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An Efficient Fault Location Algorithm for Shipboard Power Systems

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AN EFFICIENT FAULT LOCATION ALGORITHM FOR SHIPBOARD POWER SYSTEMS

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
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in

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ABSTRACT

The Shipboard Power System (SPS) supply energy to sophisticated systems for navigation, communication, weapons, and operation. Due to the ship's critical operating condition, faults can be very detrimental. Faults in the SPS may happen because of failure of electrical components or by damages that happen during a battle. These faults may interrupt the paths for supplying energy to loads that are not damaged. To enhance survivability of naval ships, SPS requires an efficient fault location algorithm in order to locate and clear the fault as well as provide an alternative path to supply energy to the loads that are not faulty or damaged.

This thesis introduces a method to generalize the Active Impedance Estimation (AIE) fault location method for Shipboard Power Systems (SPS.) In the proposed method short-duration high-frequency voltage sources are employed at selected buses and voltage/current measurements are taken for the purpose of fault location. The goal is to obtain the minimum number of voltage and current sources and measurements that observe all the faults of interest that occur in the SPS. In contrast with the conventional AIE method, in the proposed fault location method both sources and measurements are applied at multiple buses. Moreover, both voltage and current are measured at measurement buses. The proposed approach is not restricted to lateral branches and can be applied to interconnected SPSs. The fault location method does not interfere with the system's normal operation due to the applied high frequency(s) and thus superposition is used in the analysis. This approach reduces the number of measurement devices for fault location in the SPS which results in significant cost reduction. The proposed method is then applied to a SPS in simulation using MATLAB/Simulink to show the effectiveness of the approach.

CHAPTER 1
INTRODUCTION

1.1 History of Shipboard Power Systems (SPS)

The first shipboard power system was installed on the USS Trenton (figure 1.1) in 1883. The system was supplying current to 247 lamps at a voltage of 110 volts dc [1].

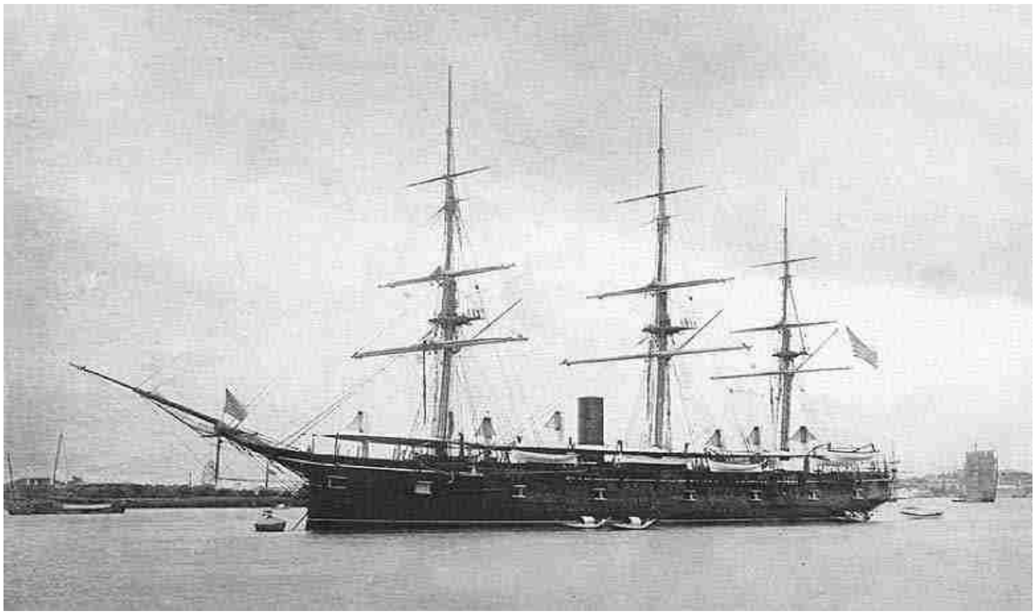

Figure 1.1 USS Trenton

Until the 1914 to 1917, the early electric power systems on ships were mostly dc with mainly motors and lighting loads. During World War I, 230 volt, 60 Hz power systems were introduced into naval vessels. Since World War II, the ship's electric systems have continued to improve, including the use of 4,160 volt power systems and the introduction of protective devices [1].

Protective devices were developed to monitor the essential parameters of electric power systems. Also, they were uses to determine the degree of configuration of the system that is necessary to limit the damage to components and equipment and to enhance the continuity of electric service for the system. While fuses were used in the past, circuit breakers were added at the end of the century. The first electronic solid-state overcurrent protective device used by the U.S. Navy was installed on the 4,160 power system in Nimitz class carriers [1].

1.2 SPS System Structure

The power in the Shipboard Power Systems (SPS) is produced by multiple generators that are normally placed in a ring configuration [1]-[7]. Usually, there are two kinds of loads in the SPS: vital loads and non-vital loads [1]-[5], [8], [9]. Navigation, communication, operation and weapons are examples of the vital loads while lighting and air conditioning systems are part of the non-vital loads [1]-[4], [6], [8], [11]. The SPS aims at supplying energy for both types of loads. In the fault condition, the system is not able to supply electrical energy to the loads. The SPS needs a comprehensive protection system in order to detect the exact location of the fault and use some alternate path to supply energy to the unfaulty loads [6], [8], [10]. It is important to note that the fault location mechanisms need not be as fast as the protective mechanisms that disconnect in milliseconds. Rather, the fault location algorithms capture the fault data quickly and try to locate the fault in a reasonably short time to redirect the electric power to the vital loads. There are three main protection schemes in power system: overcurrent, distance, and differential [7], [11].

Shipboard power system uses three-phase generators that are in a ring configuration and generally work at 60 Hz to generate AC voltage for the system. Generators are in a ring configuration in order to have alternative paths for vital loads from different generators. It enables the system to supply power to vital loads when the normal path from the main generator is defective or destroyed [1]-[5], [8].

Figure 1.2 shows single-line diagram of an 11-bus SPS [1]. This system consists of four three-phase main generators in a ring configuration operating at 60 Hz. In this system, vital loads have an alternative path in addition to the normal path from other generators to receive energy from sources in fault situations. The vital loads use either Automatic Bus Transfer (ABT) or Manual Bus Transfer (MBT) to choose an unfaulty path in order to receive the energy from the generators [1]-[6], [8]. In normal condition ABT/MBT connects the normal path to the load. When the fault occurs ABT/MBT disconnects the normal path and connects the alternative path.

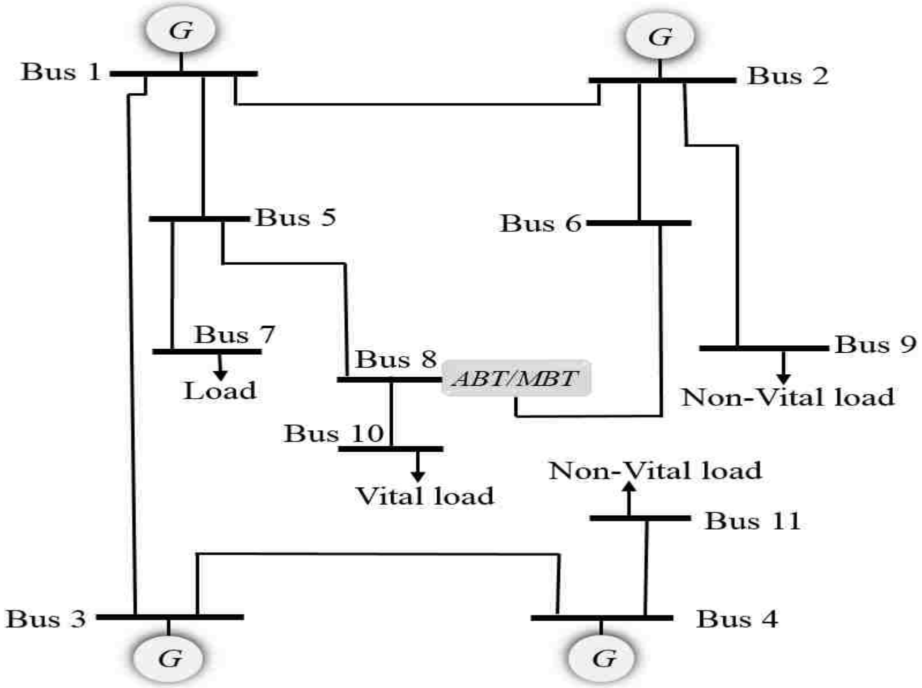

Figure 1.2 11-Bus SPS

1.3 Distance Protection and its Drawbacks

The cables lengths in the SPS are normally shorter (about 10-200 feet long [11]) than in the large distribution grids and thus the impedances of the cables are small (about 0.04/1000 feet

[11].) Using distance protection in short-length power systems is impractical because the impedances of the cables are too low to detect with small error. An improved distance scheme can be utilized to detect faults in short-length cables. The Active Impedance Estimation (AIE) fault location method [13] utilizes a high-frequency voltage at a bus in the electric system and measures the injected current followed by calculating the impedance at that frequency. A higher frequency adds resolution to the cable impedance and makes fault location easier in the shipboard power systems. In the AIE method when the system is exposed to the fault, a short duration voltage will be utilized in order to find the impedance of the system seen from the injection bus and locate the fault. This method has been applied only to radial distribution systems. This method uses the value of measured impedance to locate the fault [13], [15]. Though the available AIE can distinguish between far and close-up faults, it can be mainly utilized in lateral branches where the Thevenin equivalent impedance is equal to the cable impedance and is proportional to the fault distance. Thus, in interconnected systems, such as shipboard power systems with ring topology, the available active impedance estimation method has topological limitations.

1.4 Overcurrent Protection and its Drawbacks

Another conventional fault detection and location method is overcurrent scheme. The main power of the SPS is produced by multiple generators. Having multiple power supplies causes a complex overcurrent protection in the system that requires time delay in order to avoid over tripping [7]. Due to cables short lengths, SPS is considered a highly coupled electric system; that is, if a fault occurs in one point of the system and a quick detection and isolation is not provided by the protection system, the fault will propagate through the entire system in a short period of time and can cause catastrophic consequences [2]. Thus, using only the conventional overcurrent protection mechanisms is impractical in the shipboard power system due to short cables, time-delay requirements, and multiple supplies, that complicate the scheme [7], [11]. Overcurrent protection, however, can be employed as a safety feature to increase protection capabilities in addition to another protection scheme.

1.5 Differential Protection and its Drawbacks

Differential protection scheme, on the other hand, works properly in the system with short cables. Differential relays compare the entering current to the protected equipment with the current that leaves the equipment. If these two currents are equal, as shown in figure 1.3(a), there is no fault. However, if these two currents are not equal, as shown in figure 1.3(b), indicating that there is a fault in the protected equipment, the relay trips [7], [11]. In this method each piece of equipment in the SPS requires a differential relay to locate the fault effectively. Also, a comprehensive communication system between all protected zones and pertinent equipment is needed in order to cover the entire SPS, appropriately. Vulnerability of the communication system to fault highly reduces system reliability [9], [13]. Moreover, the approach is costly since it requires numerous differential relays and a comprehensive communication infrastructure.

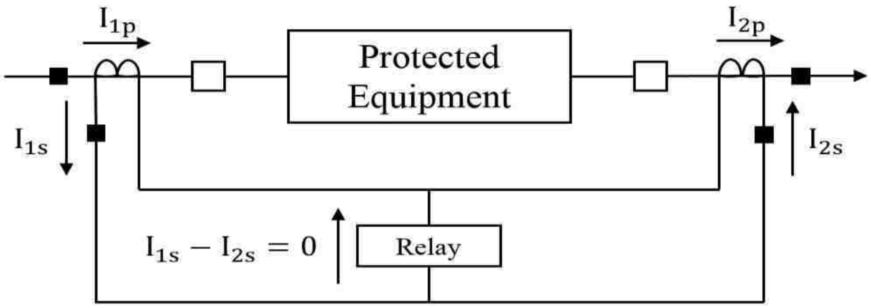

Figure 1.3(a) Differential Fault Detection (No Fault)

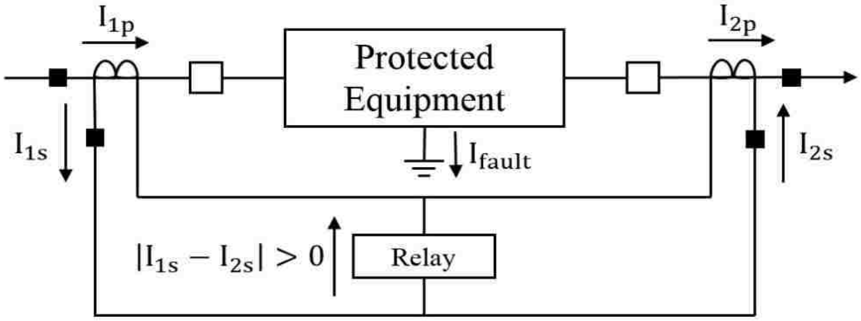

Figure 1.3(b) Differential Fault Detection (With Fault)

1.6 Proposed Method

There is a need to develop a more efficient fault location scheme to locate all the faults that occur in the shipboard power system with low cost. This thesis introduces an economical and reliable fault location scheme for the SPS. This method requires short-duration voltage application(s) with high frequency(s) in fault condition and observation of the changes in voltages and currents of the system buses due to the fault. Changes in the voltage and current in the measurement point are indicators of the location and magnitude of the fault. Different faults may have similar effects on the voltage and current at a measurement point. In this case, multi-estimation occurs [14]. Thus, the system needs multiple voltage application and/or measurement points to have unique data set for each fault. The goal is to minimize the number of the voltage applications and measurements and to find their optimal places in the system in order to uniquely identify each fault. In this paper, three-phase symmetrical faults are analyzed; however, the proposed method can be generalized to other types of faults.

1.7 References

[1] K. L. Butler-Purry, N. D. R. Sarma, C. Whitcomb, H. D. Carmo, and H. Zhang, “Shipboard Systems Deploy Automated Protection,” IEEE Computer Applications in Power, vol. 11, no. 2, pp. 31-36, Apr. 1998.

[2] S. Srivastava, and K. L. Butler-Purry, “Expert-system method for automatic reconfiguration for restoration of shipboard power sytems”, IEE Proceedings Generation, Transmission and Distribution, vol. 153, no. 3, pp. 253-260, May 2006.

[3] K. L. Butler-Purr, and N. D. R. Sarma, “Self-Healing Reconfiguration for Restoration of Naval Shipboard Power Systems,” IEEE Transactions on Power Systems, vol. 19, no. 2, pp. 754-762, May 2004.

[4] K. L. Butler-Purry, N. D. R. Sarma, and I. V. Hicks, “Service restoration in naval shipboard power systems,” IEE Proc. Gener. Transm. Distrib., vol. 151, pp. 95-102, Jan. 2004.

[5] F. M. Uriarte, and K. L. Butler-Purry, “Multicore Simulation of an AC-Radial Shipboard Power System,” Power and Energy Society General Meeting, 2010 IEEE , vol., no., pp. 1,8, 25-29 July 2010.

[6] W. M. Dahalan, and H. B. Mokhlis, “Techniques of Network Reconfiguration for Service Restoration in Shipboard Power System: A Review,” Australian Journal of Basic and Applies Science, 2010.

[7] Yanfeng Gong, Yan Huang, and N. N. Schulz, “Integrated Protection System Design for Shipboard Power System,” IEEE Transactions on Industry Applications, vol. 44, no. 6, pp. 1930-1936, Nov 2008.

[8] S. K. Srivastava, B. L. Butler, and N. D. R. Sarma, “Shipboard Power Restored for Active Duty,” IEEE Computer Application in Power, vol. 15, no. 3, pp. 16-23, July 2002.

[9] Weilin Li, A. Monti, and F. Ponci, “Fault Detection and Classification in Medium Voltage DC Shipboard Power Systems With Wavelets and Artificial Neural Network,” Instrumentation and Measurement, IEEE Transactions on ,vol. 63, no. 11, pp. 2651,2665, Nov. 2014.

[10] Quili Yu, S. Khushalani, J. Solanki, N. N. Schulz, H. L. Ginn, S. Grzybowski, A. Sirvastava, and J. Bastos, “Shipboard Power Systems Research Activities at Mississippi State University,” Electric Ship Technologies Symposium (ESTS), 2007 IEEE, vol., no., pp. 390,395, 21-23 May 2007.

[11] J. Tang, and P. G. McLaren, “A Wide Area Differential Backup Protection Scheme For Shipboard Application”, IEEE Transactions on Power Delivery, vol. 21, no. 3, July 2006.

[12] M. Islam, W. Hinton, M. McClelland, and K. Logan, “Shipboard IPS Technological Challenges-VFD and Grounding,” Electric Ship Technologies Symposium (ESTS), 2013 IEEE, vol., no., pp. 192-198, 22-24 April 2013.

[13] E. Christopher, M. Sumner, D. W. P. Thomas, Xiaohui Wang, and F. de Wildt, “Fault Location in a Zonal DC Marine Power System Using Active Impedance Estimation,” IEEE Transactions on Industry Applications, vol. 49, no. 2, pp. 860-865, March-April 2013.

[14] H. Nazaripouya, S. Mehraeen, “Optimal PMU Placement for Fault Observability in Distributed Power System by Using Simultaneous Voltage and Current Measurements,” Power and Energy Society General Meeting (PES), 2013 IEEE , vol., no., pp. 1,6, 21-25 July 2013.

[15] J. Wang, M. Sumner, D. W. P. Thomas, and R. D. Geertsma, “Active fault protection for an AC zonal marine power system,” Electrical Systems in Transportation, IET, vol. 1, no. 4, pp. 156,166, December 2011.

CHAPTER 2
METHODOLOGY

2.1 Introduction

In this thesis observation is made on the effect of each fault on the voltages and/or currents of a specific measurement set when a voltage with high frequency is applied to the system at injection buses. Pertinent terms and definitions are given:

Injection bus: For convenience, here the term injection bus is referred to the bus where the high frequency voltage is applied. There can be multiple injection buses with different frequencies in a SPS.

Measurement bus: Measurement bus is used to address the bus where the voltage and/or current is measured. It is important to mention that the injection bus may or may not be the same as the measurement bus. In addition, more than one measurement bus may be utilized. In this sense the proposed approach generalizes the conventional active impedance estimation fault location method [1], [2].

Measurement set: Measurement set is referred to the set of voltage and/or current measurement on all the buses with specified injection bus, frequency and R_{fault}. Measurement set helps to compare the results of measurement to find the unique result and the best combination of injection and measurement bus for the fault location.

Each fault may have a different effect on the voltage or current of a specific measurement bus. The goal of this paper is to find optimal places for the injection and the measurement buses to have unique measurement set for each fault. The unique measurements are then referred to a specific fault to detect the fault exact location.

Since SPS is working at 60 Hz, voltage application(s) and voltage/current measurement(s) should be applied at higher frequencies to avoid interference with the protection system. That is, superposition can be used for the analysis since the fault location algorithm is working separately from the normal operation of the system. Fault location algorithm does not see the voltage that is produced by main generators at 60 Hz but it still can detect the fault.

2.2 The Proposed Fault Location Algorithm

Suppose that I is the number of the bus that has the voltage application, M is the number of the bus that has the measurement, and F is the number of the bus that is faulty. Then, the ordered triple (I,M,F) represents a fault detection observation. In this case, $\vec{V}_{I,M,F}$ represents the voltage vector of the measurement bus when the injection is on bus I, measurement is on bus M, and fault is on bus F. If $F = 0$ it shows the initial values when there is no fault in the system. Similar definition is used for current measurement $\vec{I}_{I,M,F}$. The goal is to observe the values of $\vec{V}_{I,M,F}$ and $\vec{I}_{I,M,F}$ for all the faults, given the measurement and injection buses, and to compare the values with the normal values of currents and voltages to see if the faults are detectable. The algorithm starts from $I = 1,\ M = 1$ and applies a certain fault on each bus and observes the changes in voltages and currents at the measurement buses. For this purpose the algorithm calculates $\overrightarrow{\Delta V}_{I,M,F}$ if $I \neq M$, and $\overrightarrow{\Delta I}_{I,M,F}$ if $I = M$; that is,

$$\overrightarrow{\Delta V}_{I,M,F} = \vec{V}_{I,M,F} - \overrightarrow{V}_{I,M,0}$$

$$\overrightarrow{\Delta I}_{I,M,F} = \vec{I}_{I,M,F} - \overrightarrow{I}_{I,M,0}.$$

Figures 2.1 and 2.2 show the magnitude and phase angle of $\vec{V}_{I,M,F}$ for the simulated system described in figure 1.2 when $I = 10,\ M = 9,\ R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F $(1 \leq F \leq 11.)$ A sequence of faults with impedance $R_{fault} = 1e^{-3}$ is applied to different nodes of the systems as shown in figures and the effects are observed. Figures 2.3 and 2.4 show the magnitude and phase angle of $\vec{I}_{I,M,F}$ for the simulated system when $I = 2$, $M = 2$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F $(1 \leq F \leq 11.)$ Note that if $I = M$, algorithm considers the current values whereas for $I \neq M$ it considers the voltage values.

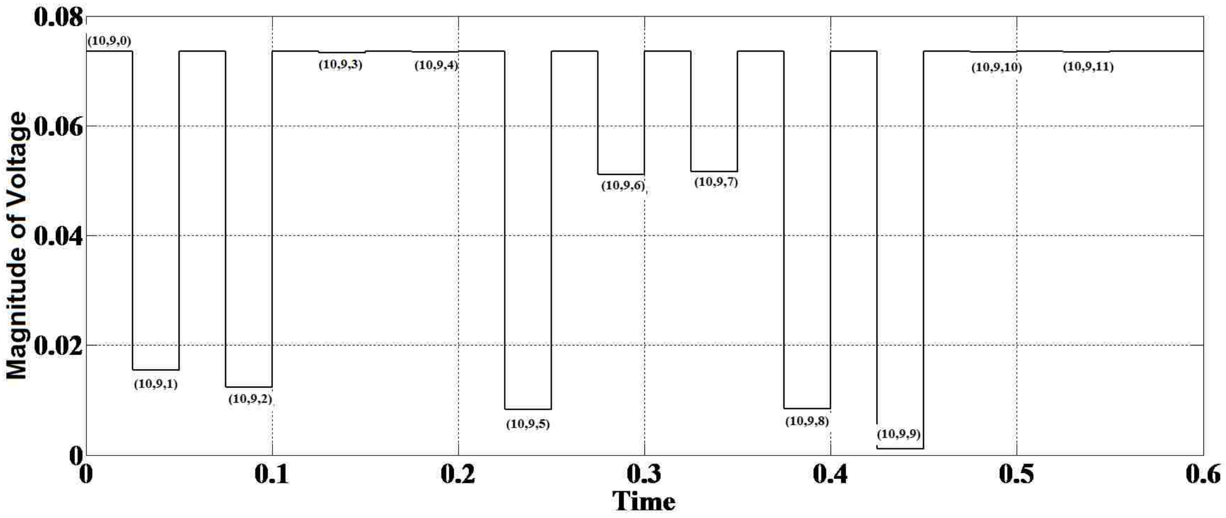

Figure 2.1 Magnitude of $\vec{V}_{I,M,F}$ when $I = 10$, $M = 9$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F

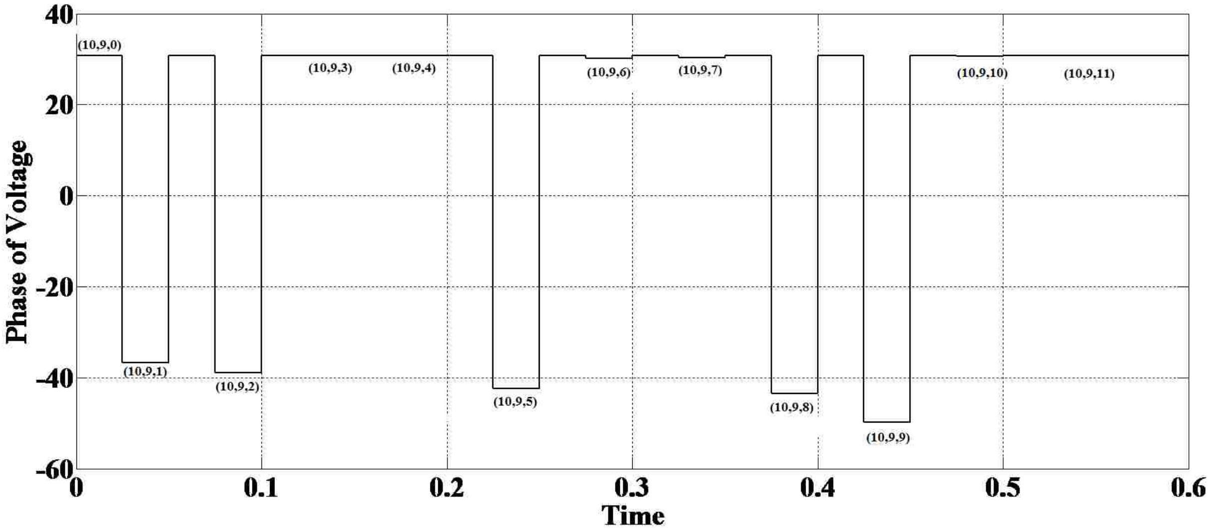

Figure 2.2 Phase angle of $\vec{V}_{I,M,F}$ when $I = 10$, $M = 9$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F

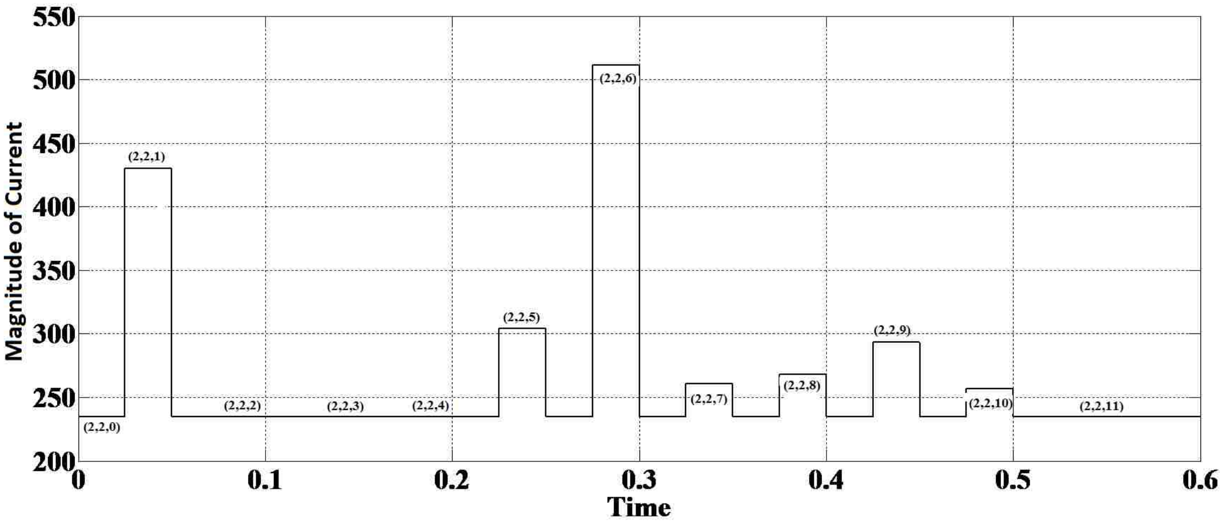

Figure 2.3 Magnitude of $\vec{I}_{I,M,F}$ when $I = 2$, $M = 2$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F

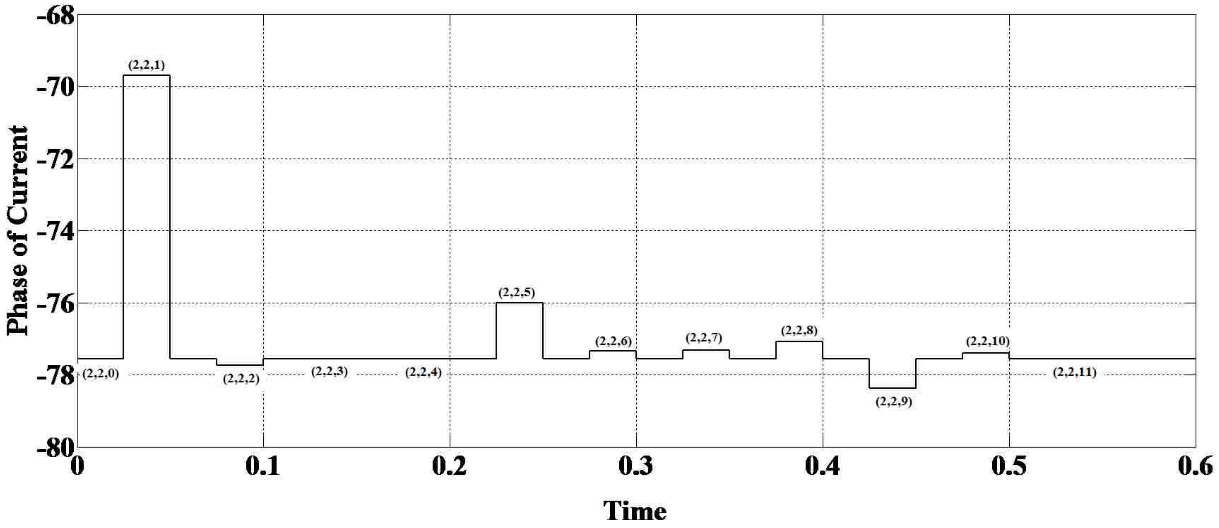

Figure 2.4 Phase angle of $\vec{I}_{I,M,F}$ when $I = 2$, $M = 2$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$ for different values of F

In the shipboard power system if $\left|\frac{\vec{V}_{I,M,F}-\vec{V}_{I,M,0}}{|\vec{V}_{I,M,0}|}\right| = \left|\frac{\overrightarrow{\Delta V}_{I,M,F}}{|\vec{V}_{I,M,0}|}\right|$ or $\left|\frac{\vec{I}_{I,M,F}-\vec{I}_{I,M,0}}{|\vec{I}_{I,M,0}|}\right| = \left|\frac{\overrightarrow{\Delta I}_{I,M,F}}{|\vec{I}_{I,M,0}|}\right| < 0.001$, $\overrightarrow{\Delta V}$ and $\overrightarrow{\Delta I}$ are difficult to detect. Table 2.1 shows the simulation results for the system presented in figure 1.2 when $I = 10$, $M = 9$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$. The numbers in Table 2.1 are the values of F that show which bus is faulty. Highlighted numbers show at which bus fault is not detectable with the selected measurement and injection buses. In other words, faults are not detectable when relative value of $\left|\overrightarrow{\Delta V}_{I,M,F}\%\right| = \frac{|\overrightarrow{\Delta V}_{I,M,F}|}{|\vec{V}_{I,M,0}|} = \frac{|\vec{V}_{I,M,F}-\vec{V}_{I,M,0}|}{|\vec{V}_{I,M,0}|}$ is smaller than 0.001 and/or $\left|\overrightarrow{\Delta I}_{I,M,F}\%\right| = \frac{|\overrightarrow{\Delta I}_{I,M,F}|}{|\vec{I}_{I,M,0}|} = \frac{|\vec{I}_{I,M,F}-\vec{I}_{I,M,0}|}{|\vec{I}_{I,M,0}|}$ is smaller than 0.001.

Table 2.1 Detectable and undetectable bus faults when $I = 10$, $M = 9$, $R_{fault} = 1e^{-3}$, and $f = 1000Hz$

Bus #	1	2	3	4	5	6	7	8	9	10	11
Delta V/I	0.06	0.066	0.02	0.0008	0.069	0.025	0.023	0.064	0.07	0.01	0.0009

The algorithm examines all the possible combinations with different values for M, I, and F to find the optimal buses for the injection and measurement that can detect all the faults (various fault impedances.) If one injection and measurement is not adequate to cover the entire system, the system requires more injection and/or measurement buses to cover all the faults. The proposed approach looks for injection and measurement buses that result in the lowest number of undetectable faults (location and impedance.) The fault is undetectable when $\left|\overrightarrow{\Delta V}_{I,M,F}\%\right| < 0.001$, or $\left|\overrightarrow{\Delta I}_{I,M,F}\%\right| < 0.001$.

Thus, the algorithm finds the cases that have the lowest number of undetectable faults with $\left|\overrightarrow{\Delta V}_{I,M,F}\%\right| < 0.001$, or $\left|\overrightarrow{\Delta I}_{I,M,F}\%\right| < 0.001$. For convenience, if the number of undetected faults are 0 or 1, the injection-measurement set comprising the selected injection and measurement buses are chosen. Then, the algorithm will check if these cases cause unique changes in $\left|\overrightarrow{\Delta V}_{I,M,F}\%\right|$ or $\left|\overrightarrow{\Delta I}_{I,M,F}\%\right|$for different faults. If $\left|\overrightarrow{\Delta V}_{I,M,F}\%\right|$ or $\left|\overrightarrow{\Delta I}_{I,M,F}\%\right|$ has the same results for different faults (i.e., that differ less than 0.001,) system faces multi-estimation. In order to check this, the algorithm evaluates $\overrightarrow{\Delta V}_{I,M,F}\%$ or $\overrightarrow{\Delta I}_{I,M,F}\%$ for all fault conditions (location and impedance) to find any similar pairs (i.e., that differ less than 0.001,) of voltage change vectors $\overrightarrow{\Delta V}_{I,M,F}\%$ and current change vectors $\overrightarrow{\Delta I}_{I,M,F}\%$, given a set of injection and measurement buses. If similarity happens, the injection-measurement set cannot offer unique results for different faults. In this case system observes multi-estimation.

Next step is to repeat the algorithm for different combinations of injection and measurement buses along with (and possibly their frequencies) to find the optimal buses for injection and measurement in order to cover all the faults with minimum number of injection and measurement buses and to avoid multi-estimation. The proposed algorithm is depicted in the flowchart of figure 2.5.

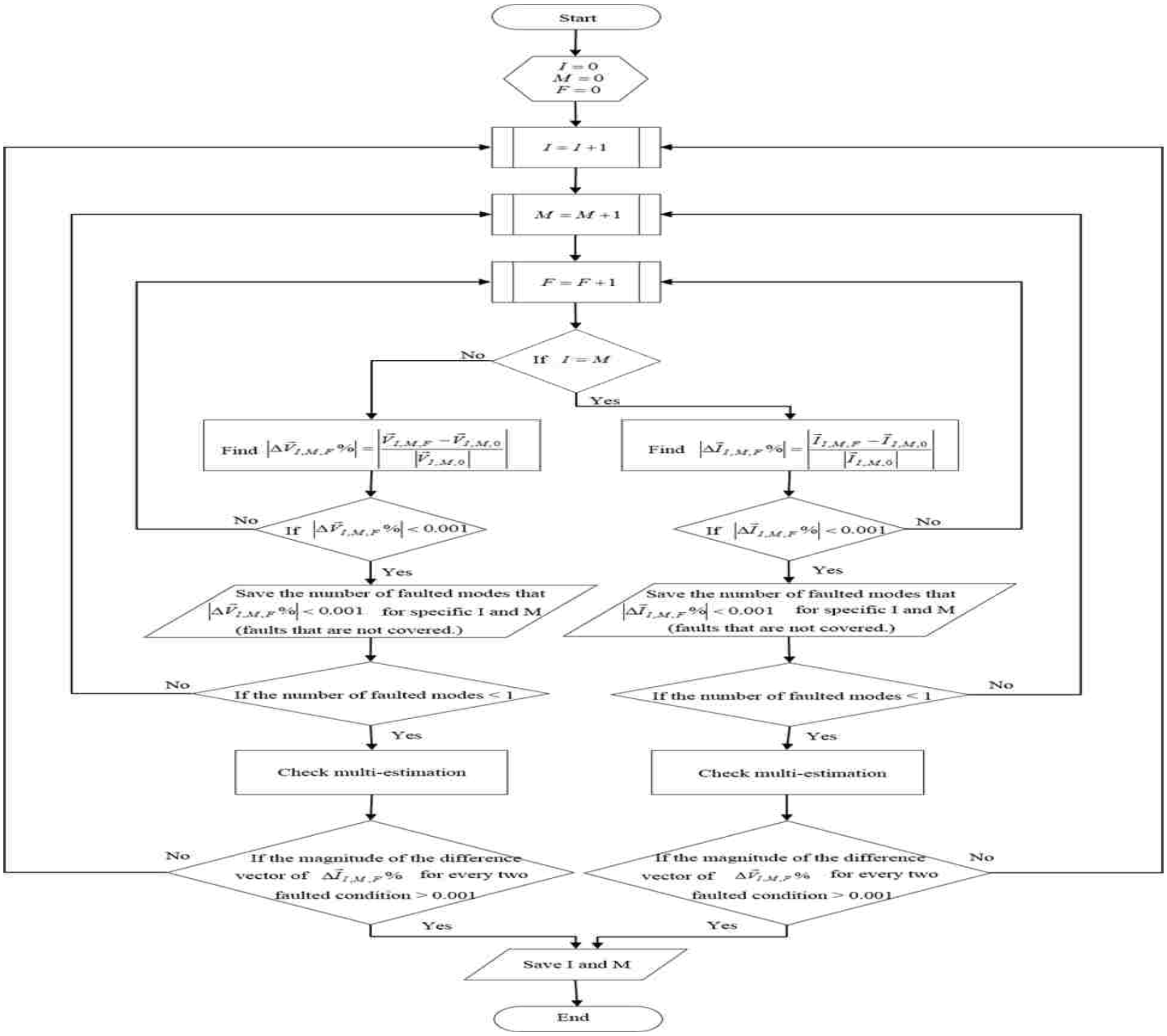

Figure 2.5 Algorithm of finding best injection and measurement placement

2.3 Fault Location Algorithm by Using Line Current Measurement

So far, current measurement has been considered only when injection and measurement buses are the same. When the injection and measurement busses are different, multiple lines may be connected to the measurement bus with different current on each line. Therefore, one must specify which line is used for the measurement. In this thesis, line current measurement is used to locate the fault based on the difference in the measured current in the faulty and unfaulty systems. Using the results from voltage and current measurements helps reduce the number of measurement equipment for fault location leading to the lowest number of undetectable faults. For this purpose one needs to measure currents on all the lines that are connected to the measurement bus to see which one has the highest variation for a range of faults under consideration.

Since each bus connects multiple lines together and each of these lines have different currents, it is important to know which line to use for the fault location resulting the lowest undetectable faults. Start with naming the lines that are connected to each measurement bus from 1 to *n* where *n* is the number of the lines connected to the selected measurement bus. Note that for buses with two lines only one current measurement is taken since the lines have the same current when the load current is ignored. This procedure is repeated for all the measurement buses (figure 2.6); i.e., all the power system buses. In the proposed method the algorithm measures currents on all the lines connected to a measurement bus for different injection buses and faulty buses, then it proceeds to the next measurement bus and repeats the same procedure for the lines connected to that bus until it covers all the measurement buses (the entire system's buses.)

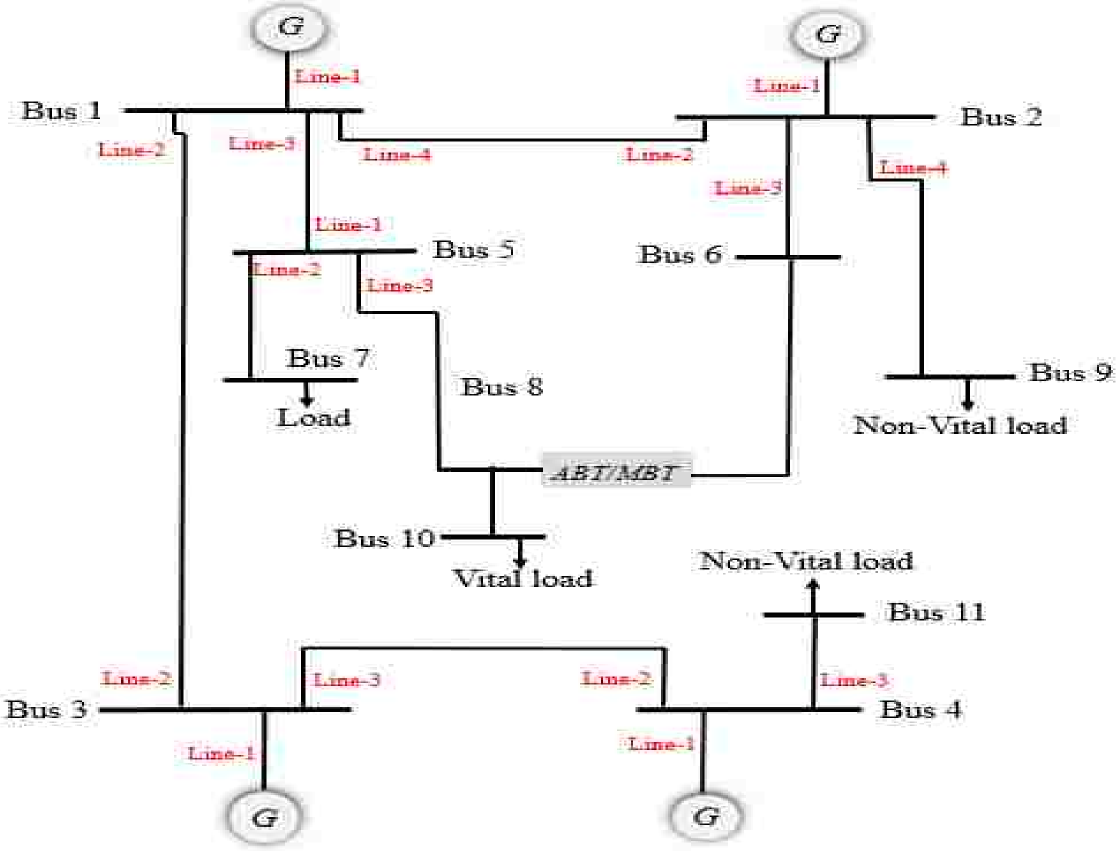

Figure 2.6 11-Bus SPS

Suppose that I is the number of the bus that has the voltage application (injection bus), B is the number of the measurement bus, M is the number of the lines connected to measurement bus B, and F is the number of the faulty bus. In this case $\vec{I}_{I,B,M,F}$ represents a current vector measured at measurement bus B. If $F = 0$ it shows the unfaulty values; that is when there is no fault in the system. The goal is to observe the values of $\vec{I}_{I,B,M,F}$ for all the faults, given the measurements, and to compare the values with the normal values of currents to see if the faults are detectable. The algorithm starts from $B = 1$, $I = 1$, $M = 1$ and applies a certain fault on each bus and observes the changes in currents at the measurement bus. For this purpose the algorithm calculates $\overrightarrow{\Delta I}_{I,B,M,F}$; that is,

$$\overrightarrow{\Delta I}_{I,B,M,F} = \vec{I}_{I,B,M,F} - \vec{I}_{I,B,M,0}.$$

In the shipboard power systems, faults are not detectable when relative value $\left|\overrightarrow{\Delta I}_{I,B,M,F}\%\right|$ that is equal to $\frac{\left|\overrightarrow{\Delta I}_{I,B,M,F}\right|}{\left|\vec{I}_{I,B,M,0}\right|} = \frac{\left|\vec{I}_{I,B,M,F} - \vec{I}_{I,B,M,0}\right|}{\left|\vec{I}_{I,B,M,0}\right|}$, is smaller than 0.001.

The algorithm examines all the possible combinations with different values for, *I*, *B*, *M*, and *F* to find the optimal bus for the injection and optimal line current measurement that can observe all the faults (various fault impedances.) If one injection bus and measurement bus are not adequate to cover the entire system, the system requires more injection buses and/or current measurements from a measurement bus or even more measurement buses to cover all the faults. The proposed approach looks for a set of injection and measurement buses that result in the lowest number of undetectable faults (location and impedance.) The fault is undetectable when $\left|\overrightarrow{\Delta I}_{I,B,M,F}\%\right| < 0.001$. The algorithm evaluates all injection and measurement buses to cover the entire system.

After the algorithm finds the cases that have the lowest number of undetectable faults with $\left|\overrightarrow{\Delta I}_{I,B,M,F}\%\right| < 0.001$, it will check if these cases cause unique changes in $\left|\overrightarrow{\Delta I}_{I,B,M,F}\%\right|$for different faults with the selected measurement and injection buses. If $\left|\overrightarrow{\Delta I}_{I,B,M,F}\%\right|$ has the same results for different faults (i.e., that differ less than 0.001,) system faces multi-estimation. In order to check this, the algorithm evaluates $\overrightarrow{\Delta I}_{I,B,M,F}\%$ for all fault conditions (location and impedance) to find any similar pairs (i.e., that differ less than 0.001,) of current change vectors $\overrightarrow{\Delta I}_{I,B,M,F}\%$, given a set of injection and measurement buses. One must repeat the algorithm for different combinations of injection and measurement buses (and possibly with different injection frequencies) to find the optimal buses for injection and measurement in order to cover all the faults with minimum number of injection and measurement buses and to avoid multi-estimation. The proposed algorithm is depicted in the flowchart of figure 2.7.

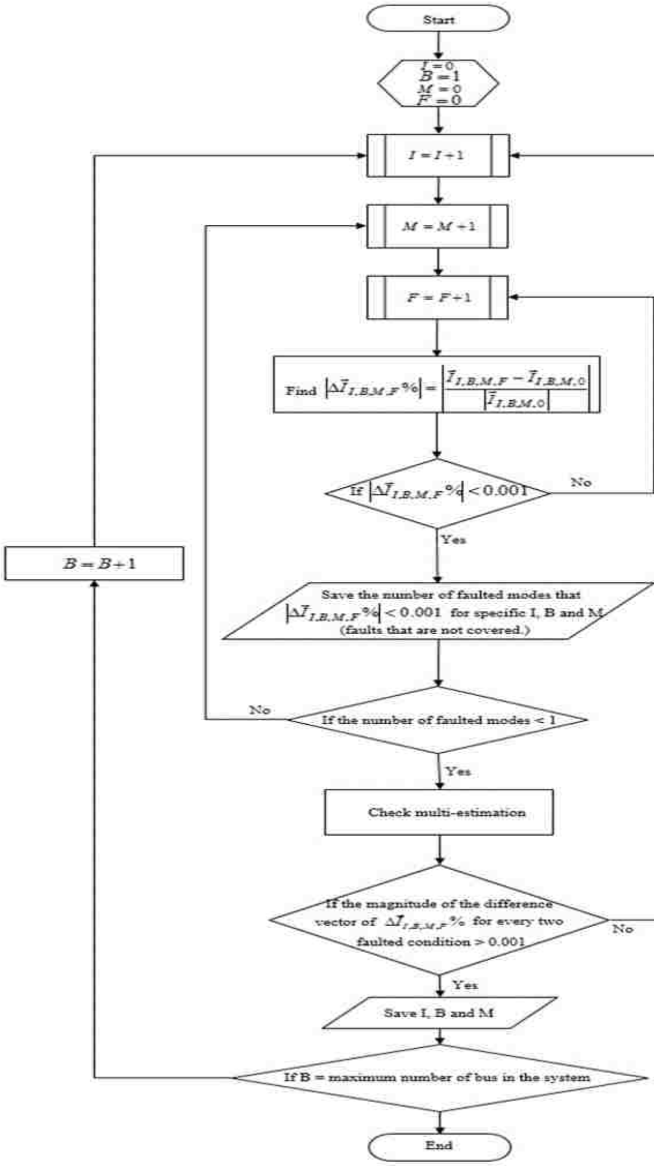

Figure 2.7 Algorithm of finding best injection and measurement placement by Using Current Measurement

2.4 References

[1] E. Christopher, M. Sumner, D. W. P. Thomas, Xiaohui Wang, and F. de Wildt, “Fault Location in a Zonal DC Marine Power System Using Active Impedance Estimation,” *IEEE Transactions on Industry Applications*, vol. 49, no. 2, pp. 860-865, March-April 2013.

[2] J. Wang, M. Sumner, D. W. P. Thomas, and R. D. Geertsma, “Active fault protection for an AC zonal marine power system,” *Electrical Systems in Transportation, IET*, vol. 1, no. 4, pp. 156,166, December 2011.

CHAPTER 3
EFFECTS OF INJECTION FREQUENCY

3.1 Introduction

Using standard fault analysis, one can find ΔV and ΔI for the proposed fault location algorithm based on the elements of Z_{bus} matrix. In order to find the relation between injection frequency and fault location one needs to track the effect of frequency in Z_{bus} matrix elements. For this purpose one needs to develop the Z_{bus} matrix.

3.2 Background-Impedance Matrix

The bus impedance matrix is an important tool for power system fault analysis [1]. There are different ways to find impedance matrix of the system. Inversion of the admittance matrix is more appropriate for small systems. In the proposed method the target is obtain the mathematical relationship between the frequency and impedance; however, inversion makes it too difficult to track the relationship. Moreover, for large systems, inversion of the admittance matrix becomes very time consuming.

The bus impedance matrix can also be directly found from power system structure [1]. In order to build the impedance matrix directly, one starts with a simple 1×1 impedance matrix between a bus and the reference node and then modifies this simple network by adding subsequent buses and lines between buses one at a time.

In order to understand how to modify impedance matrix Z_{bus}, consider notations h, i, j, and k for existing buses and m and n for the new buses, respectively, as shown in Cases 1 to 4 depicted in figures 3.1 to 3.4 below. There are four different cases that one can benefit from in modifying Z_{bus}.

Case 1. Adding branch Z_{bus} between reference node and new bus m

In order to update original impedance matrix Z_{bus}^{orig} when there is an impedance (Z_b) added between the reference node (*0*) and the new bus (*m*), one needs to add a row and column to Z_{bus}^{orig} with the values in equation (3.1) [1].

$$Z_{bus}^{new} = \left[\begin{array}{ccc|c} & & & 0 \\ & Z_{bus}^{orig} & & \vdots \\ & & & 0 \\ \hline 0 & \cdots & 0 & Z_b \end{array}\right] \tag{3.1}$$

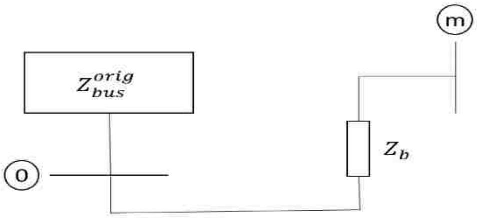

Figure 3.1 Case 1. Adding branch Z_b between reference node and new bus *m*

Case 2. Adding branch Z_b between existing bus *k* and new bus *m*

In order to update original impedance matrix Z_{bus}^{orig} when there is a new bus (*m*) connected through Z_b to an existing bus (*k*) equation (3.2) can be used [1].

$$Z_{bus}^{new} = \left[\begin{array}{cccc|c} & & & & Z_{1k} \\ & & & & Z_{2k} \\ & Z_{bus}^{orig} & & & \vdots \\ & & & & Z_{Nk} \\ \hline Z_{k1} & Z_{k2} & \ldots & Z_{kN} & Z_{kk} + Z_b \end{array}\right] \tag{3.2}$$

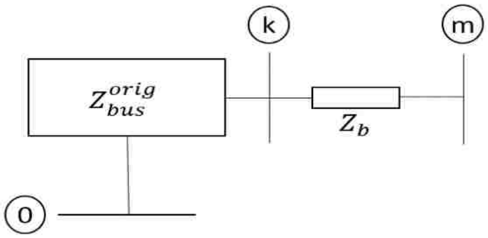

Figure 3.2 Case 2. Adding branch Z_b between existing bus *k* and new bus *m*

Case 3. Adding branch Z_b between existing bus *k* to the reference node

In this case there is an impedance Z_b between bus *k* (an existing bus) and bus (*0*) (the reference node). In order to obtain Z_{bus}^{new} one needs to add a temporary bus (*m*) connectedthrough Z_b to bus *k* (figure 3.3), then one needs to repeat case 2 and then remove row *m* and column *m* by Kron reduction. In order to use Kron reduction to find each element equation (3.3) is employed [1].

$$Z_{hi(new)} = Z_{hi} - \frac{Z_{h(N+1)}Z_{(N+1)i}}{Z_{kk}+Z_b} \tag{3.3}$$

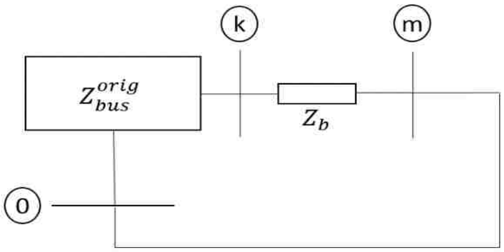

Figure 2.3 Case 3. Adding branch Z_b between existing bus *k* to the reference node

Case 4. Adding branch Z_b between existing bus *j* to existing bus *k*

In order to obtain original impedance matrix Z_{bus}^{new} for this case one needs to form the matrix using equation (3.4) [1].

$$Z_{bus}^{new} = \left[\begin{array}{c|c} Z_{bus}^{orig} & col.j - col.k \\ \hline row.j - row.k & Z_{th,\,jk} + Z_b \end{array}\right] \tag{3.4}$$

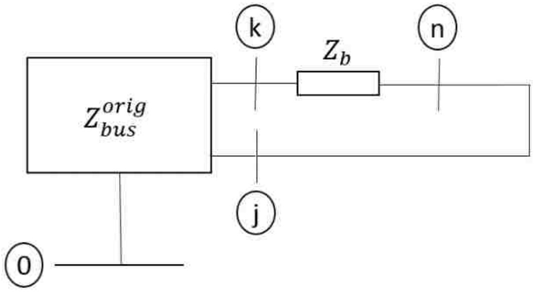

Figure 3.4 Case 4. Adding branch Z_b between existing bus *j* to existing bus *k*

where $Z_{th,jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$ and then remove row *n* and column *n* by Kroon reduction [1].

By knowing how to modify Z_{bus} using these four cases one can find Z_{bus} of the system. Impedance matrix Z_{bus} can be obtained starting from one bus connected through a branch impedance to the reference node and then expanding this simple network, based on the system topology and the four cases that mentioned above , to modify the Z_{bus} and find the large system impedance matrix. This approach is used in chapter 3.3 in order to find the relationship between injection frequency and fault location.

3.3 Effect of Injection Frequency on Fault Location

In the SPS, cables are resistive, inductive and in the form of RL which makes the impedance of each cable equals to $Z = R + jL\omega$. Note that $\omega = 2\pi f$ that makes ω depend on the frequency of the injection. Therefore, impedance of the cable also depends on the frequency. Resistance (R) and inductance (L) of the cables are also related to the length of cables in the SPS. That is, $R = rl$ where *r* is the resistance per mile and *l* is the length of the cable. With the same approach $L = al$ where a is the inductance per mile and l is the cable length. Under the assumption that the same cable is used in the entire SPS, the value for *a* and *r* remain the same for the entire system and the only parameter that is changing is the length which makes R and L different for each cable. Let each element of Z_{bus} be represented by a complex number $Z_{ij} = R_{ij} + j\omega L_{ij}$. Then, Z_{ij} can be converted to form $Z_{ij} = \Psi l$ where $\Psi \in C^1$ is a constant complex number and is equal to $\Psi = r + j\omega a$. In addition, one can consider $\frac{R}{L} = \frac{rl}{al} = const = K$. By considering this one can write:

$$Z = R + jL\omega = L\left(\frac{R}{L} + j\omega\right) = al(K + j\omega) = \overline{K}(\omega)l$$

Since *K* is considered as a constant and a as inductance per mile which is the same for all the cables used in the SPS, there are only two variables in this equation that are *l* and ω.

In the proposed approach the algorithm is supposed to look at ΔV and ΔI values in order to find the location of the fault in the system. By using standard fault analysis, the observant bus voltage changes at bus *h* (when fault occurs at bus *p*) can be described as:

$$\Delta V_h = \frac{Z(h,p)}{Z(p,p) + R_{fault}} \times V_{pref}$$

where $Z(h,p)$ is the (h,p) entree of the impedance matrix and $Z(p,p)$ is the system Thevenin impedance seen from bus p, and V_{pref} is the prefault voltage at the point of fault in the system. As shown in the equation one needs to find impedance matrix (Z_{bus}) in order to find ΔV_h. Since the proposed algorithm is related to the frequency of the injection and measurement, one needs to find the relationship between impedance matrix and frequency to find the proper frequency in order to get the best result and find the unique value of ΔV and ΔI for each fault. This will then lead to find the exact location of the fault for different values of R_{fault}.

One can find Z_{bus} by finding Y_{bus} and inverting it, but this is not convenient because it makes one unable to track the effect of frequency in the fault location formulation. For this reason the direct building algorithm of Z_{bus} is used to precisely find its relationship with the frequency of the injection. In the process of finding Z_{bus} it appears that all the elements of this matrix has $a(K+j\omega)$ in their numerator. Note that Kron reduction in this process will retain this value in the numerator of each Z_{bus} element. As mentioned, for fault analysis one needs to look at ΔV and ΔI values to find the exact location of the fault. Since each element of Z_{bus} has $\overline{K}(\omega) = a(K+j\omega)$, one can rearrange the equation as:

$$\Delta V_h = \frac{\overline{K}(\omega)\bar{Z}(h,p)}{\overline{K}(\omega)\bar{Z}(p,p) + R_{fault}} \times V_{pref}$$

where $\bar{Z}(h,p)$ and $\bar{Z}(p,p)$ are the elements of $\frac{Z_{bus}}{\overline{K}(\omega)}$ matrix.

Similarly, for the current measurement since ΔV_h is available for any h within the network according to the standard power system fault analysis, lines current changes can be expressed as:

$$\Delta I_{hu} = \frac{\Delta V_h - \Delta V_u}{Z_{hu}} = Y_{hu} \times (\Delta V_h - \Delta V_u)$$

$$Z_{hu} = \frac{1}{Y_{hu}} = \frac{-1}{Y(h,u)}$$

$$\Delta I_{hu} = Y(h,u) \times (\Delta V_u - \Delta V_h)$$

where *h* is the measurement bus and *u* is the adjacent bus connected to h by transmission line hu, Z_{hu} is the line impedance and $Y(h,u)$ is the (h,u) entree of the admittance matrix. Since $Y_{bus} = \frac{1}{Z_{bus}}$ it appears that all the elements of this matrix has $\frac{1}{(K+j\omega)}$ in their numerator. Note that Kron reduction in this process will retain this value in the numerator of each Y_{bus} element. As mentioned, for fault analysis one needs to look at ΔV and ΔI values to find the exact location of the fault. Since each element of Y_{bus} has $\frac{1}{\overline{K}(\omega)} = \frac{1}{a(K+j\omega)}$, one can rearrange the equation as:

$$\Delta I_{hu} = \frac{1}{\overline{K}(\omega)} \overline{Y}(h,u) \times (\Delta V_u - \Delta V_h)$$

where $\overline{Y}(h,u)$ is the element of $Y_{bus} \times \overline{K}(\omega)$ matrix.

From the above equations one can conclude that if R_{fault} is a small value, frequency will not have critical effects on the ΔV and ΔI values and fault location, but if R_{fault} is large, one can get higher ΔV and ΔI by increasing the frequency.

3.4 References

[1] Grainger, J. J., & Stevenson, W. D. Power system analysis (Vol. 621). New York: McGraw-Hill., 1994.

CHAPTER 4
SIMULATION RESULTS

4.1 Introduction

The 11-bus SPS in figure 1.2 is considered as the case study. The proposed algorithm is applied to this system in the Matlab/Simulink in order to find the minimum number of injections and measurements and the best place for them to cover all the faults in the network. Algorithm examines all the possible places for measurement and injection to see which faults are covered and which ones are not covered.

4.2 Simulation Results for $f = 1000Hz$

Table 4.1 shows the number of faults (occurred on buses) that are not detectable for the selected values of M and I. The algorithm is also able to show which faults are not covered in each case (numbers in the parentheses). Table 4.1 shows the results when $R_{fault} = 1e^{-4}$ and $f = 1000Hz$ in both the injections and measurements. For example, based on the Table 4.1, if we have the injection on bus 2 and the measurement on bus 3 we have two undetectable faults which are at bus 6 and bus 9.

Table 4.1 Results of applying proposed approach when $R_{fault} = 1e^{-4}$ and $f = 1000Hz$

M→ ***I↓***	**1**	**2**	**3**	**4**	**5**	**6**
1	1 (11)	7 (3,4,5,7,8,10,11)	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)
2	4 (4,6,9,11)	2 (4,11)	2 (6,9)	2 (6,9)	4 (4,6,9,11)	9 (1,3,4,5,7,8,9,10,11)
3	2 (4,11)	2 (4,11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
4	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	7 (2,5,6,7,8,9,10)	1 (11)	1 (11)
5	5 (4,7,8,10,11)	5 (4,7,8,10,11)	3 (7,8,10)	3 (7,8,10)	7 (1,2,3,4,6,9,11)	5 (4,7,8,10,11)
6	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	10 (1,2,3,4,5,7,8,9,10,11)
7	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	3 (3,4,11)
10	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
11	0	0	6 (2,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	0	0

Table 4.1 Results of applying proposed approach when $R_{fault} = 1e^{-4}$ and $f = 1000Hz$

M→ ***I↓***	***7***	***8***	***9***	***10***	***11***
1	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)	6 (2,3,4,6,9,11)	7 (2,5,6,7,8,9,10)
2	4 (4,6,9,11)	4 (4,6,9,11)	9 (1,3,4,5,6,7,8,10,11)	4 (4,6,9,11)	2 (6,9)
3	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	8 (1,2,5,6,7,8,9,10)
4	1 (11)	1 (11)	1 (11)	1 (11)	9 (1,2,3,5,6,7,8,9,10))
5	9 (1,2,3,4,6,8,9,10,11)	8 (1,2,3,4,6,7,9,11)	5 (4,7,8,10,11)	8 (1,2,3,4,6,7,9,11)	3 (7,8,10)
6	2 (4,11)	2 (4,11)	3 (3,4,11)	2 (4,11)	0
7	10 (1,2,3,4,5,6,8,9,10,11)	3 (3,4,11)	2 (4,11)	3 (3,4,11)	0
8	4 (3,4,10,11)	9 (1,2,3,4,5,6,7,9,11)	3 (4,10,11)	9 (1,2,3,4,5,6,7,9,11))	1 (10)
9	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,10,11)	2 (4,11)	0
10	3 (3,4,11)	3 (3,4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,11)	0
11	0	0	0	0	10 (1,2,3,4,5,6,7,8,9,10)

Table 4.1 shows that there are some cases with the lowest numbers (0 or 1) of undetectable faults. Zero shows that measurement on bus M can observe all the faults of the system when the injection is on bus I. If there is no zero in the table, system will consider more than one injection and/or measurement buses to observe all the possible faults of interest. Next, the cases with the lowest number of undetectable faults have to be checked for multi-estimation. In other words, these cases should prove that they have unique effects on the selected measurements for each fault. If the measurements for some faults are the same, multi-estimation has occurred, because these faults are not recognizable from one another and thus it increases the number of undetectable faults. In our simulation none of the cases in Table 4.1 involve multi-estimation. Highlighted sections in the tables are showing the cases which require multi-estimation.

Table 4.2, 4.3, 4.4, and 4.5 show that by assuming higher R_{fault} for the system with the same value for the frequency (1000 Hz) results will slightly change. In this case some of the cases involve multi-estimation.

Table 4.2 Results of applying proposed approach when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

***M*→** ***I*↓**	***1***	***2***	***3***	***4***	***5***	***6***
1	1 (11)	7 (3,4,5,7,8,10,11)	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)
2	4 (4,6,9,11)	2 (4,11)	2 (6,9)	2 (6,9)	4 (4,6,9,11)	9 (1,3,4,5,7,8,9,10,11)
3	2 (4,11)	2 (4,11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
4	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	7 (2,5,6,7,8,9,10)	1 (11)	1 (11)
5	5 (4,7,8,10,11)	5 (4,7,8,10,11)	3 (7,8,10)	3 (7,8,10)	7 (1,2,3,4,6,9,11)	5 (4,7,8,10,11)
6	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	10 (1,2,3,4,5,7,8,9,10,11)
7	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	3 (3,4,11)
10	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
11	0	0	6 (2,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	0	0

Table 4.2 Results of applying proposed approach when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

M→ ***I***↓	***7***	***8***	***9***	***10***	***11***
1	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)	6 (2,3,4,6,9,11)	7 (2,5,6,7,8,9,10)
2	4 (4,6,9,11)	4 (4,6,9,11)	9 (1,3,4,5,6,7,8,10,11)	4 (4,6,9,11)	2 (6,9)
3	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	8 (1,2,5,6,7,8,9,10)
4	1 (11)	1 (11)	1 (11)	1 (11)	9 (1,2,3,5,6,7,8,9,10))
5	9 (1,2,3,4,6,8,9,10,11)	8 (1,2,3,4,6,7,9,11)	5 (4,7,8,10,11)	8 (1,2,3,4,6,7,9,11)	3 (7,8,10)
6	2 (4,11)	2 (4,11)	3 (3,4,11)	2 (4,11)	0
7	10 (1,2,3,4,5,6,8,9,10,11)	3 (3,4,11)	2 (4,11)	3 (3,4,11)	0
8	4 (3,4,10,11)	9 (1,2,3,4,5,6,7,9,11)	3 (4,10,11)	9 (1,2,3,4,5,6,7,9,11))	1 (10)
9	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,10,11)	2 (4,11)	0
10	3 (3,4,11)	3 (3,4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,11)	0
11	0	0	0	0	10 (1,2,3,4,5,6,7,8,9,10)

Table 4.3 Results of applying proposed approach when $R_{fault} = 1e^{-2}$ and $f = 1000Hz$

M→ ***I↓***	***1***	***2***	***3***	***4***	***5***	***6***
1	2 (1,11)	8 (1,3,4,5,7,8,10,11)	8 (1,2,5,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)
2	5 (2,4,6,9,11)	3 (2,4,11)	3 (2,6,9)	3 (2,6,9)	5 (2,4,6,9,11)	10 (1,2,3,4,5,7,8,9,10,11)
3	1 (11)	1 (11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	1 (11)	1 (11)
4	0	0	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	0	0
5	6 (4,5,7,8,10,11))	6 (4,5,7,8,10,11)	4 (5,7,8,10)	4 (5,7,8,10)	8 (1,2,3,4,5,6,9,11)	6 (4,5,7,8,10,11)
6	3 (4,6,11)	4 (3,4,6,11)	1 (6)	1 (6)	3 (4,6,11)	11 (1,2,3,4,5,6,7,8,9,10,11)
7	3 (4,7,11)	3 (4,7,11)	1 (7)	1 (7)	4 (3,4,7,11)	3 (4,7,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	3 (4,9,11)	4 (3,4,9,11)	1 (9)	1 (9)	3 (4,9,11)	4 (3,4,9,11)
10	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
11	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	9 (1,2,5,6,7,8,9,10,11)	1 (11)	1 (11)

Table 4.3 Results of applying proposed approach when $R_{fault} = 1e^{-2}$ and $f = 1000Hz$

M→ ***I↓***	**7**	**8**	**9**	**10**	**11**
1	7 (1,2,3,4,6,9,11)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)	7 (1,2,3,4,6,9,11)	8 (1,2,5,6,7,8,9,10)
2	5 (2,4,6,9,11)	5 (2,4,6,9,11)	10 (1,2,3,4,5,6,7,8,10,11)	5 (2,4,6,9,11)	3 (2,6,9)
3	1 (11)	1 (11)	1 (11)	1 (11)	8 (1,2,5,6,7,8,9,10)
4	0	0	0	0	9 (1,2,3,5,6,7,8,9,10)
5	10 (1,2,3,4,5,6,8,9,10,11)	9 (1,2,3,4,5,6,7,9,11)	6 (4,5,7,8,10,11)	9 (1,2,3,4,5,6,7,9,11)	4 (5,7,8,10)
6	3 (4,6,11)	3 (4,6,11)	4 (3,4,6,11)	3 (4,6,11)	1 (6)
7	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,7,11)	3 (4,7,11)	4 (3,4,7,11)	1 (7)
8	4 (3,4,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	3 (4,10,11)	8 (1,2,3,4,6,7,9,11))	1 (10)
9	3 (4,9,11)	3 (4,9,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	3 (4,9,11)	1 (9)
10	4 (3,4,10,11)	4 (3,4,10,11)	3 (4,10,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	1 (10)
11	1 (11)	1 (11)	1 (11)	1 (11)	11 (1,2,3,4,5,6,7,8,9,10,11)

Table 4.4 Results of applying proposed approach when $R_{fault} = 1e^{-1}$ and $f = 1000Hz$

M→ I↓	***1***	***2***	***3***	***4***	***5***	***6***
1	3 (1,4,11)	8 (1,3,4,5,7,8,10,11)	8 (1,2,5,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)
2	6 (2,3,4,6,9,11)	4 (2,3,4,11)	3 (2,6,9)	3 (2,6,9)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,7,8,9,10,11)
3	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)	9 (1,2,3,5,6,7,8,9,10)	3 (3,4,11)	3 (3,4,11)
4	2 (4,11)	2 (4,11)	10 (1,2,4,5,6,7,8,9,10,11)	9 (1,2,4,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
5	7 (3,4,5,7,8,10,11)	7 (3,4,5,7,8,10,11)	4 (5,7,8,10)	4 (5,7,8,10)	8 (1,2,3,4,5,6,9,11)	7 (3,4,5,7,8,10,11)
6	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)	1 (6)	4 (3,4,6,11)	11 (1,2,3,4,5,6,7,8,9,10,11)
7	4 (3,4,7,11)	4 (3,4,7,11)	1 (7)	1 (7)	4 (3,4,7,11)	4 (3,4,7,11)
8	5 (3,4,8,10,11)	5 (3,4,8,10,11)	2 (8,10)	2 (8,10)	5 (3,4,8,10,11)	5 (3,4,8,10,11)
9	4 (3,4,9,11)	4 (3,4,9,11)	1 (9)	1 (9)	4 (3,4,9,11)	4 (3,4,10,11)
10	4 (3,4,10,11)	4 (3,4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	4 (3,4,10,11)
11	1 (11)	1 (11)	9 (1,2,5,6,7,8,9,10,11)	9 (1,2,5,6,7,8,9,10,11)	1 (11)	1 (11)

Table 4.4 Results of applying proposed approach when $R_{fault} = 1e^{-1}$ and $f = 1000Hz$

M→ ***I↓***	***7***	***8***	***9***	***10***	***11***
1	7 (1,2,3,4,6,9,11)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)	7 (1,2,3,4,6,9,11)	8 (1,2,5,6,7,8,9,10)
2	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,6,7,8,10,11)	6 (2,3,4,6,9,11)	3 (2,6,9)
3	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)
4	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,10)
5	10 (1,2,3,4,5,6,8,9,10,11)	9 (1,2,3,4,5,6,7,9,11)	7 (3,4,5,7,8,10,11)	9 (1,2,3,4,5,6,7,9,11)	4 (5,7,8,10)
6	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)
7	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,7,11)	4 (3,4,7,11)	4 (3,4,7,11)	1 (7)
8	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	2 (8,10)
9	4 (3,4,9,11)	4 (3,4,9,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,9,11)	1 (9)
10	4 (3,4,10,11)	4 (3,4,10,11)	4 (3,4,10,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	1 (10)
11	1 (11)	1 (11)	1 (11)	1 (11)	11 (1,2,3,4,5,6,7,8,9,10,11)

Table 4.5 Results of applying proposed approach when $R_{fault} = 1$ and $f = 1000Hz$

M→ I↓	**1**	**2**	**3**	**4**	**5**	**6**
1	4 (1,3,4,11)	8 (1,3,4,5,7,8,10,11)	8 (1,2,5,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)
2	6 (2,3,4,6,9,11)	4 (2,3,4,11)	3 (2,6,9)	3 (2,6,9)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,7,8,9,10,11)
3	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)	9 (1,2,3,5,6,7,8,9,10)	3 (3,4,11)	3 (3,4,11)
4	2 (4,11)	2 (4,11)	10 (1,2,4,5,6,7,8,9,10,11)	9 (1,2,4,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
5	7 (3,4,5,7,8,10,11)	7 (3,4,5,7,8,10,11)	4 (5,7,8,10)	4 (5,7,8,10)	8 (1,2,3,4,5,6,9,11)	7 (3,4,5,7,8,10,11)
6	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)	1 (6)	4 (3,4,6,11)	11 (1,2,3,4,5,6,7,8,9,10,11)
7	4 (3,4,7,11)	4 (3,4,7,11)	1 (7)	1 (7)	7 (2,3,4,6,7,9,11)	4 (3,4,7,11)
8	5 (3,4,8,10,11)	5 (3,4,8,10,11)	2 (8,10)	2 (8,10)	8 (2,3,4,6,8,9,10,11)	5 (3,4,8,10,11)
9	4 (3,4,9,11)	4 (3,4,9,11)	1 (9)	1 (9)	4 (3,4,9,11)	4 (3,4,9,11)
10	4 (3,4,10,11)	4 (3,4,10,11)	1 (10)	1 (10)	7 (2,3,4,6,9,10,11)	4 (3,4,10,11)
11	1 (11)	1 (11)	9 (1,2,5,6,7,8,9,10,11)	9 (1,2,5,6,7,8,9,10,11)	1 (11)	1 (11)

Table 4.5 Results of applying proposed approach when $R_{fault} = 1$ and $f = 1000Hz$

M→ I↓	**7**	**8**	**9**	**10**	**11**
1	7 (1,2,3,4,6,9,11)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)	7 (1,2,3,4,6,9,11)	8 (1,2,5,6,7,8,9,10)
2	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,6,7,8,10,11)	6 (2,3,4,6,9,11)	3 (2,6,9)
3	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)
4	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,10)
5	10 (1,2,3,4,5,6,8,9,10,11)	9 (1,2,3,4,5,6,7,9,11)	7 (3,4,5,7,8,10,11)	9 (1,2,3,4,5,6,7,9,11)	4 (5,7,8,10)
6	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)
7	11 (1,2,3,4,5,6,7,8,9,10,11)	7 (2,3,4,6,7,9,11)	4 (3,4,7,11)	7 (2,3,4,6,7,9,11)	1 (7)
8	8 (2,3,4,6,8,9,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	2 (8,10)
9	4 (3,4,9,11)	4 (3,4,9,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,9,11)	1 (9)
10	7 (2,3,4,6,9,10,11)	8 (1,2,3,4,6,9,10,11)	4 (3,4,10,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	1 (10)
11	1 (11)	1 (11)	1 (11)	1 (11)	11 (1,2,3,4,5,6,7,8,9,10,11)

The algorithm is also applied to the system with $R_{fault} = 1$ and $f = 1000Hz$. As shown in Table 4.5, in this scenario all cases that have the lowest number of undetectable faults suffer from multi-estimation (highlighted numbers). That is, for higher R_{fault}, combinations of injection and measurement buses with $f = 1000Hz$ cannot cover all the fault locations unless multiple injection and measurement buses are selected. It requires the algorithm to run at a higher frequency ($f = 7000Hz$.)

4.3 Simulation Results for $f = 7000Hz$

The algorithm repeats all the steps with $f = 7000Hz$ for different values of R_{fault} ($1e^{-4}$, $1e^{-3}$, $1e^{-2}$, $1e^{-1}$, and 1) to minimize the number of multi-estimations. Each R_{fault} will produce a table similar to Table 4.1 that shows the number of undetectable faults for each case (Tables 4.6, 4.7, 4.8, .49, and 4.10).

Table 4.6 Results of applying proposed approach when $R_{fault} = 1e^{-4}$ and $f = 7000Hz$

M→ I↓	**1**	**2**	**3**	**4**	**5**	**6**
1	1 (11)	7 (3,4,5,7,8,10,11)	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)
2	4 (4,6,9,11)	2 (4,11)	2 (6,9)	2 (6,9)	4 (4,6,9,11)	9 (1,3,4,5,7,8,9,10,11)
3	2 (4,11)	2 (4,11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
4	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	7 (2,5,6,7,8,9,10)	1 (11)	1 (11)
5	5 (4,7,8,10,11)	5 (4,7,8,10,11)	3 (7,8,10)	3 (7,8,10)	7 (1,2,3,4,6,9,11)	5 (4,7,8,10,11)
6	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	10 (1,2,3,4,5,7,8,9,10,11)
7	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	3 (3,4,11)
10	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
11	0	0	6 (2,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	0	0

Table 4.6 Results of applying proposed approach when $R_{fault} = 1e^{-4}$ and $f = 7000Hz$

***M*→ *I*↓**	**7**	**8**	**9**	**10**	**11**
1	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)	6 (2,3,4,6,9,11)	7 (2,5,6,7,8,9,10)
2	4 (4,6,9,11)	4 (4,6,9,11)	9 (1,3,4,5,6,7,8,10,11)	4 (4,6,9,11)	2 (6,9)
3	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	8 (1,2,5,6,7,8,9,10)
4	1 (11)	1 (11)	1 (11)	1 (11)	9 (1,2,3,5,6,7,8,9,10))
5	9 (1,2,3,4,6,8,9,10,11)	8 (1,2,3,4,6,7,9,11)	5 (4,7,8,10,11)	8 (1,2,3,4,6,7,9,11)	3 (7,8,10)
6	2 (4,11)	2 (4,11)	3 (3,4,11)	2 (4,11)	0
7	10 (1,2,3,4,5,6,8,9,10,11)	3 (3,4,11)	2 (4,11)	3 (3,4,11)	0
8	4 (3,4,10,11)	9 (1,2,3,4,5,6,7,9,11)	3 (4,10,11)	9 (1,2,3,4,5,6,7,9,11))	1 (10)
9	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,10,11)	2 (4,11)	0
10	3 (3,4,11)	3 (3,4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,11)	0
11	0	0	0	0	10 (1,2,3,4,5,6,7,8,9,10)

Table 4.7 Results of applying proposed approach when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

M→ ***I↓***	**1**	**2**	**3**	**4**	**5**	**6**
1	1 (11)	7 (3,4,5,7,8,10,11)	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)
2	4 (4,6,9,11)	2 (4,11)	2 (6,9)	2 (6,9)	4 (4,6,9,11)	9 (1,3,4,5,7,8,9,10,11)
3	2 (4,11)	2 (4,11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
4	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	7 (2,5,6,7,8,9,10)	1 (11)	1 (11)
5	5 (4,7,8,10,11)	5 (4,7,8,10,11)	3 (7,8,10)	3 (7,8,10)	7 (1,2,3,4,6,9,11)	5 (4,7,8,10,11)
6	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	10 (1,2,3,4,5,7,8,9,10,11)
7	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	3 (3,4,11)
10	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
11	0	0	6 (2,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	0	0

Table 4.7 Results of applying proposed approach when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

M→ ***I↓***	**7**	**8**	**9**	**10**	**11**
1	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)	6 (2,3,4,6,9,11)	7 (2,5,6,7,8,9,10)
2	4 (4,6,9,11)	4 (4,6,9,11)	9 (1,3,4,5,6,7,8,10,11)	4 (4,6,9,11)	2 (6,9)
3	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	8 (1,2,5,6,7,8,9,10)
4	1 (11)	1 (11)	1 (11)	1 (11)	9 (1,2,3,5,6,7,8,9,10))
5	9 (1,2,3,4,6,8,9,10,11)	8 (1,2,3,4,6,7,9,11)	5 (4,7,8,10,11)	8 (1,2,3,4,6,7,9,11)	3 (7,8,10)
6	2 (4,11)	2 (4,11)	3 (3,4,11)	2 (4,11)	0
7	10 (1,2,3,4,5,6,8,9,10,11)	3 (3,4,11)	2 (4,11)	3 (3,4,11)	0
8	4 (3,4,10,11)	9 (1,2,3,4,5,6,7,9,11)	3 (4,10,11)	9 (1,2,3,4,5,6,7,9,11))	1 (10)
9	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,10,11)	2 (4,11)	0
10	3 (3,4,11)	3 (3,4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,11)	0
11	0	0	0	0	10 (1,2,3,4,5,6,7,8,9,10)

Table 4.8 Results of applying proposed approach when $R_{fault} = 1e^{-2}$ and $f = 7000Hz$

M→ I↓	**1**	**2**	**3**	**4**	**5**	**6**
1	1 (11)	7 (3,4,5,7,8,10,11)	7 (2,5,6,7,8,9,10)	7 (2,5,6,7,8,9,10)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)
2	4 (4,6,9,11)	2 (4,11)	2 (6,9)	2 (6,9)	4 (4,6,9,11)	9 (1,3,4,5,7,8,9,10,11)
3	2 (4,11)	2 (4,11)	2 (6,9)	8 (1,2,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
4	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	7 (2,5,6,7,8,9,10)	1 (11)	1 (11)
5	5 (4,7,8,10,11)	5 (4,7,8,10,11)	3 (7,8,10)	3 (7,8,10)	7 (1,2,3,4,6,9,11)	5 (4,7,8,10,11)
6	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	10 (1,2,3,4,5,7,8,9,10,11)
7	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
8	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
9	2 (4,11)	3 (3,4,11)	0	0	2 (4,11)	3 (3,4,11)
10	2 (4,11)	2 (4,11)	0	0	3 (3,4,11)	2 (4,11)
11	0	0	6 (2,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	0	0

Table 4.8 Results of applying proposed approach when $R_{fault} = 1e^{-2}$ and $f = 7000Hz$

M→ ***I↓***	***7***	***8***	***9***	***10***	***11***
1	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	7 (3,4,5,7,8,10,11)	6 (2,3,4,6,9,11)	7 (2,5,6,7,8,9,10)
2	4 (4,6,9,11)	4 (4,6,9,11)	9 (1,3,4,5,6,7,8,10,11)	4 (4,6,9,11)	2 (6,9)
3	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	8 (1,2,5,6,7,8,9,10)
4	1 (11)	1 (11)	1 (11)	1 (11)	9 (1,2,3,5,6,7,8,9,10))
5	9 (1,2,3,4,6,8,9,10,11)	8 (1,2,3,4,6,7,9,11)	5 (4,7,8,10,11)	8 (1,2,3,4,6,7,9,11)	3 (7,8,10)
6	2 (4,11)	2 (4,11)	3 (3,4,11)	2 (4,11)	0
7	10 (1,2,3,4,5,6,8,9,10,11)	3 (3,4,11)	2 (4,11)	3 (3,4,11)	0
8	4 (3,4,10,11)	9 (1,2,3,4,5,6,7,9,11)	3 (4,10,11)	9 (1,2,3,4,5,6,7,9,11))	1 (10)
9	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,10,11)	2 (4,11)	0
10	3 (3,4,11)	3 (3,4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,11)	0
11	0	0	0	0	10 (1,2,3,4,5,6,7,8,9,10)

Table 4.9 Results of applying proposed approach when $R_{fault} = 1e^{-1}$ and $f = 7000Hz$

***M*→ *I*↓**	**1**	**2**	**3**	**4**	**5**	**6**
1	2 (1,11)	8 (1,3,4,5,7,8,10,11)	8 (1,2,5,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)
2	5 (2,4,6,9,11)	3 (2,4,11)	3 (2,6,9)	3 (2,6,9)	5 (2,4,6,9,11)	10 (1,2,3,4,5,7,8,9,10,11)
3	3 (3,4,11)	3 (3,4,11)	3 (3,6,9)	9 (1,2,3,5,6,7,8,9,10)	3 (3,4,11)	3 (3,4,11)
4	2 (4,11)	2 (4,11)	9 (2,4,5,6,7,8,9,10,11)	8 (2,4,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
5	6 (4,5,7,8,10,11)	6 (4,5,7,8,10,11)	4 (5,7,8,10)	4 (5,7,8,10)	8 (1,2,3,4,5,6,9,11)	6 (4,5,7,8,10,11)
6	3 (4,6,11)	4 (3,4,6,11)	1 (6)	1 (6)	3 (4,6,11)	11 (1,2,3,4,5,6,7,8,9,10,11)
7	3 (4,7,11)	3 (4,7,11)	1 (7)	1 (7)	4 (3,4,7,11)	3 (4,7,11)
8	4 (4,8,10,11)	4 (4,8,10,11)	2 (8,10)	2 (8,10)	5 (3,4,8,10,11)	4 (4,8,10,11)
9	3 (4,9,11)	4 (3,4,9,11)	1 (9)	1 (9)	3 (4,9,11)	4 (3,4,9,11)
10	3 (4,10,11)	3 (4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	3 (4,10,11)
11	1 (11)	1 (11)	8 (2,5,6,7,8,9,10,11)	9 (1,2,5,6,7,8,9,10,11)	1 (11)	1 (11)

Table 4.9 Results of applying proposed approach when $R_{fault} = 1e^{-1}$ and $f = 7000Hz$

***M*→** ***I*↓**	**7**	**8**	**9**	**10**	**11**
1	7 (1,2,3,4,6,9,11)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)	7 (1,2,3,4,6,9,11)	8 (1,2,5,6,7,8,9,10)
2	5 (2,4,6,9,11)	5 (2,4,6,9,11)	10 (1,2,3,4,5,6,7,8,10,11)	5 (2,4,6,9,11)	3 (2,6,9)
3	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)
4	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,10)
5	10 (1,2,3,4,5,6,8,9,10,11)	9 (1,2,3,4,5,6,7,9,11)	6 (4,5,7,8,10,11)	9 (1,2,3,4,5,6,7,9,11)	4 (5,7,8,10)
6	3 (4,6,11)	3 (4,6,11)	4 (3,4,6,11)	3 (4,6,11)	1 (6)
7	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,7,11)	3 (4,7,11)	4 (3,4,7,11)	1 (7)
8	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	4 (4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	2 (8,10)
9	3 (4,9,11)	3 (4,9,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	3 (4,9,11)	1 (9)
10	4 (3,4,10,11)	4 (3,4,10,11)	3 (4,10,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	1 (10)
11	1 (11)	1 (11)	1 (11)	1 (11)	11 (1,2,3,4,5,6,7,8,9,10,11)

Table 4.10 Results of applying proposed approach when $R_{fault} = 1$ and $f = 7000Hz$

M→ ***I↓***	***1***	***2***	***3***	***4***	***5***	***6***
1	4 (1,3,4,11)	8 (1,3,4,5,7,8,10,11)	8 (1,2,5,6,7,8,9,10)	8 (1,2,5,6,7,8,9,10)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)
2	6 (2,3,4,6,9,11)	4 (2,3,4,11)	3 (2,6,9)	3 (2,6,9)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,7,8,9,10,11)
3	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)	9 (1,2,3,5,6,7,8,9,10)	3 (3,4,11)	3 (3,4,11)
4	2 (4,11)	2 (4,11)	10 (1,2,4,5,6,7,8,9,10,11)	9 (1,2,4,5,6,7,8,9,10)	2 (4,11)	2 (4,11)
5	7 (3,4,5,7,8,10,11)	7 (3,4,5,7,8,10,11)	4 (5,7,8,10)	4 (5,7,8,10)	8 (1,2,3,4,5,6,9,11)	7 (3,4,5,7,8,10,11)
6	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)	1 (6)	4 (3,4,6,11)	11 (1,2,3,4,5,6,7,8,9,10,11)
7	4 (3,4,7,11)	4 (3,4,7,11)	1 (7)	1 (7)	4 (3,4,7,11)	4 (3,4,7,11)
8	5 (3,4,8,10,11)	5 (3,4,8,10,11)	2 (8,10)	2 (8,10)	5 (3,4,8,10,11)	5 (3,4,8,10,11)
9	4 (3,4,9,11)	4 (3,4,9,11)	1 (9)	1 (9)	4 (3,4,9,11)	4 (3,4,9,11)
10	4 (3,4,10,11)	4 (3,4,10,11)	1 (10)	1 (10)	4 (3,4,10,11)	4 (3,4,10,11)
11	1 (11)	1 (11)	9 (1,2,5,6,7,8,9,10,11)	9 (1,2,5,6,7,8,9,10,11)	1 (11)	1 (11)

Table 4.10 Results of applying proposed approach when $R_{fault} = 1$ and $f = 7000Hz$

***M*→** ***I*↓**	**7**	**8**	**9**	**10**	**11**
1	7 (1,2,3,4,6,9,11)	7 (1,2,3,4,6,9,11)	8 (1,3,4,5,7,8,10,11)	7 (1,2,3,4,6,9,11)	8 (1,2,5,6,7,8,9,10)
2	6 (2,3,4,6,9,11)	6 (2,3,4,6,9,11)	10 (1,2,3,4,5,6,7,8,10,11)	6 (2,3,4,6,9,11)	3 (2,6,9)
3	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	3 (3,4,11)	9 (1,2,3,5,6,7,8,9,10)
4	2 (4,11)	2 (4,11)	2 (4,11)	2 (4,11)	10 (1,2,3,4,5,6,7,8,9,10)
5	10 (1,2,3,4,5,6,8,9,10,11)	9 (1,2,3,4,5,6,7,9,11)	7 (3,4,5,7,8,10,11)	9 (1,2,3,4,5,6,7,9,11)	4 (5,7,8,10)
6	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	4 (3,4,6,11)	1 (6)
7	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,7,11)	4 (3,4,7,11)	4 (3,4,7,11)	1 (7)
8	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	5 (3,4,8,10,11)	10 (1,2,3,4,5,6,7,8,9,11)	2 (8,10)
9	4 (3,4,9,11)	4 (3,4,9,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	4 (3,4,9,11)	1 (9)
10	4 (3,4,10,11)	4 (3,4,10,11)	4 (3,4,10,11)	11 (1,2,3,4,5,6,7,8,9,10,11)	1 (10)
11	1 (11)	1 (11)	1 (11)	1 (11)	11 (1,2,3,4,5,6,7,8,9,10,11)

The algorithm compares all of these tables together to find the optimal buses for injection and measurement in order to find the unique location of the fault with minimal multi-estimation. Table 4.10 shows the results for $R_{fault} = 1$ when $f = 7000Hz$. This table is compared to other tables at $f = 7000Hz$ for different R_{fault} to find the optimal locations of measurement and injection. The cases with circle have the lowest number of undetectable faults for all possible R_{fault} which does not involve multi-estimation.

The optimal injection-measurement set for the analyzed system in this paper requires two injections and two measurements to cover all the faults ($1e^{-4}$, $1e^{-3}$, $1e^{-2}$, $1e^{-1}$, and 1) in the system. By comparing the tables for $f = 7000Hz$ it is shown that the first injection needs to be on bus 6 with the measurement on bus 3. The second pair of injection and measurement can be one of these cases: $I = 9$, $M = 3$, or $I = 11$, $M = 1$, or $I = 11$, $M = 2$, or $I = 11$, $M = 5$, or $I = 11$, $M = 6$, or $I = 11$, $M = 7$, or $I = 11$, $M = 8$, or $I = 11$, $M = 9$, or $I = 11$, $M = 10$.

4.4 Fault Location Results by Using Line Current Measurement

Since each bus connects multiple lines together and each of these lines have different currents one needs to know which line is used for the fault location and which line results the lowest undetectable faults.

The 11-bus SPS in figure 4.1 is considered as the case study. In this figure each line has a number used for the current measurement. The proposed algorithm is applied to this system in the Matlab/ Simulink in order to find the minimum number of injections and measurements and the best place for them to cover all the faults in the network. Algorithm examines all possible places for measurement and injection to see which faults are covered and which ones are not. The proposed algorithm is tested for $R_{fault} = 1$ and $R_{fault} = 1e^{-3}$ for both $f = 1000Hz$ and $f = 7000Hz$. In this section, each table shows the results for both current and voltage measurements. In this section, circles in the tables show the cases with better results for current measurement than voltage measurement. The highlighted tables are showing the cases that require multi-estimation.

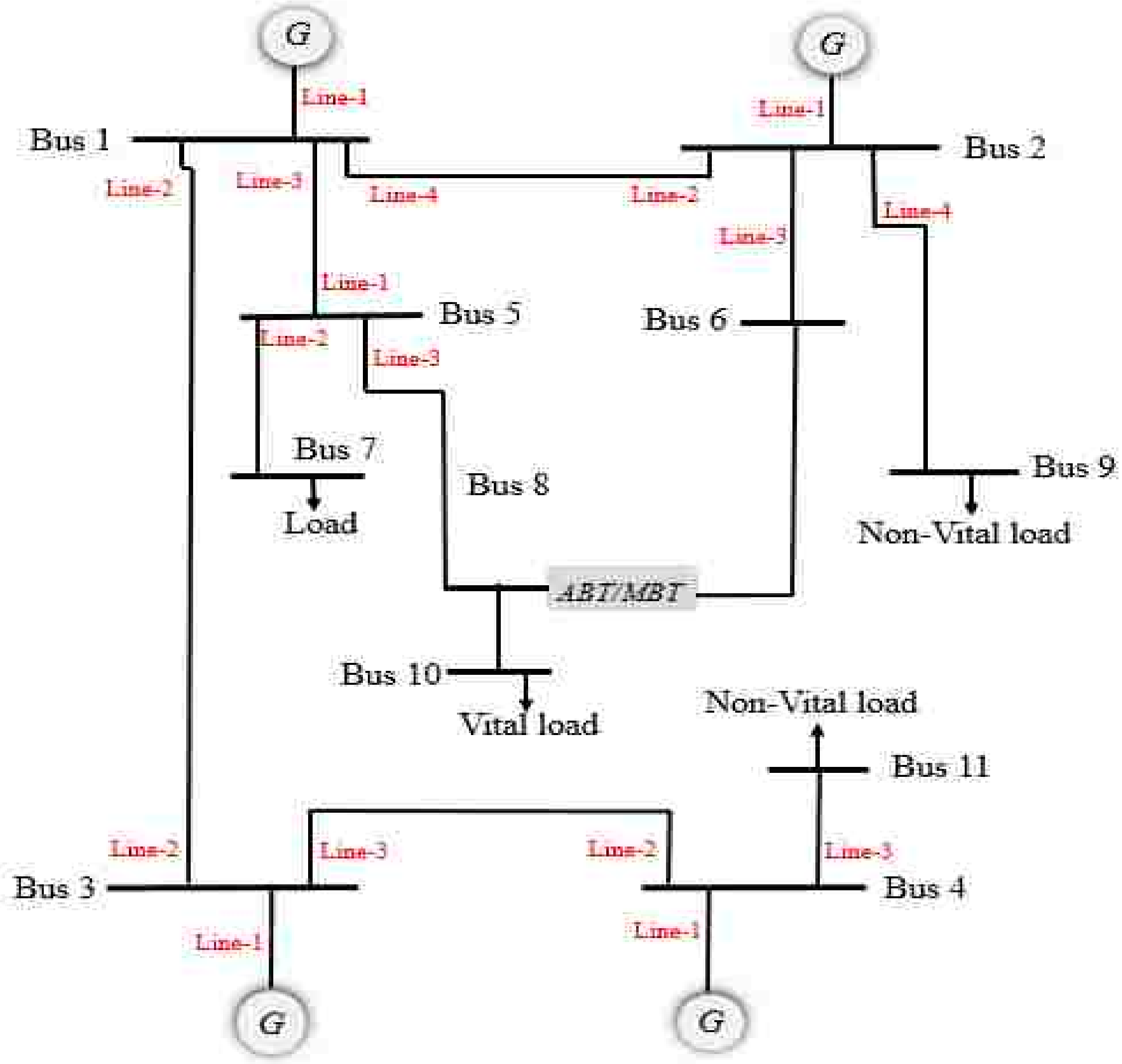

Figure 4.1 11-Bus SPS

4.5 Current Measurement Results for $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Table 4.11 Voltage and current measurement for Bus-1 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Msr / **Inj**	**Bus-1**				
	Voltage	**Current**			
	Bus-1	**Bus-1-Line-1**	**Bus-1-Line-2**	**Bus-1-Line-3**	**Bus-1-Line-4**
Bus -1	**10** (2,3,4,5,6,7,8,9,10,11)	**1** (11)	**7** (2,5,6,7,8,9,10)	**6** (2,3,4,6,9,11)	**7** (3,4,5,7,8,10,11)
Bus -2	**4** (4,6,9,11)	**4** (4,6,9,11)	**2** (6,9)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus -3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus -4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus -5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**3** (7,8,10)	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)
Bus -6	**2** (4,11)	**2** (4,11)	**0**	**2** (4,11)	**2** (4,11)
Bus -7	**2** (4,11)	**2** (4,11)	**0**	**3** (3,4,11)	**2** (4,11)
Bus -8	**3** (4,10,11)	**3** (4,10,11)	**1** (10)	**4** (3,4,10,11)	**3** (4,10,11)
Bus -9	**2** (4,11)	**2** (4,11)	**0**	**2** (4,11)	**2** (4,11)
Bus -10	**2** (4,11)	**2** (4,11)	**0**	**3** (3,4,11)	**2** (4,11)
Bus -11	**0**	**0**	**0**	**0**	**0**

Table 4.12 Voltage and current measurement for Bus-2 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Msr / **Inj**	**Bus-2**				
	Voltage	**Current**			
	Bus-2	**Bus-2-Line-1**	**Bus-2-Line-2**	**Bus-2-Line-3**	**Bus-2-Line-4**
Bus-1	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**4** (3,4,7,11)	**7** (3,4,5,7,8,10,11)
Bus-2	**10** (1,3,4,5,6,7,8,9,10,11)	**2** (4,11)	**4** (4,6,9,11)	**9** (1,3,4,5,7,8,9,10,11)	**9** (1,3,4,5,6,7,8,10,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**3** (4,7,11)	**5** (4,7,8,10,11)
Bus-6	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-7	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)
Bus-9	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-10	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-11	**0**	**0**	**0**	**0**	**0**

Table 4.13 Voltage and current measurement for Bus-3 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Msr	**Bus-3**			
	Voltage	**Current**		
Inj	**Bus-3**	**Bus-3-Line-1**	**Bus-3-Line-2**	**Bus-3-Line-3**
Bus-1	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**2** (6,9)	**2** (6,9)	**2** (6,9)	**2** (6,9)
Bus-3	**10** (1,2,4,5,6,7,8,9,10,11)	**2** (6,9)	**2** (4,11)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**8** (2,5,6,7,8,9,10,11)	**8** (2,5,6,7,8,9,10,11)	**1** (11)	**8** (1,2,6,7,8,9,10,11)
Bus-5	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**0**	**0**	**0**	**0**
Bus-7	**0**	**0**	**0**	**0**
Bus-8	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-9	**0**	**0**	**0**	**0**
Bus-10	**0**	**0**	**0**	**0**
Bus-11	**6** (2,6,7,8,9,10)	**6** (2,6,7,8,9,10)	**0**	**7** (2,5,6,7,8,9,10)

Table 4.14 Voltage and current measurement for Bus-4 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Msr / **Inj**	**Bus-4**			
	Voltage	**Current**		
	Bus-4	**Bus-4-Line-1**	**Bus-4-Line-2**	**Bus-4-Line-3**
Bus-1	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**2** (6,9)	**2** (6,9)	**2** (6,9)	**2** (6,9)
Bus-3	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**10** (1,2,3,5,6,7,8,9,10,11)	**7** (2,5,6,7,8,9,10)	**8** (2,5,6,7,8,9,10,11)	**9** (1,2,3,5,6,7,8,9,10)
Bus-5	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**0**	**0**	**0**	**0**
Bus-7	**0**	**0**	**0**	**0**
Bus-8	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-9	**0**	**0**	**0**	**0**
Bus-10	**0**	**0**	**0**	**0**
Bus-11	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)

Table 4.15 Voltage and current measurement for Bus-5 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Msr / **Inj**	**Bus-5**			
	Voltage	**Current**		
	Bus-5	**Bus-5-Line-1**	**Bus-5-Line-2**	**Bus-5-Line-3**
Bus-1	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-2	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**10** (1,2,3,4,6,7,8,9,10, 11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-6	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-7	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-8	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-9	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-10	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-11	**0**	**0**	**0**	**0**

Table 4.16 Voltage and current measurement for Bus-6, Bus-7, and Bus-8 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Measurement / **Inj**	**Bus-6**		**Bus-7**		**Bus-8**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**7** (3,4,5,7,8,10,11)	**3** (3,4,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-2	**9** (1,3,4,5,7,8,9,10,11)	**6** (3,4,7,9,10,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**5** (4,7,8,10,11)	**3** (4,7,11)	**9** (1,2,3,4,6,8,9,10,11)	**9** (1,2,3,4,6,8,9,10,11)	**8** (1,2,3,4,6,7,9,11)	**8** (1,2,3,4,6,7,9,11)
Bus-6	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-7	**2** (4,11)	**2** (4,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**10** (1,2,3,4,5,6,7,9,10,11)	**9** (1,2,3,4,5,6,7,9,11)
Bus-9	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-10	**2** (4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-11	**0**	**0**	**0**	**0**	**0**	**0**

Table 4.17 Voltage and current measurement for Bus-9, Bus-10, and Bus-11 when $R_{fault} = 1e^{-3}$ and $f = 1000Hz$

Measurement Inj	**Bus-9**		**Bus-10**		**Bus-11**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**9** (1,3,4,5,6,7,8,10,11)	**9** (1,3,4,5,6,7,8,10,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**2** (6,9)	**2** (6,9)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)
Bus-5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**8** (1,2,3,4,6,7,9,11)	**8** (1,2,3,4,6,7,9,11)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**2** (4,11)	**0**	**0**
Bus-7	**2** (4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)	**0**	**0**
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**9** (1,2,3,4,5,6,7,9,11)	**9** (1,2,3,4,5,6,7,9,11)	**1** (10)	**1** (10)
Bus-9	**10** (1,2,3,4,5,6,7,8,10,11)	**10** (1,2,3,4,5,6,7,8,10,11)	**2** (4,11)	**2** (4,11)	**0**	**0**
Bus-10	**2** (4,11)	**2** (4,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**0**	**0**
Bus-11	**0**	**0**	**0**	**0**	**10** (1,2,3,4,5,6,7,8,9,10)	**10** (1,2,3,4,5,6,7,8,9,10)

4.6 Current Measurement Results for $R_{fault} = 1$ and $f = 1000Hz$

Table 4.18 Voltage and current measurement for Bus-1 when $R_{fault} = 1$ and $f = 1000Hz$

Msr **Inj**	**Bus-1**				
	Voltage	**Current**			
	Bus-1	**Bus-1-Line-1**	**Bus-1-Line-2**	**Bus-1-Line-3**	**Bus-1-Line-4**
Bus-1	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (1,3,4,11)	**10** (1,2,4,5,6,7,8,9,10,11)	**7** (1,2,3,4,6,9,11)	**8** (1,3,4,5,7,8,10,11)
Bus-2	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**5** (2,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**7** (1,6,7,8,9,10,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**6** (4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**3** (4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**3** (4,7,11)	**7** (2,3,4,6,7,9,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**4** (4,8,10,11)	**8** (2,3,4,6,8,9,10,11)	**5** (3,4,8,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**3** (4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**3** (4,10,11)	**7** (2,3,4,6,9,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**7** (1,2,5,6,7,9,11)	**1** (11)	**1** (11)

Table 4.19 Voltage and current measurement for Bus-2 when $R_{fault} = 1$ and $f = 1000Hz$

Msr / **Inj**	**Bus-2**				
	Voltage	**Current**			
	Bus-2	**Bus-2-Line-1**	**Bus-2-Line-2**	**Bus-2-Line-3**	**Bus-2-Line-4**
Bus-1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)
Bus-2	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (2,3,4,11)	**6** (2,3,4,6,9,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,10,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**9** (1,3,4,5,6,7,8,10,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**9** (1,3,4,5,7,8,9,10,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.20 Voltage and current measurement for Bus-3 when $R_{fault} = 1$ and $f = 1000Hz$

Msr / **Inj**	**Bus-3**			
	Voltage	**Current**		
	Bus-3	**Bus-3-Line-1**	**Bus-3-Line-2**	**Bus-3-Line-3**
Bus-1	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**10** (1,2,4,5,6,7,8,9,10,11)	**8** (1,2,5,6,7,8,9,10)
Bus-2	**3** (2,6,9)	**3** (2,6,9)	**5** (2,4,6,9,11)	**3** (2,6,9)
Bus-3	**11** (1,2,3,4,5,6,7,8,9,10,11)	**9** (1,2,3,5,6,7,8,9,10)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**9** (1,2,3,5,6,7,8,9,10)
Bus-4	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,4,5,6,7,8,9,10,11)
Bus-5	**4** (5,7,8,10)	**4** (5,7,8,10)	**6** (4,5,7,8,10,11)	**4** (5,7,8,10)
Bus-6	**1** (6)	**1** (6)	**3** (4,6,11)	**1** (6)
Bus-7	**1** (7)	**1** (7)	**3** (4,7,11)	**1** (7)
Bus-8	**2** (8,10)	**2** (8,10)	**4** (4,8,10,11)	**2** (8,10)
Bus-9	**1** (9)	**1** (9)	**3** (4,9,11)	**1** (9)
Bus-10	**1** (10)	**1** (10)	**3** (4,10,11)	**1** (10)
Bus-11	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)

Table 4.21 Voltage and current measurement for Bus-4 when $R_{fault} = 1$ and $f = 1000Hz$

Msr **Inj**	**Bus-4**			
	Voltage	**Current**		
	Bus-4	**Bus-4-Line-1**	**Bus-4-Line-2**	**Bus-4-Line-3**
Bus-1	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-2	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)
Bus-3	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)
Bus-4	**11** (1,2,3,4,5,6,7,8,9,10,11)	**9** (1,2,4,5,6,7,8,9,10)	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,9,10)
Bus-5	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)
Bus-6	**1** (6)	**1** (6)	**1** (6)	**1** (6)
Bus-7	**1** (7)	**1** (7)	**1** (7)	**1** (7)
Bus-8	**2** (8,10)	**2** (8,10)	**2** (8,10)	**2** (8,10)
Bus-9	**1** (9)	**1** (9)	**1** (9)	**1** (9)
Bus-10	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-11	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)

Table 4.22 Voltage and current measurement for Bus-5 when $R_{fault} = 1$ and $f = 1000Hz$

Msr / Inj	Bus-5			
	Voltage	**Current**		
	Bus-5	**Bus-5-Line-1**	**Bus-5-Line-2**	**Bus-5-Line-3**
Bus-1	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-2	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**11** (1,2,3,4,5,6,7,8,9,10,11)	**8** (1,2,3,4,5,6,9,11)	**8** (1,2,3,4,5,6,9,11)	**8** (1,2,3,4,5,6,9,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**7** (2,3,4,6,7,9,11)	**7** (2,3,4,6,7,9,11)	**7** (2,3,4,6,7,9,11)	**7** (2,3,4,6,7,9,11)
Bus-8	**8** (2,3,4,6,8,9,10,11)	**8** (2,3,4,6,8,9,10,11)	**8** (2,3,4,6,8,9,10,11)	**8** (2,3,4,6,8,9,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**7** (2,3,4,6,9,10,11)	**7** (2,3,4,6,9,10,11)	**7** (2,3,4,6,9,10,11)	**7** (2,3,4,6,9,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.23 Voltage and current measurement for Bus-6, Bus-7, and Bus-8 when $R_{fault} = 1$ and $f = 1000Hz$

Measurement **Inj**	**Bus-6**		**Bus-7**		**Bus-8**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-2	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**9** (1,2,3,4,5,6,8,9,11)	**9** (1,2,3,4,5,6,8,9,11)
Bus-6	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**7** (2,3,4,6,7,9,11)	**7** (2,3,4,6,7,9,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**8** (2,3,4,6,8,9,10,11)	**8** (2,3,4,6,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,9,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**7** (2,3,4,6,9,10,11)	**7** (2,3,4,6,9,10,11)	**8** (1,2,3,4,6,9,10,11)	**7** (2,3,4,6,9,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.24 Voltage and current measurement for Bus-9, Bus-10, and Bus-11 when $R_{fault} = 1$ and $f = 1000Hz$

Measurement / **Injection**	**Bus-9**		**Bus-10**		**Bus-11**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-2	**10** (1,2,3,4,5,6,7,8,10,11)	**9** (1,2,3,4,5,6,7,10,11)	**5** (2,3,4,6,11)	**5** (2,3,4,6,11)	**3** (2,6,9)	**3** (2,6,9)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**6** (4,5,6,7,8,9)	**6** (1,5,6,7,8,9)
Bus-5	**4** (7,8,10,11)	**3** (7,10,11)	**5** (5,6,7,9,11)	**5** (5,6,7,9,11)	**4** (5,7,8,10)	**4** (5,7,8,10)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**1** (6)	**1** (6)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**5** (2,6,7,9,11)	**5** (2,6,7,9,11)	**1** (7)	**1** (7)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**2** (8,11)	**2** (8,11)
Bus-9	**11** (1,2,3,4,5,6,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,9,11	**4** (3,4,9,11)	**4** (3,4,9,11)	**1** (9)	**1** (9)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**1** (10)	**1** (10)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)

4.7 Current Measurement Results for $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Table 4.25 Voltage and current measurement for Bus-1 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr / **Inj**	**Bus-1**				
	Voltage	**Current**			
	Bus-1	**Bus-1-Line-1**	**Bus-1-Line-2**	**Bus-1-Line-3**	**Bus-1-Line-4**
Bus-1	**10** (2,3,4,5,6,7,8,9,10,11)	**1** (11)	**7** (2,5,6,7,8,9,10)	**6** (2,3,4,6,9,11)	**7** (3,4,5,7,8,10,11)
Bus-2	**4** (4,6,9,11)	**4** (4,6,9,11)	**2** (6,9)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**2** (9,11)	**1** (11)	**1** (11)
Bus-5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**3** (7,8,10)	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)
Bus-6	**2** (4,11)	**2** (4,11)	**0**	**2** (4,11)	**2** (4,11)
Bus-7	**2** (4,11)	**2** (4,11)	**0**	**3** (3,4,11)	**2** (4,11)
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**1** (10)	**4** (3,4,10,11)	**3** (4,10,11)
Bus-9	**2** (4,11)	**2** (4,11)	**0**	**2** (4,11)	**2** (4,11)
Bus-10	**2** (4,11)	**2** (4,11)	**0**	**3** (3,4,11)	**2** (4,11)
Bus-11	**0**	**0**	**1** (9)	**0**	**0**

Table 4.26 Voltage and current measurement for Bus-2 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr **Inj**	**Bus-2**				
	Voltage	**Current**			
	Bus-2	**Bus-2-Line-1**	**Bus-2-Line-2**	**Bus-2-Line-3**	**Bus-2-Line-4**
Bus-1	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)
Bus-2	**10** (1,3,4,5,6,7,8,9,10,11)	**2** (4,11)	**4** (4,6,9,11)	**9** (1,3,4,5,7,8,9,10,11)	**9** (1,3,4,5,6,7,8,10,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**4** (4,7,10,11)	**5** (4,7,8,10,11)
Bus-6	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-7	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)	**3** (4,10,11)
Bus-9	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-10	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-11	**0**	**0**	**0**	**0**	**0**

Table 4.27 Voltage and current measurement for Bus-3 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr / **Inj**	**Bus-3**			
	Voltage	**Current**		
	Bus-3	**Bus-3-Line-1**	**Bus-3-Line-2**	**Bus-3-Line-3**
Bus-1	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**2** (6,9)	**2** (6,9)	**2** (6,9)	**2** (6,9)
Bus-3	**10** (1,2,4,5,6,7,8,9,10,11)	**2** (6,9)	**2** (4,11)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**8** (2,5,6,7,8,9,10,11)	**8** (2,5,6,7,8,9,10,11)	**1** (11)	**8** (1,2,6,7,8,9,10,11)
Bus-5	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**0**	**0**	**0**	**0**
Bus-7	**0**	**0**	**0**	**0**
Bus-8	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-9	**0**	**0**	**0**	**0**
Bus-10	**0**	**0**	**0**	**0**
Bus-11	**6** (2,6,7,8,9,10)	**6** (2,6,7,8,9,10)	**0**	**7** (2,5,6,7,8,9,10)

Table 4.28 Voltage and current measurement for Bus-4 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr **Inj**	**Bus-4**			
	Voltage	**Current**		
	Bus-4	**Bus-4-Line-1**	**Bus-4-Line-2**	**Bus-4-Line-3**
Bus-1	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**2** (6,9)	**2** (6,9)	**2** (6,9)	**2** (6,9)
Bus-3	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**10** (1,2,3,5,6,7,8,9,10,11)	**7** (2,5,6,7,8,9,10)	**8** (2,5,6,7,8,9,10,11)	**9** (1,2,3,5,6,7,8,9,10)
Bus-5	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**0**	**0**	**0**	**0**
Bus-7	**0**	**0**	**0**	**0**
Bus-8	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-9	**0**	**0**	**0**	**0**
Bus-10	**0**	**0**	**0**	**0**
Bus-11	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)

Table 4.29 Voltage and current measurement for Bus-5 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr / **Inj**	**Bus-5**			
	Voltage	**Current**		
	Bus-5	**Bus-5-Line-1**	**Bus-5-Line-2**	**Bus-5-Line-3**
Bus-1	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-2	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**10** (1,2,3,4,6,7,8,9,10,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-6	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-7	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-8	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-9	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-10	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-11	**0**	**0**	**0**	**0**

Table 4.30 Voltage and current measurement for Bus-6, Bus-7, and Bus-8 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr **Inj**	**Bus-6**		**Bus-7**		**Bus-8**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-2	**9** (1,3,4,5,7,8,9,10,11)	**9** (1,3,4,5,7,8,9,10,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**4** (4,6,9,11)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)
Bus-5	**5** (4,7,8,10,11)	**4** (4,7,10,11)	**9** (1,2,3,4,6,8,9,10,11)	**9** (1,2,3,4,6,8,9,10,11)	**8** (1,2,3,4,6,7,9,11)	**8** (1,2,3,4,6,7,9,11)
Bus-6	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-7	**2** (4,11)	**2** (4,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**10** (1,2,3,4,5,6,7,9,10,11)	**9** (1,2,3,4,5,6,7,9,11)
Bus-9	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-10	**2** (4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-11	**0**	**0**	**0**	**0**	**0**	**0**

Table 4.31 Voltage and current measurement for Bus-9, Bus-10, and Bus-11 when $R_{fault} = 1e^{-3}$ and $f = 7000Hz$

Msr Injection	**Bus-9**		**Bus-10**		**Bus-11**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**7** (2,5,6,7,8,9,10)	**7** (2,5,6,7,8,9,10)
Bus-2	**9** (1,3,4,5,6,7,8,10,11)	**9** (1,3,4,5,6,7,8,10,11)	**4** (4,6,9,11)	**4** (4,6,9,11)	**2** (6,9)	**2** (6,9)
Bus-3	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-4	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)
Bus-5	**5** (4,7,8,10,11)	**5** (4,7,8,10,11)	**8** (1,2,3,4,6,7,9,11)	**8** (1,2,3,4,6,7,9,11)	**3** (7,8,10)	**3** (7,8,10)
Bus-6	**3** (3,4,11)	**3** (3,4,11)	**2** (4,11)	**2** (4,11)	**0**	**0**
Bus-7	**2** (4,11)	**2** (4,11)	**3** (3,4,11)	**3** (3,4,11)	**0**	**0**
Bus-8	**3** (4,10,11)	**3** (4,10,11)	**9** (1,2,3,4,5,6,7,9,11)	**9** (1,2,3,4,5,6,7,9,11)	**1** (10)	**1** (10)
Bus-9	**10** (1,2,3,4,5,6,7,8,10,11)	**10** (1,2,3,4,5,6,7,8,10,11)	**2** (4,11)	**2** (4,11)	**0**	**0**
Bus-10	**2** (4,11)	**2** (4,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**10** (1,2,3,4,5,6,7,8,9,11)	**0**	**0**
Bus-11	**0**	**0**	**0**	**0**	**10** (1,2,3,4,5,6,7,8,9,10)	**10** (1,2,3,4,5,6,7,8,9,10)

4.8 Current Measurement Results for $R_{fault} = 1$ and $f = 7000Hz$

Table 4.32 Voltage and current measurement for Bus-1 when $R_{fault} = 1$ and $f = 7000Hz$

Msr / Inj	Bus-1				
	Voltage	**Current**			
	Bus-1	**Bus-1-Line-1**	**Bus-1-Line-2**	**Bus-1-Line-3**	**Bus-1-Line-4**
Bus-1	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (1,3,4,11)	**8** (1,2,5,6,7,8,9,10)	**7** (1,2,3,4,6,9,11)	**8** (1,3,4,5,7,8,10,11)
Bus-2	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**3** (2,6,9)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**6** (2,3,4,6,9,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**5** (2,4,6,9,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**4** (5,7,8,10)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**1** (6)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**1** (7)	**4** (3,4,7,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**2** (8,10)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**1** (9)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**1** (10)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**4** (2,6,9,11)	**1** (11)	**1** (11)

Table 4.33 Voltage and current measurement for Bus-2 when $R_{fault} = 1$ and $f = 7000Hz$

Msr **Inj**	**Bus-2**				
	Voltage	**Current**			
	Bus-2	**Bus-2-Line-1**	**Bus-2-Line-2**	**Bus-2-Line-3**	**Bus-2-Line-4**
Bus-1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)
Bus-2	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (2,3,4,11)	**6** (2,3,4,6,9,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,10,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.34 Voltage and current measurement for Bus-3 when $R_{fault} = 1$ and $f = 7000Hz$

Msr / Inj	Bus-3			
	Voltage	**Current**		
	Bus-3	**Bus-3-Line-1**	**Bus-3-Line-2**	**Bus-3-Line-3**
Bus-1	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-2	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)
Bus-3	**11** (1,2,3,4,5,6,7,8,9,10,11)	**9** (1,2,3,5,6,7,8,9,10)	**5** (3,4,6,9,11)	**9** (1,2,3,5,6,7,8,9,10)
Bus-4	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,4,5,6,7,8,9,10,11)	**5** (2,4,6,9,11)	**10** (1,2,4,5,6,7,8,9,10,11)
Bus-5	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)
Bus-6	**1** (6)	**1** (6)	**1** (6)	**1** (6)
Bus-7	**1** (7)	**1** (7)	**1** (7)	**1** (7)
Bus-8	**2** (8,10)	**2** (8,10)	**2** (8,10)	**2** (8,10)
Bus-9	**1** (9)	**1** (9)	**1** (9)	**1** (9)
Bus-10	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-11	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**4** (2,6,9,11)	**9** (1,2,5,6,7,8,9,10,11)

Table 4.35 Voltage and current measurement for Bus-4 when $R_{fault} = 1$ and $f = 7000Hz$

Msr / **Inj**	**Bus-4**			
	Voltage	**Current**		
	Bus-4	**Bus-4-Line-1**	**Bus-4-Line-2**	**Bus-4-Line-3**
Bus-1	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus-2	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)	**3** (2,6,9)
Bus-3	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)	**9** (1,2,3,5,6,7,8,9,10)
Bus-4	**11** (1,2,3,4,5,6,7,8,9,10,11)	**9** (1,2,4,5,6,7,8,9,10)	**10** (1,2,4,5,6,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,9,10)
Bus-5	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)	**4** (5,7,8,10)
Bus-6	**1** (6)	**1** (6)	**1** (6)	**1** (6)
Bus-7	**1** (7)	**1** (7)	**1** (7)	**1** (7)
Bus-8	**2** (8,10)	**2** (8,10)	**2** (8,10)	**2** (8,10)
Bus-9	**1** (9)	**1** (9)	**1** (9)	**1** (9)
Bus-10	**1** (10)	**1** (10)	**1** (10)	**1** (10)
Bus-11	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)	**9** (1,2,5,6,7,8,9,10,11)

Table 4.36 Voltage and current measurement for Bus-5 when $R_{fault} = 1$ and $f = 7000Hz$

Msr / **Inj**	**Bus-5**			
	Voltage	**Current**		
	Bus-5	**Bus-5-Line-1**	**Bus-5-Line-2**	**Bus-5-Line-3**
Bus-1	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-2	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**11** (1,2,3,4,5,6,7,8,9,10,11)	**8** (1,2,3,4,5,6,9,11)	**8** (1,2,3,4,5,6,9,11)	**8** (1,2,3,4,5,6,9,11)
Bus-6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.37 Voltage and current measurement for Bus-6, Bus-7, and Bus-8 when $R_{fault} = 1$ and $f = 7000Hz$

Msr **Inj**	**Bus-6**		**Bus-7**		**Bus-8**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus-1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)
Bus-2	**10** (1,2,3,4,5,7,8,9,10,11)	**10** (1,2,3,4,5,7,8,9,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)
Bus-3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)
Bus-4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)
Bus-5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**10** (1,2,3,4,5,6,8,9,10,11)	**9** (1,2,3,4,5,6,8,9,11)	**9** (1,2,3,4,5,6,8,9,11)
Bus-6	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)
Bus-7	**4** (3,4,7,11)	**4** (3,4,7,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (3,4,7,11)	**4** (3,4,7,11)
Bus-8	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**5** (3,4,8,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**10** (1,2,3,4,5,6,7,8,9,11)
Bus-9	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**4** (3,4,9,11)
Bus-10	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)	**4** (3,4,10,11)
Bus-11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**1** (11)

Table 4.38 Voltage and current measurement for Bus-9, Bus-10, and Bus-11 when $R_{fault} = 1$ and $f = 7000Hz$

Msr Inj	**Bus-9**		**Bus-10**		**Bus-11**	
	Voltage	**Current**	**Voltage**	**Current**	**Voltage**	**Current**
Bus -1	**8** (1,3,4,5,7,8,10,11)	**8** (1,3,4,5,7,8,10,11)	**7** (1,2,3,4,6,9,11)	**7** (1,2,3,4,6,9,11)	**8** (1,2,5,6,7,8,9,10)	**8** (1,2,5,6,7,8,9,10)
Bus -2	**10** (1,2,3,4,5,6,7,8,10,11)	**10** (1,2,3,4,5,6,7,8,10,11)	**6** (2,3,4,6,9,11)	**6** (2,3,4,6,9,11)	**3** (2,6,9)	**3** (2,6,9)
Bus -3	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**3** (3,4,11)	**4** (1,2,3,10)	**9** (1,2,3,10)
Bus -4	**2** (4,11)	**2** (4,11)	**2** (4,11)	**2** (4,11)	10 (1,2,3,4,5,6,7,8,9)	10 (1,2,3,4,5,6,7,8,9)
Bus -5	**7** (3,4,5,7,8,10,11)	**7** (3,4,5,7,8,10,11)	**9** (1,2,3,4,5,6,7,9,11)	**9** (1,2,3,4,5,6,7,9,11)	**4** (5,7,8,10)	**4** (5,7,8,10)
Bus -6	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**4** (3,4,6,11)	**1** (6)	**1** (6)
Bus -7	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**4** (3,4,7,11)	**1** (7)	**1** (7)
Bus -8	**5** (3,4,8,10,11	**5** (3,4,8,10,11	**5** (5,7,8,9,11)	**5** (5,7,8,9,11)	**2** (8,11)	**2** (8,11)
Bus -9	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**4** (3,4,9,11)	**4** (3,4,9,11)	**1** (9)	**1** (9)
Bus -10	**4** (3,4,10,11)	**4** (3,4,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**11** (1,2,3,4,5,6,7,8,9,10,11)	**1** (10)	**1** (10)
Bus -11	**1** (11)	**1** (11)	**1** (11)	**1** (11)	**9** (1,3,5,6,7,8,9,10,11	**9** (1,3,5,6,7,8,9,10,11

As it shown in the tables current measurement can provides better results with less undetectable buses and less multi-estimation over voltage measurements.

CHAPTER 5
CONCLUSIONS

Conclusions

In this thesis, a new method for fault location in the shipboard power system is proposed by using applied voltages and measured voltages/current when there is a fault in the system. The system is tested for all the possible buses for the injections and measurements and the optimal points that lead to minimal numbers of injections and measurements in the system are found. The proposed approach generalizes the Active Impedance Estimation (AIE) fault location method where injection and measurements are not necessarily at the same location. Also, it is shown that by increasing the frequency of the injections multi-estimation is significantly reduced. The proposed approach is a reliable and economical method that can find the location of the fault in the shipboard power system with the minimum number of injections and measurements.

VITA

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