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ECONOMIC APPROACHES AND MARKET STRUCTURES FOR TEMPORAL-SPATIAL SPECTRUM SHARING

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To my parents, Shuxia Gao and Zhengzhi Zhang.

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My life is different, thanks to this milestone and the unforgettable memories shared with you.

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Abstract

In wireless communication systems, economic approaches can be applied to spectrum sharing and enhance spectrum utilization.

In this research, we develop a model where geographic information, including licensed areas of primary users (PUs) and locations of secondary users (SUs), plays an important role in the spectrum sharing system. We consider a multi-price policy and the pricing power of noncooperative PUs in multiple geographic areas. Meanwhile, the value assessment of a channel is price-related and the demand from the SUs is price-elastic. By applying an evolutionary procedure, we prove the existence and uniqueness of the optimal payoff for each PU selling channels without reserve. In the scenario of selling channels with reserve, we predict the channel prices for the PUs leading to the optimal supplies of the PUs and hence the optimal payoffs.

To increase spectrum utilization, the scenario of spatial spectrum reuse is considered. We consider maximizing social welfare via on-demand channel allocation, which describes the overall satisfaction of the SUs when we involve the supply and demand relationship. We design a receiver-centric spectrum reuse mechanism, in which the optimal channel allocation that maximizes social welfare can be achieved by the Vickrey-Clarke-Groves (VCG) auction for maximal independent groups (MIGs). We prove that truthful bidding is the optimal strategy for the SUs, even though the SUs do not participate in the VCG auction for MIGs directly. Therefore, the MIGs are bidding truthfully and the requirement for social welfare maximization is satisfied.

To further improve user satisfaction, user characteristics that enable heterogeneous channel valuations need to be considered in spatial spectrum reuse. We design a channel transaction mechanism for non-symmetric networks and maximize user satisfaction in consideration of multi-level flexible channel valuations of the SUs. Specifically, we introduce a constrained VCG auction. To facilitate the bid formation, we transform the constrained VCG auction to a step-by-step decision process. Meanwhile, the SUs in a coalition play a

coalitional game with transferable utilities. We use the Shapley value to realize fair payoff distribution among the SUs in a coalition.

Chapter 1
Introduction to Spectrum Sharing Using Economic Approaches

Spectrum sharing is a secondary distribution mechanism to mitigate the growing conflict between scarce spectrum resources and the explosion of wireless devices [1]. The basic assumption of spectrum sharing is a hierarchical system composed of primary users (PUs) and secondary users (SUs). When economic approaches are applied to the model of spectrum sharing, idle channels of PUs can be traded as merchandise. Channel transactions usually feature several properties. For example, channels are perishable assets. Meanwhile, channels of different PUs are differentiable products, providing different utilities. Therefore, economic approaches can be applied to study the behaviors of PUs and SUs in a spectrum sharing system.

In this research, we study the impact of flexible channel valuations of SUs in spectrum sharing. Not only are SUs benefited from our research, but also PUs can increase their payoffs by estimating the behaviors of the SUs more accurately and pricing the channels more appropriately. When a PU has multiple channels for SUs to use, the simple assumption that an SU continues to buy channels at a fixed price neglects the actual supply and demand relationship. As a result, channel transactions will be far from the expectation and payoffs of the PUs will be jeopardized. In practice, the channel valuation of an SU usually decreases when the number of obtained channels increases. In this research, we analyze the behaviors of SUs and PUs in consideration of flexible channel valuations, which helps the PUs to better estimate the behaviors of the SUs. Specifically, the research focuses on location-oriented spectrum sharing and channel auctions for spatial spectrum reuse.

Geographical information, including licensed blocks of PUs and locations of SUs, plays an important role in a spectrum sharing system. In simplified models such as the dynamic multi-band sharing in [2], utility-based cooperative game in [3], two-tier market in [4], supermodular game in [5], and spectrum trading pricing game in [6], each licensed block of PUs covers the entire system so that the relative locations of SUs with respect to PUs can be neglected. However, if each licensed block of PUs covers the system partially, permissions of channel transactions need to be granted according to the locations of SUs. In such a case, regional differences in supply and demand need to be taken into account. As a result, channel selection preference of each SU, channel selling preference of each PU, and channel prices of different regions become new parameters, which cannot be handled in the simplified models without geographical consideration.

User locations have been considered in several spectrum sharing games to model non-isotropic interference of SUs. In [7], an inter-tier spectrum sharing algorithm between a macro cell and several pico cells in consideration of cell ranges is built based on Stackelberg games. In [8], a cellular operator balances between femtocell and macrocell services in a Stackelberg game based on the coverage of femtocell services. In both models, there exists only one PU in the spectrum sharing system. When there are multiple PUs, the selling competition among PUs and the different channel selection preferences of SUs need to be considered. In [9] and [10], PUs' competition of shared bandwidth at neighboring locations has been studied, in which SUs are combined into independent sets according to mean valid graphs and floating channel prices among these sets. In [11], dynamic spectrum access of multiple PUs and multiple SUs is designed as a multiauctioneer progressive auction. The optimal channel assignment is achieved by Kuhn-Munkres algorithm. However, the channel demand models of SUs in [9–11] are based on the simplified mechanism that an SU will buy a channel when the channel price is lower than a threshold. In practice, channel demand is a function of channel price.

Among existing spectrum sharing models, few consider the licensed blocks of PUs. In [12] and [13], pricing-based decentralized spectrum access of SUs is studied in a Stackelberg game for two scenarios, monopoly PU market and multi-PU market. Although bounded licensed areas of the PUs are considered to perform admission control, channel selling preferences of the PUs are neglected due to the assumption that all SUs are within the intersection of the PUs' licensed areas. In [14], dynamic spectrum trading among multiple sellers and multiple buyers is considered under deterministic and stochastic models. In these models, the spectrum access opportunities are restricted to the licensed areas of the PUs. However, similar to the models in [10] and [11], the single price policy of the PUs and the simple channel demand model of the SUs limit the application of this approach to a large network.

To further increase spectrum utilization, we consider the scenario of spatial spectrum reuse. Spatial spectrum reuse is a spectrum sharing mechanism that enhances spectrum utilization by allowing users at different locations to access the same channel simultaneously. PUs, as authorized channel holders, can share a channel with multiple SUs who do not own the spectrum resource. The strengths of the received signal and co-channel interference are the two main factors to consider in spatial spectrum reuse [15].

Recently, more attention on spatial spectrum reuse has been paid to spectrum efficiency improvement through system throughput maximization under different network settings. In [16], enhanced spatial spectrum reuse for the coexistence of LTE and Wi-Fi systems is achieved through delicate allocation of spatial degrees of freedom. The throughput of the spatial spectrum sharing system is derived and hence can be optimized between the LTE cells and the Wi-Fi system. In [17], dynamic spectrum access is modeled as a matching game between PUs and SUs. A mechanism that raises buyers' willingness to pay and increases channel utilization is proposed to maximize the number of accessible channels of an SU given the minimum demand on the spectrum. In [18], spectrum reuse in a dense network of small cells with given traffic statistics is considered. With the proposed user association and spectrum allocation, the network capacity increases. In [19], temporal-spatial spectrum reuse in a millimeter-wave ultra-dense network is considered. To maximize the system throughput, a non-cooperative game among SUs is proposed and the existence of Nash Equilibrium (NE) is proved. In [20], a mechanism is proposed for the coexistence of massive multiple-input multiple-output (MIMO) cellular and Wi-Fi networks. The proposed method improves the MIMO cellular network throughput without significantly jeopardizing the performance of nearby Wi-Fi devices. In [21], three dynamic spectrum reuse techniques incorporating both spatial and time domains are introduced. The performance of these techniques when applied to the coexistence of LTE and WiFi networks is evaluated. In [22], the coexistence of device-to-device (D2D) and cellular networks is discussed. A mode selection scheme to manage intra-cell interference in a limited cellular network region is proposed and hence the number of successful transmissions among D2D users increases. In [23], the optimal channel allocation in a spatial spectrum reuse scenario is analyzed under centralized and decentralized policies. In [24], a resource allocation problem for spectrum reuse between uplink and D2D communications is introduced. To solve the problem, a nonconvex optimization problem is formulated and a suboptimal resource allocation algorithm is obtained via convex approximation. In [25], the efficiency of spatial spectrum reuse is studied in consideration of signal features in the THz frequency band. It is proved that there exists an optimal distance among receivers to maximize the system throughput in an ultra-dense THz wireless network. In [26], an opportunistic spectrum access problem in the MAC layer is introduced, the interference minimization game is proved to be a potential game, and two decoupled learning algorithms are proposed to approach the NE. In [27], the spatial spectrum reuse is modeled as a congestion game, in which an SU selects which channel to use and its payoff depends on the strategies of other SUs due to the co-channel interference. In the congestion game, a NE is obtained with a distributed learning algorithm. More discussions are presented in [28–31].

In large spectrum sharing systems, auction games that involve interactions between the auctioneers and bidders provide an alternative way to allocate channels in spatial spectrum reuse. By setting proper auction rules and incentives, an auction game exhibits its specific features. For example, bidders in a Vickrey auction [32] tend to submit bids that reveal their true valuations, and an English auction helps the seller to gain more profit [33]. Auction games have been applied to spatial spectrum reuse in [34], where the game design aims towards efficient and fraud-free channel allocation. In [35], the application of double auction to spectrum reuse in heterogeneous networks is studied. The proposed double auction design helps both the buyers and sellers to achieve a truthful, rational, and budget-balanced market. In [36], a spectrum auction game in a two-tier network is proposed. In between end users and channel holders, the secondary service provider (SSP) plays a role as an intermediary. The SSP obtains channels from channel holders through an auction process and distributes these channels to end users for spectrum reuse. In [37], an auction game in which channel sellers have the option to quit is proposed. Conditions on using private negotiation instead of auction are given in consideration of valuations of both buyers and sellers. In [38], the ex-post and ex-ante auctions in consideration of channel supply uncertainties are proposed. The expected payoffs to buyers and sellers and the social welfares of these auctions are compared. In [39], a truthful spectrum auction mechanism is proposed to improve spectrum utilization in consideration of both the quality of service (QoS) and spatial spectrum reuse. In addition, auction games improve spatial spectrum reuse in heterogeneous networks. In [40], a mechanism for truthful double spectrum auctions, TRUST, is proposed. The mechanism minimizes the tradeoff between spectrum reusability and economic robustness. In [41], two auction mechanisms with different charging policies are proposed for power allocation in spatial spectrum reuse. Both mechanisms obtain socially optimal outcome for a large spectrum sharing system.

The rest of the dissertation is organized as follows. In Chapter 2, we consider both the locations of SUs and the licensed blocks of PUs. By modeling the multi-price policy of the PUs and the price-elastic demand of the SUs, we specify channel selling preferences of the PUs, channel selection preferences of the SUs, and channel prices of the PUs in different regions. In Chapter 3, we propose a new mechanism for spatial spectrum reuse by considering flexible valuations of SUs' channels, which manifests supply and demand relationship. In Chapter 4, we consider multi-level flexible channel valuations of the SUs over non-identical channels, and propose a mechanism that better serve the SUs in terms of the overall satisfaction.

Chapter 2
Location-Oriented Evolutionary Games for Price-Elastic Spectrum Sharing

2.1 Introduction

In this chapter, we consider both the locations of SUs and the licensed blocks of PUs. By modeling the multi-price policy of the PUs and the price-elastic demand of the SUs, we specify channel selling preferences of the PUs, channel selection preferences of the SUs, and channel prices of the PUs in different regions. Specifically, we consider a spectrum sharing system in a large geographic region based on our preliminary work in [42], where the licensed areas of the PUs are bounded. Hence, each PU could sell idle channels only to the SUs in its licensed area. In such a scenario, the SUs have different lists of suppliers. On the other hand, based on the competition status, each PU has a chance to increase its payoff by selecting appropriate SUs to sell channels and setting appropriate prices for the SUs. To quantize the competition status, we establish a price-elastic demand model that incorporates both the double log demand (DLD) [43] and multi-nomial logit (MNL) models [44]. Our demand model is applicable to both oligopoly and monopoly markets, two common market structures for spectrum sharing [45–47].

To maximize the payoffs of the PUs, we propose a unique quota transaction process. In the process, the PUs set the number of channels that they would like to sell to each particular SU, or quota. In our individual analysis for nonhomogeneous SUs, we prove the existence and uniqueness of the evolutionary stable strategy (ESS) quota vector of each PU when the PUs sell channels without reserve. Based on the evolutionary procedure defined as replicator dynamics, we design a learning process to obtain the best integer quota (BIQ). Furthermore, we consider the scenario that the PUs sell channels with reserve. We predict the channel prices for the PUs leading to the optimal supplies of the PUs. Moreover, we

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apply a grouping mechanism when the SUs are homogeneous to simplify the process based on utility zones.

The rest of the chapter is organized as follows. In section 2.2, we give our model of the spectrum sharing system. In section 2.3, we propose our unique quota transaction process, based on which we discuss the ESS quotas for two scenarios, selling channels without reserve and selling channels with reserve. In addition, we design the learning processes for the two scenarios, respectively. Next, in section 2.4, we introduce the grouping mechanism to simplify the process. In section 2.5, we present our simulation results for the two scenarios. Finally, we draw our conclusions in section 2.6.

2.2 System Model

2.2.1 System Setup

Suppose that there are M PUs and N SUs in a spectrum sharing system. For the mth PU, it has a supply of S_m non-overlapping channels to sell to the SUs. C_m is the cost of each channel for the mth PU. We assume that a channel can be sold only once in a single transaction, and the transaction cost for each channel is C_T. For the nth SU, it has a demand of D_n channels given a budget price ψ_n, which is the price that the nth SU expects to pay for a channel.

In our system, the mth PU is licensed to use a certain bandwidth in a specific geographic block $\mathbb{A}_m$ and can only sell its idle channels in $\mathbb{A}_m$. For different SUs, their lists of suppliers are therefore not the same. In Figure 2.1, we illustrate possible circumstances regarding the geographical relationship of the PUs.

We set (x_n, y_n) as the location of the nth SU. Based on whether the nth SU is inside $\mathbb{A}_m$ or not, we use $t_{mn} = 1$ to denote the acceptance when $(x_n, y_n) \in \mathbb{A}_m$ and $t_{mn} = 0$ the denial of a transaction that the nth SU offers to buy channels from the mth PU when $(x_n, y_n) \notin \mathbb{A}_m$. We define p_{mn} as the channel price at which the mth PU expects to sell to the nth SU. If $t_{mn} = 0$, the mth PU cannot sell channels to the nth SU. In this case, p_{mn} is set to the choke price [48] to avoid possible transactions. We define d_{mn} as the number

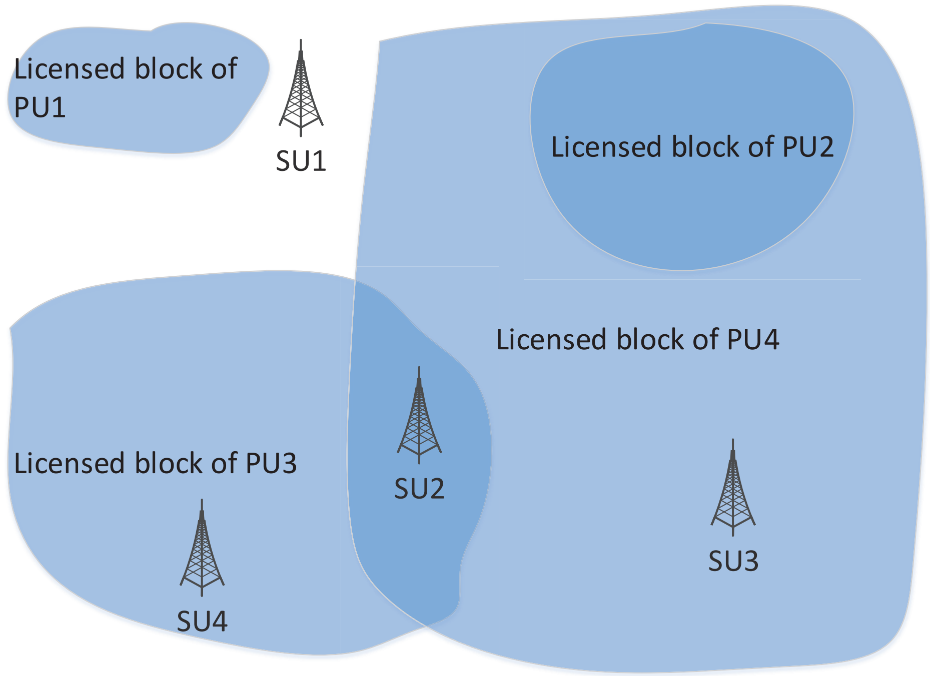

Figure 2.1. System model in location-oriented spectrum sharing.

of channels that the nth SU offers to buy from the mth PU after the nth SU obtains the information of channel prices. Meanwhile, since transaction amounts may be different from demands, we define b_{mn} as the number of channels that the nth SU actually buys from the mth PU. Note that $b_{mn} = 0$ if $t_{mn} = 0$.

2.2.2 Channel Utility

In our system, we use U_{mn} to denote the channel utility obtained by the nth SU buying a channel from the mth PU. Based on the economic model in [49],

$$U_{mn} = u_{mn} + \Xi_{mn}, \tag{2.1}$$

where u_{mn} reflects the channel capacity, and Ξ_{mn} is a linear price-income sensitivity function that describes how the channel utility is related to the channel price [50]. Specifically,

$$u_{mn} = \delta W_m \log(1 + \mathrm{SNR}_{mn}) \tag{2.2}$$

and

$$\Xi_{mn} = \gamma(\psi_n - p_{mn}), \tag{2.3}$$

where $\delta > 0$ and $\gamma > 0$ are weighting factors, W_m is the bandwidth of each channel from the mth PU, and SNR_{mn} is the signal to noise ratio that the nth SU can achieve when buying a channel from the mth PU, without breaking the interference temperature constraint on the boundary of $\mathbb{A}_m$ [51, 52].

2.2.3 Demand Model

To determine d_{mn}, we use the DLD and MNL models [43, 44] to reflect the price-elastic demand of the SUs and the market shares of the PUs, respectively. In [53], a similar concept of price-elastic demand is applied to a spectrum trading system catering to demand variations of SUs. In this chapter, we expand the application of price-elastic demand to a system considering license boundaries and location-oriented channel utilities.

Firstly, given a spectrum sharing system consisting of only the mth PU and the nth SU, the demand of the nth SU will vary according to the channel price of the mth PU. Given its budget price ψ_n, the SU would like to buy more channels to increase data rate if the channel price of the PU is low, and vice versa. A typical model to describe such price elasticity is the DLD model that has been applied to describing the demand of customers in the gasoline market [54]. Let d_{mn} denote the demand after the nth SU has the price information of the mth PU. According to the DLD model,

$$d_{mn} = \Lambda_{mn} e^{\zeta_0 \ln(\psi_n) - \zeta_1 \ln(p_{mn})}, \tag{2.4}$$

where $\zeta_0 > 1$, $\zeta_1 > 1$ are constants, and Λ_{mn} is the demand that the nth SU offers to buy from the mth PU at ψ_n. In this single-PU and single-SU system, $\Lambda_{mn} = D_n$.

Then we analyze a spectrum sharing system of multiple PUs and multiple SUs. In this case, the market shares of the PUs need to be determined. Since the channels of different PUs are different only in channel utilities, we regard the channels of different

PUs as different brands. Therefore, the attempts of selling channels to the SUs can be interpreted as brand competitions. We apply the MNL model, a well-known method for the analysis of brand choice, to estimate Λ_{mn} out of the origin D_n. Specifically,

$$\Lambda_{mn} = D_n \frac{t_{mn} e^{U_{mn}}}{\sum_{i=1}^{M} t_{in} e^{U_{in}}}, \tag{2.5}$$

where $\frac{t_{mn} e^{U_{mn}}}{\sum_{i=1}^{M} t_{in} e^{U_{in}}}$ is known as the multi-nomial logit that reflects the percentage of D_n obtained by the mth PU. In the multi-PU and multi-SU system, d_{mn} depends on not only the channel price of the mth PU but also the channel prices of all the other suppliers of the nth SU.

The MNL model determines the share of D_n that each PU can have, but does not change D_n. Meanwhile, the DLD model describes the variation of the nth SU's demand on the mth PU because of price elasticity. Our demand model is applicable to both oligopoly and monopoly markets, which can be used to stimulate channel selection preferences of the SUs and to prevent the PUs from irrational high prices.

2.2.4 Problem Formulation

For the mth PU, the satisfaction of channels requests is not guaranteed, since S_m is finite. Hence, the actual transaction number, b_{mn}, is not necessarily equal to d_{mn}. Meanwhile, the mth PU may be reluctant to sell all the channels in consideration of its potential payoff. In other words, $b_{mn} \leq d_{mn}$ and $\sum_{n=1}^{N} t_{mn} b_{mn} \leq S_m$. Therefore, the payoff of the mth PU is

$$\pi_m = \sum_{n=1}^{N} (p_{mn} - C_T) b_{mn} - S_m C_m. \tag{2.6}$$

We assume that each PU tries to achieve its maximum payoff. For the mth PU, the problem of interest is

$$\max_{\substack{(p_{m1},\cdots,p_{mN}) \\ (b_{m1},\cdots,b_{mN})}} \pi_m(p_{m1}, \cdots, p_{mN}, b_{m1}, \cdots, b_{mN}) \tag{2.7}$$

subject to

$$p_{mn} > 0, \text{ for } n = 1, 2, \cdots, N, \tag{2.8}$$

$$b_{mn} \leq d_{mn}, \text{ for } n = 1, 2, \cdots, N, \tag{2.9}$$

and

$$\sum_{n=1}^{N} b_{mn} \leq S_m. \tag{2.10}$$

Since the licensed block of the mth PU could be covered or partially overlapped by the licensed blocks of other PUs, the interest conflicts induced by competitions of selling channels in the same area are inevitable. As a result, the solution for the optimization problem in (2.7)-(2.10) is to find the best response to channel prices and channel transactions of all the other PUs which well fits a game theory framework.

2.3 Evolutionary Games of PUs with Nonhomogeneous SUs

In classical game theory, each PU must consider the strategies of other PUs to ensure its own strategy appropriate. However, the inequities of channel demands and supplies in our model add complexity for the PUs to find optimal pure strategies or mixed strategies. Meanwhile, analyzing the optimal strategies of other PUs requires more effort in a large system like ours. To solve the problem, we apply evolutionary games [55]. Therefore, each PU can adjust its strategy and achieve the ESS with its own payoff history. Furthermore, using evolutionary games can add robustness to our system when there are irrational PUs.

In general, the SUs have nonidentical budget prices. According to (2.3), the demand from each SU has to be analyzed individually. Hence, we define the SUs with nonidentical budget prices as nonhomogeneous SUs. In the following, we design evolutionary games to solve the problem in (2.7)-(2.10).

2.3.1 Quota Transaction Process

To apply evolutionary games, we design a quota transaction process, in which the PUs set the numbers of channels they would like to sell to particular SUs, or quotas. Specifically, the quota that the mth PU sets for the nth SU is k_{mn}. Note that if $t_{mn} = 0$, $k_{mn} = 0$. There are two scenarios in the quota transaction process. If the mth PU sells channels without reserve, $\sum_{n=1}^{N} k_{mn} = S_m$. If the mth PU sells channels with reserve, $\sum_{n=1}^{N} k_{mn} < S_m$. As long as a quota is set, the transaction number should be under the quota, i.e., $b_{mn} \leq k_{mn}$. Hence, $b_{mn} = \min(k_{mn}, d_{mn})$.

On the k_{mn}-d_{mn} plane, there are two possible situations.

1. When $(k_{mn}, d_{mn}) \in \{k_{mn} \geq d_{mn}\}$, we have $b_{mn} = d_{mn}$. We can view the unsold channels as reserved channels of the mth PU, without changing the transaction numbers and the payoff of the mth PU as if $k_{mn} = d_{mn}$.

2. When $(k_{mn}, d_{mn}) \in \{k_{mn} \leq d_{mn}\}$, we have $b_{mn} = k_{mn}$. Note that given b_{mn}, the higher p_{mn} is, the higher payoff can be obtained according to (2.7). Meanwhile, a higher p_{mn} indicates a lower d_{mn} from (2.5). Hence, the highest payoff implies $d_{mn} = k_{mn}$.

Therefore, we let $k_{mn} = d_{mn} = b_{mn}$ to maximize the payoff of each PU.

Given the quota information of the PUs, the payoff of each PU can be determined. For the mth PU, we denote $\mathcal{N}_m^{\diamond}$ as the set of SU indices satisfying $t_{mn} \neq 0$. Hence, the payoff of the mth PU in the quota transaction process is

$$\pi_m = \sum_{n \in \mathcal{N}_m^{\diamond}} (p_{mn} - C_T) k_{mn} - S_m C_m, \tag{2.11}$$

where p_{mn} can be determined by

$$d_{mn}(p_{1n}, \cdots, p_{Mn}) = k_{mn} \tag{2.12}$$

for all $n \in \mathcal{N}_m^{\diamond}$.

From (2.12), we notice that p_{mn} can be expressed as a function of (k_{mn}, k_{-mn}), where k_{-mn} are the quotas set for the nth SU by the PUs other than the mth PU. Meanwhile, π_m is related not only to the quotas of the mth PU, but also to the other PUs. For clarity, we further describe π_m as $\pi_m(\mathbf{k}_m, \mathbf{k}_{-m})$, where $\mathbf{k}_m = (k_{m1}, \cdots, k_{mN})$ denotes the quota vector of the mth PU and $\mathbf{k}_{-m}$ the quota vectors of the PUs other than the mth PU.

Based on the above discussion, the original optimization problem in (2.7)-(2.10) can be transformed into

$$\max_{\mathbf{k}_m} \pi_m(\mathbf{k}_m, \mathbf{k}_{-m}) \tag{2.13}$$

subject to

$$\sum_{n=1}^{N} k_{mn} \leq S_m. \tag{2.14}$$

In the quota transaction process, we eliminate transaction situations that could not maximize the payoffs of the PUs, i.e., when $d_{mn} \neq b_{mn}$. Therefore, instead of being a function of prices and transaction numbers, the payoff of the mth PU can be adjusted by changing its quotas according to (2.13). In addition, the mth PU no longer needs to consider constraint (2.9).

2.3.2 Formulation of Evolutionary Games

In our quota transaction process, each PU tries to set more quotas to the SUs with higher channel utility assessment. However, the payoff of a PU still depends on two other factors. Firstly, channel price is a decreasing function of channel supply. It does not necessarily guarantee that setting more quota can bring more payoff to the PU. Secondly, the PU has to consider the competitions of other PUs, which may reduce the expected payoff. Obviously, selling all channels to one SU is not an optimal solution. Therefore, each PU has to answer the following two questions. Which SUs to set non-zero quotas? How to set these non-zero quotas?

According to classical game theory, each PU needs to figure out the optimal strategies of other PUs, and then finds its best response of quotas. However, analyzing the optimal mixed strategies of other PUs requires more effort in a large system like ours. Instead, we suppose that the mth PU randomly sets non-zero initial quotas to the SUs in its licensed block. Afterwards, the mth PU changes its quotas according to its payoff history and gradually finds out the optimal strategy. Therefore, we formulate the optimization problem of the mth PU in (2.13)-(2.14) as evolutionary games, denoted by $\mathcal{H}_m$. If the mth PU sells channels without reserve, the virtual players are the S_m channels. If the mth PU sells channels with reserve, the virtual players are S'_m channels ($S'_m \leq S_m$). For each player, the strategy space is $\{n|n \in \mathcal{N}_m^{\diamond}\}$. Hence, the payoff for each player who chooses strategy n is $\bar{\pi}_{mn} = p_{mn} - C_m - C_T$, and the population of the players who choose strategy n is k_{mn}. Apparently, the quota vector $\mathbf{k}_m$ includes all strategies of the players. Let $\mathcal{K}_m[S_m]$ denote the set of quota vectors available to the mth PU in the scenario of selling channels without reserve, and $\mathcal{K}_m[S'_m]$ denote the set of quota vectors available to the mth PU in the scenario of selling channels with reserve. Since an ESS is an evolutionarily stable Nash equilibrium (NE) which prevents each player from alternating its strategy [55], the optimization problem in (2.13) and (2.14) is equivalent to finding an ESS quota vector in $\mathcal{K}_m[S_m]$ in the scenario of selling channels without reserve, or to finding an ESS quota vector in $\mathcal{K}_m[S'_m]$ in the scenario of selling channels with reserve.

2.3.3 ESS Quotas Without Channel Reserve

In this subsection, we discuss the existence and uniqueness of the ESS quota vector of each PU. We suppose that each PU has at least one SU to sell channels, and each PU sells channels without reserve. To obtain an ESS quota vector, the mth PU follows an evolutionary procedure defined as replicator dynamics [55]. In this procedure, to build the relationships between quotas and payoffs, the mth PU adjusts $\mathbf{k}_m(\tau)$ in each contract period τ, $\tau = 1, 2, \cdots$. Note that the contract periods mark the iteration steps in the evolutionary games. Furthermore, we use contract periods to synchronize the evolutionary games played by the PUs. Since each PU plays its evolutionary games individually, different time lengths of contract periods will cause fluctuation on channel utilities in between transactions, and therefore cause system chaos. In our model, the default length of a contract period is set by the spectrum sharing system. Additionally, the transaction cost for each channel C_T is related to the duration of a contract period. In practice, frequent channel transactions will reduce channel efficiency, and overlong duration of a contract period will decrease payoffs of PUs.

We set the replicator dynamics as

$$\begin{aligned}\Delta k_{mn}(\tau) =& \mu_m k_{mn}(\tau)\bar{\pi}_{mn}\left(k_{mn}(\tau), k_{-mn}(\tau)\right) \\ &- \mu_m k_{mn}(\tau)\bar{\pi}_m\left(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau)\right)\end{aligned} \tag{2.15}$$

for all $n \in \mathcal{N}_m^\diamond$, where $\Delta k_{mn}(\tau) = k_{mn}(\tau+1) - k_{mn}(\tau)$, $\bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau))$ is the average channel payoff of the channels sold to the nth SU for the mth PU, and $\bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau))$ is the average payoff of all the channels for the mth PU. Specifically, we have

$$\bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau)) = p_{mn}(k_{mn}(\tau), k_{-mn}(\tau)) - C_m - C_T \tag{2.16}$$

and

$$\begin{aligned}\bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau)) =& \sum_{n \in \mathcal{N}_m^\diamond} \frac{k_{mn}(\tau)}{S_m} p_{mn}(k_{mn}(\tau), k_{-mn}(\tau)) \\ &- \sum_{n \in \mathcal{N}_m^\diamond} \frac{k_{mn}(\tau)}{S_m}(C_m - C_T).\end{aligned} \tag{2.17}$$

In (2.15), $\mu_m > 0$ is a multiplier to control the growth rate $\Delta k_{mn}(\tau)/k_{mn}(\tau)$ within $(0, 1)$, which ensures the variation range of $k_{mn}(\tau)$ within $(0, S_m)$ for any $k_{mn}(1) \neq 0$ and $k_{mn}(1) \neq S_m$. Hence, negative quota will not emerge during the process.

If $\bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau))$ is larger than $\bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau))$, the mth PU will increase $k_{mn}(\tau)$ in the $(\tau + 1)$ contract period, indicating selling more channels to the nth SU, and vice versa. Although each PU changes its quotas in each contract period, the stability of $\sum_{n \in \mathcal{N}_m^\diamond} k_{mn}(\tau)$ is automatically satisfied in the replicator dynamics according to Proposition 2.1. Therefore, the constraint $\sum_{n \in \mathcal{N}_m^\diamond} k_{mn} = S_m$ is guaranteed.

Proposition 2.1. *The summation of quotas of each PU will not change during the repli-*

cator dynamics. In other words, $\sum_{n\in\mathcal{N}_m^\diamond} k_{mn}(\tau) = \sum_{n\in\mathcal{N}_m^\diamond} k_{mn}(\tau+1)$.

Proof. See Appendix A.1. □

From (2.15), a stable quota vector of the mth PU requires $\Delta k_{mn}(\tau) = 0$ for all $n \in \mathcal{N}_m^\diamond$. It can be verified that $\bar{\pi}_{mn}(k_{mn}, k_{-mn}) = \bar{\pi}_m(\mathbf{k}_m, \mathbf{k}_{-m})$ is the general solution to $\Delta k_{mn}(\tau) = 0$, while $k_{mn} = 0$ and $k_{mn} = S_m$ are special solutions. Furthermore, each PU changes its quota vector towards a stable quota vector for increased payoff according to Proposition 2.2.

Proposition 2.2. *The quota changes of each PU in each contract period generate higher payoffs. In other words,* $\sum_{n\in\mathcal{N}_m^\diamond} \bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau)) \cdot \frac{\Delta k_{mn}(\tau)}{S_m} \geq 0$.

Proof. See Appendix A.2. □

Proposition 2.2 shows that the replicator dynamics are myopic adjustment dynamics [56]. The quota variations by (2.15) can bring the mth PU a higher payoff, assuming the average channel payoff level does not change. Proposition 2.2 can be viewed as the motivation of the PUs to change quotas. Meanwhile, Proposition 2.2 also reveals that the mth PU will lose the impetus for quota change when the average channel payoffs from the SUs inside its licensed area are identical, i.e., $\bar{\pi}_{mn}(k_{mn}, k_{-mn}) = \bar{\pi}_m(\mathbf{k}_m, \mathbf{k}_{-m})$ for all $n \in \mathcal{N}_m^\diamond$.

We can determine a stable quota vector $\mathbf{k}_m^* = (k_{m1}^*, \cdots, k_{mN}^*)$ of the mth PU for the following two case:

1. The mth PU has only one SU to sell channels, i.e., $\sum_{n=1}^N t_{mn} = 1$.

   In this case, $k_{mn}^* = S_m$ when $n \in \mathcal{N}_m^\diamond$, and $k_{mn}^* = 0$ when $n \notin \mathcal{N}_m^\diamond$.

2. The mth PU has at least two SUs to sell channels, i.e., $\sum_{n=1}^N t_{mn} \geq 2$.

   In this case, $k_{mn}^* \in (0, S_m)$ satisfies $\bar{\pi}_{mn}(k_{mn}^*, k_{-mn}^*) = \bar{\pi}_m(\mathbf{k}_m^*, \mathbf{k}_{-m}^*)$ when $n \in \mathcal{N}_m^\diamond$. And $k_{mn}^* = 0$ when $n \notin \mathcal{N}_m^\diamond$.

In the second case, since

$$\bar{\pi}_{mn}(k_{mn}^*, k_{-mn}) = \bar{\pi}_m(\mathbf{k}_m^*, \mathbf{k}_{-m})$$

holds for all $n \in \mathcal{N}_m^\diamond$, the average channel payoffs from these SUs are identical. According to (2.16), the channel prices for these SUs are equal. Denote the uniform price as ρ_m^* for the SUs in $\mathcal{N}_m^\diamond$. Hence, ρ_m^* and k_{mn}^*, $n \in \mathcal{N}_m^\diamond$, are the solutions to the following equations,

$$\sum_{n \in \mathcal{N}_m^\diamond} k_{mn}^* = S_m \tag{2.18}$$

and

$$d_{mn}(\rho_1^*, \cdots, \rho_m^*, \cdots, \rho_M^*) = k_{mn}^* \tag{2.19}$$

for all $n \in \mathcal{N}_m^\diamond$. In this case, the mth PU does not adopt $k_{mn} = 0$ or $k_{mn} = S_m$ as possible quota when $n \in \mathcal{N}_m^\diamond$, since neither of them depends on the payoff obtained from the nth SU. These two possible solutions will be represented as $k_{mn} \to 0$ and $k_{mn} \to S_m$ if either one happens.

In the following, we analyze the existence and uniqueness of k_{mn}^*, $n \in \mathcal{N}_m^\diamond$.

Theorem 2.1. *If a PU is eligible for selling channels to at least one SU, the stable quota vector* $\mathbf{k}_m^* = (k_{m1}^*, \cdots, k_{mN}^*)$ *always exists and is unique.*

Proof. See Appendix A.3. □

In Theorem 2.1, we have shown the existence and uniqueness of $\mathbf{k}_m^*$. More importantly, $\mathbf{k}_m^*$ is an ESS quota vector for the mth PU according to Theorem 2.2.

Theorem 2.2. *Suppose stable quota vectors* $\mathbf{k}_{-m}^*$ *are set. Starting from any quota vector* $\mathbf{k}_m(1) \in \mathcal{K}_m[S_m]$, $\mathbf{k}_m(\tau)$ *converges to* $\mathbf{k}_m^*$ *in the replicator dynamics defined by* (2.15), *if*

$$h(\tau)h(\tau+1) \geq 0 \tag{2.20}$$

for all $n \in \mathcal{N}_m^\diamond$ *and* $\tau = 1, 2, \cdots,$ *where* $h(\tau) = \bar{\pi}_{mn}(k_{mn}(\tau), k^*_{-mn}) - \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}^*_{-m})$.

Proof. See Appendix A.4. □

Theorem 2.2 indicates that under $\mathbf{k}^*_{-m}$ and constraint (2.20), $\mathbf{k}^*_m$ is asymptotically stable in the replicator dynamics, and thus an ESS quota vector [57]. Here we use constraint (2.20) to control the quota changing speed and to ensure the convergence of the evolutionary procedure theoretically. In practice, convergence can be easily achieved by setting an appropriate μ_m in (2.15) so that $\Delta k_{mn}(\tau)$ is a small variation in comparison with $k_{mn}(\tau)$.

Since each PU tries to maximize its payoff and sells channels without reserve, $\pi_m(\mathbf{k}^*_m, \mathbf{k}^*_{-m})$ is the optimal stable payoff for the mth PU.

2.3.4 Learning Process For PUs Without Channel Reserve

In the case that the PUs sell channels without channel reserve, we propose a learning process to reach the ESS quotas. Without loss of generality, we assume that the PUs are unsophisticated users without knowing that ESS quotas indicate single-price policy in the replicator dynamics, and the PUs share quota information with others. At the same time, channel utilities and budget prices are accessible for all the PUs. In the learning process, the mth PU can obtain $\mathbf{k}^*_m$ with the payoff history of its own only. After setting an initial quota vector, the mth PU can calculate the channel price for each SU by (2.12). Then the mth PU will adjust its quota vector according to (2.15), redistributing channels among SUs to render higher average channel payoffs. When $\Delta k_{mn} = 0$ for all $n \in \mathcal{N}_m^\diamond$, the learning process will stop as the PUs have reached the ESS quota vectors.

However, the value of each ESS quota k^*_{mn} may have a fractional part and thus unrealizable for the PUs that sell each channel as a whole. Therefore, the PUs need to obtain the BIQ vectors $\tilde{\mathbf{k}}^*_m$, $m = 1, \cdots, M$, by modifying the learning process as follows. Firstly, we round the ESS quotas to the nearest integers and thus $\sum_{n \in \mathcal{N}_m^\diamond} \text{round}(k_{mn}(\tau)) - S_m = \omega$, where round$(\cdot)$ is the nearest integer function. If $\omega \geq 0$, we let $\tilde{k}_{mn}(\tau) = \text{round}(k_{mn}(\tau)) - 1$ for the ω SUs bringing the least quota payoffs while $\text{round}(k_{mn}(\tau)) \neq 0$, and $\tilde{k}_{mn}(\tau) = \text{round}(k_{mn}(\tau))$ for the other SUs. If $\omega < 0$, we let $\tilde{k}_{mn}(\tau) = \text{round}(k_{mn}(\tau)) + 1$ for the ω

SUs bringing the highest quota payoffs, and $\tilde{k}_{mn}(\tau) = \mathrm{round}(k_{mn}(\tau))$ for the other SUs. As a result, the channel price $\tilde{p}^*_{mn}$ under BIQ $\tilde{k}^*_{mn}$ is different from ρ^*_m.

The learning process can be described as in Algorithm 1. Note that the BIQs are not necessarily equal to the ESS quotas. Note that Δk_{mn}, $n \in \mathcal{N}^\diamond_m$ may not converge to zero for the learning process to stop. Therefore, we set a maximum number of contract periods τ_{max}.

Algorithm 1 Learning process for PUs without channel reserve.

Initialize: $\mathfrak{S}_m = S_m$, $\tau = 1$, $k_{mn}(1) = \mathfrak{S}_m / \sum_{n'=1}^{N} t_{mn'}$ for $n \in \mathcal{N}^\diamond_m$, and $k_{mn}(1) = 0$ for $n \notin \mathcal{N}^\diamond_m$, $n \in \mathcal{N}$. Meanwhile, channel capacity u and budget price ψ are accessible for all the PUs.
Do
The PUs share quota information with each other.
Calculate $p_{mn}(k_{mn}(\tau), k_{-mn}(\tau))$, $n \in \mathcal{N}^\diamond_m$, by (2.12).
The PUs inform the SUs of quota and price information.
Complete channel transaction.
Calculate $\bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau))$, $n \in \mathcal{N}^\diamond_m$, and $\bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau))$ by (2.16) and (2.17) respectively.
Calculate Δk_{mn}, $n \in \mathcal{N}^\diamond_m$, by (2.15).
Let $k_{mn}(\tau+1) = k_{mn}(\tau) + \Delta k_{mn}(\tau)$, $n \in \mathcal{N}^\diamond_m$.
Let $\omega = \sum_{n \in \mathcal{N}^\diamond_m} \mathrm{round}(k_{mn}(\tau)) - S_m$.
if $\omega \geq 0$ **then**
    $\tilde{k}_{mn}(\tau) = \mathrm{round}(k_{mn}(\tau)) - 1$ for the ω SUs bringing the least quota payoffs while $\mathrm{round}(k_{mn}(\tau)) \neq 0$, and $\tilde{k}_{mn}(\tau) = \mathrm{round}(k_{mn}(\tau))$ for the other SUs.
else
    $\tilde{k}_{mn}(\tau) = \mathrm{round}(k_{mn}(\tau)) + 1$ for the ω SUs bringing the highest quota payoffs, and $\tilde{k}_{mn}(\tau) = \mathrm{round}(k_{mn}(\tau))$ for the other SUs.
end if
$\tau = \tau + 1$.
Repeat do until $\Delta k_{mn} = 0$ for $n \in \mathcal{N}^\diamond_m$ or τ_{max} is reached.

In the evolutionary procedure, we let the PUs share quota information with each other to reduce the complexity and overhead of obtaining channel prices to sell allocated quotas. However, channel prices can be obtained by designing another learning process without knowing the quota information of other PUs. In a contract period, the SUs will continuously feed demands back to the PUs for given channel prices till demands equal quotas. Then we obtain the channel prices to sell allocated quotas. We omit this learning process due to space limit.

2.3.5 ESS Quotas With Channel Reserve

To achieve a higher payoff, the mth PU may sell some but not all of its channels, i.e., $\sum_{n\in\mathcal{N}_m^\diamond} k_{mn} = S'_m \leq S_m$. Similar to the scenario of selling channels without channel reserve, we denote $\mathbf{k}_m^*[S'_m]$ as the ESS quota vector constrained by $\sum_{n\in\mathcal{N}_m^\diamond} k_{mn} = S'_m$. We can determine $\mathbf{k}_m^*[S'_m]$ for the following two cases:

1. The mth PU has only one SU to sell channels, i.e., $\sum_{n=1}^{N} t_{mn} = 1$.

   In this case, $k_{mn}^*[S'_m] = S'_m$ when $n \in \mathcal{N}_m^\diamond$, and $k_{mn}^*[S'_m] = 0$ when $n \notin \mathcal{N}_m^\diamond$.

2. The mth PU has at least two SUs to sell channels, i.e., $\sum_{n=1}^{N} t_{mn} \geq 2$.

   In this case, $k_{mn}^*[S'_m] \in (0, S'_m)$ satisfies

$$\bar{\pi}_{mn}(k_{mn}^*[S'_m], k_{-mn}^*[S'_{-m}]) = \bar{\pi}_m(\mathbf{k}_m^*[S'_m], \mathbf{k}_{-m}^*[S'_{-m}])$$

   when $n \in \mathcal{N}_m^\diamond$, where S'_{-m} are the channel supplies of the PUs other than the mth PU. And $k_{mn}^*[S'_m] = 0$ when $n \notin \mathcal{N}_m^\diamond$.

Similar to (2.18) and (2.19), $\mathbf{k}_m^*[S'_m]$ and the uniform price $\rho_m^*[S'_m]$ to obtain $\mathbf{k}_m^*[S'_m]$ can be found by solving

$$\sum_{n\in\mathcal{N}_m^\diamond} k_{mn}^*[S'_m] = S'_m \tag{2.21}$$

and

$$d_{mn}(\rho_1^*[S'_1], \cdots, \rho_m^*[S'_m], \cdots, \rho_M^*[S'_M]) = k_{mn}^*[S'_m] \tag{2.22}$$

for all $n \in \mathcal{N}_m^\diamond$.

To further increase the payoff, the mth PU needs to find the optimal supply S'^*_m according to the optimal supplies of the other PUs S'^*_{-m}. In the scenario of selling channels with reserve, we can anticipate that the optimal payoff of each PU will be obtained by an ESS quota vector. In other words, the mth PU should adopt a single-price policy to search for S'^*_m.

Proposition 2.3. *If the mth PU sets the same price ρ_m for the SUs in its licensed area, $\frac{\partial \pi_m}{\partial \rho_m} = 0$ if and only if $\rho_m = \frac{\zeta_1}{\zeta_1 - 1} C_T$.*

Proof. See Appendix A.5. □

According to Proposition 2.3, $\rho_m = \frac{\zeta_1}{\zeta_1 - 1} C_T$ is a critical point for π_m. Actually, $\rho_m = \frac{\zeta_1}{\zeta_1 - 1} C_T$ is the global maximizer for π_m. If the optimal supplies of the other PUs S'^*_{-m} are set, let the total channel supply needed be S''_m when $\rho_m = \frac{\zeta_1}{\zeta_1 - 1} C_T$. Therefore, $S'^*_m = \min(S''_m, S_m)$. Accordingly, the optimal stable payoff of the mth PU is $\pi_m(\mathbf{k}^*_m[S'^*_m], \mathbf{k}^*_{-m}[S'^*_{-m}])$.

2.3.6 Learning Process For PUs With Channel Reserve

In the scenario of selling channels with reserve, the PUs need to search for the ESS quota vectors repeatedly for selected supplies of the PUs. Therefore, it is more difficult and time consuming for the mth PU to find the optimal supply S'^*_m and the ESS quota vector $\mathbf{k}^*_m[S'^*_m]$, compared with that in the scenario of selling channels without reserve.

To reach the optimal supplies $(S'^*_1, \cdots, S'^*_M)$, we propose a learning process for the PUs that sell channels with reserve. Without loss of generality, we assume that the PUs are unsophisticated users without the knowledge that $\rho^*_m[S'^*_m]$ may be equal to $\frac{\zeta_1}{\zeta_1 - 1} C_T$. The learning process consists of several iterations. The iteration sequence is indexed by β and the supplies of the PUs can be adjusted once in each iteration. Then the ESS quota vectors and the payoff levels of the PUs can be obtained by Algorithm 1 based on the supplies of the PUs. Afterwards, based on the variations of the payoff levels, the PUs can adjust the supplies in the next iteration. By applying the secant line method [58], the supply change of the mth PU is

$$\begin{aligned} \Delta S'_m(\beta) = & \phi_m \cdot \frac{\bar{\pi}_m(\mathbf{k}^*_m[S'_m(\beta)], \mathbf{k}^*_{-m}[S'_{-m}(\beta)])}{S'_m(\beta) - S'_m(\beta - 1)} \\ & - \phi_m \cdot \frac{\bar{\pi}_m(\mathbf{k}^*_m[S'_m(\beta - 1)], \mathbf{k}^*_{-m}[S'_{-m}(\beta - 1)])}{S'_m(\beta) - S'_m(\beta - 1)}, \end{aligned} \tag{2.23}$$

where ϕ_m is a constant that defines the supply variation step of the mth PU. When $|\Delta S'_m| < \sigma$ for $m = 1, \cdots, M$, where σ is a small tolerance, the learning process will stop as the PUs obtain the optimal supplies $(S_1'^*, \cdots, S_M'^*)$.

The learning process can be described as in Algorithm 2. To reduce the complexity, we assume the initial supply of the mth PU is S_m and the mth PU searches for $S_m'^*$ by decreasing channel supply. The maximum number of iterations is β_{max}.

Algorithm 2 Learning process for PUs with channel reserve.

Initialize: $S_m'(1) = S_m$, $S_m'(2) = S_m'(1) - 1$,
$\bar{\pi}_m(\mathbf{k}_m^*[S_m'(1)], \mathbf{k}_{-m}^*[S_{-m}'(1)]) = \bar{\pi}_m(\mathbf{k}_m^*, \mathbf{k}_{-m}^*)$,
and $\beta = 1$.
Do
Calculate $\bar{\pi}_m(\mathbf{k}_m^*[S_m'(\beta)], \mathbf{k}_{-m}^*[S_{-m}'(\beta)])$ according to Algorithm 1 by letting $\mathfrak{S}_m = S_m'(\beta)$.
Calculate $\Delta S_m'(\beta)$ by (2.23).
Let $S_m'(\beta+1) = S_m'(\beta) + \Delta S_m'(\beta)$.
if $S_m'(\beta+1) > S_m$ **then**
    $S_m'(\beta+1) = S_m$.
end if
$\beta = \beta + 1$.
Repeat do until $|S_m'(\beta+1) - S_m'(\beta)| < \sigma$ or β_{max} is reached.

2.4 Evolutionary Games of PUs with Homogeneous SUs

SUs using the same frequency band often have identical budget prices. In other words, the SUs are homogeneous in our model. To simplify the evolutionary procedure for homogeneous SUs, we introduce the utility zones, based on which we group the SUs and apply evolutionary games to groups of SUs.

2.4.1 Utility Zone

For homogeneous SUs, we set several utility zones for each PU. The SUs in each utility zone will obtain similar channel capacities when buying channels from the corresponding PU.

We divide $\mathbb{A}_m$ into Z_m utility zones. For the zth utility zone of the mth PU, the estimated channel capacity is $\tilde{\mathbf{u}}_{m,z}$, and $\tilde{\mathbf{u}}_{m,z+1} - \tilde{\mathbf{u}}_{m,z} = \frac{1}{Z_m} u_{m,max}$, where $u_{m,max}$ is the maximum possible channel capacity inside the licensed area $\mathbb{A}_m$. For the nth SU, if $u_{mn} \in (\frac{z-1}{Z_m} u_{m,max}, \frac{z}{Z_m} u_{m,max}]$, we can identify that the nth SU is in the zth utility zone of the mth PU with $\tilde{\mathbf{u}}_{m,z} = \frac{2z-1}{2Z_m} u_{m,max}$.

2.4.2 Grouping Mechanism

The SUs with the same PUs as their suppliers, and in the same utility zone of each supplying PU, belong to one group. In each group, the SUs have identical selection preference on suppliers and therefore can be treated as a single unit.

Suppose there are G groups of SUs in the spectrum sharing system. Similar to the nonhomogeneous scenario, we use $\hat{t}_{mg} = 1$ to denote the acceptance and $\hat{t}_{mg} = 0$ the denial of a transaction that group g offers to buy channels from the mth PU. We further define $\hat{p}_{mg}$ as the channel price at which the mth PU expects to sell to group g, $\hat{d}_{mg}$ as the number of channels that group g offers to buy from the mth PU after obtaining the information of channel prices, $\hat{b}_{mg}$ as the number of channels that group g actually buys from the mth PU, and $\hat{k}_{mg}$ as the quota that the mth PU sets for group g. We have

$$\hat{d}_{mg} = \hat{D}_g \frac{\hat{t}_{mg} e^{\hat{U}_{mg}}}{\sum_{i=1}^{M} \hat{t}_{ig} e^{\hat{U}_{ig}}} e^{\zeta_0 \ln(\hat{\psi}_g) - \zeta_1 \ln(\hat{p}_{mg})}, \tag{2.24}$$

where $\hat{D}_g = \sum_{n \in \mathcal{N}_g} D_n$, $\mathcal{N}_g$ is the set of SU indices in group g, $\hat{\psi}_g$ is the budget price of the homogeneous SUs in group g, and $\hat{U}_{mg}$ is the channel utility obtained by an SU in group g if the SU buys a channel from the mth PU. Here we suppose that the SUs in group g obtain identical channel capacity if the SUs buy channels from the same PU. Specifically,

$$\hat{U}_{mg} = \tilde{\mathfrak{u}}_{m,z_g} + \gamma(\hat{\psi}_g - \hat{p}_{mg}), \tag{2.25}$$

where z_g reveals the utility zone index in which group g is located. Moreover, the payoff of the mth PU in the quota transaction process is

$$\hat{\pi}_m = \sum_{g \in \mathcal{G}_m^{\diamond}} (\hat{p}_{mg} - C_T)\hat{k}_{mg} - S_m C_m, \tag{2.26}$$

where $\mathcal{G}_m^{\diamond}$ is the set of group indices such that $\hat{t}_{mg} \neq 0$.

We can similarly apply the quota transaction process to the spectrum sharing system with M PUs and G SU groups. The replicator dynamics is

$$\begin{aligned}\Delta\hat{k}_{mg}(\tau) =& \hat{\mu}_m \hat{k}_{mg}(\tau)\bar{\hat{\pi}}_{mg}(\hat{k}_{mg}(\tau), \hat{k}_{-mg}(\tau)) \\ &- \hat{\mu}_m \hat{k}_{mg}(\tau)\bar{\hat{\pi}}_m(\hat{\mathbf{k}}_m(\tau), \hat{\mathbf{k}}_{-m}(\tau))\end{aligned} \tag{2.27}$$

for all $g \in \mathcal{G}_m^\diamond$, where $\Delta\hat{k}_{mg}$, $\hat{\mu}_m$, $\bar{\hat{\pi}}_{mg}$, and $\bar{\hat{\pi}}_m$ are the counterparts of Δk_{mg}, μ_m, $\bar{\pi}_{mg}$, and $\bar{\pi}_m$ in (2.15). As a result, when the PUs sell channels without reserve, the ESS quotas in our grouping mechanism can be obtained by following the replicator dynamics defined in (2.27), and the BIQs in our grouping mechanism can be obtained by the learning process described by Algorithm 1. In addition, when the PUs sell channels with reserve to achieve optimal supplies, the ESS quotas that leads to the optimal payoff for the mth PU in our grouping mechanism is $\hat{\mathbf{k}}_m^*[\hat{S}_m'^*]$, and the BIQs that leads to the optimal realizable payoff for the mth PU in our grouping mechanism can be obtained by the learning process described by Algorithm 2.

Compared with the quota transaction process and the learning process in the scenario of nonhomogeneous SUs, the computational complexity in our grouping mechanism is reduced. In each contract period or iteration, only $M \times G$ variables need to be determined instead of $M \times N$ variables. On the other hand, the larger the number of utility zones is, the more accurate the estimated channel capacity in a utility zone will be. With the increase of the number of utility zones, the channel allocation results of the grouping mechanism will approach the results of the individual analysis.

2.5 Simulation Results

2.5.1 Parameters

We study a system with 3 PUs. The licensed blocks of these PUs are rectangular, similar to the geographic licensing schemes of the Federal Communications Commission (FCC) [59, 60]. In this system, homogeneous SUs whose budget price is 40 per channel are uniformly distributed. When channel price equals the budget price, each SU has a demand

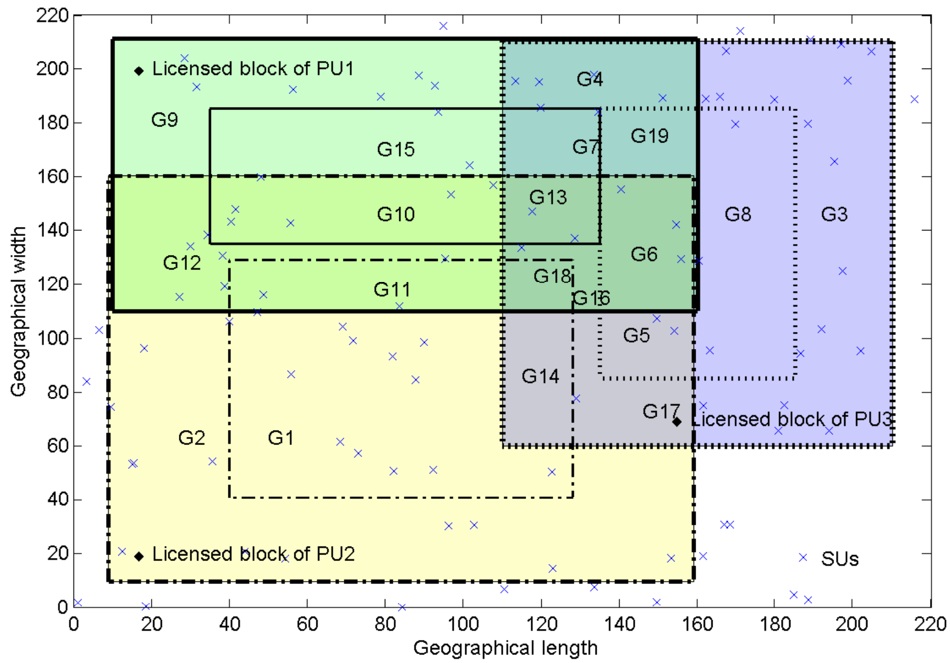

Figure 2.2. Illustration of the system in our simulation and the utility zones in grouping mechanism. $Z_1 = Z_2 = Z_3 = 2$.

of 4 channels. Figure 2.2 gives an illustration of the system in our simulation.

We suppose that the channels of the PUs are Rayleigh fading channels with additive white Gaussian noise (AWGN) [61], and the average power over each channel $\bar{\Gamma}_{Thr}$ at the licensed block boundary is 2×10^{-10} W under the interference temperature constraint [62]. According to the two-ray model [63],

$$\mathrm{SNR}_{mn} = |\omega|^2 \frac{\bar{\Gamma}_{Thr}}{\xi_n \varepsilon_0 W_m} r_{mn}^{\alpha}, \tag{2.28}$$

where ω is the channel gain that follows complex normal distribution $\mathcal{CN}(0,1)$ [61], r_{mn} is the minimum distance from (x_n, y_n) to the boundary of $\mathbb{A}_m$, α is the path loss exponent, ξ_n is a constant related to the nth SU's antenna, and $\varepsilon_0/2$ is the power spectral density of AWGN. In our simulation, we let $\alpha = 4$ to simulate typical urban areas [63], $\xi_n = 10^6$, $\varepsilon_0/2 = 3 \times 10^{-18}$ W/Hz, and the channel bandwidth $W_m = 1$ MHz. Under these assumptions, we illustrate the grouping mechanism when each PU has 2 utility zones in

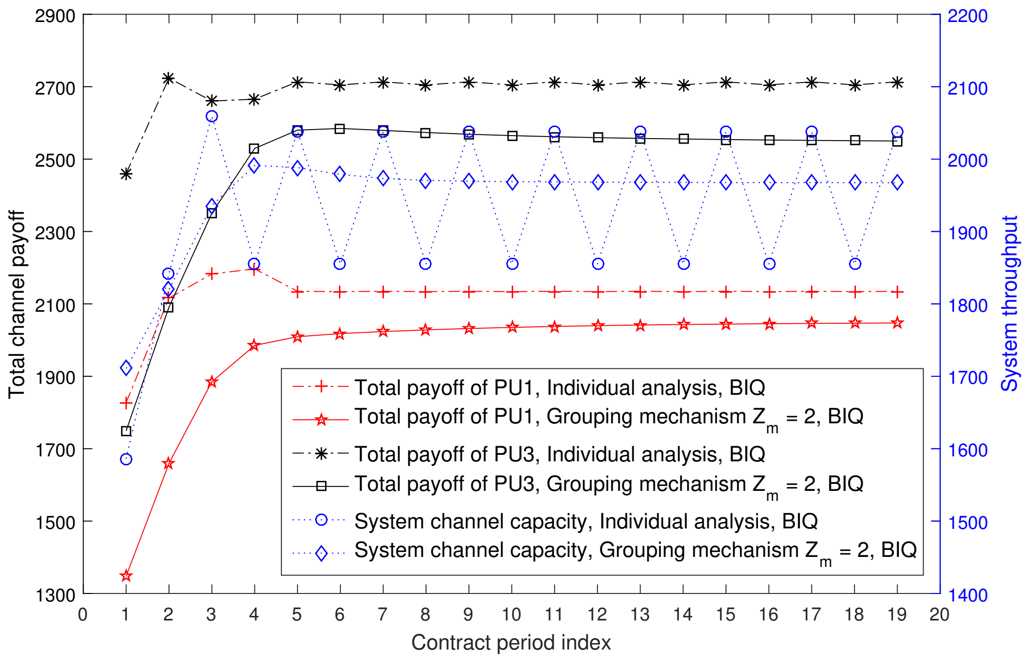

Figure 2.3. Total channel payoffs for selected 2 PUs and system throughput in individual analysis and grouping mechanism when $N = 100$.

Figure 2.2.

Meanwhile, we set $\delta = 10^{-5}$ and $\gamma = 0.1$ to make the channel capacity comparable to the price-income sensitivity. In addition, we assume that channel demand is $10D_n$ when channel price is $\frac{1}{4}\psi_n$, and channel demand is $\frac{1}{4}D_n$ when channel price is $\frac{9}{4}\psi_n$ in the DLD model. Under this assumption, $\zeta_0 = \zeta_1 = 1.66$.

2.5.2 Selling Channels Without Reserve

In this scenario, each PU has a supply of 50 idle channels, and the cost $C_m + C_T = 10$. We set μ_m for the individual analysis as

$$\begin{aligned}\frac{1}{\mu_m} &= \max_{n \in \mathcal{N}_m^\diamond} \bar{\pi}_{mn}(k_{mn}(1), k_{-mn}(1)) \\ &\quad - \min_{n \in \mathcal{N}_m^\diamond} \bar{\pi}_{mn}(k_{mn}(1), k_{-mn}(1)),\end{aligned} \tag{2.29}$$

and $\hat{\mu}_m$ similarly in grouping mechanism.

Firstly, we suppose there are 100 SUs in the system. Figure 2.3 shows the total channel payoffs in 20 contract periods for selected 2 PUs in both individual analysis and grouping mechanism. Meanwhile, Figure 2.3 shows the channel capacities obtained by all the SUs in the system in the same contract periods. The total channel payoffs in the searches for BIQs in the individual analysis fluctuate in tiny amounts after about 5 contract periods. These fluctuations come from the integerization of the ESS quotas. In contrast, the total channel payoffs in the grouping mechanism when each PU has 2 utility zones converge to constant values in about 10 contract periods. Due to a relatively large quota in the grouping mechanism, integerization of the ESS quotas does not affect a large portion of quotas. In addition, the total channel payoffs of the BIQs in the grouping mechanism are close to those in the individual analysis. Therefore, our grouping mechanism is a good way to simplify the individual analysis. Additionally, the payoff variation ranges after several contract periods are very small, indicating that quota changes do not bring significant payoff increases. If we consider the cost of switching channels of the SUs, fewer contract periods are needed for the convergence process to stop.

From Figure 2.3, the system throughput obtained by the SUs also increases to a higher level in our learning process of maximizing the payoffs of the PUs. Therefore, the quota transaction process together with the evolutionary procedure does increase the spectrum efficiency of the whole system. Note that the system throughput fluctuation in the searches for BIQs in the individual analysis also comes from the integerization of the ESS quotas.

To further study the reliability of our method, we change the number of SUs from 10 to 200. And we calculate the ESS quotas in the individual analysis and the BIQs in both individual analysis and grouping mechanism for 100 different SU distributions by following our learning processes. Figure 2.4 shows the average total channel payoffs of PU1. Meanwhile, we use the total channel payoffs of the ESS quotas in the individual analysis as our baseline to study the accuracy of the payoffs of the BIQs. Figure 2.4 also shows the normalized standard deviation (NSTD) of payoff differences of the ESS quotas in the individual analysis and the BIQs in both individual analysis and grouping mechanism.

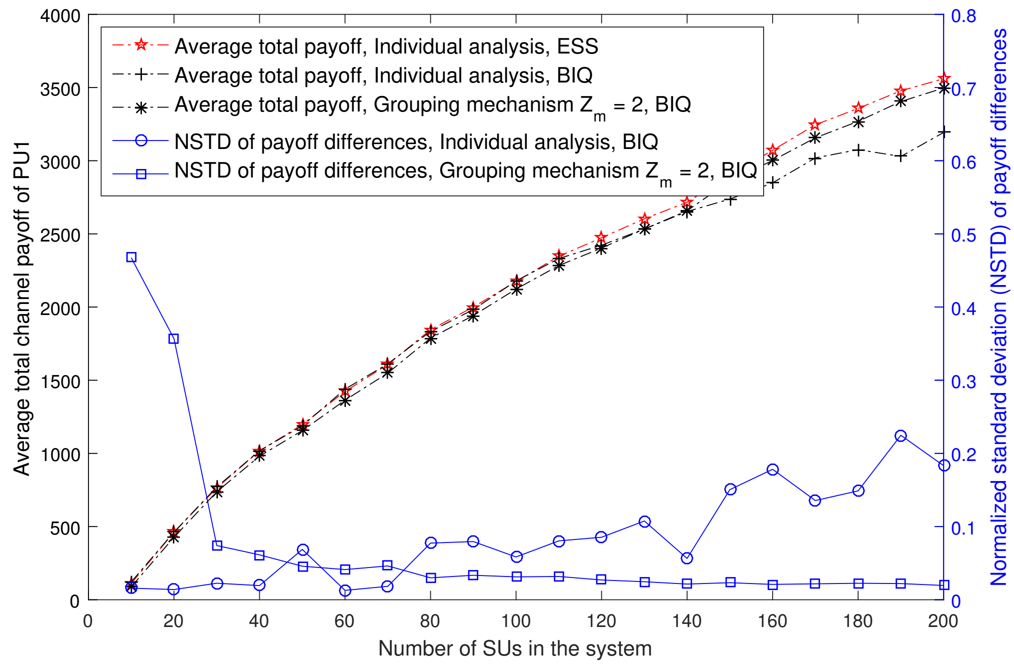

Figure 2.4. Average total channel payoff of PU1, and normalized standard deviation (NSTD) of the payoff differences between BIQs and ESS quotas.

From Figure 2.4, the average total channel payoffs of the BIQs in the individual analysis are close to those of the ESS quotas and the NSTD of payoff differences is relatively small if the number of SUs is under 140. Meanwhile, the average total channel payoffs of the BIQs in the grouping mechanism are close to those of the ESS quotas for any number of SUs, but the NSTD of payoff differences are relatively large if the number of SUs is under 30. Therefore, BIQs in the individual analysis are better choices for a small number of SUs, while BIQs in the grouping mechanism are better choices for a large number of SUs.

2.5.3 Selling Channels With Reserve

In this scenario, we analyze the trends of the total channel payoffs of the PUs with the increase of their supplies. Suppose there are 100 SUs in the system and let $C_m + C_T = 10$. Changing the supply of each PU from 10 to 910, we calculate the total channel payoffs of each PU by following Algorithm 2. Figure 2.5 shows the total channel payoffs of the ESS quotas in the individual analysis of PU2 for 5 different cost combinations in the scenario of selling channels with reserve. When PU2 only has a small number of channels to sell, the

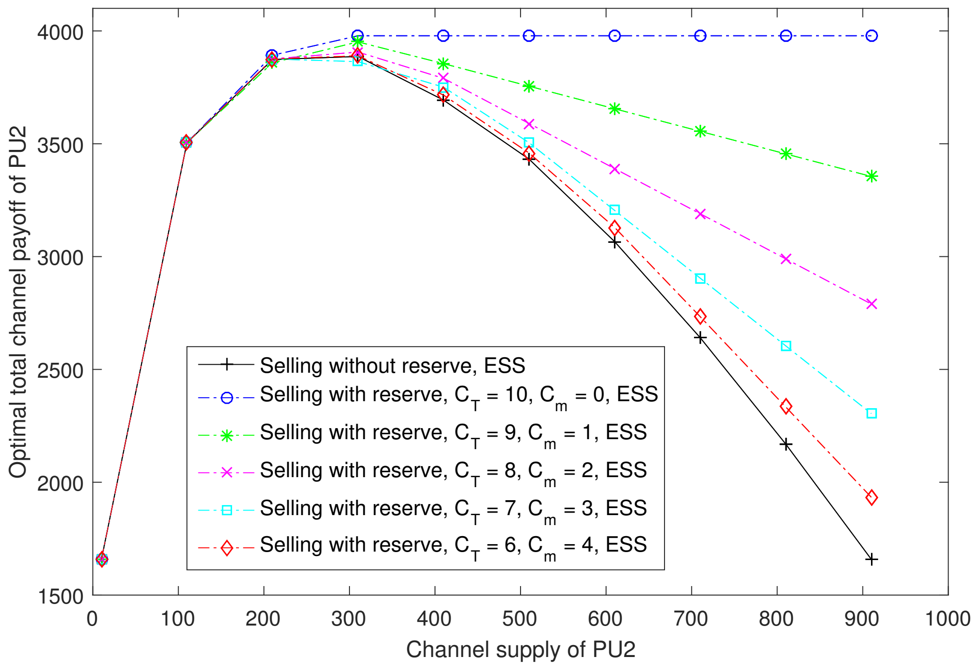

Figure 2.5. Total channel payoff of PU2 in individual analysis when selling channels with reserve and $N = 100$.

total channel payoff in the scenario of selling channels with reserve is the same as that in the scenario of selling channels without reserve. But if we increase the supply continuously, there always exists a point at which PU2 will consider selling some but not all of its channels to make higher profit. And this supply point will increase with the decrease of the channel transaction cost C_T.

2.5.4 Comparison With Centralized PUs

In a centralized system, channel transactions of all PUs are controlled by a central agent to maximize the payoff of the system [64], while non-cooperative PUs only care about their own payoffs. Suppose the parameters of the centralized system are identical to our non-cooperative system, there are 100 SUs in both systems, and let $C_m + C_T = 10$. Figure 2.6 shows the optimal total channel payoff comparison between non-cooperative PUs and centralized PUs when they sell channels without reserve. The optimal total channel payoffs of both systems are identical. In the centralized system, the PUs do not compete with each other. Therefore, the multi-nomial logit $\frac{t_{mn}e^{U_{mn}}}{\sum_{i=1}^{M} t_{in}e^{U_{in}}}$ in (2.5) always

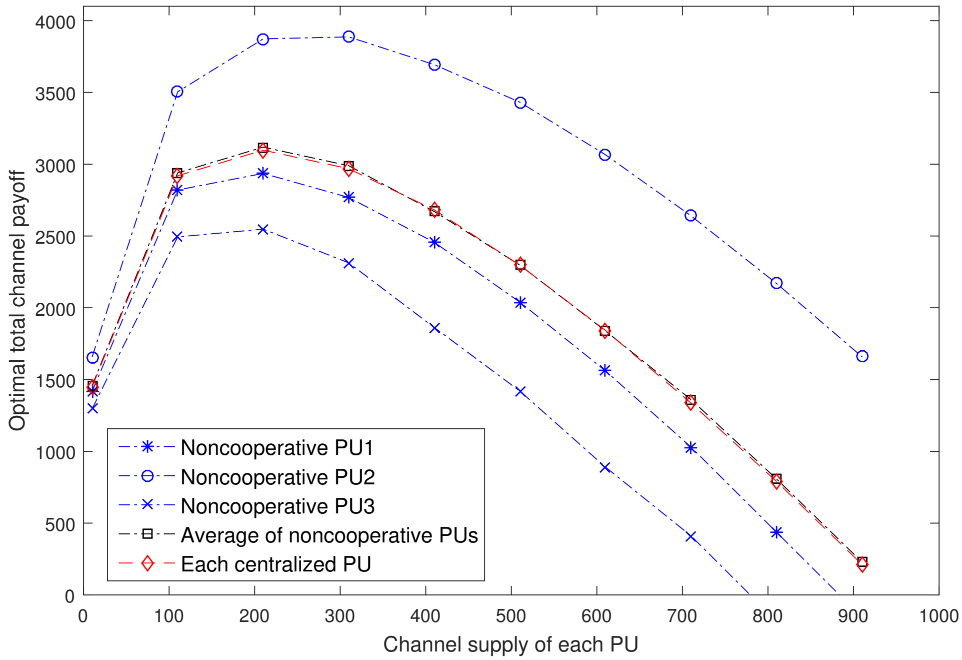

Figure 2.6. Optimal total channel payoff comparison between non-cooperative PUs and centralized PUs in individual analysis when selling channels without reserve and $N = 100$.

equals 1. In this case, the central agent randomly distributes the channels, and is reluctant to distribute the channels in an efficient way, i.e., providing more system throughput with no profit increase. In the non-cooperative system, the competitions of the PUs in different areas provide them incentives to delicately distribute the channels. Therefore, the optimal total channel payoffs of individual PUs differ from each other based on channel utilities. Furthermore, the competitiveness leads to a lower efficiency in terms of iteration steps used to achieve the optimal payoff, but at the same time increases the system throughput in comparison with the average system throughput of random channel distribution.

2.6 Conclusions

In this chapter, we consider a spectrum sharing system in which the licensed areas of the PUs and the locations of the SUs play important roles, and we seek to maximize the payoffs of the PUs in such a system. For each PU, the geographic information not only distinguishes the eligible SUs to sell channels, but also determines the potential competitors. In our model, the PUs adopt a multi-price policy and have the pricing power. We employed a generalized utility model in which both the channel capacities and the channel prices in the forms of price-income sensitivity functions are considered. Meanwhile, we established a price-elastic demand model that incorporates the DLD and MNL models and is applicable to both oligopoly and monopoly markets, two common market forms for spectrum sharing. To solve the problem, we proposed a unique transaction process and discussed two different scenarios. In the scenario of selling channels without reserve, we proved the existence and the uniqueness of the ESS quota vector of each PU by applying an evolutionary procedure defined as replicator dynamics. In the scenario of selling channels with reserve, we predicted the channel prices for the PUs to render the optimal supplies of the PUs. Meanwhile, we designed two learning processes for both scenarios. Furthermore, we introduced a grouping mechanism for homogeneous SUs to simplify the process. In our simulation, we verified the effectiveness of the learning processes and the efficiency of our spectrum sharing scheme.

Chapter 3
On-Demand Receiver-Centric Channel Allocation for Spatial Spectrum Reuse

3.1 Introduction

When maximizing the spectrum efficiency in spatial spectrum reuse under different settings, we often ignore whether channels are allocated to the users in need of them. In other words, supply and demand relationship is another factor that affects the valuation of a channel, which is omitted in the existing work on spatial spectrum reuse. In this chapter, we propose a new mechanism for spatial spectrum reuse by considering flexible valuations of channels, which manifests supply and demand relationship. In other words, the flexible channel valuations of SUs reflect the degree of desire for channels and comply with the marginal value theory [65, 66]. Different from conventional spatial spectrum reuse mechanisms solely devoted to maximizing spectrum efficiency, our goal is to allocate channels to the SUs who treasure them and to increase spectrum utilization. In this way, the social welfare, i.e., the accumulated valuation of channels, is maximized. Not only are SUs benefited from our mechanism, but also PUs can increase their payoffs by estimating the behaviors of SUs more accurately and pricing the channels more appropriately. When a PU has multiple channels for SUs to reuse, the simple assumption that an SU continues to buy channels at a fixed price neglects the actual supply and demand relationship. As a result, channel transactions will be far from the expectation and payoffs of the PUs will be jeopardized. In practice, the channel valuation of an SU usually decreases when the number of obtained channels increases. Therefore, we need to find a method to increase the social welfare in consideration of flexible channel valuations, which helps the PUs to better estimate the behaviors of the SUs.

In our model, we design a receiver-centric spatial spectrum reuse mechanism, in which

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the co-channel interference suffered by the receivers is considered. Meanwhile, the valuations of channels of an SU are decreasing when the number of supplied channels increases, which reflects the variation of marginal values. To maximize the social welfare, we consider the Vickrey-Clarke-Groves (VCG) auction process [67,68] that sells multiple identical items and leads to a socially optimal outcome. However, the application of the VCG auction to our model needs to reflect the restriction on avoiding co-channel interference when the PU allocates channels to the SUs. To solve the problem, we first group the SUs into several maximal independent groups (MIGs) by adding an interference control feature into the Bron-Kerbosch algorithm [69]. Then we introduce the VCG auction for MIGs. We prove that truthful bidding, i.e., revealing the truthful valuations on channels, is the optimal strategy for the SUs, such that the MIGs are bidding truthfully in the VCG auction for MIGs. To determine the truthful bids for the MIGs, we design a step-by-step decision process to help the MIGs to bid truthfully and the PU to allocate channels in a socially optimal manner. Meanwhile, the VCG style prices charged to the MIGs can be determined. Furthermore, we use three approximation methods to approach the optimal channel allocation.

The rest of the chapter is organized as follows. In Section 3.2, we introduce our model of the spatial spectrum sharing system and the flexible channel valuations of SUs. In Section 3.3, we introduce the constrained VCG auction for SUs and the VCG auction for MIGs. We prove that truthful bidding is the optimal strategy for the SUs in the VCG auction for MIGs, and design a decision process to implement truthful bidding for the MIGs and VCG style pricing for the PU. In Section 3.4, we approach the optimal channel allocation using a greedy algorithm, Dijkstra's algorithm, and batch allocation. In Section 3.5 we present our simulation results. Finally, we draw our conclusions in Section 3.6.

3.2 System Model

3.2.1 System Setup

Suppose that there is a PU and M SUs in the spatial spectrum sharing system. The PU has N non-overlapping idle channels to share with the SUs. Each SU consists of a transmitter and a receiver. For SU m, $m = 1, 2, \cdots, M$, its transmitter and receiver are denoted as T_m and R_m, respectively. The transmit power of T_m is p_m, which is open information for the PU. In this system, channels are tradable items and the SUs can only use channels bought from the PU. To make our model more realistic, we consider the co-channel interference suffered by the receivers rather than the transmitters to mitigate the hidden/exposed node problem, such that the channel allocation is receiver-centric. In addition, the randomly located transmitter-receiver pairs form a geographical non-symmetric network.

We consider the scenario when the spectrum resource is limited, i.e., $M \gg N$. To increase its payoff, the PU will share a channel with several SUs. On the other hand, an SU benefits from using channels, and pays the PU in return. We use $e_m^{(n)} = 1$ to denote that SU m is authorized to use channel n and $e_m^{(n)} = 0$ otherwise. In this way, we form an $M \times N$ transaction matrix $\mathbf{E}$. Specifically,

$$\mathbf{E} = \begin{bmatrix} e_1^{(1)} & \cdots & e_1^{(N)} \\ \vdots & \ddots & \vdots \\ e_M^{(1)} & \cdots & e_M^{(N)} \end{bmatrix}. \tag{3.1}$$

3.2.2 Flexible Channel Valuation

We assume that the SUs are not sensitive to the achievable rate of a channel as long as the achievable rate is larger than a threshold for an established quality of service (QoS) requirement. Instead of competing for a channel with high interference, an SU uses a more reliable way to acquire a higher data rate, that is, to buy more channels from the PU. As a result, the achievable rate of a channel does not affect the channel valuation of an SU and channels are identical items for SUs. However, the valuation for the next channel decreases according to the marginal value theory. In other words, as the tension between the channel supply and the expectation on data rate eases, the willingness of an SU to pay for an additional channel diminishes since the incremental benefit of an additional channel decreases. Let the valuation of the ith channel of SU m be $\mu_m(i)$, and $\mu_m(i) > \mu_m(i+1)$ due to such downward-sloping channel valuations. Therefore, the total channel valuation of SU m for its l_m channels is

$$\eta_m(l_m) = \sum_{i=1}^{l_m} \mu_m(i), \tag{3.2}$$

where $l_m = \sum_{n=1}^{N} e_m^{(n)}$.

3.2.3 Problem Formulation

We use $\lambda_m^{l_m}$ to denote the price charged to SU m for using l_m channels, π to denote the PU's payoff, and $\varpi_m^{l_m}$ to denote the payoff of SU m using l_m channels. Apparently, $\pi = \sum_{m=1}^{M} \lambda_m^{l_m}$ and $\varpi_m^{l_m} = \eta_m(l_m) - \lambda_m^{l_m}$. Therefore, the payoff of all the SUs is $\varpi = \sum_{m=1}^{M}(\eta_m(l_m) - \lambda_m^{l_m})$ and the social welfare of the system is $\pi + \varpi = \sum_{m=1}^{M} \eta_m(l_m)$.

In this chapter, our goal is to maximize the social welfare in the system without violating the interference limit. Specifically, we need to find an optimal transaction matrix $\mathbf{E}^*$:

$$\max_{\mathbf{E}} \quad (\pi + \varpi) = \sum_{m=1}^{M} \eta_m(l_m^*) \tag{3.3}$$

s.t.

$$\sum_{m' \neq m} e_{m'}^{(n)} p_{m'} \left| h_{m',m} \right|^2 < p_{thr}, \tag{3.4}$$

for $m = 1, \cdots, M$, $n = 1, \cdots, N$, where l_m^* is the number of channels obtained by SU m and $l_m^* = \sum_{n=1}^{N} e_m^{(n)*}$, $|h_{m',m}|^2$ is the channel gain between $T_{m'}$ and R_m, and p_{thr} is the maximum allowable interference power at the receiver. The constraint (4.6) sets a threshold for the QoS requirement of a channel, i.e., for SU m using channel n, the interference from the other SUs using channel n should be under p_{thr}.

3.3 VCG Auctions and Truthful Bidding

To maximize the social welfare, we need to find a proper channel allocation mechanism, in which the PU can avoid co-channel interference when allocating channels and the SUs reveal their truthful channel valuations. It is known that a VCG auction can maximize the social welfare by encouraging bidders to submit their bids truthfully [33]. In this section, we introduce the constrained VCG auction for SUs and the VCG auction for MIGs. To maximize the social welfare, we discuss the truthfulness of the bids of the SUs in the VCG auction for MIGs. In addition, we design a decision process to implement truthful bidding and VCG style pricing for the PU.

3.3.1 Constrained VCG Auction for SUs

In our model, we suppose that channel transactions occur according to a constrained VCG auction for SUs held by the PU, which is a sealed-bid process for selling multiple items. In other words, each SU submits its bids independently, without the knowledge of bids of other SUs. However, different from the conventional VCG auction, the PU cannot simply select SUs who hold the highest bids to sell channels in the constrained VCG auction for SUs. Co-channel interference has to be considered during the selection. In the constrained VCG auction process, the set of auction items is $\mathbf{N} = \{1, \cdots, N\}$ and the set of bidders is $\mathbf{M} = \{1, \cdots, M\}$. The main steps of the constrained VCG auction for SUs in a contract period are as follows.

1. SU m submits its bid vector $\mathbf{b}_m = (b_m(1), \cdots, b_m(N))$ to inform the PU its valuation for the ith channel, $i = 1, \cdots, N$. In a VCG auction, a bid is also the maximum acceptable price that a user is willing to pay. However, the bids are not equal to the charging prices in most circumstances.

2. The PU calculates over all cases and allocates channels in a way that maximizes the total bid of all winners, which equals the social welfare. Meanwhile, the channel allocation has to comply with the interference limit (4.6).

3. The PU charges each winner, i.e.,

$$\lambda_m^{l_m} = V_{\mathbf{M}\setminus\{m\}}^{\mathbf{M}\setminus\{m\}} - V_{\mathbf{M}}^{\mathbf{M}\setminus\{m\}}, \tag{3.5}$$

   where $\lambda_m^{l_m}$ is the price charged to SU m for using l_m channels, $V_{\mathbf{M}}^{\mathbf{M}\setminus\{m\}}$ is the accumulated bids of all winners except SU m when the set of bidders is $\mathbf{M}$, and $V_{\mathbf{M}\setminus\{m\}}^{\mathbf{M}\setminus\{m\}}$ is the accumulated bids of all winners if the set of bidders is $\mathbf{M} \setminus \{m\}$. This charging price to SU m is based on the virtual losses of the other SUs when SU m participates in the constrained VCG auction for SUs. It can also be interpreted as the harm that SU m causes to the other SUs. According to this pricing policy, the more SUs, the higher probabilities that the charging prices approach the bids of the winners.

It is impractical to apply the constrained VCG auction for SUs directly to allocating channels. The PU in this case has to explore all combinations of allocations to the SUs to avoid interference and maximize the social welfare. It is computational expensive and time consuming. The same issue exists in the pricing. The determination of $V_{\mathbf{M}\setminus\{m\}}^{\mathbf{M}\setminus\{m\}}$ requires channel allocation results when the bidder set is $\mathbf{M} \setminus \{m\}$ and thus another round of constrained VCG auction for SUs. In this case, the PU has to calculate $V_{\mathbf{M}\setminus\{m\}}^{\mathbf{M}\setminus\{m\}}$ for every user and every allocation. To simplify the constrained VCG auction for SUs, we introduce MIGs and the VCG auction for MIGs.

3.3.2 Maximal Independent Groups

In our model, the PU needs to select a group of SUs to reuse the same channel. Meanwhile, the PU is responsible to avoid unacceptable co-channel interference when allocating its channels. To maximize the social welfare, the PU needs to select a maximal group of SUs, such that any additional SU added to the selected group will break the interference limit (4.6) at the location of at least one user's receiver.

To find the maximal groups, we first build a directed interference graph $\mathscr{C}(\mathbf{M}, \mathbf{A})$, where $\mathbf{A}$ is the set of edges in $\mathscr{C}$ and $\mathbf{A} = \mathbf{M} \times \mathbf{M}$. The value of the directed edge

$(m' \to m)$ is denoted as $a_{m',m}$, representing the interference to R_m caused by $T_{m'}$ if SUs m and m' use the same channel, i.e., $a_{m',m} = p_{m'} \left| h_{m',m} \right|^2$. We define $\mathbf{I} \subset \mathbf{M}$ as an independent group in terms of interference if $\sum_{m' \in \mathbf{I},\ m' \neq m} a_{m',m} < p_{thr}$ holds for any $m \in \mathbf{I}$. In other words, SUs in $\mathbf{I}$ can use the same channel simultaneously. In addition, we define that $\mathbf{I}$ is a maximal independent group (MIG) if there does not exist any other independent group $\mathbf{I}'$ such that $\mathbf{I} \subset \mathbf{I}'$. Therefore, to find the maximal groups of SUs using the same channel without interference is equivalent to finding the MIGs in $\mathscr{C}$. Note that vertices in an MIG are still connected and MIGs are not maximal independent sets in $\mathscr{C}$ [70].

Algorithm 3 Searching MIGs in $\mathscr{C}$.

Initialize: $\mathbf{X} = \emptyset$, $\mathbf{Y} = \mathbf{M}$, $\mathbf{Z} = \emptyset$, $j = 0$.
MIG-Search($\mathbf{X}, \mathbf{Y}, \mathbf{Z}$):
if $\mathbf{Y} = \emptyset$ and $\mathbf{Z} = \emptyset$ **then**
    $j = j + 1$. $\mathbf{I}_j = \mathbf{X}$.
else
    for $\forall m \in \mathbf{Y}$ **do**
        if $\sum_{m'' \in \mathbf{X} \cup \{m\},\ m'' \neq m'} p_{m''} \left| h_{m'',m'} \right|^2 < p_{thr}$, $\forall m' \in \mathbf{X} \cup \{m\}$. **then**
            MIG-Search($\mathbf{X} \cup \{m\}$, $\mathbf{Y} \cap \mathbf{\Delta}(m)$, $\mathbf{Z} \cap \mathbf{\Delta}(m)$).
            $\mathbf{Y} = \mathbf{Y} \setminus \{m\}$. $\mathbf{Z} = \mathbf{Z} \cup \{m\}$.
        end if
    end for
end if

The algorithm to find all MIGs in $\mathscr{C}$ is described in Algorithm 6. The algorithm adds an interference control feature based on the Bron-Kerbosch algorithm, which was originally developed to search all maximal independent sets in a graph. The worst-case complexity of our modified algorithm is $O(3^{M/3}M)$, while the worst-case complexity of the Bron-Kerbosch algorithm is $O(3^{M/3})$ [71]. In Algorithm 6, $\mathbf{X}$ is a possible MIG, $\mathbf{\Delta}(m) = \{m' | p_{m'} \left| h_{m',m} \right|^2 < p_{thr}\}$, i.e., $T_{m'}$ alone does not violate the interference limit at the location of R_m if $m' \in \mathbf{\Delta}(m)$, $\mathbf{Y}$ contains SUs that each of them does not break the interference limit at the receiver of every SU currently in $\mathbf{X}$, and $\mathbf{Z}$ contains SUs that are already been considered or processed to remove duplicate MIGs. We suppose that there are J MIGs in $\mathscr{C}$, and an MIG is denoted as $\mathbf{I}_j$, $j = 1, \cdots, J$.

As a result, we have a new $J \times N$ transaction matrix $\hat{\mathbf{E}}$ in consideration of the MIGs.

Specifically,

$$\hat{\mathbf{E}} = \begin{bmatrix} \hat{e}_{\mathbf{I}_1}^{(1)} & \cdots & \hat{e}_{\mathbf{I}_1}^{(N)} \\ \vdots & \ddots & \vdots \\ \hat{e}_{\mathbf{I}_J}^{(1)} & \cdots & \hat{e}_{\mathbf{I}_J}^{(N)} \end{bmatrix}, \tag{3.6}$$

where $\hat{e}_{\mathbf{I}_j}^{(n)} = 1$ denotes that the SUs in $\mathbf{I}_j$ are authorized to use channel n, and $\hat{e}_{\mathbf{I}_j}^{(n)} = 0$ otherwise. Obviously, a channel can only be sold to one MIG. As results, we have $\sum_{j=1}^{J} \hat{e}_{\mathbf{I}_j}^{(n)} = 1$ and $\hat{\mathbf{E}}$ is a sparse matrix. In fact, since the possible combinations of the SUs are limited to the MIGs, the channel index is not needed as a parameter to differentiate SU combinations. Therefore, we focus on the number of channels that an MIG obtains, i.e., $\hat{l}_{\mathbf{I}_j} = \sum_{n=1}^{N} \hat{e}_{\mathbf{I}_j}^{(n)}$.

The MIGs simplify our optimization problem in (4.5) and (4.6). To maximize the social welfare, the optimal channel allocation $\hat{\mathbf{l}}^* = (\hat{l}_{\mathbf{I}_1}^*, \cdots, \hat{l}_{\mathbf{I}_J}^*)$ satisfies

$$\max_{\hat{\mathbf{E}}} \quad (\pi + \varpi) = \sum_{j=1}^{J} \hat{\eta}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}^*) \tag{3.7}$$

with no constraint, where $\hat{\eta}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}^*)$ is the channel valuation of $\mathbf{I}_j$ for $\hat{l}_{\mathbf{I}_j}^*$ channels. In fact,

$$\hat{\eta}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}) = \sum_{m \in \mathbf{I}_j} \eta_{m<\mathbf{I}_j>}(\hat{l}_{\mathbf{I}_j}) = \sum_{m \in \mathbf{I}_j} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \mu_{m<\mathbf{I}_j>}(\hat{i}), \tag{3.8}$$

where $\mu_{m<\mathbf{I}_j>}(\hat{i})$ is the channel valuation of SU m for the $\hat{i}$th channel obtained in $\mathbf{I}_j$, and $\eta_{m<\mathbf{I}_j>}(\hat{l}_{\mathbf{I}_j})$ is the channel valuation of SU m for the $\hat{l}_{\mathbf{I}_j}$ channels obtained in $\mathbf{I}_j$.

3.3.3 VCG Auction for MIGs

After determining the MIGs, the PU does not need to estimate interference levels when allocating channels. Therefore, the constrained VCG auction for SUs can be substituted by a standard VCG auction if the set of bidders is $\mathbf{J} = \{\mathbf{I}_1, \cdots, \mathbf{I}_J\}$, while the set of auction items is still $\mathbf{N}$. In other words, an MIG is a bidding entity and a bid of an MIG reflects the accumulated valuations by all the SUs in the MIG. The main steps of the VCG auction for MIGs are as follows.

1. MIG $\mathbf{I}_j$ submits its bid vector $\hat{\mathbf{b}}_{\mathbf{I}_j} = (\hat{b}_{\mathbf{I}_j}(1), \cdots, \hat{b}_{\mathbf{I}_j}(N))$ to inform the PU its valuation for the $\hat{i}$th channel, $\hat{i} = 1, \cdots, N$, where $\hat{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m \in \mathbf{I}_j} b_{m<\mathbf{I}_j>}(\hat{i})$, and $b_{m<\mathbf{I}_j>}(\hat{i})$ is the sub-bid of SU m for the $\hat{i}$th channel obtained in $\mathbf{I}_j$.

2. The PU calculates $(\hat{l}_{\mathbf{I}_1}, \cdots, \hat{l}_{\mathbf{I}_J})$. The PU simply chooses the largest N $\hat{b}_{\mathbf{I}_j}(\hat{i})$ such that $\sum_{j=1}^{J} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \hat{b}_{\mathbf{I}_j}(\hat{i})$ is maximized subject to $\sum_{j=1}^{J} \hat{l}_{\mathbf{I}_j} = N$.

3. The PU charges each MIG, i.e.,

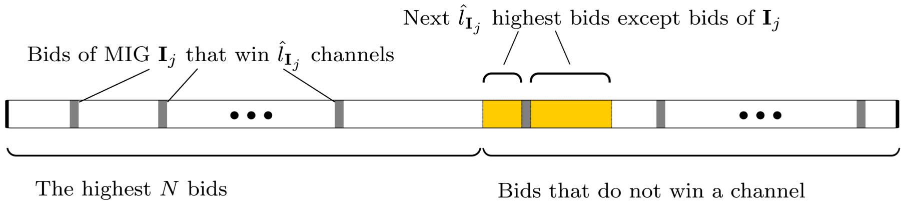
$$\hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}} = \hat{V}_{\mathbf{J}\backslash\{\mathbf{I}_j\}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}, \tag{3.9}$$

where $\hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}}$ is the price that MIG $\mathbf{I}_j$ needs to pay for using the $\hat{l}_{\mathbf{I}_j}$ channels, $\hat{V}_{\mathbf{J}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}$ is the summation of the submitted bids of the SUs who obtain channels except the bids associated with channels obtained by MIG $\mathbf{I}_j$ when the set of bidders is $\mathbf{J}$, and $\hat{V}_{\mathbf{J}\backslash\{\mathbf{I}_j\}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}$ is the summation of the submitted bids of all the SUs who obtain channels when the set of bidders is $\mathbf{J} \setminus \{\mathbf{I}_j\}$. In addition, we regulate that SUs in $\mathbf{I}_j$ will split the cost according to their sub-bids. Specifically,

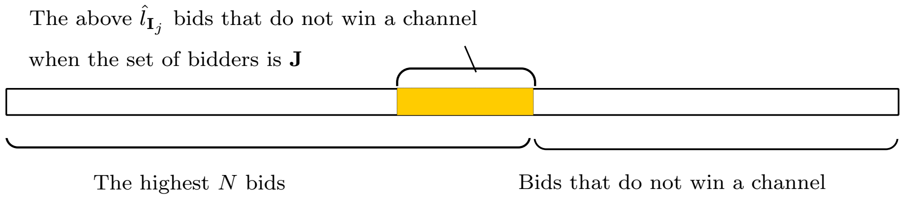
$$\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} = \frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}(\hat{i})}{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \hat{b}_{\mathbf{I}_j}(\hat{i})} \hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}}, \tag{3.10}$$

where $\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}}$ is the split cost of $\mathbf{I}_j$ to SU m.

In the VCG auction for MIGs, the charging prices to the MIGs are straightforward to evaluate according to the submitted bids. To understand the VCG style charging price to $\mathbf{I}_j$, we show the bid rankings when the set of bidders is $\mathbf{J}$ and $\mathbf{J} \setminus \{\mathbf{I}_j\}$ in Figures 3.1a and 3.1b, respectively. As shown in Figure 3.1a, the PU allocates the N channels to the MIGs that hold the highest N bids and $\mathbf{I}_j$ wins $\hat{l}_{\mathbf{I}_j}$ channels. If MIG $\mathbf{I}_j$ does not bid, the PU would reallocate the $\hat{l}_{\mathbf{I}_j}$ channels to the MIGs except $\mathbf{I}_j$ that holds the next $\hat{l}_{\mathbf{I}_j}$ bids, as



(a) Bid ranking when the set of bidders is $\mathbf{J}$.



(b) Bid ranking when the set of bidders is $\mathbf{J} \setminus \{\mathbf{I}_j\}$.

Figure 3.1. Bid ranking in VCG auction for MIGs.

highlighted in both Figures 3.1a and 3.1b. Therefore, the charging price for MIG $\mathbf{I}_j$ to use the $\hat{l}_{\mathbf{I}_j}$ channels can be determined as

$$\hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}} = \hat{V}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}_{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}_{\mathbf{J}} = \sum_{j'\neq j} \sum_{\hat{i}=\hat{l}_{\mathbf{I}_{j'}}+1}^{\hat{l}'_{\mathbf{I}_{j'}}} \hat{b}_{\mathbf{I}_{j'}}(\hat{i}), \tag{3.11}$$

where $(\hat{l}'_{\mathbf{I}_1}, \cdots, \hat{l}'_{\mathbf{I}_J})$ is the channel allocation vector if MIG $\mathbf{I}_j$ does not bid.

With the set of bidders being the MIGs, the computational complexity of the constrained VCG auction for SUs reduces significantly.

3.3.4 Truthful Bidding

It is known that bidding truthfully is Dominant-strategy incentive-compatible (DSIC) in a VCG auction [33, 72]. A strategy that is DSIC indicates that the strategy brings the optimal payoff or at least does not deteriorate the payoff irrespective of the strategies of others [73]. As a result, the social welfare can simply be maximized by choosing the highest bids in the VCG auction for MIGs. We define truthful bidding of an MIG as follows.

Definition 3.1 (Truthful Bidding of an MIG). *MIG $\mathbf{I}_j$ is bidding truthfully for the $\hat{i}$th channel if its bid of the $\hat{i}$th channel equals its channel valuation of the $\hat{i}$th channel, which is the accumulated channel valuations from the SUs in $\mathbf{I}_j$ for the $\hat{i}$th channel, i.e.,*

$\hat{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m\in\mathbf{I}_j} b_{m<\mathbf{I}_j>}(\hat{i}) = \sum_{m\in\mathbf{I}_j} \mu_{m<\mathbf{I}_j>}(\hat{i})$.

Therefore, the optimal bidding strategy of $\mathbf{I}_j$ as an entity is to reveal the truthful accumulated valuations of the SUs in $\mathbf{I}_j$, i.e., $\hat{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m\in\mathbf{I}_j} \mu_{m<\mathbf{I}_j>}(\hat{i})$, for $\hat{i} = 1, \cdots, N$, regardless of the truthfulness of the bids of other MIGs. However, the problem with the VCG auction for MIGs is how to determine $b_{m<\mathbf{I}_j>}(\hat{i})$ associated with $\hat{\mathbf{b}}_{\mathbf{I}_j}$.

In the VCG auction for MIGs, channels are evaluated by the SUs in an MIG, and the channel bid for the $\hat{i}$th channel of an MIG is the summation of the sub-bids of the SUs in the MIG, i.e., $\hat{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m\in\mathbf{I}_j} b_{m<\mathbf{I}_j>}(\hat{i})$. Therefore, the truthful bids of an MIG require truthful sub-bids of the SUs in the MIG. An ideal scenario is based on the assumption that an SU belongs to only one MIG, i.e., $\mathbf{I}_j \cap \mathbf{I}_{j'} = \emptyset$ for any $j \neq j'$, which may hold true for symmetric networks. In such a scenario, $b_{m<\mathbf{I}_j>}(\hat{i}) = b_m(\hat{i})$ for any m. If we can prove that the optimal bidding strategy of an SU in an MIG is still truthful bidding, we can draw the conclusion that $\hat{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m\in\mathbf{I}_j} \mu_m(\hat{i})$, for $\hat{i} = 1, \cdots, N$. In other words, even when $\mathbf{I}_j \cap \mathbf{I}_{j'} = \emptyset$, the truthfulness of the bids of the MIGs are not guaranteed without considering the bidding strategies of the SUs. In a general scenario that an SU belongs to different MIGs, i.e., $b_{m<\mathbf{I}_j>}(\hat{i}) \neq b_m(\hat{i})$, we need to figure out whether truthful bidding is the optimal strategy for an SU in the VCG auction for MIGs. We define truthful bidding of an SU as follows.

Definition 3.2 (Truthful Bidding of an SU). *An SU is bidding truthfully for the $\hat{i}$th channel in an MIG if its bid of the $\hat{i}$th channel equals its valuation of the $\hat{i}$th channel in the MIG, i.e., $b_{m<\mathbf{I}_j>}(\hat{i}) = \mu_{m<\mathbf{I}_j>}(\hat{i})$.*

Proposition 3.1. *An SU does not bid more than its truthful bids in the VCG auction for MIGs.*

Proof. See Appendix B.1. □

Proposition 3.2. *An SU does not bid less than its truthful bids in the VCG auction for MIGs.*

Proof. See Appendix B.2. □

We can anticipate that an SU does not bid more than its truthful bids since the increased bids only add the payment share of the SU. The proof of Proposition 3.1 provides details about this conclusion. On the other hand, SUs seem motivated to bid less than their truthful bids. According to (4.18), an SU can reduce its payment share by bidding less than its truthful bids, and the payoffs of truthful bidding SUs are jeopardized by SUs who bid less than their truthful bids. However, the SUs do not bid less than their truthful bids either, such that they do not risk losing channels according to Proposition 3.2. Therefore, truthful bidding is also the optimal strategy for an SU in the VCG auction for MIGs.

3.3.5 Decision Tree

Although the SUs are inclined to submit truthful sub-bids, i.e., $b_{m<\mathbf{I}_j>}(\hat{i}) = \mu_{m<\mathbf{I}_j>}(\hat{i})$, the problem goes back to the determination of $\mu_{m<\mathbf{I}_j>}(\hat{i})$ when an SU is involved in multiple MIGs. In other words, for an SU, providing truthful sub-bids to multiple MIGs simultaneously is contradict to our definition of flexible channel valuation, defined for the next channel only. In this subsection, we design a step-by-step decision process to provide a mechanism that determines $\mu_{m<\mathbf{I}_j>}(\hat{i})$ such that the MIGs submit truthful channel valuations. Meanwhile, we can find a solution $\hat{\mathbf{l}}^* = (\hat{l}^*_{\mathbf{I}_1}, \cdots, \hat{l}^*_{\mathbf{I}_J})$ to the optimization problem (3.7).

The decision process for channel allocation is composed of N steps. We assume that there is only one channel being sold in each step. Instead of submitting N bids at once in the VCG auction, an MIG only needs to calculate its bid for the next channel. Therefore, SUs in an MIG can update their channel valuations based on the channels that they have obtained before each step of channel allocation. We use $\hat{e}_{\mathbf{I}_j}[k] = 1$ to denote that $\mathbf{I}_j$ obtains a channel in the kth step, and $\hat{e}_{\mathbf{I}_j}[k] = 0$ otherwise. Suppose that the SUs submit their sub-bids truthfully. The bid of $\mathbf{I}_j$ for the first step is

$$\hat{b}_{\mathbf{I}_j}[1] = \sum_{m \in \mathbf{I}_j} \mu_{m<\mathbf{I}_j>}[1] = \sum_{m \in \mathbf{I}_j} \mu_m(1) \tag{3.12}$$

and the bid of $\mathbf{I}_j$ for the kth step is

$$\begin{aligned} \hat{b}_{\mathbf{I}_j}[k] &= \sum_{m \in \mathbf{I}_j} \mu_{m<\mathbf{I}_j>}[k] \\ &= \sum_{m \in \mathbf{I}_j} \mu_m(\sum_{\tau=1}^{k-1} \sum_{\{j'|m \in \mathbf{I}_{j'}\}} \hat{e}_{\mathbf{I}_{j'}}[\tau] + 1), \end{aligned} \tag{3.13}$$

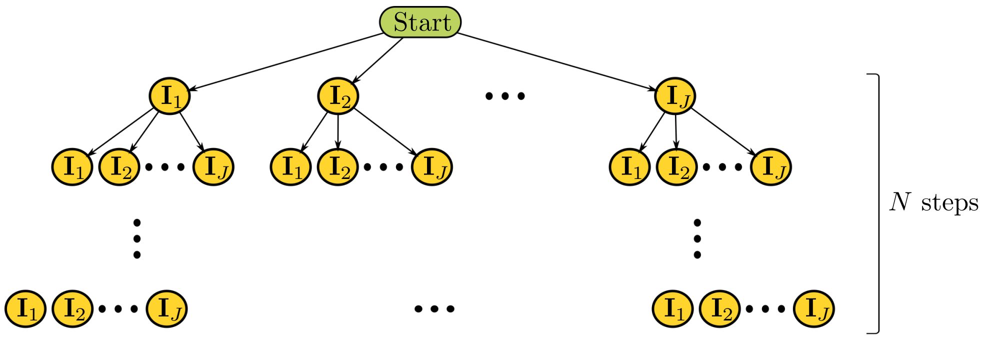
where $\mu_{m<\mathbf{I}_j>}[k]$ is the channel valuation of SU m in $\mathbf{I}_j$ for the kth step, $k = 2, \cdots, N$. Therefore, every MIG is bidding truthfully in each step.

The decision process has a tree structure illustrated in Figure 4.2a. In each step, the PU chooses an MIG from $\mathbf{J}$ to sell a channel. Therefore, each node at level N represents a possible way of channel allocation since the path that reaches the node is unique. We let the value of an edge to level k be $\hat{b}_{\mathbf{I}_j}[k]$. As a result, we can evaluate the social welfare of a path as the summation of the values of the edges that the path passes through. Therefore, the optimization problem is equivalent to finding the longest path to level N in the tree structure. Correspondingly, the optimal channel allocation $\hat{l}^*_{\mathbf{I}_j}$ to $\mathbf{I}_j$ is the frequency that the longest path passes through $\mathbf{I}_j$.

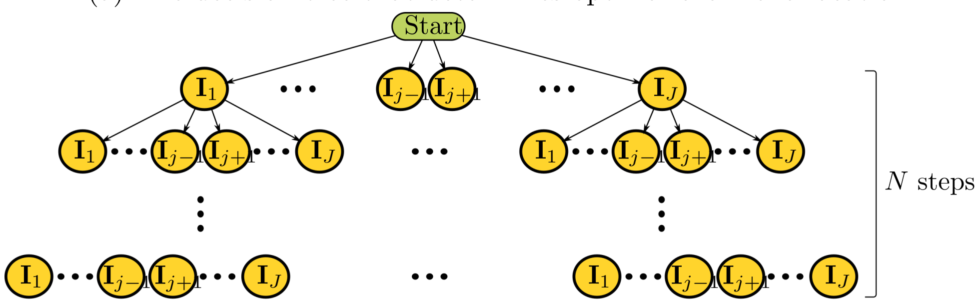
Proposition 3.3. *The optimal channel allocation* $\hat{\mathbf{l}}^* = (\hat{l}^*_{\mathbf{I}_1}, \cdots, \hat{l}^*_{\mathbf{I}_J})$ *is a weak dominant strategy for the PU.*

Proposition 3.4. *The longest path to level* N *that leads to the optimal channel allocation* $\hat{\mathbf{l}}^* = (\hat{l}^*_{\mathbf{I}_1}, \cdots, \hat{l}^*_{\mathbf{I}_J})$ *in the decision tree is not unique.*

The proofs of Propositions 3.3 and 3.4 can be combined. Based on channel allocation $\hat{\mathbf{l}}^*$, the social welfare of the system is $\pi + \varpi = \sum_{j=1}^{J} \hat{\eta}_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j}) = \sum_{m=1}^{M} \eta_m(\sum_{\{j|m \in I_j\}} \hat{l}^*_{\mathbf{I}_j})$. If there is another channel allocation $\hat{\mathbf{l}}^{*\prime} = (\hat{l}^{*\prime}_{\mathbf{I}_1}, \cdots, \hat{l}^{*\prime}_{\mathbf{I}_J})$ such that $\sum_{m=1}^{M} \eta_m(\sum_{\{j|m \in I_j\}} \hat{l}^*_{\mathbf{I}_j}) =$



(a) The decision tree that determines optimal channel allocation.



(b) The decision tree that determines optimal channel allocation when $\mathbf{I}_j$ does not bid.

Figure 3.2. Illustration of decision tree.

$\sum_{m=1}^{M}\eta_m(\sum_{\{j|m\in I_j\}}\hat{l}^{*\prime}_{\mathbf{I}_j})$, $\hat{\mathbf{l}}^{*\prime}$ will bring the same social welfare as channel allocation $\hat{\mathbf{l}}^{*}$. This scenario may occur when different SUs have identical channel valuations. Therefore, $\hat{\mathbf{l}}^{*}$ is a weak dominant strategy for the PU. Furthermore, different paths in the decision tree that lead to the same channel allocation $\hat{\mathbf{l}}^{*}$ do not change the number of channels that an SU obtains. In other words, the sequence of channel allocation does not affect the social welfare in the decision tree. Therefore, the longest path to level N that leads to the optimal channel allocation $\hat{\mathbf{l}}^{*}$ in the decision tree is not unique. In fact, there are at most $N!$ different paths in the decision tree that lead to the same channel allocation, and the uniqueness of the optimal channel allocation depends on the valuations from the SUs.

The decision tree slows down the VCG auction for MIGs and helps the MIG to submit truthful bids. In the VCG auction for MIGs, the optimal bid $\hat{b}^{*}_{\mathbf{I}_j}(\hat{i})$, $\hat{i} = 1, \cdots, \hat{l}^{*}_{\mathbf{I}_j}$ is determined according to the selected longest path in the decision tree, i.e.,

$$\hat{b}^{*}_{\mathbf{I}_j}(\hat{i}) = \hat{b}_{\mathbf{I}_j}[k_{\hat{i}}], \tag{3.14}$$

where $k_{\hat{i}}$ is the first level that satisfies $\sum_{k=1}^{k_{\hat{i}}} \hat{e}_{\mathbf{I}_j}[k] = \hat{i}$. Since $\hat{b}_{\mathbf{I}_j}[k_{\hat{i}}]$ reflects the truthful valuations of a channel from all SUs in $\mathbf{I}_j$ at the time when the transaction happens, $\hat{b}^*_{\mathbf{I}_j}(\hat{i})$, $\hat{i} = 1, \cdots, \hat{l}^*_{\mathbf{I}_j}$ is consistent with the definition of truthful bidding and thus the channel allocation of the VCG auction for MIGs maximizes the social welfare. Since the longest path that leads to $\hat{\mathbf{l}}^*$ is not unique, $\hat{b}^*_{\mathbf{I}_j}(\hat{i})$ is not unique either.

Once we find out the optimal channel allocation $\hat{\mathbf{l}}^*$, we need to determine the charging prices to the MIGs. If MIG $\mathbf{I}_j$, $j = 1, \cdots, J$, can find a proper way to set truthful bids beyond $\hat{l}^*_{\mathbf{I}_j}$ channels, i.e., $\hat{b}^*_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j} + 1), \cdots, \hat{b}^*_{\mathbf{I}_j}(N)$, we can simply apply (3.11) to evaluating the charging prices to the MIGs.

Proposition 3.5. *There do not exist truthful bidding vectors* $\hat{\mathbf{b}}^*_{\mathbf{I}_1}, \cdots, \hat{\mathbf{b}}^*_{\mathbf{I}_J}$ *in the VCG auction for MIGs, where* $\hat{\mathbf{b}}^*_{\mathbf{I}_j} = (\hat{b}^*_{\mathbf{I}_j}(1), \cdots, \hat{b}^*_{\mathbf{I}_j}(N))$*, according to which the charging prices to all MIGs bidding truthfully can be calculated by* (3.11)*.*

Proof. Suppose MIG $\mathbf{I}_j$ will have an extra channel beyond $\hat{l}^*_{\mathbf{I}_j}$ channels if $\mathbf{I}_{j'}$ or $\mathbf{I}_{j''}$ does not bid. Suppose the bid ranking of the extra channel of $\mathbf{I}_j$ when $\mathbf{I}_{j'}$ does not bid is $\sum_{\gamma \neq j'} \hat{l}^\dagger_{\mathbf{I}_\gamma}$, where $(\hat{l}^\dagger_{\mathbf{I}_1}, \cdots, \hat{l}^\dagger_{\mathbf{I}_{j'-1}}, \hat{l}^\dagger_{\mathbf{I}_{j'+1}}, \cdots, \hat{l}^\dagger_{\mathbf{I}_J})$ reflects channel allocation status before $\mathbf{I}_j$ obtains the extra channel. Similarly, the bid ranking of the extra channel of $\mathbf{I}_j$ when $\mathbf{I}_{j''}$ does not bid is $\sum_{\gamma \neq j''} \hat{l}^\ddagger_{\mathbf{I}_\gamma}$. The truthful bid of $\mathbf{I}_j$ when $\mathbf{I}_{j'}$ or $\mathbf{I}_{j''}$ does not bid is

$$\hat{b}^*_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j} + 1) = \sum_{m \in \mathbf{I}_j} \mu_m \Big(\sum_{\{\gamma | m \in \mathbf{I}_\gamma, \gamma \neq j'\}} \hat{l}^\dagger_{\mathbf{I}_\gamma} + 1 \Big)$$

or

$$\hat{b}^*_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j} + 1) = \sum_{m \in \mathbf{I}_j} \mu_m \Big(\sum_{\{\gamma | m \in \mathbf{I}_\gamma, \gamma \neq j''\}} \hat{l}^\ddagger_{\mathbf{I}_\gamma} + 1 \Big).$$

Obviously, $\hat{b}^*_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j} + 1)$ varies in different scenarios. We cannot set a fixed value for $\hat{b}^*_{\mathbf{I}_j}(\hat{l}^*_{\mathbf{I}_j} + 1)$ such that it satisfies the truthful bidding restriction for any MIG that quits bidding. Similarly, truthful bids for channels beyond the optimal allocation vary in different scenarios.

Therefore, we cannot find fixed truthful bidding vectors to calculate charging prices with (3.11). $\square$

Proposition 3.5 shows that truthful bidding brings some difficulty to the determination of charging prices, although the maximization of social welfare is benefited from bidding truthfully in the VCG auction for MIGs. Although (3.11) is not applicable, we can still use the original definition (3.9) to calculate charging prices.

We build a decision process similar to the one that we have discussed, but the selection space for the PU in each step is $\mathbf{J} \setminus \mathbf{I}_j$ instead of $\mathbf{J}$. The illustration of this decision process can be found in Figure 4.2b. The longest path of the decision tree represents the optimal channel allocation if $\mathbf{I}_j$ does not bid. Suppose that the channel allocation is $\hat{\mathbf{l}}^{*\prime}_{\mathbf{J}\setminus\mathbf{I}_j} = (\hat{l}^{*\prime}_{\mathbf{I}_1}, \cdots, \hat{l}^{*\prime}_{\mathbf{I}_{j-1}}, 0, \hat{l}^{*\prime}_{\mathbf{I}_{j+1}}, \cdots, \hat{l}^{*\prime}_{\mathbf{I}_J})$. The charging price to $\mathbf{I}_j$ is

$$\hat{\lambda}^{\hat{l}_{\mathbf{I}_j}}_{\mathbf{I}_j} = \hat{V}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}_{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}_{\mathbf{J}} = \sum_{j'\neq j} \hat{\eta}_{\mathbf{I}_{j'}}(\hat{l}^{*\prime}_{\mathbf{I}_{j'}}) - \sum_{j'\neq j} \hat{\eta}_{\mathbf{I}_{j'}}(\hat{l}^{*}_{\mathbf{I}_{j'}}). \tag{3.15}$$

To determine the charging prices to all MIGs, we need to go through J different decision processes.

The outcome of the decision tree provides the optimal channel allocation that maximizes the social welfare and the VCG style charging prices to the MIGs. Although the shape of the decision tree is similar to a sequential allocation process, no channel is allocated after each step. The optimal channel allocation is only known after the comparison of all possible allocations. In addition, the complexity of a decision process is $O(J^1 + \cdots + J^N) \sim O(J^N)$ and we have to go through $J + 1$ decision processes. Although the original problem has been simplified in comparison with the constrained VCG auction for SUs, the complexity of the decision process is still high.

3.4 Low-Complexity Channel Allocation

Instead of finding out all the optimal channel allocations in the decision tree, we consider improving the computational efficiency of the algorithm so that we can approach the optimal channel allocation with lower complexity. In this section, we approach the optimal channel allocation using a greedy algorithm, Dijkstra's algorithm, and batch allocation through approximation.

3.4.1 Greedy Algorithm

Firstly, we apply a greedy algorithm in the decision process. Intuitively, the PU would choose the MIG that holds the highest bid as the winner of each step in the decision tree. Correspondingly, the allocation result is $\hat{\mathbf{l}}_{Greedy} = (\hat{l}_{\mathbf{I}_1,Greedy}, \cdots, \hat{l}_{\mathbf{I}_J,Greedy})$. According to (3.9), the charging price to $\mathbf{I}_j$ is evaluated according to the channel allocation when $\mathbf{I}_j$ does not bid, which is also determined by the greedy algorithm.

Proposition 3.6. *In the decision process, the greedy algorithm may not maximize the social welfare in the spatial spectrum sharing system.*

Proof. See Appendix B.3. □

Proposition 3.6 shows that choosing the MIG with the highest bid in each step cannot guarantee the maximization of social welfare in the decision tree. Furthermore, since channel valuations in the kth step are related to the channel allocation in the previous $k-1$ steps, we anticipate that a myopic strategy [74] may fail to maximize the social welfare in the decision tree. However, maximizing the bid in every step significantly reduces the complexity to $O(N)$. The greedy algorithm is straightforward and requires less computation for the PU.

3.4.2 Dijkstra's Algorithm

We build a directed graph $\mathscr{G} = (\Phi, \Psi)$ to represent a simplified N-step channel allocation, where Φ is the set of $N \times J + 2$ vertices and Ψ is the set of edges. Specifically,

$$\Phi = \{\mathbf{I}[0], \mathbf{I}_1[1], \cdots, \mathbf{I}_J[1], \cdots, \mathbf{I}_1[N], \cdots, \mathbf{I}_J[N], \mathbf{I}[N+1]\},$$

where $\mathbf{I}_j[k]$ is the MIG $\mathbf{I}_j$ in the kth step, and $\mathbf{I}[0]$ and $\mathbf{I}[N+1]$ are the virtual source and target. The directed edge $(\mathbf{I}_j[k] \to \mathbf{I}_{j'}[k+1])$ exists for any $\mathbf{I}_j, \mathbf{I}_{j'} \in \mathbf{J}$, $k = 1, \cdots, N-1$, and

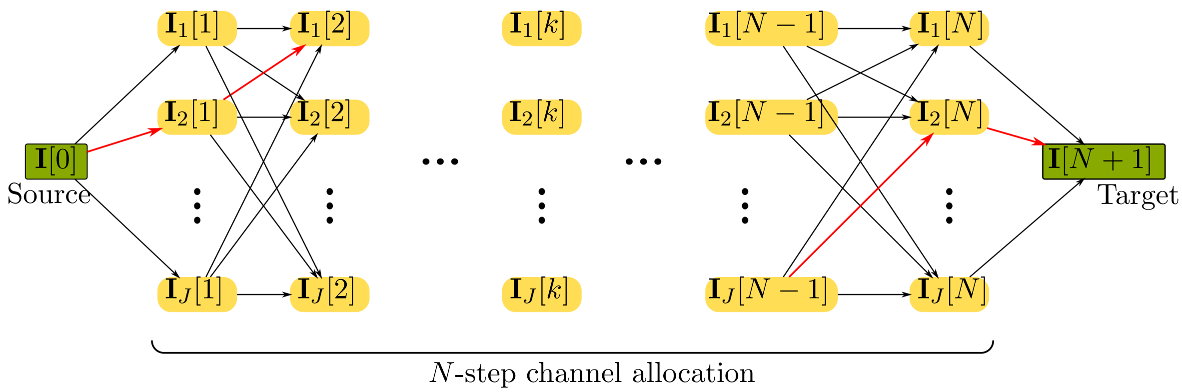


Figure 3.3. Longest path problem in Dijkstra's algorithm.

the weight of the edge is $\hat{b}_{\mathbf{I}_{j'}}[k+1]$. Note that $\hat{b}_{\mathbf{I}_{j'}}[k+1]$ depends on the channel allocation of the previous k steps. Additionally, the directed edges $(\mathbf{I}[0] \to \mathbf{I}_j[1])$ and $(\mathbf{I}_j[N] \to \mathbf{I}[N+1])$ are present for any $\mathbf{I}_j \in \mathbf{J}$, and the weight of $(\mathbf{I}[0] \to \mathbf{I}_j[1])$ is $\hat{b}_{\mathbf{I}_j}[1]$, while the weight of $(\mathbf{I}_j[N] \to \mathbf{I}[N+1])$ is zero. We illustrate graph $\mathscr{G}$ in Figure 4.3.

Algorithm 4 The application of Dijkstra's algorithm in $\mathscr{G}$.

Initialize: $dist[\mathbf{I}_j[k]] = 0$ for every vertex in $\mathscr{G}$, $\mathbf{Path}[\mathbf{I}_j[1]] = \{\mathbf{I}[0]\}$ for $j = 0, \cdots, J$.
for $k = 1$ to $N-1$ **do**
  for $\mathbf{I}_j \in \mathbf{J}$ **do**
    for $\mathbf{I}_{j'} \in \mathbf{J}$ **do**
      $alt = dist[\mathbf{I}_j[k]] + \hat{b}_{\mathbf{I}_{j'}}[k+1]$.
      if $alt > dist[\mathbf{I}_{j'}[k+1]]$ **then**
        $dist[\mathbf{I}_{j'}[k+1]] = alt$.
        $\mathbf{Path}[\mathbf{I}_{j'}[k+1]] = \mathbf{Path}[\mathbf{I}_j[k]] \cup \mathbf{I}_j[k]$
      end if
    end for
  end for
end for
$\mathbf{Path}[\mathbf{I}[N+1]] = \mathbf{Path}[\text{argmax}_{\mathbf{I}_j[N]}(dist[\mathbf{I}_j[N]])] \cup \text{argmax}_{\mathbf{I}_j[N]}(dist[\mathbf{I}_j[N]])$.

We apply Dijkstra's algorithm [75] to finding the longest path between the source and target in graph $\mathscr{G}$. The process of Dijkstra's algorithm is described in Algorithm 4, where $dist[\mathbf{I}_j[k]]$ is the longest distance from the source node to $\mathbf{I}_j[k]$, and $\mathbf{Path}[\mathbf{I}_j[k]]$ records the vertices that the longest path to $\mathbf{I}_j[k]$ passes through. Correspondingly, the allocation result of Dijkstra's algorithm is $\hat{\mathbf{l}}_{Dijkstra} = (\hat{l}_{\mathbf{I}_1,Dijkstra}, \cdots, \hat{l}_{\mathbf{I}_J,Dijkstra})$, where $\hat{l}_{\mathbf{I}_j,Dijkstra}$ is the frequency of the longest path to $\mathbf{I}[N+1]$ passing through $\mathbf{I}_j$ in $\mathscr{G}$. To determine the charging price to $\mathbf{I}_j$, we need to find the longest path in $\mathscr{G}' = (\Phi \setminus \mathbf{I}_j, \Psi')$, where Ψ' does not contain any edge towards $\mathbf{I}_j$ or from $\mathbf{I}_j$. Afterwards, we are able to find the channel allocation if $\mathbf{I}_j$ does not bid and thus the charging price to $\mathbf{I}_j$.

The application of Dijkstra's algorithm assumes

$$dist[\mathbf{I}_{j'}[k+1]] = \max_{\mathbf{I}_j[k]} \quad (dist[\mathbf{I}_j[k]] + \hat{b}_{\mathbf{I}_{j'}}[k+1]). \tag{3.16}$$

Note that $\hat{b}_{\mathbf{I}_{j'}}[k+1]$ is related to the channel allocation of the previous k steps. In other words, we eliminate the possibility that there is a non-longest path to $\mathbf{I}_j[k]$ that turns out to be the longest path to $\mathbf{I}_{j'}[k+1]$. The complexity of Dijkstra's algorithm is $O(|\Phi|+|\Psi|) = O(NJ + NJ^2)$. Dijkstra's algorithm searches more pathes in the decision tree and hence is more reliable than the greedy algorithm.

3.4.3 Batch Allocation

Algorithm 5 Batch allocation.

Initialize: SU m submits its bid vector $\mathbf{b}_m = (b_m(1), \cdots, b_m(N))$ to the PU, $m = 1, \cdots, M$. Let the untruthful bid vector of MIG $\mathbf{I}_j$ be $\tilde{\mathbf{b}}_{\mathbf{I}_j} = (\tilde{b}_{\mathbf{I}_j}(1), \cdots, \tilde{b}_{\mathbf{I}_j}(N))$, where $\tilde{b}_{\mathbf{I}_j}(\hat{i}) = \sum_{m \in \mathbf{I}_j} b_m(\hat{i})$. The MIGs that hold the highest N $\tilde{b}_{\mathbf{I}_j}(\hat{i})$ are selected to allocate channels. The allocation result is $\hat{\mathbf{l}}_{Batch}$.
Do
In the decision tree, find the path that allocates channels in the same way as $\hat{\mathbf{l}}_{Batch}$ and the corresponding truthful bids of the MIGs. The lowest truthful bid for the current allocation is $\hat{b}[N]$.
Calculate the truthful bid of $\mathbf{I}_j$ for the next channel, i.e., $\hat{b}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j,Batch} + 1)$, for $j = 1, \cdots, J$. There are Θ MIGs whose $\hat{b}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j,Batch} + 1) > \hat{b}[N]$.
if $\Theta \leq \Theta_{max}$ **then**
    Substitute the Θ MIGs for the last Θ nodes in the path.
else
    Substitute the MIGs that holds the highest Θ_{max} $\hat{b}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j,Batch} + 1)$ for the last Θ_{max} nodes in the path.
end if
Update the allocation result $\hat{\mathbf{l}}_{Batch}$.
Repeat do until $\Theta = 0$ or the maximum number of iteration steps is reached.

In batch allocation, the PU allocates all channels to selected MIGs in the first step to improve allocation efficiency, and then tries to increase the social welfare by replacing MIGs in a learning process. The process of batch allocation is described in Algorithm 8,

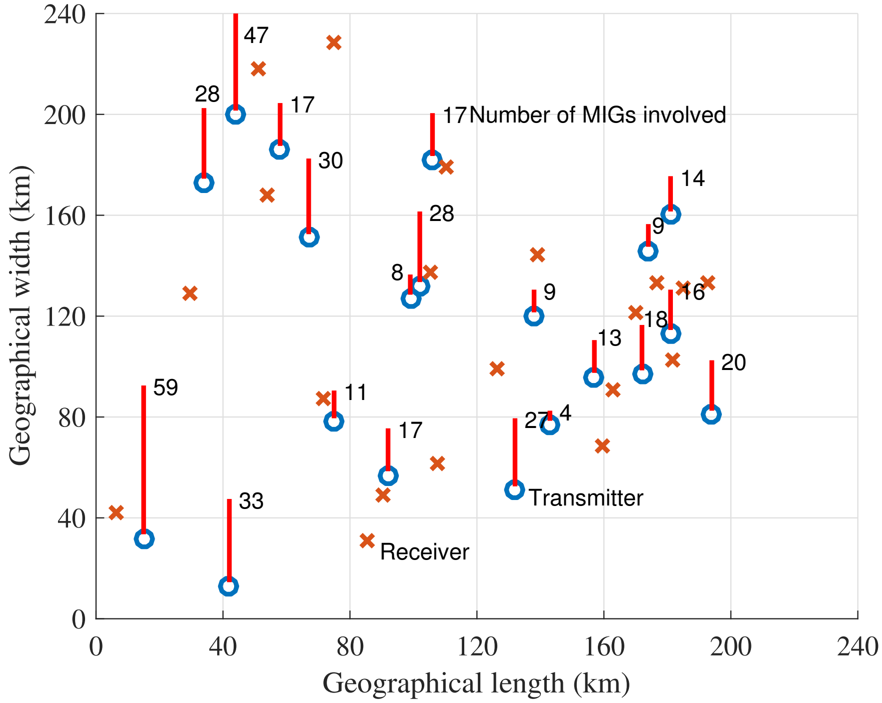


Figure 3.4. A random SU distribution and the number of MIGs involved with each SU.

where Θ_{max} controls the number of MIGs being replaced in each step to maintain stability. The channel allocation result is $\hat{\mathbf{l}}_{Batch} = (\hat{l}_{\mathbf{I}_1,Batch}, \cdots, \hat{l}_{\mathbf{I}_J,Batch})$.

3.5 Simulation Results

In our simulation, we consider additive white Gaussian noise (AWGN) channels with power spectrum density $\varepsilon/2 = 3 \times 10^{-18}$ W/Hz. We let the interference threshold be 1×10^{-10} W and the transmit power of the SUs be identical with $p_m = 1$ W. Using the two-ray model [63], we let $|h_{m',m}|^2 = \frac{\xi_{m',m}}{d^{\alpha}_{m',m}}$, where $\xi_{m',m}$ is a constant related to the antennas of $T_{m'}$ and R_m, which is set to $\xi_{m',m} = 10^6$, $d_{m',m}$ is the distance between $T_{m'}$ and R_m, and α is the path loss exponent, which is set to $\alpha = 4$ to simulate typical urban areas [63]. Under this setting, we calculate the number of MIGs involved with each SU based on a random SU distribution shown in Figure 3.4. Transceivers of 20 SUs are located in the area and there are 92 MIGs in total. The SUs located in a dense area are involved with a relatively small number of MIGs. According to the modified Bron-Kerbosch algorithm, an SU in a dense area is more likely to be excluded from an MIG to avoid co-channel interference.

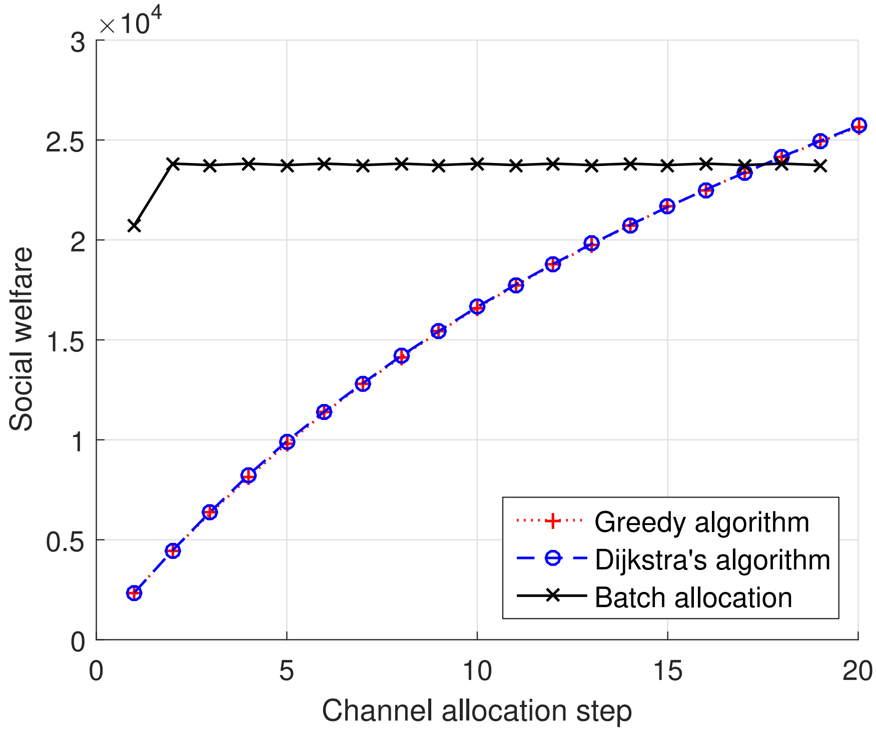


Figure 3.5. Social welfare of each channel allocation step when $M = 30$ and $N = 20$.

As a result, an SU located in a sparse area has more opportunities to obtain channels in comparison with an SU in a dense area.

Considering the uncertainty and irregularity of the channel valuations by the SUs, we let the valuations from an SU be random descending numbers and follow uniform distribution between 0 and 1000. Based on the SU distribution in Figure 3.4, we show the social welfare of our approximation methods along channel allocation steps when $M = 30$ and $N = 20$ in Figure 3.5. The step-wise performance curves of the greedy algorithm and Dijkstra's algorithm are very close to each other, as both algorithms allocate all channels and obtain the channel allocation results after N steps. In batch allocation, all channels are allocated in the first step and thus the social welfare is higher than the other two algorithms before the final steps. Meanwhile, the social welfare of batch allocation fluctuates for about two steps. Although the greedy algorithm and Dijkstra's algorithm can achieve higher social welfare, batch allocation needs fewer allocation steps, especially when there are a large number of channels to allocate. In Figure 3.6, we show the channel allocation to the SUs

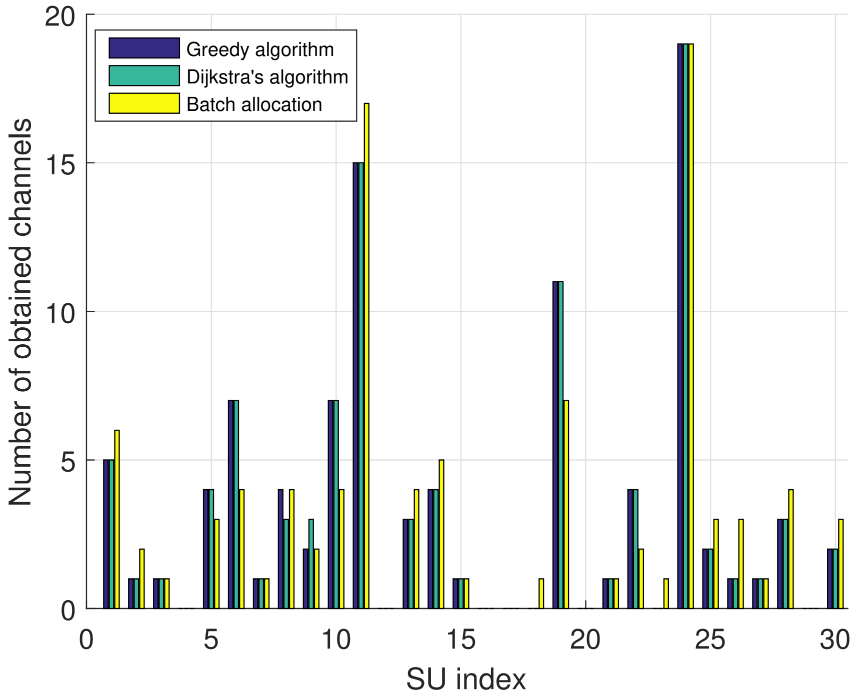


Figure 3.6. Channel allocation to the SUs when $M = 30$ and $N = 20$.

using different approximation methods when $M = 30$ and $N = 20$. The greedy algorithm and Dijkstra's algorithm allocate channels to the SUs in almost the same way, while their allocations to the MIGs are quite different, which can be well-explained by Proposition 4.1. In comparison with the greedy algorithm and Dijkstra's algorithm, channel allocation of batch allocation shows the same trend.

To further study the performance of the greedy algorithm, Dijkstra's algorithm, and batch allocation, we change the number of channels from 5 to 40 while fixing the number of users to 20. In an ideal case, we do not consider the possibility that the charging price could be higher than the channel valuation. We run the geographical distributions and channel valuations of the SUs 2,000 times and obtain the average social welfare and average system throughput with a specific number of channels. The results are shown in Figure 3.7. In terms of social welfare in the ideal case, the greedy algorithm and Dijkstra's algorithm are very close to each other, which is difficult to differentiate. Dijkstra's algorithm obtains slightly higher social welfare than the greedy algorithm. In comparison, the social welfare

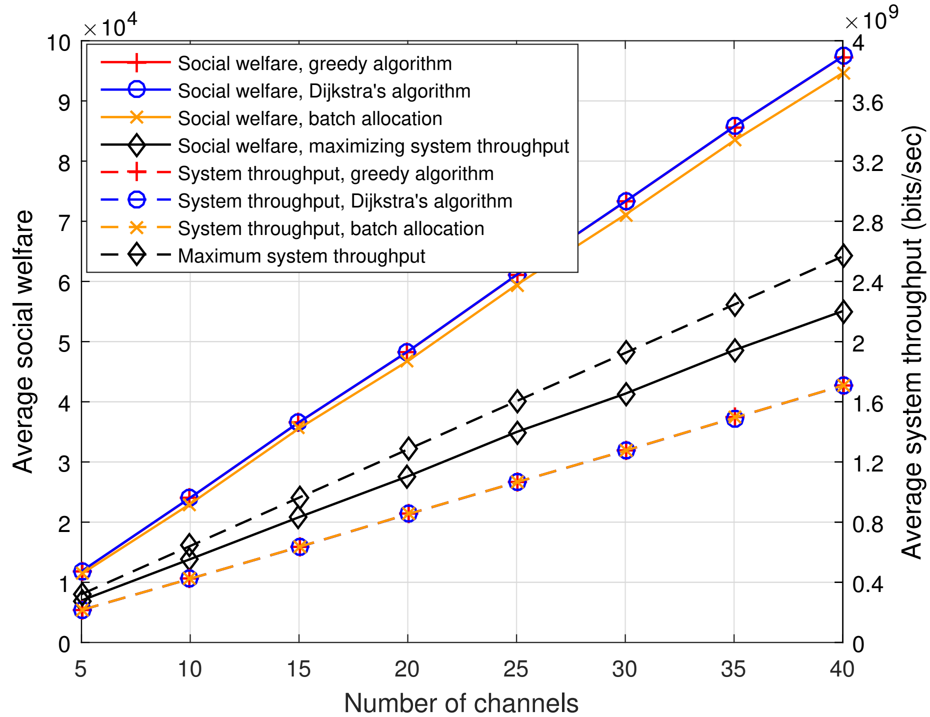


Figure 3.7. Average social welfare and average system throughput vs. number of channels when $M = 20$.

gap between batch allocation and Dijkstra's algorithm enlarges with the increasing number of channels. Additionally, we consider the case that every channel is allocated to the MIG with the highest system throughput, which maximizes system throughput. In comparison with the social welfare of the channel allocation that maximizes system throughput, the social welfares of the channel allocations obtained by our algorithms are significantly higher. The system throughputs of all of our three algorithms are at the same level and within an acceptable range of the maximum system throughput.

Similarly, we change the number of users from 10 to 35 while fixing the number of channels to 20. We run the distributions and valuations of the SUs 2,000 times and obtain the average social welfare and average number of MIGs with a specific number of SUs in the ideal case. In Figure 3.8, we show the average social welfare and average number of MIGs with different numbers of SUs. In consideration of social welfare, similar conclusions can be drawn as in the case of varying the number of channels. In comparison with

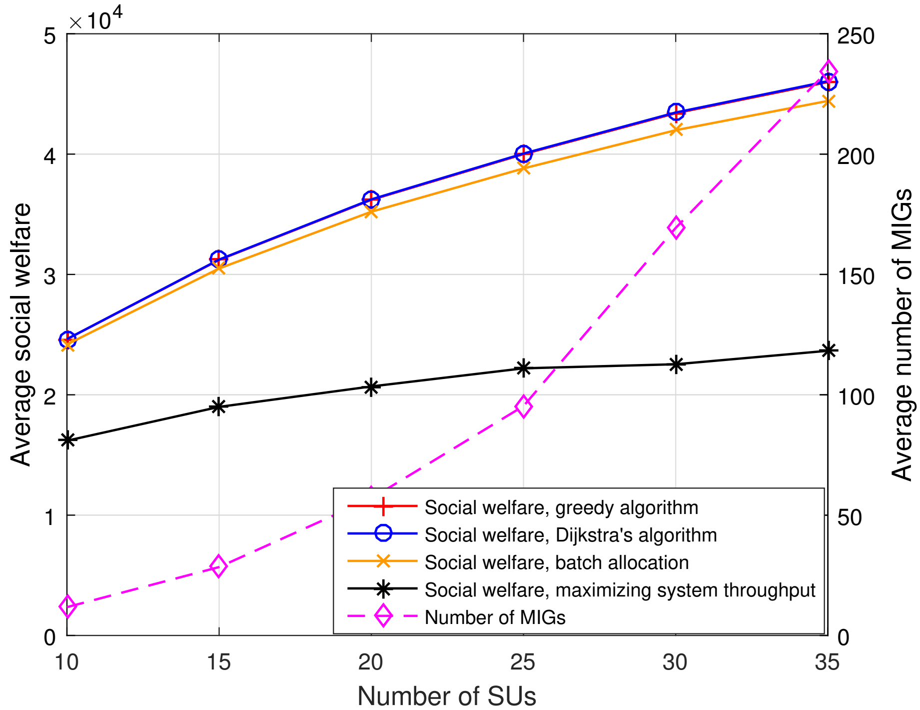


Figure 3.8. Average social welfare and average number of MIGs vs. number of SUs when $N = 20$.

the social welfare of the channel allocation that maximizes system throughput, the social welfare of the channel allocation obtained by our algorithms is significantly higher. With the increasing number of SUs, the number of MIGs soars according to the modified Bron-Kerbosch algorithm. As a result, applying Dijkstra's algorithm is time-consuming when there is a large number of SUs.

Furthermore, as the complexity of the decision process is $O(J^N)$, it is time-consuming to find the optimal channel allocation by following the decision process, especially when there are a large number of channels to allocate. To evaluate the performance of the three approximation methods in comparison with the decision process, we examine the adjusted average social welfare after charging prices are determined. In practice, an MIG will refuse to buy channels if the VCG style charging price is higher than its valuation of these channels. As a result, the number of channel transactions may be smaller than the number of channels, which lowers the social welfare. This case occurs when an algorithm fails to find the optimal

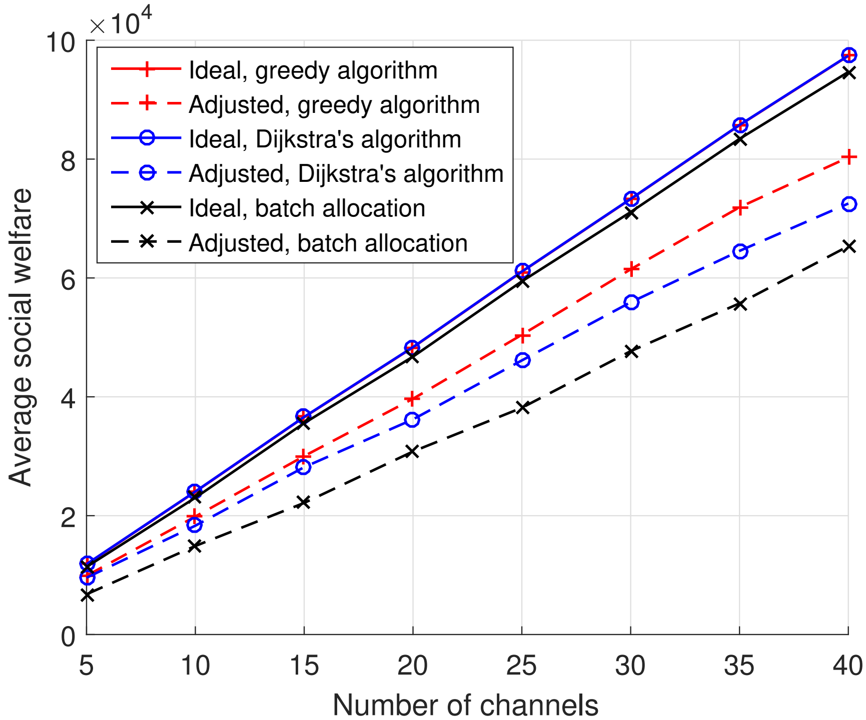


Figure 3.9. Adjusted average social welfare in consideration of charging prices when $M = 20$.

channel allocation, but successfully finds a better channel allocation when determining the charging price to an MIG. Therefore, the gap between the social welfare in the ideal case and the adjusted average social welfare in consideration of charging prices reveals the accuracy of an approximation method. In our simulation, we change the number of channels from 5 to 40 while fixing the number of users to 20, run the distributions and valuations of the SUs 2,000 times, and obtain the adjusted average social welfare in consideration of charging prices with a specific number of channels. The results are shown in Figure 4.9. The three approximation methods are more accurate when there are fewer channels.

3.6 Conclusions

In this chapter, we design a receiver-centric spatial spectrum reuse mechanism with downward-sloping channel valuations of the SUs. The PU conducts on-demand channel allocation and aims to maximize social welfare. We adopt the VCG auction to sell multiple identical channels under the constraint of co-channel interference. We group the SUs into several interference-free MIGs, and then simplify the constrained VCG auction for SUs to the VCG auction for MIGs. We also prove that bidding truthfully is the optimal bidding strategy for the SUs, such that the MIGs are bidding truthfully in the VCG auction for MIGs. Since truthful bids are difficult to determine without knowing the sequence of channel allocation, we build a decision process such that the MIGs as representatives of the SUs can update their channel valuations in each step and submit truthful bids. Therefore, the PU can determine the optimal channel allocation that maximizes social welfare and the charging prices to the MIGs accordingly. We prove that the optimal channel allocation is not unique and thus a weakly-dominant strategy for the PU. Furthermore, we use a greedy algorithm, Dijkstra's algorithm, and batch allocation to approach the optimal channel allocation. In our simulation, we compare the proposed methods and demonstrate that our on-demand channel allocation increases social welfare.

Chapter 4
Multi-Level Channel Valuations for Spatial Spectrum Reuse

4.1 Introduction

Although existing studies of spatial spectrum reuse focus more on the system throughput, system throughput maximization may not result in the maximum number of SUs being satisfied. In some cases, the SUs who anticipate relatively small data rates may be in urgent need of channels, and thus systems need to take user satisfaction into account. To address the conflict between system throughput and user satisfaction, we consider in channel allocation a neglected factor, the supply and demand relationship, in addition to the data rates. Focusing on such user characteristics, we for the first time enable heterogeneous channel valuations in spatial spectrum reuse. In consideration of this factor, user satisfaction can be achieved and spatial spectrum reuse is more efficient and practical.

In this chapter, we maximize the social welfare, which better describes the overall satisfaction of the SUs when we involve the supply and demand relationship. In addition, we consider multi-level flexible channel valuations of the SUs over non-identical channels. Specifically, channel valuations are related to both data rates and marginal values [65, 66]. Therefore, we need to reveal the channel supply and demand relationship in consideration of the impact of data rates on channel valuations. Meanwhile, we need to find a mechanism to better serve the SUs in terms of the overall satisfaction. In practice, the channel valuation of an SU decreases when the number of obtained channels at the same data rate level increases. In addition, the channel valuations of SUs are different due to the differences in the demands of the SUs. Therefore, a channel transaction mechanism that motivates the SUs to submit their truthful valuations to the PU is necessary.

In our model, the PU and SUs form a non-symmetric network in a spatial spectrum sharing system. Channel valuations of the SUs are related to both data rates and existing

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channel supplies. For a specific data rate level valuated by an SU, the willingness to pay for an additional channel decreases since the incremental benefit of an additional channel diminishes. In the system, the PU maximizes the social welfare through channel allocation to delicately selected SUs. The maximization of the social welfare in our model increases system throughput to an appropriate level that satisfies the demands of the SUs. In other words, the PU not only considers providing channels to the SUs at the data rate levels that meet the urgent demands of the SUs, which are reflected by the channel valuations of the SUs, but also needs to avoid channel oversupply to specific SUs or at specific data rate levels.

To maximize the social welfare, we group the SUs into allowable user crowds (AUCs) through a modified Bron-Kerbosch algorithm [69]. The AUCs are candidates for channel allocation of the PU, and at least the lowest data rate level is guaranteed in the AUCs. Then we introduce a Vickrey-Clarke-Groves (VCG) auction [67, 68] that sells multiple items in a socially optimal manner, in which the participants are limited to the AUCs. Different from conventional VCG auctions, the bids of the AUCs are determined according to channel supply status and cannot be submitted at once. Therefore, we transform the constrained VCG auction for the AUCs to a step-by-step decision tree, which slows down the auction but maintains its social optimality. We further define truthful bids of an AUC as the accumulated valuation of the coalition of the AUC in each step. A coalition of an AUC represents the highest accumulated valuation among all the subsets of the AUC in a specific step, according to the valuations of the SUs in the AUC based on channel supply status. Furthermore, we introduce channel cost split among coalition members as a coalitional game with transferable utilities. The proposed cost split mechanism based on the Shapley value is a fair payoff distribution and guarantees the formation of the coalitions. To approach the optimal channel allocation, we provide three low-complexity algorithms, finding the longest path in a directed acyclic graph (DAG), a greedy algorithm, and batch allocation.

The rest of the chapter is organized as follows. In Section 4.2, we model the spatial spectrum sharing system and the channel valuations related to different data rate levels and supply status. In Section 4.3, we present a modified Bron-Kerbosch algorithm to find all AUCs, design a constrained VCG auction for the AUCs to maximize the social welfare, and later transform the auction to a decision tree. In addition, we discuss truthful bidding of the AUCs and the Shapley value to distribute payoffs to the SUs. In Section 4.4, we simplify the decision process using three low-complexity algorithms. In Section 4.5, we provide our simulation results. Finally, we draw our conclusions in Section 4.6.

4.2 System Model

4.2.1 System Setup

We consider a spatial spectrum sharing system with one PU and M SUs, in which N non-overlapping idle channels of the PU are treated as merchandise and the PU is the only channel provider for the SUs in the system. In our system, the spectrum resource is scarce, i.e., $M \gg N$, such that the PU will sell a channel to multiple SUs.

In our model, each SU represents a transmitter-receiver pair. T_m and R_m denote the transmitter and receiver of SU m, $m = 1, 2, \cdots, M$, respectively. The transmit power of T_m is p_m, which is known to the system. To make our model more realistic, we consider the co-channel interference suffered by the receiver rather than the transmitter to mitigate the hidden/exposed node problem, such that the positions of the transmitter and receiver in a transmitter-receiver pair are not interchangeable. In addition, the randomly located transmitter-receiver pairs form in a geographically non-symmetric shape. Therefore, the network composed of the transmitter-receiver pairs is referred to as a non-symmetric network. Figure 4.1 is an illustration of our model.

An $M \times N$ matrix $\mathbf{E}$ can be used to describe the channel transactions between the PU

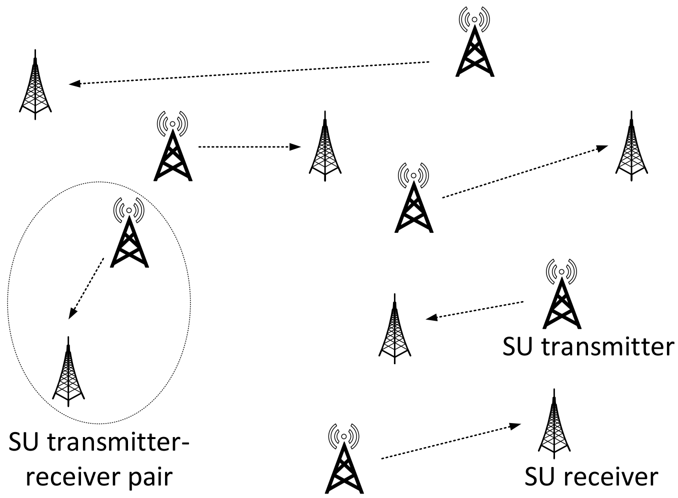


Figure 4.1. System model.

and SUs. Specifically,

$$\mathbf{E} = \begin{bmatrix} e_1^{(1)} & \cdots & e_1^{(N)} \\ \vdots & \ddots & \vdots \\ e_M^{(1)} & \cdots & e_M^{(N)} \end{bmatrix}, \tag{4.1}$$

where $e_m^{(n)} = 1$ indicates that SU m obtains channel n and $e_m^{(n)} = 0$ otherwise.

4.2.2 Channel Rate

Let $v_m^{(n)}$ denote the achievable channel rate of SU m when using channel n. Specifically,

$$v_m^{(n)} = W \log(1 + \frac{p_m \left|h_m\right|^2}{\mathscr{N} + \sum_{m' \neq m} e_{m'}^{(n)} p_{m'} \left|h_{m',m}\right|^2}), \tag{4.2}$$

where W is the channel bandwidth, $\mathscr{N}$ is the background noise power, $\left|h_m\right|^2$ is the channel power gain between T_m and R_m, $\left|h_{m',m}\right|^2$ is the channel power gain between $T_{m'}$ and R_m, such that $\sum_{m' \neq m} e_{m'}^{(n)} p_{m'} \left|h_{m',m}\right|^2$ denotes the interference power suffered by R_m when there are other SUs using channel n.

4.2.3 Multi-Level Channel Valuations

Channel valuations are heterogeneous among SUs due to several reasons. Firstly, interference levels of a channel vary at different locations, which results in different channel valuations among the SUs. Secondly, supply and demand status of the SUs also adds uncertainty to the channel valuations according to the marginal value theory. Therefore, channel valuations are flexible in our model, which is specifically related to channel rate levels and existing channel supply.

According to the achievable rate, a channel can be labeled by different levels according to the requirements of different services. We suppose that there are totally Q levels, i.e., $\Upsilon_1, \cdots, \Upsilon_Q$, where Υ_Q indicates the highest channel rate level in the system while Υ_1 indicates the lowest one. The number of levels Q is pre-determined based on the trade-off between accuracy and complexity.

We define $v_m^{(n)} \sim \Upsilon_q$ if $v_m^{(n)}$ is beyond the requirement of level Υ_q but does not meet the requirement of level Υ_{q+1}, indicating that channel n is labeled as Υ_q by SU m. Note that a channel can be labeled differently by the SUs because of different interference levels. For an SU, channels with the same label are identical items. However, the willingness of an SU to pay for an additional channel decreases since the incremental benefit of an additional channel diminishes as the tension between the channel supply and channel demand diminishes. Therefore, we model the valuation for a channel as the marginal valuation for the next channel with the same label. Let the valuation for the ith channel with label Υ_q of SU m be $\mu_m(q,i)$, and $\mu_m(q,i) > \mu_m(q,i+1)$ due to such downward-sloping channel valuations. In addition, we have $\mu_m(q,N) > \mu_m(q',1)$, if $q > q'$. Therefore, the channel valuation of SU m for its $l_{m,q}$ channels with label Υ_q is

$$\eta_{m,q}(l_{m,q}) = \sum_{i=1}^{l_{m,q}} \mu_m(q,i), \tag{4.3}$$

and the total channel valuation of SU m for all its obtained channels is

$$\eta_m(\mathbf{l}_m) = \sum_{q=1}^{Q} \sum_{i=1}^{l_{m,q}} \mu_m(q,i), \tag{4.4}$$

where $\mathbf{l}_m = (l_{m,1}, \cdots, l_{m,Q})$. Although channels are evaluated independently, the SUs share their channel valuations with each other.

4.2.4 Problem Formulation

The payoff the the PU is $\pi = \sum_{m=1}^{M} \lambda_m^{\mathbf{l}_m}$, where $\lambda_m^{\mathbf{l}_m}$ is the price charged to SU m for using $\mathbf{l}_m$ channels. The payoff of SU m when using $\mathbf{l}_m$ channels is $\varpi_m^{\mathbf{l}_m} = \eta_m(\mathbf{l}_m) - \lambda_m^{\mathbf{l}_m}$. Therefore, the payoff of all the SUs is $\varpi = \sum_{m=1}^{M}(\eta_m(\mathbf{l}_m) - \lambda_m^{\mathbf{l}_m})$, and the social welfare is $\pi + \varpi = \sum_{m=1}^{M} \eta_m(\mathbf{l}_m)$.

In this chapter, our goal is to maximize the social welfare under the interference limit. Specifically, we need to determine the optimal transaction matrix $\mathbf{E}^*$ such that

$$\max_{\mathbf{E}} \quad (\pi + \varpi) = \sum_{m=1}^{M} \eta_m(\mathbf{l}_m^*), \tag{4.5}$$

s.t.

$$\sum_{m' \neq m} e_{m'}^{(n)} p_{m'} \left|h_{m',m}\right|^2 < p_{thr} \tag{4.6}$$

for $m = 1, \cdots, M$, $n = 1, \cdots, N$, where $\mathbf{l}_m^* = (l_{m,1}^*, \cdots, l_{m,Q}^*)$ and $\sum_{q=1}^{Q} l_{m,q}^* = \sum_{n=1}^{N} e_m^{(n)*}$. Note that the rate level of a channel for SU m is affected by the interference from the other SUs using the same channel, i.e., $\{m' | e_{m'}^{(n)} = 1\}$. Therefore, the optimization of $\mathbf{E}$ translates to the optimization of $\mathbf{l}_m$. The constraint in (4.6) reflects the QoS requirement, i.e., the interference power suffered by SU m using channel n cannot exceed a threshold p_{thr}. Satisfying the constraint indicates that a channel is at least at level Υ_1 for an SU.

4.3 Constrained VCG Auctions and Shapley Value

In this section, we discuss the solution to social welfare maximization in spatial spectrum reuse in consideration of multi-level channel valuations. To maximize the social welfare, the PU needs to consider both the channel rate levels for the SUs in channel allocation and the descending feature of the channel valuations of the SUs. The strategy space of the PU is composed of the combinations of the SUs, and its channel allocation is to select among different combinations of the SUs. Therefore, we first eliminate the SU combinations that render unacceptable co-channel interference among the SUs by finding all

AUCs. Based on the AUCs, we apply the constrained VCG auction as the channel allocation mechanism, which is well known by its socially optimal transactions. The constrained VCG auction guarantees the bidding truthfulness of the AUCs and hence maximizes the social welfare. In the later part of this section, we discuss the bid formation process of the AUCs and the payoff distribution among the SUs in an AUC.

4.3.1 Allowable User Crowd

According to our model, a group of SUs is formed to reuse a channel. At the same time, unacceptable interference defined by (4.6) has to be avoided among the SUs using the same channel. To eliminate the SU combinations that render unacceptable co-channel interference, we need to figure out all the maximal allowable SU combinations. Here a maximal allowable SU combination indicates that any additional SU participating in the combination will breach the interference upper bound at the location of at least one user's receiver. Therefore, any subset of a maximal allowable SU combination can use a channel simultaneously.

To find all the maximal allowable SU combinations, we use a directed complete graph $\mathscr{G}(\mathbf{M}, \mathbf{M} \times \mathbf{M})$. A directed edge $(m' \to m)$ represents the co-channel interference to R_m caused by $T_{m'}$, i.e., $\epsilon_{m',m} = p_{m'} \left|h_{m',m}\right|^2$. Let $\mathbf{I} \subset \mathbf{M}$ be an allowable SU combination if $\sum_{m' \in \mathbf{I},\ m' \neq m} \epsilon_{m',m} < p_{thr}$ holds for any $m \in \mathbf{I}$. Therefore, the SUs in $\mathbf{I}$ are allowed to use the same channel simultaneously. Furthermore, we define that $\mathbf{I}$ is a maximal allowable SU combination, or an allowable user crowd (AUC), if there does not exist any other allowable SU combination such that $\mathbf{I} \subset \mathbf{I}'$. Note that an allowable SU combination is determined according to the values of the connected edges, and hence an AUC is different from the definition of an independent set in graph theory [70].

Algorithm 6 is a modified Bron-Kerbosch algorithm to find all AUCs in $\mathscr{G}$. Specifically, we add an interference control function to the Bron-Kerbosch algorithm, such that the formation of a vertex set is not based on the existences of edges, but through decision functions related to the accumulated value of edges. The Bron-Kerbosch algorithm is used

Algorithm 6 AUC formation in $\mathscr{G}$.

Initialize: $\mathbf{A} = \emptyset$, $\mathbf{B} = \mathbf{M}$, $\mathbf{C} = \emptyset$, $j = 0$.
AUC-Search$(\mathbf{A}, \mathbf{B}, \mathbf{C})$:
if $\mathbf{B} = \emptyset$ and $\mathbf{C} = \emptyset$ **then**
  $\mathbf{I}_j = \mathbf{A}$. $j = j + 1$.
else
  for $\forall m \in \mathbf{B}$ **do**
    if $\forall m' \in \mathbf{A} \cup \{m\}$, $\sum_{m'' \in \mathbf{A} \cup \{m\},\ m'' \neq m'} p_{m''} \left| h_{m'',m'} \right|^2 < p_{thr}$ **then**
      AUC-Search$(\mathbf{A} \cup \{m\}, \mathbf{B} \cap \mathbf{\Delta}(m), \mathbf{C} \cap \mathbf{\Delta}(m))$.
      $\mathbf{B} = \mathbf{B} \setminus \{m\}$. $\mathbf{C} = \mathbf{C} \cup \{m\}$.
    end if
  end for
end if

to identify all maximal independent sets in a graph. The worst-case complexity of our modified algorithm is $O(3^{M/3}M)$, while the worst-case complexity of the Bron-Kerbosch algorithm is $O(3^{M/3})$ [71]. In Algorithm 6, $\mathbf{A}$ is a possible AUC, vertices in $\mathbf{B}$ indicate that each of the corresponding SUs does not cause unacceptable interference to the receiver of any SU currently in $\mathbf{A}$, and vertices in $\mathbf{C}$ indicate that the corresponding SUs have already been excluded. We let $\mathbf{\Delta}(m) = \{m' | p_{m'} \left| h_{m',m} \right|^2 < p_{thr}\}$, i.e., $T_{m'}$ alone does not cause unacceptable interference to R_m if $m' \in \mathbf{\Delta}(m)$. The number of AUCs in $\mathscr{G}$ is J, and an AUC is denoted as $\mathbf{I}_j$, $j = 1, \cdots, J$.

A $J \times N$ matrix $\hat{\mathbf{E}}$ can be used to describe the channel transactions between the PU and AUCs. Specifically,

$$\hat{\mathbf{E}} = \begin{bmatrix} \hat{e}_1^{(1)} & \cdots & \hat{e}_1^{(N)} \\ \vdots & \ddots & \vdots \\ \hat{e}_J^{(1)} & \cdots & \hat{e}_J^{(N)} \end{bmatrix}, \tag{4.7}$$

where $\hat{e}_j^{(n)} = 1$ indicates that AUC $\mathbf{I}_j$ obtains channel n, and $\hat{e}_j^{(n)} = 0$ otherwise. To avoid unacceptable co-channel interference, two AUCs cannot share the same channel. Therefore, $\sum_{j=1}^{J} \hat{e}_j^{(n)} = 1$, i.e., $\hat{\mathbf{E}}$ is a sparse matrix. The number of channels that AUC $\mathbf{I}_j$ obtains is $\hat{l}_j = \sum_{n=1}^{N} \hat{e}_j^{(n)}$. Note that we use $\hat{\cdot}$ to indicate the notation related to an AUC in this chapter.

To maximize the social welfare, the optimal channel allocation $\hat{\mathbf{l}}^* = (\hat{l}_1^*, \cdots, \hat{l}_J^*)$ satisfies

$$\max_{\hat{\mathbf{E}}} \quad (\pi + \varpi) = \sum_{j=1}^{J} \hat{\eta}_j(\hat{l}_j^*), \tag{4.8}$$

where $\hat{\eta}_j(\hat{l}_j^*)$ is the channel valuation of $\mathbf{I}_j$ for $\hat{l}_j^*$ channels. Similar to $\eta_m(\mathbf{l}_m)$, we have $\hat{\eta}_j(\hat{l}_j) = \sum_{\hat{i}=1}^{\hat{l}_j} \hat{\mu}_j(\hat{i})$, where $\hat{\mu}_j(\hat{i})$ is the valuation of the $\hat{i}$th channel of $\mathbf{I}_j$. The relationship between $\hat{\mu}_j(\hat{i})$ and $\mu_m(q, i)$ will be explained later. Note that an SU may be excluded from an AUC to maximize the social welfare.

4.3.2 Constrained VCG Auction

In the transaction process, the SUs are authorized to use channels through a constrained VCG auction of the PU. The participants of the auction are limited to the AUCs, instead of the individual SUs. As a result, the PU can simply select the AUCs submitting the highest bids to allocate channels, while the PU has to estimate channel rate levels and avoid unacceptable interference when allocating channels if the participants are the individual SUs. In the auction, an AUC is a bidding entity whose bid is the accumulated valuation by all or some of the SUs in the AUC. The VCG auction is a sealed-bid process. In other words, an AUC only knows its own bid information. The auction is defined as the constrained VCG auction for AUCs. The set of auction items is $\mathbf{N} = \{1, \cdots, N\}$ and the set of bidders is $\mathbf{J} = \{\mathbf{I}_1, \cdots, \mathbf{I}_J\}$. The constrained VCG auction follows three main steps: bid submission, channel allocation, and price determination.

1. Bid submission. AUC $\mathbf{I}_j$ informs the PU its bid vector $\hat{\mathbf{b}}_j = (\hat{b}_j(1), \cdots, \hat{b}_j(N))$, where $\hat{b}_j(n)$ is the channel valuation of $\mathbf{I}_j$ for the $\hat{i}$th channel.

2. Channel allocation. The PU determines the number of channels allocated to each AUC, i.e., $(\hat{l}_1, \cdots, \hat{l}_J)$. Specifically, the PU selects the largest N $\hat{b}_j(\hat{i})$, where $\hat{l}_j$ is the number of selected bids of $\mathbf{I}_j$.

3. Price determination. The price $\hat{\lambda}_j^{\hat{l}_j}$ that AUC $\mathbf{I}_j$ needs to pay for the obtained $\hat{l}_j$

channels is

$$\hat{\lambda}_j^{\hat{l}_j} = \hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}, \tag{4.9}$$

where $\hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}$ sums the bids of all winners except AUC $\mathbf{I}_j$ when the set of bidders is $\mathbf{J}$, and $\hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}$ sums the bids of all winners when the set of bidders is $\mathbf{J} \setminus \{\mathbf{I}_j\}$.

Note that in the VCG auction, a bid submitted by an AUC is the upper bound of charging price given its existing channel supply status. An AUC will refuse a channel transaction if the charging price exceeds the bid or the channel valuation. This price charged to AUC $\mathbf{I}_j$ is interpreted as the loss of the other AUCs caused by the participation of AUC $\mathbf{I}_j$ in the VCG auction. According to the constrained VCG auction, the payoff $\hat{\varpi}_j^{\hat{l}_j}$ of AUC $\mathbf{I}_j$ obtained by using $\hat{l}_j$ is $\hat{\varpi}_j^{\hat{l}_j} = \hat{\eta}_j(\hat{l}_j) - \hat{\lambda}_j^{\hat{l}_j}$.

It is known that bidding truthfully is Dominant-strategy incentive-compatible (DSIC) in a VCG auction [33, 72]. A strategy that is DSIC indicates that the strategy brings the optimal payoff or at least maintain the payoff level regardless of the strategies of others [73]. As a result, the social welfare can simply be maximized by the choice of the highest bids in the constrained VCG auction.

4.3.3 Decision Tree

The optimal bidding strategy of $\mathbf{I}_j$ as an entity is to submit the truthful channel valuations, irrespective of the truthfulness of other competitors. However, it is problematic to determine truthful bids for the AUCs in the constrained VCG auction. A bid of an AUC can reflect the accumulated valuations by all or some of the SUs in the AUC, and the involvement of an SU can affect channel rate level evaluations of other SUs in the AUC. Meanwhile, an AUC has to submit N truthful bids at the beginning of the constrained VCG auction. Note that generally an SU belongs to different AUCs. In this situation, the submission of N truthful bids indicates that the channel allocation result is known to the AUCs before the auction, which is in contradiction to the procedure of the constrained VCG auction.

In this subsection, a decision process is designed for the PU such that the AUCs can determine their truthful bids in the constrained VCG auction. The decision process consists of N steps. In each step, the PU only allocates one channel to a selected AUC. As a result, an AUC does not need to determine N bids at the beginning of the constrained VCG auction. Instead, an AUC can calculate its channel valuation in each step based on the channels obtained in the previous steps. We use $\hat{e}_j[n] = 1$ to denote that $\mathbf{I}_j$ wins a channel in the nth step, and $\hat{e}_j[n] = 0$ otherwise. Similarly, we use $e_{m\langle \mathbf{I}_j \rangle}[n] = 1$ to imply that SU m in $\mathbf{I}_j$ wins a channel in the nth step, and $e_{m\langle \mathbf{I}_j \rangle}[n] = 0$ otherwise. The bid of $\mathbf{I}_j$ in the nth step is $\hat{b}_j[n]$.

In Figure 4.2a, we illustrate our decision process, which involves a tree structure. In each step, an AUC is selected by the PU. As a result, each leaf node in step N represents a possible channel allocation outcome. Note that a leaf node can only be reached through a specific path. We further assign the value of an edge to node $\mathbf{I}_j$ in the nth step as $\hat{b}_j[n]$, and hence the social welfare of channel allocation is the accumulated value of the edges on the corresponding path. Therefore, the optimal channel allocation that maximizes the social welfare in the constrained VCG auction is equivalent to the path with the highest accumulated value in the decision tree. In addition, the optimal number of channels $\hat{l}_j^*$ allocated to $\mathbf{I}_j$ is the number of times that the longest path crosses $\mathbf{I}_j$.

The decision tree separates the bid submission process in the constrained VCG auction into N steps. Meanwhile, the winning N bids in the constrained auction are the values of the edges on the longest path in the decision tree. Specifically,

$$\hat{b}_j^*(\hat{\imath}) = \hat{b}_j[n_{\hat{\imath}}] \tag{4.10}$$

for $j = 1, \cdots, J$, $\hat{\imath} = 1, \cdots, \hat{l}_j^*$, where $n_{\hat{\imath}}$ is the first step when $\sum_{n=1}^{n_{\hat{\imath}}} \hat{e}_j[n] = \hat{\imath}$.

Proposition 4.1. *The optimal channel allocation $\hat{\mathbf{l}}^* = (\hat{l}_1^*, \cdots, \hat{l}_J^*)$ is a weak dominant strategy for the PU in the decision tree.*

The proof of Propositions 4.1 is straightforward. The optimal channel allocation $(\hat{l}_1^*, \cdots, \hat{l}_J^*)$ to the AUCs corresponds to the optimal channel allocation $(\mathbf{l}_1^*, \cdots, \mathbf{l}_M^*)$ to the SUs. There may exists another channel allocation to the AUCs, $(\hat{l}_1^{*\prime}, \cdots, \hat{l}_J^{*\prime})$, and the corresponding channel allocation to the SUs is $(\mathbf{l}_1^{*\prime}, \cdots, \mathbf{l}_M^{*\prime})$. As long as $\sum_{m=1}^{M} \eta_m(\mathbf{l}_m^*) = \sum_{m=1}^{M} \eta_m(\mathbf{l}_m^{*\prime})$, the two different allocations result in the same social welfare. Therefore, $(\hat{l}_1^*, \cdots, \hat{l}_J^*)$ is a weak dominant strategy for the PU in the decision tree.

Once we find out $(\hat{l}_1^*, \cdots, \hat{l}_J^*)$, we determine the VCG style charging prices to $\mathbf{I}_j$ in another decision tree, which is similar to the one that we have discussed, but the selection space for the PU in each step is $\mathbf{J} \setminus \{\mathbf{I}_j\}$ instead of $\mathbf{J}$. The illustration of this decision process can be found in Figure 4.2b. The longest path of the decision tree represents the channel allocation if $\mathbf{I}_j$ does not bid. Suppose that the channel allocation is $(\hat{l}_1', \cdots, \hat{l}_{j-1}', 0, \hat{l}_{j+1}', \cdots, \hat{l}_J')$. The charging price to $\mathbf{I}_j$ is

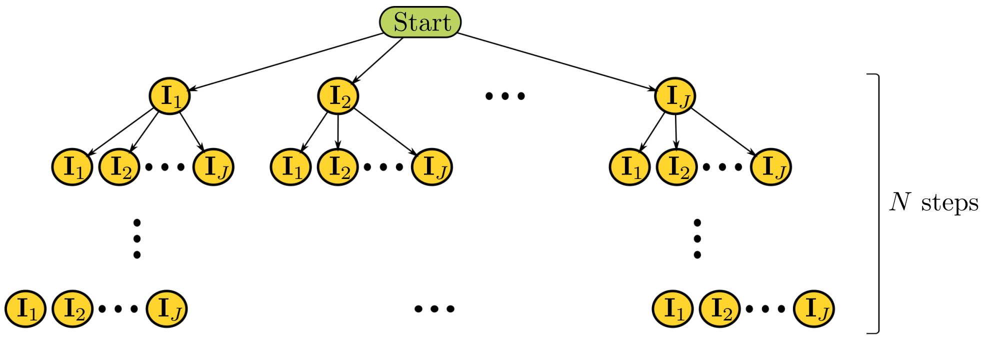
$$\hat{\lambda}_j^{\hat{l}_j} = \hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} = \sum_{j' \neq j} \hat{\eta}_{j'}(\hat{l}_{j'}') - \sum_{j' \neq j} \hat{\eta}_{j'}(\hat{l}_{j'}^*). \tag{4.11}$$

To determine the charging prices to all AUCs, we need to go through J different decision processes.

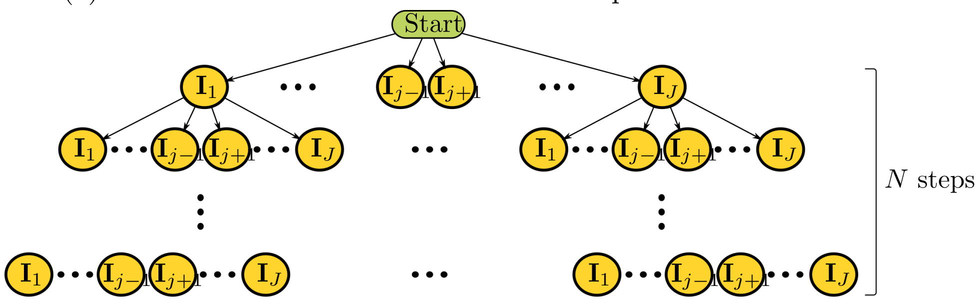
The decision tree provides the AUCs a practical method to determine their bids and keeps the VCG style charging prices to the AUCs. The VCG style charging prices motivate the AUCs to reveal their truthful biddings and hence help to maximize the social welfare. Although the shape of the decision tree is similar to that of a sequential allocation process, no channel is allocated in each step and the channel allocation is only known after the comparison of all possible allocations.

4.3.4 Truthful Bidding of an AUC

The decision tree provides a feasible way for the AUCs to submit bids in each step and guarantees the truthfulness of the bids. However, the decision process allocates channels to the AUCs instead of directly to the SUs. As a result, channel allocation to specific SUs



(a) The decision tree that determines the optimal channel allocation.



(b) The decision tree that determines the charging price to $\mathbf{I}_j$.

Figure 4.2. Illustration of decision tree.

is still unknown because whether an SU will be included in an AUC cannot be determined yet. On the other hand, the realization of the optimal channel allocation in the decision tree is based on the truthful bidding of the AUCs. However, the definition of truthful bidding of the AUCs in each step is still missing. To address these issues, we regulate the truthful bidding of the AUCs in this subsection.

For the AUCs in a step of the decision tree, a bid that reflects the accumulated valuations by any subset of the SUs in $\mathbf{I}_j$ is considered a truthful bid, as long as the channel use is confined in the subset. Note that the valuation of an SU varies in different subsets. While an AUC has many options to perform truthful bidding, only the largest truthful bid, i.e., the maximum accumulated valuation, can maximize the chance of obtaining the channel for the AUC. Let the SUs in the subset of $\mathbf{I}_j$ that hold the maximum accumulated valuation in the nth step form the coalition $\Theta_j[n]$ of $\mathbf{I}_j$ in the nth step, i.e.,

$$\Theta_j[n] = \underset{\theta_j \subseteq \mathbf{I}_j}{\operatorname{argmax}}\Big(\sum_{m\in\theta_j} \mu_m(q(\theta_j), k_{m,n-1}+1\Big), \tag{4.12}$$

where θ_j is a subset of $\mathbf{I}_j$, $q(\theta_j)$ is the evaluated channel rate level of SU m in θ_j, and $k_{m,n-1}$ is the number of obtained channels of SU m in the previous $n-1$ steps. Specifically, $k_{m,0} = 0$, $k_{m,n-1} = \sum_{\tau=1}^{n-1} e_{m\langle \mathbf{I}_j \rangle}[\tau]$, $q(\theta_j)$ is related to the locations of the other SUs in θ_j, and $\hat{v}_m(\theta_j) \sim \Upsilon_{q(\theta_j)}$, where

$$\hat{v}_m(\theta_j) = W \log(1 + \frac{p_m \left|h_m\right|^2}{\mathscr{N} + \sum_{m' \in \theta_j,\ m' \neq m} p_{m'} \left|h_{m',m}\right|^2}). \tag{4.13}$$

In a simplified scenario, the coalition of $\mathbf{I}_j$ in each step is $\mathbf{I}_j$ itself. The grand coalition usually forms when an additional channel for an SU is always considered more valuable than the channel rate level upgrade of another SU. For example, the gap between the highest and lowest channel rate levels may not be significant, especially when $\max_{m,i}(\mu_m(Q,i) - \mu_m(1,i)) < \mu_{m'}(q,i')$ for $m' = 1, \cdots, M$, $i, i' = 1, \cdots, N$ and $q = 1, \cdots, Q$. In a more general scenario, AUC $\mathbf{I}_j$ has to explore all the subsets in $\mathbf{I}_j$ to determine $\Theta_j[n]$ since the valuations of the SUs are not regulated to follow any pattern. In practice, channel valuations for the next channel are exchanged among the SUs in an AUC in each step of the decision tree, and the channel levels of the SUs can be estimated according to system specifications or via historical data. This requires little computational power and can be implemented through a common control channel among the SUs.

We define the truthful bidding of an AUC in a step of the decision tree as follows.

Definition 4.1 (**Truthful Bidding of an AUC**). *AUC $\mathbf{I}_j$ is bidding truthfully in the nth step of the decision tree if its bid equals the maximum accumulated valuation of all or some of the SUs in $\mathbf{I}_j$, i.e.,*

$$\hat{b}_j[n] = \sum_{m \in \Theta_j[n]} \mu_m(q(\Theta_j[n]), k_{m,n-1} + 1). \tag{4.14}$$

According to the definition, some SUs in $\mathbf{I}_j$ may be excluded from joining the coalition in a step to increase the bid of $\mathbf{I}_j$ and hence the chance to obtain a channel. For example, there may exist some SUs in an AUC in urgent need of channels in higher rate levels and the exclusion of some other SUs can increase the accumulated valuation of the AUC. Since the SUs can be involved in multiple AUCs, the exclusion of the SUs from the coalition in the step is acceptable for them.

With the defined truthful bidding, the outcome of the decision tree is

$$\max_{j[1],\cdots,j[N]}(\sum_{n=1}^{N}\hat{b}_{j[n]}[n]) = \max_{j[1],\cdots,j[N]}\sum_{n=1}^{N}\sum_{m\in\Theta_{j[n]}[n]}\mu_m(q(\Theta_{j[n]}[n]),k_{m,n-1}+1), \tag{4.15}$$

where $j[n]$ is the AUC that obtains a channel in the nth step, i.e., $j[n]=j$ if $\hat{e}_j[n]=1$. In comparison with the optimization problem in (4.5) and (4.6), the outcome through truthful bidding of the AUCs maximizes the social welfare. Therefore, the channel allocation based on our regulated truthful bidding in the decision tree is the optimal one, i.e., $\hat{\mathbf{l}}^* = (\hat{l}_1^*,\cdots,\hat{l}_J^*)$ for the AUCs and $\mathbf{l}_m^* = (l_{m,1}^*,\cdots,l_{m,Q}^*)$ for the SUs, $m=1,\cdots,M$.

In addition, the truthful bidding of an AUC also defines the relationship between $\hat{\mu}_j(\hat{i})$ and $\mu_m(q,i)$. Specifically, we have $\hat{\mu}_j(\hat{i}) = \sum_{m\in\Theta_j[n_{\hat{i}}]}\mu_m(q(\Theta_j),k_{m,n_{\hat{i}}-1}+1)$, where $n_{\hat{i}}$ is the first level that satisfies $\sum_{n=1}^{n_{\hat{i}}}\hat{e}_j[n]=\hat{i}$.

4.3.5 Shapley Value

We have transformed the constrained VCG auction for AUCs to the decision tree, such that the AUCs bid truthfully in a feasible way. In the nth step, AUC $\mathbf{I}_j$ bids as an entity and submits the accumulated valuation of all the SUs in the coalition $\Theta_j[n]$ in $\mathbf{I}_j$. However, the SUs cannot form coalitions without appropriate rules. At this stage, we consider the factor that can bond SUs together, which is the payoff distribution in a coalition.

We suppose that the SUs in $\Theta_j[n]$ play a coalitional game with complete information [74], denoted as $\kappa_j[n] = (\Theta_j[n],\hat{\phi}_j[n])$, where $\hat{\phi}_j[n]$ is the payoff function associated with each subset of $\Theta_j[n]$ using a channel obtained in the nth step, i.e., $\hat{\phi}_j[n](\theta_j)$ for any $\theta_j \subseteq$

$\Theta_j[n]$. We assume

$$\hat{\phi}_j[n](\theta_j) = f_{\Theta_j[n]}(\sum_{m\in\theta_j} \mu_m(q(\theta_j), k_{m,n-1}+1)) \cdot \sum_{m\in\theta_j} \mu_m(q(\theta_j), k_{m,n-1}+1), \tag{4.16}$$

where $f_{\Theta_j[n]}(\cdot)$ is an increasing convex function that determines the winning probability of a bid from non-coalitional SU groups. Note that $f_{\Theta_j[n]}$, as well as the winning probability, is a hypothetical consensus among the SUs in coalition $\Theta_j[n]$, which exists only in the process of payoff distribution. This function may be inaccurate, but is accepted by all the SUs in $\Theta_j[n]$. Specifically, $\hat{\phi}_j[n](\emptyset) = 0$ and $\hat{\phi}_j[n](\Theta_j[n]) = \hat{\varpi}_j[n]$, where $\hat{\varpi}_j[n]$ is the payoff of $\mathbf{I}_j$ in the nth step.

In the coalitional game $\kappa_j[n]$, we can easily prove that $\hat{\phi}_j[n](\theta_j + \theta'_j) > \hat{\phi}_j[n](\theta_j)$ for any $\theta_j, \theta'_j \subset \Theta_j[n]$ and $\theta_j \cap \theta'_j = \emptyset$. Meanwhile, only one coalition in $\mathbf{I}_j$ is allowed in each step since an AUC can only submit one bid in each step. Therefore, the coalitional game $\kappa_j[n] = (\Theta_j[n], \hat{\phi}_j[n])$ can be considered superadditive, i.e., $\hat{\phi}_j[n](\theta_j + \theta'_j) \geq \hat{\phi}_j[n](\theta_j) + \hat{\phi}_j[n](\theta'_j)$ for any $\theta_j, \theta'_j \subset \Theta_j[n]$ and $\theta_j \cap \theta'_j = \emptyset$. Note that $\hat{\phi}_j[n](\theta'_j)$ is assumed to be zero since only one coalition is allowed.

In addition, the SUs in $\Theta_j[n]$ will split channel cost $\hat{\lambda}_j[n]$ if they obtain a channel in the nth step. The cost split process can be interpreted as utility transfer among the SUs. Therefore, $\kappa_j[n]$ can be modeled as a coalitional game with transferable utilities. It is well known that the Shapley value provides solutions to coalitional games with transferable utilities [76]. The Shapley value is considered as a fair and unique way to distribute the payoff of a coalition among members in a coalitional game [77]. The main concept of the Shapley value is to charge more to the players who are benefited more from the coalition.

In the coalitional game $\kappa_j[n]$, the SUs in $\Theta_j[n]$ who value a channel more can be desperate to obtain the channel, and hence they need to pay more for the obtained channel according to the Shapley value. The Shapley value assigns the payoff $\hat{\varpi}_j[n]$ to each SU in

$\Theta_j[n]$. Specifically,

$$\varpi_{m\langle \mathbf{I}_j \rangle}[n] = \frac{1}{|\Theta_j[n]|} \sum_{\theta_j \subseteq \Theta_j[n] \backslash \{m\}} |\theta_j|!(|\Theta_j[n]| - |\theta_j| - 1)![\hat{\phi}_j[n](\theta_j \cup \{m\}) - \hat{\phi}_j[n](\theta_j)] \quad (4.17)$$

for all $m \in \Theta_j[n]$, where $\varpi_{m\langle \mathbf{I}_j \rangle}[n]$ is the payoff of SU m in $\mathbf{I}_j$ in the nth step and $|\cdot|$ reveals the cardinality of a set. The expression $\hat{\phi}_j[n](\theta_j \cup \{m\}) - \hat{\phi}_j[n](\theta_j)$ is the marginal contribution of SU m to SU combination $\theta_j \cup \{m\}$. Therefore, the Shapley value is the expected marginal contribution of SU m to coalition $\Theta_j[n]$. The Shapley value satisfies the efficiency condition [76], i.e., $\sum_{m \in \Theta_j[n]} \varpi_{m\langle \mathbf{I}_j \rangle}[n] = \hat{\varpi}_j[n]$. Meanwhile, the Shapley value always exists if a coalitional game with transferable utilities is superadditive [76] and this condition is satisfied in $\kappa_j[n]$.

According to the payoff distributed by the Shapley value, we can figure out the charging prices to the SUs in each step. Specifically,

$$\lambda_{m\langle \mathbf{I}_j \rangle}[n] = \mu_m(q(\Theta_j), k_{m,n-1} + 1) - \varpi_{m\langle \mathbf{I}_j \rangle}[n] \quad (4.18)$$

for all $m \in \Theta_j[n]$, where $\lambda_{m\langle \mathbf{I}_j \rangle}[n]$ is the charging price to SU m in the nth step.

Proposition 4.2. *Under incomplete information, an SU may lower its shared channel cost $\lambda_{m\langle \mathbf{I}_j \rangle}[n]$ in step n through channel devaluation, i.e., reporting $\mu_m(q(\theta_j), k_{m,n-1} + 1) - \varrho(\theta_j)$ instead of $\mu_m(q(\theta_j), k_{m,n-1} + 1)$ to the coalition, where $\varrho(\theta_j)$ is the devaluation in θ_j.*

Proof. See Appendix C.1. □

In this chapter, we assume that the SUs will share their channel valuations with each other under the promise of fair payoff distribution in a coalitional game. Without the assumption, i.e., under incomplete information, an SU does have the chance to lower its shared charging price for a channel through channel devaluation according to Proposition 4.2. However, the restraint of the SU providing devalued bid under incomplete information is that the coalition in which the SU is included may end up not winning a channel because of the devaluation, or the SU may be excluded from the coalition.

The prerequisite for applying (4.17) and (4.18) is the knowledge of $\hat{\varpi}_j[n]$, where $\hat{\varpi}_j[n] = \hat{b}_j[n] - \hat{\lambda}_j[n]$ and $\hat{\lambda}_j[n]$ is the charging price to $\mathbf{I}_j$ in the nth step. However, $\hat{\lambda}_j[n]$ cannot be determined according to the decision tree since the decision process only feeds the charging price $\hat{\lambda}_j^{\hat{l}_j}$ back for all the $\hat{l}_j$ channels obtained by $\mathbf{I}_j$.

To address the problem, we assume that each coalition of $\mathbf{I}_j$ that obtains a channel enjoys the payoff $\hat{\varpi}_j^{\hat{l}_j}$ equally, i.e., $\hat{\varpi}_j[n] = \frac{1}{\hat{l}_j}(\sum_{\{n|\hat{e}_j[n]=1\}} \hat{b}_j[n] - \hat{\lambda}_j^{\hat{l}_j})$. Therefore, the charging prices to the SUs in each coalition, determined by the Shapley value, are believed to be fair, and hence the SUs prefer to join the coalitions rather than being excluded. The charging price to SU m in $\mathbf{I}_j$ is

$$\lambda_{m\langle\mathbf{I}_j\rangle} = \sum_{\{n|e_{m\langle\mathbf{I}_j\rangle}[n]=1\}} \lambda_{m\langle\mathbf{I}_j\rangle}[n]. \tag{4.19}$$

Therefore, SU m needs to pay $\sum_{j=1}^{J} \lambda_{m\langle\mathbf{I}_j\rangle}$ for all its obtained channels.

In a simplified scenario where the grand coalition forms in each step, i.e., $\Theta_j = \mathbf{I}_j$, the charging prices to the SUs can be calculated precisely. When the grand coalition forms, the payoff distribution to SU m in $\Theta_j = \mathbf{I}_j$ in the nth step is

$$\varpi_{m\langle\mathbf{I}_j\rangle}[n] = \frac{1}{|\mathbf{I}_j|} \sum_{\theta_j \subseteq \mathbf{I}_j \backslash \{m\}} |\theta_j|!(|\mathbf{I}_j| - |\theta_j| - 1)![\hat{\phi}_j[n](\theta_j \cup \{m\}) - \hat{\phi}_j[n](\theta_j)] \tag{4.20}$$

for all $m \in \Theta_j[n]$. Accumulating the payoffs of SU m in the steps that $\mathbf{I}_j$ obtains some channels, we have

$$\sum_{\{n|\hat{e}_j[n]=1\}} \varpi_{m\langle\mathbf{I}_j\rangle}[n] = \frac{\hat{l}_j}{|\mathbf{I}_j|} \sum_{\theta_j \subseteq \mathbf{I}_j \backslash \{m\}} |\theta_j|!(|\mathbf{I}_j| - |\theta_j| - 1)![\hat{\phi}_j[n](\theta_j \cup \{m\}) - \hat{\phi}_j[n](\theta_j)] \tag{4.21}$$

since every SU in $\mathbf{I}_j$ obtains $\hat{l}_j$ channels. Let $\hat{\phi}_j^{\hat{l}_j}(\theta_j) = \hat{l}_j \hat{\phi}_j[n](\theta_j)$ and $\hat{\phi}_j^{\hat{l}_j}(\mathbf{I}_j) = \hat{\varpi}_j^{\hat{l}_j}$, we have

$$\varpi_{m\langle\mathbf{I}_j\rangle} = \frac{1}{|\mathbf{I}_j|} \sum_{\theta_j \subseteq \mathbf{I}_j \backslash \{m\}} |\theta_j|!(|\mathbf{I}_j| - |\theta_j| - 1)![\hat{\phi}_j^{\hat{l}_j}(\theta_j \cup \{m\}) - \hat{\phi}_j^{\hat{l}_j}(\theta_j)], \tag{4.22}$$

where $\varpi_{m\langle \mathbf{I}_j \rangle} = \sum_{\{n|\hat{e}_j[n]=1\}} \varpi_{m\langle \mathbf{I}_j \rangle}[n]$. Note that (4.22) is the payoff distribution to SU m in $\mathbf{I}_j$ in terms of the Shapley value in the decision tree. Therefore, the charging price to SU m in $\mathbf{I}_j$ is

$$\lambda_{m\langle \mathbf{I}_j \rangle} = \sum_{\{n|\hat{e}_j[n]=1\}} \mu_m(q(\mathbf{I}_j), k_{m,n-1}+1) - \varpi_{m\langle \mathbf{I}_j \rangle}. \tag{4.23}$$

The interaction among SUs in an AUC in each step of the decision tree to obtain a channel is cooperation rather than competition. The Shapley value as a payoff distribution mechanism does not affect channel allocation results in the decision tree. Instead, the Shapley value helps to form the coalition that holds the maximum accumulated valuation in each step through fair payoff distribution among the SUs in the coalition.

4.4 Low-Complexity Channel Allocation

The decision tree provides the PU a way to achieve the optimal channel allocation. However, the complexity of a decision tree is $O(J^1 + \cdots + J^N) \sim O(J^N)$ and we have to go through another J decision trees to determine the charging prices to the AUCs. In this section, we consider low-complexity algorithms to approach the optimal channel allocation. Since the decision tree can be easily transformed to a directed acyclic graph (DAG), we try to find the longest path in the DAG, which is a problem with linear time complexity. Then we compare the algorithm with a straightforward greedy algorithm and batch allocation.

4.4.1 Directed Acyclic Graph

We simplify the N-step decision tree to a directed graph $\mathscr{D} = (\Phi, \Psi)$. Specifically,

$$\Phi = \{\mathbf{I}[0], \mathbf{I}_1[1], \cdots, \mathbf{I}_J[1], \cdots, \mathbf{I}_1[N], \cdots, \mathbf{I}_J[N], \mathbf{I}[N+1]\}, \tag{4.24}$$

where $\mathbf{I}[0]$ and $\mathbf{I}[N+1]$ are the virtual source and end vertices in steps 0 and $N+1$, and $\mathbf{I}_j[k]$ represents AUC $\mathbf{I}_j$ in the nth step. The directed edge $(\mathbf{I}_j[n] \to \mathbf{I}_{j'}[n+1])$ exists for any $j, j' = 1, \cdots, J$, $n = 1, \cdots, N-1$, and the weight of the edge is $\hat{b}_{j'}[n+1]$. In addition, the directed edge from the source vertex $(\mathbf{I}[0] \to \mathbf{I}_j[1])$ and the directed edge to the end vertex $(\mathbf{I}_j[N] \to \mathbf{I}[N+1])$ are present for any $j = 1, \cdots, J$. The weight of $(\mathbf{I}[0] \to \mathbf{I}_j[1])$

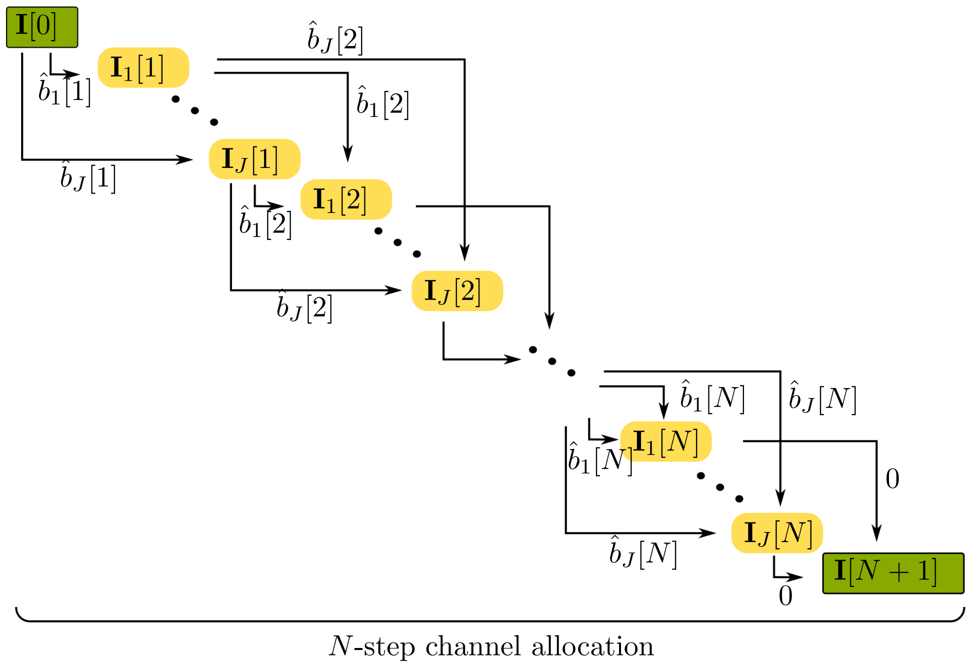


Figure 4.3. Topological sorting of the directed acyclic graph $\mathscr{D}$.

is $\hat{b}_j[1]$ and the weight of $(\mathbf{I}_j[N] \to \mathbf{I}[N+1])$ is zero. Note that there is no way to start from any vertex and return to the same vertex by following the directed edges in graph $\mathscr{D}$. Therefore, graph $\mathscr{D}$ is a DAG [78]. Instead of following the procedure of topological ordering, we can easily arrange all the vertices in a row from step 0 to $N+1$ to form a topological sorting of graph $\mathscr{D}$ [79], which is illustrated in Figure 4.3.

To approach the optimal channel allocation, we find the longest path from the source to end vertices in the DAG according to Algorithm 7. In Algorithm 7, $\text{dist}[\mathbf{I}_j[n]]$ is the longest distance from $\mathbf{I}[0]$ to $\mathbf{I}_j[n]$, $\text{path}[\mathbf{I}_j[n]]$ logs the vertices on the longest path to $\mathbf{I}_j[n]$, and $\text{weight}(\mathbf{I}_j[n], \mathbf{I}_{j'}[n+1])$ is the weight of the edge $(\mathbf{I}_j[n] \to \mathbf{I}_{j'}[n+1])$. The allocation result of Algorithm 7 is $\hat{\mathbf{l}}_{Acyclic} = (\hat{l}_{1,Acyclic}, \cdots, \hat{l}_{J,Acyclic})$, where $\hat{l}_{j,Acyclic}$ is the number of times that the longest path to $\mathbf{I}[N+1]$ crosses $\mathbf{I}_j$ in $\mathscr{D}$. The time complexity of Algorithm 7 is $O(\Psi) = O(NJ^2)$.

The application of the algorithm that finds the longest path in a DAG assumes

$$\text{dist}[\mathbf{I}_{j'}[n+1]] = \max_{\mathbf{I}_j[n]} \quad (\text{dist}[\mathbf{I}_j[n]] + \text{weight}(\mathbf{I}_j[n], \mathbf{I}_{j'}[n+1])). \tag{4.25}$$

In other words, we eliminate the non-longest paths to $\mathbf{I}_j[n]$ when we search for the longest path to $\mathbf{I}_{j'}[n+1]$, which makes the algorithm less complex than the decision tree. However, if there is a non-longest path to $\mathbf{I}_j[n]$ that turns out to be the longest path to $\mathbf{I}_{j'}[n+1]$, channel allocation of the algorithm that finds the longest path in a DAG will be sub-optimal. As long as the algorithm that finds the longest path in a DAG allocates channels optimally most of the time, the AUCs will still bid truthfully in the long run according to Proposition 4.3.

Proposition 4.3. *If sub-optimal channel allocation results occur occasionally in a series of constrained VCG auctions based on a low-complexity algorithm, the optimal strategy for the AUCs is still to bid truthfully.*

Proof. See Appendix C.2. □

Algorithm 7 Finding the longest path in graph $\mathscr{D}$.

Initialize: $\text{dist}[\beta] = 0$ for every vertex $\beta \in \Phi$, $\text{path}[\beta] = \{\mathbf{I}[0]\}$ for $\beta = \mathbf{I}_j[1]$, $j = 1, \cdots, J$.
for every vertex $\beta \in \Phi$ in topological order **do**
    for every adjacent vertex ζ of β **do**
        if $\text{dist}[\zeta] < \text{dist}[\beta] + \text{weight}(\beta, \zeta)$ **then**
            $\text{dist}[\zeta] = \text{dist}[\beta] + \text{weight}(\beta, \zeta)$
            $\text{path}[\zeta] = \text{path}[\beta] \cup \{\beta\}$
        end if
    end for
end for

4.4.2 Greedy Algorithm

The greedy algorithm can be applied to searching the longest path in the decision tree. In each step, the PU selects the AUC that submits the largest bid. Accordingly, the allocation outcome is $\hat{\mathbf{l}}_{Greedy} = (\hat{l}_{1,Greedy}, \cdots, \hat{l}_{J,Greedy})$. The time complexity of the greedy algorithm is $O(N)$. The greedy algorithm is straightforward and hence requires less computation. However, the greedy algorithm searches fewer paths in the decision tree in comparison with the algorithm that finds the longest path in a DAG, and hence is less reliable for the PU to approach the optimal channel allocation.

4.4.3 Batch Allocation

In batch allocation, the PU allocates all channels to selected AUCs in the first step, and then tries to increase the social welfare by replacing AUCs in a learning process. The process of batch allocation is described in Algorithm 8, where Λ_{max} controls the number of AUCs being replaced in each step to maintain stability. The channel allocation result is $\hat{\mathbf{l}}_{Batch} = (\hat{l}_{1,Batch}, \cdots, \hat{l}_{J,Batch})$. The batch allocation allocates all channels in the first step and therefore improves the computational efficiency. However, different from the algorithm that finds the longest path in a DAG and the greedy algorithm, untruthful bids are submitted in batch allocation. Here the batch allocation is introduced as a comparison with the algorithms in which the AUCs bid truthfully.

Algorithm 8 Batch allocation.

1: **Initialize**: AUC $\mathbf{I}_j$ submits its untruthful bid vector $\tilde{\mathbf{b}}_j = (\tilde{b}_j(1), \cdots, \tilde{b}_j(N))$ assuming the grand coalition will always form, i.e., $\tilde{b}_j(\hat{i}) = \sum_{m \in \mathbf{I}_j} \mu_m(q(\mathbf{I}_j), \hat{i})$. The PU chooses AUCs that hold the largest N $\tilde{b}_j(\hat{i})$ to allocate channels. The allocation result is $\hat{\mathbf{l}}_{Batch}$.
2: **Do**
3: Find the path in the decision tree that leads to $\hat{\mathbf{l}}_{Batch}$ and the truthful bids of the AUCs along the path. Let the lowest truthful bid for the current allocation be $\hat{b}[N]$.
4: The truthful bid of AUC j for the next channel is $\hat{b}_j(\hat{l}_{j,Batch} + 1)$, $j = 1, \cdots, J$. There are Λ AUCs whose $\hat{b}_j(\hat{l}_{j,Batch} + 1) > \hat{b}[N]$.
5: **if** $\Lambda \leq \Lambda_{max}$ **then**
6:     Substitute the last Λ nodes in the path for the Λ AUCs.
7: **else**
8:     Substitute the last Λ_{max} nodes in the path for the AUCs that hold the highest Λ_{max} $\hat{b}_j(\hat{l}_{\mathrm{j},Batch} + 1)$.
9: **end if**
10: Update the allocation result $\hat{\mathbf{l}}_{Batch}$.
11: **Repeat do until** $\Lambda = 0$ or the maximum number of steps is reached.

4.5 Simulation Results

In the simulation, we consider additive white Gaussian noise (AWGN) channels. The noise power spectrum density is $\varepsilon/2 = 3 \times 10^{-18}$ W/Hz. The transmit power of SU m is $p_m = 1$ W, for $m = 1, 2, \cdots, M$ and the interference threshold p_{thr} is 1×10^{-10} W. We adopt the two-ray model [63] such that $|h_{m',m}|^2 = \frac{\xi_{m',m}}{d^{\alpha}_{m',m}}$, where $\xi_{m',m}$ is a constant determined by the antenna parameters including $T_{m'}$ and R_m, $d_{m',m}$ is the distance between $T_{m'}$ and R_m,

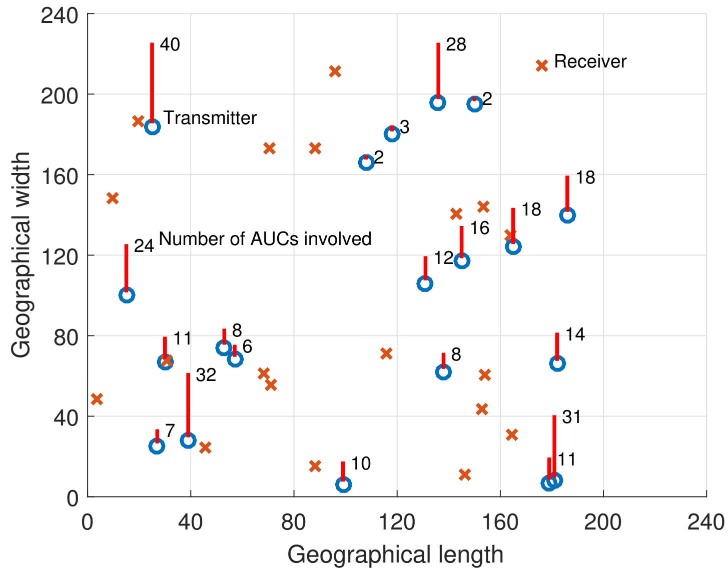


Figure 4.4. An illustration of random SU distribution in the simulation.

and α is the path loss exponent. To simulate the urban environment, we let $\xi_{m',m} = 10^6$ and $\alpha = 4$ [63].

In Figure 4.4, we illustrate a random SU distribution. There are 20 SU transmitter-receiver pairs in the area. The number of AUCs in the system is 64. At the upper right corner of each SU, we label the number of AUCs involving the SU. To avoid unacceptable co-channel interference, an SU in a crowded area is less likely to participate in an AUC. Therefore, the SUs in crowded areas have less chance to obtain channels in comparison with the SUs located in sparse areas.

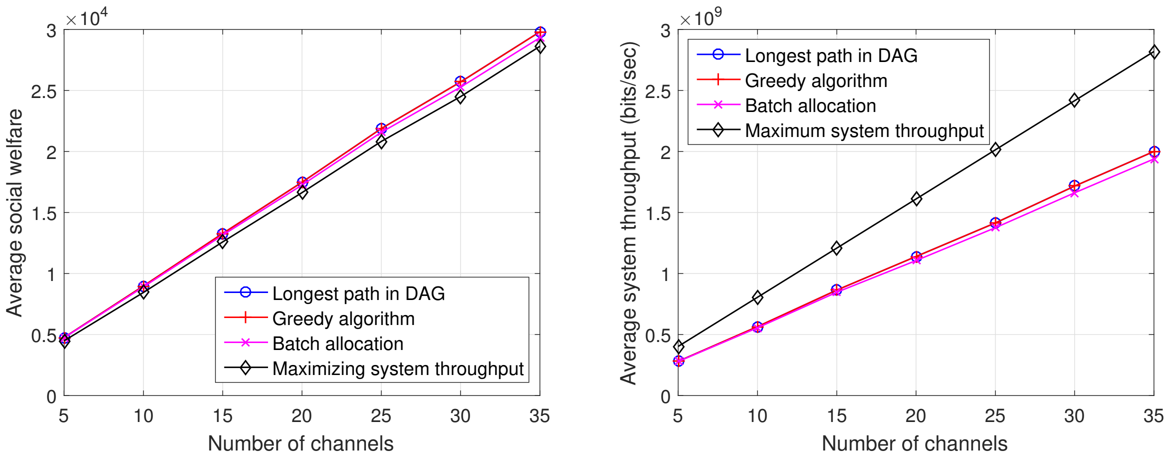
There are $Q-1$ data rate boundaries in the system with Q rate levels, and the $Q-1$ data rate boundaries are $\upsilon_{bound,1}, \cdots, \upsilon_{bound,Q-1}$, where $\upsilon_{bound,q} = \frac{q}{Q}\upsilon_{step}$ and

$$\upsilon_{step} = \frac{1}{M}(\sum_{m=1}^{M} \upsilon_{m,max} + \sum_{m=1}^{M} \upsilon_{m,min}), \tag{4.26}$$

where $\upsilon_{m,max} = W\log(1 + \frac{p_m|h_m|^2}{\mathcal{N}})$ indicates that SU m occupies a channel alone, and $\upsilon_{m,min} = W\log(1 + \frac{p_m|h_m|^2}{\mathcal{N}+p_{thr}})$ indicates the lowest achievable rate of SU m in the system. In consideration of the uncertainty and irregularity of the channel valuations of the SUs, we suppose that the descending valuations of an SU are random and follow uniform distribution. The valuations for channels at level q is between $100q$ and $100(q+1)$.

To study the performance of our low-complexity algorithms, we consider the case that the goal of the PU is to maximize the system throughput. Specifically, the PU allocates all channels to an SU combination that generates the highest achievable rate. We suppose that the channels are categorized by the SUs into high rate and low rate ones, i.e., $Q = 2$. We fix the number of SUs to 20 while changing the number of channels in the market from 5 to 35. The geographical distributions and channel valuations of the SUs are simulated 1,000 times. Figure 4.5a shows the average social welfare of each of the three low-complexity algorithms with respect to the number of channels. Compared with the social welfare associated with the maximum system throughput, the social welfares associated with all our low-complexity algorithms are higher. The algorithm that finds the longest path in the DAG and the greedy algorithm are very close to each other, and they both perform better than the batch allocation in terms of the social welfare. However, the algorithm that finds the longest path in the DAG and the greedy algorithm both need N steps to allocate all the channels while the batch allocation roughly allocates all channels in one step. In addition, Figure 4.5b shows the average system throughput of the three low-complexity algorithms with respect to the number of channels. When we pursue the maximization of the social welfare using our low-complexity algorithms, we sacrifice the system throughput. In comparison with the maximum system throughput, the decreases in system throughputs of all our low-complexity algorithms are within an acceptable range. Similar to the case when we consider the social welfare, the performance of the algorithm that finds the longest path in the DAG and that of the greedy algorithm are close, and they are both better than the batch allocation in terms of the system throughput.

Additionally, we set $Q = 2$ and fix the number of channels to 15, while changing the number of SUs from 10 to 30. Again, the geographical distributions and channel valuations of the SUs are simulated 1,000 times. Figure 4.6 shows the average social welfare and

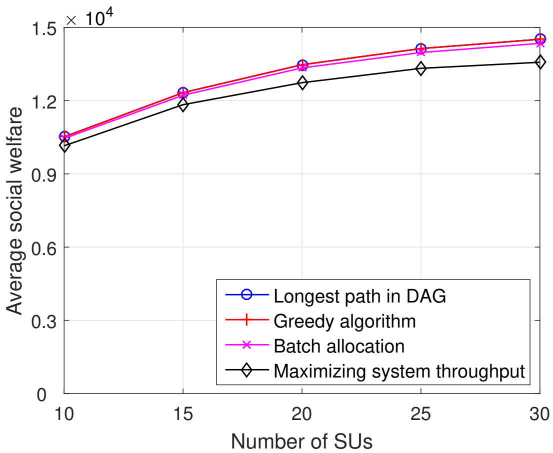


(a) Average social welfare. (b) Average system throughput.

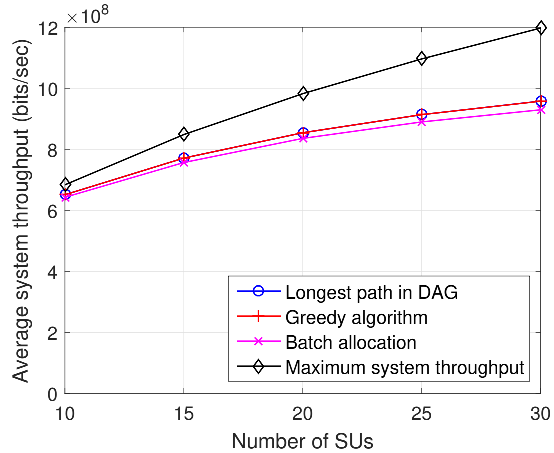
Figure 4.5. Average social welfare and average system throughput vs. number of channels when $M = 20$ and $Q = 2$.

average system throughput with respect to the number of SUs. In Figure 4.6a, the average social welfare of the channel allocation of the low-complexity algorithms is higher than the social welfare of the channel allocation that maximizes the system throughput. In Figure 4.6b, the average system throughput of the channel allocation of the low-complexity algorithms is lower but in an acceptable range in comparison with the maximum system throughput. In addition, the increase of both the social welfare and system throughput slows down with the increasing number of SUs in the system. When there are more SUs in the system, the number of AUCs increases rapidly according to the modified Bron-Kerbosch algorithm, while the average number of SUs included in each AUC is limited due to the co-channel interference constraint. Therefore, the social welfare and system throughput are both limited by the number of channels shared by the PU.

The social welfares with different numbers of total channel rate levels are not comparable since channel valuations of the SUs need to be adjusted for a particular number of total channel rate levels. On the other hand, changing the number of total channel rate levels in the system does not affect the measure of system throughput. Therefore, we compare the system throughputs with different numbers of total channel rate levels. In Figure 4.7, we show the average system throughput calculated by the algorithm that finds the longest



(a) Average social welfare.



(b) Average system throughput.

Figure 4.6. Average social welfare and average system throughput vs. number of SUs when $N = 15$ and $Q = 2$.

path in the DAG with respect to the number of channels while changing the number of total channel rate levels from 1 to 4, i.e., $Q = 1, 2, 3, 4$. For each setting, we change the number of channels in the market from 5 to 35, and run the random geographical distributions and channel valuations of the SUs 1,000 times, while setting the number of SUs to 20. The case $Q = 1$ indicates that the SUs are not sensitive to the data rates and hence channels are identical items, which is discussed in our previous work [80]. In comparison with the case $Q = 1$, the average system throughput increases with the increasing number of total channel rate levels. Therefore, the more channel rate levels in a system, the more emphasis the PU puts on the system throughput in the channel allocation. On the other hand, the fewer channel rate levels in a system, the more the PU focuses on the on-demand channel allocation. In addition, the channel allocation is also related to the number of total channel rate levels. In Figure 4.8a, we show the average number of winning SUs out of 20 SUs with respect to the number of channels. The channel allocation is obtained with the algorithm that finds the longest path in the DAG. In Figure 4.8b, we show the average number of channels obtained by each winning SU with respect to the number of channels. From these two figures, channels are allocated to a small amount of SUs in the case of $Q = 4$, while about two thirds of the SUs can obtain channels in the case of $Q = 1$. With the increasing

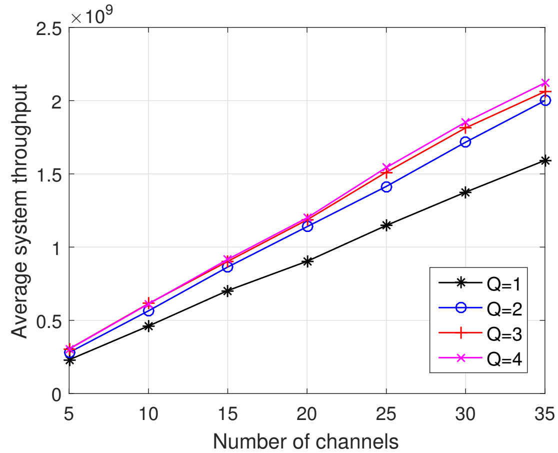
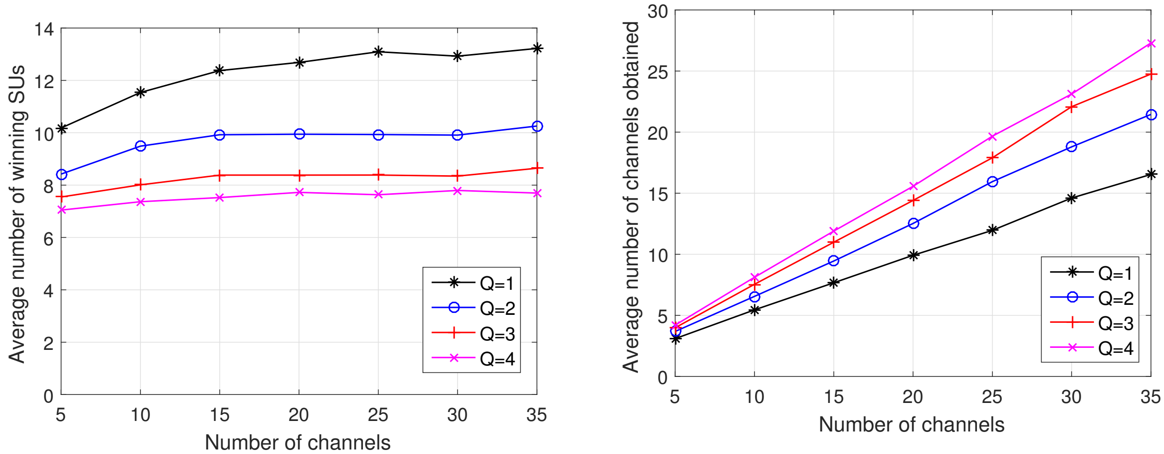


Figure 4.7. Average system throughput vs. number of channels calculated by the algorithm that finds the longest path in the DAG with different numbers of total channel rate levels. $M = 20$.

number of total channel rate levels, the channel allocation of the PU is concentrated to a smaller group of SUs who generate more system throughput.

The algorithms that we select are low-complexity ones to approach the optimal channel allocation, in comparison with searching the whole decision tree that has a time complexity $O(J^N)$. Whether the channel allocation outcome using a selected algorithm is optimal, i.e., the accuracy, is therefore our concern. To evaluate the accuracy of the algorithm that finds the longest path in the DAG in comparison with searching the decision tree, we examine the adjusted average social welfare after charging prices are determined. Ideally, positive utility is guaranteed in optimal channel allocation by the VCG auction process. In non-optimal channel allocation, payments required by the PU may exceed valuations of some AUCs under a certain condition, i.e., a low-complexity algorithm obtains a sub-optimal solution rather the optimal channel allocation when determining the charging price to an AUC. However, an AUC will refuse a channel transaction if the charging price exceeds the bid or the channel valuation. Therefore, the social welfare will be adjusted accordingly and become smaller than that in the ideal case. The gap between the social welfare in the ideal case and the adjusted average social welfare in consideration of charging prices is related to the accuracies of the low-complexity algorithms. In Figure 4.9, we show the ideal and



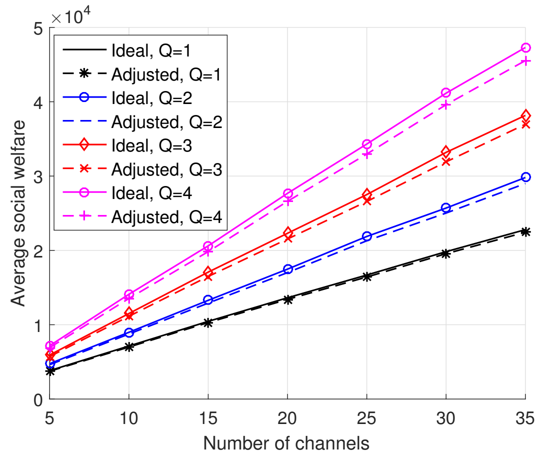
(a) Average number of winning SUs.

(b) Average number of channels of each winning SU.

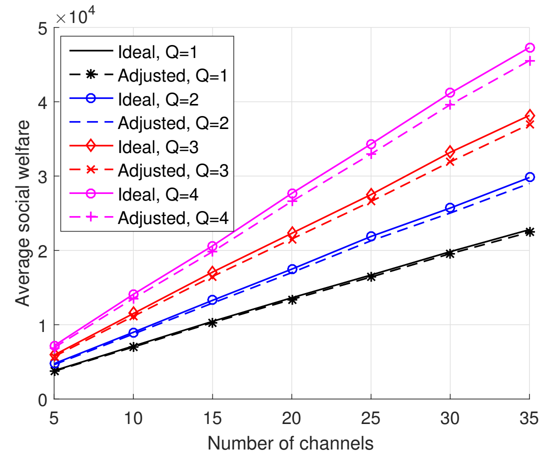
Figure 4.8. Average numbers of winning SUs and their obtained channels vs. number of channels when $M = 20$.

adjusted average social welfares of the algorithm that finds the longest path in the DAG and the greedy algorithm with respect to the number of channels when $Q = 1, 2, 3, 4$. For each setting, we fix the number of SUs to 20 while changing the number of channels in the market from 5 to 35. The geographical distributions and channel valuations of the SUs are simulated 1,000 times. The accuracies of the algorithm that finds the longest path in the DAG and the greedy algorithm are both acceptable in terms of the gaps between the ideal and adjusted social welfares with different Q. With the increasing numbers of channels and total channel rate levels, the accuracies of the algorithms decrease.

In a VCG auction, the VCG pricing mechanism motivates the AUCs to submit truthful bids and the social welfare maximization is to select the largest bids. In sub-optimal channel allocation schemes, the charging prices to the AUCs deviate from the VCG pricing. As a result, the AUCs lose the motivation to bid truthfully if sub-optimal channel allocation results occur frequently. However, sub-optimal channel allocation results only occur occasionally in a series of constrained VCG auctions based on the algorithm that finds the longest path in the DAG and the greedy algorithm as shown in Figure 9. Therefore, the AUCs are still motivated to bid truthfully in the long run in both low-complexity



(a) Longest path in the DAG.



(b) Greedy algorithm.

Figure 4.9. Ideal and adjusted average social welfares vs. number of channels. $M = 20$.

algorithms according to Proposition 4.3. In fact, this can be explained by the concept of Panopticon [81]. Auctions that result in optimal allocation outcomes can be understood as observations over the truthfulness of the AUCs, while auctions that result in sub-optimal allocation outcomes rarely occur. In other words, the AUCs are being watched continuously, in which untruthful AUCs will be punished frequently, i.e., losing potential payoffs, while most auctions allocate channels optimally in the algorithm that finds the longest path in the DAG and the greedy algorithm. Therefore, the AUCs always bid truthfully in the two low-complexity algorithms as if the auctions would result in optimal allocation outcomes at all times.

4.6 Conclusions

In this chapter, we design a secondary market for spatial spectrum reuse in a non-symmetric network, where satisfying the demands of the SUs is more important than the system throughput. Therefore, we consider the multi-level channel valuations of the SUs and maximize the social welfare accordingly. The PU allocates channels to the combinations of SUs at the channel rate levels that best satisfy the demands of the SUs, and avoids channel oversupply to specific SUs or at specific channel rate levels. We first find all AUCs of the SUs as candidates for the channel allocation of the PU. We also adopt the concept of the VCG auction process to maximize the social welfare. To cater to the bid formation process in the spatial spectrum sharing system, we transform the constrained VCG auction for AUCs to a step-by-step decision process, in which the AUCs can update their bids based on realtime channel supply status. To maximize the probability of obtaining a channel, a coalition of SUs forms in an AUC in each step. We then define the truthful bidding of an AUC as submitting the accumulated valuation of the coalition, which reflects the highest accumulated valuation of the subsets of the AUC in each step. The relationship of the SUs in a coalition is modeled as a coalitional game. We apply the Shapley value to fair payoff distribution among the SUs in coalitions. Furthermore, we approach the optimal channel allocation through finding the longest path in a directed acyclic graph, a greedy algorithm, and batch allocation. In our simulation, we compare the low-complexity algorithms and demonstrate the efficiency of our proposed channel transaction mechanism.

Chapter 5
Summary and Conclusions

To build a secondary market for spatial spectrum reuse, flexible channel valuations of SUs as channel users have to be considered and reflect different user locations, data rate requirements, and supply status. According to these factors, PUs as channel sellers decide their spectrum sharing strategies, including market structure, channel allocation, and pricing policy.

In this research, we study the impact of flexible channel valuations of SUs in spectrum sharing. Not only are SUs benefited from our research, but also PUs can increase their payoffs by estimating the behaviors of the SUs more accurately and pricing the channels more appropriately. We analyze the behaviors of SUs and PUs in consideration of flexible channel valuations, which helps the PUs to better estimate the behaviors of the SUs. Specifically, the research focuses on location-oriented spectrum sharing and channel auctions for spatial spectrum reuse.

For a spectrum sharing system using economic approaches, conventional models without geographic considerations are oversimplified. In Chapter 2, we develop a model where geographic information, including licensed areas of PUs and locations of SUs, plays an important role in the spectrum sharing system. We consider a multi-price policy and the pricing power of noncooperative PUs in multiple geographic areas. Meanwhile, the value assessment of a channel is price-related and the demand from the SUs is price-elastic. To maximize the payoffs of the PUs, we propose a unique quota transaction process. By applying an evolutionary procedure defined as replicator dynamics, we prove the existence and uniqueness of the evolutionary stable strategy quota vector of each PU, which leads to the optimal payoff for each PU selling channels without reserve. In the scenario of selling channels with reserve, we predict the channel prices for the PUs leading to the optimal supplies of the PUs and hence the optimal payoffs. Furthermore, we introduce a grouping mechanism to simplify the process. In the simulation, the effectiveness of the learning processes designed for the two scenarios is verified and our spectrum sharing scheme is shown efficient in utilizing the frequency resources.

To further increase spectrum utilization, the scenario of spatial spectrum reuse is considered in Chapter 3. Spatial spectrum reuse enables better utilization of limited spectral resources to achieve higher system throughput. However, improving the system throughput or spectrum efficiency does not necessarily translate to the satisfaction of more SUs according to their demands. Instead of focusing solely on spectrum efficiency, we consider maximizing social welfare via on-demand channel allocation, which better describes the overall satisfaction of the SUs when we involve the supply and demand relationship. We design a receiver-centric spectrum reuse mechanism, in which the optimal channel allocation that maximizes social welfare can be achieved by the constrained Vickrey-Clarke-Groves (VCG) auction for SUs. To simplify the constrained VCG auction for SUs, we group the SUs into maximal independent groups (MIGs) using a modified Bron-Kerbosch algorithm. We prove that truthful bidding is the optimal strategy for the SUs, even though the SUs do not participate in the VCG auction for MIGs directly. Therefore, the MIGs are bidding truthfully and the requirement for social welfare maximization is satisfied. We also prove that the optimal channel allocation is not unique and thus a weakly-dominant strategy for the primary user, and the VCG style pricing based on truthful bidding can be implemented by using a decision tree repeatedly. Furthermore, we approximate and simplify the optimal channel allocation with a greedy algorithm, Dijkstra's algorithm, and batch allocation. In our simulation, we compare the proposed methods and demonstrate that our on-demand channel allocation increases social welfare.

To further improve user satisfaction, user characteristics that enable heterogeneous channel valuations need to be considered in spatial spectrum reuse. In Chapter 4, we design a channel transaction mechanism for non-symmetric networks and maximize user satisfaction in consideration of multi-level flexible channel valuations of the SUs. Specifically, we introduce a constrained VCG auction, in which the participants are limited to the allowable user crowds (AUCs). To facilitate the bid formation, we transform the constrained VCG auction to a step-by-step decision process. Meanwhile, the SUs in a coalition play a coalitional game with transferable utilities. We use the Shapley value to realize fair payoff distribution among the SUs in a coalition. Furthermore, we approach the optimal channel allocation via finding the longest path in a directed acyclic graph, a greedy algorithm, and batch allocation. In our simulation, we compare the low-complexity algorithms and demonstrate the efficiency of the channel transaction mechanism.

Appendix A
Proofs in Chapter 2

A.1 Proof of Proposition 2.1

Proof. We know that $k_{mn}(\tau+1) = k_{mn}(\tau) + \Delta k_{mn}(\tau)$ and $\sum_{n \in \mathcal{N}_m^{\diamond}} k_{mn}(\tau) = S_m$. Hence,

$$\sum_{n \in \mathcal{N}_m^{\diamond}} k_{mn}(\tau+1) = \sum_{n \in \mathcal{N}_m^{\diamond}} k_{mn}(\tau) + \sum_{n \in \mathcal{N}_m^{\diamond}} \Delta k_{mn}(\tau).$$

According to (2.15),

$$\begin{aligned}\sum_{n \in \mathcal{N}_m^{\diamond}} \frac{\Delta k_{mn}(\tau)}{S_m} =& \mu_m \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau)) \\ & - 1 \cdot \mu_m \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}(\tau)) \\ =& 0\end{aligned}$$

so that $\sum_{n \in \mathcal{N}_m^{\diamond}} \Delta k_{mn}(\tau) = 0$. Therefore,

$$\sum_{n \in \mathcal{N}_m^{\diamond}} k_{mn}(\tau+1) = \sum_{n \in \mathcal{N}_m^{\diamond}} k_{mn}(\tau).$$

□

A.2 Proof of Proposition 2.2

Proof.

$$\begin{aligned}&\sum_{n \in \mathcal{N}_m^{\diamond}} \bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau)) \cdot \frac{\Delta k_{mn}(\tau)}{S_m} \\ =& \mu_m \cdot \sum_{n \in \mathcal{N}_m^{\diamond}} \frac{k_{mn}(\tau)}{S_m} \bar{\pi}_{mn}^2(k_{mn}(\tau), k_{-mn}(\tau)) \\ & - \mu_m [\sum_{n \in \mathcal{N}_m^{\diamond}} \frac{k_{mn}(\tau)}{S_m} \bar{\pi}_{mn}(k_{mn}(\tau), k_{-mn}(\tau))]^2.\end{aligned}$$

Note that $\sum_{n\in\mathcal{N}_m^\diamond}\frac{k_{mn}(\tau)}{S_m}=1$. From Jensen's inequality, we have

$$\begin{aligned}&\mu_m\cdot\sum_{n\in\mathcal{N}_m^\diamond}\frac{k_{mn}(\tau)}{S_m}\bar{\pi}_{mn}^2(k_{mn}(\tau),k_{-mn}(\tau))\\ \geq&\mu_m[\sum_{n\in\mathcal{N}_m^\diamond}\frac{k_{mn}(\tau)}{S_m}\bar{\pi}_{mn}(k_{mn}(\tau),k_{-mn}(\tau))]^2,\end{aligned}$$

and equality holds if and only if $\bar{\pi}_{mn}(k_{mn}(\tau),k_{-mn}(\tau))$, $n\in\mathcal{N}_m^\diamond$, equals each other. Therefore,

$$\sum_{n\in\mathcal{N}_m^\diamond}\bar{\pi}_{mn}(k_{mn}(\tau),k_{-mn}(\tau))\cdot\frac{\Delta k_{mn}(\tau)}{S_m}\geq 0.$$

□

A.3 Proof of Theorem 2.1

Proof. We prove the existence and uniqueness of $\mathbf{k}_m^*$ in the following two cases, respectively.

1. $\sum_{n=1}^N t_{mn}=1$.

   In this case, the mth PU has no choice but to sell all its channels to the only SU inside the licensed area. Apparently, $\mathbf{k}_m^*$ exists and is unique.

2. $\sum_{n=1}^N t_{mn}\geq 2$.

   Suppose $\mathbf{k}_{-m}^*$ are already set. According to (2.19), we have

   $$D_n\frac{e^{u_{mn}+\gamma(\psi_n-\rho_m)}}{\sum_{i=1}^M e^{u_{in}+\gamma(\psi_n-\rho_i^*)}}e^{\zeta_0\ln(\psi_n)-\zeta_1\ln(\rho_m)}-k_{mn}=0,\tag{A.1}$$

   where ρ_m is a uniform channel price set by the mth PU. Apparently, for any $k_{mn}\in(0,S_m)$, $\frac{\partial\rho_m}{\partial k_{mn}}<0$. Hence, the mth PU can always find $\rho_m^+>0$, and when $0<\rho_m<\rho_m^+$, $\sum_{n\in\mathcal{N}_m^\diamond}k_{mn}>S_m$ always holds. Similarly, the mth PU can always find $\rho_m^->\rho_m^+$, and when $\rho_m>\rho_m^-$, $\sum_{n\in\mathcal{N}_m^\diamond}k_{mn}<S_m$ always holds.

   Because of the continuity of (A.1), there always exists a price ρ_m^*, and when $\rho_m=\rho_m^*$, $\sum_{n\in\mathcal{N}_m^\diamond}k_{mn}=S_m$. Furthermore, because of the monotony of ρ_m on $k_{mn}\in(0,S_m)$, $n\in\mathcal{N}_m^\diamond$, ρ_m^* is the unique positive solution that satisfies (2.18) and (2.19).

Correspondingly, k^*_{mn}, $n \in \mathcal{N}^{\diamond}_m$, is the unique solution to (2.18) and (2.19). Therefore, we conclude that $\mathbf{k}^*_m$ always exists and is unique.

□

A.4 Proof of Theorem 2.2

Proof. In contract period τ, we denote $\mathcal{N}^+_m$ as the set of SU indices whose quotas will be increased, and $\mathcal{N}^-_m$ as the set of SU indices whose quotas will be decreased, and $\mathcal{N}^+_m \subset \mathcal{N}^{\diamond}_m$ and $\mathcal{N}^-_m \subset \mathcal{N}^{\diamond}_m$. According to Proposition 2.1,

$$\sum_{n^+ \in \mathcal{N}^+_m} \Delta k_{mn^+}(\tau) + \sum_{n^- \in \mathcal{N}^-_m} \Delta k_{mn^-}(\tau) = 0. \tag{A.2}$$

Apparently,

$$\bar{\pi}^-_m(\tau) < \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}^*_{-m}) < \bar{\pi}^+_m(\tau),$$

where $\bar{\pi}^-_m$ is the average channel payoff for the SUs in $\mathcal{N}^-_m$ and $\bar{\pi}^+_m$ is the average channel payoff for the SUs in $\mathcal{N}^+_m$. Specifically,

$$\bar{\pi}^-_m(\tau) = \frac{\sum_{n^- \in \mathcal{N}^-_m} k_{mn^-}(\tau) \cdot \bar{\pi}_{mn^-}(k_{mn^-}(\tau), k^*_{-mn^-})}{\sum_{n^- \in \mathcal{N}^-_m} k_{mn^-}(\tau)}$$

and

$$\bar{\pi}^+_m(\tau) = \frac{\sum_{n^+ \in \mathcal{N}^+_m} k_{mg^+}(\tau) \cdot \bar{\pi}_{mn^+}(k_{mn^+}(\tau), k^*_{-mn^+})}{\sum_{n^+ \in \mathcal{N}^+_m} k_{mn^+}(\tau)}.$$

If (2.20) holds, the quotas for the SUs in $\mathcal{N}^+_m$ will continue to be increased, and the quotas for the SUs in $\mathcal{N}^-_m$ will continue to be decreased. Therefore, we have $\bar{\pi}^+_m(\tau+1) < \bar{\pi}^+_m(\tau)$ and $\bar{\pi}^-_m(\tau+1) > \bar{\pi}^-_m(\tau)$. Meanwhile,

$$\bar{\pi}^-_m(\tau+1) < \bar{\pi}_m(\mathbf{k}_m(\tau+1), \mathbf{k}^*_{-m}) < \bar{\pi}^+_m(\tau+1).$$

Obviously, the value range of $\bar{\pi}_m$ is narrowed in contract period $\tau+1$.

Since (2.20) always holds, $\bar{\pi}_m^+$ and $\bar{\pi}_m^-$ both converge to $\bar{\pi}_m$. Specifically,

$$\lim_{\tau\to\infty} \bar{\pi}_m^-(\tau) = \lim_{\tau\to\infty} \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}_{-m}) = \lim_{\tau\to\infty} \bar{\pi}_m^+(\tau).$$

In the replicator dynamics, it is equivalent to

$$\begin{aligned}
&\lim_{\tau\to\infty} \bar{\pi}_{mn^-}(k_{mn^-}(\tau), k^*_{-mn^-}) \\
&= \lim_{\tau\to\infty} \bar{\pi}_m(\mathbf{k}_m(\tau), \mathbf{k}^*_{-m}) \\
&= \lim_{\tau\to\infty} \bar{\pi}_{mn^+}(k_{mn^+}(\tau), k^*_{-mn^+}),
\end{aligned}$$

i.e., $\lim_{\tau\to\infty} k_{mn^-}(\tau) = k^*_{mn^-}$ and $\lim_{\tau\to\infty} k_{mn^+}(\tau) = k^*_{mn^+}$, $n^+ \in \mathcal{N}_m^+$, and $n^- \in \mathcal{N}_m^-$. Therefore, $\mathbf{k}_m$ converges to $\mathbf{k}_m^*$ if (2.20) holds. □

A.5 Proof of Proposition 2.3

Proof. By simplifying (A.1), we have

$$k_{mn} = D_n \psi_n^{\zeta_0} \rho_m^{-\zeta_1} \frac{1}{1 + \mathcal{F}_{mn} e^{\gamma \rho_m}},$$

where

$$\mathcal{F}_{mn} = \frac{\sum_{\substack{i=1 \\ i\neq m}}^{M} t_{in} e^{u_{in} - \gamma p_{in}}}{t_{mn} e^{u_{mn}}}.$$

According to (2.11),

$$\begin{aligned}
\pi_m &= \sum_{n\in\mathcal{N}_m^\diamond} (\rho_m - C_T) k_{mn} - S_m C_m \\
&= \sum_{n\in\mathcal{N}_m^\diamond} D_n \psi_n^{\zeta_0} \frac{\rho_m^{-\zeta_1}(\rho_m - C_T)}{1 + \mathcal{F}_{mn} e^{\gamma \rho_m}} - S_m C_m.
\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial \pi_m}{\partial \rho_m} =& \rho_m^{-\zeta_1} \sum_{n \in \mathcal{N}_m^{\diamond}} D_n \psi_n^{\zeta_0} \frac{-\gamma \mathcal{F}_{mn} e^{\gamma \rho_m}}{(1+\mathcal{F}_{mn} e^{\gamma \rho_m})^2} \\ &+ \zeta_1 \rho_m^{-\zeta_1 - 1} C_T \sum_{n \in \mathcal{N}_m^{\diamond}} D_n \psi_n^{\zeta_0} \frac{-\gamma \mathcal{F}_{mn} e^{\gamma \rho_m}}{(1+\mathcal{F}_{mn} e^{\gamma \rho_m})^2} \\ &- \zeta_1 \rho_m^{-\zeta_1} \sum_{n \in \mathcal{N}_m^{\diamond}} D_n \psi_n^{\zeta_0} \frac{-\gamma \mathcal{F}_{mn} e^{\gamma \rho_m}}{(1+\mathcal{F}_{mn} e^{\gamma \rho_m})^2}.\end{aligned}$$

Let $\frac{\partial \pi_m}{\partial \rho_m} = 0$, we have

$$-\zeta_1 \rho_m^{-\zeta_1 - 1}(\rho_m - C_T) + \rho_m^{-\zeta_1} = 0,$$

or $\rho_m = \frac{\zeta_1}{\zeta_1 - 1} C_T$, which completes the proof. $\square$

Appendix B
Proofs in Chapter 3

B.1 Proof of Proposition 3.1

Proof. Suppose that MIG $\mathbf{I}_j$ submits a truthful bidding vector, i.e., $\hat{\mathbf{b}}_{\mathbf{I}_j} = (\hat{\mu}_{\mathbf{I}_j}(1), \cdots, \hat{\mu}_{\mathbf{I}_j}(N))$, and obtains $\hat{l}_{\mathbf{I}_j}$ channels in the VCG auction for MIGs. The truthful bids of MIG $\mathbf{I}_j$ indicate that each SU in $\mathbf{I}_j$ is bidding truthfully, i.e., $\mathbf{b}_{m<\mathbf{I}_j>} = (\mu_{m<\mathbf{I}_j>}(1), \cdots, \mu_{m<\mathbf{I}_j>}(N))$ for any $m \in \mathbf{I}_j$. The buyer's payoff of $\mathbf{I}_j$ by bidding truthfully is

$$\begin{aligned}
\hat{\varpi}_{\mathbf{I}_j} &= \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \hat{\mu}_{\mathbf{I}_j}(\hat{i}) - \hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}} \\
&= \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \hat{\mu}_{\mathbf{I}_j}(\hat{i}) - [\hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}].
\end{aligned} \tag{B.1}$$

Let $\varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}}$ denote the buyer's payoff of SU m in $\mathbf{I}_j$ by bidding truthfully. Specifically,

$$\begin{aligned}
\varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} &= \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \mu_{m<\mathbf{I}_j>}(\hat{i}) - \lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} = \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} \mu_{m<\mathbf{I}_j>}(\hat{i}) \\
&- \frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}(\hat{i})}{\sum_{m\in\mathbf{I}_j} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}(\hat{i})} [\hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}].
\end{aligned} \tag{B.2}$$

We then slightly increase the bids of SU m and set the bid vector as $\mathbf{b}_{m<\mathbf{I}_j>}^{+}$, while the bids of other SUs in $\mathbf{I}_j$ remain the same. As a result, the bidding vector of $\mathbf{I}_j$ also increases. Accordingly, $\mathbf{I}_j$ will obtain $\hat{l}_{\mathbf{I}_j}^{+}$ channels when the bids of the other MIGs are unchanged.

1. If $\hat{l}_{\mathbf{I}_j}^{+} = \hat{l}_{\mathbf{I}_j}$, the buyer's payoff of SU m is

$$\varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^{+}} = \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}^{+}} \mu_{m<\mathbf{I}_j>}(\hat{i}) - \lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^{+}},$$

where

$$\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^{+}} = \frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}^{+}(\hat{i})}{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}^{+}(\hat{i}) + \sum_{\substack{m'\in\mathbf{I}_j\\ m'\neq m}} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m'<\mathbf{I}_j>}(\hat{i})} \cdot [\hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}}].$$

Therefore, $\varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^{+}} < \varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}}$. In this case, the bid increment of SU m does not bring an extra channel but adds its own cost.

2. If $\hat{l}_{\mathbf{I}_j}^{+} = \hat{l}_{\mathbf{I}_j} + 1$, the buyer's payoff of MIG $\mathbf{I}_j$ is

$$\begin{aligned}\hat{\varpi}_{\mathbf{I}_j}^{+} &= \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}^{+}} \hat{\mu}_{\mathbf{I}_j}(\hat{i}) - \hat{\lambda}_{\mathbf{I}_j}^{\hat{l}_{\mathbf{I}_j}^{+}} \\ &= \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}+1} \hat{\mu}_{\mathbf{I}_j}(\hat{i}) - [\hat{V}_{\mathbf{J}\setminus\{\mathbf{I}_j\}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} - (\hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}})'].\end{aligned}$$

The payoff difference of $\mathbf{I}_j$ is

$$\begin{aligned}\hat{\varpi}_{\mathbf{I}_j}^{+} - \hat{\varpi}_{\mathbf{I}_j} &= \hat{\mu}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}+1) + (\hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}})' - \hat{V}_{\mathbf{J}}^{\mathbf{J}\setminus\{\mathbf{I}_j\}} \\ =&\hat{\mu}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}+1) + \sum_{j'\neq j}\sum_{\hat{i}=1}^{\hat{l}'_{\mathbf{I}_{j'}}} \hat{b}_{\mathbf{I}_{j'}}(\hat{i}) - \sum_{j'\neq j}\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_{j'}}} \hat{b}_{\mathbf{I}_{j'}}(\hat{i}),\end{aligned}$$

where $\hat{l}_{\mathbf{I}_{j'}}$ is the number of channels that $\mathbf{I}_{j'}$ has obtained when $\mathbf{I}_j$ is bidding truthfully, and $\hat{l}'_{\mathbf{I}_{j'}}$ is the number of channels that $\mathbf{I}_{j'}$ obtains after the bid change of $\mathbf{I}_j$. We assume that the extra channel obtained by MIG $\mathbf{I}_j$ has originally been allocated to MIG $\mathbf{I}_\beta$, i.e., $\hat{l}'_{\mathbf{I}_\beta} = \hat{l}_{\mathbf{I}_\beta} - 1$. We have

$$\hat{\varpi}_{\mathbf{I}_j}^{+} - \hat{\varpi}_{\mathbf{I}_j} = \hat{\mu}_{\mathbf{I}_j}(\hat{l}_{\mathbf{I}_j}+1) - \hat{b}_{\mathbf{I}_\beta}(\hat{l}_{\mathbf{I}_\beta}) < 0.$$

Otherwise $\mathbf{I}_j$ would obtain $\hat{l}_{\mathbf{I}_j}+1$ channels when bidding truthfully. The payoff of $\mathbf{I}_j$ decreases when SU m increases its bids. Meanwhile, only the payment share of SU m increases according to (4.18). Therefore, we have $\varpi_{m<\mathbf{I}_j>}^{\hat{l}^{+}_{\mathbf{I}_j}} < \varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}}$. In other words, the bid increment reduces the payoff of SU m, despite an extra channel.

It is straightforward to prove that if we increase the bids of SU m further, the payoff gap between truthful bidding and non-truthful bidding will be larger. Therefore, SU m does not bid more than its truthful bids. □

B.2 Proof of Proposition 3.2

Proof. Under the same set-up as in Appendix B.1, MIG $\mathbf{I}_j$ obtains $\hat{l}_{\mathbf{I}_j}$ channels through truthful bidding, and the buyer's payoff of $\mathbf{I}_j$ and that of SU m in $\mathbf{I}_j$ by bidding truthfully is (B.1) and (B.2), respectively. We then slightly decrease the bids of SU m and set the bid vector as $\mathbf{b}^{-}_{m<\mathbf{I}_j>} = (b^{-}_{m<\mathbf{I}_j>}(1), \cdots, b^{-}_{m<\mathbf{I}_j>}(N))$, while the bids of other SUs in $\mathbf{I}_j$ remain the same. As a result, the bidding vector of $\mathbf{I}_j$ also decreases. Accordingly, $\mathbf{I}_j$ will obtain $\hat{l}^{-}_{\mathbf{I}_j}$ channels when the bids of the other MIGs are unchanged. Let $\hat{l}^{-}_{\mathbf{I}_j} = \hat{l}_{\mathbf{I}_j} - \delta$, $\delta = 0, \cdots, \hat{l}_{\mathbf{I}_j}$. The buyer's payoff of SU m is

$$\varpi_{m<\mathbf{I}_j>}^{\hat{l}^{-}_{\mathbf{I}_j}} = \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}-\delta} \mu_{m<\mathbf{I}_j>}(\hat{i}) - \lambda_{m<\mathbf{I}_j>}^{\hat{l}^{-}_{\mathbf{I}_j}},$$

where

$$\lambda_{m<\mathbf{I}_j>}^{\hat{l}^{-}_{\mathbf{I}_j}} = \frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b^{-}_{m<\mathbf{I}_j>}(\hat{i})}{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b^{-}_{m<\mathbf{I}_j>}(\hat{i}) + \sum_{\substack{m'\in\mathbf{I}_j \\ m'\neq m}} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m'<\mathbf{I}_j>}(\hat{i})} \cdot [\hat{V}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}_{\mathbf{J}\backslash\{\mathbf{I}_j\}} - (\hat{V}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}_{\mathbf{J}})''].$$

The payoff difference of SU m is

$$\begin{aligned}
&\varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^-} - \varpi_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} \\
&= - \sum_{\hat{i}=\hat{l}_{\mathbf{I}_j}-\delta}^{\hat{l}_{\mathbf{I}_j}} \mu_{m<\mathbf{I}_j>}(\hat{i}) + (\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} - \lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^-}).
\end{aligned}$$

Since the virtual losses of the other MIGs decrease when $\mathbf{I}_j$ obtains fewer channels, $[\hat{V}_{\mathbf{J}\backslash\{\mathbf{I}_j\}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}} - (\hat{V}_{\mathbf{J}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}})'']< [\hat{V}_{\mathbf{J}\backslash\{\mathbf{I}_j\}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}} - \hat{V}_{\mathbf{J}}^{\mathbf{J}\backslash\{\mathbf{I}_j\}}]$. In addition, we have

$$\begin{aligned}
&\frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}^-(\hat{i})}{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}^-(\hat{i}) + \sum_{\substack{m'\in\mathbf{I}_j \\ m'\neq m}} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m'<\mathbf{I}_j>}(\hat{i})} \\
<&\frac{\sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}(\hat{i})}{\sum_{m\in\mathbf{I}_j} \sum_{\hat{i}=1}^{\hat{l}_{\mathbf{I}_j}} b_{m<\mathbf{I}_j>}(\hat{i})}.
\end{aligned}$$

Therefore, $\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} - \lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^-} > 0$, i.e., the channel cost of SU m reduces. However, it is not guaranteed that SU m can benefit from untruthful bidding. While bidding less than the truthful bids reduces channel cost, SU m does risk losing some or all of its obtained channels. The payoff of SU m can either increase or decrease through untruthful bidding, depending on if the reduced channel cost exceeds the channel valuations of the lost channels.

In the VCG auction for MIGs, the gaps between the winning bids and close-to-winning bids are only known to the PU, since the VCG auction for MIGs is a sealed-bid process. Most importantly, the winning bids and close-to-winning bids are very close due to the severe competition among the MIGs. As a result, the space for SU m to lower its bids while keeping its obtained channels is limited. Therefore, the reduced channel cost is always less than the channel valuations of the lost channels, i.e., $\lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}} - \lambda_{m<\mathbf{I}_j>}^{\hat{l}_{\mathbf{I}_j}^-} < \sum_{\hat{i}=\hat{l}_{\mathbf{I}_j}-\delta}^{\hat{l}_{\mathbf{I}_j}} \mu_{m<\mathbf{I}_j>}(\hat{i})$. As a result, an SU does not bid less than its truthful bids in the VCG auction for MIGs.

□

B.3 Proof of Proposition 3.6

Proof. We examine a two-step decision tree, which corresponds to $N = 2$. $\hat{b}_{\mathbf{I}_j}[1]$ is the highest bid when $k = 1$. After $\mathbf{I}_j$ is selected, $\hat{b}_{\mathbf{I}_{j'}}[2]$ is the highest bid when $k = 2$. The social welfare is

$$(\pi + \varpi) = \sum_{m \in \mathbf{I}_j \setminus \mathbf{I}_{j'}} \mu_m(1) + \sum_{m \in \mathbf{I}_{j'} \setminus \mathbf{I}_j} \mu_m(1) + \sum_{m \in \mathbf{I}_j \cap \mathbf{I}_{j'}} (\mu_m(1) + \mu_m(2)),$$

where $\mathbf{I}_j \setminus \mathbf{I}_{j'}$ is the set of SUs in $\mathbf{I}_j$ but not in $\mathbf{I}_{j'}$.

Different from choosing the MIG with the highest bid, we select $\mathbf{I}_{j'}$ when $k = 1$, and $\mathbf{I}_{j''}$ when $k = 2$. Suppose $\mathbf{I}_{j'} \cap \mathbf{I}_{j''} = \emptyset$. The social welfare is

$$(\pi + \varpi)' = \sum_{m \in \mathbf{I}_{j'}} \mu_m(1) + \sum_{m \in \mathbf{I}_{j''}} \mu_m(1).$$

We have

$$(\pi + \varpi) - (\pi + \varpi)' = \sum_{m \in \mathbf{I}_j \setminus (\mathbf{I}_{j'} \cup \mathbf{I}_{j''})} \mu_m(1) + \sum_{m \in \mathbf{I}_j \cap \mathbf{I}_{j'}} \mu_m(2) - \sum_{m \in \mathbf{I}_{j''} \setminus \mathbf{I}_j} \mu_m(1).$$

Meanwhile, since $\hat{b}_{\mathbf{I}_j}[1] > \hat{b}_{\mathbf{I}_{j''}}[1]$, we have

$$\sum_{m \in \mathbf{I}_j \setminus (\mathbf{I}_{j'} \cup \mathbf{I}_{j''})} \mu_m(1) + \sum_{m \in \mathbf{I}_j \cap \mathbf{I}_{j'}} \mu_m(1) > \sum_{m \in \mathbf{I}_{j''} \setminus \mathbf{I}_j} \mu_m(1).$$

Therefore, when

$$
\begin{aligned}
&\sum_{m \in \mathbf{I}_j \setminus (\mathbf{I}_{j'} \cup \mathbf{I}_{j''})} \mu_m(1) + \sum_{m \in \mathbf{I}_j \cap \mathbf{I}_{j'}} \mu_m(2) \\
&< \sum_{m \in \mathbf{I}_{j''} \setminus \mathbf{I}_j} \mu_m(1) < \sum_{m \in \mathbf{I}_j \setminus (\mathbf{I}_{j'} \cup \mathbf{I}_{j''})} \mu_m(1) + \sum_{m \in \mathbf{I}_j \cap \mathbf{I}_{j'}} \mu_m(1),
\end{aligned}
$$

choosing the MIG with the highest bid does not maximize the social welfare.

When $N > 2$ and $\mathbf{I}_{j'} \cap \mathbf{I}_{j''} \neq \emptyset$, similar cases can be found, which completes the proof of Proposition 3.6. □

Appendix C
Proofs in Chapter 4

C.1 Proof of Proposition 4.2

Proof. We suppose that there is a coalition $\Theta_j[n]$ in $\mathbf{I}_j$ composed of two SUs in the nth step. If both SUs report their truthful channel valuations, the payoff functions associated with all subsets of $\Theta_j[n] = \{1, 2\}$ are

$$\hat{\phi}_j[n](\{1\}) = f_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1} + 1)) \cdot \mu_1(q(\{1\}), k_{1,n-1} + 1),$$

$$\hat{\phi}_j[n](\{2\}) = f_{\{1,2\}}(\mu_2(q(\{2\}), k_{2,n-1} + 1)) \cdot \mu_2(q(\{2\}), k_{2,n-1} + 1),$$

$$\hat{\phi}_j[n](\{1, 2\}) = \mu_1(q(\{1, 2\}), k_{1,n-1} + 1) + \mu_2(q(\{1, 2\}), k_{2,n-1} + 1),$$

and $\hat{\phi}_j[n](\emptyset) = 0$. Therefore, the payoff distribution to SU 1 according to the Shapley value is

$$\begin{aligned}\varpi_{1\langle\mathbf{I}_j\rangle}[n] =& \frac{1}{2}(\hat{\phi}_j[n](\{1\}) - \hat{\phi}_j[n](\emptyset)) + \frac{1}{2}(\hat{\phi}_j[n](\{1, 2\}) - \hat{\phi}_j[n](\{2\}))\\ =& \frac{1}{2}[\mu_1(q(\{1, 2\}), k_{1,n-1} + 1) + \mu_2(q(\{1, 2\}), k_{2,n-1} + 1)\\ &+ f_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1} + 1)) \cdot \mu_1(q(\{1\}), k_{1,n-1} + 1)\\ &- f_{\{1,2\}}(\mu_2(q(\{2\}), k_{2,n-1} + 1)) \cdot \mu_2(q(\{2\}), k_{2,n-1} + 1)],\end{aligned}$$

and the charging price to SU 1 is

$$\lambda_{1\langle\mathbf{I}_j\rangle}[n] = \mu_1(q(\{1, 2\}), k_{1,n-1} + 1) - \varpi_{1\langle\mathbf{I}_j\rangle}[n].$$

Then we suppose that SU 1 will devalue its channel valuation, while SU 2 continues to report truthful channel valuation. In this case, the payoff functions associated with all subsets of $\Theta_j[n]$ are

$$\hat{\phi}'_j[n](\{1\}) = f'_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1}+1) - \varrho(\{1\})) \cdot \mu_1(q(\{1\}), k_{1,n-1}+1),$$

$$\hat{\phi}'_j[n](\{2\}) = f'_{\{1,2\}}(\mu_2(q(\{2\}), k_{2,n-1}+1)) \cdot \mu_2(q(\{2\}), k_{2,n-1}+1),$$

$$\hat{\phi}'_j[n](\{1,2\}) = \mu_1(q(\{1,2\}), k_{1,n-1}+1)) - \varrho(\{1,2\}) + \mu_2(q(\{1,2\}), k_{2,n-1}+1),$$

and $\hat{\phi}'_j[n](\varnothing) = 0$, where $f'_{\{1,2\}}(\cdot)$ is the adjusted winning probability function. Since $\hat{\phi}_j[n](\Theta_j[n]) = \hat{\varpi}_j[n]$, we have

$$\begin{aligned}
&f_{\{1,2\}}(\mu_1(q(\{1,2\}), k_{1,n-1}+1)) + \mu_2(q(\{1,2\}), k_{2,n-1}+1)) \\
=&f'_{\{1,2\}}(\mu_1(q(\{1,2\}), k_{1,n-1}+1)) - \varrho(\{1,2\}) + \mu_2(q(\{1,2\}), k_{2,n-1}+1)) = 1.
\end{aligned}$$

Note that $f'_{\{1,2\}}$ is adjusted based on $f_{\{1,2\}}$, and hence $f_{\{1,2\}}$ and $f'_{\{1,2\}}$ are both increasing convex functions that belong to the same family of functions. Therefore, we have $f'_{\{1,2\}}(\mu) > f_{\{1,2\}}(\mu)$. In this case, the payoff distribution to SU 1 when SU 1 devalues its valuation is

$$\begin{aligned}
\varpi'_{1\langle \mathbf{I}_j \rangle}[n] =&\frac{1}{2}(\hat{\phi}'_j[n](\{1\}) - \hat{\phi}'_j[n](\varnothing)) + \frac{1}{2}(\hat{\phi}'_j[n](\{1,2\}) - \hat{\phi}'_j[n](\{2\})) \\
=&\frac{1}{2}[\mu_1(q(\{1,2\}), k_{1,n-1}+1) - \varrho(\{1,2\}) + \mu_2(q(\{1,2\}), k_{2,n-1}+1) \\
&+f'_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1}+1) - \varrho(\{1\})) \cdot [\mu_1(q(\{1\}), k_{1,n-1}+1) - \varrho(\{1\})] \\
&-f'_{\{1,2\}}(\mu_2(q(\{2\}), k_{2,n-1}+1)) \cdot \mu_2(q(\{2\}), k_{2,n-1}+1)],
\end{aligned}$$

and the charging price to SU 1 is

$$\lambda'_{1\langle \mathbf{I}_j \rangle}[n] = \mu_1(q(\{1,2\}), k_{1,n-1}+1) - \varrho(\{1,2\}) - \varpi'_{1\langle \mathbf{I}_j \rangle}[n].$$

The charging price difference is

$$\lambda_{1\langle \mathbf{I}_j \rangle}[n] - \lambda'_{1\langle \mathbf{I}_j \rangle}[n] = \frac{1}{2}[\varrho(\{1,2\}) + \delta\mu_1(q(\{1,2\}), k_{1,n-1}+1) \\ -\delta\varrho(\{1\}) - \varrho(\{1\}) f_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1}+1)], \tag{C.1}$$

where $\delta = f'_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1}+1) - \varrho(\{1\})) - f_{\{1,2\}}(\mu_1(q(\{1\}), k_{1,n-1}+1))$. According to (C.1), SU 1 can manipulate its shared charging price through channel devaluation in every subset of $\Theta_j[n]$ where SU 1 is involved. When $\lambda_{1\langle \mathbf{I}_j \rangle}[n] - \lambda'_{1\langle \mathbf{I}_j \rangle}[n] > 0$, SU 1 is benefited from channel devaluation. For example, when $\varrho(\{1\}) = 0$ such that $\delta > 0$, we have $\lambda_{1\langle \mathbf{I}_j \rangle}[n] - \lambda'_{1\langle \mathbf{I}_j \rangle}[n] = \frac{1}{2}[\varrho(\{1,2\}) + \delta\mu_1(q(\{1,2\}), k_{1,n-1}+1)] > 0$.

When more SUs are included in a coalition, an SU can still decrease its shared charging price through channel devaluation. However, it takes effort for the SU providing untruthful bid information in a large coalition to determine the profitable channel devaluation in every subset of the coalition. □

C.2 Proof of Proposition 4.3

Proof. For AUCs that cannot obtain any channel through truthful bidding, untruthful bidding only leads to negative payoffs, i.e., the payment exceeds channel valuation. Therefore, only the AUCs that can obtain channels are considered and there are four different scenarios.

1. AUC $\mathbf{I}_j$ bids truthfully, and the channel allocation outcome is optimal. The payoff of $\mathbf{I}_j$ is $\hat{\varpi}_j$ and $\hat{\varpi}_j > 0$.

2. AUC $\mathbf{I}_j$ bids truthfully, but the channel allocation outcome is sub-optimal. The payoff of $\mathbf{I}_j$ is $\hat{\varpi}_j + \Delta\varpi_{j,1}$, where $\Delta\varpi_{j,1}$ is the adjusted payoff in this scenario.

3. AUC $\mathbf{I}_j$ bids untruthfully, and the channel allocation outcome is optimal according to the submitted bids. The payoff of $\mathbf{I}_j$ is $\hat{\varpi}_j + \Delta\varpi_{j,2}$, where $\Delta\varpi_{j,2}$ is the adjusted payoff in this scenario. Since untruthful bidding is not DSIC strategy in the constrained

VCG auction, $\Delta\varpi_{j,2} \leq 0$.

4. AUC $\mathbf{I}_j$ bids untruthfully, and the channel allocation outcome is sub-optimal. The payoff of $\mathbf{I}_j$ is $\hat{\varpi}_j + \Delta\varpi_{j,3}$, where $\Delta\varpi_{j,3}$ is the adjusted payoff in this scenario.

Denote ϑ the probability that a low-complexity algorithm results in the optimal channel allocation. If AUC $\mathbf{I}_j$ bids truthfully, the expected payoff in a low-complexity algorithm is

$$\bar{\varpi}_{j,truthful} = \hat{\varpi}_j + (1-\vartheta)\Delta\varpi_{j,1}.$$

If AUC $\mathbf{I}_j$ bids untruthfully, the expected payoff in a low-complexity algorithm is

$$\bar{\varpi}_{j,untruthful} = \hat{\varpi}_j + \vartheta\Delta\varpi_{j,2} + (1-\vartheta)\Delta\varpi_{j,3}.$$

Since $\Delta\varpi_{j,2} \leq 0$, we have $\bar{\varpi}_{j,truthful} \geq \bar{\varpi}_{j,untruthful}$ if ϑ approaches 1. In other words, if sub-optimal channel allocation results only occur occasionally in a series of constrained VCG auctions based on a low-complexity algorithm, the optimal strategy for the AUCs is still to bid truthfully.

In addition, we have

$$\vartheta = \vartheta_o = \frac{\Delta\varpi_{j,1} - \Delta\varpi_{j,3}}{\Delta\varpi_{j,1} - \Delta\varpi_{j,3} + \Delta\varpi_{j,2}}$$

by setting $\bar{\varpi}_{j,truthful} = \bar{\varpi}_{j,untruthful}$. If $\Delta\varpi_{j,1} - \Delta\varpi_{j,3} \geq 0$, we have $\vartheta_o \geq 1$ or $\vartheta_o \leq 0$. In other words, the optimal strategy for the AUCs is always to bid truthfully regardless of the sub-optimal allocation results of a low-complexity algorithm. If $\Delta\varpi_{j,1} - \Delta\varpi_{j,3} < 0$, the value of ϑ_o depends on the ratio of $\frac{\Delta\varpi_{j,2}}{\Delta\varpi_{j,1} - \Delta\varpi_{j,3}}$. With a higher adjusted payoff in scenario 3 or a smaller difference between the adjusted payoffs in scenarios 2 and 4, the optimal strategy for the AUCs is to bid truthfully even when sub-optimal channel allocation results occur more often, as long as $\vartheta > \vartheta_o$. $\square$

Appendix D
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- F. Zhang, X. Zhou, and X. Cao, "Location-oriented evolutionary games for price-elastic spectrum sharing," *IEEE Transactions on Communications*, vol. 64, no. 9, pp. 3958-3969, Sept. 2016.

- F. Zhang, X. Zhou, and M. Sun, "Constrained VCG auction for spatial spectrum reuse with flexible channel evaluations," in *Proceedings of IEEE Global Communications Conference*, Dec. 2017.

Sincerely,

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Vita

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