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Powers and Compensation in Three-Phase Systems with Nonsinusoidal and Asymmetrical Voltages and Currents

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POWERS AND COMPENSATION IN THREE-PHASE SYSTEMS WITH NONSINUSOIDAL AND ASYMMETRICAL VOLTAGES AND CURRENTS

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This dissertation is dedicated to my parents Manju and Kalyan Bhattarai

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ABSTRACT

A contribution to power theory development of three-phase three-wire systems with asymmetrical and nonsinusoidal supply voltages is presented in this dissertation.

It includes:

- contribution to explanation of power related phenomena
- contribution to methods of compensation

The power equation of unbalanced Linear Time Invariant (LTI) loads at sinusoidal but asymmetrical voltage is first presented. The different current components of such a load and the phenomenon associated with these current components are described. The load current decomposition is used for the design of reactive balancing compensators for power factor improvement. Next, the current of LTI loads operating at nonsinusoidal asymmetrical voltage is decomposed, and the power equation of such a load is developed. Methods of the design of reactive compensators for the complete compensation of the reactive and unbalanced current components, as well as the design of optimized compensator for minimization of these currents are also presented.

Next, the power equation of Harmonics Generating Loads (HGLs) connected to nonsinusoidal asymmetrical voltage is developed. The voltage and current harmonics are divided into two subsets, namely the subset of the harmonic orders originating in the supply, and the subset of the harmonic orders originating in the load. The load current is decomposed based on the Currents' Physical Components (CPC) power theory, and the theory is also used for reference signal generation for the control of Switching Compensators used for power factor improvement. Results of simulation in MATLAB Simulink are presented as well.

CHAPTER 1: INTRODUCTION

1.1 Dissertation background

The studies on the power properties of electrical systems with nonsinusoidal voltages and currents were initiated by Steinmetz in [1] towards the end of the 19th century and they are still on going. Description of these properties is shortly called a ‘power theory’. It is one of the most controversial areas of research in electrical engineering.

Many scientists have dedicated their scientific life to power theory development. The most known are Budeanu [2], Fryze [3], Shepherd [4], Kusters [5], Moore [5], Czarnecki [6, 17 - 19], Nabae [33], Akagi [33] and Tenti [46]. This research over the past century has been mainly focused on power properties of systems with nonsinusoidal, but symmetrical voltages. The power equations of both single-phase as well as three-phase loads supplied with nonsinusoidal and symmetrical voltages are now known. The internal voltage of the distribution system can often be asymmetrical, however. Unfortunately, it is still not yet known how to describe power properties of such systems with asymmetrical voltages and currents. It is also not clear how to compensate loads operating at asymmetrical and nonsinusoidal voltage.

The commonly known power equation relates the apparent, active and reactive powers *S*, *P* and *Q* as follows

$$S^2 = P^2 + Q^2 \ . \tag{1.1}$$

It is valid only if the load is linear time invariant (LTI) and balanced, and if the supply voltage is sinusoidal and symmetrical, however. This is a major deficiency of the state of the knowledge on power properties of electrical systems. The effectiveness of the utilization of the energy supply capability of the energy provider is specified by the power factor, defined as

$$\lambda = \frac{P}{S} \; . \tag{1.2}$$

Consequently, in a situation where (1.1) does not describe the power properties of electrical loads correctly, the same applies to the power factor calculation. Moreover, in the lack of right power equation it is not clear how the power properties of the distribution system at low power factor could be improved.

A three-phase voltage/current waveform is said to be nonsinusoidal if it cannot be described by a sinusoidal function. The following factors are the main contributors to voltage and current distortion:

- Nonlinear devices of small ratings, consisting mainly of fluorescent bulbs, computer and TV supplies, as well as power supplies used in low power appliances.
- Static power converters used in industries such as three-phase rectifiers, AC to DC converters, inverters, cycloconverters.
- Power electronics devices used for the interfacing of renewable energy sources like wind farms and photovoltaic sources with the grid.
- Arc furnaces.

A three-phase voltage is said to be symmetrical if the phase voltages are mutually shifted by one-third of the period, otherwise voltages are asymmetrical. The following factors contribute to voltage and current asymmetry

- Structural asymmetry of the transmission system
- Unequal residential loading on the individual phases and imbalance of industrial loads such as arc furnaces and traction loads.

1.2 Dissertation subject

Power properties of three-phase loads supplied from three-phase, three-wire sources of asymmetrical nonsinusoidal voltage, as well as the methods of compensator design for the power factor improvement of such loads are the subject of studies reported in this dissertation.

1.3 Dissertation objective and approach

Development of the power equation of Linear Time Invariant (LTI) and Harmonic Generating Loads (HGLs) supplied with nonsinusoidal and/or asymmetrical voltage as well as methods of the design of compensators for power factor improvement of such loads is the objective of this dissertation. The analysis will be done by decomposing the load current into physical components, each associated with a distinct physical phenomenon. This approach is based on the Currents Physical Components (CPC) concept and it differs from the traditional approach of the power theory development in that it considers the current as the fundamental quantity and focusses on the decomposition of the load current instead of decomposing the load power.

1.4 Dissertation chapters breakdown

Chapter 2 of this dissertation will provide the background on power theory development as well as the shortcomings of the power theory at present. Development of the power equation of LTI loads and the methods of its reactive compensation at asymmetrical and sinusoidal voltage will be presented in Chapter 3, while the same will be presented for asymmetrical but nonsinusoidal voltage in Chapter 4. The power equation of HGLs at asymmetrical and nonsinusoidal voltage will be developed in Chapter 5, while compensation of HGLs using Switching Compensators will be presented in Chapter 6. The conclusions of this dissertation as well as the potential directions for continuation of this research will be presented in Chapter 7.

CHAPTER 2: BACKGROUND OF POWER THEORY DEVELOPMENT

2.1 Introduction

At the beginning the AC power systems were built of synchronous generators and linear loads such as incandescent lamps, resistive heating appliances and induction machines. Power properties of such systems were described in terms of only the active and reactive powers.

Over the course of time, the power system has undergone a lot of changes. Fluorescent lamps have replaced the incandescent bulbs and a lot of power electronics based equipment has been added to the system. These non-linear and/or periodically switched loads also referred to as harmonic generating loads (HGLs), cause current and voltage waveform distortion. In addition, AC arc furnaces used in industries are non-linear and could be highly unbalanced. Consequently, they cause voltage and current waveform distortion as well as asymmetry.

Moreover, in addition to synchronous generators, other types of energy sources have been introduced to the system. These are wind generators, photovoltaic sources, etc. which require a power electronics interface before they can be connected to the AC system. These interfaces can cause distortion of the system supply voltage.

The power theory used for describing power properties of present day power systems should be capable of describing the system with nonsinusoidal and asymmetrical voltages and currents. The traditional power theory, based only on the active and the reactive powers, was developed at the assumption that the voltages and currents are sinusoidal and symmetrical. Therefore, in presence of the asymmetry and distortion in the power system, the abovementioned assumptions need to be removed, and the power theory should describe power properties of electrical systems in the presence of the voltage and current asymmetry and distortion.

2.2 Traditional definitions of the apparent power

Most of the residential and commercial loads are single-phase loads supplied from a three-phase transformer in the Δ / Y configuration as shown in Fig. 2.1.

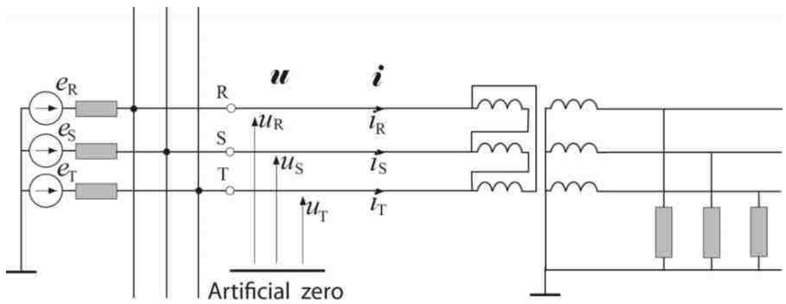


Fig. 2.1 Single-phase load connected to a three-phase distribution system

A considerable amount of energy produced in power systems is distributed in circuits as shown in Fig 2.1, where some level of the load imbalance can occur. For such a system built for energy delivery, the power properties are crucial for the evaluation of the effectiveness of this delivery as well as its improvement by compensation.

The power equation

$$S^2 = P^2 + Q^2 \tag{2.1}$$

was used traditionally to describe both the single-phase and three-phase circuits in terms of power where

$$P = \sum_{\mathrm{X=R,S,T}} U_\mathrm{X}\, I_\mathrm{X} \cos\varphi_\mathrm{X}\,, \quad Q = \sum_{\mathrm{X=R,S,T}} U_\mathrm{X}\, I_\mathrm{X} \sin\varphi_\mathrm{X}\,, \tag{2.2}$$

are the active and the reactive powers respectively, and S is the apparent power.

The studies on power theory were initiated with the observation by Steinmetz [1] in 1892 that the power equation (2.1) is not valid in single-phase circuits with electric arcs . Some of the important literature on this subject is [2-10], which is mostly focused on single-phase systems with nonsinusoidal voltages and currents.

It was concluded by Lyon [11] in 1920 that the imbalance of three-phase loads reduces the power factor, even if the voltages and currents are sinusoidal. This observation was not formulated in quantitative terms, however. At that time, it was not clear how the apparent power S should be defined in three-phase systems. It is because the apparent power is not a physical quantity, but a conventional quantity. It is used to specify the power ratings of transmission equipment and for the calculation of the power factor

$$\lambda = \frac{P}{S} \ , \tag{2.3}$$

which specifies the effectiveness of the energy delivery to the load.

After some inconclusive debate [12, 13], two different definitions, namely

$$S \ = \ S_{\mathrm{A}} = U_{\mathrm{R}} I_{\mathrm{R}} + U_{\mathrm{S}} I_{\mathrm{S}} + U_{\mathrm{T}} I_{\mathrm{T}} \tag{2.4}$$

referenced as ***an arithmetic apparent power*** and

$$S = S_{\mathrm{G}} \ = \sqrt{P^2 + Q^2} \tag{2.5}$$

referenced as ***geometric apparent power***, were adopted. Both these definitions were used for several decades and were supported by the IEEE Standard Dictionary of Electrical and Electronics Terms [14]. There is also another definition of the apparent power, as suggested by Buchholtz in [16], but not commonly known in the United States; that uses the sum of squares of the line voltage and current rms values, namely

$$S = S_{\mathrm{B}} \ = \sqrt{U_{\mathrm{R}}^2 + U_{\mathrm{S}}^2 + U_{\mathrm{T}}^2} \ \sqrt{I_{\mathrm{R}}^2 + I_{\mathrm{S}}^2 + I_{\mathrm{T}}^2} \tag{2.6}$$

In circuits with sinusoidal and symmetrical voltages and currents, these three definitions result in the same numerical values. However, as demonstrated in [17], at current and voltage asymmetry and/or distortion, these definitions result in different values of the apparent power, thereby leading to different values of the power factor λ. It is unclear which one is correct.

Illustration 2.1 Let us calculate the apparent power and the power factor of an unbalanced load in Fig. 2.2 using the different definitions of the apparent power.

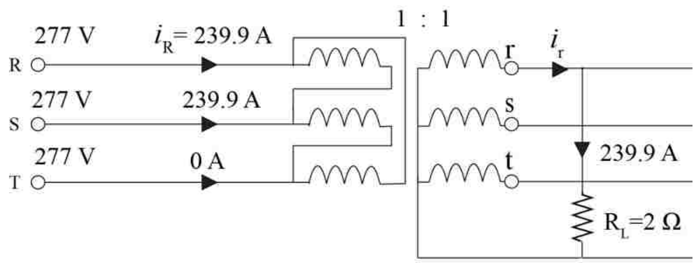


Fig. 2.2 Three-phase supply feeding a single-phase load

The line-to-ground voltage rms value is 277 V, while the transformer turns ratio is chosen for simplicity to be 1:1. The load current rms value,

$$||i_{\mathrm{r}}|| = \frac{277\sqrt{3}}{2} = 239.9 \ \mathrm{A}.$$

The active power of the load

$$P = ||i_{\mathrm{r}}||^2 \times R_{\mathrm{L}} = 239.9^2 \times 2 = 115.09 \ \mathrm{kW}$$

The line current rms values are $||i_{\mathrm{R}}|| = 239.9$ A, $||i_{\mathrm{S}}|| = 239.9$ A, $||i_{\mathrm{T}}|| = 0$ A.

Depending upon the definition, the apparent power of such a load is

$$S_{\mathrm{A}} = 132.9 \ \mathrm{kVA}, \quad S_{\mathrm{G}} = 115.1 \ \mathrm{kVA}, \quad S_{\mathrm{B}} = 162.8 \ \mathrm{kVA}.$$

Hence, the power factor corresponding to the different values of the apparent powers is,

$$\lambda_{\mathrm{A}} = 0.86, \qquad \lambda_{\mathrm{G}} = 1\ , \qquad \lambda_{\mathrm{B}} = 0.71.$$

The reactive power of the load shown in Fig. 2.2 is zero. Therefore, the power equation in (2.1) is valid only for the geometric definition of the apparent power.

Definitions of the apparent power were investigated in [17] and it was demonstrated that in the presence of load imbalance both the arithmetical and the geometrical apparent powers result in erroneous value of the power factor. The correct value of the power factor is obtained when the apparent power S is calculated according to the Buchholtz definition given by (2.6). In other words, the correct value of the power factor of the load in illustration 2.1 is $\lambda_{\mathrm{B}} = 0.71$.

Unfortunately, even at such a definition of the apparent power, the power equation (2.1) with the active and reactive powers calculated using formula (2.2) in the presence of current asymmetry is not satisfied [18]. This is also evident in illustration 2.1. The problem was solved in [19] using the Currents' Physical Component (CPC) concept by introduction of a new power quantity, referred to as an unbalanced power.

2.3 The original concept of unbalanced power and the CPC power theory

Any three-phase LTI load as seen from the primary side of a Δ / Y transformer, as shown in Fig. 2.1, has an infinite number of equivalent circuits with respect to load currents [18]. These equivalent circuits can be in a Y or in a Δ configuration. For analysis of a load with a three-wire supply, as shown in Fig. 2.1, it is more convenient if the equivalent configuration is in a Δ configuration as depicted in Fig. 2.3, and therefore, such a configuration of the equivalent circuit is chosen for the following analysis. Since there is an infinite number of equivalent circuits with respect to load currents, one of the line-to-line admittances $\boldsymbol{Y}_{\mathrm{RS}}, \boldsymbol{Y}_{\mathrm{ST}}$ or $\boldsymbol{Y}_{\mathrm{ST}}$ can be chosen and the remaining two can be calculated accordingly.

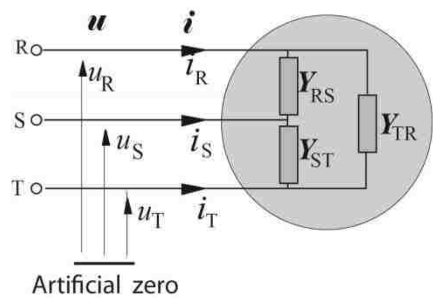


Fig. 2.3 Equivalent circuit of three-phase load in delta configuration

The association between a sinusoidal quantity $x(t)$

$$x(t) = \sqrt{2}\ X \cos(\omega t + \alpha) \tag{2.7}$$

and its complex rms (crms) value $\boldsymbol{X} = X\ \mathrm{e}^{j\alpha}$, has the form

$$x(t) = \sqrt{2}\ \mathrm{Re}\{\boldsymbol{X}\ \mathrm{e}^{j\omega t}\} \tag{2.8}$$

and can be generalized to three-phase vectors of the supply voltages and the load line currents as follows

$$\boldsymbol{u}(t) \stackrel{\mathrm{df}}{=} \boldsymbol{u} \stackrel{\mathrm{df}}{=} \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix} \mathrm{e}^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{U}\,\mathrm{e}^{j\omega t}\} \tag{2.9}$$

and,

$$\boldsymbol{i}(t) \stackrel{\mathrm{df}}{=} \boldsymbol{i} \stackrel{\mathrm{df}}{=} \begin{bmatrix} i_R(t) \\ i_S(t) \\ i_T(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} \mathrm{e}^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}\,\mathrm{e}^{j\omega t}\} \tag{2.10}$$

Symbols $\boldsymbol{U}$ and $\boldsymbol{I}$ denote three-phase vectors of complex rms (crms) values $\boldsymbol{U}_R$, $\boldsymbol{U}_S$, and $\boldsymbol{U}_T$ of line voltages, measured with respect to an artificial zero, and line currents $\boldsymbol{I}_R$, $\boldsymbol{I}_S$, and $\boldsymbol{I}_T$.

The current of the load shown in Fig. 2.3 can be expressed as

$$\boldsymbol{i}(t) = \sqrt{2}\ \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{e}} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{S}} \\ \boldsymbol{U}_{\mathrm{T}} \end{bmatrix} + \boldsymbol{A} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}} \\ \boldsymbol{U}_{\mathrm{T}} \\ \boldsymbol{U}_{\mathrm{S}} \end{bmatrix} \mathrm{e}^{j\omega t}\}. \tag{2.11}$$

The admittances $\boldsymbol{Y}_{\mathrm{e}}$ is referred to as an ***equivalent admittance*** of the load and is equal to

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + j\,B_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}} \tag{2.12}$$

while the admittance $\boldsymbol{A}$ is referred to as an ***unbalanced admittance*** of the load and is equal to

$$\boldsymbol{A} = Ae^{j\psi} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha \boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{RS}})\,. \tag{2.13}$$

Taking the equations (2.12) and (2.13) into account, the vector of the load current in (2.11) can be written as

$$\boldsymbol{i}(t) = \boldsymbol{i}_{\mathrm{a}}(t) + \boldsymbol{i}_{\mathrm{r}}(t) + \boldsymbol{i}_{\mathrm{u}}(t) \tag{2.14}$$

where the current vectors $\boldsymbol{i}_{\mathrm{a}}(t)$, $\boldsymbol{i}_{\mathrm{r}}(t)$ and $\boldsymbol{i}_{\mathrm{u}}(t)$ are defined as

$$\begin{aligned} \boldsymbol{i}_{\mathrm{a}}(t) &\overset{\mathrm{df}}{=} \sqrt{2}\ \mathrm{Re}\,\{G_{\mathrm{e}} \boldsymbol{U}\, \mathrm{e}^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{r}}(t) &\overset{\mathrm{df}}{=} \sqrt{2}\ \mathrm{Re}\,\{\,j\,B_{\mathrm{e}} \boldsymbol{U}\, \mathrm{e}^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{u}}(t) &\overset{\mathrm{df}}{=} \sqrt{2}\ \mathrm{Re}\,\{\boldsymbol{A} \boldsymbol{U}^{\#}\, \mathrm{e}^{j\omega t}\} \end{aligned} \tag{2.15}$$

while the voltage vector $\boldsymbol{U}^{\#}$ is equal to,

$$\boldsymbol{U}^{\#} = [\,\boldsymbol{U}_{\mathrm{R}} \quad \boldsymbol{U}_{\mathrm{T}} \quad \boldsymbol{U}_{\mathrm{S}}\,]^{\mathrm{T}} \tag{2.16}$$

The active power of the load at symmetrical voltage is

$$\begin{aligned} P &= \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{RS}}\}U_{\mathrm{RS}}^{2} + \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{ST}}\}U_{\mathrm{ST}}^{2} + \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{TR}}\}U_{\mathrm{TR}}^{2} \\ &= \mathrm{Re}\{\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}\}3U_{\mathrm{R}}^{2} \quad = G_{\mathrm{e}}\,||\boldsymbol{u}||^{2} \end{aligned} \tag{2.17}$$

where $||\boldsymbol{u}||$ is the three-phase rms value of the supply voltage defined as,

$$||\boldsymbol{u}|| = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{u}^{\mathrm{T}}(t)\,\boldsymbol{u}(t)\,dt} = \sqrt{U_{\mathrm{R}}^2 + U_{\mathrm{S}}^2 + U_{\mathrm{T}}^2} = \sqrt{3}U_{\mathrm{R}}\,. \tag{2.18}$$

Thus, the equivalent conductance G_{e} defined in the above equation is equal to

$$G_{\mathrm{e}} = \frac{P}{||\boldsymbol{u}||^2} \tag{2.19}$$

Therefore, the current $\boldsymbol{i}_{\mathrm{a}}(t)$, which is proportional to the equivalent conductance G_{e}, is associated with the phenomenon of permanent energy flow from the supply to the load. It is the ***active current*** of the load.

The reactive power of the load at symmetrical voltage is

$$\begin{aligned} Q &= -\mathrm{Im}\{\boldsymbol{Y}_{\mathrm{RS}}\}\,U_{\mathrm{RS}}^2 - \mathrm{Im}\{\boldsymbol{Y}_{\mathrm{ST}}\}\,U_{\mathrm{ST}}^2 - \mathrm{Im}\{\boldsymbol{Y}_{\mathrm{TR}}\}\,U_{\mathrm{TR}}^2 \\ &= -\mathrm{Im}\{\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}\}3U_{\mathrm{R}}^2 \quad = -B_{\mathrm{e}}\,||\boldsymbol{u}||^2\,. \end{aligned} \tag{2.20}$$

Hence the equivalent susceptance B_{e} is equal to,

$$B_{\mathrm{e}} = -\frac{Q}{||\boldsymbol{u}||^2}\,. \tag{2.21}$$

The current $\boldsymbol{i}_{\mathrm{r}}(t)$ is associated with the phenomenon of the phase shift of the load current with respect to the supply voltage and consequently, the presence of the reactive power. It is the ***reactive current*** of the load.

When the load is balanced, meaning that admittances $\boldsymbol{Y}_{\mathrm{RS}}, \boldsymbol{Y}_{\mathrm{ST}}$ and $\boldsymbol{Y}_{\mathrm{ST}}$ are equal, then the unbalanced admittance $\boldsymbol{A}$ defined by (2.13) is zero. The current $\boldsymbol{i}_{\mathrm{u}}(t)$ occurs only due to the load imbalance. Therefore, this current is referred to as the ***unbalanced current*** of the load. It is to be noted though that the equality of $\boldsymbol{Y}_{\mathrm{RS}}, \boldsymbol{Y}_{\mathrm{ST}}$ and $\boldsymbol{Y}_{\mathrm{ST}}$ is only a sufficient condition for $\boldsymbol{i}_{\mathrm{u}}(t)$ to be zero, but not the necessary condition.

Thus the active, reactive and unbalanced are associated with three different physical phenomena in the circuit and are referred to as the Currents' Physical Components (CPC).

If we define the unit three-phase vectors of the positive and negative sequence as

$$\mathbf{1}^{\mathrm{p}} = \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{-j2\pi/3} \\ 1e^{j2\pi/3} \end{bmatrix}, \quad \mathbf{1}^{\mathrm{n}} = \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix} \tag{2.22}$$

and illustrated in Fig. 2.4,

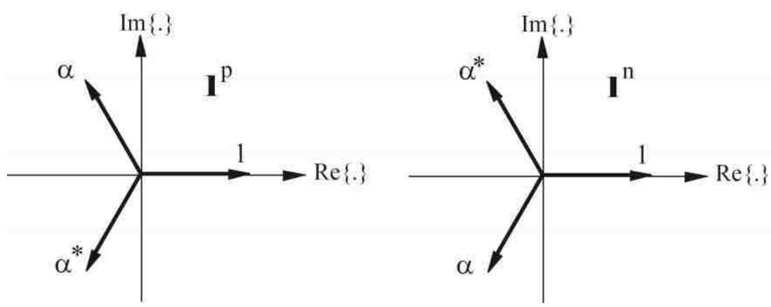


Fig. 2.4 Symmetrical three-phase unit vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$

then using these vectors, the current vectors defined in (2.15) can be rewritten as

$$\begin{aligned} \boldsymbol{i}_{\mathrm{a}}(t) &= \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{e}}\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}}\,\mathrm{e}^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{r}}(t) &= \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{e}}\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}}\,\mathrm{e}^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{u}}(t) &= \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\,\mathbf{1}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}}\,\mathrm{e}^{j\omega t}\}. \end{aligned} \tag{2.23}$$

The above equations emphasize that if the supply voltage is sinusoidal and symmetrical with a positive sequence, then the active and the reactive currents are of the positive sequence, while the unbalanced current is of the negative sequence.

The three-phase rms values of the currents in (2.23) are

$$\begin{aligned} ||\boldsymbol{i}_{\mathrm{a}}|| &= G_{\mathrm{e}}\,||\boldsymbol{u}|| \\ ||\boldsymbol{i}_{\mathrm{r}}|| &= |B_{\mathrm{e}}|\,||\boldsymbol{u}|| \\ ||\boldsymbol{i}_{\mathrm{u}}|| &= A\,||\boldsymbol{u}|| \end{aligned} \tag{2.24}$$

The current components in (2.14) are mutually orthogonal on the condition that their scalar products, defined for three-phase vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ of the same frequency as

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt \tag{2.25}$$

are equal to zero. The scalar product defined by (2.25) can be calculated using the vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$ of crms values of $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ as following

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{y}(t)\,dt = \mathrm{Re}\{\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y}^{*}\} \tag{2.26}$$

If these vectors are orthogonal, then their three-phase rms value, defined as

$$||\boldsymbol{x}|| \overset{\mathrm{df}}{=} \sqrt{(\boldsymbol{x},\boldsymbol{x})} = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\,\boldsymbol{x}(t)\,dt} \tag{2.27}$$

satisfy the relationship

$$||\boldsymbol{x}+\boldsymbol{y}||^2 = ||\boldsymbol{x}||^2 + ||\boldsymbol{y}||^2 \ . \tag{2.28}$$

The scalar product of the active and the reactive current

$$\begin{aligned} (\boldsymbol{i}_{\mathrm{a}},\boldsymbol{i}_{\mathrm{r}}) &= \mathrm{Re}\{\boldsymbol{I}_{\mathrm{a}}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{r}}^{*}\} = \mathrm{Re}\{[(G_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}_{\mathrm{R}})^{\mathrm{T}}][(j\,B_{\mathrm{e}}\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}_{\mathrm{R}})^{*}]\} \\ &= G_{\mathrm{e}}\,B_{\mathrm{e}}\,U_{\mathrm{R}}^2\,\mathrm{Re}\{-j\,(\mathbf{1}^{\mathrm{p}})^{\mathrm{T}}\,\mathbf{1}^{\mathrm{n}}\} = \end{aligned}$$

$$= G_{\mathrm{e}}\, B_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{-j[1,\ \alpha^{*},\ \alpha]\begin{bmatrix} 1 \\ \alpha \\ \alpha^{*} \end{bmatrix}\}$$

$$= G_{\mathrm{e}}\, B_{\mathrm{e}} U_{\mathrm{R}}^{2}\ \mathrm{Re}\{-j3\} = 0. \tag{2.29}$$

Similarly, the scalar product of the active and the unbalanced currents

$$(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{u}}) = \mathrm{Re}\{\boldsymbol{I}_{\mathrm{a}}^{\mathrm{T}} \boldsymbol{I}_{\mathrm{u}}^{*}\} = \mathrm{Re}\{[(G_{\mathrm{e}}\, \mathbf{1}^{\mathrm{p}}\, \boldsymbol{U}_{\mathrm{R}})^{\mathrm{T}}][(\boldsymbol{A}\mathbf{1}^{\mathrm{n}}\, \boldsymbol{U}_{\mathrm{R}})^{*}]\}$$

$$= G_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\boldsymbol{A}^{*}\, (\mathbf{1}^{\mathrm{p}})^{\mathrm{T}}\, \mathbf{1}^{\mathrm{p}}\} = G_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\boldsymbol{A}^{*}[1,\ \alpha^{*},\ \alpha]\begin{bmatrix} 1 \\ \alpha^{*} \\ \alpha \end{bmatrix}\} \tag{2.30}$$

$$= G_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\boldsymbol{A}^{*}(1+\alpha+\alpha^{*})\} = 0\,.$$

The scalar product of the unbalanced and reactive currents

$$(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{u}}) = \mathrm{Re}\{\boldsymbol{I}_{\mathrm{r}}^{\mathrm{T}} \boldsymbol{I}_{\mathrm{u}}^{*}\} = \mathrm{Re}\{[(\,jB_{\mathrm{e}}\, \mathbf{1}^{\mathrm{p}}\, \boldsymbol{U}_{\mathrm{R}})^{\mathrm{T}}][(\boldsymbol{A}\mathbf{1}^{\mathrm{n}}\, \boldsymbol{U}_{\mathrm{R}})^{*}]\}$$

$$= B_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\,j\boldsymbol{A}^{*}\, (\mathbf{1}^{\mathrm{p}})^{\mathrm{T}}\, \mathbf{1}^{\mathrm{p}}\} = B_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\,j\boldsymbol{A}^{*}[1,\ \alpha^{*},\ \alpha]\begin{bmatrix} 1 \\ \alpha^{*} \\ \alpha \end{bmatrix}\} \tag{2.31}$$

$$= B_{\mathrm{e}} U_{\mathrm{R}}^{2}\, \mathrm{Re}\{\,j\boldsymbol{A}^{*}(1+\alpha+\alpha^{*})\} = 0.$$

Thus, the three current components of the current in (2.14) are mutually orthogonal and therefore their three-phase rms values satisfy the relationship

$$||\boldsymbol{i}||^{2} = ||\boldsymbol{i}_{\mathrm{a}}||^{2} + ||\boldsymbol{i}_{\mathrm{r}}||^{2} + ||\boldsymbol{i}_{\mathrm{u}}||^{2} \tag{2.32}$$

Multiplying (2.32) by the square of the three-phase rms value of the supply voltage,

$$||\boldsymbol{i}||^{2} \times ||\boldsymbol{u}||^{2} = [\,||\boldsymbol{i}_{\mathrm{a}}||^{2} + ||\boldsymbol{i}_{\mathrm{r}}||^{2} + ||\boldsymbol{i}_{\mathrm{u}}||^{2}\,] \times ||\boldsymbol{u}||^{2} \tag{2.33}$$

yields the power equation

$$S^{2} = P^{2} + Q^{2} + D_{\mathrm{u}}^{2} \tag{2.34}$$

where,

$$S = ||\boldsymbol{u}|| \times ||\boldsymbol{i}|| \tag{2.35}$$

is the apparent power of the load,

$$Q = -B_{\mathrm{e}}\ ||\boldsymbol{u}||^2 \tag{2.36}$$

is the reactive power of the load and

$$D_{\mathrm{u}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{u}}|| = A\,||\boldsymbol{u}||^2 \tag{2.37}$$

is the unbalanced power of the load.

The power factor

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2 + D_{\mathrm{u}}^2}} = \frac{||\boldsymbol{i}_{\mathrm{a}}||}{\sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2}} = \frac{G_{\mathrm{e}}}{\sqrt{G_{\mathrm{e}}^2 + B_{\mathrm{e}}^2 + A^2}}\ . \tag{2.38}$$

It is evident from (2.38) that both the reactive and the unbalanced currents contribute to the increase of the supply current rms value and the apparent power and consequently, to the decline of the power factor. The reduction of these currents will lead to the improvement of the power factor. These currents can be reduced using a shunt balancing compensator. Complete compensation of the load occurs when the compensator current is equal to the negative of the sum of the reactive and unbalanced currents. Such a balancing compensator can be built as a reactive compensator, composed of inductors and capacitors, or as a switching compensator, composed of a three-phase inverter with a measurement and a control system. A reactive compensator, as shown in Fig. 2.5, can be used for the compensation of the reactive and the unbalance currents of LTI loads at sinusoidal and symmetrical voltage.

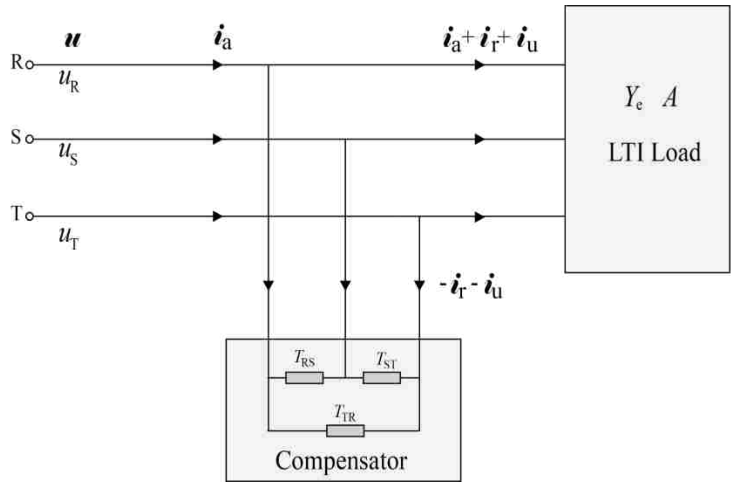


Fig. 2.5 Circuit with shunt reactive compensator

The compensator in Fig. 2.5 is of the delta structure and it is assumed to be composed of ideal lossless reactance elements. It can have an inductor or a capacitor in each branch with susceptances T_{RS}, T_{ST} and T_{TR}. Such a lossless compensator modifies the reactive and the unbalanced current to

$$\boldsymbol{i}_r' = \sqrt{2}\ \mathrm{Re}\,\{\, j[\, B_e + (T_{ST} + T_{TR} + T_{RS})\,]\,\boldsymbol{U}\,e^{j\omega t}\},$$
$$\boldsymbol{i}_u' = \sqrt{2}\ \mathrm{Re}\,\{\,[\,\boldsymbol{A} - j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})\,]\,\mathbf{1}^n \boldsymbol{U}_R^P\, e^{j\omega t}\}. \tag{2.39}$$

The reactive current is compensated to zero if

$$[\, B_e + (T_{ST} + T_{TR} + T_{RS})] = 0 \tag{2.40}$$

while the unbalanced current is compensated to zero if

$$[\,\boldsymbol{A} - j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})\,] = 0. \tag{2.41}$$

Equation (2.41) contains complex quantities and therefore it has to be satisfied for both the real and the imaginary parts, i.e.,

$$\mathrm{Re}\{\,[\,\boldsymbol{A} - j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})\,]\,\} = 0.$$
$$\mathrm{Im}\{\,[\,\boldsymbol{A} - j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})\,]\,\} = 0. \tag{2.42}$$

Hence, the two equations in (2.42) and equation (2.40) provide three linear equations which can be used to solve for the three unknowns namely T_{RS}, T_{ST} and T_{TR}.

Solving the three equations yields

$$\begin{aligned} T_{RS} &= (\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A} - \mathrm{Im}\,\boldsymbol{A} - B_e)/3, \\ T_{ST} &= (2\,\mathrm{Im}\,\boldsymbol{A} - B_e)/3, \\ T_{TR} &= (-\sqrt{3}\,\mathrm{Re}\,\boldsymbol{A} - \mathrm{Im}\,\boldsymbol{A} - B_e)/3. \end{aligned} \tag{2.43}$$

If the susceptance T_{XY} obtained from (2.43) above is positive, then a capacitor of capacitance

$$C_{XY} = \frac{T_{XY}}{\omega_1}$$

should be selected for the branch XY. If the susceptance T_{XY} obtained from (2.43) above is negative, then an inductor of inductance

$$L_{XY} = -\frac{1}{\omega_1 T_{XY}}$$

should be selected for the branch XY. Such a compensator will compensate entirely the reactive and the unbalanced currents. Thus, it will improve the power factor to unity.

Illustration 2.2 Application of CPC Theory and compensation techniques to an unbalanced LTI load.

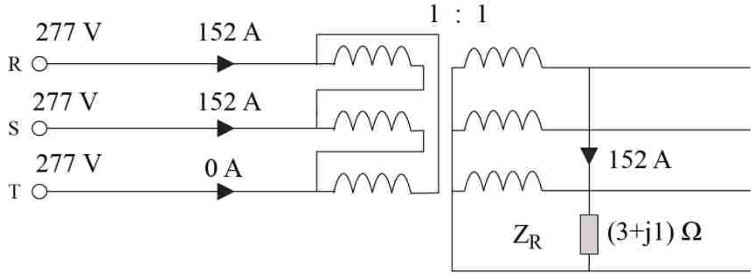


Fig. 2.6 Three-phase distribution system with an unbalanced load

An unbalanced LTI load as shown in Fig. 2.6 is supplied from a source of sinusoidal symmetrical voltage. The line-to-ground rms value of the supply voltage $U = 277\ \mathrm{V}$. If the load impedance $\boldsymbol{Z}_{\mathrm{L}} = 3 + j\,1\ \Omega$, then the line-line admittance Y_{RS} from the point of view of the supply side is

$$\boldsymbol{Y}_{\mathrm{RS}} = \frac{1}{\boldsymbol{Z}_{\mathrm{RS}}} = \frac{1}{3 + j\,1} = 0.3 - j\,0.1 = 0.316 e^{-j18.4°}\,\mathrm{S}.$$

Thus the equivalent and the unbalanced admittances are equal to

$$\begin{aligned}
\boldsymbol{Y}_{\mathrm{e}} &= G_{\mathrm{e}} + j\,B_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}} = 0.3 - j\,0.1 = 0.316 e^{-j18.4°}\,\mathrm{S}\\
\boldsymbol{A} &= A e^{j\psi} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha \boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{RS}}) = -\alpha^{*} \boldsymbol{Y}_{\mathrm{RS}} = 0.316 e^{j\,41.6°}\,\mathrm{S}.
\end{aligned}$$

The three-phase rms value of the supply voltage $||\boldsymbol{u}|| = \sqrt{3}\,U = \sqrt{3}\;277 = 480\,\mathrm{V}$. Therefore, the current’s physical components (CPC) are

$$\begin{aligned}
||\boldsymbol{i}_{\mathrm{a}}|| &= G_{\mathrm{e}}\,||\boldsymbol{u}|| = 0.3 \times 480 = 144\ \mathrm{A},\\
||\boldsymbol{i}_{\mathrm{r}}|| &= |B_{\mathrm{e}}|\,||\boldsymbol{u}|| = 0.1 \times 480 = 48\ \mathrm{A},\\
||\boldsymbol{i}_{\mathrm{u}}|| &= \boldsymbol{A}\,||\boldsymbol{u}|| = 0.316 \times 480 = 152\ \mathrm{A},
\end{aligned}$$

and the supply current rms value is

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2} = \sqrt{144^2 + 48^2 + 152^2} = 215\,\mathrm{A}.$$

The three-phase rms value of the current is indeed the same as calculated using the line currents,

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{R}}||^2 + ||\boldsymbol{i}_{\mathrm{S}}||^2 + ||\boldsymbol{i}_{\mathrm{T}}||^2} = \sqrt{152^2 + 152^2} = 215\,\mathrm{A},$$

as expected. The load powers are

$$P = G_{\mathrm{e}}\,||\boldsymbol{u}||^2 = 0.3 \times 480^{\,2} = 69\ \mathrm{kW}$$

$$Q = B_{\rm e}\ ||\boldsymbol{u}||^2 = 0.1\times 480^{\,2} = 23\ \text{kVA}$$
$$D = A||\boldsymbol{u}||^2 = 0.316\times 480^{\,2} = 73\ \text{kVA.}$$

Thus, the apparent power of the load, calculated using the load powers is

$$S = \sqrt{P^2 + Q^2 + D_{\rm u}^2} = \sqrt{69^2 + 23^2 + 73^2} = 103\ \text{kVA}$$

which is indeed the same as that calculated using rms values of the supply line voltage and currents,

$$S = ||\boldsymbol{u}||\times||\boldsymbol{i}|| = 480\times 215 = 103\ \text{kVA.}$$

The power factor is equal to

$$\lambda = \frac{P}{S} = \frac{69}{103} = 0.67\ .$$

The balancing compensator shown in Fig. 2.5 can be used for the compensation of the reactive and the unbalanced currents and the improvement of the power factor.

The equivalent susceptance of the load in Fig. 2.6 is $B_{\rm e} = -\,0.1\,\text{S}$ while the real and the imaginary parts of the unbalance admittance $\boldsymbol{A}$ are equal to $\text{Re}\{\boldsymbol{A}\} = 0.236\ \text{S}$ and $\text{Im}\{\boldsymbol{A}\} = 0.210\ \text{S}$. Thus, the compensator susceptances are equal to

$$T_{\rm RS} = (\ \sqrt{3}\,\text{Re}\,\boldsymbol{A} - \text{Im}\,\boldsymbol{A} - B_{\rm e}\,)/3\ = (\ \sqrt{3}\times 0.236 - 0.210 - \times - 0.10)/3 = 0.10\ \ \text{S}$$
$$T_{\rm ST} = (\ 2\ \text{Im}\,\boldsymbol{A} - B_{\rm e}\,)/3\ = (\ 2\times 0.210 - \times - 0.10)/3 = 0.173\ \text{S}$$
$$T_{\rm TR} = (\ -\sqrt{3}\,\text{Re}\,\boldsymbol{A} - \text{Im}\,\boldsymbol{A} - B_{\rm e}\,)/3 = (-\ \sqrt{3}\times 0.236 - 0.210 - \times - 0.10)/3 = -0.173\ \ \text{S}$$

Therefore, an inductor should be connected between the lines T and R while capacitors should be connected between the lines R and S and the lines S and T as shown in Fig. 2.7.

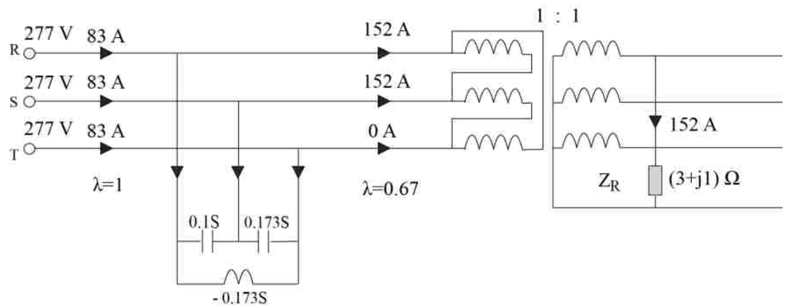


Fig. 2.7 Unbalanced LTI load with a reactive balancing compensator

Such a compensator reduces the supply current three-phase rms value from 215 A to 144 A. It restores the supply current symmetry and improves the power factor from 0.67 to unity.

2.4 Conclusion

The apparent power *S* calculated using the traditional power equation consisting of just the active power *P* and the reactive power *Q* is incorrect at load imbalance. The Currents' Physical Components (CPC) Power Theory, where the original concept of the unbalanced power was introduced, enables the development of the correct power equation of unbalanced LTI loads at sinusoidal symmetrical voltages. The CPC power theory also enables the design of a balancing reactive compensator for the improvement of the power factor to unity.

CHAPTER 3: POWERS AND REACTIVE COMPENSATION OF UNBALANCED LTI LOADS WITH SINUSOIDAL BUT ASYMMETRICAL VOLTAGES AND CURRENTS

3.1 Introduction

The power equation which describes power properties of unbalanced Linear Time Invariant (LTI) loads with sinusoidal and symmetrical (S&S) voltages and currents was presented in the previous chapter. It was discussed that traditional definitions, as supported by IEEE Standard Dictionary of Electrical Engineering Terms [14], of the apparent power do not provide the right value of the power factor of unbalanced loads. Traditional power theories and definitions were presented in Chapter 2, along with their shortcomings that made them inadequate for description of unbalanced loads. A new power equation of such loads based on the Currents' Physical Components (CPC) concept was also presented Fundamentals of design of reactive compensators which enable entire reduction of the reactive and unbalanced powers were presented as well.

Generally, the distribution system voltage is not S&S. It could be asymmetrical and/or distorted. Unfortunately, the power equation which describes an LTI load at asymmetrical voltage correctly is not known yet. Development of such an equation is just the objective of this chapter. The previous chapter provided only a starting point for the development of the power theory of unbalanced loads.

Power theory does not describe power properties of real systems, but only their models, simplified by various assumptions. We should approach description of real systems by progressively abandoning these assumptions step by step, which makes the power theory more accurate, but unfortunately, more and more complex. Therefore it is reasonable to abandon these assumptions only one by one, after verification that the power theory describes the system with previous assumptions correctly.

3.2 Brief description of the system at asymmetrical and sinusoidal voltage

It is assumed in this chapter that the internal voltage of the distribution system, as shown in Fig. 3.1 is asymmetrical and sinusoidal (A&S). It can be expressed in the form of a three-phase vector $\boldsymbol{e} = [\boldsymbol{e}_{\mathrm{R}}, \boldsymbol{e}_{\mathrm{S}}, \boldsymbol{e}_{\mathrm{T}}]^{\mathrm{T}}$.

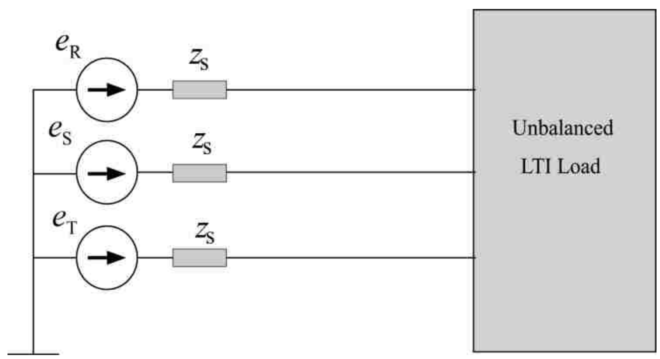


Fig. 3.1 Unbalanced LTI connected to three-phase supply

Such an asymmetrical voltage can be decomposed into symmetrical components of the positive, negative and zero sequence, $\boldsymbol{e}^{\mathrm{p}}, \boldsymbol{e}^{\mathrm{n}}$ and $\boldsymbol{e}^{\mathrm{z}}$, respectively. Since our analysis is limited to three-wire systems, the zero sequence component of the supply voltage cannot cause any current flow in the circuit. Consequently, it does not contribute to the power related phenomena in the load. This component of voltage does contribute to the three-phase rms value $||\boldsymbol{u}||$ of the voltage at load terminals, however, leading to erroneous value of the power factor. Thus, the zero sequence component should be eliminated from analysis by referencing the voltages to an artificial zero.

If the line voltages and currents are known, then an unbalanced load, as shown in Fig. 3.1, can be represented by an equivalent load connected in delta configuration. Such an equivalent load supplied with a voltage referred to an artificial zero is shown in Fig. 3.2. The voltage $\boldsymbol{u}$ in the figure is the vector of the three-phase voltages at the load terminals.

It is assumed in this chapter that the supply voltage is sinusoidal and the load is Linear Time Invariant (LTI). Such LTI loads can be analyzed using the Superposition Principle.

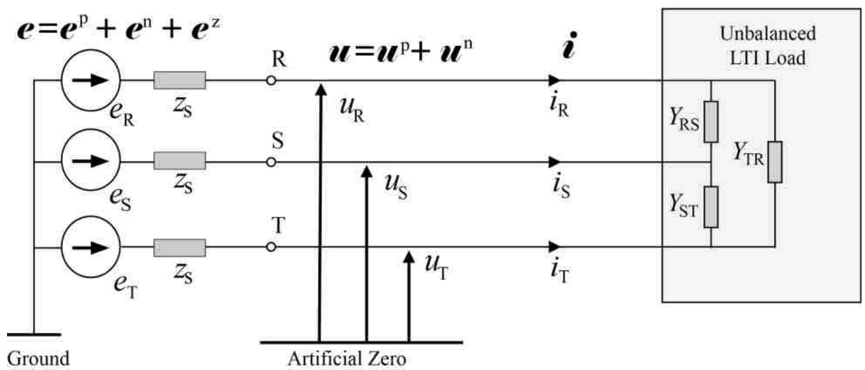


Fig. 3.2 Unbalanced three-phase load supplied with a voltage referred to artificial zero

3.3 Symbols of the apparent and complex powers

Let us assume that the load shown in Fig. 3.2 has active power P and reactive power Q. If the load is balanced and supplied with a sinusoidal symmetrical voltage, then the apparent power of the load is

$$S = \sqrt{P^2 + Q^2}. \tag{3.1}$$

The active and the reactive powers can be calculated using the line voltages and currents using traditional definitions, namely

$$P = \sum_{\mathrm{X=R,S,T}} U_{\mathrm{X}}\, I_{\mathrm{X}} \cos\varphi_{\mathrm{X}}\,, \quad Q = \sum_{\mathrm{X=R,S,T}} U_{\mathrm{X}}\, I_{\mathrm{X}} \sin\varphi_{\mathrm{X}}\,, \tag{3.2}$$

and the apparent power S is the magnitude of the ***complex apparent power***, commonly denoted by $\boldsymbol{S}$, and defined as

$$\boldsymbol{S} = P + jQ. \tag{3.3}$$

The load can be unbalanced, however. It was illustrated in the previous chapter that when the supply voltage is sinusoidal and symmetrical, but the load is unbalanced, then, in addition to the active and the reactive powers, such a load has also an unbalanced power. Due to the presence of the unbalanced power, the square of the apparent power of the load is higher than the sum of the squares of the active and reactive powers, viz.

$$S^2 = ||\boldsymbol{u}|| \times ||\boldsymbol{i}|| > P^2 + Q^2 \ . \tag{3.4}$$

Consequently, the apparent power S of such load is not equal to the magnitude of the complex apparent power ***S.*** Therefore, to avoid misinterpretations, we will denote the apparent power using the symbol S, while a different symbol is needed for denoting the complex power of the load, and, henceforth it will be denoted by $\boldsymbol{C}$, such that,

$$\boldsymbol{C} = C e^{j\varphi} \stackrel{\text{df}}{=} P + jQ \ . \tag{3.5}$$

We assume in this chapter that in addition to the load imbalance, the supply voltage is also asymmetrical.

3.4 Load current decomposition at asymmetrical but sinusoidal supply voltage

3.4.1 Superposition based current decomposition

It was demonstrated in Chapter 2 that the current of unbalanced LTI loads connected to sinusoidal symmetrical three-phase voltage, consist of the active, reactive and unbalanced current components. The load in Fig. 3.2 is an example of such a load. Also, the aforementioned active and reactive currents are of the same sequence as the supply voltage, while the unbalanced current is of the opposite sequence. At voltage asymmetry, the positive sequence component of the supply voltage causes the active and reactive currents of the positive sequence and an unbalanced current of the negative sequence in the supply lines. On the other hand, the negative sequence component of the supply voltage causes the active and reactive currents of the negative sequence and an unbalanced current of the positive sequence in the supply lines. i.e,

$$\begin{aligned} \boldsymbol{u}^{\mathrm{p}} &\rightarrow (\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}) \\ \boldsymbol{u}^{\mathrm{n}} &\rightarrow (\boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}) \end{aligned} \tag{3.6}$$

Since the positive and the negative sequence voltage components are symmetrical, the information of one phase is sufficient to calculate these voltages on the remaining lines. For simplicity, $\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}}$ can be denoted by $\boldsymbol{U}^{\mathrm{p}}$ and $\boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}$ can be denoted by $\boldsymbol{U}^{\mathrm{n}}$. Based on the analysis presented in Chapter 2,

$$\begin{aligned} \boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}(t) &= G_{\mathrm{e}}^{\mathrm{p}}\ \boldsymbol{u}^{\mathrm{p}}(\mathrm{t}) = \sqrt{2}\ \mathrm{Re}\{G_{\mathrm{e}}^{\mathrm{p}}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})e^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}}(t) &= B_{\mathrm{e}}^{\mathrm{p}}\ \boldsymbol{u}^{\mathrm{p}}(t+\frac{T}{4}) = \sqrt{2}\ \mathrm{Re}\{j\,B_{\mathrm{e}}^{\mathrm{p}}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})e^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}(t) &= \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}^{\mathrm{p}}\,(\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}})e^{j\omega t}\} \end{aligned} \tag{3.7}$$

and

$$\begin{aligned} \boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}}(t) &= G_{\mathrm{e}}^{\mathrm{n}}\ \boldsymbol{u}^{\mathrm{n}}(t) = \sqrt{2}\ \mathrm{Re}\{G_{\mathrm{e}}^{\mathrm{n}}\,(\mathbf{1}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}})e^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}}(t) &= B_{\mathrm{e}}^{\mathrm{n}}\ \boldsymbol{u}^{\mathrm{n}}(t+\frac{T}{4}) = \sqrt{2}\ \mathrm{Re}\{j\,B_{\mathrm{e}}^{\mathrm{n}}\,(\mathbf{1}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}})e^{j\omega t}\} \\ \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}(t) &= \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}^{\mathrm{n}}\,(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}})\ e^{j\omega t}\} \end{aligned} \tag{3.8}$$

where, $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$ are symmetrical three-phase unit vectors defined as

$$\mathbf{1}^{\mathrm{p}} = \begin{bmatrix} 1 \\ \alpha^{*} \\ \alpha \end{bmatrix}, \qquad \mathbf{1}^{\mathrm{n}} = \begin{bmatrix} 1 \\ \alpha \\ \alpha^{*} \end{bmatrix} \tag{3.9}$$

and illustrated in Fig. 3.3.

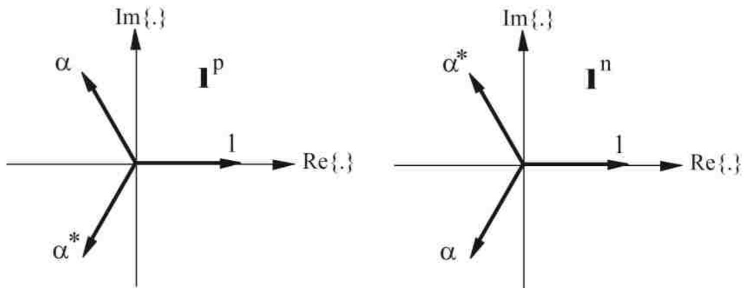


Fig. 3.3 Symmetrical three-phase unit vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$

As assumed earlier, the load is linear. Hence, based on the Superposition Principle, the total current of the LTI load caused by the voltage $\boldsymbol{u}$, is equal to the sum of the currents caused by the voltages $\boldsymbol{u}^{\mathrm{p}}$ and $\boldsymbol{u}^{\mathrm{n}}$ separately, i.e.,

$$\boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} \rightarrow (\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}) + (\boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}) \; . \tag{3.10}$$

The load current $\boldsymbol{i}$ is the sum of the currents caused by the positive and the negative sequence component of the supply voltages. Therefore,

$$\boldsymbol{i} \; = \boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} \; . \tag{3.11}$$

The three-phase rms value of the load current, $||\boldsymbol{i}||$, is equal to the square root of the sum of squares of the current components on the condition that these components are mutually orthogonal. The active and reactive currents of the same sequence are orthogonal to each other. Similarly, currents of the opposite sequences are orthogonal to one another. Thus, the orthogonality of the currents $\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}}$ and $\boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}}$ in (3.11) is straightforward. However, orthogonality of the unbalanced current with the active and reactive currents needs to be verified. The scalar product of $\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$ is

$$\begin{aligned}(\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}, \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}) &= \mathrm{Re}\{G_{\mathrm{e}}^{\mathrm{p}}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})^{\mathrm{T}}(\boldsymbol{A}^{\mathrm{n}}\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}})^{*}\} \\ &= \mathrm{Re}\,\{\,G_{\mathrm{e}}^{\mathrm{p}}\,\boldsymbol{A}^{\mathrm{n}*}(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}})^{\mathrm{T}}(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}})^{*}\} =\end{aligned}$$

$$= \mathrm{Re}\{G_{\mathrm{e}}^{\mathrm{p}}\,\boldsymbol{A}^{\mathrm{n}*}[\boldsymbol{U}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}*} + \alpha^{*}\boldsymbol{U}^{\mathrm{p}}(\alpha^{*}\boldsymbol{U}^{\mathrm{n}})^{*} + \alpha\boldsymbol{U}^{\mathrm{p}}(\alpha\boldsymbol{U}^{\mathrm{n}})^{*}]\}$$
$$= \mathrm{Re}\{G_{\mathrm{e}}^{\mathrm{p}}\,\boldsymbol{A}^{\mathrm{n}*}[\boldsymbol{U}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}*} + \alpha^{*}\alpha\boldsymbol{U}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{n}*} + \alpha\alpha^{*}\boldsymbol{U}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{n}*}]\}$$
$$= \mathrm{Re}\{3G_{\mathrm{e}}^{\mathrm{p}}\,\boldsymbol{A}^{\mathrm{n}*}\boldsymbol{U}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}*}\}. \tag{3.12}$$

It means that the scalar product of the currents $\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$ can have any value depending on the unbalanced admittances and the voltage crms values and consequently, it can be nonzero. Thus, the unbalanced current of the positive sequence is not necessarily orthogonal to the active current component of the positive sequence. Therefore, the components of the current decomposition in (3.11) may not be mutually orthogonal. For such a case, the three-phase rms value of the load current is not the sum of the squares of the three-phase rms values of the current components, viz,

$$||\boldsymbol{i}||^2 \neq ||\boldsymbol{i}_{\mathrm{a}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{a}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2\ . \tag{3.13}$$

Superposition based current decomposition does not enable decomposition of the current into orthogonal components and a different approach is needed to develop the power equation of LTI load at S&A supply voltage.

3.4.2 Decomposition of the load current into orthogonal components

Let us consider a balanced load supplied with voltage $\boldsymbol{u}$ as shown in Fig. 3.4. It has the same active power P and reactive power Q as the original load shown in Fig. 3.2. Such a load is therefore equivalent to the original load with respect to the active and reactive powers P and Q, and is referred to as the ***equivalent balanced load***.

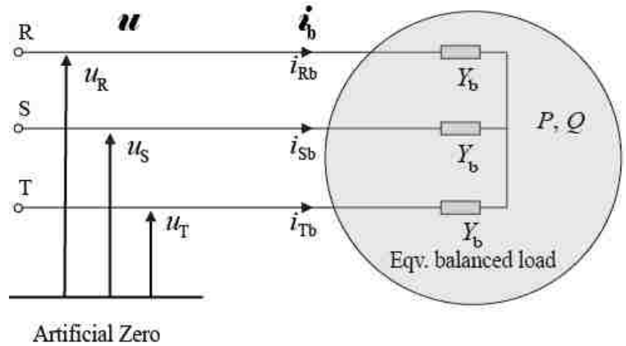


Fig. 3.4 Balanced load equivalent to the original load with respect to *P* and *Q*

Phase admittance of such an equivalent load is

$$\boldsymbol{Y}_b = G_b + jB_b = \frac{P - jQ}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{C}_b^*}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{C}^*}{||\boldsymbol{u}||^2}\ , \tag{3.14}$$

where $\boldsymbol{C}_b$ is the complex power of the equivalent balanced load and $\boldsymbol{C}$ is the complex power of the original unbalanced load. The admittance $\boldsymbol{Y}_b$ is referred to as the ***equivalent balanced admittance*** and draws the current

$$\boldsymbol{i}_b = \boldsymbol{i}_a + \boldsymbol{i}_r = \sqrt{2}\mathrm{Re}\{\ \boldsymbol{I}_b\ e^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\boldsymbol{Y}_b\ \boldsymbol{U}\ e^{j\omega t}\}. \tag{3.15}$$

The current $\boldsymbol{i}_b$, also referred to as the balanced current, consists of the active current

$$\boldsymbol{i}_a = G_b\,\boldsymbol{u} = \sqrt{2}\mathrm{Re}\{G_b\,(\boldsymbol{U}^p + \boldsymbol{U}^n)e^{j\omega t}\}, \tag{3.16}$$

and the reactive current

$$\boldsymbol{i}_r(t) = B_b\,\boldsymbol{u}(t + T/4) = \sqrt{2}\mathrm{Re}\{\ jB_b\,(\boldsymbol{U}^p + \boldsymbol{U}^n)\ e^{j\omega t}\}. \tag{3.17}$$

It is important to observe that the terms *symmetrical* and *asymmetrical*, and the terms *balanced* and *unbalanced*, have different meanings. The terms *symmetrical* and/or *asymmetrical* are used exclusively for the three-phase voltages and currents, based upon the symmetry of the three-phase quantities. The terms *balanced* and/or *unbalanced* on the other hand, are used for the load, based upon the values of the three-phase impedances.

The balanced current $\boldsymbol{i}_{\mathrm{b}}$ acquires its name based on its association with the equivalent balanced load. It is important to remember that it does not provide any information on its symmetry. The term *balanced* is therefore an indication of the load property. It simply represents the portion of the load current that is drawn by the equivalent balanced load, in order to have the active power P and the reactive power Q.

If $\boldsymbol{I}_{\mathrm{b}}$ is a vector of the crms value of the current of such a balanced load, then

$$\boldsymbol{C}_{\mathrm{b}} = \boldsymbol{U}^{\mathrm{T}}\, \boldsymbol{I}_{\mathrm{b}}^{*} = \boldsymbol{U}^{\mathrm{T}}\,(Y_{\mathrm{b}}\, \boldsymbol{U})^{*} = Y_{\mathrm{b}}^{*}\,||\boldsymbol{u}||^{2}\,. \tag{3.18}$$

Since the balanced load discussed above draws the balanced current $\boldsymbol{i}_{\mathrm{b}}$, the remaining component of the original current is due to the load imbalance. It is called the ***unbalanced current*** and is equal to

$$\boldsymbol{i}_{\mathrm{u}} = \boldsymbol{i} - \boldsymbol{i}_{\mathrm{b}} = \sqrt{2}\mathrm{Re}\{(\boldsymbol{I} - \boldsymbol{I}_{\mathrm{b}})\, \mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\boldsymbol{I}_{\mathrm{u}}\, \mathrm{e}^{j\omega t}\}\ . \tag{3.19}$$

The unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ acquires its name from its association with the load imbalance, and the term *unbalance* does not indicate its asymmetry. In fact, for symmetrical supply voltage, the unbalanced current is symmetrical. When the supply voltage is asymmetrical, the unbalance current is also asymmetrical, however. The name unbalanced only indicates that this is the portion of the total current $\boldsymbol{i}$ which is not associated with the equivalent balanced load discussed above. Rearranging the first part of (3.19) yields,

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{b}} + \boldsymbol{i}_{\mathrm{u}} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}\,, \tag{3.20}$$

meaning decomposition of the load current into the active, reactive and unbalanced currents.

If the current components are mutually orthogonal, then, the square of the three-phase rms value of the current $\boldsymbol{i}$ in (3.20) is equal to the sum of squares of the three-phase rms values of the current components. The active and the reactive currents are mutually orthogonal to each other because of their mutual shift by 90 degrees. The scalar product of the balanced and the unbalanced currents,

$$
\begin{aligned}
(\boldsymbol{i}_\mathrm{b}, \boldsymbol{i}_\mathrm{u}) &= \mathrm{Re}\{\boldsymbol{I}_\mathrm{b}^\mathrm{T}(\boldsymbol{I}-\boldsymbol{I}_\mathrm{b})^*\}\\
&= \mathrm{Re}\{Y_\mathrm{b}\boldsymbol{U}^\mathrm{T}\boldsymbol{I}^* - Y_\mathrm{b}\boldsymbol{U}^\mathrm{T}Y_\mathrm{b}^*\boldsymbol{U}^*\}\\
&= \mathrm{Re}\{\boldsymbol{Y}_\mathrm{b}(\boldsymbol{U}^\mathrm{T}\boldsymbol{I}^* - \boldsymbol{U}^\mathrm{T}Y_\mathrm{b}^*\boldsymbol{U}^*)\}\\
&= \mathrm{Re}\{\boldsymbol{Y}_\mathrm{b}(\boldsymbol{C}-\boldsymbol{C}_\mathrm{b})\}\\
&= 0.
\end{aligned}
\tag{3.21}
$$

Thus the balanced and the unbalanced currents are orthogonal as well. Therefore, the current three-phase rms values in (3.20) satisfy the relationship,

$$
||\boldsymbol{i}||^2 = ||\boldsymbol{i}_\mathrm{a}||^2 + ||\boldsymbol{i}_\mathrm{r}||^2 + ||\boldsymbol{i}_\mathrm{u}||^2 \ . \tag{3.22}
$$

The load current can thus be decomposed into three orthogonal components associated with a distinctive physical phenomenon. The active current $\boldsymbol{i}_\mathrm{a}$ is associated with the permanent flow of the energy from the supply to the load, the reactive current $\boldsymbol{i}_\mathrm{r}$ is associated with a phase shift between the supply voltage and the load current and the unbalanced current $\boldsymbol{i}_\mathrm{u}$ is associated with the load imbalance. Multiplying (3.22) by the square of the three-phase rms value of the supply voltage,

$$
||\boldsymbol{i}||^2 \times ||\boldsymbol{u}||^2 = [\,||\boldsymbol{i}_\mathrm{a}||^2 + ||\boldsymbol{i}_\mathrm{r}||^2 + ||\boldsymbol{i}_\mathrm{u}||^2\,] \times ||\boldsymbol{u}||^2 \tag{3.23}
$$

yields the power equation

$$
S^2 = P^2 + Q^2 + D_\mathrm{u}^2 \tag{3.24}
$$

where S is the apparent power of the load, P is the active power, Q is the reactive power and D_{u} is the unbalanced power, defined as,

$$D_{\mathrm{u}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{u}}|| \ . \tag{3.25}$$

Equation (3.24) shows that although the superposition approach at the asymmetrical supply voltage failed to provide the orthogonal decomposition of the load current, the CPC approach enabled development of the power equations for unbalanced LTI loads supplied with asymmetrical voltage. Furthermore, comparing (3.24) with (2.34) in Chapter 2, it is evident that the form of the power equation does not change when the supply voltage is asymmetrical. The power equation developed in this section is also discussed in [20]

Illustration 3.1 An unbalanced load is supplied from a source of asymmetrical voltage as shown in Fig. 3.5. Let us calculate the powers and currents of such a load.

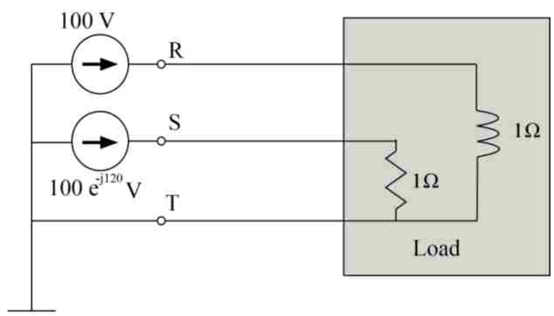


Fig. 3.5 Unbalanced load supplied with asymmetrical voltage

In the circuit given above, the crms values of the supply voltages are $\boldsymbol{E}_{\mathrm{R}} = 100\,\mathrm{V}$, $\boldsymbol{E}_{\mathrm{S}} = 100 e^{-j120°}\,\mathrm{V}$ and $\boldsymbol{E}_{\mathrm{T}} = 0\,\mathrm{V}$. Also $\boldsymbol{Z}_{\mathrm{RS}} = inf$, $\boldsymbol{Z}_{\mathrm{ST}} = 1\Omega$, and $\boldsymbol{Z}_{\mathrm{TR}} = j1\Omega$. For such a supply voltage, the crms value of the positive and negative sequence components are

$$\begin{bmatrix} \boldsymbol{U}^{\mathrm{p}} \\ \boldsymbol{U}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} \boldsymbol{E}_{\mathrm{R}} \\ \boldsymbol{E}_{\mathrm{S}} \\ \boldsymbol{E}_{\mathrm{T}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} 100 \\ 100e^{-j120°} \\ 0 \end{bmatrix} = \begin{bmatrix} 66.66 \\ 33.33e^{j60°} \end{bmatrix} \mathrm{V}\,.$$

The three-phase rms values of these symmetrical components are

$$||\boldsymbol{e}^{\mathrm{p}}|| = \sqrt{3}\boldsymbol{U}^{\mathrm{p}} = \sqrt{3}\times 66.66 = 115.47\,\mathrm{V} = ||\boldsymbol{u}^{\mathrm{p}}||$$
$$||\boldsymbol{e}^{\mathrm{n}}|| = \sqrt{3}\boldsymbol{U}^{\mathrm{n}} = \sqrt{3}\times 33.33 = 57.73\,\mathrm{V} = ||\boldsymbol{u}^{\mathrm{n}}||.$$

Symbols $||\boldsymbol{u}^{\mathrm{p}}||$ and $||\boldsymbol{u}^{\mathrm{n}}||$ denote three-phase rms values of the symmetrical components with respect to artificial zero. They are the same as the positive and negative sequence components of the internal voltage $\boldsymbol{e}$ of the distribution system under the assumption that the supply voltage is ideal. Thus, the three-phase rms value $||\boldsymbol{u}||$ of the supply voltage with respect to artificial zero is

$$||\boldsymbol{u}|| = \sqrt{||\boldsymbol{u}^{\mathrm{p}}||^2 + ||\boldsymbol{u}^{\mathrm{n}}||^2} = \sqrt{115.47^2 + 57.73^2} = 129.1\ \mathrm{V}\,.$$

The three-phase vector of the load current is

$$\boldsymbol{i} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 100e^{-j90°} \\ 100e^{-j120°} \\ 193.19e^{j75°} \end{bmatrix}\mathrm{e}^{j\omega t}\}\ \mathrm{A}.$$

The three-phase rms value of the line current $||\boldsymbol{i}|| = \sqrt{100^2 + 100^2 + 193.19^2} = 239.4\ \mathrm{A}$.

The apparent power of the load is $S = ||\boldsymbol{u}|| \times ||\boldsymbol{i}|| = 129.1\times 239.4 = 30.9\ \mathrm{kVA}$.

The crms value of the line voltages with respect to artificial zero are

$$\boldsymbol{U}_{\mathrm{R}} = \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}} = 66.66 + 33.33e^{j60°} = 88.19e^{j19.1°}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{S}} = \alpha^*\boldsymbol{U}^{\mathrm{p}} + \alpha\boldsymbol{U}^{\mathrm{n}} = (1e^{-j120°})\times 66.66 + (1e^{j120°})\times 33.33e^{j60°} = 88.19e^{-j139.1°}\ \mathrm{V}$$

$$\boldsymbol{U}_{\mathrm{T}} = \alpha \boldsymbol{U}^{\mathrm{p}} + \alpha^{*}\boldsymbol{U}^{\mathrm{n}} = (1e^{j120^\circ}) \times 66.66 + (1e^{-j120^\circ}) \times 33.33e^{j60^\circ} = 33.33e^{j120^\circ} \text{ V.}$$

The active and reactive powers of such a load are

$$P = 100^2 \times 1 = 10 \text{ kW}$$
$$Q = 100^2 \times 1 = 10 \text{ kvar}$$

Thus, the equivalent balanced admittance of the load

$$\boldsymbol{Y}_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{||\boldsymbol{u}||^2} = 0.6 - j0.6 \text{ S}.$$

The active current is

$$\boldsymbol{i}_{\mathrm{a}} = G_{\mathrm{b}}\,\boldsymbol{u} = \sqrt{2}\mathrm{Re}\{G_{\mathrm{b}}\,(\boldsymbol{U})\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{0.6\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{S}}\\ \boldsymbol{U}_{\mathrm{T}}\end{bmatrix}\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{0.6\begin{bmatrix}88.19e^{j19.1^\circ}\\ 88.19e^{-j139.1^\circ}\\ 33.33e^{j120^\circ}\end{bmatrix}\mathrm{e}^{j\omega t}\}$$

$$= \sqrt{2}\mathrm{Re}\{\begin{bmatrix}52.91e^{j19.1^\circ}\\ 52.91e^{-j139.1^\circ}\\ 20e^{j120^\circ}\end{bmatrix}\mathrm{e}^{j\omega t}\} \text{ A}$$

with the three-phase rms value $||\boldsymbol{i}_{\mathrm{a}}|| = G_{\mathrm{b}}\,||\boldsymbol{u}|| = 0.6 \times 129.1 = 77.46$ A. Similarly the reactive current is

$$\boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\mathrm{Re}\{\,jB_{\mathrm{b}}\,(\boldsymbol{U})\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\,j0.6\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{S}}\\ \boldsymbol{U}_{\mathrm{T}}\end{bmatrix}\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix}52.91e^{-j70.9^\circ}\\ 52.91e^{j130.9^\circ}\\ 20e^{j30^\circ}\end{bmatrix}\mathrm{e}^{j\omega t}\} \text{ A},$$

with the three-phase rms value $||\boldsymbol{i}_{\mathrm{r}}|| = |B_{\mathrm{b}}| \times ||\boldsymbol{u}|| = 0.6 \times 129.1 = 77.46$ A and the unbalanced current is,

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\mathrm{Re}\{(\boldsymbol{I} - \boldsymbol{I}_{\mathrm{a}} - \boldsymbol{I}_{\mathrm{r}})\,\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 95.2e^{-j135°} \\ 95.2e^{-j75°} \\ 164.9e^{j75°} \end{bmatrix}\mathrm{e}^{j\omega t}\}\mathrm{A} \;,$$

with the three-phase rms value $||\boldsymbol{i}_{\mathrm{u}}|| = \sqrt{95.2^2 + 95.2^2 + 164.9^2} = 212.88\ \mathrm{A}$.

The three-phase rms value of the supply current can be calculated using the three-phase rms values of the active, reactive and unbalanced currents as,

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2} = \sqrt{77.46^2 + 77.46^2 + 212.9^2} = 239.4\ \mathrm{A}\,,$$

which is indeed the same as the current rms value calculated from the line currents, in the earlier part of the illustration.

The active power,

$$P = ||\boldsymbol{i}_{\mathrm{a}}|| \times ||\boldsymbol{u}|| = 77.46 \times 129.1 = 10\ \mathrm{kW}\,,$$

the reactive power,

$$Q = ||\boldsymbol{i}_{\mathrm{r}}|| \times ||\boldsymbol{u}|| = 77.46 \times 129.1 = 10\ \mathrm{kvar}\,,$$

and the unbalanced power

$$D_{\mathrm{u}} = ||\boldsymbol{i}_{\mathrm{u}}|| \times ||\boldsymbol{u}|| = 212.9 \times 129.1 = 27.5\ \mathrm{kVA}\,.$$

The apparent power

$$S = \sqrt{P^2 + Q^2 + D_{\mathrm{u}}^2} = \sqrt{10^2 + 10^2 + 27.5^2} = 30.9\ \mathrm{kVA}\,,$$

which has the same magnitude as the apparent power calculated directly using the line voltages and currents rms values.

The power factor of the load $\lambda = \frac{P}{S} = \frac{10}{30.9} = 0.32$.

3.5 Dependence of the load current components on the load parameters

The power equation of an LTI load with asymmetrical voltage and currents was presented in the previous section. It would be desirable that powers and currents be expressed in terms of the load parameters, so that they can be calculated based on the knowledge of the load. More importantly, the design of a reactive compensator for the load balancing and reactive power compensation requires that the equivalent admittance be expressed in terms of the load equivalent parameters. Recalling CPC for LTI loads with sinusoidal and symmetrical voltages and currents, presented in Chapter 2, the active and reactive currents are expressed in terms of the equivalent admittance $\boldsymbol{Y}_{\rm e}$. At asymmetrical voltages, these quantities are expressed in terms of the *equivalent balanced admittance* $\boldsymbol{Y}_{\rm b}$. The question arises what is the difference between these two admittances?

The complex power of the load in Fig. 3.4 is

$$\boldsymbol{C} = P + jQ = \boldsymbol{C}_{\rm RS} + \boldsymbol{C}_{\rm ST} + \boldsymbol{C}_{\rm TR}\ , \tag{3.26}$$

where $\boldsymbol{C}_{\rm RS}, \boldsymbol{C}_{\rm ST}$ and $\boldsymbol{C}_{\rm TR}$ are the complex powers for the individual load branches.

$$\boldsymbol{C}_{\rm RS} = \boldsymbol{U}_{\rm RS}\,\boldsymbol{I}^*_{\rm RS} = \boldsymbol{U}_{\rm RS}\,\boldsymbol{I}^*_{\rm RS}\boldsymbol{U}^*_{\rm RS} = \boldsymbol{Y}^*_{\rm RS}(U^2_{\rm R} + U^2_{\rm S} - 2{\rm Re}\{\boldsymbol{U}_{\rm R}\boldsymbol{U}^*_{\rm S}\}). \tag{3.27}$$

For a three-phase three wire system,

$$U^2_{\rm T} = (-\boldsymbol{U}_{\rm R} - \boldsymbol{U}_{\rm S})(-\boldsymbol{U}_{\rm R} - \boldsymbol{U}_{\rm S})^* = U^2_{\rm R} + U^2_{\rm S} + 2{\rm Re}\{\boldsymbol{U}_{\rm R}\boldsymbol{U}^*_{\rm S}\}\ . \tag{3.28}$$

Replacing the results of (3.28) in (3.27), we get

$$\boldsymbol{C}_{\rm RS} = \boldsymbol{Y}^*_{\rm RS}(2U^2_{\rm R} + 2U^2_{\rm S} - U^2_{\rm T}) = \boldsymbol{Y}^*_{\rm RS}(2\,||\boldsymbol{u}||^2 - 3U^2_{\rm T}). \tag{3.29}$$

Similarly,

$$\boldsymbol{C}_{\mathrm{ST}} = \boldsymbol{U}_{\mathrm{ST}}\,\boldsymbol{I}_{\mathrm{ST}}^{*}\boldsymbol{U}_{\mathrm{ST}}^{*} = \boldsymbol{Y}_{\mathrm{ST}}^{*}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{R}}^{2}) \tag{3.30}$$

and,

$$\boldsymbol{C}_{\mathrm{TR}} = \boldsymbol{U}_{\mathrm{RT}}\,\boldsymbol{I}_{\mathrm{TR}}^{*}\boldsymbol{U}_{\mathrm{TR}}^{*} = \boldsymbol{Y}_{\mathrm{TR}}^{*}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{S}}^{2})\ . \tag{3.31}$$

Using these results, the equivalent balanced admittance can be written as

$$\begin{aligned}
\boldsymbol{Y}_{\mathrm{b}} &= \frac{\boldsymbol{C}^{*}}{||\boldsymbol{u}||^{2}} = \frac{\boldsymbol{C}_{\mathrm{RS}}^{*} + \boldsymbol{C}_{\mathrm{ST}}^{*} + \boldsymbol{C}_{\mathrm{TR}}^{*}}{||\boldsymbol{u}||^{2}} \\
&= \frac{\boldsymbol{Y}_{\mathrm{RS}}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{T}}^{2}) + \boldsymbol{Y}_{\mathrm{ST}}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{R}}^{2}) + \boldsymbol{Y}_{\mathrm{TR}}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{S}}^{2})}{||\boldsymbol{u}||^{2}} \\
&= \frac{2\,||\boldsymbol{u}||^{2}\,(\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}) - 3(\boldsymbol{Y}_{\mathrm{RS}}\,U_{\mathrm{T}}^{2} + \boldsymbol{Y}_{\mathrm{ST}}\,U_{\mathrm{R}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}\,U_{\mathrm{S}}^{2})}{||\boldsymbol{u}||^{2}} \\
&= 2\boldsymbol{Y}_{\mathrm{e}} - \frac{3(\boldsymbol{Y}_{\mathrm{ST}}\,U_{\mathrm{R}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}\,U_{\mathrm{S}}^{2} + \boldsymbol{Y}_{\mathrm{RS}}\,U_{\mathrm{T}}^{2})}{||\boldsymbol{u}||^{2}}.
\end{aligned} \tag{3.32}$$

The admittance $\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + j\,B_{\mathrm{e}}$ is the equivalent admittance of the load and is equal to $\boldsymbol{Y}_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}}$. Equation (3.32) can be rearranged as

$$\boldsymbol{Y}_{\mathrm{b}} = \boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{d}}, \tag{3.33}$$

where

$$\boldsymbol{Y}_{\mathrm{d}} = \frac{3(\boldsymbol{Y}_{\mathrm{ST}}\,U_{\mathrm{R}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}\,U_{\mathrm{S}}^{2} + \boldsymbol{Y}_{\mathrm{RS}}\,U_{\mathrm{T}}^{2})}{||\boldsymbol{u}||^{2}} - \boldsymbol{Y}_{\mathrm{e}} \tag{3.34}$$

is called the ***asymmetry dependent unbalanced admittance***. Observing (3.34) it is clear that $\boldsymbol{Y}_{\mathrm{d}}$ can have a non-zero value only when the load is unbalanced and the voltage is asymmetrical simultaneously. Otherwise $\boldsymbol{Y}_{\mathrm{d}}$ will be zero. Evidently, when the load is balanced,

$$\boldsymbol{Y}_{\mathrm{RS}} = \boldsymbol{Y}_{\mathrm{ST}} = \boldsymbol{Y}_{\mathrm{TR}} = \frac{\boldsymbol{Y}_{\mathrm{e}}}{3},$$

and therefore, $\boldsymbol{Y}_{\mathrm{d}} = 0$. Also when the voltages are symmetrical,

$$U_{\mathrm{R}}^2 = U_{\mathrm{S}}^2 = U_{\mathrm{T}}^2 = \frac{||\boldsymbol{u}||^2}{3},$$

and $\boldsymbol{Y}_{\mathrm{d}} = 0$.

The equivalent balanced admittance $\boldsymbol{Y}_{\mathrm{b}}$ for the asymmetrical sinusoidal supply voltage differs from the equivalent admittance $\boldsymbol{Y}_{\mathrm{e}}$ for the sinusoidal symmetrical supply by the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\mathrm{d}}$. When the supply voltage is symmetrical $\boldsymbol{Y}_{\mathrm{d}}$ becomes zero and $\boldsymbol{Y}_{\mathrm{b}}$ becomes equal to $\boldsymbol{Y}_{\mathrm{e}}$.

The supply voltages can be expressed in terms of the crms values of the positive and the negative sequence symmetrical components as

$$\boldsymbol{U}_{\mathrm{R}} = \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}}, \quad \boldsymbol{U}_{\mathrm{S}} = \alpha^* \boldsymbol{U}^{\mathrm{p}} + \alpha \boldsymbol{U}^{\mathrm{n}}, \quad \boldsymbol{U}_{\mathrm{T}} = \alpha \boldsymbol{U}^{\mathrm{p}} + \alpha^* \boldsymbol{U}^{\mathrm{n}}. \tag{3.35}$$

Using these relations we can write,

$$\begin{aligned} U_{\mathrm{R}}^2 &= (U^{\mathrm{p}})^2 + (U^{\mathrm{n}})^2 + 2\mathrm{Re}\left\{\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\}, \\ U_{\mathrm{S}}^2 &= (U^{\mathrm{p}})^2 + (U^{\mathrm{n}})^2 + 2\mathrm{Re}\left\{\alpha^*\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\}, \\ U_{\mathrm{T}}^2 &= (U^{\mathrm{p}})^2 + (U^{\mathrm{n}})^2 + 2\mathrm{Re}\left\{\alpha\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\}, \end{aligned} \tag{3.36}$$

Since $\boldsymbol{U}^{\mathrm{p}} = U^{\mathrm{p}} e^{\mathrm{j}\varphi}$ and $\boldsymbol{U}^{\mathrm{n}} = U^{\mathrm{n}} e^{\mathrm{j}\phi}$, we can rewrite expression for $\boldsymbol{Y}_{\mathrm{d}}$ in (3.34) as

$$\boldsymbol{Y}_{\mathrm{d}} = \frac{2\left(\boldsymbol{Y}_{\mathrm{ST}}\mathrm{Re}\left\{\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\} + \boldsymbol{Y}_{\mathrm{TR}}\mathrm{Re}\left\{\alpha^*\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\} + \boldsymbol{Y}_{\mathrm{RS}}\mathrm{Re}\left\{\alpha\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\right\}\right)}{U^{\mathrm{p}2} + U^{\mathrm{n}2}} \; . \tag{3.37}$$

Let us define the ratio of the negative and the positive sequence components of the supply voltages, called the ***complex coefficient of the supply voltage asymmetry*** as

$$\boldsymbol{a} = a\,e^{j\psi} = \frac{\boldsymbol{U}^{\rm n}}{\boldsymbol{U}^{\rm p}} = \frac{U^{\rm n}\,e^{{\rm j}\alpha^{\rm n}}}{U^{\rm p}\,e^{{\rm j}\alpha^{\rm p}}} = \frac{U^{\rm n}}{U^{\rm p}}\,e^{j(\alpha^{\rm n}-\alpha^{\rm p})}\,. \tag{3.38}$$

Then,

$$\begin{aligned}
&\frac{{\rm Re}\{\boldsymbol{U}^{{\rm p}*}\boldsymbol{U}^{\rm n}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{U^{\rm p}U^{\rm n}\,{\rm Re}\{e^{j(\alpha^{\rm n}-\alpha^{\rm p})}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{a}{1+a^2}\cos\psi\,,\\
&\frac{{\rm Re}\{\alpha^*\boldsymbol{U}^{{\rm p}*}\boldsymbol{U}^{\rm n}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{U^{\rm p}U^{\rm n}\,{\rm Re}\{e^{-j2\pi/3}\;e^{j(\alpha^{\rm n}-\alpha^{\rm p})}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{a}{1+a^2}\cos(\psi-\frac{2\pi}{3}),\\
&\frac{{\rm Re}\{\alpha\boldsymbol{U}^{{\rm p}*}\boldsymbol{U}^{\rm n}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{U^{\rm p}U^{\rm n}\,{\rm Re}\{e^{j2\pi/3}\;e^{j(\alpha^{\rm n}-\alpha^{\rm p})}\}}{U^{{\rm p}2}+U^{{\rm n}2}} = \frac{a}{1+a^2}\cos(\psi+\frac{2\pi}{3})
\end{aligned} \tag{3.39}$$

Using the formulae in (3.39), $\boldsymbol{Y}_{\rm d}$ can be rewritten as

$$\boldsymbol{Y}_{\rm d} = \frac{2a}{1+a^2}[\boldsymbol{Y}_{\rm ST}\cos\psi+\boldsymbol{Y}_{\rm TR}\cos(\psi-\frac{2\pi}{3})+\boldsymbol{Y}_{\rm RS}\cos(\psi+\frac{2\pi}{3})]\,. \tag{3.40}$$

The crms value of the current in line R is equal to

$$\begin{aligned}
\boldsymbol{I}_{\rm R} &= \boldsymbol{Y}_{\rm RS}(\boldsymbol{U}_{\rm R}-\boldsymbol{U}_{\rm S})-\boldsymbol{Y}_{\rm TR}(\boldsymbol{U}_{\rm T}-\boldsymbol{U}_{\rm R})\\
&= \boldsymbol{Y}_{\rm RS}\boldsymbol{U}_{\rm R}-\boldsymbol{Y}_{\rm RS}\boldsymbol{U}_{\rm S}-\boldsymbol{Y}_{\rm TR}\boldsymbol{U}_{\rm T}+\boldsymbol{Y}_{\rm TR}\boldsymbol{U}_{\rm R}+\boldsymbol{Y}_{\rm ST}\boldsymbol{U}_{\rm R}-\boldsymbol{Y}_{\rm ST}\boldsymbol{U}_{\rm R}\\
&= (\boldsymbol{Y}_{\rm RS}+\boldsymbol{Y}_{\rm ST}+\boldsymbol{Y}_{\rm TR})\boldsymbol{U}_{\rm R}-\boldsymbol{Y}_{\rm RS}\boldsymbol{U}_{\rm S}-\boldsymbol{Y}_{\rm TR}\boldsymbol{U}_{\rm T}-\boldsymbol{Y}_{\rm ST}\boldsymbol{U}_{\rm R}\\
&= \boldsymbol{Y}_{\rm e}\boldsymbol{U}_{\rm R}-(\boldsymbol{Y}_{\rm ST}\boldsymbol{U}_{\rm R}+\boldsymbol{Y}_{\rm TR}\boldsymbol{U}_{\rm T}+\boldsymbol{Y}_{\rm RS}\boldsymbol{U}_{\rm S}).
\end{aligned} \tag{3.41}$$

If we express the line voltages in terms of the symmetrical components as shown in (3.35), then (3.41) can be rearranged as follows

$$\boldsymbol{I}_{\rm R} = \boldsymbol{Y}_{\rm e}\boldsymbol{U}_{\rm R}+\boldsymbol{A}^{\rm p}\boldsymbol{U}_{\rm R}^{\rm p}+\boldsymbol{A}^{\rm n}\boldsymbol{U}_{\rm R}^{\rm n} \tag{3.42}$$

where,

$$\boldsymbol{A}^{\mathrm{p}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha \boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{RS}}), \qquad \boldsymbol{A}^{\mathrm{n}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{TR}} + \alpha \boldsymbol{Y}_{\mathrm{RS}}). \tag{3.43}$$

Using similar approach, the crms values of the line currents $\boldsymbol{I}_{\mathrm{S}}$ and $\boldsymbol{I}_{\mathrm{T}}$ can be written as

$$\begin{aligned} \boldsymbol{I}_{\mathrm{S}} &= \boldsymbol{Y}_{\mathrm{e}} \boldsymbol{U}_{\mathrm{S}} + \boldsymbol{A}^{\mathrm{p}} \boldsymbol{U}_{\mathrm{T}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}_{\mathrm{T}}^{\mathrm{n}} \\ \boldsymbol{I}_{\mathrm{T}} &= \boldsymbol{Y}_{\mathrm{e}} \boldsymbol{U}_{\mathrm{T}} + \boldsymbol{A}^{\mathrm{p}} \boldsymbol{U}_{\mathrm{S}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \boldsymbol{U}_{\mathrm{S}}^{\mathrm{n}}. \end{aligned} \tag{3.44}$$

The crms values of currents given in (3.42) and (3.44) can be expressed in vector form as

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}} \end{bmatrix} = \boldsymbol{Y}_{\mathrm{e}} \boldsymbol{U} + \boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}}. \tag{3.45}$$

Thus the vector of the unbalanced current is

$$\begin{aligned} \boldsymbol{I}_{\mathrm{u}} = \boldsymbol{I} - \boldsymbol{I}_{\mathrm{b}} &= \boldsymbol{Y}_{\mathrm{e}} \boldsymbol{U} + \boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} - \boldsymbol{Y}_{\mathrm{b}} \boldsymbol{U} \\ &= (\boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{b}}) \boldsymbol{U} + \boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} \\ &= \boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U} + \boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} \\ &= \boldsymbol{Y}_{\mathrm{d}} (\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}}) + \boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} \\ &= (\boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{n}}) \\ &= \boldsymbol{I}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{I}_{\mathrm{u}}^{\mathrm{n}}. \end{aligned} \tag{3.46}$$

The analysis presented above shows that although the form of the power equation for LTI loads at asymmetrical supply voltage remains the same as compared to the case with symmetrical supply voltage, the equivalent load parameters are affected by the voltage asymmetry. In the case of asymmetrical supply, in addition to the equivalent admittance $\boldsymbol{Y}_{\mathrm{e}}$, the load also has an asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\mathrm{d}}$, which is not constant, but it is dependent on the supply voltage asymmetry.

Illustration 3.2 Let us calculate the unbalanced current of the load shown in Fig. 3.5 using the relations developed in the section above.

The positive and the negative sequence voltage symmetrical components for the circuit in Fig. 3.5 were calculated earlier in the chapter. We obtained

$$\boldsymbol{U}^{\mathrm{p}} = 66.66\ \mathrm{V}, \quad \boldsymbol{U}^{\mathrm{n}} = 33.33 e^{j60^\circ}\ \mathrm{V}\,.$$

The complex coefficient of asymmetry

$$\boldsymbol{a} = a e^{j\psi} = \frac{\boldsymbol{U}^{\mathrm{n}}}{\boldsymbol{U}^{\mathrm{p}}} = \frac{33.33 e^{j60^\circ}}{66.66} = 0.5 e^{j60^\circ}\,.$$

The unbalance admittances,

$$\boldsymbol{A}^{\mathrm{p}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha \boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{RS}}) = -(1 + \alpha(-j1) + 0) = 1.932 e^{-j165^\circ}\,\mathrm{S}$$
$$\boldsymbol{A}^{\mathrm{n}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha^{*} \boldsymbol{Y}_{\mathrm{TR}} + \alpha \boldsymbol{Y}_{\mathrm{RS}}) = -(1 + \alpha^{*}(-j1) + 0) = 0.518 e^{-j105^\circ}\,\mathrm{S}.$$

The asymmetry dependent unbalanced admittance is equal to

$$\boldsymbol{Y}_{\mathrm{d}} = \frac{2a}{1+a^2}\left[\boldsymbol{Y}_{\mathrm{ST}}\cos\psi + \boldsymbol{Y}_{\mathrm{TR}}\cos(\psi - \frac{2\pi}{3}) + \boldsymbol{Y}_{\mathrm{RS}}\cos(\psi + \frac{2\pi}{3})\right]$$
$$= \frac{2\times 0.5}{1+0.5^2}\left[1\cos(60^\circ) - j\cos(60^\circ - 120^\circ) + 0\right] = 0.566 e^{-j45^\circ}\,\mathrm{S}$$

Thus, the crms value of the load unbalanced current is

$$\boldsymbol{I}_{\mathrm{u}} = \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \boldsymbol{A}^{\mathrm{p}}\mathbf{1}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}}\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{n}}$$
$$= 0.566 e^{-j45^\circ}\begin{bmatrix} 88.19 e^{j19.1^\circ} \\ 88.19 e^{-j139.1^\circ} \\ 33.33 e^{j120^\circ} \end{bmatrix} + 1.932 e^{-j165^\circ}\begin{bmatrix} 66.66 \\ 66.66 e^{j120^\circ} \\ 66.66 e^{-j120^\circ} \end{bmatrix} + 0.518 e^{-j105^\circ}\begin{bmatrix} 33.33 e^{j60^\circ} \\ 33.33 e^{-j60^\circ} \\ 33.33 e^{j180^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 95.28e^{-j134.9°} \\ 95.28e^{-j75°} \\ 164.9e^{j75°} \end{bmatrix} \text{A}$$

and the three-phase rms value of the unbalanced current

$$||\boldsymbol{i}_{\text{u}}|| = \sqrt{95.28^2 + 95.28^2 + 164.9^2} = 212.95\text{A} ,$$

which is the same as the value calculated in Illustration 3.1.

3.6 Reactive compensation

The active current is associated with the permanent energy transfer between the supply and the load. In other words, it is the portion of the load current related to the active power of the load. The reactive and the unbalanced currents are superfluous currents which increase the supply current rms value and consequently, increase the energy losses at delivery. Thereby, they cause a decline of the power factor. Compensation of these currents is needed for an improvement of the power factor. Similar to the case when the supply voltage was symmetrical, a lossless shunt compensator of the delta structure, as shown in Fig. 3.6, can be used for compensation of the reactive and unbalanced currents.

Let us assume that such a balancing compensator is composed of lossless reactance elements. It can have a either an inductor or a capacitor in each branch, with susceptances T_{RS}, T_{ST} and T_{TR}. An inductor may be added in series to the capacitor to avoid resonance with the source inductance. The balancing compensator serves two main purposes:

- balances the system

- compensates the reactive power of the load.

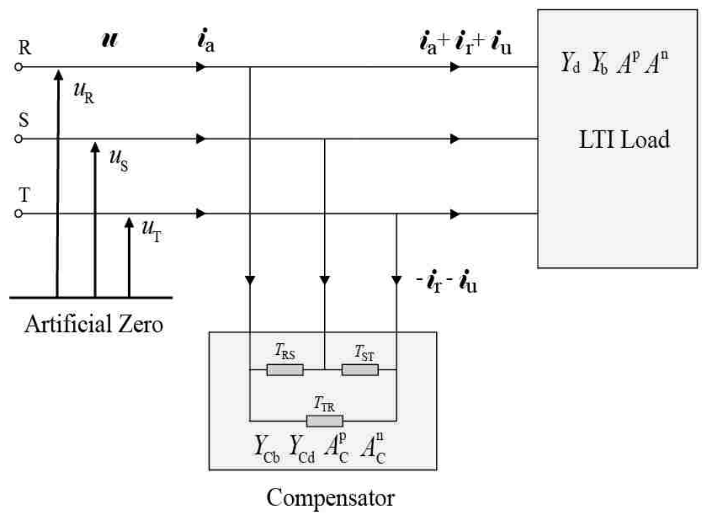


Fig. 3.6 Three-phase LTI load with balancing reactive compensator

The compensator reactances are chosen such that the load with the compensator is balanced and purely resistive.

If the compensator branch susceptances are T_{RS}, T_{ST} and T_{TR}, then the unbalanced admittances $\boldsymbol{A}_C^p$, $\boldsymbol{A}_C^n$ and $\boldsymbol{Y}_{Cd}$ of the compensator can be found using similar approach as that used for the calculation of the unbalanced admittances of the load, namely

$$\begin{aligned}
\boldsymbol{A}_C^p &= -j(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS}) \\
\boldsymbol{A}_C^n &= -j(T_{ST} + \alpha^* T_{TR} + \alpha T_{RS}) \\
\boldsymbol{Y}_{Cd} &= \frac{j2a}{1+a^2}\left[T_{ST}\cos\psi + T_{TR}\cos(\psi - \frac{2\pi}{3}) + T_{RS}\cos(\psi + \frac{2\pi}{3})\right]
\end{aligned} \tag{3.47}$$

The total reactive power of such a compensator is

$$Q_{Cb} = -(T_{RS}U_{RS}^2 + T_{ST}U_{ST}^2 + T_{TR}U_{TR}^2)\ . \tag{3.48}$$

The negative sign in (3.48) is in accordance to the convention that the reactive power of the inductor is positive and that of the capacitor is negative. Also, if the element in the branch is capacitive, its susceptance is positive while if the element is an inductor, the susceptance is negative.

If B_{Cb} is the equivalent susceptance of the compensator, then the total reactive power of the compensator, $Q_{Cb} = -B_{Cb}||\boldsymbol{u}||^2$. Using this relation, (3.48) can be re written as

$$B_{Cb} = \frac{T_{RS}U_{RS}^2 + T_{ST}U_{ST}^2 + T_{TR}U_{TR}^2}{||\boldsymbol{u}||^2} \quad . \tag{3.49}$$

Such a compensator draws reactive current with crms value

$$\boldsymbol{I}_{Cr} = j\,B_{Cb}\boldsymbol{U} \quad . \tag{3.50}$$

Recalling equation (3.17), the reactive current of the load is associated with the equivalent balanced admittance $\boldsymbol{Y}_b$, in particular the equivalent balanced susceptance B_b. Since the compensator is in parallel with the load, the equivalent susceptance of the load and the compensator is $\left(B_{Cb} + B_b\right)$. The crms value of the total reactive current of the load and the compensator is

$$\boldsymbol{I}' = \boldsymbol{I}_{Cr} + \boldsymbol{I}_r = j\,B_{Cb}\boldsymbol{U} + jB_b\boldsymbol{U} \quad . \tag{3.51}$$

Such a compensator reduces the reactive current to zero under the condition that

$$B_{Cb} + B_b = 0 \tag{3.52}$$

The susceptance related equations (3.49) and (3.52) can be combined and rearranged as

$$T_{RS}\,U_{RS}^2 + T_{ST}\,U_{ST}^2 + T_{TR}\,U_{TR}^2 = -B_b\,||\boldsymbol{u}||^2 \quad . \tag{3.53}$$

The vector of crms values of the unbalanced current of the compensator is

$$\boldsymbol{I}_{Cu} = (A_C^n \boldsymbol{1}^p \boldsymbol{U}^n + \boldsymbol{Y}_{Cd}\boldsymbol{U}^p) + (A_C^p \boldsymbol{1}^n \boldsymbol{U}^p + \boldsymbol{Y}_{Cd}\boldsymbol{U}^n) \quad , \tag{3.54}$$

and the unbalanced current of the source is the sum of the unbalanced currents of the load and the compensator, i.e.,

$$\begin{aligned}
\boldsymbol{I}'_{\mathrm{u}} &= \boldsymbol{I}_{\mathrm{u}} + \boldsymbol{I}_{\mathrm{Cu}} \\
&= (\boldsymbol{A}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{d}} \boldsymbol{U}^{\mathrm{n}}) + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} \mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{Cd}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} \mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{Cd}} \boldsymbol{U}^{\mathrm{n}}) \\
&= (\boldsymbol{Y}_{\mathrm{d}} + \boldsymbol{Y}_{\mathrm{Cd}})(\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}})(\mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}})(\mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}}) \\
&= (\boldsymbol{Y}_{\mathrm{d}} + \boldsymbol{Y}_{\mathrm{Cd}}) \boldsymbol{U} + (\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}})(\mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}})(\mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}}).
\end{aligned} \tag{3.55}$$

Hence, the unbalanced current of the source given in (3.55) is reduced to zero under the condition that

$$(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}}) \boldsymbol{U} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}})(\mathbf{1}^{\mathrm{n}} \boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}})(\mathbf{1}^{\mathrm{p}} \boldsymbol{U}^{\mathrm{n}}) = 0 \ . \tag{3.56}$$

This equation for the current three-phase vector components has to be satisfied for each phase separately. In particular, for phase R, the following equation has to be satisfied,

$$(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}}) \boldsymbol{U}_{\mathrm{R}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}) \boldsymbol{U}^{\mathrm{p}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}}) \boldsymbol{U}^{\mathrm{n}} = 0 \ . \tag{3.57}$$

Equation (3.57) contains complex quantities and therefore, to be valid, it has to satisfy the condition for both the real and the imaginary parts, thereby leading to two equations. These two equations combined with (3.53) provide three linear equations with three unknowns, namely T_{RS}, T_{ST} and T_{TR}. Solution of these equations provide the compensator branch susceptance values which will reduce both the reactive and unbalanced currents to zero, and improve the power factor to unity.

The three abovementioned equations are

$$\begin{aligned}
&T_{\mathrm{RS}} U_{\mathrm{RS}}^2 + T_{\mathrm{ST}} U_{\mathrm{ST}}^2 + T_{\mathrm{TR}} U_{\mathrm{TR}}^2 = -B_{\mathrm{b}} \, ||\boldsymbol{u}||^2 \\
&\mathrm{Re}\{(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}}) \boldsymbol{U}_{\mathrm{R}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}) \boldsymbol{U}^{\mathrm{p}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}}) \boldsymbol{U}^{\mathrm{n}}\} = 0 \\
&\mathrm{Im}\{(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}}) \boldsymbol{U}_{\mathrm{R}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}) \boldsymbol{U}^{\mathrm{p}} + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}}) \boldsymbol{U}^{\mathrm{n}}\} = 0.
\end{aligned} \tag{3.58}$$

The last two equations of (3.58) contain complex quantities and some rearrangement is needed before they can be easily used for the calculation of the compensator susceptances.

Dividing (3.57) by $\boldsymbol{U}^{\mathrm{p}}$ yields,

$$(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}})(1 + ae^{j\psi}) + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}) + (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}})\, ae^{j\psi} = 0. \tag{3.59}$$

The parameters $\boldsymbol{Y}_{\mathrm{d}}$, $\boldsymbol{A}^{\mathrm{p}}$ and $\boldsymbol{A}^{\mathrm{n}}$ in the above equation can be obtained if the voltages, currents and the load impedances are given. On the other hand, $\boldsymbol{Y}_{\mathrm{Cd}}$, $\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}}$ and $\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}}$ are unknowns that we need to solve for. To reduce the complexity of the analysis, we can break (3.59) into two portions such that one of them consists the unknown compensators parameters and the other contains the given load parameters. Rearranging (3.59) yields,

$$\boldsymbol{Y}_{\mathrm{Cd}}(1 + ae^{j\psi}) + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}}\, ae^{j\psi} + \boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{d}}(1 + ae^{j\psi}) + \boldsymbol{A}^{\mathrm{n}}\, ae^{j\psi} = 0\,. \tag{3.60}$$

The first part of equation (3.60)

$$\boldsymbol{Y}_{\mathrm{Cd}}(1 + ae^{j\psi}) = (c_1 T_{\mathrm{ST}} + c_2 T_{\mathrm{TR}} + c_3 T_{\mathrm{RS}})(1 + ae^{j\psi}) \tag{3.61}$$

where,

$$\begin{aligned} c_1 &= \frac{j\,2a\ \cos\psi}{1 + a^2} \\ c_2 &= \frac{j\,2a\ \cos(\psi - 2\pi/3)}{1 + a^2} \\ c_3 &= \frac{j\,2a\ \cos(\psi + 2\pi/3)}{1 + a^2}. \end{aligned} \tag{3.62}$$

Similarly,

$$\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}}\, a\, e^{j\psi} = -j(T_{\mathrm{ST}} + \alpha^* T_{\mathrm{TR}} + \alpha T_{\mathrm{RS}})\, a\, e^{j\psi} \tag{3.63}$$

and,

$$A_C^p = -j\,(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS})\ . \tag{3.64}$$

Using equations (3.61) to (3.64) , equation (3.60) can be rearranged to the form

$$\boldsymbol{F}_1 T_{RS} + \boldsymbol{F}_2 T_{ST} + \boldsymbol{F}_3 T_{TR} + \boldsymbol{F}_4 = 0 \tag{3.65}$$

where,

$$\begin{aligned}
\boldsymbol{F}_1 &= c_3\,(1 + ae^{j\psi}) - j\,(\alpha^* + \alpha\, ae^{j\psi}) \\
\boldsymbol{F}_2 &= c_1\,(1 + ae^{j\psi}) - j\,(1 + ae^{j\psi}) \\
\boldsymbol{F}_3 &= c_2\,(1 + ae^{j\psi}) - j\,(\alpha + \alpha^*\, ae^{j\psi}) \\
\boldsymbol{F}_4 &= \boldsymbol{Y}_d\,(1 + ae^{j\psi}) + \boldsymbol{A}^p + \boldsymbol{A}^n\, ae^{j\psi}\ .
\end{aligned} \tag{3.66}$$

Equation (3.65) has to be satisfied for both the real and the imaginary parts, thus,

$$\begin{aligned}
T_{RS}\ \mathrm{Re}\boldsymbol{F}_1 + T_{ST}\ \mathrm{Re}\boldsymbol{F}_2 + T_{TR}\ \mathrm{Re}\boldsymbol{F}_3 + \mathrm{Re}\boldsymbol{F}_4 = 0 \\
T_{RS}\ \mathrm{Im}\boldsymbol{F}_1 + T_{ST}\ \mathrm{Im}\boldsymbol{F}_2 + T_{TR}\ \mathrm{Im}\boldsymbol{F}_3 + \mathrm{Im}\boldsymbol{F}_4 = 0
\end{aligned} \tag{3.67}$$

Equation (3.53) and the two equations in (3.67) can be used as the three linear equations to solve for the three unknown variables T_{RS}, T_{ST} and T_{TR} . They can be rewritten in matrix form as

$$\begin{bmatrix} U_{RS}^2 & U_{ST}^2 & U_{TR}^2 \\ \mathrm{Re}\boldsymbol{F}_1 & \mathrm{Re}\boldsymbol{F}_2 & \mathrm{Re}\boldsymbol{F}_3 \\ \mathrm{Im}\boldsymbol{F}_1 & \mathrm{Im}\boldsymbol{F}_2 & \mathrm{Im}\boldsymbol{F}_3 \end{bmatrix} \begin{bmatrix} T_{RS} \\ T_{ST} \\ T_{TR} \end{bmatrix} = \begin{bmatrix} -B_b\,||\boldsymbol{u}||^2 \\ -\mathrm{Re}\boldsymbol{F}_4 \\ -\mathrm{Im}\boldsymbol{F}_4 \end{bmatrix} \tag{3.68}$$

Equation (3.68) is referred to as the **compensator equation** and can be used to solve for the compensator susceptances T_{RS}, T_{ST} and T_{TR} .

Illustration 3.3 Let us design a compensator to improve the power factor of the load shown in illustration 3.2. A shunt compensator of the delta structure as shown in Fig. 3.6 can be used for compensation of the power factor. The compensator branch susceptances T_{RS}, T_{ST} and T_{TR} can be calculated using (3.68).

The line to line voltages with respect to artificial zero are

$$\boldsymbol{U}_{\mathrm{RS}} = \boldsymbol{U}_{\mathrm{R}} - \boldsymbol{U}_{\mathrm{S}} = 173.2\,e^{j30^\circ}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{ST}} = \boldsymbol{U}_{\mathrm{S}} - \boldsymbol{U}_{\mathrm{T}} = 100\,e^{-j90^\circ}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{TR}} = \boldsymbol{U}_{\mathrm{T}} - \boldsymbol{U}_{\mathrm{R}} = 100\,e^{j180^\circ}\ \mathrm{V}$$

The square of the magnitudes of these line to line voltage crms values are

$$U_{\mathrm{RS}}^2 = 30000\ \mathrm{V}^2, \quad U_{\mathrm{ST}}^2 = 10000\ \mathrm{V}^2, \quad U_{\mathrm{TR}}^2 = 10000\ \mathrm{V}^2.$$

We obtained earlier that $||\boldsymbol{u}|| = 129.1$ V. Thus $||\boldsymbol{u}||^2 = 16667\ \mathrm{V}^2$.

Therefore,

$$c_1 = \frac{j\,2\,a\cos\psi}{1+a^2} = \frac{(j\,2\times 0.5\cos(60))}{1+0.5^2} = j\,0.4$$
$$c_2 = \frac{j\,2\,a\cos(\psi - 2\pi/3)}{1+a^2} = j\,0.4$$
$$c_3 = \frac{j\,2\,a\cos(\psi + 2\pi/3)}{1+a^2} = -j\,0.8.$$

Next,

$$\boldsymbol{F}_1 = c_3(1+ae^{j\psi}) - j(\alpha^* + \alpha\, ae^{j\psi}) = -j\,0.8(1+0.5e^{j60^\circ}) - j(\alpha^* + \alpha \times 0.5e^{j60^\circ}) = -0.519$$
$$\boldsymbol{F}_2 = c_1(1+ae^{j\psi}) - j(1+ae^{j\psi}) = 0.2598 - j\,0.75$$
$$\boldsymbol{F}_3 = c_2(1+ae^{j\psi}) - j(\alpha + \alpha^*\, ae^{j\psi}) = 0.2598 + j\,0.75$$
$$\boldsymbol{F}_4 = \boldsymbol{Y}_{\mathrm{d}}(1+ae^{j\psi}) + \boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}}\, ae^{j\psi} = -1.0098 - j1.0098.$$

The compensator equation,

$$\begin{bmatrix} U_{\mathrm{RS}}^2 & U_{\mathrm{ST}}^2 & U_{\mathrm{TR}}^2 \\ \mathrm{Re}\boldsymbol{F}_1 & \mathrm{Re}\boldsymbol{F}_2 & \mathrm{Re}\boldsymbol{F}_3 \\ \mathrm{Im}\boldsymbol{F}_1 & \mathrm{Im}\boldsymbol{F}_2 & \mathrm{Im}\boldsymbol{F}_3 \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS}} \\ T_{\mathrm{ST}} \\ T_{\mathrm{TR}} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b}}\,||\boldsymbol{u}||^2 \\ -\mathrm{Re}\boldsymbol{F}_4 \\ -\mathrm{Im}\boldsymbol{F}_4 \end{bmatrix}$$

has the values,

$$\begin{bmatrix} 29998.24 & 10000 & 10000 \\ -0.5196 & 0.2598 & 0.2598 \\ 0 & -0.75 & 0.75 \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS}} \\ T_{\mathrm{ST}} \\ T_{\mathrm{TR}} \end{bmatrix} = \begin{bmatrix} 10000 \\ 1.0098 \\ 1.0098 \end{bmatrix}$$

Solving the above equations we get,

$$\begin{bmatrix} T_{\mathrm{RS}} \\ T_{\mathrm{ST}} \\ T_{\mathrm{TR}} \end{bmatrix} = \begin{bmatrix} -0.578 \\ 0.693 \\ 2.039 \end{bmatrix} \mathrm{S}.$$

The structure of the compensator should be as shown in Fig 3.5. If $\omega = 377$ rad/sec, then the compensator branch RS has an inductor L_{RS} = 4.59 mH, branch ST has a capacitor C_{ST} = 1.84 mF and branch TR has a capacitor C_{TR} = 6.35 mF.

The equivalent admittance of the compensator plus the load in parallel is $\boldsymbol{Y}_{\mathrm{e}}^{'} = 1 + j1.155\,\mathrm{S}$

Equivalent balanced admittance after compensation $\boldsymbol{Y}_{\mathrm{b}}^{'} = 0.6\mathrm{S}$.

Therefore, $G_{\mathrm{b}}^{'} = 0.6$ S, $B_{\mathrm{b}}^{'} = 0$ S. Hence,

$$||\boldsymbol{i}_{\mathrm{a}}^{'}|| = G_{\mathrm{b}}^{'} \times ||\boldsymbol{u}|| = 77.46\,\mathrm{A}, \qquad \left\|\boldsymbol{i}_{\mathrm{r}}^{'}\right\| = B_{\mathrm{b}}^{'} \times ||\boldsymbol{u}|| = 0\,\mathrm{A}.$$

The unbalance admittances of the compensator,

$$\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} = 2.27\,e^{j.96^\circ}\ \mathrm{S}, \qquad \boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} = 2.27\,e^{j179.04^\circ}\ \mathrm{S},$$

$$\begin{aligned} \boldsymbol{Y}_{\mathrm{Cd}} &= \frac{j\,2\,a}{1+a^2}\left[T_{\mathrm{ST}}\cos\psi + T_{\mathrm{TR}}\cos(\psi - 2\pi/3) + T_{\mathrm{RS}}\cos(\psi + 2\pi/3)\right] \\ &= \frac{j\,2\mathrm{x}0.5}{1+0.5^2}\left[0.693\times\cos(60^\circ) + 2.039\times\cos(60^\circ - 120^\circ) - 1\times -0.578\right] = j1.5547 = 1.5547\,e^{j90^\circ}\ \mathrm{S} \end{aligned}$$

$$\therefore (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}) = 0.611e^{-j49.1^\circ}\,\mathrm{S},\ (\boldsymbol{A}_{\mathrm{C}}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}}) = 2.44e^{-j169.1^\circ}\,\mathrm{S}\ \text{ and }\ \left(\boldsymbol{Y}_{\mathrm{Cd}} + \boldsymbol{Y}_{\mathrm{d}}\right)() = 1.22e^{j70.9^\circ}\,\mathrm{S}.$$

The unbalanced current after compensation,

$$\boldsymbol{I}'_{\mathrm{u}} = (\boldsymbol{Y}_{\mathrm{d}} + \boldsymbol{Y}_{\mathrm{Cd}})\boldsymbol{U} + (\boldsymbol{A}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}})(\mathbf{1}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{p}}) + (\boldsymbol{A}^{\mathrm{n}} + \boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}})(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{n}})$$

$$= (1.22e^{j70.9^\circ})\begin{bmatrix} 88.19e^{j19.1^\circ} \\ 88.19e^{-j139.1^\circ} \\ 33.33e^{j120^\circ} \end{bmatrix} + (0.611e^{-j49.1^\circ})\begin{bmatrix} 66.66 \\ 66.66e^{j120^\circ} \\ 66.66e^{-j120^\circ} \end{bmatrix} + (2.44e^{-j169.1^\circ})\begin{bmatrix} 33.33e^{j60^\circ} \\ 33.33e^{-j60^\circ} \\ 33.33e^{j180^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ A}$$

$$\therefore \|\boldsymbol{i}'_{\mathrm{u}}\| = 0 \text{ A},$$

and this result confirms numerical correctness of the compensator design.

The powers after compensation are equal to,

$$P' = \|\boldsymbol{i}'_{\mathrm{a}}\| \times \|\boldsymbol{u}\| = 10\mathrm{kW},$$
$$Q' = \|\boldsymbol{i}'_{\mathrm{r}}\| \times \|\boldsymbol{u}\| = 0$$
$$D'_{\mathrm{u}} = \|\boldsymbol{i}'_{\mathrm{u}}\| \times \|\boldsymbol{u}\| = 0.$$

Thus, $S' = 10\mathrm{KVA}$ and the power factor after compensation $\lambda' = \dfrac{P'}{S'} = 1$

Hence, the compensator completely compensates the reactive and the unbalanced currents and improves the power factor to unity.

The results obtained above are illustrated in Fig. 3.7. Note that the supply current after compensation is not symmetrical. In fact, it has the same asymmetry as that of the supply voltage referred to an artificial zero. The load and compensator draw just the active current from the source, and the current is proportional to the supply voltages referenced to artificial zero. The design of the reactive compensator presented above is also discussed in [21] and its industrial application is discussed in [22].

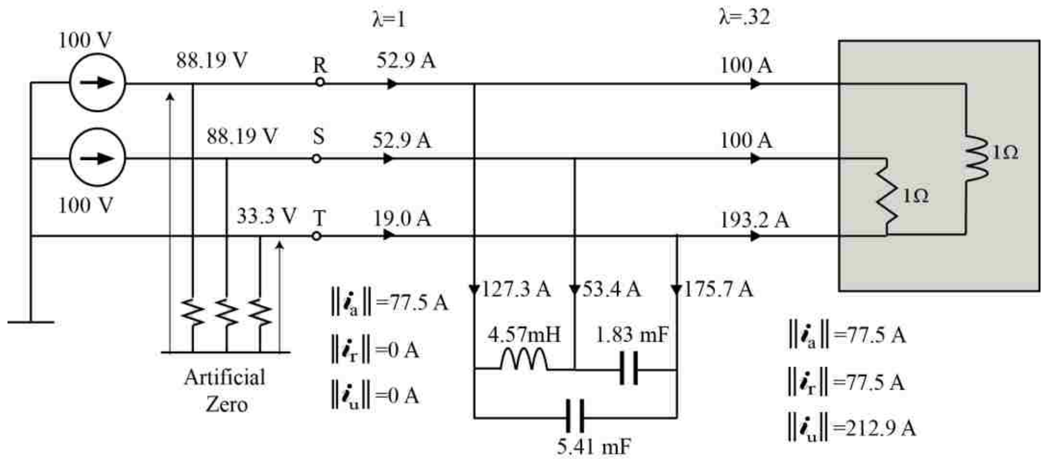


Fig. 3.7 Unbalanced load with a reactive compensator

3.7 Conclusion

The power equation of LTI loads at sinusoidal but asymmetrical supply voltage was developed using the Currents Physical Components (CPC) based load current decomposition in this chapter. The form of the power equation at asymmetrical voltage is the same as the one at symmetrical voltage. Moreover, the load current contains the same current components. However, the parameters on which the powers and the current components depend upon have changed. At symmetrical voltage, the currents were dependent on just the equivalent load admittance $\boldsymbol{Y}_{\rm e}$, while at voltage asymmetry, they are dependent on the equivalent balanced admittance $\boldsymbol{Y}_{\rm b}$, and the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\rm d}$, both of which are dependent on the supply voltage asymmetry. The CPC based current decomposition also enabled the design of a reactive compensator which completely compensates the reactive and unbalanced currents and improves the power factor to unity. It is seen that the structure of such a reactive compensator is not affected by voltage asymmetry.

CHAPTER 4: POWERS AND REACTIVE COMPENSATION OF UNBALANCED LTI LOADS WITH NONSINUSOIDAL AND ASYMMETRICAL VOLTAGES AND CURRENTS

4.1 Introduction

A power theory describes only properties of the energy flow in the cross section of a power system with a model simplified with various reasonable assumptions. The accuracy of such a model can be improved by removing the assumptions step by step. The commonly known power equation

$$S^2 = P^2 + Q^2 \tag{4.1}$$

is valid only at the assumption that the voltages and currents are sinusoidal and symmetrical. Such an assumption is not always valid because the internal voltage of the modern distribution system can be asymmetrical and distorted. In order to correctly describe power properties of any load, or for accurate metering, the correct power equation of the load has to be developed for the given operating conditions. Therefore, it is important to develop the power equation to describe the power properties of LTI loads at voltage asymmetry and distortion. The power equation of LTI loads at asymmetrical but sinusoidal voltage was developed in Chapter 3. In this chapter, the assumption that the supply voltage is sinusoidal is also abandoned, and the power equation of LTI loads is developed at nonsinusoidal asymmetrical (N&A) supply voltage.

Recalling Chapter 3, load current decomposition was first done based on the Superposition Principle. That approach was unsuccessful, however, because such a current decomposition did not result in orthogonal components associated with a distinct physical phenomenon. The next approach was the Current's Physical Components (CPC) based load current decomposition, which enabled the development of the power equation for such a load. It also enabled the design of a reactance compensator that reduced completely the reactive and unbalanced currents and improved the power factor to unity. This chapter is a continuation in the same direction as the previous one, except it is assumed that the supply voltage can be nonsinusoidal. The power theory of LTI loads at N&A voltages and currents is presented here and the theory is used for the design of a reactance compensator for power factor improvement.

4.2 Power equation of LTI loads at nonsinusoidal and asymmetrical supply voltage

The internal voltage of the distribution system, expressed in a form of a three-phase vector $\boldsymbol{e} = [\boldsymbol{e}_{\mathrm{R}}, \boldsymbol{e}_{\mathrm{S}}, \boldsymbol{e}_{\mathrm{T}}]^{\mathrm{T}}$, is assumed to be N&A. Therefore, in addition to the voltage symmetrical components of the positive and the negative sequence, it can also have a component of the zero sequence, i.e. it can be expressed in the form

$$\boldsymbol{e}(t) = \sum_{n\in \boldsymbol{N}} \boldsymbol{e}_{\mathrm{n}} = \sum_{n\in \boldsymbol{N}} (\boldsymbol{e}_n^{\mathrm{p}} + \boldsymbol{e}_n^{\mathrm{n}} + \boldsymbol{e}_n^{\mathrm{z}}) = \boldsymbol{e}^{\mathrm{p}} + \boldsymbol{e}^{\mathrm{n}} + \boldsymbol{e}^{\mathrm{z}} \tag{4.2}$$

The zero sequence component $\boldsymbol{e}^{\mathrm{z}}$ of the internal voltage cannot cause current in three-wire system, but nevertheless, it increases the supply voltage rms value; thereby increasing the apparent power S and reducing the power factor. Even a balanced resistive load supplied with a voltage containing the symmetrical component of the zero sequence will have a power factor lower than one. To avoid this, the zero sequence component of the supply voltage has to be eliminated by referencing the voltage to an artificial zero as shown in Fig. 4.1. Such a voltage referred to an artificial zero contains symmetrical components only of the positive and the negative sequence at the load terminal.

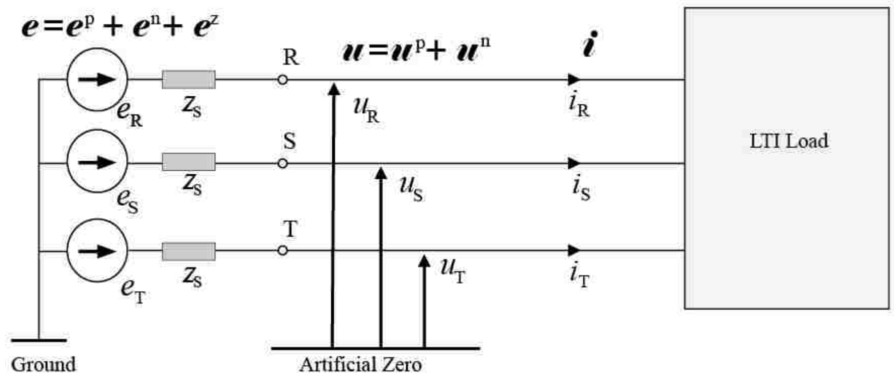


Fig. 4.1 LTI load supplied from a source with the voltage referenced to artificial zero

Let $\boldsymbol{u}$ denote a three-phase vector of the line voltages at the load terminals, such that

$$\boldsymbol{u}(t) = \sum_{n\in\boldsymbol{N}} \boldsymbol{u}_n = \sum_{n\in\boldsymbol{N}} (\boldsymbol{u}_n^{\mathrm{p}} + \boldsymbol{u}_n^{\mathrm{n}}) = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} \ . \tag{4.3}$$

The crms values of the symmetrical components of the supply voltage harmonics $\boldsymbol{U}_n^{\mathrm{p}}$ and $\boldsymbol{U}_n^{\mathrm{n}}$ are calculated using the formula,

$$\begin{bmatrix} \boldsymbol{U}_n^{\mathrm{p}} \\ \boldsymbol{U}_n^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix}. \tag{4.4}$$

When the supply voltage is symmetrical, then the constituent harmonics are also symmetrical and of a specified sequence. Harmonics of the order $n = 3k + 1$ are of the positive sequence; harmonics of the order $n = 3k - 1$ are of the negative sequence and the harmonics of the order $n = 3k$ are of the zero sequence, which are eliminated if the supply voltage is referred to an artificial zero. When the supply voltage is asymmetrical, this property is no longer valid, however. In particular, the third order harmonic can exist both in the supply voltage and in the load current, because when the supply voltage is asymmetrical, the third order harmonic is not exclusively zero sequence. It can contain symmetrical components of the positive and the negative sequence.

A N&A voltage can be represented in the form of a three-phase vector as

$$\boldsymbol{u}(t) = \begin{bmatrix} u_{\mathrm{R}}(t) \\ u_{\mathrm{S}}(t) \\ u_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in \boldsymbol{N}} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in \boldsymbol{N}} \begin{bmatrix} \boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in \boldsymbol{N}} \boldsymbol{U}_n\, e^{jn\omega_1 t}\,. \tag{4.5}$$

The above vector can be expressed in terms of the crms values of the positive and the negative sequence voltage symmetrical components $\boldsymbol{U}_n^{\mathrm{p}}$ and $\boldsymbol{U}_n^{\mathrm{n}}$ as follows,

$$\boldsymbol{u}(t) = \sum_{n\in \boldsymbol{N}} \boldsymbol{u}_n = \sqrt{2}\,\mathrm{Re}\sum_{n\in \boldsymbol{N}} \boldsymbol{U}_n\, e^{jn\omega_1 t}$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in \boldsymbol{N}} (\boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}}) e^{jn\omega_1 t} = \sqrt{2}\,\mathrm{Re}\sum_{n\in \boldsymbol{N}} (\mathbf{1}^{\mathrm{p}} U_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}} U_n^{\mathrm{n}}) e^{jn\omega_1 t}\,. \tag{4.6}$$

The vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$ are three-phase vectors defined as

$$\mathbf{1}^{\mathrm{p}} \overset{\mathrm{df.}}{=} \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix}, \qquad \mathbf{1}^{\mathrm{n}} \overset{\mathrm{df.}}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} \tag{4.7}$$

and illustrated as

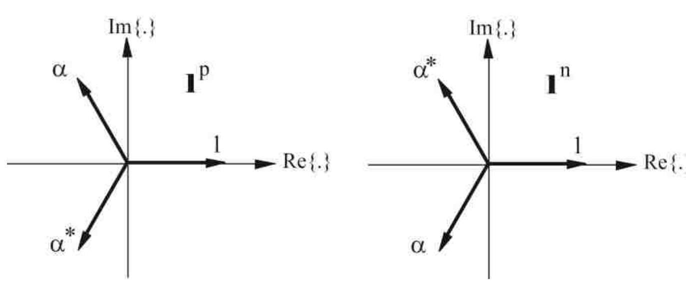


Fig. 4.2 Symmetrical three-phase unit vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$

It is to be noticed that the superscripts p and n are used to denote the sequence of a quantity in this text, while the subscript n is used to denote the harmonic order. The symbol $\boldsymbol{u}_n^{\mathrm{n}}$, for example denotes the negative sequence voltage symmetrical components of the n^{th} harmonic order while $\boldsymbol{u}_n^{\mathrm{p}}$ denotes the positive sequence voltage symmetrical component of n^{th} harmonic order.

Let us assume that an unbalanced LTI load supplied from an asymmetrical and distorted voltage has the active power P. Such a load as shown in Fig. 4.3(a) is equivalent with respect to active power P to a balanced resistive load as the one shown in Fig. 4.3(b).

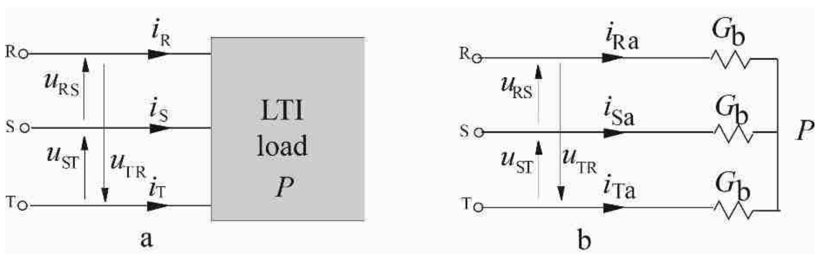


Fig. 4.3 A three-phase load (a) and a balanced resistive load (b) equivalent with respect to the active power P

The phase conductance of such a balanced resistive load is

$$G_{\mathrm{b}} = \frac{P}{||\boldsymbol{u}||^2} \tag{4.8}$$

where $\|\boldsymbol{u}\|$ denotes the three-phase rms value of the supply voltage and can be calculated as

$$||\boldsymbol{u}|| = \sqrt{||u_{\mathrm{R}}||^2 + ||u_{\mathrm{S}}||^2 + ||u_{\mathrm{T}}||^2}\ . \tag{4.9}$$

The current of such an equivalent load is

$$\boldsymbol{i}_{\mathrm{a}} = G_{\mathrm{b}}\boldsymbol{u} = \sqrt{2}\ \mathrm{Re}\sum_{n\in K} G_{\mathrm{b}}(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})e^{jn\omega_1 t}\ , \tag{4.10}$$

and is referred to as the active current, due to its association with the active power P of the load.

Analysis of LTI loads presented in Chapter 3 was confined to asymmetrical and sinusoidal (A&S) voltages and currents. The only difference from that to this chapter is that the voltage is not only asymmetrical but can also be distorted. Since the different order harmonics are orthogonal to one another, the current response of the load to such an asymmetrical and distorted voltage can be calculated independently for each harmonic order; for which the supply voltage is sinusoidal and of the n^{th} harmonic frequency.

At each harmonic frequency, the load has active and reactive powers that can be calculated using the line currents and voltages of that particular frequency. In general, for the n^{th} harmonic order,

$$P_n = \mathrm{Re}\{\boldsymbol{U}_n^{\mathrm{T}} \boldsymbol{I}_n^*\}, \qquad Q_n = \mathrm{Im}\{\boldsymbol{U}_n^{\mathrm{T}} \boldsymbol{I}_n^*\} \tag{4.11}$$

Although the load can be unbalanced for the n^{th} order harmonic, but with respect to the active and reactive powers P_n and Q_n at voltage $\boldsymbol{u}_n$, such a load is equivalent to a balanced load as shown in Fig. 4.4.

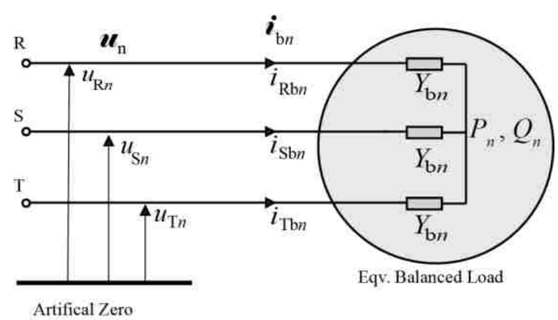


Fig. 4.4 A balanced load equivalent to the original load with respect to P_n and Q_n for the n^{th} harmonic order

and has the phase admittance

$$\boldsymbol{Y}_{bn} = G_{bn} + jB_{bn} = \frac{P_n - jQ_n}{||\boldsymbol{u}_n||^2} = \frac{\boldsymbol{C}_n^*}{||\boldsymbol{u}_n||^2} \tag{4.12}$$

where $||\boldsymbol{u}_n||$ is the three-phase rms value of the n^{th} order harmonic of the supply voltage and is equal to

$$||\boldsymbol{u}_n|| = \sqrt{U_{\text{R}n}^2 + U_{\text{S}n}^2 + U_{\text{T}n}^2}\ . \tag{4.13}$$

The line current of the balanced load shown in Fig. 4.4 is composed of the active current

$$\begin{aligned} \boldsymbol{i}_{\text{a}n} = G_{\text{b}n}\,\boldsymbol{u}_n = \sqrt{2}\,\text{Re}\{G_{\text{b}n}\,(\boldsymbol{U}_n^{\text{p}} + \boldsymbol{U}_n^{\text{n}})\,e^{jn\omega_1 t}\} = \\ = \sqrt{2}\,\text{Re}\{G_{\text{b}n}\,(\mathbf{1}^{\text{p}}\,U_n^{\text{p}} + \mathbf{1}^{\text{n}}\,U_n^{\text{n}})\,e^{jn\omega_1 t}\} \end{aligned} \tag{4.14}$$

and the reactive current

$$\begin{aligned} \boldsymbol{i}_{\text{r}n} = B_{\text{b}n}\,\boldsymbol{u}_n(t + \frac{T}{4n}) = \sqrt{2}\,\text{Re}\{jB_{\text{b}n}\,(\boldsymbol{U}_n^{\text{p}} + \boldsymbol{U}_n^{\text{n}})\,e^{jn\omega t}\} = \\ = \sqrt{2}\,\text{Re}\{jB_{\text{b}n}\,(\mathbf{1}^{\text{p}}\,U_n^{\text{p}} + \mathbf{1}^{\text{n}}\,U_n^{\text{n}})\,e^{jn\omega t}\}. \end{aligned} \tag{4.15}$$

The admittance $\boldsymbol{Y}_{\text{b}n}$ defined above is the admittance of the equivalent balanced load for the n^{th} order harmonic, equivalent to the original load for that harmonic order with respect to the powers P_n and Q_n. However, the load for the n^{th} harmonic order can be unbalanced, and consequently, the current of such a load will also contain the unbalanced current

$$\begin{aligned} \boldsymbol{i}_{\text{u}n} = \boldsymbol{i}_n - \boldsymbol{i}_{\text{b}n} = \boldsymbol{i}_n - (\boldsymbol{i}_{\text{a}n} + \boldsymbol{i}_{\text{r}n}) = \\ = \sqrt{2}\,\text{Re}\{[\boldsymbol{I}_{\text{n}} - \boldsymbol{Y}_{\text{b}n}\,(\mathbf{1}^{\text{p}}\,U_n^{\text{p}} + \mathbf{1}^{\text{n}}\,U_n^{\text{n}})]e^{jn\omega_1 t}\}. \end{aligned} \tag{4.16}$$

Thus the total current for the n^{th} order harmonic, is the sum of the active, reactive and unbalanced currents mentioned above, namely,

$$\boldsymbol{i}_n = \boldsymbol{i}_{\text{a}n} + \boldsymbol{i}_{\text{r}n} + \boldsymbol{i}_{\text{u}n} \tag{4.17}$$

and the total load current, which is the sum of the currents for all the harmonic orders, is equal to

$$\boldsymbol{i}(t) = \begin{bmatrix} i_{\mathrm{R}}(t) \\ i_{\mathrm{S}}(t) \\ i_{\mathrm{T}}(t) \end{bmatrix} = \sum_{n\in N} \boldsymbol{i}_n(t) = \sum_{n\in N} (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}). \tag{4.18}$$

Since the active current defined in (4.10) is responsible for permanent energy transfer the remainder of the current in (4.18) is undesirable, and can be calculated as

$$\begin{aligned} \boldsymbol{i} - \boldsymbol{i}_{\mathrm{a}} &= \Big(\sum_{n\in N} (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n})\Big) - \boldsymbol{i}_{\mathrm{a}} \\ &= \sum_{n\in \boldsymbol{N}} \boldsymbol{i}_{\mathrm{a}n} - \boldsymbol{i}_{\mathrm{a}} + \sum_{n\in \boldsymbol{N}} \boldsymbol{i}_{\mathrm{r}n} + \sum_{n\in \boldsymbol{N}} \boldsymbol{i}_{\mathrm{u}n} \\ &= \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}. \end{aligned} \tag{4.19}$$

The current

$$\sum_{n\in N} \boldsymbol{i}_{\mathrm{r}n} = \boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} jB_{\mathrm{b}n}\,(\mathbf{1}^{\mathrm{p}} \boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}} \boldsymbol{U}_n^{\mathrm{n}})\, e^{jn\omega t} \tag{4.20}$$

is a reactive current which occurs in the lines because of the mutual phase shift of the load current with respect to the supply voltage. Similarly the current,

$$\sum_{n\in N} \boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}_{\mathrm{u}} \tag{4.21}$$

is an unbalanced current. It occurs due to the unbalance of the load for the harmonic frequencies. The current

$$\sum_{n\in N} \boldsymbol{i}_{\mathrm{a}n} - \boldsymbol{i}_{\mathrm{a}} = \sqrt{2}\,\mathrm{Re} \sum_{n\in N} (G_{\mathrm{b}n} - G_{\mathrm{b}})(\mathbf{1}^{\mathrm{p}} \boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}} \boldsymbol{U}_n^{\mathrm{n}})\, e^{jn\omega t} = \boldsymbol{i}_{\mathrm{s}} \tag{4.22}$$

is the scattered current and it occurs because the equivalent balanced conductance G_{b} of the load differs from the equivalent balanced conductance $G_{\mathrm{b}n}$ for the harmonic frequencies. Formula (4.19) can be written as

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}. \tag{4.23}$$

The load current is decomposed into four components, each associated with a distinct physical phenomena in the load. The active current $\boldsymbol{i}_{\mathrm{a}}$ is associated with the permanent energy delivery to the load with active power P. The scattered current $\boldsymbol{i}_{\mathrm{s}}$ is associated with the change of the load conductance with the harmonic order. The reactive current $\boldsymbol{i}_{\mathrm{r}}$ is associated with the phase shift between the load current and the supply voltage, and the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ is associated with the load unbalance.

The square of the rms value of the load current $\boldsymbol{i}$ in (4.23) is equal to the sum of squares of the rms values of the current components if the current components in (4.23) are mutually orthogonal.

Mutual orthogonality of two quantities can be verified based on the value of their scalar product. To be more specific, two currents are mutually orthogonal if their scalar product

$$(\boldsymbol{i}_{\mathrm{x}}, \boldsymbol{i}_{\mathrm{y}}) = \frac{1}{T}\int_0^T \boldsymbol{i}_{\mathrm{x}}{}^{\mathrm{T}}(t)\,\boldsymbol{i}_{\mathrm{y}}(t)\,dt \tag{4.24}$$

is equal to zero. Because quantities of different harmonic orders are mutually orthogonal, the three-phase rms value of each of the currents components above can be calculated using the sum squares of the particular current component for all the harmonics, viz.,

$$\begin{aligned}
||\boldsymbol{i}_{\mathrm{r}}|| &= \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{\mathrm{r}n}||^2} = \sqrt{\sum_{n\in N} B_{\mathrm{b}n}^2 ||\boldsymbol{u}_n||^2}\,,\\
||\boldsymbol{i}_{\mathrm{s}}|| &= \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{\mathrm{s}n}||^2} = \sqrt{\sum_{n\in N} (G_{\mathrm{b}n} - G_{\mathrm{b}})^2 ||\boldsymbol{u}_n||^2}\\
||\boldsymbol{i}_{\mathrm{u}}|| &= \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{\mathrm{u}n}||^2}
\end{aligned} \tag{4.25}$$

The mutual orthogonality of the active, reactive and unbalanced currents for sinusoidal system was proven in Chapter 3. These proofs also apply to the different harmonic orders. Therefore, the active, reactive and unbalanced current components of the n^{th} order harmonic of the load current, specified in (4.17) are mutually orthogonal, i.e.,

$$(\boldsymbol{i}_{\mathrm{a}n}, \boldsymbol{i}_{\mathrm{r}n}) = 0, \qquad (\boldsymbol{i}_{\mathrm{a}n}, \boldsymbol{i}_{\mathrm{u}n}) = 0, \qquad (\boldsymbol{i}_{\mathrm{r}n}, \boldsymbol{i}_{\mathrm{u}n}) = 0 \tag{4.26}$$

The current in (4.18) is the sum of the harmonics in (4.17). Since the currents of different harmonic orders are orthogonal, it implies that the scalar product of two currents is the sum of the scalar product of the current harmonics, namely,

$$(\boldsymbol{i}_{\mathrm{x}}, \boldsymbol{i}_{\mathrm{v}}) = \sum_{n\in \mathrm{N}} (\boldsymbol{i}_{\mathrm{x}n}, \boldsymbol{i}_{\mathrm{v}n}) \ . \tag{4.27}$$

This means that if two current components are orthogonal for all harmonics of orders *n*, these current components are orthogonal to one another, viz.,

$$(\boldsymbol{i}_{\mathrm{x}n}, \boldsymbol{i}_{\mathrm{v}n}) = 0 \ \rightarrow (\boldsymbol{i}_{\mathrm{x}}, \boldsymbol{i}_{\mathrm{v}}) = 0 \, . \tag{4.28}$$

The relations in (4.26) and (4.28) imply that the current components in (4.18) are mutually orthogonal. Hence,

$$||\boldsymbol{i}||^2 = ||\sum_{n\in N} \boldsymbol{i}_{\mathrm{a}n}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2 \ . \tag{4.29}$$

Since

$$\sum_{n\in N} \boldsymbol{i}_{\mathrm{a}n} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} \tag{4.30}$$

the currents in (4.23) are mutually orthogonal if the currents $\boldsymbol{i}_{\mathrm{a}}$ and $\boldsymbol{i}_{\mathrm{s}}$ are orthogonal. Indeed, the scalar product of these two currents,

$$
\begin{aligned}
(\boldsymbol{i}_{\mathrm{a}},\boldsymbol{i}_{\mathrm{s}}) &= \mathrm{Re}\{\boldsymbol{I}_{\mathrm{a}}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{s}}^{*}\} = \mathrm{Re}\sum_{\mathrm{n}\in\boldsymbol{N}}\{G_{\mathrm{b}}\boldsymbol{U}_{n}^{\mathrm{T}}(G_{\mathrm{b}n}-G_{\mathrm{b}})\boldsymbol{U}_{n}^{*}\} \\
&= G_{\mathrm{b}}\,\mathrm{Re}\sum_{n\in\boldsymbol{N}}\{(G_{\mathrm{b}n}-G_{\mathrm{b}})\,||\boldsymbol{u}_{n}||^{2}\} \\
&= G_{\mathrm{b}}\,\mathrm{Re}\sum_{n\in\boldsymbol{N}}\{G_{\mathrm{b}n}\,||\boldsymbol{u}_{n}||^{2}-G_{\mathrm{b}}\,||\boldsymbol{u}_{n}||^{2}\} \\
&= G_{\mathrm{b}}\{\mathrm{Re}\sum_{n\in\boldsymbol{N}}G_{\mathrm{b}n}\,||\boldsymbol{u}_{n}||^{2}-G_{\mathrm{b}}\,\mathrm{Re}\sum_{\mathrm{n}\in\boldsymbol{N}}||\boldsymbol{u}_{n}||^{2}\} \\
&= G_{\mathrm{b}}\{\mathrm{Re}\sum_{n\in\boldsymbol{N}}P_{n}-G_{\mathrm{b}}||\boldsymbol{u}_{n}||^{2}\} \\
&= G_{\mathrm{b}}\,(P-P)=0.
\end{aligned} \tag{4.31}
$$

Therefore,

$$||\boldsymbol{i}||^{2}=||\boldsymbol{i}_{\mathrm{a}}||^{2}+||\boldsymbol{i}_{\mathrm{s}}||^{2}+||\boldsymbol{i}_{\mathrm{r}}||^{2}+||\boldsymbol{i}_{\mathrm{u}}||^{2}\ . \tag{4.32}$$

Multiplying (4.32) by the square of the three-phase rms $||\boldsymbol{u}||^{2}$ of the supply voltage

$$||\boldsymbol{u}||^{2}\times||\boldsymbol{i}||^{2}=||\boldsymbol{u}||^{2}\times[\,||\boldsymbol{i}_{\mathrm{a}}||^{2}+||\boldsymbol{i}_{\mathrm{s}}||^{2}+||\boldsymbol{i}_{\mathrm{r}}||^{2}+||\boldsymbol{i}_{\mathrm{u}}||^{2}\,]\ ,$$

yields the power equation

$$S^{2}=P^{2}+D_{\mathrm{s}}^{2}+Q^{2}+D_{\mathrm{u}}^{2} \tag{4.33}$$

where,

$S=||\boldsymbol{u}||\times||\boldsymbol{i}||$ is the apparent power of the load,

$P\ =||\boldsymbol{u}||\times||\boldsymbol{i}_{\mathrm{a}}||$ is the active power of the load and is associated with permanent energy transfer between the supply and the load,

$D_{\mathrm{s}}=||\boldsymbol{u}||\times||\boldsymbol{i}_{\mathrm{s}}||$ is the scattered power. It is associated with the change in the load conductances with the harmonic order,

$Q = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{r}}||$ is the reactive power of the load. It is associated with the phase difference between the supply voltage and load current harmonics

and $D_{\mathrm{u}} = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{u}}||$ is the unbalanced power. It is associated with the load imbalance.

The power factor λ of the load shown in Fig. with the different powers as depicted in (4.33) is

$$\lambda = \frac{P}{S} = \frac{P}{\sqrt{P^2 + D_{\mathrm{s}}^2 + Q^2 + D_{\mathrm{u}}^2}} \ . \tag{4.34}$$

Illustration 4.1 An unbalanced load is supplied from a source of asymmetrical and nonsinusoidal voltage as shown in Fig. 4.5.

The supply voltages are equal to

$$\begin{aligned}
\boldsymbol{e}_{\mathrm{R}} &= \sqrt{2}\mathrm{Re}\{100\ \mathrm{e}^{j\omega_1 t} + 2.5\ \mathrm{e}^{j5\omega_1 t} + 2\ \mathrm{e}^{j7\omega_1 t}\}\ \mathrm{V} \\
\boldsymbol{e}_{\mathrm{S}} &= \sqrt{2}\mathrm{Re}\{100\ \mathrm{e}^{-j120}\ \mathrm{e}^{j\omega_1 t} + 3.3\mathrm{e}^{j127^\circ}\ \mathrm{e}^{j5\omega_1 t} + 4\ \mathrm{e}^{-j120}\ \mathrm{e}^{j7\omega_1 t}\}\ \mathrm{V} \\
\boldsymbol{e}_{\mathrm{T}} &= 0.
\end{aligned}$$

Let us calculate the powers and currents of such a load.

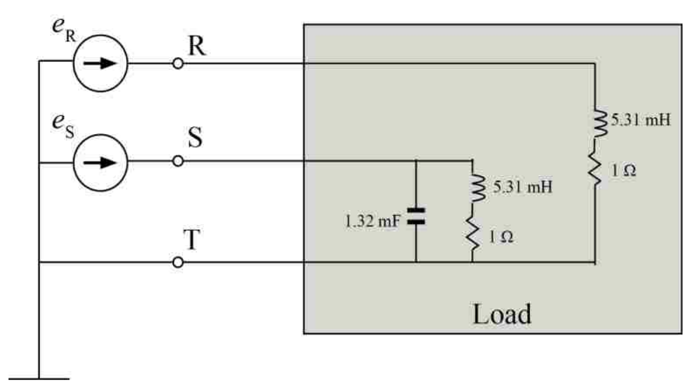


Fig. 4.5 Unbalanced LTI load supplied from a three-phase source of nonsinusoidal and asymmetrical voltage

The supply voltage is asymmetrical and it contains the 5th and the 7th order voltage harmonic components. The crms values of the supply voltage harmonics are as following

$$\begin{bmatrix} \boldsymbol{E}_{\mathrm{R1}} \\ \boldsymbol{E}_{\mathrm{S1}} \\ \boldsymbol{E}_{\mathrm{T1}} \end{bmatrix} = \begin{bmatrix} 100 \\ 100\,\mathrm{e}^{-j120°} \\ 0 \end{bmatrix} \mathrm{V}, \quad \begin{bmatrix} \boldsymbol{E}_{\mathrm{R5}} \\ \boldsymbol{E}_{\mathrm{S5}} \\ \boldsymbol{E}_{\mathrm{T5}} \end{bmatrix} = \begin{bmatrix} 2.5 \\ 3.3\,\mathrm{e}^{j127°} \\ 0 \end{bmatrix} \mathrm{V}, \quad \begin{bmatrix} \boldsymbol{E}_{\mathrm{R7}} \\ \boldsymbol{E}_{\mathrm{S7}} \\ \boldsymbol{E}_{\mathrm{T7}} \end{bmatrix} = \begin{bmatrix} 2 \\ 4\mathrm{e}^{-j120°} \\ 0 \end{bmatrix} \mathrm{V} \, .$$

The impedances for the different harmonic orders are as following:

$$\begin{bmatrix} \boldsymbol{Z}_{\mathrm{RS1}} \\ \boldsymbol{Z}_{\mathrm{ST1}} \\ \boldsymbol{Z}_{\mathrm{TR1}} \end{bmatrix} = \begin{bmatrix} inf \\ 2 \\ 1+j1 \end{bmatrix} \Omega, \quad \begin{bmatrix} \boldsymbol{Z}_{\mathrm{RS5}} \\ \boldsymbol{Z}_{\mathrm{ST5}} \\ \boldsymbol{Z}_{\mathrm{TR5}} \end{bmatrix} = \begin{bmatrix} inf \\ 0.007-j0.43 \\ 1+j5 \end{bmatrix} \Omega, \quad \begin{bmatrix} \boldsymbol{Z}_{\mathrm{RS7}} \\ \boldsymbol{Z}_{\mathrm{ST7}} \\ \boldsymbol{Z}_{\mathrm{TR7}} \end{bmatrix} = \begin{bmatrix} inf \\ 0.002-j0.29 \\ 1+j7 \end{bmatrix} \Omega \, .$$

The positive and the negative sequence voltage components are equal to

$$\begin{bmatrix} \boldsymbol{E}_1^{\mathrm{p}} \\ \boldsymbol{E}_1^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_{\mathrm{R1}} \\ \boldsymbol{E}_{\mathrm{S1}} \\ \boldsymbol{E}_{\mathrm{T1}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix} \begin{bmatrix} 100 \\ 100e^{-j120°} \\ 0 \end{bmatrix} = \begin{bmatrix} 66.67 \\ 33.33e^{j60°} \end{bmatrix} \mathrm{V} = \begin{bmatrix} \boldsymbol{U}_1^{\mathrm{p}} \\ \boldsymbol{U}_1^{\mathrm{n}} \end{bmatrix} .$$

Similarly,

$$\begin{bmatrix} \boldsymbol{U}_5^{\mathrm{p}} \\ \boldsymbol{U}_5^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 1.1e^{-j67.6°} \\ 1.93e^{j4.3°} \end{bmatrix} \mathrm{V}, \qquad \begin{bmatrix} \boldsymbol{U}_7^{\mathrm{p}} \\ \boldsymbol{U}_7^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.15e^{j90°} \end{bmatrix} \mathrm{V},$$

The three-phase rms values of the supply voltage harmonics w.r.t. artificial zero can be calculated as following

$$\begin{aligned} ||\boldsymbol{u}_1^{\mathrm{p}}|| &= ||\boldsymbol{e}_1^{\mathrm{p}}|| = \sqrt{3}\,\boldsymbol{U}_1^{\mathrm{p}} = \sqrt{3}\times 66.66 = 115.5\,\mathrm{V} \\ ||\boldsymbol{u}_1^{\mathrm{n}}|| &= ||\boldsymbol{e}_1^{\mathrm{n}}|| = \sqrt{3}\,\boldsymbol{U}_1^{\mathrm{n}} = \sqrt{3}\times 33.33 = 57.7\,\mathrm{V} \\ ||\boldsymbol{u}_1|| &= \sqrt{||\boldsymbol{u}_1^{\mathrm{p}}||^2 + ||\boldsymbol{u}_1^{\mathrm{n}}||^2} = \sqrt{115.5^2 + 57.7^2} = 129.1\,\mathrm{V} \end{aligned}$$

and similarly,

$$||\boldsymbol{u}_5|| = 3.83\ \mathrm{V}, \qquad ||\boldsymbol{u}_7|| = 4.0\ \mathrm{V}\,.$$

Thus, the three-phase rms value of the supply voltage is equal to

$$||\boldsymbol{u}|| = \sqrt{||\boldsymbol{u}_1||^2 + ||\boldsymbol{u}_5||^2 + ||\boldsymbol{u}_7||^2} = 129.22 \text{ V} .$$

The waveform the line currents with respect to artificial zero is equal to

$$\boldsymbol{i} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 70.7\ \mathrm{e}^{-j45°} \\ 50\ \mathrm{e}^{-j120°} \\ 96.6\ \mathrm{e}^{j105°} \end{bmatrix} \mathrm{e}^{j\omega_1 t} + \begin{bmatrix} 0.5\ \mathrm{e}^{-j78.7°} \\ 7.6\ \mathrm{e}^{-j143.4°} \\ 7.8\ \mathrm{e}^{j40°} \end{bmatrix} \mathrm{e}^{j5\omega_1 t} + \begin{bmatrix} 0.24\ \mathrm{e}^{-j80.54°} \\ 13.5\ \mathrm{e}^{-j30.3°} \\ 13.6\ \mathrm{e}^{j148.7°} \end{bmatrix} \mathrm{e}^{j7\omega_1 t}\ \} \text{ A} .$$

Hence, the three-phase current rms values for the different harmonic orders are

$||\boldsymbol{i}_1|| = 129.7\,\text{A}, \quad ||\boldsymbol{i}_5|| = 10.9\text{A}, \quad ||\boldsymbol{i}_7|| = 19.14\,\text{A}.$

Thus, $||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_1||^2 + ||\boldsymbol{i}_5||^2 + ||\boldsymbol{i}_7||^2} = 131.6 \text{ A}$ and,

the apparent power of the load $S = ||\boldsymbol{u}|| \times ||\boldsymbol{i}|| = 129.2 \times 131.6 = 17 \text{ kVA}.$

The active power P of the load is 10 kW.

Thus, the equivalent conductance of the load, $G_\mathrm{b} = \dfrac{P}{||\boldsymbol{u}||^2} = \dfrac{10000}{129.2^2} = 0.599 \text{ S}.$

The equivalent balance admittance for the n harmonic orders are

$$\boldsymbol{Y}_{\mathrm{b}1} = G_{\mathrm{b}1} + jB_{\mathrm{b}1} = \frac{P_1 - jQ_1}{||\boldsymbol{u}_1||^2} = 0.6 - j0.3 \text{ S},$$

$$\boldsymbol{Y}_{\mathrm{b}5} = 0.045 + j1.61\text{S}, \quad \boldsymbol{Y}_{\mathrm{b}7} = 0.025 + j3.33 \text{ S}.$$

For the given circuit, we have

$$||\boldsymbol{i}_\mathrm{a}|| = 77.4 \text{ A}, \quad ||\boldsymbol{i}_\mathrm{s}|| = 3.13\text{A}, \quad ||\boldsymbol{i}_\mathrm{r}|| = 41.41 \text{ A}, \quad ||\boldsymbol{i}_\mathrm{u}|| = 97.98\text{A}.$$

Therefore, the load current three-phase rms value calculated using the CPC currents is

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_\mathrm{a}||^2 + ||\boldsymbol{i}_\mathrm{s}||^2 + ||\boldsymbol{i}_\mathrm{r}||^2 + ||\boldsymbol{i}_\mathrm{u}||^2} = 131.58\text{A}$$

which is the same as the current three-phase rms value calculated using the line currents above. Therefore,

the active power $P = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{a}}|| = 129.22 \times 77.4 = 10$ kW.

the scattered power $D_{\mathrm{s}} = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{s}}|| = 129.22 \times 3.13 = 0.41$ kVA

the reactive power $Q = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{r}}|| = 129.22 \times 41.4 = 5.35$ kVars

the unbalance power $D_{\mathrm{u}} = ||\boldsymbol{u}|| \times ||\boldsymbol{i}_{\mathrm{u}}|| = 129.22 \times 97.98 = 12.67$ kVA

and the apparent power is

$$S = \sqrt{P^2 + D_{\mathrm{s}}^2 + Q^2 + D_{\mathrm{u}}^2} = \sqrt{10^2 + 0.41^2 + 5.35^2 + 12.67^2} = 17 \text{ kVA} .$$

The power factor of the load, $\lambda = \dfrac{P}{S} = \dfrac{10}{17} = 0.59$.

4.3 Dependence of powers on load parameters

The power equation (4.33) describes how the apparent power constitutes of various powers each associated with a distinct physical phenomenon. It is adequate for describing the power properties of a circuit. Since some of the powers in (4.33) and their associated currents are undesired, it is logical to view the equation from the perspective of compensation, however. In that regard, (4.33) does not provide the necessary information about the circuit. To enable the design of a compensator, the currents and powers should be represented in terms of the load parameters.

The complex power of the branch RS for the n^{th} harmonic order is equal to

$$\begin{aligned} \boldsymbol{C}_{\mathrm{RS}n} &= \boldsymbol{U}_{\mathrm{RS}n}\, \boldsymbol{I}^*_{\mathrm{RS}n} = \boldsymbol{Y}^*_{\mathrm{RS}n}\, U^2_{\mathrm{RS}n} \\ &= \boldsymbol{Y}^*_{\mathrm{RS}n}\, (U^2_{\mathrm{R}n} + U^2_{\mathrm{S}n} - 2\mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}n}\, \boldsymbol{U}^*_{\mathrm{S}n}\}). \end{aligned} \tag{4.35}$$

For a three wire system,

$$U_{\mathrm{T}n}^2 = (-\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{U}_{\mathrm{R}n})(-\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{U}_{\mathrm{R}n})^* = U_{\mathrm{R}n}^2 + U_{\mathrm{S}n}^2 + 2\mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}n}\, \boldsymbol{U}_{\mathrm{S}n}^*\} \;. \tag{4.36}$$

Replacing the value of $2\mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}n}\, \boldsymbol{U}_{\mathrm{S}n}^*\}$ in (4.35) we get,

$$\begin{aligned} \boldsymbol{C}_{\mathrm{RS}n} &= \boldsymbol{Y}_{\mathrm{RS}n}^* \;(2U_{\mathrm{R}n}^2 + 2U_{\mathrm{S}n}^2 - U_{\mathrm{T}n}^2) \\ &= \boldsymbol{Y}_{\mathrm{RS}n}^*(2\,||\boldsymbol{u}_n||^2 - 3U_{\mathrm{T}n}^2). \end{aligned} \tag{4.37}$$

Similarly,

$$\begin{aligned} \boldsymbol{C}_{\mathrm{ST}n} &= \boldsymbol{Y}_{\mathrm{ST}n}^*(2\;||\boldsymbol{u}_n||^2 - 3U_{\mathrm{R}n}^2) \\ \boldsymbol{C}_{\mathrm{TR}n} &= \boldsymbol{Y}_{\mathrm{TR}n}^*(2\;||\boldsymbol{u}_n||^2 - 3U_{\mathrm{S}n}^2). \end{aligned} \tag{4.38}$$

The total complex power of the nth harmonic order is the sum of the complex powers of each of the branches in the equivalent delta configuration, namely

$$\boldsymbol{C}_n = \boldsymbol{C}_{\mathrm{RS}n} + \boldsymbol{C}_{\mathrm{ST}n} + \boldsymbol{C}_{\mathrm{TR}n} \;. \tag{4.39}$$

The admittance of the equivalent balanced load for the n^{th} harmonic order, which has the same active and reactive powers as the original load for the same harmonic is

$$\begin{aligned} \boldsymbol{Y}_{\mathrm{b}n} &= G_{\mathrm{b}n} + j\,B_{\mathrm{b}n} = \frac{\boldsymbol{C}_n^*}{||\boldsymbol{u}_n||^2} = \frac{\boldsymbol{C}_{\mathrm{RS}n}^* + \boldsymbol{C}_{\mathrm{ST}n}^* + \boldsymbol{C}_{\mathrm{TR}n}^*}{||\boldsymbol{u}_n||^2} \\ &= \frac{\boldsymbol{Y}_{\mathrm{RS}n}(2||\boldsymbol{u}_n||^2 - 3U_{\mathrm{T}n}^2) + \boldsymbol{Y}_{\mathrm{ST}n}(2||\boldsymbol{u}_n||^2 - 3U_{\mathrm{R}n}^2) + \boldsymbol{Y}_{\mathrm{TR}n}(2||\boldsymbol{u}_n||^2 - 3U_{\mathrm{S}n}^2)}{||\boldsymbol{u}_n||^2} \\ &= \frac{2||\boldsymbol{u}_n||^2\;(\boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{ST}n} + \boldsymbol{Y}_{\mathrm{TR}n}) - 3(\boldsymbol{Y}_{\mathrm{RS}n}U_{\mathrm{T}n}^2 + \boldsymbol{Y}_{\mathrm{ST}n}U_{\mathrm{R}n}^2 + \boldsymbol{Y}_{\mathrm{TR}n}U_{\mathrm{S}n}^2)}{||\boldsymbol{u}_n||^2} \\ &= 2\boldsymbol{Y}_{\mathrm{e}n} - \frac{3(\boldsymbol{Y}_{\mathrm{ST}n}U_{\mathrm{R}n}^2 + \boldsymbol{Y}_{\mathrm{TR}n}U_{\mathrm{S}n}^2 + \boldsymbol{Y}_{\mathrm{RS}n}U_{\mathrm{T}n}^2)}{||\boldsymbol{u}_n||^2} \\ &= \boldsymbol{Y}_{\mathrm{e}n} - \boldsymbol{Y}_{\mathrm{d}n} \end{aligned} \tag{4.40}$$

where,

$$\boldsymbol{Y}_{\mathrm{d}n} = \frac{3}{||\boldsymbol{u}_n||^2}(\boldsymbol{Y}_{\mathrm{ST}n}U_{\mathrm{R}n}^2 + \boldsymbol{Y}_{\mathrm{TR}n}U_{\mathrm{S}n}^2 + \boldsymbol{Y}_{\mathrm{RS}n}U_{\mathrm{T}n}^2) - \boldsymbol{Y}_{\mathrm{e}n} \tag{4.41}$$

is called the **asymmetry dependent unbalanced admittance for the n^{th} harmonic order.** It can have a nonzero value only when the supply voltage for the n^{th} order is asymmetrical and simultaneously the equivalent load for the n^{th} order is unbalanced.

The crms values of the line voltages for the n^{th} order harmonic, expressed in terms of the crms values of the symmetrical components of positive and negative sequence are

$$\begin{aligned} \boldsymbol{U}_{\mathrm{R}n} &= \boldsymbol{U}_n^{\mathrm{p}} + \boldsymbol{U}_n^{\mathrm{n}}, \\ \boldsymbol{U}_{\mathrm{S}n} &= \alpha^* \boldsymbol{U}_n^{\mathrm{p}} + \alpha \boldsymbol{U}_n^{\mathrm{n}} \quad \text{and} \\ \boldsymbol{U}_{\mathrm{T}n} &= \alpha \boldsymbol{U}_n^{\mathrm{p}} + \alpha^* \boldsymbol{U}_n^{\mathrm{n}}. \end{aligned} \tag{4.42}$$

This gives,

$$\begin{aligned} U_{\mathrm{R}n}^2 &= (U_n^{\mathrm{p}})^2 + (U_n^{\mathrm{n}})^2 + 2\mathrm{Re}\{(\boldsymbol{U}_n^{\mathrm{p}})^* \boldsymbol{U}_n^{\mathrm{n}}\} \\ U_{\mathrm{S}n}^2 &= (U_n^{\mathrm{p}})^2 + (U_n^{\mathrm{n}})^2 + 2\mathrm{Re}\{\alpha^* (\boldsymbol{U}_n^{\mathrm{p}})^* \boldsymbol{U}_n^{\mathrm{n}}\} \\ U_{\mathrm{T}n}^2 &= (U_n^{\mathrm{p}})^2 + (U_n^{\mathrm{n}})^2 + 2\mathrm{Re}\{\alpha (\boldsymbol{U}_n^{\mathrm{p}})^* \boldsymbol{U}_n^{\mathrm{n}}\} \end{aligned} \tag{4.43}$$

where $\boldsymbol{U}_n^{\mathrm{p}} = U_n^{\mathrm{p}} e^{j\alpha_{\mathrm{p}n}}$ and $\boldsymbol{U}_{\mathrm{n}}^{\mathrm{n}} = U_{\mathrm{n}}^{\mathrm{n}} e^{j\alpha_{\mathrm{n}n}}$.

Let $\boldsymbol{a}_n$ be defined as the complex coefficient of the supply voltage asymmetry, such that

$$\boldsymbol{a}_n = a_n e^{j\psi_n} = \frac{\boldsymbol{U}_n^{\mathrm{n}}}{\boldsymbol{U}_n^{\mathrm{p}}}. \tag{4.44}$$

Using (4.43) and (4.44),formula (4.41) can be rearranged as

$$\boldsymbol{Y}_{\mathrm{d}n} = \frac{2a_n}{1+a_n^2}[\boldsymbol{Y}_{\mathrm{ST}n} \cos\psi_n + \boldsymbol{Y}_{\mathrm{TR}n} \cos(\psi_n - \frac{2\pi}{3}) + \boldsymbol{Y}_{\mathrm{RS}n} \cos(\psi_n + \frac{2\pi}{3})] \ . \tag{4.45}$$

If coefficient a_{n} is zero in the above equation, meaning that the supply voltage for the n^{th} order is symmetrical, then $\boldsymbol{Y}_{\mathrm{d}n}$ is zero. Similarly if $\boldsymbol{Y}_{\mathrm{ST}n} = \boldsymbol{Y}_{\mathrm{TR}n} = \boldsymbol{Y}_{\mathrm{RS}n}$, meaning that the equivalent delta load for the n^{th} order is balanced, then also $\boldsymbol{Y}_{\mathrm{d}n}$ is zero. Hence the name asymmetry dependent unbalanced admittance.

The crms of the line current in R for the n^{th} order harmonic is

$$\begin{aligned}
\boldsymbol{I}_{\mathrm{R}n} &= \boldsymbol{I}_{\mathrm{RS}n} - \boldsymbol{I}_{\mathrm{ST}n} \\
&= \boldsymbol{Y}_{\mathrm{RS}n}(\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{U}_{\mathrm{S}n}) - \boldsymbol{Y}_{\mathrm{TR}n}(\boldsymbol{U}_{\mathrm{T}n} - \boldsymbol{U}_{\mathrm{R}n}) \\
&= \boldsymbol{Y}_{\mathrm{RS}n}\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{RS}n}\boldsymbol{U}_{\mathrm{S}n} - \boldsymbol{Y}_{\mathrm{TR}n}\boldsymbol{U}_{\mathrm{T}n} + \boldsymbol{Y}_{\mathrm{TR}n}\boldsymbol{U}_{\mathrm{R}n} + \boldsymbol{Y}_{\mathrm{ST}n}\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{ST}n}\boldsymbol{U}_{\mathrm{R}n} \\
&= (\boldsymbol{Y}_{\mathrm{RS}n} + \boldsymbol{Y}_{\mathrm{TR}n} + \boldsymbol{Y}_{\mathrm{ST}n})\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{RS}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{S}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{S}n}) - \boldsymbol{Y}_{\mathrm{TR}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{T}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{ST}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{R}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{R}n}) \\
&= \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{RS}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{S}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{S}n}) - \boldsymbol{Y}_{\mathrm{TR}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{T}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{ST}n}(\boldsymbol{U}^{\mathrm{p}}_{\mathrm{R}n} + \boldsymbol{U}^{\mathrm{n}}_{\mathrm{R}n}) \\
&= \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_{\mathrm{R}n} - (\boldsymbol{Y}_{\mathrm{ST}n} + \alpha\boldsymbol{Y}_{\mathrm{TR}n} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}n})\boldsymbol{U}^{\mathrm{p}}_{\mathrm{R}n} - (\boldsymbol{Y}_{\mathrm{ST}n} + \alpha^{*}\boldsymbol{Y}_{\mathrm{TR}n} + \alpha\boldsymbol{Y}_{\mathrm{RS}n})\boldsymbol{U}^{\mathrm{n}}_{\mathrm{R}n} \\
&= \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_{\mathrm{R}n} + \boldsymbol{A}^{\mathrm{p}}_{n}\boldsymbol{U}^{\mathrm{p}}_{\mathrm{R}n} + \boldsymbol{A}^{\mathrm{n}}_{n}\boldsymbol{U}^{\mathrm{n}}_{\mathrm{R}n}
\end{aligned} \tag{4.46}$$

where,

$$\boldsymbol{A}^{\mathrm{p}}_{n} = -(\boldsymbol{Y}_{\mathrm{ST}n} + \alpha\boldsymbol{Y}_{\mathrm{TR}n} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}n}), \qquad \boldsymbol{A}^{\mathrm{n}}_{n} = -(\boldsymbol{Y}_{\mathrm{ST}n} + \alpha^{*}\boldsymbol{Y}_{\mathrm{TR}n} + \alpha\boldsymbol{Y}_{\mathrm{RS}n}) \ . \tag{4.47}$$

Similarly,

$$\begin{aligned}
\boldsymbol{I}_{\mathrm{S}n} &= \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_{\mathrm{S}n} + \boldsymbol{A}^{\mathrm{p}}_{n}\boldsymbol{U}^{\mathrm{p}}_{\mathrm{T}n} + \boldsymbol{A}^{\mathrm{n}}_{n}\boldsymbol{U}^{\mathrm{n}}_{\mathrm{T}n} \\
\boldsymbol{I}_{\mathrm{T}n} &= \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_{\mathrm{T}n} + \boldsymbol{A}^{\mathrm{p}}_{n}\boldsymbol{U}^{\mathrm{p}}_{\mathrm{S}n} + \boldsymbol{A}^{\mathrm{n}}_{n}\boldsymbol{U}^{\mathrm{n}}_{\mathrm{S}n}.
\end{aligned} \tag{4.48}$$

The crms of the currents in equations (4.46) and (4.48) can be presented in the vector form as

$$\boldsymbol{I}_n = \begin{bmatrix} \boldsymbol{I}_{\mathrm{R}n} \\ \boldsymbol{I}_{\mathrm{S}n} \\ \boldsymbol{I}_{\mathrm{T}n} \end{bmatrix} = \boldsymbol{Y}_{\mathrm{e}n}\boldsymbol{U}_n + \boldsymbol{A}^{\mathrm{p}}_{n}\ \mathbf{1}^{\mathrm{n}}\ \boldsymbol{U}^{\mathrm{p}}_{n} + \boldsymbol{A}^{\mathrm{n}}_{n}\ \mathbf{1}^{\mathrm{p}}\ \boldsymbol{U}^{\mathrm{n}}_{n} \tag{4.49}$$

where $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$ are the **unit vectors of the positive and the negative sequences** respectively, defined earlier in the chapter.

The vector of the crms of the unbalanced current is

$$\boldsymbol{I}_{\mathrm{u}n} = \boldsymbol{I} - \boldsymbol{I}_{\mathrm{b}n} = (Y_{\mathrm{e}n} - Y_{\mathrm{b}n})\boldsymbol{U}_n + A_n^{\mathrm{p}}\, \mathbf{1}^{\mathrm{n}}\, U_n^{\mathrm{p}} + A_n^{\mathrm{n}}\, \mathbf{1}^{\mathrm{p}}\, U_n^{\mathrm{n}} \;. \tag{4.50}$$

4.4 Reactive compensation

4.4.1 Design of shunt reactive compensator

Recalling equation (4.17), the current for the n^{th} harmonic order is

$$\boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}\,, \tag{4.51}$$

where $\boldsymbol{i}_{\mathrm{a}n}$, $\boldsymbol{i}_{\mathrm{r}n}$ and $\boldsymbol{i}_{\mathrm{u}n}$ are the active reactive and unbalanced components of the n^{th} order load current harmonic respectively. The active current $\boldsymbol{i}_{\mathrm{a}n}$ is responsible for the permanent energy transfer between the source and the load. The currents $\boldsymbol{i}_{\mathrm{r}n}$ and $\boldsymbol{i}_{\mathrm{u}n}$ are surplus currents. To minimize the losses in the lines and to eventually improve the power factor it is desirable that only the active current be supplied from the distribution system. This can be done by connecting a shunt compensator of the delta structure as shown in Fig. 4.6.

The shunt compensator in Fig. 4.6 is composed of lossless reactance elements and can have either an inductor or a capacitor in each branch. The elements of the compensator are chosen such that the load with the compensator is balanced and purely resistive.

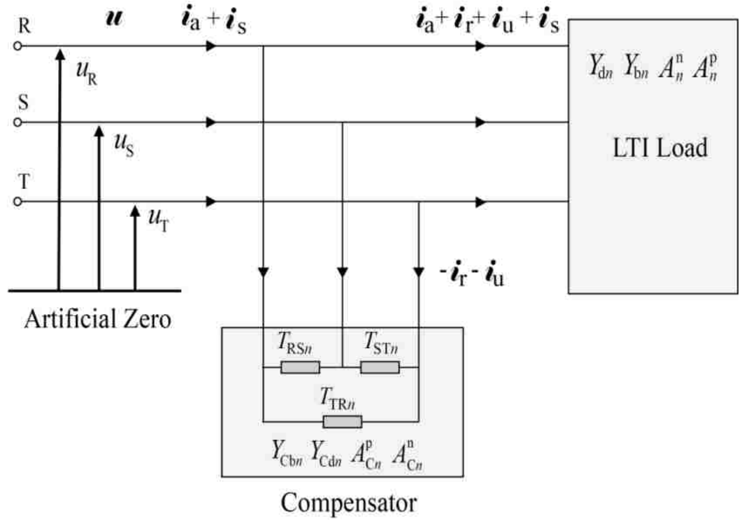


Fig. 4.6 LTI load with a reactive compensator

If the compensator branch susceptances are $T_{\mathrm{RS}n}$, $T_{\mathrm{ST}n}$ and $T_{\mathrm{TR}n}$ for the n^{th} harmonic order, then it has the unbalanced admittances

$$\begin{aligned} \boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n} &= -j\,(T_{\mathrm{ST}n} + \alpha\, T_{\mathrm{TR}n} + \alpha^{*}\, T_{\mathrm{RS}n}) \\ \boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n} &= -j\,(T_{\mathrm{ST}n} + \alpha^{*}\, T_{\mathrm{TR}n} + \alpha\, T_{\mathrm{RS}n}) \\ \boldsymbol{Y}_{\mathrm{Cd}n} &= \frac{j\,2a}{1+a^2}\left[T_{\mathrm{ST}n}\,\cos\psi + T_{\mathrm{TR}n}\,\cos(\psi - \frac{2\pi}{3}) + T_{\mathrm{RS}n}\,\cos(\psi + \frac{2\pi}{3})\right]. \end{aligned} \tag{4.52}$$

The reactive power of the compensator for the n^{th} harmonic order is

$$Q_{\mathrm{Cb}n} = -\,B_{\mathrm{Cb}n}\,||\boldsymbol{u}_n||^2 \tag{4.53}$$

where $B_{\mathrm{Cb}n}$ is the equivalent susceptance of the compensator for the n^{th} harmonic order. Thus,

$$B_{\mathrm{Cb}n} = \frac{-Q_{\mathrm{Cb}n}}{||\boldsymbol{u}_n||^2} = \frac{T_{\mathrm{RS}n}\,U^2_{\mathrm{RS}n} + T_{\mathrm{ST}n}\,U^2_{\mathrm{ST}n} + T_{\mathrm{TR}n}\,U^2_{\mathrm{TR}n}}{||\boldsymbol{u}_n||^2}. \tag{4.54}$$

The compensator draws the reactive current with the crms value

$$\boldsymbol{I}_{\mathrm{Cr}n} = j\,B_{\mathrm{Cb}n}\,\boldsymbol{U}_n. \tag{4.55}$$

The crms value of the reactive current of the load and the shunt compensator is

$$\boldsymbol{I}' = \boldsymbol{I}_{\mathrm{Cr}n} + \boldsymbol{I}_{\mathrm{r}n} = j\,B_{\mathrm{Cb}n}\ \ \boldsymbol{U}_n + j\ B_{\mathrm{b}n}\boldsymbol{U}_n \tag{4.56}$$

and it is reduced to zero under the condition

$$B_{\mathrm{Cb}n} + B_{\mathrm{b}n} = 0. \tag{4.57}$$

Combining the equations (4.54) and (4.57) yields,

$$T_{\mathrm{RS}n}\ U^2_{\mathrm{RS}n} + T_{\mathrm{ST}\,n}\ U^2_{\mathrm{ST}\,n} + T_{\mathrm{TR}\,n}\ U^2_{\mathrm{TR}\,n} = -B_{\mathrm{b}n}\,||\,\boldsymbol{u}_n\,||^2\ \ . \tag{4.58}$$

The vector of the crms values of the unbalanced current of the compensator is

$$\boldsymbol{I}_{\mathrm{Cu}\,n} = (\boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n}\,\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}}_n + \boldsymbol{Y}_{\mathrm{Cd}\,n}\,\boldsymbol{U}^{\mathrm{p}}_n) + (\boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n}\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}}_n + \boldsymbol{Y}_{\mathrm{Cd}\,n}\,\boldsymbol{U}^{\mathrm{n}}_n)\ \ . \tag{4.59}$$

The crms value of the unbalanced current of the load and the shunt compensator is

$$\begin{aligned}
\boldsymbol{I}' &= \boldsymbol{I}_{\mathrm{u}\,n} + \boldsymbol{I}_{\mathrm{Cu}\,n}\\
&= (\boldsymbol{A}^{\mathrm{p}}_n\,\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}}_n + \boldsymbol{Y}_{\mathrm{d}\,n}\,\boldsymbol{U}^{\mathrm{p}}_n) + (\boldsymbol{A}^{\mathrm{n}}_n\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}}_n + \boldsymbol{Y}_{\mathrm{d}\,n}\,\boldsymbol{U}^{\mathrm{n}}_n) +\\
&\quad (\boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n}\,\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}}_n + \boldsymbol{Y}_{\mathrm{Cd}\,n}\,\boldsymbol{U}^{\mathrm{p}}_n) + (\boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n}\,\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}}_n + \boldsymbol{Y}_{\mathrm{Cd}\,n}\,\boldsymbol{U}^{\mathrm{n}}_n)\\
&= (\boldsymbol{A}^{\mathrm{p}}_n + \boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n})(\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}}_n) + (\boldsymbol{A}^{\mathrm{n}}_n + \boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n})(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}}_n) + (\boldsymbol{Y}_{\mathrm{d}\,n} + \boldsymbol{Y}_{\mathrm{Cd}\,n})\boldsymbol{U}_n
\end{aligned} \tag{4.60}$$

and is reduced to zero under the condition

$$(\boldsymbol{Y}_{\mathrm{Cd}\,n} + \boldsymbol{Y}_{\mathrm{d}\,n})\boldsymbol{U}_n + (\boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n} + \boldsymbol{A}^{\mathrm{p}}_n)(\mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}^{\mathrm{p}}_n) + (\boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n} + \boldsymbol{A}^{\mathrm{n}}_n)(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}^{\mathrm{n}}_n) = 0\ \ . \tag{4.61}$$

This equation has to be satisfied for all three-phases, therefore, in particular for phase R,

$$(\boldsymbol{Y}_{\mathrm{Cd}\,n} + \boldsymbol{Y}_{\mathrm{d}\,n})\boldsymbol{U}_{\mathrm{R}\,n} + (\boldsymbol{A}^{\mathrm{p}}_{\mathrm{C}n} + \boldsymbol{A}^{\mathrm{p}}_n)\ \boldsymbol{U}^{\mathrm{p}}_n + (\boldsymbol{A}^{\mathrm{n}}_{\mathrm{C}n} + \boldsymbol{A}^{\mathrm{n}}_n)\boldsymbol{U}^{\mathrm{n}}_n = 0\ \ . \tag{4.62}$$

The above equation contains complex quantities and therefore it provides two equations, one for the real parts and the other for the imaginary parts. These two equations combined with equation (4.58) provide three linear equations with three unknowns $T_{\mathrm{RS}n}$, $T_{\mathrm{ST}\,n}$ and $T_{\mathrm{TR}\,n}$, viz.

$$
\begin{aligned}
&T_{\mathrm{RS}n}\, U_{\mathrm{RS}n}^2 + T_{\mathrm{ST}n}\, U_{\mathrm{ST}n}^2 + T_{\mathrm{TR}\,n}\, U_{\mathrm{TR}\,n}^2 = -B_{\mathrm{b}n}\, \|\boldsymbol{u}_n\|^2 \\
&\mathrm{Re}\,\{(\boldsymbol{Y}_{\mathrm{Cd}n} + \boldsymbol{Y}_{\mathrm{d}n})\boldsymbol{U}_{\mathrm{R}\,n} + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{p}})\,\boldsymbol{U}_n^{\mathrm{p}} + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{n}})\boldsymbol{U}_n^{\mathrm{n}}\} = 0 \\
&\mathrm{Im}\,\{(\boldsymbol{Y}_{\mathrm{Cd}n} + \boldsymbol{Y}_{\mathrm{d}n})\boldsymbol{U}_{\mathrm{R}\,n} + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{p}})\,\boldsymbol{U}_n^{\mathrm{p}} + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{n}})\boldsymbol{U}_n^{\mathrm{n}}\} = 0.
\end{aligned}
\tag{4.63}
$$

The last two equations above and can be further simplified. Dividing (4.62) by $\boldsymbol{U}_n^{\mathrm{p}}$ yields,

$$
(\boldsymbol{Y}_{\mathrm{Cd}n} + \boldsymbol{Y}_{\mathrm{d}n})(1 + a_n e^{\mathrm{j}\psi_n}) + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{p}}) + (\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{n}} + \boldsymbol{A}_n^{\mathrm{n}})\, a_n e^{\mathrm{j}\psi_n} = 0\ . \tag{4.64}
$$

The equation can be rearranged by separating the known and the unknown quantities as

$$
\boldsymbol{Y}_{\mathrm{Cd}n}\ (1 + a_n e^{\mathrm{j}\psi_n}) + \boldsymbol{A}_{\mathrm{C}n}^{\mathrm{p}} + \boldsymbol{A}_{\mathrm{C}n}^{\mathrm{n}}(a_n e^{\mathrm{j}\psi_n}) + \boldsymbol{Y}_{\mathrm{d}n}\ (1 + a_n e^{\mathrm{j}\psi_n}) + \boldsymbol{A}_n^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{n}}(a_n e^{\mathrm{j}\psi_n}) = 0\ . \tag{4.65}
$$

The first part of the equation above can be rewritten as

$$
\boldsymbol{Y}_{\mathrm{Cd}n}\ (1 + a_n e^{j\psi_n}) = (c_{1n}\, T_{\mathrm{ST}n} + c_{2n}\, T_{\mathrm{TR}\,n} + c_{3n}\, T_{\mathrm{RS}n})(1 + a_n e^{j\psi_n}) \tag{4.66}
$$

where,

$$
\begin{aligned}
&\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{n}}\, a_n\, e^{j\psi_n} = -j\,(T_{\mathrm{ST}n} + \alpha^* T_{\mathrm{TR}\,n} + \alpha T_{\mathrm{RS}n})\, a_n\, e^{j\psi_n}, \\
&\boldsymbol{A}_{\mathrm{C}n}^{\mathrm{p}} = -j\,(T_{\mathrm{ST}n} + \alpha T_{\mathrm{TR}\,n} + \alpha^* T_{\mathrm{RS}n})
\end{aligned}
\tag{4.67}
$$

and,

$$
\begin{aligned}
c_{1n} &= \frac{j\,2\,a_n\ \cos\psi_n}{1 + a_n^2} \\
c_{2n} &= \frac{j\,2\,a_n\ \cos(\psi_n - 2\pi/3)}{1 + a_n^2} \\
c_{3n} &= \frac{j\,2\,a_n\ \cos(\psi_n + 2\pi/3)}{1 + a_n^2}.
\end{aligned}
\tag{4.68}
$$

Equation (4.65) can therefore be rewritten as

$$
\boldsymbol{F}_{1n}\, T_{\mathrm{RS}n} + \boldsymbol{F}_{2n}\, T_{\mathrm{ST}n} + \boldsymbol{F}_{3n}\, T_{\mathrm{TR}\,n} + \boldsymbol{F}_{4n} = 0 \tag{4.69}
$$

with,

$$\begin{aligned}
\boldsymbol{F}_{1n} &= c_{3n}(1+a_n e^{j\psi_n}) - j(\alpha^* + \alpha\ a_n\ e^{j\psi_n}) \\
\boldsymbol{F}_{2n} &= c_{1n}(1+a_n e^{j\psi_n}) - j(1+ a_n\ e^{j\psi_n}) \\
\boldsymbol{F}_{3n} &= c_{2n}(1+a_n e^{j\psi_n}) - j(\alpha + \alpha^*\ a_n\ e^{j\psi_n}) \\
\boldsymbol{F}_{4n} &= \boldsymbol{Y}_{\mathrm{d}n}(1+a_n e^{j\psi_n}) + \boldsymbol{A}_n^{\mathrm{p}} + \boldsymbol{A}_n^{\mathrm{n}}\ a_n\ e^{j\psi_n}\,.
\end{aligned} \tag{4.70}$$

Equation (4.69) contains both real and imaginary terms, and therefore, we can write

$$\begin{aligned}
&\mathrm{Re}\{\boldsymbol{F}_{1n}\ T_{\mathrm{RS}n} + \boldsymbol{F}_{2n}\ T_{\mathrm{ST}n} + \boldsymbol{F}_{3n}\ T_{\mathrm{TR}n} + \boldsymbol{F}_{4n}\} = 0 \\
&\mathrm{Im}\{\boldsymbol{F}_{1n}\ T_{\mathrm{RS}n} + \boldsymbol{F}_{2n}\ T_{\mathrm{ST}n} + \boldsymbol{F}_{3n}\ T_{\mathrm{TR}n} + \boldsymbol{F}_{4n}\} = 0.
\end{aligned} \tag{4.71}$$

The equations in (4.71) along with (4.58) provide the three linear equations with three unknown quantities. These equations can be written in matrix form as

$$\begin{bmatrix} U_{\mathrm{RS}n}^2 & U_{\mathrm{ST}n}^2 & U_{\mathrm{TR}n}^2 \\ \mathrm{Re}(\boldsymbol{F}_{1n}) & \mathrm{Re}(\boldsymbol{F}_{2n}) & \mathrm{Re}(\boldsymbol{F}_{3n}) \\ \mathrm{Im}(\boldsymbol{F}_{1n}) & \mathrm{Im}(\boldsymbol{F}_{2n}) & \mathrm{Im}(\boldsymbol{F}_{3n}) \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS}n} \\ T_{\mathrm{ST}n} \\ T_{\mathrm{TR}n} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b}n}\,\|\boldsymbol{u}_n\|^2 \\ -\mathrm{Re}(\boldsymbol{F}_{4n}) \\ -\mathrm{Im}(\boldsymbol{F}_{4n}) \end{bmatrix}, \tag{4.72}$$

also known as the **compensator equation** and it can be used to solve for the compensator susceptances $T_{\mathrm{RS}n}$, $T_{\mathrm{ST}n}$ and $T_{\mathrm{TR}n}$. It is to be noted that such a balancing compensator only compensates the reactive and the unbalanced currents. It does not affect the scattered currents in any way. Hence, such a shunt compensator cannot improve the power factor to unity.

Illustration 4.2 Design of a reactive compensator.

Let us design a reactive compensator to compensate the reactive and the unbalanced currents of the load shown in Fig. 4.5 in Illustration 4.1.

We obtained from the previous illustration that

$$\begin{bmatrix} \boldsymbol{U}_1^{\mathrm{p}} \\ \boldsymbol{U}_1^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 66.66 \\ 33.33e^{j60^\circ} \end{bmatrix} \mathrm{V}, \quad \begin{bmatrix} \boldsymbol{U}_5^{\mathrm{p}} \\ \boldsymbol{U}_5^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 1.1e^{-j67.6^\circ} \\ 1.93e^{j4.3^\circ} \end{bmatrix} \mathrm{V}, \quad \begin{bmatrix} \boldsymbol{U}_7^{\mathrm{p}} \\ \boldsymbol{U}_7^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 2 \\ 1.15e^{j90^\circ} \end{bmatrix} \mathrm{V}.$$

Thus, $\boldsymbol{a}_1 = a_1 e^{j\psi_1} = 0.5e^{j60^\circ}$, $\quad \boldsymbol{a}_5 = 1.76e^{j71.86^\circ}$, $\quad \boldsymbol{a}_7 = 0.578e^{j90^\circ}$.

The unbalance admittances

$$\begin{bmatrix} \boldsymbol{A}_1^{\mathrm{p}} \\ \boldsymbol{A}_1^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 0.97e^{-j135^\circ} \\ 0.26e^{j45^\circ} \end{bmatrix} \mathrm{S}, \quad \begin{bmatrix} \boldsymbol{A}_5^{\mathrm{p}} \\ \boldsymbol{A}_5^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 2.45e^{-j94.35^\circ} \\ 2.37e^{-j86.4^\circ} \end{bmatrix} \mathrm{S}, \quad \begin{bmatrix} \boldsymbol{A}_7^{\mathrm{p}} \\ \boldsymbol{A}_7^{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} 3.44e^{-j92.2^\circ} \\ 3.42e^{-j88.2^\circ} \end{bmatrix} \mathrm{S}\ .$$

The asymmetry dependent unbalance admittance

$$\boldsymbol{Y}_{\mathrm{d}1} = 0.45e^{-j26.6^\circ}\,\mathrm{S}, \quad \boldsymbol{Y}_{\mathrm{d}5} = 0.51e^{j86.4^\circ}\,\mathrm{S} \quad \boldsymbol{Y}_{\mathrm{d}7} = 0.015e^{-j45^\circ}\,\mathrm{S}\ .$$

The line to line voltages with respect to artificial zero are

$$\boldsymbol{U}_{\mathrm{RS}1} = \boldsymbol{U}_{\mathrm{R}1} - \boldsymbol{U}_{\mathrm{S}1} = 173.2e^{j30^\circ}\,\mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{ST}1} = \boldsymbol{U}_{\mathrm{S}1} - \boldsymbol{U}_{\mathrm{T}1} = 100e^{-j120^\circ}\,\mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{TR}1} = \boldsymbol{U}_{\mathrm{T}1} - \boldsymbol{U}_{\mathrm{R}1} = 100e^{j180^\circ}\,\mathrm{V}.$$

Similarly,

$$\begin{bmatrix} \boldsymbol{U}_{\mathrm{RS}5} \\ \boldsymbol{U}_{\mathrm{ST}5} \\ \boldsymbol{U}_{\mathrm{TR}5} \end{bmatrix} = \begin{bmatrix} 5.2e^{-j30^\circ} \\ 3.3^{j127.6^\circ} \\ 2.5e^{j180^\circ} \end{bmatrix} \mathrm{V}, \quad \begin{bmatrix} \boldsymbol{U}_{\mathrm{RS}7} \\ \boldsymbol{U}_{\mathrm{ST}7} \\ \boldsymbol{U}_{\mathrm{TR}7} \end{bmatrix} = \begin{bmatrix} 5.3e^{j41.2^\circ} \\ 4e^{-j120^\circ} \\ 2e^{j180^\circ} \end{bmatrix} \mathrm{V}\ .$$

The square of the magnitudes of these line to line voltage crms values are

$$U_{\mathrm{RS}1}^2 = 30000\ \mathrm{V}^2, \quad U_{\mathrm{ST}1}^2 = 10000\ \mathrm{V}^2, \quad U_{\mathrm{TR}1}^2 = 10000\ \mathrm{V}^2,$$
$$U_{\mathrm{RS}5}^2 = 27.05\ \mathrm{V}^2, \quad U_{\mathrm{ST}5}^2 = 10.89\ \mathrm{V}^2, \quad U_{\mathrm{TR}5}^2 = 6.25\ \mathrm{V}^2,\ .$$
$$U_{\mathrm{RS}7}^2 = 28.1\ \mathrm{V}^2, \quad U_{\mathrm{ST}7}^2 = 16.0\ \mathrm{V}^2, \quad U_{\mathrm{TR}7}^2 = 4.0\ \mathrm{V}^2,$$

We obtained earlier that $||\boldsymbol{u}_1||$=129.1 V. Thus $||\boldsymbol{u}_1||^2$=16667 V^2.

Similarly $||\boldsymbol{u}_5||^2 = 14.67\ \mathrm{V}^2$ and $||\boldsymbol{u}_7||^2 = 16.0\ \mathrm{V}^2$.

The coefficients,

$$\begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} = \begin{bmatrix} j0.4 \\ j0.4 \\ -j0.8 \end{bmatrix}, \quad \begin{bmatrix} c_{15} \\ c_{25} \\ c_{35} \end{bmatrix} = \begin{bmatrix} j0.27 \\ j0.57 \\ -j0.84 \end{bmatrix} \quad \begin{bmatrix} c_{17} \\ c_{27} \\ c_{37} \end{bmatrix} = \begin{bmatrix} 0 \\ j0.75 \\ -j0.75 \end{bmatrix}$$

and,

$$\begin{bmatrix} \boldsymbol{F}_{11} \\ \boldsymbol{F}_{21} \\ \boldsymbol{F}_{31} \\ \boldsymbol{F}_{41} \end{bmatrix} = \begin{bmatrix} -0.519 \\ 0.2598 - j0.75 \\ 0.2598 + j0.75 \\ -0.13 - j0.64 \end{bmatrix}, \begin{bmatrix} \boldsymbol{F}_{15} \\ \boldsymbol{F}_{25} \\ \boldsymbol{F}_{35} \\ \boldsymbol{F}_{45} \end{bmatrix} = \begin{bmatrix} 0.177 + j0.92 \\ 1.23 - j1.13 \\ -1.4 + j0.21 \\ -3.06 - j2.65 \end{bmatrix}, \begin{bmatrix} \boldsymbol{F}_{17} \\ \boldsymbol{F}_{27} \\ \boldsymbol{F}_{37} \\ \boldsymbol{F}_{47} \end{bmatrix} = \begin{bmatrix} -0.72 + j0.25 \\ 0.58 - j1 \\ 0.14 + j0.75 \\ 1.92 - j3.48 \end{bmatrix}.$$

The compensator equation for the fundamental frequency,

$$\begin{bmatrix} U^2_{\mathrm{RS1}} & U^2_{\mathrm{ST1}} & U^2_{\mathrm{TR1}} \\ \mathrm{Re}\,(\boldsymbol{F}_{11}) & \mathrm{Re}\,(\boldsymbol{F}_{21}) & \mathrm{Re}\,(\boldsymbol{F}_{31}) \\ \mathrm{Im}\,(\boldsymbol{F}_{11}) & \mathrm{Im}\,(\boldsymbol{F}_{21}) & \mathrm{Im}\,(\boldsymbol{F}_{31}) \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS1}} \\ T_{\mathrm{ST1}} \\ T_{\mathrm{TR1}} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b1}} \,||\, \boldsymbol{u}_1 \,||^2 \\ -\mathrm{Re}(\boldsymbol{F}_{41}) \\ -\mathrm{Im}(\boldsymbol{F}_{41}) \end{bmatrix}$$

has the values,

$$\begin{bmatrix} 30000 & 10000 & 10000 \\ -0.519 & 0.2598 & 0.2598 \\ 0 & -0.75 & 0.75 \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS}} \\ T_{\mathrm{ST}} \\ T_{\mathrm{TR}} \end{bmatrix} = \begin{bmatrix} 10000 \\ -0.13 \\ 0.64 \end{bmatrix}$$

Solving the above equations we get,

$$\begin{bmatrix} T_{\mathrm{RS1}} \\ T_{\mathrm{ST1}} \\ T_{\mathrm{TR1}} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.173 \\ 0.67 \end{bmatrix} \mathrm{S}\,.$$

Similarly, solving the compensator equation

$$\begin{bmatrix} U^2_{\mathrm{RS5}} & U^2_{\mathrm{ST5}} & U^2_{\mathrm{TR5}} \\ \mathrm{Re}\,(\boldsymbol{F}_{15}) & \mathrm{Re}\,(\boldsymbol{F}_{25}) & \mathrm{Re}\,(\boldsymbol{F}_{35}) \\ \mathrm{Im}\,(\boldsymbol{F}_{15}) & \mathrm{Im}\,(\boldsymbol{F}_{25}) & \mathrm{Im}\,(\boldsymbol{F}_{35}) \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS5}} \\ T_{\mathrm{ST5}} \\ T_{\mathrm{TR5}} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b5}} \,||\, \boldsymbol{u}_5 \,||^2 \\ -\mathrm{Re}(\boldsymbol{F}_{45}) \\ -\mathrm{Im}(\boldsymbol{F}_{45}) \end{bmatrix}$$

for the fifth order harmonic yields,

$$\begin{bmatrix} T_{\mathrm{RS5}} \\ T_{\mathrm{ST5}} \\ T_{\mathrm{TR5}} \end{bmatrix} = \begin{bmatrix} 0.003 \\ -2.31 \\ 0.171 \end{bmatrix} \mathrm{S},$$

and solving the compensator equation for the 7th order harmonic

$$\begin{bmatrix} U_{\mathrm{RS7}}^2 & U_{\mathrm{ST7}}^2 & U_{\mathrm{TR7}}^2 \\ \mathrm{Re}\,(\boldsymbol{F}_{17}) & \mathrm{Re}\,(\boldsymbol{F}_{27}) & \mathrm{Re}\,(\boldsymbol{F}_{37}) \\ \mathrm{Im}\,(\boldsymbol{F}_{17}) & \mathrm{Im}\,(\boldsymbol{F}_{27}) & \mathrm{Im}\,(\boldsymbol{F}_{37}) \end{bmatrix} \begin{bmatrix} T_{\mathrm{RS7}} \\ T_{\mathrm{ST7}} \\ T_{\mathrm{TR7}} \end{bmatrix} = \begin{bmatrix} -B_{\mathrm{b7}} \,||\boldsymbol{u}_7||^2 \\ -\mathrm{Re}(\boldsymbol{F}_{47}) \\ -\mathrm{Im}(\boldsymbol{F}_{47}) \end{bmatrix}$$

we get,

$$\begin{bmatrix} T_{\mathrm{RS7}} \\ T_{\mathrm{ST7}} \\ T_{\mathrm{TR7}} \end{bmatrix} = \begin{bmatrix} -0.003 \\ -3.36 \\ 0.16 \end{bmatrix} \mathrm{S}.$$

The reactance elements in the branch RS should be chosen in such a way that the susceptance of the branch is $0\,\mathrm{S}$ for the fundamental frequency, $0.003\,\mathrm{S}$ for the 5th order harmonic and $-0.003\,\mathrm{S}$ for the 7th order harmonics. Likewise, the reactance elements in the branch ST should be chosen in such a way that the susceptance of the branch is $-0.173\,\mathrm{S}$ for the fundamental frequency, $-2.31\,\mathrm{S}$ for the 5th order harmonic and $-3.36\,\mathrm{S}$ for the 7th order harmonics. Similarly the reactance elements in the branch TR should be chosen in such a way that the susceptance of the branch is $0.67\,\mathrm{S}$ for the fundamental frequency, $0.171\,\mathrm{S}$ for the 5th order harmonic and $0.16\,\mathrm{S}$ for the 7th order harmonics. Such a reactive compensator will completely compensate the reactive and the unbalanced currents and significantly improve the power factor. It cannot improve the power factor to unity, however. This is because the scattered current cannot be compensated using reactive compensators.

4.4.2 Properties of shunt reactive compensator

Reactive compensators consist of inductors and capacitors and are regarded as lossless. Thus, such LC devices are subsets of RLC circuits with R = 0. The immitance of such LC one-ports is also referred to as Reactance Functions (RFs) and they are a subset of the set of Positive Real Functions. The detailed theory of RFs and Positive Real Functions is given in [23]. Only those concepts that are relevant to the design of reactive compensators are briefly discussed here.

RFs have POLEs p_r and ZEROs z_k on the s-plane. It means that the value of $s = p_r$ when the function $F(s)$ approaches infinity, while the value of $s = z_k$ when the function is equal to zero. The POLEs and ZEROs of RFs are exclusively on the imaginary axis of the s-plane.

A RF is an odd function of s and is represented as a rational function as the ratio of two polynomials, such that

$$F(s) = \frac{N(s)}{D(s)} = \frac{W_n(s)}{W_m(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \ldots . a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \ldots . b_1 s + b_0} \quad . \tag{4.73}$$

Therefore, the two polynomials cannot have the same order, or in other words, $m \neq n$. Also, since the orders of the polynomials of Positive Real Functions cannot differ by more than 1,

$$|n - m| = 1 \tag{4.74}$$

It means that RFs are ratio of odd and even polynomials. When the odd polynomial is in the numerator, then the function $F(s)$ approaches zero as s approaches zero, while if the odd polynomial is in the denominator, the function $F(s)$ approaches infinity as s approaches zero. Also, if the polynomial of the higher order is in the numerator, then the function $F(s)$ approaches infinity as s approaches infinity, while if the polynomial of the higher order is in the denominator, then the function $F(s)$ approaches zero as s approaches infinity.

The derivative of RFs with respect to frequency on the imaginary axis is always positive, i.e.,

$$\frac{d}{d(j\omega)}F(j\omega) > 0 \ , \tag{4.75}$$

which means that the susceptance $B(\omega)$ and the reactance $X(\omega)$ of a reactance one-port can only with increase with the increase in frequency. This also implies that when a RF has multiple POLEs and ZEROs, they have to interlace each other. This is because the reactance can increase between two ZEROs only if they are separated by a POLE. Also, the reactance between two POLEs can only increase if they are separated by a ZERO. This is illustrated in the plot of susceptance against the frequency in Fig. 4.7.

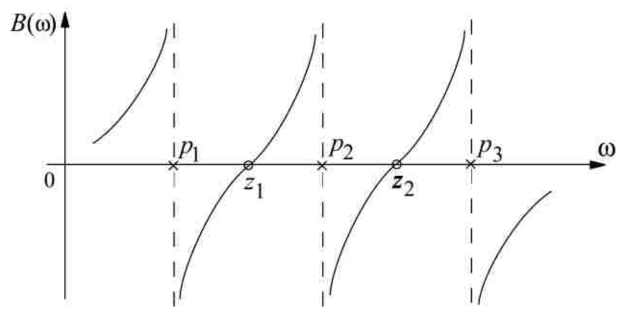


Fig. 4.7 Plot of susceptance $B(\omega)$ against frequency

When a reactance one-port is used as a shunt compensator, then it has to have specified susceptances for each of the supply voltage harmonics. In particular, for a harmonic order n or frequency $n\omega_1$, the compensator susceptance has to be equal to B_{Cn}. For such a case, the general form of the admittance $Y_C(s)$ has to be found. Such a form is specified by the number of POLEs and ZEROs which in turn can be found using a constantly increasing susceptance $B_C(\omega)$, which has the value B_{Cn} for harmonic order n. Finally, the calculated POLEs and/or ZEROs have to be added at zero and/or infinity in such a way so as to have a Reactance Function. Mathematically, such a compensator admittance can be expressed in the form

$$Y_C(s) = f(s)\frac{(s^2+z_1^2)...(s^2+z_n^2)}{(s^2+p_1^2)...(s^2+p_m^2)}, \quad \text{with } f(s) = \begin{cases} As \\ \dfrac{A}{s} \end{cases} \quad \text{and } A > 0. \tag{4.76}$$

Illustration 4.3 Let us calculate the compensator branch parameters for the load shown in Fig. 4.8 supplied from a source with the voltages:

$$\boldsymbol{u}_\mathrm{R} = \sqrt{2}\mathrm{Re}\{100\,\mathrm{e}^{j\omega_1 t} + 50\,\mathrm{e}^{j5\omega_1 t} + 25\,\mathrm{e}^{j7\omega_1 t}\}\ \mathrm{V}$$
$$\boldsymbol{u}_\mathrm{S} = \sqrt{2}\mathrm{Re}\{100\,\mathrm{e}^{-j120}\,\mathrm{e}^{j\omega_1 t} + 50\,\mathrm{e}^{j120}\,\mathrm{e}^{j5\omega_1 t} + 25\,\mathrm{e}^{-j120}\,\mathrm{e}^{j7\omega_1 t}\}\ \mathrm{V}$$
$$\boldsymbol{u}_\mathrm{T} = 0$$

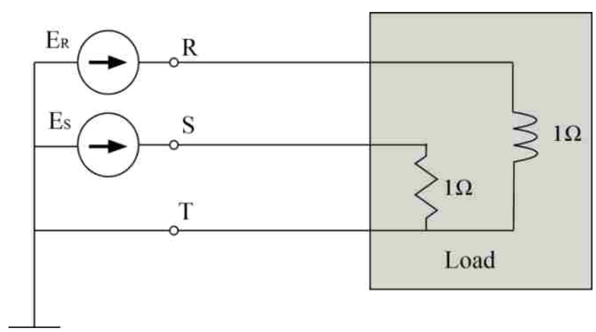


Fig. 4.8 Unbalanced load supplied from a source of asymmetrical nonsinusoidal voltage

For such a load, the compensator branch susceptances are equal to

$$\begin{bmatrix} T_{\mathrm{RS1}} \\ T_{\mathrm{ST1}} \\ T_{\mathrm{TR1}} \end{bmatrix} = \begin{bmatrix} -0.578 \\ 0.693 \\ 2.039 \end{bmatrix} \mathrm{S}, \quad \begin{bmatrix} T_{\mathrm{RS5}} \\ T_{\mathrm{ST5}} \\ T_{\mathrm{TR5}} \end{bmatrix} = \begin{bmatrix} 0.578 \\ -0.693 \\ -.839 \end{bmatrix} \mathrm{S} \quad \text{and} \quad \begin{bmatrix} T_{\mathrm{RS7}} \\ T_{\mathrm{ST7}} \\ T_{\mathrm{TR7}} \end{bmatrix} = \begin{bmatrix} -0.578 \\ 0.693 \\ 1.182 \end{bmatrix} \mathrm{S}.$$

Let us compute the admittance function $Y_C(s)$ for the branch TR. We know that the compensator branch TR has to compensate the load for the voltage fundamental harmonic, as well as the fifth and the seventh order harmonics. Also, for such a purpose, it requires that the compensator branch susceptances are $B_{C1} = 2.039\,\mathrm{S}$, $B_{C5} = -0.839\,\mathrm{S}$

and $B_{C7} = 1.182$ S. The plot of the compensator susceptance $B_C(\omega)$ corresponding to these values is as shown in Fig. 4.9.

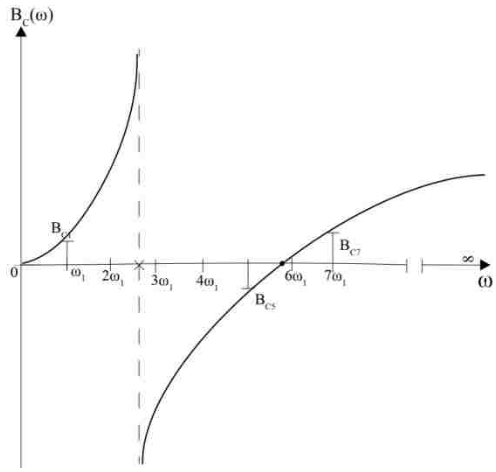


Fig. 4.9 Plot of susceptance $B_C(\omega)$

The following conclusions can be made about the admittance function $Y_C(s)$

- it has a ZERO at s = 0, hence the odd polynomial is in the numerator
- it has a POLE at s = infinity, hence the higher order polynomial is in the numerator
- it has one other ZERO z_1
- it has one other POLE p_1

Therefore, the admittance function has the form

$$Y_C(s) = As\frac{(s^2 + z_1^2)}{(s^2 + p_1^2)}$$

There are three unknowns namely A, z_1^2 and p_1^2 the expression above.

Before we proceed to solve for the compensator parameters, let us normalize the fundamental frequency ω_1 to 1 rad/s for simplicity of calculations. The calculated values can be converted to correspond to the system frequency at the end. We have the three equations

$$A(j1)\frac{[(j1)^2+z_1^2]}{[(j1)^2+p_1^2)]} = jB_{C1}$$

$$A(j5)\frac{[(j5)^2+z_1^2]}{[(j5)^2+p_1^2]} = jB_{C5}$$

$$A(j7)\frac{[(j7)^2+z_1^2]}{[(j7)^2+p_1^2]} = jB_{C7}$$

Observing Fig. 4.9, it is clear that the admittance has a POLE between s = 1 and s = 5. If we choose the pole p_1 to be at s = 2.7115, then, we will have,

$$A(1)\frac{(z_1^2-1)}{(p_1^2-1)} = 2.039$$

$$A(5)\frac{(z_1^2-25)}{(p_1^2-25)} = -0.839$$

$$A(7)\frac{(z_1^2-49)}{(p_1^2-47)} = 1.182$$

The set of equation have a solution $A = 0.416$, $z_1 = 5.67$ and $p_1 = 2.71$. Thus the admittance function is

$$Y_C(s) = 0.4165\,s\,\frac{(s^2+32.11)}{(s^2+7.35)}.$$

There are two main methods, namely the **Foster procedures** and the **Cauer procedures**, that can be used to develop the reactance one-port structure when the admittance is known. Each of these two procedures have two sub procedures, one of which has to be chosen based on the admittance function. The numerator polynomial of the admittance function that we obtained is of the higher degree, and therefore, the **Cauer First procedure** is used here for the calculation of the reactance one-port structure.

$$Y_C(s) = 0.4165s\frac{(s^2+32.11)}{(s^2+7.35)} = \frac{0.4165s^3+13.37s}{(s^2+7.35)}$$

$$= \frac{0.4165s(s^2+7.35) - 0.4165\times 7.35s + 13.37s}{(s^2+7.35)}$$

$$= 0.4165s + \frac{10.31s}{(s^2+7.35)}$$

$$= 0.4165s + \frac{1}{0.097s + \frac{0.7131}{s}}.$$

As per the **Cauer First procedure,** $C_1 = d_1$, $L_2 = d_2$, $C_3 = d_3$, etc., therefore, the structure of the reactance one port corresponding to the above calculated admittance function is as shown in Fig. 4.10.

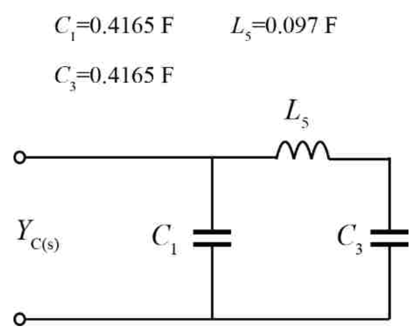


Fig. 4.10 Branch TR of compensator with Cauer First structure

It is to be noted that these values are corresponding to the normalized frequency. Fig. 4.10 corresponds to the branch TR. The structures of the compensator branches RS and ST can be found using a similar method.

It is evident from illustration 4.3 that the process of finding the structure of the reactance compensator branches is complex. In this particular case, a very simple circuit was chosen where the supply voltage contains harmonics of only three different orders. As the number of harmonic orders increases, the filter complexity, as well as the cost of the compensator increases as well. Therefore, design of such a compensator may become impractical from an application perspective. One solution to that problem is to design a compensator with reduced complexity, for example, by assuming that the compensator branch cannot have more elements than two. The parameters of such an optimized reactance one-port compensator are chosen so that it can have the highest power factor at no more than two elements per compensator branch. Such a compensator generally does not improve the power factor to unity, however.

4.4.3 Design of an optimized compensator for the minimization of the unbalanced and reactive currents

A shunt Two Element Series LC compensator, or simply TESLC compensator, can be used for the minimization of the supply current three-phase rms value [24]. Such a compensator has a significantly reduced complexity and the goal of such a TESLC compensator is to reduce the three-phase rms value of the supply current to its minimum possible value with no more than two elements per compensator branch. Such a compensator is depicted in Fig. 4.11.

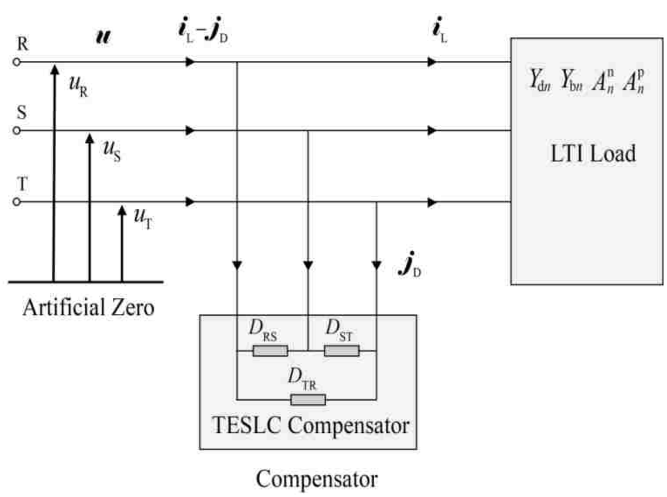


Fig. 4.11 Load with an optimized compensator

Let us recall the compensator shown in Fig. 4.6 above, which has the branch susceptances $T_{\mathrm{RS}n}, T_{\mathrm{ST}n}$ and $T_{\mathrm{TR}n}$, and, completely compensates the reactive and the unbalanced currents. Its susceptances are specified by equation(4.72), and has a current specified by the three-phase current vector $\boldsymbol{j}_{\mathrm{T}}$ at the supply phase to phase voltage $\boldsymbol{U}_{\mathrm{XY}n}$.

Let $D_{\mathrm{RS}n}, D_{\mathrm{ST}n}$ and $D_{\mathrm{TR}n}$ be the branch susceptances of the optimized compensator for the nth harmonic order. Let the current vector of the optimized compensator be $\boldsymbol{j}_{\mathrm{D}}$, such that

$$\boldsymbol{j}_{\mathrm{D}} = [\boldsymbol{j}_{\mathrm{RS}} \;\; \boldsymbol{j}_{\mathrm{ST}} \;\; \boldsymbol{j}_{\mathrm{TR}}]^{\mathrm{T}}. \tag{4.77}$$

The effectiveness of the current minimization of the optimized compensator is measured by the deviation of the current $\boldsymbol{j}_{\mathrm{D}}$ of the optimized compensator from the current $\boldsymbol{j}_{\mathrm{T}}$ of the ideal compensator. Ideally, the deviation should be zero. The sum of the squares of the deviation of the currents on each of the phases, can be written as

$$d^2 = \sum_{n \in \boldsymbol{N}} [(T_{\mathrm{RS}n} - D_{\mathrm{RS}n})U_{\mathrm{RS}n}]^2 + [(T_{\mathrm{ST}n} - D_{\mathrm{ST}n})U_{\mathrm{ST}n}]^2 + [(T_{\mathrm{TR}n} - D_{\mathrm{TR}n})U_{\mathrm{TR}n}]^2 \tag{4.78}$$

which comprises of the components of the form

$$d_{\mathrm{xy}}^2 = \sum_{n \in N} [(T_{\mathrm{XY}n} - D_{\mathrm{XY}n})U_{\mathrm{XY}n}]^2 = \sum_{n \in N} d_{\mathrm{xy}n}^2 \,. \tag{4.79}$$

Due of the presence of the square on the right hand side of the equation, each of the components is a positive value, and, therefore, for the total deviation defined in (4.78) to be minimum, each of the three components d_{XY} should be minimum.

The square of the deviation specified in (4.79) is dependent on three components, namely the susceptances $T_{\mathrm{XY}n}$, the susceptances $D_{\mathrm{XY}n}$ and the supply line to line voltage harmonic rms value $U_{\mathrm{XY}n}$. The susceptances $T_{\mathrm{XY}n}$ of the ideal compensator are in turn dependent on the load parameters as well as the supply voltage rms values, as indicated in equation (4.72). Due to the source impedance, the supply voltage harmonics are also affected by the change in the source current after the addition of the compensator. However, if the parameters are chosen so as to avoid resonance, then the change in the voltage is much smaller compared to the change in the currents. This means that it is reasonable to assume that the minimization of the deviation d_{XY} under the assumption that $U_{\mathrm{XY}n}$ is constant, will not lead to substantial errors in results of the minimization. Therefore, it is assumed for the following analysis that the source impedance is zero and that the source voltage is not affected by the compensator.

The supply line to line voltage rms value U_{XY1} of the fundamental harmonic is generally much higher than the supply line to line voltage rms value $U_{\mathrm{XY}n}$ (n>1) of the higher order harmonics. Thus, the term $(T_{\mathrm{XY1}} - D_{\mathrm{XY1}})$ is the main contributor to the deviation d_{XY}, and, therefore, a reduction in the term $(T_{\mathrm{XY1}} - D_{\mathrm{XY1}})$ leads to a reduction in d_{XY}. For this purpose, the signs of the susceptances T_{XY1} and D_{XY1} should be the same. Hence, if $T_{\mathrm{XY1}} < 0$, then, a L-type branch should be chosen so as to make $D_{\mathrm{XY1}} < 0$, while, if $T_{\mathrm{XY1}} > 0$, then, a LC type branch should be chosen so as to make $D_{\mathrm{XY1}} > 0$. Note that in the latter case, a LC type branch is chosen instead of a C type branch in order to avoid the resonance with the source impedance at a frequency of one of the supply voltage harmonics. In the first case, if the optimum inductance is L_{XY}, then the square of the deviation,

$$d_{\mathrm{xy}}^2 = \sum_{n \in \boldsymbol{N}} (T_{\mathrm{XY}n} + \frac{1}{n\omega_1 L_{\mathrm{XY}}})^2 U_{\mathrm{XY}n}^2 . \tag{4.80}$$

On the other hand, for case two, if the optimum capacitance is C_{XY} and the optimum inductance is L_{XY}, then the square of the deviation,

$$d_{\text{xy}}^2 = \sum_{n\in \boldsymbol{N}} (T_{\text{XY}\,n} - \frac{n\,\omega_1\,C_{\text{XY}}}{1-n^2\,\omega_1^2\,C_{\text{XY}}\,L_{\text{XY}}})^2\,U_{\text{XY}\,n}^2\,. \tag{4.81}$$

The rms value of the supply current is minimum when the value of the square of the deviation mentioned above is minimum. Differentiating equation (4.80) with respect to L_{XY} yields,

$$\begin{aligned}
\frac{\partial(d_{\text{xy}}^2)}{\partial L_{\text{XY}}} &= \frac{\partial}{\partial L_{\text{XY}}}\sum_{n\in \boldsymbol{N}} [(T_{\text{XY}\,n} + \frac{1}{n\,\omega_1\,L_{\text{XY}}})U_{\text{XY}\,n}]^2 \\
&= \sum_{n\in \boldsymbol{N}} 2(T_{\text{XY}\,n} + \frac{1}{n\,\omega_1\,L_{\text{XY}}})U_{\text{XY}\,n} \times (0 - \frac{1}{n\,\omega_1\,L_{\text{XY}}^2})U_{\text{XY}\,n}] \\
&= \sum_{n\in \boldsymbol{N}} 2(T_{\text{XY}\,n} + \frac{1}{n\,\omega_1\,L_{\text{XY}}})U_{\text{XY}\,n}] \times (\frac{-U_{\text{XY}\,n}}{n\,\omega_1\,L_{\text{XY}}^2})
\end{aligned} \tag{4.82}$$

For minima,

$$\frac{\partial(d_{\text{xy}}^2)}{\partial L_{\text{XY}}} = \sum_{n\in N} 2(T_{\text{XY}\,n} + \frac{1}{n\,\omega_1\,L_{\text{XY}}})\frac{-U_{\text{XY}\,n}^2}{n\,\omega_1\,L_{\text{XY}}^2} = 0$$

$$\sum_{n\in N} (T_{\text{XY}\,n} + \frac{1}{n\,\omega_1\,L_{\text{XY}}})\frac{-U_{\text{XY}\,n}^2}{n} = 0$$

$$\sum_{n\in N} \frac{-T_{\text{XY}\,n} \times U_{\text{XY}\,n}^2}{n} = \sum_{n\in N} \frac{U_{\text{XY}\,n}^2}{n^2\,\omega_1\,L_{\text{XY}}}.$$

Hence,

$$L_{\text{XY}} = -\frac{\sum_{n\in N} \frac{U_{\text{XY}\,n}^2}{n^2\,\omega_1}}{\sum_{n\in N} \frac{T_{\text{XY}\,n} \times U_{\text{XY}\,n}^2}{n}} \tag{4.83}$$

The inductor L_{XY} specified in (4.83) is the optimal inductor for the compensator branch XY if $T_{\text{XY1}} < 0$.

For the second case specified above, i.e., when the susceptance $T_{\mathrm{XY}1} > 0$, then, the compensator should have a series LC branch between the lines X and Y. In this case, the deviation in the compensator current rms values is given by

$$d_{\mathrm{xy}}^2 = \sum_{n \in N} (T_{\mathrm{XY}n} - \frac{n\omega_1 C_{\mathrm{XY}}}{1 - n^2 \omega_1^2 C_{\mathrm{XY}} L_{\mathrm{XY}}})^2 U_{\mathrm{XY}n}^2 \tag{4.84}$$

The inductor L_{XY} is added in series to the compensator capacitor so as to shift the resonance with the source impedance to a frequency not present in the supply voltage harmonics. The resonant frequency,

$$\omega_r = 1/\sqrt{\mathrm{L_{XY} C_{XY}}}$$

Hence,

$$\mathrm{L_{XY} C_{XY}} = 1/\omega_r^2 \tag{4.85}$$

If we define

$$M_{\mathrm{LC}} = \omega_1^2 C_{\mathrm{XY}} L_{\mathrm{XY}} = \omega_1^2 / \omega_r^2 \tag{4.86}$$

we can rewrite (4.84) as,

$$d_{\mathrm{xy}}^2 = \sum_{n \in N} [(T_{\mathrm{XY}n} - \frac{n\omega_1 C_{\mathrm{XY}}}{1 - M_{\mathrm{LC}} n^2}) U_{\mathrm{XY}n}]^2 \tag{4.87}$$

The optimal value of the capacitor can be found by differentiating (4.87) with respect to C_{XY} and equating it to zero. That is,

$$\frac{\partial (d_{\mathrm{xy}}^2)}{\partial C_{\mathrm{XY}}} = \frac{\partial}{\partial C_{\mathrm{XY}}} \sum_{n \in N} \{[(T_{\mathrm{XY}n} - \frac{n\omega_1 C_{\mathrm{XY}}}{1 - M_{\mathrm{LC}} n^2}) U_{\mathrm{XY}n}]^2\}$$

$$= \sum_{n \in N} 2(T_{\mathrm{XY}n} - \frac{n\omega_1 C_{\mathrm{XY}}}{1 - M_{\mathrm{LC}} n^2}) U_{\mathrm{XY}n}] \times (0 - \frac{n\omega_1}{1 - M_{\mathrm{LC}} n^2}) U_{\mathrm{XY}n} =$$

$$= \sum_{n \in N} 2(T_{\mathrm{XY}n} - \frac{n\omega_1 C_{\mathrm{XY}}}{1 - M_{\mathrm{LC}}\, n^2}) \frac{n\omega_1 U_{\mathrm{XYn}}^2}{1 - M_{\mathrm{LC}}\, n^2} = 0. \tag{4.88}$$

Rearranging (4.88) we get,

$$\sum_{n \in N} [(\frac{n\omega_1\, T_{\mathrm{XY}n} U_{\mathrm{XY}n}^2}{1 - N_{\mathrm{LC}}\, n^2} - \frac{n^2\, \omega_1^2\, C_{\mathrm{XY}}\, U_{\mathrm{XY}n}^2}{(1 - M_{\mathrm{LC}}\, n^2)^2})] = 0.$$

$$\text{Or,} \quad \sum_{n \in N} \frac{n\omega_1\, T_{\mathrm{XY}n} U_{\mathrm{XY}n}^2}{1 - M_{\mathrm{LC}}\, n^2} = \sum_{n \in \boldsymbol{N}} \frac{n^2\, \omega_1^2\, C_{\mathrm{XY}}\, U_{\mathrm{XY}n}^2}{(1 - M_{\mathrm{LC}}\, n^2)^2}$$

Hence,

$$C_{\mathrm{XY}} = \frac{\sum_{n \in N} \frac{n\omega_1\, T_{\mathrm{XY}n} U_{\mathrm{XY}n}^2}{1 - M_{\mathrm{LC}}\, n^2}}{\sum_{n \in N} \frac{n^2\, \omega_1^2\, U_{\mathrm{XY}n}^2}{(1 - M_{\mathrm{LC}}\, n^2)^2}} \tag{4.89}$$

The capacitor C_{XY} specified in (4.89) is the optimal capacitor for the compensator branch XY under the scenario that T_{XY1}>0.

The above mentioned steps should be repeated for all three phases based on the sign of the compensator fundamental harmonic susceptance T_{XY1} , and the optimal L- Branch or the optimal LC- Branch should be connected between the phases X and Y. Such an optimal compensator will reduce the three-phase rms value of the supply current to its lowest possible value for the given number of elements per branch.

Illustration 4.4 Design of an optimized compensator for an unbalanced load supplied from a source of asymmetrical and nonsinusoidal voltage.

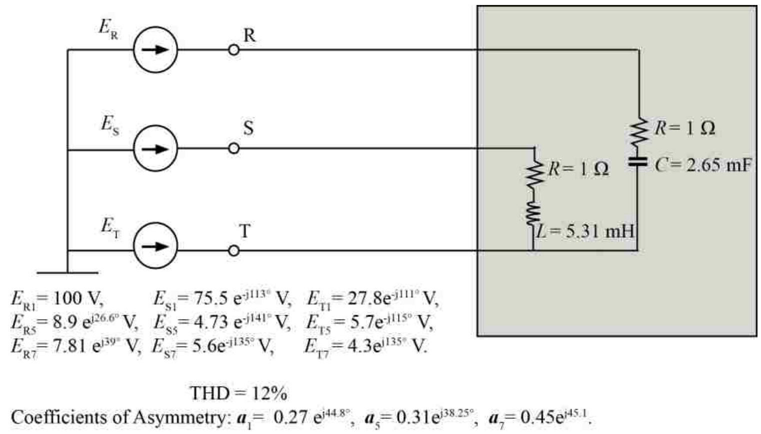


Fig. 4.12 Unbalanced load supplied from a source of asymmetrical and distorted voltage

Fig. 4.12 above depicts an unbalanced LTI load supplied from a source with high distortion and asymmetry.

The three-phase rms value of the supply voltage $||\boldsymbol{u}|| = 121.96$ V.

The rms values of the line currents,

$$||i_{\mathrm{R}}|| = 80 \text{ A},$$
$$||i_{\mathrm{S}}|| = 43.5 \text{ A},$$
$$||i_{\mathrm{T}}|| = 40.4 \text{ A}.$$

and the currents three-phase rms value $||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2} = 99.62$ A.

The rms values of Currents' Physical Components (CPC) are as following:

active current $||\boldsymbol{i}_{\mathrm{a}}|| = 67.97$ A,

scattered current $||\boldsymbol{i}_{\mathrm{s}}|| = 7.99$ A,

reactive current $||\boldsymbol{i}_{\mathrm{r}}|| = 21.52$ A,

and unbalanced current $||\boldsymbol{i}_{\mathrm{u}}|| = 69.12$ A.

Similarly,

active power $P = 8.29$ kW,

scattered power $D_S = 0.98$ kVA,

reactive power $Q = 2.63$ kVar,

and unbalanced power $D_u = 8.43$ kVA.

The apparent Power

$$S = \sqrt{P^2 + D_s{}^2 + Q^2 + D_u{}^2} = \sqrt{8.29^2 + 0.98^2 + 2.63^2 + 8.43^2} = 12.15 \text{ kVA}$$

and the power factor $\lambda = \dfrac{P}{S} = 0.68$.

The results are shown in Fig. 4.13.

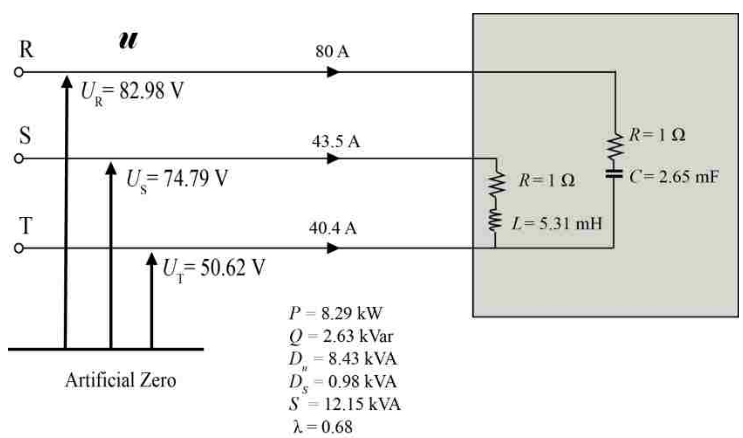


Fig. 4.13 Powers and line currents of the load

Design of a Reactive Compensator:

The load has a power factor of 0.68. This can be improved by adding a reactive compensator of the delta structure shown in Fig. 4.6. Such an ideal reactive compensator will completely compensate the reactive and the unbalanced currents.

The steps needed for the calculation of the compensator parameters is presented in detail in illustration 4.2 above. A similar approach is used in this illustration and only the final results are presented here. For the parameters of this illustration, the compensator equations given in (4.72) above leads to the following susceptances for the compensator branches:

$$\begin{bmatrix} T_{RS1} \\ T_{ST1} \\ T_{TR1} \end{bmatrix} = \begin{bmatrix} 0.148 \\ 0.038 \\ -0.482 \end{bmatrix} \text{ S}, \quad \begin{bmatrix} T_{RS5} \\ T_{ST5} \\ T_{TR5} \end{bmatrix} = \begin{bmatrix} 0.014 \\ 0.067 \\ -0.196 \end{bmatrix} \text{ S} \quad \text{and} \quad \begin{bmatrix} T_{RS7} \\ T_{ST7} \\ T_{TR7} \end{bmatrix} = \begin{bmatrix} 0.008 \\ 0.052 \\ -0.146 \end{bmatrix} \text{ S.}$$

The reactance elements in the branch RS should be chosen in such a way that the susceptance of the branch is 0.148 S for the fundamental frequency, 0.014 S for the 5^{th} order harmonic and 0.008 S for the 7^{th} order harmonic. Likewise, the reactance elements in the branch ST should be chosen in such a way that the susceptance of the branch is 0.038 S, 0.067 S and 0.052 S for the fundamental, fifth and the seventh harmonic orders respectively. Similarly, the reactance elements in the branch ST should be chosen such that the susceptance of the branch is -0.482 S, -0.196 S and -0.146 S for the fundamental, fifth and the seventh harmonic orders respectively. The load with the given compensator shown as a block is depicted in Fig. 4.14.

The addition of such a compensator leads to the following results:

The rms values of the line currents,

$$\begin{aligned} ||i_R|| &= 45.96 \text{ A}, \\ ||i_S|| &= 41.96 \text{ A}, \\ ||i_T|| &= 28.45 \text{ A} \end{aligned}$$

and the currents three-phase rms value $||\boldsymbol{i}|| = \sqrt{||i_R||^2 + ||i_S||^2 + ||i_T||^2} = 68.43$ A.

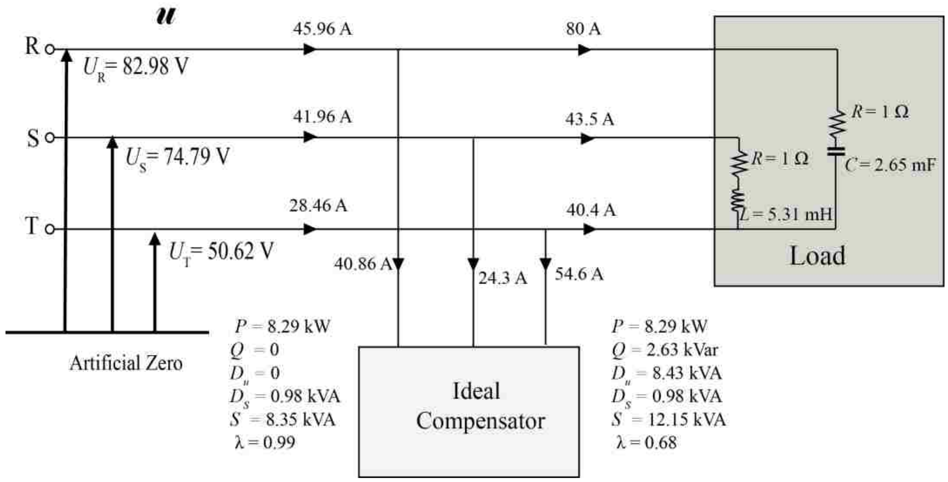


Fig. 4.14 Results after the addition of an ideal LC Compensator

Note that due the asymmetry in the supply voltage, the line currents after compensation are still asymmetrical. The line currents are proportional, and in phase to the line voltages, however.

The rms values of Currents' Physical Components are as following:

active current $||\boldsymbol{i}_{\mathrm{a}}|| = 67.96$ A,

scattered current $||\boldsymbol{i}_{\mathrm{s}}|| = 7.99$ A,

reactive current $||\boldsymbol{i}_{\mathrm{r}}|| = 0$,

and unbalanced current $||\boldsymbol{i}_{\mathrm{u}}|| = 0$.

Similarly,

active power $P = 8.29$ kW,

scattered power $D_{\mathrm{S}} = 0.98$ kVA,

reactive power $Q = 0$,

and unbalanced power $D_{\mathrm{u}} = 0$.

The apparent Power $S = \sqrt{P^2 + D_{\mathrm{s}}^2 + Q^2 + D_{\mathrm{u}}^2} = 8.35$ kVA

and the power factor $\lambda = \dfrac{P}{S} = 0.99$

Thus, an ideal reactive balancing compensator as shown in Fig. 4.14 completely compensates the reactive and the unbalanced currents and improves the power factor to almost unity. In order to be able to implement the compensator in a real system, we have to design the compensator branches so as to have the desired reactance properties. For example, the compensator branch RS should be designed in such a way that its susceptances for the fundamental, fifth and the seventh harmonic orders are 0.148 S, 0.014 S and 0.008 S respectively. As it was evident for a very similar case in illustration 4.3 earlier in the chapter, the design as well as the structure of such an ideal compensator is very complex, and, more importantly, it does not have a practical value. Therefore, the design of the branches of the ideal compensator is not presented here. Rather, let us try to design an optimized TESLC compensator for the load above, which significantly reduces the cost and the complexity of the structure of the compensator branches, without making a significant compromise on the power factor.

Design of a TESLC Compensator:

As described in section 4.4.3 above, a TESLC compensator contains no more than two elements per compensator branch. Each branch can either have an inductor, or a capacitor in series with an inductor, depending upon the susceptance of the compensator branch for the fundamental harmonic. In this case, since $T_{\mathrm{RS1}} > 0$ we will choose a LC branch between the lines R and S. Since the supply voltage does not contain any components of the third harmonic order, we can choose the value of the series inductor such that the resonance is just past the third harmonic. Then,

$$M_{\mathrm{LC}} = \omega_1^2 / \omega_r^2 = 0.1$$

Using the equation of the optimal capacitor,

$$C_{\mathrm{XY}} = \frac{\sum_{n \in N} \frac{n\,\omega_1\, T_{\mathrm{XY}\,n} U_{\mathrm{XY}\,n}^2}{1 - M_{\mathrm{LC}}\; n^2}}{\sum_{n \in N} \frac{n^2\,\omega_1^2\, U_{\mathrm{XY}\,n}^2}{(1 - M_{\mathrm{LC}}\; n^2)^2}}$$

to calculate the value of the capacitor for the branch RS, with N = {1,5,7} yields,

$$C_{\mathrm{RS}} = \frac{\sum_{n \in N} \frac{n\,\omega_1\, T_{\mathrm{RS1}} U_{\mathrm{RSn}}^2}{1 - 0.1\, n^2}}{\sum_{n \in N} \frac{n^2\,\omega_1^2\, U_{\mathrm{RSn}}^2}{(1 - 0.1\, n^2)^2}} = 0.319 \text{ mF}\,.$$

Then, the series inductor

$$L_{\mathrm{RS}} = 2.19 \;\; \text{mH}\,.$$

Thus, the compensator branch RS should have a capacitor of 0.319 mF in series with an inductor of 2.19 mH.

Similarly, since $T_{\mathrm{ST1}} > 0$, we choose a LC branch between the compensator terminal S and T. Substituting in the formula for the optimal capacitor,

$$C_{\mathrm{ST}} = \frac{\sum_{n \in N} \frac{n\,\omega_1\, T_{\mathrm{ST1}} U_{\mathrm{STn}}^2}{1 - 0.1\, n^2}}{\sum_{n \in N} \frac{n^2\,\omega_1^2\, U_{\mathrm{STn}}^2}{(1 - 0.1\, n^2)^2}} = 0.079 \text{ mF}$$

and, the series inductor $L_{ST} = 8.84$ mH. Thus, the compensator branch ST should consist of a capacitor of 0.079 mF in series with an inductor of 8.84 mH.

Finally, since $T_{TR1} < 0$, the compensator branch TR consists of an inductor of the value

$$L_{TR} = -\frac{\sum_{n \in \boldsymbol{N}} \frac{U_{TRn}^2}{\omega_1 n^2}}{\sum_{n \in \boldsymbol{N}} \frac{T_{TR1} \times U_{TRn}^2}{n}} = 5.49 \text{ mH}$$

Thus, a compensator with the parameters calculated above and as shown in Fig. 4.15 is the optimized compensator that minimizes the supply current three-phase rms $\|\boldsymbol{i}\|$ to the lowest possible value while using no more than two reactive elements per compensator branch.

The addition of the optimized TESLC compensator leads to the results as shown in Fig. 4.15.

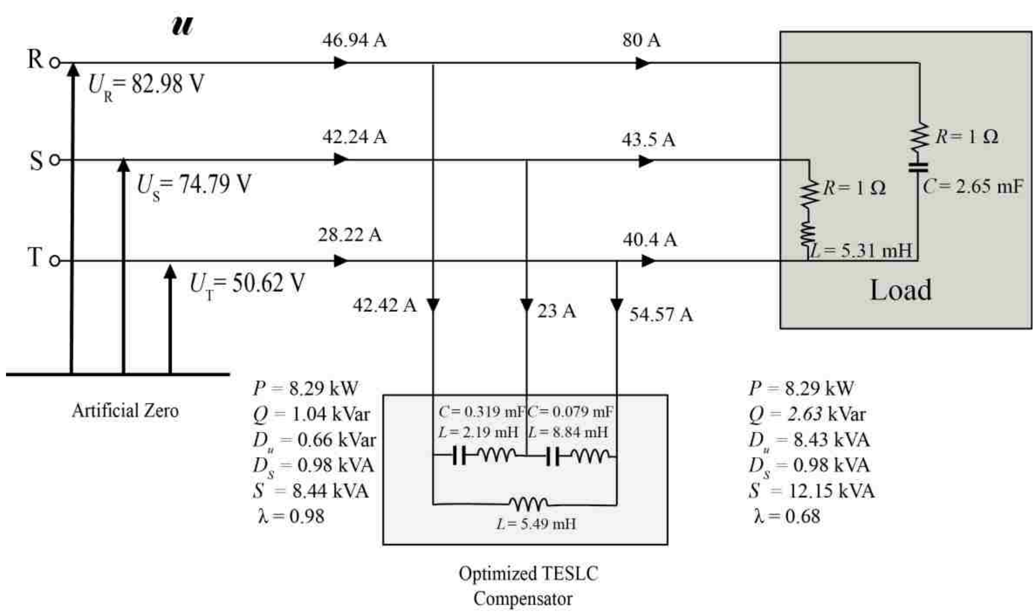


Fig. 4.15 Results after the addition of an Optimized TESLC Compensator

The rms values of the line currents,

$$||i_{\mathrm{R}}|| = 46.94 \text{ A},$$
$$||i_{\mathrm{S}}|| = 42.24 \text{ A},$$
$$||i_{\mathrm{T}}|| = 28.22 \text{ A}$$

and the currents three-phase rms value $||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2} = 69.17 \text{ A}$.

The three-phase rms values of Currents' Physical Components (CPC) are as following :

active current $||\boldsymbol{i}_{\mathrm{a}}|| = 67.96 \text{ A}$,

scattered current $||\boldsymbol{i}_{\mathrm{s}}|| = 7.99 \text{ A}$,

reactive current $||\boldsymbol{i}_{\mathrm{r}}|| = 8.47 \text{ A}$,

and unbalanced current $||\boldsymbol{i}_{\mathrm{u}}|| = 5.38 \text{ A}$.

Similarly,

active power $P = 8.29 \text{ kW}$,

scattered power $D_{\mathrm{S}} = 0.98 \text{ kVA}$,

reactive power $Q = 1.04 \text{ kVAR}$,

and unbalanced power $D_{\mathrm{u}} = 0.66 \text{ kVA}$.

The apparent power $S = \sqrt{P^2 + D_{\mathrm{s}}{}^2 + Q^2 + D_{\mathrm{u}}{}^2} = 8.44 \text{ kVA}$

and the power factor $\lambda = \dfrac{P}{S} = 0.98$

Thus, the optimized TESLC compensator improves the power factor from 0.68 to 0.98. It is very close to 0.99, which is the value of the power factor after the addition of an ideal compensator. The TESLC compensator significantly reduces the cost and complexity of the compensator, with a very small compromise in the power factor improvement.

4.5 Conclusion

The power equation of LTI loads at nonsinusoidal and asymmetrical three-phase three-wire supply voltage was presented in this chapter. The Currents' Physical Components (CPC) based load current decomposition enabled the development of the power equation. Comparing the power equation of LTI loads at sinusoidal and asymmetrical voltages presented in Chapter 3, now the load also has a scattered power; in addition to the active, reactive and unbalanced powers. The scattered power is associated with the change of the equivalent conductance of the load with the harmonic order. The CPC concept also enabled the design of a reactive compensator for the compensation of the reactive and the unbalanced currents. Such a reactive compensator cannot compensate the scattered current, however. Therefore, when the supply voltage is N&A, and the load has a scattered power, then the power factor cannot be improved to unity using reactive compensation. Moreover, it was also demonstrated that the design, as well as the structure of such a compensator is very complex. As the number of the harmonic orders present in the supply voltage increases, such a compensator may become impractical. To solve the problem, an optimized compensator with no more than two elements per compensator branch (TESLC) was designed and implemented. Such a compensator has a significantly reduced complexity in design and structure. It cannot compensate the reactive and the unbalanced currents completely, however. Nonetheless, it was shown that such an optimized compensator works very well under practical situations and improves the power factor to close to unity. The significant reduction in cost and complexity of such a compensator outweighs the slight compromise in the power factor.

CHAPTER 5: CURRENTS PHYSICAL COMPONENTS (CPC) OF UNBALANCED HARMONICS GENERATING LOADS AT ASYMMETRICAL VOLTAGES

5.1 Introduction

Earlier chapters of this dissertation were confined to power properties of systems with Linear Time Invariant (LTI) loads. The development of the power equation and methods of reactive compensation of such loads at asymmetrical but sinusoidal voltage were presented in Chapter 3, while Chapter 4 was focused on the development of the power equation and methods of reactive compensation at nonsinusoidal and asymmetrical voltage.

A power theory describes power properties of distribution systems simplified by various assumptions. The accuracy of such a description can be increased by abandoning step by step these assumptions. The analysis in Chapter 3 was presented at the assumption that the supply voltage was sinusoidal. That assumption was abandoned in Chapter 4, where, in addition to the supply voltage asymmetry, also the supply voltage distortion was taken into account. The next step in power theory development is the description of the power properties of three-phase three-wire systems with Harmonics Generating Loads (HGLs) at asymmetrical and nonsinusoidal supply voltage, as presented in this chapter.

5.2 Background on Harmonic Generating Loads (HGLs)

Non-linear and periodically switched loads are commonly referred to as Harmonic Generating Loads (HGLs). These loads include fluorescent lamps, micro-waves, video and computer-like equipment, power electronics devices, arc furnaces etc. HGLs are increasingly common in the commercial and industrial systems, primarily because of the power electronics equipment used for the control of the energy flow in such systems. HGLs have parameters that vary with the voltage or current, and consequently cause periodic distortion of the supply current. This distortion of the load current is specified in terms of the load generated harmonics.

Transformers were initially considered to be the main sources of harmonics since they consist of non-linear magnetic cores. Proliferation of power electronics based switching loads in the last few decades causes that the contribution of transformers to the voltage and current waveform distortion nowadays is usually much lower as compared to the distortion due to the power electronics devices.

In general, HGLs can be divided into three categories [25, 26]:

- Nonlinear devices of small ratings consisting mainly of fluorescent bulbs, computer and TV supplies, and power supplies used in low power appliances
- Static power converters used in industry such as rectifiers, AC to DC converters, inverters or cycloconverters
- Electric arc furnaces.

A load is non-linear if its parameter changes with a change in the applied voltage or current, such as it is with diodes or electric arc furnaces. The current of non-linear loads is nonsinusoidal, even at sinusoidal supply voltages. The *v*-*i* relationship of a diode is shown in Fig. 5.1.

On the other hand, periodically switched devices refer to circuits consisting of switches for the control of energy flow. Examples of periodically switched devices are AC to DC converters, variable speed drives, static power converters (SPCs) etc. The current of these devices depends upon the switching of the power semiconductor devices. The current of a thyristor controlled resistive load at sinusoidal voltage is shown in Fig. 5.2.

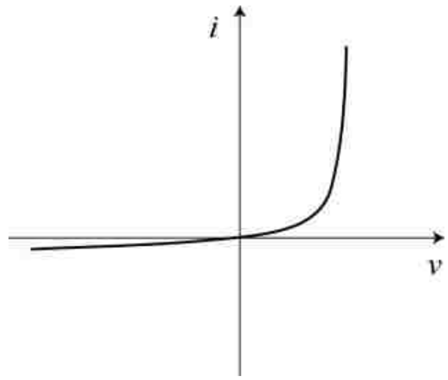


Fig. 5.1 Voltage-current relationship of a diode

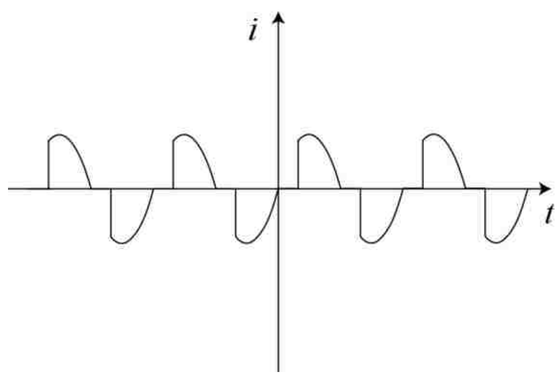


Fig. 5.2 Current of a thyristor controlled resistive load

5.3 Equivalent circuit of Harmonics Generating Loads

Let us consider a single phase fluorescent lamp as an example of a HGL as shown in Fig. 5.3.

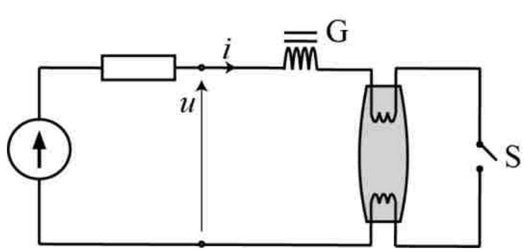


Fig. 5.3 A fluorescent lamp circuit with a thermal starter S and ballast G

A fluorescent lamp starts to conduct when the supply voltage is close to its rated operating voltage [27]. The relationship between voltage and current rms values of such a lamp is shown in Fig. 5.4.

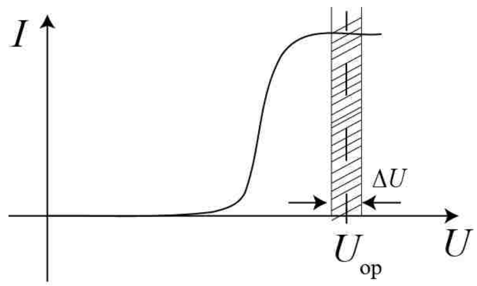


Fig. 5.4 Relationship between the voltage and current rms values of a fluorescent lamp

When the voltage rms value is in a vicinity of the operating point, the current rms value does not change with small changes ΔU in the voltage shown in Fig. 5.4. When the voltage rms value is kept constant in the vicinity of the operating point, the lamp operates like a linear load with a shunt current source of higher order harmonics. The equivalent circuit of a fluorescent lamp can be drawn as Fig. 5.5

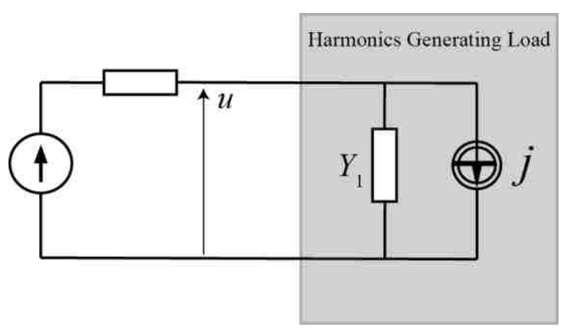


Fig. 5.5 Equivalent circuit of single-phase fluorescent lamp

The voltage and current waveforms of the fluorescent lamp operating close to its operating point looks like 5.6.

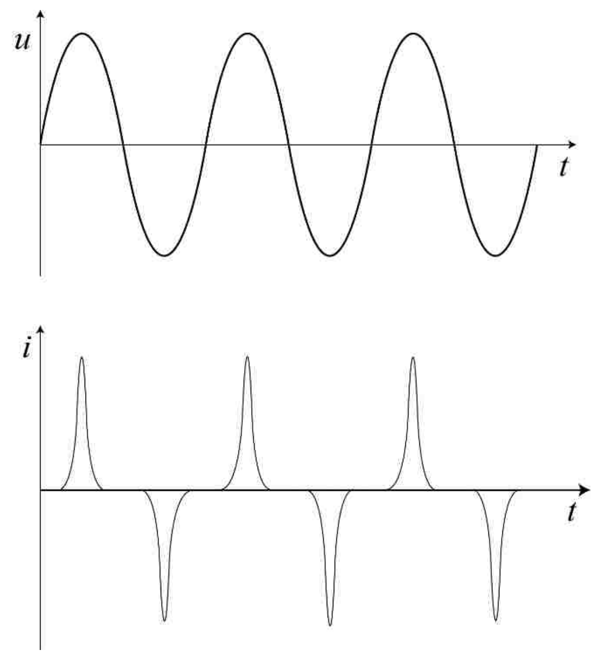


Fig. 5.6 Voltage and current waveforms of fluorescent lamp at the operating point

The current waveform in Fig. 5.6 is equivalent to the sum of the fundamental component and higher order harmonics,

$$\boldsymbol{i} = \boldsymbol{i}_1 + \sum_{n=2}^{\infty} \boldsymbol{i}_n = \boldsymbol{i}_1 + \boldsymbol{j} \ . \tag{5.1}$$

The admittance Y_1 is equal to,

$$\boldsymbol{Y}_1 = \frac{\boldsymbol{I}_1}{\boldsymbol{U}}, \tag{5.2}$$

and the load generated current harmonics are equal to

$$\boldsymbol{j} \stackrel{\text{df.}}{=} \sum_{n=2}^{\infty} \boldsymbol{i}_n \ . \tag{5.3}$$

It is necessary to remember that Fig. 5.5 is the correct representation of the fluorescent lamp shown in 5.3 only when it is operating at a voltage close to its rated operating voltage. If there is signification deviation in the supply voltage rms value, then the equivalent parameters $\boldsymbol{j}$ and Y_1

will vary. The equivalent circuit of any HGL can be drawn in similar manner using a linear approximation around a fixed operating point.

5.4 CPC based current decomposition of HGLs at nonsinusoidal and asymmetrical voltage

Let us consider a HGL supplied from a source of nonsinusoidal and asymmetrical voltage as shown in Fig. 5.7. Such a load causes periodic distortion of the supply currents and consequently introduces current harmonics into the system.

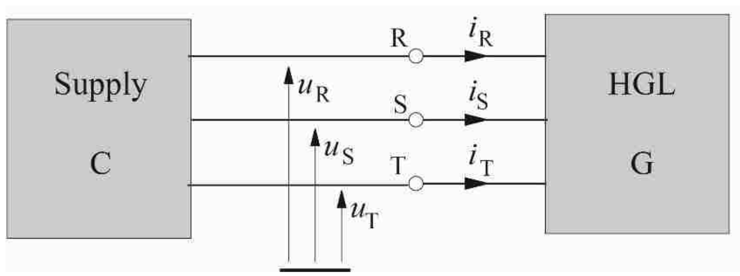


Fig. 5.7 Three-phase three-wire system with a harmonic generating load

HGLs introduce current harmonics of the order that could be not present in the supply voltage [17, 19]. As a result, the flow of energy for these harmonics originates in the load, and the load becomes the source of energy. It means that the active power for these harmonics is negative [28]. The energy of such a harmonic dissipates in the resistance of the distribution systems. Since the load is passive, this energy has to be delivered from supply, mainly by the fundamental components of the voltage and current. The equivalent linear model of a three-phase HGL is shown in Fig. 5.8.

The active power of the n^{th} order harmonic is equal to

$$P_n = \mathrm{Re}\{\boldsymbol{C}_n\} = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}n}\boldsymbol{I}^*_{\mathrm{R}n} + \boldsymbol{U}_{\mathrm{S}n}\boldsymbol{I}^*_{\mathrm{S}n} + \boldsymbol{U}_{\mathrm{T}n}\boldsymbol{I}^*_{\mathrm{T}n}\}\ . \tag{5.4}$$

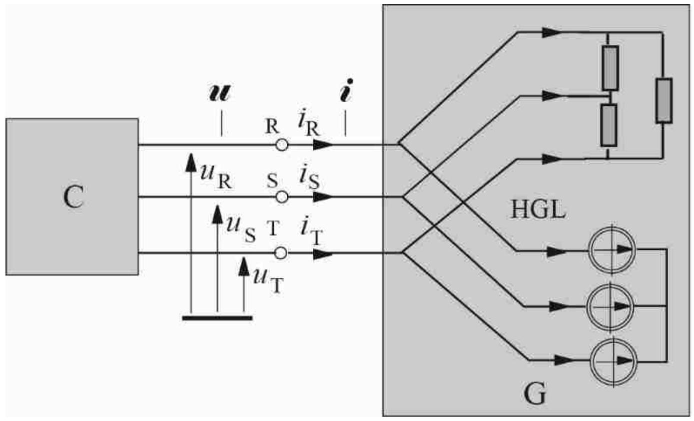


Fig. 5.8 Equivalent circuit of a three-phase system with HGL

The active power P_n can be positive or negative [28]. Depending upon the sign of P_n, the set N of all harmonic orders can be decomposed into two subsets, N_C and N_G. When the harmonic active power P_n is positive or zero, we assume that the distribution voltage harmonic $\boldsymbol{u}_n$ is the cause of the energy flow from the source to the load. In such a case, the harmonic order n belongs to the set N_C. On the other hand, if the harmonic active power P_n is negative, we assume that the current harmonic $\boldsymbol{j}_n$ generated in the load is the cause of the energy flow from the load back to the supply. In such a case, the harmonic order n belongs to the set N_G, i.e.,

$$\begin{aligned} P_n \geq 0, &\rightarrow\ n \in N_C \\ P_n < 0, &\rightarrow\ n \in N_G\,. \end{aligned} \tag{5.5}$$

After the sets N_C and N_G have been defined, we can associate the voltages, currents, and the active power components of the various harmonic orders with the direction of energy flow. All the harmonic orders belonging to the set N_C specify the voltage and current components $\boldsymbol{u}_C$ and $\boldsymbol{i}_C$, associated with the energy flow from the supply to the load, such that

$$\sum_{n \in N_C} \boldsymbol{i}_n = \boldsymbol{i}_C, \quad \sum_{n \in N_C} \boldsymbol{u}_n = \boldsymbol{u}_C, \quad \sum_{n \in N_C} P_n = P_C\,. \tag{5.6}$$

Likewise, the harmonic orders belonging to the set N_{G} specify the voltage and the current components $\boldsymbol{i}_{\mathrm{G}}$ and $\boldsymbol{u}_{\mathrm{G}}$, associated with the energy flow from the load to the supply, such that

$$\sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{G}}, \quad \sum_{n\in N_{\mathrm{G}}} \boldsymbol{u}_n = -\boldsymbol{u}_{\mathrm{G}}, \quad \sum_{n\in N_{\mathrm{G}}} P_n = -P_{\mathrm{G}}. \tag{5.7}$$

Thus, the total current, voltage, and the active power can be written as,

$$\sum_{n\in N} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{C}} + \boldsymbol{i}_{\mathrm{G}}, \quad \sum_{n\in N} \boldsymbol{u}_n = \boldsymbol{u}_{\mathrm{C}} - \boldsymbol{u}_{\mathrm{G}}, \quad \sum_{n\in N} P_n = P_{\mathrm{C}} - P_{\mathrm{G}}. \tag{5.8}$$

The current components $\boldsymbol{i}_{\mathrm{C}}$ and $\boldsymbol{i}_{\mathrm{G}}$ do not contain the harmonics of the same order and are therefore mutually orthogonal. The voltage components $\boldsymbol{u}_{\mathrm{C}}$ and $\boldsymbol{u}_{\mathrm{G}}$ are also orthogonal for the same reason. Thus, the scalar products of the currents $\boldsymbol{i}_{\mathrm{C}}$ and $\boldsymbol{i}_{\mathrm{G}}$, and the voltages $\boldsymbol{u}_{\mathrm{C}}$ and $\boldsymbol{u}_{\mathrm{G}}$ are equal to zero, viz,

$$(\boldsymbol{i}_{\mathrm{C}}, \boldsymbol{i}_{\mathrm{G}}) = 0 \ , \qquad\qquad (\boldsymbol{u}_{\mathrm{C}}, \boldsymbol{u}_{\mathrm{G}}) = 0 \ . \tag{5.9}$$

Hence, the three-phase rms value of the load current

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{C}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2, \tag{5.10}$$

and the three-phase rms value of the voltage

$$||\boldsymbol{u}||^2 = ||\boldsymbol{u}_{\mathrm{C}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2 \ . \tag{5.11}$$

For the harmonic orders n belonging to the set N_{C}, the original system can be regarded as a system with a passive load. Consequently, for such harmonics, the system is equivalent to a linear load supplied from a source of nonsinusoidal and asymmetrical voltage. The analysis for these harmonic orders remains the same as presented in Chapter 4. The equivalent circuit of the system for these harmonics is as shown in Fig. 5.9.

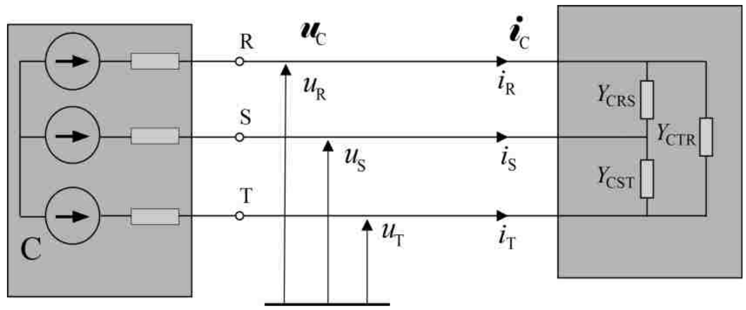


Fig. 5.9 Equivalent circuit of the system for harmonics of the order n from subset N_C

On the other hand, for the harmonic orders n belonging to the set N_G, the same system can be regarded as a system with current sources $\boldsymbol{j}_G$ on the load side and a passive distribution system on the supply side, as shown in Fig. 5.10. Also for these harmonic orders, the voltage response of the distribution system to the load generated harmonics $\boldsymbol{u}_G$, has an opposite direction as compared to the direction of the distribution voltage $\boldsymbol{u}_C$.

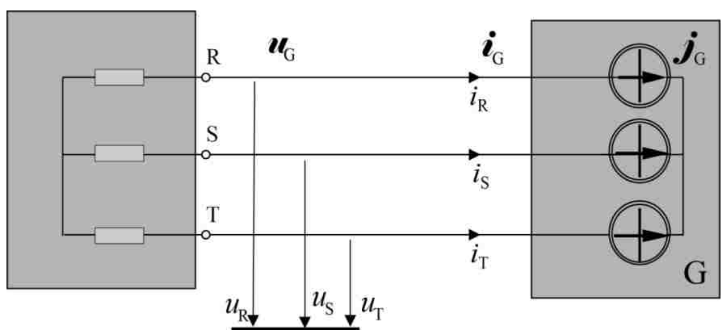


Fig. 5.10 Equivalent circuit of the system for harmonics of the order n from subset N_G

The system comprising of the harmonic orders n belonging to N_C and shown in Fig. 5.10 has the voltage $\boldsymbol{u}_C$, the current $\boldsymbol{i}_C$, and an LTI load with the active power P_C. Such a load is equivalent to a balanced resistive load as shown in Fig. 5.11.

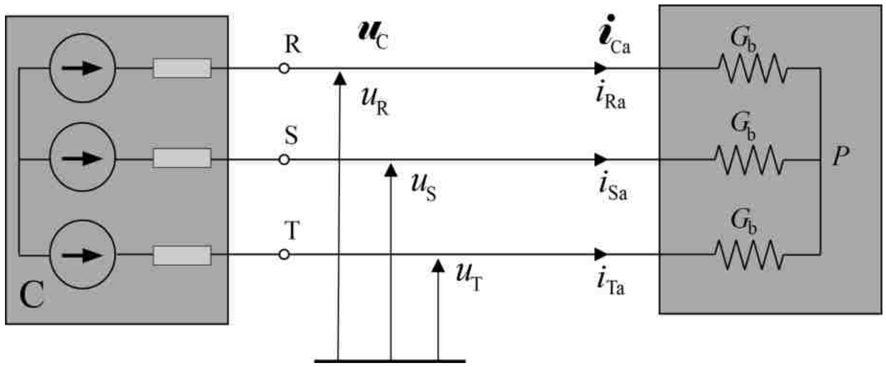


Fig. 5.11 Balanced resistive load equivalent to the original load for subset N_c with respect to active power P

The phase conductance of such a balanced resistive load is

$$G_{\mathrm{Cb}} = \frac{P_{\mathrm{C}}}{||\boldsymbol{u}_{\mathrm{C}}||^2} \tag{5.12}$$

and it draws the current

$$\boldsymbol{i}_{\mathrm{Ca}} = G_{\mathrm{Cb}}\,\boldsymbol{u}_{\mathrm{C}} = \sqrt{2}\ \mathrm{Re}\sum_{n\in \boldsymbol{N}_{\mathrm{C}}} G_{\mathrm{Cb}}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})e^{jn\omega_1 t} \tag{5.13}$$

referred to as the active current, from the source.

Let $P_{\mathrm{C}n}$ and $Q_{\mathrm{C}n}$ be the active and the reactive powers of the n^{th} harmonic order of the subset $n \in N_{\mathrm{C}}$. We can imagine a balanced load for the n^{th} harmonic order which is equivalent to the original load with respect to $P_{\mathrm{C}n}$ and $Q_{\mathrm{C}n}$. It has the equivalent balanced admittance

$$\boldsymbol{Y}_{\mathrm{Cb}n} = G_{\mathrm{Cb}n} + jB_{\mathrm{Cb}n} = \frac{P_{\mathrm{C}n} - jQ_{\mathrm{C}n}}{||\boldsymbol{u}_n||^2} = \frac{\boldsymbol{C}_{\mathrm{C}n}^{*}}{||\boldsymbol{u}_n||^2} \tag{5.14}$$

and draws the active current

$$\boldsymbol{i}_{\mathrm{Ca}n}(t) = G_{\mathrm{Cb}n}\,\boldsymbol{u}_n(t) = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{Cb}n}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})e^{jn\omega_1 t}\} \tag{5.15}$$

and the reactive current,

$$\boldsymbol{i}_{\mathrm{Cr}n}(t) = B_{\mathrm{Cb}n}\,\boldsymbol{u}_n(t+\frac{T}{4n}) = \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{Cb}n}\,(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}_n^{\mathrm{n}})\,e^{jn\omega t}\} \tag{5.16}$$

and is shown in Fig. 5.12.

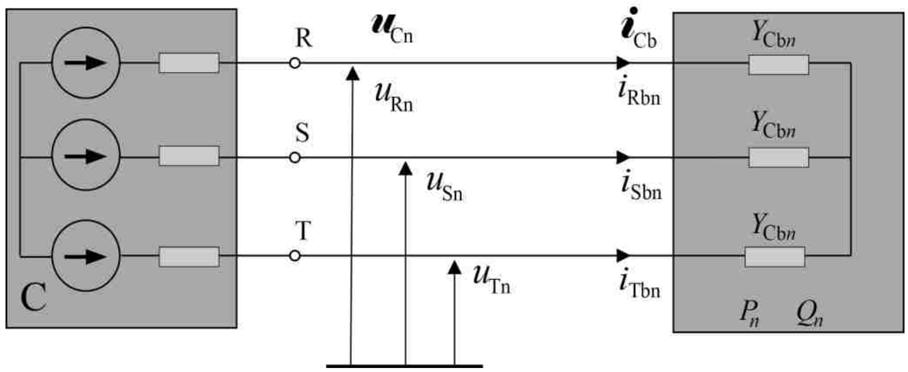


Fig. 5.12 Equivalent balanced load for the n^{th} order harmonic of the order n from the subset N_{C} with respect to the active power $P_{\mathrm{C}n}$ and reactive power $Q_{\mathrm{C}n}$

In addition to the active and reactive currents of the equivalent balanced load, the load for the n^{th} order harmonic also draws the unbalanced current, namely,

$$\boldsymbol{i}_{\mathrm{Cu}n} = \boldsymbol{i}_{\mathrm{C}n} - \boldsymbol{i}_{\mathrm{Cb}n} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}_{\mathrm{C}n} - Y_{\mathrm{Cb}n}\,(\mathbf{1}^{\mathrm{p}}\,\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\,\boldsymbol{U}_n^{\mathrm{n}})\,e^{jn\omega_1 t}\}\ . \tag{5.17}$$

The total current for the n^{th} order harmonic belonging to the subset N_{C} is equal to

$$\boldsymbol{i}_{\mathrm{C}n} = \boldsymbol{i}_{\mathrm{Ca}n} + \boldsymbol{i}_{\mathrm{Cr}n} + \boldsymbol{i}_{\mathrm{Cu}n}\ . \tag{5.18}$$

Therefore, the total current of all the harmonic orders of the subset $n \in N_{\mathrm{C}}$ is equal to

$$\begin{aligned} \boldsymbol{i}_{\mathrm{C}} &= \sum_{n\in N_{\mathrm{C}}} \boldsymbol{i}_{\mathrm{C}n} = \sum_{n\in N_{\mathrm{C}}} (\boldsymbol{i}_{\mathrm{Ca}n} + \boldsymbol{i}_{\mathrm{Cr}n} + \boldsymbol{i}_{\mathrm{Cu}n}) \\ &= \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} \end{aligned} \tag{5.19}$$

where,

$$\boldsymbol{i}_{\mathrm{Ca}} = G_{\mathrm{Cb}}\,\boldsymbol{u}_{\mathrm{C}} \tag{5.20}$$

is the active current of the load,

$$\boldsymbol{i}_{\mathrm{Cs}} = \sum_{n\in N_{\mathrm{C}}} \boldsymbol{i}_{\mathrm{Ca}\,n} - \boldsymbol{i}_{\mathrm{Ca}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_c} (G_{\mathrm{Cb}\,n} - G_{\mathrm{Cb}})(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})\,e^{jn\omega_1 t} \tag{5.21}$$

is the scattered current,

$$\sum_{n\in N_c} \boldsymbol{i}_{\mathrm{Cr}\,n} = \boldsymbol{i}_{\mathrm{Cr}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N_c} jB_{\mathrm{Cb}\,n}\,(\mathbf{1}^{\mathrm{p}}\boldsymbol{U}_n^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}\boldsymbol{U}_n^{\mathrm{n}})\,e^{jn\omega_1 t} \tag{5.22}$$

is the reactive current, and,

$$\sum_{n\in N_c} \boldsymbol{i}_{\mathrm{Cu}\,n} = \boldsymbol{i}_{\mathrm{Cu}} \tag{5.23}$$

is the unbalanced current.

On the other hand, as described by (5.7) above, the current of the load corresponding to the harmonic orders n belonging to the subset $n \in N_{\mathrm{G}}$, and shown in Fig. 5.10, is equal to

$$\sum_{n\in N_{\mathrm{G}}} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{G}} \tag{5.24}$$

Therefore, the total current of the load is equal to

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{C}} + \boldsymbol{i}_{\mathrm{G}}, \tag{5.25}$$

Using the relations obtained in (5.19) above, the total current of the load can be written as

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{Ca}} + \boldsymbol{i}_{\mathrm{Cs}} + \boldsymbol{i}_{\mathrm{Cr}} + \boldsymbol{i}_{\mathrm{Cu}} + \boldsymbol{i}_{\mathrm{G}}. \tag{5.26}$$

Based on the analysis of LTI loads presented in Chapter 4, we can write that the currents $\boldsymbol{i}_{\mathrm{Ca}}, \boldsymbol{i}_{\mathrm{Cs}}, \boldsymbol{i}_{\mathrm{Cr}}$ and $\boldsymbol{i}_{\mathrm{Cu}}$ are mutually orthogonal. Moreover, the currents $\boldsymbol{i}_{\mathrm{C}}$ and $\boldsymbol{i}_{\mathrm{G}}$ are comprised of different harmonic orders, and therefore they are mutually orthogonal. Hence, the five current components of the HGL are mutually orthogonal. Thus, the square of the three-phase rms value of the current of the HGL is equal to

$$||\boldsymbol{i}||^2 = ||\boldsymbol{i}_{\mathrm{Ca}}||^2 + ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2 \,. \tag{5.27}$$

The current of a HGL supplied from a source of nonsinusoidal and asymmetrical voltage is composed of five mutually orthogonal components, each associated with a distinct physical phenomenon. These are the Currents Physical Components of a Harmonic Generating Load.

The active, reactive, unbalanced and scattered currents are similar to and associated with the same phenomenon as those in systems with Linear Time Invariant (LTI) loads. It is interesting to be noted that only a portion of the supply voltage $\boldsymbol{u}$, namely the voltage $\boldsymbol{u}_{\mathrm{C}}$ affects these currents, however. The remaining portion of the supply voltage, namely the voltage $\boldsymbol{u}_{\mathrm{G}}$, occurring as a result of the response of the distribution system to the load generated harmonics $\boldsymbol{j}_{\mathrm{G}}$, does not affect these currents.

The generation of the current harmonics in the load and the consequent presence of the negative active power P_n in systems with HGLs is a new physical phenomena as compared to the systems with LTI Loads. Owing to this phenomenon, the energy flows from the load to the source at certain frequencies. Since the energy has to come from the distribution system, this further increases the three-phase rms value of the supply current, thereby leading to the degradation of the power factor.

The apparent power of the load is equal to

$$\begin{aligned} S &= ||\boldsymbol{u}||\,||\boldsymbol{i}|| \\ &= \sqrt{||\boldsymbol{u}_{\mathrm{C}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2}\sqrt{||\boldsymbol{i}_{\mathrm{Ca}}||^2 + ||\boldsymbol{i}_{\mathrm{Cs}}||^2 + ||\boldsymbol{i}_{\mathrm{Cr}}||^2 + ||\boldsymbol{i}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}_{\mathrm{G}}||^2} \\ &= \sqrt{S_{\mathrm{C}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2} \end{aligned} \tag{5.28}$$

where,

$$S_{\mathrm{C}} = ||\boldsymbol{u}_{\mathrm{C}}||\ ||\boldsymbol{i}_{\mathrm{C}}|| = \sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2}\ , \tag{5.29}$$

$$S_{\mathrm{G}} = |||\boldsymbol{u}_{\mathrm{G}}||\ ||\boldsymbol{i}_{\mathrm{G}}||\ , \tag{5.30}$$

$$S_{\mathrm{E}} = \sqrt{||\boldsymbol{u}_{\mathrm{C}}||^2\ ||\boldsymbol{i}_{\mathrm{G}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2\ ||\boldsymbol{i}_{\mathrm{C}}||^2}\ . \tag{5.31}$$

Comparing the apparent power of LTI loads with that of the HGLs, it is evident that in case of HGLs, the apparent power contains additional components which contribute to its increase. The apparent power S of the HGLs contains the components S_{C}, S_{G} and S_{E}. The component S_{C} is the same as the apparent power in systems with LTI loads, while the components S_{G} and S_{E} are not present in systems with LTI loads. The power S_{G} depends on the load generated harmonics $\boldsymbol{j}_{\mathrm{G}}$ as well as the distribution system impedance. It is similar to the apparent power S_{C} in terms of the physical phenomenon that it is associated with, except that it is originates in the load. It is therefore referred to as ***load generated apparent power***.

The power component S_{E} differs fundamentally from the apparent powers S_{G} and S_{C} in the regard that it is not associated with any physical phenomenon. Its square is merely the product of the rms values of the voltages and currents of exclusively different harmonic orders from the subsets N_{C} and N_{G}. It only contributes to the increase of the voltage and current rms values and is referred to as ***cross apparent power***.

The apparent power of an unbalanced HGL specified in (5.28) can be rewritten using the various power components as

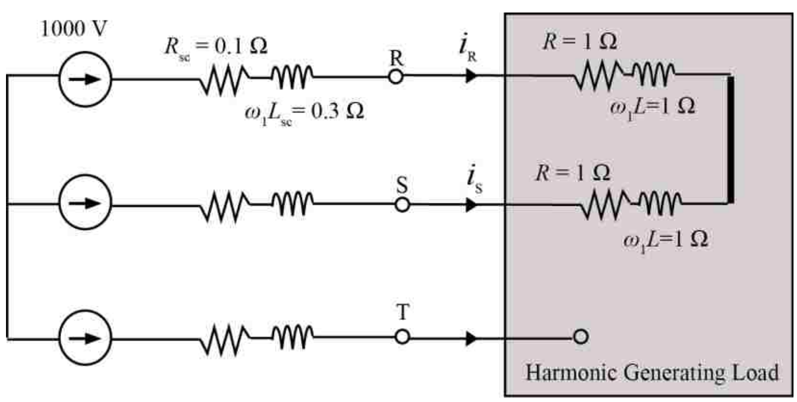
$$S = \sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2} \tag{5.32}$$

and the power factor, which specifies the effectiveness of energy delivery from the source to the load, of such a load is equal to

$$\lambda = \frac{P}{S} = \frac{P_{\mathrm{C}} - P_{\mathrm{G}}}{\sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2}} \quad . \tag{5.33}$$

Equation (5.33) reveals how the different components of the power contribute to the degradation of the power factor in systems with HGLs. It is also important to observe that the load generated apparent power not only increases the total apparent power of the load, but also reduces the active power of the load, evident in the numerator of (5.33).

Illustration 5.1 Let us calculate the CPC currents and the various powers of an industrial arc furnace load approximated by a harmonic generating load. It is assumed that the furnace has an extinguished arc in phase T and that the internal voltage of the supply is sinusoidal and symmetrical. The furnace is supplied from a transformer with relatively low power. The short circuit parameters of the transformer calculated to the secondary side and the line currents are given. The voltage asymmetry and distortion at the furnace terminal is caused due to the asymmetry and distortion in the furnace current.



$$i_{\mathrm{R}} = \sqrt{2}\,\mathrm{Re}\{508.55\, e^{-j19.76^\circ} e^{j\omega_1 t} + 100\, e^{j2\omega_1 t} + 100\, e^{j5\omega_1 t} + 100\, e^{j7\omega_1 t}\}\ \mathrm{A}$$

Fig. 5.13 A Harmonic Generating Load supplied from a source of sinusoidal symmetrical voltage

The internal voltage of the distribution system is sinusoidal and symmetrical and is equal to

$$\boldsymbol{E}_{\mathrm{R1}} = 1000 \text{ V}, \qquad \boldsymbol{E}_{\mathrm{S1}} = 1000 e^{-j120°} \text{ V}, \qquad \boldsymbol{E}_{\mathrm{T1}} = 1000 e^{j120°} \text{ V}.$$

The source impedance consists of the impedances

$$R_{\mathrm{sc}} = 0.1\Omega, \qquad \omega_1 L_{\mathrm{sc}} = 0.3\ \Omega.$$

The current on line R is equal to

$$i_{\mathrm{R}} = \sqrt{2}\mathrm{Re}\{508.5\ e^{-j19.76°}\ e^{j\omega_1 t} + 100\ e^{j2\omega_1 t} + 100\ e^{j5\omega_1 t} + 100 e^{j7\omega_1 t}\} \text{ A}.$$

The current contains the components of the fundamental, 2nd, 5th and 7th order harmonics. Since the supply voltage is sinusoidal, the higher order harmonics are due to the nonlinearity of the load. Therefore, for this particular illustration the different harmonics can be categorized into the following subsets:

$$n \in N_{\mathrm{C}} \quad \text{with } N_{\mathrm{C}} = \{1\}$$
$$n \in N_{\mathrm{G}} \quad \text{with } N_{\mathrm{G}} = \{2,5,7\}.$$

For the subset $n \in N_{\mathrm{C}}$, the system is equivalent to one depicted in Fig. 5.14.

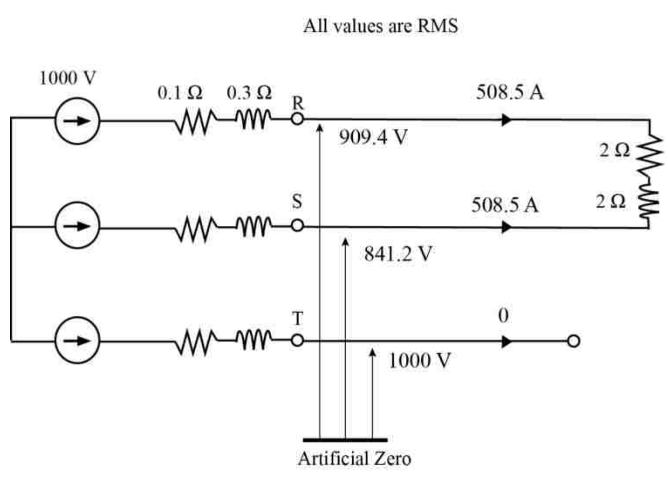


Fig. 5.14 The equivalent circuit of the system for n = 1

The load line to line admittances for the fundamental frequency are equal to

$$\boldsymbol{Y}_{\mathrm{RS}1} = 0.25 - j0.25\ \mathrm{S}\,, \quad \boldsymbol{Y}_{\mathrm{ST}1} = 0\,, \quad \boldsymbol{Y}_{\mathrm{TR}1} = 0\,,$$

while and the line currents rms values are equal to

$$||i_{\mathrm{R}1}|| = 508.6\ \mathrm{A}, \qquad ||i_{\mathrm{S}1}|| = 508.6\ \mathrm{A}, \qquad ||i_{\mathrm{T}1}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_1|| = 719.2\ \mathrm{A}$.

These currents cause a voltage drop in the source impedances. The voltages across the load terminals after referring to artificial zero are equal to

$$\boldsymbol{U}_{\mathrm{R}1} = 909.4e^{-j7.98°}\ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S}1} = 841.1e^{-j118.44°}\ \mathrm{V}, \quad \boldsymbol{U}_{\mathrm{T}1} = 1000e^{j120°}\ \mathrm{V}\ .$$

The voltage three-phase rms value is equal to $||\boldsymbol{u}_1|| = 1592\ \mathrm{V}$.

Next, we need to consider the system for the harmonic orders of the subset $n \in N_{\mathrm{G}}$. For the approximation of the arc furnace used in this illustration, the Harmonic Generating Load injects currents of the 2nd, 5th and the 7th harmonic order. Let us analyze the system for each of these harmonics one by one.

The system corresponding to the 2nd harmonic order is equivalent to the one shown in Fig. 5.15.

The line current crms values for the 2nd order harmonics are as follows,

$$\boldsymbol{I}_{\mathrm{R}2} = 100\ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S}2} = 100\ e^{j180°}\ \mathrm{A}$$

and the current three-phase rms value $||\boldsymbol{i}_2|| = 141.4\ \mathrm{A}$.

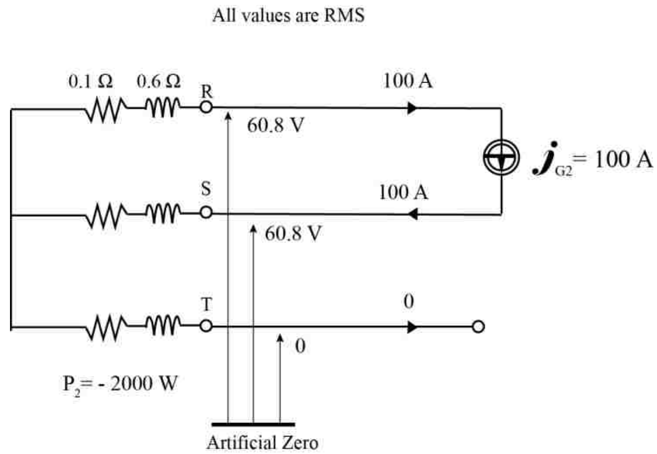


Fig. 5.15 The equivalent circuit of the system for the 2nd order harmonic

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{\mathrm{R2}} = 60.8\mathrm{e}^{-j99.5^\circ}\ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S2}} = 60.8\ \mathrm{e}^{j80.54^\circ}\ \mathrm{V}$$

with the three-phase rms value $||\boldsymbol{u}_2|| = 86.0$ V

The active power corresponding to the 2nd harmonic order is equal to

$$P_2 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R2}}\,\boldsymbol{I}^*_{\mathrm{R2}} + \boldsymbol{U}_{\mathrm{S2}}\boldsymbol{I}^*_{\mathrm{S2}}\} = -2000\ \mathrm{W}\,.$$

Likewise, the system corresponding to the 5th harmonic order is equivalent to Fig. 5.16.

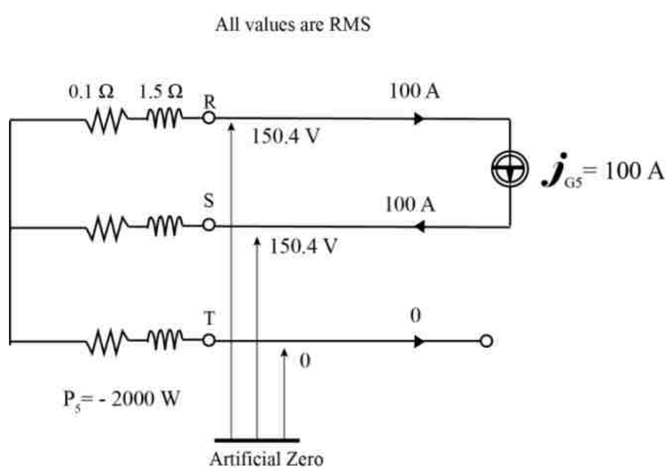


Fig. 5.16 The equivalent circuit of the system for the 5th order harmonic

Therefore, the crms values of the line currents for the 5th harmonic order are,

$$\boldsymbol{I}_{R5} = 100 \text{ A}, \qquad \boldsymbol{I}_{S5} = 100\, e^{j180°} \text{ A}$$

and the current three-phase rms value $||\boldsymbol{i}_5|| = 141.42$ A

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{R5} = 150.4\, \mathrm{e}^{-j93.8°} \text{ V}, \qquad \boldsymbol{U}_{S5} = 150.4\, \mathrm{e}^{j86.2°} \text{ V}$$

with the three-phase rms value $||\boldsymbol{u}_5|| = 212.6$ V

The active power corresponding to the 5th harmonic order is equal to

$$P_5 = \mathrm{Re}\{\boldsymbol{U}_{R5}\, \boldsymbol{I}^*_{R5} + \boldsymbol{U}_{S5}\, \boldsymbol{I}^*_{S5}\} = -2000 \text{ W}.$$

Finally, the system corresponding to the 7th harmonic order is as shown in Fig. 5.17.

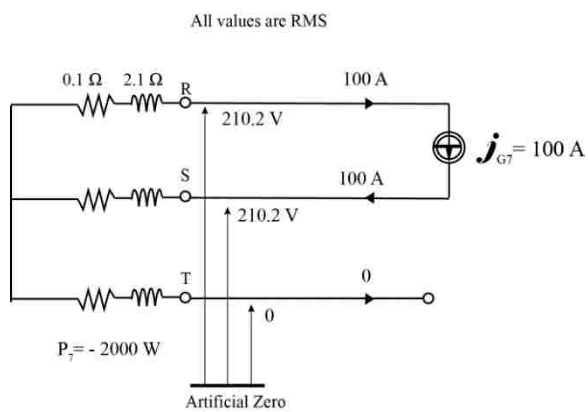


Fig. 5.17 The equivalent circuit of the system for the 7th harmonic order

The crms values of the line currents for the 7th harmonic order are:

$$\boldsymbol{I}_{R7} = 100 \text{ A}, \qquad \boldsymbol{I}_{S7} = 100\, e^{j180°} \text{ A}$$

and the current three-phase rms value $||\boldsymbol{i}_7|| = 141.42$ A

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{R7} = 210.2\mathrm{e}^{-j92.73°} \text{ V}, \qquad \boldsymbol{U}_{S7} = 210.2\mathrm{e}^{j87.3°} \text{ V}$$

with the three-phase rms value $||\boldsymbol{u}_7|| = 297.3$ V

The active power corresponding to the 7th harmonic order is equal to

$$P_7 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}7}\boldsymbol{I}^*_{\mathrm{R}7} + \boldsymbol{U}_{\mathrm{S}7}\boldsymbol{I}^*_{\mathrm{R}7}\} = -2000 \text{ W}.$$

Since the voltages and the currents of different harmonic orders are orthogonal to one another, the total three-phase voltage rms value is equal to

$$||\boldsymbol{u}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{u}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{u}_1||^2 + ||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_5||^2 + ||\boldsymbol{u}_7||^2} = 1635.6 \text{ V}$$

while the current three-phase rms value is equal to

$$||\boldsymbol{i}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{i}_1||^2 + ||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_5||^2 + ||\boldsymbol{i}_7||^2} = 759.8 \text{A}$$

Thus, the apparent power of the load is equal to

$$S = ||\boldsymbol{u}||\,||\boldsymbol{i}|| = 1242.7 \text{ kVA} = 1.243 \text{ MVA}.$$

Apparent power calculated using the CPC power components

The rms values of the active and the reactive current components for the subset $n \in N_{\mathrm{C}}$ are equal to

$$||\boldsymbol{i}_{\mathrm{Ca}}|| = 324.9 \text{ A}, \qquad ||\boldsymbol{i}_{\mathrm{Cr}}|| = 324.9 \text{ A}.$$

The positive and the negative sequence components of the unbalanced current are

$$||\boldsymbol{i}^{\mathrm{p}}_{\mathrm{Cu}}|| = 32.3 \text{ A}, \qquad ||\boldsymbol{i}^{\mathrm{n}}_{\mathrm{Cu}}|| = 317.8 \text{A}$$

Hence, the three-phase rms value of the unbalanced current is equal to,

$$||\boldsymbol{i}_{\mathrm{Cu}}|| = \sqrt{3}\times\sqrt{||\boldsymbol{i}^{\mathrm{p}}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}^{\mathrm{n}}_{\mathrm{Cu}}||^2} = \sqrt{3}\times\sqrt{32.3^2 + 317.8^2} = 553.4 \text{ A}.$$

The different powers that these currents are associated with are equal to

$$P_{\mathrm{C}} = 517.2 \text{ kW}, \qquad Q_{\mathrm{C}} = 517.2 \text{ kVars}, \qquad D_{\mathrm{Cu}} = 880.8 \text{ kVA}.$$

The apparent power for the subset $n \in N_{\mathrm{C}}$ is equal to

$$S_{\mathrm{C}} = \sqrt{P_{\mathrm{C}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{uC}}^2} = 1145 \text{ kVA}$$

The voltage three-phase rms value for the subset $n \in N_{\mathrm{G}}$ is equal to

$$||\boldsymbol{u}_{\mathrm{G}}|| = \sqrt{\sum_{n \in N_{\mathrm{G}}} ||\boldsymbol{u}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_5||^2 + ||\boldsymbol{u}_7||^2} = 375.5 \text{ V}$$

while the three-phase rms value of the current harmonic $\boldsymbol{j}_{\mathrm{G}}$ is equal to

$$||\boldsymbol{i}_{\mathrm{G}}|| = \sqrt{\sum_{n \in N_{\mathrm{G}}} ||\boldsymbol{i}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_5||^2 + ||\boldsymbol{i}_7||^2} = 244.9 \text{ A}$$

Thus, the apparent power corresponding to all the harmonics belonging to the subset $n \in N_{\mathrm{G}}$ is

$$S_{\mathrm{G}} = ||\boldsymbol{u}_{\mathrm{G}}|| \; ||\boldsymbol{i}_{\mathrm{G}}|| = 92 \text{ kVA}.$$

Likewise, the cross harmonic apparent power

$$S_{\mathrm{E}} = \sqrt{||\boldsymbol{u}_{\mathrm{C}}||^2 \; ||\boldsymbol{i}_{\mathrm{G}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2 \; ||\boldsymbol{i}_{\mathrm{C}}||^2} = 474.4 \text{ kVA}.$$

Thus, the total apparent power calculated using the different components of the apparent power is equal to,

$$S = \sqrt{S_{\mathrm{C}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2} = 1242.7 \text{ kVA} \;,$$

which is the same as value of the apparent power calculated earlier using the three-phase rms values of the line currents and voltages, calculated earlier in the illustration. This verifies that the CPC decomposition of the load currents and the powers calculated using these currents are correct.

The total active power of the load is equal to:

$$P = \sum_{n\in N_{\rm C}} P_{\rm n} + \sum_{n\in N_{\rm G}} P_{\rm n} = (517242 - 2000 - 2000 - 2000)\ \mathrm{W} = 511.2\ \mathrm{kW}$$

while the overall power factor of the load is equal to,

$$\lambda = \frac{P}{S} = \frac{P_{\rm C} - P_{\rm G}}{\sqrt{P_{\rm C}^2 + D_{\rm Cs}^2 + Q_{\rm C}^2 + D_{\rm Cu}^2 + S_{\rm G}^2 + S_{\rm E}^2}} = \frac{511.2}{1242.7} = 0.41$$

Despite the fact that the internal voltage of the distribution system is sinusoidal and symmetrical, the voltage across the load terminal is highly asymmetrical and distorted, owing the load generated harmonics as well as the load asymmetry. The vectors of the line currents and the vector of the line voltages are given below, followed by the voltage and current waveforms in Fig. 5.18.

The vector of the line currents is equal to

$$\boldsymbol{i} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 508.6\mathrm{e}^{-j19.76°}\mathrm{e}^{\,j\omega_1 \mathrm{t}} + 100\ \mathrm{e}^{\,j2\omega_1 \mathrm{t}} + 100\ \mathrm{e}^{\,j5\omega_1 \mathrm{t}} + 100\ \mathrm{e}^{\,j7\omega_1 \mathrm{t}} \\ 508.6\mathrm{e}^{j160.24°}\mathrm{e}^{\,j\omega_1 \mathrm{t}} + 100\ \mathrm{e}^{\,j2\omega_1 \mathrm{t}}\ \mathrm{e}^{j180°} + 100\ \mathrm{e}^{\,j5\omega_1 \mathrm{t}}\ \mathrm{e}^{j180°} + 100\ \mathrm{e}^{\,j7\omega_1 \mathrm{t}}\ \mathrm{e}^{j180°} \\ 0 \end{bmatrix}\}\mathrm{A}$$

Similarly, the vector of the load voltages is equal to

$$\boldsymbol{u} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 909.4e^{-j7.98°}\ \mathrm{e}^{\,j\omega_1 \mathrm{t}} + 60.8\ \mathrm{e}^{-j99.5°}\mathrm{e}^{\,j2\omega_1 \mathrm{t}} + 150.4\mathrm{e}^{-j93.8°}\mathrm{e}^{\,j5\omega_1 \mathrm{t}} + 210.3\ \mathrm{e}^{-j92.73°}\mathrm{e}^{\,j7\omega_1 \mathrm{t}} \\ 841.2e^{-j118.44°}\ \mathrm{e}^{\,j\omega_1 \mathrm{t}} + 60.8\ \mathrm{e}^{j80.54°}\ \mathrm{e}^{\,j2\omega_1 \mathrm{t}} + 150.4\ \mathrm{e}^{j86.2°}\mathrm{e}^{\,j5\omega_1 \mathrm{t}} + 210.3\ \mathrm{e}^{j87.3°}\mathrm{e}^{\,j7\omega_1 \mathrm{t}} \\ 1000e^{j120°}\ \mathrm{e}^{\,j\omega_1 \mathrm{t}} \end{bmatrix}\}\mathrm{V}$$

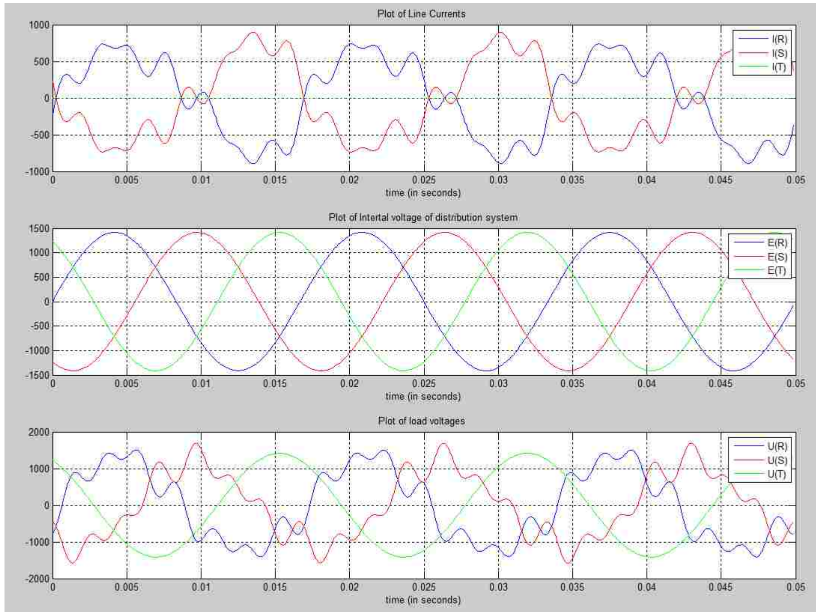
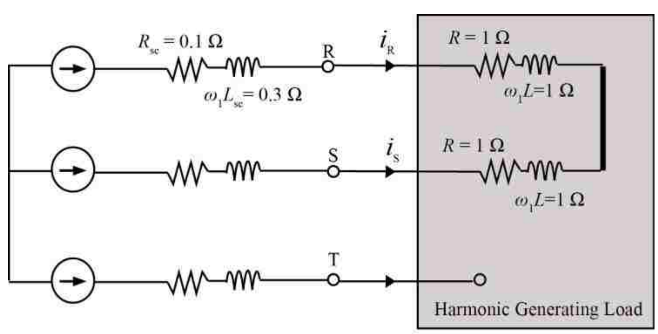


Fig. 5.18 Plot of the line currents, the distribution voltage and the load voltages

Illustration 5.2 Let us calculate the CPC currents and the various powers of the HGL in the previous illustration when the supply voltage is nonsinusoidal as shown in Fig. 5.19.



$$e_R = \sqrt{2}\,\mathrm{Re}\{100\, e^{j\omega_1 t} + 100\, e^{j3\omega_1 t} + 100\, e^{j5\omega_1 t}\}\ \mathrm{V}$$

$$j_{RG} = \sqrt{2}\,\mathrm{Re}\{100\, e^{j2\omega_1 t} + 100\, e^{j4\omega_1 t} + 100\, e^{j7\omega_1 t}\}\ \mathrm{V}$$

Fig. 5.19 A Harmonic Generating Load supplied from a source of nonsinusoidal symmetrical voltage

The supply voltage is equal to

$$\boldsymbol{e} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 1000\, e^{\,j\omega_1 \mathrm{t}} + 100\, e^{\,j3\omega_1 \mathrm{t}} + 100\, e^{\,j5\omega_1 \mathrm{t}} \\ 1000\, e^{-j120^\circ} e^{\,j\omega_1 \mathrm{t}} + 100\, e^{\,j3\omega_1 \mathrm{t}} + 100\, e^{\,j120^\circ} e^{\,j5\omega_1 \mathrm{t}} \\ 1000\, e^{\,j120^\circ} e^{\,j\omega_1 \mathrm{t}} + 100\, e^{\,j3\omega_1 \mathrm{t}} + 100\, e^{-j120^\circ} e^{\,j5\omega_1 \mathrm{t}} \end{bmatrix}\}\mathrm{V}$$

while the load generated harmonics in the current on line R are equal to

$$j_{RG} = \sqrt{2}\mathrm{Re}\{100\, e^{j2\omega_1 t} + 100\, e^{j4\omega_1 t} + 100 e^{j7\omega_1 t}\}\ \mathrm{A}\,.$$

In a similar manner as the previous illustration, the voltage and the current contains the harmonics of two subsets, namely,

$$n \in N_C \quad \text{with } N_C = \{1,3,5\}$$
$$n \in N_G \quad \text{with } N_G = \{2,4,7\}$$

For the harmonic orders n belonging to the subset N_C, the source of energy flow is the distribution system, while, for the harmonic orders n belonging to the subset N_G, the source of the energy flow is the load. Thus, the system has to be analyzed differently for these two subsets.

Analysis for the subset $n \in N_C$:

For the fundamental component, the system is equivalent to the one depicted in Fig. 5.20

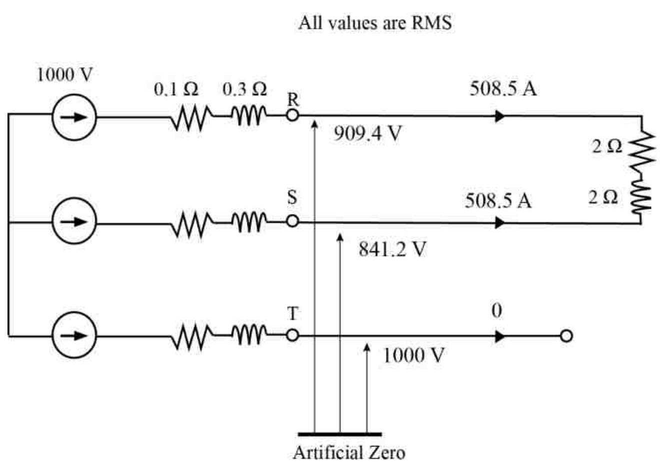


Fig. 5.20 Equivalent circuit of the system for to n = 1

At the fundamental frequency, the sources impedance

$$R_{sc} = 0.1\,\Omega, \qquad \omega_1 L_{sc} = 0.3\,\Omega$$

while the load impedance is

$$R_{\rm RS} = 2\Omega, \qquad \omega_1 L_{\rm RS} = 2\ \Omega\,.$$

Thus, the load line to line admittances for the fundamental frequency are

$$\boldsymbol{Y}_{\rm RS1} = 0.25 - j0.25\ \text{S}, \quad \boldsymbol{Y}_{\rm ST1} = 0, \quad \boldsymbol{Y}_{\rm TR1} = 0\,.$$

The line currents rms values are equal to

$$||i_{\rm R1}|| = 508.6\ \text{A}, \qquad ||i_{\rm S1}|| = 508.6\ \text{A}, \qquad ||i_{\rm T1}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_1|| = 719.4\ \text{A}$.

These currents cause a voltage drop in the source impedances, and the line voltages after the drop in the source impedance and after referencing to an artificial zero are equal to

$$\boldsymbol{U}_{\rm R1} = 909.4e^{-j7.98^\circ}\ \text{V}, \quad \boldsymbol{U}_{\rm S1} = 841.1e^{-j118.44^\circ}\ \text{V}, \quad \boldsymbol{U}_{\rm T1} = 1000e^{j120^\circ}\ \text{V}.$$

The voltage three-phase rms value is equal to $||\boldsymbol{u}_1|| = 1592\ \text{V}$.

The active power of the fundamental harmonic is equal to

$$P_1 = \text{Re}\{\boldsymbol{U}_{\rm R1}\boldsymbol{I}^*_{\rm R1} + \boldsymbol{U}_{\rm S1}\boldsymbol{I}^*_{\rm S1}\} = 517.6\ \text{kW}\,.$$

For the 3rd harmonic order, the system is equivalent to the one shown in Fig. 5.21.

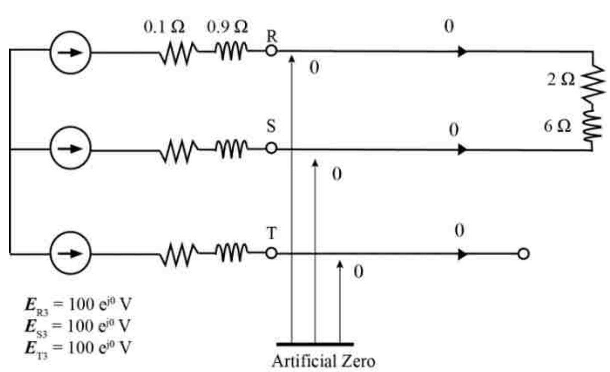


Fig. 5.21 The equivalent circuit of the system for n = 3

The 3rd harmonic component of the internal voltage of the distribution is

$$\boldsymbol{E}_3 = \begin{bmatrix} 100\, e^{j0°} \\ 100\, e^{j0°} \\ 100\, e^{j0°} \end{bmatrix} \text{ V}$$

while the load line to line admittances for this frequency are

$$\boldsymbol{Y}_{\text{RS3}} = 0.05 - j0.15 \text{ S}, \quad \boldsymbol{Y}_{\text{ST3}} = 0\,, \quad \boldsymbol{Y}_{\text{TR3}} = 0\,.$$

Note that the internal voltage of the distribution system is nonsinusoidal but symmetrical. Hence, the third order voltage harmonics are composed exclusively of the zero sequence. Therefore, the load voltages after referring to artificial zero are also 0, i.e,

$$\boldsymbol{U}_{\text{R3}} = 0, \quad \boldsymbol{U}_{\text{S3}} = 0\,, \quad \boldsymbol{U}_{\text{T3}} = 0$$

For a three-wire system, like the one used for analysis in this illustration, a voltage of the zero sequence cannot cause any current to flow in the system. Hence, the line currents for the third harmonic order are

$$||i_{\text{R3}}|| = 0, \qquad ||i_{\text{S3}}|| = 0, \qquad ||i_{\text{T3}}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_3|| = 0$.

Thus the active power of the 3rd harmonic is

$$P_3 = \text{Re}\{\boldsymbol{U}_{\text{R3}}\boldsymbol{I}^*_{\text{R3}} + \boldsymbol{U}_{\text{S3}}\boldsymbol{I}^*_{\text{S3}}\} = 0\ .$$

Finally, for the 5th harmonic order, the system is equivalent to Fig. 5.22. The crms values of the 5th harmonic component of the distribution system voltage are

$$\boldsymbol{E}_5 = \begin{bmatrix} 100 \\ 100\ \text{e}^{j120°} \\ 100\ \text{e}^{-j120°} \end{bmatrix} \text{ V}$$

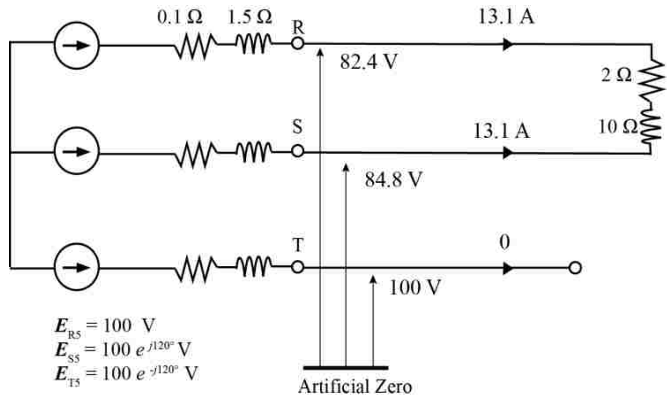


Fig. 5.22 The equivalent circuit of the system corresponding to n = 5

The load line to line admittances for this frequency are

$$\boldsymbol{Y}_{\mathrm{RS5}} = 0.0192 - j\,0.0962\ \mathrm{S},\quad \boldsymbol{Y}_{\mathrm{ST5}} = 0,\quad \boldsymbol{Y}_{\mathrm{TR5}} = 0\,.$$

The line current rms values for the 5th harmonic order are

$$||i_{\mathrm{R5}}|| = 13.1\ \mathrm{A},\qquad ||i_{\mathrm{S5}}|| = 13.1\ \mathrm{A},\qquad ||i_{\mathrm{T5}}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_5|| = 18.6\ \mathrm{A}$.

Therefore, the load voltages after referring to artificial zero are equal to,

$$\boldsymbol{U}_{\mathrm{R5}} = 82.4\ \mathrm{e}^{j5.6°}\ \mathrm{V},\ \boldsymbol{U}_{\mathrm{S5}} = 84.8\ \mathrm{e}^{j112.2°}\ \mathrm{V},\ \boldsymbol{U}_{\mathrm{T5}} = 100\ \mathrm{e}^{-j120°}\ \mathrm{V}\,.$$

Thus the active power of the 5th order harmonic is

$$P_5 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R5}}\boldsymbol{I}^*_{\mathrm{R5}} + \boldsymbol{U}_{\mathrm{S5}}\boldsymbol{I}^*_{\mathrm{S5}}\} = 350\ \mathrm{W}\ .$$

Analysis for the harmonic orders n belonging to the subset N_G:

Next, we need to consider the system for the harmonic orders of subset $n \in N_{\mathrm{G}}$. For the approximation of the arc furnace used in this illustration, the Harmonic Generating Load injects currents of the 2nd, 4th and the 7th harmonic orders. Let us analyze the system for each of these harmonics one by one.

The system corresponding to the 2nd harmonic order is equivalent to the one shown in Fig. 5.23.

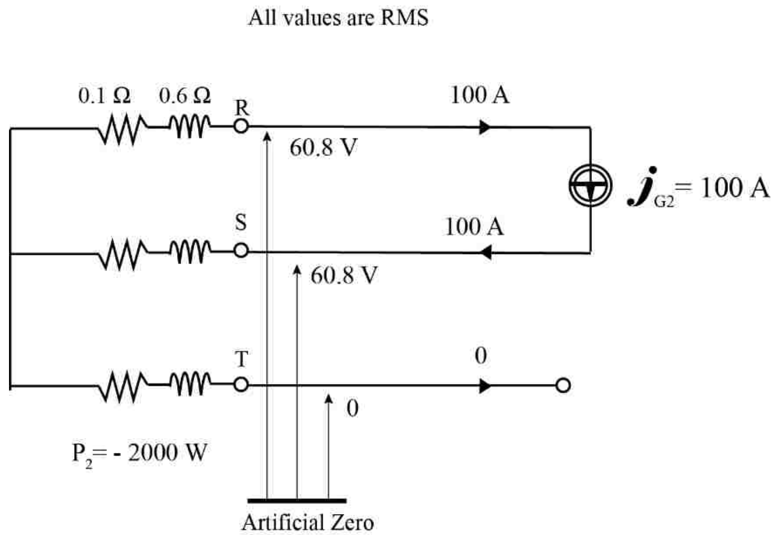


Fig. 5.23 The equivalent circuit of the system for the 2nd order harmonic

The line current crms values for the 2nd order harmonics are as follows,

$$\boldsymbol{I}_{\mathrm{R2}} = 100 \ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S2}} = 100 \ e^{j180°} \ \mathrm{A}$$

and the current three-phase rms value $||\boldsymbol{i}_2|| = 141.42 \ \mathrm{A}$

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{\mathrm{R2}} = 60.8 \mathrm{e}^{-j99.5°} \ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S2}} = 60.8 \ \mathrm{e}^{j80.54°} \ \mathrm{V}$$

with the three-phase rms value $||\boldsymbol{u}_2|| = 86.0 \ \mathrm{V}$.

The active power corresponding to the 2nd harmonic order is equal to

$$P_2 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R2}}\boldsymbol{I}^*_{\mathrm{R2}} + \boldsymbol{U}_{\mathrm{S2}}\boldsymbol{I}^*_{\mathrm{S2}}\} = -2000 \ \mathrm{W}.$$

Likewise, the system corresponding to the 4th harmonic order is equivalent to Fig. 5.24.

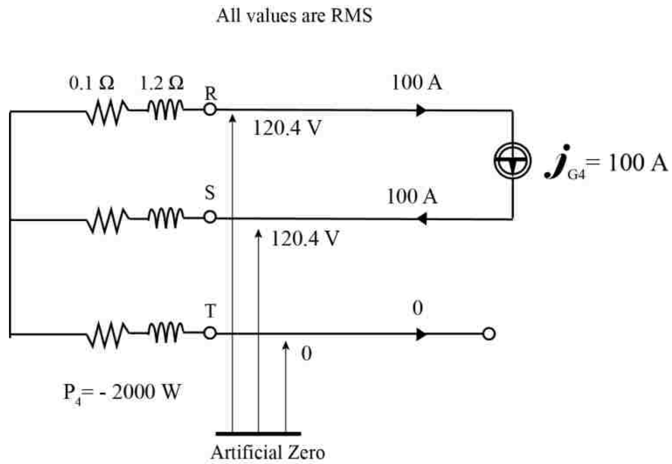


Fig. 5.24 The equivalent circuit of the system for the 4th order harmonic

Therefore, the crms values of the line currents for the 4th harmonic order are,

$$\boldsymbol{I}_{\mathrm{R4}} = 100 \ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S4}} = 100\ e^{j180^\circ}\ \mathrm{A}$$

and the current three-phase rms value $||\boldsymbol{i}_4|| = 141.4$ A

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{\mathrm{R4}} = 120.4\ \mathrm{e}^{-j94.7^\circ}\ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S4}} = 120.4\ \mathrm{e}^{j85.24^\circ}\ \mathrm{V}$$

with the three-phase rms value $||\boldsymbol{u}_4|| = 170.3\ \mathrm{V}$

The active power corresponding to the 4th harmonic order is equal to

$$P_4 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R4}}\boldsymbol{I}^*_{\mathrm{R4}} + \boldsymbol{U}_{\mathrm{S4}}\boldsymbol{I}^*_{\mathrm{S4}}\} = -\,2000\ \mathrm{W}\,.$$

Finally, the system corresponding to the 7th harmonic order is as shown in Fig. 5.25.

The crms values of the line currents for the 7th harmonic order are:

$$\boldsymbol{I}_{\mathrm{R7}} = 100\ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S7}} = 100\ e^{j180^\circ}\ \mathrm{A}$$

and the current three-phase rms value $||\boldsymbol{i}_7|| = 141.4\ \mathrm{A}\,.$

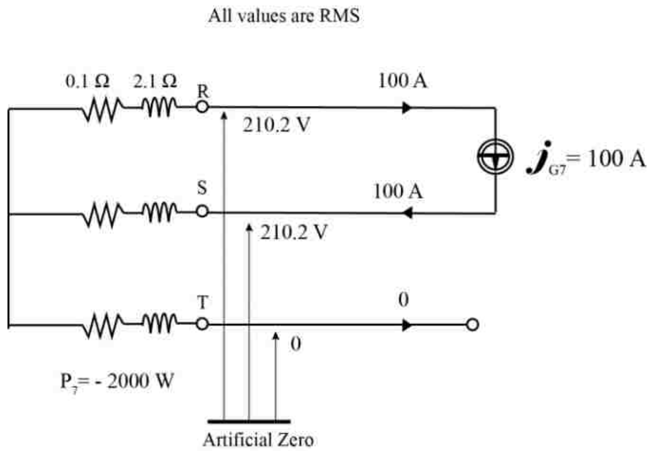


Fig. 5.25 The equivalent circuit of the system corresponding to the 7th harmonic order

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{R7} = 210.2\mathrm{e}^{-j92.73°}\ \mathrm{V}, \qquad \boldsymbol{U}_{S7} = 210.2\mathrm{e}^{j87.3°}\ \mathrm{V}$$

with the three-phase rms value $||\boldsymbol{u}_7|| = 297.3$ V

The active power corresponding to the 7th harmonic order is equal to

$$P_7 = \mathrm{Re}\{\boldsymbol{U}_{R7}\boldsymbol{I}^*_{R7} + \boldsymbol{U}_{S7}\boldsymbol{I}^*_{S7}\} = -2000\ \mathrm{W}\,.$$

Since the voltages and the currents of different harmonic orders are orthogonal to one another, the total three-phase voltage rms value is equal to

$$||\boldsymbol{u}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{u}_n||^2} = \sqrt{||\boldsymbol{u}_1||^2 + ||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_3||^2 + ||\boldsymbol{u}_4||^2 + ||\boldsymbol{u}_5||^2 + ||\boldsymbol{u}_7||^2} = 1638.1\ \mathrm{V}$$

while the current three-phase rms value is equal to

$$||\boldsymbol{i}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_n||^2} = \sqrt{||\boldsymbol{i}_1||^2 + ||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_3||^2 + ||\boldsymbol{i}_4||^2 + ||\boldsymbol{i}_5||^2 + ||\boldsymbol{i}_7||^2} = 759.9\ \mathrm{A}$$

Thus, the apparent power of the load is equal to

$$S = ||\boldsymbol{u}||\ ||\boldsymbol{i}|| = 1244.9\ \mathrm{kVA} = 1.25\ \mathrm{MVA}\,.$$

Apparent power calculated using the CPC power components:

The rms values of the active, scattered and the reactive current components for the subset $n \in N_{\mathrm{C}}$ are equal to

$$||\boldsymbol{i}_{\mathrm{Ca}}|| = 323.6 \text{ A}, \qquad ||\boldsymbol{i}_{\mathrm{Cs}}|| = 29.1 \text{ A}, \qquad ||\boldsymbol{i}_{\mathrm{Cr}}|| = 323.6 \text{ A}.$$

The positive and the negative sequence components of the unbalanced current are

$$||\boldsymbol{i}^{\mathrm{p}}_{\mathrm{Cu}}|| = 33.4 \text{ A}, \qquad\qquad ||\boldsymbol{i}^{\mathrm{n}}_{\mathrm{Cu}}|| = 317.8 \text{ A}$$

Hence, the three-phase rms value of the unbalanced current is equal to,

$$||\boldsymbol{i}_{\mathrm{Cu}}|| = \sqrt{3}\,\sqrt{||\boldsymbol{i}^{\mathrm{p}}_{\mathrm{Cu}}||^2 + ||\boldsymbol{i}^{\mathrm{n}}_{\mathrm{Cu}}||^2} = \sqrt{3}\sqrt{32.3^2 + 317.8^2} = 553.5 \text{ A}.$$

The different powers that these currents are associated with are equal to

$$P_{\mathrm{C}} = 517.6 \text{ kW}, \qquad D_{\mathrm{sC}} = 46.8 \text{ kVA}, \qquad Q_{\mathrm{C}} = 520 \text{ kVars}, \qquad D_{\mathrm{uC}} = 885.3 \text{ kVA}$$

The apparent power for the subset $n \in N_{\mathrm{C}}$ is equal to

$$S_{\mathrm{C}} = \sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2} = 1150.8 \text{ kVA}$$

Similarly, the voltage three-phase rms value for the subset $n \in N_{\mathrm{G}}$ is equal to

$$||\boldsymbol{u}_{\mathrm{G}}|| = \sqrt{\sum_{n \in N_{\mathrm{G}}} ||\boldsymbol{u}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_4||^2 + ||\boldsymbol{u}_7||^2} = 353.3 \text{ V}$$

while the three-phase rms value of the current harmonic $\boldsymbol{j}_{\mathrm{G}}$ is equal to

$$||\boldsymbol{i}_{\mathrm{G}}|| = \sqrt{\sum_{n \in N_{\mathrm{G}}} ||\boldsymbol{i}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_4||^2 + ||\boldsymbol{i}_7||^2} = 245 \text{ A}$$

Thus, the apparent power corresponding to all the harmonics belonging to the subset $n \in N_{\mathrm{G}}$ is

$$S_{\mathrm{G}} = ||\boldsymbol{u}_{\mathrm{G}}||\ \ ||\boldsymbol{i}_{\mathrm{G}}|| = \ 86.5\ \mathrm{kVA}$$

Next, the cross harmonic apparent power

$$S_{\mathrm{E}} = \sqrt{||\boldsymbol{u}_{\mathrm{C}}||^2\ ||\boldsymbol{i}_{\mathrm{G}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2\ ||\boldsymbol{i}_{\mathrm{C}}||^2} = 467\ \mathrm{kVA}$$

Thus, the total apparent power calculated using the different components of the apparent power is equal to,

$$S = \sqrt{S_{\mathrm{C}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2} = 1244.9\ \ \mathrm{kVA} = \ 1.25\ \ \mathrm{MVA}\,,$$

which is the same as value of the apparent power calculated earlier using the three-phase rms values of the line currents and voltages, calculated earlier in the illustration. This verifies that the CPC decomposition of the load currents and the powers calculated using these currents are correct.

The total active power of the load is equal to:

$$P = \sum_{n \in N_{\mathrm{C}}} P_{\mathrm{n}} + \sum_{n \in N_{\mathrm{G}}} P_{\mathrm{n}} = (517589 - 6000)\ \ \mathrm{W}\ = 511.6\ \mathrm{kW}$$

while the overall power factor of the load is equal to,

$$\lambda = \frac{P}{S} = \frac{P_{\mathrm{C}} - P_{\mathrm{G}}}{\sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2}} = \frac{511.6}{1245} = 0.41$$

The waveforms of the line currents as well as the supply and the load voltages are as following.

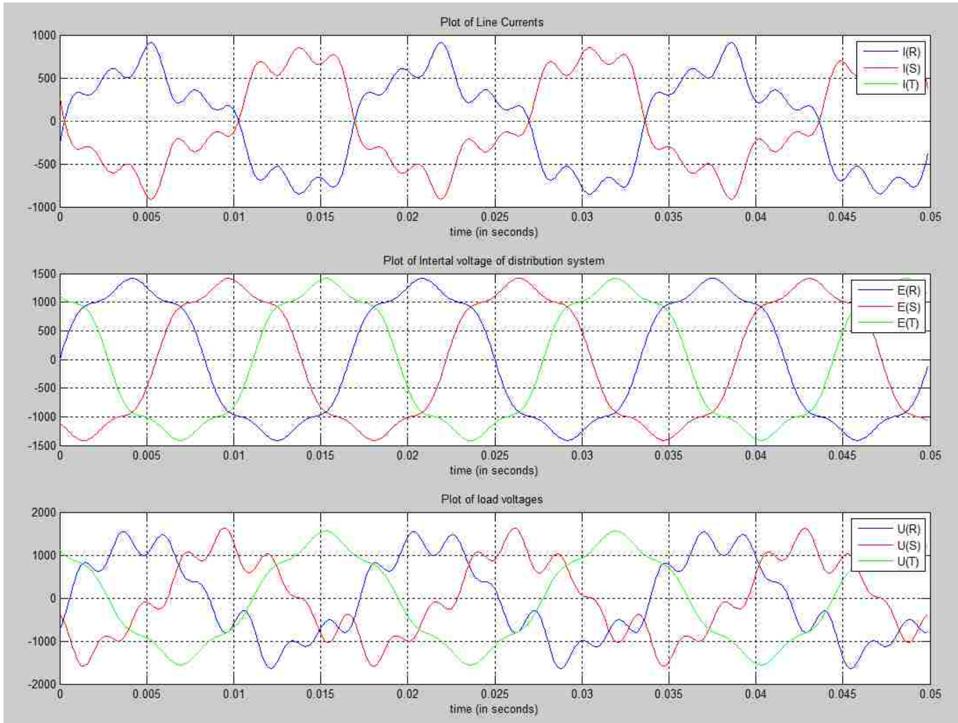


Fig. 5.26 Plot of the line currents, the distribution voltage and the load voltages

Illustration 5.3 Let us calculate the CPC currents and the various powers of the load in the previous illustration when the supply voltage is asymmetrical and nonsinusoidal and is equal to

$$\boldsymbol{e} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 1000\, e^{\,j\omega_1 t} + 100\, e^{\,j3\omega_1 t} \\ 500\, e^{-j120^\circ}\, e^{\,j\omega_1 t} + 100\, e^{\,j120^\circ}\, e^{\,j3\omega_1 t} \\ 0 \end{bmatrix}\}\mathrm{V}$$

while the load generated harmonics in the current on line R are equal to

$$j_{\mathrm{RG}} = \sqrt{2}\mathrm{Re}\{100\, e^{j2\omega_1 t} + 100\, e^{j4\omega_1 t} + 100 e^{j7\omega_1 t}\}\ \mathrm{A}\,.$$

In this case the voltage and the current contains the harmonics of two subsets, namely,

$$n \in N_{\mathrm{C}} \quad \text{with } N_{\mathrm{C}} = \{1,3\}$$
$$n \in N_{\mathrm{G}} \quad \text{with } N_{\mathrm{G}} = \{2,4,7\}$$

Analysis for the subset $n \in N_C$ *:*

For the fundamental component, the system is equivalent to the one depicted in Fig. 5.27.

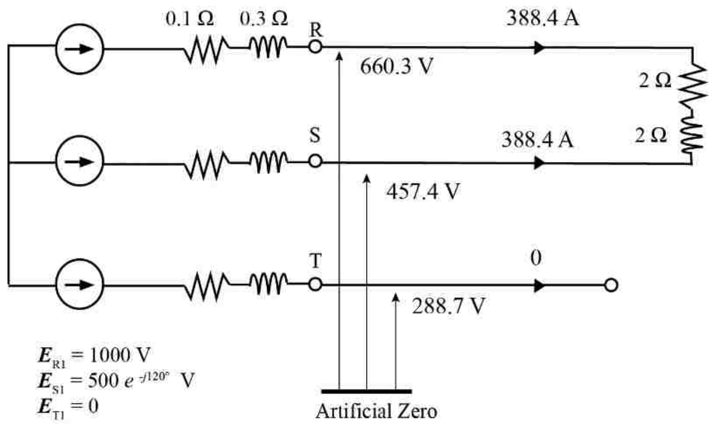


Fig. 5.27 The equivalent circuit of the system for n = 1

The internal voltage of the distribution system for the fundamental frequency is

$$\boldsymbol{E} = \begin{bmatrix} 1000 \\ 500\ e^{-j120°} \\ 0 \end{bmatrix} \text{V}$$

At this frequency, the sources impedance is

$$R_{SC} = 0.1\,\Omega\ , \qquad \omega_1 L_{SC} = 0.3\,\Omega$$

while the load impedance is

$$R_{RS} = 2\Omega, \qquad \omega_1 L_{RS} = 2\ \Omega\,.$$

Thus, the load line to line admittances for the fundamental frequency are

$$\boldsymbol{Y}_{RS1} = 0.25 - j0.25\ \text{S}, \quad \boldsymbol{Y}_{ST1} = 0, \quad \boldsymbol{Y}_{TR1} = 0\,.$$

The line currents rms values are equal to

$$||i_{R1}||\ = 388.41\ \text{A}, \qquad ||i_{S1}|| = 388.41\text{A}, \qquad ||i_{T1}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_1|| = 549$ A .

These currents cause a voltage drop in the source impedances, and the line voltages after the drop in the source impedance and after referencing to an artificial zero are equal to

$$\boldsymbol{U}_{\mathrm{R}1} = 660.3e^{j5.55°}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{S}1} = 457.4e^{-j152.9°}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{T}1} = 288.7e^{j150°}\ \mathrm{V}$$

The voltage three-phase rms value is equal to $||\boldsymbol{u}_1|| = 853.2$ V .

The active power of the fundamental harmonic is equal to

$$P_1 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}1}\boldsymbol{I}^*_{\mathrm{R}1} + \boldsymbol{U}_{\mathrm{S}1}\boldsymbol{I}^*_{\mathrm{S}1}\} = 301.7\ \ \mathrm{kW}\ .$$

For the 3rd harmonic order, the system is equivalent to the one shown in Fig. 5.28.

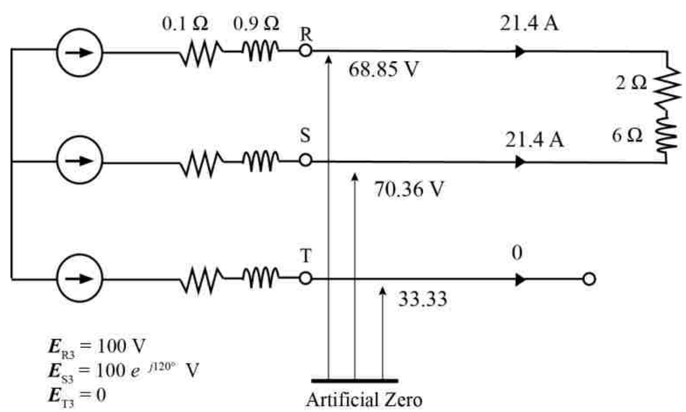


Fig. 5.28 The equivalent circuit of the system for $n = 3$

The crms value of the 3rd order harmonic of the internal voltage of the distribution is

$$\boldsymbol{E}_3 = \begin{bmatrix} 100 \\ 100\ e^{j120°} \\ 0 \end{bmatrix}\ \mathrm{V}$$

while the load line to line admittances for this frequency are

$$\boldsymbol{Y}_{\mathrm{RS}3} = 0.05 - j0.15\ \mathrm{S},\quad \boldsymbol{Y}_{\mathrm{ST}3} = 0\,,\quad \boldsymbol{Y}_{\mathrm{TR}3} = 0\ .$$

Note that unlike the previous case, the internal voltage of the distribution system in this illustration is nonsinusoidal and asymmetrical. Hence, the third order voltage harmonics are contain the positive as well as the negative sequence voltage components in addition to the zero sequence components. Therefore, the load voltages after referring to artificial zero is,

$$\boldsymbol{U}_{\mathrm{R3}} = 68.8\, e^{-j18.7^\circ}\ \mathrm{V}, \quad \boldsymbol{U}_{\mathrm{S3}} = 70.4\, e^{j133.6^\circ}\ \mathrm{V}, \quad \boldsymbol{U}_{\mathrm{T3}} = 33.3\, e^{-j120^\circ}\ \mathrm{V}\ .$$

The line currents for the third harmonic order are

$$||i_{\mathrm{R3}}|| = 21.4, \qquad ||i_{\mathrm{S3}}|| = 21.4, \qquad ||i_{\mathrm{T3}}|| = 0$$

with the three-phase rms value equal to $||\boldsymbol{i}_3|| = 30.2\ \mathrm{A}$.

Thus the active power of the 3rd harmonic is

$$P_3 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R3}}\boldsymbol{I}^*_{\mathrm{R3}} + \boldsymbol{U}_{\mathrm{S3}}\boldsymbol{I}^*_{\mathrm{S3}}\} = 910\ \mathrm{W}\ .$$

Analysis for the harmonic orders n belonging to the subset N_G :

Next, we need to consider the system for the harmonic orders of subset $n \in N_{\mathrm{G}}$. Since, the load generated harmonics in this case are the same as the previous illustration and the fact that the change in the supply parameters do not affect the analysis for the subset $n \in N_{\mathrm{G}}$. The analysis for this subset will be the same as it was in the previous illustration. Therefore, only the results are presented here.

The line current crms values for the 2nd order harmonics are

$$\boldsymbol{I}_{\mathrm{R2}} = 100\ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S2}} = 100\ e^{j180^\circ}\ \mathrm{A}$$

and the current three-phase rms value $||\boldsymbol{i}_2|| = 141.4\ \mathrm{A}$

The line voltages resulting from the drop in the source impedance are equal to

$$\boldsymbol{U}_{\mathrm{R2}} = 60.8\mathrm{e}^{-j99.5^\circ}\ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S2}} = 60.8\ \mathrm{e}^{j80.54^\circ}\ \mathrm{V}$$

with the three-phase rms value $||\boldsymbol{u}_2|| = 86.0$ V

The active power corresponding to the 2nd harmonic order is equal to

$$P_2 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R2}}\boldsymbol{I}^*_{\mathrm{R2}} + \boldsymbol{U}_{\mathrm{S2}}\boldsymbol{I}^*_{\mathrm{S2}}\} = -2000\ \mathrm{W}.$$

Likewise, for the 4th harmonic order, the line current crms values are

$$\boldsymbol{I}_{\mathrm{R4}} = 100\ \mathrm{A}, \quad \boldsymbol{I}_{\mathrm{S4}} = 100\ e^{j180^\circ}\ \mathrm{A}, \quad ||\boldsymbol{i}_4|| = 141.42\ \mathrm{A}.$$

The voltage corms values are equal to,

$$\boldsymbol{U}_{\mathrm{R4}} = 120.4\ \mathrm{e}^{-j94.7^\circ}\ \mathrm{V}, \quad \boldsymbol{U}_{\mathrm{S4}} = 120.4\ \mathrm{e}^{j85.24^\circ}\ \mathrm{V}, \quad ||\boldsymbol{u}_4|| = 170.3\ \mathrm{V}$$

The active power, $P_4 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R4}}\boldsymbol{I}^*_{\mathrm{R4}} + \boldsymbol{U}_{\mathrm{S4}}\boldsymbol{I}^*_{\mathrm{S4}}\} = -2000\ \mathrm{W}$.

Finally, for the 7th harmonic order, the currents

$$\boldsymbol{I}_{\mathrm{R7}} = 100\ \mathrm{A}, \qquad \boldsymbol{I}_{\mathrm{S7}} = 100\ e^{j180^\circ}\ \mathrm{A} \quad \text{and} \quad ||\boldsymbol{i}_7|| = 141.4\ \mathrm{A}$$

while the voltages

$$\boldsymbol{U}_{\mathrm{R7}} = 210.2\mathrm{e}^{-j92.73^\circ}\ \mathrm{V}, \qquad \boldsymbol{U}_{\mathrm{S7}} = 210.2\mathrm{e}^{j87.3^\circ}\ \mathrm{V} \quad \text{and} \qquad ||\boldsymbol{u}_7|| = 297.3\ \mathrm{V}$$

The active power $P_7 = \mathrm{Re}\{\boldsymbol{U}_{\mathrm{R7}}\boldsymbol{I}^*_{\mathrm{R7}} + \boldsymbol{U}_{\mathrm{S7}}\boldsymbol{I}^*_{\mathrm{S7}}\} = -2000\ \mathrm{W}$.

The total three-phase voltage rms value is equal to

$$||\boldsymbol{u}|| = \sqrt{\sum_{n \in N} ||\boldsymbol{u}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{u}_1||^2 + ||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_3||^2 + ||\boldsymbol{u}_4||^2 + ||\boldsymbol{u}_7||^2} = 929.5\ \mathrm{V}$$

while the current three-phase rms value is equal to

$$||\boldsymbol{i}|| = \sqrt{\sum_{n\in N} ||\boldsymbol{i}_{\rm n}||^2} = \sqrt{||\boldsymbol{i}_1||^2 + ||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_3||^2 + ||\boldsymbol{i}_4||^2 + ||\boldsymbol{i}_7||^2} = 602.2 \text{ A}$$

Thus, the apparent power of the load is equal to

$$S = ||\boldsymbol{u}||\ ||\boldsymbol{i}|| = 559.8 \text{ kVA} \ .$$

Apparent power calculated using the CPC power components

The rms values of the active, reactive and the scattered current components for the subset $n \in N_{\rm C}$ are equal to

$$||\boldsymbol{i}_{\rm Ca}|| = 352 \text{ A}, \qquad ||\boldsymbol{i}_{\rm Cs}|| = 34.0 \text{ A}, \qquad ||\boldsymbol{i}_{\rm Cr}|| = 354.5 \text{ A}.$$

The positive and the negative sequence components of the unbalanced current are

$$||\boldsymbol{i}^{\rm p}_{\rm Cu}|| = 64.2 \text{ A}, \qquad\qquad ||\boldsymbol{i}^{\rm n}_{\rm Cu}|| = 114.8 \text{A}$$

Hence, the three-phase rms value of the unbalanced current is equal to,

$$||\boldsymbol{i}_{\rm Cu}|| = \sqrt{3}\sqrt{||\boldsymbol{i}^{\rm p}_{\rm Cu}||^2 + ||\boldsymbol{i}^{\rm n}_{\rm Cu}||^2} = \sqrt{3}\sqrt{64.2^2 + 114.8^2} = 227.8 \text{ A} \ .$$

The different powers that these currents are associated with are equal to

$$P_{\rm C} = 302.6 \text{ kW}, \qquad D_{\rm Cs} = 29.2 \text{ kVA} \qquad Q_{\rm C} = 304.8 \text{ kVars}, \qquad D_{\rm Cu} = 195.9 \text{ kVA}$$

The apparent power for the subset $n \in N_{\rm C}$ is equal to

$$S_{\rm C} = \sqrt{P_{\rm C}^2 + D_{\rm Cs}^2 + Q_{\rm C}^2 + D_{\rm Cu}^2} = 473 \text{ kVA}$$

Similarly, the voltage three-phase rms value for the subset $n \in N_{\rm G}$ is equal to

$$||\boldsymbol{u}_{\rm G}|| = \sqrt{\sum_{n\in N_{\rm G}} ||\boldsymbol{u}_{\rm n}||^2} = \sqrt{||\boldsymbol{u}_2||^2 + ||\boldsymbol{u}_4||^2 + ||\boldsymbol{u}_7||^2} = 353.3 \text{V}$$

while the three-phase rms value of the current harmonic $\boldsymbol{j}_{\mathrm{G}}$ is equal to

$$||\boldsymbol{i}_{\mathrm{G}}|| = \sqrt{\sum_{n \in N_{\mathrm{G}}} ||\boldsymbol{i}_{\mathrm{n}}||^2} = \sqrt{||\boldsymbol{i}_2||^2 + ||\boldsymbol{i}_4||^2 + ||\boldsymbol{i}_7||^2} = 245 \text{ A}$$

Thus, the apparent power corresponding to all the harmonics belonging to the subset $n \in N_{\mathrm{G}}$ is

$$S_{\mathrm{G}} = ||\boldsymbol{u}_{\mathrm{G}}||\ ||\boldsymbol{i}_{\mathrm{G}}|| = 86.5 \text{ kVA}$$

Next, the cross harmonic apparent power

$$S_{\mathrm{E}} = \sqrt{||\boldsymbol{u}_{\mathrm{C}}||^2\ ||\boldsymbol{i}_{\mathrm{G}}||^2 + ||\boldsymbol{u}_{\mathrm{G}}||^2\ ||\boldsymbol{i}_{\mathrm{C}}||^2} = 286.8 \text{ kVA}$$

Thus, the total apparent power calculated using the different components of the apparent power is equal to,

$$S = \sqrt{S_{\mathrm{C}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2} = 559.8 \text{ kVA},$$

which is the same as value of the apparent power calculated earlier using the three-phase rms values of the line currents and voltages.

The total active power of the load is equal to:

$$P = \sum_{n \in N_{\mathrm{C}}} P_{\mathrm{n}} + \sum_{n \in N_{\mathrm{G}}} P_{\mathrm{n}} = 296.7 \text{ kW}$$

while the overall power factor of the load is equal to,

$$\lambda = \frac{P}{S} = \frac{P_{\mathrm{C}} - P_{\mathrm{G}}}{\sqrt{P_{\mathrm{C}}^2 + D_{\mathrm{Cs}}^2 + Q_{\mathrm{C}}^2 + D_{\mathrm{Cu}}^2 + S_{\mathrm{G}}^2 + S_{\mathrm{E}}^2}} = \frac{296.7}{559.8} = 0.53$$

The waveforms of the line currents as well as the supply and the load voltages are as following.

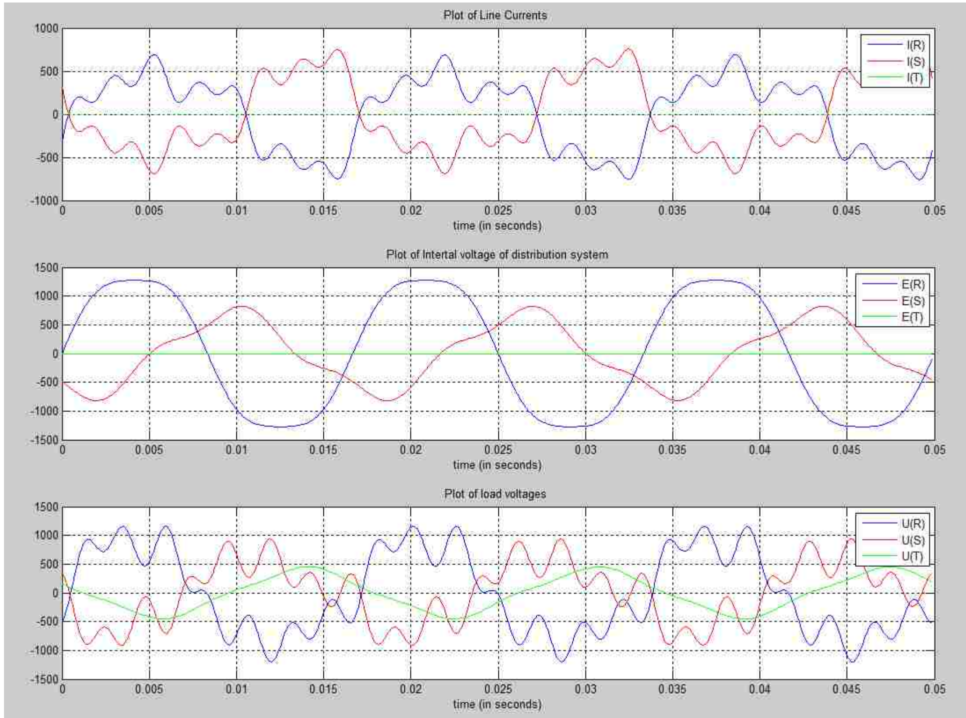


Fig. 5.29 Plot of the line currents, the distribution voltage and the load voltages

5.5 Conclusion

The analysis of circuits with Harmonic Generating Loads (HGLs) supplied from a source of nonsinusoidal and asymmetrical voltage was presented in this chapter. The load current of a HGL was decomposed into orthogonal components, also known as Currents' Physical Components, each of which is associated with a distinct physical phenomenon. The apparent power of the HGL is composed of the apparent powers S_{C}, S_{G} and S_{E}. The apparent power S_{C} is the same as the apparent power of the load for LTI systems and is associated with the same physical phenomena as that in LTI systems. The power S_{G} is a new component of the apparent power and originates as a result of the load generated current harmonics. In addition to these two powers, the apparent power also contains the power S_{E}, which stems from the increment in the voltage and current three-phase rms values. The reactive, unbalanced and scattered power components of the power S_{C} as well as the powers S_{G} and S_{E} all contribute the degradation of the power factor of HGLs. Moreover, the harmonic generated apparent power S_G not only increases the apparent power of the load, but also reduces the active power of the load.

CHAPTER 6: REFERENCE SIGNAL GENERATION FOR SHUNT SWITCHING COMPENSATORS IN THREE-WIRE SYSTEMS AT ASYMMETRICAL VOLTAGE

6.1 Introduction

Chapters 3 and 4 of this dissertation were focused on description of the power properties of linear time invariant (LTI) loads, as well as on the methods of the design of reactive compensators for power factor improvement. Chapter 5 was dedicated to the development of the power equation of Harmonic Generating Loads (HGLs) at asymmetrical and nonsinusoidal supply voltage. The methods of compensation of HGLs were not discussed, however, because reactive balancing compensators are not as effective for HGLs as compared to LTI loads. Switching Compensators, also commonly known as Active Power Filters, are used for this purpose instead.

Switching Compensators (SCs) are power electronics devices that inject the compensating current into the distribution system. In essence, SCs are controlled current sources which reproduce the waveform of a reference signal, which in turn depends on the goals of compensation.

Operating principle of SCs is not the subject of power theory, however. The subject of this chapter is the compensation of HGLs using SCs as well as the methods used for the generation of the reference signal of SCs. Therefore the focus of this chapter is on the algorithms used for the generation of the reference signal, while SCs are modelled as controlled current sources in this chapter.

Two terms that will be used frequently in this chapter are Supply Quality (SQ) and Loading Quality (LQ). Supply Quality [29] refers to the properties of the supply voltage. An ideal three-phase voltage is one which is symmetrical, sinusoidal, and has a constant RMS value. Any deviation from these standards leads to the degradation of the Supply Quality. On the other hand, Loading Quality [29] refers to properties of a load as seen from the supply side. An ideal three-phase load is balanced, resistive and linear. Any deviation from these properties leads to the degradation of the Loading Quality. The scope of this chapter, as well as this dissertation, is limited to the improvement of the Loading Quality.

6.2 Issues with compensation of HGLs at nonsinusoidal voltage

Studies on compensation in the earlier chapters of this dissertation were confined to reactive compensators, applied mainly to LTI loads. In the case of HGLs, harmonics generated in such loads can be reduced by Resonant Harmonic Filters (RHFs)[30], which are a kind of reactive compensators. Branches of such filters provide a short-circuit path to harmonics to which they are tuned to. Consequently, the load generated harmonics are filtered out and the supply current waveform is prevented from distortion. Unfortunately, when the supply voltage is distorted, and in particular, it contains the voltage harmonics of the same order as the load generated current harmonics, the filter branches tuned for the load generated harmonics inadvertently amplifies the supply current harmonics. Consequently, efficiency of RHFs in reduction of distortion declines. Even at relatively low voltage distortion, RHFs can lose effectiveness [31].

6.3 Background on Shunt Switching Compensators (Active Power Filters)

Distribution system can be protected against harmonics generated in the load by Switching Compensators (SCs) known mainly as ‘Active Filters’ [32]. Shunt SCs are devices capable of injecting the compensating current into the distribution system. Loads with degraded Loading Quality may draw the active, reactive, unbalanced, and scattered currents from the source but only the active current contributes to the energy delivery from the source to the load. Hence, the non-active components of the load current are considered as useless currents as they increase the supply current RMS value, lead to losses, and reduce the effectiveness of energy delivery. If the SC injects a current, equal to the difference between the active and the load current, into the system - the load draws its normal current while the supply sees an ideal load and is only loaded with the active current. This reduces the losses and improves the effectiveness of energy delivery.

A SC consists of a PWM Inverter, a Data Acquisition System, and a Digital Signal Processing System as shown in the Fig. 6.1. Information about the line voltages and currents is acquired through the Data Acquisition System, using voltage and current sensors. These voltage and current signals are then fed to the Digital Signal Processing system. An algorithm based on a specific power theory is then used to generate a reference signal. The reference signal is fed to a Pulse Width Modulation (PWM) generator which controls the inverter that injects the desired current into the power system.

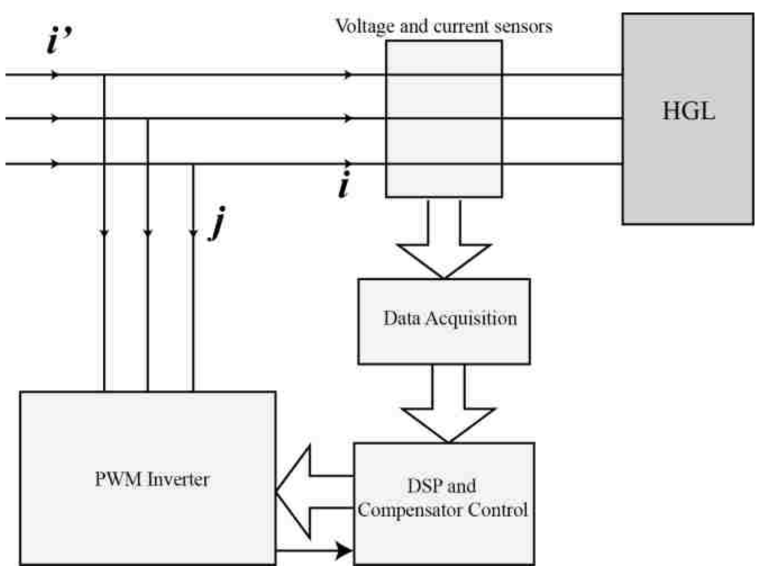


Fig. 6.1 Block diagram of PWM Inverter based Switching Compensator

6.4 Algorithms used for reference signal generation

Reference signal carries the information about the waveform of the compensator current which is to be produced by PWM Inverter. Once the voltage and current signals are obtained from the Data Acquisition System, an algorithm, based on a power theory as per the goals of compensation, is used to generate the reference signal. The eventual purpose of the SC, regardless of the algorithm that it is being used, is to improve the Loading Quality. Different approaches have different goals of compensation, however. When the source has ideal Supply Quality, the goals of compensation of all the approaches should converge. On the other hand, when the source has degraded Supply Quality, the results of compensation may depend on the used power theory. Some of the most commonly used power theories for reference signal generation are discussed below.

6.4.1 Instantaneous Reactive Power (IRP) *p*-*q* Theory

The Instantaneous Reactive Power (IRP) *p*-*q* Theory [33] is the most commonly used power theory [34] for the reference signal generation for switching compensators. It was introduced by Akagi, Kanazawa and Nabae in [33] and has since gone through a lot of modifications and development, some of which include [35-38]. Although the theory is sometimes criticized [39, 40] as a power theory due to its shortcomings in the description of the power phenomenon in the load, it is commonly used for the control of switching compensators.

This theory is based upon two power p and q, which are defined by the instantaneous values of the voltages and currents. According to the *p*-*q* theory the instantaneous active power of ideal loads should be constant. The control algorithm is also designed on the same principle [41] and its goal is to compensate the components $\tilde{p}$ and q of the instantaneous power.

The IRP *p-q* is built upon the Park and Clark Transforms of the voltages and currents into orthogonal α and β coordinates, namely,

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{3}{\sqrt{2}} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \\ u_\mathrm{T} \end{bmatrix} . \tag{6.1}$$

For three-phase three-wire systems, the information about two line voltages is sufficient to calculate the Clark Transforms. Formula (6.1) can be simplified to the form

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \end{bmatrix} = \mathbf{C} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \end{bmatrix} . \tag{6.2}$$

Similarly, the Clark Transform of the line currents is equal to

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_\mathrm{R} \\ i_\mathrm{S} \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_\mathrm{R} \\ i_\mathrm{S} \end{bmatrix} . \tag{6.3}$$

After these transformations, the load is described in terms of two powers, namely,

$$p = u_\alpha\, i_\alpha + u_\beta\, i_\beta \tag{6.4}$$

referred to as the *instantaneous active power*, and

$$q = u_\alpha\, i_\beta - u_\beta\, i_\alpha \tag{6.5}$$

referred to as the *instantaneous reactive power*.

Having these powers the *instantaneous active current*

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} p, \qquad i_{\beta p} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2} p \tag{6.6}$$

and the *instantaneous reactive current*

$$i_{\alpha q} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2} q, \qquad i_{\beta q} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2} q \tag{6.7}$$

are defined.

The names *instantaneous active current* and *instantaneous reactive current* can be misleading, however [39]. These currents have nothing in common with active and reactive currents known traditionally in electrical systems. The *instantaneous reactive current* is not related to the reactive power Q. Likewise, the *instantaneous active current* is different than the active current defined by Fryze, which is associated with permanent energy transfer. Also, from the point of view of compensation, the *instantaneous active current* is not the current that should be supplied from the distribution system after compensation. The goal of compensation of the *p-q* theory is to compensate the power q as well as the alternating component of the *instantaneous active power p*.

The compensation current can be calculated as

$$\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \frac{1}{u_\alpha^2 + u_\beta^2} \begin{bmatrix} u_\alpha & -u_\beta \\ u_\beta & u_\alpha \end{bmatrix} \begin{bmatrix} \tilde{p} \\ q \end{bmatrix}. \tag{6.8}$$

The compensator currents expressed in terms of the α and β coordinates are then converted into phase quantities using Inverse Clark Transform, namely,

$$\begin{bmatrix} j_R \\ j_S \end{bmatrix} = \mathbf{C}^{-1}\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{2}{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} j_\alpha \\ j_\beta \end{bmatrix} \tag{6.9}$$

This algorithm used for reference signal generation works well when the supply has ideal quality because in such a case the *instantaneous active power* of ideal loads is constant. Unfortunately, supply voltage distortion or asymmetry often results in an alternating component in the *instantaneous active power*, even if the load is ideal. If the goal of compensation is to compensate the alternating component, then the compensator is often ineffective, while it can sometimes even be detrimental.

Illustration 6.1 Let us consider a system with a balanced resistive load supplied from a source of sinusoidal asymmetrical voltage as shown in Fig. 6.2.

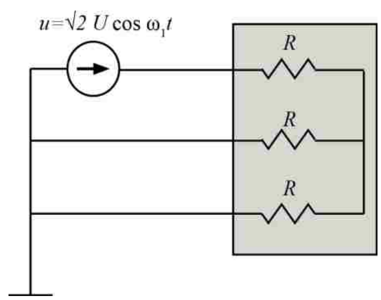


Fig. 6.2 Balanced resistive supplied from a source of asymmetrical voltage

We have,

$$u_R = \sqrt{2}U\cos\omega_1 t, \qquad u_S = 0, \qquad u_T = 0.$$

Therefore,

$$i_R = \frac{2\sqrt{2}U\cos\omega_1 t}{3R}, \qquad i_S = \frac{-\sqrt{2}U\cos\omega_1 t}{3R}.$$

Thus,

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \end{bmatrix} = \mathbf{C} \begin{bmatrix} u_\mathrm{R} \\ u_\mathrm{S} \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, U \cos \omega_1 t \\ \sqrt{3}\, U \cos \omega_1 t \end{bmatrix}$$

and,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{3}{2}} & 0 \\ \frac{1}{\sqrt{2}} & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_\mathrm{R} \\ i_\mathrm{S} \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_\mathrm{R} \\ i_\mathrm{S} \end{bmatrix} = \begin{bmatrix} \frac{2GU \cos \omega_1 t}{\sqrt{3}} \\ 0 \end{bmatrix}.$$

The *instantaneous active power,*

$$p = u_\alpha\, i_\alpha + u_\beta\, i_\beta = U^2 G + U^2 G \cos 2\omega_1 t$$

and the *instantaneous reactive power*

$$q = u_\alpha\, i_\beta - u_\beta\, i_\alpha = -U^2 G - U^2 G \cos 2\omega_1 t \,.$$

Thus a balanced resistive load, which has ideal Loading Quality, has an alternating *instantaneous active power* component $\tilde{p}$ equal to $U^2 G \cos 2\omega_1 t$ as well as the *instantaneous reactive power* component q equal to $-U^2 G - U^2 G \cos 2\omega_1 t$. The goal of the IRP *p-q* Theory based algorithm is to produce the compensator current to compensate the powers $\tilde{p}$ and q. Unfortunately, such a compensator current will degrade the power quality. Moreover, since the load is balanced and resistive, it already has ideal Loading Quality and shunt compensation cannot improve the condition any further. This simple illustration demonstrates how the failure to distinguish degraded Loading Quality from degraded Supply Quality can lead to erroneous results.

The main argument in favor of the IRP *p-q* Theory is that it supposedly provides the theoretical fundamentals for the design of control algorithms - and somehow it is not that important whether or not it interprets the power phenomenon in electrical systems correctly. Unfortunately, this fundamental deficiency in the *p-q* theory renders it ineffective when there is supply voltage asymmetry or distortion. Moreover, since the theory interprets the power phenomena incorrectly, it can rather lead to the degradation, as opposed to the improvement of power quality. Examples of a few situations have been presented in [39, 42, 43] where compensation using the *p-q* approach leads to detrimental results.

6.4.2 CPC Power theory

Unlike the IRP *p-q* Theory and the Fryze Power Theory [3, 44] (not discussed in this chapter), which are based on the time-domain, the CPC power theory is formulated in the frequency-domain. It is based on the decomposition of the load current into orthogonal components, each associated with a distinct physical phenomenon. It enables the description of the power properties of the load.

Although the CPC power theory is based on the frequency-domain, the CPC based compensation algorithms utilize both the frequency-domain, and the time-domain. The time-domain is used to expedite the computation of the compensator current. In this approach, the active current is computed using the frequency-domain, while the remainder of the current, which can be calculated by subtracting the active current from the load current, is calculated in time-domain and used for reference signal generation.

The non-active current can be calculated as a combined quantity in order to expedite the process. The CPC theory does enable the calculation of the physical components of the current, however. This is particularly important because the CPC theory is the only approach in published literature that enables decomposition of the load current into components based on physical phenomena, thereby enabling the design of hybrid compensators, which compensate these components separately. A hybrid compensator utilizes reactive compensator for the compensation of the bulk of the reactive and unbalanced currents, and switching compensators for the compensation of harmonic currents as well as the scattered current. Such a hybrid compensator could be capable of handling industrial loads such as arc furnaces with power rating in the range of hundreds of MVAs. Compensation of such loads is well beyond the capability of any approach that relies only on Switching Compensators.

The goal of compensation is to make the load as seen from the supply source as resistive, balanced and linear as much as possible. In order to do this, first the voltages and currents have to be sampled and then used to calculate the active current. Next, the non-active current is calculated and the negative of this current is generated and then injected into the system. Consequently, the load and the compensator together will only draw the active current from the supply.

6.5 Reference signal generation based on Currents Physical Component Power Theory

The two main advantages of using CPC based algorithm for reference signal generation is that it enables compensation at voltage asymmetry and distortion, and provides the flexibility to choose the current components that are to be compensated. Consequently, CPC based algorithms enable design of hybrid compensators which can combine SCs of low power fast switching capabilities with reactive compensators of high power but without adaptive properties.

The first step in the generation of the reference signal is to acquire the line voltages and currents. For a three-phase three-wire system, the phase voltages and currents have the given relationships:

$$u_{\mathrm{R}}(t)+u_{\mathrm{S}}(t)+u_{\mathrm{T}}(t)\equiv 0$$
$$i_{\mathrm{R}}(t)+i_{\mathrm{S}}(t)+i_{\mathrm{T}}(t)\equiv 0. \tag{6.10}$$

Therefore, the information on voltages and currents of two phases is sufficient to calculate them in the third phase. Hence, only two voltages and currents need to be sampled. Once the samples of these quantities are obtained, the Fourier Transform of these quantities is done in order to transform these quantities to the frequency domain. Next, the active power of the load can be calculated by integrating the instantaneous power over time

$$P=\frac{1}{N}\sum_{\mathrm{k}=1}^{N}u_{\mathrm{k}}\, i_{\mathrm{k}} \tag{6.11}$$

or by using the crms values of the line voltages and currents

$$P=\frac{1}{N}\sum_{\mathrm{X=R,S,T}}\sum_{\mathrm{k}}u_{\mathrm{kx}}\, i_{\mathrm{kx}} \tag{6.12}$$

The equivalent conductance of such a load is equal to

$$G_{\mathrm{b}}=\frac{P}{||\boldsymbol{u}||^{2}} \tag{6.13}$$

where $||\boldsymbol{u}||$ is the three-phase rms value of the supply voltage defined earlier in the dissertation and can be calculated as

$$||\boldsymbol{u}||=\sqrt{\frac{1}{N}\sum_{\mathrm{X=R,S,T}}\sum_{\mathrm{k}}u_{\mathrm{kx}}^{2}} \tag{6.14}$$

The active current of the load is equal to

$$\boldsymbol{i}_{\mathrm{ak}}=G_{\mathrm{b}}\,\boldsymbol{u}_{\mathrm{k}}\ . \tag{6.15}$$

Once the active current is known, the compensator current can be calculated by subtracting the active current from the load current,

$$\boldsymbol{i}_j = \boldsymbol{i}_{\mathrm{k}} - \boldsymbol{i}_{\mathrm{ak}} \tag{6.16}$$

6.6 Implementation of Reference signal in PWM Inverter based Switching Compensator

Although the methods used for the calculation of the active current was described in section 6.4 above, the details of its implementation using a shunt Switching Compensator were not explained. Since SCs are not the subject of this chapter, and are, moreover, modeled using controlled current sources in this chapter, these considerations are only shortly described below. The details of the implementation of SCs can be found in [34, 45].

In addition to the energy transferred to the load, the active current after compensation should also carry the energy that is dissipated in the compensator in order to maintain the voltage level of the capacitor used in the PWM Inverter.

After the correct reference signal is calculated in the digital form, the current has to be produced by the PWM Inverter. In order to do this, the inverter has to be controlled utilizing a method known as the Space Vector Pulse Width Modulation (SV-PWM) approach [45]. This method is used to operate the switches of the inverter, so that the desired current output is obtained across the compensator output terminals.

6.7 Simulation and Results

The simulations were carried out in the Simpowersystem Toolkit of MATLAB. The voltages and currents were first measured using a Three-Phase V-I Measurement Block. Then the active power and the active current were calculated and then used to calculate the reference signal for compensation. The goal of compensation was to reduce the supply current to just the active current. Therefore, the supply current after compensation should be in phase with the supply voltages. The reference signal was then fed to the controlled current sources which were used to model the Switching Compensators. Based on the reference signal, the current sources injected the compensator current into the system and improved the Loading Quality. CPC based algorithm was used for the compensation of both linear as well as Harmonic Generating loads, operating at nonsinusoidal as well as asymmetrical supply voltage. The model shown in Fig. 6.3 was used for simulation. Details of the various parts of the model are given in the Appendix section.

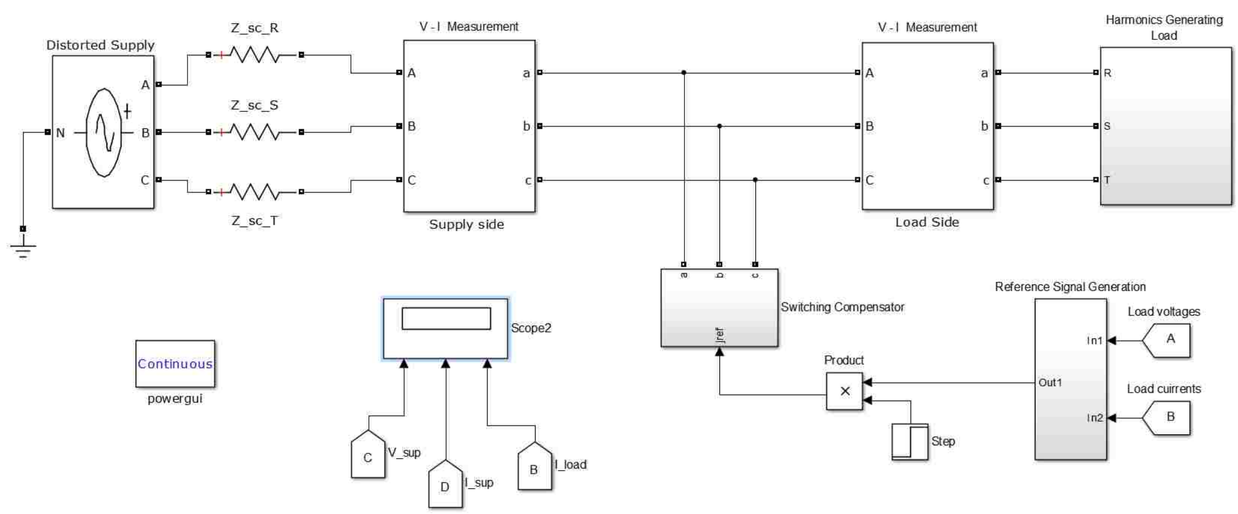


Fig. 6.3 Matlab Simulink model used for simulation

Two V-I Measurement Blocks were used in the model. The first block, namely “Supply Side” measured the supply voltage and currents while the second block, namely “Load Side” measured the load side voltages and currents.

1. Balanced resistive LTI load supplied from a source of sinusoidal symmetrical voltage

In this case a balanced resistive load of 1 Ohm was supplied from a sinusoidal symmetrical voltage source with rms value 100 Volts.

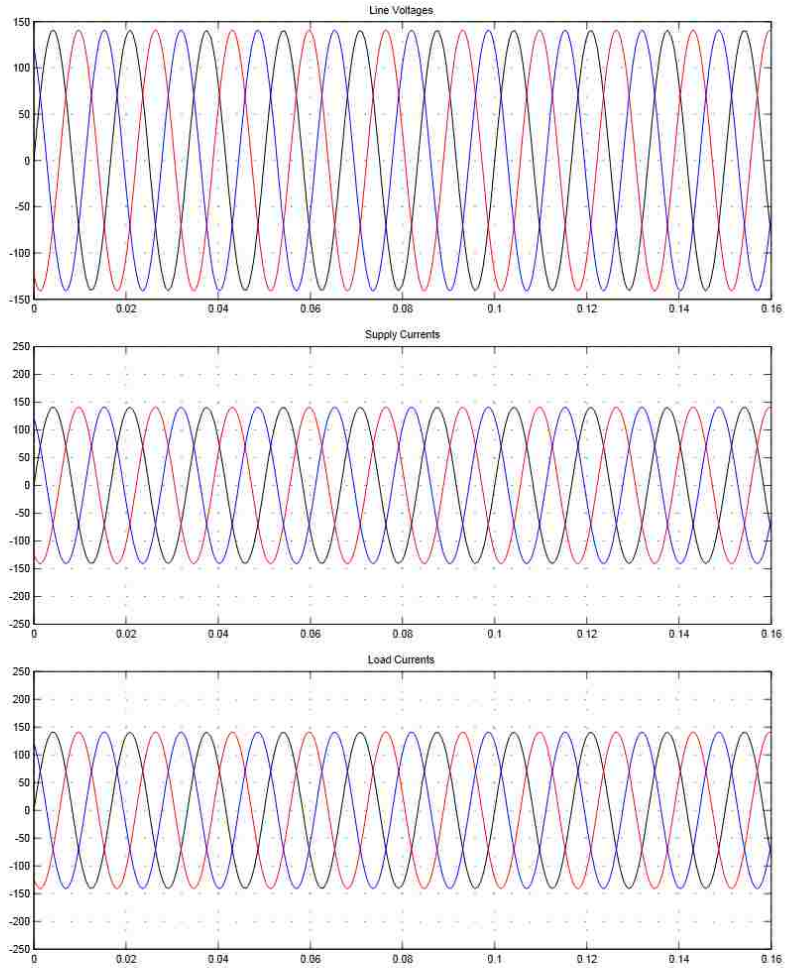


Fig. 6.4 Simulation results of balanced LTI load supplied with sinusoidal symmetrical voltage

This case is included to verify that the model is operating correctly. As shown in Fig. 6.4, the line currents and voltages are in phase with one another while the supply current is the same as the load current. As expected, a balanced resistive load draws sinusoidal symmetrical current from a source of sinusoidal symmetrical voltage.

2. Unbalanced LTI load supplied from a source of sinusoidal symmetrical voltage

A sinusoidal symmetrical voltage of rms value 100 volts was connected to an unbalanced LTI load as shown in Fig. 6.5. The results of simulation are shown in Fig. 6.6.

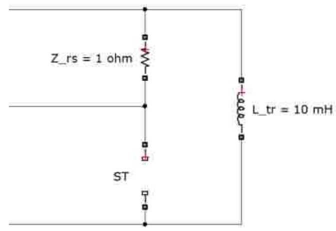


Fig. 6.5 Unbalanced LTI load

Unbalanced load draws three-phase currents which are also unbalanced. The compensator is turned on at t = 0.05 s, after which the supply is only loaded with the active current, while the load still draws its normal current. Supply current after compensation is in phase with the supply voltage. Also, there is a significant reduction in the supply current rms value after compensation.

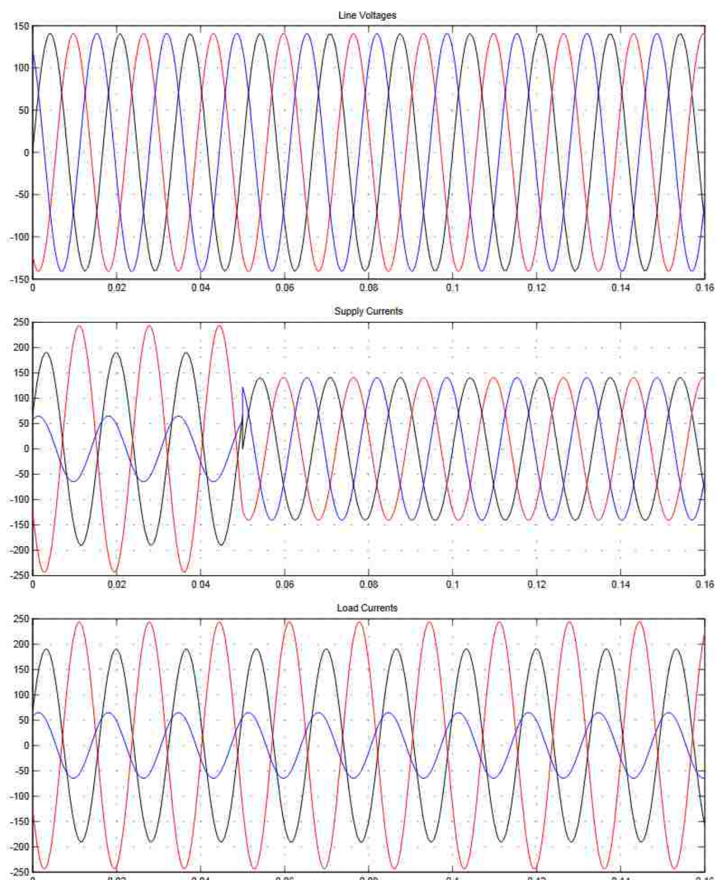


Fig. 6.6 Simulation results of unbalanced LTI load with sinusoidal symmetrical voltage

3. Unbalanced LTI load supplied from a source of sinusoidal asymmetrical voltage

The same unbalanced load from the previous case and shown in Fig 6.5 was next connected to a sinusoidal but asymmetrical voltage source. The line voltages were equal to $\boldsymbol{U}_{\mathrm{R}} = 100\, e^{j0°}$ V, $\boldsymbol{U}_{\mathrm{S}} = 84.8\, e^{-j100°}$ V and $\boldsymbol{U}_{\mathrm{T}} = 120\, e^{j135°}$ V. The simulation results are shown in Fig. 6.7.

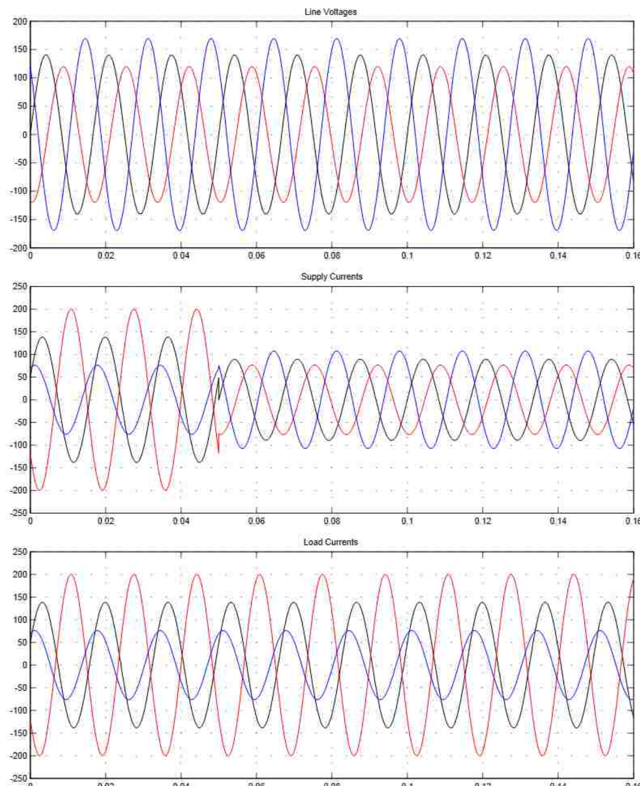


Fig. 6.7 Simulation results of unbalanced LTI load connected to sinusoidal asymmetrical voltage

The supply current before compensation is asymmetrical. After the compensator is turned on at t t = 0.05 s, the compensator compensates the unbalanced and reactive currents. The supply current contains only the active current and hence it is in phase with the supply voltage. There is a reduction in the supply current rms value and the supply currents are proportional to the supply voltages after compensation. The load current is not affected by the addition of the compensator.

4. Unbalanced LTI load supplied from a source of nonsinusoidal symmetrical voltage

Next, the unbalanced LTI load was connected to a source of symmetrical, but nonsinusoidal voltage with the crms values of harmonics

$$\boldsymbol{U}_{\mathrm{R1}} = 100\, e^{j0^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{R5}} = 20\, e^{j0^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{R7}} = 20\, e^{j0^\circ}\ \mathrm{V},$$

$$\boldsymbol{U}_{\mathrm{S1}} = 100\, e^{-j120^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{S5}} = 20\, e^{j120^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{S7}} = 20\, e^{-j120^\circ}\ \mathrm{V},$$

$$\boldsymbol{U}_{\mathrm{T1}} = 100\, e^{j120^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{T5}} = 20\, e^{-j120^\circ}\ \mathrm{V},\quad \boldsymbol{U}_{\mathrm{T7}} = 20\, e^{j120^\circ}\ \mathrm{V}.$$

The results of simulation are shown in Fig. 6.8. The simulation can be divided into three sections. The first section is from the beginning to t_1=0.05 s, when the supply voltage is sinusoidal and symmetrical, and the compensator is off. From 0.05 to 0.1 s, the voltage is the same but the compensator is turned on. The voltage harmonics are introduced at t_2 = 0.1 s with the compensator remaining on till the end. From the beginning to t_1=0.05 s, the supply voltage is sinusoidal and symmetrical. Since the load is unbalanced, it draws unbalanced current during this period of time. The supply current is the same as the load current. The compensator is turned on at t_1=0.05 s and kept on until the end of the simulation. After the compensator is in turned on, it compensates the reactive and unbalanced currents and the supply current is loaded with only the active current. As the voltage is sinusoidal and symmetrical, the supply current after compensation is also sinusoidal and symmetrical. The load current is not affected by the addition of the compensator. Next, voltage harmonics of the fifth and the seventh harmonic orders are added to the supply voltage at t_2 = 0.1 s. After the supply is loaded with the voltage harmonics, there is a change in the load current, as it now draws distorted current from the source. The compensator adjusts to the change in the voltage and injects the necessary current into the lines so that the supply current is equal to the active current. As a result, supply current is proportional to the supply voltage after compensation.

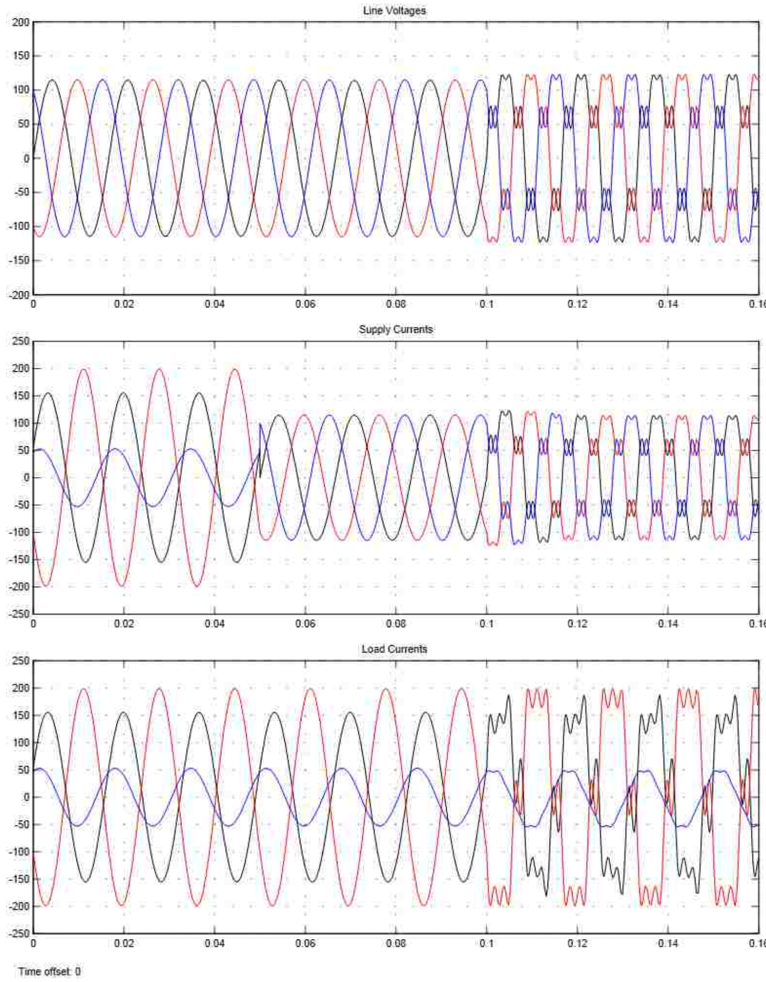


Fig. 6.8 Simulation results of unbalanced LTI load supplied with nonsinusoidal voltage

5. HGL supplied from a source of sinusoidal asymmetrical voltage

Next, a Harmonic Generating Load (HGL) as shown in Fig. 6.9 was connected to a sinusoidal symmetrical voltage with line voltage rms value equal to 100 volts. This particular case was included to verify that the model with the HGL is working as per the expectations. The results of simulation are shown in Fig. 6.10.

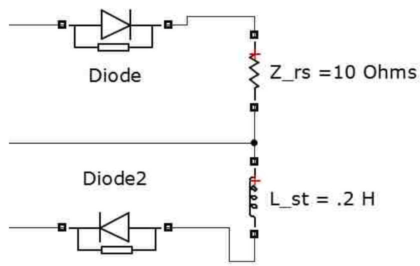


Fig. 6.9 Harmonics Generating Load

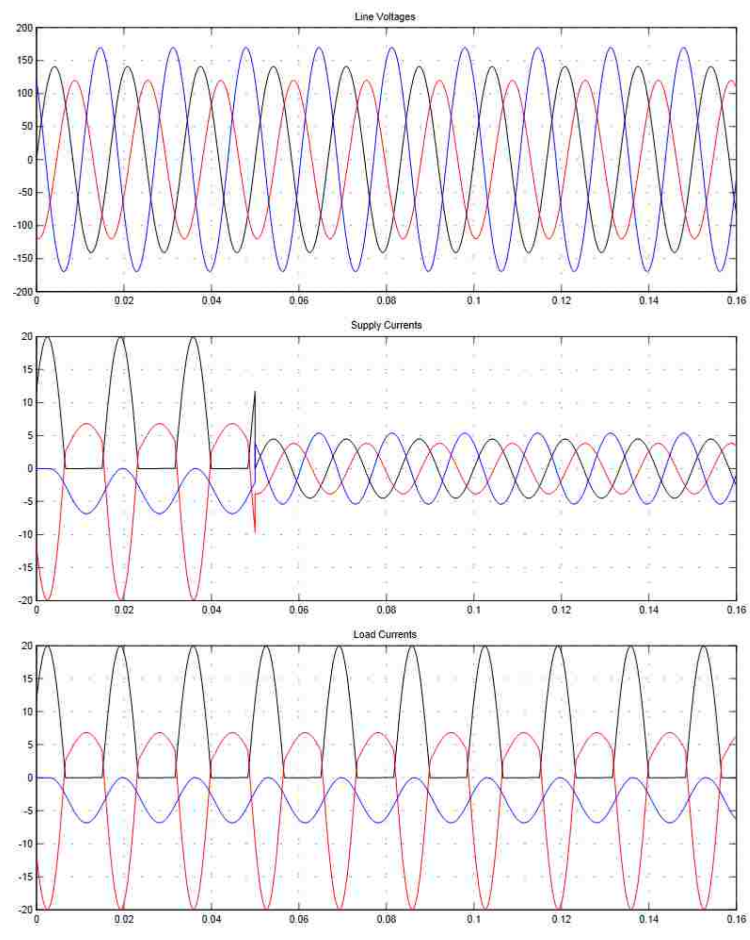


Fig. 6.10 Simulation results of HGL connected to sinusoidal asymmetrical voltage

Before the compensator is turned on, the load draws nonsinusoidal current from the source. The load current is equal to the supply current. This can be seen in the plots from the beginning to t = 0.05 s. After the compensation is turned on at t = 0.05 s, the compensator injects the non-active components of the load current into the distribution system and the supply is loaded with only the active current. As a result, the supply current is in phase with the supply voltage. The peak and rms values of the supply current waveforms are much smaller compared to those of the load current after compensation. The load current is not naffected by the compensator.

6. HGL supplied from a source of nonsinusoidal and symmetrical voltage

The HGL shown in Fig. 6.9 was connected to nonsinusoidal voltage with the crms values of the harmonics

$\boldsymbol{U}_{\mathrm{R1}} = 100\ e^{j0°}$ V, $\boldsymbol{U}_{\mathrm{R5}} = 20\ e^{j0°}$ V, $\boldsymbol{U}_{\mathrm{R7}} = 20\ e^{j0°}$ V,,

$\boldsymbol{U}_{\mathrm{S1}} = 100\ e^{-j120°}$ V, $\boldsymbol{U}_{\mathrm{S5}} = 20\ e^{j120°}$ V, $\boldsymbol{U}_{\mathrm{S7}} = 20\ e^{-j120°}$ V,

$\boldsymbol{U}_{\mathrm{T1}} = 100\ e^{j120°}$ V, $\boldsymbol{U}_{\mathrm{T5}} = 20\ e^{-j120°}$ V, $\boldsymbol{U}_{\mathrm{T7}} = 20\ e^{j120°}$ V .

The voltage harmonics were introduced at t_2 = 0.1 s while the compensator was turned on at t_1 = 0.1 s. The result of compensation is shown in Fig. 6.10. Even when the voltage is sinusoidal and symmetrical, the HGL load draws nonsinusoidal current from the source. This is seen between the start to t_1 = 0.05 s in the figure. The compensator current is zero in this period. When the compensator is turned on at t_1 = 0.05 s, it injects the non-active components of the current into the distribution system. As a result, the supply is only loaded with the active currents. The supply current is proportional to the supply voltage. Since the supply voltage is sinusoidal and symmetrical, the supply current after compensation is also sinusoidal and symmetrical in this period of time. The load current on the other hand is unaffected by the compensator and draws its normal current, which is nonsinusoidal. Next, the voltage harmonics are introduced at t_2 = 0.1 s. The load current also changes as a result. Since the compensator is already on, it adjusts to the change in the supply voltage and injects a different compensator current into the distribution system. As a result, the supply current is still in phase with the supply voltage. Observe that the supply current after compensation is proportional to the supply voltage and since the voltage is distorted, so is the supply current after compensation. The load with the compensator has ideal loading quality.

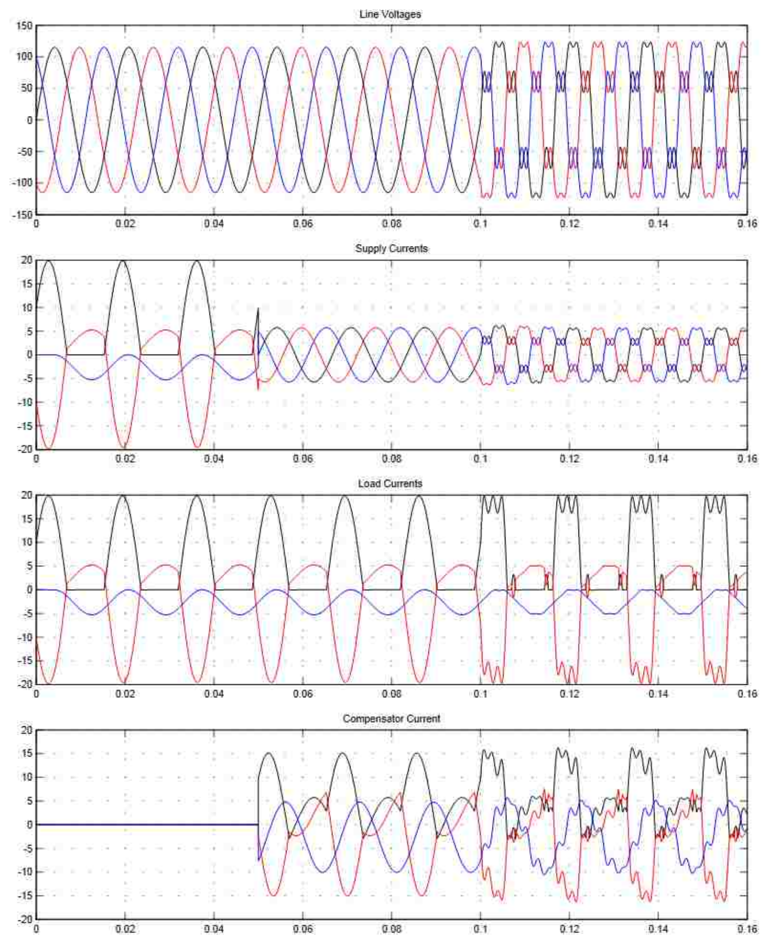


Fig. 6.11 Simulation results of HGL connected to nonsinusoidal voltage

Observe that for each of the cases given above, the supply current is proportional to the supply voltage after compensation. This result may seem strange, in particular because the current after compensation in most of the above given cases is either asymmetrical, or distorted, or both. This is because the goal of compensation of the algorithm used above is to reduce the supply current after compensation to the active current. As such, whatever the nature of the voltage waveform, is reflected on the supply current waveform after compensation. There are some approaches where the goal of compensation is to get sinusoidal symmetrical current even at nonsinusoidal or asymmetrical supply voltage. Although achieving such a goal is possible, the rms of the resulting supply current after compensation is higher than the rms value of the active current, which is the minimum current necessary for the energy transfer from the supply to the load.

6.8 Conclusion

Some of the most commonly used algorithms for the generations of the reference current signal in Switching Compensators do not produce the desired results at voltage asymmetry and distortion. This is mainly because such algorithms are not able to distinguish the degradation of Loading Quality from that of Supply Quality. That shortcoming is overcome in this chapter by developing the compensator reference current algorithm based on the Currents Physical Components (CPC) power theory which distinguishes degraded Supply Quality from degraded Loading Quality. As a result, the CPC based algorithm, with the goal of compensation to reduce the supply current to the active current after compensation, improves the power factor to unity of both linear as well as Harmonic Generating Loads, at supply voltage asymmetry as well as distortion. The waveform supply current after compensation is in phase and proportional to the supply voltage.

CHAPTER 7: CONCLUSION AND FUTURE WORK

7.1 Conclusion

This dissertation presents a solution of one of the unsolved problems of electrical engineering, namely, how to describe power properties and how to compensate three-phase loads supplied with asymmetrical and nonsinusoidal voltage.

Results obtained in the research and reported in this dissertation enable:

- description of loads supplied with asymmetrical and nonsinusoidal voltage and currents in power terms
- design of compensator for power factor improvement of such loads.

These results apply to Linear Time Invariant (LTI) as well as to Harmonic Generating Loads (HGLs).

It is also now known that voltage asymmetry does affect the form of the power equation of LTI loads. The parameters that the load powers are dependent on are affected, however.

The results presented in this dissertation contribute to closing the chapter on power theory development of three-phase three-wire systems with periodic voltages and currents. Results of these studies also provide the answer to one of the most important questions on compensation, namely the power factor of LTI as well as HGL loads can be improved to unity even at the supply voltage asymmetry.

This dissertation also demonstrates that the Currents' Physical Components (CPC) framework can be extended to three-phase systems with asymmetrical voltages and currents, and to Harmonic Generating Loads (HGLs).

7.2 Suggestions for future work

Results in this dissertation were obtained based on theoretical analysis. Their application to compensator control could be seem as a next step in research. Also, the scope of the research presented in this dissertation was limited to three-phase three-wire systems. It can be extended to three-phase four-wire systems. Similarly, this dissertation can be used as a platform for the description of the power properties of three-phase loads with non-periodic voltages and currents. The CPC based algorithm can also be used for the design and control of hybrid compensators.

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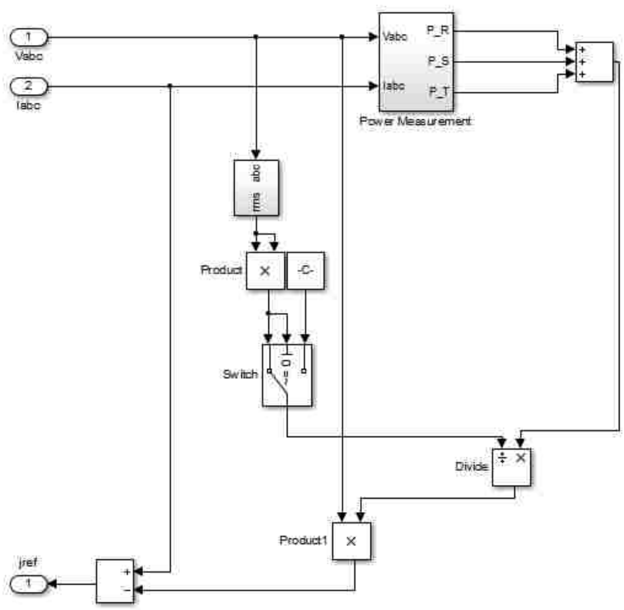
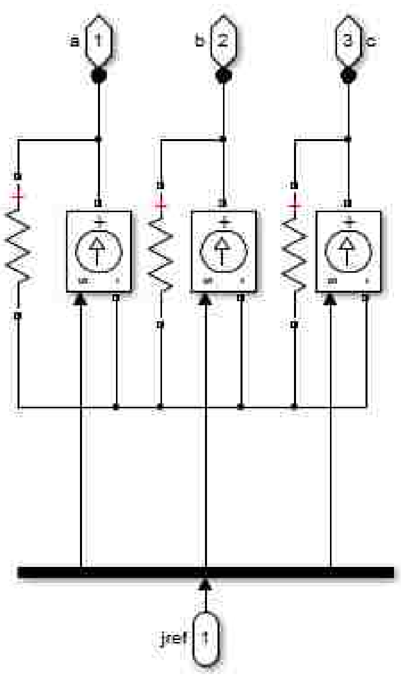
APPENDIX A – SIMULINK BLOCK DIAGRAMS

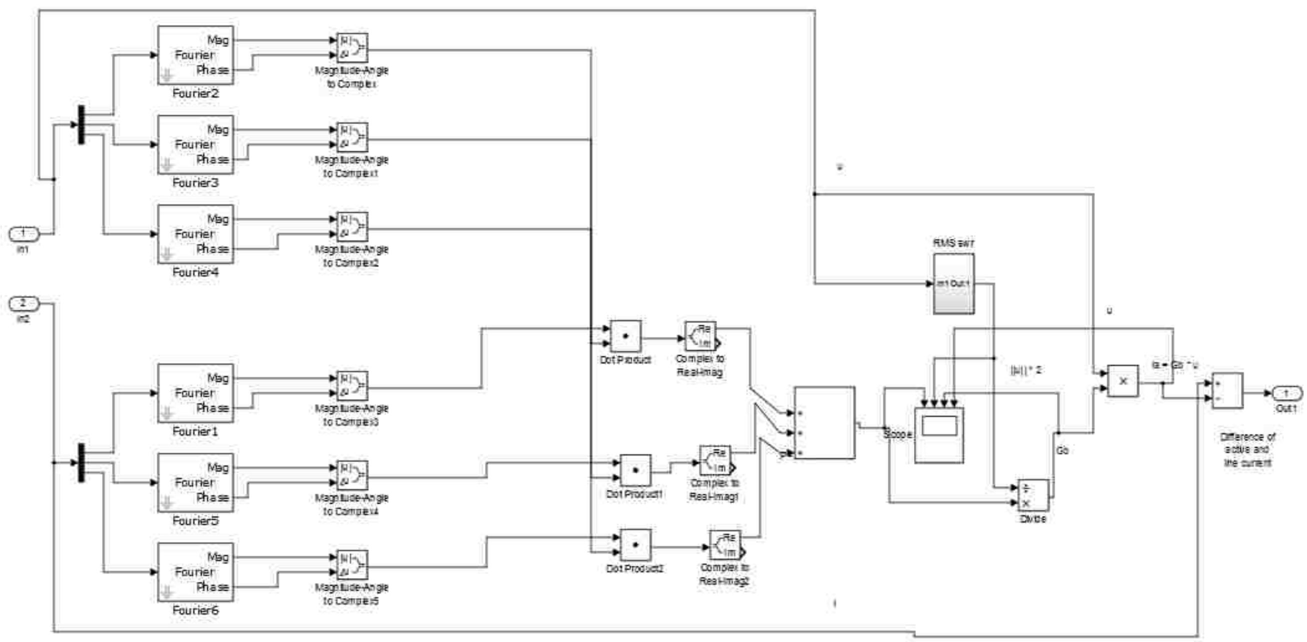
Block for active current calculation



Block for compensator current generation



Block for harmonic current components



APPENDIX B – MATLAB CODES

Code used for Asymmetrical Sinusoidal Voltage and LTI Load

```
% IT solves A&S for LTI
% Problem of incorrect Yd corrected in this file

% This part is for initialization for constants
clear all
clc
alp = -.5 + ((sqrt(3))/2) * 1i ;
f=60;
omega = 2*pi * f ;

% % Ex.1: Supply is symmetrical and load is balanced and resistive
% %LOAD PARAMETERS
% Z_rs = 1 ;
% Z_st = 1;
% Z_tr =  1;
% %SUPPLY PARAMETERS
% U_r = 100+0i;
% U_s = -50 - (50 * sqrt(3))*1i;
% U_t = -50 + (50 * sqrt(3))*1i;

%%Ex:2, Refer illust 4,pg 28 onwards of Ch.7 of Professor's book
% % LOAD PARAMETERS
Z_rs = inf ;
Z_st = 1;
Z_tr = 0+1*1i;

% %% Unbalanced Resistive Load
% Z_rs = 140;
% Z_st = 15;
% Z_tr = 25;
%%
% %SUPPLY PARAMETERS S&S
% U_r = 100+0i;
% U_s = -50 - (50 * sqrt(3))*1i;
% U_t = -50 + (50 * sqrt(3))*1i;

% SUPPLY PARAMETERS : Used for illustration and in General Exam
U_r =100;
U_s= -50 -sqrt(3)*50i;
U_t =0;


% ######################################################################
% The actual program starts from here.... everything before this is given
% Calculations of basic paramters based on input

Y_rs = 1 / ( Z_rs);
Y_st = 1 / ( Z_st);
Y_tr = 1 / ( Z_tr);
```

```
%U_rms = sqrt((U_r * conj(U_r))+(U_s * conj(U_s))+(U_t * conj(U_t))) ; % This
line is key !!!


% Calculation of more specific parameters

U_p = (1/3) * ( U_r + alp * U_s + (power(alp,2)) * U_t);% Pos seq voltage
U_n = (1/3) * ( U_r + (power(alp,2)) * U_s+ alp * U_t );%Neg seq vol
U_z = (1/3) * ( U_r +  U_s + U_t ); % zero seq voltage
U_rms_art = sqrt(3) * sqrt((U_p * conj(U_p))+(U_n * conj(U_n))) ;

% VOLTAGES
U_r_art = U_p + U_n ;
U_s_art= U_p * conj(alp) + alp * U_n;
U_t_art= U_p * (alp) + conj(alp) * U_n;
U_rs_art = U_r_art - U_s_art ;
U_st_art = U_s_art - U_t_art ;
U_tr_art = U_t_art - U_r_art ;
% BRANCHE CURRENT
i_rs = Y_rs * U_rs_art ;
i_st = Y_st * U_st_art ;
i_tr = Y_tr * U_tr_art ;
%LINE CURRENTS
i_R = i_rs-i_tr;
i_S = i_st-i_rs;
i_T = i_tr-i_st;
i_rms1 = sqrt( power(abs(i_R),2) + power(abs(i_S),2) + power(abs(i_T),2));
%COMPLEZ POWER
CP = U_r_art * conj(i_R)+ U_s_art*conj(i_S)+U_t_art*conj(i_T);
% POWERS CACULATED FROM COMPLEX POWER
P = real (CP) ;
Q = imag (CP) ;
% EQUIVALENT PARAMETERS FOR THE NEW BALANCED CIRCUIT
Y_b = conj(CP) / power (U_rms_art,2) ;
G_b = real(Y_b); % real of Y_b or eqv balanced conductance
B_b = imag(Y_b); % eqv balanced susceptance

Y_e = Y_rs + Y_st + Y_tr; % eqv admittance
G_e = real(Y_e);% eqv conductance
B_e = imag(Y_e);%eqv susceptance

Y_d = Y_e - Y_b ;
A_p = -1 * ( Y_st + alp * Y_tr + (conj(alp)) * Y_rs );  % posi. seq unbalance
admittance
A_n = -1 * ( Y_st + conj(alp) * Y_tr + alp * Y_rs ) ; % neg seq unbalance
admittance

i_act = G_b * U_rms_art;
i_rea = abs(B_b) * U_rms_art;
I_Ru_P = A_n * U_n + Y_d * U_p;
I_Ru_N = A_p * U_p + Y_d * U_n;
i_unb = sqrt(3) * sqrt( power(abs(I_Ru_P),2) + power(abs(I_Ru_N),2) ) ;

i_rms = sqrt( power(i_act,2) + power(i_rea,2) + power(i_unb,2) );
```

```
P_2 = U_rms_art * i_act ;
Q_2 = U_rms_art * i_rea ;
D_u = U_rms_art * i_unb ;

S= sqrt( power(P,2) + power(Q,2) + power(D_u,2));

pf = P_2 / S ;

a_complex = (U_n /U_p);  % this is complex quantity "a"
a=abs(a_complex);
theta=angle(a_complex);

Y_d1 = (( 3 / power(U_rms_art,2) ) * ( ( Y_st * (U_r_art * conj(U_r_art)))+ (
Y_tr * (U_s_art * conj(U_s_art)))+( Y_rs * (U_t_art * conj(U_t_art))))) -
Y_e;
% Y_d is Asymmetry dependent unbalance admittance
B_c_b = -1 * B_b ;

% the following are terms that i used in my analysis to simplify
% expressions... they will be easier to follow if my paperwork is referenced
Ycd_tst = (1i * 2*a*cos(theta)) / ( 1 + power(a,2)) ;% this variable is c1 in
paper
Ycd_ttr = (1i * 2*a*cos(theta-(2*pi/3))) / ( 1 + power(a,2)); % this is c2
Ycd_trs = (1i * 2*a*cos(theta+(2*pi/3))) / ( 1 + power(a,2)); % this is c3

%calculated coeffieceints of Trs, Tst and Ttr in equation

% again, A B C and D calcualted below are there to simplify analysis. They
% will be easier to follow if my paperwork is referenced.

% I am calling the coef or Trs as A , Tst as B and Ttr as C....
A = (1+a_complex)* Ycd_trs - 1i*(conj(alp)+a_complex*alp );
B = (1+a_complex)* ( Ycd_tst - 1i);
C = (1+a_complex)* Ycd_ttr - 1i*(alp+a_complex*conj(alp));
D = Y_d * ( 1 + a_complex) + A_p + a_complex * A_n ;

% calculation of coeffieceints in final equation
 % real and imaginary parts of the coefficeints...
A_real = real(A);
B_real = real(B);
C_real = real(C);
D_real = real(D);
A_imag = imag(A);
B_imag = imag(B);
C_imag = imag(C);
D_imag = imag(D);

% Declaring matrices coef_mat and cnst_mat for solving
% Dont confuse these elements A_mn and B_mn with the earlier A B C and D

% This is to declare matrix as coefficeints of Trs, Tst and Ttr in eqns
A_11 = U_rs_art*conj(U_rs_art);
A_12 = U_st_art*conj(U_st_art);
```

```
A_13 = U_tr_art*conj(U_tr_art);
A_21 = A_real;
A_22 = B_real;
A_23 = C_real;
A_31 = A_imag;
A_32 = B_imag;
A_33 = C_imag;

B_11 = -1 *B_b *  power(U_rms_art,2);
B_21 = -1* D_real;
B_31 = -1* D_imag;

% declaring matrix

coef_mat = [A_11,A_12,A_13;A_21,A_22,A_23;A_31,A_32,A_33];
cons_mat = [B_11;B_21;B_31];

%% Alternative Compensator

K_con = Y_d * U_s_art + alp * A_p * U_p + A_n * conj(alp) * U_n;
K_trs = Ycd_trs * U_s_art- 1i * U_p - 1i * U_n;
K_tst = Ycd_tst * U_s_art- 1i * alp * U_p - 1i * conj(alp) * U_n;
K_ttr = Ycd_ttr * U_s_art- 1i * conj(alp) * U_p - 1i *alp *  U_n;

coef_mat_alt =
[A_11,A_12,A_13;real(K_trs),real(K_tst),real(K_ttr);imag(K_trs),imag(K_tst),i
mag(K_ttr)];
cons_mat_alt = [B_11;-real(K_con);-imag(K_con)];
ans_mat_alt = inv(coef_mat_alt) * cons_mat_alt;


coeff_new =
[A_11,A_12,A_13;real(K_trs),real(K_tst),real(K_ttr);A_31,A_32,A_33];
cons_new = [B_11;-real(K_con);B_31];




ans_mat_new = inv(coeff_new) * cons_new;  % the values of compnesator
susceptances
T_rs_new = ans_mat_new(1,1);
T_st_new = ans_mat_new(2,1);
T_tr_new = ans_mat_new(3,1);
%%
ans_mat = inv(coef_mat) * cons_mat;  % the values of compnesator susceptances
T_rs = ans_mat(1,1);
T_st = ans_mat(2,1);
T_tr = ans_mat(3,1);

% n = 1 for only original , n= 2 for only alternative, n=0 or other for
% both but the remaining program has original trs

n_sel = 0;
% Section to Choose
if n_sel ==1
```

```
     T_rs = ans_mat(1,1);
     T_st = ans_mat(2,1);
     T_tr = ans_mat(3,1);
    disp( ' ');
    disp( ' ORIGINAL COMPENSATOR VALUES IN SIEMENS');
    fprintf(' T_rs = %d \n',T_rs);
    fprintf(' T_st = %d \n',T_st);
    fprintf(' T_tr = %d \n',T_tr);
    disp(' The Structure of Compensator : ');
else if n_sel==2
    T_rs = ans_mat_alt(1,1);
        T_st = ans_mat_alt(2,1);
        T_tr = ans_mat_alt(3,1);

disp( ' ---------      ----------');
disp( ' ');
disp( ' ALTERNATIVE COMPENSATOR VALUES IN SIEMENS : these used for the
remained of the program: ');
fprintf(' T_rs = %d \n',T_rs);
fprintf(' T_st = %d \n',T_st);
fprintf(' T_tr = %d \n',T_tr);

      else
        T_rs_alt = ans_mat_alt(1,1);
        T_st_alt = ans_mat_alt(2,1);
        T_tr_alt = ans_mat_alt(3,1);

disp( ' ---------      ----------');
disp( ' ');
disp( ' ALTERNATIVE COMPENSATOR VALUES IN SIEMENS');
fprintf(' T_rs_ALT = %d \n',T_rs_alt);
fprintf(' T_st_ALT = %d \n',T_st_alt);
fprintf(' T_tr_ALT = %d \n',T_tr_alt);
    T_rs = ans_mat(1,1);
     T_st = ans_mat(2,1);
     T_tr = ans_mat(3,1);
    disp( ' ');
    disp( ' ORIGINAL COMPENSATOR VALUES IN SIEMENS');
    fprintf(' T_rs = %d \n',T_rs);
    fprintf(' T_st = %d \n',T_st);
    fprintf(' T_tr = %d \n',T_tr);
    disp(' The Structure of Compensator : ');

   end
end


if T_rs < 0
    X_rs = -1 / ( omega * T_rs ) ;
    fprintf(' Branch RS contains inductor of value ( Henry) : %d \n',X_rs);
else
    X_rs = T_rs / omega ;
    fprintf(' Branch RS contains capacitor of value ( Farads) : %d \n',X_rs);
end
```

```
if T_st < 0
    X_st = -1 / ( omega * T_st ) ;
    fprintf(' Branch ST contains inductor of value ( Henry) : %d \n',X_st);
else
    X_st = T_st / omega ;
    fprintf(' Branch ST contains capacitor of value ( Farads) : %d \n',X_st);
end

if T_tr < 0
    X_tr = -1 / ( omega * T_tr ) ;
    fprintf(' Branch TR contains inductor of value ( Henry) : %d \n',X_tr);
else
    X_tr = T_tr / omega ;
    fprintf(' Branch TR contains capacitor of value ( Farads) : %d \n',X_tr);
end


% THE FOLLOWING PORTION CALCULATES CURRENTS AND VOLTAGES BEFORE AND AFTER
% THE COMPENSATOR AND CALCULATES THE PF AND VERIFIES IF COMPENS. WORKS

%The following are the values before compensation



val =[P,Q,D_u,S,pf];
disp( ' ');
disp('              ');
disp( ' ALL VALUES ARE W.R.T. ARTIFICAL ZERO ' );

disp( ' VALUES BEFORE COMPENSATION : ' );
disp( '                               ' );
fprintf(' Active current = %d \n',i_act);
fprintf(' Reactive current  = %d \n',i_rea);
fprintf(' unbalance current  = %d \n',i_unb);
fprintf(' Total RMS = %d \n',i_rms);
disp( '                               ' );
fprintf(' Current in R Phase  = %d \n',abs(i_R));
fprintf(' Current in S Phase  = %d \n',abs(i_S));
fprintf(' Current in T Phase  = %d \n',abs(i_T));
fprintf(' RMS Current =  %d \n',i_rms1);

disp('                   ');
fprintf('Active Power P = %d \n',P);
fprintf(' Reactive Power Q = %d \n',Q);
fprintf(' Unbalanced Power Du = %d \n',D_u);
fprintf(' Apparent Power S = %d \n',S);
fprintf(' Power Factor PF = %d \n',pf);

% The following are the values after compensation :

%% Calculation of equivalent admittances after compensation

Y_rs_new = Y_rs + T_rs * 1i ;
Y_st_new = Y_st + T_st * 1i ;
Y_tr_new = Y_tr + T_tr * 1i;
```

```
% Imp of load and comp in delta structure
Z_rs_n_d = 1 / Y_rs_new;
Z_st_n_d = 1 / Y_st_new;
Z_tr_n_d = 1 / Y_tr_new;

Z_r_n_d = ( Z_rs_n_d * Z_tr_n_d ) / ( Z_rs_n_d +Z_st_n_d + Z_tr_n_d);
Z_s_n_d = ( Z_st_n_d * Z_rs_n_d ) / ( Z_rs_n_d +Z_st_n_d + Z_tr_n_d);
Z_t_n_d = ( Z_st_n_d * Z_tr_n_d ) / ( Z_rs_n_d +Z_st_n_d + Z_tr_n_d);

i_rs_comp = T_rs*1i * U_rs_art ;
i_st_comp = T_st*1i * U_st_art ;
i_tr_comp = T_tr*1i * U_tr_art ;

i_r_comp = i_rs_comp-i_tr_comp;
i_s_comp = i_st_comp- i_rs_comp;
i_t_comp = i_tr_comp- i_st_comp;

i_R_new = i_R + i_r_comp;
i_S_new = i_S + i_s_comp;
i_T_new = i_T + i_t_comp;

i_R_new_rms = abs(i_R_new);
i_S_new_rms = abs(i_S_new);
i_T_new_rms = abs(i_T_new);
% CUrrent may still not be equal... need to calculate reactive and unbl cur

Y_e_new = Y_rs_new +Y_st_new+Y_tr_new;

G_e_new = real(Y_e_new);
B_e_new = imag(Y_e_new);

Y_d_new = (( 3 / power(U_rms_art,2) ) * ( ( Y_st_new * (U_r_art *
conj(U_r_art)))+ ( Y_tr_new * (U_s_art * conj(U_s_art)))+( Y_rs_new *
(U_t_art * conj(U_t_art))))) - Y_e_new;
Y_b_new = Y_e_new - Y_d_new;

G_b_new = real(Y_b_new);
B_b_new = imag(Y_b_new);

% I tihnk this is where the problem is ...
A_p_new = -1 * ( Y_st_new + alp * Y_tr_new + (conj(alp)) * Y_rs_new ) ; %
posi. seq unbalance admittance
A_n_new = -1 * ( Y_st_new + conj(alp) * Y_tr_new + alp * Y_rs_new ) ; % neg
seq unbalance admittance

i_act_new = G_b_new * U_rms_art;
i_rea_new = (B_b_new) * U_rms_art;

I_Ru_P_new = A_n_new * U_n + Y_d_new * U_p;
I_Ru_N_new = A_p_new * U_p + Y_d_new * U_n;

i_unb_new = sqrt(3) * sqrt( power(abs(I_Ru_P_new),2) +
power(abs(I_Ru_N_new),2) ) ;
```

```
% % to check for erros
%
% err2 = A_n - 1i * ( T_st + T_tr * conj(alp) + alp * T_tr);
% err1 = A_p - 1i * ( T_st + T_tr * (alp) + conj(alp) * T_tr);
% err3 = Y_d + ((2*a)/(1+power(a,2)))*1i * ( T_st

% ERROR CACLULATIONS PART ....

% although A_c_p and A_c_n not used in this code, these values used for
% checking ...

A_c_p = -1i * ( T_st + alp * T_tr + (conj(alp)) * T_rs );  % posi. seq
unbalance admittance
A_c_n = -1i * ( T_st + conj(alp) * T_tr + alp * T_rs )  ;% neg seq unbalance
admittance
Y_c_d =   ( 2*a*1i / ( 1 + power(a,2))) * ( T_st * cos (theta) + T_tr * cos
(theta - 2*pi/3 ) + T_rs * cos(theta+2*pi/3));


err1 = T_rs * A_11 + T_st * A_12 + T_tr * A_13 + B_b * power (U_rms_art,2);
err11 = T_rs * power(abs(U_rs_art),2) + T_st * power(abs(U_st_art),2) + T_tr
* power(abs(U_tr_art),2) + power(U_rms_art,2)* B_b;
err2 = T_rs * real(A) + T_st * real(B) + T_tr * real(C)+ real(D);

err3 = T_rs * imag(A) + T_st * imag(B) + T_tr * imag(C)+ imag(D);
err4 = A * T_rs + B * T_st * C * T_tr + D;

err22 = real(T_rs * A + T_st * B + T_tr * C + D);
err33 = imag(T_rs * A + T_st * B + T_tr * C + D);
% all athe above were found to be almost or practiacally 0

err5 = (A_c_p + A_p ) + ( A_c_n + A_n) * a_complex + ( Y_c_d + Y_d )*(1 +
a_complex);

% checking to see if the voltages wrt aritifical zeros are in some
% rations..
U_r_art_rms = abs(U_r_art);
U_s_art_rms = abs(U_s_art);
U_t_art_rms = abs(U_t_art);

Rat_vol_sr = U_s_art_rms / U_r_art_rms ;
Rat_vol_tr = U_t_art_rms / U_r_art_rms ;

Rat_cur_sr = i_S_new_rms / i_R_new_rms ;
Rat_cur_tr = i_T_new_rms / i_R_new_rms ;

% dISPLAYING    results...

i_rms_new = sqrt( power(i_act_new,2) + power(i_rea_new,2) +
power(i_unb_new,2) );

P_new = U_rms_art * i_act_new ;
Q_new = U_rms_art * i_rea_new ;
```

```
D_u_new = U_rms_art * i_unb_new ;
S_new= sqrt( power(P_new,2) + power(Q_new,2) + power(D_u_new,2));

pf_new = P_new / S_new ;

val =[P_new,Q_new,D_u_new,S_new,pf_new];
disp( ' ');
disp('              ');
disp( ' ALL VALUES ARE W.R.T. ARTIFICAL ZERO ' );

disp( ' VALUES AFTER COMPENSATION : ' );
disp( '                               ' );
fprintf(' Active current = %d \n',i_act_new);
fprintf(' Reactive current  = %d \n',i_rea_new);
fprintf(' unbalance current  = %d \n',i_unb_new);
fprintf(' Total RMS = %d \n',i_rms_new);
disp( '                               ' );
fprintf(' Current in R Phase after compensation ( wrt art zero ) = %d
\n',abs(i_R_new_rms));
fprintf(' Current in S Phase after compensation ( wrt art zero ) = %d
\n',abs(i_S_new_rms));
fprintf(' Current in T Phase after compensation ( wrt art zero ) = %d
\n',abs(i_T_new_rms));
fprintf(' RMS Current after compensation ( wrt art zero ) =  %d
\n',i_rms_new);

disp('                  ');
fprintf('Active Power after compensation P = %d \n',P_new);
fprintf(' Reactive Power after compensation Q = %d \n',Q_new);
fprintf(' Unbalanced after compensation Power Du = %d \n',D_u_new);
fprintf(' Apparent Power after compensation S = %d \n',S_new);
fprintf(' Power Factor after compensation PF = %d \n',pf_new);

disp ( '                  ' );
fprintf(' RMS of voltage Ur wrt art zero = %d \n',U_r_art_rms);
fprintf(' Angle of voltage Ur wrt art zero = %d \n',angle(U_r_art));
fprintf(' RMS of voltage Us wrt art zero = %d \n',U_s_art_rms);
fprintf(' Angle of voltage Us wrt art zero = %d \n',angle(U_s_art));
fprintf(' RMS of voltage Ut wrt art zero = %d \n',U_t_art_rms);
fprintf(' Angle of voltage Ut wrt art zero = %d \n',angle(U_t_art));

fprintf(' RMS of Line current Ir after compensation wrt art zero = %d
\n',i_R_new_rms);
fprintf(' Angle of Current Ir wrt art zero after compensation = %d
\n',angle(i_R_new));
fprintf(' RMS of Line current Is after compensation wrt art zero = %d
\n',i_S_new_rms);
fprintf(' Angle of Current Is wrt art zero after compensation = %d
\n',angle(i_S_new));
fprintf(' RMS of Line current It after compensation wrt art zero = %d
\n',i_T_new_rms);
fprintf(' Angle of Current It wrt art zero after compensation = %d
\n',angle(i_T_new));
disp(' ');
disp(' Ir, I_s and I_t after compensation wrt art zero are in phase with U_r,
U_s and U_t ');
```

```
disp(' ');
fprintf(' Ratio of voltage Us_art and Ur_art  = %d \n',Rat_vol_sr);
fprintf(' Ratio of current Is_art and Ir_art  = %d \n',Rat_cur_sr);
fprintf(' Ratio of voltage Ut_art and Ur_art  = %d \n',Rat_vol_tr);
fprintf(' Ratio of current It_art and Ir_art  = %d \n',Rat_cur_tr);

disp (' new test : ' );
Y_cd = Ycd_tst * T_st + Ycd_ttr * T_tr +Ycd_trs * T_rs ;
Acp = - 1i * ( T_st + alp * T_tr + conj(alp)*T_rs);
Acn = - 1i * ( T_st + conj(alp) * T_tr + alp*T_rs);

(Y_cd + Y_d);
(Acp + A_p);
(Acn+A_n);
eq_val = (Y_cd + Y_d)* U_s_art + (Acp + A_p)*alp*U_p +
(Acn+A_n)*conj(alp)*U_n;
```

Code for Asymmetrical nonsinusoidal system

```
%% INITIAL PARAMETER INITIALIZATION
clc
clear all
alp = -.5 + ((sqrt(3))/2) * 1i ;
% % This is voltage is similar to illustration
%
% U_r = [100,0,0,0,50,0,25];
% U_s = [ -50 - 86.6i,0,0,0,-25+43.3j,0,-12.5-21.65i];
% U_t = [0,0,0,0,0,0,0];

% This is where the voltage is defined, change here
U_r = [100,0,0,0,2.5,0,2];
U_s = [ -50 - (50 * sqrt(3))*1i,0,0,0,-2+(1.5*sqrt(3)*1i),0,-2-(2 *
sqrt(3))*1i];
U_t = [0,0,0,0,0,0,0];

% % Symmetrical Distorted supply
% U_r = [90+0i,0,0,0,9,0,9];
% U_s = [ -45 - (45 * sqrt(3))*1i,0,0,0,-4.5 + (45 * sqrt(3))*1i,0,-4.5 -
(4.5 * sqrt(3))*1i];
% U_t = [-45+45*sqrt(3)*1i,0,0,0,-4.5-4.5*sqrt(3)*1i,0,-4.5+4.5*sqrt(3)*1i];

% % % S&S
% U_r = [90+0i,0,0,0,0,0,0];
% U_s = [-45-45*sqrt(3)*1i,0,0,0,0,0,0];
% U_t = [-45+45*sqrt(3)*1i,0,0,0,0,0,0];

% % Zero Seq
% U_r = [90,0,0,0,0,0,0];
% U_s = [90,0,0,0,0,0,0];
```

```
% U_t = [90,0,0,0,0,0,0];

U_bold = [U_r',U_s',U_t'];

THD_r =sqrt( power(abs(U_r(1,5)),2) + power(abs(U_r(1,7)),2) )/
abs(U_r(1,1));
U_bold = [norm(U_r);norm(U_s);norm(U_t)];
U_bold_rms = norm(U_bold);
U_rms_tot_sq=0;P_tot_sq=0;C_n_tot=0;i_r_rms_sq=0;i_a_rms_sq=0;i_u_rms_sq=0;
Q_tot_sq=0;Du_tot_sq=0;i_rms_tot_sq=0;S_ind_sq=0;D_scat_sq=0;C_n1_tot=0;i_rms
_har_sq=0;i_a_h_sq=0;
i_scat_rms_sq=0;i_rms1_sq=0;i_reac_rms_sq=0;i_act_rms_sq=0;i_unb_rms_sq=0;i_a
ct_n_rms_sq=0;i_R_rms_sq = 0 ;i_S_rms_sq = 0;i_T_rms_sq = 0;
i_R_new_rms_sq=0;i_S_new_rms_sq=0;i_T_new_rms_sq=0;U_r_rms_sq=0;U_s_rms_sq=0;
U_t_rms_sq=0;
disp( ' ALL VALUES ARE W.R.T. ARTIFICAL ZERO ' );
disp( ' ---------       ----------');
%% CALCULATION OF ACTIVE POWER AND INITIAL BALANCED
for n=[1,5,7]

    % THIS PART HAS THE ACTUAL LOAD PARAMETERS... CHANGE HERE ...
    Z_rs(1,n)=inf;%
    Z_st(1,n)=(1+1*n*1i)*(-2i/n)/ ((1+1*n*1i)-2i/n);%
    Z_tr(1,n)=1+1*n*1i;%
    Y_rs(1,n)=1/ Z_rs(1,n);
    Y_st(1,n)=1/Z_st(1,n);
    Y_tr(1,n)=1/Z_tr(1,n);

% Calculations of Various voltages ( wrt to Art. Zero )
U_p(1,n) = (1/3) * ( U_r(1,n) + alp * U_s(1,n) + (power(alp,2)) * U_t(1,n));%
Pos seq voltage
U_n(1,n) = (1/3) * ( U_r(1,n) + (power(alp,2)) * U_s(1,n)+ alp * U_t(1,n)
);%Neg seq vol
U_z(1,n) = (1/3) * ( U_r(1,n) +  U_s(1,n) + U_t(1,n) ); % zero seq voltage
U_rms_art(1,n) = sqrt(3) * sqrt((U_p(1,n) * conj(U_p(1,n)))+(U_n(1,n) *
conj(U_n(1,n))));
U_rms_tot_sq=U_rms_tot_sq+power(U_rms_art(1,n),2); % adding up squares of RMS
for later
U_r_art(1,n) = U_p(1,n) + U_n(1,n) ;  % Ur wrt art zero
U_s_art(1,n)=U_p(1,n) * conj(alp) + alp * U_n(1,n);
U_t_art(1,n)=U_p(1,n) * (alp) + conj(alp) * U_n(1,n);
U_rs_art(1,n) = U_r_art(1,n) - U_s_art(1,n) ;
U_st_art(1,n) = U_s_art(1,n) - U_t_art(1,n) ;
U_tr_art(1,n) = U_t_art(1,n) - U_r_art(1,n) ;
U_r_rms_sq = U_r_rms_sq + power(abs(U_r_art(1,n)),2);
U_s_rms_sq = U_s_rms_sq + power(abs(U_s_art(1,n)),2);
U_t_rms_sq = U_t_rms_sq + power(abs(U_t_art(1,n)),2);
U_rms_art1(1,n) = sqrt(power(abs(U_r_art(1,n)),2) +
power(abs(U_s_art(1,n)),2)+power(abs(U_t_art(1,n)),2));
% ****** BRANCH CURRENTS ******
i_rs(1,n) = Y_rs(1,n) * U_rs_art(1,n) ;
i_st(1,n) = Y_st(1,n) * U_st_art(1,n) ;
i_tr(1,n) = Y_tr(1,n) * U_tr_art(1,n) ;
% ****** LINE CURRENTS *******
i_R(1,n) = i_rs(1,n)-i_tr(1,n);
i_S(1,n) = i_st(1,n)-i_rs(1,n);
```

```
i_T(1,n) = i_tr(1,n)-i_st(1,n);
i_R_rms_sq = i_R_rms_sq + power(abs(i_R(1,n)),2);
i_S_rms_sq = i_S_rms_sq + power(abs(i_S(1,n)),2);
i_T_rms_sq = i_T_rms_sq + power(abs(i_T(1,n)),2);
%Complex Powers for Harmonic and Balanced Admittances
C_n(1,n)=U_r_art(1,n)*conj(i_R(1,n))+
U_s_art(1,n)*conj(i_S(1,n))+U_t_art(1,n)*conj(i_T(1,n));
C_n_tot=C_n_tot+C_n(1,n); % The sum of Complex powers for each harmonic.. Is
this mistake like Budanue ?

Y_b(1,n) = (conj(C_n(1,n)))/power(U_rms_art(1,n),2); % equivalent balanced
admittance or eqv admittance of balance load
G_b(1,n) = real(Y_b(1,n)); % real of Y_b or eqv balanced conductance
B_b(1,n) = imag(Y_b(1,n)); % eqv balanced susceptance
i_rms1(1,n) = sqrt( power(abs(i_R(1,n)),2) + power(abs(i_S(1,n)),2) +
power(abs(i_T(1,n)),2));
i_rms1_sq = i_rms1_sq + power (i_rms1(1,n),2);
end
P_n_tot= real(C_n_tot);
G_b_tot= P_n_tot/U_rms_tot_sq;
i_act_orig = G_b_tot * sqrt(U_rms_tot_sq); % TO find the total orginial
conductacne
fprintf(' Supply voltage rms from harmonics  : %d \n',sqrt(U_rms_tot_sq));
fprintf (' Line R voltage rms : %d \n', sqrt(U_r_rms_sq));
fprintf (' Line S voltage rms : %d \n', sqrt(U_s_rms_sq));
fprintf (' Line T voltage rms : %d \n', sqrt(U_t_rms_sq));
fprintf(' Supply voltage rms from line voltages : %d
\n',sqrt(U_r_rms_sq+U_s_rms_sq+U_t_rms_sq));
disp(' Line Current RMS values : ');
fprintf(' ||ir|| = %d \n',sqrt(i_R_rms_sq));
fprintf(' ||is|| = %d \n',sqrt(i_S_rms_sq));
fprintf(' ||it|| = %d \n',sqrt(i_T_rms_sq));
fprintf(' total RMS calculated from line currents, ||i|| = %d \n',
sqrt(i_R_rms_sq+i_S_rms_sq+i_T_rms_sq));
Z_mat = [Z_rs(1,1);Z_st(1,1);Z_tr(1,1)];

%% CALCULATION OF UNBALANCE PARAMTERS AND OTHER POWERS
i_a = 0 ;
for n=[1,5,7]
a_complex(1,n) = (U_n(1,n) /U_p(1,n));  % this is complex quantity "a"
Y_e(1,n) = Y_rs(1,n) + Y_st(1,n) + Y_tr(1,n); % eqv admittance
G_e(1,n) = real(Y_e(1,n));% eqv conductance
B_e(1,n) = imag(Y_e(1,n));%eqv susceptance
Y_d(1,n)= Y_e(1,n) - Y_b(1,n);
% ******* UNBALANCE PARAMETERS********
A_p(1,n) = -1 * ( Y_st(1,n) + alp * Y_tr(1,n) + (conj(alp)) * Y_rs(1,n) ) ; %
posi. seq unbalance admittance
A_n(1,n) = -1 * ( Y_st(1,n) + conj(alp) * Y_tr(1,n) + alp * Y_rs(1,n) ) ; %
neg seq unbalance admittance
a(1,n)=abs(a_complex(1,n));
theta(1,n)=angle(a_complex(1,n));
deg = theta(1,n)*180/pi;
%***********************************************
% ******** DIFFERENT CURRENTS FOR THE HARMONICS**********

%Section below uses rms for harmonics..which is wrong.
```

```
%i_a_h(1,n) = G_b(1,n) * U_rms_art(1,n);
i_a_h(1,n) = G_b_tot * U_rms_art(1,n); %

i_scat_rms(1,n) = abs(-G_b_tot + G_b(1,n))* U_rms_art(1,n);
i_rea(1,n) = abs(B_b(1,n)) * U_rms_art(1,n);
I_Ru_P(1,n) = A_n(1,n) * U_n(1,n) + Y_d(1,n) * U_p(1,n);
I_Ru_N(1,n) = A_p(1,n) * U_p(1,n) + Y_d(1,n) * U_n(1,n);
i_unb(1,n) = sqrt(3) * sqrt( power(abs(I_Ru_P(1,n)),2) +
power(abs(I_Ru_N(1,n)),2) ) ;

%****SQUARING ALL CURRENTS OF EACH HARMONIC TO FIND POWERS LATER*****
i_reac_rms_sq=i_reac_rms_sq+power(i_rea(1,n),2);
i_unb_rms_sq=i_unb_rms_sq+power(i_unb(1,n),2);
i_scat_rms_sq = i_scat_rms_sq + power(i_scat_rms(1,n),2);
% THE FOLLOWING ACTIVE CURRENT IS NOT HTE CORRECT ONE THOUGH.. ITS NOT USED
i_a_h_sq=i_a_h_sq+power(i_a_h(1,n),2);

% TOTAL RMS ..CALCULATED FOR REFERENCE .... TO COMPARE BOTH WAYS
i_rms(1,n) = sqrt( power(i_a_h(1,n),2) + power(i_rea(1,n),2) +
power(i_unb(1,n),2) + power(i_scat_rms(1,n),2));
i_rms_har_sq = i_rms_har_sq + power(i_rms(1,n),2);
end
i_rms_tot_sq = power (i_act_orig,2)+  i_scat_rms_sq + i_unb_rms_sq +
i_reac_rms_sq ;
P = sqrt(U_rms_tot_sq * i_a_h_sq); % This P is using active currents for
each.. which looks like is useless
P1 = sqrt(U_rms_tot_sq) * i_a_h ; % does not look different from above
P_act = sqrt(U_rms_tot_sq) * i_act_orig;
D_scat = sqrt(U_rms_tot_sq * i_scat_rms_sq);
Q = sqrt(U_rms_tot_sq * i_reac_rms_sq);
D_unb = sqrt(U_rms_tot_sq * i_unb_rms_sq);
%power(P_n_tot,2) - power(P,2)%+ power(D_scat,2) not useful after error
S_1_sq = i_rms_har_sq * U_rms_tot_sq;% Using act, reac , .. currents
S_2_sq = i_rms1_sq * U_rms_tot_sq; % Using Line currents
S_3_sq = i_rms_tot_sq * U_rms_tot_sq;
%Pow_diff=(S_1_sq-  S_2_sq-power(D_scat,2));  % This gave D scatered
S_4_sq = power(P,2) + power(D_scat,2) + power(Q,2) + power(D_unb,2);  % Using
squares of powers for reference
U_p;U_n;Y_e; Y_b;Y_d;
pf = P / sqrt(S_4_sq);
disp ('CPC currents rms before compensation: ')
fprintf('Active Current ||ia||  = %d \n',i_act_orig);
fprintf('Scattred current ||is|| = %d \n',sqrt(i_scat_rms_sq));
fprintf('Reactive current ||ir|| = %d \n',sqrt(i_reac_rms_sq));
fprintf('Unbalanced current ||iu|| = %d \n',sqrt(i_unb_rms_sq));
fprintf('Total RMS ||i|| = %d \n',sqrt(i_rms_tot_sq));
disp ('Powers before compensation: ')
fprintf('Active power P  = %d \n',P);
fprintf('Reactive power Q = %d \n',Q);
fprintf('Unbalanced power Du = %d \n',D_unb);
fprintf('Scattered power Ds = %d \n',D_scat);
fprintf('Apparent power S = %d \n',sqrt(S_2_sq));
fprintf(' Power factor  pf = %d \n',pf);

%% COMPENSTOR RELATED SECTIONS - TOTAL COMPENSATION WITH ideal COMPENSATOR
for n = [1,5,7]
```

```
% the following are THE THREE TERMS of YcD
Ycd_tst(1,n) = (1i * 2*a(1,n)*cos(theta(1,n))) / ( 1 + power(a(1,n),2));
Ycd_ttr(1,n) = (1i * 2*a(1,n)*cos(theta(1,n)-(2*pi/3))) / ( 1 +
power(a(1,n),2));
Ycd_trs(1,n) = (1i * 2*a(1,n)*cos(theta(1,n)+(2*pi/3))) / ( 1 +
power(a(1,n),2));

%calculated coeffieceints of Trs, Tst and Ttr in equation A B C and D
calcualted below are there to simplify analysis. They
% will be easier to follow if my paperwork is referenced.
% I am calling the coef or Trs as A , Tst as B and Ttr as C....
A(1,n) = (1+a_complex(1,n))* Ycd_trs(1,n) - 1i*(conj(alp)+a_complex(1,n)*alp
);
B(1,n) = (1+a_complex(1,n))* ( Ycd_tst(1,n) - 1i);
C(1,n) = (1+a_complex(1,n))* Ycd_ttr(1,n) -
1i*(alp+a_complex(1,n)*conj(alp));
D(1,n) = Y_d(1,n) * ( 1 + a_complex(1,n)) + A_p(1,n) + a_complex(1,n) *
A_n(1,n) ;

% calculation of coeffieceints in final equation
 % real and imaginary parts of the coefficeints...
A_real(1,n) = real(A(1,n));
B_real(1,n) = real(B(1,n));
C_real(1,n) = real(C(1,n));
D_real(1,n) = real(D(1,n));
A_imag(1,n) = imag(A(1,n));
B_imag(1,n) = imag(B(1,n));
C_imag(1,n) = imag(C(1,n));
D_imag(1,n) = imag(D(1,n));

% Declaring matrices coef_mat and cnst_mat for solving
% Dont confuse these elements A_mn and B_mn with the earlier A B C and D

% This is to declare matrix as coefficeints of Trs, Tst and Ttr in eqns
A_11(1,n) = U_rs_art(1,n)*conj(U_rs_art(1,n));
A_12(1,n) = U_st_art(1,n)*conj(U_st_art(1,n));
A_13(1,n) = U_tr_art(1,n)*conj(U_tr_art(1,n));
A_21(1,n) = A_real(1,n);
A_22(1,n) = B_real(1,n);
A_23(1,n) = C_real(1,n);
A_31(1,n) = A_imag(1,n);
A_32(1,n) = B_imag(1,n);
A_33(1,n) = C_imag(1,n);

B_11(1,n) = -1 *B_b(1,n) *  power(U_rms_art(1,n),2);
B_21(1,n) = -1* D_real(1,n);
B_31(1,n) = -1* D_imag(1,n);

% declaring matrix

coef_mat =
[A_11(1,n),A_12(1,n),A_13(1,n);A_21(1,n),A_22(1,n),A_23(1,n);A_31(1,n),A_32(1
,n),A_33(1,n)];
cons_mat = [B_11(1,n);B_21(1,n);B_31(1,n)];
```

```
ans_mat = inv(coef_mat) * cons_mat;  % the values of compnesator susceptances
T_rs(1,n) = ans_mat(1,1);
T_st(1,n) = ans_mat(2,1);
T_tr(1,n) = ans_mat(3,1);


disp( ' ');
disp('              ');

fprintf(' HARMONIC ORDER = %d \n',n);
disp( ' ');
disp( ' COMPENSATOR VALUES IN SIEMENS');
fprintf(' T_rs = %d \n',T_rs(1,n));
fprintf(' T_st = %d \n',T_st(1,n));
fprintf(' T_tr = %d \n',T_tr(1,n));
disp( '                              ' );

if T_rs(1,n) > 0
    E_rs(1,n)= T_rs(1,n) / ( 2 * pi * 60 * n);
    fprintf(' Compensator element T_rs capacitor( Farads) = %d
\n',E_rs(1,n));
else
    E_rs(1,n)= -1 / (T_rs(1,n)* 2 * pi * 60 * n)  ;
    fprintf(' Compensator element T_rs inductor(H) = %d \n',E_rs(1,n));
end

if T_st(1,n) > 0
    E_st(1,n)= T_st(1,n) / ( 2 * pi * 60 * n);
    fprintf(' Compensator element T_st capacitor( Farads) = %d
\n',E_st(1,n));
else
    E_st(1,n)= -1 / (T_st(1,n)* 2 * pi * 60 * n)  ;
    fprintf(' Compensator element T_st inductor(H) = %d \n',E_st(1,n));
end

if T_tr(1,n) > 0
    E_tr(1,n)= T_tr(1,n) / ( 2 * pi * 60 * n);
    fprintf(' Compensator element T_tr capacitor( Farads) = %d
\n',E_tr(1,n));
else
    E_tr(1,n)= -1 / (T_tr(1,n)* 2 * pi * 60 * n)  ;
    fprintf(' Compensator element T_tr inductor(H) = %d \n',E_tr(1,n));
end

end


%%
i_R_comp_rms_sq=0;i_S_comp_rms_sq=0;i_T_comp_rms_sq=0;
%% Section to calculate the currents and powers overall after comp--
C_n_tot_new=0;i_rms1_sq_new = 0;
for n=[1,5,7]

Y_rs_new(1,n) = Y_rs(1,n) + T_rs(1,n) * 1i ;
Y_st_new(1,n) = Y_st(1,n) + T_st(1,n) * 1i ;
```

```
Y_tr_new(1,n) = Y_tr(1,n) + T_tr(1,n) * 1i ;

i_rs_comp(1,n) = T_rs(1,n)*1i * U_rs_art(1,n) ;
i_st_comp(1,n) = T_st(1,n)*1i * U_st_art(1,n) ;
i_tr_comp(1,n) = T_tr(1,n)*1i * U_tr_art(1,n) ;

i_R_comp(1,n) = i_rs_comp(1,n)-i_tr_comp(1,n);
i_S_comp(1,n) = i_st_comp(1,n)-i_rs_comp(1,n);
i_T_comp(1,n) = i_tr_comp(1,n)-i_st_comp(1,n);

i_R_comp_rms(1,n) = abs(i_R_comp(1,n));
i_S_comp_rms(1,n) = abs(i_S_comp(1,n));
i_T_comp_rms(1,n) = abs(i_T_comp(1,n));

i_R_comp_rms_sq = i_R_comp_rms_sq + power(i_R_comp_rms(1,n),2);
i_S_comp_rms_sq = i_S_comp_rms_sq + power(i_S_comp_rms(1,n),2);
i_T_comp_rms_sq = i_T_comp_rms_sq + power(i_T_comp_rms(1,n),2);


i_R_new(1,n) = i_R(1,n) + i_R_comp(1,n);
i_S_new(1,n) = i_S(1,n) + i_S_comp(1,n);
i_T_new(1,n) = i_T(1,n) + i_T_comp(1,n);

i_R_new_rms(1,n) = abs(i_R_new(1,n));
i_S_new_rms(1,n) = abs(i_S_new(1,n));
i_T_new_rms(1,n) = abs(i_T_new(1,n));
i_R_new_rms_sq = i_R_new_rms_sq + power(i_R_new_rms(1,n),2);
i_S_new_rms_sq = i_S_new_rms_sq + power(i_S_new_rms(1,n),2);
i_T_new_rms_sq = i_T_new_rms_sq + power(i_T_new_rms(1,n),2);
% CUrrent may still not be equal... need to calculate reactive and unbl cur
%Complex Powers for Harmonic and Balanced Admittances
C_n_new(1,n)=U_r_art(1,n)*conj(i_R_new(1,n))+
U_s_art(1,n)*conj(i_S_new(1,n))+U_t_art(1,n)*conj(i_T_new(1,n));
C_n_tot_new=C_n_tot_new+C_n_new(1,n); % The sum of Complex powers for each
harmonic.. Is this mistake like Budanue ?

Y_b_new(1,n) = (conj(C_n_new(1,n)))/power(U_rms_art(1,n),2); % equivalent
balanced admittance or eqv admittance of balance load
G_b_new(1,n) = real(Y_b_new(1,n)); % real of Y_b or eqv balanced conductance
B_b_new(1,n) = imag(Y_b_new(1,n)); % eqv balanced susceptance
i_rms1_new(1,n) = sqrt( power(abs(i_R_new(1,n)),2) +
power(abs(i_S_new(1,n)),2) + power(abs(i_T_new(1,n)),2));
i_rms1_sq_new = i_rms1_sq_new + power (i_rms1_new(1,n),2);
end
P_n_tot_new= real(C_n_tot_new);
G_b_tot_new= P_n_tot_new/U_rms_tot_sq;
i_act_orig_new = G_b_tot_new * sqrt(U_rms_tot_sq);
i_rms_har_sq_new=0;i_reac_rms_sq_new=0;i_unb_rms_sq_new=0;i_scat_rms_sq_new
=0;i_a_h_sq_new=0;

 for n=[1,5,7]
a_complex(1,n) = (U_n(1,n) /U_p(1,n)) ; % this is complex quantity "a"
Y_e_new(1,n) = Y_rs_new(1,n) + Y_st_new(1,n) + Y_tr_new(1,n); % eqv
admittance
G_e_new(1,n) = real(Y_e_new(1,n));% eqv conductance
```

```
B_e_new(1,n) = imag(Y_e_new(1,n));%eqv susceptance
Y_d_new(1,n)= Y_e_new(1,n) - Y_b_new(1,n);
% ******* UNBALANCE PARAMETERS********
A_p_new(1,n) = -1 * ( Y_st_new(1,n) + alp * Y_tr_new(1,n) + (conj(alp)) *
Y_rs_new(1,n) ) ; % posi. seq unbalance admittance
A_n_new(1,n) = -1 * ( Y_st_new(1,n) + conj(alp) * Y_tr_new(1,n) + alp *
Y_rs_new(1,n) ) ; % neg seq unbalance admittance
a(1,n)=abs(a_complex(1,n));
theta(1,n)=angle(a_complex(1,n));

%**********************************************
% ******** DIFFERENT CURRENTS FOR THE HARMONICS**********

%Section below uses rms for harmonics..which is wrong.
%i_a_h(1,n) = G_b(1,n) * U_rms_art(1,n);
i_a_h_new(1,n) = G_b_tot_new * U_rms_art(1,n); %

i_scat_rms_new(1,n) = abs(-G_b_tot_new + G_b_new(1,n))* U_rms_art(1,n);
i_rea_new(1,n) = abs(B_b_new(1,n)) * U_rms_art(1,n);
I_Ru_P_new(1,n) = A_n_new(1,n) * U_n(1,n) + Y_d_new(1,n) * U_p(1,n);
I_Ru_N_new(1,n) = A_p_new(1,n) * U_p(1,n) + Y_d_new(1,n) * U_n(1,n);
i_unb_new(1,n) = sqrt(3) * sqrt( power(abs(I_Ru_P_new(1,n)),2) +
power(abs(I_Ru_N_new(1,n)),2) ) ;

%****SQUARING ALL CURRENTS OF EACH HARMONIC TO FIND POWERS LATER*****
i_reac_rms_sq_new=i_reac_rms_sq_new+power(i_rea_new(1,n),2);
i_unb_rms_sq_new=i_unb_rms_sq_new+power(i_unb_new(1,n),2);
i_scat_rms_sq_new = i_scat_rms_sq_new + power(i_scat_rms_new(1,n),2);
% THE FOLLOWING ACTIVE CURRENT IS NOT HTE CORRECT ONE THOUGH.. ITS NOT USED
i_a_h_sq_new=i_a_h_sq_new+power(i_a_h_new(1,n),2);

% TOTAL RMS ..CALCULATED FOR REFERENCE .... TO COMPARE BOTH WAYS
i_rms_new(1,n) = sqrt( power(i_a_h_new(1,n),2) + power(i_rea_new(1,n),2) +
power(i_unb_new(1,n),2) + power(i_scat_rms_new(1,n),2));
i_rms_har_sq_new = i_rms_har_sq_new + power(i_rms_new(1,n),2);
end
i_rms_tot_sq_new = power (i_act_orig_new,2)+  i_scat_rms_sq_new +
i_unb_rms_sq_new + i_reac_rms_sq_new ;
P_new = sqrt(U_rms_tot_sq * i_a_h_sq_new); % This P is using active currents
for each.. which looks like is useless
P1_new = sqrt(U_rms_tot_sq) * i_a_h_new ; % does not look different from
above
P_act_new = sqrt(U_rms_tot_sq) * i_act_orig_new;
D_scat_new = sqrt(U_rms_tot_sq * i_scat_rms_sq_new);
Q_new = sqrt(U_rms_tot_sq * i_reac_rms_sq_new);
D_unb_new = sqrt(U_rms_tot_sq * i_unb_rms_sq_new);
%power(P_n_tot,2) - power(P,2)%+ power(D_scat,2) not useful after error
S_1_sq_new = i_rms_har_sq_new * U_rms_tot_sq;% Using act, reac , .. currents
S_2_sq_new = i_rms1_sq_new * U_rms_tot_sq; % Using Line currents
S_3_sq_new = i_rms_tot_sq_new * U_rms_tot_sq ;
%Pow_diff=(S_1_sq-  S_2_sq-power(D_scat,2));  % This gave D scatered
S_4_sq_new = power(P_new,2) + power(D_scat_new,2) + power(Q_new,2) +
power(D_unb_new,2);
pf_new = P_new / sqrt(S_4_sq_new);
disp ('                                              ')
disp(' Values after complete compensation using ideal complex compensator ')
```

```
disp(' Line Current RMS values : ');
fprintf(' ||ir|| = %d \n',sqrt(i_R_new_rms_sq));
fprintf(' ||is|| = %d \n',sqrt(i_S_new_rms_sq));
fprintf(' ||it|| = %d \n',sqrt(i_T_new_rms_sq));
fprintf(' total RMS calculated from line currents, ||i|| = %d \n',
sqrt(i_R_new_rms_sq+i_S_new_rms_sq+i_T_new_rms_sq));

disp(' Compensator Current RMS values : ');
fprintf(' ||ir_com|| = %d \n',sqrt(i_R_comp_rms_sq));
fprintf(' ||is_com|| = %d \n',sqrt(i_S_comp_rms_sq));
fprintf(' ||it_com|| = %d \n',sqrt(i_T_comp_rms_sq));
disp ('CPC currents rms after compensation: ')
fprintf('Active Current ||ia||  = %d \n',i_act_orig_new);
fprintf('Scattred current ||is|| = %d \n',sqrt(i_scat_rms_sq_new));
fprintf('Reactive current ||ir|| = %d \n',sqrt(i_reac_rms_sq_new));
fprintf('Unbalanced current ||iu|| = %d \n',sqrt(i_unb_rms_sq_new));
fprintf('Total RMS ||i|| = %d \n',sqrt(i_rms_tot_sq_new));
disp ('Powers after compensation: ')
fprintf('Active power P_com = %d \n',P_new);
fprintf('Reactive power Q_com = %d \n',Q_new);
fprintf('Unbalanced power Du_com = %d \n',D_unb_new);
fprintf('Scattered power Ds_com = %d \n',D_scat_new);
fprintf('Apparent Power S_com = %d \n',sqrt(S_4_sq_new));
fprintf('Power factor pf_com = %d \n',pf_new);
% ---------- end of section for powers and pf calculations after comp---

  % OPTIMIZATION PORTION
disp( '                               ' );
disp( ' SECTION FOR OPTIMIZATION ' );
disp( '                               ' );

% % ===================================================================
% OPTIMIATION PART for branch T_rs
lcf = .1 ; % the fraction the inductive reactnace is of the capacitive
if T_rs(1,1)<0 % IF FUNDAMENTAL NEEDS INDUCTOR
     fprintf(' Since fund. T_rs requires inductor, we choose L branch \n ');
    L_nume=0;L_deno =0;
    Lrs = -1 / (T_rs(1,1)* 2 * pi * 60 * 1); % used when Lrs started as
fundamental
% Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_deno = L_deno + T_rs(1,n)* power(abs(U_rs_art(1,n)),2)/n;
                 L_nume = L_nume + power(abs(U_rs_art(1,n)),2)/( 2
*pi*60*n*n);
    end
    L_rs_op = -1* L_nume / L_deno;
    if L_rs_op <0
         fprintf(' Error in design because reactance is negative !!!! /n')
    end
    dis1=0;
    for n = [ 1,5,7]
                 dis1 = dis1 + power(((T_rs(1,n) + 1 / ( 2 * pi * 60 * n *
L_rs_op ) )* abs(U_rs_art(1,n))),2);
    end
    dis1;
%%END of nume and deno section
```

```
% % This following section is to compare if the optimized value is the best
dis2=0;
    for n = [ 1,5,7]
                 dis2 = dis2 + power(((T_rs(1,n) + 1 / ( 2 * pi * 60 * n *
Lrs ) )* abs(U_rs_art(1,n))),2);
    end
    dis2;
  %disp('                    %%%%%%%%%');
   fprintf(' The inductor element for branch T_rs = %d \n',L_rs_op);
%   fprintf(' The current dis sqr using this value = %d \n',dis1);
  %fprintf(' The current dis sqr using this funda value = %d \n',dis2);
  disp('                  -----                         ');

  for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 T_rs_op(1,n)=-1/ (2*pi*60*n*L_rs_op);
  end
end

if T_rs(1,1)>0 % IF FUNDAMENTAL NEEDS CAPACITOR
     fprintf(' Since fund. T_rs needs capacitor, we choose LC Branch \n');
    L_nume=0;L_deno =0;
    % Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_nume = L_nume + ((2*pi*60)*n*T_rs(1,n)*
power(abs(U_rs_art(1,n)),2))/((1-lcf*n*n));
                 L_deno = L_deno +
power(abs(U_rs_art(1,n)),2)*((2*pi*60*n)^2)/(( 1-lcf*n*n)^2);
    end
    C_rs_op = 1* L_nume / L_deno;
    if C_rs_op <0
         fprintf(' Error in design because reactance is negative !!!! /n')
    end
    L_rs_op = lcf / (376.99*376.99*C_rs_op );
    dis_C1=0;
    for n = [ 1,5,7]
                 dis_C1 = dis_C1 + power( (  ( T_rs(1,n)-
(2*pi*60*n*C_rs_op)/(1-lcf*n*n*((2*pi*60)^2) ) )* abs(U_rs_art(1,n)) ),2);

    end
    dis_C1;
     fprintf(' The capa element for branch T_rs = %d \n',C_rs_op);
     fprintf(' The inductor in series for branch T_rs = %d \n',L_rs_op);
%     fprintf(' The current dis sqr using this value = %d \n',dis_C1);
    disp('                  -----                         ');


    for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 %T_rs_op(1,n)= ( n *
2*pi*60*C_rs_op)/(1+n*n*((2*pi*60)^2)*C_rs_op*L_rs_op)
                 T_rs_op(1,n)= ( n * 2*pi*60*C_rs_op)/(1-n*n*lcf);

    end

end
```

```
% % END OF OPTIMIZATION PART FOR T_RS
% % =======================================================================



% % ===================OPTIMIATION PART for branch T_st===================
if T_st(1,1)<0
     fprintf(' Since fund. T_st requires inductor, we choose L branch \n');
    L_nume=0;L_deno =0;
    Lst = -1 / (T_st(1,1)* 2 * pi * 60 * 1); % used when Lst started as
fundamental
% Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_deno = L_deno + T_st(1,n)* power(abs(U_st_art(1,n)),2)/n;
                 L_nume = L_nume + power(abs(U_st_art(1,n)),2)/( 2
*pi*60*n*n);
    end
    L_st_op = -1* L_nume / L_deno;
    if L_st_op <0
        fprintf(' Error in design because reactance is negative !!!! \n')
    end
    dis3=0;
    for n = [ 1,5,7]
                 dis3 = dis3 + power(((T_st(1,n) + 1 / ( 2 * pi * 60 * n *
L_st_op ) )* abs(U_st_art(1,n))),2);
    end
    dis3;
%%END of nume and deno section
% % % This following section is to compare if the optimized value is the best
dis4=0;
    for n = [ 1,5,7]
                 dis4 = dis4 + power(((T_st(1,n) + 1 / ( 2 * pi * 60 * n *
Lst ) )* abs(U_st_art(1,n))),2);
    end
    dis4;
   fprintf(' The inductor element for branch T_st = %d \n',L_st_op);
%   fprintf(' The current dis sqr using this value = %d \n',dis3);
  %fprintf(' The current dis sqr using this funda value = %d \n',dis4);
  disp('                   -----                         ');
  for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 T_st_op(1,n)=-1/ (2*pi*60*n*L_st_op);

  end
end

if T_st(1,1)>0
     fprintf(' Since fund. T_st needs capacitor, we choose LC Branch \n');
    L_nume=0;L_deno =0;
    % Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_nume = L_nume + (n*T_st(1,n)*
power(abs(U_st_art(1,n)),2))/((1-lcf*n*n));
                 L_deno = L_deno +
power(abs(U_st_art(1,n)),2)*(2*pi*60*n*n)/(( 1-lcf*n*n)^2);
    end
    C_st_op = 1* L_nume / L_deno;
    if C_st_op <0
```

```
        fprintf(' Error in design because reactance is negative !!!! /n')
    end
    dis_C3=0;
    L_st_op = lcf / ( 376.99*376.99*C_st_op );
    for n = [ 1,5,7]
                 dis_C3 = dis_C3 + power( (   ( T_st(1,n)-
(2*pi*60*n*C_st_op)/(1-lcf*n*n*((2*pi*60)^2) ) )* abs(U_st_art(1,n)) ),2);
    end
    dis_C3;
     fprintf(' The capa element for branch T_st = %d \n',C_st_op);
     fprintf(' The inductor in series for branch T_st = %d \n',L_st_op);
%     fprintf(' The current dis sqr using this value = %d \n',dis_C3);
%     disp('              -----                        ');
    L_st_op = lcf / ( 376.99*376.99*C_st_op );
    for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 %T_st_op(1,n)= ( n *
2*pi*60*C_st_op)/(1+n*n*((2*pi*60)^2)*C_st_op*L_st_op);
                 T_st_op(1,n)= ( n * 2*pi*60*C_st_op)/(1-n*n*lcf);
    end
end
%%END of nume and deno section

% % END OF OPTIMIZATION PART FOR T_st
%
%==========================================================================

% % ===================OPTIMIATION PART for branch T_tr===================
if T_tr(1,1)<0
%     fprintf(' Since fund. T_tr requires inductor, we choose L branch \n');
    L_nume=0;L_deno =0;
    %Ltr = -1 / (T_tr(1,1)* 2 * pi * 60 * 1); % used when Ltr started as
fundamental
% Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_deno = L_deno + T_tr(1,n)* power(abs(U_tr_art(1,n)),2)/n;
                 L_nume = L_nume + power(abs(U_tr_art(1,n)),2)/( 2
*pi*60*n*n);
    end
    L_tr_op = -1* L_nume / L_deno;
    if L_tr_op <0
        fprintf(' Error in design because reactance is negative !!!! /n')
    end
    dis5=0;
    for n = [ 1,5,7]
                 dis5 = dis5 + power(((T_tr(1,n) + 1 / ( 2 * pi * 60 * n *
L_tr_op ) )* abs(U_tr_art(1,n))),2);
    end
    dis5;
   fprintf(' The inductor element for branch T_tr = %d \n',L_tr_op);
%   fprintf(' The current dis sqr using this value = %d \n',dis5);
  disp('              -----                        ');
  for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 T_tr_op(1,n)=-1/ (2*pi*60*n*L_tr_op);
  end
end
```

```
if T_tr(1,1)>0
     fprintf(' Since fund. T_tr needs capacitor, we choose LC Branch \n');
    L_nume=0;L_deno =0;
    % Section used to calculated the nume and deno of the summaation
    for n = [ 1,5,7]
                 L_nume = L_nume + (n*T_tr(1,n)*
power(abs(U_tr_art(1,n)),2))/((1-lcf*n*n));
                 L_deno = L_deno +
power(abs(U_tr_art(1,n)),2)*(2*pi*60*n*n)/(( 1-lcf*n*n)^2);
    end
    C_tr_op = 1* L_nume / L_deno;
    if C_tr_op <0
        fprintf(' Error in design because reactance is negative !!!! /n')
    end
    L_tr_op = lcf / ( 376.99*376.99*C_tr_op );
    dis_C5=0;
    for n = [ 1,5,7]
                 dis_C5 = dis_C5 + power( (   ( T_tr(1,n)-
(2*pi*60*n*C_tr_op)/(1-lcf*n*n*((2*pi*60)^2) ) )* abs(U_tr_art(1,n)) ),2);
    end
%     dis_C5;
     fprintf(' The capa element for branch T_tr = %d \n',C_tr_op);
     fprintf(' The inductor in series for branch T_tr = %d \n',L_tr_op);
%     fprintf(' The current dis sqr using this value = %d \n',dis_C5);
%     disp('                   -----                          ');

    for n = [ 1,5,7] % finding the susceptance of the optimized comp.
                 %T_tr_op(1,n)= ( n *
2*pi*60*C_tr_op)/(1+n*n*((2*pi*60)^2)*C_tr_op*L_tr_op);
                 T_tr_op(1,n)= ( n * 2*pi*60*C_tr_op)/(1-n*n*lcf);
    end
end
%T_rs_op,T_st_op,T_tr_op

% % END OF OPTIMIZATION PART FOR T_tr
%
%============================================================================
i_R_c_rms_sq=0;i_S_c_rms_sq=0;i_T_c_rms_sq=0;

%% POWERS ETC OF JUST THE OPT COMP
C_n_tot_c=0;i_rms1_sq_c=0;
for n=[1,5,7]

    Y_rs_c(1,n)= T_rs_op(1,n) * 1i  ;
    Y_st_c(1,n)= T_st_op(1,n) * 1i  ;
    Y_tr_c(1,n)= T_tr_op(1,n) * 1i  ;

% ****** BRANCH CURRENTS ******
i_rs_c(1,n) = Y_rs_c(1,n) * U_rs_art(1,n) ;
i_st_c(1,n) = Y_st_c(1,n) * U_st_art(1,n) ;
i_tr_c(1,n) = Y_tr_c(1,n) * U_tr_art(1,n) ;
% ****** LINE CURRENTS *******
i_R_c(1,n) = i_rs_c(1,n)-i_tr_c(1,n);
i_S_c(1,n) = i_st_c(1,n)-i_rs_c(1,n);
i_T_c(1,n) = i_tr_c(1,n)-i_st_c(1,n);
```

```
i_R_c_rms(1,n)=abs(i_R_c(1,n));
i_S_c_rms(1,n)=abs(i_S_c(1,n));
i_T_c_rms(1,n)=abs(i_T_c(1,n));

i_R_c_rms_sq = i_R_c_rms_sq + power(i_R_c_rms(1,n),2);
i_S_c_rms_sq = i_S_c_rms_sq + power(i_S_c_rms(1,n),2);
i_T_c_rms_sq = i_T_c_rms_sq + power(i_T_c_rms(1,n),2);

%Complex Powers for Harmonic and Balanced Admittances
C_n_c(1,n)=U_r_art(1,n)*conj(i_R_c(1,n))+
U_s_art(1,n)*conj(i_S_c(1,n))+U_t_art(1,n)*conj(i_T_c(1,n));
C_n_tot_c=C_n_tot_c+C_n_c(1,n); % The sum of Complex powers for each
harmonic.. Is this mistake like Budanue ?
Y_b_c(1,n) = (conj(C_n_c(1,n)))/power(U_rms_art(1,n),2); % equivalent
balanced admittance or eqv admittance of balance load
G_b_c(1,n) = real(Y_b_c(1,n)); % real of Y_b or eqv balanced conductance
B_b_c(1,n) = imag(Y_b_c(1,n)); % eqv balanced susceptance
i_rms1_c(1,n) = sqrt( power(abs(i_R_c(1,n)),2) + power(abs(i_S_c(1,n)),2) +
power(abs(i_T_c(1,n)),2));
i_rms1_sq_c = i_rms1_sq_c + power (i_rms1_c(1,n),2);
end
U_rms_art;
P_n_tot_c= real(C_n_tot_c);
G_b_tot_c= P_n_tot_c/U_rms_tot_sq;
i_act_orig_c = G_b_tot_c * sqrt(U_rms_tot_sq); % TO find the total orginial
conductacne


% ----------------- END OF COMPENSATOR POWER SECTION -------------------

%% SECTION TO CALCULATE POEWRS AFTER OPTIMIZATION

C_n_tot_f=0;i_rms1_sq_f=0;i_R_f_rms_sq =0;i_S_f_rms_sq =0;i_T_f_rms_sq =0;
for n = [1,5,7]

    Y_rs_f(1,n)=Y_rs(1,n)+ T_rs_op(1,n) * 1i  ;
    Y_st_f(1,n)=Y_st(1,n)+ T_st_op(1,n) * 1i  ;
    Y_tr_f(1,n)=Y_tr(1,n)+ T_tr_op(1,n) * 1i  ;

%     Section to compare if fundamental element does better than optimal
%     Y_rs_f(1,n)=Y_rs(1,n)+ T_rs(1,1) * 1i  ;
%     Y_st_f(1,n)=Y_st(1,n)+ T_st(1,1) * 1i  ;
%     Y_tr_f(1,n)=Y_tr(1,n)+ T_tr(1,1) * 1i  ;
% %
% ****** BRANCH CURRENTS ******
i_rs_f(1,n) = Y_rs_f(1,n) * U_rs_art(1,n) ;
i_st_f(1,n) = Y_st_f(1,n) * U_st_art(1,n) ;
i_tr_f(1,n) = Y_tr_f(1,n) * U_tr_art(1,n) ;
% ****** LINE CURRENTS *******
i_R_f(1,n) = i_rs_f(1,n)-i_tr_f(1,n);
i_S_f(1,n) = i_st_f(1,n)-i_rs_f(1,n);
i_T_f(1,n) = i_tr_f(1,n)-i_st_f(1,n);
i_R_f_rms_sq = i_R_f_rms_sq + power(abs(i_R_f(1,n)),2);
i_S_f_rms_sq = i_S_f_rms_sq + power(abs(i_S_f(1,n)),2);
i_T_f_rms_sq = i_T_f_rms_sq + power(abs(i_T_f(1,n)),2);
```

```
%Complex Powers for Harmonic and Balanced Admittances
C_n_f(1,n)=U_r_art(1,n)*conj(i_R_f(1,n))+
U_s_art(1,n)*conj(i_S_f(1,n))+U_t_art(1,n)*conj(i_T_f(1,n));
C_n_tot_f=C_n_tot_f+C_n_f(1,n); % The sum of Complex powers for each
harmonic.. Is this mistake like Budanue ?
Y_b_f(1,n) = (conj(C_n_f(1,n)))/power(U_rms_art(1,n),2); % equivalent
balanced admittance or eqv admittance of balance load
G_b_f(1,n) = real(Y_b_f(1,n)); % real of Y_b or eqv balanced conductance
B_b_f(1,n) = imag(Y_b_f(1,n)); % eqv balanced susceptance
i_rms1_f(1,n) = sqrt( power(abs(i_R_f(1,n)),2) + power(abs(i_S_f(1,n)),2) +
power(abs(i_T_f(1,n)),2));
i_rms1_sq_f = i_rms1_sq_f + power (i_rms1_f(1,n),2);

end

P_n_tot_f= real(C_n_tot_f);
G_b_tot_f= P_n_tot_f/U_rms_tot_sq;

i_act_orig_f = G_b_tot_f * sqrt(U_rms_tot_sq);
i_rms_har_sq_f=0;i_reac_rms_sq_f=0;i_unb_rms_sq_f=0;i_scat_rms_sq_f
=0;i_a_h_sq_f=0;

disp ('                                              ')
disp(' Values after optimized compensator  ')
fprintf('Supply current in line R rms ||iR|| = %d \n',sqrt(i_R_f_rms_sq));
fprintf('Supply current in line S rms ||iS|| = %d \n',sqrt(i_S_f_rms_sq));
fprintf('Supply current in line T rms ||iT|| = %d \n',sqrt(i_T_f_rms_sq));
fprintf('Opt Comp current in line R rms ||iR_op|| = %d
\n',sqrt(i_R_c_rms_sq));
fprintf('OPt Comp current in line S rms ||iS_op|| = %d
\n',sqrt(i_S_c_rms_sq));
fprintf('Opt Comp current in line T rms ||iT_op|| = %d
\n',sqrt(i_T_c_rms_sq));



 for n=[1,5,7]
a_complex(1,n) = (U_n(1,n) /U_p(1,n)) ; % this is complex quantity "a"
Y_e_f(1,n) = Y_rs_f(1,n) + Y_st_f(1,n) + Y_tr_f(1,n); % eqv admittance
G_e_f(1,n) = real(Y_e_f(1,n));% eqv conductance
B_e_f(1,n) = imag(Y_e_f(1,n));%eqv susceptance
Y_d_f(1,n)= Y_e_f(1,n) - Y_b_f(1,n);
% ******* UNBALANCE PARAMETERS********
A_p_f(1,n) = -1 * ( Y_st_f(1,n) + alp * Y_tr_f(1,n) + (conj(alp)) *
Y_rs_f(1,n) ) ; % posi. seq unbalance admittance
A_n_f(1,n) = -1 * ( Y_st_f(1,n) + conj(alp) * Y_tr_f(1,n) + alp * Y_rs_f(1,n)
) ; % neg seq unbalance admittance
a(1,n)=abs(a_complex(1,n));
theta(1,n)=angle(a_complex(1,n));
%**********************************************
% ******** DIFFERENT CURRENTS FOR THE HARMONICS**********

%Section below uses rms for harmonics..which is wrong.
%i_a_h(1,n) = G_b(1,n) * U_rms_art(1,n);
i_a_h_f(1,n) = G_b_tot_f * U_rms_art(1,n); %
```

```
i_scat_rms_f(1,n) = abs(-G_b_tot_f + G_b_f(1,n))* U_rms_art(1,n);
i_rea_f(1,n) = abs(B_b_f(1,n)) * U_rms_art(1,n);
I_Ru_P_f(1,n) = A_n_f(1,n) * U_n(1,n) + Y_d_f(1,n) * U_p(1,n);
I_Ru_N_f(1,n) = A_p_f(1,n) * U_p(1,n) + Y_d_f(1,n) * U_n(1,n);
i_unb_f(1,n) = sqrt(3) * sqrt( power(abs(I_Ru_P_f(1,n)),2) +
power(abs(I_Ru_N_f(1,n)),2) ) ;

%****SQUARING ALL CURRENTS OF EACH HARMONIC TO FIND POWERS LATER*****
i_reac_rms_sq_f=i_reac_rms_sq_f+power(i_rea_f(1,n),2);
i_unb_rms_sq_f=i_unb_rms_sq_f+power(i_unb_f(1,n),2);
i_scat_rms_sq_f = i_scat_rms_sq_f + power(i_scat_rms_f(1,n),2);
% THE FOLLOWING ACTIVE CURRENT IS NOT HTE CORRECT ONE THOUGH.. ITS NOT USED
i_a_h_sq_f=i_a_h_sq_f+power(i_a_h_f(1,n),2);

% TOTAL RMS ..CALCULATED FOR REFERENCE .... TO COMPARE BOTH WAYS
i_rms_f(1,n) = sqrt( power(i_a_h_f(1,n),2) + power(i_rea_f(1,n),2) +
power(i_unb_f(1,n),2) + power(i_scat_rms_f(1,n),2));
i_rms_har_sq_f = i_rms_har_sq_f + power(i_rms_f(1,n),2);
end
i_rms_tot_sq_f = power (i_act_orig_f,2)+  i_scat_rms_sq_f + i_unb_rms_sq_f +
i_reac_rms_sq_f ;
P_f = sqrt(U_rms_tot_sq * i_a_h_sq_f); % This P is using active currents for
each.. which looks like is useless
P1_f = sqrt(U_rms_tot_sq) * i_a_h_f ; % does not look different from above
P_act_f = sqrt(U_rms_tot_sq) * i_act_orig_f;
D_scat_f = sqrt(U_rms_tot_sq * i_scat_rms_sq_f);
Q_f = sqrt(U_rms_tot_sq * i_reac_rms_sq_f);
D_unb_f = sqrt(U_rms_tot_sq * i_unb_rms_sq_f);
%power(P_n_tot,2) - power(P,2)%+ power(D_scat,2) not useful after error
S_1_sq_f = i_rms_har_sq_f * U_rms_tot_sq;% Using act, reac , .. currents
S_2_sq_f = i_rms1_sq_f * U_rms_tot_sq; % Using Line currents
S_3_sq_f = i_rms_tot_sq_f * U_rms_tot_sq ;
%Pow_diff=(S_1_sq-  S_2_sq-power(D_scat,2));  % This gave D scatered
S_4_sq_f = power(P_f,2) + power(D_scat_f,2) + power(Q_f,2) +
power(D_unb_f,2);
pf_f = P_f / sqrt(S_4_sq_f);


% % ----------- end of section for powers and pf calculations after opt---
%% PRINTING SOME VALUES FOR CHECKING
%P,Q,D_unb,D_scat,pf,P_new,Q_new,D_unb_new,D_scat_new,pf_new,P_f,Q_f,D_unb_f,
D_scat_f,pf_f
%Y_rs,Y_rs_f,Y_rs_new
% Y_st,Y_st_f,Y_st_new
% Y_tr,Y_tr_f,Y_tr_new

fprintf('Supply Current active component rms ||ia||= %d
\n',sqrt(i_a_h_sq_f));
fprintf('Supply Current Scattered component rms ||is||= %d
\n',sqrt(i_scat_rms_sq_f));
fprintf('Supply Current reactive component rms ||ir||= %d
\n',sqrt(i_reac_rms_sq_f));
fprintf('Supply Current unbalanced component rms ||iu||= %d
\n',sqrt(i_unb_rms_sq_f));
```

```
fprintf('Active power P_op = %d \n',P_f);
fprintf('Reactive power Q_op = %d \n',Q_f);
fprintf('Unbalanced power Du_op = %d \n',D_unb_f);
fprintf('Scattered power Ds_op = %d \n',D_scat_f);
fprintf('Apparent Power S_op = %d \n',sqrt(S_4_sq_f));
fprintf('Power factor pf_op = %d \n',pf_f);
```

Code of HGL

```
% This is modified for latest version.. measured at line terminals ..

% START FROM THIS POINT ONWARDS TO CHECK IF THE CIRCUIT CONFGURATIONS IS THE
SAME AND FOLLOW DOWNWARDS...
clc
clear all
alp = -.5 + ((sqrt(3))/2) * 1i ;

% % SUPPLY PARAMETERS

% % %% Symmetrical and Sinusoidal
% % e_r = [1000,0,0,0,0,0,0];
% % e_s = [-500-866.03i,0,0,0,0,0,0];
% % e_t = [-500+866.03i,0,0,0,0,0,0];
% %
% % i_h_rs_par = [0,100,0,0,100,0,100];
% % i_h_st_par = [0,0,0,0,0,0,0];
% % i_h_tr_par = [0,0,0,0,0,0,0];

% %% Symmetrical but Nonsinusoidal
% e_r = [1000,0,100,0,100,0,0];
% e_s = [-500-866.03i,0,100,0,-50+86.6i,0,0];
% e_t = [-500+866.03i,0,100,0,-50-86.6i,0,0];
%
% i_h_rs_par = [0,100,0,100,0,0,100];
% i_h_st_par = [0,0,0,0,0,0,0];
% i_h_tr_par = [0,0,0,0,0,0,0];

% %% Asymmetrical but Sinusoidal
% e_r = [1000,0,0,0,0,0,0];
% e_s = [-353.56 - 353.56i,0,0,0,0,0,0];
% e_t = [-353.56 + 353.56i,0,0,0,0,0,0];
%
```

```
% i_h_rs_par = [0,100,0,0,100,0,100];
% i_h_st_par = [0,0,0,0,0,0,0];
% i_h_tr_par = [0,0,0,0,0,0,0];

%% Asymmetrical but Nonsinusoidal
e_r = [1000,0,100,0,0,0,0];
e_s = [-250-433.02i,0,-50+86.6i,0,0,0,0];
e_t = [0,0,0,0,0,0,0];

i_h_rs_par = [0,100,0,100,0,0,100];
i_h_st_par = [0,0,0,0,0,0,0];
i_h_tr_par = [0,0,0,0,0,0,0];

%% Declare here
U_rms_tot_sq=0;P_tot_sq=0;C_n_tot=0;i_r_rms_sq=0;i_a_rms_sq=0;i_u_rms_sq=0;
Q_tot_sq=0;Du_tot_sq=0;i_rms_tot_sq=0;S_ind_sq=0;D_scat_sq=0;C_n1_tot=0;
i_scat_rms_sq=0;i_rms1_sq=0;i_reac_rms_sq=0;i_act_rms_sq=0;i_unb_rms_sq=0;
i_unb_p_sq = 0;i_unb_n_sq=0;
%%

fprintf(' Results for the n belonging to Nc \n');
for n= 1 : 1 : 7
    %fprintf(' Results for n = %d \n',n);
    % THIS PART HAS THE ACTUAL LOAD PARAMETERS... CHANGE HERE ...
    Z_sou(1,n)= 0.1+0.3*n*1i ; % Source inductance is 3*.000265 H
    Z_rs(1,n)=2+n*2i;%
    Z_st(1,n)=inf;%
    Z_tr(1,n)=inf;%;
    Y_rs(1,n)=1/Z_rs(1,n);
    Y_st(1,n)=1/Z_st(1,n);
    Y_tr(1,n)=1/Z_tr(1,n);

   i_r(1,n) = ( e_r(1,n) - e_s(1,n) ) / ( 2 * Z_sou(1,n) + Z_rs(1,n) );
   i_s(1,n) = ( e_s(1,n) - e_r(1,n) ) / ( 2 * Z_sou(1,n) + Z_rs(1,n) );
   i_t(1,n) = 0 ;



U_r(1,n) = e_r(1,n) - Z_sou(1,n) * i_r(1,n);
U_s(1,n) = e_s(1,n) - Z_sou(1,n) * i_s(1,n);
U_t(1,n) = e_t(1,n) - Z_sou(1,n) * i_t(1,n);

    % Calculations of Various voltages ( wrt to Art. Zero )
U_p(1,n) = (1/3) * ( U_r(1,n) + alp * U_s(1,n) + (power(alp,2)) * U_t(1,n));%
Pos seq voltage
U_n(1,n) = (1/3) * ( U_r(1,n) + (power(alp,2)) * U_s(1,n)+ alp * U_t(1,n)
);%Neg seq vol
U_z(1,n) = (1/3) * ( U_r(1,n) +  U_s(1,n) + U_t(1,n)); % zero seq voltage
U_rms_art(1,n) = sqrt(3) * sqrt((U_p(1,n) * conj(U_p(1,n)))+(U_n(1,n) *
conj(U_n(1,n)))) ;
U_rms_tot_sq=U_rms_tot_sq+power(U_rms_art(1,n),2); % adding up squares of RMS
for later
U_r_art(1,n) = U_p(1,n) + U_n(1,n)    % Ur wrt art zero
U_s_art(1,n)=U_p(1,n) * conj(alp) + alp * U_n(1,n)
U_t_art(1,n)=U_p(1,n) * (alp) + conj(alp) * U_n(1,n)
U_rs_art(1,n) = U_r_art(1,n) - U_s_art(1,n) ;
```

```
U_st_art(1,n) = U_s_art(1,n) - U_t_art(1,n) ;
U_tr_art(1,n) = U_t_art(1,n) - U_r_art(1,n) ;
U_rms_art1(1,n) = sqrt(power(abs(U_r_art(1,n)),2) +
power(abs(U_s_art(1,n)),2)+power(abs(U_t_art(1,n)),2));
% ****** BRANCH CURRENTS ******
i_rs(1,n) = Y_rs(1,n) * U_rs_art(1,n) ;
i_st(1,n) = Y_st(1,n) * U_st_art(1,n) ;
i_tr(1,n) = Y_tr(1,n) * U_tr_art(1,n) ;
% ****** LINE CURRENTS *******
i_R(1,n) = i_rs(1,n)-i_tr(1,n);
i_S(1,n) = i_st(1,n)-i_rs(1,n);
i_T(1,n) = i_tr(1,n)-i_st(1,n);
i_r_rms = abs(i_R(1,n))
i_s_rms= abs(i_S(1,n))
i_t_rms= abs(i_T(1,n))

%Complex Powers for Harmonic and Balanced Admittances
C_n(1,n)=U_r_art(1,n)*conj(i_R(1,n))+
U_s_art(1,n)*conj(i_S(1,n))+U_t_art(1,n)*conj(i_T(1,n))
C_n_tot=C_n_tot+C_n(1,n); % The sum of Complex powers for each harmonic..
Mistake like Budanue do not use this

        if U_rms_art(1,n)== 0
                Y_b(1,n)=0;G_b(1,n)=0;B_b(1,n)=0;
        else
                Y_b(1,n) = (conj(C_n(1,n)))/power(U_rms_art(1,n),2); %
equivalent balanced admittance or eqv admittance of balance load
                G_b(1,n) = real(Y_b(1,n)); % real of Y_b or eqv balanced
conductance
                B_b(1,n) = imag(Y_b(1,n)) ;% eqv balanced susceptance
        end


% Y_b(1,n) = (conj(C_n(1,n)))/power(U_rms_art(1,n),2) % equivalent balanced
admittance or eqv admittance of balance load
% G_b(1,n) = real(Y_b(1,n)); % real of Y_b or eqv balanced conductance
% B_b(1,n) = imag(Y_b(1,n)); % eqv balanced susceptance

i_rms1(1,n) = sqrt( power(abs(i_R(1,n)),2) + power(abs(i_S(1,n)),2) +
power(abs(i_T(1,n)),2));
i_rms1_sq = i_rms1_sq + power (i_rms1(1,n),2);
% fprintf(' Positive sequence voltage Up = %d at %d  \n',
abs(U_p(1,n)),angle(U_p(1,n))*180/pi);
% fprintf(' Negative sequence voltage Un = %d at %d  \n',
abs(U_n(1,n)),angle(U_n(1,n))*180/pi);
% fprintf(' Zero sequence voltage Uz = %d at %d  \n',
abs(U_z(1,n)),angle(U_z(1,n))*180/pi);

end
% U_rms_art;
P_n_tot= real(C_n_tot);
G_b_tot= P_n_tot/U_rms_tot_sq;

for n= 1 :1 : 7
a_complex(1,n) = (U_n(1,n) /U_p(1,n)) ; % this is complex quantity "a"
```

```
Y_e(1,n) = Y_rs(1,n) + Y_st(1,n) + Y_tr(1,n); % eqv admittance
G_e(1,n) = real(Y_e(1,n));% eqv conductance
B_e(1,n) = imag(Y_e(1,n));%eqv susceptance
Y_d(1,n)= Y_e(1,n) - Y_b(1,n);
% ******* UNBALANCE PARAMETERS********
A_p(1,n) = -1 * ( Y_st(1,n) + alp * Y_tr(1,n) + (conj(alp)) * Y_rs(1,n) ) ; %
posi. seq unbalance admittance
A_n(1,n) = -1 * ( Y_st(1,n) + conj(alp) * Y_tr(1,n) + alp * Y_rs(1,n) ) ; %
neg seq unbalance admittance
a(1,n)=abs(a_complex(1,n));
theta(1,n)=angle(a_complex(1,n));
%***********************************************
% ******** DIFFERENT CURRENTS FOR THE HARMONICS**********
% i_act(1,n) = G_b(1,n) * U_rms_art(1,n);
i_act(1,n) = G_b_tot * U_rms_art(1,n);
i_scat_rms(1,n) = abs(-G_b_tot + G_b(1,n))* U_rms_art(1,n);
i_rea(1,n) = abs(B_b(1,n)) * U_rms_art(1,n);
I_Ru_P(1,n) = A_n(1,n) * U_n(1,n) + Y_d(1,n) * U_p(1,n);
I_Ru_N(1,n) = A_p(1,n) * U_p(1,n) + Y_d(1,n) * U_n(1,n);
i_unb_p_sq = i_unb_p_sq + power(abs(I_Ru_P(1,n)),2);
i_unb_n_sq = i_unb_n_sq + power(abs(I_Ru_N(1,n)),2);
i_unb(1,n) = sqrt(3) * sqrt( power(abs(I_Ru_P(1,n)),2) +
power(abs(I_Ru_N(1,n)),2) ) ;
%****SQUARING ALL CURRENTS OF EACH HARMONIC TO FIND POWERS LATER*****
i_reac_rms_sq=i_reac_rms_sq+power(i_rea(1,n),2);
i_act_rms_sq=i_act_rms_sq+power(i_act(1,n),2);
i_unb_rms_sq=i_unb_rms_sq+power(i_unb(1,n),2);
i_scat_rms_sq = i_scat_rms_sq + power(i_scat_rms(1,n),2);
% TOTAL RMS ..CALCULATED FOR REFERENCE .... TO COMPARE BOTH WAYS
i_rms(1,n) = sqrt( power(i_act(1,n),2) + power(i_rea(1,n),2) +
power(i_unb(1,n),2) + power(i_scat_rms(1,n),2));
i_rms_tot_sq = i_rms_tot_sq + power(i_rms(1,n),2);
end
Com_pow = C_n;
i_rms = sqrt(i_rms_tot_sq);
i_acti = sqrt(i_act_rms_sq);
i_react = sqrt(i_reac_rms_sq);
i_unbal = sqrt(i_unb_rms_sq);
i_scater = sqrt(i_scat_rms_sq);
i_diff1 = i_rms1_sq - i_rms_tot_sq ;% This suggests that the first RMS
current calculated usING Ir Is It doesnt give scattered
i_diff2 = i_rms_tot_sq  - i_reac_rms_sq- i_act_rms_sq-i_unb_rms_sq-
i_scat_rms_sq;
P = sqrt(U_rms_tot_sq * i_act_rms_sq);
D_scat = sqrt(U_rms_tot_sq * i_scat_rms_sq);
Q = sqrt(U_rms_tot_sq * i_reac_rms_sq);
D_unb = sqrt(U_rms_tot_sq * i_unb_rms_sq);
%power(P_n_tot,2) - power(P,2)%+ power(D_scat,2) not useful after error
S_1_sq = i_rms_tot_sq * U_rms_tot_sq; % Using act, reac , .. currents
S_2_sq = i_rms1_sq * U_rms_tot_sq ; % Using Line currents
%Pow_diff=(S_1_sq-  S_2_sq-power(D_scat,2));  % This gave D scatered
S_3_sq = power(P,2) + power(D_scat,2) + power(Q,2) + power(D_unb,2); % Using
squares of powers for reference
AppPow=sqrt(S_2_sq);
U_p;U_n;Y_e; Y_b;Y_d;
```

```
fprintf(' || u || total Voltgae three phase rms : %d \n',sqrt(U_rms_tot_sq));
fprintf(' || i || total current three phase rms : %d \n',
sqrt(i_rms_tot_sq));
fprintf(' || i_a|| Active Current RMS %d \n',i_acti )
fprintf(' || i_r|| reactove Current RMS %d \n',i_react )
fprintf(' || i_s|| Scattered Current RMS %d \n',i_scater )
fprintf(' ||i_u_p|| Unbalanced Current pos seq RMS %d \n',sqrt(i_unb_p_sq));
fprintf(' ||i_u_n|| Unbalanced Current neg seq RMS %d \n',sqrt(i_unb_n_sq));
fprintf(' || i_u|| Unbalanced Current RMS %d \n',i_unbal )
fprintf(' || i_a|| Active Power : %d \n',P);
fprintf(' Q Reactive Power : %d \n',Q);
fprintf(' D_s Scattered Power : %d \n',D_scat);
fprintf(' D_u Unbalanced Power : %d \n',D_unb);
fprintf(' S_c Apparent Power  : %d \n',AppPow);

%% Section for the HGL
% It is assumed that the generated harmonics are of the 2,4 and 7 sequence

u_hgl_rms_sq = 0 ; i_rms_hgl_sq=0;

P_h_n_tot=0;U_h_rms_tot_sq=0;i_h_rms1_sq=0;S_h_tot_sq=0;
for n = [1:1:7]
disp('               ');
fprintf('    ******   For HGL Order n = %d ********       \n',n )

Z_sou(1,n)= 0.1+0.3*n*1i ;

i_h_rs(1,n) = i_h_rs_par(1,n) ;%* (2+n*2i) / (2+n*2i+Z_sou(1,n));
i_h_st(1,n) = i_h_st_par(1,n) ;%* (2+n*2i) / (2+n*2i+Z_sou(1,n));
i_h_tr(1,n) = i_h_tr_par(1,n) ;%* (2+n*2i) / (2+n*2i+Z_sou(1,n));

%i_load(1,n)= i_h_rs_par(1,n) - i_h_rs(1,n);
%abs(i_load(1,n));

% ****** LINE CURRENTS *******
i_h_R(1,n) = i_h_rs(1,n)-i_h_tr(1,n);
i_h_S(1,n) = i_h_st(1,n)-i_h_rs(1,n);
i_h_T(1,n) = i_h_tr(1,n)-i_h_st(1,n);

abs(i_h_R(1,n));abs(i_h_S(1,n));abs(i_h_T(1,n));

U_h_r(1,n) =  - Z_sou(1,n) * i_h_R(1,n);
U_h_s(1,n) =  - Z_sou(1,n) * i_h_S(1,n);
U_h_t(1,n) =  - Z_sou(1,n) * i_h_T(1,n);

   % Calculations of Various voltages ( wrt to Art. Zero )
U_h_p(1,n) = (1/3) * ( U_h_r(1,n) + alp * U_h_s(1,n) + (power(alp,2)) *
U_h_t(1,n));% Pos seq voltage
U_h_n(1,n) = (1/3) * ( U_h_r(1,n) + (power(alp,2)) * U_h_s(1,n)+ alp *
U_h_t(1,n) );%Neg seq vol
U_h_z(1,n) = (1/3) * ( U_h_r(1,n) +  U_h_s(1,n) + U_h_t(1,n)) ;% zero seq
voltage
U_h_rms_art(1,n) = sqrt(3) * sqrt((U_h_p(1,n) * conj(U_h_p(1,n)))+(U_h_n(1,n)
* conj(U_h_n(1,n)))) ;
```

```
U_h_rms_tot_sq=U_h_rms_tot_sq+power(U_h_rms_art(1,n),2); % adding up squares
of RMS for later

U_h_r_art(1,n) = U_h_p(1,n) + U_h_n(1,n)   % Ur wrt art zero
U_h_s_art(1,n)=U_h_p(1,n) * conj(alp) + alp * U_h_n(1,n)
U_h_t_art(1,n)=U_h_p(1,n) * (alp) + conj(alp) * U_h_n(1,n)
U_h_rs_art(1,n) = U_h_r_art(1,n) - U_h_s_art(1,n) ;
U_h_st_art(1,n) = U_h_s_art(1,n) - U_h_t_art(1,n) ;
U_h_tr_art(1,n) = U_h_t_art(1,n) - U_h_r_art(1,n) ;
U_h_rms_art1(1,n) = sqrt(power(abs(U_h_r_art(1,n)),2) +
power(abs(U_h_s_art(1,n)),2)+power(abs(U_h_t_art(1,n)),2));

i_h_rms1(1,n) = sqrt( power(abs(i_h_R(1,n)),2) + power(abs(i_h_S(1,n)),2) +
power(abs(i_h_T(1,n)),2));
S_n(1,n) = U_h_rms_art1(1,n)*i_h_rms1(1,n);
i_h_rms1_sq = i_h_rms1_sq + power (i_h_rms1(1,n),2);

%Complex Powers for Harmonic and Balanced Admittances
U_h_r_art(1,n)*conj(i_h_R(1,n));
U_h_s_art(1,n)*conj(i_h_S(1,n));
U_h_t_art(1,n)*conj(i_h_T(1,n));

C_h_n(1,n)=U_h_r_art(1,n)*conj(i_h_R(1,n))+
U_h_s_art(1,n)*conj(i_h_S(1,n))+U_h_t_art(1,n)*conj(i_h_T(1,n));
P_h_n(1,n)= real(C_h_n(1,n));

P_h_n_tot = P_h_n_tot + P_h_n(1,n);
fprintf('Harmonic Current:%d at %d
\n',abs(i_h_rs(1,n)),angle(i_h_rs(1,n))*180/pi);
fprintf('Line R Current:%d at %d
\n',abs(i_h_R(1,n)),angle(i_h_R(1,n))*180/pi);
fprintf('Line S Current:%d at %d
\n',abs(i_h_S(1,n)),angle(i_h_S(1,n))*180/pi);
fprintf('Current 3 phase rms :%d  \n',i_h_rms1(1,n));
fprintf('Voltage Ur :%d at %d
\n',abs(U_h_r_art(1,n)),angle(U_h_r_art(1,n))*180/pi);
fprintf('Voltage Us :%d at %d
\n',abs(U_h_s_art(1,n)),angle(U_h_s_art(1,n))*180/pi);
fprintf('Voltage 3 phase rms :%d  \n',U_h_rms_art1(1,n));
fprintf('Active Power:%d \n',P_h_n(1,n));
end
P;
Com_pow_har = C_h_n;
P_all = P + P_h_n_tot;

U_all_rms_sq = U_rms_tot_sq + U_h_rms_tot_sq;  % Total VOltage RMS sq
i_all_rms_sq = i_rms_tot_sq + i_h_rms1_sq;
S_all_sq = U_all_rms_sq * i_all_rms_sq;
%
S_h1_sq = U_h_rms_tot_sq * i_h_rms1_sq;

S_E_sq = U_rms_tot_sq * i_h_rms1_sq + U_h_rms_tot_sq * i_rms_tot_sq;
S_1_all_sq = S_1_sq + S_h1_sq + S_E_sq;
S_1_all = sqrt(S_1_all_sq) ;
pf_all = P_all / S_1_all;
```

```
pf_1 = P / AppPow;
disp('                ');
disp(' Final Results of Power     ');
disp('                ');
fprintf('Uh -  Harmonic Voltage 3 phase rms :%d  \n',sqrt(U_h_rms_tot_sq));
fprintf('Ih  Harmonic Geenrated Current 3 phase rms :%d
\n',sqrt(i_h_rms1_sq));
fprintf('SG -  Harmonic Apparent Power SG =  :%d  \n',sqrt(S_h1_sq));
fprintf('SE  -Cross Harmonic Apparent Power SE =  :%d  \n',sqrt(S_E_sq));
fprintf('SC - Supp;ly Apparent Power SC =  :%d  \n',AppPow);
fprintf('S - Apparent Power from diff power S =  :%d  \n',S_1_all);
fprintf('U rms 3 ph voltage three phase RMS value :%d
\n',sqrt(U_all_rms_sq));
fprintf('I rms 3 ph - current three phase RMS value :%d
\n',sqrt(i_all_rms_sq));
fprintf('S - Apparent Power voltage and currents S =  :%d
\n',sqrt(S_all_sq));
fprintf('Total Active Power P =  :%d  \n',P_all);
fprintf(' pf  Power Factor overall %d \n',pf_all);

%% Section to calculate things to Plot
i_h_R;
i_R;
disp (' Current waveforms values : ');
for n = 1 : 1 : 7
i_h_R(1,n) = i_R(1,n)+i_h_R(1,n);
i_h_S(1,n) = i_S(1,n)+i_h_S(1,n);
U_h_r_art(1,n) = U_r_art(1,n)+U_h_r_art(1,n);
U_h_s_art(1,n) = U_s_art(1,n)+U_h_s_art(1,n);
U_h_t_art(1,n) = U_t_art(1,n)+U_h_t_art(1,n);

% fprintf(' U_r - n = %d : %d at %d \n ',
n,abs(U_h_r_art(1,n)),angle(U_h_r_art(1,n))*180/pi);
% fprintf(' U_s - n = %d : %d at %d \n ',
n,abs(U_h_s_art(1,n)),angle(U_h_s_art(1,n))*180/pi);
% fprintf(' U_t - n = %d : %d at %d \n ',
n,abs(U_h_t_art(1,n)),angle(U_h_t_art(1,n))*180/pi);
% fprintf(' i_r - n = %d : %d at %d \n ',
n,abs(i_h_R(1,n)),angle(i_h_R(1,n))*180/pi);
% fprintf(' i_s - n = %d : %d at %d \n ',
n,abs(i_h_S(1,n)),angle(i_h_S(1,n))*180/pi);
end
i_h_R;
U_h_r_art;
Fc=60;
Fs = 8000;                    % samples per second
dt = 1/Fs;                    % seconds per sample
StopTime = .05;
nh=[1,2,3,5,7];
i_R_plot=0;i_S_plot=0;i_T_plot=0;
E_R_plot = 0; E_S_plot = 0; E_T_plot = 0;
V_R = 0; V_S = 0; V_T = 0;
for n = 1:1:7
 t = (0:dt:StopTime-dt)';      % seconds
    i_R_plot = i_R_plot + abs(i_h_R(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(i_h_R(1,n)));
```

```
    i_S_plot = i_S_plot + abs(i_h_S(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(i_h_S(1,n)));
    E_R_plot = E_R_plot + abs(e_r(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(e_r(1,n)));
    E_S_plot = E_S_plot + abs(e_s(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(e_s(1,n)));
    E_T_plot = E_T_plot + abs(e_t(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(e_t(1,n)));

    V_R = V_R + abs(U_h_r_art(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(U_h_r_art(1,n)));
    V_S = V_S + abs(U_h_s_art(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(U_h_s_art(1,n)));
    V_T = V_T + abs(U_h_t_art(1,n))*sqrt(2) * sin(2*pi*n*Fc*t+
angle(U_h_t_art(1,n)));
end



   figure;
   axis;
   subplot(3,1,1)
   axis;
   plot(t,i_R_plot,'b',t,i_S_plot,'r',t,i_T_plot,'g');
   xlabel('time (in seconds)');
   title('Plot of Line Currents ');
   grid on ;
   zoom xon;
   legend('I(R)','I(S)','I(T)')

   subplot(3,1,2)
   plot(t,E_R_plot,'b',t,E_S_plot,'r',t,E_T_plot,'g');
   xlabel('time (in seconds)');
   title('Plot of Intertal voltage of distribution system');
   legend('E(R)','E(S)','E(T)')
   grid on ;
   zoom xon;
   subplot(3,1,3)
   plot(t,V_R,'b',t,V_S,'r',t,V_T,'g');
   xlabel('time (in seconds)');
   title('Plot of load voltages ');
   legend('U(R)','U(S)','U(T)')
   grid on ;
   zoom xon;
%
```

VITA

Prashanna Dev Bhattarai was born in Kathmandu, Nepal in April 1984. He finished his high school from St. Xavier's Campus in 2003 and obtained Bachelor's degree in Electrical Engineering from Pulchowk Engineering Campus, Nepal in 2008. After working part time for a year, travelling to India, and trekking in the Himalayas, he joined the Department of Electrical Engineering at Louisiana State University for graduate studies in Fall 2009. He completed Masters of Science in Electrical Engineering in 2013, and is expected to complete his PhD in Electrical Engineering in 2016, both under the supervision of Dr. Leszek Czarnecki. Outside of academia his interests include trekking, soccer, and music.