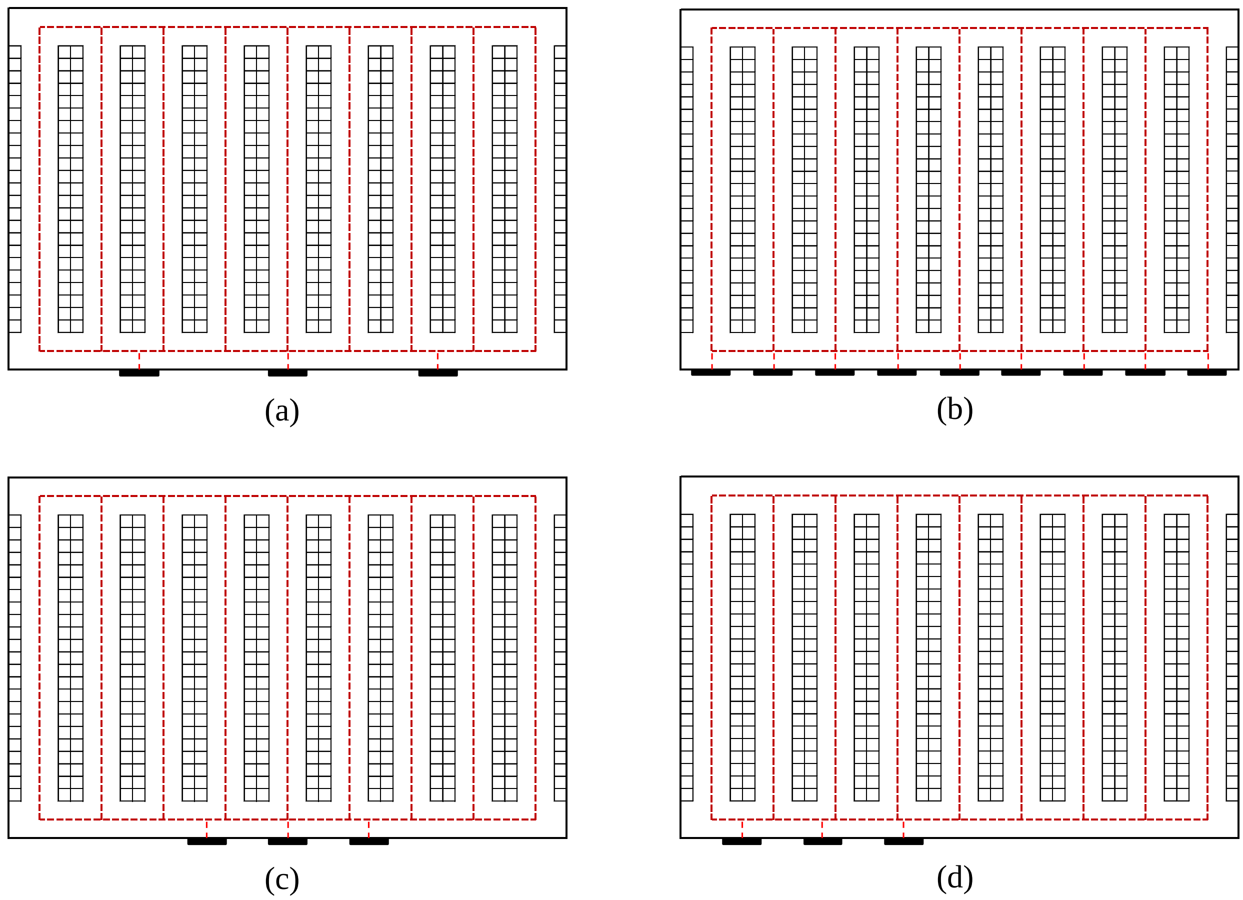

Figure 2: (a) $k = 3$ uniformly distributed docks, (b) k docks located across the entire wall, (c) $k = 3$ centrally located on the wall with a fixed distance between adjacent docks, and (d) $k = 3$ docks located along a wall with a fixed distance between adjacent docks

Relaxing the single dock limitation, we treat four cases for the location of *k* docks: (a) a given value of *k* uniformly distributed along a wall (see Figure 2.a), (b) a flexible value of *k* with docks covering entirely a wall (see Figure 2.b), (c) centrally located on a wall with a fixed distance

[2] The second derivative is positive, so the function is convex and the value obtained is the optimal shape factor. The convexity of the expected distance function holds throughout the thesis for subsequent calculations of the optimal shape factor.

between adjacent docks (see Figure 2.c), and (d) not centrally located on a wall, but with a fixed distance between adjacent docks (see Figure 2.d). In addition to the cases shown in Figure 2, we considered the case of a single dock that is located a specified distance from the centerline of the warehouse.

In continuous space, we do not attempt to approximate the expected dual-command distance. To do so, we would need to sum the expected single-command distance and the expected travel-between distance. From Bozer (1985), the expected parallel distance between two independent uniformly distributed points over the interval (0, W) is W / 3. Likewise, the expected perpendicular distance is D / 3. However, as evident from Figure 3, given the rack structure in the warehouse, assuming rectilinear travel can occur between two random points is not realistic.

Specifically, because travel must occur along aisles, rectilinear travel cannot occur between two points in different picking aisles; instead, it requires perpendicular travel to the end of the picking aisle, plus parallel travel to the next picking aisle, plus perpendicular travel to the destination point in the picking aisle. Hence, a continuous space approximation of travel-between distance significantly underestimates the actual distance traveled.

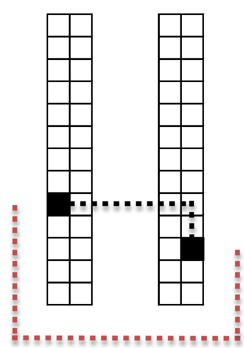

Figure 3: Continuous (black) and discrete (red) travel distances

4.1 *k* docks uniformly located on one wall

Because perpendicular travel does not depend on the number and location(s) of dock(s), adding another dock to the warehouse does not affect the perpendicular travel distance. Therefore, we are interested in measuring the expected parallel distance traveled. We begin by assuming two docks (k = 2) are located along one wall in such a way that one-third of the wall is to the left of the centerline of the leftmost dock and one-third of the wall is to the centerline of the right of the rightmost dock (hereafter, we refer to this distribution of docks as being uniformly distributed along the wall). Assuming the leftmost dock is the first dock, others are numbered sequentially through to the rightmost dock. The average roundtrip parallel distance traveled to the left of the centerline of the first dock is W / 3 and the probability travel will be to the left is 1 / 3; also, the average parallel distance traveled to the right of the centerline of the first dock is $2W$ / 3 and the probability travel will be to the right is 2 / 3. Therefore, the expected parallel distance traveled from the centerline of the first dock to a random point is[3]

$$E[D_1] = (1/3)(W/3) + (2/3)(2W/3) = 5W/9 \tag{3}$$

With the dock locations being symmetric about the center of the warehouse, the expected roundtrip distance expression for the second dock is the same as the first dock, $E[D_1] = E[D_2]$. Assuming each dock is equally likely to be used, the expected single-command travel distance can be written as

$$E[SC] = 5W/9 + D \tag{4}$$

Substituting shape factor and storage area into Equation 4 yields

[3] Hereafter, distances to, from, and between docks are based on centerlines of the docks.

$$E[SC] = 5\sqrt{A\,S}\Big/9 + \sqrt{A/S} \tag{5}$$

Taking the first derivative of Equation 5 with respect to *S* and setting it equal to zero, we obtain

$$d\,E[SC]/d\;S = 5\sqrt{A/S}\Big/18 - \sqrt{A/S}\Big/2S = 0 \tag{6}$$

Solving for S_{SC}^{*}, we obtain a value of 1.8.

Using similar steps, we can develop a general expression for *k* uniformly located docks along one wall. The average roundtrip parallel distance traveled to the left of the i^{th} dock is $(i * W)/(k+1)$ and the probability travel will be to the left is $i/(k+1)$; also, the average roundtrip parallel distance to the right of the i^{th} dock is $((k+1-i)W)/(k+1)$ and the probability travel will be to the right is $(k+1-i)/(k+1)$. Therefore, the expected single-command distance for *k* docks is given by

$$E[SC] = \frac{1}{k}\sum_{i=1}^{k}\left\{\left(\frac{i}{k+1}\right)\left(\frac{i\,W}{k+1}\right) + \left(\frac{k+1-i}{k+1}\right)\left(\frac{(k+1-i)W}{k+1}\right)\right\} + D$$

$$= \frac{(2k+1)W}{3(k+1)} + D \tag{7}$$

Following the same steps as for *k* = 1 and *k* = 2, we obtain S_{SC}^{*} as

$$S_{SC}^{*} = \frac{3(k+1)}{(2k+1)} \tag{8}$$

As *k* approaches infinity, S_{SC}^{*} approaches 1.5. A similar calculation by Thomas and Meller (2014), with uniform dock usage and random storage, also yields an optimal shape factor of 1.5.

Based on Equation 8, we can determine the shape factor for any given number of uniformly distributed docks.

From Equation 8, the optimal width (W^*) and depth (D^*) for the warehouse are given by $W^* = \sqrt{A[3(k+1)/(2k+1)]}$ and $D^* = \sqrt{A[(2k+1)/3(k+1)]}$. The minimum expected single-command distance is given by $E^*[SC] = \sqrt{A[(2k+1)/3(k+1)]} + \sqrt{A[(2k+1)/3(k+1)]}$. Therefore, when optimally configured, the expected parallel distance equals the expected perpendicular distance, and the warehouse is "balanced" insofar as parallel and perpendicular travels are concerned.

4.2 *k* docks located across an entire wall

Thomas and Meller (2014) considered locating docks across an entire wall. We can interpret their approach as locating the maximum number (k) of docks along one wall, as in Figure 2.b, and letting the distance between adjacent docks equal δ. In such a case, the width of the warehouse (W) equals $(k + 1)\delta$, and the expected single-command distance is given by

$$E[SC] = \frac{(k+1)\delta}{2} + D \tag{9}$$

Letting $D = A / (k + 1)\delta$ taking the first derivative of Equation 9 with respect to k, setting the result equal to zero and solving for k gives $k^* = \sqrt{2A/\delta^2} - 1$, $D^* = \sqrt{A/2}$ and $W^* = (k+1)\delta = \sqrt{2A}$. Solving for the optimal shape factor, we obtain a value of S^*_{SC} = 2. Therefore, dock configurations similar to those in Figures 2.a and 2.b yield the same optimal shape factor for single-command travel. Likewise, when designed optimally, the expected parallel travel equals the expected perpendicular travel.

4.3 *k* docks centrally located on the wall with a fixed distance between adjacent docks

Another relaxation of the single dock assumption is to locate *k* docks centrally, but with a fixed distance between adjacent docks. As noted previously, perpendicular travel is not affected by the number and locations of docks. Therefore, we are again interested in formulating the expected parallel distance. Assuming two docks are located along one wall with a fixed distance (δ) between adjacent docks, the average roundtrip parallel distance to the left of the first dock is $(W - \delta) / 2$ and the probability travel will be to the left is $(W - \delta) / 2W$; also, the average roundtrip parallel distance to the right of the first dock is $(W + \delta) / 2$ and the probability travel will be to the right is $(W + \delta) / 2W$. Therefore, the expected roundtrip parallel distance between the first dock and a random point is as follows

$$E[D_1] = \left(\frac{W-\delta}{2}\right)\left(\frac{W-\delta}{2W}\right) + \left(\frac{W+\delta}{2}\right)\left(\frac{W+\delta}{2W}\right) = \frac{W^2+\delta^2}{2W} \tag{10}$$

Because the expected roundtrip distance expression for the second dock is the same as the first dock, $E[D_1] = E[D_2]$. With the assumption of docks being used equally, the expected single-command distance can be written as

$$E[SC] = \frac{W^2+\delta^2}{2W} + D \tag{11}$$

As before, we substitute $\sqrt{A\,S}$ and $\sqrt{A/S}$ in Equation 11 for W and D, respectively. Then, taking the first derivative with respect to S, setting it equal to zero and solving for S yields

$$S_{SC}^{*} = 2 + \frac{\delta^2}{A} \tag{12}$$

Following the same steps used for k = 2, we develop a general expression for k docks with a fixed distance (δ) between adjacent docks. The average roundtrip parallel distance to the left of the i^{th} dock is $\{W - [k-(2i-1)]\delta\}/2$ and the probability travel will be to the left is $\{W - [k-(2i-1)]\delta\}/2W$; also, the average roundtrip parallel distance to the right of the i^{th} dock is $\{W + [k-(2i-1)]\delta\}/2$ and the probability travel will be to the right is $\{W + [k-(2i-1)]\delta\}/2W$. Therefore, the expected single-command distance for k docks with a fixed value δ is

$$E[SC] = \frac{1}{k}\sum_{i=1}^{k}\left\{\left(\frac{W-[k-(2i-1)]\delta}{2}\right)\left(\frac{W-[k-(2i-1)]\delta}{2W}\right) + \left(\frac{W+[k-(2i-1)]\delta}{2}\right)\left(\frac{W+[k-(2i-1)]\delta}{2W}\right)\right\} + D$$

$$E[SC] = \frac{W}{2} + \frac{\delta^2(k^2-1)}{6W} + D \tag{13}$$

By using the same steps as used for k = 2, we obtain S_{SC}^{*} as

$$S_{SC}^{*} = 2 + \frac{\delta^2(k^2-1)}{3A} \tag{14}$$

In contrast to earlier results, with multiple docks separated by a fixed distance, the optimal shape factor is greater than 2. Unlike the case with uniformly distributed docks, the optimal shape factor increases with an increasing number of docks. This result is important because warehouses having a fixed distance between adjacent docks are quite common. Obtaining an optimal shape factor greater than 2 is an unexpected, but useful result.

To illustrate the impact of the number of docks on the optimal shape factor, we consider the following values for the design parameters: A = 60,000 ft^2 and δ = 12 ft. As shown in Figure 4, with single-command operations, the optimal shape factor increases with an increasing number of docks when the distance between docks is fixed, whereas it decreases with an increasing number of uniformly distributed docks.

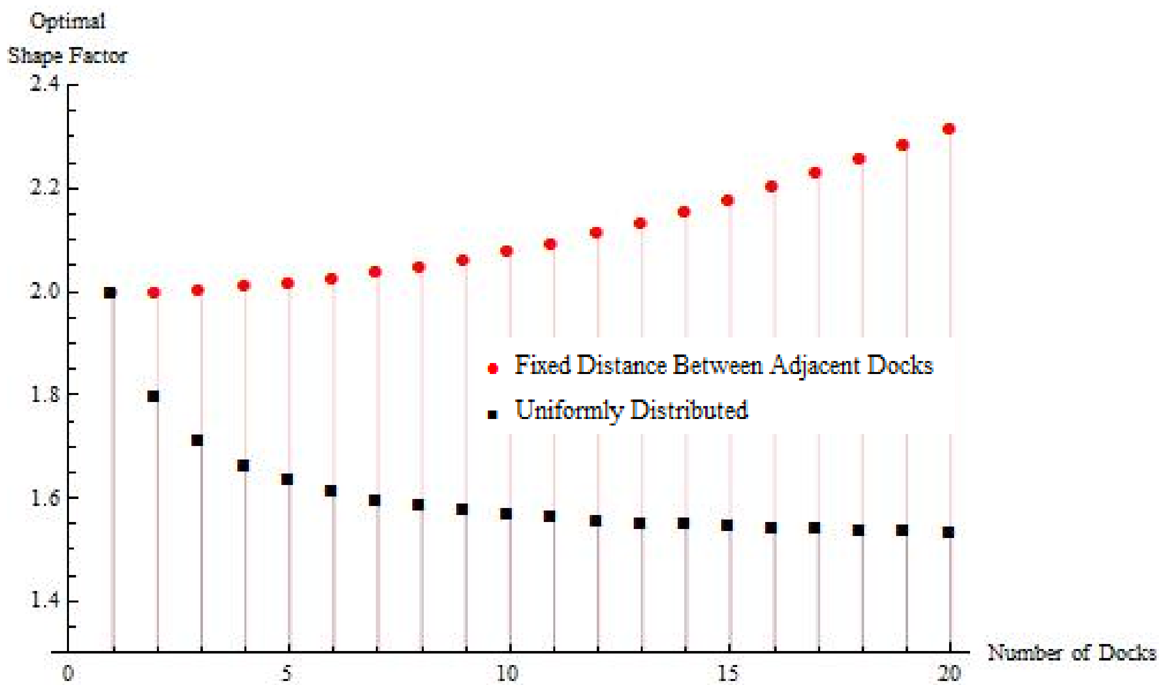

Figure 4: Optimal shape factor values for single-command operations with different dock distributions when A = 60,000 ft^2.

Why do the differences reflected in Figure 2.a and 2.c yield such different optimal shape factors? Because the proportionality among D, W, and δ is lost when the spacing between docks is fixed. We recognize the result is counter intuitive. As noted, with a specified number of uniformly distributed docks along one wall, when optimally configured the expected parallel distance equals the expected perpendicular distance. That is not the case with a specified number of docks and a specified distance between adjacent docks, when they are centrally located along one wall.

From Equations 13 and 14, with, say, $k = 2$, $E[SC] = W/2 + \delta^2/2W + D$ and $S_{SC}^* = 2 + \delta^2/A = W/D$. Therefore, $D^* = W/\left(2 + \delta^2/A\right)$. Hence, the optimal parallel distance is $W/2 + \delta^2/2W$ and the optimal perpendicular distance is $AW/\left(2A + \delta^2\right)$. It can be shown that the expected parallel distance is greater than the expected perpendicular distance when the warehouse is optimally shaped. Therefore, in contrast to the case with uniformly distributed docks, having specified distances between adjacent docks creates an imbalance in parallel and perpendicular expected distances and results in the warehouse being wider and shallower (less deep) than for the uniformly distributed case. Basically, if W is a function of either the number or spacing between docks, as in Figure 2.a and 2.b, then the optimal shape factor for single-command operations is less than or equal to 2.0. However, if the number of and spacing between docks is fixed, then the optimal single-command shape factor for single-command operations is greater than 2.0.

4.4 *k* docks not centrally located on the wall with a fixed distance between adjacent docks

Next, we relax the assumption that the docks are centrally located. As noted previously, perpendicular travel does not depend on the number and locations of docks. Therefore, we are again interested in formulating the expected parallel distance. We begin by assuming a single dock is located with a fixed distance (ϕ) from the left wall. The average roundtrip parallel distance to the left of the dock is ϕ and the probability travel will be to the left is ϕ / W; also, the average roundtrip parallel distance to the right of the dock is ($W - \phi$) and the probability travel will be to the right is ($W - \phi$) / W. Therefore, the expected single-command distance is

$$E[SC] = \frac{\phi^2}{W} + \frac{(W-\phi)^2}{W} + D = W + \frac{6\phi^2}{3W} - 2\phi + D \tag{15}$$

As before, substituting $\sqrt{A\,S}$ and $\sqrt{A/S}$ in Equation 15 for W and D, respectively, then, taking the first derivative with respect to S, setting it equal to zero and solving for S yields

$$S_{SC}^{*} = 1 + \frac{2\phi^2}{A} \tag{16}$$

Assume two docks are located along one wall in such a way that the first dock is located a distance ϕ from the left wall and there is a fixed distance δ between the docks. For the first dock, the average roundtrip parallel distance to the left of the dock is ϕ and the probability travel will be to the left is ϕ / W; also, the average roundtrip parallel distance to the right of the first dock is ($W - \phi$) and the probability travel will be to the right is ($W - \phi$) / W. Unlike models developed in previous sub-sections (Because $E[D_1] \neq E[D_2]$), we must also explicitly account for the location of the second dock. So, for the second dock, the average roundtrip parallel distance to the left of the dock is ($\phi + \delta$) and the probability travel will be to the left is ($\phi + \delta$) / W; also, the average roundtrip parallel distance to the right of the second dock is ($W - \phi - \delta$) and the probability travel will be to the right is ($W - \phi - \delta$) / W. Therefore, the expected single-command distance between a randomly selected dock and a random point in the warehouse is

$$E[SC] = \frac{1}{2}\left[\frac{\phi^2}{W} + \frac{(W-\phi)^2}{W}\right] + \frac{1}{2}\left[\frac{(\phi+\delta)^2}{W} + \frac{(W-\phi-\delta)^2}{W}\right] + D$$

$$= W + \frac{6\phi^2 + 6\phi\delta + 3\delta^2}{3W} - (2\phi + \delta) + D \tag{17}$$

Substituting *S* and *A* in Equation 17, taking the first derivative for shape factor, setting it equal to zero and then solving in terms of *S* yields

$$S_{SC}^* = 1 + \frac{2\phi^2 + 2\phi\delta + \delta^2}{A} \tag{18}$$

Using steps similar to those employed previously, we develop a general expression for *k* docks where the leftmost dock is located a fixed distance ϕ from the left wall and a fixed distance δ between adjacent docks as illustrated in Figure 2.c. The average roundtrip parallel distance to the left of the i^{th} dock is $[\phi + (i-1)\delta]$ and the probability travel will be to the left is $[\phi + (i-1)\delta]/W$; also, the average roundtrip parallel distance to the right of the i^{th} dock is $[W - \phi - (i-1)\delta]$ and the probability travel will be to the right is $[W - \phi - (i-1)\delta]/W$. Therefore, the expected single-command distance between *k* docks and a random point in the warehouse is

$$E[SC] = \frac{1}{k}\sum_{i=1}^{k}\left\{\frac{[\phi + (i-1)\delta]^2}{W} + \frac{[W - \phi - (i-1)\delta]^2}{W}\right\} + D$$

Reducing, we obtain

$$E[SC] = W + \frac{6\phi^2 + 6(k-1)\phi\delta + (2k^2 - 3k + 1)\delta^2}{3W} - (2\phi + (k-1)\delta) + D \tag{19}$$

Using steps similar to those employed for $k = 1$, we obtain S_{SC}^* as

$$S_{SC}^* = 1 + \frac{6\phi^2 + 6(k-1)\phi\delta + (2k^2 - 3k + 1)\delta^2}{3A} \tag{20}$$

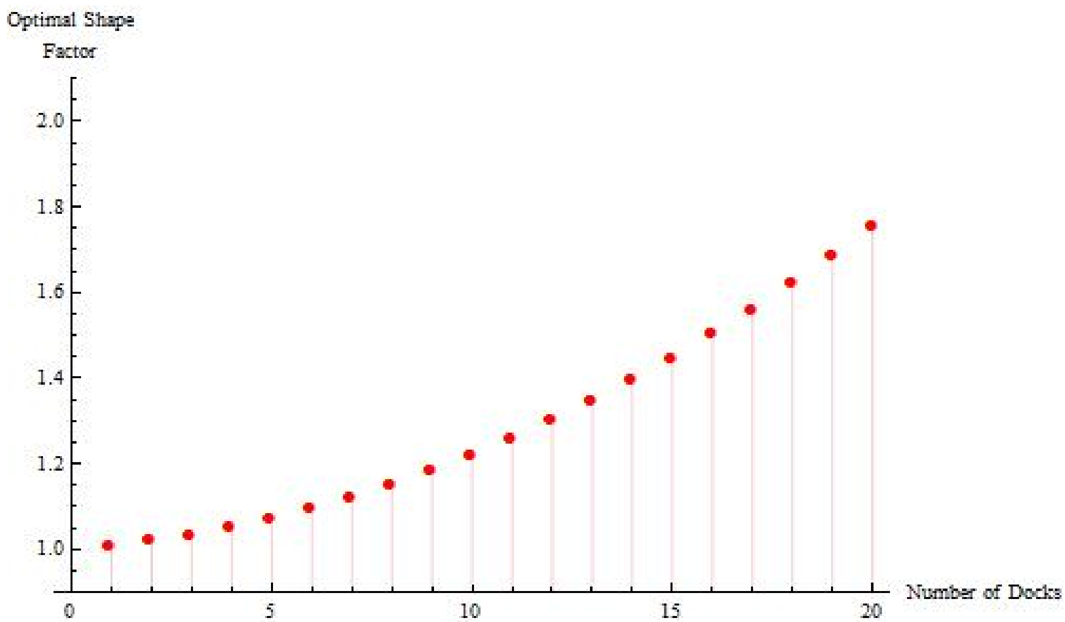

Figure 6: Optimal shape factor with ϕ = 20 ft for single-command operations

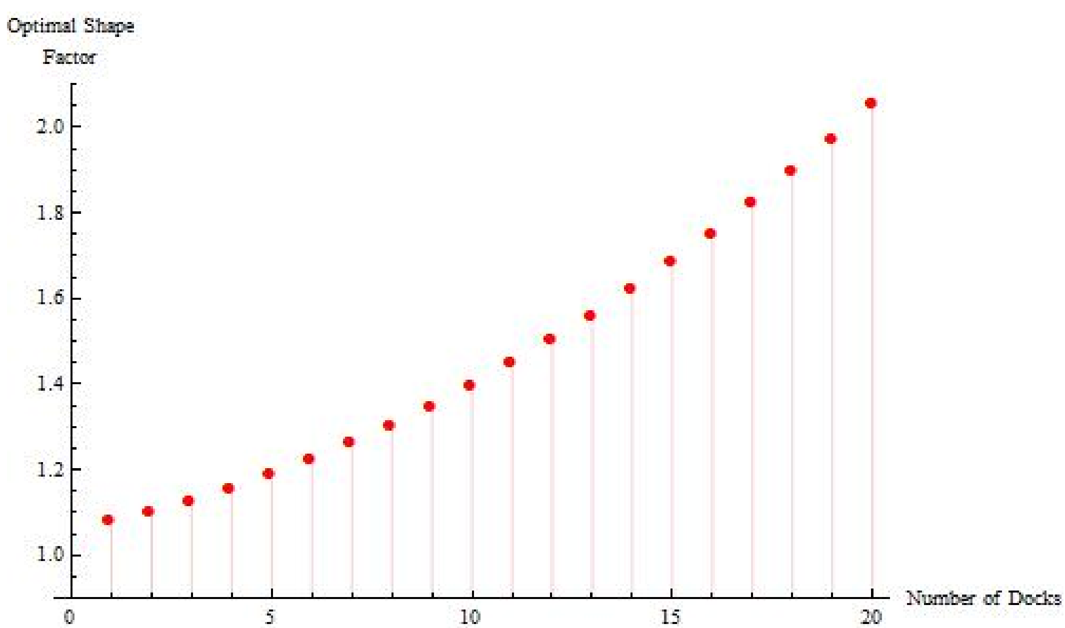

Figure 6: Optimal shape factor with ϕ = 50 ft for single-command operations

As expected, increasing k, δ, and/or ϕ, increases S^*_{SC}. Also, increasing A decreases S^*_{SC}. For example, with A = 60,000 ft^2 and δ = 12 ft, Figures 5 and 6 illustrates the impact of the number of docks on the optimal shape factor for different values of ϕ. Notice, the optimal shape factor for single-command operations is less than 2.0 for the smaller value of ϕ and, for the larger value of ϕ is less than 2.0 for a small number of docks.

4.5 Single dock offset by θ from centerline of the warehouse

Unlike Section 4.4, in this section we assume the location of a single dock is measured from the centerline of the warehouse instead of the left wall of the warehouse. We develop a general expression for a single dock located a fixed value θ from the centerline of the warehouse. For the dock, the average roundtrip parallel distance to the left of the dock is (W / 2 – θ) and the probability travel will be to the left is (W / 2 – θ) / W; also, the average roundtrip parallel distance to the right of the dock is (W / 2 + θ) and the probability travel will be to the right is (W / 2 – θ) / W. Therefore, the expected single-command distance between a single dock and a random point in the warehouse is

$$E[SC] = \frac{(W/2-\theta)^2}{W} + \frac{(W/2+\theta)^2}{W} + D = \frac{W}{2} + \frac{2\theta^2}{W} + D \tag{21}$$

As before, substituting $\sqrt{A\,S}$ and $\sqrt{A/S}$ in Equation 21 for W and D, respectively, taking the first derivative with respect to S, setting it equal to zero and solving for S yields

$$S_{SC}^{*} = 2 + \frac{(2\theta)^2}{A} \tag{22}$$

From Equation 22, if the non-central location of the dock is determined by measuring the offset distance from the centerline of the warehouse, rather than measuring the distance from the left wall, the optimal shape factor is greater than 2.0. [4]

5 Discrete Space

Although continuous space formulations can be used to gain useful insights regarding the effect of the number and location of docks on the optimal shape factor, they do not accurately measure the travel that occurs in a warehouse. This is particularly true for dual-command operations. Specifically, as noted previously, the expected rectilinear travel-between distance calculation is not a realistic measurement of travel between two points in different picking aisles.

In this section, we develop a general formulation of the expected parallel distance traveled from docks located at any points along a wall to random points in storage. When continuous space formulations were used, we ignored the locations of docks relative to centerlines of picking aisles and back-to-back rack locations. Here, we wish to calculate the exact distance between the centerline of a dock and the nearest centerline of a picking aisle. Knowing that distance, the distances to all other picking aisles are easily calculated.

To facilitate the development of the general formulation of the expected parallel distance traveled from docks located at any points along the wall, we let d_i denote the distance from the left end of the wall containing the docks to the centerline of dock i and t_i denote the distance from the left end of the wall containing the docks to the back-to-back rack location that is closest to dock i. In so doing, we consider four cases involving d_i and t_i: (a) $d_i > t_i$, (b) $d_i < t_i$, (c) $d_i = t_i$, and (d) $| d_i – t_i | = a / 2$.

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[4] Letting θ be the distance from the centerline to the dock, the optimal single-command shape factor is $2 + (2\theta)^2 / A$ when there is a single dock. Whereas, from Equation 27, with a single dock the optimal single-command shape factor is $1 + 2\phi^2 / A$.

For the case shown in Figure 7.a, because $d_i > t_i$, dock i is located to the left of the closest picking aisle. Therefore, the distance to the closest picking aisle is $a / 2 + t_i - d_i$, which means the closest picking aisle is to the right of dock i. To the left of dock i the number of picking aisles equals t_i / a, and the distance to the closest picking aisle to the left of dock i equals $a / 2 - t_i + d_i$. Therefore, the distance traveled from dock i to picking aisle j to the left equals $(j-1)a + a/2 - t_i + d_i$ for $j = 1, 2, \ldots, t_i / a$. To the right of dock i, there are $n - t_i / a$ picking aisles, and the distance to the closest picking aisle to the right of dock i equals $a / 2 + t_i - d_i$, so the distance traveled from dock i to picking aisle j to the right equals

$(j - t_i/a - 1)a + a/2 + t_i - d_i$ for $j = t_i / a + 1, t_i / a + 2, \ldots, n$.

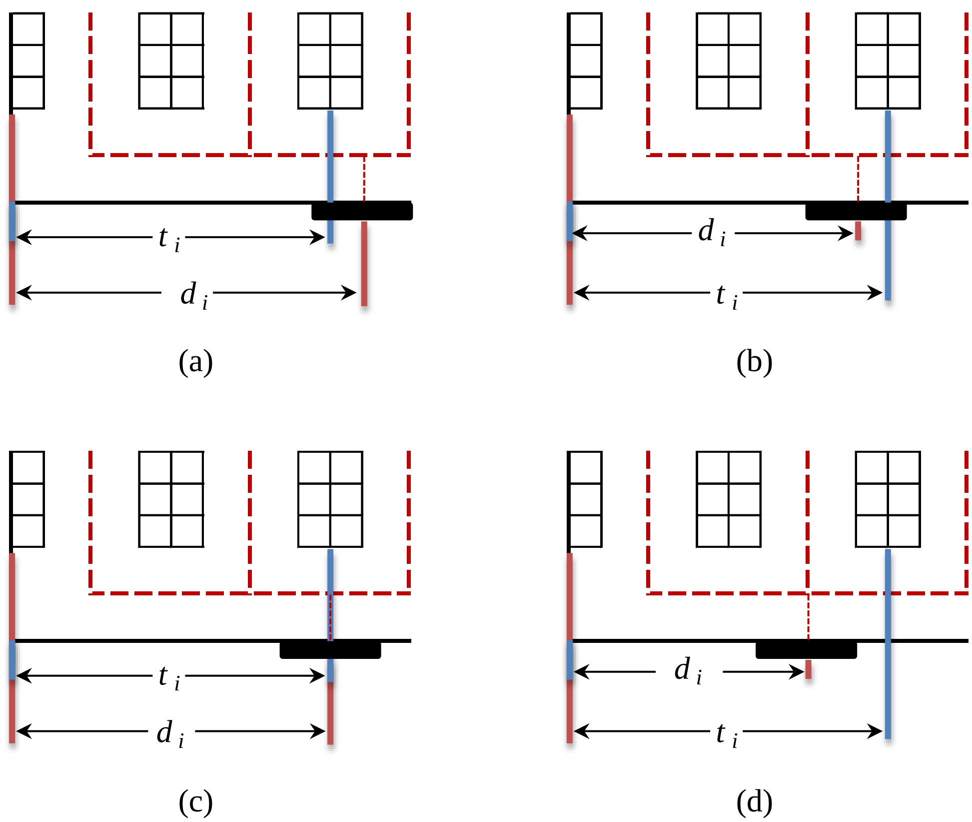

Figure 7: Dock locations (a) $d_i > t_i$, (b) $d_i < t_i$, (c) $d_i = t_i$ and (d) $| d_i - t_i | = a / 2$

For the case illustrated in Figure 7.b, because $d_i < t_i$, dock i is located to the right of the closest picking aisle. Therefore, the distance to the closest picking aisle is $a / 2 - t_i + d_i$, which is to the left of dock i. The number of picking aisles to the left of t_i still equals t_i / a, and the number of picking aisles to the right of t_i still equals $n - t_i / a$, because we are measuring t_i from the left end of the wall containing the docks. Consequently, the equations for case (a) apply for case (b).

For the case shown in the Figure 7.c, because $d_i = t_i$, the location of dock i coincides with a back-to-back rack location. Therefore, the distance to the closest picking aisle is $a / 2$, and the number of picking aisles to the left of t_i continues to equal t_i / a and the number of picking aisles to the right of t_i continues to equal $n - t_i / a$. Again, the equations for case (a) apply, because $(t_i - d_i)$ equals zero.

For the case illustrated in Figure 7.d, because $|d_i - t_i| = a / 2$, dock i is located at a centerline of a picking aisle. Therefore, no parallel travel occurs to reach the nearest picking aisle. To resolve the choice of traveling to the right or to left to reach the "nearest" adjacent picking aisle, we resolve the decision by using a *Mathematica* equation, Round[x, y]. Specifically, we let t_i = Round [d_i, a]. Alternately, using *Excel*, we can let $t_i = a$ * ROUND (d_i/a, 0) or t_i = MROUND (d_i, a). Defining t_i in this way results in the equations for case (a) applying to case (d).

With random storage, each picking aisle is equally likely to be visited. Therefore, to determine the expected parallel roundtrip distance for dock i, we add the two equations shown above, multiply the sum by 2 and divide by the number of picking aisles. For k docks, the expected parallel roundtrip distance is given by

$$E[SC]=\left(\frac{2}{n\,k}\right)\sum_{i=1}^{k}\left(\sum_{j=1}^{\frac{t_i}{a}}\left((j-1)a+\frac{a}{2}-t_i+d_i\right)+\sum_{j=\frac{t_i}{a}+1}^{n}\left(\left(j-\frac{t_i}{a}-1\right)a+\frac{a}{2}+t_i-d_i\right)\right) \qquad (23)$$

Simplifying Equation 23, the expected parallel roundtrip follows

$$E[SC]=\left(\frac{1}{a\,n\,k}\right)\sum_{i=1}^{k}\left(a^2n^2-2a\,n\,d_i+4d_i\,t_i-2t_i^{\,2}\right) \qquad (24)$$

Because perpendicular distance does not depend on the location of the dock(s), to obtain the expected perpendicular distance expression for any dock we calculate the expected number of storage locations visited in a storage row and multiply the value obtained by the product of 2 and the width of a storage location

$$E[SC]=2w\left(\frac{1}{m}\right)\sum_{j=1}^{m-1}j+2(v+f) \qquad (25)$$

where m is the number of storage locations along one side of a picking aisle, w is the width of a storage location and f is the distance from the centerline of the first storage location to the centerline of cross-aisle located between the docks and storage racks ($f = v + w/2$).

The summation in Equation 25 reduces to $w\ (m - 1)$. Substituting $v + w/2$ for f and $D – 4v$ for wm, we obtain the expected perpendicular distance

$$E[SC]=D \qquad (26)$$

To obtain the expected single-command distance, we sum Equation 24 and Equation 26. A general discrete expression for the expected single-command distance follows

$$E[SC] = D + \left(\frac{1}{a\,n\,k}\right)\sum_{i=1}^{k}\left(a^2n^2 - 2a\,n\,d_i + 4d_i\,t_i - 2t_i^{\,2}\right) \tag{27}$$

letting $C = \left(\frac{1}{a\,n\,k}\right)\sum_{i=1}^{k}\left(a^2n^2 - 2a\,n\,d_i + 4d_i\,t_i - 2t_i^{\,2}\right)$

$$E[SC] = D + C \tag{28}$$

where D is the expected perpendicular distance and C is the expected parallel distance.

To determine the expected travel-between distance with discrete space, we again consider two parts: (1) parallel travel, and (2) perpendicular travel. Because all picking aisles have the same length, all storage locations in the picking aisles are equally likely to be chosen. Note that the probability of the two storage locations being in the same aisle is not the same as the probability of the two storage locations being in different aisles. The probability the two locations are in the same aisle is 1 / n and the expected perpendicular distance between the two locations is given by

$$E[TB_{sa}] = \frac{2w}{m^2}\sum_{i=0}^{m-1}\sum_{j=0}^{i} j = \frac{w\left(m^2 - 1\right)}{3m} \tag{29}$$

The probability of two locations being in different aisles is (1 – 1 / n) and the expected perpendicular distance between the two locations is given by

$$E[TB_{da}] = \frac{2w}{m^2}\left[\left(\sum_{i=1}^{m}\sum_{k=i-1}^{m-1} k\right) - m\left(m-1\right)\right] + 2\,f \;= \frac{w}{3m}\left(2m-1\right)\left(m-1\right) + 2\,f \tag{30}$$

The expected parallel travel is given by Pohl et al. (2009) as follows

$$\frac{a\left(n^2 - 1\right)}{3n} \tag{31}$$

Combining the parallel and perpendicular components, a discrete formulation for the expected travel-between distance is given by

$$E[TB] = \frac{1}{n}\left[\frac{w\left(m^2-1\right)}{3m} + \left(n-1\right)\left(\frac{w\left(2m^2-3m+1\right)}{3m} + 2f\right)\right] + \frac{a\left(n^2-1\right)}{3n} \tag{32}$$

5.1 *k* docks uniformly located on one wall

As defined before, d_i is the distance between the left wall and dock i; it is measured along the bottom cross-aisle of the warehouse. When docks are located uniformly, we can develop an expression for the locations of docks to facilitate subsequent calculations. With k docks, the width of the warehouse is divided by (k + 1) to obtain the spacing between adjacent docks. Hence, the distance between adjacent docks and between an end-wall and its closest dock equals $W / (k + 1)$. Therefore, the distance between the left wall and the i^{th} dock is given by

$$d_i = \frac{W}{k+1} + \left(i-1\right)\frac{W}{k+1} = \frac{iW}{k+1} \tag{33}$$

Substituting Equation 33 in Equation 27 for d_i, we verify the accuracy of the continuous formulation derived in Equation 7 for expected single-command distance. Table 1 provides values of the error introduced by using a continuous approximation instead of the discrete formulation provided in Equation 27. The errors shown are based on the following parameter values: m = 75 ft, w = 4 ft, $mw = L$ = 300 ft, v = 6 ft, f = 8 ft, a = 20 ft, and δ = 12 ft. Values ranging from 1 to 30 are used for k, depending on the value of n, and values ranging from 3 to18 are used for n.

The continuous approximation overestimated the expected single-command distance for all cases considered. We calculate the percentage error for the continuous formulation by subtracting the expected distance obtained using a discrete formulation from the expected distance obtained using a continuous formulation, dividing the result by the discrete formulation result, and multiplying the quotient obtained by 100. Therefore, the percentage error is $\left(\left(E\left[SC^{C}\right]-E\left[SC^{D}\right]\right)/E\left[SC^{D}\right]\right)*100$.[5]

Table 1: The percent error for a continuous approximation of single-command distance with uniformly distributed docks

Number of docks (k)	Number of aisles (n): 3	6	9	12	15	18
1	0.9506	0.0000	0.2691	0.0000	0.1408	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.4664	0.2828	0.1297	0.0000	0.0668	0.0694
4	0.3717	0.1686	0.1030	0.0713	0.0000	0.0412
5		0.0000	0.1538	0.0000	0.0789	0.0000
6		0.1597	0.0973	0.0672	0.0499	0.0387
7		0.1793	0.1001	0.1006	0.0512	0.0434
8		0.1392	0.0000	0.0585	0.0433	0.0000
9		0.1484	0.0959	0.0623	0.0721	0.0358
10			0.0922	0.0636	0.0471	0.0365
11			0.1036	0.0000	0.0529	0.0547
12			0.0909	0.0626	0.0464	0.0360
13			0.0917	0.0613	0.0468	0.0352
14			0.0867	0.0597	0.0000	0.0343
15				0.0696	0.0462	0.0366
16				0.0614	0.0454	0.0352
17				0.0545	0.0479	0.0000
18				0.0610	0.0451	0.0350
19				0.0585	0.0507	0.0356
20					0.0440	0.0341

[5] Superscript *C* denotes continuous approximation and superscript *D* denotes discrete formulation.

From Table 1, the error percentage resulting from using a continuous approximation for single-command operations varies from 0.0 to 0.95 percent, with an average value of 0.078 percent. Because very little error is introduced by using the continuous approximation, we employ it in subsequent calculations. As shown, increasing the number of aisles results in decreasing the error for the continuous approximation.

5.2 *k* docks centrally located on a wall with a fixed distance between adjacent docks

When docks are located centrally, but with a fixed distance between adjacent docks, we develop a different expression for d_i. If we have k docks with a fixed distance (δ) between adjacent docks, the distance from the left wall to the first dock (d_1) will be $\{W - (k - 1)\,\delta\} / 2$, and the distance from the left wall to subsequent dock locations is obtained by adding multiples of the fixed distance (δ). Thus, the distance to the i^{th} dock from the left wall is given by:

$$d_i = \frac{W - (k-1)\delta}{2} + (i-1)\delta \tag{34}$$

Substituting Equation 34 into Equation 27, we verified the accuracy of the continuous formulation derived in Equation 13 for expected single-command distance by comparing results with those obtained from the discrete formulation. Values of the error percentage introduced by using a continuous approximation are provided in Table 2, based on the same parameter values used in the previous section.

From Table 2, the error percentage resulting from using a continuous formulation for single-command operations varies from 0.0 to 0.95 percent, with an average value of 0.083 percent. As shown, increasing the number of aisles results in decreasing the error percentage for the continuous approximation.

Table 2: The percent error for a continuous approximation of single-command distance with a fixed distance between adjacent docks

		Number of aisles (n)					
		3	6	9	12	15	18
Number of docks (k)	1	0.9506	0.0000	0.2691	0.0000	0.1408	0.0000
	2	0.1504	0.1563	0.0429	0.0676	0.0225	0.0397
	3	0.3371	0.1848	0.0965	0.0800	0.0506	0.0470
	4	0.3717	0.0862	0.1070	0.0374	0.0561	0.0220
	5		0.1374	0.0960	0.0598	0.0505	0.0352
	6		0.1993	0.0708	0.0870	0.0373	0.0512
	7		0.1163	0.0952	0.0510	0.0503	0.0301
	8		0.1515	0.0948	0.0667	0.0501	0.0394
	9		0.1484	0.0756	0.0656	0.0401	0.0388
	10			0.0833	0.0661	0.0443	0.0392
	11			0.1082	0.0532	0.0577	0.0316
	12			0.0753	0.0655	0.0403	0.0390
	13			0.0917	0.0624	0.0493	0.0372
	14			0.0867	0.0565	0.0467	0.0338
	15				0.0572	0.0488	0.0343
	16				0.0710	0.0405	0.0427
	17				0.0531	0.0483	0.0320
	18				0.0630	0.0451	0.0381
	19				0.0585	0.0433	0.0355
	20					0.0422	0.0377

5.3 *k* docks not centrally located on a wall with a fixed distance between adjacent docks

Relaxing the assumption that the docks are centrally located, we assume the first dock is located a distance ϕ from the left wall. Again, we update the d_i expression in Equation 27. Now, the distance from the left wall to the i^{th} dock is given by

$$d_i = \phi + (i-1)\delta \tag{35}$$

Substituting Equation 35 into Equation 27, we compared the results obtained from the continuous approximation using Equation19 with the results obtained from the discrete formulation in Equation 27. Values of the error percentage introduced by using a continuous approximation are provided in Table 3, based on the parameter values used in the previous two sections and letting $\phi = d_1 = 20$ ft.

Table 3: The percent error for a continuous approximation of single-command distance for not centrally located docks on a wall with a fixed distance between adjacent docks

		Number of aisles (n)					
		3	6	9	12	15	18
Number of docks (k)	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.3007	0.1322	0.0774	0.0515	0.0370	0.0280
	3	0.2494	0.1116	0.0655	0.0436	0.0313	0.0237
	4		0.1014	0.0598	0.0399	0.0286	0.0216
	5		0.1359	0.0807	0.0540	0.0387	0.0292
	6		0.1133	0.0679	0.0455	0.0327	0.0247
	7		0.1356	0.0821	0.0552	0.0398	0.0300
	8		0.1262	0.0774	0.0523	0.0377	0.0285
	9			0.0735	0.0500	0.0361	0.0273
	10			0.0827	0.0566	0.0410	0.0310
	11			0.0750	0.0517	0.0376	0.0285
	12			0.0821	0.0570	0.0416	0.0316
	13			0.0784	0.0549	0.0402	0.0306
	14				0.0530	0.0390	0.0298
	15				0.0569	0.0422	0.0323
	16				0.0532	0.0396	0.0304
	17				0.0564	0.0423	0.0326
	18				0.0545	0.0411	0.0318
	19					0.0400	0.0310
	20					0.0420	0.0328

From Table 3, the error percentage resulting from using a continuous formulation for single-command operations varies from 0.0 to 0.30 percent, with an average value of 0.054 percent. As shown, increasing the number of aisles results in decreasing the error percentage for the continuous approximation.

6 Quasi-Discrete Space

With a desire to calculate the optimal shape factor for the discrete formulations, we employed approximations to facilitate calculations. Specifically, we employed a continuous approximation for $E[TB]$. As a result, in this section, formulations are partially discrete and partially continuous.

As noted previously, travel-between distance is not affected by the number or locations of docks. Therefore, it is the same for all cases considered in the previous sections. To illustrate the approach, we consider the case in which a fixed distance exists between adjacent docks, which are centrally located.

Specifically, in Equation 32, we replace $m^2 - 1$ with m^2 and replace $(2m^2 - 3m + 1)/m$ with $(2m - 3)$. The resulting approximation for expected travel-between distance is

$$E[TB] \approx \frac{1}{n}\left[\frac{wm}{3} + (n-1)\left(\frac{w(2m-3)}{3} + 2f\right)\right] + \frac{a(n^2-1)}{3n} \qquad (36)$$

Table 4 provides values for the percent error of the approximation for travel-between distance using the values of the parameter values stated previously. As indicated, the error introduced by using a continuous approximation varies from 0.76 percent to 0.92 percent, with an average of 0.83 percent.

Table 4 : The percent error for a continuous approximation of travel-between distance

Number of aisles (n)					
3	6	9	12	15	18
0.9167	0.948	0.9064	0.857	0.809	0.7646

Combining Equations 13 and 36 and simplifying, a continuous approximation of the expected distance for a dual-command operation is given by

$$E[DC] \approx D\left(\frac{5n-1}{3n}\right) - 2v\left(\frac{n+1}{3n}\right) + a\left(\frac{5n^2-2}{6n}\right) + \frac{\delta^2(k^2-1)}{6an} \tag{37}$$

Replacing n and D in Equation 37 with $\sqrt{AS/a^2}$ and $\sqrt{A/S}$, respectively, and taking the first derivative with respect to S yields

$$\frac{dE[DC]}{dS} = \frac{a}{3\sqrt{S}} + \left(\frac{5\sqrt{A}}{12}\right)S - \frac{5\sqrt{A}}{6} + \frac{av}{3\sqrt{A}} + \frac{a^2}{6\sqrt{A}} - \frac{\delta^2(k^2-1)}{12\sqrt{A}} \tag{38}$$

Setting Equation 38 equal to zero and solving with the *Mathematica* (2013) software package for S^*_{DC} yields the following result containing complex numbers

$$S^*_{DC} \approx \frac{1}{15}\left[-\frac{2e}{A} - \frac{(1-i\sqrt{3})Ae^2}{2^{2/3}R^{1/3}} - \frac{(1+i\sqrt{3})R^{1/3}}{2^{1/3}A}\right] \tag{39}$$

where $R = 135A^6(4aA)^2 + 2A^6e^3 + 3\sqrt{15}\sqrt{4A^{12}e^3(4aA)^2 + 135A^{12}(4aA)^4}$ and

$e = 2a^2 - 10A + 4av - \delta^2(k^2-1)$

To illustrate the impact of the number of docks on the optimal shape factor, we consider design parameters having the following values: A = 60,000 ft^2, a = 20 ft, v = 6 ft and δ = 12 ft. As shown in Figure 8, with single-command operations, the optimal shape factor increases the number of docks increases when the distance between docks is fixed. As expected, with dual-command operations, the optimal shape factor increases slightly with an increasing number of docks when a fixed distance exists between adjacent docks.

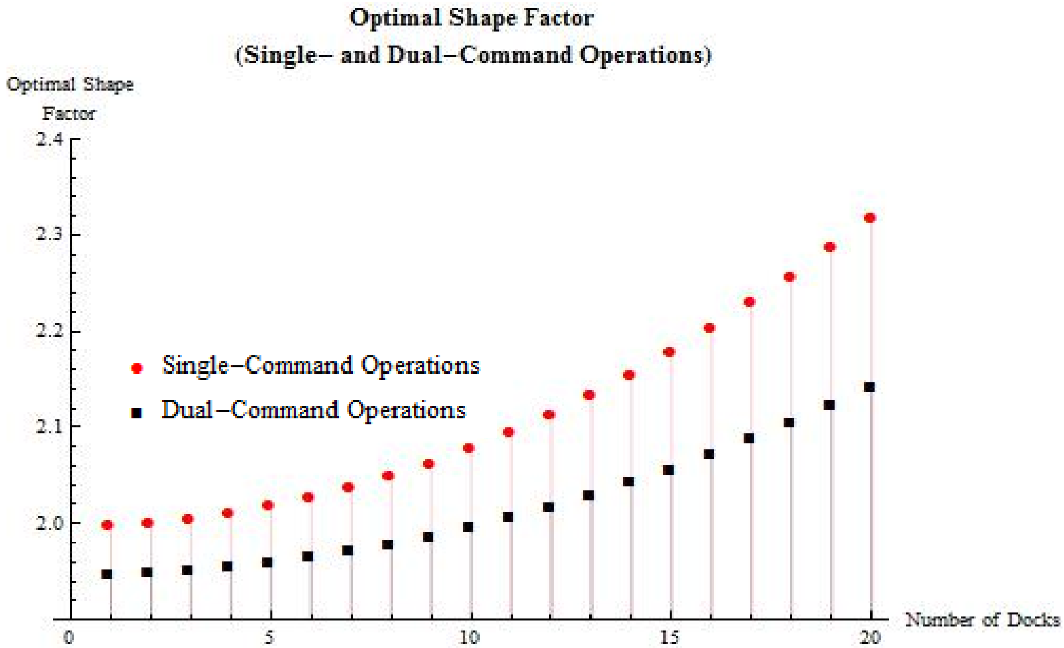

Figure 8: The effect on the optimal shape factor of increasing the number of docks with single- and dual-command operations

7 Using Different Docks for Receiving and Shipping

In the previous sections, we did not distinguish between docks insofar as their functions were concerned. In fact, we assumed docks were equally likely to be used. Now, we extend our analysis to allow a cluster of docks to be designated as receiving docks and another cluster be designated as shipping docks. To facilitate the development of the formulations, we introduce new notation and add subscripts to distinguish the two clusters of docks.

We assume the cluster of docks located to the left of the warehouse centerline are devoted to receiving, whereas shipping docks are located to the right of the warehouse centerline. For the receiving docks, we use ϕ_R to denote the distance from the left wall to the closest receiving dock, δ_R to denote the distance between adjacent receiving docks, and k_R to denote the number of receiving docks. Similarly, we use ϕ_S, δ_S and k_S for the distance from the right wall to the closest shipping dock, the distance between adjacent shipping docks, and the number of shipping docks, respectively. As shown in Figure 9, the only difference in measuring distances for the two clusters is that we measure the distance for the shipping docks from the right wall instead of the left wall. From Equation 19, the expected single-command expression is given by

$$E[SC] = \left[\sum_{i=1}^{k_R} \left\{ \frac{\left[\phi_R + (i-1)\delta_R\right]^2}{W} + \frac{\left[W - \phi_R - (i-1)\delta_R\right]^2}{W} \right\} \right.$$

$$\left. + \sum_{i=1}^{k_S} \left\{ \frac{\left[\phi_S + (i-1)\delta_S\right]^2}{W} + \frac{\left[W - \phi_S - (i-1)\delta_S\right]^2}{W} \right\} \right] \Big/ (k_R + k_S) + D \qquad (40)$$

Employing the same procedure as previously, we obtain the formula for the optimal shape factor

$$S_{SC}^{*} = \frac{1}{3A(k_R + k_S)} \left\{ \left[k_R \left(3A + 6\phi_R{}^2 + 6\phi_R \delta_R (k_R - 1) + \delta_R{}^2 \left(2k_R{}^2 - 3k_R + 1 \right) \right) \right] \right.$$

$$\left. + \left[k_S \left(3A + 6\phi_S{}^2 + 6\phi_S \delta_S (k_S - 1) + \delta_S{}^2 \left(2k_S{}^2 - 3k_S + 1 \right) \right) \right] \right\} \qquad (41)$$

Based on Equation 41, increasing the area (A) decreases the value of the optimal shape factor. In addition, S_{SC}^{*} is a convex function of ϕ_R, ϕ_S, δ_R, δ_S, k_R, and k_S. Therefore, an increase in the value of each of the parameters can increase or decrease the value of the optimal shape factor depending on the stationary points of each. However, it appears that the stationary point occurs in a non-feasible region, namely for negative-valued distances and a fractional number of docks. This is a subject we plan to explore further in the future.

Previously, we found that an increase in the number of docks decreased the value of the optimal shape factor. Hence, having separate receiving and shipping docks with fixed distances between adjacent docks and the nearest walls yields an interesting result insofar as the optimal shape factor is concerned. This, too, is a subject for further research.

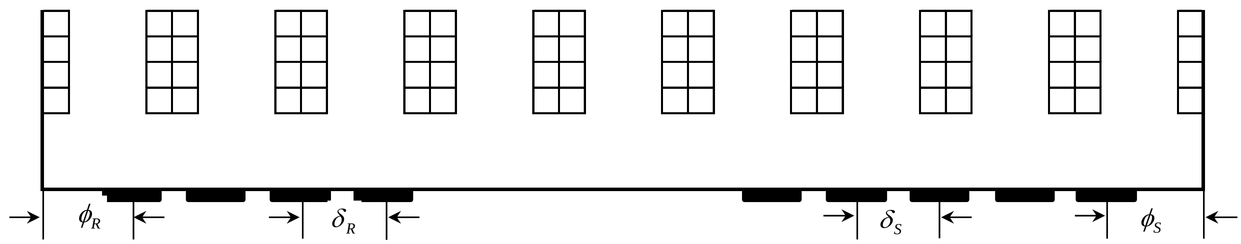

Figure 9: Different docks used for receiving and shipping

8 Cross-Docking Travel

In this section, we consider a mixture of single-command and cross-docking travel in the warehouse and its impact on the shape factor.

As in the previous section, we designate the cluster of docks located to the left of the warehouse centerline for receiving, whereas shipping docks are located to the right of the warehouse centerline. There exist k_R docks on the left side (the leftmost dock is the first dock) and k_S docks on the right side (the rightmost dock is the first dock). The total number of docks is k ($k = k_R + k_S$). The distance between the two clusters of docks is B. If the distance between adjacent docks is not the same for receiving and shipping, then the expected distance for cross-docking is

$$E[CD] = 2B + 4v + \frac{2}{k_R k_S}\sum_{i=1}^{k_R}\sum_{j=1}^{k_S}\left[(k_R - i)\delta_R + (k_S - j)\delta_S\right] = 2B + 4v + (k_R - 1)\delta_R + (k_S - 1)\delta_S \quad (42)$$

where $B = W - \phi_R - \phi_L - (k_R - 1)\delta_R - (k_S - 1)\delta_S$

Simplifying Equation 59 yields the expected roundtrip distance to perform cross-docking

$$E[CD] = 2(W + 2v - \phi_R - \phi_S) - (k_R - 1)\delta_R - (k_S - 1)\delta_S \tag{43}$$

For a unit-load warehouse that only performs single-command operations, including cross-docking, three distinct unit-load moves occur: transporting a unit-load to storage from receiving, transporting a unit-load from storage to shipping, and transporting a unit-load from receiving to shipping (cross-docking). From the perspective of shipping, unit-loads arrive either from storage or, via cross-docking, from receiving.

We let the probability a unit-load arrives at shipping from storage be p_S and the probability be p_C that a unit-load arrives at shipping via cross-docking, such that $p_S + p_C = 1.00$. Before a unit-load can arrive at shipping from storage, it is transported to storage from receiving. Therefore, with probability p_S, the expected distance traveled is equal to the sum of the expected distance from receiving to storage and the expected distance from storage to shipping. With probability p_C , the expected distance traveled equals the expected cross-dock distance. Drawing on the results in Section 7, the overall expected distance traveled by a unit-load is given by

$$E[SC] = p_S(E + F) + p_C(G) \tag{44}$$

where $E = W + \dfrac{6\phi_R{}^2 + 6(k_R - 1)\phi_R\delta_R + (2k_R{}^2 - 3k_R + 1)\delta_R{}^2}{3W} - (2\phi_R + (k_R - 1)\delta_R) + D$,

$F = W + \dfrac{6\phi_S{}^2 + 6(k_S - 1)\phi_S\delta_S + (2k_S{}^2 - 3k_S + 1)\delta_S{}^2}{3W} - (2\phi_S + (k_S - 1)\delta_S) + D$ and

$G = p_C\left[2(W + 2v - \phi_R - \phi_S) - (k_R - 1)\delta_R - (k_S - 1)\delta_S\right]$

Replacing *W* and *D* with the warehouse area and shape factor, we obtain

$$E[SC] = p_S(E+F) + p_C(G) \tag{45}$$

where $E = \sqrt{AS} + \dfrac{6\phi_R^{\ 2} + 6(k_R - 1)\phi_R\delta_R + (2k_R^{\ 2} - 3k_R + 1)\delta_R^{\ 2}}{3\sqrt{AS}} - (2\phi_R + (k_R - 1)\delta_R) + \sqrt{\dfrac{A}{S}}$,

$F = \sqrt{AS} + \dfrac{6\phi_S^{\ 2} + 6(k_S - 1)\phi_S\delta_S + (2k_S^{\ 2} - 3k_S + 1)\delta_S^{\ 2}}{3\sqrt{AS}} - (2\phi_S + (k_S - 1)\delta_S) + \sqrt{\dfrac{A}{S}}$ and

$G = p_C\left[2\left(\sqrt{AS} + 2v - \phi_R - \phi_S\right) - (k_R - 1)\delta_R - (k_S - 1)\delta_S\right]$

Taking the first derivative with respect to *S*, setting it equal to zero, and solving for S_{SC}^* gives

$$S_{SC}^* = \frac{(1-p_C)}{6A}\left\{6A + \delta_R^{\ 2}\left(2k_R^{\ 2} - 3k_R + 1\right) + \delta_S^{\ 2}\left(2k_S^{\ 2} - 3k_S + 1\right)\right.$$

$$\left. + 6\phi_R\left(\phi_R + k_R\delta_R - \delta_R\right) + 6\phi_S\left(\phi_S + k_S\delta_S - \delta_S\right)\right\} \tag{46}$$

To illustrate the effect cross-docking can have on the optimal shape factor, we assign the following values to the parameters: A = 60,000 ft^2, k_R = 5, k_S = 7, ϕ_R = 20 ft, ϕ_S = 30 ft, δ_R = 12 ft, δ_S = 14 ft. The probability of cross-docking ranges from 0 to 1. As indicated in Equation 46, there exists a linear relationship between the optimal shape factor and the probability of cross-docking. As expected, cross-docking tends to decrease the optimal shape factor. Also, the optimal shape factor is less than 1.5 for all cases considered.

Given the convexity of S_{SC}^* with respect to the distance related parameters and the number of docks, in the feasible region for the values of the parameters, increasing the number of docks increases the optimal shape factor. Moreover, increasing the distance the closest dock is to either the left or right wall also increases the optimal shape factor. Conversely, increasing the area (A) or the probability of cross-docking (p_C) increases the optimal shape factor.

9 Summary, Conclusions, Significance and Recommendations

9.1 Summary

Although warehouses generally have more than one dock, researchers have focused primarily on one centrally located dock. In the thesis, we extended studies conducted by Francis (1967), Pohl et al. (2009) and Thomas and Meller (2014) by considering multiple docks with fixed distances between adjacent docks and by examining the effect of additional docks on the optimal shape factor.

In the first of the five continuous space cases considered, we found that increasing the number of docks decreases the spacing between adjacent docks and the optimal shape factor is less than 2.0. Letting the docks be spread uniformly over the wall, we verified the result given by Thomas and Meller (2014) that increasing the number of docks with a fixed distance between adjacent docks increases the width of the warehouse and results in the optimal shape factor being less than 2.0. Subsequently, we found that having a fixed number of docks and a fixed distance between adjacent docks causes the optimal shape factor to be greater than 2.0. Then, we found, for non-centrally located docks, that specifying the offset distance when measured from the left wall of the warehouse can result in the optimal shape factor being either less than 2.0 or greater than 2.0, depending on the magnitude of the offset distance and the number of docks. Finally, we found that specifying the offset distance when measured from the centerline of the warehouse results in the optimal shape factor being greater than 2.0.

Using a continuous approximation for single-command operations resulted in an error of approximately 0.08 percent. Because of an approximate error of 56.86% for travel-between operations in continuous space, we developed a discrete formulation to obtain more accurate expected distances for dual-command operations. Because the discrete formulation was cumbersome, to obtain expected dual-command distance, we combined the continuous space, single-command formulation with a continuous approximation based on our discrete formulation of expected travel-between distance. The combination yielded an average error of 0.5 percent for the cases considered.

9.2 Conclusions

Based on the research performed, we concluded that the number and locations of docks significantly affects the expected distance traveled in warehouses. Also, when the distance between adjacent docks is fixed, the magnitude of the distance affects the expected distance traveled in warehouses. Finally, when docks are offset from the centerline of the warehouse, the optimal shape factor is significantly impacted by the decision to measure the offset distance from the left wall or the centerline of the warehouse.

9.3 Significance

Having developed a variety of mathematical models of travel distances in a unit-load warehouse, we now address the significance of the research insofar as the design and operation of unit-load warehouses are concerned. Consider the design of a new unit-load warehouse. The designer must determine the number of docks to be included in the design, as well as their relative locations. If a fixed number of docks are needed and the distance between adjacent docks is to be specified, then the width of the warehouse should be more than twice its depth; our results can guide the designer in determining how much greater than 2.0 the shape factor should be.

On the other hand, if the designer wants to include docks over the entire width of a wall because it is cheaper to include them during construction than to add them later, if needed, then it is important to know if all docks will, in fact, be used and, if so, will they be used equally. If so, then a shape factor less than 2.0 should be used in designing the warehouse. However, if docks are included across the entire width of a wall and only a subset of the docks will be used, then a shape factor greater than 2.0 should be evaluated.

For an existing warehouse with docks installed across the entire width of a wall, if not all of the warehouse is needed for the storage and retrieval of unit-loads, then the storage area should be configured in such a way that the width of the storage area is more than double the depth of the storage area, i.e., a shape factor greater than 2.0. The balance of the space in the warehouse can be used for offices, restrooms, break areas, battery charging stations for industrial trucks, etc.

The most significant contribution of the study is identifying conditions for which the optimal shape factor should be greater than 2.0 for a unit-load warehouse. A secondary contribution is gaining an understanding of the effect on the optimal shape factor of the number and location of docks and cross-docking. Models produced by the study can be used to obtain exact values for design parameters, instead of relying on rules of thumb or intuition.

9.4 Recommendations

Recommendations for future research include:

1. consideration of other traditional layout configurations with multiple docks,

2. consideration of the effect of dock locations on the expected distance for single- and dual-command operations in warehouses having non-traditional aisle structure,
3. consideration of the effects of multiple docks on expected distance traveled and the optimal shape factor when class-based storage and/or turnover-based storage is used,
4. consideration of having docks with unequal probabilities of usage,
5. further consideration of cross-docking with receiving and shipping docks physically separated, and
6. determination of the optimal number of docks and their locations by considering congestion, dock cost, and travel cost.

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