8-2013

Parameterizing Analytical Models to Support an Empirically Based Warehouse Design Methodology

Lisa Thomas
University of Arkansas, Fayetteville

PARAMETERIZING ANALYTICAL MODELS TO SUPPORT AN EMPIRICALLY BASED WAREHOUSE DESIGN METHODOLOGY

PARAMETERIZING ANALYTICAL MODELS TO SUPPORT
AN EMPIRICALLLY BASED WAREHOUSE DESIGN METHODOLOGY

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in Industrial Engineering

By

Lisa Thomas
University of Arkansas
Bachelor of Science in Computer Systems Engineering, 1990
University of Arkansas
Master of Science in Industrial Engineering, 1993

August 2013
University of Arkansas

This dissertation is approved for recommendation to the Graduate Council.

Dissertation Director:

____

Dr. Russell D. Meller

Dissertation Committee:

________________________________ ________________________________

Dr. John A. White, Jr. Dr. Richard E. Webb

____

Dr. Sarah E. Root

Abstract

Research in the area of warehouse design is characterized by a myriad of analytical models that address one, typically small and isolated, area of the warehouse. These models, although important in gaining insight into one question of warehouse design, are of limited value when one considers the larger question of overall warehouse design. Thus, research in the area of overall warehouse design typically consists of procedure-driven processes based on qualitative factors and not the quantifiable results of analytical models.

In contrast, practitioners have significant empirical data related to how a design alternative performs in an industry, a company, or a particular warehouse. However, because practitioners lack a means for comparing the performance of competing alternatives over multiple facilities, they may adopt a sub-optimal design for a given facility.

A valuable tool for depicting a design is the functional flow network, where nodes represent the functional areas in the warehouse and arcs connecting the nodes define the product flow between functional areas. We propose a design methodology that employs the use of functional flow networks, as well as analytical models and empirical data for quantifying design performance. First, we develop a complete set of analytical models for a manual, case-picking warehouse, and we use the models to investigate the optimal warehouse shape. Next, we implement the design methodology using the analytical models. We then parameterize the analytical models to create lookup tables to demonstrate the design methodology using empirical data. We use an example to show that the two methods lead to the same solutions, thus providing a proof-of-concept for using empirical data to design a warehouse. Finally, we present a preliminary search heuristic for designing a manual, case-picking warehouse. The search heuristic is based on warehouse operating characteristics and provides an initial design that can be further analyzed and optimized.

We believe that our design methodology provides two key features that are typically

missing from existing overall warehouse design methodologies: comparing design alternatives through quantifiable output from analytical models and empirical observations, and therefore, considering a broad range of design alternatives.

Acknowledgements

This dissertation embodies the central theme of my research endeavors over the last five years. I am especially grateful to my advisor, Dr. Russell D. Meller, for the time that he invested in me. He challenged and inspired me to become a better researcher, thinker and writer, and he provided research opportunities to prepare me for my overall career objective of obtaining a position in industry. Along with Dr. Meller, my committee members, Dr. John A. White, Jr., Dr. Richard E. Webb, and Dr. Sarah E. Root, provided me with invaluable insight and direction. I consider myself blessed to have benefited from their expertise and wisdom.

I would like to express my gratitude to the Material Handling Education Foundation in supporting me through scholarships and to the University of Arkansas Graduate School for the Doctoral Academy Fellowship. In addition, I am fortunate to have worked on research projects through the Center for Innovation in Healthcare Logistics (CIHL) and the Center for Excellence in Logistics and Distribution (CELDi). I learned a great deal about the healthcare supply chain through two research projects with CIHL. I am also grateful to CELDi and its industry partners for allowing me to work on industry-based research projects in the area of warehouse design and to the National Science Foundation in supporting CELDi.

I am also thankful for the friendships of Tish, Jen, Angelica and Barb throughout my academic journey.

I am blessed to have a wonderful family. My pursuit of a Ph.D. would not have been possible without my husband, Greg. I am eternally grateful for his love, support and dedication to our children and for fulfilling many of my responsibilities as a mother so that I could pursue a dream. To my children, Daniel and Rachel, thank you for being so understanding when I had to miss family time in order to study. And, I will always cherish the lifelong friendship of my sister, Sheila. To the rest of my family, thank you for your prayers, support and encouragement.

Dedication

This is dedicated to my parents, Wayne and Sara Irwin, who have encouraged me to believe in myself and more importantly, raised me to love and fear the Lord.

Contents

List of Figures

List of Tables

List of Published Papers

Chapter 4:

Thomas, L. M., and Meller, R. D., "Analytical Models for Warehouse Configuration," *IIE Transactions* (in review) (2013).

Chapter 5:

Thomas, L. M., and Meller, R. D., "Using Analytical Models to Assess Performance in Overall Warehouse Design," in *Proceedings of the 2013 Industrial and Systems Engineering Research Conference*, San Juan, Puerto Rico (2013).

Chapter 6:

Thomas, L. M., and Meller, R. D., "Using Empirical Data to Assess Performance in Overall Warehouse Design," *in Proceedings of the 2013 Industrial and Systems Engineering Research Conference*, San Juan, Puerto Rico (2013).

Chapter 1

Introduction

According to the results of the *2011 Warehouse and Distribution Center Operations Survey* [10], most companies are taking steps to reduce warehouse operating costs by improving processes or changing storage racks and layouts. Moreover, the design of the functional areas within the warehouse directly affects the operational costs, so a systematic approach to overall design that considers such costs is desirable. Nonetheless, overall warehouse design is a complex process with many interrelated components, and coordinating these processes makes systematic design challenging.

In general, practitioners use empirical observations when designing a warehouse [2], while academic research in this area typically focuses on analytical models for one or two design components [11]. The two approaches taken by researchers and practitioners each have their own merits, as analytical models yield quantitative results that can be used to compare design alternatives, and empirical observations provide discrete realizations that might be overlooked by using analytical models alone. Rouwenhorst et al. [11] state that to a large extent, the design phase determines the logistical costs associated with a warehouse, and the authors emphasize the need to integrate the models and methods for specific design components in order to develop a methodology for systematic warehouse design. Further, the methods and models presented in the academic literature significantly outperform the methods used in practice, yet warehouse management systems still utilize simple heuristics to solve problems [12]. The interrelationships that exist among design components make overall warehouse design a daunting task, and this is likely the reason that no design methodology has been widely adopted by industry practitioners or academicians.

The five basic considerations in overall warehouse design include: overall structure, sizing and dimensioning, departmental layout, equipment selection, and operational strategies [6].

That is, the design process must also consider the operations in the areas of receiving, storage, order picking and shipping [5]. Figure 1.1 illustrates a traditional warehouse layout with areas designated for receiving and shipping and with pallet rack for storage. The overall relationship between the design and operational components depicted by Gu et al. are illustrated in Figure 1.2 [5].

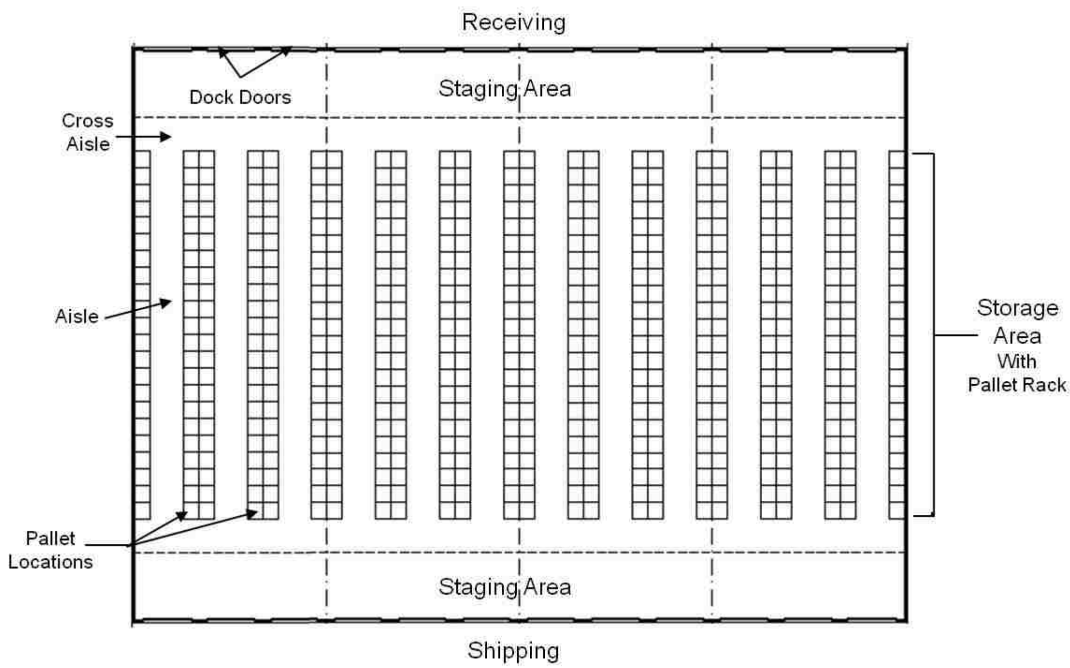


Figure 1.1: Warehouse layout.

From Figure 1.2 one can see the complexity of the overall design process and the effect of design decisions on operational performance. For example, consider the design goal of sizing and dimensioning the warehouse in order to accommodate a given number of dock doors. As the warehouse shape changes (becoming more or less elongated to achieve a given number of doors), the travel distance required for put-away, order-picking and replenishment operations changes as well. Thus, altering even a single design parameter can significantly affect the operational performance of the warehouse. The operational components of warehouse design include the following:

Receiving

Operational decisions in receiving include scheduling the arrival of inbound trucks and

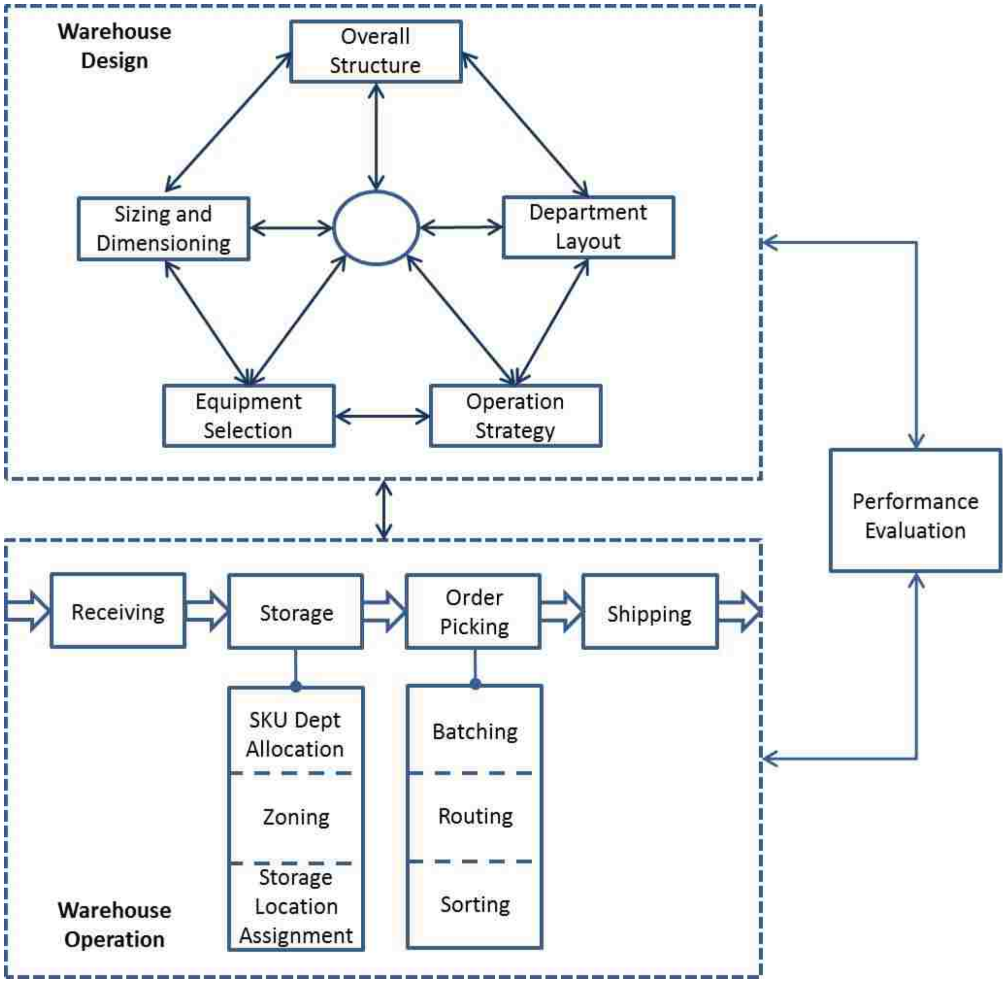


Figure 1.2: Warehouse design and operational decisions adapted from [5].

assigning trucks to docks. Both of these decisions impact the utilization of material handling equipment, as well as the flow of material within the warehouse. In addition to impacting flow, the assignment of trucks to docks can affect the travel distance required for the put-away operation.

Storage

Storage-related decisions can have a huge impact on warehouse performance, as storage affects both warehouse utilization and labor. Typical storage policies include random storage, dedicated storage and class-based storage. Other storage decisions involve determining the amount of inventory to be stored and whether or not there will be a forward reserve or fast-pick area. Still other storage determinations include allocating products to departments and creating zones within the departments.

Order Picking

Because order picking accounts for a large part of warehouse operational costs, it is an area that has received considerable attention from the academic community. Order-picking strategies include operations such as batching, routing and sequencing, and sorting. Inherent in this decision is the level and type of automation, if any. Warehouses that utilize automated order-picking systems are often referred to as part-to-picker warehouses, whereas manual warehouses are referred to as picker-to-part warehouses.

Shipping

Shipping decisions are related to receiving in that there must be an assignment of outbound trucks to docks, as well as the scheduling of trucks arriving for pickup. Again, these decisions affect the flow of material within the warehouse.

The number of available dock doors affects both shipping and receiving, and consideration should be given to the number of trucks arriving during a receiving or shipping time period. If the storage area results in too few dock doors to meet truck throughput requirements, either the warehouse will have to become larger or the facility will have to be configured with doors on two sides to accommodate additional doors. Both of these options result in added labor and building costs. In order to avoid these added costs, additional workers can be scheduled to turn trucks quickly, and coordinating the arrival of trucks may be necessary as well.

Various strategies and methodologies have been developed to improve the operational performance of warehouses. For example, because order picking is the most labor-intensive component of warehouse operations, research typically focuses on improving storage and routing policies to reduce the cost associated with order-picking labor. The random storage policy makes the most efficient use of storage because incoming products can be put away to any open storage location, as there is no specific assignment of product to location. Volume-based storage, on the other hand, places fast-moving items closest to the pickup and deposit

(P&D) location in order to minimize order-picking travel. Items are assigned to dedicated storage areas according to their level of activity. As such, incoming products can only be put away to their designated storage locations, thus requiring more storage locations as compared to random storage. Consequently, there is a tradeoff in storage space and order-picking efficiency when choosing between random and volume-based storage. This tradeoff illustrates one of the many interactions that exist among the operational components in a warehouse.

Given the large number of design decisions and operational policies requiring consideration in overall warehouse design, the number of possible designs can be quite large. To illustrate, consider the following design options for a new warehouse:

- Case picks from reserve storage or two possible forward areas (lower level of pallet rack or case flow rack).
- Piece picks from reserve storage or two possible forward areas (lower level of pallet rack or case flow rack).
- Dock doors located on one or both sides of the facility, implying two possible layouts for class-based storage.
- Warehouse shape ratios of 1.5, 2.0, 2.5, 3.0, or 3.5.
- Four or five levels of pallet rack.

This results in 180 ($3 \times 3 \times 2 \times 5 \times 2$) possible warehouse designs. Further, this list is not exhaustive, and there are operational policies such as batching, zoning, and sorting that must be considered for each design because these policies affect the operational performance of the warehouse. Moreover, because of the interrelationships between design components and operational performance, comparing alternatives is not an easy task, as there are often tradeoffs that must be considered. For example, moving from random to volume-based storage results in an increase in order-picking efficiency but a decrease in storage utilization. Thus, it is easy to see that the number of designs grows rapidly.

Valuable research contributions have been made in warehousing literature, and many analytical models are available that quantify specific warehouse functions in terms of operational labor. For instance, order-picking models have been developed for various routing strategies for random storage [7, 4] and volume-based storage [3, 8]. Yet, these models have never been integrated into a holistic methodology for warehouse design. Therefore, a systematic approach to overall warehouse design that allows a quantitative comparison of alternatives would benefit practitioners.

Apple et al. [1] proposed a design methodology to expolit the benefits of both the practitioner's approach (empirical observations) and the academic approach (analytical methods). The first step in their design methodology is to generate a list of relevant design factors (e.g., type of appropriate storage rack based on product characterisitcs). Then, a discrete matrix solution is constructed for each design factor to allow a quantitative comparison of design alternatives. The authors' vision is to motivate the developement of numerical matrices related to labor, space and capital investment for each warehouse function using empirical data.

Like Apple et al. [1], the objective of our research is to provide a means by which to implement a warehouse design methodology that allows a comparison of alternatives in terms of operational labor and storage requirements. However, we propose two methods for achieving this goal: analytical models and empirical data. As stated above, analytical models have been developed by the research community and can be "parameterized" to obtain quantitative solutions in terms of labor and space requirements for various warehouse functions. When analytical models are not available, empirical data can be used to quantify labor and space requirements. Similar to McGinnis et al. [9] and Apple et al. [1], our methodology characterizes design alternatives using functional flow networks.

A functional flow network (FFN) is a network of nodes, where each node represents a functional area in the warehouse. The nodes are connected with arcs that represent the flow of material from one functional area to another. Figure 1.3 illustrates three functional flow networks, from a very simple design (Figure 1.3(a)) to more complex designs (Figures 1.3(b) and 1.3(c)).

The FFN can be used to compare various warehouse designs by first sizing the nodes to accommodate a given number of storage locations and then balancing the flow of product along the arcs. From a systematic design perspective, a designer would begin with a very simple design and then move to more complex designs, evaluating the space and labor requirements for each. In general, moving to different designs requires a resizing of the storage areas, and the product flow must be allocated accordingly. For example, consider a simple design where 6,000 cases per day are picked from a reserve storage area with pallet rack that is five levels high and contains 10,000 pallet locations. A more complex design might include pallet rack that is six levels high with a forward area for picking 80 percent of the cases, where the forward area is 1,200 pallet locations on the bottom level of the centermost aisles of pallet rack. The pallet rack area now must be resized to six levels of pallet rack (a smaller footprint), and the forward area must be sized to accommodate 1,200 pallet locations in the centermost aisles on the bottom level. With this configuration, product flow is such that 4,800 cases are picked from the bottom-level forward area, and the remaining 1,200 cases are picked from the upper levels of pallet rack and/or the outermost aisles.

The key to supporting this design approach, and the objective of this research, is to develop a means to convert the product flow in the FFN to labor requirements. We propose two approaches to achieve this objective. The first approach parameterizes analytical models and uses product flow to quantify labor requirements. For example, an existing model for order-picking distance with random storage has input parameters for the number of aisles, number of pick lines, width of the picking area, and depth of the picking area [7]. Travel-time requirements can be determined by applying the input parameters (number of aisles and pick lines, along with the picking area width and depth) specified by a particular design

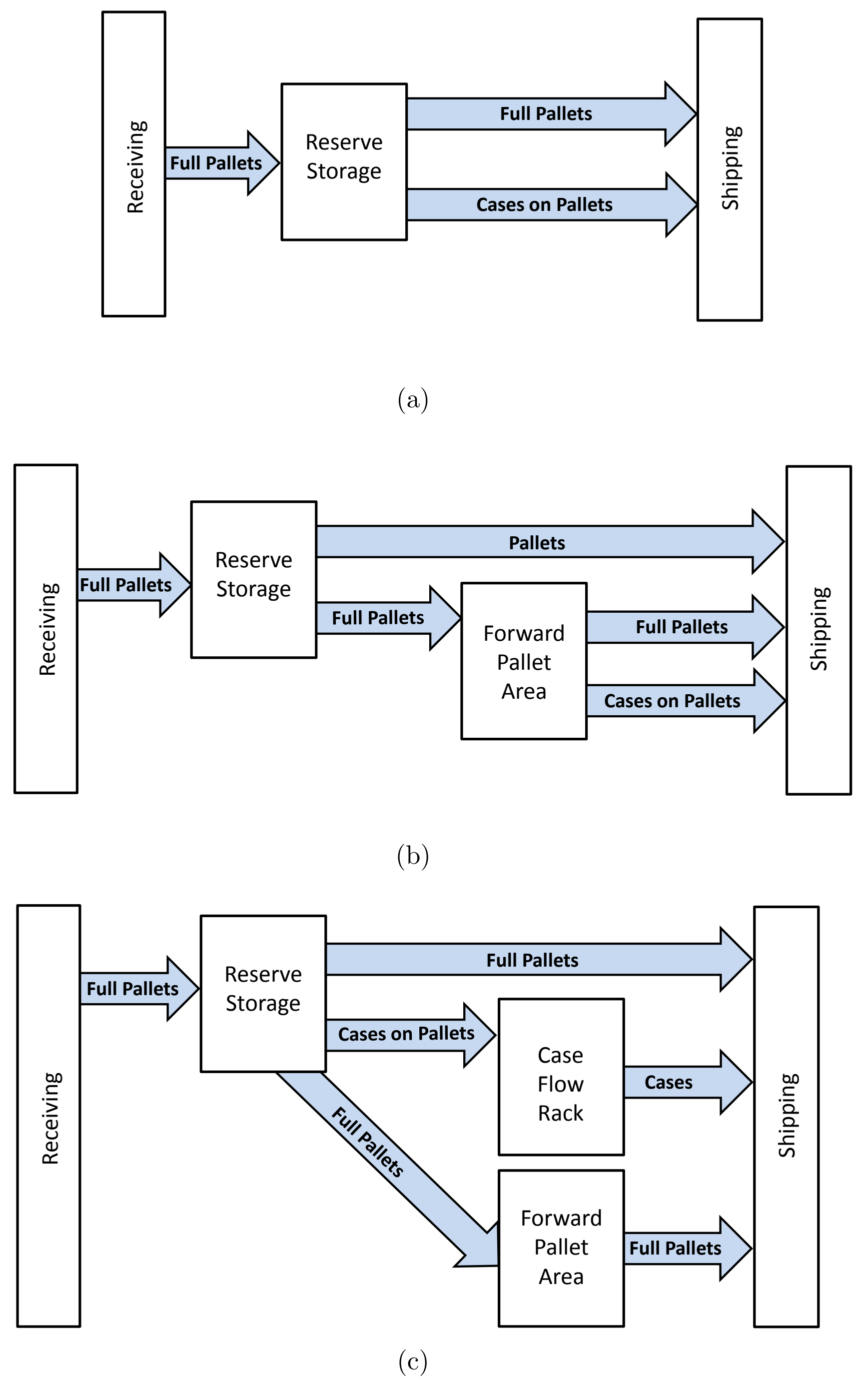


Figure 1.3: (a) FFN with reserve storage; (b) FFN with reserve storage and pallet rack; (c) FFN with reserve storage, pallet rack, and case flow rack.

to the order-picking model, thus “parameterizing” the model. The resulting distance is then divided by the appropriate travel speed, hours per day, etc., to determine the labor required.

The second approach uses empirical data in the form of lookup tables to estimate labor requirements, thus addressing the case where analytical models are not available. Undoubtedly, these tables cannot accommodate all possible combinations for storage size and labor rates. Accordingly, the tables contain discrete data points, and interpolation is used to determine the storage and labor requirements for values that lie between given data points.

We illustrate both approaches for quantifying labor requirements through a detailed example. For the analytical model approach, our example includes the put-away, order-picking and replenishment operations in a manual warehouse. Models for the put-away and order-picking operations for a random storage warehouse can be found in existing literature. The existing models include a central point for pickup and deposit (i.e., a single, centrally located dock door), so we modify these models to include a uniform distribution of dock doors. In addition, we develop a model for replenishment, as there are no existing models for replenishment in warehousing literature. We also develop models for put-away, order-picking and replenishment operations for two class-based storage layouts. We then utilize the models in our detailed example to illustrate the design process for a manual (picker-to-part) warehouse.

For the empirical data approach, we construct data tables for sizing the pallet rack area of a case-picking warehouse. We employ a sizing algorithm that determines the aisle length and number of aisles necessary for a pallet area of a given size and shape to construct the sizing tables. In addition, we construct distance tables by parameterizing the analytical models that were developed for the put-away, order-picking and replenishment operations. We illustrate how interpolation can be used to obtain values for the storage area size and travel distances for parameters that are not explicitly listed in the tables. Further, we present a full example to demonstrate how the tables can be used to design a manual, case-picking warehouse. Then, in order to illustrate how the empirical data approach compares to the anlaytical model approach, we provide the results of both approaches for our detailed example. The comparison provides insight into the validity of the empirical data approach as a means of quantifying storage and labor requirements for warehouse design.

This design approach supports a decision process that combines the invaluable insights of practitioners (in defining base warehouse components such as choosing the appropriate racking for various product types and choosing among design options) with a quantitative comparison of design alternatives through the use of analytical models and empirical data. We have implemented the design approach for a manual warehouse through a computer tool that includes both case- and piece-picking operations. The tool can be used to evaluate and compare various designs.

The motivation for this research stemmed from two research projects through the Center for Excellence in Logistics and Distribution (CELDi) with a member organization pertaining to warehouse design. The first project involved modeling the member organization's forward-reserve problem. Their forward area for case picking consists of the bottom level of pallet rack (with the upper levels serving as reserve storage). A tool was developed to evaluate the effect of the size of the forward area on operational efficiency. The second project provided the member organization with a tool to determine the best shape of a facility for a given number of pallet locations while considering the number of dock doors available, the labor for storage and retrieval of product, as well as construction costs. To emphasize the relevance of the proposed research, we include a quote from the CELDi Industrial Advisory Board member representative from this organization:

> The two CELDi projects referred to above have been implemented by our organization with positive results. Project 1 is the CELDi project focused on determining the optimal size of our Zone 10, forward picking area. The output of this project was an Excel-based Tool that we have used in eight distribution centers so far to re-size our Zone 10, forward picking area. We are consistently trying to minimize our travel within our warehousing activities and have im-

plemented a travel time per line metric to measure our performance. Whereas the aforementioned tool was utilized, we have seen a significant drop (>10%) in travel time per line after implementation.

In addition, Project 2 is the CELDi project focused on the optimal distribution center facility shape and dock-door configuration. The output of this project (which incorporated the output of Project 1) was an Excel-based Tool that we have used to help analyze three distribution centers to date. We have not necessarily implemented all of the findings as the buildings were already standing, but we have been able to have discussions with our building/real estate group on the additional costs in building the facility in the non-optimal way. The Tool will be instrumental in helping to arrive at a final building configuration for future greenfield buildings, which we believe will have noteworthy savings of tens of thousands of dollars.

With these tools, we are able to make easy recommendations on forward pick zone sizing and dock usage without going through an entire project cycle. More importantly, having these two tools—we are now able to rely on empirical data versus emotional "assumptions" to drive key business decisions in facility layout design.

In the next chapter we present the literature related to overall warehouse design, as well as the literature pertaining to storage policies and routing heuristics for order picking, as our design approach utilizes order-picking models to quantify labor requirements and because order picking comprises a large portion of the operational expenses in most warehouses. In Chapter 3, we formally present the problem statement of our research, emphasizing the contributions of our research. In Chapter 4, travel-time models are presented for the put-away, order-picking, and replenishment operations in a manual warehouse. The models consider a centrally located pickup point and uniformly distributed dock doors for both random storage and volume-based storage policies. For each of these models, we investigate the optimal warehouse shape factor.

In Chapter 5 we present the design methodology and illustrate its use through parameterized analytical models for the put-away, order-picking and replenishment operations in a manual, case-picking warehouse. In Chapter 6, we use the analytical models presented in Chapter 4 to generate "discretized lookup tables" to illustrate the design methodology using empirical data. We then compare the results using the lookup tables to the results obtained using analytical models to investigate the feasibility of empirical data for quantifying labor requirements in warehouse design. In Chapter 7, we use order data from six existing warehouses, along with fourteen derived data sets, to analyze the effect of the warehouse operating environment on the optimal design that is obtained through complete enumeration. Using the results from the analysis, we present a heuristic search procedure to prune the solution space in order to provide a good design that can be further analyzed and improved. Finally, we summarize our research involving overall warehouse design and present our thoughts on future research in Chapter 8.

Bibliography

[1] Apple, J. M., Meller, R. D., and White, J. A., "Empirically-Based Warehouse Design: Can Academics Accept Such an Approach?," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[2] Baker, P., and Canessa, M., "Warehouse Design: A Structured Approach," *European Journal of Operational Research*, 193, 245–258 (2009).

[3] Caron, F., Marchet, G., and Perego, A., "Routing Policies and COI-Based Storage Policies in Picker-to-Part Systems," *International Journal of Production Research*, 36, 713–732 (1998).

[4] Chew, E. P., and Tang, L. C., "Travel Time Analysis for General Item Location Assignment in a Rectangular Warehouse," *European Journal of Operational Research*, 112, 582–597 (1999).

[5] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Operation: A Comprehensive Review," *European Journal of Operational Research*, 177, 1–21 (2007).

[6] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[7] Hall, R. W., "Distance Approximations for Routing Manual Pickers in a Warehouse," *IIE Transactions*, 25, 4, 76–87 (1993).

[8] Le-Duc, T., and de Koster, R., "Travel Distance Estimation and Storage Zone Optimization in a 2-Block Class-Based Storage Strategy Warehouse," *International Journal of Production Research*, 43, 17, 3561–3581 (2005).

[9] McGinnis, L. F., Goetschalckx, M., Sharp, G., Bodner, D., and Govindaraj, T., "Rethinking Warehouse Design Research," in *Proceedings of the 2000 International Material Handling Research Colloquium*, Charlotte, NC (2000).

[10] Modern Materials Handling, "2011 Warehouse/DC Operations Survey," `http://www.mmh.com/article/2011_warehouse_dc_operations_survey`, accessed November 22, 2011 (2011).

[11] Rouwenhorst, B., Reuter, B., Stockrahm, V., van Houtum, G., Mantel, R., and Zijm, W., "Warehouse Design and Control: Framework and Literature Review," *European Journal of Operational Research*, 122, 515–533 (2000).

[12] van den Berg, J. P., "A literature survey on planning and control of warehousing systems," *IIE Transactions*, 31, 751–762 (1999).

Chapter 2

Literature Review

The focus of this research is concerned with overall warehouse design. We present the methodologies that have been proposed in the academic literature in this area. One of the key objectives in warehouse design is to minimize the operational costs inherent in the design. Operational costs include receiving, put away, order picking, checking and packing, and shipping [4]. Order picking accounts for a significant portion of operational expenses, with travel time being the dominant cost factor [4]. In order to reduce travel, activity-based storage layouts have been presented in the literature that place fast-moving items in close proximity to the pickup and deposit (P&D) point. Consequently, in addition to overall warehouse design, we present the academic literature concerned with order picking and storage layout, as routing and storage policies represent key decisions in overall warehouse design.

2.1. Overall Warehouse Design

Two factors make overall warehouse design a challenging problem: the number of possible designs and the interactions among the functional components of the warehouse. In a review of warehouse design papers, Gu et al. [14] state that published research in overall warehouse design consists of qualitative models with simplifying assumptions. Further, the authors assert that a simple, validated model that provides results for guiding overall structural design would be a valuable research contribution. Currently, simulation is the most common method for assessing warehouse performance in both research and industry, and more computational tools for warehouse design and operation may encourage a closer alignment of academic research with practical application [14].

A survey paper by Rouwenhorst et al. [30] characterizes warehouses in terms of processes, resources and organization, and the authors classify design problems at the strategic, tactical and operational levels. The authors contend that design decisions at the strategic and tactical level are often interrelated and require joint consideration. Further, the authors conclude that the majority of research papers address isolated subproblems, and they recognize the need for an integration of models and methods in order to develop a methodology for systematic warehouse design. Van den Berg et al. [31] present a hierarchy of warehousing decisions for operational planning and control, and the authors present the methods and models that have appeared in the literature for each area.

According to the survey papers by van den Berg et al. [31], Rouwenhorst et al. [30], and Gu et al. [13], research in the area of overall warehouse design is limited. Furthermore, no comprehensive synthesis of models and techniques for overall warehouse design has been developed [30, 13]. Thus, an overall warehouse design methodology should provide the following:

- a quantitative comparison of design alternatives, and
- an initial design that can be further optimized to meet specific design requirements.

Research in the area of overall warehouse design generally falls into one of two categories: 1) solution procedures that provide a general, qualitative design framework, and 2) detailed models that provide a quantitative comparison of design alternatives. The papers that provide a quantitative comparison of solutions to the design problem often include models that require an extensive number of input parameters and are not general enough to apply to a broad range of warehouses. We first present the research papers that include general design frameworks and then discuss design methodologies that provide quantitative solution procedures.

Baker and Canessa [3] assert that no comprehensive, systematic methodology has been achieved for warehouse design. The authors performed a survey of research papers in the area of overall warehouse design, grouping them as those that examine tools and techniques and those that address overall steps in the design process. The authors then compared the papers and formulated a general framework of steps in order to assist practitioners and researchers in a more comprehensive warehouse design methodology. To validate the framework, twelve warehouse design companies were interviewed and responses from seven of the contacts were used to improve the framework. The steps in the proposed framework are as follows: 1) Define system requirements, 2) Define and obtain data, 3) Analyze data, 4) Establish unit loads to be used, 5) Determine operating procedures and methods, 6) Consider possible equipment types and characteristics, 7) Calculate equipment capacities and quantities, 8) Define services and ancillary operations, 9) Prepare possible layouts, 10) Evaluate and assess, and 11) Identify the preferred design. The paper offers tools and references for each of the steps, but the authors note that there is less consensus on the tools to be used for each step.

Ashayeri and Gelders [2] categorize solution procedures as analytical, simulation or heuristic and identify the research in each area. The authors suggest a two-step technique for system design that first uses analytical models to prune the decision space, and then introduces simulation to capture the dynamic aspects of the simplified analytical models. Yoon and Sharp [32] presented a systematic design procedure for order-picking systems with functional areas for order picking to assist designers in determining alternatives for order-picking configurations. The structured design procedure occurs in three stages including: analysis of order transaction data, selection of equipment types and operating strategies, and evaluation in terms of a performance analysis for each subsystem. The selection stage includes four design tasks that require joint consideration: calculation of storage capacity, equipment and operating policy specifications, physical transformations of load types, and sorting area specifications. The evaluation stage includes a performance analysis for each subsystem. No specific models are presented for throughput calculations for each subsystem, though the authors reference previous research for these calculations.

Three papers provide solution procedures that provide a quantitative comparison of design alternatives. First, Gray et al. [12] developed a model for overall design with the objective of minimizing initial incremental costs and operating costs including labor and inventory holding costs. In order to reduce the complexity of the formulation, the authors propose a hierarchical decomposition of the problem with three decision levels: facility design and technology selection, item allocation and assignment, and operating policy (number of pickers and zones, number of orders per batch, number of sorters, etc.). Analytical models were developed for a specific company for the assignment of items to zones, item facings and aisle length, assignment of items to storage types, order batch size, number of pickers and zones, and pick cycle time. The analytical models were coordinated to prune the decision space, and simulation was used to evaluate the alternative designs and to validate the analytical models. The solution procedure involved iteration among the three decision levels, and the authors estimated a labor savings of close to 50% with the new design. The authors acknowledge that a detailed formulation for general use is not viable because specific features would have to be considered that are not necessarily applicable to other problems.

Next, Park and Webster [22] formulated a design model for a unit-load warehouse. Analytical models were developed to determine land, building, equipment, labor and operating costs. The solution procedure requires input parameters for product flow and equipment characteristics and costs. An iterative process is used to determine the maximum inventory levels, initial investment and annual costs, and storage and equipment requirements. The procedure enumerates all possible storage rules, equipment types, control procedures and equipment patterns. The authors illustrate the solution procedure through a case study that considers three alternative designs: a fully automated AS/RS, narrow-aisle lift trucks, or counter-balanced lift trucks. The design model produced three superior alternatives that all employed an AS/RS with a dual-command control procedure and with simultaneous, two-dimensional movement. The final step involved choosing among random, two-class or three-class storage rules. The authors acknowledge that obtaining cost and model parameters for individual firms would require considerable effort.

Finally, Apple et al. [1] proposed an empirically based warehouse design methodology that uses a qualitative list of factors to consider (usually in the form of checklists), as well as quantitative matrix solution guides. Pareto charts are suggested to subdivide the warehouse activities in terms of storage and activity for each handling unit, and FFNs are used to represent each conceptual design. Each functional area is then sized (using available tools), and trial block layouts are developed that seek to minimize handling distances. Finally, product flows are synchronized and connecting processes, slotting, and zoning/batching procedures are developed in order to estimate material handling and labor costs. The paper also lists situations that may warrant moving from a manual to an automated warehouse. The authors acknowledge that in order to implement such a methodology, work in two areas must be accomplished. First, standardized definitions for process descriptions must be developed, and second, the quantitative matrix solution tables must be populated.

2.2. Storage Layout

Rosenblatt and Roll [29] introduced a twelve-step procedure to determine the optimal storage design in terms of warehouse capacity and extent of randomness in order to minimize the costs associated with shortage of space (resulting in rejection of incoming shipments), as well as costs for construction, handling, and storage policy, where the cost models are assumed to be known. The authors point out that random storage in larger zones results in better utilization of storage within the warehouse. Thus, for a class-based storage system, the storage utilization would decrease as the number of classes increases.

Frazelle et al. [10] developed a procedure to first determine the best size of the forward area and then determine which products should be included in the forward area, as well as in what quantities. A case study was used to illustrate the procedure, resulting in a 40 percent reduction in order-picking costs.

Jarvis and McDowell [18] developed a stochastic model to allocate product to storage locations based on the traversal routing strategy. Using the results from the model, the authors developed an assignment algorithm to minimize the within-aisle travel by grouping products with the lowest demand in an aisle and the products with the next lowest demand in another aisle, until all products have been assigned to an aisle. The algorithm assigns the products with the highest demand to the centermost aisles, while the products with the lowest demand are placed in the outermost aisles, such that the aisles in the warehouse are symmetrical. The authors determined that the factors that influence the optimal product location in a warehouse include the average number of picks per order, the number of items in the warehouse, and the shape of the ABC curve. Based on their research, the shape of the ABC curve has the greatest impact on the number of aisles traversed, and as the number of picks per order increases, the optimal product layout has less of an impact on the number of aisles crossed.

As noted by Le-Duc and de Koster [20], there is no general rule for determining the storage boundaries in the diagonal layout. In the diagonal layout, each aisle contains different storage classes, and storage boundaries for each class differ across aisles. Thus, the authors developed a heuristic to determine the optimal (or near optimal) storage boundaries using a travel distance approximation for the diagonal layout, where each aisle is assigned a storage boundary for each product class. Their results indicated that for a large number of picks, the storage boundaries for each class should be the same across all aisles. This layout is known as the identical-aisle or across-aisle layout. They found that for a small number of picks, the warehouse shape plays a role in determining the optimal storage boundaries for each class. Further, their results indicated that the identical-aisle layout provided very near optimal results, regardless of the number of picks.

Heskett [16] was the first to introduce the concept of Cube-Per-Order Index (COI) that locates items based on the space required per cubic feet divided by the order frequency.

2.3. Travel-Time Models

2.3.1 Put-away Operation

Francis [9] was the first to develop a model for the expected travel in a warehouse. The distance approximation assumed a single pickup and deposit location for a single item (akin to the put-away operation). The resulting model was used to show that the optimal shape of a warehouse is one that is twice as wide as it is long.

Bassan et al. [5] developed expressions for optimal design parameters for two aisle layouts with random storage when considering the costs associated with handling, warehouse area, and building perimeter. The handling costs are based on the expected annual travel distance for the storage and retrieval of an item (a put-away operation). The authors also consider the optimal warehouse shape if random access to any door is allowed. In their analysis, they considered each door individually and found that the minimum distance is achieved from the middle of the longitudinal wall. Thus, like Francis [9], they concluded that a warehouse that is twice as wide as it is deep is optimal.

2.3.2 Order-Picking Operation

Order picking has received considerable attention in the warehousing literature because manual systems tend to be highly labor-intensive, while automated systems can be very capital-intensive [19]. Manual warehouses are often referred to as "picker-to-part" systems, and automated warehouses are characterized as "part-to-picker" systems. Ratliff and Rosenthal [27] developed an algorithm to determine the optimal order-picking tour in a picker-to-part warehouse; however, the optimal tour is often confusing to the order picker and difficult to implement with current warehouse management system software. Thus, heuristic strategies are primarily used in practice. However, Bartholdi and Hackman [4] point out that advances in technology will eventually allow order pickers to receive precise travel instructions, allowing implementation of optimal pick paths.

Hall [15] developed analytical models for the traversal, midpoint, and largest gap strategies in a random storage warehouse and presented rules of thumb for choosing between these strategies. The models that Hall [15] developed indicated that the largest gap and midpoint strategies are close to optimal when the number of aisles is larger than the number of picks. Further, Hall determined that the largest gap strategy is preferred over the traversal strategy when the number of picks per aisle is less than 3.8. In terms of warehouse shape, Hall [15] found that as the number of stops per tour increases, the traversal strategy favors a wider warehouse (with a higher width-to-depth shape ratio).

Petersen [23] compared the results of four routing policies (traversal, return, largest gap, and composite) to optimal routing for a random storage warehouse with 1,000 storage locations. The parameters for warehouse shape, P&D location, and number of picks were varied to determine the effect on the performance of the routing strategy. Results from the experiment indicated that as the warehouse becomes wider (the number of aisles increases), the performance of the routing strategies becomes more consistent and similar. Further, the performance of the traversal strategy improves as the number of stops increases, while the composite, largest gap and midpoint strategies produce results that are similar to the optimal route. Petersen [23] also compared the mean route lengths for a centrally located P&D point and a corner P&D point and found that a centrally located P&D point results in a travel savings of only 0.9 percent over a corner P&D point.

Caron et al. [6] considered the optimization of a COI-based storage layout and two routing policies for a warehouse with two sections separated with a cross-aisle. The aisles are assumed to be parallel to the front of the warehouse with two sections of aisles separated by a center cross aisle and the P&D point. For the return strategy, the items with the lowest COI value were placed at the ends of the aisles nearest the centermost cross aisle; for the traversal strategy, the items with the lowest COI value were placed in the aisles nearest the front of the warehouse, leaving the back aisles for the items with the highest COI values. For the return policy, the expected number of picks in an aisle is calculated based on the total picks divided by the number of aisles. This results in an overestimation of the expected within-aisle travel, with a maximum error for low values of n ($n < 1$, where n is the number of picks per aisle). The model for the traversal policy overestimates the within-aisle travel due to the return travel to the front of the warehouse (which is calculated as half of the aisle length). The results indicate that the return policy outperforms the traversal policy only when the number of picks per aisle is less than one or for highly skewed COI-based ABC curves. Also, a frequent relocation of items is necessary to maintain the strict COI storage policy with the return strategy.

Petersen and Schmenner [25] evaluated the various routing heuristics and compared them to the optimal route for volume-based storage, as well as the impact of pick list size and demand skewness on routing policies. An experimental design was conducted for a warehouse with 1,000 storage locations and 10 aisles. The factors considered in the analysis included six routing policies, eight storage layouts and P&D combinations, five pick list sizes and three levels of demand skewness. The authors found that within-aisle storage is the best overall volume-based storage policy. Based on their experimental results, the authors concluded that the perimeter and across-aisle layouts do not perform well, but they may work well for a warehouse where congestion is a problem. The results also indicated that as the demand skewness increases, there is less of a difference in performance among the routing policies. Further, the return policy works well with the diagonal and across-aisle storage layouts, and traversal routing performs the best for the within-aisle storage layout. Their research also shows that the return policy works well for a small number of picks.

Petersen and Aase [24] used a simulation model to compare class-based storage to a strict volume-based storage policy, as well as a random storage policy, for order-picking travel. Their results indicated that class-based storage saves travel as compared to random storage, and the performance approaches that of a strict volume-based storage policy as the number of storage classes increases. With two classes the results showed a 78 percent improvement over random storage, while three classes yielded a 90 percent improvement, and four classes improved travel by 94 percent. In their experiment the authors also found that the traversal routing policy produced routes that were nine percent longer than the optimal route, and only six percent above optimal when using class-based storage and volume-based storage. The level of savings depends on the number of picks, as large pick lists have a greater probability of containing less popular items.

Hwang et al. [17] developed analytical models to determine order-picking travel based on a COI storage policy. The models included the return policy for the across-aisle layout, the traversal policy for the within-aisle layout, and the midpoint policy for the perimeter layout (where the fastest-moving items are placed in the outermost perimeter of the warehouse, and the slowest-moving items are located in the innermost storage locations). The performance of the three policies were compared by varying the number of picks, the skewness parameter for the COI-based ABC curve, and the shape ratio of the warehouse. Based on the models, the return policy performed well for a small number of picks (N=4), and the traversal policy performed the best for a large number of picks (N=64 to 80). In general, however, the midpoint policy outperformed both the traversal and return policies in terms of minimizing order-picking travel. Hwang et al. [17] also found that a highly skewed ABC curve can significantly reduce travel, regardless of the routing policy. Further, the authors concluded that the best warehouse shape is such that the length is twice as long as the width of the warehouse.

Le-Duc and de Koster [20] developed a model to estimate the travel required for an order-picking tour using the diagonal or across-aisle layout with the return policy. The model accounts for the fact that the storage boundaries for each class within an aisle may not be identical across all aisles. First, the expected number of picks within an aisle is calculated, and the expected value is then used to determine the number of picks within each class in the aisle. The model allows for a cross aisle between storage classes, where the order picker visits the farthest pick in an aisle and then returns only as far as the cross aisle. This can result in a travel reduction in aisles where there are picks for slower-moving items but no picks for fast-moving items.

Goetschalckx and Ratliff [11] considered the side-to-side travel in an aisle where the aisles are at least twelve feet wide, and they develop an optimal-traversal algorithm that was shown to save as much as 30 percent in travel time over commonly used policies. Their optimal algorithm was found to perform well except in cases where the pick density is low and aisles are narrow. The authors found that the breakpoint for the return policy versus the traversal policy depends on the density of picks and the width of the aisle, but for most practical densities, an all-traversal policy is better than a return policy.

Parikh and Meller [21] were the first to consider the vertical travel component of order picking. They developed throughput models that consider both Tchebychev and rectilinear travel for a random storage warehouse with single-deep pallet storage. The resulting model can be used to determine the optimal system configuration in terms of the number, length, and height of the storage aisles to meet storage and throughput requirements. A simple cost-based optimization model that considers the cost of pickers, equipment and space was used to evaluate the optimal height of the storage system. The authors concluded that the optimal storage height tends to decrease for a system with a high throughput requirement but increases as the cost of storage space increases.

Roodbergen et al. [28] developed analytical models for the S-shape heuristic that minimizes travel for order picking by identifying a layout structure with one or more blocks of parallel aisles. A layout optimization model is presented that finds the best balance between cross-aisle and within-aisle travel such that the total travel distance is minimized. The authors found that if aisles are very long, an additional cross aisle will significantly reduce the within-aisle travel (while only slightly increasing the cross-aisle travel.) However, for short subaisles, the extra cross aisle still reduces the within-aisle travel, but the gain is smaller due to the increased travel in the cross aisles. The authors contend that it is always better to have a multiple-block layout (at least one center cross aisle), unless the pick density is high and the cross aisles are wide.

Chew and Tang [7] model the order-picking travel for a traversal policy with a corner P&D point. The model is based on the occupancy problem and is applicable to any item-location assignment. The occupancy problem [8] involves determining the probability of filling exactly J urns (out of M possible urns) with at least one of n balls such that $J \leq M$ (i.e., determining the number of aisles that have at least one pick). However, the model does not apply to the diagonal or identical-aisle layouts, as it assumes a single probability for visiting a given aisle. The authors then model the total order-picking system as a queueing system that includes picking time, as well as sorting time.

2.3.3 Replenishment Operation

Very little research has considered the distance required for a replenishment operation. However, Pohl et al. [26] developed travel-time models for dual-command cycles in a unit-load, random-storage warehouse. The travel-between portion of the dual-command cycle is akin to the travel required for a replenishment operation. In this segment of the dual-command cycle, the worker performs a storage operation and then travels to a retrieval location before returning to the P&D point. With replenishment travel, however, instead of returning to a P&D location, the worker would travel to the reserve storage location for the next replenishment.

2.4. Summary of Literature Review

The published papers in the area of overall warehouse design are limited, and the models that have been proposed tend to be more qualitative, rather than quantitative. The numerous design options and interrelationships among functional areas of the warehouse make this a difficult problem to solve. Thus, a comprehensive model that provides quantitative results is needed.

Research in the area of storage layout has shown that order-picking travel can be reduced by using a volume-based layout versus a random storage layout. This is important because order picking is the most labor-intensive operation in most warehouses. Consequently, order picking has received considerable attention in warehousing literature. Models have been developed for various routing strategies, but most assume a random storage policy and a single P&D point. Two papers have considered travel models for COI-based storage, but this storage policy can require a frequent relocation of items if the order frequency is not consistent. Only one paper considers the travel for order picking in a class-based storage layout where each aisle can contain more than one class. The put-away operation has been modeled for a random storage warehouse with a central P&D point, but the replenishment operation has received little attention in the literature.

Bibliography

[1] Apple, J. M., Meller, R. D., and White, J. A., "Empirically-Based Warehouse Design: Can Academics Accept Such an Approach?," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[2] Ashayeri, J., and Gelders, L., "Warehouse Design Optimization," *European Journal of Operational Research*, 21, 285–294 (1985).

[3] Baker, P., and Canessa, M., "Warehouse Design: A Structured Approach," *European Journal of Operational Research*, 193, 245–258 (2009).

[4] Bartholdi, J. J., and Hackman, S. T., *Warehouse & Distribution Science*, Version 0.95 (2011).

[5] Bassan, Y., Roll, Y., and Rosenblatt, M. J., "Internal Layout Design of a Warehouse," *IIE Transactions*, 12, 4, 317–322 (1980).

[6] Caron, F., Marchet, G., and Perego, A., "Routing Policies and COI-Based Storage Policies in Picker-to-Part Systems," *International Journal of Production Research*, 36, 713–732 (1998).

[7] Chew, E. P., and Tang, L. C., "Travel Time Analysis for General Item Location Assignment in a Rectangular Warehouse," *European Journal of Operational Research*, 112, 582–597 (1999).

[8] Feller, W., *An Introduction to Probability Theory and Its Applications: Volume I*, Wiley, New York, NY (1968).

[9] Francis, R. L., "On Some Problems of Rectangular Warehouse Design and Layout," *The Journal of Industrial Engineering*, 18, 10, 595–604 (1967).

[10] Frazelle, E. H., Hackman, S. T., and Platzman, L. K., "Improving Order Picking Productivity through Intelligent Stock Assignment Planning," in *Proceedings of the Council of Logistics Management*, 353–371 (1989).

[11] Goetschalckx, M., and Ratliff, H. D., "Order Picking in an Aisle," *IIE Transactions*, 20, 53–62 (1988).

[12] Gray, A. E., Karmarkar, U. S., and Seidmann, A., "Design and Operation of an Order-Consolidation Warehouse: Models and Application," *European Journal of Operational Research*, 58, 14–36 (1992).

[13] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Operation: A Comprehensive Review," *European Journal of Operational Research*, 177, 1–21 (2007).

[14] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[15] Hall, R. W., "Distance Approximations for Routing Manual Pickers in a Warehouse," *IIE Transactions*, 25, 4, 76–87 (1993).

[16] Heskett, J. L., "Cube-Per-Order Index–A Key to Warehouse Stock Location," *Transportation and Distribution Management*, 3, 27–31 (1963).

[17] Hwang, H., Oh, Y. H., and Lee, Y. K., "An Evaluation of Routing Policies for Order-Picking Operation in Low-Level Picker-to-Part System," *International Journal of Production Research*, 42, 3873–3889 (2004).

[18] Jarvis, J. M., and McDowell, E. D., "Optimal Product Layout in an Order Picking Warehouse," *IIE Transactions*, 23, 93–102 (1988).

[19] Koster, R. D., Le-Duc, T., and Roodbergen, K. J., "Design and Control of Warehouse Order Picking: A Literature Review," *European Journal of Operational Research*, 182, 2, 481–501 (2007).

[20] Le-Duc, T., and de Koster, R., "Travel Distance Estimation and Storage Zone Optimization in a 2-Block Class-Based Storage Strategy Warehouse," *International Journal of Production Research*, 43, 17, 3561–3581 (2005).

[21] Parikh, P., and Meller, R. D., "A Travel-Time Model for a Person-Onboard Order Picking System," *European Journal of Operational Research*, 200, 2, 385–394 (2010).

[22] Park, Y. H., and Webster, D. B., "Modelling of Three-Dimensional Warehouse Systems," *International Journal of Production Research*, 27, 6, 985–1003 (1989).

[23] Petersen, C. G., "An Evaluation of Order Picking Routeing Policies," *International Journal of Operations & Production Management*, 17, 11, 1098–1111 (1997).

[24] Petersen, C. G., and Aase, G. R., "Improving Order-Picking Performance Through the Implementation of Class-Based Storage," *International Journal of Physical Distribution & Logistics Management*, 34, 7, 534–544 (2004).

[25] Petersen, C. G., and Schmenner, R. W., "An Evaluation of Routing and Volume-based Storage Policies in an Order Picking Operation," *Decision Sciences*, 30, 2, 481–501 (1999).

[26] Pohl, L. M., Meller, R. D., and Gue, K. R., "An Analysis of Dual-Command Operations in Common Warehouse Designs," *Transportation Research Part E: Logistics and Transportation Review*, 45E, 3, 367–379 (2009).

[27] Ratliff, H. D., and Rosenthal, A. S., "Order-Picking in a Rectangular Warehouse: A Solvable Case of the Traveling Salesman Problem," *Operations Research*, 31, 3, 507–521 (1983).

[28] Roodbergen, K. J., Sharp, G. P., and Vis, I. F. A., “Designing the Layout Structure of Manual Order Picking Areas in Warehouses,” *IIE Transactions*, 40, 1032–1045 (2008).

[29] Rosenblatt, M. J., and Roll, Y., “Warehouse Design with Storage Policy Considerations,” *International Journal of Production Research*, 22, 809–821 (1984).

[30] Rouwenhorst, B., Reuter, B., Stockrahm, V., van Houtum, G., Mantel, R., and Zijm, W., “Warehouse Design and Control: Framework and Literature Review,” *European Journal of Operational Research*, 122, 515–533 (2000).

[31] van den Berg, J. P., and Gademann, A. J. R. M. N., “A Literature Survey on Planning and Control of Warehousing Systems,” *IIE Transactions on Scheduling & Logistics*, 31, 8, 751–762 (1999).

[32] Yoon, C. S., and Sharp, G. P., “A Structured Procedure for Analysis and Design of Order Pick Systems,” *IIE Transactions*, 28, 379–389 (1994).

Chapter 3

Problem Statement

Overall warehouse design is a challenging problem because the combination of possible options for each functional area in the warehouse results in an overwhelming number of potential facility designs. Further, interrelationships exist among functional areas such that changing one functional area can affect many other areas, often resulting in a tradeoff in operational performance or space requirements between designs. The scope of the problem taken together with the interaction among functional components makes overall warehouse design a daunting task. Thus, it is not surprising that research in the area of overall warehouse design has received less attention than individual design components. Research that does address overall design typically proposes a theoretical framework, rather than providing an inclusive model that can be implemented.

Nonetheless, research that focuses on individual design components provides us with analytical models that allow a quantitative comparison of different solutions. Industry practitioners, however, often use discrete empirical observations when designing warehouses, and so the design results may differ considerably between practitioners. Both approaches have merit, though there is no holistic design methodology for either technique. Thus, a comprehensive, structured approach to overall warehouse design would be of value in the initial design phase of a warehouse.

Our objective is to present a structured approach to overall warehouse design and to provide a method for implementing the approach. The design methodology utilizes functional flow networks (FFNs) to characterize each design under consideration, where the nodes denote the functional areas and the arcs indicate the flow of product between the functional areas. Each node in the FFN first must be sized to accommodate the desired number of storage locations, thus providing a "footprint" of the functional area, and the flow of product must be rationalized between the functional areas. The product flow is then translated into labor requirements to allow a comparison of designs. The key to this approach, and a major focus of this research, is to provide a method for translating the product flow into labor requirements.

In our research we focus on two methods for accomplishing this task: analytical models and empirical data. Analytical models can be used to determine travel-time requirements for put-away, order-picking and replenishment operations. When analytical models are not available, empirical data in the form of lookup tables with discrete data points can be used to estimate distance requirements. For data points that are not specified in the table, linear interpolation can be used.

Once the space and labor requirements have been defined, the solution is evaluated to determine if the design is acceptable for consideration. This process is implemented until all reasonable designs have been considered. The candidate designs can then be further evaluated and compared. The complete design process is illustrated in Figure 3.1.

Our design methodology integrates analytical models and empirical data for specific warehouse functions into a model that can be used for overall warehouse design. Some analytical models can be found in the warehousing literature, but a complete set of models is not available. Thus, the first contribution of our research focuses on travel-time models for put away, order picking, and replenishment for random storage, as well as two class-based storage layouts. Our models include a uniform distribution of dock doors. We use these models to assess the effect of warehouse shape (width-to-depth ratio) on the put-away, order-picking and replenishment operations, as warehouse shape can significantly affect the travel distance for these operations.

Warehousing literature commonly refers to a 2:1 width-to-depth ratio as the optimal shape for a warehouse, as first shown by Francis [1]. However, this finding assumes a centrally located dock door and applies only to a single-stop tour (i.e., a put-away operation). Other papers have investigated the impact of warehouse shape on order picking, but only for a

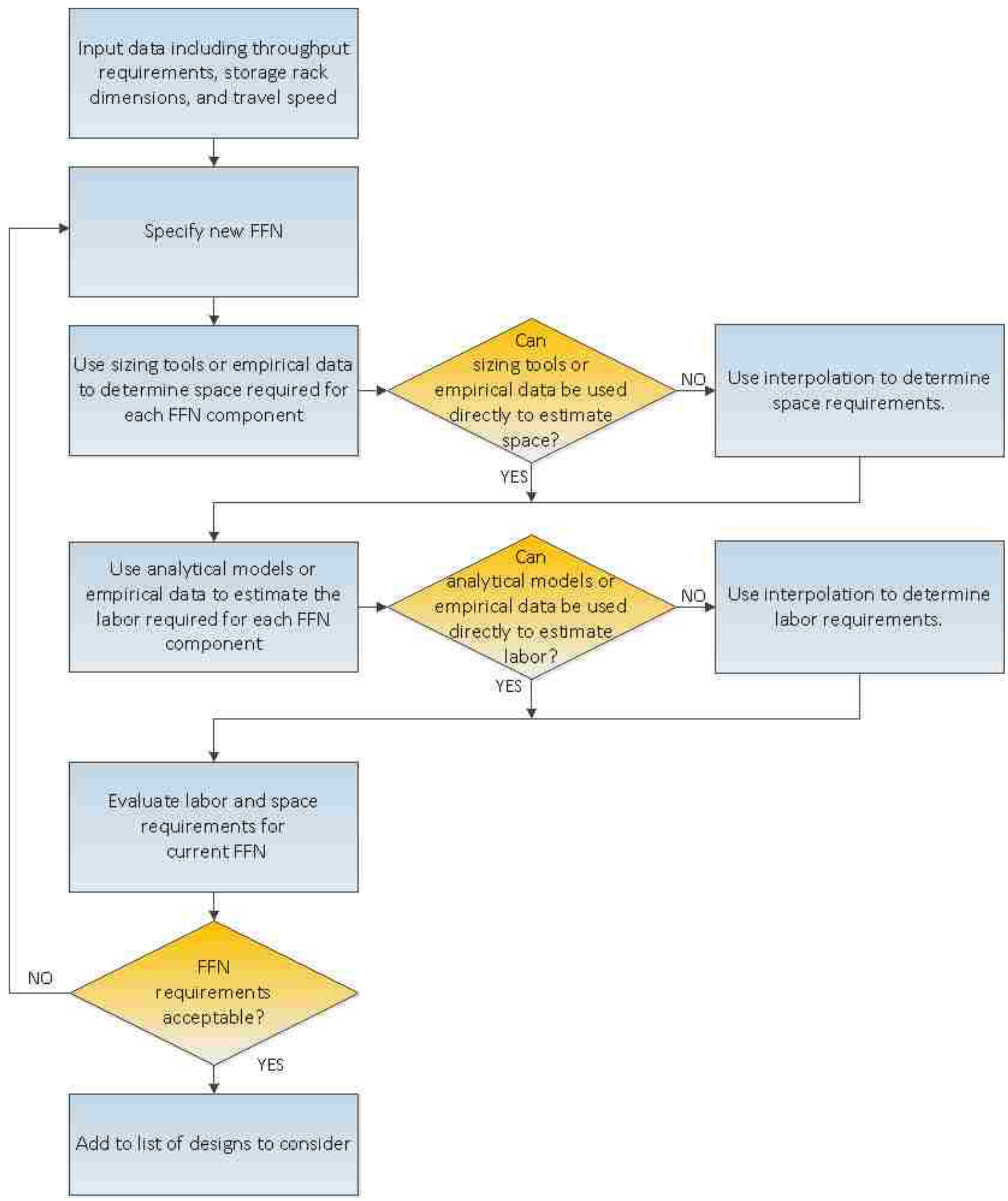


Figure 3.1: Empirically based warehouse design methodology.

random storage warehouse. Further, there are no research papers that consider the optimal warehouse shape for all travel-time operations, namely, the put-away, order-picking, and replenishment operations. Thus, we investigate the optimal warehouse shape that considers all of these operations.

The second contribution of our research involves implementing the warehouse design methodology for a manual, case-picking warehouse using the models that were developed for put away, order picking, and replenishment. We provide a detailed example to illustrate how the analytical models can be used to compare alternative designs.

Next, we apply the design methodology for the case where only empirical data are available. We use a sizing algorithm to populate a table with lookup values for space requirements. We parameterize the analytical models for put away, order picking and replenishment to generate lookup tables with discrete data points for travel distance requirements. Finally, we present an example problem and implement the design methodology for each approach, comparing the results from the use of analytical models to the results from the lookup tables.

A comprehensive design methodology that incorporates analytical models and empirical data to allow a quantitative comparison of design alternatives before making any design changes would be beneficial to practitioners. Practitioners generally know "what works" and "what doesn't work," but we believe that incorporating quantitative results in terms of storage and labor requirements will provide a solid basis for comparing design alternatives and will provide a broader base of solutions from which to choose.

The objective of this research is to provide a model for overall warehouse design that can be applied to existing as well as new warehouses. Although new warehouses offer more design flexibility in terms of warehouse size and shape, existing warehouses can be improved significantly by considering class-based storage and separate forward areas for picking. Our design methodology focuses primarily on manual warehouses, yet the design approach can be expanded through future research to include new or existing technologies, as well as improved models.

3.1. Travel-Time Models for a Manual Warehouse

Order-picking travel accounts for a large portion of warehouse operating expenses, and as a result, order picking has received considerable attention in the literature. Specifically, models have been developed to approximate the distance required for various order-picking routing strategies. However, most of the research has focused on random storage warehouses, with less attention on models for volume-based storage.

Our objective is to present models for the order-picking, put-away and replenishment operations for class-based storage, as well as random storage. We chose class-based storage over other volume-based policies because it is easier to characterize in terms of input parameters as compared to strict dedicated policies. Also, class-based storage requires less storage space than dedicated storage, as the storage is random within each storage class. For the order-picking operation, we consider the case where there is a centrally located pickup point and a uniformly distributed deposit point (i.e., uniformly distributed dock doors and dock door usage). We consider two class-based storage layouts. The across-aisle layout is useful when dock doors are located on only one side of the facility, and the within-aisle layout is conducive to dock doors on both sides of the facility. We include models for each of these layouts to determine the impact of warehouse shape on order-picking travel. Warehouse shape is an important design decision, as it affects not only the travel distance required for put away, order picking, and replenishment, but also the number of dock doors that are available.

The put-away operation was the first travel component to be considered in evaluating the optimal warehouse shape. In particular, Francis [1] determined that a warehouse with a 2:1 width-to-depth ratio is optimal for the travel required for a pallet put away or retrieval. However, the model assumes a single, centrally located P&D point, rather than a uniform distribution of dock doors. We consider uniformly distributed dock doors in our models, and we examine the effect of class-based storage on the put-away operation, as well well its impact on the optimal warehouse shape.

Hall [2] developed a model to determine the order-picking travel distance for the traversal routing strategy with a random storage layout. We modified Hall's model to include a uniformly distributed deposit point (i.e., depositing at a uniformly distributed dock door), instead of a single, central deposit point. In addition, we developed new order-picking models for two class-based storage layouts. In our research we use the models to show how the optimal warehouse shape is affected by the skewness of the ABC curve.

Replenishment has received limited attention in academic research. We present new models for the replenishment operation for random storage, as well as class-based storage. We use the models to determine how the replenishment operation is affected by different warehouse shapes. Finally, we conclude this research with an example that illustrates how to determine the optimal warehouse shape when considering all three travel components, namely, the put-away, order-picking and replenishment operations.

3.2. The Design Methodology Using Analytical Models

Our next research objective is to demonstrate how the design methodology can be implemented using analytical models. We focus on a manual, case-picking warehouse, and use the complete set of models that we developed for put away, order picking and replenishment for both random storage and two class-based storage layouts. We present an example that considers several designs, namely a reserve storage area consisting of pallet rack from which all order picking occurs, as well as designs with a forward area of various sizes for picking (such as the top 10% or top 20% of the SKUs for a given ABC curve). Our example considers a forward picking area with random and class-based storage, where the bottom level of the centermost aisles of pallet rack comprise the forward area. Finally, we demonstrate how a comparison can be made of the various design solutions.

3.3. The Design Methodology Using Discrete Empirical Data

With this research our objective is to demonstrate how the design methodology can be used with empirical data that quantifies space and labor requirements. Again, we consider a manual, case-picking warehouse with put-away, order-picking and replenishment operations. We present a table for sizing the pallet rack area, and we parameterize our analytical models to generate a set of tables that contain discrete data points for labor requirements. The sizing table provides the parameters for a pallet area of a given width and number of aisles. These parameters are then used as lookup values for the tables that contain distance requirements for put away, order picking and replenishment. The resulting travel distance is then divided by the appropriate travel speed to determine the time per operation.

Using an example, we compare the two methods for quantifying space and labor requirements. We believe that our research confirms the feasibility of using empirical data for warehouse design when analytical models are not available.

3.4. A Search Heuristic for a Manual, Case-Picking Warehouse

Our final research objective is to develop a heuristic search procedure for designing a manual, case-picking warehouse. We evaluate the performance of various designs based on the warehouse operating environment. We present warehouse parameters for 20 data sets, and we use complete enumeration to determine the best design for each data set. Again, we use the analytical models that we developed for put away, order picking and replenishment to quantify the space and associated labor requirements for each design. To determine the effect of warehouse parameters on design decisions, we compute correlation coefficients between the warehouse parameters and the optimal forward area size, as well as the optimal pallet area shape. In addition, we vary the warehouse parameters for each data set. Based on our analysis, we present a heuristic search procedure to determine a good design that can be further analyzed and optimized. We test the heuristic search procedure on ten independent data sets to illustrate the heuristic's performance.

Our heuristic is the first, comprehensive approach to warehouse design, albeit for the limited scope of a manual, case-picking situation. We believe this shows promise for our overall approach of using parameterized analytical models to support an empirically based warehouse design methodology.

Bibliography

[1] Francis, R. L., “On Some Problems of Rectangular Warehouse Design and Layout,” *The Journal of Industrial Engineering*, 18, 10, 595–604 (1967).

[2] Hall, R. W., “Distance Approximations for Routing Manual Pickers in a Warehouse,” *IIE Transactions*, 25, 4, 76–87 (1993).

Chapter 4

Contribution 1: A Paper on, "Analytical Models for Warehouse Configuration"

Abstract: The performance of a warehouse is impacted by how it is configured, yet there is no optimization model in the literature to answer the question of how to best configure the warehouse in terms of warehouse shape and the configuration of the dock doors. Moreover, the building blocks for such a model (put-away, replenishment and order picking models that can be combined in an optimization model) are either not available (in the case of replenishment) or built on a set of inconsistent assumptions (in the case of put-away and order picking). Therefore, we lay the foundation for more sophisticated warehouse configuration optimization models by developing the first analytical model for replenishment operation performance and extending put-away and order picking performance models. We then use these new models to address a question motivated by industry: the optimal configuration of a case-picking warehouse in terms of the shape of the facility and whether the facility is configured with dock doors on one or both sides. We present an example to demonstrate the use of our models in answering such a question, quantifying the benefit of using an integrated approach to warehouse configuration.

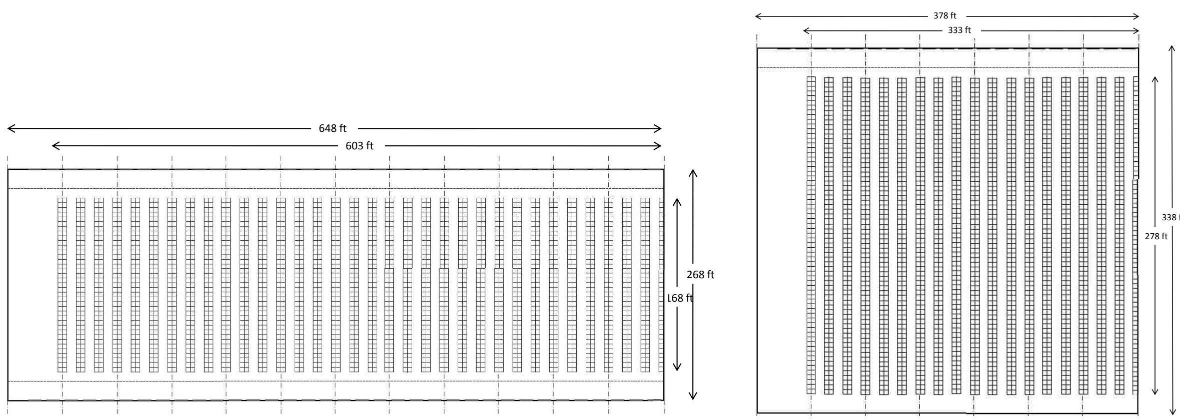
4.1. Warehouse Shape and Door Configuration

Warehouse design is a complex process that involves both structural and operational decisions that ultimately affect the overall performance of the warehouse. To further complicate the process, many of the design decisions are interrelated, leading to several design alternatives. Two such design decisions are warehouse shape and dock door configuration. We refer to the joint problem of these two design decisions as *warehouse configuration*.

The warehouse that we consider fulfills orders for cases and product is stored in pallet rack. The order-picking locations for fast-moving items reside on the bottom level of pallet rack and the upper levels are designated as reserve storage. Warehouses where an order picker travels along aisles to pick items represent the majority of warehouses [14], and the most common forward area for picking cases is the ground floor of pallet rack [1]. Accordingly, we believe that our research is applicable to a broad range of warehouses.

The warehouse shape factor is defined as the width-to-depth ratio of the storage-rack area, which also impacts the overall shape of the building itself [8]. Figure 4.1 illustrates two traditional warehouse layouts with parallel aisles where travel is rectilinear. The storage areas in the two layouts accommodate the same number of storage locations, but the shapes of the storage areas are very different. The shape of a warehouse directly impacts the number and length of the picking aisles. With rectilinear travel, it is clear from Figures 4.1(a) and 4.1(b) that the shape of the warehouse affects the travel distance for put-away, order picking and replenishment. Consequently, warehouse shape is an important design consideration.

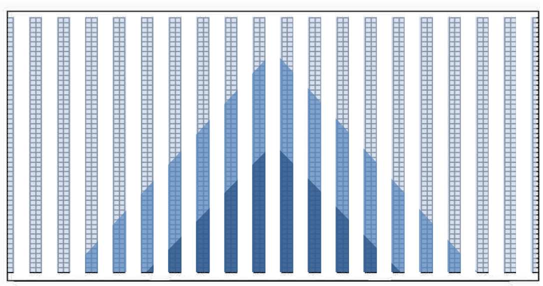
Francis [8] determined that for a centrally located pickup and deposit (P&D) point (or dock door) and single stops, as in a unit-load put-away operation, the optimal warehouse shape is such that the warehouse is twice as wide as it is deep. In fact, the 2:1 shape ratio has been accepted as "optimal" [8, 2]. However, we will show that removing the assumption of a single P&D point results in a different optimal shape. For a unit-load warehouse that utilizes all dock doors and performs only put-away and retrieval operations, this finding is significant. In addition to put away, one must also consider the order-picking and replenishment operations in determining the optimal warehouse shape because these three operations represent the major travel components in the warehouse that we consider. The replenishment operation is necessary when there is a forward area for picking, as only fast-moving products are stored here in order to reduce the size of the area, and, as a result, reduce travel. Replenishment occurs when a location in the forward area nears depletion.



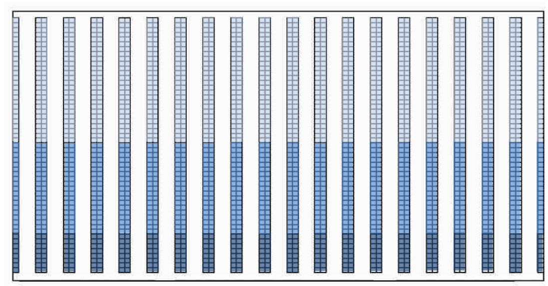
(a) (b)

Figure 4.1: (a) Width-to-depth shape ratio 3.6, doors on two sides; (b) Width-to-depth ratio 1.2, doors on one side.

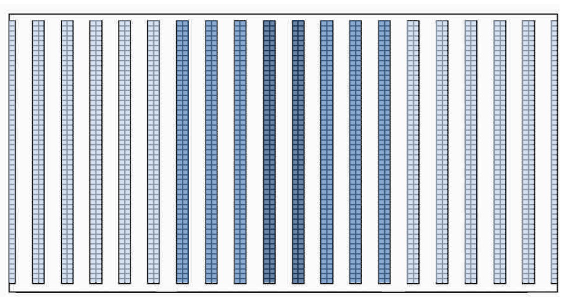
Other factors affecting travel distance are the storage and routing policies. In order to reduce travel, fast-moving items are often stored in locations that are convenient to P&D points. Class-based storage groups items into classes based on their level of activity, where the fastest-moving items are located closest to the P&D points in order to reduce travel and storage within each class is random. Class-based storage is preferred to full-turnover-based storage, as full-turnover-based storage requires a repositioning of items when demand frequencies change [22, 14]. Thus, considering class-based storage in the design phase helps to produce designs that are not overly sensitive to assumptions about demand patterns. Figure 4.2 displays three layouts for three classes of storage where the darker shades of color represent the fastest-moving items. When dock doors are located on only one side of the facility, the fast-moving items are located near the ends of the aisles closest to the dock doors as in Figures 4.2(a) and 4.2(b). The diagonal layout in Figure 4.2(a) places the fastest moving items closest to a single-central (assumed) P&D point at the front of the warehouse, and the slower items are placed in locations that are farthest from the P&D point. The identical-aisle layout in Figure 4.2(b) is a special case of the diagonal layout in Figure 4.2(a), where the boundary for each storage class does not vary from one aisle to the next but is identical across all aisles. This configuration aims to reduce the distance traveled along the aisles. If dock doors are located on opposite sides of the facility, the fast-moving items are generally located in the centermost aisles (near a central pickup point) as illustrated in Figure 4.2(c). This configuration seeks to reduce the number of aisles traveled, as well as the cross-aisle travel, and is often referred to as the within-aisle layout.



(a)



(b)



(c)

Figure 4.2: Class-based layouts: (a) Diagonal, 1-sided dock doors; (b) Identical-aisle, 1-sided dock doors; (c) Within-aisle, 2-sided dock doors.

The travel distance for each class-based layout depends on the amount of storage and the activity level for each class of items, as well as the warehouse shape. Further, the class-based storage layouts entail two different strategies for reducing travel (i.e., reducing cross-aisle travel versus within-aisle travel). Thus, the optimal warehouse shape is not obvious, and it would appear that practitioners would benefit by understanding how the warehouse configuration affects its performance.

To reduce the complexity of the models that we develop and to form the basis for validating our methodology, we restrict our consideration to layouts without center cross aisles. Cross aisles may reduce travel by allowing more opportunities for order pickers to change aisles, but inserting cross aisles requires additional warehouse space. Also, in cases where the pick density is high, additional cross aisles can result in longer picking tours. See Roodbergen et al. [20] for a model to estimate travel for cross aisles with a random storage layout and Berglund and Batta [4] for the optimal placement of cross aisles in a warehouse with class-based storage.

We were able to confirm that warehouse configuration is of interest to industry through two projects in the Center for Excellence in Logistics and Distribution (CELDi). A CELDi member organization was interested in determining how warehouse configuration affects the overall performance of their case-picking warehouse. Their only insight into this question was the above "optimal" 2:1 warehouse shape result. In addition, the company had an informal policy that more dock doors are preferred to fewer dock doors in designing a facility. Thus, most of their current facilities are configured with dock doors along both sides of the facility and as close to a 2:1 ratio as is permissible given the site plan. Generalization of the work for this member organization was funded by the other members of CELDi.

Thus, the objective of our research is to help such organizations. We do so by first developing analytical models to estimate the expected travel for the put-away, order-picking and replenishment operations for random storage, as well as the class-based storage layouts in Figures 4.2(b) and 4.2(c). Then, we use the models to investigate the optimal warehouse shape based on each operation. We believe that analytical models for overall warehouse design are preferred over simulation in evaluating design performance, as simulation is less conducive to generalization [10]. A primary result of this investigation is that the optimal shape varies considerably by the operation considered. Finally, we illustrate how to determine the optimal shape of a case-picking warehouse that considers the combined travel for put-away, order-picking and replenishment operations over a period of time. Even though we focus on the optimal warehouse shape as a design consideration, our models can be used collectively to asses overall warehouse design performance with regard to forward area size and layout, pallet rack height, as well as the shape of the pallet rack area.

4.2. Literature Review

Francis [8] was the first to consider the optimal shape of the warehouse and concluded that a 2:1 shape ratio is optimal for a single P&D point and a single stop. Bassan et al. [2] considered multiple dock doors and concluded that the distance to a single point is minimized from the most centrally located door and that all doors should be as near as possible to the center of the warehouse. Although this is correct, they (incorrectly) concluded that a 2:1 warehouse shape is optimal for the case of a unit-load warehouse with multiple doors, as well as for a single P&D point.

Several routing policies have been suggested in the literature for order picking. The simplest strategy is the traversal policy, where the order picker enters every aisle that contains at least one pick location, traverses the entire aisle, and exits at the opposite end of the aisle [11]. With the return policy, the order picker enters an aisle, travels to the farthest pick, and returns to the same end of the aisle that was entered [14]. With the midpoint strategy the order picker travels only as far as the midpoint of the aisle before returning; picks past the midpoint are obtained from the back cross aisle [11]. With the largest gap policy, the order picker enters an aisle as far as the largest gap between two adjacent pick locations or between the end of the aisle and the closest pick, thus avoiding the largest gap that does not contain picks [11]. Finally, the composite strategy combines the traversal and return policies; an aisle is not traversed if returning results in less travel for a given aisle [14].

Hall [11] developed models to approximate the expected travel distances in an order-picking warehouse for the traversal, midpoint and largest-gap routing strategies based on a fixed area with a random storage policy and a centrally located P&D point. He found that for random storage, elongated (wider) warehouses are favorable as the number of picks increases. This finding is intuitive because if there are as many picks as there are aisles, then it is likely that all aisles would be traversed. In this situation, a more elongated warehouse would increase the number of aisles while also making the aisles shorter. By elongating the warehouse, the within-aisle travel is then reduced.

Petersen [17] used simulation to compare the performance of the traversal, return, midpoint, largest-gap, composite (a hybrid of the return and traversal policies) and optimal routes in a random storage warehouse with 1,000 storage locations by generating pick lists of 5, 15, 25, 35 and 45 picks. Petersen concluded that narrow, deeper warehouses are more effective at minimizing order-picking travel for all of the strategies except the return policy. However, we attribute these results to the fact that Petersen [17] performed his analysis for 1,000 storage locations and only considered shape ratios of 3:1, 2:1, 1:1, and 1:2. Furthermore, with only 1,000 storage locations, it is conceivable (depending on the dimensions of the storage locations) that a pick list with more than 20 lines would result in more than one stop per aisle for the shape ratios considered. With more than one stop per aisle, narrow warehouses would indeed reduce travel, requiring fewer (but longer) aisles to be entered. Petersen [17] also concluded that the largest gap strategy is preferred for a smaller number of pick lines.

In Petersen's [18] simulation-based evaluation of storage layouts and routing policies, the within-aisle layout was favorable to the diagonal layout, regardless of the pick list size in a warehouse with ten aisles. Petersen's work (which assumed a warehouse with 10 aisles and 1,000 items) did not consider warehouses of varying shapes.

An activity-based strategy that assigns items to storage locations based on a ratio of the required space to the order frequency is the cube-per-order-index (COI), as first introduced by Heskett [12]. Caron et al. [6] developed analytical models for order-picking travel with a COI-based storage strategy where the warehouse is divided into two sections separated by a cross-aisle. The first model estimated the expected travel for a return routing policy using a modified version of the identical-aisle layout, and the second model estimated travel for the traversal routing policy using a modified version of the within-aisle layout. For the return strategy model, the authors acknowledge that the within-aisle travel is overestimated because it is based on the average number of picks per aisle.

Hwang et al. [13] also developed analytical models to determine order-picking travel based on a COI storage policy. Their models included the return policy for the identical-aisle layout, the traversal policy for the within-aisle layout, and the midpoint policy for a perimeter layout (where the fastest-moving items are placed at both ends of an aisle, and the slowest-moving items are located in the innermost storage locations in the aisle). The return policy performed well for a small number of picks, and the traversal policy performed the best for a large number of picks (N=64 to 80). In general, however, the midpoint policy outperformed both the traversal and return policies in terms of minimizing order-picking travel. Hwang et al. [13] also found that a highly skewed ABC curve can significantly reduce travel regardless of the routing policy and that the best shape for a warehouse is a 2:1 width-to-depth ratio. However, only five shape ratios were considered, ranging from 0.45 to 1.75.

Class-based storage, on the other hand, groups items into classes based on their activity level, where the storage within a class is assumed to be random. Le-Duc and de Koster [15] developed a model to estimate order-picking travel in a class-based storage warehouse with the diagonal layout, where the percent of storage for each class can vary across aisles. The model is based on the return strategy and utilizes expected values to determine the number of picks in an aisle, as well as the number of picks in each class. The authors acknowledge that using expected values results in an overestimation of within-aisle travel. Le-Duc and de Koster [15] were the first to consider the storage zone optimization problem for the diagonal-aisle layout. They developed a heuristic procedure to determine the optimal boundaries for each storage zone in an aisle and found that the identical-aisle layout is a robust layout for minimizing travel, regardless of the number of stops per tour.

Chew and Tang [7] modeled the order-picking travel for the traversal policy based on the occupancy problem for any item-to-location assignment. Their model assumes a corner P&D location, and because it uses a single probability for visiting a given aisle, the model does not apply to the diagonal or identical-aisle layout.

Despite extensive research on the forward-reserve problem [9], no prior research exists that considers the expected travel for replenishments (Thomas and Meller [21] recently presented a limited replenishment model). Pohl et al. [19] modeled the expected dual-command travel distance in a unit-load warehouse with random storage. The travel-between portion of their model estimates the expected distance between two random points in a warehouse. However, in a replenishment operation there are two travel legs: the first is travel between the last replenishment location and the storage location for the current replenishment operation (and is similar to the travel-between portion of dual-command); the second leg is travel between the storage location and the picking location for that product. As we discuss later, this second leg typically does not occur between two random points due to put-away strategies.

In summary, the research that has considered warehouse shape has been limited to the put-away operation or the order-picking operation in isolation, and only in the context of a random storage warehouse. Further, most of the previous research has considered the travel to and from a single P&D point rather than multiple dock doors. In taking all of this research together, the best shape of a warehouse is not obvious and appears to be dependent upon which operations are considered [21].

4.3. Optimal Warehouse Shape

In our travel-time models, we assume a rectangular warehouse with aisles that are orthogonal to the side(s) of the facility containing dock doors, where the doors are located at points along the entire width of the warehouse. We consider both a 1-sided and 2-sided configuration of dock doors. Our models assume that aisles are continuously located, that storage locations are continuous, and that the side-to-side movement within an aisle is negligible. We assume a uniform usage of dock doors for put-away operations, rather than a single, centrally located dock door.

For order picking, the pick list is obtained from a centrally located pickup point, and the completed order is deposited at a uniformly distributed dock door before returning to the pickup point. We consider a traversal routing policy for random storage. Even though the largest-gap strategy outperforms the traversal strategy, we acknowledge that it is less commonly used in practice due to its complexity. The forward area for picking is the bottom level of storage within the picking aisles, with the reserve storage locations in the upper levels of storage. Thus, replenishment to the forward area occurs within the storage aisles. We model the expected distance for replenishment such that the worker enters or exits an aisle from the end that results in the least travel.

For the 1-sided class-based storage layout, the shipping and receiving doors are located on only one side of the facility, and we utilize the identical-aisle layout in Figure 4.2(b). With the fast-moving items concentrated at the ends of the aisles, it is likely that most of the picks in an aisle will not require travel past the center of the aisle (for a skewed ABC curve), and the return policy minimizes the within-aisle travel for this layout. Le-Duc and de Koster [15] investigated the optimal storage boundaries for a class-based storage warehouse using the diagonal layout, and they found that the identical-aisle layout is optimal for a large number of picks using the return policy (where the picker enters and exits from the same end of the aisle, traveling as far as the farthest pick). Furthermore, Le-Duc and De Koster found that the identical-aisle layout provided very good results for the return policy, regardless of the pick list size. Because this layout places the fast-moving items along the entire width of the facility that contains doors, it may also result in less congestion than the diagonal layout.

For the 2-sided layout, the dock doors are located along both sides of the facility. Because there is a centrally located pickup point for obtaining the pick list and an effort to concentrate the pick locations of the fast-moving items, we propose the layout in Figure 4.2(c) with the traversal routing strategy. This layout will significantly reduce both the across-aisle travel and the number of aisles that require traversal, as most of the picks will occur in the centermost aisles. Additionally, because all of the items in an aisle are the same class, this layout will allow a flow-through of product, making the storage locations convenient to both the shipping and receiving doors [1].

We now present models that extend current research in three areas:

1. We present travel-time models for put away and order picking that consider more than one P&D point (multiple dock doors) for both random storage, as well as the 1-sided and 2-sided class-based storage layouts.

2. We present new travel-time models for replenishment operations for both random and class-based storage layouts. These models may provide useful insight to future research on the forward-reserve problem because forward-reserve models assume that the cost of replenishment can be specified *a priori*.

3. We consider warehouse shape for layouts with random and class-based storage and include an example that illustrates how the three warehouse operations (put away, order picking, and replenishment) can be weighted to determine the best overall warehouse shape.

The remainder of this paper is organized as follows. In Section 4.4 we present our models for the put-away operation for both random and class-based storage layouts. Models to determine the expected distance for order picking for random and class-based storage are presented in Section 4.5. In Section 4.6 we introduce models for the replenishment operation for both random and class-based storage, and in Section 4.7 we provide an example that illustrates how to determine the optimal warehouse shape when considering the put-away, order-picking and replenishment operations. Finally, in Section 4.8 we summarize our findings regarding warehouse shape for both random and class-based storage.

4.4. Put-Away Travel

In our models we denote the width of the warehouse as W, the depth of the warehouse (parallel to the picking aisles) as L as shown in Figure 4.3, and the number of levels of pallet rack as H. Accordingly, the two horizontal travel components x and y denote the across-aisle travel and within-aisle travel, respectively. Vertical travel, denoted by z, is required to access pallet locations above the bottom level of pallet rack. We refer to the expected horizontal distance required for a put-away operation as $E[D_{x,y}]$.

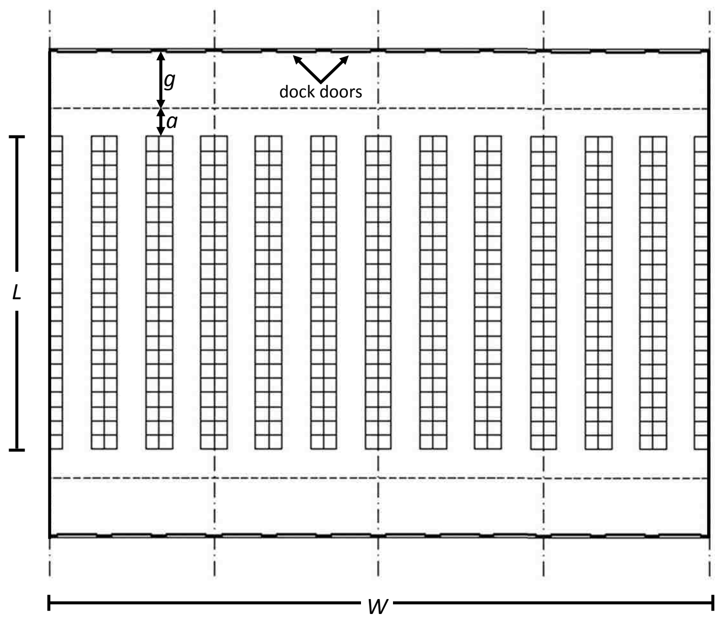


Figure 4.3: Warehouse parameters.

In our modeling we assume the height of the warehouse is fixed (see [16] for a model to determine this parameter). In Section 4.4.4 we include an expression for the expected vertical travel to put away a pallet and then explain how to calculate the total travel time that includes both the horizontal and vertical travel components.

4.4.1 Uniform (1-Sided or 2-Sided) Doors with Random Storage

Using well-known results [5] for the expected values of the maximum and minimum of two continuous uniform [0,1] random variables (and treating aisle locations as continuous random variables), the expected one-way horizontal travel for a put-away operation with random storage can be expressed as:

$$E[D_{x,y}] = W/3 + L/2 + g + a,$$

where g is the depth of the staging area and a is the width of the end cross-aisle in Figure 4.3.

Theorem 4.1. *The optimal warehouse shape for a put-away operation with uniform door usage and random storage is 3/2.*

Proof. We use the relationship for area ($A = WL$) and warehouse shape ($r = W/L$) to express the warehouse length and width in terms of the area and shape factor, r:

$$L = \sqrt{A/r} \qquad \text{and} \qquad W = \sqrt{Ar}.$$

Thus, the expression for expected travel can be written as:

$$E[D_{x,y}] = \frac{\sqrt{Ar}}{3} + \frac{\sqrt{A/r}}{2} + g + a.$$

Taking the derivative with respect to r and setting it equal to 0, we have:

$$\frac{1}{2}\left(\frac{\sqrt{A}}{3}\right) r^{-\frac{1}{2}} - \frac{1}{2}\left(\frac{\sqrt{A}}{2}\right) r^{-\frac{3}{2}} = 0.$$

Solving for r yields the optimal width-to-depth shape ratio:

$$r^* = 3/2.$$

□

With random storage, the optimal r value is the same regardless of a 1-sided or 2-sided configuration of dock doors. A similar exercise with one, centrally located door yields an optimal ratio of 2.0 [8]. Thus, we can clearly see that warehouse design is sensitive to the door usage assumption; with the use of all doors the optimal shape is 1.5, but with a single, centrally located dock door the optimal shape is 2.0.

4.4.2 Uniform (1-Sided) Doors with Class-Based Storage

The expected distance required for a put-away operation with class-based storage is different from the random storage model in that now we must consider the percent of storage for each class, as well as the frequency of put aways for each class. For the 1-sided layout, the objective is to determine how far into an aisle the operator must travel for the put away. The expected one-way travel for a 1-sided configuration with class-based storage is:

$$E[D_{x,y}] = W/3 + pL + g + a, \tag{4.1}$$

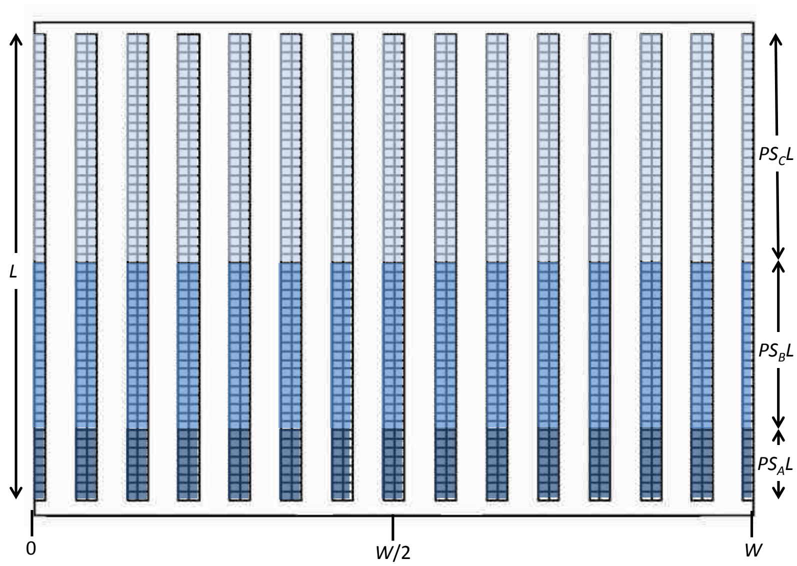
where p is the fraction of the aisle that is traveled for a given turnover, or activity profile. Here we use a class-based ABC curve. In Figure 4.4(a) the distance for each class within the aisle is shown as PS_AL, PS_BL and PS_CL, where PS is the percent of storage and the subscript is the storage class (such that $PS_A + PS_B + PS_C = 1$). Of interest is the optimal warehouse shape for the put-away operation in such a warehouse (the following result can be shown by convex analysis; a detailed proof can be found in Appendix A.1).

Result 1. *The optimal warehouse shape for a put-away operation with uniform doors and a 1-sided storage layout is* $3p$*.*

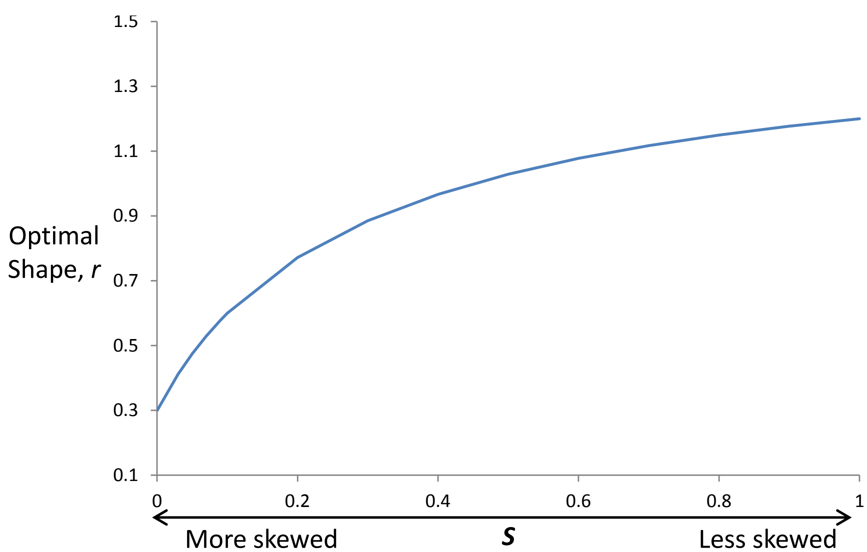
The value of p can be determined using the following:

$$p = PA_A(0.5PS_A) + PA_B(PS_A + 0.5PS_B) + PA_C(PS_A + PS_B + 0.5PS_C), \tag{4.2}$$

where PA is the percent of activity and the subscript is the storage class. For a particular ABC curve where the percent of storage is 20/30/50 for class-A, class-B, and class-C items



(a)



(b)

Figure 4.4: (a) 1-Sided, class-based storage; (b) Optimal r values.

and the percent of activity is 80/15/5, the value of p is:

$$p = 0.17.$$

Therefore, the optimal value for r for an 80/15/5 ABC curve is:

$$r = 3(0.17) = 0.51.$$

Note that for an ABC curve where the percent of activity is $\frac{1}{3}/\frac{1}{3}/\frac{1}{3}$ and the percent of storage is $\frac{1}{3}/\frac{1}{3}/\frac{1}{3}$ for class-A, class-B and class-C items, the value for p is:

$$p = 0.5\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + 0.5\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + 0.5\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{2}$$

and therefore the optimal r for uniform storage is $\frac{3}{2}$, which is the equivalent result from Theorem 4.1.

Thus far, we have used the ABC curve to determine the value p. In subsequent sections where we consider order picking and replenishment, we will use the parameter S from Bender [3] to refer to the skewness of the ABC curve,

$$Y = \frac{(1+S)X}{S+X},$$

where X is the percent of storage, Y is the percent of activity and S is the skewness factor. For a set of points (X_i, Y_i), where the values for X_i represent the percent of storage and Y_i represents the percent of activity for each item i, the value of S can be determined with the least squares method using the following expression:

$$\sum Y_i - (1-S)\sum \frac{X_i}{S - X_i} = 0.$$

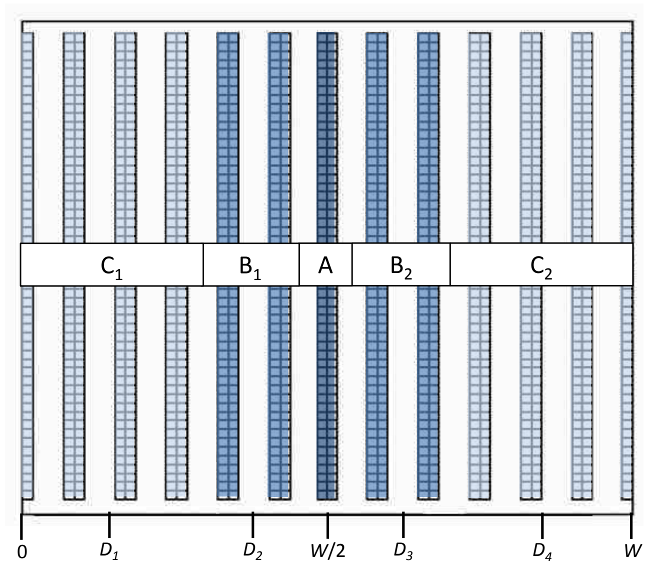
We note here that S is positive and approaches infinity for the case where all items approach an equal activity level (uniform distribution). Figure 4.4(b) displays the optimal values for r for a range of ABC curve parameters, denoted by S. The table in Appendix B lists values for S and p for a range of ABC curves.

4.4.3 Uniform (2-Sided) Doors with Class-Based Storage

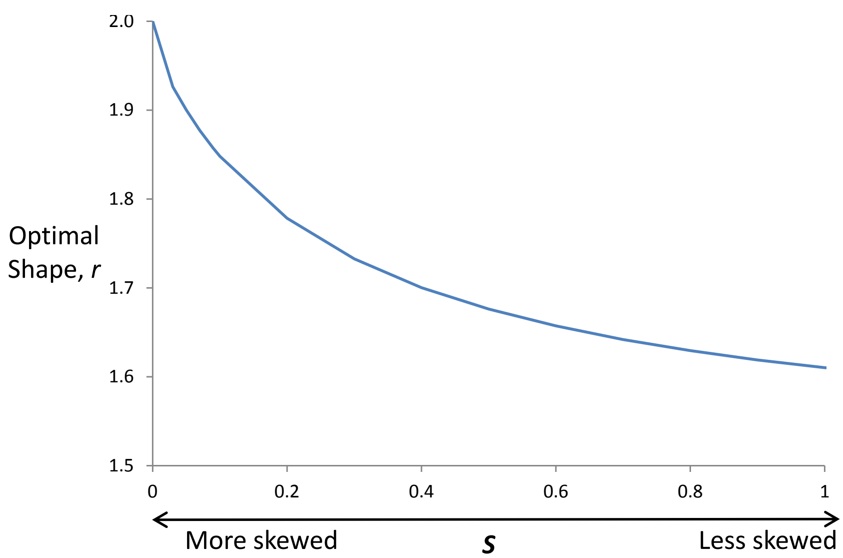
For a put-away operation with the 2-sided, class-based layout, the across-aisle distance must be determined based on the frequency and layout of each class, whereas the distance into an aisle is equivalent to that of random storage (i.e., half the length of the aisle). The layout of class-A, class-B and class-C items for a 2-sided configuration is displayed in Figure 4.5(a), and the distances to the center of each storage class can be defined as:

$$
\begin{aligned}
D_1 &= 0.25(PS_C)W, && (4.3)\\
D_2 &= 0.5(PS_C)W + 0.25(PS_B)W, && (4.4)\\
D_3 &= W/2 + 0.5(PS_A)W + 0.25(PS_B)W, && (4.5)\\
D_4 &= W - 0.25(PS_C)W. && (4.6)
\end{aligned}
$$

The percent of activity for each storage class can be used to determine the across-aisle travel from a dock door (within a storage class) to an aisle in any storage class. Given the symmetry of the layout, the expected distance traveled from a door within C_1 is the same as the expected distance from a door within C_2; the expected distance from a door within B_1 is the same as the expected distance from a door within B_2. If the put-away aisle resides within the same class section as the door, then the expected travel is one-third of the width of the section.



(a)



(b)

Figure 4.5: (a) 2-Sided, class-based storage; (b) optimal r values.

The expected x-distance from a door within C_1 (or C_2) to some aisle is:

$$\begin{aligned} E[D_x^C] =& PA_C \left[\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) PS_C W + \left(\frac{1}{2}\right)(D_4 - D_1) \right] \\ &+ PA_B \left[\left(\frac{1}{2}\right)(D_2 - D_1) + \left(\frac{1}{2}\right)(D_3 - D_1) \right] + PA_A \left[\frac{W}{2} - D_1 \right]. \end{aligned} \tag{4.7}$$

The expected x-distance from a door within B_1 (or B_2) is:

$$\begin{aligned} E[D_x^B] =& PA_C \left[\left(\frac{1}{2}\right)(D_2 - D_1) + \left(\frac{1}{2}\right)(D_4 - D_2) \right] \\ &+ PA_B \left[\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) PS_B W + \left(\frac{1}{2}\right)(D_3 - D_2) \right] + PA_A \left[\frac{W}{2} - D_2 \right]. \end{aligned} \tag{4.8}$$

The expected x-distance from a door within A is:

$$E[D_x^A] = PA_A \left[\frac{1}{3} PS_A W \right] + PA_B \left[\frac{W}{2} - D_2 \right] + PA_C \left[\frac{W}{2} - D_1 \right]. \tag{4.9}$$

Multiplying the three previous equations by the corresponding percent of storage for the dock door (probability of using the dock door) and then summing these equations, we have the total expected x-distance: The expected one-way horizontal travel can be expressed as:

$$E[D_{x,y}] = E[D_x] + L/2 + g + a. \tag{4.10}$$

In the following result (proved via convex analysis; see Appendix A.2), we use the parameter q to denote the coefficient of W (embedded in $E[D_x]$) as defined by (4.7)–(4.9). The derivation of this parameter is included in Appendix C.

Result 2. *The optimal warehouse shape for a put-away operation with uniform doors and a 2-sided storage layout is* $1/(2q)$.

If the percent of storage for class-A, class-B and class-C items is 20/30/50 and the percent of activity is 80/15/5, the optimal ratio is:

$$r = 1.89.$$

Figure 4.5(b) displays the optimal values for r for a range of ABC curves. As the value of S decreases (a more skewed ABC curve), the optimal value of r approaches 2.0; as the value of S increases (toward a uniform distribution), the optimal value of r is 1.5.

4.4.4 Summary

We have considered the optimal shape for the put-away operation for a random storage warehouse, as well as class-based storage layouts for two dock door configurations. The optimal shape for a random storage layout with uniform dock door usage is 1.5. For the 1-sided, class-based storage layout, the optimal shape decreases below 1.5 as the skewness of the ABC curve increases. The optimal shape for the 2-sided, class-based storage layout is between 1.5 and 2.0, where it approaches 2.0 as the skewness of the ABC curve increases.

Thus far we have only considered the horizontal travel for the put-away operation. The expected one-way vertical distance to put away a pallet in one of the $(H-1)$ levels of storage rack (above level one) can be expressed as:

$$E[D_z] = P_h \left(\frac{1 + (H-1)}{2} \right) = P_h \left(\frac{H}{2} \right),$$

where P_h is the height of a pallet opening. The total two-way travel time for a put-away operation, then, can be expressed as:

$$E[T] = 2 \left(\frac{E[D_{x,y}]}{v_{x,y}} + \frac{E[D_z]}{v_z} \right),$$

where $v_{x,y}$ and v_z are the horizontal and vertical speeds of the lift truck, respectively.

4.5. Order-Picking Travel

Several strategies have been developed for routing order pickers in a warehouse. Hall [11] presents analytical models for three strategies: traversal, midpoint and largest gap. Because the traversal strategy is the most common policy used in practice, we consider this strategy for random storage and the 2-sided, class-based storage layout. For the 1-sided class-based storage layout, we consider a return policy to take advantage of the reduction in within-aisle travel. We do not consider vertical travel in the pick tour because our problem definition states that all case picks come from the bottom level of storage.

4.5.1 Random Storage Policy

With a random storage policy, all pick locations are equally likely to be visited on a pick tour. The models presented by Hall [11] assume that storage is random and travel occurs from a centrally located P&D point. Because we assume that the order will be deposited at a uniformly distributed dock door before returning to the central pickup point, we modify Hall's equation to include the extra distance required for the dropoff. Hall's work shows that for uniformly distributed pick locations, increasing the number of picks per route favors elongated warehouses (i.e., larger values of r) for a fixed storage area.

Traversal Strategy with a Centrally Located Pickup Point and Uniformly Distributed Dropoff Point

For values of N greater than or equal to 5, Hall [11] models the expected length of an order-picking tour from a centrally located P&D point as:

$$E[D_{x,y}] = 2W\left[\frac{(N-1)}{(N+1)}\right] + ML\left[1 - \left[\frac{(M-1)}{M}\right]^N\right] + 0.5L,$$

where N is the number of picks and M is the number of aisles in the warehouse. The first term in the expression is the expected x-distance (across-aisle travel) between the outermost picks, where the picks are uniformly distributed. The second term is the y-distance (within-aisle travel) times the number of aisles and the probability that an aisle contains at least one pick. The final term accounts for the expected travel required to return to the front of the warehouse.

If the order is dropped off at a uniformly distributed dock door, the probability that the door is located within the x-distance traveled to fulfill the order (i.e., the range of aisles containing the items picked) is $(N-1)/(N+1)$. Figure 4.6 shows the distance from a dock door outside of the pick range to the closest pick.

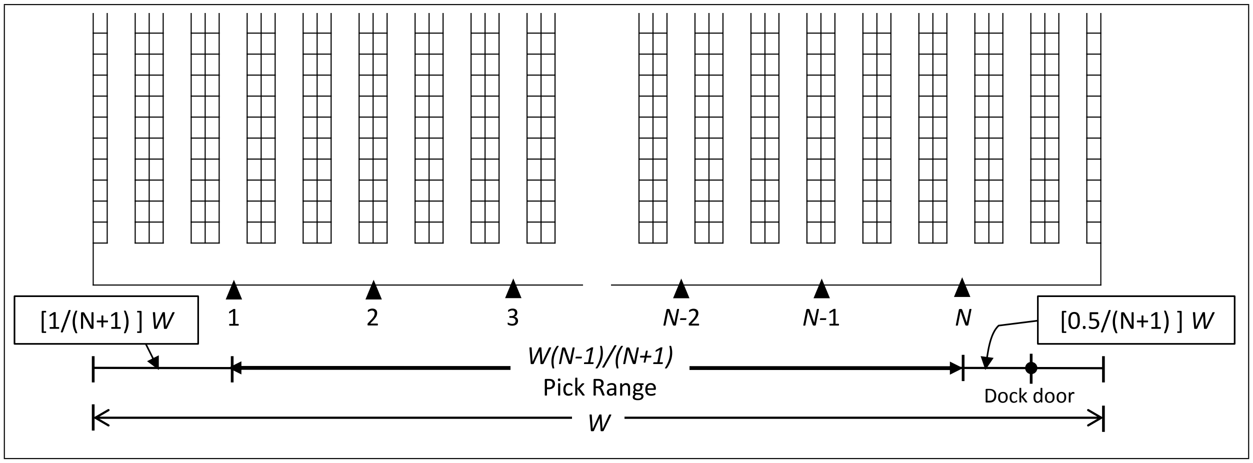


Figure 4.6: Distance to a uniform dock door.

The probability that a door lies outside of the pick range is:

$$\Pr(\text{drop-off door is outside the pick range}) = 1 - \left[\frac{N-1}{N+1}\right] = \frac{2}{N+1}.$$

If the dock door for the drop-off does lie outside the pick range, the expected one-way distance from the farthest pick position to the dock door is:

$$D_{\text{drop-off}} = 0.5\left(\frac{1}{N+1}\right)W.$$

Multiplying the probability that the dock door lies outside of the pick range by the expected distance to the door outside the pick range, we have the expected one-way distance to a dock door outside the pick range:

$$E[D_{\text{drop-off}}] = \frac{W}{(N+1)^2}.$$

The expected tour length (that includes the additional travel to a door located outside the pick range) is:

$$E[D_{x,y}] = 2W\left[\frac{(N-1)}{(N+1)} + \frac{1}{(N+1)^2}\right] + ML\left[1 - \left[\frac{(M-1)}{M}\right]^N\right] + 0.5L. \qquad (4.11)$$

This distance assumes that the order picker would first travel in the direction that is opposite the dock door where the order will be deposited (so as to complete the tour by heading towards the order's dock door). Figure 4.7 displays the optimal values of r, where the number of picks range from 5 to 40 (for areas of 100,000 ft^2 to 300,000 ft^2, where the center-to-center aisle width is 20 feet).

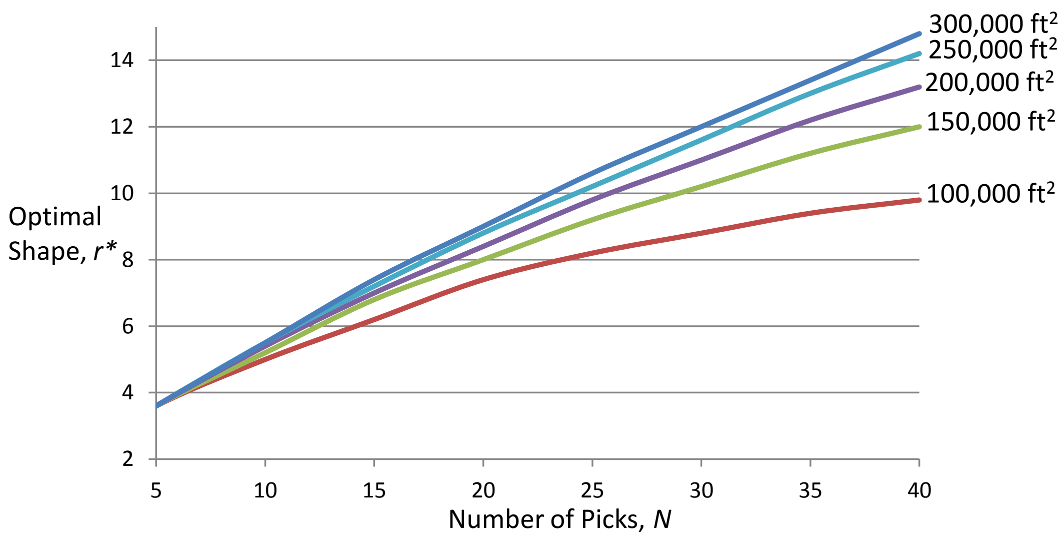


Figure 4.7: Optimal r values for the traversal strategy with random storage and a central pickup point and uniform deposit points.

From Figure 4.7, as the number of picks increases, so does the optimal value for r. This is consistent with Hall's recommendation that an elongated warehouse is advantageous when there are several picks per tour [11]. Further, note that these values of r are much greater than 1.5 (the optimal shape for put-away operations in a random storage warehouse).

4.5.2 Class-Based Storage

For a class-based storage policy, fast-moving items are located in such a way to take advantage of shorter distances to the P&D points. Thus, the layouts for a 1-sided and 2-sided configuration employ two different routing strategies in order to achieve a minimum expected tour distance.

Return Policy for 1-Sided Layout and a Centrally Located Pickup and Uniform Deposit Point

The return strategy involves entering and exiting from the same end of the aisle to retrieve items, traveling only as far as the farthest pick in the aisle. For a highly skewed class-based ABC curve, a return policy would require travel through only a small percentage of the total aisle length. As suggested by Le-Duc and de Koster [15], only the return policy is relevant for this layout. The across-aisle (x-travel) required for the return policy is the same as the first term in (4.11):

$$E[D_x] = 2W\left[\frac{(N-1)}{(N+1)} + \frac{1}{(N+1)^2}\right].$$

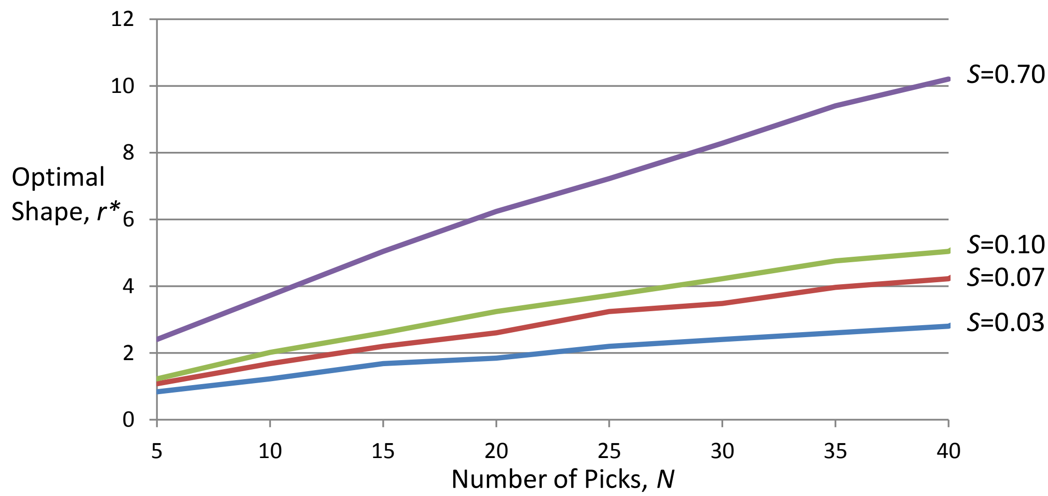
For the within-aisle travel (y-distance), however, instead of traversing the entire aisle, a picker would only travel as far as the farthest pick in the aisle. To determine this distance, we consider the possibility of n picks in an aisle, ranging from 1 to N, and calculate the associated probabilities. The n picks may be any combination of classes, so we enumerate over every possibility (i.e., 2 class-A picks and $(n-2)$ class-B picks, etc.), multiplying the probability and distance for each. We sum the distances for each class combination of n picks and multiply by the probability of n picks. Finally, we sum over all possible picks in an aisle to get the expected distance into an aisle:

$$
\begin{aligned}
E[D_y] \quad = \quad & 2M \sum_{n=1}^{N} \binom{N}{n} \left(\frac{1}{M}\right)^n \left(\frac{M-1}{M}\right)^{N-n} \times \\
& \Bigg[(PA_A)^n \left(\frac{n}{n+1}\right) PS_A L + (PA_B)^n \left[PS_A + \left(\frac{n}{n+1}\right) PS_B \right] L + \\
& (PA_C)^n \left[PS_A + PS_B + \left(\frac{n}{n+1}\right) PS_C \right] L + \\
& \sum_{i=1}^{n-1} \binom{n}{i} \Bigg[(PA_A)^{n-i} (PA_B)^i \left[PS_A + \left(\frac{i}{i+1}\right) PS_B \right] L + \\
& (PA_C)^i (1 - PA_C)^{n-i} \left[PS_A + PS_B + \left(\frac{i}{i+1}\right) PS_C \right] L \Bigg] \Bigg].
\end{aligned}
$$

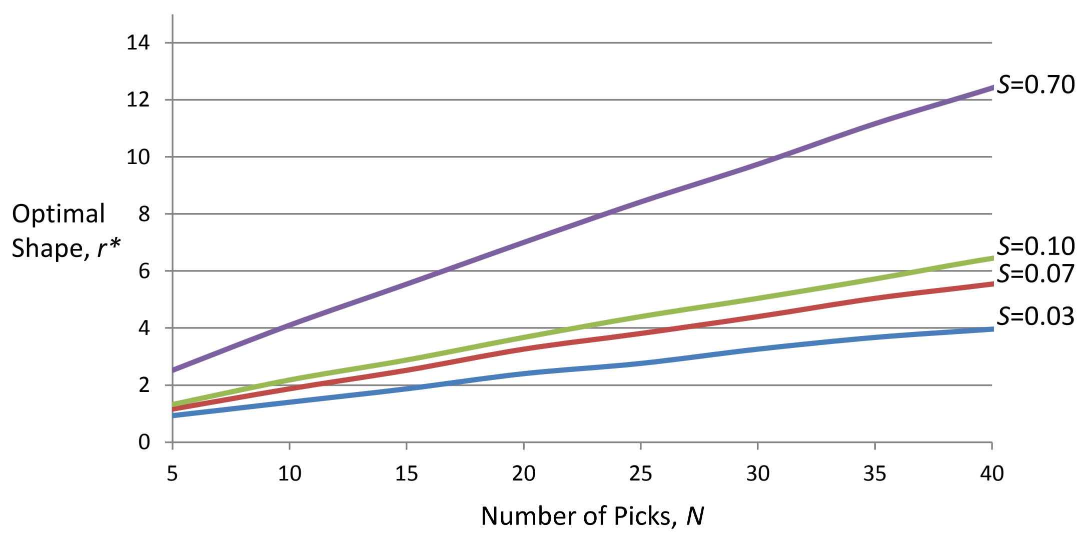
Thus, the total expected distance for the order-picking tour is:

$$E[D_{x,y}] = E[D_x] + E[D_y]. \tag{4.12}$$

Figures 4.8(a) and 4.8(b) show the optimal shape for ABC curves with S parameter values between 0.03 (very skewed) to 0.7 (hardly skewed) for warehouses with picking areas of 100,000 ft^2 and 300,000 ft^2. Warehouses with larger picking areas favor slightly more elongated warehouses compared to those with smaller picking areas, as with the random storage layout. The optimal shape becomes more elongated as the number of picks increases and as the skewness of the ABC curve decreases (toward a more uniform distribution). In general, the optimal values of r for the order-picking operation are greater than the optimal values for the put-away operation (0.3 to 1.5) with the 1-sided layout. However, the return policy with the 1-sided, class-based storage layout favors a less-elongated (i.e., less wide but with longer aisles) warehouse in comparison to the traversal strategy in a random storage warehouse.



(a)



(b)

Figure 4.8: Optimal r values for a 1-sided layout with a return policy and a central pickup point and uniform deposit point: (a) 100,000 ft^2; (b) 300,000 ft^2.

Traversal Strategy for 2-Sided Layout and a Centrally Located Pickup Point and Uniform Deposit Point

With a centrally located pickup point, the two-sided layout is symmetric about the pickup point, and consideration must be given to individual picking scenarios. For example, if a tour contains class-A items and class-B items, but only one class-C item, travel on one side of the pickup point extends, on average, as far as the center of the class-C aisles (to pick the class-C item), but travel on the other side of the pickup point extends only as far as the farthest class-B pick on that side. Thus, one can see how the expected across-aisle distance varies according to the number of class-A, class-B and class-C picks. For a tour with N pick locations, we can use the multinomial distribution to determine the probability of each scenario. For each pick, there are three possible types of picks (class-A, class-B, or class-C). The probability of n_A class-A picks, n_B class-B picks and n_C class-C picks is:

$$Pr(n_A, n_B, n_C) = \frac{N!}{n_A! n_B! n_C!}(PA_A)^{n_A}(PA_B)^{n_B}(PA_C)^{n_C}.$$

Figure 4.9 illustrates the twelve combinations of picks that result in different x-distances, and the respective distances (d_1 to d_{12}) are inclued in Appendix D. The total expected across-aisle distance is then the sum of each of the d_i distances plus the probability that the dock door for the drop-off of the order is outside the picking range times half of the distance outside the picking range:

$$E[D_x] = \sum_{i=1}^{12} d_i + \left(1 - \frac{\sum_{i=1}^{12} d_i}{2W}\right)\left[0.5\left(W - 0.5\sum_{i=1}^{12} d_i\right)\right].$$

The expected within-aisle distance is simply the length of an aisle times the expected number of aisles traveled. The expected number of aisles is less than that of random storage because most of the picks are concentrated in the class-A aisles (which account for only a small percentage of the total number of aisles for a fairly skewed ABC curve). First we consider

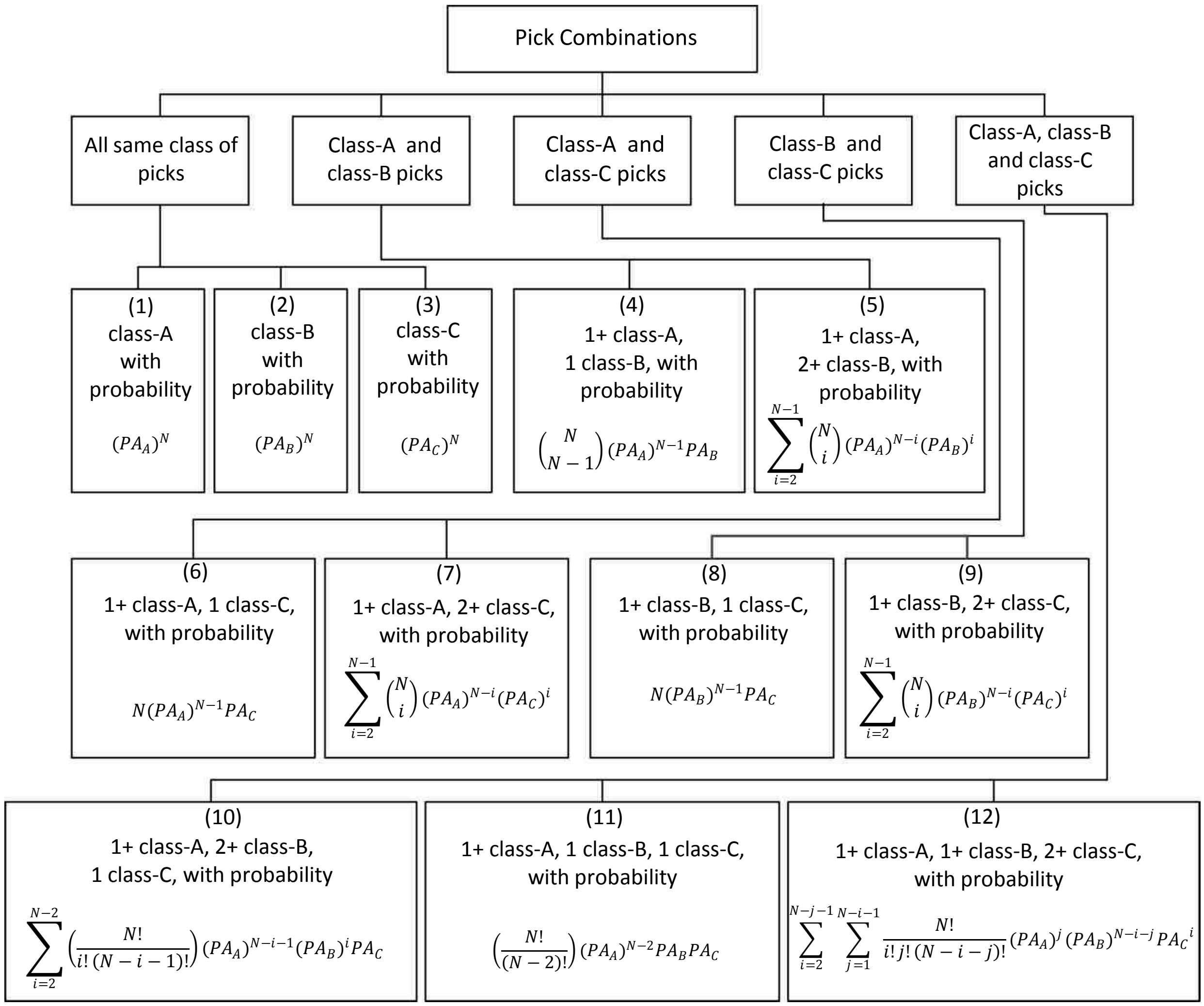


Figure 4.9: Combinations of picks for the 2-sided layout.

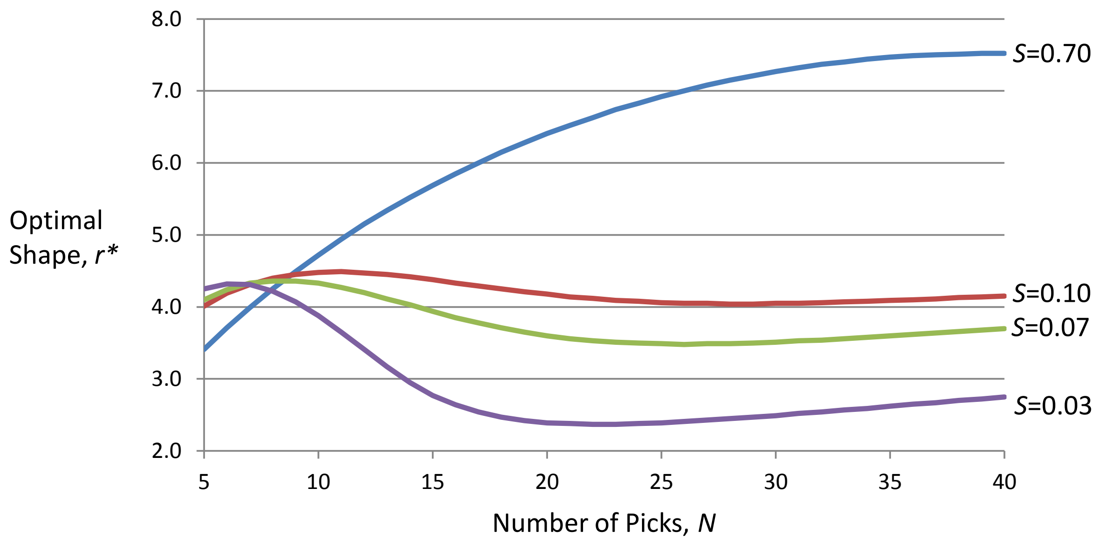
the probability for every possible class-combination of picks. Then, for each possibility, we calculate and total the expected number of class-A, class-B, and class-C aisles. (We approximate the number of aisles for each class by multiplying the total number of aisles (M) by the percent of storage for each class.) Finally, we sum over all combinations of picks to determine the expected number of aisles and multiply by the aisle length. The additional half-aisle accounts for the possibility of return travel to the front of the warehouse. The resulting distance approximation for the within-aisle travel is then:

$$\begin{aligned} E[D_y] &= LM\sum_{i=0}^{N}\sum_{j=0}^{N-i}(PA_A)^i(PA_B)^j(PA_C)^{N-i-j}\left(\frac{N!}{i!j!\,(N-i-j)!}\right) \\ & \left[PS_A\left[1-\left(1-\frac{1}{MPS_A}\right)^i\right]+PS_B\left[1-\left(1-\frac{1}{MPS_B}\right)^j\right]\right. \\ & \left.+PS_C\left[1-\left(1-\frac{1}{MPS_C}\right)^{N-i-j}\right]\right]+0.5L. \end{aligned}$$

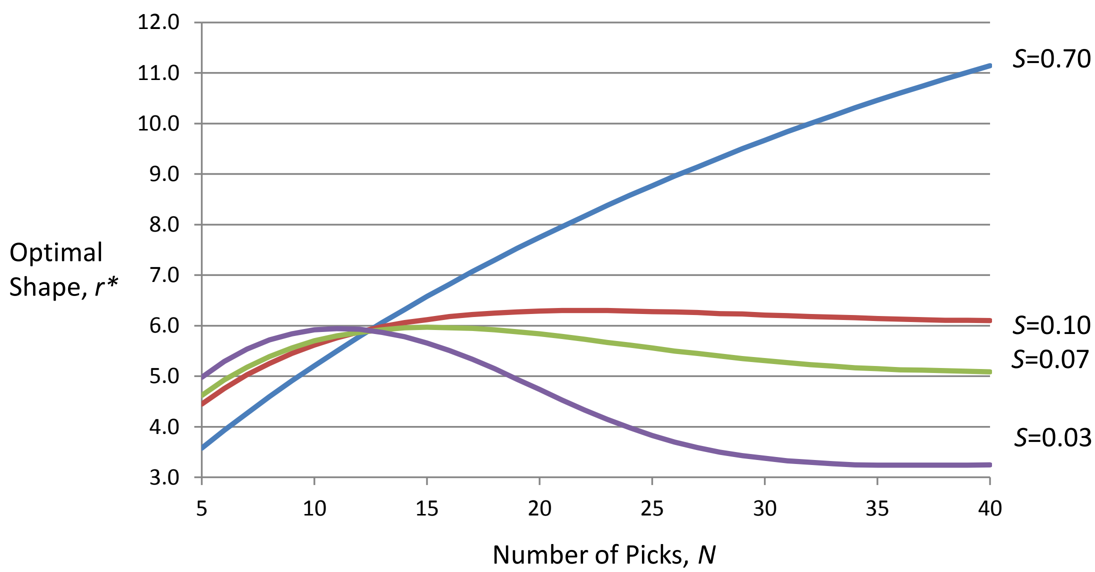
The total expected tour length is then:

$$E[D_{x,y}] = E[D_x] + E[D_y].$$

Figures 4.10(a) and 4.10(b) illustrate the optimal shape for ABC curves with S parameter values from 0.03 (very skewed) to 0.7 (hardly skewed) for warehouses with picking areas of 100,000 ft^2 and 300,000 ft^2. Again, warehouses with smaller picking areas favor slightly less elongated warehouses than those with larger picking areas. For the more skewed ABC curves (S = 0.03, 0.07 and 0.10), the optimal shape reaches a peak when the number of class-A picks is close to the number of class-A aisles. (For the 100,000 ft^2 picking area, this peak occurs for a smaller number of picks, as compared to the 300,000 ft^2 picking area, because there are fewer class-A aisles.) Then, as the number of picks increases, the shape decreases to achieve fewer (longer) aisles, as there are multiple class-A picks per aisle. However, when the number of class-B picks becomes a factor, the optimal shape increases such that the class-B aisles become shorter. This increase is very gradual because even though the number of picks is increasing, very few of these picks are class-B items for fairly skewed ABC curves. In addition, picking areas that are larger result in higher shape ratios, and the peak occurs at much higher values of N because there are more aisles for each class of items. For the hardly skewed curve, the optimal shape increases steadily as the number of picks increases because there are significantly more class-B and class-C items.



(a)



(b)

Figure 4.10: Optimal r values for a 2-sided layout with the traversal policy and a central pickup point and uniform deposit point: (a) 100,000 ft^2; (b) 300,000 ft^2.

The optimal shapes for the 2-sided traversal strategy are much greater than the optimal shapes for the 2-sided put-away operation (1.5 to 2.0). Further, as the number of stops per tour increases, the optimal warehouse shape for the 2-sided layout is significantly less as compared to traversal with random storage. With random storage, increasing the number of pick lines would result in more aisles traversed, but with the 2-sided layout, likely the additional lines include primarily class-A aisles (with multiple pick lines).

Summary

The optimal warehouse shape for order picking with a random storage layout and a 1-sided, class-based layout increases as the number of stops per tour increases. Both the 1-sided and 2-sided class-based layouts favor less-elongated warehouses as compared to random storage for a large number of picks, especially for more skewed ABC curves. This is intuitive because both class-based storage layouts aim to reduce the within-aisle travel component. For a small number of stops per tour (e.g., $N < 10$), the 1-sided layout favors less elongated warehouses than the 2-sided layout. As the number of stops per tour increases (with more class-B picks), there is less of an impact on the optimal warehouse shape for the 2-sided layout. Further, warehouse shape is a more significant factor for a random storage warehouse than one with class-based storage.

4.6. Replenishment Travel

The replenishment operation begins at the picking location that was last replenished and involves travel to the reserve storage location for the next item to be replenished, followed by travel to the next replenishment location. We define α as the probability that the replenishment location for an item resides within the same aisle as the reserve storage location for the item. If put aways are truly random, $\alpha = 1/M$. However, we contend that even in a random storage warehouse, some effort is made to place the reserve storage location for an item in the same aisle as the picking location of the item. Therefore, in general, $\alpha \geq 1/M$.

For simplicity, we use locations 1, 2 and 3 to denote the location of the last replenishment, the reserve storage location for the next replenishment, and the location for the next replenishment, respectively. In our models, we assume that the worker exits toward the end of the aisle that minimizes travel. The distance models that we use are based on the probabilities shown in Figure 4.11. As illustrated in Figure 4.11, the replenishment distance depends on

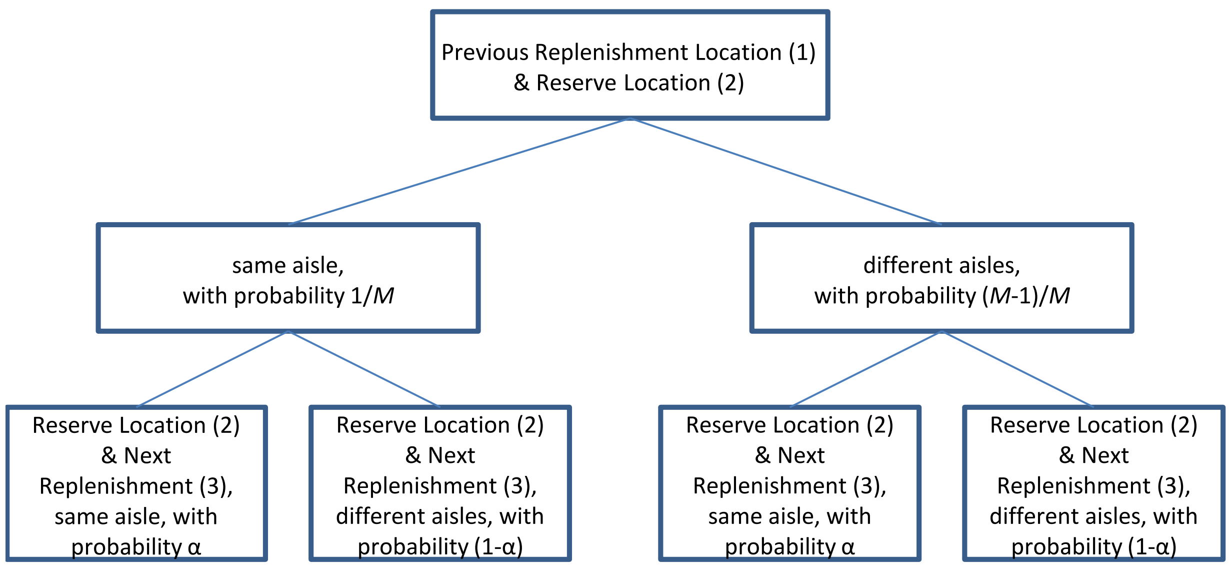


Figure 4.11: Probabilities for replenishment travel.

the aisle locations of the last replenishment, reserve storage for the next replenishment, and the next replenishment. Because location 3 is visited after location 2, the total expected distance for the case where locations 1 and 3 reside in the same aisle is the same as if three aisles were visited. This probability is included in the last scenario in Figure 4.11. In the following sections, we present models to estimate the distance required for a replenishment for both random and class-based storage layouts.

4.6.1 Replenishment Travel for Random Storage

For random storage, the replenishment locations and reserve storage locations are uniformly distributed within the aisle. In modeling the within-aisle travel for random storage, we calculate the expected distances such that if two (or all three) of the points are located in the same aisle, they can be in any order. The expected distance for each replenishment scenario is as follows:

Case 1 (all three locations are in the same aisle): Locations 1 and 2 are in the same aisle with probability $1/M$, and location 3 is in the same aisle as location 2 with probability α. Thus, all travel is in the same aisle as shown in Figure 4.12, with three uniformly distributed points (labeled a, b, and c, where any point can represent any of the three locations).

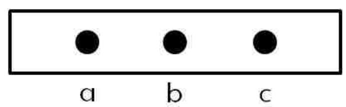


Figure 4.12: Three locations in the same aisle.

From Figure 4.12, if location 1 is in position a or c, the expected horizontal distance is:

$$0.5\left(\frac{1}{2}L\right)+0.5\left(\frac{3}{4}L\right).$$

If location 1 is in position b, the expected distance is:

$$0.5\left(\frac{3}{4}L\right)+0.5\left(\frac{3}{4}L\right).$$

With an equal probability of location 1 being in any of the three positions, the total expected distance is:

$$\frac{2}{3}L.$$

Multiplying the probability of three locations in the same aisle by the expected distance, we have:

$$\frac{\alpha}{M}\left[\frac{2}{3}L\right].$$

Case 2 (locations 1 and 2 are in the same aisle): Locations 1 and 2 are in the same aisle with probability $1/M$, and location 3 is in a different aisle than location 2 with probability $1 - \alpha$. Thus, two uniformly distributed points are located in the same aisle, and the third location is in a different aisle. Figure 4.13 illustrates the two possibilities for within-aisle travel. In each case, the worker exits toward the end of the

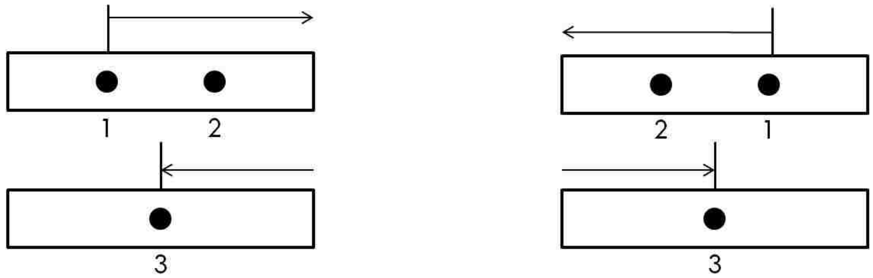


Figure 4.13: Possible routes with locations 1 and 2 in same aisle.

aisle that minimizes travel, such that backtracking does not occur between locations 1 and 2. Thus the total within-aisle distance for the case of three locations in two aisles is:

$$\frac{2}{3}L + \frac{1}{2}L = \frac{7}{6}L. \tag{4.13}$$

The expected across-aisle distance between the two aisles of interest is $(1/3)W$. Multiplying the probability by the expected travel, we have:

$$\frac{1-\alpha}{M}\left[\frac{7}{6}L + \frac{1}{3}W\right].$$

Case 3 (locations 1 and 2 are in different aisles, but locations 2 and 3 are in the same aisle): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are in the same aisle with probability α. Again, the across-aisle travel is between two aisles as depicted in Figure 4.14. The total within-aisle travel can be determined from (4.13).

Thus, the expected across-aisle distance and within-aisle distance is the same as for

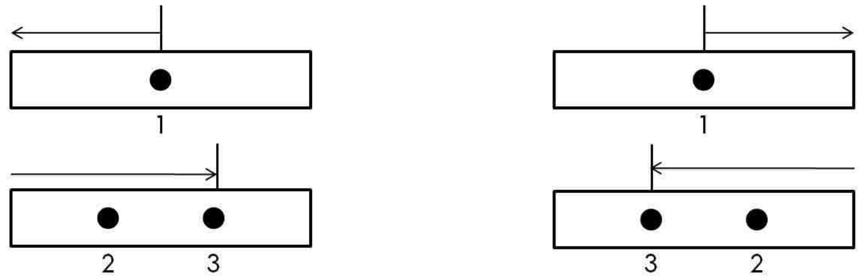


Figure 4.14: Possible routes with locations 2 and 3 in same aisle.

Case 2. Multiplying the probability by the expected distance yields the following:

$$\frac{\alpha(M-1)}{M}\left[\frac{7}{6}L+\frac{1}{3}W\right].$$

Case 4 (three locations are in three different aisles): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are also in different aisles with probability $(1-\alpha)$. Thus, travel involves entering/exiting three aisles. Note that we include the case where locations 1 and 3 are in the same aisle here. The within-aisle travel to a uniform point in three different aisles is $4(\frac{1}{2}L) = 2L$, and the across-aisle travel for three aisles is $\frac{2}{3}W$:

$$\frac{(1-\alpha)(M-1)}{M}\left[2L+\frac{2}{3}W\right].$$

Taking into consideration all possible scenarios, we have the total expected horizontal distance for a replenishment operation in a random storage warehouse:

$$\begin{aligned}E[D_{x,y}] =& \frac{\alpha}{M}\left[\frac{2}{3}L\right]+\frac{\alpha(M-1)}{M}\left[\frac{7}{6}L+\frac{1}{3}W\right]+\frac{(1-\alpha)}{M}\left[\frac{7}{6}L+\frac{1}{3}W\right]\\ &+\frac{(1-\alpha)(M-1)}{M}\left[2L+\frac{2}{3}W\right].\end{aligned}$$

As stated previously, if put aways are completely random, then α can be expressed as $1/M$. Accordingly, the expected horizontal distance can be expressed as:

$$\begin{aligned} E[D_{x,y}] =& \frac{1}{M^2}\left[\frac{2}{3}L\right] + \frac{M-1}{M^2}\left[\frac{7}{6}L + \frac{1}{3}W\right] + \frac{M-1}{M^2}\left[\frac{7}{6}L + \frac{1}{3}W\right] \\ &+ \frac{(M-1)^2}{M^2}\left[2L + \frac{2}{3}W\right], \end{aligned}$$

and combining like terms, we have:

$$E[D_{x,y}] = \frac{1}{M^2}\left[\frac{2}{3}L\right] + \frac{2M-2}{M^2}\left[\frac{7}{6}L + \frac{1}{3}W\right] + \frac{(M-1)^2}{M^2}\left[2L + \frac{2}{3}W\right].$$

Figure 4.15 depicts the optimal shape for different values of α for picking areas of 100,000 ft^2 and 300,000 ft^2. The lower bound for α is $1/M$, and the optimal warehouse shape for this case is 2.81 and 2.88 for the 100,000 ft^2 and 300,000 ft^2 areas, respectively. The shape of the warehouse becomes more elongated for increasing values of α with maximum optimal shapes of 3.40 and 3.44 for the picking areas considered. The optimal shape for the replenishment operation is greater than the optimal shape for the put-away operation but less than the optimal shape for order picking in a random storage warehouse.

4.6.2 Replenishment Travel for 1-Sided Layout with Class-Based Storage

With the 1-sided layout, we again use α to represent the probability that the reserve storage location (location 2) is in the same aisle as the next replenishment location (location 3), and as before, the reserve storage location is still randomly located within an aisle. However, the replenishment locations are not uniformly distributed, and we assume that a worker exits toward the end of the aisle that minimizes travel. Thus, now we must consider the storage class of the item being replenished (location 3) and the storage class of the previous replenishment (location 1).

For each of the four cases presented above for random storage, we now also include all

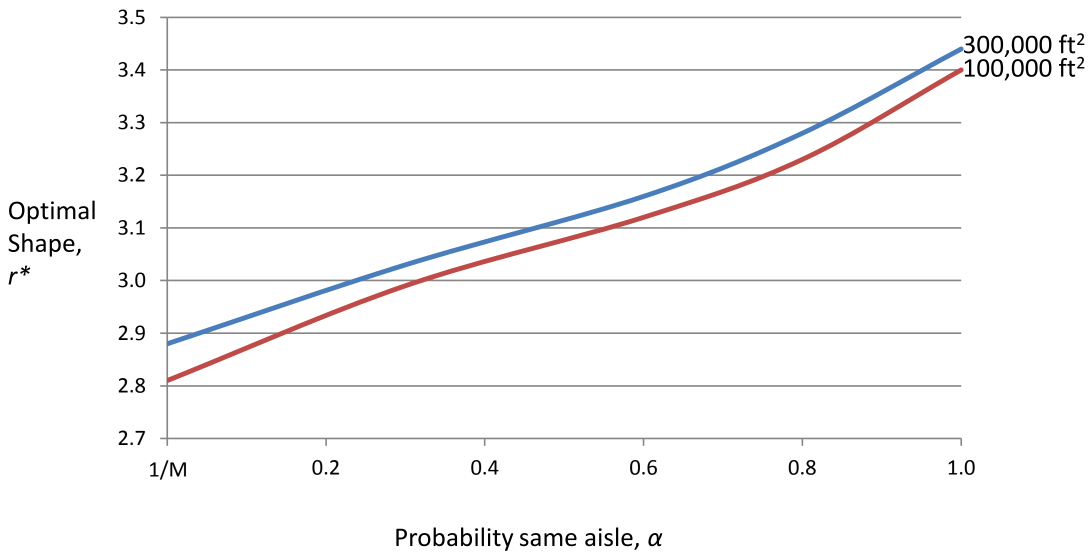


Figure 4.15: Optimal r values for replenishment with random storage (100,000 ft^2 and 300,000 ft^2).

possible combinations of storage classes for locations 1 and 3 (e.g., location 1 was a class-A item and location 3 is a class-A item; or, location 1 was a class-A item and location 3 is a class-B item, etc.). After the distances for each of these scenarios have been determined, we then multiply each distance by its probability of occurrence and sum over all scenarios to determine the total expected distance for a replenishment operation. For example, if location 1 is a class-A item and location 3 is a class-B item, the probability of occurrence is $PA_A \times PA_B$; this probability is then multiplied by the expected distance in traveling from a class-A replenishment location to a class-B replenishment location. The distance equations are included in Appendix E.

Figure 4.16 illustrates the optimal r values for a 1-sided warehouse with picking areas of 100,000 ft^2 and 300,000 ft^2 for an 80/20 ABC curve. For ABC curves with different levels of skewness, there is no appreciable difference in optimal shape.

The optimal shape increases slightly as α increases, but the shape is less elongated than for replenishments in a random storage warehouse. This is due to the reduced within-aisle travel for the replenishment operation with the 1-sided, class-based storage layout. The optimal shape of the warehouse has a lower bound ($\alpha = 1/M$) of approximately 1.9 and an

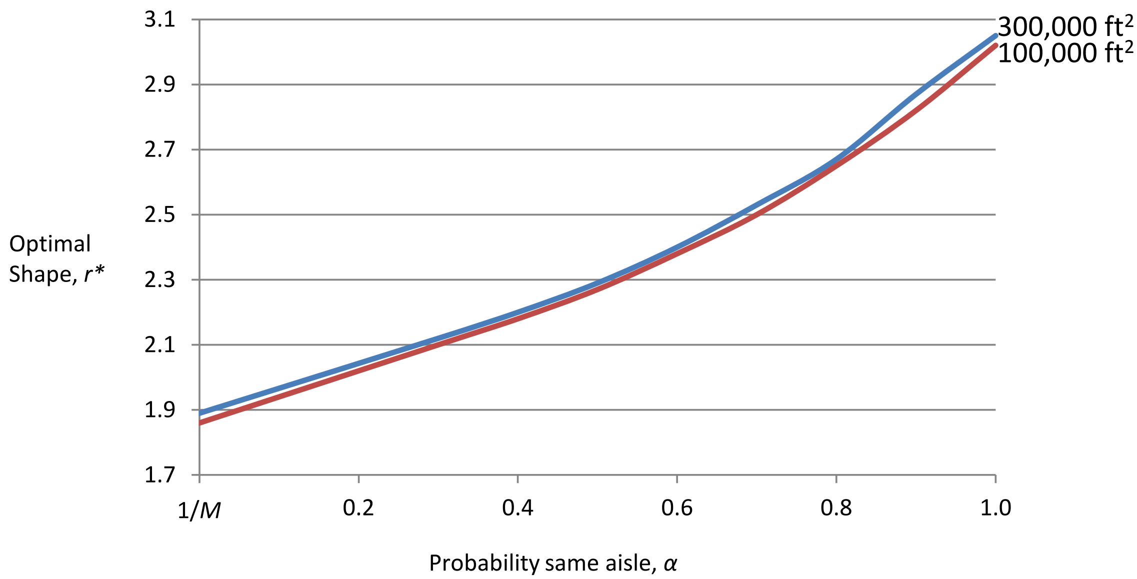


Figure 4.16: Optimal r values for replenishment with a 1-sided layout (100,000 ft^2 and 300,000 ft^2).

upper bound ($\alpha = 1$) of 3.0 for the picking areas considered. Also, the optimal shape for replenishment in the 1-sided warehouse is greater than the optimal shape for the put-away operation, but less than the optimal shape for order picking for an 80/20 ABC curve.

4.6.3 Replenishment Travel for 2-Sided Layout with Class-Based Storage

The expected distance of the replenishment operation in a 2-sided layout is also dependent on the storage class of the previous and next replenishment. However, for the 2-sided layout, the storage class is no longer defined within the aisle; instead, each aisle contains a given storage class, and we assume that the reserve storage locations within the aisle are uniformly distributed. Travel across aisles, on the other hand, depends on the number of aisles in each storage class and on the class of the previous and last replenishment. Consequently, we present the distance equations according to the storage classes of locations 1 and 3, instead of ordering by the four cases defined previously. We do so because not all cases apply for a given pair of storage classes for locations 1 and 3. For example, if location 1 resides within class-A storage and location 3 resides within class-B storage, then it is not possible for all locations to reside in the same aisle.

Because the aisles are not identical in terms of the storage class (as was the case with the 1-sided layout), the across-aisle travel will result in different distances, depending on the storage class of the aisle that contains the reserve storage location. For example, consider the case where location 1 is a class-A item and location 3 is a class-A item. For the case where the three locations are in three different aisles, the three aisles could all be class-A aisles, or the aisle with the reserve location could be a class-B or class-C aisle. Hence, the expected distances for these two scenarios (for the case where all locations are in different aisles) are different. The equations for the expected distance for the replenishment operation with the 2-sided layout are included in Appendix F.

Because the aisles are not identical, in terms of the activity profile, the optimal warehouse shape is now more dependent on the skewness of the ABC curve. Figure 4.17 displays the optimal shape for a fairly skewed ABC curve (80/20) and a hardly skewed ABC curve for picking areas of 100,000 ft^2 and 300,000 ft^2. The optimal shapes for the fairly skewed ABC curve range from approximately 3.5 to 8.6, and the optimal shapes for the hardly skewed ABC curve range from 3.0 to 4.3 for the picking areas considered. Thus, as the skewness of the ABC curve increases and as α increases, more elongated warehouses are preferred. Further, the 2-sided layout results in optimal warehouse shapes that are significantly higher than for the 1-sided layout. This is intuitive because the within-aisle travel for the 2-sided layout has more of an impact on total travel than the across-aisle travel, especially for highly skewed ABC curves (resulting in significantly less across-aisle travel for the fast-moving items that span across a small number of aisles).

4.6.4 Summary

The optimal warehouse shape for the replenishment operation results in warehouses that are generally more elongated compared to the put-away operation. For random storage warehouses with picking areas between 100,000 ft^2 and 300,000 ft^2, the optimal warehouse

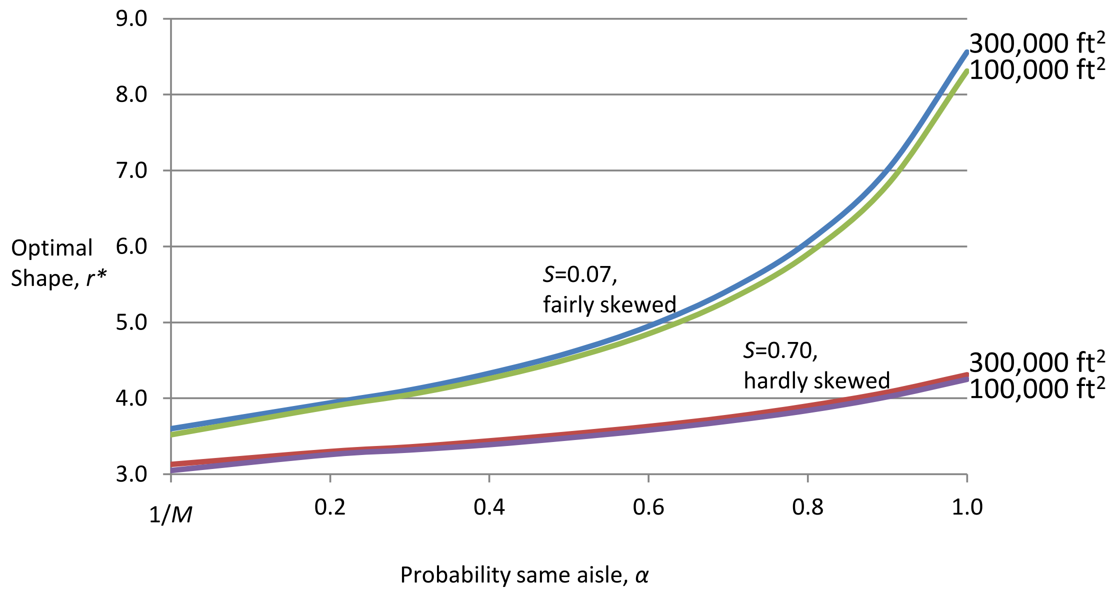


Figure 4.17: Optimal r values for replenishment with a 2-sided ayout (100,000 ft^2 and 300,000 ft^2).

shape ranges from approximately 2.9 to 3.4 for increasing values of α, and the optimal shapes for the 1-sided warehouse range from 1.9 to 3.0. The 1-sided layout is very insensitive to the skewness of the ABC curve, as compared to the 2-sided layout. As the skewness of the ABC curve increases, the optimal warehouse shape for the 2-sided warehouse becomes significantly more elongated. In addition, the 1-sided layout results in warehouse shapes that are less-elongated than the 2-sided and random storage layouts. In the next section we demonstrate how to determine the optimal warehouse shape that considers put-away, order-picking and replenishment travel.

4.7. Warehouse Shape Example

Thus far we have presented the optimal warehouse shape for individual warehouse operations. However, the optimal warehouse shape should reflect all travel operations, so we will present an example to demonstrate how to determine the optimal warehouse shape that takes into account the horizontal travel for put away, case-based order picking, and replenishment to the forward area.

Consider a warehouse with dock doors on one side only and with a storage area of 300,000 ft^2. The staging area (including the end cross aisles) is 50 feet. Items are stored according to the 1-sided, class-based layout. The percent of activity and percent of storage are 65/20/15 and 20/30/50, respectively, for class-A, class-B and class-C items. Pallet put aways are such that 80 percent of the time, reserve storage locations are in the same aisle as their forward picking location. In addition, batches for order picking average 22 lines (or stops) per batch with approximately 1.2 picks per line. Incoming pallets, on average, consist of 30 cases.

An average day consists of 700 pallet put aways, 50 pallet picks, and 19,500 case picks. With an average of 1.2 case picks per line (and 22 lines), the number of batches is approximately 738 (19,500/(22$\times$ 1.2)). This results in 650 replenishments per day (assuming 19,500 picks and 30 cases per pallet).

4.7.1 Optimal Warehouse Shape

Using (4.2), the value for p is 0.25, and the optimal warehouse shape for the put-away operation is 0.75. This results in a storage area with a width of 474 feet and depth of 632 feet. Using (4.1), the one-way distance for a put-away operation is 364.65 ft; thus two-way travel requires 729.3 feet. The optimal shape for a pallet pick requires the same travel as for a pallet put away, and again the optimal shape is 0.75.

The optimal warehouse shape for order picking with 22 lines per batch (using the return policy) results in an optimal warehouse shape of 5.2, with a width of 1249.0 feet and depth of 240.2 feet. Using (4.12), the travel per batch is 4,698.3 feet. For replenishment, the optimal warehouse shape for the 1-sided layout with an α value of 0.8 is 2.7, and the corresponding distance is 725.9 feet using a storage area width of 900.0 feet and depth of 333.3 feet.

Table 4.1 displays the optimal shape for each operation and the total travel distance per day. Thus, the total distance for all operations represents a lower bound (LB) for the total minimum travel distance.

Table 4.1: Daily Travel Distance by Operation

Operation (equation number)	Optimal Shape, r^*	Distance per Operation (ft)	Number of Operations	Total Travel Distance (ft)
Put away (4.1)	0.75	729.3	700	510,510.0
Pallet pick (4.1)	0.75	729.3	50	36,465.0
Case picking tour (4.12)	5.20	4,698.3	738	3,467,345.4
Replenishment (4.19)	2.70	725.9	650	471,835.0
Total				4,486,155.4

From Table 4.1, the optimal warehouse shape varies from 0.75 to 5.20 for the put-away, order-picking and replenishment operations. The optimal warehouse shape is not a linear function for all operations, and the optimal shape that considers all operations is not clear. To determine a composite warehouse shape, we plot the total travel distance of all operations for shapes ranging from 0.5 to 5.2 as shown in Figure 4.18.

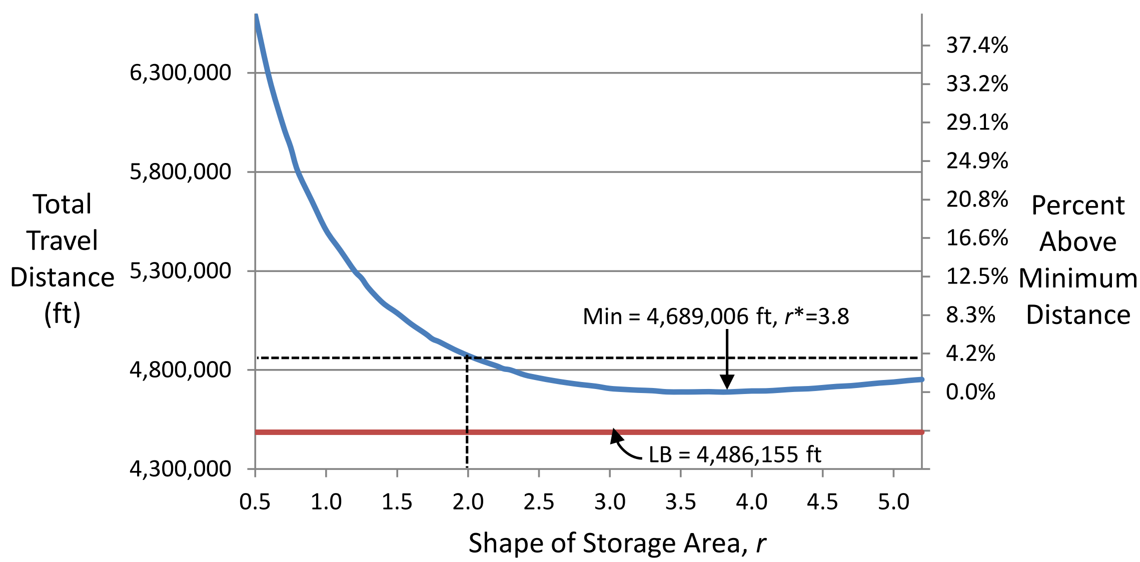


Figure 4.18: Total distance traveled for shapes ranging from 0.5 to 5.0.

The minimum distance of 4,689,006 feet occurs at a shape of 3.8, a difference of 202,851 feet more than the lower bound. From Figure 4.18 it is clear that choosing an optimal warehouse shape that considers all operations can result in significant labor savings. For example, consider a company that uses a warehouse shape ratio of 2.0 as a rule-of-thumb to design its warehouses. This would result in a warehouse that is 3.9% above the optimal daily travel, resulting in $55K of additional labor per year (using a horizontal equipment speed of 250 fpm, $18 per hour labor rate, and 250 operational days per year).

4.8. Conclusions

A warehouse's configuration affects the travel distances for put away, order picking, and replenishment; thus, it is an important design consideration. We have presented new expressions for put-away, order-picking and replenishment operations for random storage and two class-based storage layouts. Our models include a uniform distribution of dock doors instead of a single P&D location that is commonly assumed in the warehousing literature. We presented structural results on the optimal shape of a warehouse under specific assumptions and graphical illustrations that could lead to useful rules-of-thumb for industry going forward. In terms of the overall optimization problem, we presented numerical results for the put-away, order-picking and replenishment distances in an example warehouse with a 1-sided, class-based storage layout. The numerical results from our example illustrate that the optimal warehouse configuration is not consistent for all operations, and consideration must be given to the number of operations and distances associated with each operation. The total distance for all operations and dock-door configurations can be evaluated over the range of optimal shapes for individual operations to determine the optimal warehouse configuration that minimizes the total travel distance. Though we confined our results to an analysis of warehouse shape, the models presented can be used to evaluate a broad range of warehouses including varying design parameters such as the size of the forward area for random and class-based layouts, as well as the effect of activity distributions that change over time.

We believe our research, which covers warehouses that fulfill orders at the case level by picking cases from pallet rack, provides a foundation for a more sophisticated examination of this problem. That is, additional aspects of this type of warehouse could be modeled and combined with our models to enlarge the solution space (e.g., travel-time models for layouts with additional cross aisles). Also, the types of warehouses considered could be extended in the same vein. In particular, item-level picking would require a new set of order-picking and replenishment performance models to be incorporated into the overall optimization framework. In addition, our optimization framework is based on enumerating over a range of warehouse shapes and dock door configurations. A more sophisticated treatment of this non-linear optimization problem may provide additional structural results of use to industry.

Acknowledgements

This work was funded in part by the member organizations of the Center for Excellence in Logistics and Distribution (CELDi), an NSF Industry-University Cooperative Research Center. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Bibliography

[1] Bartholdi, J. J., and Hackman, S. T., *Warehouse & Distribution Science*, Version 0.95 (2011).

[2] Bassan, Y., Roll, Y., and Rosenblatt, M. J., "Internal Layout Design of a Warehouse," *IIE Transactions*, 12, 4, 317–322 (1980).

[3] Bender, P. S., "Mathematical Modeling of the 20/80 Rule: Theory and Practice," *Journal of Business Logistics*, 2, 2, 139–157 (1981).

[4] Berglund, P., and Batta, R., "Optimal Placement of Warehouse Cross-Aisles in a Picker-to-Part Warehouse with Class-Based Storage," *IIE Transactions*, 44, 107–120 (2012).

[5] Bozer, Y. A., *Optimizing Throughput Performance in Designing Order Picking Systems*, PhD thesis, Georgia Institute of Technology (1985).

[6] Caron, F., Marchet, G., and Perego, A., "Routing Policies and COI-Based Storage Policies in Picker-to-Part Systems," *International Journal of Production Research*, 36, 713–732 (1998).

[7] Chew, E. P., and Tang, L. C., "Travel Time Analysis for General Item Location Assignment in a Rectangular Warehouse," *European Journal of Operational Research*, 112, 582–597 (1999).

[8] Francis, R. L., "On Some Problems of Rectangular Warehouse Design and Layout," *The Journal of Industrial Engineering*, 18, 10, 595–604 (1967).

[9] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Operation: A Comprehensive Review," *European Journal of Operational Research*, 177, 1–21 (2007).

[10] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[11] Hall, R. W., "Distance Approximations for Routing Manual Pickers in a Warehouse," *IIE Transactions*, 25, 4, 76–87 (1993).

[12] Heskett, J. L., "Cube-Per-Order Index–A Key to Warehouse Stock Location," *Transportation and Distribution Management*, 3, 27–31 (1963).

[13] Hwang, H., Oh, Y. H., and Lee, Y. K., "An Evaluation of Routing Policies for Order-Picking Operation in Low-Level Picker-to-Part System," *International Journal of Production Research*, 42, 3873–3889 (2004).

[14] Koster, R. D., Le-Duc, T., and Roodbergen, K. J., "Design and Control of Warehouse Order Picking: A Literature Review," *European Journal of Operational Research*, 182, 2, 481–501 (2007).

[15] Le-Duc, T., and de Koster, R., "Travel Distance Estimation and Storage Zone Optimization in a 2-Block Class-Based Storage Strategy Warehouse," *International Journal of Production Research*, 43, 17, 3561–3581 (2005).

[16] Parikh, P., and Meller, R. D., "A Travel-Time Model for a Person-Onboard Order Picking System," *European Journal of Operational Research*, 200, 2, 385–394 (2010).

[17] Petersen, C. G., "An Evaluation of Order Picking Routeing Policies," *International Journal of Operations & Production Management*, 17, 11, 1098–1111 (1997).

[18] Petersen, C. G., "The Impact of Routing and Storage Policies on Warehouse Efficiency," *International Journal of Operations & Production Management*, 19, 9-10, 1053–1064 (1999).

[19] Pohl, L. M., Meller, R. D., and Gue, K. R., "Turnover-Based Storage in New Unit-Load Warehouse Designs," Technical report, Department of Industrial Engineering, University of Arkansas (2009).

[20] Roodbergen, K. J., Sharp, G. P., and Vis, I. F. A., "Designing the Layout Structure of Manual Order Picking Areas in Warehouses," *IIE Transactions*, 40, 1032–1045 (2008).

[21] Thomas, L. M., and Meller, R. D., "A Warehouse Model for Replenishment to a Bottom-Level Forward Area with Random Storage," in *Proceedings of the 2012 Industrial and Systems Engineering Research Conference*, Orlando, FL (2012).

[22] Tompkins, J. A., White, J. A., Bozer, Y. A., and Tanchoco, J. M. A., *Facilities Planning*, Wiley, New York, New York, 4th edition (2010).

A. Proofs for Optimal r Values for the Put-Away Operation

A.1 Result 1: 1-Sided Layout

Proof. Using the relationship for area and warehouse shape in (4.1), we have:

$$E[D_{x,y}] = \frac{\sqrt{Ar}}{3} + p\sqrt{A/r} + g + a. \tag{4.14}$$

Taking the derivative with respect to r and setting it equal to 0 yields:

$$\frac{1}{2}\left(\frac{\sqrt{A}}{3}\right) r^{-\frac{1}{2}} - \frac{1}{2}\left(p\sqrt{A}\right) r^{-\frac{3}{2}} = 0. \tag{4.15}$$

Solving for r yields the optimal width-to-depth ratio for a 1-sided facility with a general ABC curve:

$$r^* = 3p.$$

□

A.2 Result 2: 2-Sided Layout

Proof. Using the relationship for area and warehouse shape in (4.10), we have:

$$E[D_{x,y}] = q\sqrt{Ar} + \frac{\sqrt{A/r}}{2} + g + a. \tag{4.16}$$

Taking the derivative with respect to r and solving for r yields the following expression for the optimal r:

$$r^* = 1/\left(2q\right). \tag{4.17}$$

□

B. Values for S and p

Table 4.2: ABC Curve Parameters

S Parameter	% Activity			% Storage			p Parameter
	class-A	class-B	class-C	class-A	class-B	class-C	
0.0001	0.9996	0.0003	0.0001	0.2	0.3	0.5	0.1001
0.0300	0.8957	0.0760	0.0283	0.2	0.3	0.5	0.1374
0.0500	0.8400	0.1145	0.0455	0.2	0.3	0.5	0.1582
0.0700	0.7926	0.1460	0.0614	0.2	0.3	0.5	0.1764
0.0900	0.7517	0.1720	0.0763	0.2	0.3	0.5	0.1926
0.1000	0.7333	0.1833	0.0833	0.2	0.3	0.5	0.2000
0.2000	0.6000	0.2571	0.1429	0.2	0.3	0.5	0.2571
0.3000	0.5200	0.2925	0.1875	0.2	0.3	0.5	0.2950
0.4000	0.4667	0.3111	0.2222	0.2	0.3	0.5	0.3222
0.5000	0.4286	0.3214	0.2500	0.2	0.3	0.5	0.3429
0.6000	0.4000	0.3273	0.2727	0.2	0.3	0.5	0.3591
0.7000	0.3778	0.3306	0.2917	0.2	0.3	0.5	0.3722

C. Definition of Parameter q for the Put-Away Operation with the 2-Sided Layout

Using (4.3)–(4.6), we substitute the expressions for D_1, D_2, D_3 and D_4 into the expected across-aisle travel in (4.7)–(4.9) and multiply by the percent of storage for the dock door (probability of using the dock door):

$$\begin{aligned} q_1 =& \frac{1}{2}PA_C PS_C \left[W - \frac{1}{3}PS_C W\right] + \frac{1}{2}PA_B PS_C \left[\frac{1}{2}PS_B W + \frac{W}{2} + \frac{1}{2}PS_A W\right] \\ &+ PA_A PS_C \left[\frac{W}{2} - \frac{1}{4}PS_C W\right] \end{aligned}$$

$$\begin{aligned} q_2 =& \frac{1}{2}PA_C PS_B \left[W - \frac{1}{2}PS_C W\right] + \frac{1}{2}PA_B PS_B \left[\frac{W}{2} + \frac{1}{2}PS_A W + \frac{1}{6}PS_B W - \frac{1}{2}PS_C W\right] \\ &+ PA_A PS_B \left[\frac{W}{2} - \frac{1}{2}PS_C W - \frac{1}{4}PS_B W\right] \end{aligned}$$

$$
\begin{aligned}
q_3 = & PA_C PS_A \left[\frac{W}{2} - \frac{1}{4}PS_C W\right] + PA_B PS_A \left[\frac{W}{2} - \frac{1}{2}PS_C W - \frac{1}{4}PS_B W\right] \\
& + PA_A PS_A \left[\frac{1}{3}PS_A W\right].
\end{aligned}
$$

Summing the coefficients of W from the previous three equations, we define the quantity q as:

$$
\begin{aligned}
q = & \frac{1}{2}PA_C PS_C - \frac{1}{6}PA_C PS_C^2 + \frac{1}{4}PA_B PS_C + \frac{1}{2}PA_A PS_C - \frac{1}{4}PA_A PS_C^2 + \frac{1}{2}PA_C PS_B \\
& - \frac{1}{4}PA_C PS_B PS_C + \frac{1}{4}PA_B PS_B + \frac{1}{12}PA_B PS_B^2 + \frac{1}{2}PA_A PS_B - \frac{1}{2}PA_A PS_B PS_C \\
& - \frac{1}{4}PA_A PS_B^2 + \frac{1}{3}PA_A PS_A^2 + \frac{1}{2}PA_B PS_A + \frac{1}{2}PA_C PS_A - \frac{1}{4}PA_C PS_A PS_C \\
& - \frac{1}{4}PA_B PS_A PS_C.
\end{aligned}
\tag{4.18}
$$

D. Equations for Across-Aisle, Order-Picking Travel for the 2-Sided Layout

In the following scenarios, we consider the x-distance traveled for the traversal strategy with the 2-sided layout where the term in the expression before the left square bracket is the probability of occurrence, and the expression in the square bracket is the expected two-way x-distance (d_i) for the particular scenario.

1. *All class-A picks:*

$$d_1 = (PA_A)^N \left[\left(\frac{N-1}{N+1}\right) PS_A\right] 2W.$$

2. *All class-B picks:*

$$d_2 = (PA_B)^N \left[PS_A + \left(\frac{N-1}{N+1}\right) PS_B\right] 2W.$$

3. *All class-C picks:*

$$d_3 = (PA_C)^N \left[PS_A + PS_B + \left(\frac{N-1}{N+1}\right) PS_C\right] 2W.$$

4. *At least 1 class-A pick, and exactly 1 class-B pick:*

$$d_4 = N\left(PA_A\right)^{N-1} PA_B \left[\left(\frac{N-1}{N}\right) PS_A + 0.25 PS_B\right] 2W.$$

5. *At least 1 class-A pick and 2 or more class-B picks:*

$$d_5 = \sum_{i=2}^{N-1} \binom{N}{i} \left(PA_A\right)^{N-i} \left(PA_B\right)^{i} \left[PS_A + \left(\frac{i-1}{i+1}\right) PS_B\right] 2W.$$

6. *One or more class-A picks and exactly 1 class-C pick:*

$$d_6 = N\left(PA_A\right)^{N-1} PA_C \left[\left(\frac{N-1}{N}\right) PS_A + 0.5 PS_B + 0.25 PS_C\right] 2W.$$

7. *At least one class-A pick and 2 or more class-C picks:*

$$d_7 = \sum_{i=2}^{N-1} \binom{N}{i} \left(PA_A\right)^{N-i} \left(PA_C\right)^{i} \left[PS_A + PS_B + \left(\frac{i-1}{i+1}\right) PS_C\right] 2W.$$

8. *At least 1 class-B pick and exactly 1 class-C pick:*

$$d_8 = N\left(PA_B\right)^{N-1} PA_C \left[PS_A + \left(\frac{N-1}{N}\right) PS_B + 0.25 PS_C\right] 2W.$$

9. *At least 1 class-B pick and 2 or more class-C picks:*

$$d_9 = \sum_{i=2}^{N-1} \binom{N}{i} \left(PA_B\right)^{N-i} \left(PA_C\right)^{i} \left[PS_A + PS_B + \left(\frac{i-1}{i+1}\right) PS_C\right] 2W.$$

10. *At least 1 class-A pick, 2 or more class-B picks, and exactly 1 class-C pick:*

$$\begin{aligned} d_{10} &= \sum_{i=2}^{N-2} \left(\frac{N!}{i!\,(N-i-1)!} \right) (PA_A)^{N-i-1} (PA_B)^i \, PA_C \times \\ &\quad \left[PS_A + \left(\frac{i}{i+1} \right) PS_B + 0.25 PS_C \right] 2W. \end{aligned}$$

11. *At least 1 class-A pick, exactly 1 class-B pick, and exactly 1 class-C pick (where the class-B and class-C picks could be on the same side of the pickup point or on opposite sides of the pickup point):*

$$\begin{aligned} d_{11} &= N(N-1)(PA_A)^{N-2} PA_B PA_C \left[0.5 \left[\left(\frac{N-2}{N-1} \right) PS_A + 0.5 PS_B + 0.25 PS_C \right] \right. \\ &\quad \left. +0.5 \left[PS_A + 0.75 PS_B + 0.25 PS_C \right] \right] 2W. \end{aligned}$$

12. *At least 1 class-A pick, 2 or more class-C picks, at least one class-B pick:*

$$\begin{aligned} d_{12} &= \sum_{i=2}^{N-j-1} \sum_{j=1}^{N-i-1} \left(\frac{N!}{i!j!\,(N-i-j)!} \right) (PA_A)^j (PA_B)^{N-i-j} (PA_C)^i \times \\ &\quad \left[PS_A + PS_B + \left(\frac{i-1}{i+1} \right) PS_C \right] 2W. \end{aligned}$$

E. Equations for Replenishment Travel with the 1-Sided Layout

The expected distance for the replenishment operation with a 1-sided layout is included below. We begin with a detailed explanation of the expected travel where locations 1 and 3 are both class-A items. Then we present the distance equations for the remaining scenarios.

Case 1 (all 3 locations are in the same aisle): Location 1 is in the same aisle as location 2 with probability $1/M$, and locations 2 and 3 are in the same aisle with probability α.

From A to A:

Location 2 (randomly located in the aisle) can be to the left of location 1, between locations 1 and 3, or to the right of location 3 as shown in Figure 4.19:

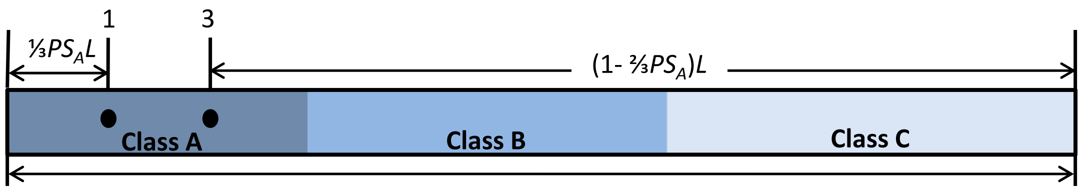


Figure 4.19: Reserve location uniformly distributed along L.

The distance traveled from location 1 to location 3 is $\frac{1}{3}PS_A L$. If location 2 is between locations 1 and 3, no additional travel is required. The probability that location 2 is to the left of location 1 is $\frac{1}{3}PS_A$, and the two-way distance from location 2 back to location 1 is $\frac{1}{3}PS_A L$. The probability that location 2 is to the right of location 3 is $(1-\frac{2}{3}PS_A)$, and the two-way distance from location 2 back to location 3 is $(1-\frac{2}{3}PS_A)L$. Therefore, we can express the expected distance for this scenario as:

$$\begin{aligned} d &= \frac{1}{3}PS_A L + \left(\frac{1}{3}PS_A\right)^2 L + \left(1 - \frac{2}{3}PS_A\right)^2 L \\ &= \left(\frac{5}{9}PS_A{}^2 - PS_A + 1\right) L. \end{aligned}$$

The total expected distance is then:

$$d_1^{R1} = (PA_A)^2 \frac{\alpha}{M} \left[\frac{5}{9}(PS_A)^2 - PS_A + 1\right] L.$$

Case 2 (locations 1 and 2 are in the same aisle): Locations 1 and 2 are in the same aisle with probability $1/M$, and locations 2 and 3 are in different aisles with probability $1 - \alpha$.

From A to A:

$$d_2^{R1} = (PA_A)^2 \left(\frac{1-\alpha}{M}\right) \left[\left(0.25\,(PS_A)^2 + 1\right) L + \frac{1}{3}W\right].$$

Case 3 (locations 1 and 2 are in different aisles; locations 2 and 3 are in the same aisle): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are in the same aisle with probability α.

From A to A:

$$d_3^{R1} = (PA_A)^2 \frac{\alpha (M-1)}{M} \left[\left(0.25 (PS_A)^2 + 1\right) L + \frac{1}{3} W \right].$$

Case 4 (three locations are in three different aisles): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are in different aisles with probability $1-\alpha$.

From A to A:

$$d_4^{R1} = (PA_A)^2 \frac{(1-\alpha)(M-1)}{M} \left[(PS_A + 1) L + \frac{2}{3} W \right].$$

Case 1 (all 3 locations are in the same aisle): Location 1 is in the same aisle as location 2 with probability $1/M$, and locations 2 and 3 are in the same aisle with probability α.

From (A to B) or (B to A):

$$\begin{aligned} d_5^{R1} =& 2(PA_A)(PA_B) \frac{\alpha}{M} \times \\ & \left[0.25 (PS_A)^2 + 0.25 (PS_B)^2 + (PS_C)^2 + 0.5 PS_A + 0.5 PS_B + PS_B PS_C \right] L. \end{aligned}$$

From (A to C) or (C to A):

$$d_6^{R1} = 2(PA_A)(PA_C) \frac{\alpha}{M} \left[0.25 (PS_A)^2 + 0.25 (PS_C)^2 + 0.5 PS_A + PS_B + 0.5 PS_C \right] L.$$

From B to B:

$$d_7^{R1} = (PA_B)^2 \frac{\alpha}{M}\left[(PS_A)^2 + \frac{2}{9}(PS_B)^2 + (PS_C)^2 + \frac{2}{3}PS_A PS_B + \frac{2}{3}PS_B PS_C + \frac{1}{3}PS_B \right] L.$$

From (B to C) or (C to B):

$$\begin{aligned} d_8^{R1} =& 2(PA_B)(PA_C)\frac{\alpha}{M}\times \\ & \left[(PS_A)^2 + 0.25(PS_B)^2 + 0.25(PS_C)^2 + PS_A PS_B + 0.5PS_B + 0.5PS_C \right] L. \end{aligned}$$

From C to C:

$$d_9^{R1} = (PA_C)^2 \frac{\alpha}{M}\left[\frac{5}{9}(PS_C)^2 - PS_C + 1 \right] L.$$

Case 2 (locations 1 and 2 are in the same aisle): Locations 1 and 2 are in the same aisle with probability $1/M$, and locations 2 and 3 are in different aisles with probability $1-\alpha$.

From A to B:

$$d_{10}^{R1} = (PA_A)(PA_B)\left(\frac{1-\alpha}{M}\right)\left[\left(0.25(PS_A)^2 + 0.5PS_A + 0.5PS_B + 1\right) L + \frac{1}{3}W \right].$$

From A to C:

$$d_{11}^{R1} = (PA_A)(PA_C)\left(\frac{1-\alpha}{M}\right)\left[\left(0.25(PS_A)^2 - 0.5PS_A + 0.5PS_C + 1\right) L + \frac{1}{3}W \right].$$

From B to A:

$$\begin{aligned} d_{12}^{R1} =& (PA_B)(PA_A)\left(\frac{1-\alpha}{M}\right)\times \\ & \left[\left((PS_A)^2 + 0.25(PS_B)^2 + PS_A PS_B - 0.5PS_A - 0.5PS_B + 1\right) L + \frac{1}{3}W \right]. \end{aligned}$$

From B to B:

$$d_{13}^{R1} = (PA_B)^2\left(\frac{1-\alpha}{M}\right)\left[\left((PS_A)^2 + 0.25\,(PS_B)^2 + PS_A PS_B + 1\right)L + \frac{1}{3}W\right].$$

From B to C (exit toward class-C end):

$$\begin{aligned} d_{14}^{R1} =&(PA_B)(PA_C)\left(\frac{1-\alpha}{M}\right)\times \\ &\left[\left(0.25\,(PS_B)^2 + (PS_C)^2 + PS_B PS_C + PS_A + 0.5PS_B + 0.5PS_C\right)L + \frac{1}{3}W\right]. \end{aligned}$$

From C to A:

$$d_{15}^{R1} = \quad (PA_C)(PA_A)\left(\frac{1-\alpha}{M}\right)\left[\left(0.25\,(PS_C)^2 + 0.5PS_A - 0.5PS_C + 1\right)L + \frac{1}{3}W\right].$$

From C to B:

$$d_{16}^{R1} = (PA_C)(PA_B)\left(\frac{1-\alpha}{M}\right)\left[\left(0.25\,(PS_C)^2 + PS_A + 0.5PS_B - 0.5PS_C + 1\right)L + \frac{1}{3}W\right].$$

From C to C (exit toward class-C end):

$$d_{17}^{R1} = (PA_C)^2\left(\frac{1-\alpha}{M}\right)\left[\left(0.25\,(PS_C)^2 + 1\right)L + \frac{1}{3}W\right].$$

Case 3 (locations 1 and 2 are in different aisles; locations 2 and 3 are in the same aisle): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are in the same aisle with probability α.

From A to B:

$$\begin{aligned} d_{18}^{R1} =&(PA_A)(PA_B)\frac{\alpha\,(M-1)}{M}\times \\ &\left[\left((PS_A)^2 + 0.25\,(PS_B)^2 + PS_A PS_B - 0.5PS_A - 0.5PS_B + 1\right)L + \frac{1}{3}W\right]. \end{aligned}$$

From A to C:

$$d_{19}^{R1} = (PA_A)(PA_C)\frac{\alpha(M-1)}{M}\left[\Big(0.25\,(PS_C)^2 + 0.5PS_A - 0.5PS_C + 1\Big)\,L + \frac{1}{3}W\right].$$

From B to A:

$$d_{20}^{R1} = (PA_B)(PA_A)\frac{\alpha(M-1)}{M}\left[(0.25(PS_A)^2 + 0.5PS_A + 0.5PS_B + 1)L + \frac{1}{3}W\right].$$

From B to B:

$$d_{21}^{R1} = (PA_B)^2\,\frac{\alpha(M-1)}{M}\left[((PS_A)^2 + 0.25\,(PS_B)^2 + PS_A PS_B + 1)L + \frac{1}{3}W\right].$$

From B to C:

$$d_{22}^{R1} = (PA_B)(PA_C)\frac{\alpha(M-1)}{M}\left[(0.25(PS_C)^2 + PS_A + 0.5PS_B - 0.5PS_C + 1)L + \frac{1}{3}W\right].$$

From C to A:

$$d_{23}^{R1} = (PA_C)(PA_A)\frac{\alpha(M-1)}{M}\left[(0.25(PS_A)^2 - 0.5PS_A + 0.5PS_C + 1)L + \frac{1}{3}W\right].$$

From C to B:

$$\begin{aligned} d_{24}^{R1} = &(PA_C)(PA_B)\frac{\alpha(M-1)}{M}\times \\ &\left[(0.25(PS_B)^2 + (PS_C)^2 + PS_B PS_C - 0.5PS_B - 0.5PS_C + 1)L + \frac{1}{3}W\right]. \end{aligned}$$

From C to C:

$$d_{25}^{R1} = (PA_C)^2\,\frac{\alpha(M-1)}{M}\left[\Big(0.25\,(PS_C)^2 + 1\Big)\,L + \frac{1}{3}W\right].$$

Case 4 (three locations are in three different aisles): Locations 1 and 2 are in different aisles with probability $(M-1)/M$, and locations 2 and 3 are in different aisles with probability $1-\alpha$.

From (A to B) or (B to A):

$$d_{26}^{R1} = 2(PA_A)(PA_B)\frac{(1-\alpha)(M-1)}{M}\left[(1.5PS_A + 0.5PS_B + 1)\,L + \frac{2}{3}W\right].$$

From (A to C) or (C to A), exit toward class-C, traverse reserve aisle:

$$d_{27}^{R1} = 2(PA_A)(PA_C)\frac{(1-\alpha)(M-1)}{M}\left[(0.5PS_A + 0.5PS_C + 1)\,L + \frac{2}{3}W\right].$$

From B to B:

$$d_{28}^{R1} = (PA_B)^2\,\frac{(1-\alpha)(M-1)}{M}\left[(2PS_A + PS_B + 1)\,L + \frac{2}{3}W\right].$$

From (B to C) or (C to B), traverse reserve aisle:

$$d_{29}^{R1} = 2(PA_B)(PA_C)\frac{(1-\alpha)(M-1)}{M}\left[(PS_A + 0.5PS_B + 0.5PS_C + 1)\,L + \frac{2}{3}W\right].$$

From C to C, exit toward class-C:

$$d_{30}^{R1} = (PA_C)^2\,\frac{(1-\alpha)(M-1)}{M}\left[(PS_C + 1)\,L + \frac{2}{3}W\right].$$

The total distance for a replenishment operation with the 1-sided layout can be determined by summing each of these distances:

$$E[D_{x,y}] = \sum_{i=1}^{30} d_i^{R1}. \tag{4.19}$$

F. Equations for Replenishment Travel with the 2-Sided Layout

Below we provide a detailed example of how the probabilities and expected distances are determined using the case where locations 1 and 3 are both class-A items. Then we provide the expected distance equations for each of the remaining scenarios. As with the random storage and 1-sided layouts, α is used to represent the probability that the reserve storage location is in the same aisle as the replenishment location. We also use M_A, M_B and M_C to represent the number of aisles for storage classes A, B and C, respectively.

- *Location 1 is class-A, location 3 is class-A:*

  When location 1 is a class-A item and location 3 is a class-A item, only two possibilities allow locations 2 and 3 to be in the same aisle. First, all three locations could be located in the same aisle. Second, locations 2 and 3 could be located in the same aisle, but in a different aisle from location 1.

  For the first case, there are M_A possible ways that all three locations are within the same aisle. For the second case, we establish the number of possibilities of occurrence by first considering that there are M_A choose two combinations of two aisles within the class-A aisles. Given the two aisles, it matters now that the remaining two points are ordered together in the same aisle, and there are two choose one ways for this to occur (${}_2C_1 = 2$). Thus, it matters whether location 2 is in the same aisle as location 1 (which contributes to $1 - \alpha$), or if location 2 is in the same aisle as location 3 (which contributes to α).

For the first case with all locations in the same aisle, the probability of occurrence is:

$$\begin{aligned} p &= \frac{M_A}{M_A + \binom{M_A}{2}\binom{2}{1}} \\ &= \frac{M_A}{M_A + \frac{M_A(M_A-1)(M_A-2)!2!}{(M_A-2)!2!}} \\ &= \frac{M_A}{M_A(1+M_A-1)} \\ &= \frac{1}{M_A}. \end{aligned}$$

For the second case, where there are two aisles and locations 2 and 3 are in the same aisle, the probability is:

$$\begin{aligned} p &= \frac{\binom{M_A}{2}\binom{2}{1}}{M_A + \binom{M_A}{2}\binom{2}{1}} \\ &= 1 - \frac{1}{M_A}. \end{aligned}$$

There are three different scenarios to consider when locations 2 and 3 are not in the same aisle $(1-\alpha)$. We note here that because locations 1 and 3 *must* reside in class-A aisles, there are M_A possible aisles for these locations. Location 2 (the reserve location), on the other hand, can reside in *any* aisle. However, locations 2 and 3 cannot reside in the same aisle by definition of $(1-\alpha)$, so location 2 can reside in $(M-1)$ possible aisles.

First, the three locations could be in three different class-A aisles. Given that location 1 is in a class-A aisle, location 2 must be in a different class-A aisle $((M_A-1)/(M-1))$, and location 3 must be in a class-A aisle that is different from locations 1 and 2

$((M_A - 2)/M_A)$:

$$p = \frac{(M_A - 1)(M_A - 2)}{M_A(M - 1)}.$$

Second, location 2 could be in a class-B or class-C aisle, with $M - M_A$ possible aisles. If location 1 resides in the same aisle as location 3, the probability is:

$$p = \frac{M - M_A}{M_A(M - 1)},$$

and if locations 1 and 3 are in different aisles, the probability is:

$$p = \frac{(M - M_A)(M_A - 1)}{M_A(M - 1)}.$$

Third, locations 1 and 2 could reside in the same aisle, with location 3 in a different aisle:

$$p = \frac{M_A - 1}{M_A(M - 1)}.$$

Now that the probabilities have been defined, we multiply these by the expected distance for each scenario:

All locations are in the same aisle:

$$d_1^{R2} = (PA_A)^2 \frac{\alpha}{M_A} \left[\frac{2}{3} L \right].$$

Locations 2 and 3 are in same aisle; location 1 is in a different aisle:

$$d_2^{R2} = (PA_A)^2 \alpha \left(1 - \frac{1}{M_A} \right) \left[\frac{1}{3} PS_A W + \frac{7}{6} L \right].$$

The three locations are in three different class-A aisles:

$$d_3^{R2} = (PA_A)^2\,(1-\alpha)\,\frac{(M_A-1)(M_A-2)}{M_A(M-1)}\left[\frac{2}{3}PS_AW + 2L\right].$$

Location 2 resides in a class-B or class-C aisle (with locations 1 and 3 in the same or different aisles):

$$d_4^{R2} = (PA_A)^2\,(1-\alpha)\,\frac{M_A(M-M_A)}{M_A(M-1)}\left[(PS_A + 0.5PS_B + 0.5PS_C)W + 2L\right].$$

Locations 1 and 2 are in the same class-A aisle; location 3 is in a different class-A aisle:

$$d_5^{R2} = (PA_A)^2\,(1-\alpha)\,\frac{M_A-1}{M_A(M-1)}\left[\frac{1}{3}PS_AW + \frac{7}{6}L\right].$$

Locations 1 and 3 are in the same class-A aisle, with location 2 in a different class-A aisle:

$$d_6^{R2} = (PA_A)^2\,(1-\alpha)\,\frac{M_A-1}{M_A(M-1)}\left[\frac{1}{3}PS_AW + 2L\right].$$

- *Location 1 is class-A, location 3 is class-B:*

  When location 1 is in a class-A aisle and location 3 is in a class-B aisle, only one scenario contributes to α.

  Location 1 is in a class-A aisle, and locations 2 and 3 are in the same class-B aisle:

$$d_7^{R2} = (PA_A)(PA_B)\alpha\left[(0.5PS_A + 0.25PS_B)\,W + \frac{7}{6}L\right].$$

  There are four scenarios (with different distances) that contribute to the case where locations 2 and 3 are not in the same aisle.

  Location 2 is in a class-B aisle, different from location 3 (where location 2 can be *in either of the two sections of class-B aisles:*

$$d_8^{R2} = (PA_A)(PA_B)\,(1-\alpha)\,\frac{M_B - 1}{M-1}\left[(PS_A + \frac{7}{12}PS_B)W + 2L\right].$$

Location 2 is in a class-A aisle, where:

locations 1 and 2 in the same aisle:

$$d_9^{R2} = (PA_A)(PA_B)\,(1-\alpha)\,\frac{1}{M-1}\left[(0.5PS_A + 0.25PS_B)W + \frac{7}{6}L\right],$$

or, locations 1 and 2 are in different aisles:

$$d_{10}^{R2} = (PA_A)(PA_B)\,(1-\alpha)\,\frac{M_A - 1}{M-1}\left[\left(\frac{5}{6}PS_A + 0.5PS_B\right)W + 2L\right].$$

Location 2 resides in a class-C aisle:

$$d_{11}^{R2} = (PA_A)(PA_B)\,(1-\alpha)\,\frac{M_C}{M-1}\big[\,(PS_A + PS_B + 0.5PS_C)\,W + 2L\big].$$

- *Location 1 is class-A, location 3 is class-C:*

There is only way that locations 2 and 3 can be in the same aisle.

Locations 2 and 3 must reside in the same class-C aisle:

$$d_{12}^{R2} = (PA_A)(PA_C)\alpha\left[(0.5PS_A + 0.5PS_B + 0.25PS_C)\,W + \frac{7}{6}L\right].$$

If location 2 is in a class-A aisle, then:

If locations 1 and 2 are in the same aisle, the expected distance is:

$$d_{13}^{R2} = (PA_A)(PA_C)\,(1-\alpha)\,\frac{1}{M-1}\left[(0.5PS_A + 0.5PS_B + 0.25PS_C)\,W + \frac{7}{6}L\right],$$

and if locations 1 and 2 are in different aisles, the expected distance is:

$$d_{14}^{R2} = (PA_A)(PA_C)\,(1-\alpha)\,\frac{M_A - 1}{M-1}\left[\left(\frac{5}{6}PS_A + 0.5PS_B + 0.25PS_C\right)W + 2L\right].$$

If location 2 is in a class-B aisle, the expected distance is:

$$d_{15}^{R2} = (PA_A)(PA_C)\,(1-\alpha)\,\frac{M_B}{M-1}\left[(PS_A + 0.5PS_B + 0.25PS_C)\,W + 2L\right].$$

If location 2 is in a class-C aisle (different from location 3) the expected distance is:

$$d_{16}^{R2} = (PA_A)(PA_C)\,(1-\alpha)\,\frac{M_C - 1}{M-1}\left[\left(1.5PS_A + PS_B + \frac{7}{12}PS_C\right)W + 2L\right].$$

- *Location 1 is class-B, location 3 is class-B:*

There are two scenarios where locations 2 and 3 can be in the same aisle:

All locations are in the same aisle:

$$d_{17}^{R2} = (PA_B)^2\frac{\alpha}{M_B}\left[\frac{2}{3}L\right].$$

Locations 2 and 3 are in same aisle, but different from location 1, where the location 1 can be in either section of class-B aisles:

$$d_{18}^{R2} = (PA_B)^2\alpha\left(1 - \frac{1}{M_B}\right)\left[\left(0.5PS_A + \frac{1}{3}PS_B\right)W + \frac{7}{6}L\right].$$

The possibilities for $(1-\alpha)$ include:

Locations 1 and 2 are in the same class-B aisle, with location 3 in a different class-B aisle:

$$d_{19}^{R2} = \quad (PA_B)^2\,(1-\alpha)\,\frac{M_B-1}{M_B(M-1)}\left[\left(0.5PS_A+\frac{1}{3}PS_B\right)W+\frac{7}{6}L\right].$$

Locations 1 and 3 are in the same class-B aisle, with location 2 in a different class-B aisle:

$$d_{20}^{R2} = \quad (PA_B)^2\,(1-\alpha)\,\frac{M_B-1}{M_B(M-1)}\left[\left(PS_A+\frac{7}{12}PS_B\right)W+2L\right].$$

All locations reside in different class-B aisles:

$$d_{21}^{R2} = \quad (PA_B)^2\,(1-\alpha)\,\frac{(M_B-1)(M_B-2)}{M_B(M-1)}\left[\left(PS_A+\frac{2}{3}PS_B\right)W+2L\right].$$

Location 2 is in a class-A aisle:

$$d_{22}^{R2} = \quad (PA_B)^2\,(1-\alpha)\,\frac{M_A}{M-1}\left[(PS_A+0.5PS_B)\,W+2L\right].$$

Locaton 2 is in a class-C aisle:

$$d_{23}^{R2} = (PA_B)^2\,(1-\alpha)\,\frac{M_C}{M-1}\left[(PS_A+PS_B+0.5PS_C)\,W+2L\right].$$

- *Location 1 is class-B, location 3 is class-C:*

There is one way for locations 2 and 3 to reside in the same aisle:

$$d_{24}^{R2} = (PA_B)(PA_C)\alpha \left[(0.5PS_A + 0.5PS_B + 0.25PS_C)\, W + \frac{7}{6}L \right].$$

Four scenarios contribute to $(1-\alpha)$:

Locations 1 and 2 are in the same class-B aisle:

$$\begin{aligned} d_{25}^{R2} = & (PA_B)(PA_C)\,(1-\alpha)\,\frac{1}{M-1} \times \\ & \left[(0.5PS_A + 0.5PS_B + 0.25PS_C)\, W + \frac{7}{6}L \right]. \end{aligned}$$

Locations 1 and 2 are in different class-B aisles:

$$\begin{aligned} d_{26}^{R2} = & (PA_B)(PA_C)\,(1-\alpha)\,\frac{M_B-1}{M-1} \times \\ & \left[\left(PS_A + \frac{23}{24}PS_B + 0.25PS_C \right) W + 2L \right]. \end{aligned}$$

Location 2 is in a class-A aisle:

$$d_{27}^{R2} = \quad (PA_B)(PA_C)\,(1-\alpha)\,\frac{M_A}{M-1}\left[(PS_A + PS_B + 0.25PS_C)\, W + 2L \right].$$

Locations 2 and 3 are in different class-C aisles:

$$d_{28}^{R2} = \quad (PA_B)(PA_C)\,(1-\alpha)\,\frac{M_C-1}{M-1}\left[\left(PS_A + PS_B + \frac{5}{12}PS_C \right) W + 2L \right].$$

- *Location 1 is class-C, location 3 is class-C:*

There are two ways for locations 2 and 3 to be in the same aisle:

All locations are in the same class-C aisle:

$$d_{29}^{R2} = (PA_C)^2 \frac{\alpha}{M_C} \left[\frac{2}{3}L\right].$$

Locations 2 and 3 are in same class-C aisle, but different from location 1, where location 1 can be in either section of class-C aisles:

$$d_{30}^{R2} = (PA_C)^2 \alpha \left(1 - \frac{1}{M_C}\right) \left[\left(0.5PS_A + 0.5PS_B + \frac{1}{3}PS_C\right) W + \frac{7}{6}L\right].$$

Five scenarios contribute to $(1 - \alpha)$:

The three locations are in three different class-C aisles:

$$\begin{aligned} d_{31}^{R2} = &(PA_C)^2 (1 - \alpha) \frac{(M_C - 1)(M_C - 2)}{M_C(M - 1)} \times \\ &\left[\left(PS_A + PS_B + \frac{33}{48}PS_C\right) W + 2L\right]. \end{aligned}$$

Locations 1 and 2 are located in the same class-C aisle, but different from location 3:

$$\begin{aligned} d_{32}^{R2} = &(PA_C)^2 (1 - \alpha) \frac{M_C - 1}{M_C(M - 1)} \times \\ &\left[\left(0.5PS_A + 0.5PS_B + \frac{1}{3}PS_C\right) W + \frac{7}{6}L\right]. \end{aligned}$$

Locations 1 and 3 are located in the same class-C aisle, with location 2 in a different class-C aisle:

$$d_{33}^{R2} = \quad (PA_C)^2 (1 - \alpha) \frac{M_C - 1}{M_C(M - 1)} \left[\left(PS_A + PS_B + \frac{7}{12}PS_C\right) W + 2L\right].$$

Location 2 is in a class-B aisle:

$$d_{34}^{R2} = \quad (PA_C)^2 (1-\alpha) \frac{M_B}{M-1} \Big[(PS_A + PS_B + 0.5PS_C) W + 2L \Big].$$

Location 2 is in a class-A aisle:

$$d_{35}^{R2} = \quad (PA_C)^2 (1-\alpha) \frac{M_A}{M-1} \Big[(PS_A + PS_B + 0.5PS_C) W + 2L \Big].$$

The total distance for a replenishment operation with the 1-sided layout can be determined by summing each of these distances:

$$E[D_{x,y}] = \sum_{i=1}^{35} d_i^{R2}.$$

G. Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Russell D. Meller, Professor and Holder of the Hefley Professorship

DATE: June 18, 2013

SUBJECT: Certification of student effort

I certify that greater than 51% of the work conducted for this chapter entitled, "Contribution 1: A Paper on, 'Analytical Models for Warehouse Configuration'," was conducted by Lisa M. Thomas.

4207 Bell Engineering Center • Fayetteville, Arkansas 72701 • 479-575-3156 • 479-575-8431 Fax

Chapter 5

Contribution 2: A Paper on, "Using Analytical Models to Assess Performance in Overall Warehouse Design"

Abstract: Overall warehouse design is a complex and challenging process. Because warehouse functions are interrelated, changing even one design parameter can affect several functional areas, and the combination of possible parameters results in a large number of designs to consider. Thus, having a means to compare designs in terms of operational performance is crucial, yet there is no comprehensive model for quantifying the labor requirements of a given design. Many design methodologies suggest simulation for comparing different designs. We demonstrate how a set of analytical models can be used to assess warehouse performance using the example of a manual, case-picking warehouse. We use functional flow networks to drive the design process, beginning with the most basic design and progressing to more complex designs. This methodology has been tested using students as warehouse designers with positive results.

5.1. Introduction

One of the greatest challenges of overall warehouse design is determining which design is best in terms of cost and performance. Comparing designs is difficult because there is no comprehensive model for quantifying the labor requirements for a given design. Two factors that complicate the design process are: 1) the number of possible designs and 2) the interactions among the functional components of the warehouse. For a manual, case-picking warehouse, the design parameters may include: the shape (width-to-depth ratio) of the storage area, the number of levels of pallet rack, the size for the forward area (if any), the layout of the forward area, and dock doors on one or both sides of the facility. The number of combinations of possible parameters can result in a large number of designs to consider. For example, if design parameters are such that storage area shapes can range from 1.0 to 4.0, with two possible levels of storage, a range of 5–50% of the SKUs in the forward area with random or class-based storage, and with dock doors on one or both sides of the facility, hundreds of designs are possible. Further, the functional areas of the warehouse are interrelated; changing one design parameter can affect other functional areas in the warehouse. For example, changing the size of the forward area can affect the overall storage requirements along with the number of pick lines, as a smaller forward area contains the fastest-moving items that likely have more picks per line, allowing fewer total lines per batch in the forward area. In addition, the size of the forward area affects replenishment travel as well as order-picking travel in both the forward and reserve areas. Thus, a mechanism for comparing designs is essential in order to determine the best design.

Some overall design methodologies suggest simulation for comparing different designs, but we demonstrate how a set of analytical models can be used to compare designs for a case-picking warehouse with pallet rack. We consider the labor required for put-away, order-picking and replenishment operations. We begin the design process with the most basic design (pallet rack with no forward area) and progress to more complex designs (forward areas of different sizes and with different layouts), using functional flow networks (FFNs) to represent various designs. A FFN is a series of nodes with connecting arcs, where the nodes represent the functional areas in the warehouse and the arcs represent the flow of material between functional areas [8, 4]. We use analytical models to size each functional area and to convert the flow of product into labor requirements. The goal of this paper is to illustrate the impact of design decisions on the operational cost of the warehouse using an integrated set of models. As we enumerate over a subset of design parameters, computational and search issues related to the "optimal design" are left for a full investigation of the topic.

5.2. Literature Review

According to survey papers by van den Berg and Gademann [13], Rouwenhorst et al. [11], and Gu et al. [6], research in the area of overall warehouse design is limited. Moreover, no comprehensive synthesis of models has been developed [11] [6]. Gu et al. [7] assert that a simple, validated model that provides results for guiding overall structural design would be a valuable research contribution. Currently, simulation is the most common method for assessing warehouse performance in both research and industry, and more computational tools for warehouse design and operation may encourage a closer alignment of academic research with practical application [7].

The survey paper by Rouwenhorst et al. [11] classifies design problems at the strategic, tactical and operational levels. The authors contend that design decisions at the strategic and tactical level are often interrelated and require joint consideration. In addition, the authors conclude that the majority of research papers address isolated subproblems.Van den Berg and Gademann [13] present a hierarchy of warehousing decisions for operational planning and control, and the authors present the methods and models that have appeared in the literature for each area.

Research in the area of overall warehouse design generally falls into one of two categories: 1) solution procedures that provide a general, qualitative design framework, and 2) detailed models that provide a quantitative comparison of design alternatives. The papers that provide a quantitative comparison of solutions to the design problem often include models that require an extensive number of input parameters and are not general enough to apply to a broad range of warehouses. We first present the research papers that include general design frameworks and then discuss design methodologies that provide quantitative solution procedures.

Baker and Canessa [3] performed a survey of research papers in the area of overall warehouse design, grouping them as those that examine tools and techniques and those that address overall steps in the design process. The authors formulated a general framework of steps in order to assist practitioners and researchers in a more comprehensive warehouse design methodology.

Ashayeri and Gelders [2] categorized solution procedures as analytical, simulation or heuristic and identified the research in each area. The authors suggest a two-step technique for system design that first uses analytical models to prune the decision space, and then introduces simulation to capture the dynamic aspects of the simplified analytical models. Yoon and Sharp [14] presented a systematic design procedure for order-picking systems with functional areas for order picking to assist designers in determining alternatives for order-picking configurations.

Four papers provide solution procedures that provide a quantitative comparison of design alternatives. First, Gray et al. [5] developed a model for overall design with the objective of minimizing initial incremental costs and operating costs including labor and inventory holding costs. In order to reduce the complexity of the formulation, the authors propose a hierarchical decomposition of the problem with three decision levels: facility design and technology selection, item allocation and assignment, and operating policy. Analytical models were developed for a specific company to prune the decision space, and simulation was used to evaluate the alternative designs and to validate the analytical models. The solution procedure involved iteration between the three decision levels, and the authors estimated a labor savings of close to 50% with the new design. The authors acknowledge that a detailed formulation for general use is not viable because specific features would have to be considered that are not necessarily applicable to other problems.

Next, Park and Webster [10] formulated a design model for a unit-load warehouse. Analytical models were developed to determine land, building, equipment, labor and operating costs. An iterative process is used to determine the maximum inventory levels, initial investment and annual costs, and storage and equipment requirements. The authors illustrate the solution procedure through a case study that considers three alternative designs: a fully automated AS/RS, narrow-aisle lift trucks, or counter-balanced lift trucks. The authors acknowledge that obtaining cost and model parameters for individual firms would require considerable effort.

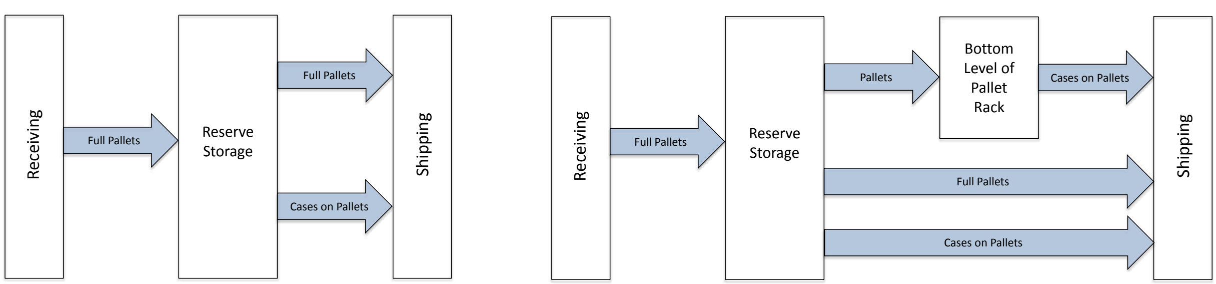
McGinnis et al. [8] first introduced the FFN concept, and subsequent papers detail how warehouse design workflows can be used in a systematic, integrated design procedure for overall warehouse design [12, 4]. Seven modeling principles are introduced that lay the foundation for creating integrated warehouse designs [4].

Finally, Apple et al. [1] proposed an empirically based warehouse design methodology that uses a qualitative list of factors to consider (usually in the form of checklists), as well as quantitative matrix solution guides. Pareto charts are suggested to subdivide the warehouse activities in terms of storage and activity for each handling unit, and FFNs are used to represent each conceptual design. Each functional area is then sized (using available tools), and trial block layouts are developed that seek to minimize handling distances. The authors assert that in order to implement such a methodology, work in two areas must be accomplished. First, standardized definitions for process descriptions must be developed, and second, the quantitative matrix solution tables must be populated.

5.3. Design Methodology

We consider a manual, case-picking warehouse in which pallets are received and cases and full pallets are shipped. For simplicity, we assume that cases are picked and loaded onto a pallet and outgoing cases are floor loaded in the trailer, such that no palletizing is required. Further, we assume that orders are batched to maximize pallet utilization and that sorting is negligible. We evaluate designs with dock doors on one and both sides of the facility, as well as designs with varying shape ratios for the reserve storage area. We also consider pallet rack of different heights (e.g., five or six levels of racking).

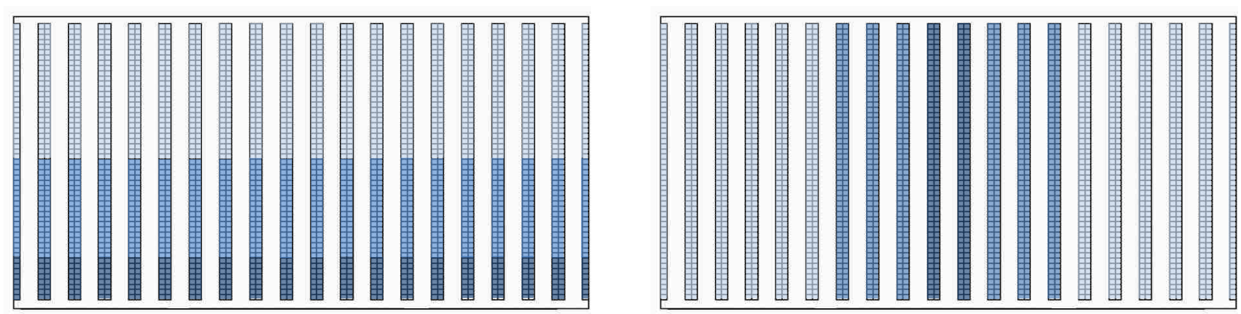
First, we begin with a basic design (pallet rack with no forward area) as depicted in Figure 5.1(a). For these designs, picking occurs over the entire pallet rack area, where all locations are equally likely to contain a pick. Next, we consider a co-located forward area, where the bottom level of the centermost aisles of pallet rack serves as the forward area and the upper levels serve as reserve storage locations as illustrated in Figure 5.1(b). A forward area for picking fast-moving items can reduce order-picking travel, as a small percentage of items often accounts for a large percentage of picks. Thus, the majority of the picks require travel through only a subset of the aisles and picking from the bottom level eliminates the vertical travel component and requires less sophisticated equipment. The cost associated with implementing a forward area is the replenishment travel incurred to move items from reserve storage to the forward area (and, as we discuss later, including a high number of SKUs in the forward area may increase the size of the warehouse to accommodate all of the SKUs on the bottom level).



(a) (b)

Figure 5.1: Functional flow networks: (a) Basic FFN with all picks from reserve storage; (b) FFN including a co-located forward area with case picks from the forward area (bottom level), pallet and case picks from the reserve area, and with replenishments in pallet quantities from the reserve area to the forward area.

Initially, we consider random storage within the aisles of the forward area followed by two class-based storage layouts for doors on one and both sides of the facility. Figures 5.2(a) and 5.2(b) illustrate the class-based storage layouts, where the darker shades represent the fastest moving items. With the identical-aisle layout, the single-sided dock doors are located nearest to the fast-moving items, and the within-aisle layout places fast-moving items in a central location that is convenient to doors on both sides of the facility.



(a) (b)

Figure 5.2: Class-based layouts: (a) Identical-aisle, 1-sided dock doors; (b) Within-aisle, 2-sided dock doors.

We consider a range of 5–100% of the SKUs for random storage forward areas and 20–100% of the SKUs for class-based forward areas. Figure 5.3 illustrates the designs that we evaluate. For each design we use an existing algorithm [9] to size a storage area to meet a target number of pallet positions, and we use the put-away, order-picking and replenishment models in Chapter 4 to determine the labor required for each design. In the following section, we define the design parameters for a particular warehouse to illustrate the design process.

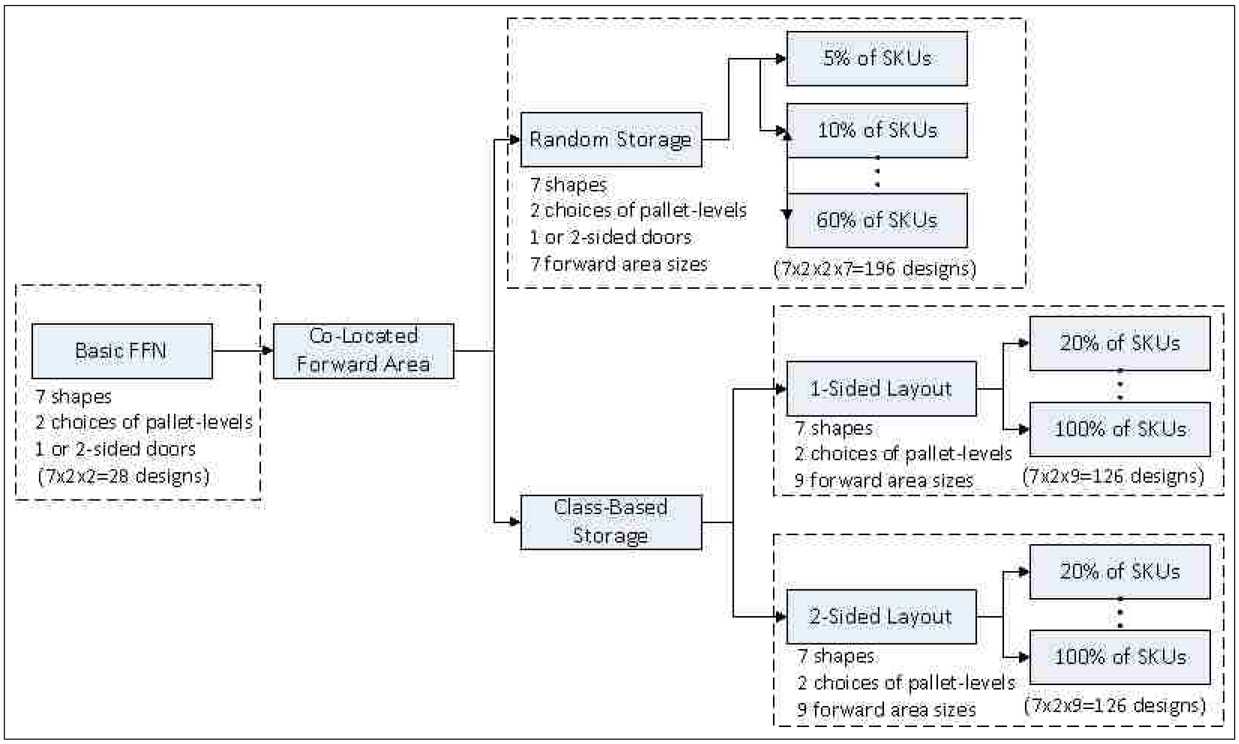


Figure 5.3: Designs considered.

5.4. Example

We evaluate designs for a manual warehouse with pallet rack for storage and conventional aisles that are orthogonal to the side(s) of the warehouse with dock doors. We assume that picking locations reside within the pallet rack storage area, where each location is the same size and contains the same number of cases per pallet location. The forward area for picking fast-moving cases is the bottom level of pallet rack, located within the centermost aisles, where one bottom-level pallet location is designated for each SKU in the forward area.

5.4.1 Warehouse Parameters

Consider a company with a warehouse that picks in pallet and case quantities and requires approximately 35,000 pallet positions to ensure an adequate supply of an inventory for its 10,000 SKUs. On average, incoming pallets contain 50 cases and the warehouse receives 1,200 pallets of product per day. On a given work day orders entail 200 pallet picks and 50,000 case picks. A typical batch of orders has approximately 17.5 lines and 2.0 picks per line, requiring approximately 1,429 $(50,000/(17.5 \times 2.0))$ order-picking batches per day. The warehouse requires a staging area that is 40-feet deep, storage aisles that are 9.5-feet deep and end cross-aisles that are 10-feet deep. The pallet rack openings are 100-inches wide, 48-inches deep and 60-inches high, where each pallet opening can store two pallets. The horizontal and vertical rack members are 4 inches, with a flue space of 6 inches between back-to-back pallet positions. Supporting columns in the warehouse are 54 feet apart, and the trucks for put-away and order-picking operations have a horizontal travel speed of 264 fpm and a vertical travel speed of 44 fpm.

The activity profile is such that 20% of the SKUs account for 80% of the picks. Thus, the size of the forward area (% of SKUs assigned to bottom locations) also affects the activity in the forward area according to the shape of the ABC curve. Likewise, faster-moving SKUs are typically ordered in larger quantities (on a per order basis) than slower-moving SKUs. Table 5.1 lists the parameters for the forward area sizes considered in terms of pick lines, picks per line and number of batches.

For the forward areas, we also list the average number of replenishments required to move pallets from the reserve area to the forward area. For example, 5% of the SKUs in the forward area represents 45% of the daily activity, or 22,500 case picks $(50,000 \times 0.45)$, and with approximately 50 cases per pallet, the number of daily replenishments is 450 (22,500/50). When the forward area is less than 50% of the SKUs, put-away strategies are such that approximately 60 percent of the reserve storage locations reside within the same aisle as their bottom-level forward locations; 80 percent of the reserve locations reside in the aisle of the picking location for 50% or more SKUs in the forward area. This parameter (α) is used in the replenishment models (see Chapter 4 for a detailed description of this parameter).

Table 5.1: Pick Lines and Batches for Forward and Reserve Areas

Forward Area						Reserve Area		
% of SKUs	% of Activity	Lines/ Batch	Picks/ Line	# Batches	# Replens	Lines/ Batch	Picks/ Line	# Batches
5	45	9.7	3.60	643	450	18.2	1.92	786
10	60	10.8	3.24	857	600	18.8	1.86	571
20	80	12.3	2.85	1,143	800	19.6	1.79	286
30	87	13.1	2.67	1,243	870	20.5	1.71	186
40	92	13.8	2.53	1,314	920	21.2	1.65	114
50	94	14.4	2.43	1,343	940	22.3	1.57	86
60	96	15.1	2.32	1,371	960	23.2	1.51	57
70	97	15.7	2.23	1,386	970	24.1	1.45	43
80	98	16.3	2.15	1,400	980	25.0	1.40	29
90	99	16.9	2.07	1,414	990	25.5	1.37	14
100	100	17.5	2.00	1,429	1,000	–	–	–

For forward areas with class-based storage, the put-away, order-picking and replenishment models that we use assume that SKUs are sub-divided into three storage classes. Thus, for any size forward area we designate the top-20% of SKUs included in the forward area as A-items and the next-30% as B-items (remainder as C-items). Table 5.2 lists the percent of activity for each storage class for the various class-based forward areas considered.

Table 5.2: Forward Activity for Class-Based Layouts

% of SKUs	Class-A	Class-B	Class-C	% of SKUs	Class-A	Class-B	Class-C
20	50%	30%	20%	70	75%	17%	8%
30	59%	26%	15%	80	77%	16%	7%
40	64%	24%	12%	90	79%	15%	6%
50	68%	21%	11%	100	80%	15%	5%
60	72%	19%	9%				

5.4.2 Sizing the Pallet Rack Area

Thus far we have provided some of the parameters necessary for the models that will be used to determine the labor requirements for put-away, order-picking and replenishment operations. However, these models also depend on the size of the storage area, namely the aisle length, number of aisles and warehouse width. We use the algorithm in [9] to size a pallet area for a specific shape ratio to meet a target number of pallet positions. The results are given in Table 5.3.

Note that the number of pallet locations available is within a range of the required locations, due to discrete numbers of aisles and racks within aisles. On average, the facilities with 5 levels require an additional 55,000–60,000 ft^2 footprint compared to those with 6 levels. These results hold for all designs where the number of SKUs in the forward area are 50% or less. For larger forward areas, the footprint of the warehouse may require additional sections (as defined by the spacing between adjacent supporting columns) in order to accommodate all of the forward SKUs on the bottom level of pallet rack. Figures 5.4(a) and 5.4(b) illustrate how the pallet rack area grows as the size of the forward area increases for five and six levels of pallet rack (with a shape of 2.0). The additional footprint can be significant when greater than 80% of the SKUs are included in the forward area in this example.

Table 5.3: Sizing Results for Example Warehouse

Levels	Shape	Aisles	Aisle Length (ft)	Pallets Available	Warehouse Width (ft)	Area 1-Sided (ft^2)	Area 2-Sided (ft^2)
5	1.0	30	540.0	36,000	540	324,000	345,600
5	1.5	36	432.0	34,560	648	318,816	344,736
5	2.0	42	378.0	35,280	756	331,128	361,368
5	2.5	48	345.6	36,480	864	350,438	384,998
5	3.0	51	306.0	34,680	918	335,988	372,708
5	3.5	57	293.1	36,480	1,026	362,325	403,365
5	4.0	60	270.0	36,000	1,080	356,400	399,600
6	1.0	27	486.0	34,992	486	265,356	284,796
6	1.5	33	396.0	34,848	594	270,864	294,624
6	2.0	39	351.0	36,504	702	288,522	316,602
6	2.5	42	302.4	33,264	756	273,974	304,214
6	3.0	48	288.0	36,864	864	300,672	335,232
6	3.5	51	262.3	35,496	918	295,858	332,578
6	4.0	54	243.0	34,992	972	294,516	333,396

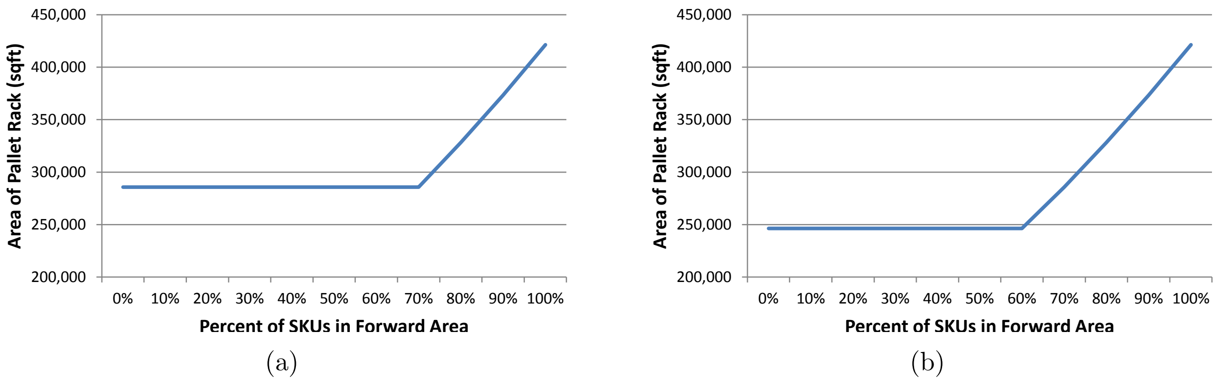


Figure 5.4: Pallet rack area as the forward area grows: (a) 5 levels of rack, shape 2.0; (b) 6 levels of rack, shape 2.0.

5.4.3 Evaluation of Labor Requirements

Now that the pallet rack area has been sized to meet a target number of pallet locations, we use the dimensions of the pallet rack area (width of pallet area, number of aisles, and aisle length) as well as the previously defined parameters (number of lines, picks per line, and batches; activity profile; α value; and horizontal and vertical speeds) to determine the labor requirements using the distance models in Chapter 4. Table 5.4 lists the distance requirements for the designs with 5 levels and with no forward area (top half) and designs with 5% of the SKUs in a forward area with random storage (bottom half).

Table 5.4: Distance Requirements by Operation for Selected Designs

			Reserve Area		Forward Area			
	Put-Away Distance (ft)		Order-Picking Dist (ft)		Order-Picking Dist (ft)		Replen Dist (ft)	
Shape	Horiz	Vert	Horiz	Vert	Horiz	Vert	Horiz	Vert
1.0	1,000.0	21.3	8,485.7	21.3	-	-	-	-
1.5	964.0	21.3	7,428.6	21.3	-	-	-	-
2.0	982.0	21.3	7,004.5	21.3	-	-	-	-
2.5	1,021.6	21.3	6,831.4	21.3	-	-	-	-
3.0	1,018.0	21.3	6,366.5	21.3	-	-	-	-
3.5	1,077.1	21.3	6,433.4	21.3	-	-	-	-
4.0	1,090.0	21.3	6,195.8	21.3	-	-	-	-
1.0	1,000.0	21.3	8,708.4	21.3	1,947.4	0.0	715.2	21.3
1.5	964.0	21.3	7,626.7	21.3	1,575.6	0.0	576.0	21.3
2.0	982.0	21.3	7,191.7	21.3	1,389.8	0.0	506.4	21.3
2.5	1,021.6	21.3	7,013.0	21.3	1,589.3	0.0	491.3	21.3
3.0	1,018.0	21.3	6,531.8	21.3	1,420.7	0.0	438.2	21.3
3.5	1,077.1	21.3	6,598.6	21.3	1,366.0	0.0	420.9	21.3
4.0	1,090.0	21.3	6,351.2	21.3	1,478.8	0.0	406.8	21.3

The distance for picking from all levels of pallet rack (reserve storage) increases when moving to the forward area designs because the top 5% of the SKUs are now located in the bottom-level forward area, resulting in fewer picks per line in the reserve area (and requiring more stops per batch to fill the pallet). Despite the increased travel for reserve batches and the additional travel for replenishments, the overall labor requirement for forward area designs is still less than designs with no forward area due to the reduced travel for batches from the forward area.

To illustrate how the shape of the pallet area affects different operations, we show the total labor hours required for different shapes of the pallet area (for designs with no forward area) for put away and pallet picks in Figure 5.5(a) and for case picking from reserve storage in Figure 5.5(b). For put-away and pallet-pick operations, a shape of 1.5 results in the least travel (Figure 5.5(a)), and the order-picking distance decreases as the shape of the pallet area increases (Figure 5.5(b)); the optimal shapes in the graphs apply to this example problem (see Chapter 4 for a detailed explanation of optimal shape for each operation). Figure 5.6 depicts the travel for each operation for designs with 5 levels of pallet rack, where 10% of the SKUs are in a forward area with random storage. Thus, Figures 5.5–5.6 indicate that the optimal warehouse shape varies by operation. From Figure 5.6, picking from the reserve area dominates the labor, and for this set of designs, labor is minimized for the highest (4.0) shape ratio considered.

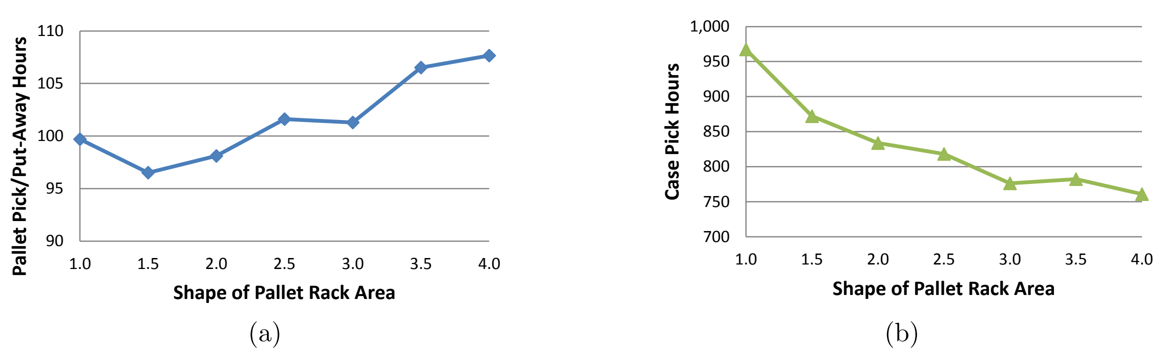


Figure 5.5: Hours required for different pallet area shapes: (a) Put away and pallet picks; (b) Case picks from reserve.

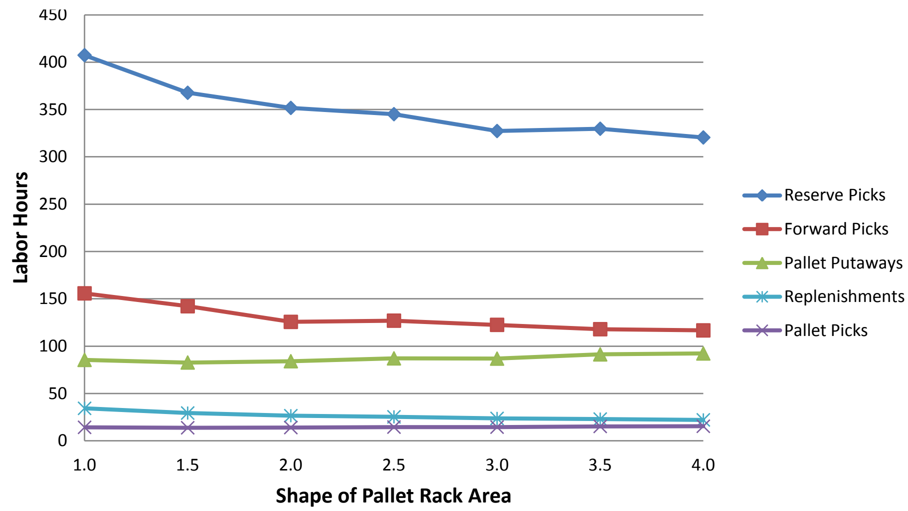


Figure 5.6: Hours required for different pallet area shapes using random-storage forward areas with 20% of SKUs.

From Table 5.4, the designs with a forward area outperform the designs with no forward area; however, designs with different forward area sizes result in different labor requirements. Figure 5.7(a) shows the designs with 5 levels and with random storage in the forward area. From these results, the designs with 20% of the SKUs in the forward area outperform the designs with 5% of the SKUs in the forward area. However, increasing the forward area beyond 20% of the SKUs results in an increase in labor because the size of the forward area is increasing. In addition, adding more SKUs to the forward area results in more replenishment operations. The best designs with a random-storage forward area (20% of the SKUs) outperform the designs with no forward area by 30–37%.

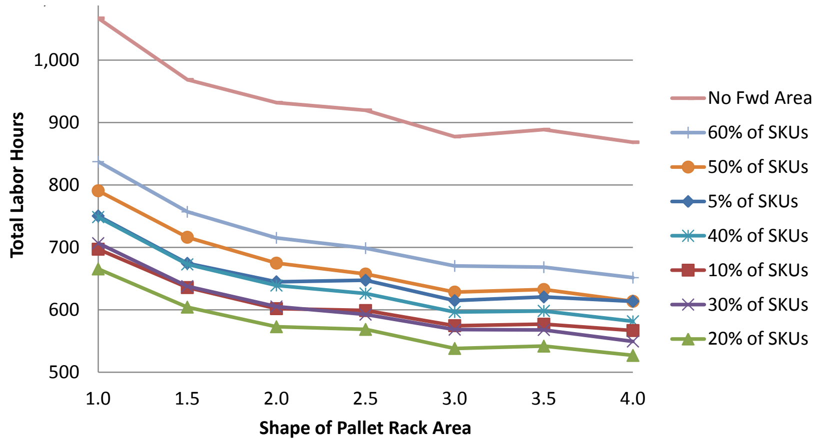
Next, we consider class-based storage in the forward area for the 1-sided and 2-sided layouts. Figures 5.7(b) and 5.7(c) illustrate how the total labor changes as the size of the forward area changes and as the shape of the overall pallet area changes. The designs with the 1-sided layout perform best when the size of the forward area contains approximately 40% of the SKUs, and the 2-sided layout designs perform best when the size of the forward area comprises 20% of the SKUs. The 1-sided layout results in a 20–24% improvement compared to the best forward-area design (20% of SKUs in the forward area) with random storage; the 2-sided layouts perform 7–9% better than the best forward area designs with random storage.

On average, both of the class-based storage designs perform better with more SKUs in the forward area as compared to the designs with random storage in the forward area. This can be attributed to the additional savings that is achieved by further sub-dividing the forward SKUs into classes based on activity. Notice also that there is less variation in the travel distance for the 1-sided layouts as compared to the 2-sided layouts. The 1-sided layout reduces the within-aisle travel by concentrating all of the fastest-moving SKUs at the end of the aisle so that only a portion of the aisle requires travel (using the return routing strategy). The 2-sided layout reduces within-aisle travel by concentrating the fastest-moving SKUs in the centermost aisles of the forward area so that only a subset of the forward aisles requires travel (using the traversal strategy). Thus, more SKUs can be included in the forward areas with class-based storage layouts.

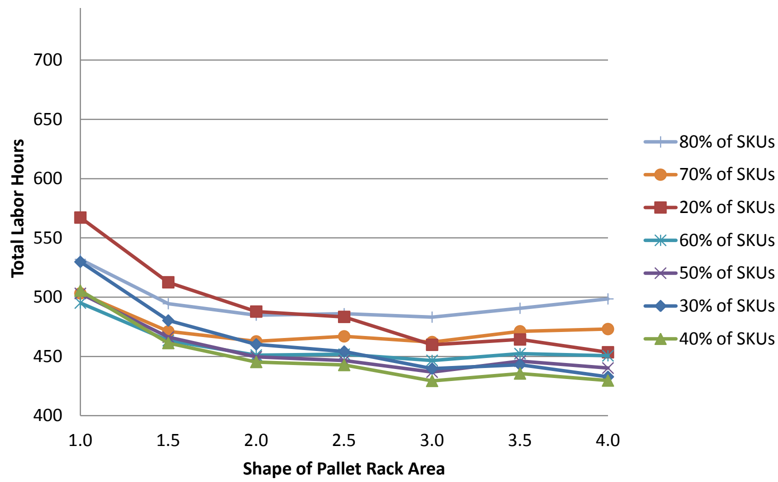
The number of levels of pallet rack affects the vertical travel component of the put-away, pallet-picking and replenishment operations, as more levels results in more vertical travel. However, increasing the levels of pallet rack also reduces the footprint of the pallet rack area, resulting in less horizontal travel. Figure 5.8 illustrates how the total labor changes as the number of levels of pallet rack increases for designs with a random-storage forward area comprising the top 20% of the SKUs and for the pallet area shapes considered.

For this subset of designs, higher levels of pallet rack result in less labor (as well as a smaller building footprint). Thus, the smaller storage area reduces horizontal travel enough to compensate for the additional vertical travel. However, higher levels may not always yield the best performance, as warehouse parameters play an important role in determining the optimal number of levels. For example, if the number of cases per pallet is low, more replenishments (and pallet put aways) may be required, resulting in more vertical travel.

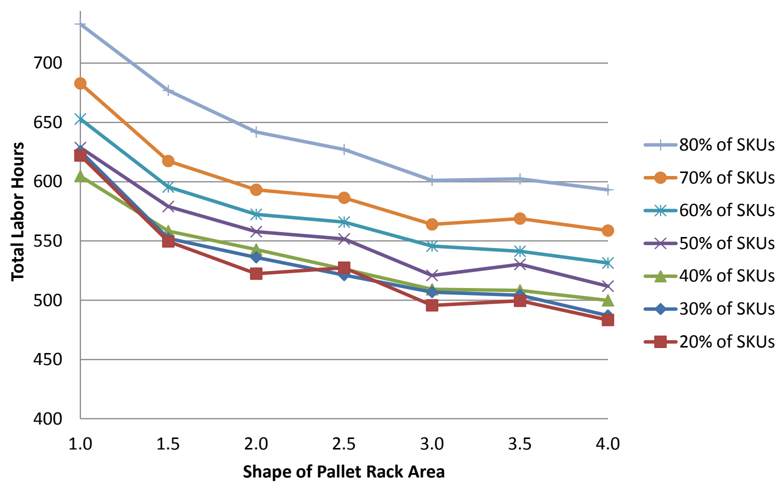
From a labor perspective, the overall best design for our example warehouse places the top 50% of the SKUs in a forward area with the 1-sided layout and utilizes a shape ratio of 2.5 and 6 levels of pallet rack. Clearly, other factors play a role in determining the best



(a)



(b)



(c)

Figure 5.7: Hours required for different pallet area shapes for: (a) random storage (b) 1-sided layout; (c) 2-sided layout.

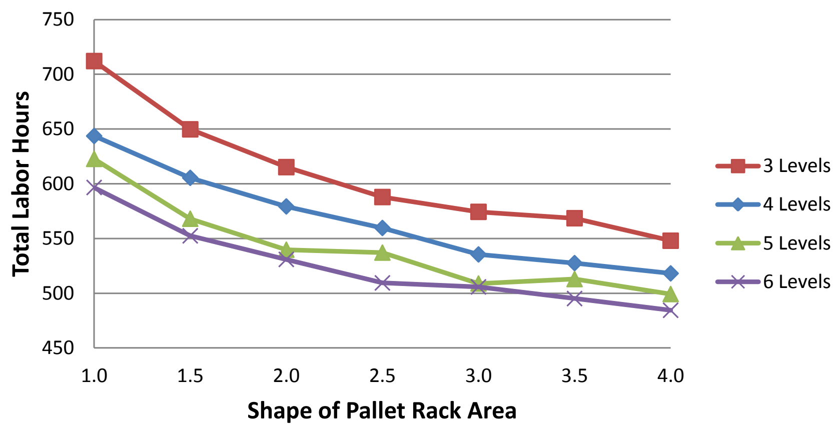


Figure 5.8: Hours required for different pallet area shapes and levels of pallet rack, with random storage in the forward area and 20% of SKUs.

design, including building cost, dock door availability, and congestion, to name a few, but a comprehensive set of analytical models is useful for comparing various designs and for evaluating how changing parameters affects overall design performance.

Figure 5.9 illustrates the labor requirements in hours for all 476 designs, where designs are ordered from most to least hours required. As illustrated in the figure, the designs with the 1-sided, class-based layout perform the best for the example warehouse, and the worst designs are those with no forward area.

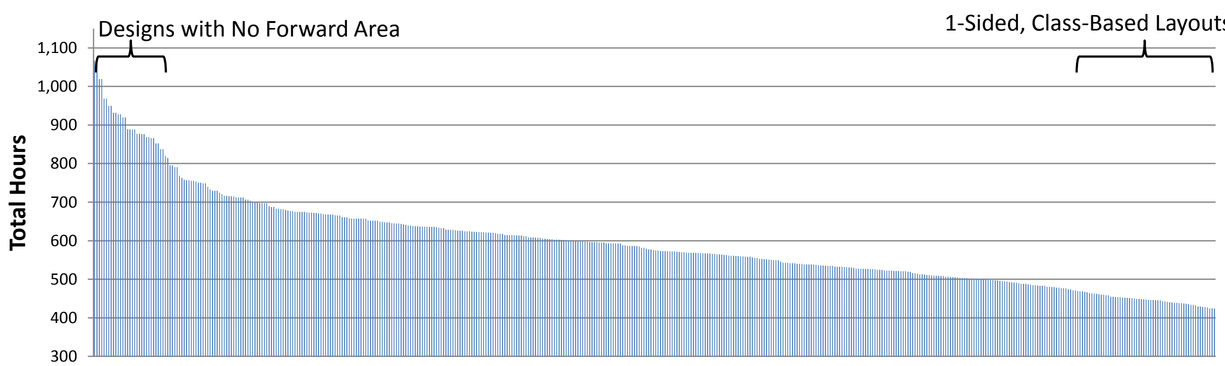


Figure 5.9: Total labor hours (designs ordered from most to least hours required).

5.4.4 Summary of Results

For this example, labor can be reduced by as much as 37% by placing fast-moving items in a forward area. Further improvements can be seen with designs that utilize class-based storage in the forward area. The 1-sided layout outperformed the 2-sided layout in terms of labor, and the 1-sided designs yielded improvements of 20–24% over the best random-storage forward area designs. The number of storage levels impacts the horizontal and vertical travel, as well as the building footprint. A higher number of levels performed the best for the warehouse parameters considered.

Students from a facility logistics class were asked to use this methodology in a case study for designing a warehouse. The students were placed in groups to form design teams. All groups were able to quantify the benefit of using a forward area for random and class-based storage. In addition, students experimented with design parameters that otherwise would not have been considered (number of warehouse levels, many shapes of warehouses, etc.). Using this methodology, the students generated significantly more potential designs as compared to previous classes.

5.5. Conclusions and Future Research

Overall warehouse design is complex due to the number of designs as well as the interrelationships that exist among design parameters and functional areas. Analytical models can be useful in determining a base design that can be further analyzed and optimized. For the example warehouse presented, the operational costs for the designs considered varied by as much as 152% $((1,067 - 424)/424)$ when considering the best design (424 hours) and worst design (1,067 hours). Thus, having a means to compare designs can result in significant savings in labor and may help designers gain a better understanding of how changing various design parameters affects overall performance.

This research can be extended to include a broader range of warehouses such as warehouses that have piece-picking operations and/or warehouses that include automation. Existing models can be incorporated into this overall design methodology and additional models can be developed for the sizing and labor quantification necessary for such an approach. In addition, research on how to search the design space would benefit the designer in two ways: 1) instead of feeling the need to evaluate each of the hundreds of designs, the designer could evaluate the appropriate subset, which would save time; 2) this would allow the designer to incorporate non-quantifiable factors into the optimization process. Given that there are currently no comprehensive models to assist designers in quantifying labor requirements in order to compare designs, extending this research would benefit designers in the initial design phase.

Acknowledgements

This research was supported, in part, by the National Science Foundation Industry-University Cooperative Research Center for Excellence in Logistics and Distribution (CELDi). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Bibliography

[1] Apple, J. M., Meller, R. D., and White, J. A., "Empirically-Based Warehouse Design: Can Academics Accept Such an Approach?," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[2] Ashayeri, J., and Gelders, L., "Warehouse Design Optimization," *European Journal of Operational Research*, 21, 285–294 (1985).

[3] Baker, P., and Canessa, M., "Warehouse Design: A Structured Approach," *European Journal of Operational Research*, 193, 245–258 (2009).

[4] Goetschalckx, M., McGinnis, L. F., and Sharp, G., "Modeling Foundations for Formal Warehouse Design," in *Proceedings of the 2008 International Material Handling Research Colloquium*, Dortmund, Germany (2008).

[5] Gray, A. E., Karmarkar, U. S., and Seidmann, A., "Design and Operation of an Order-Consolidation Warehouse: Models and Application," *European Journal of Operational Research*, 58, 14–36 (1992).

[6] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Operation: A Comprehensive Review," *European Journal of Operational Research*, 177, 1–21 (2007).

[7] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[8] McGinnis, L. F., Goetschalckx, M., Sharp, G., Bodner, D., and Govindaraj, T., "Rethinking Warehouse Design Research," in *Proceedings of the 2000 International Material Handling Research Colloquium*, Charlotte, NC (2000).

[9] Meller, R. D., and Thomas, L. M., "Optimizing Distribution Center Configuration: A Practical View of a Multi-Objective Problem," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[10] Park, Y. H., and Webster, D. B., "Modelling of Three-Dimensional Warehouse Systems," *International Journal of Production Research*, 27, 6, 985–1003 (1989).

[11] Rouwenhorst, B., Reuter, B., Stockrahm, V., van Houtum, G., Mantel, R., and Zijm, W., "Warehouse Design and Control: Framework and Literature Review," *European Journal of Operational Research*, 122, 515–533 (2000).

[12] Sharp, G., Goetschalckx, M., and McGinnis, L. F., "A Systematic Warehouse Design Workflow," in *Proceedings of the 2008 International Material Handling Research Colloquium*, Dortmund, Germany (2008).

[13] van den Berg, J. P., and Gademann, A. J. R. M. N., "A Literature Survey on Planning and Control of Warehousing Systems," *IIE Transactions on Scheduling & Logistics*, 31, 8, 751–762 (1999).

[14] Yoon, C. S., and Sharp, G. P., "A Structured Procedure for Analysis and Design of Order Pick Systems," *IIE Transactions*, 28, 379–389 (1994).

A. Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Russell D. Meller, Professor and Holder of the Hefley Professorship

DATE: June 18, 2013

SUBJECT: Certification of student effort

I certify that greater than 51% of the work conducted for this chapter entitled, "Contribution 2: A Paper on, 'Using Analytical Models to Assess Performance in Overall Warehouse Design'," was conducted by Lisa M. Thomas.

4207 Bell Engineering Center • Fayetteville, Arkansas 72701 • 479-575-3156 • 479-575-8431 Fax

B. Release from the Institute of Industrial Engineers

The student, Lisa M. Thomas, has permission to include a paper in her dissertation with acknowledgment to the *Proceedings of the 2013 Industrial and Systems Engineering Research Conference*. The title of the paper is "Using Analytical Models to Assess Performance in Overall Warehouse Design" and it was co-authored by her advisor, Dr. Russell D. Meller.

The Institute of Industrial Engineers approves of the inclusion of the paper in this dissertation based on an e-mail dated June 19, 2013.

From: Monica Elliott <melliott@iienet.org>
Date: June 19, 2013 8:13:21 AM CDT
To: "rmeller@uark.edu" <rmeller@uark.edu>
Subject: RE: Request for a student

Hi Russ,

Your student has permission to use her papers that appeared in the ISERC 2013 proceedings in her dissertation project. She should include the acknowledgment/reference that it appeared in the proceedings as you have indicated. We do not provide formal letters of permission, so please consider this email as permission.

Best regards,
Monica

MONICA ELLIOTT
Director of Communications | Institute of Industrial Engineers
3577 Parkway Lane, Suite 200 | Norcross, GA 30092
(770) 449-0461, x116 | melliott@iienet.org
IIE on Facebook | IIE on LinkedIn | IIE (iienet) on Twitter

IIE Engineering Lean & Six Sigma Conference 2013
Sept. 23-25 | Atlanta | www.iienet.org/leansixsigma

Chapter 6

Contribution 3: A Paper on, "Using Empirical Data to Assess Performance in Overall Warehouse Design"

Abstract: From an academic perspective, industry practitioners overly rely on a single observation of performance to design warehouses versus using analytical models that can be scaled to estimate the performance in other settings. But what if multiple empirical observations were combined in a way to aid in warehouse design? Could the set of empirical observations be used effectively to design warehouses? We propose an approach for assessing the operational performance of a given design that uses empirical data in the form of lookup tables. We demonstrate this approach for a manual, case-picking warehouse by populating tables based on existing analytical models and using the tables to quantify the space and labor requirements for various designs. We begin with basic designs and move to more complex designs through the use of functional flow networks. Issues related to how to use the populated tables will be explored.

6.1. Introduction

Mathematical tables have existed for hundreds of years, serving as an impetus to scientific advancements by allowing complex information to be represented in a two-dimensional format [6] . Through the decades engineering disciplines have utilized tables such as compound interest tables, statistical tables, steam tables, and specific gravity tables, to name a few. In fact, the *CRC Handbook of Engineering Tables* includes 450 tables and figures with important data that is widely used by engineering practitioners [4]. Dynamic tables in the form of spreadsheets are ever present today. Accordingly, Campbell-Kelly et al. [6] note an interesting paradox regarding the introduction of computers: "On the one hand computers have been the death of the printed table-as-calculating-aid, but conversely computerized spreadsheets have given new and vigorous life to the still ubiquitous table-as-data-presentation format."

Tables can be used to show both empirical and derived data. As improved practices and new order-fulfillment technologies emerge, analytical models do not always exist, but multiple empirical observations can be combined to aid in warehouse design. The ability to relate design parameters to performance measures using empirical data is invaluable in complex and incompletely modeled situations [7]. In this paper we show how empirical data in the form of lookup tables can be used to assess various warehouse designs in terms of space and labor requirements. For demonstration purposes, we use existing analytical models for a manual, case-picking warehouse to populate lookup tables for space and labor requirements.

We use functional flow networks (FFNs) to drive the design process, beginning with the most basic design (pallet rack with random storage and no forward area) and moving to more complex designs (forward areas of different sizes). A functional flow network consists of a series of nodes and arcs, where nodes represent the functional areas in the warehouse and arcs represent the flow of product from one functional area to another. Each node in the FFN is sized to accommodate a given number of storage locations, and each arc is translated into labor requirements. The sizing and labor conversions are determined based on values in the lookup tables. The goal of this paper is to illustrate how lookup tables can be used to obtain values for sizing and distance requirements when designing a warehouse. In an example, we compare design results in using lookup tables versus applying analytical models directly.

6.2. Literature Review

Two research papers consider empirical data in tables for warehouse design. Bozer and Sharp [5] developed a simulation model for order accumulation and sortation systems and presented the results in tabular form. Different tables were constructed for various design considerations including the number of lanes, lane capacity, throughput capacity and whether or not recirculation occurs. The authors used the tables to determine the effect of the induction capacity and the number of lanes on throughput capacity.

Apple et al. [1] proposed an empirically based warehouse design methodology that would utilize quantitative matrix solution guides with numerical equivalencies related to labor, space and capital investment for various ABC ratings and operational parameters. Pareto charts are suggested to subdivide the warehouse activities in terms of storage and activity for each handling unit, and FFNs are used to represent each conceptual design. Each functional area is then sized (using available tools), and trial block layouts are developed that seek to minimize handling distances. Finally, product rows are synchronized and connecting processes, slotting, and zoning/batching procedures are developed in order to estimate material handling and labor costs. The authors acknowledge that in order to implement such a methodology, standardized definitions for process descriptions must be developed and quantitative matrix solution tables must be populated.

Research in the area of overall warehouse design is limited [14] [13] [9], and no comprehensive synthesis of models and techniques for overall warehouse design has been developed [13] [9]. Thus, a simple, validated model that provides results in order to direct the overall design process would be a valuable research contribution [10]. Currently, simulation is the most common means for assessing warehouse performance in both research and industry [10].

A survey paper by Rouwenhorst et al. [13] characterizes warehouses in terms of processes, resources and organization, and the authors classify design problems at the strategic, tactical and operational levels. The authors contend that design decisions at the strategic and tactical level are often interrelated and require joint consideration. Van den Berg and Gademann [14] present a hierarchy of warehousing decisions for operational planning and control and outline the methods and models that have appeared in the literature for each area.

Most research papers that consider overall warehouse design generally provide solution procedures with a general, qualitative design framework. Papers that provide a quantitative comparison of design alternatives include detailed models that require an extensive number of input parameters and are not general enough to apply to a broad range of warehouses. First, we present the research papers that include general design frameworks.

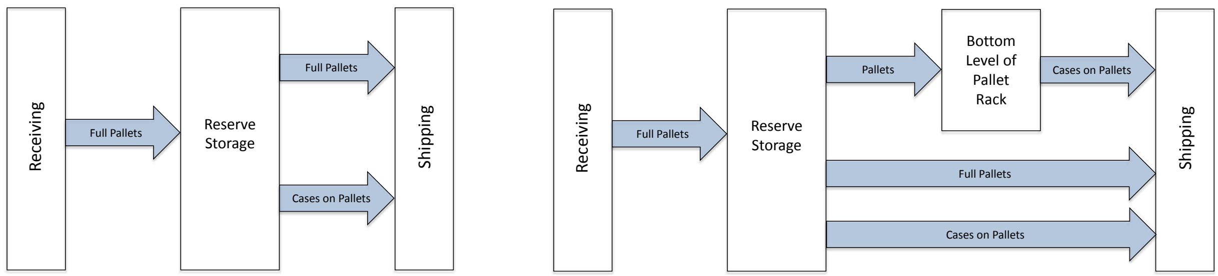
Baker and Canessa [3] compared research papers in the area of overall warehouse design, and formulated a general framework of steps in order to assist practitioners and researchers in a more comprehensive warehouse design methodology. Ashayeri and Gelders [2] categorized solution procedures in warehousing literature as analytical, simulation or heuristic and suggested a two-step technique for system design that uses analytical models to prune the decision space and simulation to capture the dynamic aspects of the simplified analytical models. Yoon and Sharp [15] presented a systematic design procedure for order-picking systems with functional areas for order picking to assist designers in determining alternatives for order-picking configurations. The structured design procedure occurs in three stages including: analysis of order transaction data, selection of equipment types and operating strategies, and evaluation in terms of a performance analysis for each subsystem. No specific models are presented for throughput calculations for each subsystem, though the authors reference previous research for these calculations.

Along with [1], two other papers present design methodologies with quantitative solution procedures for comparing design alternatives. Gray et al. [8] developed a model for overall design with the objective of minimizing initial incremental costs and operating costs including labor and inventory holding costs. The authors propose a hierarchical decomposition of the problem in order to reduce the complexity of the formulation. Company-specific analytical models were developed to prune the decision space, and simulation was used to evaluate the alternative designs and to validate the analytical models. The solution procedure involved iteration among the three decision levels, and the authors estimated a labor savings of close to 50% with the new design. Park and Webster [12] formulated a design model for a unit-load warehouse and developed analytical models to determine land, building, equipment, labor and operating costs. A case study was presented that considered three alternative designs: a fully automated AS/RS, narrow-aisle lift trucks, or counter-balanced lift trucks. The authors acknowledge that obtaining cost and model parameters for individual firms would require considerable effort.

6.3. Design Assumptions

In this paper, we construct tables for warehouse design based on a manual, case-picking warehouse where items are received in pallet quantities and stored in pallet rack. For simplicity, we assume that cases are picked onto pallets and then floor loaded into a trailer, where no palletizing is required and sorting is negligible. In the pallet rack area, we consider traditional aisles that are orthogonal to the side(s) of the warehouse with dock doors, where each storage location is the same size and contains the same number of cases per pallet.

For designs with a forward area for picking fast-moving items, the forward locations are concentrated in the centermost aisles on the bottom level of pallet rack, and the reserve storage locations are stored in the upper levels of pallet rack. Replenishments from reserve storage locations to forward locations are implemented in pallet quantities. In this paper, we focus on designs with random storage in the forward area; however, our methodology could be applied to designs with class-based storage as well.



(a) (b)

Figure 6.1: Functional flow networks: (a) basic FFN with picks from all levels of pallet rack; (b) FFN with co-located forward area, with picking from the reserve and forward areas and with replenishments in pallet quantities to the forward area.

6.4. Design Methodology

In our design methodology, we begin with a basic design (pallet rack with no forward area) as depicted in Figure 6.1(a), where picking occurs over the entire pallet rack area and where each location has an equal probability of containing a pick. Next, we consider a co-located forward area as illustrated in Figure 6.1(b).

In order to determine the labor requirements for the FFNs in Figures 6.1(a) and 6.1(b), we must first size the functional area (pallet rack area) and then convert the product flow across the arcs into labor requirements.

We consider a range of SKUs in the forward area (0%, 5%, 10%, 20%, 30%, 40% and 50% of the SKUs, 7 possible sizes). In addition to the size of the forward area, we consider designs with five or six levels of pallet rack, seven width-to-depth shape ratios for the pallet area, and doors on one or both sides of the facility. We evaluate a total of 196 designs ($7 \times 2 \times 7 \times 2 = 196$). Given that the expected vertical travel is one of two values (corresponding to 5 or 6 levels), we use the vertical travel model in Chapter 4 for put-away, order-picking from all levels, and replenishment operations.

Table 6.1: Pick Lines and Batches for Forward and Reserve Areas

Forward Area						Reserve Area		
% of SKUs	% of Activity	Lines/ Batch	Picks/ Line	# Batches	# Replens	Lines/ Batch	Picks/ Line	# Batches
5	45	9.7	3.60	643	450	18.2	1.92	786
10	60	10.8	3.24	857	600	18.8	1.86	571
20	80	12.3	2.85	1,143	800	19.6	1.79	286
30	87	13.1	2.67	1,243	870	20.5	1.71	186
40	92	13.8	2.53	1,314	920	21.2	1.65	114
50	94	14.4	2.43	1,343	940	22.3	1.57	86
60	96	15.1	2.32	1,371	960	23.2	1.51	57
70	97	15.7	2.23	1,386	970	24.1	1.45	43
80	98	16.3	2.15	1,400	980	25.0	1.40	29
90	99	16.9	2.07	1,414	990	25.5	1.37	14
100	100	17.5	2.00	1,429	1,000	–	–	–

6.5. Example

Consider a company with a warehouse that picks in pallet and case quantities and requires approximately 35,000 pallet positions to ensure an adequate supply of an inventory for its 10,000 SKUs. On average, incoming pallets contain 50 cases and the warehouse receives 1,200 pallets of product per day. On a given work day orders entail 200 pallet picks and 50,000 case picks. A typical batch of orders has approximately 17.5 lines and 2.0 picks per line, requiring approximately 1,429 ($50,000/(17.5 \times 2.0)$) order-picking batches per day. The warehouse requires a staging area that is 40-feet deep, storage aisles that are 9.5-feet deep and end cross-aisles that are 10-feet deep. The pallet rack openings are 100-inches wide, 48-inches deep and 60-inches high, where each pallet opening can store two pallets. The horizontal and vertical rack members are 4 inches, with a flue space of 6 inches between back-to-back pallet positions. Supporting columns in the warehouse are 54-feet apart, and the trucks for put-away and order-picking operations have a horizontal travel speed of 264 fpm and a vertical travel speed of 44 fpm. Five or six levels of pallet rack are viable options for the warehouse.

The activity profile is such that 20% of the SKUs account for 80% of the picks. Thus, the size of the forward area (% of SKUs assigned to bottom locations) also affects the activity in the forward area according to the shape of the ABC curve. Likewise, the top percentage of SKUs accounts for more picks per line than the slower moving items. Table 6.1 lists the parameters for the forward area sizes considered in terms of pick lines, picks per line and number of batches. For the forward areas, we also list the average number of replenishments required to move pallets from the reserve area to the forward area. For example, 5% of the SKUs in the forward area represents 45% of the activity or 22,500 case picks ($50,000 \times 0.45$), and with approximately 50 cases per pallet, the number of replenishments is 450 (22,500/50). When the forward area is less than 50% of the SKUs, put-away strategies are such that approximately 60 percent of the reserve storage locations reside within the same aisle as their bottom-level forward locations; 80 percent of the reserve locations reside in the aisle of the picking location for 50% or more SKUs in the forward area. This parameter (α) is used in the replenishment models (see Chapter 4 for a detailed description of this parameter).

6.5.1 Empirical Data

We populate a table for sizing the pallet rack area by using an existing algorithm [11] that determines the required dimensions (for a given width-to-depth shape ratio) to meet a target number of pallet positions. Next, we create tables to determine the required distances for put away, order picking and replenishment for a range of parameters using the models in Chapter 4.

Sizing Tables

For our example warehouse, approximately 35,000 pallet locations are needed, where pallet rack levels of five and six are under consideration. Thus, the target number of bottom-level pallets is 7,000 (35,000/5 levels) or 5,834 (35,000/6 levels). In addition, the warehouse shape can vary from 1.0–4.0. We construct a table with pairs of a bottom-level pallet value (three levels) and warehouse shape (in 0.5 increments) and present the results in Table 6.2.

Table 6.2: Sizing Table for a Targeted Number of Bottom-Level Pallets

	6,000 Bottom-Level Pallets			6,500 Bottom-Level Pallets			7,000 Bottom-Level Pallets		
Shape	Dims* (ft)	Aisles	Pallets	Dims* (ft)	Aisles	Pallets	Dims* (ft)	Aisles	Pallets
1.0	486 x 486	27	5,832	540 x 540	30	7,200	540 x 540	30	7,200
1.5	594 x 396	33	5,808	648 x 432	36	6,912	648 x 432	36	6,912
2.0	702 x 351	39	6,084	756 x 378	42	7,056	756 x 378	42	7,056
2.5	810 x 324	45	6,480	810 x 324	45	6,480	864 x 346	48	7,296
3.0	864 x 288	48	6,144	918 x 306	51	6,936	918 x 306	51	6,936
3.5	918 x 263	51	5,916	972 x 278	54	6,480	1026 x 294	57	7,296
4.0	972 x 243	54	5,832	1026 x 257	57	6,384	1080 x 270	60	7,200

* Dimensions correspond to column spacing of 54 ft, 18-ft center-to-center aisles, and 100-inch pallet openings with 4-inch horizontal rack members and two pallets per opening.

Note that the number of pallet locations available varies due to discrete numbers of aisles and racks within aisles. Also, the aisle length is defined as the second dimension of the pallet area (columns 2, 5 and 8 in Table 6.2). For designs with six levels of pallet rack, the maximum number of forward locations is approximately 6,000; thus, including more than 60 percent of the 10,000 SKUs in the forward area would require a larger warehouse. Similarly, a larger warehouse is necessary for designs with five levels of pallet rack and 80 percent or more of the SKUs in the forward area.

The results from the sizing tables can be adjusted by adding or removing one or more sections of pallet rack openings (thus obtaining a shape ratio not explicitly listed in the table). This increased precision in the sizing of the warehouse may be advantageous in providing table values of the number of pallets that are closer to the targeted value.

Labor Tables

We use the analytical models for put away, order picking and replenishment in Chapter 4 to construct tables that contain travel distance requirements for the design parameters under consideration. Table 6.3 lists horizontal put-away travel, Tables 6.4 and 6.5 list horizontal travel for order picking from the forward area and from all levels of pallet rack, and Table 6.6 lists horizontal travel for replenishment. We list only partial tables (eg., put-away and order-picking travel distances truncated at 52 aisles and only a subset of possible aisle lengths) in order to be succinct.

Table 6.3: Horizontal Two-Way Travel for Put Away with Random Storage*

Aisle	Number of Aisles															
Length	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52
240	604	628	652	676	700	724	748	772	796	820	844	868	892	916	940	964
260	624	648	672	696	720	744	768	792	816	840	864	888	912	936	960	984
280	644	668	692	716	740	764	788	812	836	860	884	908	932	956	980	1004
300	664	688	712	736	760	784	808	832	856	880	904	928	952	976	1000	1024
320	684	708	732	756	780	804	828	852	876	900	924	948	972	996	1020	1044
340	704	728	752	776	800	824	848	872	896	920	944	968	992	1016	1040	1064
360	724	748	772	796	820	844	868	892	916	940	964	988	1012	1036	1060	1084
380	744	768	792	816	840	864	888	912	936	960	984	1008	1032	1056	1080	1104
400	764	788	812	836	860	884	908	932	956	980	1004	1028	1052	1076	1100	1124
420	784	808	832	856	880	904	928	952	976	1000	1024	1048	1072	1096	1120	1144
440	804	828	852	876	900	924	948	972	996	1020	1044	1068	1092	1116	1140	1164
460	824	848	872	896	920	944	968	992	1016	1040	1064	1088	1112	1136	1160	1184
480	844	868	892	916	940	964	988	1012	1036	1060	1084	1108	1132	1156	1180	1204
500	864	888	912	936	960	984	1008	1032	1056	1080	1104	1128	1152	1176	1200	1224

* Put-away distances in feet include two-way travel across a 40-ft staging area and 10-ft end cross-aisle, and center-to-center aisles are 18 feet.

Table 6.4: Horizontal Travel for Picking from Reserve Storage*

Aisle	Pick	Number of Aisles															
Length	Lines	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52
240	17	3712	3857	3991	4118	4237	4351	4459	4564	4665	4762	4857	4949	5039	5127	5214	5299
	18	3825	3978	4120	4253	4378	4497	4611	4720	4825	4926	5024	5120	5213	5305	5394	5481
	19	3933	4094	4243	4382	4514	4638	4757	4871	4980	5085	5188	5287	5383	5478	5570	5660
	20	4036	4205	4361	4507	4645	4775	4899	5017	5131	5240	5346	5449	5549	5646	5741	5834
	21	4134	4311	4475	4627	4771	4907	5036	5159	5277	5391	5501	5607	5710	5811	5909	6005
	22	4227	4412	4583	4743	4893	5034	5169	5297	5419	5537	5651	5761	5868	5972	6073	6172
	23	4316	4509	4688	4854	5010	5158	5297	5430	5558	5680	5798	5911	6022	6129	6233	6335
	24	4401	4602	4788	4962	5124	5277	5422	5560	5692	5819	5940	6058	6172	6282	6390	6494
	25	4482	4691	4885	5065	5234	5392	5543	5686	5823	5954	6079	6201	6318	6432	6543	6650
	26	4559	4776	4977	5164	5339	5504	5660	5808	5950	6085	6215	6340	6461	6579	6692	6803
260	17	3963	4114	4254	4386	4510	4628	4740	4848	4952	5052	5149	5244	5336	5426	5514	5601
	18	4085	4245	4393	4532	4662	4786	4903	5016	5124	5229	5330	5428	5524	5617	5709	5798
	19	4201	4370	4526	4672	4809	4938	5061	5179	5292	5401	5506	5608	5707	5804	5898	5991
	20	4312	4490	4654	4807	4950	5086	5214	5337	5455	5568	5677	5783	5886	5986	6084	6179
	21	4418	4604	4776	4936	5087	5228	5363	5491	5613	5731	5844	5954	6060	6164	6265	6363
	22	4519	4714	4894	5061	5218	5366	5506	5639	5767	5889	6007	6120	6231	6338	6442	6543
	23	4615	4819	5007	5182	5345	5499	5645	5784	5916	6043	6165	6283	6397	6507	6615	6719
	24	4707	4919	5115	5298	5468	5628	5780	5924	6061	6193	6319	6441	6559	6673	6784	6892

	25	4794	5016	5220	5409	5586	5753	5911	6060	6202	6339	6470	6595	6717	6835	6949	7060
	26	4878	5108	5320	5517	5701	5874	6037	6192	6340	6481	6616	6746	6872	6993	7111	7225
280	17	4213	4371	4518	4654	4783	4905	5021	5132	5239	5342	5441	5538	5633	5725	5815	5903
	18	4344	4512	4666	4811	4946	5074	5196	5312	5424	5532	5636	5737	5835	5930	6024	6115
	19	4470	4646	4809	4961	5104	5238	5366	5488	5604	5716	5825	5930	6031	6131	6227	6322
	20	4589	4775	4946	5106	5256	5397	5530	5657	5779	5896	6009	6118	6223	6326	6426	6524
	21	4703	4898	5078	5246	5402	5550	5689	5822	5949	6071	6188	6301	6411	6517	6621	6721
	22	4811	5016	5205	5380	5544	5698	5844	5982	6114	6241	6362	6480	6593	6703	6810	6915
	23	4914	5129	5326	5509	5680	5841	5993	6137	6275	6406	6532	6654	6772	6886	6996	7104
	24	5013	5237	5443	5634	5812	5980	6138	6288	6431	6567	6698	6824	6946	7064	7178	7289
	25	5107	5340	5555	5754	5939	6114	6278	6434	6582	6724	6860	6990	7116	7238	7356	7470
	26	5197	5439	5662	5869	6062	6243	6414	6576	6730	6877	7017	7152	7282	7408	7530	7648
300	17	4464	4628	4781	4922	5056	5182	5301	5416	5526	5631	5734	5833	5930	6024	6116	6206
	18	4604	4779	4940	5090	5230	5363	5489	5609	5724	5835	5941	6045	6145	6243	6338	6432
	19	4738	4922	5093	5251	5399	5538	5670	5796	5916	6032	6143	6251	6355	6457	6556	6653
	20	4865	5060	5239	5406	5561	5707	5846	5977	6103	6224	6340	6452	6561	6666	6769	6869
	21	4987	5192	5380	5555	5718	5871	6016	6154	6285	6411	6531	6648	6761	6870	6976	7080
	22	5103	5318	5515	5698	5869	6029	6181	6325	6461	6592	6718	6839	6956	7069	7179	7286
	23	5213	5438	5645	5837	6015	6183	6341	6490	6633	6769	6900	7026	7147	7264	7378	7489
	24	5319	5554	5770	5970	6156	6331	6496	6652	6800	6941	7077	7207	7333	7454	7572	7686
	25	5419	5664	5889	6098	6292	6474	6646	6808	6962	7109	7250	7385	7515	7641	7762	7880

	26	5515	5770	6005	6222	6424	6613	6791	6960	7120	7272	7418	7558	7693	7823	7948	8070

* Order-picking distances are in feet and center-to-center aisles are assumed to be 18 feet.

Table 6.5: Horizontal Travel for Picking from a Forward Area*

Aisle	Pick	Number of Aisles															
Length	Lines	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
240	9	657	1125	1456	1696	1882	2034	2164	2278	2382	2478	2568	2653	2734	2812	2888	2962
	10	659	1145	1506	1773	1981	2151	2295	2422	2536	2641	2739	2831	2918	3002	3083	3161
	11	660	1160	1548	1840	2069	2257	2417	2556	2681	2795	2900	2999	3093	3183	3269	3352
	12	661	1172	1583	1899	2149	2354	2529	2681	2816	2940	3054	3160	3260	3355	3447	3535
	13	662	1181	1612	1950	2220	2443	2632	2797	2944	3077	3199	3313	3419	3521	3618	3710
	14	663	1188	1636	1995	2285	2524	2728	2906	3064	3206	3337	3458	3572	3679	3782	3879
	15	663	1194	1656	2034	2342	2599	2817	3008	3177	3329	3468	3597	3718	3831	3939	4042
	16	664	1198	1673	2068	2394	2667	2900	3103	3283	3445	3593	3730	3858	3977	4091	4199
	17	664	1201	1688	2099	2441	2729	2976	3192	3383	3555	3712	3857	3991	4118	4237	4351
	18	665	1204	1700	2125	2483	2786	3047	3275	3478	3660	3825	3978	4120	4253	4378	4497
260	9	707	1209	1563	1818	2014	2174	2310	2429	2537	2636	2728	2815	2899	2979	3056	3131
	10	709	1230	1617	1901	2121	2300	2452	2584	2703	2812	2912	3007	3097	3183	3265	3345
	11	710	1247	1662	1973	2217	2415	2583	2729	2859	2977	3087	3189	3285	3377	3465	3551
	12	711	1260	1699	2036	2302	2520	2704	2863	3005	3134	3252	3362	3465	3563	3657	3747
	13	712	1269	1730	2092	2380	2616	2816	2989	3143	3281	3409	3527	3637	3742	3841	3937
	14	713	1277	1757	2140	2449	2704	2919	3106	3272	3421	3558	3684	3802	3913	4018	4119
	15	713	1283	1779	2182	2511	2784	3015	3216	3394	3554	3699	3834	3959	4077	4188	4295
	16	714	1287	1797	2220	2567	2857	3104	3319	3509	3679	3834	3977	4110	4235	4352	4464

	17	714	1291	1812	2252	2618	2925	3187	3415	3617	3798	3963	4114	4254	4386	4510	4628
	18	715	1293	1825	2280	2663	2986	3263	3505	3719	3911	4085	4245	4393	4532	4662	4786
280	9	757	1293	1669	1940	2147	2314	2456	2580	2692	2794	2889	2978	3063	3145	3224	3300
	10	759	1316	1727	2029	2261	2450	2608	2746	2870	2982	3086	3183	3275	3363	3448	3529
	11	760	1334	1775	2106	2364	2573	2749	2901	3037	3160	3273	3378	3478	3572	3662	3749
	12	761	1347	1816	2174	2456	2685	2879	3046	3194	3327	3450	3564	3670	3771	3868	3960
	13	762	1358	1849	2234	2539	2788	2999	3181	3341	3486	3618	3741	3855	3963	4065	4163
	14	763	1365	1877	2285	2613	2883	3110	3307	3480	3636	3778	3909	4031	4146	4255	4359
	15	763	1372	1901	2331	2680	2969	3213	3425	3611	3778	3930	4070	4200	4322	4438	4547
	16	764	1376	1920	2371	2740	3048	3309	3535	3734	3913	4075	4224	4362	4492	4613	4729
	17	764	1380	1937	2405	2794	3120	3397	3638	3851	4041	4213	4371	4518	4654	4783	4905
	18	765	1383	1951	2436	2843	3186	3480	3735	3960	4162	4344	4512	4666	4811	4946	5074

* Order-picking distances are in feet and center-to-center aisles are assumed to be 18 feet.

Table 6.6: Horizontal Travel for Replenishment

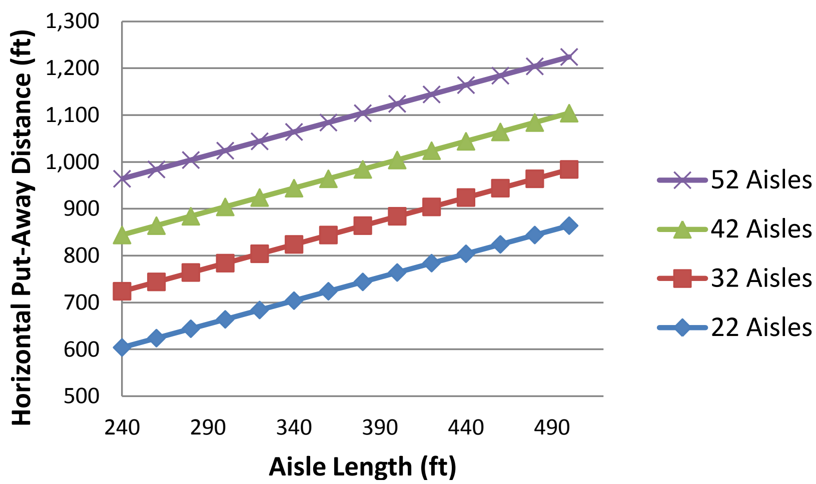
Aisle	Number of Forward Aisles															
Length	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
240	260	309	335	355	372	389	405	421	436	451	466	481	496	511	525	540
260	281	333	359	380	398	415	431	447	462	477	492	507	522	537	552	566
280	302	356	384	405	423	441	457	473	488	503	519	534	548	563	578	593
300	323	380	409	430	449	466	483	499	514	530	545	560	575	590	604	619
320	344	404	434	456	475	492	509	525	540	556	571	586	601	616	631	645
340	365	428	458	481	500	518	534	550	566	582	597	612	627	642	657	672
360	386	452	483	506	526	543	560	576	592	608	623	638	653	668	683	698
380	407	476	508	531	551	569	586	602	618	634	649	664	680	695	709	724
400	428	499	533	557	577	595	612	628	644	660	675	691	706	721	736	751
420	449	523	558	582	602	621	638	654	670	686	702	717	732	747	762	777
440	470	547	582	607	628	646	664	680	696	712	728	743	758	773	788	803
460	491	571	607	632	653	672	690	706	722	738	754	769	785	800	815	830
480	512	595	632	658	679	698	715	732	748	764	780	795	811	826	841	856
500	533	619	657	683	704	723	741	758	775	791	806	822	837	852	867	882

* Replenishment distances are in feet and center-to-center aisles are assumed to be 18 feet, with $\alpha = 0.8$.

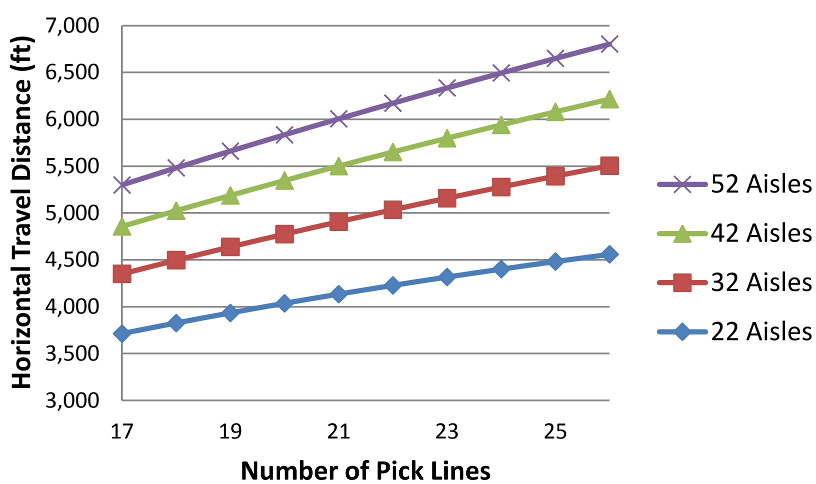
6.5.2 Using the Empirical Data to Determine Labor Requirements

Now that tables have been constructed for sizing the pallet rack area and for determining distance requirements for various sizes of the pallet rack area, we can translate the flow of product into labor requirements. Given that the tables contain discrete lookup values, interpolation is likely necessary. For example, consider the pallet rack area with 5 levels (7,000 bottom-level pallets) with a pallet-area shape of 3.0. From Table 6.2, this layout is comprised of 51 aisles that are 306 feet in length. To determine the put-away distance required, we utilize Table 6.3. However, this table does not contain a discrete value for 306 feet or for 51 aisles. Figure 6.2(a) illustrates the horizontal put-away distances for a different number of aisles and different aisle lengths. The graphs in the figure imply a nearly linear relationship between aisle length and put-away distance, as well as a nearly linear relationship between number of aisles and put-away distance. Linear interpolation between aisle lengths of 300 and 320 feet, followed by a subsequent linear interpolation between 50 and 52 aisles, results in a put-away distance of 1,016 feet. Using the analytical model for put away directly (with 51 aisles and an aisle length of 306 feet) yields the same result.

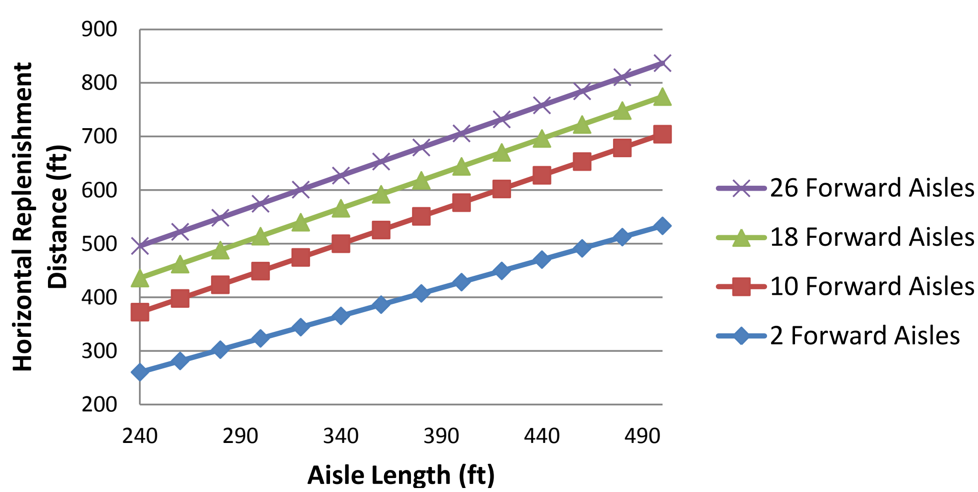
Next we consider interpolation for the order-picking operation. Figure 6.2(b) depicts the order-picking travel for a discrete number of picks and for a different number of aisles (with an aisle length of 240 feet). From the shape of the graphs in Figure 6.2(b), linear interpolation is a viable option for determining the horizontal distance required for order picking from the random storage reserve area. Thus, in order to find the order-picking distance for 17.5 pick lines and 22 aisles, we use the distance for 17 and 18 pick lines and linearly interpolate to obtain a distance of 3,768.50 feet $(3,712 + 0.5 \times (3,825 - 3,712))$, compared to 3,769.45 feet using the analytical model directly. For order picking, as many as three interpolations may be necessary (for pick lines, aisle length, and/or odd aisles). Finally, replenishment travel distances for a range of forward aisles is depicted in Figure 6.2(c). Again, we use linear interpolation to determine the horizontal distance for replenishments that are not represented by discrete values in the replenishment tables.



(a)



(b)



(c)

Figure 6.2: Travel distances: (a) horizontal put-away travel; (b) horizontal order-picking travel, aisle length = 240 ft; (c) horizontal replenishment travel, $\alpha = 0.8$.

After the travel distances for each operation (and for each design) have been determined, the total travel for each design is computed by multiplying the number of daily trips for each operation. Next, we divide the resulting travel distances by the respective horizontal and vertical travel speeds in order to determine the number of hours required for each functional area.

6.6. Results

We compare the results for the example using emprical data versus applying analytical models directly in assessing warehouse design performance. Figure 6.3 contains two bars for each design. The first bar corresponds to the cost from the analytical-model method and the second bar corresponds to the estimate from the empirical-data method. As one looks from left to right in the chart, it is clear that although the two methods do not produce identical results, the overall trend is extremely similar.

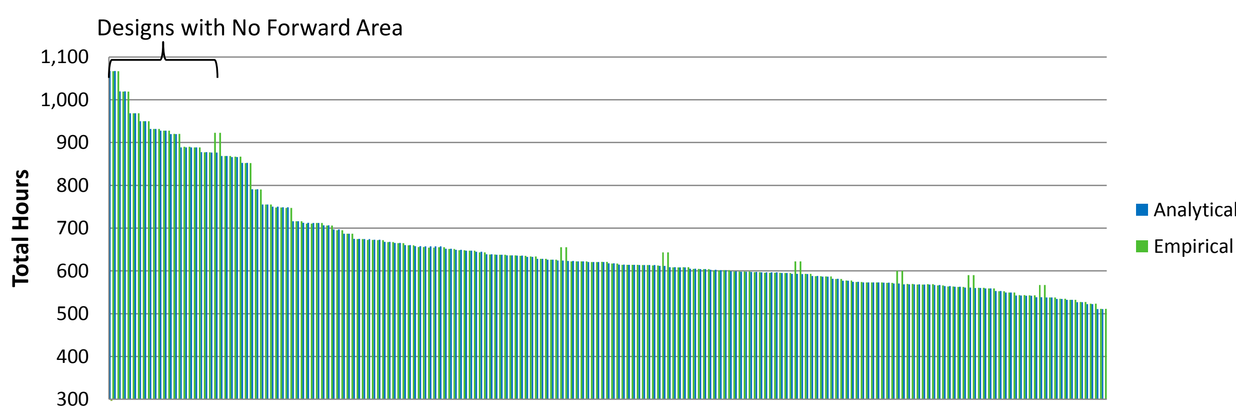


Figure 6.3: Total labor, designs ordered from most to least hours with alternating results for analytical-model hours and empirical-data hours.

The percent difference in travel requirements for each operation as a result of using the analytical models directly versus using lookup tables with empirical data is displayed in Table 6.6. The results indicate that using empirical data for assessing warehouse design is promising. Note that the percent difference for the designs with a shape of 2.5, with 5% or more SKUs in the forward area and 6 levels of pallet rack, is high compared to other designs.

A closer look at the design parameters reveals that the sizing table led to a slightly larger warehouse than using the sizing model directly (45 aisles with a length of 324 ft compared to 42 aisles with a length of 302 ft). The outliers in Figure 6.3 are attributed to these designs as well. Nevertheless, a comparison of the results for the two methods in Figure 6.4 reveals that despite the sizing difference, the relative difference among the forward area sizes is consistent.

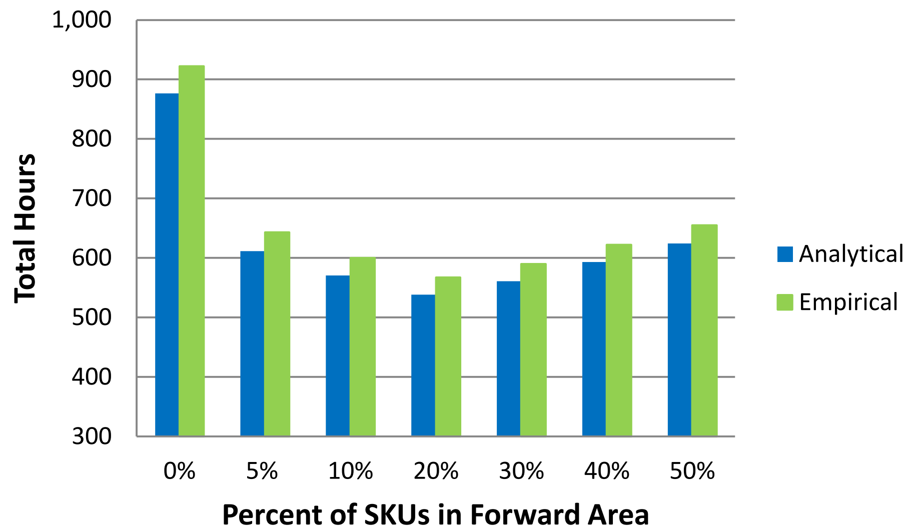


Figure 6.4: Comparison of random-storage forward area designs using empirical data versus analytical models for designs with a pallet-area shape of 2.5 and 6 levels of pallet rack.

Despite this anomaly, both methods for assessing warehouse design revealed that the design with a pallet-area shape of 4.0 with six levels of racking and 20% of the SKUs in the forward area was superior to other designs in terms of total labor required. Further, ranking the designs (in terms of total labor hours) revealed that 66 out of the 196 designs considered received the same ranking by each method, and 91% of the designs were within 3 rankings when comparing the two methods.

Table 6.7: Comparison of Results for Empirical Data and Analytical Models*

Forward SKUs	Shape	5 Levels				6 levels			
		Put Away	Pick All Levels	Forward Picking	Replenishment	Put Away	Pick All Levels	Forward Picking	Replenishment
0%	1.0	0.0%	0.0%	–	–	0.0%	0.0%	–	–
	1.5	0.0%	0.0%	–	–	0.0%	0.0%	–	–
	2.0	0.0%	0.0%	–	–	0.0%	0.0%	–	–
	2.5	0.2%	0.1%	–	–	3.7%	8.1%	–	–
	3.0	-0.2%	0.0%	–	–	0.0%	0.0%	–	–
	3.5	0.1%	0.2%	–	–	0.1%	0.2%	–	–
	4.0	0.0%	0.0%	–	–	0.0%	0.0%	–	–
5%	1.0	0.0%	-0.1%	-1.9%	-2.0%	0.0%	-0.1%	-1.9%	-2.0%
	1.5	0.0%	-0.1%	-1.8%	-2.0%	0.0%	-0.1%	-1.8%	-2.0%
	2.0	0.0%	-0.1%	-1.8%	-2.0%	0.0%	-0.1%	-0.1%	0.0%
	2.5	0.2%	0.0%	0.0%	0.1%	3.7%	8.1%	6.5%	6.7%
	3.0	-0.2%	-0.1%	-0.1%	0.0%	0.0%	-0.1%	-0.1%	0.0%
	3.5	0.1%	0.1%	0.2%	0.3%	0.1%	0.1%	-1.0%	-0.1%
	4.0	0.0%	-0.1%	-1.2%	-0.3%	0.0%	-0.1%	-1.2%	-0.4%
10%	1.0	0.0%	-0.1%	-1.2%	-0.4%	0.0%	-0.1%	-1.1%	-0.4%
	1.5	0.0%	-0.1%	0.0%	0.0%	0.0%	-0.1%	0.0%	0.0%
	2.0	0.0%	-0.1%	0.0%	0.0%	0.0%	-0.1%	-0.6%	-0.1%
	2.5	0.2%	0.0%	-0.5%	0.0%	3.7%	8.1%	6.3%	6.3%
	3.0	-0.2%	-0.1%	0.0%	0.0%	0.0%	-0.1%	0.0%	0.0%
	3.5	0.1%	0.1%	0.2%	0.2%	0.1%	0.1%	-0.1%	0.2%
	4.0	0.0%	-0.1%	-0.4%	0.0%	0.0%	-0.1%	0.0%	0.0%
20%	1.0	0.0%	0.1%	-0.3%	-0.1%	0.0%	0.1%	0.0%	0.0%
	1.5	0.0%	0.1%	-0.2%	0.0%	0.0%	0.1%	0.0%	0.0%
	2.0	0.0%	0.1%	0.0%	0.0%	0.0%	0.1%	-0.1%	0.0%
	2.5	0.2%	0.2%	0.2%	0.1%	3.7%	8.4%	6.1%	5.5%
	3.0	-0.2%	0.1%	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%
	3.5	0.1%	0.4%	0.3%	0.2%	0.1%	0.3%	0.3%	0.2%
	4.0	0.0%	0.1%	0.0%	0.0%	0.0%	0.2%	0.0%	0.0%

* Percentages denote difference in analytical model versus empirical tables.

6.7. Conclusions and Future Research

Overall warehouse design presents a challenging problem, due to the number of possible designs and the interrelationships that exist among design parameters and functional areas. Using empirical data to quantify space and labor requirements in overall warehouse design would allow designers to include new technologies (that are incompletely modeled) in the set of possible designs to consider. Further, empirical data in a tabular format can be of value to practitioners in understanding how changing design parameters affects the performance of a functional area in the warehouse, especially as new technologies emerge.

To expedite the lookup process, we envision that the lookup tables for sizing and labor requirements would be implemented through a graphical user interface (GUI). A menu-driven GUI would provide a comparison of different design parameters and may prove even more valuable than printed tables by allowing for an increased number of comparisons.

This research can be extended by constructing sizing and labor tables for designs that consider automation or additional functional areas within the warehouse such as piece picking. Existing models can be utilized to construct tables for this overall design methodology, and data from industry can be incorporated as well. In addition, experimenting with fewer parameters (i.e., increments of 10 aisles instead of 2 aisles) in the tables would provide insight into the performance of lookup tables with fewer discrete values. Finally, research on how to search the design space would benefit the designer in two ways: 1) reducing the time spent in searching the lookup tables, and 2) allowing the designer to incorporate other non-quantifiable factors into design assessment. Given that there are currently no comprehensive models to assist designers in quantifying labor requirements in order to compare designs, extending this research would benefit practitioners in the initial design phase.

Acknowledgements

This research was supported, in part, by the National Science Foundation Industry-University Cooperative Research Center for Excellence in Logistics and Distribution (CELDi). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Bibliography

[1] Apple, J. M., Meller, R. D., and White, J. A., "Empirically-Based Warehouse Design: Can Academics Accept Such an Approach?," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[2] Ashayeri, J., and Gelders, L., "Warehouse Design Optimization," *European Journal of Operational Research*, 21, 285–294 (1985).

[3] Baker, P., and Canessa, M., "Warehouse Design: A Structured Approach," *European Journal of Operational Research*, 193, 245–258 (2009).

[4] Bolz, R. E., and Tuve, G. L., editors, *CRC Handbook of Tables for Applied Engineering Science*, CRC Press, Cleveland, OH (1973).

[5] Bozer, Y., and Sharp, G., "An Empirical Evaluation of General Purpose Automated Order Accumulation and Sortation System Used in Batch Picking," *Material Flow*, 2, 111–131 (1985).

[6] Campbell-Kelly, M., Croarken, M., Flood, R., and Robson, E., editors, *The History of Mathematical Tables: From Sumer to Spreadsheets*, Oxford University Press, New York, NY (2003).

[7] Committee on Engineering Design Theory and Methodology, *Improving Engineering Design: Designing for Competitive Advantage*, National Academy Press, Washington, DC (1991).

[8] Gray, A. E., Karmarkar, U. S., and Seidmann, A., "Design and Operation of an Order-Consolidation Warehouse: Models and Application," *European Journal of Operational Research*, 58, 14–36 (1992).

[9] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Operation: A Comprehensive Review," *European Journal of Operational Research*, 177, 1–21 (2007).

[10] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[11] Meller, R. D., and Thomas, L. M., "Optimizing Distribution Center Configuration: A Practical View of a Multi-Objective Problem," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[12] Park, Y. H., and Webster, D. B., "Modelling of Three-Dimensional Warehouse Systems," *International Journal of Production Research*, 27, 6, 985–1003 (1989).

[13] Rouwenhorst, B., Reuter, B., Stockrahm, V., van Houtum, G., Mantel, R., and Zijm, W., "Warehouse Design and Control: Framework and Literature Review," *European Journal of Operational Research*, 122, 515–533 (2000).

[14] van den Berg, J. P., and Gademann, A. J. R. M. N., "A Literature Survey on Planning and Control of Warehousing Systems," *IIE Transactions on Scheduling & Logistics*, 31, 8, 751–762 (1999).

[15] Yoon, C. S., and Sharp, G. P., "A Structured Procedure for Analysis and Design of Order Pick Systems," *IIE Transactions*, 28, 379–389 (1994).

A. Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Russell D. Meller, Professor and Holder of the Hefley Professorship

DATE: June 18, 2013

SUBJECT: Certification of student effort

I certify that greater than 51% of the work conducted for this chapter entitled, "Contribution 3: A Paper on, 'Using Empirical Data to Assess Performance in Overall Warehouse Design'," was conducted by Lisa M. Thomas.

4207 Bell Engineering Center • Fayetteville, Arkansas 72701 • 479-575-3156 • 479-575-8431 Fax

B. Release from the Institute of Industrial Engineers

The student, Lisa M. Thomas, has permission to include a paper in her dissertation with acknowledgment to the *Proceedings of the 2013 Industrial and Systems Engineering Research Conference*. The title of the paper is "Using Empirical Data to Assess Performance in Overall Warehouse Design" and it was co-authored by her advisor, Dr. Russell D. Meller.

The Institute of Industrial Engineers approves of the inclusion of the paper in this dissertation based on an e-mail dated June 19, 2013.

From: Monica Elliott <melliott@iienet.org>
Date: June 19, 2013 8:13:21 AM CDT
To: "rmeller@uark.edu" <rmeller@uark.edu>
Subject: RE: Request for a student

Hi Russ,

Your student has permission to use her papers that appeared in the ISERC 2013 proceedings in her dissertation project. She should include the acknowledgment/reference that it appeared in the proceedings as you have indicated. We do not provide formal letters of permission, so please consider this email as permission.

Best regards,
Monica

MONICA ELLIOTT
Director of Communications | Institute of Industrial Engineers
3577 Parkway Lane, Suite 200 | Norcross, GA 30092
(770) 449-0461, x116 | melliott@iienet.org
IIE on Facebook | IIE on LinkedIn | IIE (iienet) on Twitter

Chapter 7

Contribution 4: A Paper on, "A Search Heuristic for Designing a Case-Picking Warehouse"

Abstract: Warehouses can be characterized in many ways, including the number of items stored, the average number of cases per pallet, throughput and inventory requirements, and demand profile, to name a few. Thus, there is no one-size-fits-all design for case-picking warehouses, and hundreds of designs are possible. Moreover, the decision variables in warehouse design are interrelated and this further complicates the design process. The purpose of this paper is to provide a preliminary search heuristic for a good design configuration for a manual, case-picking warehouse. Our goal in designing the heuristic is that it would provide a design that is close to the optimal solution, which could then be further analyzed and improved. We limit the decision variables considered in our heuristic to include the size and layout of the forward area, dock door configuration, pallet area shape, and pallet rack height. To design our heuristic we employ a statistical-based methodology, whereby we use one set of data to develop the heuristic and an independent set of data to evaluate the performance of the heuristic. Our results indicate that our heuristic would be of value in the search for a good case-picking warehouse design that minimizes labor hours.

7.1. Introduction

According to the 2012 "DC Measures" study conducted by the Warehousing Education and Research Council and *DC Velocity*, the number of distribution centers (DCs) with primarily full case-picking operations has increased over the last four years [14]. Approximately one third of the 2012 survey respondents characterized their facilities as having primarily case-picking operations, and based on previous years' surveys, the number of DCs with full case-picking operations has increased over the last four years [14].

The study lists key benchmark metrics for warehouse operations such as inventory turns, put aways per hour, lines picked and shipped per hour, and cases picked and shipped per hour [14]. These metrics are highly dependent on the layout and design of the warehouse. In order to improve such metrics, the overall design of the warehouse should be considered.

Analytical models can predict performance metrics such as put aways per hour and lines picked per hour for a given warehouse design, yet the best design is not always apparent, as hundreds of solutions are possible. The warehouse operating environment can be characterized in many ways, including the number of pallet locations, the number of SKUs, the number of cases per pallet, throughput requirements, and product activity, to name a few. Moreover, the decision variables in warehouse design are interrelated, and this further complicates the design process. Because warehouse design entails a vast solution space, practitioners would benefit in having a search algorithm that points to designs that are close to the optimal solution for its set of characteristics.

For a manual, case-picking warehouse that employs picking from pallet rack, two decision variables are the shape of the pallet rack area and the number of levels of pallet rack. The shape of the pallet area can be characterized using a ratio of the width-to-depth of the pallet area, where the depth refers to the distance along an aisle. Both the shape and number of levels impact the footprint of the pallet area, as higher levels of pallet rack require a smaller footprint for a given number of storage locations.

Dock door configuration is another decision in warehouse design. A one-sided configuration involves a single staging area with shipping and receiving along only one side of the facility, whereas a two-sided configuration entails two staging areas and dock doors along opposite sides of the facility. Determining the dock door requirements involves consideration of the number of trucks arriving during a receiving or shipping time period. In general, the number of pallet locations and pallet area shape define the number of available dock doors. However, if the storage area results in a number of doors that is not sufficient to meet truck throughput requirements, the designer has two choices, configuring the facility with dock doors on two sides or increasing the size of the warehouse. Both choices increase labor and building costs. To overcome this issue, additional workers can be scheduled during the arrival of trucks in order to turn doors faster. Alternatively, scheduling the arrival of trucks may alleviate truck traffic and prevent worker congestion within the warehouse.

Another important decision variable involves the question of whether or not to include a forward area for picking. A forward area can increase picking efficiency by placing fast-moving items in a smaller area, so as to decrease the travel required for picking these items. However, an additional cost is involved in replenishing the items in the forward area. And, as the forward area grows in size, the picking efficiency decreases due to increased travel. Thus, the size of the forward area is another decision variable requiring consideration. In addition, the layout of the forward area can include random or volume-based storage such as class-based storage. If class-based storage is utilized, the dock door configuration implies either a 1-sided or 2-sided layout. With class-based storage, the storage classes are often based on product popularity; however, classes may also be defined by other product characteristics, like the weight of the product or the suitability of a case of the product to form the base of a pallet. In a case-picking warehouse, a class of heavy and base products are often defined and placed near the P&D point so that they are generally picked first in an order-picking tour.

Given the number of decision variables, the number of possible designs to consider can be overwhelming. For example, if one considers 7 possible pallet area shapes, 2 choices for pallet rack height, a one- or two-sided dock door configuration, 7 possible sizes for a random storage forward area, and 9 possible sizes for the 1-sided and 2-sided class-based forward area layouts, the number of possible designs is 476, as illustrated in Figure 7.1.

With a large number of possible designs, some designs will perform better than others.

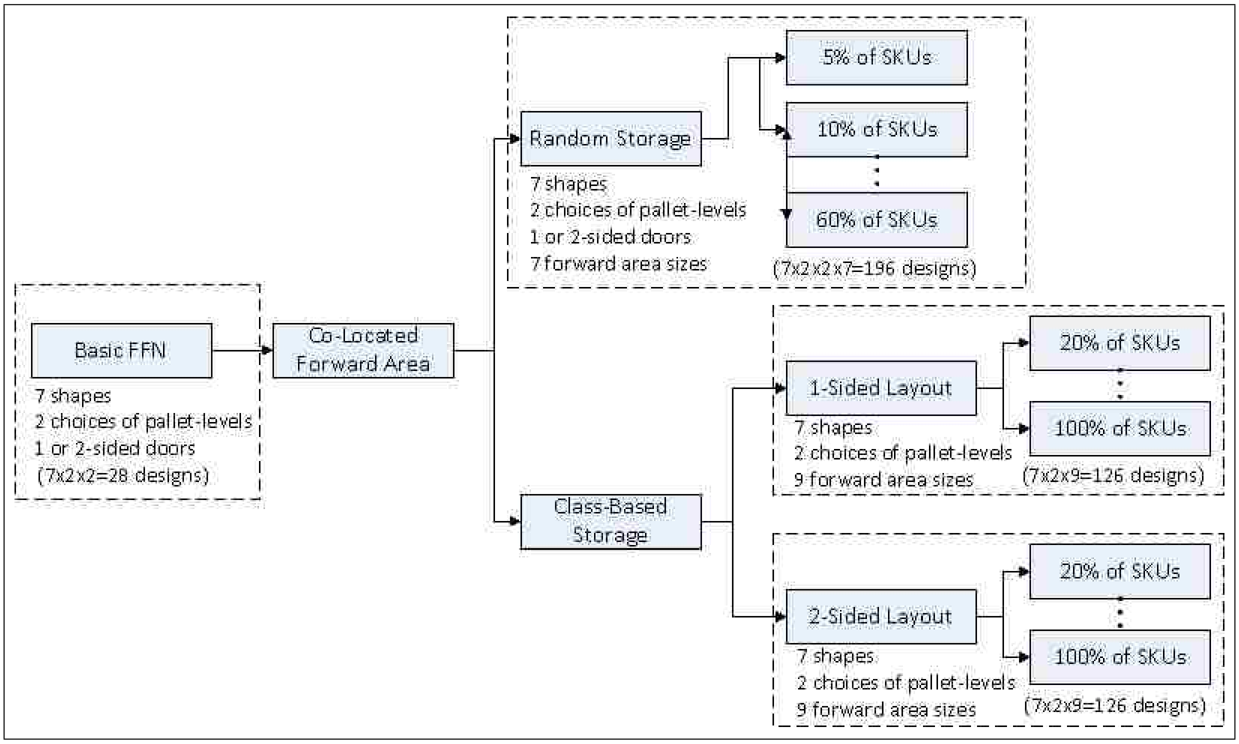


Figure 7.1: Example of possible designs to consider.

For example, consider the distribution of labor requirements for each of the 476 designs (from the range of choices in Figure 7.1) in Figure 7.2 for a particular set of warehouse operating characteristics. For this warehouse environment, the designs with the 1-sided, class-based layout perform the best, and the worst designs are those with no forward area. Consequently, a heuristic search procedure that narrowed the solution space with these data to design choices with 1-sided, class-based layouts would benefit a practitioner.

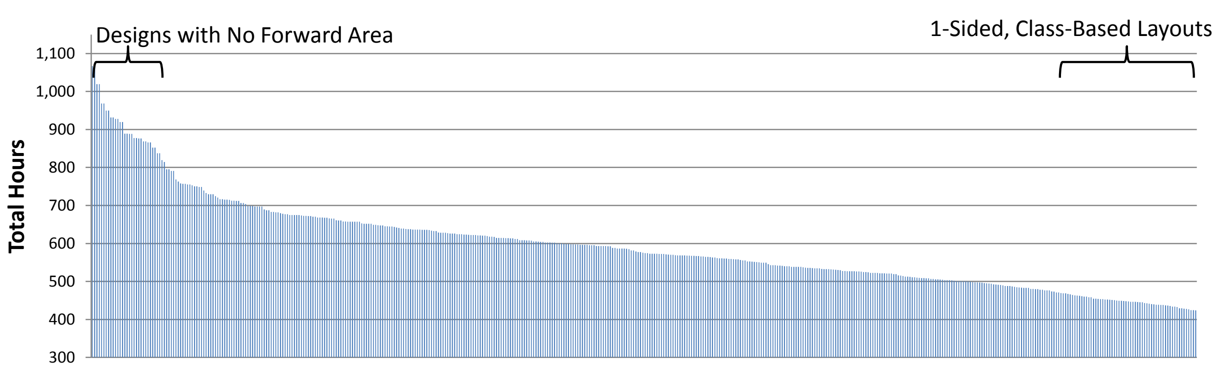


Figure 7.2: Total labor hours (designs ordered from most to least hours required).

The purpose of this paper is to provide a search algorithm for a manual, case-picking warehouse design configuration that considers the warehouse environment characteristics in minimizing the total labor required. The resulting design configuration can be further analyzed and improved. We define a design configuration to include the following decision variables: the size and layout of the forward area, the dock door configuration, pallet area shape, and pallet rack height. We examine the warehouse operating environment data for a number of data sets, as well as the best warehouse that results for each data set from a complete enumeration of the solution space, to derive a heuristic search procedure that considers the warehouse operating environment.

In the next section we include a review of literature related to the aforementioned design variables. In Section 7.3, we provide a problem statement, and Section 7.4 includes the methodology for the heuristic search procedure described in Section 7.5. Then, in Section 7.6 we summarize our findings related to designing a case-picking warehouse that are embedded in our heuristic, and we also provide a comparison of the results of the heuristic to the best design that is obtained through complete enumeration.

7.2. Literature Review

The forward-reserve problem has received considerable attention in the literature [7]. In a forward-reserve configuration, fast-moving items are stored in a forward area that is smaller than the reserve storage area that contains all items. Items that do not have a location in the forward area are picked from the reserve area. Thus, by placing items in the smaller forward area, order-picking travel is reduced, but at the cost of replenishing items in the forward area from the reserve area. As more items are placed in the forward area, the forward area increases in size, and as a result, the order-picking savings decreases. Consequently, the forward-reserve problem aims to determine which items to include in the forward area, as well as the quantity stored for each item in order to minimize the overall picking and replenishment time.

Various forward area configurations have been studied in the literature. One such configuration entails a forward area that consists of case flow rack and/or bin shelving and a reserve area that is comprised of pallet rack. Bozer [3] was the first to consider a configuration with a co-located forward area, where the bottom level of pallet rack serves as the forward area, and the upper levels include the reserve storage locations. This configuration is ideal for a case-picking operation where cases are received in pallet quantities, as no additional storage space is required for the forward area.

Frazelle et al. [6] developed a procedure for determining the best size of the forward area, the set of SKUs (stock keeping units, or items) to be included in the forward area, as well as the quantity of each SKU in the forward area. The procedure uses input data including the activity profile, pick and replenishment productivity, labor, occupancy index, and forward area size. Clusters of SKUs that are typically ordered together and that warrant a location in the forward area are assigned to locations in the forward area. In a case study, the procedure resulted in a 40% decrease in annual operating costs compared to the current policy that included all SKUs (in equal quantities) in a forward area that consisted of bin shelving and flow rack.

Hackman and Rosenblatt [8] developed a heuristic procedure to determine the items and the quantity to be stored in an AS/RS when the AS/RS capacity is limited. Accordingly, the AS/RS serves as a forward area, and the reserve storage is located in a manual material handling area. After the items and quantities to be stored in the AS/RS have been determined, then items are assigned to locations. The relevant factors considered in the heuristic approach include the time savings in retrieving an item from the AS/RS as compared to a manual retrieval and the cost of replenishment.

Frazelle et al. [5] developed an aggregate model that includes the size of the forward area and the effect on the forward area size on picking costs. An economic assignment quotient (EAQ) is used to rank items according to their inclusion in the forward area, as first used by Hackman and Rosenblatt [8].

Van der Berg et al. [13] consider the case where order picking is performed during a busy period, and replenishments occur during a preceding idle period. By placing more than one unit load in the forward area, replenishments can be deferred until after the busy picking period. The authors present heuristics to determine the items that should have more than one unit load in the forward area in order to minimize extra replenishment labor during picking, so as to increase throughput. They present a general model that minimizes labor during the busy period and a restricted model that limits the number of replenishments during the busy order-picking period.

Bartholdi and Hackman [1] consider storage units in less-than-pallet quantities as in a distribution center that stocks small parts. The authors showed that storing the same amount of space for each SKU is equivalent to storing an equal time supply for each SKU. In addition, the authors showed that a three to six percent reduction in restocks can be achieved by changing from equal space-time allocations to optimal allocations that use the mean lead-time demand and safety stock information to re-allocate space in the forward area.

Bartholdi and Hackman [2] developed a model for case picking from a forward area within bottom-level pallet locations. The model determines the number of locations to allocate to each SKU such that the maximum benefit is achieved from the forward area. However, the labor savings per pick achieved by including a SKU in the forward area (as opposed to picking from reserve) is fixed and independent of the size of the forward area.

Another decision in DC design is the number of pallet rack levels. Parikh and Meller [11] were the first to consider the optimal height of a single-deep pallet rack storage system that employs order-picking trucks with both Tchebychev and rectilinear travel. They presented a model that can be used to determine the number, length and height of storage aisles in order to meet both storage and throughput requirements. In evaluating the optimal height of the pallet rack, a simple cost-based optimization model is used that considers the cost of picking, equipment and space. The authors conclude that the optimal storage height decreases for a system with a high throughput requirement, but increases as the cost of storage space increases.

In terms of the width and depth of the pallet rack area, Francis [4] developed a model for the expected travel in a random storage warehouse and determined that for unit-load retrievals and a single pickup and deposit (P&D) point, the optimal width-to-depth ratio of the storage area is two to one (the depth refers to the side of the warehouse that is parallel to the picking aisles). Hall [9] developed models for order picking in a random storage warehouse and determined that the optimal shape of the pallet area increases (with more aisles that are shorter) as the number of pick lines per tour increases.

Petersen [12] used simulation to compare the performance of various routing strategies in a random storage warehouse with 1,000 storage locations, while considering pick lists of with 5, 15, 25, 35 and 45 pick lines. Petersen found that narrow, deeper warehouses (lower shape ratios) are more effective at minimizing order-picking travel for all of the strategies except the return policy. Nevertheless, depending on the dimensions of the storage locations, a pick list with more than 20 lines may result in more than one stop per aisle for the shape ratios considered. With more than one stop per aisle, narrow warehouses would be preferred to reduce travel, requiring fewer (but longer) aisles.

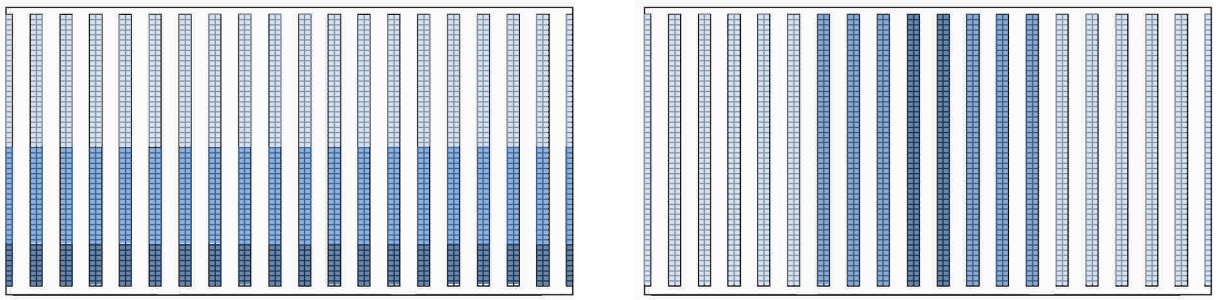
7.3. Problem Statement

Despite the extensive literature related to warehouse design, there are no methodologies that provide a search procedure to identify designs that perform well. Because warehouse operating environments are different in terms of the number of pallet locations, the number of SKUs, the number of cases per pallet, throughput requirements, and product activity, the search process should consider these characteristics. In this paper we focus on a manual, case-picking warehouse and develop a heuristic procedure to search the solution space for preferred designs in terms of minimizing the labor required for put away, order picking and replenishment. In developing our heuristic search procedure, we consider a range of warehouse data sets and the performance of various designs for the data sets. We make special note of the designs that work best for a given data set and attempt to characterize the interaction.

In our analysis, we explore a wide range of designs. The designs that we consider include a traditional layout with aisles of pallet rack that are orthogonal to the side(s) of the warehouse with dock doors, and without center cross aisles. We consider designs with and without a forward area. For designs without a forward area, we assume a shared storage layout, where picks are equally likely to occur from any pallet location in the warehouse (we call this a "random" storage layout). For designs that include a forward area, the forward locations are at the bottom level of the centermost aisles of pallet rack, and the reserve storage locations are randomly located throughout the warehouse in the upper levels of pallet rack. The size of the forward area depends on the number of SKUs assigned to it, and any SKUs that are not represented in the forward area are picked from the reserve area. For simplicity, we assume that each SKU in the forward area is allotted exactly one pallet location on the bottom level.

The forward area layouts that we consider include random storage and the two class-based storage layouts as depicted in Figure 7.3. With the identical-aisle layout in Figure 7.3(a), the fast-moving items (represented with darker shading) are located nearest to the dock doors along one side of the facility. The within-aisle layout in Figure 7.3(b) places fast-moving items in the centermost aisles, such that the locations are convenient to dock doors on opposite sides of the facility. Thus, design decisions related to the forward area include: whether or not to have a forward area, and for designs with a forward area, both the number of SKUs to include, as well as the storage layout.

In addition to the design decisions related to the forward area, the other decision variables that we consider include the dock door configuration (i.e., doors on one or both sides of the facility), the shape of the pallet area, and the number of levels of pallet rack. Again, we note that the decisions pertaining to dock door configuration and forward area layout are related, in that the door configuration determines the storage area layout for class-based forward



(a) (b)

Figure 7.3: Class-based layouts: (a) Identical-aisle, 1-sided dock doors; (b) Within-aisle, 2-sided dock doors.

area designs. Likewise, decision variables for pallet area shape and dock door configuration are related, as higher shape ratios allow more dock doors, and a 2-sided door configuration provides twice as many dock doors as the 1-sided configuration. The number of dock doors should be sufficient to meet throughput requirements. Figure 7.4 represents the decision variables related to the designs considered.

To assess the performance of each design on a given data set, we use a pallet-area sizing algorithm [10] along with analytical models (from Chapter 4) to quantify the space and labor requirements for a given design. In our analysis, we assume that cases are received in pallet quantities, and we use an average number of cases per pallet for determining the number of pallet put aways. Cases are picked and loaded onto pallets, such that the case quantity per order-picking tour is 80 percent of the number of cases on incoming pallets. The number of lines per tour is dependent on the number of picks per line, as well as the pallet capacity. The operational labor that we consider includes pallet put away, order picking from a forward area (if applicable) and the reserve storage area, as well as replenishment to the forward area (if any). Factors such as blocking and congestion are not considered, nor are cost factors (like labor, land, and construction costs), which we assume will be taken into consideration in the final design process. The following section provides our methodology for developing a search heuristic for designing a manual, case-picking warehouse.

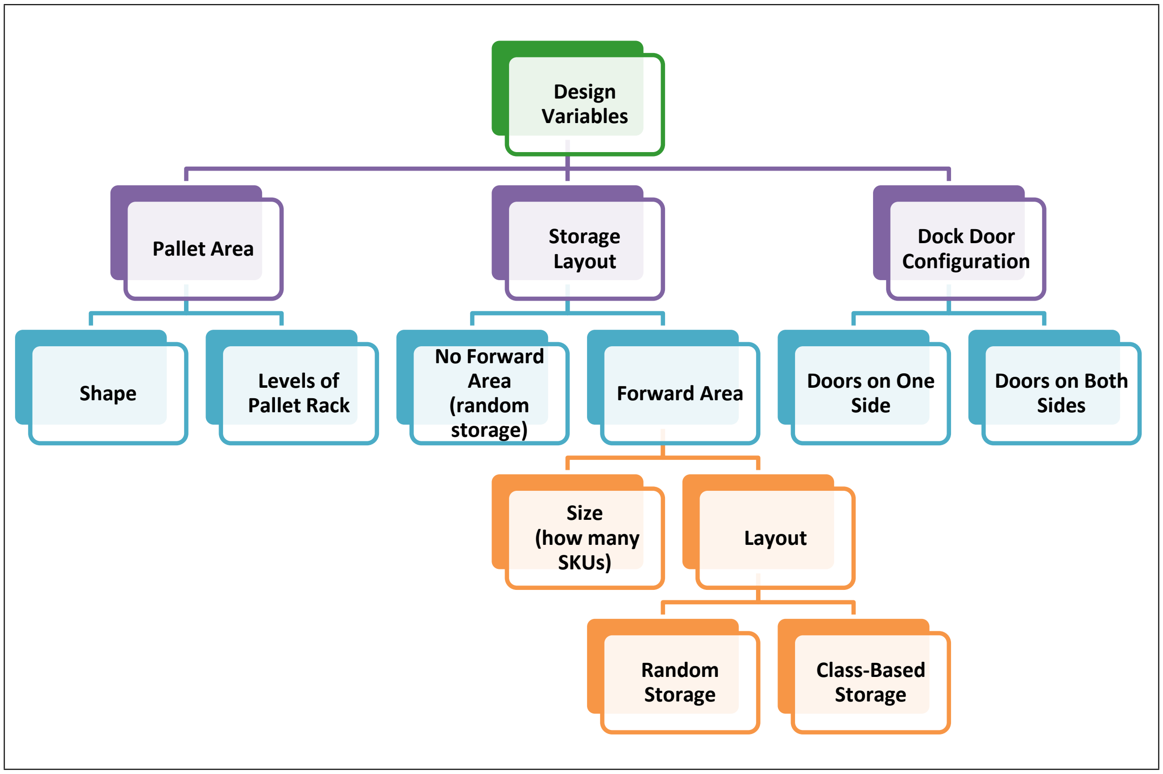


Figure 7.4: Design variables considered.

7.4. Methodology

Specifying a warehouse involves two sets of values. We refer to the first set that describe a particular warehouse as the *warehouse parameters*. These parameters pertain to the characteristics of the warehouse including the required number of pallet locations, the number of SKUs, the average number of cases per pallet, throughput requirements, and activity profile. The warehouse parameters are fixed for a given problem.

The second set of values to specify a warehouse are the values associated with the design choices, and we refer to these as *decision variables*. The decision variables involve design decisions related to the pallet area shape, the number of levels of pallet rack, the dock door configuration, and the forward area size and layout. Many of these decision variables are interrelated. For example, the pallet rack height and pallet area shape affect the size and shape of the storage area for a given number of pallet locations, as well as the number of locations available on the bottom level. Thus, these factors also impact the travel time for put away, order picking and replenishment. Accordingly, the output from the pallet-area sizing algorithm is used as input for the models used to determine labor requirements.

A list of warehouse parameters and decision variables is included in Appendix A for the layouts that we consider. Included in Appendix A are calculations for the number of aisles and dock doors per column section, as feasibility relationships exist between some of the warehouse parameters. These relationships need to be considered in choosing warehouse parameters in order to obtain an integer number of aisles and dock doors per column section. For our analysis, we use a typical rack configuration with the parameters listed in Table 7.1. The rack parameters are consistent with the relationships described in Appendix A, such that the rack depth and aisle width result in an integer number of aisles per column section in the warehouse. We assume that columns are buried in the pallet rack, though we do not account for any loss of pallet positions due to columns in our analysis. Table 7.1 also includes our assumptions for the horizontal and vertical travel speeds of order-picking equipment, as well as the probability of forward and reserve storage locations residing in the same aisle.

Table 7.1: Warehouse Parameter Assumptions

Parameter	Value
Column spacing	54 ft
Center-to-center aisle distance	18 ft
Pallet opening width	100 in
Width of vertical rack member	4 in
Pallet opening height	60 in
Horizontal travel speed	264 fpm
Vertical travel speed	44 fpm
Probability of forward & reserve locations in same aisle	0.4

Table 7.2: Data Sets Based on Order Data

Data Set	Pallet locns	SKUs	Incoming cases per pallet	Case picks per day	Avg picks per line	Skewness	
						A	%Lines/%SKUs
DS1	60,000	5,286	96.0	389,396	11.65	0.097	65/20
DS2	24,000	10,831	12.7	4,293	1.50	0.071	77/20
DS3	23,600	10,612	10.6	3,384	1.28	0.079	75/20
DS4	14,000	5,493	11.8	2,401	1.19	0.053	81/20
DS5	8,650	5,574	12.5	1,251	1.29	0.122	68/20
DS6	35,000	8,539	48	44,097	3.69	0.024	92/20
DS7	50,000	8,000	25	60,000	4.00	0.079	75/20
DS8	45,000	6,000	30	45,000	1.50	0.071	77/20
DS9	30,000	7,000	35	40,000	2.25	0.053	81/20
DS10	10,000	4,000	20	11,000	4.50	0.122	68/20
DS11	50,000	8,333	70	126,000	3.00	0.068	80/20
DS12	40,000	6,700	100	160,000	8.00	0.253	55/20
DS13	30,000	5,000	30	32,400	1.00	0.146	66/20
DS14	10,000	6,000	15	6,600	3.00	0.096	74/20
DS15	40,000	26,000	100	96,000	8.00	0.253	55/20
DS16	30,000	25,000	80	76,800	1.00	0.107	72/20
DS17	10,000	5,000	8	2,880	2.00	0.079	75/20
DS18	20,000	6,000	30	21,600	1.50	0.097	65/20
DS19	45,000	4,500	20	36,500	3.50	0.024	92/20
DS20	5,000	500	15	2,400	1.00	0.117	71/20

Next, we provide the warehouse parameters associated with the order data of six existing warehouses and 14 example data sets listed in Table 7.2. DS1 and DS6 were provided by Fortna (`www.fortna.com`), a multi-national provider of supply chain professional services, who removed any client-identifying information before supplying the data. DS2–4 were provided by a member organization of CELDi (`www.celdi.org`). These data sets are used to determine the performance of various designs on warehouse parameters. The twenty data sets represent a range of warehouses of varying sizes, demand skewness levels (12 different ABC curves), and throughput requirements. Our objective is to use the data sets to evaluate how various warehouse parameters affect the best design choices. For each data set, we determine the labor requirements for put away, order picking and replenishment (using the models in Chapter 4) by completely enumerating over all designs. Then we determine the designs that result in the least labor hours for each layout, as well as the optimal forward area size and pallet area shape for each layout.

In order to understand how the warehouse parameters affect design performance, we first calculate correlation coefficients between the warehouse parameters and the design variables of forward area size and pallet area shape that result in the least labor hours. We focus on these two decision variables, as they can have a significant impact on design performance (see Chapter 5). (The variable for pallet area height was set at 6 levels because, as we show later, designs with higher levels of pallet rack result in lower labor hours than lower levels of pallet rack in most cases.) Table 7.3 lists the correlation coefficients for each warehouse parameter with the optimal forward area size for each layout, and Table 7.4 contains the correlation coefficients for each warehouse parameter with the optimal pallet area shape.

Table 7.3: Correlation Coefficients of Warehouse Parameters with Forward Area Size

Warehouse	Optimal % SKUs		
Parameter	1-sided	2-sided	Random
Pallet locations	0.4897	0.0124	0.4012
Cases per pallet	0.2815	0.0788	0.3223
SKUs	-0.3317	-0.4077	-0.4607
Picks per line	0.2045	0.0819	0.3740
Skewness	-0.0973	-0.2284	-0.2834
SKUs-to-bottom-level-pallets	-0.7443	-0.5589	-0.7333
Lines per batch	0.1414	0.1047	0.0865

From Table 7.3 we see that as the number of pallet locations increases, the size of the forward area increases so as to offset the increased travel due to a larger warehouse area. The number of cases per pallet also has a positive correlation with the size of the forward area. Increasing the number of cases per pallet decreases the number of required replenishment trips, thus reducing the overall cost of the forward area. On the other hand, the number of SKUs has a negative correlation with forward area size; as the number of SKUs increases, it is advantageous to have a smaller forward area with only the fastest moving SKUs. Next, the benefit of having a forward area increases with more picks per line, as productivity increases due to more picks at no additional travel costs. The skewness of the ABC curve has a negative correlation with the forward area size. As skewness increases, a smaller forward area is warranted so as to take advantage of the small number of SKUs that represent a large percentage of pick lines.

The next parameter that we consider is the number of SKUs-to-bottom-level-pallets. This ratio represents a comparison of the number of SKUs to bottom-level pallet locations and can be calculated from the second and third columns in Table 7.2, while considering the number of levels of pallet rack. A ratio of less than one indicates that there are enough bottom-level locations to allocate all SKUs a bottom-level, forward area location (if warranted). For values greater than 1.0, the footprint of the pallet area would have to grow in order to accommodate designs with all SKUs on the bottom level. This ratio has a negative correlation with the size of the forward area. It is more advantageous to have a smaller forward area with fewer SKUs than to have a larger footprint that can accommodate more SKUs. Finally, the number of lines per batch increases the benefit of having a forward area, as more stops per order-picking tour results in fewer trips through the forward area and less overall travel. Of all of the factors considered, the SKUs-to-bottom-level-pallets ratio has the highest correlation with the size of the forward area.

Table 7.4: Correlation Coefficients of Warehouse Parameters with Pallet Area Shape

Warehouse	Optimal Shape		
Parameter	1-sided	2-sided	Random
Pallet locations	0.4080	0.6564	0.6558
Cases per pallet	0.7111	0.3741	0.3629
SKUs	0.2216	0.3544	0.0991
Picks per line	0.2303	0.1947	0.1628
Skewness	-0.3839	0.1409	0.1657
SKUs-to-bottom-level pallets	0.0796	-0.1543	-0.3831
Lines per batch	0.5937	0.2118	0.3018

Next, we consider the correlation of the warehouse parameters with the optimal shape of the pallet area as listed in Table 7.4. The shape factor represents the width-to-depth ratio of the warehouse, where higher ratios indicate more elongated warehouses with shorter aisles. Again, increasing the number of pallet locations results in a larger warehouse. For a large warehouse, it is more advantageous to have shorter aisles (with a higher shape ratio), especially in a warehouse with random storage or the 2-sided class-based layout that utilize a traversal routing strategy. The traversal strategy involves traveling the entire length of the aisle for all aisles that contain at least one pick location. The number of cases per pallet also has a positive correlation with the shape factor; a larger pallet capacity allows more pick lines per batch, such that shorter aisles are desired in order to reduce the within-aisle travel. The number of SKUs and picks per line has a small correlation with the shape factor, again indicating that shorter aisles are preferred for these two factors. For the skewness parameter, the 1-sided layout favors a more elongated warehouse, whereas the random storage and 1-sided forward areas favor longer aisles. An increase in skewness results in more class-A picks that are located at the end of the aisle for the 1-sided layout, such that only a small portion of the aisle is entered. In this case fewer, longer aisles (resulting in a lower shape ratio) are more advantageous. However, with random storage and the two-sided layout, shorter aisles are warranted because the picks are uniformly distributed in the aisle for these layouts. As the SKUs-to-bottom-level-pallets ratio increases (with less available forward locations), the 2-sided and random storage layouts prefer fewer, longer aisles, whereas the 1-sided layout performs better with shorter aisles in this case. The number of lines per batch has a positive correlation with shape for all of the layouts, indicating an advantage to shorter aisles. For this analysis, the number of pallet locations has the greatest impact on the pallet area shape factor.

In our next analysis, we again consider each data set from Table 7.2 and the performance of each data set for each design (by enumerating over all possible values for the design variables). Hence, we evaluate the data sets to determine any trends in design performance associated with the parameters of the warehouse. In other words, if certain designs perform well for a given range of warehouse parameters, then generalizations can be made about the preferred design for the range of (fixed) warehouse parameters. In order to be thorough in our analysis, we also vary the warehouse parameters for each of the 20 data sets to determine their impact on design performance.

Forward Area Layout and Pallet Area Shape

Of the design variables considered, the forward area size and layout, as well as the shape of the pallet area have the greatest impact on travel times (see Chapters 4 and 5). First we consider these variables for each of the data sets listed in Table 7.2. Tables 7.5–7.6 list pallet area shapes ranging from 0.5 to 7.0 for data sets 1 and 2, as well as the percent of SKUs in the forward area *that results in the least labor hours* for four layouts: no forward area, a forward area with random storage, a forward area with the 1-sided class-based layout, and a forward area with the 2-sided class-based layout. The daily hours listed in the tables total the travel times for put away, order picking and replenishment that meet the throughput requirements for each data set.

Table 7.5: DS1 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	3223	30%	2411	100%	1662	50%	2329
1.0	2554	20%	1942	100%	1342	50%	1906
1.5	2403	30%	1806	100%	1272	50%	1726
2.0	2271	30%	1680	100%	1214	40%	1601
3.0	2212	40%	1580	100%	1208	50%	1487
4.0	2180*	40%	1511	70%	1203*	70%	1432
5.0	3205	40%	1493	70%	1211	50%	1398
6.0	2196	50%	1459*	70%	1212	50%	1369*
7.0	2268	40%	1475	70%	1251	50%	1382

Table 7.6: DS2 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	175	5%	145	40%	112	20%	152
1.0	142	5%	120	30%	94	20%	115
1.5	136	10%	113	30%	91	20%	109
2.0	126	10%	106	20%	86	20%	100
3.0	120*	10%	98	20%	83*	20%	93
4.0	126	10%	100	20%	86	20%	95
5.0	122	10%	97*	20%	86	20%	93
6.0	126	10%	98	20%	87	20%	93
7.0	125	10%	97*	20%	88	20%	92*

From Tables 7.5–7.6 we observe that, in general, the 1-sided layout outperforms the other layouts (lower total hours), followed by the 2-sided layout. Also, we observe that for these two data sets the percent of SKUs included in the forward area varies by the type of forward area as well. These two observations hold for the remaining data sets, which are presented in Appendix B.

Table 7.7 provides a summary for all twenty data sets, listing the best shape and forward area size for each of the three layouts. From this set of twenty examples and their associated skewness levels, a forward area is warranted in all the data sets. That is, the savings in order picking from the forward area outweighs the extra labor in replenishment, even for data sets 2–5, 14, 17 and 20, where the number of cases per pallet is relatively low (approximately 8–15). In addition, the impact of pallet-area shape is the greatest for shapes of 0.5–2.0, but shapes higher than 2.0 yield more similar results in terms of required labor hours. In general, pallet-area shapes of 3.0 or higher result in the least labor. The examples also reveal that the one-sided, class-based storage layouts favor more SKUs in the forward area as compared to the random-storage forward area layout. As more SKUs are included in the class-based layouts, they are generally assigned to less favorable locations (i.e., class-C locations). Consequently, although the total area increases, the location of the fastest-moving SKUs does not change. Thus, the performance of class-based storage layouts do not deteriorate by adding additional SKUs, except in those cases where adding more SKUs necessitates an increase in the footprint of the pallet area.

In comparing the results presented in Tables 7.5, 7.6, and 7.21–7.38 in Appendix B, most of the data sets result in the least labor with ∼10% of the SKUs in the random storage forward area, ∼20-30% of the SKUs in the one-sided, class-based layout, and ∼10-20% in the two-sided, class based layout. However, the data sets with a low SKUs-to-bottom-level-pallets ratio (<= 1.0) favor higher percentages of SKUs in the forward area compared to the others. Thus, further investigation is necessary in order to determine the impact of this parameter on the best design.

Table 7.7: Summary of Best Shape and Forward Area Size

Warehouse	Random		1-Sided		2-Sided	
	Best Shape	SKUs	Best Shape	SKUs	Best Shape	SKUs
DS1	6.0	50%	4.0	70%	6.0	50%
DS2	3.0	10%	3.0	20%	3.0	20%
DS3	3.0	10%	3.0	20%	3.0	20%
DS4	3.0	10%	3.0	20%	3.0	20%
DS5	3.0	10%	3.0	20%	3.0	20%
DS6	3.0	10%	3.0	30%	6.0	20%
DS7	6.0	20%	3.0	30%	6.0	20%
DS8	6.0	30%	3.0	90%	6.0	40%
DS9	6.0	20%	4.0	30%	6.0	20%
DS10	4.0	10%	3.0	20%	3.0	20%
DS11	6.0	20%	6.0	50%	6.0	20%
DS12	6.0	30%	6.0	50%	6.0	40%
DS13	6.0	30%	6.0	60%	6.0	50%
DS14	4.0	5%	2.0	20%	4.0	20%
DS15	5.0	10%	5.0	20%	6.0	10%
DS16	6.0	10%	6.0	20%	6.0	20%
DS17	4.0	10%	2.0	10%	5.0	10%
DS18	6.0	30%	4.0	40%	6.0	20%
DS19	4.0	10%	3.0	40%	6.0	20%
DS20	4.0	40%	3.0	70%	3.0	90%

SKUs to Bottom-Level Pallets

In considering the impact of the SKUs-to-bottom-level-pallets ratio, data sets 1, 8 and 19–20 have SKUs-to-bottom-level-pallets ratios of less than 1.0, and data sets 7 and 11–13 have ratios of 1.0. The remaining data sets have ratios higher than 1.0. To determine if this ratio affects the optimal number of SKUs in the forward area, this ratio is adjusted (by varying the number of pallet locations). Table 7.8 lists the optimal percentage of SKUs in the random-storage forward area, and Tables 7.9 and 7.10 list the optimal percentage of SKUs in the forward areas for the 1-sided and 2-sided class-based storage layouts for a range of SKUs-to-bottom-level-pallets ratios. (Again, we fix the number of pallet levels to 6 and the pallet area shape to 3.0, as these values generally perform well as compared to other values for these variables.)

Table 7.8: Optimal SKUs in Random Forward Area*

Example	SKUs-to-bottom-pallets					
	0.3	0.5	1.0	2.0	3.0	4.0
DS1	50%	40%	30%	20%	20%	20%
DS2	20-40%	20-30%	20%	10-20%	10%	5-10%
DS3	20%	20%	10-20%	10%	10%	10%
DS4	20%	10-20%	10-20%	10-20%	10%	5-10%
DS5	20-40%	20-50%	20-40%	10-20%	5-20%	10%
DS6	10%	10%	10%	5%	5%	5%
DS7	10%	10%	10%	10%	5%	5%
DS8	30%	30%	20%	20%	20%	20%
DS9	20%	20%	20%	20%	10%	10%
DS10	30%	20%	20%	20%	20%	20%
DS11	20%	20%	20%	20%	20%	20%
DS12	40%	40%	30%	20%	20%	20%
DS13	30%	30%	30%	20%	20%	20%
DS14	30%	20-30%	10-20%	10%	10%	50%
DS15	30%	30%	20%	20%	10%	10%
DS16	30%	20%	20%	20%	10%	10%
DS17	20-30%	20-30%	10-20%	10%	5-10%	5-10%
DS18	30%	30%	40%	30%	20%	20%
DS19	10%	10%	10%	10%	10%	10%
DS20	40%	30%	40%	30%	30%	30%

* Results assume 6 pallet levels and a pallet area shape of 3.0.

Table 7.9: Optimal SKUs in 1-Sided, Class-Based Forward Area*

Example	SKUs-to-bottom-pallets					
	0.3	0.5	1.0	2.0	3.0	4.0
DS1	100%	100%	70%	40%	20%	20%
DS2	80-100%	70-80%	40-50%	20-30%	20%	10-20%
DS3	90-100%	70%	40-50%	20%	20%	10-20%
DS4	80-100%	50-80%	30-60%	20-30%	20%	20%
DS5	80-100%	60-100%	40-70%	20-40%	20%	20%
DS6	50%	50%	40%	30%	30%	10%
DS7	90%	70%	30%	20%	20%	20%
DS8	100%	90%	70%	40%	30%	20%
DS9	90%	80%	50%	30%	20%	20%
DS10	70%	60%	20%	20%	20%	20%
DS11	100%	100%	60%	40%	20%	20%
DS12	90%	90%	60%	40%	30%	20%
DS13	100%	100%	90-100%	50%	30%	30%
DS14	60-100%	50-70%	30-40%	20%	20%	10%
DS15	90%	90%	60%	30%	30%	20%
DS16	100%	100%	100%	50%	30%	20%
DS17	60-90%	50-60%	30-40%	20%	10-20%	10%
DS18	100%	100%	70%	40%	40%	20%
DS19	60%	60%	30%	20%	20%	10%
DS20	100%	100%	60%	40%	40%	40%

* Results assume 6 pallet levels and a pallet area shape of 3.0.

Table 7.10: Optimal SKUs in 2-Sided, Class-Based Forward Area*

Example	SKUs-to-bottom-pallets					
	0.3	0.5	1.0	2.0	3.0	4.0
DS1	70%	50%	20%	20%	20%	20%
DS2	20%	50%	20-40%	20%	10-20%	10-20%
DS3	20%	30%	20-30%	20%	10-20%	10-20%
DS4	40%	20-30%	20-30%	20%	20%	20%
DS5	60%	50-60%	20%	20%	20%	20%
DS6	20%	30%	30%	20%	20%	20%
DS7	40%	30%	20%	20%	20%	20%
DS8	60%	50%	40%	30%	20%	20%
DS9	40%	30%	30%	20%	20%	20%
DS10	30%	20%	20%	20%	20%	20%
DS11	40%	20%	20%	20%	20%	20%
DS12	60%	60%	30%	30%	20%	20%
DS13	90%	50%	40%	40%	30%	20%
DS14	40%	20-30%	20%	20%	20%	20%
DS15	60%	50%	30%	20%	20%	10%
DS16	50%	30%	20%	20%	20%	20%
DS17	30%	20%	20%	20%	20%	20%
DS18	80%	50%	20%	20%	20%	20%
DS19	30%	20%	20%	20%	20%	10%
DS20	100%	70%	60%	40%	30%	30%

* Results assume 6 pallet levels and a pallet area shape of 3.0.

Again, a SKUs-to-bottom-level-pallets ratio of 1.0 or less implies that all of the SKUs can be located in bottom-level pallet positions, and values greater than 1.0 indicate that the footprint of the pallet area would have to grow in order to accommodate all SKUs on the bottom level. Thus, in moving from left to right in Tables 7.8 to 7.10, it is not surprising that the optimal number of forward SKUs decreases as the number of available bottom-level locations decreases. When there are few SKUs compared to bottom-level pallets, intuitively, more SKUs should be placed in the forward area to minimize travel. Also, note that the 1-sided forward area layout favors significantly more SKUs in the forward area for SKUs-to-bottom-level-pallets ratios of 1.0 or less as compared to the random storage and 2-sided forward area layouts.

Demand Skewness

Next, we consider the effect of demand skewness on the optimal size of the forward area. We evaluate the example data sets using three levels of demand skewness as depicted in Figure 7.5: average skewness (80/20, such that 20% of the items represent 80% of the demand), moderately skewed (60%/20%), and hardly skewed (40%/20%). In comparing the percent of SKUs in the forward area across all data sets, we assume that the number of pallet positions is such that all SKUs can have a bottom-level location (with the number of bottom-level locations approximately equal to the number of SKUs).

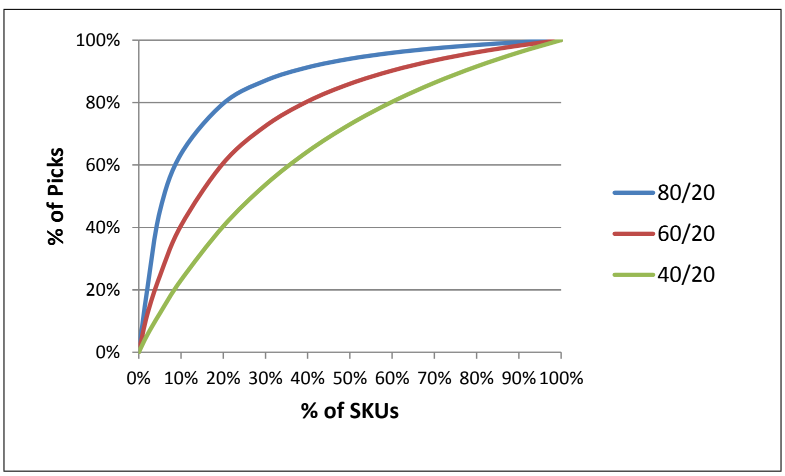


Figure 7.5: Demand skewness levels.

The results for the example data sets are listed in Tables 7.11–7.13, with the forward area size that results in the least amount of travel for each layout.

Table 7.11: Random Forward Area Sizes for Different Skewness Levels*

Example	No Forward Area	Random Forward Area					
		80/20		60/20		40/20	
		% SKUs	Hours	% SKUs	Hours	% SKUs	Hours
DS1	1695	20%	1164	20-30%	1309	40-50%	1406
DS2	182	20%	138	30%	155	50-60%	169
DS3	169	20%	127	30%	139	30-50%	150
DS4	93	20%	63	20-30%	70	40%	75
DS5	45	20%	31	20-30%	35	40-70%	38
DS6	473	10%	332	20%	373	30%	400
DS7	961	20%	731	20%	786	30%	832
DS8	1178	20%	659	40%	782	70%	859
DS9	686	20%	520	40%	607	80%	660
DS10	104	20%	87	20%	96	20%	105
DS11	1554	20%	858	20%	1025	40%	1146
DS12	886	10%	575	30%	652	40%	708
DS13	971	20%	477	30%	575	50-60%	650
DS14	103	10%	87	20%	92	20%	96
DS15	921	10%	675	20%	746	30%	796
DS16	3510	20%	1745	20%	2168	40%	2512
DS17	98	20%	78	20-30%	83	30-40%	87
DS18	544	20%	323	30%	374	40-50%	409
DS19	699	20%	492	30%	537	40%	575
DS20	48	20%	21	40%	24	70%	26

* Results assume 6 pallet levels, a pallet area shape of 3.0, and a SKUs-to-bottom-level-pallets ratio of 1.0.

Table 7.12: 1-Sided Forward Area Sizes for Different Skewness Levels

Example	No Forward Area	1-Sided Forward Area					
		80/20		60/20		40/20	
		% SKUs	Hours	% SKUs	Hours	% SKUs	Hours
DS1	1695	40%	931	60%	1124	50%	1295
DS2	182	50%	105	60%	127	60-90%	149
DS3	169	40-50%	98	60%	117	60-70%	136
DS4	93	40-50%	50	60%	59	60-90%	69
DS5	45	40-50%	24	60-70%	29	60-100%	34
DS6	473	40%	269	40%	325	40%	375
DS7	961	40%	545	60%	637	60%	720
DS8	1178	50%	507	100%	602	100%	704
DS9	686	50%	391	90%	479	100%	558
DS10	104	20%	74	20%	88	20%	110
DS11	1554	60%	713	100%	877	100%	1049
DS12	886	50%	442	60%	535	70%	624
DS13	971	60%	390	100%	480	100%	576
DS14	103	20%	72	40%	81	40%	89
DS15	921	50%	496	60%	603	60%	703
DS16	3510	100%	1405	100%	1819	100%	2320
DS17	98	40%	62	40%	71	40-50%	79
DS18	544	50%	252	70%	312	80%	369
DS19	699	40%	388	60%	461	60%	526
DS20	48	40%	18	60%	21	100%	23

* Results assume 6 pallet levels, a pallet area shape of 3.0, and a SKUs-to-bottom-level-pallets ratio of 1.0.

Table 7.13: 2-Sided Forward Area Sizes for Different Skewness Levels

Example	No Forward Area	2-Sided Forward Area					
		80/20		60/20		40/20	
		% SKUs	Hours	% SKUs	Hours	% SKUs	Hours
DS1	1695	20%	1084	40%	1256	50%	1382
DS2	182	20%	130	40%	149	50-80%	164
DS3	169	20%	119	40%	135	40-50%	148
DS4	93	20%	58	30-60%	68	40-60%	74
DS5	45	20%	29	30-70%	34	50-80%	37
DS6	473	20%	306	30%	361	30%	396
DS7	961	20%	678	30%	768	40%	824
DS8	1178	40%	601	60%	738	100%	821
DS9	686	20%	477	60%	574	80%	636
DS10	104	20%	83	20%	95	20%	105
DS11	1554	30%	768	40%	977	40%	1128
DS12	886	20%	529	40%	630	40%	698
DS13	971	40%	429	40%	540	60%	634
DS14	103	20%	84	20%	91	20%	95
DS15	921	20%	629	20%	732	40%	788
DS16	3510	20%	1600	40%	2121	40%	2482
DS17	98	20%	74	40%	78	20-30%	86
DS18	544	20%	292	40%	356	50%	402
DS19	699	20%	461	40%	517	40%	567
DS20	48	40%	18	60%	22	70%	25

* Results assume 6 pallet levels, a pallet area shape of 3.0, and a SKUs-to-bottom-level-pallets ratio of 1.0.

In general, the optimal forward area size increases as the skewness decreases, as indicated in Tables 7.11–7.13. For an ABC curve with average skewness, the optimal percentage of SKUs in the random storage forward area is approximately 20%; a moderately skewed curve favors 20–40% of the SKUs, and the hardly skewed curve performs well with 40–70% of the SKUs in the forward area.

Again, the 1-sided layout in Table 7.12 outperforms the other layouts. The curve with an average skewness results in the least travel for approximately 40–50% of the SKUs in the forward area; the moderately skewed curve favors about 60% of the SKUs, and the hardly skewed curve performs best with 60% or more of the SKUs in the forward area.

For the 2-sided forward area layout, the curve with an average skewness performs well with 20% of the SKUs; the moderately skewed curve favors about 40-60% of the SKUs, and the hardly skewed curve performs the best with around 40–80% of the SKUs in the forward area. Note that for a hardly skewed curve, the random storage forward area and 2-sided forward area have similar performance, especially for the data sets with a lower throughput requirement. Thus, for lower demand skewness, the random storage forward layout may be preferred for a doors-on-two-sides configuration, as random storage is generally easier to maintain than class-based storage.

Cases Per Pallet

The number of cases per pallet, along with the number of picks per line, affect the number of replenishments. Next, we evaluate various combinations of cases-per-pallet and picks-per-line (average values) to determine if there are any situations where a forward area is not preferred. The number of cases per pallet for order picking is assumed to be approximately 80 percent of the number of cases on incoming pallets.

Table 7.14 lists the labor hours for an ABC curve with average skewness, and Table 7.15 lists results for a hardly skewed ABC curve for DS2 for various cases-per-pallet and picks-per-line combinations. (The travel-time model for the 2-sided, class-based layout requires at least three pick lines per tour, so travel times are blank for pick lines of less than three.)

Table 7.14: DS2 Labor Hours for an ABC Curve with Average Skewness

Incoming cases/pallet	Picks/ Line	Avg Lines	No Forward	Random Forward	1-Sided Forward	2-Sided Forward
10	1	8	241	167	128	153
10	5	1.6	99	87	72	–
10	10	0.8	69	68	59	–
20	1	16	202	130	101	117
20	5	3.2	88	73	61	68
20	10	1.6	66	59	51	–
50	1	40	165	92	78	83
50	5	8	74	59	48	54
50	10	4	58	50	42	47

Table 7.15: DS2 Labor Hours for an ABC Curve with Low Skewness

Incoming cases/pallet	Picks/ Line	Avg Lines	No Forward	Random Forward	1-Sided Forward	2-Sided Forward
10	1	8	241	198	176	196
10	5	1.6	99	94	88	–
10	10	0.8	69	69	66	–
20	1	16	202	160	143	157
20	5	3.2	88	80	75	79
20	10	1.6	66	63	60	–
50	1	40	165	120	110	117
50	5	8	74	65	60	64
50	10	4	58	54	49	53

As expected, the benefit of having a forward area is diminished as the number of picks per line is high relative to the capacity of the pallet, especially for a hardly skewed ABC curve. Also, in situations where the savings of having a forward area is very small, failure to choose the optimal percentage of SKUs in the forward area may actually result in higher labor for the forward area layouts as compared to a random storage layout with no forward area. However, if the number of bottom-level pallets is much greater than the number of bottom level SKUs, a forward area may be justified, even for a high number of picks per line. Note also that even though we only consider one pallet for each SKU in the forward area, including all of the reserve locations in the forward area for fast-moving SKUs that have a very high number of picks per line may be beneficial.

Pallet Rack Height

Finally, we investigate the variable for pallet rack height by evaluating the labor required for pallet rack levels of 4, 5, and 6. A pallet area shape of 3.0 is considered for the example data sets. The results for data sets 1 and 2 are listed in Tables 7.16 and 7.17, respectively, with the results for the remaining data sets in Appendix C.

Table 7.16: DS1: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	2388	2300	2212	–	–	–	–	–	–
20	1785	1700	1647	1667	1591	1536	1677	1601	1540
40	1724	1654	1580	1462	1405	1346	1627	1563	1519
60	1766	1704	1619	1352	1312	1256	1625	1586	1507
80	1854	1765	1706	1287	1239	1217	1641	1587	1521
100	1936	1858	1792	1254	1225	1208	1659	1611	1541

For DS1 all of the SKUs can be accommodated on the bottom level of pallet rack for the three levels of pallet rack considered. Table 7.16 lists the labor required for the different levels of pallet rack for various percentages of SKUs in the forward area (note that the random storage layout with no forward area is included as 0% of the SKUs with the random storage forward area). For each layout in Table 7.16, the travel times decrease as the number of

Table 7.17: DS2: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	127	127	120	–	–	–	–	–	–
20	107	107	101	86	86	83	99	99	93
40	120	119	118	90	91	92	106	105	105
60	138	139	141	100	102	104	118	120	122
80	160	161	163	111	113	114	134	136	137
100	177	178	180	119	121	122	145	146	148

pallet levels increase. For this example data set, the decrease in the footprint of the pallet area results in labor savings that are more than the labor increases associated with the extra vertical travel for higher levels of pallet rack.

For DS2 listed in Table 7.17, the labor hours decrease as the pallet rack height increases for forward areas that have 20% or less SKUs in the forward area. However, with 10,831 SKUs and at most 6,000 bottom-level locations (for 4 levels of pallet rack), not all SKUs can receive a bottom-level location without increasing the footprint of the warehouse. Thus, for more than 40% of the SKUs in the forward area, the travel times increase for higher levels of pallet rack due to the larger footprint of the pallet area.

From this analysis, a smaller pallet rack footprint (with higher levels of pallet rack) is preferred if all SKUs can be accommodated on the bottom level. Further, we observe that having less SKUs in the forward area is preferred to increasing the footprint of the warehouse in order to make room for more bottom-level SKUs.

7.5. Heuristic for Designing Manual, Case-Picking Warehouses

We use the results from our analysis of the examples in the previous section to develop a heuristic search procedure for determining a base design for a given set of data. We envision that the base design could be further improved through additional analyses. The first step in the design process is to perform an analysis of data to determine the warehouse parameters including the number of SKUs, inventory requirements (number of pallet locations), an activity profile of the SKUs, the average number of cases per pallet and picks per line, as well as an estimated number of dock doors required to meet throughput requirements.

Because keeping the footprint of the pallet rack area smaller results in less travel (and determines how many SKUs can be allocated to the forward area designs), this decision should be considered first. The number of pallet levels should be as high as possible to allow a smaller pallet area footprint such that the horizontal travel component is minimized. In addition, the warehouse should be sized such that the pallet area shape is approximately 3.0 or higher (though higher shape ratios result in larger warehouses due to the additional staging area for dock doors). The next decision is to determine if implementing a forward area is justified. In general, unless a high number of picks per lines results in an extremely low number of lines per batch (e.g., <= 2 lines per batch), a forward area is likely justified.

If a forward area is desirable, the forward area layout should be chosen next. The 1-sided class-based layout should be chosen if a doors-on-one-side configuration allows a sufficient number of dock doors to meet throughput requirements. Otherwise, the random storage forward area or 2-sided class-based layout should be chosen. For an average to moderately skewed ABC curve, the 2-sided layout is preferred, but a random-storage forward area may be desirable for a hardly skewed ABC curve. Finally, the size of the forward area can be determined based on the SKUs-to-bottom-level-locations ratio and the skewness of the ABC curve.

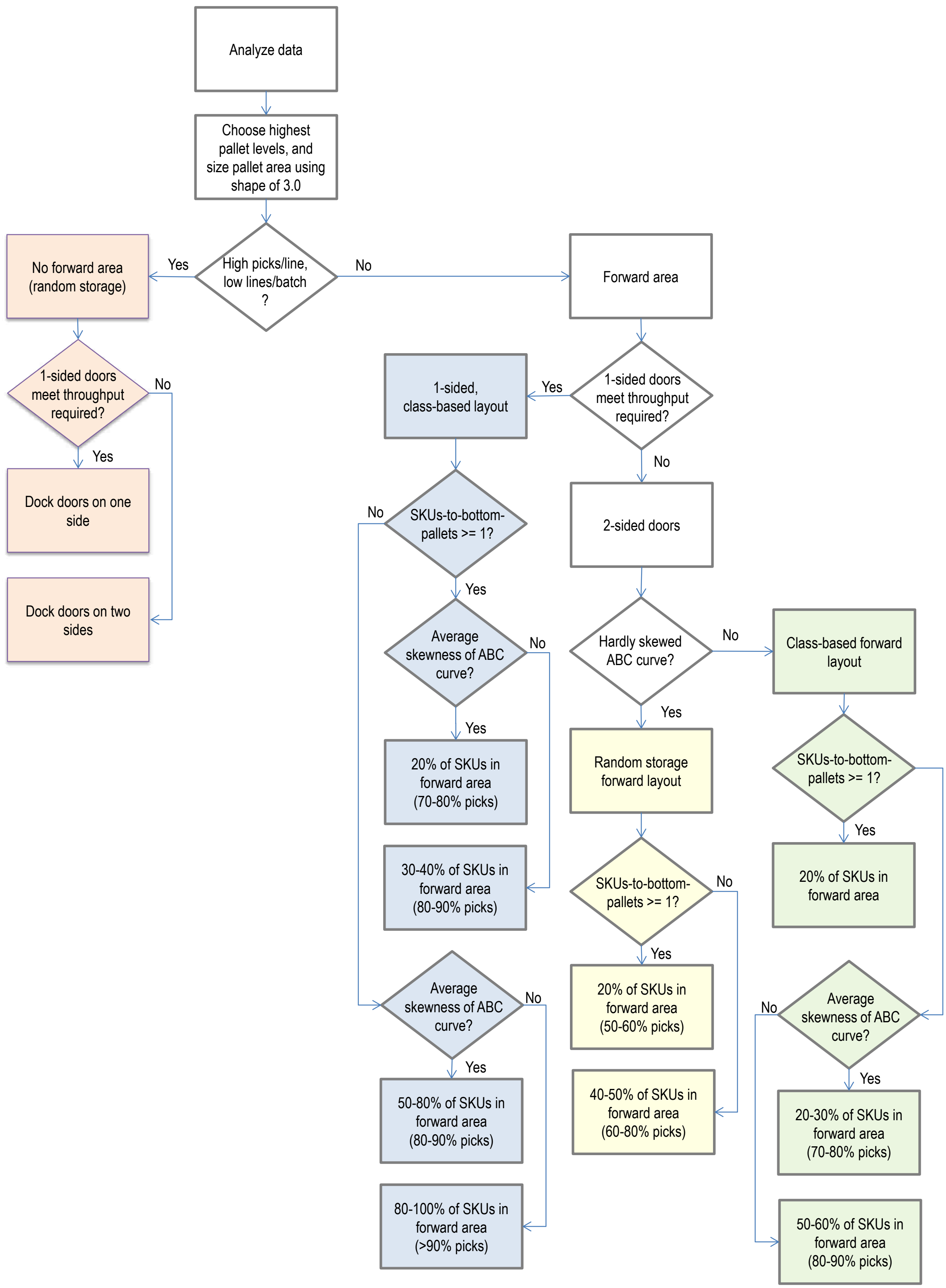


Figure 7.6: Heuristic procedure for choosing a design.

The heuristic search procedure depicted in the flowchart in Figure 7.6 is further described as follows:

1. Perform a data analysis to determine the warehouse parameters.

2. Choose the highest number of pallet levels possible, and size the pallet area using a shape of 3.0.

3. Evaluate the number of case picks per line relative to the pallet capacity for order picking. For a high number of case picks per line (for most of the SKUs) that results in *very few* pick lines per tour, use a random storage layout without a forward area, and continue with Step 4. Otherwise, go to Step 5.

4. Determine if the number of dock doors available with a doors-on-one-side configuration meets throughput requirements. If not, choose the doors-on-two-sides configuration, requiring a larger footprint due to staging areas on both sides of the warehouse. Otherwise, choose the doors-on-one-side configuration. The base warehouse design has been determined, and the search procedure can be terminated.

5. Determine if the number of dock doors on one side of the facility is sufficient to meet throughput requirements. If so, choose the 1-sided, class-based forward area layout and continue with Step 6; otherwise, go to Step 9.

6. Evaluate the number of SKUs to bottom-level pallets. For values greater than or equal to 1.0, continue with Step 7; otherwise go to Step 8.

7. Size the forward area with 30% of the SKUs for a less skewed ABC curve (i.e., 70%/20% or less) or with 20% of the SKUs for moderately to highly skewed curve. Terminate the search procedure.

8. For SKUs to bottom-level pallets of less than one (such that there are more bottom-level pallets than SKUs), size the forward area with 50–80% of the SKUs for a more

skewed ABC curve (i.e., more than 70%/20%) or with 80–100% of the SKUs for a less skewed curve. Terminate the search procedure.

9. For a hardly skewed ABC curve (i.e., 40%/20% or less), choose the random storage forward area, and continue with Step 10; otherwise go to Step 11.

10. Evaluate the number of SKUs to bottom-level pallets. For values greater than or equal to 1.0, size the forward area with 20% of the SKUs; otherwise, size the forward area with 40–50% of the SKUs. Terminate the search procedure.

11. Choose the 2-sided class-based forward area layout, and evaluate the SKUs to bottom-level pallets. For values greater than or equal to 1.0, size the forward area with 20% of the SKUs and terminate the search procedure; otherwise, continue with Step 12.

12. For highly skewed ABC curves (70%/20% or more), size the forward area with 20–30% of the SKUs; otherwise, size the forward area with 50–60% of the SKUs. Terminate the search procedure.

Next, we apply the heuristic to test data sets and compare the results to the best design obtained by enumerating over a large range of parameters (pallet area shapes of 0.5 to 7.0 and forward area sizes from 5 to 100 percent of the SKUs). The test data is provided in Table 7.18.

Table 7.19 lists the results for the 1-sided dock door configuration, and Table 7.20 displays the results for the 2-sided dock door configuration. The designs with a doors/layout of 1S, 2S and 2R refer to the 1-sided layout, the 2-sided layout, and the random storage layout (with 2-sided dock doors), respectively.

For designs with 1-sided dock doors, the heuristic solution resulted in six of the test data sets that are within 3% of the best design (obtained through complete enumeration). For the remaining 4 data sets, the heuristic resulted in designs within 6% of the best design. For the designs with 2-sided dock doors, the heuristic solution resulted in solutions that were

Table 7.18: Test Data Sets for Order Data

Data Set	Pallet locns	SKUs	Incoming cases per pallet	Case picks per day	Avg picks per line	Skewness	
						A	%Lines/%SKUs
TS1	10,000	4,200	30	9,600	2.2	0.071	77/20
TS2	25,000	2,500	60	72,000	3.0	0.053	81/20
TS3	30,000	7,500	50	120,000	5.0	0.024	91/20
TS4	15,000	8,000	15	6,300	1.0	0.079	75/20
TS5	35,000	11,100	25	38,500	1.5	0.122	68/20
TS6	40,000	6,600	40	48,000	1.8	0.107	72/20
TS7	50,000	15,000	50	90,000	3.2	0.096	74/20
TS8	25,000	10,000	20	21,000	1.2	0.253	55/20
TS9	32,000	10,000	70	80,640	2.6	0.024	91/20
TS10	28,000	22,000	24	25,536	1.3	0.053	81/20

Table 7.19: Comparison of Best Design to Heuristic for 1-Sided Door Configuration

WH	SKUs-to-Bottom Pallets	ABC Skewness	Best Design (Doors, shape, fwd SKUs, hours)	Heuristic (Doors/layout, shape, fwd SKUs, hours)	% Diff
TS1	2.5	Mod	1S, 4.0, 20%, 69	1S, 3.0, 20%, 71	3%
TS2	0.6	Avg	1S, 6.0, 50%, 298	1S, 3.0, 50-80%, 303-311	2-4%
TS3	1.5	Avg	1S, 4.0, 20%, 542	1S, 3.0, 20%,552	2%
TS4	3.2	Mod	1S, 4.0, 20%, 120	1S, 3.0, 20%,124	3%
TS5	1.9	Mod	1S, 5.0, 30%, 653	1S, 3.0, 20%,668	2%
TS6	1.0	Mod	1S, 6.0, 50%, 501	1S, 3.0, 50-80%, 508-515	1-3%
TS7	1.8	Mod	1S, 4.0, 30%, 788	1S, 3.0, 20%, 825	5%
TS8	2.4	Hardly	1S, 6.0, 30%, 454	1S, 3.0, 20%, 483	6%
TS9	1.9	Avg	1S, 3.0, 30%, 425	1S, 3.0, 20%, 442	4%
TS10	2.4	Avg	1S, 6.0, 10%, 443	1S, 3.0, 20%, 456	3%

Table 7.20: Comparison of Best Design to Heuristic for 2-Sided Door Configuration

WH	SKUs-to-Bottom Pallets	ABC Skewness	Best Design (Doors, shape, fwd SKUs, hours)	Heuristic (Doors, shape, fwd SKUs, hours)	% Diff
TS1	2.5	Mod	2S, 4.0, 20%, 74	2S, 3.0, 20%, 76	3%
TS2	0.6	Avg	2S, 6.0, 50%, 318	2S, 3.0, 20-30%, 336-368	6-15%
TS3	1.5	Avg	2S, 6.0, 20%, 623	2S, 3.0, 20%, 673	8%
TS4	3.2	Mod	2S, 6.0, 20%, 129	2S, 3.0, 20%, 136	5%
TS5	1.9	Mod	2S, 6.0, 20%, 676	2S, 3.0, 20%, 704	4%
TS6	1.0	Mod	2S, 6.0, 30%, 552	2S, 3.0, 50-60%, 607-618	9-12%
TS7	1.8	Mod	2S, 6.0, 20%, 886	2S, 3.0, 20%, 940	6%
TS8	2.4	Hardly	2S, 6.0, 20%, 471	2R, 3.0, 20%, 506	7%
TS9	1.9	Avg	2S, 5.0, 20%, 470	2S, 3.0, 20%, 482	3%
TS10	2.4	Avg	2S, 6.0, 20%, 503	2S, 3.0, 20-30%, 519-602	3-15%

within 8% for seven of the 10 test cases. The heuristic results for TS2, TS6, and TS10 were within 6–15%, 9–12% and 3–15%, respectively. For the 2-sided warehouses, the heuristic results in slightly higher differences overall, ranging from 3–15%. In many of the problems the differences are mostly attributed to warehouse shape. That is, our heuristic sets the shape to 3.0. Extending the heuristic to also choose warehouse shape can be considered to improve its performance.

7.6. Conclusions

A heuristic was presented for designing a manual, case-picking warehouse. Embedded in the heuristic are our findings related to the effect of warehouse parameters and design variables on design performance. First, the footprint of the pallet area significantly impacts the travel time for put away, order picking and replenishment, as one might expect. Our analysis assumed fixed warehouse parameters for the number of pallet locations, yet the days-on-hand inventory is an important decision that should be carefully evaluated prior to the design process. In other words, an increase in inventory results in more storage space, and more storage space translates to increased travel, and thus higher labor hours. Also, for the range of levels that we examined, the highest possible pallet rack height resulted in reduced travel due to the reduction in the footprint of the warehouse. However, our labor hour models do not consider worker blocking, and this factor should be included in the final design process.

In most cases, a forward area is preferred for even slightly skewed ABC curves. However, if the number of picks per line is high compared to the batch capacity, a random storage warehouse without a forward area may be justified. The 1-sided forward area layout outperforms the random storage and 2-sided forward area layouts, but the number of dock doors is a constraint that needs to be considered. If a doors-on-two-sides configuration is necessary to meet the required number of doors (based on dock-door throughput requirements), the 2-sided layout is preferred to the random storage layout, except for cases where the ABC curve is hardly skewed.

In general, a pallet area shape of 3.0 or more performs well for all layouts, and the performance of the design diminishes for pallet area shapes of less than 2.0. We limited our heuristic to include only pallet area shapes of 3.0, and based on the results of our test data sets, many of the differences between the optimal design and the heuristic solution are attributed to this limitation. Extending our heuristic to include higher shape ratios may improve its performance, and this is an area of future research.

In determining the size of the forward area, the SKUs-to-bottom-level-pallets ratio should be considered, along with the ABC curve skewness. SKUs-to-bottom-level-pallets ratios of 1.0 or more favor less SKUs in the forward area than ratios of less than 1.0, as increasing the number of SKUs would require a larger footprint. In addition, ABC curves with average skewness perform better with fewer SKUs in the forward area as compared to hardly skewed curves.

The heuristic search procedure presented in Section 7.5 provides a base design that can be further analyzed and optimized. The heuristic focuses on the labor hours required for put away, order picking and replenishment. Additonal factors such as congestion and blocking should be considered as well. But more importantly, building and equipment costs should be considered in the subsequent analysis. Accordingly, the warehouse design problem is multi-objective, as the best design in terms of minimizing labor may not be optimal. For example, considering higher shape ratios may lead to designs with lower labor hours, yet higher shape ratios also result in buildings with higher construction costs due to the additional staging area.

There are other fine-tuning aspects of a design that should be considered. For example, our base designs assume only one pallet allocation for each SKU in the forward area. If the number of pick lines is high for some SKUs, inclusion of all stored pallets in the forward area may be warranted for these SKUs. Accordingly, this may affect the optimal number of SKUs in the forward area. Other aspects of design can be considered as well during a fine-tuning step.

In summary, this paper provides a search heuristic that reduces the number of designs that require consideration for a case-picking warehouse. In addition, insights concerning the effect of warehouse parameters on design performance presented in this paper may prove useful to practitioners in the overall design process.

Acknowledgements

This research was supported, in part, by the National Science Foundation Industry-University Cooperative Research Center for Excellence in Logistics and Distribution (CELDi). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

The authors would like to thank Fortna (`www.fortna.com`), a multi-national provider of supply chain professional services, for supplying us with data. In addition, we thank the CELDi member organization (`www.celdi.org`) for providing data.

Bibliography

[1] Bartholdi, J. J., and Hackman, S. T., "Allocating Space in a Forward Pick Area of a Distribution Center for Small Parts," *IIE Transactions*, 40, 1046–1053 (2008).

[2] Bartholdi, J. J., and Hackman, S. T., *Warehouse & Distribution Science*, Version 0.95 (2011).

[3] Bozer, Y. A., *Optimizing Throughput Performance in Designing Order Picking Systems*, PhD thesis, Georgia Institute of Technology (1985).

[4] Francis, R. L., "On Some Problems of Rectangular Warehouse Design and Layout," *The Journal of Industrial Engineering*, 18, 10, 595–604 (1967).

[5] Frazelle, E. H., Hackman, S. T., Passy, U., and Platzman, L. K., "The Forward-Reserve Problem," in Ciriani, T. A., and Leachman, R. C., editors, *Optimization in Industry 2*, 43–61. John Wiley & Sons Ltd. (1994).

[6] Frazelle, E. H., Hackman, S. T., and Platzman, L. K., "Improving Order Picking Productivity through Intelligent Stock Assignment Planning," in *Proceedings of the Council of Logistics Management*, 353–371 (1989).

[7] Gu, J., Goetschalckx, M., and McGinnis, L. F., "Research on Warehouse Design and Performance Evaluation: A Comprehensive Review," *European Journal of Operational Research*, 203, 539–549 (2010).

[8] Hackman, S. T., and Rosenblatt, M. J., "Allocating Items to an Automated Storage and Retrieval System," *IIE Transactions*, 22, 7–14 (1990).

[9] Hall, R. W., "Distance Approximations for Routing Manual Pickers in a Warehouse," *IIE Transactions*, 25, 4, 76–87 (1993).

[10] Meller, R. D., and Thomas, L. M., "Optimizing Distribution Center Configuration: A Practical View of a Multi-Objective Problem," in *Progress in Material Handling Research: 2010*, Charlotte, NC (2010).

[11] Parikh, P., and Meller, R. D., "A Travel-Time Model for a Person-Onboard Order Picking System," *European Journal of Operational Research*, 200, 2, 385–394 (2010).

[12] Petersen, C. G., "An Evaluation of Order Picking Routeing Policies," *International Journal of Operations & Production Management*, 17, 11, 1098–1111 (1997).

[13] van den Berg, J. P., Sharp, G. P., Gademann, A. N., and Pochet, Y., "Forward-Reserve Allocation in a Warehouse with Unit-Load Replenishments," *European Journal of Operational Research*, 111, 98–113 (1998).

A. Parameters for Case-Picking Warehouse Analysis

Parameter Class	Parameter		No Fwd Area	Fwd Area, Random Storage	Fwd Area, Class-based Storage
Throughput Requirements	Required pallet locations		✓	✓	✓
	Cases per pallet (avg.)		✓	✓	✓
	Case picks per day		✓	✓	✓
	Lines per batch		✓	✓	✓
	Picks per line		✓	✓	✓
Pallet Area Sizing	Column spacing[1]	W^C	✓	✓	✓
	Staging area depth[1]	s	✓	✓	✓
	Aisle width[1]	a	✓	✓	✓
	End aisle depth[1]	v	✓	✓	✓
	Pallet opening width[2]	P_w	✓	✓	✓
	Pallets per opening		✓	✓	✓
	Pallet opening depth[2]	P_d	✓	✓	✓
	Pallet opening height[2]	P_h	✓	✓	✓
	Width of vertical rack member[2]	h	✓	✓	✓
	Height of load beam		✓	✓	✓
	Flue space[2]	v	✓	✓	✓
	Dock door width[1]	d_w	✓	✓	✓
	Distance between doors[1]	b	✓	✓	✓
Activity Profile	Number of SKUs			✓	✓
	ABC Curve			✓	✓
	Partition of ABC Curve				✓
Travel Speed	Horizontal travel speed		✓	✓	✓
	Vertical travel speed		✓	✓	✓
Decision Variables	Levels of pallet rack		✓	✓	✓
	Pallet area shape		✓	✓	✓
	Forward area SKUs			✓	✓
	Dock door configuration		✓	✓	✓

[1] See Figure 7.8.
[2] See Figure 7.9.

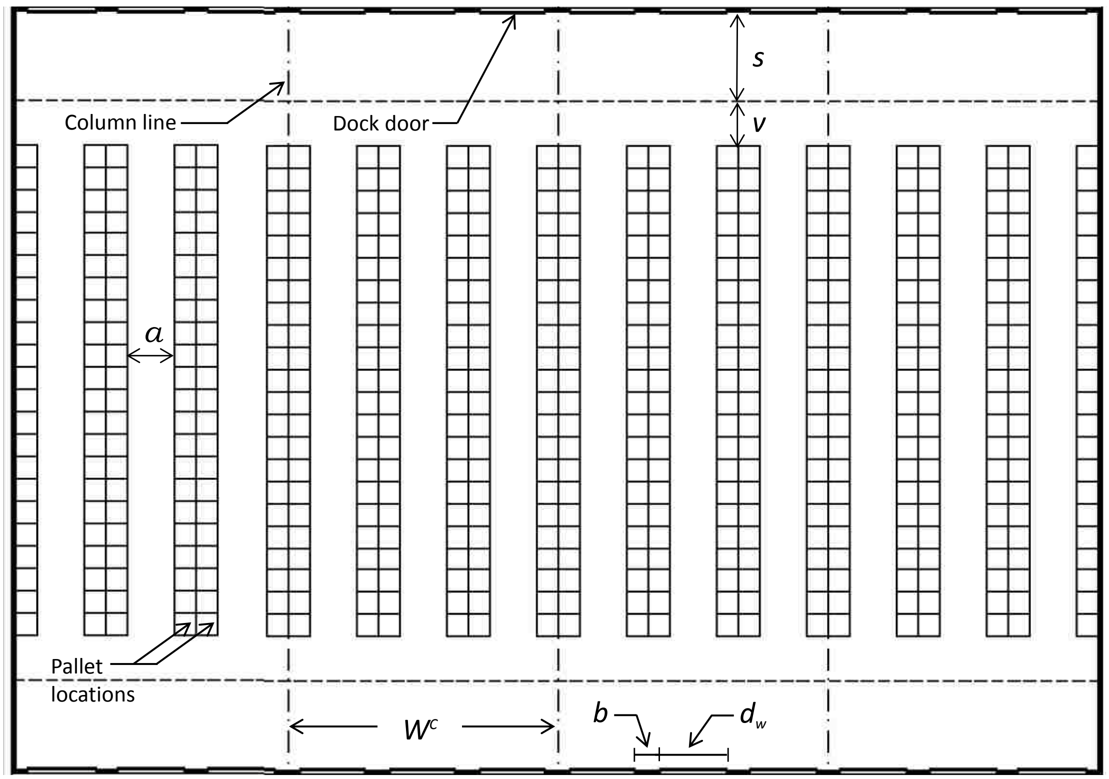


Figure 7.8: Warehouse Parameters

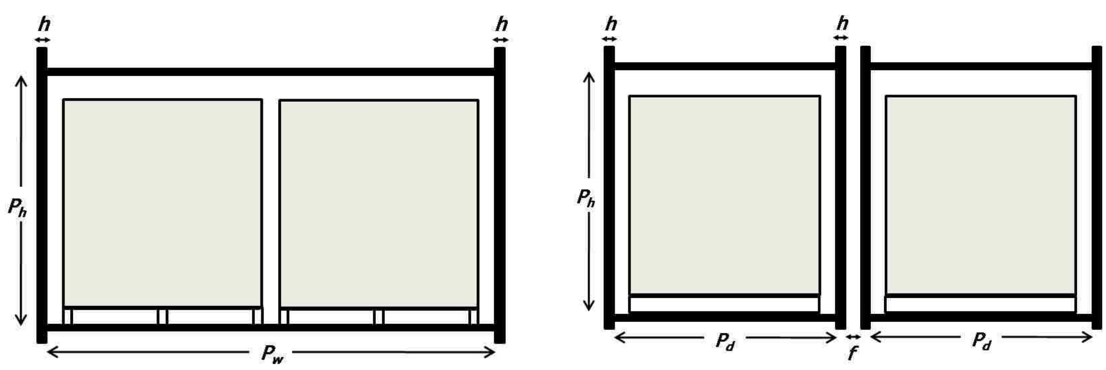


Figure 7.9: Pallet opening: (a) Front view; (b) Side view.

Calculation for Number of Aisles per Column Section, n_c:

$$n_c = \left\lfloor \frac{W^c}{2P_d + f + a} \right\rfloor .$$

Constraint: The aisle width, a, should be adjusted to ensure that $(W^c/(2P_d + f + a))$ is an integer.

Calculation for Number of Dock Doors per Column Section, d_c:

$$d_c = \left\lfloor \frac{W^c}{d_w + b} \right\rfloor .$$

Constraint: The distance between adjacent dock doors, b, should be adjusted to ensure that $(W^c/(d_w + b))$ is an integer.

B. Tables for Forward Area Layout and Pallet Area Shape

Table 7.21: DS3 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	162	5%	130	40%	106	20%	136
1.0	132	5%	108	30%	88	20%	107
1.5	126	5%	104	30%	84	20%	101
2.0	118	5%	97	30%	81	20%	92
3.0	112*	10%	90	20%	78*	20%	87
4.0	112*	10%	89*	20%	78*	20%	86
5.0	114	10%	89	20%	80	20%	85*
6.0	117	10%	90	20%	81	20%	86
7.0	117	10%	89	10%	82	20%	86

Table 7.22: DS4 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	99	10%	70	60%	53	30%	64
1.0	76	5%	52	30%	44	20%	52
1.5	69	5%	49	20%	41	20%	46
2.0	73	5%	51	20%	42	20%	47
3.0	66*	10%	46	20%	39*	20%	43
4.0	68	10%	46	20%	40	20%	43
5.0	67	10%	46	20%	40	20%	43
6.0	67	10%	45*	20%	40	20%	42*
7.0	70	10%	47	20%	42	20%	44

Table 7.23: DS5 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	40	10%	32	40%	27	20%	30
1.0	32	10%	26	20%	23	20%	25
1.5	30	10%	24	20%	21	20%	23
2.0	30	10%	24	20%	21	20%	23
3.0	28*	10%	22*	20%	20*	20%	21*
4.0	29	10%	23	20%	21	20%	22
5.0	29	10%	23	20%	21	20%	22
6.0	28*	10%	22*	20%	21	20%	21*
7.0	30	10%	23	20%	22	20%	22

Table 7.24: DS6 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	687	10%	366	70%	277	20%	369
1.0	593	10%	322	50%	243	30%	309
1.5	540	10%	301	40%	227	20%	286
2.0	491	5%	278	40%	215	20%	253
3.0	473	10%	268	30%	214	20%	249
4.0	475	10%	269	30%	214	20%	244
5.0	468	10%	266	30%	216	20%	243
6.0	469	10%	266	30%	218	20%	238
7.0	474	10%	269	30%	225	20%	238

Table 7.25: DS7 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1314	5%	1083	100%	674	30%	1036
1.0	1083	5%	865	70%	601	20%	830
1.5	1045	5%	818	70%	584	30%	799
2.0	1004	10%	797	50%	569	20%	757
3.0	961	10%	743	30%	556	20%	688
4.0	969	20%	730	30%	563	30%	683
5.0	998	20%	729	30%	572	30%	675
6.0	1006	20%	719	30%	583	20%	668
7.0	1023	20%	724	30%	595	20%	670

Table 7.26: DS8 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1791	30%	1069	100%	745	70%	956
1.0	1483	20%	913	100%	619	30%	806
1.5	1045	20%	825	90%	569	40%	729
2.0	1263	20%	794	90%	549	50%	692
3.0	1178	20%	750	90%	538	40%	645
4.0	1158	20%	730	70%	545	40%	643
5.0	1164	20%	728	70%	554	30%	632
6.0	1117	30%	690	60%	539	40%	610
7.0	1148	30%	700	60%	552	40%	614

Table 7.27: DS9 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	912	10%	622	70%	449	20%	573
1.0	825	10%	569	80%	395	20%	496
1.5	766	10%	512	50%	375	30%	456
2.0	702	10%	476	50%	358	20%	431
3.0	686	20%	459	30%	355	20%	407
4.0	667	20%	440	30%	347	30%	396
5.0	665	20%	434	30%	350	20%	391
6.0	673	20%	431	30%	350	20%	387
7.0	684	20%	436	30%	361	20%	397

Table 7.28: DS10 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	134	5%	123	40%	96	20%	121
1.0	108	5%	94	20%	77	20%	91
1.5	102	5%	89	20%	73	20%	86
2.0	101	10%	85	20%	72	20%	83
3.0	104	10%	85	20%	73	20%	82
4.0	102	10%	82	20%	73	20%	80
5.0	103	10%	82	20%	74	20%	79
6.0	105	10%	83	20%	75	20%	80
7.0	108	10%	84	20%	78	20%	81

Table 7.29: DS11 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	2208	20%	1158	100%	970	20%	1224
1.0	1865	20%	1011	100%	817	20%	919
1.5	1790	20%	967	100%	782	20%	893
2.0	1688	20%	918	100%	755	20%	855
3.0	1554	20%	858	60%	713	20%	768
4.0	1510	20%	846	50%	698	20%	776
5.0	1503	20%	836	50%	696	20%	758
6.0	1473	20%	830	50%	693	20%	749
7.0	1461	20%	830	40%	702	20%	754

Table 7.30: DS12 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1231	20%	929	90%	744	30%	922
1.0	1054	20%	815	90%	636	40%	809
1.5	960	20%	738	90%	588	30%	721
2.0	928	30%	709	90%	572	30%	686
3.0	886	30%	667	60%	557	30%	647
4.0	887	30%	656	50%	557	30%	635
5.0	876	30%	644	50%	558	40%	619
6.0	878	30%	635	50%	556	40%	612
7.0	888	30%	638	50%	568	40%	615

Table 7.31: DS13 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1224	40%	688	100%	621	40%	639
1.0	1164	30%	624	100%	542	30%	597
1.5	1091	30%	604	100%	498	40%	574
2.0	1001	20%	564	100%	467	40%	534
3.0	971	30%	550	90%	455	40%	508
4.0	934	30%	530	80%	447	40%	497
5.0	921	30%	533	70%	449	40%	497
6.0	919	30%	520	60%	443	50%	485
7.0	925	30%	526	60%	451	40%	490

Table 7.32: DS14 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	85	5%	82	30%	69	20%	88
1.0	69	5%	65	20%	57	20%	68
1.5	65	5%	61	20%	55	20%	64
2.0	64	5%	60	20%	55	20%	62
3.0	67	10%	60	10%	56	20%	62
4.0	66	5%	59	10%	55	20%	61
5.0	67	10%	59	10%	56	20%	62
6.0	69	10%	60	10%	57	20%	63
7.0	71	10%	61	10%	58	20%	65

Table 7.33: DS15 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	738	5%	671	20%	632	20%	692
1.0	632	10%	571	20%	531	20%	580
1.5	576	10%	519	20%	483	10%	517
2.0	557	10%	499	20%	463	10%	497
3.0	532	10%	473	20%	443	10%	470
4.0	532	10%	468	20%	442	10%	466
5.0	525	10%	460	20%	439	10%	458
6.0	527	10%	460	20%	443	10%	457
7.0	533	10%	464	20%	450	10%	461

Table 7.34: DS16 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1979	10%	1325	10%	1445	20%	1422
1.0	2015	10%	1331	10%	1423	10%	1323
1.5	1948	10%	1299	20%	1327	10%	1280
2.0	1814	10%	1224	20%	1221	10%	1212
3.0	1787	10%	1199	20%	1166	20%	1182
4.0	1727	10%	1168	20%	1117	20%	1148
5.0	1702	10%	1161	20%	1097	20%	1132
6.0	1695	10%	1146	20%	1077	20%	1115
7.0	1697	10%	1153	20%	1081	20%	1122

Table 7.35: DS17 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	86	5%	79	30%	61	20%	80
1.0	69	5%	62	20%	51	10%	61
1.5	65	5%	57	20%	48	10%	56
2.0	64	5%	55	10%	48	10%	53
3.0	67	5%	56	10%	49	10%	53
4.0	66	10%	54	10%	49	10%	53
5.0	67	10%	54	10%	49	10%	52
6.0	68	5%	55	10%	50	10%	53
7.0	70	5%	56	10%	52	10%	54

Table 7.36: DS18 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	580	20%	406	60%	323	20%	374
1.0	507	20%	368	50%	320	20%	335
1.5	496	20%	353	60%	299	20%	323
2.0	464	20%	327	60%	284	20%	298
3.0	441	30%	309	40%	274	20%	284
4.0	438	30%	303	40%	272	20%	279
5.0	442	30%	302	40%	274	20%	279
6.0	434	30%	297	40%	273	20%	274
7.0	446	30%	303	40%	278	20%	280

Table 7.37: DS19 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	1228	5%	791	90%	468	50%	720
1.0	1009	10%	624	60%	411	40%	569
1.5	913	10%	595	60%	379	30%	510
2.0	883	10%	553	60%	372	20%	501
3.0	851	10%	530	40%	368	20%	455
4.0	861	10%	512	30%	370	30%	449
5.0	889	20%	524	30%	383	40%	463
6.0	869	20%	491	30%	374	20%	438
7.0	913	20%	505	30%	389	30%	445

Table 7.38: DS20 Labor for Varying Shapes and Layouts; $\alpha = 0.4$, 6 levels

Shape	No Forward Area	Forward Area Layouts					
		Random		1-sided		2-sided	
		SKUs	Hours	SKUs	Hours	SKUs	Hours
0.5	69	50%	31	80%	26	–	–
1.0	61	30%	28	90%	22	90%	25
1.5	60	30%	26	90%	21	90%	23
2.0	54	30%	25	60%	20	80%	21
3.0	52	30%	25	70%	20	90%	21
4.0	53	40%	24	70%	20	50%	21
5.0	53	50%	24	50%	21	50%	21
6.0	55	50%	24	50%	21	50%	21
7.0	56	50%	25	50%	22	60%	22

C. Tables Listing Daily Travel Time for Different Levels of Pallet Rack

Table 7.39: DS3: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	119	119	112	–	–	–	–	–	–
20	100	98	93	81	80	78	92	92	87
40	112	111	109	85	86	86	100	99	98
60	128	129	131	94	96	97	110	112	113
80	146	147	148	104	105	107	123	124	126
100	165	166	167	112	114	115	136	138	139

Table 7.40: DS4: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	71	66	66	–	–	–	–	–	–
20	52	49	48	42	40	39	46	44	43
40	59	55	55	43	42	42	50	48	48
60	65	65	66	46	46	47	54	54	54
80	73	74	75	50	51	52	59	60	61
100	82	83	84	53	54	55	65	66	67

Table 7.41: DS5: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	28	28	28	–	–	–	–	–	–
20	23	23	23	20	20	20	21	21	21
40	26	27	27	21	22	23	24	24	25
60	31	32	32	24	25	25	27	28	28
80	36	36	37	26	27	27	31	31	32
100	40	41	41	28	29	29	34	34	35

Table 7.42: DS6: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	511	492	473	–	–	–	–	–	–
20	315	300	289	253	241	233	263	251	249
40	372	353	337	226	219	215	283	278	263
60	421	402	388	239	236	237	315	298	290
80	438	424	420	255	257	261	326	316	311
100	456	452	456	269	273	277	339	334	337

Table 7.43: DS7: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	1056	1008	961	–	–	–	–	–	–
20	819	782	744	629	605	580	763	726	688
40	859	824	788	589	574	558	783	748	712
60	911	874	843	592	580	573	806	777	744
80	962	937	912	609	611	613	837	813	786
100	1021	996	976	641	644	654	869	842	817

Table 7.44: DS8: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	1256	1217	1178	–	–	–	–	–	–
20	784	773	750	700	680	657	720	722	700
40	826	790	764	629	603	583	710	673	645
60	880	843	802	594	573	551	705	699	661
80	940	894	860	582	562	551	748	707	692
100	998	953	913	567	554	546	764	744	706

Table 7.45: DS9: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	713	682	686	–	–	–	–	–	–
20	486	462	459	385	368	368	434	409	407
40	530	502	497	368	355	356	448	422	427
60	577	549	540	370	362	363	471	454	444
80	619	597	602	382	383	388	494	477	482
100	665	670	675	408	412	417	519	523	528

Table 7.46: DS10: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	107	105	104	–	–	–	–	–	–
20	91	88	87	74	73	72	85	83	83
40	100	98	98	80	79	81	92	90	90
60	111	110	113	87	88	91	100	98	101
80	125	127	130	97	99	101	109	112	114
100	138	140	142	105	107	110	118	120	123

Table 7.47: DS11: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	1638	1596	1554	–	–	–	–	–	–
20	931	882	858	842	800	773	845	803	768
40	1067	1013	975	793	758	729	888	840	803
60	1183	1130	1084	764	736	713	924	878	861
80	1282	1227	1177	761	740	723	945	927	903
100	1366	1311	1259	763	748	738	986	964	937

Table 7.48: DS12: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	961	923	886	–	–	–	–	–	–
20	732	704	674	698	670	640	722	696	–
40	736	706	673	627	602	577	705	680	648
60	766	737	704	595	576	557	719	687	660
80	805	777	747	587	575	563	735	712	684
100	846	821	795	600	594	588	768	745	719

Table 7.49: DS13: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	982	952	971	–	–	–	–	–	–
20	561	548	560	572	554	562	548	538	551
40	582	566	562	516	496	492	534	515	508
60	638	616	603	489	470	462	538	531	521
80	693	667	654	477	462	457	569	542	543
100	742	714	696	473	462	455	585	571	555

Table 7.50: DS14: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	70	68	67	–	–	–	–	–	–
20	66	65	64	57	56	56	64	63	62
40	77	76	78	63	64	66	71	71	73
60	92	94	96	72	74	76	83	85	86
80	105	107	109	80	82	84	93	95	96
100	116	117	119	87	89	91	100	102	104

Table 7.51: DS15: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	576	554	532	–	–	–	–	–	–
20	520	501	481	476	459	443	512	493	474
40	580	589	598	488	497	506	556	565	574
60	712	718	725	558	565	571	668	675	681
80	824	829	834	613	618	623	760	765	770
100	918	922	926	664	668	671	841	845	849

Table 7.52: DS16: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	1749	1721	1787	–	–	–	–	–	–
20	1273	1247	1246	1207	1168	1166	1203	1182	1182
40	1666	1689	1712	1300	1324	1347	1459	1482	1505
60	2066	2079	2093	1392	1405	1419	1698	1711	1724
80	2455	2463	2470	1461	1468	1476	1897	1905	1912
100	2815	2819	2823	1561	1565	1569	2134	2138	2142

Table 7.53: DS17: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	70	68	67	–	–	–	–	–	–
20	61	60	58	52	49	49	57	56	55
40	69	68	70	54	55	56	63	62	64
60	80	82	83	62	63	65	71	73	74
80	90	92	93	69	70	72	78	80	81
100	101	102	104	74	76	77	86	88	89

Table 7.54: DS18: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	457	436	441						
20	326	313	311	311	297	295	296	283	284
40	337	323	315	291	280	274	319	307	300
60	365	350	354	286	278	282	333	320	324
80	398	400	403	286	289	292	347	350	353
100	439	442	445	295	298	301	364	367	370

Table 7.55: DS19: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	937	894	851	–	–	–	–	–	–
20	602	563	539	435	411	396	513	481	455
40	651	618	584	394	381	368	519	492	491
60	693	656	627	389	379	374	543	513	506
80	730	699	666	402	398	395	570	558	528
100	764	738	711	423	426	428	598	584	555

Table 7.56: DS20: Daily Travel Time for Different Levels of Pallet Rack

% Forward SKUs	Random Layout			1-Sided Layout			2-Sided Layout		
	Levels			Levels			Levels		
	4	5	6	4	5	6	4	5	6
0	52	52	52	–	–	–	–	–	–
20	26	26	27	26	26	27	–	–	–
40	25	24	25	23	22	22	–	–	23
60	25	25	25	21	21	21	–	22	21
80	28	27	26	20	20	20	21	22	22
100	28	28	27	20	20	20	22	22	21

Chapter 8

Conclusions and Future Research

8.1. Conclusions

In achieving our objective of presenting a structured approach to overall warehouse design and providing a method for implementing the approach we formed a number of conclusions.

Forward area size and pallet area shape are important design considerations.

This research focused on the design of a manual, case-picking warehouse. Using the models that we developed for put away, order picking and replenishment, we determined that forward area size and pallet area shape are important design considerations. The optimal forward area size varies depending on the layout of the forward area, as well as the warehouse operating environment. In general, the 1-sided, class-based layout performs better with more SKUs in the forward area as compared to the random and 2-sided layouts. Further, a SKUs-to-bottom-level-pallets ratio of more than one implies that not all of the SKUs can be allocated to a bottom-level forward location without increasing the footprint of the pallet area.

We determined that the optimal pallet area shape varies by operation, and the optimal shape that considers all operations is not straightforward. However, the analytical models that we developed can be used to determine the optimal warehouse shape that considers all operations. We proved that the optimal shape for the put-away operation in a random storage warehouse with a uniform distribution of dock doors is 3:2. The well-known optimal shape of 2:1 for the put-away operation only applies to a single, centrally located P&D point. For the order-picking operation, the two class-based storage layouts that we considered favor smaller shape ratios than a random storage layout. Also, warehouse shape is a more significant design consideration for random storage as compared to class-based storage. In general, designs with a shape ratio of 3.0 or higher perform better (i.e., lower labor hours) than designs with lower shape ratios.

It is possible to narrow the search space quite effectively using the observations from our work.

The design variables that we considered, including pallet area shape and number of pallet rack levels, forward area size and layout, and dock door configuration can result in a large number of designs to consider. The heuristic search procedure that we developed aims to provide a good design that can be further analyzed and improved.

Empirical data is effective in characterizing performance in the search process.

Analytical models can be used to quantify labor requirements in assessing design performance. Likewise, our research shows that empirical data in the form of lookup tables are also effective in comparing designs. More importantly, the two methods can be combined to evaluate designs that consider new technologies that are incompletely modeled. The problem of warehouse design is a multi-objective function. The design that minimizes labor hours is not necessarily the optimal design, as other cost factors such as land and construction costs must be considered as well. This and other observations lead to the need for future research.

8.2. Future Research

The analytical models that we developed assume a pallet area without center cross-aisles. Adapting the models to include cross-aisles would be useful to practitioners in quantifying the labor savings associated with cross-aisles. In addition, models that consider piece-picking operations could be incorporated into our design methodology to include a more comprehensive warehouse design model, as many warehouses include both case- and piece-picking operations. Initial modeling of a U-shaped pick module has been completed [1], and this model as well as a model for picking from case flow aisles have been incorporated in a warehouse design tool [2].

In terms of our design methodology, we populated lookup tables based on parameterized analytical models in order to illustrate the use of empirical data in warehouse design. However, extending the types of warehouses that we consider would require us to include data that is truly derived from empirical sources. Such an extension would further validate our design methodology.

The search heuristic that we developed for designing a case-picking warehouse is limited in that it only considers designs with a pallet area shape of 3.0. The heuristic can be extended to consider designs with higher shape ratios. In addition, further testing may reveal other factors that should be incorporated into our heuristic as well. Another extension of this research is to focus on issues not addressed in our work, including factors such as picker blocking due to congestion and dynamic issues related to seasonality. To do so, it may prove necessary to group designs with similar performance in terms of required labor so that distinctions on other metrics (e.g., congestion, performance during the peak season, etc.) can be measured and/or the impact of various constraints can be evaluated so as to arrive at an improved solution to a more-encompassing objective. Such an approach may also form the basis for evaluating the risks associated with making wrong decisions in reconfiguring an existing warehouse, which is likely to be a significant benefit to practitioners.

Bibliography

[1] Russell D. Meller and Lisa M. Thomas. Piece-picking module design for medline. Final Report, Center for Excellence in Logistics and Distribution, University of Arkansas (2013).

[2] Russell D. Meller and Lisa M. Thomas. Decision support for warehouse design. Final Report, Center for Excellence in Logistics and Distribution, University of Arkansas, 2013, 2013.