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Relay Selection Strategies for Multi-hop Cooperative Networks

Hui Sun
Louisiana State University and Agricultural and Mechanical College

RELAY SELECTION STRATEGIES
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Hui Sun
B.Sc., Nanjing University of Posts and Telecommunications, 2008
M.Sc., Nanjing University of Science and Technology, 2010
M.Sc., Louisiana State University, 2012
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To my family

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Table of Contents

List of Figures

Abstract

In this dissertation we consider several relay selection strategies for multi-hop cooperative networks. The relay selection strategies we propose do not require a central controller (CC). Instead, the relay selection is on a hop-by-hop basis. As such, these strategies can be implemented in a distributed manner. Therefore, increasing the number of hops in the network would not increase the complexity or time consumed for the relay selection procedure of each hop.

We first investigate the performance of a hop-by-hop relay selection strategy for multi-hop decode-and-forward (DF) cooperative networks. In each relay cluster, relays that successfully receive and decode the message from the previous hop form a *decoding set* for relaying, and the relay which has the highest signal-to-noise ratio (SNR) link to the next hop is then selected for retransmission. We analyze the performance of this method in terms of end-to-end outage probability, and we derive approximations for the ergodic capacity and the effective ergodic capacity of this strategy.

Next we propose a novel hop-by-hop relay selection strategy where the relay in the decoding set with the largest number of "good" channels to the next stage is selected for retransmission. We analyze the performance of this method in terms of end-to-end outage probability in the case of perfect and imperfect channel state information (CSI).

We also investigate relay selection strategies in underlay spectrum sharing cognitive relay networks. We consider a two-hop DF cognitive relay network with a constraint on the interference to the primary user. The outage probability of the secondary user and the interference probability at the primary user are analyzed under imperfect CSI scenario.

Finally we introduce a hop-by-hop relay selection strategy for underlay spectrum sharing multi-hop relay networks. Relay selection in each stage is only based on the CSI in that hop. It is shown that in terms of outage probability, the performance of this method is nearly optimal.

Chapter 1
Introduction

1.1 Wireless Cooperative Communication

The rapid development of wireless communication technologies in the last several decades has profoundly affected people's daily lives. The performance of wireless communication is limited by the propagation characteristic of the wireless channel, which can be further categorized into large-scale fading and small-scale fading [1]. Large-scale fading includes path loss, which is the signal attenuation due to the large distance between transmitter and receiver, and shadowing due to large-scale obstacles in the propagation path. Small-scale fading is mainly caused due to the constructive and destructive addition of multipath signal components arriving at the receiver.

Cooperative relay networks have proven to be an efficient method to overcome the detrimental effects of fading, extend the coverage area, and improve capacity of wireless communication systems [2, 3, 4, 5]. Compared to MIMO systems, cooperative relay networks do not need to employ multiple antennas at transmitter or receiver, and thus, reduce the size of terminals [6, 7]. The basic concept of cooperative relay networks as a three-terminal network was first introduced in [8]. The capacity of relay networks was studied for the first time by Cover and El Gamal in [9]. Based on the three terminal model, a basic cooperative relay network in which a source node transmits information to a destination node with the help of a single half-duplex relay node is shown in Figure 1.1. Transmission from source to destination takes two orthogonal phases. In phase 1, the source transmits its message to the destination, and the relay also receives the information transmitted from the source. In phase 2, the relay transmits the received message to the destination.

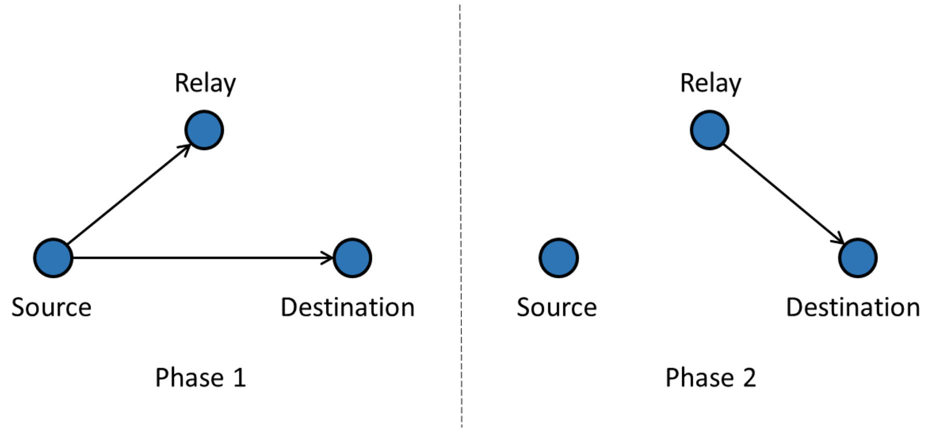

FIGURE 1.1. Cooperative Relay Network.

The relaying mechanism can be categorized as fixed relaying, selective DF relaying and incremental relaying [4, 10]. Fixed relaying can be mainly categorized as amplify-and-forward (AF) and decode-and-forward (DF) requiring substantially different amount of processing in the relays [4, 11, 12, 13, 14]. In AF relaying, very little processing in relays is needed. Relays simply amplify the received message, and retransmit the scaled message to the destination. The problem with AF relaying is that the noise is also amplified along with the message and retransmitted to the destination. In DF relaying, relays first decode the received message from the source, and then forward the (re-encoded) message to the destination.

Selective DF relaying and incremental relaying are two methods in adaptive relaying, in which the relays may or may not participate in relaying depending on the channel condition [10]. In selective relaying, whether the relay decodes the received message transmitted from the source and retransmits to the destination is determined by the signal-to-noise ratio (SNR) of the source-relay link. If the SNR of the source-relay link is above the SNR threshold, the relay is able to correctly decode the message from the source, and retransmits the message to the destination. Destination can decode the message from the source and the relay by applying maximal ratio combining (MRC). If the SNR of the source-relay link is below the SNR threshold, the relay remains idle [15, 16, 17, 10, 18]. Incremental relaying can be performed if a feedback channel from the destination to the relay is available [4, 19, 20, 21]. Destination receives the message transmitted from the source in phase 1, and notifies the relay if it is able to correctly decode the message. Relay will only transmit the message to the destination in phase 2 if the source-destination transmission in phase 1 has failed.

1.2 Opportunistic Relay Selection

In cooperative relay networks, when there are multiple relays which are willing to forward the received message from the source to the destination, relay selection is a key aspect which directly affects the performance of the network. Among the relay selection strategies in two-hop relay networks, opportunistic relay selection (ORS) proposed by Bletsas [22, 23] is a good scheme for implementation. ORS is a low complexity strategy in which the best relay is selected for retransmitting the message. With this strategy, synchronization among the relays is not needed and power consumption of terminals can be reduced. ORS is based on instantaneous channel conditions to select the best relay for retransmission, which can be performed without the knowledge of global channel state information (CSI) at each relay [24]. More specifically it is assumed that the channel gains between the source and the relays are measured from the request-to-send (RTS) message from the source. Similarly the channel gains between the relays and the destination are measured from the clear-to-send (CTS) message transmitted by the destination. Each relay now starts a timer which is inversely proportional to the end-to-end channel quality. Therefore the relay whose timer expires first has the best end-to-end channel quality. That relay transmits a short "flag" message indicating that it has the best channel and that relay will be the one retransmitting the packet from the source to the destination while all other relays remain silent [24].

In DF relaying mode, ORS can be categorized into proactive DF ORS and reactive DF ORS depending on whether the relay selection is before source transmission or after source transmission [23]. In proactive ORS shown in Figure 1.2, the best relay is selected before the source transmits. The best relay is normally the relay which has the highest SNR bottleneck of the source-relay link and relay-destination link. Once the best relay is selected, the source transmits the message to the selected relay. The selected relay decodes the message and retransmits the re-encoded message to the destination. In reactive ORS in Figure 1.3, the best relay is selected after the source

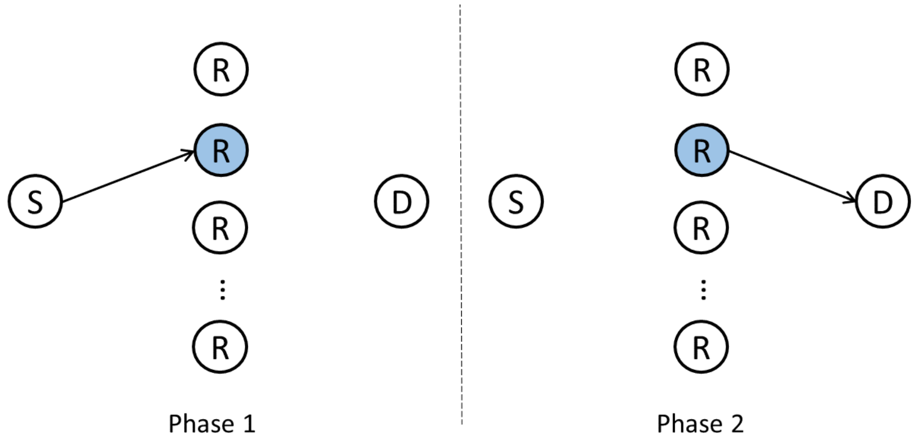

FIGURE 1.2. Proactive Opportunistic Relay Selection.

transmits. The source first broadcasts the message to all the relays. The relays which are able to correctly decode the message from the source form a decoding set. Among all relays in the decoding set, the relay which has the highest relay-to-destination SNR is considered as the best relay, and is selected for retransmission.

A key challenge in the implementation of the cooperative relay systems is that in a practical system the exact CSI is not available. CSI needs to be estimated at the receivers and fed back to the transmitters. Consequently there are two sources of uncertainty in CSI. First, due to the estimation error there will be a discrepancy between the estimated CSI which is used for relay selection and the actual CSI. Secondly, in

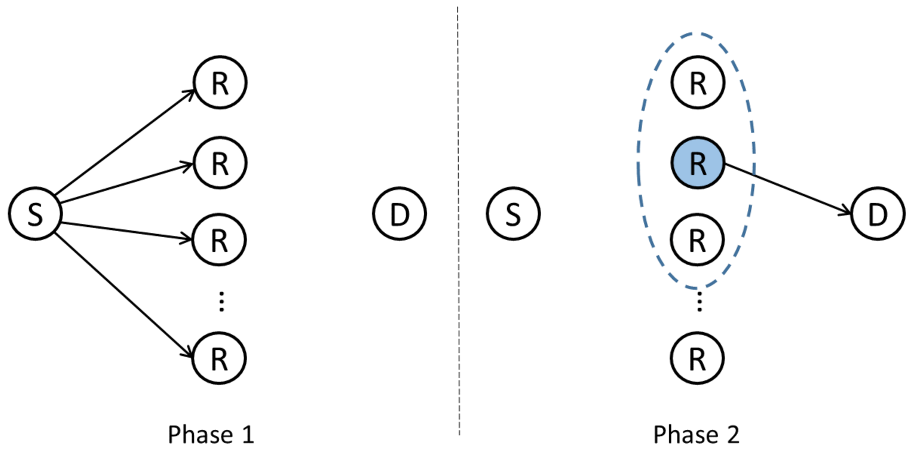

FIGURE 1.3. Reactive Opportunistic Relay Selection.

mobile systems, due to Doppler shifts, the channel coefficients are time-varying. Since some time elapses between channel estimation and relays' transmissions, even in the absence of CSI estimation errors, the CSI available to the relay network is outdated.

There have been several studies dealing with the relay selection problem in the case of imperfect CSI. In [25], the outage probability of DF ORS with outdated CSI is studied and in [26], the effect of outdated CSI on outage of AF relay selection is considered. In [27], the performance of outdated CSI on partial and opportunistic AF relay selection is analyzed. A multiple relay selection strategy for improving the performance of ORS with imperfect CSI is proposed in [28]. And in [29], the effects of imperfect CSI from both source-relay and relay-destination links of ORS, in which relay is selected before source transmission, is investigated.

1.3 Multi-hop Relay Networks

Recent years have witnessed extensive interest in multi-hop relay networks, and a standard, IEEE 802.16J, referred to as Mobile Multi-hop Relay (MMR) has been developed which allows for fixed, nomadic, and mobile relays [30]. Multi-hop networks have the potential to further extend the coverage, enhance the throughput (due to shorter hops), and extend battery life due to lower power transmission. In [31], the end-to-end outage probability of multi-hop networks over independent Nakagami fading channels is evaluated. The average outage duration of multi-hop communication systems is derived in [32].

In multi-hop relay networks, cooperative diversity can be achieved by employing a number of relays in each hop, and therefore, improving the system performance. However, multi-hop networks require a relay selection strategy and path management and introduce extra delay due to multi-hop relaying. An optimal relay selection strategy was proposed in [33] which requires a central controller (CC) to collect the CSI for all the links in the network. The path which has the highest SNR bottleneck is selected as the best path for transmission. In [34], a relay selection strategy called last-n-hop selection was proposed, whereby a CC or a combination of a CC and a distributed protocol is needed.

The complexity of the relay selection protocols that require a CC is very high. The CC must collect the CSI of all the links in order to select the desired path and must inform all the relays along the selected path so that they can participate in relaying the data. Another difficulty is that the CSI of all the links in the network is required before the desired path can be computed. Although several authors have proposed novel channel estimation techniques for multi-hop relay networks [35, 36, 37], obtaining CSI for all the links is a time consuming process. Clearly the channel estimation time as well as the time for end-to-end transmission of the message must not exceed the channel coherence time, or else the estimated CSI used for path selection will be irrelevant. For fast fading channels with a short coherence time, this implies that channel estimation has to be performed very frequently and only a short data segment can be transmitted before another channel estimation is required resulting in diminished system throughput.

Two new relay selection strategies were introduced in [38] and [39] and their outage probabilities were evaluated. In [38] all the relays in a cluster which are able to decode the signal (referred to as the *Decoding Set*) broadcast the signal to the next relay cluster. The relays in this cluster use MRC to decode the message. Requiring all the relays in the decoding set to retransmit the message demands a great deal of channel resources[1] and results in reduced throughput. In addition, this approach increases the hardware complexity of the relays since each relay is required to have multiple RF chains, correlators or matched filters, and an MRC combiner. Moreover, higher power resources are required since instead of a single relay, all the relays in the decoding set must transmit. In [39], a cooperative multi-hop parallel relay network is investigated. This approach is not a hop-by-hop strategy as it requires the CSI of all the links in order to select the best path among a set of parallel paths.

The ad-hoc relay selection strategy proposed in [33] is a low-complexity method in which in each stage, the relay with the highest SNR to the node in the previous hop is selected for retransmission, except that the relay in the last cluster is selected by considering the last two hops together. This approach does not have the drawbacks of strategies described above since the CSI for the links in each hop can be obtained just prior to retransmission and only a single relay will forward the signal. However, compared to the methods discussed above, this approach has significantly lower performance in terms of outage probability.

It can be seen that although relay networks have been the subject of many studies, to date, very few viable relay selection strategies for multi-hop relay networks have been introduced and/or analyzed.

[1]Depending on the multi-access scheme, this would be in the form of TDMA slots, FDMA frequency bands, OFDM sub-carriers, etc.

1.4 Cognitive Radio Networks

The overcrowding of the radio spectrum which has created a bottleneck for introduction of new services along with the inefficient use of currently allocated spectrum has prompted the regulatory agencies to seek alternative methods for spectrum access [40, 41]. Cognitive radio networks (CRNs) are envisioned to alleviate this problem by allowing secondary users (SUs) to dynamically access and utilize a frequency band as long as they do not cause harmful interference to primary users (PUs) [42, 43]. This approach is often referred to as dynamic (or opportunistic) spectrum access (DSA). For example, the IEEE 802.22 standard referred to as Wireless Regional Area Network (WRAN) is designed to operate in the TV broadcast frequency bands [44]. It aims at bringing broadband access to hard-to-reach, low population-density areas, typical of rural environments and developing countries.

Three different paradigms for DSA have been studied in recent years which involve different degrees of interaction between the primary and secondary networks; namely interweave, underlay and overlay [45, 46]. Interweave model is proposed based on the original idea of cognitive radios. Licensed spectrum is not fully utilized by PUs, in the interweave model, SUs employ spectrum sensing to detect the presence or absence of the PUs and only transmit if the PUs are absent [43, 47, 48, 49, 50, 51, 52]. On the other hand, both underlay and overlay schemes allow for simultaneous transmissions by the SUs and PUs. In the underlay model, SUs can share the spectrum with the PUs as long as the interference they cause to the PUs remains below a predefined threshold [53]. In the overlay model, non-causal/causal knowledge of the PU message and/or the codebook at the SUs is used to mitigate or cancel the interference at SUs [54]. Among these paradigms for DSA, the underlay model is considered to be an efficient transmission scheme for spectrum sharing without a great deal of system complexity.

Since the radios in a secondary underlay network must not cause undue interference to the primary user, they are expected to practice stringent power control requiring hard limits on their transmit power. In such cases multi-hop relay networks are well suited to carry the message from the source to the destination with low transmit powers using the relay nodes. Some authors have investigated the problem of cognitive spectrum sharing relay networks. In [55] the outage performance of cognitive relay network under spectrum-sharing constraint is investigated. In [56] the end-to-end signal-to-interference plus noise ratio is used for relay selection. The exact outage probability of an underlay spectrum sharing relay network over non-identical Rayleigh fading channels is derived in [57]. In [58] the authors evaluate the outage performance of the relay system where it is shown that due to the interference power constraint, the received SNRs are dependent. Joint relay selection and power allocation is studied in [59] to maximize the system throughput with a constraint on the interference caused to the PU. In [60] the outage performance of the relay network is evaluated taking into account the interference from the PU to SU network, the interference from the SU to PU network and the dependence resulting from the interference constraint. In [61] the authors evaluate the outage probability in the relay system under two different constraints: the peak interference power at the PU, and the peak interference power at the PU and the maximum transmit power at the SU-source and SU-relays. And in [62], the outage performance of two-hop multiuser and multirelay networks underlay spectrum sharing is investigated.

Multi-hop underlay cognitive spectrum sharing systems have also been investigated in [63, 64, 65, 66, 67]. In [63], outage probability of a cognitive radio based multi-hop network with underlay paradigm is derived. The outage probability, bit error rate, symbol error rate and ergodic capacity of underlay cognitive multi-hop regenerative relaying systems with multiple primary receivers in independent, non-identically distributed Nakagami-m fading channels are derived in [64]. In [65], the outage probability of a cognitive multi-hop relay network under multiple primary users interference is studied, in which both non-identical fading parameters as well as signal to interference plus noise ratio statistics are considered. In [66], closed-form and asymptotic expressions for the outage probability of cognitive multi-hop relay networks over Nakagami-m fading channels in the presence of multiple primary transmitters and receivers are derived. The exact outage probability and bit error rate, and approximate expressions for ergodic capacity of spectrum sharing-based multi-hop DF relay networks in non-identical Rayleigh fading channels are derived in [67].

Since protection of PUs is of utmost importance, the imperfect CSI problem is more significant in spectrum sharing networks [68, 69, 70, 71]. In the underlay paradigm, the transmit power is determined by the CSI between SU transmitters (source and relays) and PU receiver so that the interference to PU remains below a predefined threshold. If the CSI is outdated, this interference constraint may be violated. In [68], the capacity gains of opportunistic spectrum sharing channels in a Rayleigh fading environment with imperfect CSI is analyzed. By considering peak interference power constraint and maximum SU transmit power constraint, a closed-form expression for the mean SU capacity in cognitive radio systems with imperfect CSI is derived [69]. In [70], the impact of imperfect CSI on the partial relay selection in AF relaying cooperative communications systems is studied. By considering interference power constraints, imperfect CSI, and interference from PU transmitter, the performance of SU is investigated in [71].

In multi-hop underlay cognitive spectrum sharing networks, relay selection strategies of multi-hop networks are not applicable to underlay spectrum sharing cognitive networks since the interference to the primary user is of paramount concern. The relay selection strategies which require the CSI of the links in several (or all of the) hops in the network may cause interference to the primary user well beyond the specified threshold. Therefore, hop-by-hop relay selection strategies should be much preferable for multi-hop underlay cognitive spectrum sharing networks.

1.5 Outline of the Dissertation

In Chapter 2, we analyze the performance of a hop-by-hop relay selection strategy for multi-hop DF cooperative relay networks. In each relay cluster, relays that successfully receive and decode the message from the previous hop form the candidate set for relaying, and the relay which has the highest channel gain to the next stage is selected for retransmission. Therefore in this method, a CC is not required, and relay selection of each relay cluster is only based on the CSI to the next hop. We evaluate the performance of this relay selection method in terms of end-to-end outage probability through analysis and simulation. Accurate approximations for the ergodic capacity and effective ergodic capacity of this relay selection strategy are also derived.

In Chapter 3, a novel hop-by-hop relay selection strategy for multi-hop DF cooperative relay networks is proposed where relay selection at each hop is only based on the CSI to relays in the next stage. In each stage, relays that successfully receive and decode the message from the previous stage form the candidate set for relaying, and among them, the relay with the largest number of "good" channels to the next stage is selected for retransmission. We analyze the performance of the proposed method in terms of end-to-end outage probability for the cases of perfect and imperfect CSI. Numerical results from analysis closely match those obtained from simulation.

In Chapter 4, we consider cognitive relay networks with imperfect CSI under interference power constraint. Reactive DF and ORS are assumed whereby SU relays that successfully receive and decode the message from the SU source form the candidate set for relaying, and the best relay among them is selected to retransmit to the SU destination. We investigate the performance of DF-ORS in terms of outage probability of the SU and the interference probability at the PU. In order to allow the secondary network to back-off its peak transmit power, two power margin factors are considered for the SU source and relays. Numerical results show that with the proper selection of the power margin factors the desired values of outage and interference probabilities can be achieved.

In Chapter 5, we introduce a hop-by-hop relay selection strategy for multi-hop underlay cognitive spectrum sharing systems. In each stage, relays that successfully decode the message from previous hop form a candidate set. Each relay in this candidate set calculates its available transmit power and evaluates its instantaneous SNR to relays in the next stage. Then one relay which has the largest number of channels with an acceptable SNR level to relays in the next stage is selected for retransmission. Therefore, relay selection is only based on the CSI of the channels of one hop. This strategy can be implemented in a distributed manner, and a CC is not required. We analyze the performance of the introduced strategy in terms of end-to-end outage probability, and show that the results match those obtained from simulation closely.

Finally, the conclusions are given in Chapter 6.

Chapter 2

Performance Analysis of a Hop-By-Hop Relay Selection Strategy in Multi-hop Networks

2.1 Introduction

In this chapter, we introduce and evaluate the performance of a hop-by-hop relay selection strategy for multi-hop DF networks which does not require a CC. For each relay cluster the relay selection is only based on the CSI of the channels to the next stage. In each relay cluster, relays that successfully receive and decode the message from the previous hop form a decoding set for relaying, and the best relay among them is then selected for retransmission to the next hop. We analyze the performance of this relay selection method in terms of end-to-end outage probability, and show that the results closely match those from simulations. Approximations for the ergodic capacity and effective ergodic capacity are also derived which closely match the simulation results.

Notations: Our notations and some of our modeling assumptions for this chapter are introduced here. S, RC_m, and D refer to source, relay cluster m, and destination, respectively. $h^{(m)}_{\mathrm{A,B}}$ and $\gamma^{(m)}_{\mathrm{A,B}}$ denote the instantaneous CSI and the instantaneous SNR of link A $\rightarrow$ B of hop m, respectively. $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. The transmit power of each node is denoted by P_s and for each hop m, all channel coefficients are assumed to be independent and identically distributed (*iid*). Moreover, $h^{(m)}_{\mathrm{A,B}} \sim \mathcal{CN}(0, \lambda_m)$, and denote $g^{(m)}_{\mathrm{A,B}} \triangleq |h^{(m)}_{\mathrm{A,B}}|^2$. The noise random variable at receiver B is denoted by n_{B} and the noise variables at all receivers are assumed to be *iid* with $n_{\mathrm{B}} \sim \mathcal{CN}(0, \sigma_n^2)$. We denote $X_{1\sim n} \triangleq (X_1, X_2, \cdots, X_n)$. An event such as $\{X_1 < T, X_2 < T, \cdots, X_n < T\}$ is denoted as $\{X_{1\sim n} < T\}$. Finally the cardinality of the set $\mathcal{D}$ is denoted by $c(\mathcal{D})$.

2.2 System Model

As shown in Figure 2.1, we consider a multi-hop wireless relay network consisting of one source (S), one destination (D) and M relay clusters (RC_m, $m = 1, \cdots, M$) in between the source and destination. Each relay cluster RC_m includes L_m single-antenna half-duplex relay nodes. There are totally $M+1$ hops (from hop 0 to hop M) from S to D. We denote the first hop from S to RC_1 as hop 0, RC_1 to RC_2 as hop 1, and so on. Moreover to simplify our discussion we use the convention that RC_{M+1} denotes the destination D.

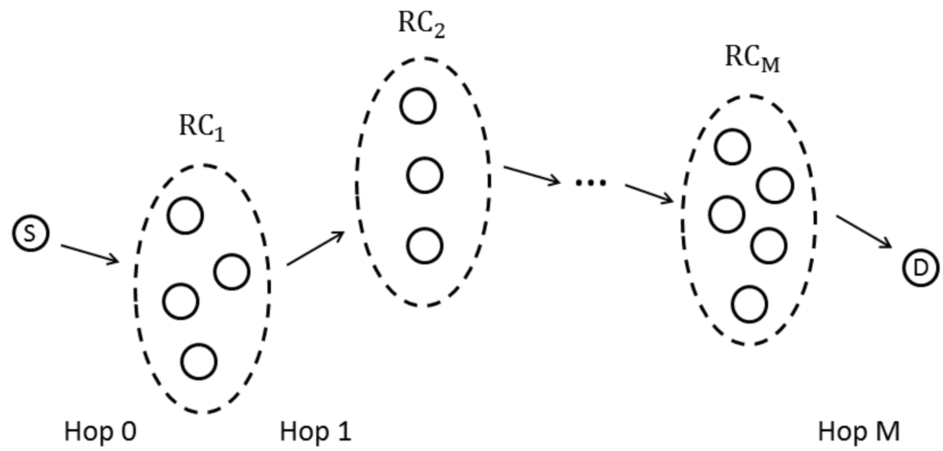

FIGURE 2.1. System Model of Multi-hop Relay Networks.

The instantaneous SNR of link A $\rightarrow$ B at hop m is given by

$$\gamma_{\text{A,B}}^{(m)} = P_s |h_{\text{A,B}}^{(m)}|^2 / \sigma_n^2 = \bar{\gamma} g_{\text{A,B}}^{(m)}, \tag{2.1}$$

where $\bar{\gamma} \triangleq P_s/\sigma_n^2$. We have

$$P(\gamma_{\text{A,B}}^{(m)} < x) = P(g_{\text{A,B}}^{(m)} < \frac{x}{\bar{\gamma}}) = 1 - e^{-\frac{x}{\bar{\gamma}\lambda_m}} \tag{2.2}$$

All the relays are assumed to use the DF relaying protocol. The relay selection strategy is as follows. In the first hop, S broadcasts its signal to the first relay cluster (RC_1). In any stage $m = 1, 2, \cdots, M$, the relays in RC_m which are able to correctly decode the information from the previous stage form a decoding set denoted by $\mathcal{D}_m$. Using a pilot or training sequence, each relay in $\mathcal{D}_m$ obtains estimates of its channel coefficients to the relays in RC_{m+1}. Then each relay in $\mathcal{D}_m$ calculates its maximum channel gain to all the relays in RC_{m+1} and starts a timer inversely proportional to its maximum gain. The relay whose timer expires first has the largest channel gain among all the links from relays in $\mathcal{D}_m$ to relays in RC_{m+1}. This relay will retransmit the message. All the other relays in $\mathcal{D}_m$ will hear this transmission and will not transmit[1]. For any hop m between relay clusters RC_m and RC_{m+1} $(m = 1, 2, \cdots, M-1)$, $l_m = c(\mathcal{D}_m)$, there are $l_m L_{m+1}$ links between l_m relays in $\mathcal{D}_m$ and the L_{m+1} relays in RC_{m+1}. Among the l_m relays in $\mathcal{D}_m$, we select the relay which has the link with the highest instantaneous SNR among all these $l_m L_{m+1}$ links to retransmit the message. For the last hop between RC_M and D, each relay candidate in $\mathcal{D}_M$ evaluates its channel coefficient to D, and the one which has highest instantaneous SNR to D is selected to retransmit. It can be seen that in this method, relay selection and transmission is on a hop-by-hop basis. In the following we evaluate the end-to-end outage probability and approximations for the ergodic capacity and effective ergodic capacity of this relay selection method.

2.3 Outage Probability

For $n = 1, 2, \cdots, M$ let $\mathcal{O}_n = \{c(\mathcal{D}_n) = 0\}$, i.e., $\mathcal{O}_n$ is the event that no relay in the nth cluster can decode the message, and $\mathcal{O}_{M+1}$ is the event that destination D cannot decode the message. Then for the end-to-end outage event $\mathcal{O}$ we can write

$$P(\mathcal{O}) = P(\cup_{n=1}^{M+1} \mathcal{O}_n) \tag{2.3}$$

Denote by $\mathcal{R}$ the required end-to-end spectral efficiency in bps/Hz. Then for $m = 1, 2, \cdots, M$, $\mathcal{D}_m$ consists of those relays whose link capacity from the previous stage

[1]Note that this strategy does not require any communication among the relays in a given cluster or a CC.

exceeds $\mathcal{R}$, that is,

$$\mathcal{D}_m = \{j : \frac{1}{M+1}\log_2(1+\gamma_{i^*,j}^{(m-1)}) \geq \mathcal{R}\} = \{j : \gamma_{i^*,j}^{(m-1)} \geq 2^{(M+1)\mathcal{R}} - 1\} \tag{2.4}$$

where $\gamma_{i^*,j}^{(0)} \triangleq \gamma_{\mathrm{S},j}^{(0)}$ is the SNR from S to Relay j in RC_1, and for $m = 2, 3, \cdots, M$, $\gamma_{i^*,j}^{(m-1)}$ is the SNR from selected relay i^* in $\mathcal{D}_{m-1}$[2] to relay j in RC_m. In the following we denote $T \triangleq 2^{(M+1)\mathcal{R}} - 1$. Now we can write

$$\begin{aligned} P(\cup_{n=1}^{M+1}\mathcal{O}_n) &= \sum_{l_1=0}^{L_1} P(\cup_{n=1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1) \\ &= P(\mathcal{O}_1) + \sum_{l_1=1}^{L_1} P(\cup_{n=2}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1) \end{aligned} \tag{2.5}$$

Clearly, $P(\mathcal{O}_1) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{L_1}$. Moreover, we can calculate the probability that there are l_1 relays in $\mathcal{D}_1$, which is

$$P(c(\mathcal{D}_1) = l_1) = \binom{L_1}{l_1}(1 - e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{L_1 - l_1}(e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{l_1} \tag{2.6}$$

Now we consider $P(\cup_{n=m+1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_m) = l_m)$ for any $1 \leq m \leq M-1$. For $1 \leq l_m \leq L_m$, we have

$$\begin{aligned} &P(\cup_{n=m+1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_m) = l_m) \\ &= \sum_{l_{m+1}=0}^{L_{m+1}} P(\cup_{n=m+1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_{m+1}) = l_{m+1}, c(\mathcal{D}_m) = l_m) \\ &\quad \times P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m) \\ &= P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) \\ &\quad + \sum_{l_{m+1}=1}^{L_{m+1}} P(\cup_{n=m+2}^{M+1}\mathcal{O}_n | c(\mathcal{D}_{m+1}) = l_{m+1}) P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m) \end{aligned} \tag{2.7}$$

Note that in the above

$$P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) = P(c(\mathcal{D}_{m+1}) = 0 | c(\mathcal{D}_m) = l_m) \tag{2.8}$$

[2]Which has the highest channel gain or SNR to the relays in RC_m.

Therefore we need to evaluate $P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m)$ for $1 \leq m \leq M$, and $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$ for $1 \leq m \leq M-1$.

For $m = 1, 2, \cdots, M-1$, $P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m)$ is the probability that from the selected relay in $\mathcal{D}_m$, all L_{m+1} links are in outage. According to the relay selection strategy, the link which has the highest instantaneous SNR among all $l_m L_{m+1}$ links from $\mathcal{D}_m$ to RC_{m+1} is among these L_{m+1} links from the selected relay. This is equivalent to the fact that the SNR of all $l_m L_{m+1}$ links from $\mathcal{D}_m$ to RC_{m+1} are below the threshold T. Therefore,

$$\begin{aligned} & P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m) \\ & = P(\max_{i \in \mathcal{D}_m, j \in \mathrm{RC}_{m+1}} \gamma_{i,j}^{(m)} < T|c(\mathcal{D}_m) = l_m) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}} \end{aligned} \tag{2.9}$$

Similarly, since for the last hop (when $m = M$), the selected relay in $\mathcal{D}_M$ which has the highest instantaneous SNR among $c(\mathcal{D}_M) = l_M$ links is selected for retransmission, we have

$$\begin{aligned} & P(\mathcal{O}_{M+1}|c(\mathcal{D}_M) = l_M) \\ & = P(\max_{i \in \mathcal{D}_M} \{\gamma_{i,\mathrm{D}}^{(M)}\} < T|c(\mathcal{D}_M) = l_M) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_M}})^{l_M} \end{aligned} \tag{2.10}$$

To calculate $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$, we first consider the case when $l_m = 1$. When $l_m = 1$, there is only a single relay in $\mathcal{D}_m$, that relay would retransmit to next hop. Similar to (2.6), we have

$$P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = 1) = \binom{L_{m+1}}{l_{m+1}} (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1} - l_{m+1}} (e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_{m+1}} \tag{2.11}$$

When $2 \leq l_m \leq L_m$, we would discuss several cases according to different values of l_{m+1}. Denote $A_{i^*,j^*}^{l_m}$ as the event that link $i^* \to j^*$ is the link which has highest SNR among all the links from l_m relays in $\mathcal{D}_m$ to L_{m+1} relays in RC_{m+1}. For notational convenience, and without loss of generality we assume that $j^* = L_{m+1}$. That is, we assume that the link from Relay i^* in $\mathcal{D}_m$ to Relay L_{m+1} in RC_{m+1} has the highest SNR at hop m. Note that since all the channels at hop m are *iid*, all the nodes $j \in \mathrm{RC}_{m+1}$ are equally likely to belong to the highest SNR link. Let

$$\Gamma_{\max}^{(m)}(l, L) \triangleq \max\{\gamma_{i,j}^{(m)}; i = 1, \cdots, l, j = 1, \cdots, L\}$$

The details of our derivations are given below.

When $2 \leq l_m \leq L_m$ and $l_{m+1} = 1$, among all L_{m+1} links from the selected relay to relays in RC_{m+1}, only a single link is not in outage. This is the link which has the highest SNR among all $l_m L_{m+1}$ links. The remaining $L_{m+1} - 1$ links from the selected relay are all in outage. The following lemma whose proof is given in Appendix A is used to derive the result in this case.

Lemma 1. *Given N random variables $X_1, X_2, \cdots, X_N$ which are iid, let $X_{max} \triangleq \max\{X_1, X_2, \cdots, X_N\}$. Then for $1 \leq n < N$,*

$$P(X_{1\sim n} < y | X_{1\sim n} \neq X_{max}) = \frac{P(X_{1\sim n} < y) - \frac{n}{N} P(X_{max} < y)}{P(X_{1\sim n} \neq X_{max})} \tag{2.12}$$

Theorem 2.

$$\begin{aligned} &P(c(\mathcal{D}_{m+1}) = 1 | c(\mathcal{D}_m) = l_m) \\ &= \frac{l_m L_{m+1}[(1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-1} - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}]}{l_m L_{m+1} - L_{m+1} + 1} \end{aligned} \tag{2.13}$$

Proof. We can write

$$\begin{aligned} &P(c(\mathcal{D}_{m+1}) = 1 | c(\mathcal{D}_m) = l_m) \\ &= P(\gamma_{i^*,1\sim L_{m+1}-1}^{(m)} < T, \gamma_{i^*,L_{m+1}}^{(m)} > T | A_{i^*,L_{m+1}}^{l_m}) \\ &= P(\gamma_{i^*,1\sim L_{m+1}-1}^{(m)} < T | A_{i^*,L_{m+1}}^{l_m}) - P(\gamma_{i^*,1\sim L_{m+1}-1}^{(m)} < T, \gamma_{i^*,L_{m+1}}^{(m)} < T | A_{i^*,L_{m+1}}^{l_m}) \end{aligned} \tag{2.14}$$

For the second term in (2.14) we have

$$
\begin{aligned}
&P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} < T, \gamma^{(m)}_{i^*,L_{m+1}} < T | A^{l_m}_{i^*,L_{m+1}}) \\
&= P(\Gamma^{(m)}_{\max}(l_m, L_{m+1}) < T) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}
\end{aligned} \tag{2.15}
$$

Moreover, according to Lemma 1, we have

$$
\begin{aligned}
&P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} < T | A^{l_m}_{i^*,L_{m+1}}) \\
&= P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} < T | \gamma^{(m)}_{i^*,1\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1})) \\
&= \frac{l_m L_{m+1}}{l_m L_{m+1} - L_{m+1} + 1} P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} < T) \\
&\quad - \frac{L_{m+1} - 1}{l_m L_{m+1} - L_{m+1} + 1} P(\Gamma^{(m)}_{\max}(l_m, L_{m+1}) < T)
\end{aligned} \tag{2.16}
$$

in which

$$
P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} < T) = (1 - e^{\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-1} \tag{2.17}
$$

and where we have used the fact that

$$
P(\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1})) = \frac{l_m L_{m+1} - L_{m+1} + 1}{l_m L_{m+1}} \tag{2.18}
$$

Putting (2.14)-(2.17) together we get (2.13). □

When $2 \leq l_m \leq L_m$ and $2 \leq l_{m+1} = L_{m+1}$, the link which has the highest SNR is not in outage, and all the links from the selected relay in $\mathcal{D}_m$ to the other relays in RC_{m+1} are not in outage either. That is, all the links from selected relay in $\mathcal{D}_m$ to relays in RC_{m+1} are not in outage. We first discuss a special scenario that $l_{m+1} = L_{m+1} = 2$. Using Lemma 1 it can be shown that

$$
\begin{aligned}
&P(\gamma^{(m)}_{i^*,1} > T | \gamma^{(m)}_{i^*,1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1})) \\
&= 1 - P(\gamma^{(m)}_{i^*,1} < T | \gamma^{(m)}_{i^*,1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1})) \\
&= 1 - \frac{l_m L_{m+1}(1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}}) - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}}{l_m L_{m+1} - 1}
\end{aligned} \tag{2.19}
$$

Letting $L_{m+1} = 2$ in (2.19) we can get

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = 2 | c(\mathcal{D}_m) = l_m) \\
&= P(\gamma_{i^*,1}^{(m)} > T, \gamma_{i^*,2}^{(m)} > T | A_{i^*,2}^{l_m}) \\
&= P(\gamma_{i^*,1}^{(m)} > T | \gamma_{i^*,1}^{(m)} \neq \Gamma_{\max}^{(m)}(l_m, 2)) \\
&= 1 - \frac{2l_m(1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}}) - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{2l_m}}{2l_m - 1}
\end{aligned} \tag{2.20}
$$

We now consider the case that $2 < l_{m+1} = L_{m+1}$. To derive $P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m)$, we introduce the following recursive lemma whose proof is given in Appendix B.

Lemma 3. *Given N iid random variables $X_1, X_2, \cdots, X_N$, let $X_{max} \triangleq \max\{X_1, X_2, \cdots, X_N\}$. Then for $1 < n < N$, we have*

$$
\begin{aligned}
&P(X_{1\sim n} > y | X_{1\sim n} \neq X_{max}) \\
&= \frac{P(X_{1\sim n} > y) - \frac{n}{N} P(X_{1\sim n-1} > y | X_{1\sim n-1} \neq X_{max})}{P(X_{1\sim n} \neq X_{max})}
\end{aligned} \tag{2.21}
$$

Theorem 4. *The probability $P(c(\mathcal{D}_{m+1}) = L_{m+1} | c(\mathcal{D}_m) = l_m)$ can be calculated as*

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = L_{m+1} | c(\mathcal{D}_m) = l_m) \\
&= P(\gamma_{i^*,1\sim L_{m+1}-1}^{(m)} > T, \gamma_{i^*,L_{m+1}}^{(m)} > T | A_{i^*,L_{m+1}}^{l_m}) \\
&= P(\gamma_{i^*,1\sim L_{m+1}-1}^{(m)} > T | \gamma_{i^*,1\sim L_{m+1}-1}^{(m)} \neq \Gamma_{\max}^{(m)}(l_m, L_{m+1}))
\end{aligned} \tag{2.22}
$$

where (2.22) is evaluated recursively for any $2 \leq l' \leq L_{m+1} - 1$, from

$$
\begin{aligned}
&P(\gamma_{i^*,1\sim l'}^{(m)} > T | \gamma_{i^*,1\sim l'}^{(m)} \neq \Gamma_{\max}^{(m)}(l_m, L_{m+1})) \\
&= \frac{l_m L_{m+1} (e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l'} - l' P(\gamma_{i^*,1\sim l'-1}^{(m)} > T | \gamma_{i^*,1\sim l'-1}^{(m)} \neq \Gamma_{\max}^{(m)}(l_m, L_{m+1}))}{l_m L_{m+1} - l'}
\end{aligned} \tag{2.23}
$$

Proof. Proof of (2.23) follows from Lemma 3, where for $l' = 2$ the initial term of the recursion is given in (2.19). □

When $2 \leq l_m \leq L_m$ and $2 \leq l_{m+1} < L_{m+1}$, the highest SNR link from the selected relay is not in outage. In addition, $l_{m+1}-1$ links from the selected relay are not in outage, while the remaining $L_{m+1}-l_{m+1}$ links are in outage. As before we assume that the link from Relay i^* in $\mathcal{D}_m$ to Relay L_{m+1} in RC_{m+1} is the highest SNR link at hop m. Consider the event that among the remaining $L_{m+1}-1$ relays in RC_{m+1}, Relays $\{1, 2, \cdots, l_{m+1}-1\}$ are able to decode the message, while Relays $\{l_{m+1}, l_{m+1}+1, \cdots, L_{m+1}-1\}$ are not. For these $L_{m+1}-1$ relays, this is only one of the combinations which results in $l_{m+1}-1$ relays out of $L_{m+1}-1$ being able to correctly decode the message. Totally, there are $\binom{L_{m+1}-1}{l_{m+1}-1}$ combinations with all the events having the same probability. Therefore we get

$$\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)\\
&= \binom{L_{m+1}-1}{l_{m+1}-1} P(\gamma^{(m)}_{i^*,1\sim l_{m+1}-1} > T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T, \gamma^{(m)}_{i^*,L_{m+1}} > T|A^{l_m}_{i^*,L_{m+1}})\\
&= \binom{L_{m+1}-1}{l_{m+1}-1}\\
&\quad \times P(\gamma^{(m)}_{i^*,1\sim l_{m+1}-1} > T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T|\gamma^{(m)}_{i^*,1\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1}))
\end{aligned} \tag{2.24}$$

For the case of $l_{m+1} = 2$, the probability in (2.24) is obtained from

$$\begin{aligned}
&P(\gamma^{(m)}_{i^*,1} > T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T|\gamma^{(m)}_{i^*,1}, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1}))\\
&= P(\gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T|\gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1}))\\
&\quad - P(\gamma^{(m)}_{i^*,1} < T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T|\gamma^{(m)}_{i^*,1}, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1}))\\
&= \frac{l_m L_{m+1}(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-l_{m+1}} - (L_{m+1}-l_{m+1})(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}}{l_m L_{m+1} - L_{m+1} + l_{m+1}}\\
&\quad - \frac{l_m L_{m+1}(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-l_{m+1}+1} - (L_{m+1}-l_{m+1}+1)(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}}{l_m L_{m+1} - L_{m+1} + l_{m+1} - 1}
\end{aligned} \tag{2.25}$$

Then the final result for $l_{m+1} = 2$ is given as

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = 2|c(\mathcal{D}_m) = l_m) \\
&= \frac{(L_{m+1}-1)l_m L_{m+1}(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-2} - (L_{m+1}-2)(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}}{l_m L_{m+1} - L_{m+1} + 2} \\
&\quad - \frac{(L_{m+1}-1)l_m L_{m+1}(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-1} - (L_{m+1}-1)(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}}}{l_m L_{m+1} - L_{m+1} + 1}
\end{aligned} \tag{2.26}
$$

For the case of $2 < l_{m+1} < L_{m+1}$, we evaluate the probability in (2.24) recursively using the following lemma whose proof is given in Appendix C.

Lemma 5. *Given N random variables $X_1, X_2, \cdots, X_N$ which are iid, let $X_{max} \triangleq \max\{X_1, X_2, \cdots, X_N\}$. Let $1 < n \leq N_a < N_b < N$. Then $P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y | X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})$ can be recursively calculated from*

$$
\begin{aligned}
&P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y | X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max}) \\
&= \frac{P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&\quad - \frac{\frac{n}{N} P(X_{1\sim n-1} > y, X_{N_a+1\sim N_b} < y | X_{1\sim n-1} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})}
\end{aligned} \tag{2.27}
$$

For any $2 \leq l' \leq l_{m+1} - 1$, the recursion is given in

$$
\begin{aligned}
&P(\gamma^{(m)}_{i^*,1\sim l'} > T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T | \gamma^{(m)}_{i^*,1\sim l'}, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1})) \\
&= \frac{l_m L_{m+1}(1-e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-l_{m+1}}(e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l'}}{l_m L_{m+1} - (l' + L_{m+1} - l_{m+1})} \\
&\quad - \frac{l' P(\gamma^{(m)}_{i^*,1\sim l'-1} > T, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} < T | \gamma^{(m)}_{i^*,1\sim l'-1}, \gamma^{(m)}_{i^*,l_{m+1}\sim L_{m+1}-1} \neq \Gamma^{(m)}_{\max}(l_m, L_{m+1}))}{l_m L_{m+1} - (l' + L_{m+1} - l_{m+1})}
\end{aligned} \tag{2.28}
$$

The initial condition for the recursion in (2.28) corresponding to $l' = 1$ is given in (2.25).

2.4 Ergodic Capacity

In this section we derive approximations for the end-to-end ergodic capacity and the effective ergodic capacity of the multi-hop relay selection strategy.

2.4.1 Approximation of the Ergodic Capacity

To evaluate the end-to-end ergodic capacity of the relay system, we need to evaluate the CDF of the end-to-end SNR. However, the computation of ergodic capacity from the CDF of the end-to-end SNR is mathematically intractable. Therefore, we evaluate an approximation for the end-to-end ergodic capacity. The numerical results in Section 2.5 show that the approximation is tight.

Let i_m^* denote the relay selected in $\mathcal{D}_m$ in cluster RC_m $(m = 1, \cdots, M)$ for retransmission. Then, the CDF of $\gamma_{\mathrm{S},i_1^*}^{(0)}$ is given by

$$F_{\gamma_{\mathrm{S},i_1^*}^{(0)}}(t) = (1 - e^{-\frac{t-T}{\bar{\gamma}\lambda_0}})u(t-T) \tag{2.29}$$

Furthermore, given that $c(\mathcal{D}_M) = l_M$, the conditional CDF of $\gamma_{i_M^*,\mathrm{D}}^{(M)}$ is given by

$$F_{\gamma_{i_M^*,\mathrm{D}}^{(M)}}(t|c(\mathcal{D}_M) = l_M) = (1 - e^{-\frac{t}{\bar{\gamma}\lambda_M}})^{l_M} \tag{2.30}$$

In addition, we need the CDF of $\gamma_{i_m^*,i_{m+1}^*}^{(m)}$ for $m = 1, \cdots, M-1$ to evaluate end-to-end ergodic capacity. Unfortunately the computation of the end-to-end ergodic capacity based on the CDFs above is not mathematically tractable. Therefore we assume that the Relay i_m is randomly selected among all the relays in $\mathcal{D}_m$, and use $\gamma_{i_m,i_{m+1}}^{(m)}$, the SNR of the link $i_m \to i_{m+1}$, instead of $\gamma_{i_m^*,i_{m+1}^*}^{(m)}$, the SNR of link $i_m^* \to i_{m+1}^*$. The CDF of $\gamma_{i_m,i_{m+1}}^{(m)}$ is given by

$$F_{\gamma_{i_m,i_{m+1}}^{(m)}}(t) = (1 - e^{-\frac{t-T}{\bar{\gamma}\lambda_m}})u(t-T) \tag{2.31}$$

With the above assumption, the CDF of the end-to-end SNR, Γ, given that $c(\mathcal{D}_M) = l_M$, is given by

$$
\begin{aligned}
&F_\Gamma(t|c(\mathcal{D}_M) = l_M) \\
&= 1 - P(\min(\gamma^{(0)}_{\mathrm{S},i_1}, \underset{m=1,\cdots,M-2}{\gamma^{(m)}_{i_m,i_{m+1}}}, \gamma^{(M-1)}_{i_{M-1},i^*_M}, \gamma^{(M)}_{i^*_M,\mathrm{D}}) > t|c(\mathcal{D}_M) = l_M) \\
&= (1 - e^{-\frac{t}{\bar{\gamma}\lambda_M}})^{l_M} + (1 - e^{-\frac{t-T}{\bar{\gamma}}\sum_{m=0}^{M-1}\frac{1}{\lambda_m}})[1 - (1 - e^{-\frac{t}{\bar{\gamma}\lambda_M}})^{l_M}]u(t - T) \qquad (2.32)
\end{aligned}
$$

The end-to-end capacity is defined as $C = \frac{1}{M+1}\log_2(1+\Gamma)$. Therefore, the conditional CDF of C is given by

$$
\begin{aligned}
&F_C(y|c(\mathcal{D}_M) = l_M) = F_\Gamma(2^{(M+1)y} - 1|c(\mathcal{D}_M) = l_M) \\
&= (1 - e^{-\frac{2^{(M+1)y}-1}{\bar{\gamma}\lambda_M}})^{l_M} + (1 - e^{-\frac{2^{(M+1)y}-1-T}{\bar{\gamma}}\sum_{m=0}^{M-1}\frac{1}{\lambda_m}}) \\
&\quad \times [1 - (1 - e^{-\frac{2^{(M+1)y}-1}{\bar{\gamma}\lambda_M}})^{l_M}]u(y - \mathcal{R}) \qquad (2.33)
\end{aligned}
$$

where, as mentioned before, $\mathcal{R} = \frac{1}{M+1}\log_2(1+T)$. Using integration by parts, and after some manipulations, the approximation on the end-to-end ergodic capacity, given that $c(\mathcal{D}_M) = l_M$, can be expressed as

$$
\begin{aligned}
&\bar{C}_{\{\mathrm{approx}|l_M\}} \\
&= \sum_{k=1}^{l_M}\binom{l_M}{k}(-1)^k e^{\frac{k}{\bar{\gamma}\lambda_M}}\frac{1}{(M+1)\ln 2} \\
&\quad \times [-E_1(\frac{k}{\bar{\gamma}\lambda_M}) + E_1(\frac{k(T+1)}{\bar{\gamma}\lambda_M}) - e^{\frac{T+1}{\bar{\gamma}}(\sum_{i=0}^{M-1}\frac{1}{\lambda_i})}E_1(\frac{1}{\bar{\gamma}}(\sum_{i=0}^{M-1}\frac{1}{\lambda_i} + \frac{k}{\lambda_M})(T+1))]
\end{aligned}
$$
(2.34)

where $E_1(z)$ is the exponential integral function defined as $E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt$.

The probability of the event $\{c(\mathcal{D}_M) = l_M\}$ can be computed from

$$
\begin{aligned}
&P(c(\mathcal{D}_m) = l_m) \\
&= \sum_{l_{m-1}=1}^{L_{m-1}} P(c(\mathcal{D}_m) = l_m|c(\mathcal{D}_{m-1}) = l_{m-1})P(c(\mathcal{D}_{m-1}) = l_{m-1}) \qquad (2.35)
\end{aligned}
$$

where $P(c(\mathcal{D}_1) = l_1)$ and $P(c(\mathcal{D}_m) = l_m | c(\mathcal{D}_{m-1}) = l_{m-1})$ for any m $(1 < m \leq M)$ were derived previously. Now the approximation on the end-to-end ergodic capacity can be obtained from

$$\bar{C}_{\text{approx}} = \sum_{l_M=1}^{L_M} P(c(\mathcal{D}_M) = l_M) \bar{C}_{\{\text{approx}|l_M\}} \tag{2.36}$$

2.4.2 Approximation of the Effective Ergodic Capacity

In the above we derived an approximation $\bar{C}_{\text{approx}}$ on the end-to-end ergodic capacity. However, it is possible that the transmission from the last Relay i_M^* to D fails, and no reliable information is delivered to D. The computation of $\bar{C}_{\text{approx}}$ does not exclude this scenario. Therefore, we adopt the concept of effective ergodic capacity, which is defined as the average spectral efficiency for end-to-end successful transmissions [72]. Since $\mathcal{R} = \frac{1}{M+1} \log_2(1+T)$ is the spectral efficiency required to successfully decode the message, the average effective ergodic capacity is calculated within the range of $[\mathcal{R}, \infty)$ instead of $[0, \infty)$. Therefore, only successful end-to-end transmissions are included in the calculation. The approximation for the end-to-end effective ergodic capacity, given that $c(\mathcal{D}_M) = l_M$, can be expressed as

$$\begin{aligned}
&\bar{C}^{\text{eff}}_{\{\text{approx}|l_M\}} = \int_{y=\mathcal{R}}^{\infty} y \; dF_C(y|c(\mathcal{D}_M) = l_M) \\
&= \mathcal{R}[1 - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_M}})^{l_M}] \\
&\quad - \frac{1}{(M+1)\ln 2} e^{\frac{T+1}{\bar{\gamma}}(\sum_{i=0}^{M-1} \frac{1}{\lambda_i})} \sum_{k=1}^{l_M} \binom{l_M}{k} (-1)^k e^{\frac{k}{\bar{\gamma}\lambda_M}} E_1(\frac{1}{\bar{\gamma}}(\sum_{i=0}^{M-1} \frac{1}{\lambda_i} + \frac{k}{\lambda_M})(T+1))
\end{aligned} \tag{2.37}$$

Finally, the approximation on the end-to-end effective ergodic capacity is obtained from

$$\bar{C}^{\text{eff}}_{\text{approx}} = \sum_{l_M=1}^{L_M} P(c(\mathcal{D}_M) = l_M) \bar{C}^{\text{eff}}_{\{\text{approx}|l_M\}} \tag{2.38}$$

2.5 Numerical Results

In this section we present our numerical results from analysis and compare to those obtained from simulation. Moreover, we compare the proposed hop-by-hop relay selection strategy to optimal and ad-hoc relay selection strategies in [33] in terms of outage probability.

In Figure 2.2, we show the outage probability vs. average SNR[3] for the case that $M = 3$ (4 hops), $\sigma_n^2 = 1$, $\lambda_m = 1$, and a target rate of $C = 2/(M+1)$ bps/Hz. We compare the outage probability of hop-by-hop relay selection strategy with the results from optimal and ad-hoc strategies. As expected, under the same relay distribution (L_1, L_2, L_3), optimal relay selection strategy has the lowest outage probability, and hop-by-hop relay selection strategy outperforms ad-hoc relay selection strategy. It can be seen that for hop-by-hop relay selection strategy, as we increase the number of relays in the first and last clusters the performance improves significantly. This effect can be explained in terms of the number of links available for each hop. In particular, in the first hop the diversity order is L_1, while in last hop it is $c(\mathcal{D}_M) = l_M$. For any hop m between two relay clusters, the link with the highest SNR among $l_m L_{m+1}$ links is selected. When all relay clusters have the same number of relays, it can be seen that from the source to destination, the last hop is the bottleneck hop, followed by the first hop which is the second bottleneck. Therefore, increasing the number of relays in clusters RC_1 and RC_M can improve the performance. In contrast, ad-hoc relay selection strategy does not have this characteristic. Even if we do not consider the last two hops, for any hop m excepting the last two hops, ad-hoc relay selection strategy selects the link with the highest SNR among L_{m+1} links. When L_{m+1} varies with m, the hop with the smallest number of links becomes the bottleneck hop. Increasing the number of relays in the first and last clusters will not only decrease, but rather

[3]Average SNR is defined as $10\log(P_s\lambda_m/\sigma_n^2)$.

increase the outage probability. Moreover, we can see that hop-by-hop relay selection strategy with $(L_1, L_2, L_3) = (6, 2, 7)$ outperforms optimal relay selection strategy with $(L_1, L_2, L_3) = (5, 5, 5)$. Finally, the results from analysis closely match those from simulation verifying the accuracy of our analysis.

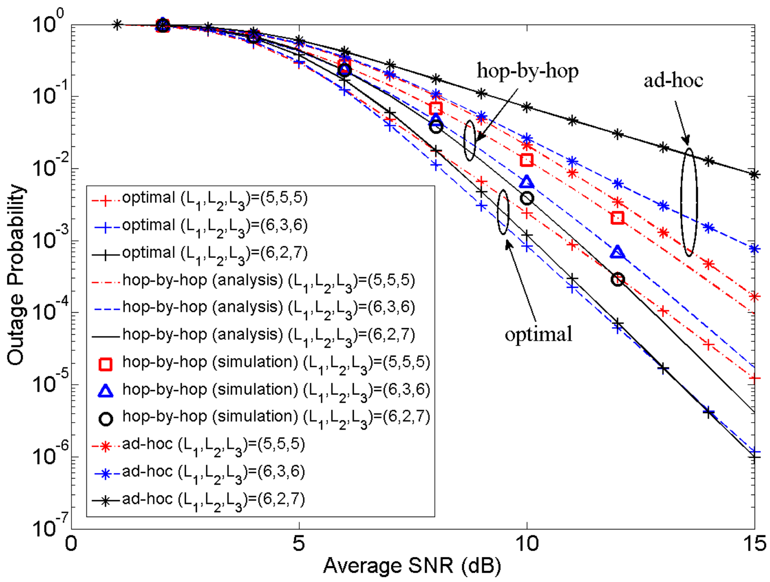

FIGURE 2.2. Outage Probability vs. Average SNR with different relay distributions.

As mentioned previously, when the clusters have the same number of relays, the bottleneck of hop-by-hop strategy is in the last hop, since the diversity order of the last hop is $c(\mathcal{D}_M) = l_M$. As mentioned in [33] that for optimal relay selection, the first hop and the last hop are the main constraints for outage probability. In Figure 2.3, we consider $(L_1, L_2, L_3) = (5, 5, 5)$, and $\lambda_m = \lambda$ for $m = 0, \cdots, M-1$, where $M = 3$. We then plot the outage probability vs. $10\log(\lambda_M/\lambda)$. As we can see in the figure, as λ_M increases, the outage probability of hop-by-hop strategy approaches that of the optimal strategy. That shows that when the channel gains (or average SNR) of the last hop increases, the hop-by-hop strategy can achieve the same performance as the optimal strategy. The figure also shows that the ad-hoc strategy cannot achieve the same performance even for large channel gains in the last hop.

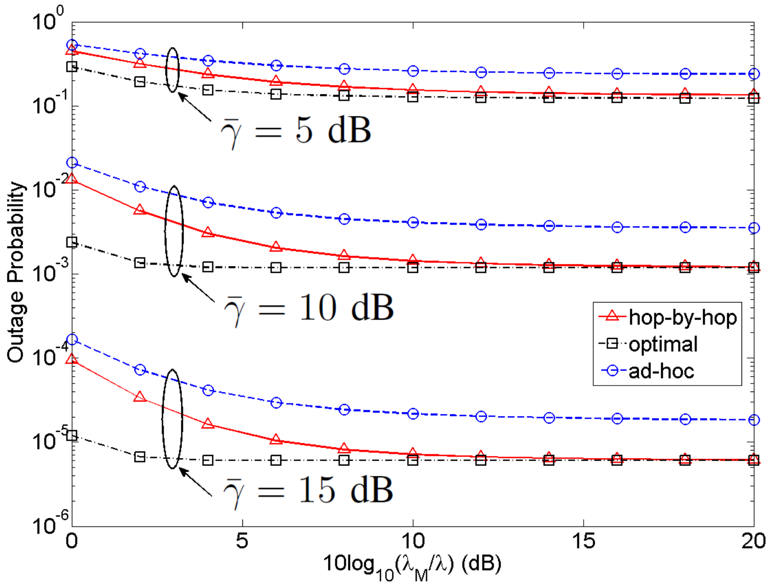

FIGURE 2.3. Outage Probability vs. different channel gains of last hop.

A key challenge in the implementation of the cooperative relay systems is that in a practical system the exact CSI is not available. In mobile systems, the channel coefficients are time-varying due to Doppler shifts. That is, the CSI collected for relay selection may be different from the instantaneous CSI at the instant of relay transmissions. This affects the system performance. Denote by $\tilde{h}^{(m)}_{\text{A,B}}$ the CSI at relay selection time, and by $h^{(m)}_{\text{A,B}}$ the CSI at relay transmission time. According to [73], to model the channel state information uncertainty, we adopt a first-order autoregressive model given by

$$h^{(m)}_{\text{A,B}} = \rho \tilde{h}^{(m)}_{\text{A,B}} + \sqrt{1-\rho^2}\, w^{(m)}_{\text{A,B}} \tag{2.39}$$

where $w^{(m)}_{\text{A,B}} \sim \mathcal{CN}(0, \lambda_m)$ is independent of $h^{(m)}_{\text{A,B}}$.

The parameter ρ is the correlation coefficient between $\tilde{h}^{(m)}_{\text{A,B}}$ and $h^{(m)}_{\text{A,B}}$, which can be determined from Jakes' model, [74], namely $\rho = J_0(2\pi f_d T_d)$, where f_d is the Doppler frequency and T_d is the time delay between relay selection and transmission[4]. Clearly the time delay T_d will be significantly larger for the optimal strategy than for the

[4]We note that the model in (2.39) is also suitable to represent the CSI estimation errors [75].

proposed hop-by-hop method. Therefore, for a fair comparison, the value of ρ for the optimal strategy should be chosen to be smaller than that for the proposed method. However, to determine the exact value of time delay and ρ, one must carefully examine the specific wireless technology involved. Therefore here we take a very optimistic view of the optimal strategy and assume that it has the same value of ρ as the hop-by-hop method. As the numerical results show, even under this scenario, the hop-by-hop method outperforms the optimal strategy.

In Figure 2.4, we show the simulation results for a four-hop network with $L_i = 3$, $i = 1, 2, 3$ as well as a seven-hop network with $L_i = 3$, $i = 1, 2, \cdots, 6$ for the cases of perfect CSI and imperfect CSI with $\rho = 0.95$. In the case of perfect CSI, both optimal and hop-by-hop strategies have almost no performance loss as the number of hops increases from four to seven. However, in the case of the ad-hoc strategy, the seven-hop network has a higher outage probability than the four-hop network. In the case of imperfect CSI, the hop-by-hop strategy has almost no performance loss as the number of hops increases from four to seven. On the other hand for both optimal and ad-hoc strategies, the outage probability increases. Moreover, in the case of imperfect CSI, the hop-by-hop strategy outperforms the optimal[5] and ah-hoc strategies. In particular, for an outage probability of 10^{-2}, the hop-by-hop strategy outperforms the optimal strategy by nearly 4 dB in SNR and this improvement increases for smaller outage probabilities. The reason for this improvement is that the path selected by the optimal strategy may fail at transmission time due to the fact that the (outdated) CSI at the time of transmission may be significantly different from the CSI used for relay selection, although a working path may be available at the time of transmission. In contrast, in the hop-by-hop method relays are not pre-selected but on a hop-by-hop basis right before transmission.

[5]Note that the optimal strategy is only optimal in the case of perfect CSI.

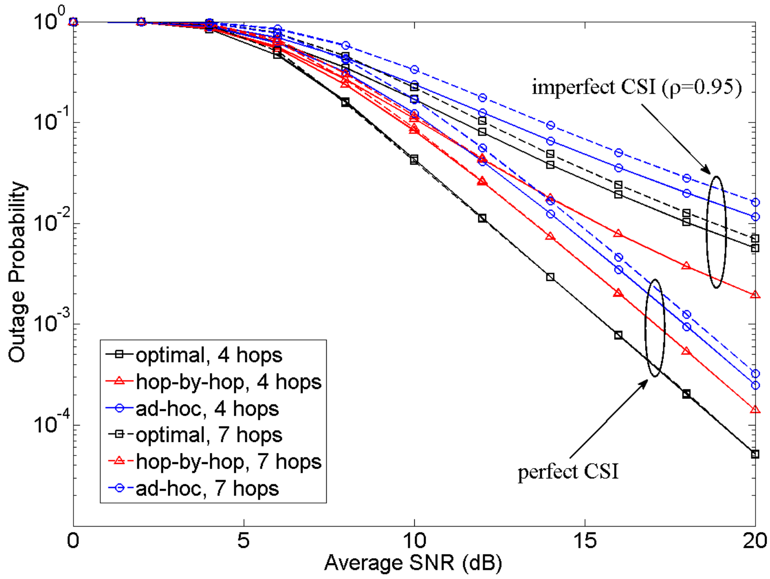

FIGURE 2.4. Outage Probability vs. Average SNR with ρ.

Figure 2.5 shows the end-to-end ergodic capacity vs. the average SNR for networks with different number of relays. It can be seen that for small values of SNR, the increased diversity resulting from a larger number relays improves the ergodic capacity. However, as SNR increases the effect of the higher diversity diminishes and the ergodic capacities are the same for different number of relays. Moreover, at low SNR values, a higher required rate $\mathcal{R}$ leads to a lower ergodic capacity. This is due to the fact that for low SNR values and a large rate $\mathcal{R}$, the decoding sets are likely to be empty, and therefore, the ergodic capacity is nearly zero. The figure also shows that the approximation derived from analysis closely matches the results obtained form simulations.

In Figure 2.6 we show the effective ergodic capacity vs. average SNR for different number or relays. The same conclusions as those for Figure 2.5 can be drawn.

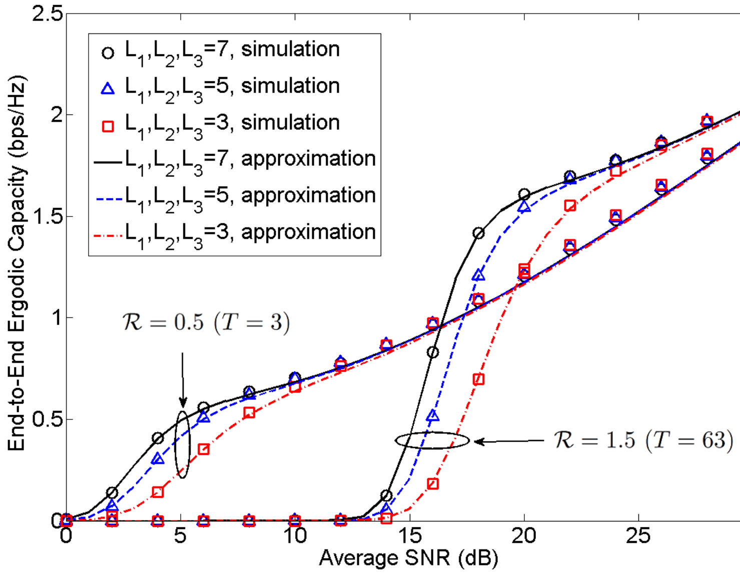

FIGURE 2.5. End-to-End Ergodic Capacity vs. Average SNR.

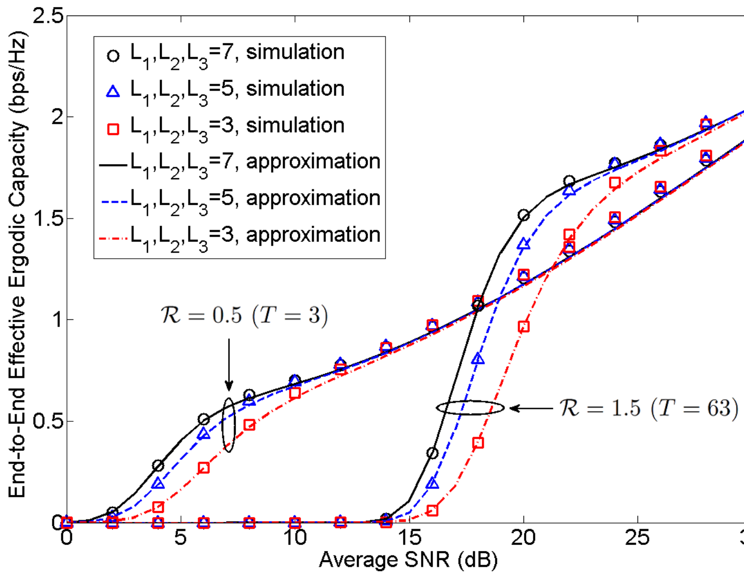

FIGURE 2.6. End-to-End Effective Ergodic Capacity vs. Average SNR.

Chapter 3
A Novel Hop-By-Hop Relay Selection Strategy for Multi-hop Relay Networks

3.1 Introduction

A key challenge in the implementation of the cooperative relay systems is that in a practical system the exact CSI is not available. This problem is further exacerbated for strategies which need to collect the CSI of all or most of the links before they calculate a path, as in the case of optimal [33] and last-n-hop [34] relay selection strategies, specially when the number of hops increases.

In this chapter, we propose a hop-by-hop relay selection strategy for multi-hop DF networks. In each relay cluster, relays that successfully decode the message from the previous hop form a candidate set for relaying. The relay in this candidate set which has the largest number of channels with an acceptable SNR level to the relays in the next hop is selected for retransmission. Therefore, relay selection is only based on the CSI of the channels in the following hop. Hence this method does not require a CC and can be implemented in a distributed manner. We analyze the performance of the proposed method in terms of end-to-end outage probability in both perfect and imperfect CSI scenarios and show that the results closely match those obtained from simulation. We also compare our results with those from ad-hoc and optimal relay selection [33], and show that in the case of perfect CSI the results are close to the optimal relay selection and in the case of imperfect CSI, the proposed method outperforms optimal relay selection. Moreover, our method outperforms ad-hoc in both perfect CSI and imperfect CSI cases.

Notations: Our notations and some of our modeling assumptions for this chapter are introduced here. S, RC_m, and D refer to source, relay cluster m, and destination, respectively. $\mathrm{R}_i^{(m)}$ denotes relay i in the mth relay cluster. $h_{\mathrm{A,B}}^{(m)}$ and $\gamma_{\mathrm{A,B}}^{(m)}$ denote the instantaneous CSI and the instantaneous SNR of link $\mathrm{A} \rightarrow \mathrm{B}$ of hop m, respectively. And we let $g_{\mathrm{A,B}}^{(m)} \triangleq |h_{\mathrm{A,B}}^{(m)}|^2$. We denote the transmit power of each node by P_s and the noise random variable at receiver B by n_{B}. $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2.

3.2 System Model

As shown in Figure 3.1, We consider a multi-hop wireless relay network consisting of one source (S), one destination (D), and M relay clusters (RC_m, $m = 1, \cdots, M$) located between the source and destination. Each relay cluster RC_m includes L_m single-antenna half-duplex relay nodes. There are totally $M + 1$ hops (from hop 0 to hop M) from S to D. We denote the first hop from S to RC_1 as hop 0, RC_1 to RC_2 as hop 1, and so on. We assume $n_{\mathrm{B}} \sim \mathcal{CN}(0, \sigma_n^2)$, and for each hop m, $h_{\mathrm{A,B}}^{(m)} \sim \mathcal{CN}(0, \lambda_m)$.

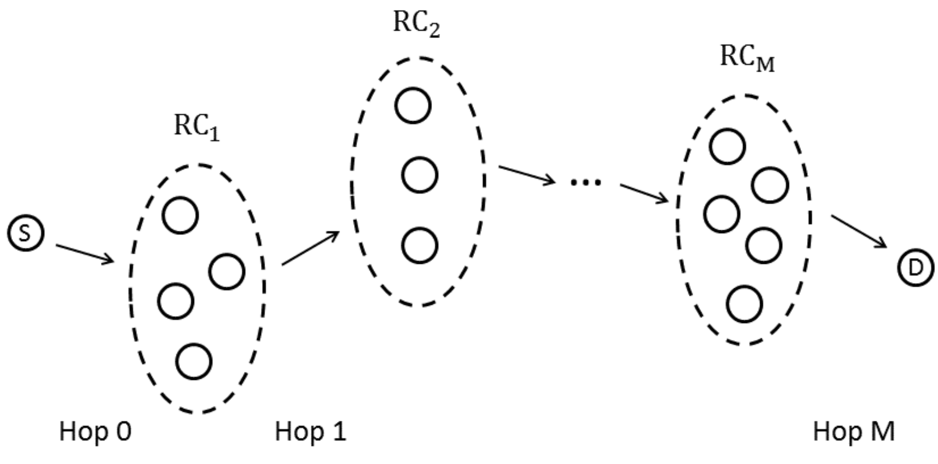

FIGURE 3.1. System Model of Multi-hop Relay Networks.

The instantaneous SNR of link $\mathrm{A} \rightarrow \mathrm{B}$ at hop m is given by

$$\gamma_{\mathrm{A,B}}^{(m)} = P_s |h_{\mathrm{A,B}}^{(m)}|^2 / \sigma_n^2 = \bar{\gamma} g_{\mathrm{A,B}}^{(m)} \tag{3.1}$$

where $\bar{\gamma} = P_s / \sigma_n^2$.

Reactive DF relaying scheme is used where in each relay cluster only a single relay is selected for retransmission. The proposed relay selection strategy is as follows. At the first hop, S broadcasts its signal to the first relay cluster (RC_1). At any hop $m = 1, 2, \cdots, M-1$ the relays in RC_m which are able to correctly decode the received signal form a decoding set denoted by $\mathcal{D}_m$. The decoding set, defined formally in (3.3), consists of all those relays whose SNR exceeds a predefined threshold T[1]. Each relay in $\mathcal{D}_m$ estimates the channel coefficients from itself to all the L_{m+1} relays in RC_{m+1}, computes the corresponding instantaneous SNRs from (3.1), and compares these SNRs to the threshold T. For $\mathrm{R}_i^{(m)}$ in $\mathcal{D}_m$, let $N_i^{(m)}$ denote the number of channels to relays in RC_{m+1} for which the instantaneous SNR exceeds T. $\mathrm{R}_i^{(m)}$ now starts a timer inversely proportional to $N_i^{(m)}$. $\mathrm{R}_{i^*}^{(m)}$ whose timer expires first has the largest number of "good" channels, i.e., $i^* = \arg\max_i N_i^{(m)}$, and will retransmit[2]. All the other relays in $\mathcal{D}_m$ hear this transmission and remain silent. We define

$$N_{\max}^{(m)} \triangleq \max\{N_i^{(m)}; i \in \mathcal{D}_m\} \tag{3.2}$$

We should point out that if $N_{\max}^{(m)} = 0$, then outage is declared. Finally at the last hop, the relay in $\mathcal{D}_M$ which has the highest instantaneous SNR is selected for transmission to D.

Denote by C the required end-to-end spectral efficiency in bps/Hz. Then for $m = 1, 2, \cdots, M$, $\mathcal{D}_m$ consists of those relays whose link capacity from the previous stage exceeds C, i.e.

$$\mathcal{D}_m = \{j : \frac{1}{M+1}\log_2(1 + \gamma_{i^*,j}^{(m-1)}) \geq C\} = \{j : \gamma_{i^*,j}^{(m-1)} \geq 2^{(M+1)C} - 1\} \tag{3.3}$$

where $\gamma_{i^*,j}^{(0)} \triangleq \gamma_{\mathrm{S},j}^{(0)}$ is the SNR from S to $\mathrm{R}_j^{(m)}$ in RC_1, and for $m = 2, 3, \cdots, M$, $\gamma_{i^*,j}^{(m-1)}$ is the SNR from selected $\mathrm{R}_{i^*}^{(m-1)} \in \mathrm{RC}_{m-1}$[3] to $\mathrm{R}_j^{(m)}$ in RC_m. Then the SNR threshold

[1] This is the minimum SNR required for correct decoding of the message.
[2] A small randomization can be introduced into the timer to avoid collisions in the case of ties.
[3] This relay has the largest number of SNR qualified channels to the relays in RC_m.

T is defined as $T \triangleq 2^{(M+1)C} - 1$. In the Sections 3.3 and 3.4 we derive the outage probability of the proposed method for perfect and imperfect CSI, respectively.

3.3 Outage Probability in the Case of Perfect CSI

When CSI is perfect, the CSI used in relay selection is the same as the CSI at the time of retransmission. Therefore for any hop $m = 1, 2, \cdots, M-1$, the number of relays in $\mathcal{D}_{m+1}$ is equal to $N_{\max}^{(m)}$. Let $c(\mathcal{D}_n)$ denote the cardinality of the set $\mathcal{D}_n$, and let $\mathcal{O}_n = \{c(\mathcal{D}_n) = 0\}$, i.e., $\mathcal{O}_n$ is the event that no relay in the nth cluster can decode the message, and $\mathcal{O}_{M+1}$ is the event that destination D cannot decode the message. Then for the end-to-end outage event $\mathcal{O}$ we can write

$$\begin{aligned} P(\mathcal{O}) &= P(\cup_{n=1}^{M+1}\mathcal{O}_n) = \sum_{l_1=0}^{L_1} P(\cup_{n=1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1) \\ &= P(\mathcal{O}_1) + \sum_{l_1=1}^{L_1} P(\cup_{n=2}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1) \end{aligned} \tag{3.4}$$

Clearly, we have

$$P(\mathcal{O}_1) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{L_1} \tag{3.5}$$

Moreover, we can calculate the probability that there are l_1 relays in $\mathcal{D}_1$, which is

$$P(c(\mathcal{D}_1) = l_1) = \binom{L_1}{l_1} (1 - e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{L_1 - l_1} (e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{l_1} \tag{3.6}$$

Now what we need to calculate is $P(\cup_{n=m+1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_m) = l_m)$ for any $1 \le m < M$. For $1 \le l_m \le L_m$, we have

$$\begin{aligned} &P(\cup_{n=m+1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_m) = l_m) \\ =&P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) \\ &+ \sum_{l_{m+1}=1}^{L_{m+1}} P(\cup_{n=m+2}^{M+1}\mathcal{O}_n | c(\mathcal{D}_{m+1}) = l_{m+1}) P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m) \end{aligned} \tag{3.7}$$

Note that in the above

$$P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) = P(c(\mathcal{D}_{m+1}) = 0 | c(\mathcal{D}_m) = l_m)$$

Therefore we need to evaluate $P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m)$ and $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$.

For $m = 1, 2, \cdots, M-1$, $P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m)$ is the probability that from any relay in $\mathcal{D}_m$, the SNRs of all L_{m+1} links to the relays in RC_{m+1} are below the threshold T. Therefore,

$$P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}} \tag{3.8}$$

Similarly, for the last hop (when $m = M$), since the relay in $\mathcal{D}_M$ with the highest SNR among the $c(\mathcal{D}_M) = l_M$ links is selected for retransmission, outage occurs when the SNR of all these l_M links are below the threshold T. Therefore,

$$P(\mathcal{O}_{M+1}|c(\mathcal{D}_M) = l_M) = P(\max_{i\in\mathcal{D}_M}\{\gamma_{i,\mathrm{D}}^{(M)}\} < T|c(\mathcal{D}_M) = l_M) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_M}})^{l_M} \tag{3.9}$$

Let $A_k^{(m)}$ denote the event that from a relay in $\mathcal{D}_m$, there are k channels to relays in RC_{m+1} whose instantaneous SNRs are above the threshold T, and let $B_k^{(m)} = \bigcup_{l'=0}^{k-1} A_{l'}^{(m)}$. Then we have

$$P(A_k^{(m)}) = \binom{L_{m+1}}{k}(e^{-\frac{T}{\bar{\gamma}\lambda_m}})^k(1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}-k} \tag{3.10}$$

Also $P(B_k^{(m)}) = \sum_{l'=0}^{k-1} P(A_{l'}^{(m)})$.

To evaluate $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$, we note that there must be l ($1 \le l \le l_m$) relays in $\mathcal{D}_m$ which have l_{m+1} channels to relays in RC_{m+1}, whose SNRs exceed the threshold T, while the remaining $l_m - l$ relays in $\mathcal{D}_m$, have fewer than l_{m+1} channels with SNRs above the threshold T. Therefore we can write

$$P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m) = \sum_{l=1}^{l_m}\binom{l_m}{l}\left[P(A_{l_{m+1}}^{(m)})\right]^l\left[P(B_{l_{m+1}}^{(m)})\right]^{l_m-l} \tag{3.11}$$

Finally by putting(3.5) (3.6), (3.8)-(3.11) into (3.4), we get the outage probability for the case of perfect CSI.

3.4 Outage Probability in the Presence of CSI Uncertainty

In mobile systems, the exact CSI is not available. Therefore, the CSI used for relay selection may be different from the CSI at the time of retransmission. This implies that the relay selected for retransmission may not satisfy the criterion of the relay selection strategy, which would degrade the outage performance of the relay selection strategy.

Therefore, in this section we evaluate the performance of the proposed strategy in the presence of uncertainty in the CSI.

Similar to (2.39), we denote by $\tilde{h}_{\mathrm{A,B}}^{(m)}$ and $h_{\mathrm{A,B}}^{(m)}$ the CSIs at the instants of relay selection and relay transmission, respectively. To model the CSI uncertainty, we adopt a first-order autoregressive model given by

$$h_{\mathrm{A,B}}^{(m)} = \rho \tilde{h}_{\mathrm{A,B}}^{(m)} + \sqrt{1-\rho^2} w_{\mathrm{A,B}}^{(m)} \tag{3.12}$$

where $w_{\mathrm{A,B}}^{(m)} \sim \mathcal{CN}(0, \lambda_m)$ is independent of $\tilde{h}_{\mathrm{A,B}}^{(m)}$.

To evaluate the outage probability in this case we again start with (3.4). We first consider the transmission from S to RC_1 at hop 0. Since S simply broadcasts its message to all the relays in RC_1 without any CSI collection or relay selection procedure, CSI uncertainty is not an issue here. Therefore $P(\mathcal{O}_1)$ and $P(c(\mathcal{D}_1) = l_1)$ are exactly the same as in the case of perfect CSI.

Next, to evaluate $P(\cup_{n=m+1}^{M+1} \mathcal{O}_n | c(\mathcal{D}_m) = l_m)$ for any $1 \leq m < M$ and $1 \leq l_m \leq L_m$, we use (3.7) for which we need to compute $P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m)$ for $1 \leq m \leq M$, and $P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m)$ for $1 \leq m \leq M-1$. We start by evaluating $P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m)$ for $1 \leq m \leq M-1$. When $1 \leq l_m \leq L_m$, one relay, say $\mathrm{R}_{i^*}^{(m)} \in \mathcal{D}_m$, among these l_m relays is selected for retransmission. In this case the outage probability in this stage can be expressed as

$$
\begin{aligned}
&P(\mathcal{O}_{m+1}|c(\mathcal{D}_m) = l_m) \\
&= \sum_{\tilde{l}_{m+1}=1}^{L_{m+1}} P(c(\mathcal{D}_{m+1}) = 0|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}, c(\mathcal{D}_m) = l_m) \\
&\quad \times P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m) \\
&\quad + P(c(\tilde{\mathcal{D}}_{m+1}) = 0|c(\mathcal{D}_m) = l_m) \\
&= \sum_{\tilde{l}_{m+1}=1}^{L_{m+1}} P(c(\mathcal{D}_{m+1}) = 0|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}) P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m) \\
&\quad + P(c(\tilde{\mathcal{D}}_{m+1}) = 0|c(\mathcal{D}_m) = l_m)
\end{aligned} \tag{3.13}
$$

where $\tilde{\mathcal{D}}_{m+1}$ denotes the decoding set assumed in RC_{m+1} when $\mathrm{R}_{i^*}^{(m)}$ is selected, which might be different from the actual decoding set $\mathcal{D}_{m+1}$ formed after transmission from $\mathrm{R}_{i^*}^{(m)}$. To briefly explain (3.13), we note that if $c(\tilde{\mathcal{D}}_{m+1}) = 0$, then an outage is declared and no retransmission is attempted at this stage. On the other hand, when $c(\tilde{\mathcal{D}}_{m+1}) > 0$, an outage occurs if after retransmission, no relays in relay cluster RC_{m+1} can decode the message, i.e., $c(\mathcal{D}_{m+1}) = 0$.

Similar to (3.11), when $0 < \tilde{l}_{m+1} \le L_{m+1}$, we can write

$$
P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m) = \sum_{l=1}^{l_m} \binom{l_m}{l} \left[P(A_{\tilde{l}_{m+1}}^{(m)})\right]^l \left[P(B_{\tilde{l}_{m+1}}^{(m)})\right]^{l_m - l} \tag{3.14}
$$

and similar to (3.8), we have

$$
P(c(\tilde{\mathcal{D}}_{m+1}) = 0|c(\mathcal{D}_m) = l_m) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{l_m L_{m+1}} \tag{3.15}
$$

Since CSI used for relay selection is imperfect, it is possible the actual decoding set formed after transmission is actually an empty set. With some thought it can be seen that

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = 0|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}) \\
&= [P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} > T)]^{\tilde{l}_{m+1}} [P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} < T)]^{L_{m+1} - \tilde{l}_{m+1}}
\end{aligned} \tag{3.16}
$$

The following lemmas whose proofs are given in Appendix D and E are used to evaluate the two probabilities involved in (3.16).

Lemma 6. *$P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} < T)$ is given by*

$$
\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} < T) \\
&= 1 - \frac{e^{-\frac{T}{\bar{\gamma}\lambda_m}}[Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) - Q_1(\rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})]}{1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}}} \quad (3.17)
\end{aligned}
$$

where $Q_1(a,b) = \int_b^\infty xe^{-\frac{x^2+a^2}{2}} I_0(ax)dx$ is the first-order Marcum Q-function [76].

Lemma 7. *$P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} > T)$ is given by*

$$
\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} > T) \\
&= Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) - Q_1(\rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})
\end{aligned}
\quad (3.18)
$$

Finally for the last hop when $m = M$, and for $1 \leq l_M \leq L_M$, according to Eq. (8) in [25], we have

$$
P(\mathcal{O}_{M+1}|c(\mathcal{D}_M) = l_M) = l_M \sum_{s=0}^{l_M-1} \binom{l_M - 1}{s} (-1)^s \frac{1}{1+s}(1 - e^{\frac{-(1+s)T}{(1+s-\rho^2 s)\bar{\gamma}\lambda_M}}) \quad (3.19)
$$

Next, we need to evaluate $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$. When $1 \leq l_m \leq L_m$ and $0 < l_{m+1} \leq L_{m+1}$, $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$ can be written as

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m) \\
&= \sum_{\tilde{l}_{m+1}=1}^{L_{m+1}} P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}, c(\mathcal{D}_m) = l_m) \\
&\quad \times P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m) \\
&= \sum_{\tilde{l}_{m+1}=1}^{L_{m+1}} P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}) P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m) \quad (3.20)
\end{aligned}
$$

in which $P(c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}|c(\mathcal{D}_m) = l_m)$ is calculated in (3.14) and $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1})$ can be derived as

$$
\begin{aligned}
&P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\tilde{\mathcal{D}}_{m+1}) = \tilde{l}_{m+1}) \\
&= \sum_{k=\max\{l_{m+1}-(L_{m+1}-\tilde{l}_{m+1}),0\}}^{\min\{l_{m+1},\tilde{l}_{m+1}\}} \binom{\tilde{l}_{m+1}}{k} \\
&\quad \times [P(\gamma_{i^*,j}^{(m)} > T|\tilde{\gamma}_{i^*,j}^{(m)} > T)]^k [P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} > T)]^{\tilde{l}_{m+1}-k} \\
&\quad \times \binom{L_{m+1}-\tilde{l}_{m+1}}{l_{m+1}-k} [P(\gamma_{i^*,j}^{(m)} > T|\tilde{\gamma}_{i^*,j}^{(m)} < T)]^{l_{m+1}-k} \\
&\quad \times [P(\gamma_{i^*,j}^{(m)} < T|\tilde{\gamma}_{i^*,j}^{(m)} < T)]^{L_{m+1}-\tilde{l}_{m+1}-(l_{m+1}-k)}
\end{aligned}
\tag{3.21}
$$

We would like to note that (3.16) is a special case of (3.21) when $l_{m+1} = 0$. The probabilities in (3.21) are evaluated from Lemmas 6 and 7.

From Lemmas 6 and 7, we can evaluate the left hand side of (3.16) and (3.21). Then using (3.13) (3.16) (3.19) and (3.21) into (3.7), we get $P(\cup_{n=m+1}^{M+1}\mathcal{O}_n|c(\mathcal{D}_m) = l_m)$ for the case of imperfect CSI. Then the end-to-end outage probability can be computed from (3.4).

3.5 Numerical Results

We start by comparing the complexity of our proposed method with that of optimal and ad-hoc relay selection strategies. Optimal relay selection requires the CSI of all the links to be collected by a CC before the best path can be selected. The total number of CSIs in this case is $\Lambda_1 = L_1 + L_M + \sum_{m=1}^{M-1} L_m L_{m+1}$. In the proposed method the number of CSI needed to select the relay in RC_m (from the set $\mathcal{D}_m$), $(m = 1, 2, \cdots, M-1)$ is $l_m L_{m+1}$, and for the last hop it is l_M. Therefore the total number of CSIs needed is $\Lambda_2 = l_M + \sum_{m=1}^{M-1} l_m L_{m+1}$. Clearly $\Lambda_2 < \Lambda_1$. In cases when channel conditions are not favorable we have $l_m << L_m$ and $\Lambda_2 << \Lambda_1$. More importantly however, the proposed method is a hop-by-hop relay selection strategy. It does not require a CC for end-to-end path selection and therefore its algorithmic complexity and communication overhead are significantly lower than optimal relay selection. In the ad-hoc relay selection strategy the total number of CSIs needed for relay selection is $\Lambda_3 = \sum_{m=1}^{M} L_m$, which is much less than our proposed method. However, as shown below, the performance of the ad-hoc strategy is significantly inferior to our method.

It is not possible to determine the time delay between relay selection and transmission without careful examination of the specific wireless technology involved. Clearly the time delay for optimal relay selection will be significantly larger than that of our proposed method and ad-hoc relay selection since relay selection and transmission in optimal relay selection is not on hop-by-hop basis. However, in the results presented here we take a very optimistic view of optimal relay selection and assume that it has the same time delay as hop-by-hop relay selection strategies. Therefore we compare our results with those from optimal relay selection for the same value of ρ. As the numerical results for the case of imperfect CSI show, even in this scenario, our method outperforms optimal relay selection. The reason for this improvement is explained in the following when we discuss the figures. However, for fair comparison, the value of ρ for optimal relay selection should be smaller than that for the proposed method resulting in even larger improvements for our method over optimal relay selection.

In Figure 3.2, we show the outage probability vs. SNR for 4-hop ($M = 3$) relay networks, where SNR is defined as $\bar{\gamma} = P_s/\sigma_n^2$, and we have assumed that $\lambda_m = 1$. We assume $\sigma_n^2 = 1$, and a target rate of $C = 2/(M + 1)$ bps/Hz. We compare the outage probability of our proposed relay selection strategy with the results from optimal and ad-hoc relay selection strategies in the case of perfect CSI. When $(L_1, L_2, L_3) = (5, 5, 5)$, all relay clusters have the same number of relays. As we can see, optimal relay selection has the lowest outage probability, but the outage probability of our proposed method approaches and converges to that of optimal relay selection at high

SNR region. Meanwhile, ad-hoc relay selection has a much higher outage probability compared to our proposed method.

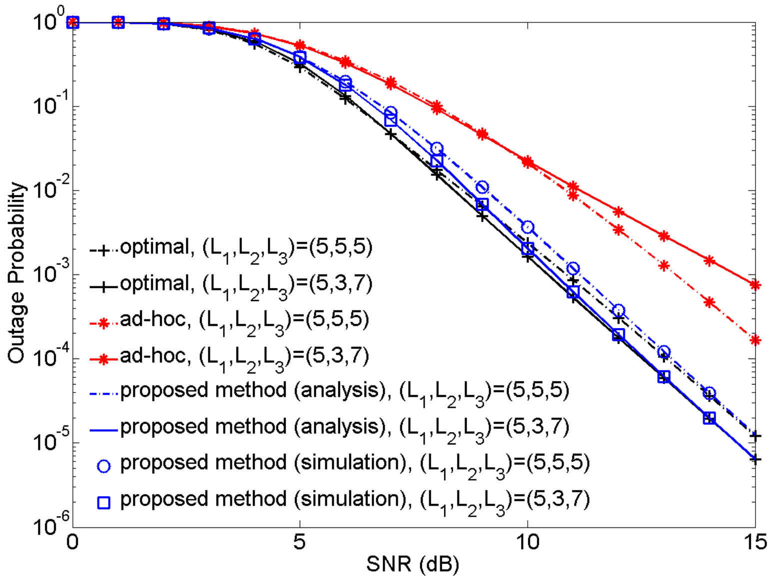

FIGURE 3.2. Outage Probability vs. SNR with different relay distributions.

In the optimal relay selection strategy in [33], all relay clusters have the same number of relays, the first hop and the last hop are the main constraints for outage probability. In our proposed method, in the first hop the diversity order is L_1, while in last hop it is $c(\mathcal{D}_M) = l_M$. For any hop m between two relay clusters, there are $l_m L_{m+1}$ links to be considered for relay selection. Therefore, in this case the last hop which has the smallest diversity order becomes the bottleneck. As SNR increases, with high probability, all relays in the last relay cluster are included in the decoding set. As a result the last hop has the same diversity order as the first hop. This is the reason that the outage probability of the proposed method approaches that of optimal relay selection at high SNR region. When $(L_1, L_2, L_3) = (5, 3, 7)$, we can see that both optimal relay selection and our proposed method have lower outage probabilities compared to $(L_1, L_2, L_3) = (5, 5, 5)$ case, since the diversity order of last hop increases, Unlike optimal relay selection and our proposed method, ad-hoc relay selection has even higher outage probability compared to $(L_1, L_2, L_3) = (5, 5, 5)$ case. For ad-hoc relay selection the diversity order of hop m is L_m and reducing L_2 from 5 to 3, causes a bottleneck at this hop resulting in higher outage probability.

In Figure 3.3, we set $(L_1, L_2, L_3) = (5, 5, 5)$. We also use the same parameters as in Figure 3.2 except that we consider both perfect CSI and imperfect CSI cases for comparison. In the case of perfect CSI, as we discussed above, the outage probability of our proposed method approaches that of optimal relay selection at high SNR region. When $\rho = 0.99$, it is clear that our proposed method has the lowest outage probability among the three relay selection strategys. When ρ reduces from 0.99 to 0.9, the difference between outage probabilities of the proposed method and optimal relay selection is even larger. The reason is that in the case of imperfect CSI, optimal relay selection does not fully explore the broadcast nature of the relays. Once the path from source to destination is selected, all the relays participating in transmission are determined. In the case of perfect CSI, this path is the best path. However, between any two relay clusters, one pair of relays and their link are selected and fixed for the path. When this link is down due to the fact that the CSI at transmission time is different from the CSI that was used for path selection, there is no back-up channel available and end-to-end outage occurs. In contrast, in our proposed relay selection, relays are not pre-selected globally but on a hop-by-hop basis right before transmission. For any hop m ($m = 1, 2, \cdots, M-1$), the best relay selected in $\mathcal{D}_m$ transmits to all the relays in RC_{m+1}. All the relays in RC_{m+1} which are able to correctly decode the message form $\mathcal{D}_{m+1}$ are candidates for forwarding the message. Due to this broadcast nature of relays at every hop, our proposed method is less vulnerable and more robust to imperfect CSI issue.

In Figure 3.4, we plot the outage probability vs. ρ for $(L_1, L_2, L_3) = (5, 5, 5)$. We also use the same parameters as in Figure 3.2 except that $\bar{\gamma} = P_s/\sigma_n^2 = 15$ dB. In addition

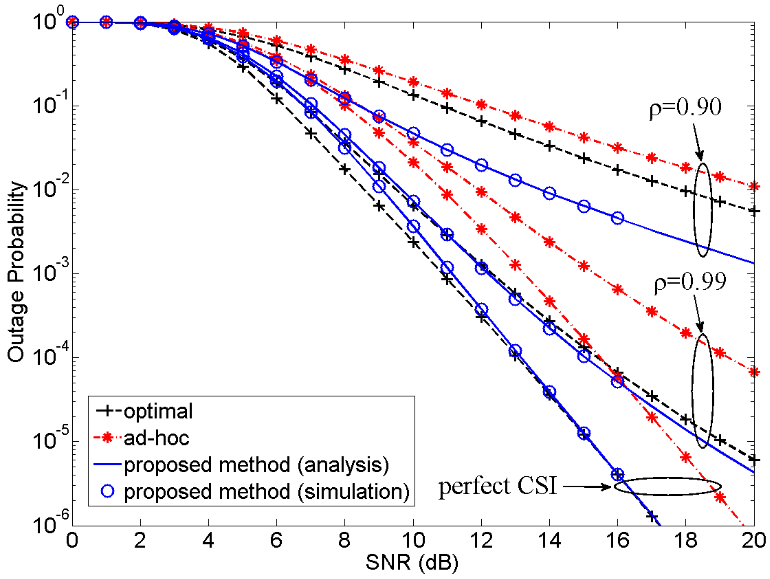

FIGURE 3.3. Outage Probability vs. SNR in perfect CSI and imperfect CSI cases.

to the proposed method, the optimal and the ad-hoc relay selection strategies, we have plotted the outage probabilities for two other simple relay selection methods.

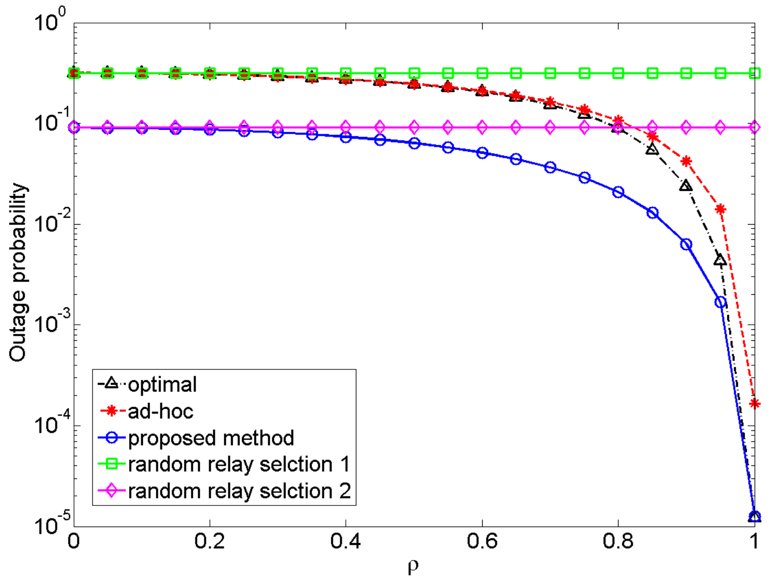

FIGURE 3.4. Outage Probability vs. ρ.

The "random relay selection 1" refers to a method in which a random end-to-end path is selected for transmission. In other words, in each relay cluster, one relay will receive, decode, and retransmit to the selected relay/destination in the following stage. Since all the relays are randomly selected without considering any CSI/SNR, there is no imperfect CSI issue. The performance of this scheme is the same as the case when there is only a single relay in each stage. For an $M+1$-hop network, the end-to-end outage probability is given by

$$\begin{aligned} P(\mathcal{O}_{rrs1}) &= 1 - \sum_{m=1}^{M+1} [1 - P(\mathcal{O}_m)] \\ &= 1 - \sum_{m=1}^{M+1} e^{-\frac{T}{\bar{\gamma}\lambda_{m-1}}} \end{aligned} \tag{3.22}$$

The "random relay selection 2" refers to the method where in each relay cluster, the relays which are able to correctly decode the received signal from the previous stage form a decoding set, and one relay among them is chosen at random to retransmit to relays/destination in the following stage. The end-to-end outage probability of this method can be expressed as

$$\begin{aligned} P(\mathcal{O}_{rrs2}) &= P(\cup_{n=1}^{M+1} \mathcal{O}_n) \\ &= P(\mathcal{O}_1) + P(\cup_{n=2}^{M+1} \mathcal{O}_n | c(D_1) > 0) P(c(D_1) > 0) \end{aligned} \tag{3.23}$$

in which $P(\mathcal{O}_1)$ is given in (3.5). $P(c(D_1) > 0)$ can be directly calculated as

$$\begin{aligned} P(c(D_1) > 0) &= 1 - P(\mathcal{O}_1) \\ &= 1 - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_0}})^{L_1} \end{aligned} \tag{3.24}$$

For any $1 \leq m < M$,

$$\begin{aligned} & P(\cup_{n=m+1}^{M+1} \mathcal{O}_n | c(D_m) > 0) \\ & = P(\mathcal{O}_{m+1} | c(D_m) > 0) \\ & \quad + P(\cup_{n=m+2}^{M+1} \mathcal{O}_n | c(D_{m+1}) > 0) P(c(D_{m+1}) > 0 | c(D_m) > 0) \end{aligned} \tag{3.25}$$

in which

$$P(\mathcal{O}_{m+1} | c(D_m) > 0) = (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}} \tag{3.26}$$

and

$$P(c(D_{m+1}) > 0|c(D_m) > 0) = 1 - P(\mathcal{O}_{m+1}|c(D_m) > 0)$$
$$= 1 - (1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}})^{L_{m+1}} \quad (3.27)$$

And for the last hop, one random relay in D_M is selected to transmit to D, the diversity order is one, and we have

$$P(\mathcal{O}_{M+1}|c(D_M) > 0) = 1 - e^{-\frac{T}{\bar{\gamma}\lambda_M}} \quad (3.28)$$

When $\rho = 0$, the instantaneous CSI at the time of relay transmission is unrelated to the instantaneous CSI at the time of relay selection. In the optimal relay selection strategy, this is equivalent to the scenario that a random end-to-end path is selected for transmission, which is the same as "random relay selection 1". Similarly for the ad-hoc method, $\rho = 0$ is equivalent to 'random relay selection 1". Figure 3.4 verifies that at $\rho = 0$, these three methods have the same performance. In contrast, when $\rho = 0$, the proposed method would be the same as "random relay selection 2" and this is also verified in Figure 3.4. As ρ increases, the outage probabilities of the optimal, ad-hoc and the proposed methods are reduced. However, as the figure shows, the ad-hoc strategy can never achieve the same performance as the proposed method, and the optimal strategy can only achieve the same performance as our proposed method when $\rho = 1$. Figure 3.4 also shows that for values of $\rho < 0.8$, the improvement of the outage probability vs. ρ is very slow and that acceptable performance can only be achieved for values of ρ close to 1. This trend is also valid for other configurations of the network. From figures such as Figure 3.4 we can obtain the minimum value of ρ which guarantees an upper bound on the resulting outage probability. Using the Jakes' model, this can then be translated into an upper limit on the time duration between CSI measurement and relay transmission.

In Figure 3.5, we show the results of a 4-hop $(L_1, L_2, L_3) = (3, 3, 3)$ network and a 7-hop $(L_1, L_2, L_3, L_4, L_5, L_6) = (3, 3, 3, 3, 3, 3)$ network in perfect CSI case and imperfect CSI ($\rho = 0.90$) case, respectively. In the case of perfect CSI, both optimal relay selection and our method have almost no performance loss as the number of hops increases from 4 to 7. However, it is clear that for ad-hoc relay selection, the 7-hop network has a higher outage probability than the 4-hop network. In the case of imperfect CSI ($\rho = 0.90$), our method has almost no performance loss as the number of hops increases from 4 to 7. Therefore, in the case of imperfect CSI, our method is robust to increases in the number of hops in the network. However, for both optimal relay selection and ad-hoc relay selection, the 7-hop network has a higher outage probability than the 4-hop network.

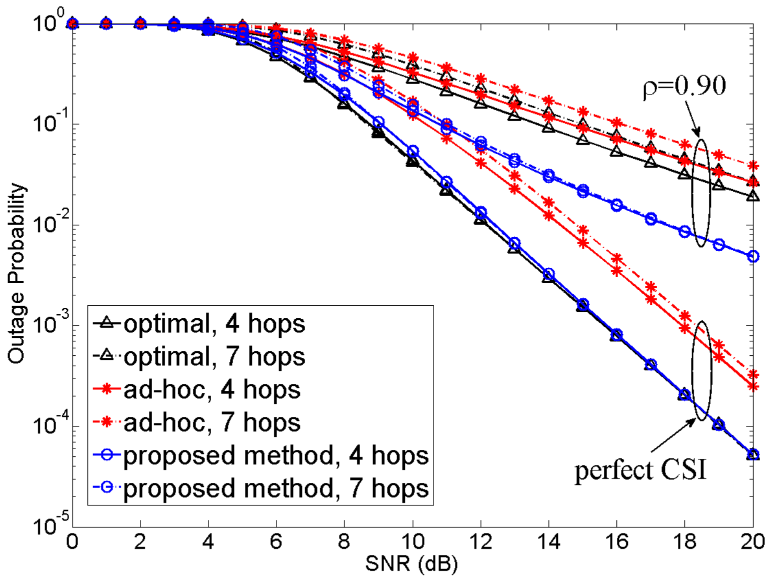

FIGURE 3.5. Outage Probability vs. SNR with different number of hops.

Chapter 4

Decode-and-Forward Relay Selection with Imperfect CSI in Two-hop Cognitive Relay Networks

4.1 Introduction

In this chapter[1], we consider cognitive relay networks with imperfect CSI under interference power constraint. Reactive DF and ORS are assumed whereby SU relays that successfully receive and decode the message from the SU source form the candidate set for relaying, and the best relay among them is selected to retransmit to the SU destination. We investigate the performance of DF-ORS in terms of outage probability of the SU and the interference probability at the PU. In order to allow the secondary network to back-off its peak transmit power, two power margin factors are considered for the SU source and relays. Numerical results show that with the proper selection of the power margin factors the desired values of outage and interference probabilities can be achieved. The results here can also be used to select other system parameters in order to achieve the desired system performance.

Notations: Here we introduce the notation used in the rest of the chapter. S, P, R(k) and D refer to SU source, PU (receiver), relay k, and SU destination, respectively. $\tilde{h}_{\mathrm{A,B}}$ denotes the imperfect channel coefficient for link A $\rightarrow$ B. $h_{\mathrm{A,B}}$ denotes the current CSI of link A $\rightarrow$ B. $\tilde{\gamma}_{\mathrm{A,B}}$ and $\gamma_{\mathrm{A,B}}$ denote the imperfect SNR and the current SNR of link A $\rightarrow$ B, respectively. We denote by P_{A} the transmit power from node A. $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2, and $\mathrm{Exp}(1/\lambda)$ denotes an exponential distribution with mean λ. We

[1]Sections of this chapter previously appeared as Hui Sun, Mort Naraghi-Pour, Decode-and-Forward Relay Selection with Imperfect CSI in Cognitive Relay Networks, at the 2014 IEEE Military Communications Conference ©2014 IEEE. It is reprinted by permission of IEEE.

use the notations $f_X(\cdot)$ and $F_X(\cdot)$ to refer to the probability density function (PDF) and the cumulative distribution function (CDF) of random variable X, respectively.

4.2 System Model

Consider an SU in a cognitive relay network with a single source and destination and a set of K relays as shown in Figure 4.1.

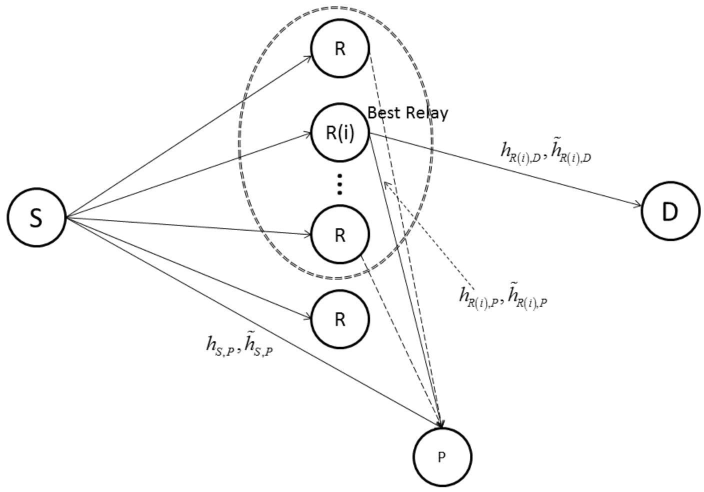

FIGURE 4.1. System Model of Two-hop Underlay Spectrum Sharing Cognitive Relay Networks.

For any link from node A to node B, the received signal at B is given by

$$r_{\mathrm{B}} = h_{\mathrm{A,B}} x_{\mathrm{A}} + n_{\mathrm{B}} \tag{4.1}$$

where x_{A} is the transmitted symbol from A, $n_{\mathrm{B}} \sim \mathcal{CN}(0, \sigma_{\mathrm{B}}^2)$ is the noise random variable at node B. We assume that the noise variables at all the receivers are *iid* at all SU relays. All channel coefficients are also assumed to be independent. For transmitter A and receiver B, the channel coefficient $h_{\mathrm{A,B}} \sim \mathcal{CN}(0, \lambda_{\mathrm{A,B}})$ with $\lambda_{\mathrm{A,B}} = (\frac{d_{\mathrm{A,B}}}{d_0})^{-\eta}$, where η is the path-loss exponent and $d_{\mathrm{A,B}}$ is the distance between A and B, and d_0 is the close-in reference distance. Hence the channel gain $g_{\mathrm{A,B}} = |h_{\mathrm{A,B}}|^2 \sim \mathrm{Exp}(1/\lambda_{\mathrm{A,B}})$ with PDF $f_{g_{\mathrm{A,B}}}(x) = \frac{1}{\lambda_{\mathrm{A,B}}} e^{\frac{-x}{\lambda_{\mathrm{A,B}}}}$. Assuming that the relays are approximately equidistant from the SU source, the SU destination and the PU. Therefore, $\lambda_{\mathrm{S,R}(k)} = \lambda_{\mathrm{S,R}}$,

$\lambda_{\mathrm{R}(k),\mathrm{D}} = \lambda_{\mathrm{R,D}}$, and $\lambda_{\mathrm{R}(k),\mathrm{P}} = \lambda_{\mathrm{R,P}}$, for $k = 1, \cdots, K$[2]. The instantaneous SNR of link $\mathrm{A} \to \mathrm{B}$, denoted by $\gamma_{\mathrm{A,B}}$, is given by $\gamma_{\mathrm{A,B}} = P_{\mathrm{A}}|h_{\mathrm{A,B}}|^2/\sigma_{\mathrm{B}}^2$.

Let I_p denote the maximum interference power that the PU can tolerate. It is required that the interference at the PU receiver remain below I_p. Therefore the transmit power at S, denoted by P_s, is limited by $P_s|\tilde{h}_{\mathrm{S,P}}|^2 \le I_p$ based on imperfect $\mathrm{S} \to \mathrm{P}$ channel. To get the maximum SNR from source to relay, we may choose $P_s = I_p/|\tilde{h}_{\mathrm{S,P}}|^2$. However, we introduce a power margin factor α $(0 < \alpha \le 1)$ and let $P_s = \alpha I_p/|\tilde{h}_{\mathrm{S,P}}|^2$. When transmit power at the SU source is decided, the SU source broadcasts its data to all the relays. At this time, CSI of $\mathrm{S} \to \mathrm{P}$ is $h_{\mathrm{S,P}}$, which may be different from the outdated CSI $\tilde{h}_{\mathrm{S,P}}$.

All the relays which are able to decode the source information form a decoding set ($\mathcal{D}$). From among these, the relay which has the best relay-to-destination channel is selected to retransmit. Also, we introduce a power margin factor β $(0 < \beta \le 1)$ in order to lower the interference probability from the relays to the PU. Therefore, for relay k in $\mathcal{D}$, the retransmit power $P_{\mathrm{R}(k)}$ is given by $P_{\mathrm{R}(k)} = \beta I_p/|\tilde{h}_{\mathrm{R}(k),\mathrm{P}}|^2$, and the SNR in the link $\mathrm{R}(k) \to \mathrm{D}$ at the relay selection time is given by $\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} = P_{\mathrm{R}(k)}|\tilde{h}_{\mathrm{R}(k),\mathrm{D}}|^2/\sigma_{\mathrm{D}}^2$. From the $\mathcal{D}$, we choose relay $\mathrm{R}(i)$ for retransmission if

$$i = \arg\max_{k \in \mathcal{D}} \ \tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \tag{4.2}$$

Note that the SNR in the link $\mathrm{R}(i) \to \mathrm{D}$ at the retransmission time is given by $\gamma_{\mathrm{R}(i),\mathrm{D}} = P_{\mathrm{R}(i)}|h_{\mathrm{R}(i),\mathrm{D}}|^2/\sigma_{\mathrm{D}}^2$. Similar to (2.39), to model the CSI uncertainty, we adopt a first-order autoregressive model given by

$$h_{\mathrm{A,B}} = \rho \tilde{h}_{\mathrm{A,B}} + \sqrt{1-\rho^2}\, w_{\mathrm{A,B}} \tag{4.3}$$

where $w_{\mathrm{A,B}} \sim \mathcal{CN}(0, \lambda_{\mathrm{A,B}})$ is independent of $\tilde{h}_{\mathrm{A,B}}$.

[2]This assumption is justified by the proximity of the relays to each other and their large separation from the other entities in the network.

4.3 Outage Probability

Let C denote the end-to-end spectral efficiency in bps/Hz required for the SU. Then the set $\mathcal{D}$ consists of those relays whose link capacity to the source exceeds C, that is,

$$\begin{aligned}\mathcal{D} &= \{k : \frac{1}{2}\log_2(1+\gamma_{\mathrm{S,R}(k)}) \geq C\} \\ &= \{k : \gamma_{\mathrm{S,R}(k)} \geq 2^{2C} - 1\} \end{aligned} \tag{4.4}$$

Let $T \triangleq 2^{2C} - 1$. Then $\mathcal{D} = \{k : \gamma_{\mathrm{S,R}(k)} \geq T\}$. We denote by $c(\mathcal{D})$ the cardinality of the set $\mathcal{D}$. Then outage probability can be written as

$$P_{out}(T) = \sum_{l=0}^{K} P(outage|c(\mathcal{D}) = l)P(c(\mathcal{D}) = l) \tag{4.5}$$

In the following, we compute $P(c(\mathcal{D}) = l)$ and $P(outage|c(\mathcal{D}) = l)$, respectively.

We can write

$$P(c(\mathcal{D}) = l) = \int_0^\infty P(c(\mathcal{D}) = l|\tilde{g}_{\mathrm{S,P}} = y)f_{\tilde{g}_{\mathrm{S,P}}}(y)\ dy. \tag{4.6}$$

Thus we need to evaluate $P(c(\mathcal{D}) = l|\tilde{g}_{\mathrm{S,P}} = y)$ and $f_{\tilde{g}_{\mathrm{S,P}}}(y)$. Since $P_S = \alpha I_p/\tilde{g}_{\mathrm{S,P}}$, we have

$$\gamma_{\mathrm{S,R}(k)} = \frac{P_S|h_{\mathrm{S,R}(k)}|^2}{\sigma_R^2} = \frac{\alpha I_p g_{\mathrm{S,R}(k)}}{\sigma_R^2 \tilde{g}_{\mathrm{S,P}}} \tag{4.7}$$

Let $\bar{\gamma}_{\mathrm{S,R}} \triangleq \frac{I_p}{\sigma_R^2}$. Then

$$P(\gamma_{\mathrm{S,R}(k)} < x|\tilde{g}_{\mathrm{S,P}} = y) = P(g_{\mathrm{S,R}(k)} < \frac{xy}{\alpha\bar{\gamma}_{\mathrm{S,R}}}) = 1 - e^{-\frac{xy}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}} \tag{4.8}$$

from which and the fact that all the source-to-relay channels are *iid*, we get

$$P(c(\mathcal{D}) = l|\tilde{g}_{\mathrm{S,P}} = y) = \binom{K}{l}(1 - e^{-\frac{Ty}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^{K-l}(e^{-\frac{Ty}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^l \tag{4.9}$$

Finally noting that $f_{\tilde{g}_{\mathrm{S,P}}}(x) = f_{g_{\mathrm{S,P}}}(x) = \frac{1}{\lambda_{\mathrm{S,P}}} e^{-\frac{x}{\lambda_{\mathrm{S,P}}}}$ and using (4.9) in (4.6) we get

$$
\begin{aligned}
&P(c(\mathcal{D}) = l) \\
&= \int_0^\infty \binom{K}{l} (1 - e^{-\frac{Ty}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^{K-l} (e^{-\frac{Ty}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^l \ \frac{1}{\lambda_{\mathrm{S,P}}} e^{-\frac{y}{\lambda_{\mathrm{S,P}}}} \ dy \\
&= \binom{K}{l} \frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(l + \frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K - l + 1) \qquad (4.10)
\end{aligned}
$$

where $B(\mu, \nu) = \int_0^1 t^{\mu-1}(1-t)^{\nu-1} \ dt$ is the Beta function [77].

We now need to calculate $P(outage|c(\mathcal{D}) = l)$. Since $P_{\mathrm{R}(k)} = \beta I_p / \tilde{g}_{\mathrm{R}(k),\mathrm{P}}$, letting $\bar{\gamma}_{\mathrm{R,D}} = \frac{I_p}{\sigma_{\mathrm{D}}^2}$, we have

$$
\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} = \frac{P_{\mathrm{R}(k)} |\tilde{h}_{\mathrm{R}(k),\mathrm{D}}|^2}{\sigma_{\mathrm{D}}^2} = \beta\bar{\gamma}_{\mathrm{R,D}} \frac{\tilde{g}_{\mathrm{R}(k),\mathrm{D}}}{\tilde{g}_{\mathrm{R}(k),\mathrm{P}}} \qquad (4.11)
$$

Moreover, the SNR for relay k at retransmission time is given by

$$
\gamma_{\mathrm{R}(k),\mathrm{D}} = \frac{P_{\mathrm{R}(k)} |h_{\mathrm{R}(k),\mathrm{D}}|^2}{\sigma_{\mathrm{D}}^2} = \beta\bar{\gamma}_{\mathrm{R,D}} \frac{g_{\mathrm{R}(k),\mathrm{D}}}{\tilde{g}_{\mathrm{R}(k),\mathrm{P}}} \qquad (4.12)
$$

When $l = 0$, due to the fact that the set $\mathcal{D}$ is empty, $P(outage|c(\mathcal{D}) = 0) = 1$. When $l = 1$, there is only one relay in $\mathcal{D}$, so there is no relay selection process. As a result, CSI uncertainty is not an issue since $\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}$ and $\gamma_{\mathrm{R}(k),\mathrm{D}}$ have the same distribution. Define $\gamma_{1(k)} \triangleq \beta I_p g_{\mathrm{R}(k),\mathrm{D}}$, and $\tilde{\gamma}_{2(k)} \triangleq \sigma_{\mathrm{D}}^2 \tilde{g}_{\mathrm{R}(k),\mathrm{P}}$. The distributions of $\gamma_{1(k)}$ and $\gamma_{2(k)}$ are given by

$$
\gamma_{1(k)} \ \sim \mathrm{Exp}(\frac{1}{\beta I_p \lambda_{\mathrm{R,D}}}), \quad \tilde{\gamma}_{2(k)} \ \sim \mathrm{Exp}(\frac{1}{\sigma_{\mathrm{D}}^2 \lambda_{\mathrm{R,P}}}) \qquad (4.13)
$$

From these the PDF and CDF of $\gamma_{\mathrm{R}(k),\mathrm{D}} = \gamma_{1(k)} / \tilde{\gamma}_{2(k)}$ are given by

$$
f_{\gamma_{\mathrm{R}(k),\mathrm{D}}}(x) = \frac{\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}} \frac{1}{(\frac{x\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}} + 1)^2}, \qquad k = 1, 2, \cdots, K, \qquad (4.14)
$$

and

$$
F_{\gamma_{\mathrm{R}(k),\mathrm{D}}}(x) = 1 - \frac{1}{1 + \frac{x\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}}}, \qquad (4.15)
$$

respectively. From (4.15) we get

$$P(outage|c(\mathcal{D}) = 1) = P(\gamma_{\mathrm{R}(k),\mathrm{D}} < T) = 1 - \frac{1}{1 + \frac{T\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}}} \tag{4.16}$$

When $l > 1$, we define $A_i(l)$ as the event that relay i from the set $\mathcal{D}$ is selected for retransmission, i.e., given that $c(\mathcal{D}) = l$, $l = 1, 2, \cdots, K$,

$$A_i(l) = \{\tilde{\gamma}_{\mathrm{R}(i),\mathrm{D}} = \max_{k\in\mathcal{D}} \tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}\}. \tag{4.17}$$

Since we assume that all the relay-to-destination channels are *iid*, each relay in the $\mathcal{D}$ has the same probability of being selected for retransmission (to have the highest relay-to-destination SNR). Therefore, $P(A_i(l)|c(\mathcal{D}) = l) = \frac{1}{l}$. Then we have

$$\begin{aligned} &P(outage|c(\mathcal{D}) = l) \\ &\quad = \sum_{i=1}^{l} P(\gamma_{\mathrm{R}(i),\mathrm{D}} < T|c(\mathcal{D}) = l, A_i(l))P(A_i(l)|c(\mathcal{D}) = l) \\ &\quad = P(\gamma_{\mathrm{R}(i),\mathrm{D}} < T|c(\mathcal{D}) = l, A_i(l)) \end{aligned} \tag{4.18}$$

Remark 8. *In order to simplify our notation, with some abuse of notation we drop the conditioning on the event $c(\mathcal{D}) = l$ from all of subsequent derivations, understanding that when conditioning on the event $A_i(l)$, it is given that $c(\mathcal{D}) = l$. For example instead of $P(\gamma_{R(i),D} < T|c(\mathcal{D}) = l, A_i(l))$ we write $P(\gamma_{R(i),D} < T|A_i(l))$.*

In Appendix F it is shown that

$$\begin{aligned} &P(\gamma_{\mathrm{R}(i),\mathrm{D}} < T|A_i(l)) = \\ &l \int_{x=0}^{\infty}\int_{y=0}^{\infty}\int_{z=0}^{T} \frac{x}{\beta\bar{\gamma}_{\mathrm{R,D}}(1-\rho^2)\lambda_{\mathrm{R,D}}} e^{\frac{-(\frac{xz}{\beta\bar{\gamma}_{\mathrm{R,D}}}+\rho^2 y)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}} \\ &\quad \times I_0\left(\frac{2\sqrt{\rho^2 xyz}}{(1-\rho^2)\lambda_{\mathrm{R,D}}\sqrt{\beta\bar{\gamma}_{\mathrm{R,D}}}}\right)\left(\frac{\lambda_{\mathrm{R,P}}y}{\lambda_{\mathrm{R,D}}x + \lambda_{\mathrm{R,P}}y}\right)^{l-1} \\ &\quad \times \frac{1}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}} dzdydx \end{aligned} \tag{4.19}$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Now combining (4.5), (4.10), (4.18) and (4.19), we get the outage probability

$$\begin{aligned}
&P_{out}(T) \\
&= \frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K+1) \\
&\quad + K\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(1+\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K)(1-\frac{1}{1+\frac{T\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}}}) \\
&\quad + \sum_{l=2}^{K}\binom{K}{l}\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(l+\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K-l+1) \\
&\quad \times \int_{x=0}^{\infty}\int_{y=0}^{\infty}\int_{z=0}^{T} \frac{x}{\beta\bar{\gamma}_{\mathrm{R,D}}(1-\rho^2)\lambda_{\mathrm{R,D}}} e^{\frac{-(\frac{xz}{\beta\bar{\gamma}_{\mathrm{R,D}}}+\rho^2 y)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}} I_0(\frac{2\sqrt{\rho^2 xyz}}{(1-\rho^2)\lambda_{\mathrm{R,D}}\sqrt{\beta\bar{\gamma}_{\mathrm{R,D}}}})l \\
&\quad \times (\frac{\lambda_{\mathrm{R,P}}y}{\lambda_{\mathrm{R,D}}x+\lambda_{\mathrm{R,P}}y})^{l-1}\frac{1}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}} dzdydx
\end{aligned} \tag{4.20}$$

Finally using the first-order Marcum Q-function $Q_1(a,b) \triangleq \int_b^\infty x e^{-\frac{x^2+a^2}{2}} I_0(ax)dx$ [76], we can simplify (4.20) to

$$\begin{aligned}
&P_{out}(T) \\
&= \frac{\alpha\bar{\gamma}_{S,R}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K+1) \\
&\quad + K\frac{\alpha\bar{\gamma}_{S,R}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(1+\frac{\alpha\bar{\gamma}_{S,R}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K)(1-\frac{1}{1+\frac{T\lambda_{\mathrm{R,P}}}{\beta\bar{\gamma}_{\mathrm{R,D}}\lambda_{\mathrm{R,D}}}}) \\
&\quad + \sum_{l=2}^{K}\binom{K}{l}\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(l+\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K-l+1) \\
&\quad \times \int_{x=0}^{\infty}\int_{y=0}^{\infty}[1-Q_1(\rho\sqrt{\frac{2y}{(1-\rho^2)\lambda_{\mathrm{R,D}}}}, \sqrt{\frac{2xT}{(1-\rho^2)\lambda_{\mathrm{R,D}}\beta\bar{\gamma}_{\mathrm{R,D}}}})] \\
&\quad \times l(\frac{\lambda_{\mathrm{R,P}}y}{\lambda_{\mathrm{R,D}}x+\lambda_{\mathrm{R,P}}y})^{l-1}\frac{1}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}} dydx
\end{aligned} \tag{4.21}$$

Remark 9. *As indicated in* (4.3) *when* $\rho = 1$*, there is no uncertainty in the channel coefficients. In other words perfect knowledge of CSI is available. In this case we have*

$$\begin{aligned} & P(outage|c(\mathcal{D}) = l) \\ & \quad = P(\gamma_{R(i),D} \le T | i = \arg\max_{k=1,\cdots,l}\{\gamma_{R(k),D}\}) \\ & \quad = [P(\gamma_{R(i),D} \le T)]^l \\ & \quad = (1 - \frac{1}{1 + \frac{T\lambda_{R,P}}{\beta\bar{\gamma}_{R,D}\lambda_{R,D}}})^l \end{aligned} \tag{4.22}$$

Therefore the outage probability is given by

$$\begin{aligned} P_{out}(T) = & \frac{\alpha\bar{\gamma}_{S,R}\lambda_{S,R}}{T\lambda_{S,P}} \sum_{l=0}^{K} (1 - \frac{1}{1 + \frac{T\lambda_{R,P}}{\beta\bar{\gamma}_{R,D}\lambda_{R,D}}})^l \\ & \times \binom{K}{l} B(l + \frac{\alpha\bar{\gamma}_{S,R}\lambda_{S,R}}{T\lambda_{S,P}}, K - l + 1) \end{aligned} \tag{4.23}$$

4.4 Interference Probability

Interference probability is defined as the probability that the interference inflicted upon the PU by the SU network exceeds the maximum interference power I_p. This may come either from the SU source or from the selected SU relay. Since CSI is not perfect, interference to the PU cannot be prevented. Let $\bar{\mathcal{I}}_S$ denote the event that the interference from SU source does not exceed the threshold I_p, and let $\bar{\mathcal{I}}_R$ denote the event that the interference from the selected relay does not exceed the threshold I_p. The interference probability, denoted by P_{int}, is given by

$$P_{int} = 1 - \sum_{l=0}^{K} P(c(\mathcal{D}) = l) P(\bar{\mathcal{I}}_S | c(\mathcal{D}) = l) P(\bar{\mathcal{I}}_R | c(\mathcal{D}) = l) \tag{4.24}$$

We now derive $P(\bar{\mathcal{I}}_S|c(\mathcal{D}) = l)$. Let $I_S \triangleq P_S|h_{\mathrm{S,P}}|^2$ denote the interference power received at the PU from the SU source, where, as mentioned previously, $P_S = \alpha I_p / |\tilde{h}_{\mathrm{S,P}}|^2$ is determined using the imperfect CSI $\tilde{h}_{\mathrm{S,P}}$. Then

$$I_S = \frac{\alpha I_p |h_{\mathrm{S,P}}|^2}{|\tilde{h}_{\mathrm{S,P}}|^2} = \frac{\alpha I_p g_{\mathrm{S,P}}}{\tilde{g}_{\mathrm{S,P}}} \tag{4.25}$$

There will not be any interference from the SU source to PU if $I_S \leq I_p$, which is equivalent to $\alpha g_{\mathrm{S,P}} \leq \tilde{g}_{\mathrm{S,P}}$. Thus

$$\begin{aligned}
&P(\bar{\mathcal{I}}_S | c(\mathcal{D}) = l) \\
&= \int_{x_1=0}^{\infty} \int_{x_2=0}^{\frac{x_1}{\alpha}} f_{g_{\mathrm{S,P}}, \tilde{g}_{\mathrm{S,P}}}(x_2, x_1 | c(\mathcal{D}) = l) \; dx_2 dx_1 \\
&= \int_{x_1=0}^{\infty} \int_{x_2=0}^{\frac{x_1}{\alpha}} P(c(\mathcal{D}) = l | g_{\mathrm{S,P}} = x_2, \tilde{g}_{\mathrm{S,P}} = x_1) \frac{f_{g_{\mathrm{S,P}}, \tilde{g}_{\mathrm{S,P}}}(x_2, x_1)}{P(c(\mathcal{D}) = l)} \; dx_2 dx_1 \qquad (4.26)
\end{aligned}$$

Now we have

$$P(c(\mathcal{D}) = l | g_{\mathrm{S,P}} = x_2, \tilde{g}_{\mathrm{S,P}} = x_1) = P(c(\mathcal{D}) = l | \tilde{g}_{\mathrm{S,P}} = x_1) \qquad (4.27)$$

which is given in (4.9). The distribution of $g_{\mathrm{S,P}}$ conditioned on $\tilde{g}_{\mathrm{S,P}}$, follows a non-central chi-square distribution with 2 degrees of freedom given by [73]

$$f_{g_{\mathrm{S,P}} | \tilde{g}_{\mathrm{S,P}}}(x_2 | x_1) = \frac{1}{(1-\rho^2)\lambda_{\mathrm{S,P}}} e^{\frac{-(x_2 + \rho^2 x_1)}{(1-\rho^2)\lambda_{\mathrm{S,P}}}} \cdot I_0\left(\frac{2\sqrt{\rho^2 x_2 x_1}}{(1-\rho^2)\lambda_{\mathrm{S,P}}}\right) \qquad (4.28)$$

From (4.28) and the fact that $\tilde{g}_{\mathrm{S,P}} \sim \mathrm{Exp}(1/\lambda_{\mathrm{S,P}})$, we get

$$f_{g_{\mathrm{S,P}} \tilde{g}_{\mathrm{S,P}}}(x_2, x_1) = \frac{1}{(1-\rho^2)\lambda_{\mathrm{S,P}}^2} e^{\frac{-(x_2 + x_1)}{(1-\rho^2)\lambda_{\mathrm{S,P}}}} I_0\left(\frac{2\sqrt{\rho^2 x_2 x_1}}{(1-\rho^2)\lambda_{\mathrm{S,P}}}\right) \qquad (4.29)$$

Combining (4.27), (4.29) and (4.10) into (4.26), we have

$$\begin{aligned}
&P(\bar{\mathcal{I}}_S | c(\mathcal{D}) = l) \\
&= \int_{x_1=0}^{\infty} \int_{x_2=0}^{\frac{x_1}{\alpha}} (1 - e^{-\frac{T x_1}{\alpha \bar{\gamma}_{\mathrm{S,R}} \lambda_{\mathrm{S,R}}}})^{K-l} (e^{-\frac{T x_1}{\alpha \bar{\gamma}_{\mathrm{S,R}} \lambda_{\mathrm{S,R}}}})^l \\
&\quad \times \frac{\frac{1}{(1-\rho^2)\lambda_{\mathrm{S,P}}^2} e^{\frac{-(x_2 + x_1)}{(1-\rho^2)\lambda_{\mathrm{S,P}}}} I_0\left(\frac{2\sqrt{\rho^2 x_2 x_1}}{(1-\rho^2)\lambda_{\mathrm{S,P}}}\right)}{\frac{\alpha \bar{\gamma}_{\mathrm{S,R}} \lambda_{\mathrm{S,R}}}{T \lambda_{\mathrm{S,P}}} B(l + \frac{\alpha \bar{\gamma}_{\mathrm{S,R}} \lambda_{\mathrm{S,R}}}{T \lambda_{\mathrm{S,P}}}, K - l + 1)} dx_2 dx_1 \qquad (4.30)
\end{aligned}$$

Now using the first-order Marcum Q-function $Q_1(a,b) = \int_b^\infty xe^{-\frac{x^2+a^2}{2}} I_0(ax)dx$ [76], we can simplify (4.30) to (4.31).

$$P(\bar{\mathcal{I}}_S|c(\mathcal{D})=l) = \int_{x_1=0}^{\infty} \frac{(1-e^{-\frac{Tx_1}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^{K-l}(e^{-\frac{Tx_1}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^l}{\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}} B(l+\frac{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}{T\lambda_{\mathrm{S,P}}}, K-l+1)} \frac{1}{\lambda_{\mathrm{S,P}}} e^{-\frac{x_1}{\lambda_{\mathrm{S,P}}}} \times [1-Q_1(\sqrt{\frac{2\rho^2 x_1}{(1-\rho^2)\lambda_{\mathrm{S,P}}}}, \sqrt{\frac{2x_1}{(1-\rho^2)\alpha\lambda_{\mathrm{S,P}}}})]dx_1 \tag{4.31}$$

Next we evaluate $P(\bar{\mathcal{I}}_R|c(\mathcal{D})=l)$. Clearly when there is no relay for retransmission ($l=0$), no interference occurs from the SU relays. Then we have

$$P(\bar{\mathcal{I}}_R|c(\mathcal{D})=0) = 1 \tag{4.32}$$

For $l>0$, $P(\bar{\mathcal{I}}_R|c(\mathcal{D})=l)$ is derived in Appendix G. Now using (4.10), (4.31) and (6.30) in (4.24), and using z instead of x_1 we get the interference probability

$$\begin{aligned}
P_{int} &\\
= 1 &- \int_{z=0}^{\infty} (1-e^{-\frac{Tz}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^K \frac{1}{\lambda_{\mathrm{S,P}}} e^{-\frac{z}{\lambda_{\mathrm{S,P}}}} [1-Q_1(\sqrt{\frac{2\rho^2 z}{(1-\rho^2)\lambda_{\mathrm{S,P}}}}, \sqrt{\frac{2z}{(1-\rho^2)\alpha\lambda_{\mathrm{S,P}}}})]\ dz \\
&- \sum_{l=1}^{K} \binom{K}{l} \int_{z=0}^{\infty} \frac{(1-e^{-\frac{Tz}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^{K-l}(e^{-\frac{Tz}{\alpha\bar{\gamma}_{\mathrm{S,R}}\lambda_{\mathrm{S,R}}}})^l}{\lambda_{\mathrm{S,P}}} e^{-\frac{z}{\lambda_{\mathrm{S,P}}}} \\
&\times [1-Q_1(\sqrt{\frac{2\rho^2 z}{(1-\rho^2)\lambda_{\mathrm{S,P}}}}, \sqrt{\frac{2z}{(1-\rho^2)\alpha\lambda_{\mathrm{S,P}}}})]\ dz \\
&\times \int_{x=0}^{\infty}\int_{y=0}^{\infty} l(\frac{y\lambda_{\mathrm{R,P}}}{x\lambda_{\mathrm{R,D}}+y\lambda_{\mathrm{R,P}}})^{l-1} \frac{1}{\lambda_{\mathrm{R,D}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} \frac{1}{\lambda_{\mathrm{R,P}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}} \\
&\times [1-Q_1(\sqrt{\frac{2\rho^2 x}{(1-\rho^2)\lambda_{\mathrm{R,P}}}}, \sqrt{\frac{2x}{(1-\rho^2)\beta\lambda_{\mathrm{R,P}}}})]\ dydx
\end{aligned} \tag{4.33}$$

We should note that when $\rho=1$, there is no interference to the PU, i.e., $P_{int}=0$.

4.5 Numerical Results

In Figure 4.2 we show the relationship between outage probability and the distance between SU and PU for the case of $K=5$ relays, target rate of $C=1$ bps/Hz, $\sigma_R^2 =$

$\sigma_{\mathrm{D}}^2 = 1$, and the maximum interference tolerance power $I_p = 10$ dB obtained from analysis and simulations. Let the distance between SU and PU be $d = d_{\mathrm{S,P}} = d_{\mathrm{R,P}}$, let $d_{\mathrm{S,R}} = d_{\mathrm{R,D}} = d_0$, $\eta = 3$. The values of power margin factors α and β ($\alpha = \beta$) are chosen so that $P_{int} = 0.1$. As expected, larger distances between SU and PU lead to lower outage probabilities. This is because larger distances between SU and PU allow the SU (source and relay) to choose a larger transmit power and still satisfy the interference threshold. Also, as expected, larger values of ρ lead to lower outage probabilities. Since relay selection is based on the imperfect CSI, which is different from the CSI at retransmission time, as ρ increases, it is more likely that the selected relay is in fact the best relay. Finally the figure shows a close match between the results from analysis and simulation.

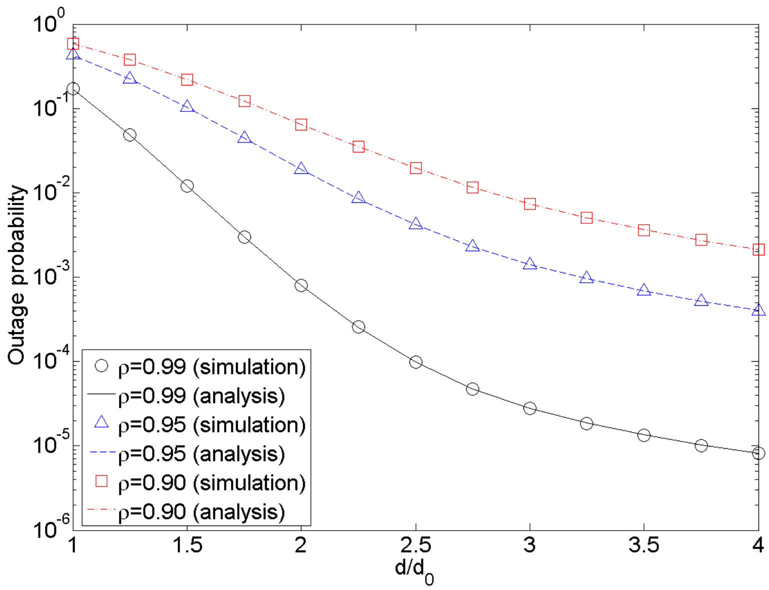

FIGURE 4.2. Outage Probability vs. d/d_0 with K=5.

In Figure 4.3 we show the relationship between outage probability and the distance between SU and PU for the case of $\rho = 0.99$, for different number of relays K. The other parameters are the same as those in Figure 4.2. As expected larger number of relays K lead to lower outage probabilities. As the number of relays increases, the diversity in the system increases leading to improved outage probability. Again we see a close match between the results from simulation and analysis.

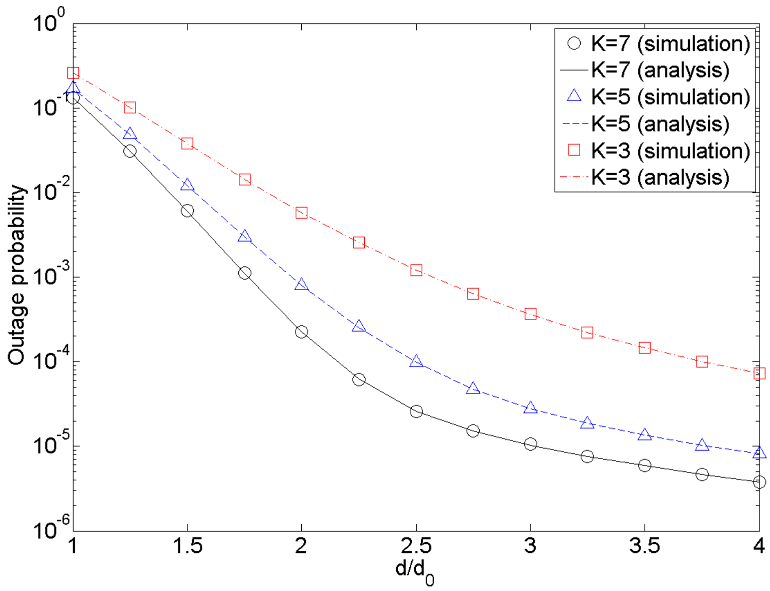

FIGURE 4.3. Outage Probability vs. d/d_0 with ρ=0.99.

In Figure 4.4 we show the relationship between interference probability and the distance between SU and PU for the case of $K = 5$ relays, where $\alpha = \beta$ is set so as to get a fixed outage probability of 0.05. The remaining parameters are the same as those in Figure 4.2. It can be seen that interference probability decreases with distance between SU and PU. This is due to the fact that larger distances between SU and PU would allow the SU to have smaller power margin factors α and β while still satisfying the required outage probability. Also, as expected larger values of coefficient ρ lead to lower interference probabilities. The reason is that when α and β are less than 1, for the same outage probability, larger values of ρ lead to smaller α and β. Also, when ρ is larger, the value of the actual channel gain is closer to the outdated channel gain. Therefore, when actual channel gain is multiplied by the power margin factor, the probability that this product exceeds the outdated channel gain is smaller, leading to a lower interference probability.

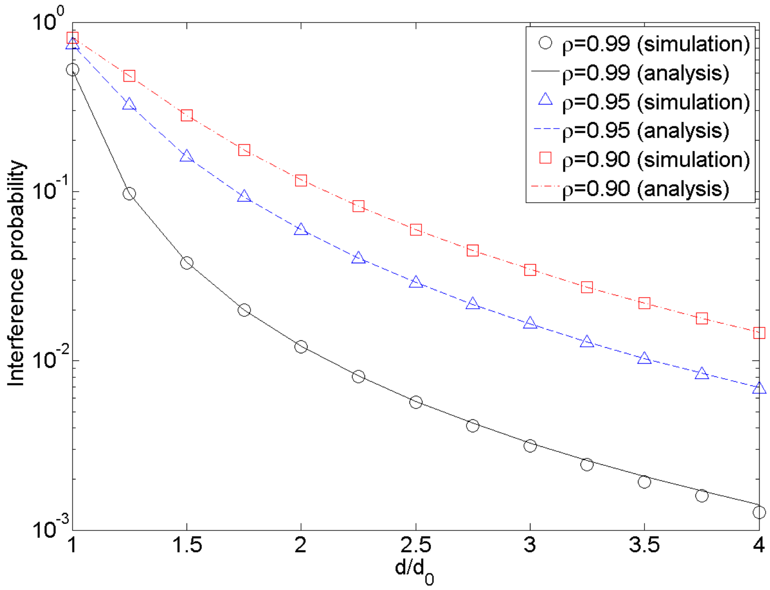

FIGURE 4.4. Interference Probability vs. d/d_0 with K=5.

Using the results in this chapter the systems parameters can be designed in order to guarantee that the desired interference and outage probabilities are satisfied. For example, for given interference and outage probabilities, from Figures such as 4.2 and 4.4 one can determine the minimum separation between the PU and the secondary network.

Chapter 5

Hop-By-Hop Relay Selection in Multi-hop Relay Networks under Spectrum Sharing Constraint

5.1 Introduction

In this chapter, we introduce a hop-by-hop relay selection strategy for multi-hop underlay cognitive spectrum sharing systems. In each stage, relays that successfully decode the message from previous hop form a candidate set. Each relay in this candidate set calculates its available transmit power and evaluates its instantaneous SNR to relays in the next stage. Then one relay which has the largest number of channels with an acceptable SNR level to relays in the next stage is selected for retransmission. Therefore, relay selection is only based on the CSI of the channels of one hop. This strategy can be implemented in a distributed manner, and a CC is not required. We analyze the performance of the introduced strategy in terms of end-to-end outage probability, and show that the results match those obtained from simulation closely.

Notations: Our notations and some of our modeling assumptions for this chapter are introduced here. S, RC_m, and D refer to source, relay cluster m, and destination, respectively. $\mathrm{R}_i^{(m)}$ denotes relay i in RC_m. PU-Rx denotes the primary user (receiver). $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. $h_{\mathrm{A,B}}^{(m)}$ and $\gamma_{\mathrm{A,B}}^{(m)}$ denote the instantaneous CSI and the instantaneous SNR of link A $\rightarrow$ B of hop m, respectively, where $h_{\mathrm{A,B}}^{(m)} \sim \mathcal{CN}(0, \lambda_{\mathrm{A,B}}^{(m)})$. Letting $g_{\mathrm{A,B}}^{(m)} \triangleq |h_{\mathrm{A,B}}^{(m)}|^2$, then $g_{\mathrm{A,B}}^{(m)}$ is exponentially distributed with mean $\lambda_{\mathrm{A,B}}^{(m)}$. We denote the transmit power of S by P_{S}, $\mathrm{R}_i^{(m)}$ by $P_i^{(m)}$, and the noise random variable at receiver B by n_{B}. The noise variables at all receivers are assumed to be *iid* with $n_{\mathrm{B}} \sim \mathcal{CN}(0, \sigma_n^2)$.

5.2 System Model

As shown in Figure 5.1, we consider a multi-hop underlay cognitive secondary network with the source S, the destination D, and M relay clusters (RC_m, $m = 1, \cdots, M$) in between the source and destination. Each relay cluster RC_m includes L_m single-antenna half-duplex relay nodes. Message transmission from S to D is implemented indirectly with the help of the M relay clusters. Therefore, there are totally $M + 1$ hops from S to D. We denote the first hop from S to RC_1 as hop 0, RC_1 to RC_2 as hop 1, and so on. A primary user receiver, PU-Rx, is also in the vicinity of the cooperative relay system and may experience interference from the source S and/or relays.

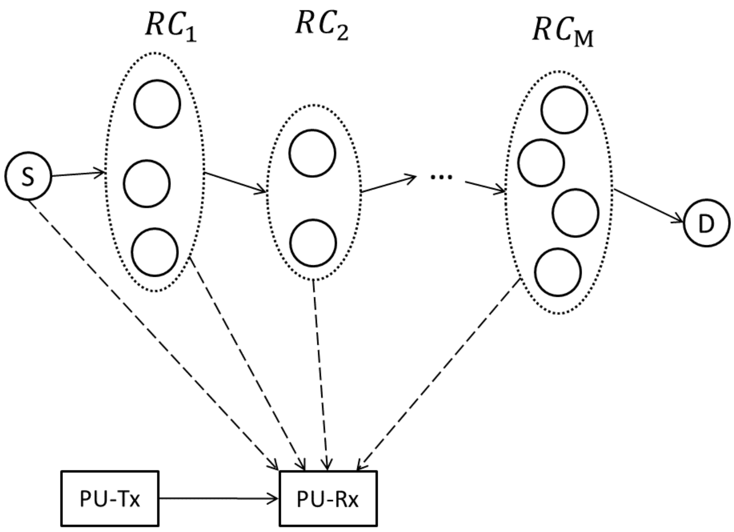

FIGURE 5.1. System Model of Multi-hop Underlay Spectrum Sharing Cognitive Relay Networks.

Transmissions of secondary network are allowed as long as the resulting interference at PU-Rx remains below a given threshold level. Let I_p denote the maximum interference power that PU-Rx can tolerate. It is required that the interference at PU-Rx does not exceed I_p. Therefore the transmit power at S, P_{S}, is limited by $P_{\mathrm{S}} g_{\mathrm{S,P}}^{(0)} \leq I_p$. Similarly, the transmit power at $\mathrm{R}_i^{(m)}$, $P_i^{(m)}$, is limited by $P_i^{(m)} g_{i,\mathrm{P}}^{(m)} \leq I_p$. Also, the transmit power of each node is limited by a maximum transmit power $P_{\max}$.

Therefore, the transmit power at S is given as

$$P_{\mathrm{S}} = \min\{\frac{I_p}{g^{(0)}_{\mathrm{S,P}}}, P_{\max}\} \tag{5.1}$$

and the transmit power at $\mathrm{R}_i^{(m)}$ is given as

$$P_i^{(m)} = \min\{\frac{I_p}{g^{(m)}_{i,\mathrm{P}}}, P_{\max}\} \tag{5.2}$$

The instantaneous SNR of link A $\rightarrow$ B at hop m is given by

$$\gamma^{(m)}_{\mathrm{A,B}} = P^{(m)}_{\mathrm{A}}|h^{(m)}_{\mathrm{A,B}}|^2/\sigma_n^2 = P^{(m)}_{\mathrm{A}} g^{(m)}_{\mathrm{A,B}}/\sigma_n^2, \tag{5.3}$$

Reactive DF relaying scheme is used where in each relay cluster a single relay is selected for retransmission. The proposed path selection strategy is as follows. At the first hop, S estimates its channel coefficient to PU-Rx and determines its transmit power P_{S} according to (5.1). Then S broadcasts its signal to RC_1. In any stage $m = 1, 2, \cdots, M-1$ the relays in RC_m which are able to correctly decode the received signal from previous stage form a decoding set denoted by $\mathcal{D}_m$. The decoding set, defined formally later in (5.5), consists of all those relays whose SNR exceeds a predefined threshold T, which is the minimum required SNR for successful decoding of the message. Each relay in $\mathcal{D}_m$ estimates its channel coefficient to PU-Rx, as well as the channel coefficients from itself to all the relays in RC_{m+1}. The transmit power $P_i^{(m)}$ is determined based on (5.2), and the corresponding instantaneous SNR to each relays in RC_{m+1} is calculated from (5.3). The calculated instantaneous SNR of each link is compared to the SNR threshold T. For $\mathrm{R}_i^{(m)}$ in $\mathcal{D}_m$, let $N_i^{(m)}$ denote the number of channels to relays in RC_{m+1} for which the instantaneous SNR exceeds T. $\mathrm{R}_i^{(m)}$ now starts a timer inversely proportional to $N_i^{(m)}$. The relay whose timer expires first, denoted by $\mathrm{R}_{i^*}^{(m)} \in \mathcal{D}_m$, will transmit[1]. This relay has the largest number of "good"

[1]A small randomization can be introduced into the timer to avoid collisions in the case of ties.

channels, i.e., $i^* = \arg\max_i N_i^{(m)}$. All the other relays in $\mathcal{D}_m$ hear this transmission and remain silent. We define

$$N_{\max}^{(m)} \triangleq \max\{N_i^{(m)}; i \in \mathcal{D}_m\} \tag{5.4}$$

We should point out that if $N_{\max}^{(m)} = 0$, then outage is declared. Finally in the last hop, the relay in $\mathcal{D}_M$ which has the highest instantaneous SNR is selected for transmission to D. Note that this protocol can be implemented in a distributed manner and does not require a CC.

For any $j = 1, 2, \cdots, L_1$, the instantaneous channel gain of $\mathrm{S} \to R_j^{(1)}$ denoted by $g_{\mathrm{S},j}^{(0)}$ is exponentially distributed. We denote its mean by $\lambda_{\mathrm{S,R}}^{(0)}$. In other words, we assume that the channels from S to all the relays in the first stage are *iid*. Similarly, the gains for the channels $\mathrm{S} \to$ PU-Rx, $\mathrm{R}_i^{(m)} \to \mathrm{R}_j^{(m+1)}$, $\mathrm{R}_i^{(m)} \to$ PU-Rx, $\mathrm{R}_i^{(M)} \to \mathrm{D}$ are denoted by $g_{\mathrm{S,P}}^{(0)}$, $g_{i,j}^{(m)}$, $g_{i,\mathrm{P}}^{(m)}$, $g_{i,\mathrm{D}}^{(M)}$, respectively. These channel gains are all exponentially distributed and are assumed to have means $\lambda_{\mathrm{S,P}}^{(0)}$, $\lambda_{\mathrm{R,R}}^{(m)}$, $\lambda_{\mathrm{R,P}}^{(m)}$, $\lambda_{\mathrm{R,D}}^{(M)}$, respectively.

5.3 Outage Probability

Denote by C the required end-to-end spectral efficiency in bps/Hz. Then for $m = 1, 2, \cdots, M$, $\mathcal{D}_m$ consists of those relays whose link capacity from the previous stage exceeds C, i.e.

$$\begin{aligned}\mathcal{D}_m &= \{j : \frac{1}{M+1}\log_2(1 + \gamma_{i^*,j}^{(m-1)}) \geq C\} \\ &= \{j : \gamma_{i^*,j}^{(m-1)} \geq 2^{(M+1)C} - 1\}\end{aligned} \tag{5.5}$$

where $\gamma_{i^*,j}^{(0)} \triangleq \gamma_{\mathrm{S},j}^{(0)}$ is the SNR from S to $\mathrm{R}_j^{(1)}$, and for $m = 2, 3, \cdots, M$, $\gamma_{i^*,j}^{(m-1)}$ is the SNR from the selected relay $\mathrm{R}_{i^*}^{(m-1)}$ in RC_{m-1} to $\mathrm{R}_j^{(m)}$. Then the SNR threshold T is defined as $T \triangleq 2^{(M+1)C} - 1$.

For any hop $m = 1, 2, \cdots, M-1$, the number of relays in $\mathcal{D}_{m+1}$ is equal to $N_{\max}^{(m)}$. Let $c(\mathcal{D}_n)$ denote the cardinality of the set $\mathcal{D}_n$, and let $\mathcal{O}_n = \{c(\mathcal{D}_n) = 0\}$, i.e., $\mathcal{O}_n$ is the event that no relay in the nth cluster can decode the message, and $\mathcal{O}_{M+1}$ is the event that D cannot decode the message. Then for the end-to-end outage event $\mathcal{O}$ we can write

$$
\begin{aligned}
P(\mathcal{O}) &= P(\cup_{n=1}^{M+1}\mathcal{O}_n) \\
&= \sum_{l_1=0}^{L_1} P(\cup_{n=1}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1) \\
&= P(\mathcal{O}_1) + \sum_{l_1=1}^{L_1} P(\cup_{n=2}^{M+1}\mathcal{O}_n | c(\mathcal{D}_1) = l_1) P(c(\mathcal{D}_1) = l_1)
\end{aligned}
\tag{5.6}
$$

$P(\mathcal{O}_1)$ is the probability that no relays in RC_1 can successfully decode the message transmitted from S and can be calculated as follows.

$$
\begin{aligned}
&P(\mathcal{O}_1) \\
&= \int_{x=0}^{\infty} P(\mathcal{O}_1 | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&= \int_{x=0}^{\infty} P(\max_{j \in \mathrm{RC}_1}\{\gamma_{\mathrm{S},j}^{(0)}\} < T | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&= \int_{x=0}^{\frac{I_p}{P_{\max}}} P(\max_{j \in \mathrm{RC}_1}\{\frac{P_{\max} g_{\mathrm{S},j}^{(0)}}{\sigma_n^2}\} < T | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&\quad + \int_{\frac{I_p}{P_{\max}}}^{\infty} P(\max_{j \in \mathrm{RC}_1}\{\frac{I_p g_{\mathrm{S},j}^{(0)}}{g_{\mathrm{S,P}}^{(0)} \sigma_n^2}\} < T | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&= [P(g_{\mathrm{S},j}^{(0)} < \frac{\sigma_n^2 T}{P_{\max}})]^{L_1} F_{g_{\mathrm{S,P}}^{(0)}}(\frac{I_p}{P_{\max}}) + \int_{\frac{I_p}{P_{\max}}}^{\infty} [P(g_{\mathrm{S},j}^{(0)} < \frac{\sigma_n^2 T x}{I_p})]^{L_1} \frac{1}{\lambda_{\mathrm{S,P}}^{(0)}} e^{-\frac{x}{\lambda_{\mathrm{S,P}}^{(0)}}} dx \\
&= (1 - e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{S,R}}^{(0)}}})^{L_1} (1 - e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{S,P}}^{(0)}}}) \\
&\quad + \sum_{k=0}^{L_1} \binom{L_1}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{S,P}}^{(0)} k}{I_p \lambda_{\mathrm{S,R}}^{(0)}} + 1} e^{-(\frac{\sigma_n^2 T k}{I_p \lambda_{\mathrm{S,R}}^{(0)}} + \frac{1}{\lambda_{\mathrm{S,P}}^{(0)}}) \frac{I_p}{P_{\max}}}
\end{aligned}
\tag{5.7}
$$

Moreover, we can calculate the probability that there are l_1 relays in $\mathcal{D}_1$ as

$$
\begin{aligned}
&P(c(\mathcal{D}_1) = l_1) \\
&= \int_{x=0}^{\infty} P(c(\mathcal{D}_1) = l_1 | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&= \int_{x=0}^{\frac{I_p}{P_{\max}}} P(c(\mathcal{D}_1) = l_1 | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&\quad + \int_{\frac{I_p}{P_{\max}}}^{\infty} P(c(\mathcal{D}_1) = l_1 | g_{\mathrm{S,P}}^{(0)} = x) f_{g_{\mathrm{S,P}}^{(0)}}(x) dx \\
&= \binom{L_1}{l_1} (1 - e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{S,R}}^{(0)}}})^{L_1 - l_1} (e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{S,R}}^{(0)}}})^{l_1} (1 - e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{S,P}}^{(0)}}}) \\
&\quad + \int_{\frac{I_p}{P_{\max}}}^{\infty} \binom{L_1}{l_1} (1 - e^{-\frac{\sigma_n^2 T x}{I_p \lambda_{\mathrm{S,R}}^{(0)}}})^{L_1 - l_1} (e^{-\frac{\sigma_n^2 T x}{I_p \lambda_{\mathrm{S,R}}^{(0)}}})^{l_1} \frac{1}{\lambda_{\mathrm{S,P}}^{(0)}} e^{-\frac{x}{\lambda_{\mathrm{S,P}}^{(0)}}} dx \\
&= \binom{L_1}{l_1} (1 - e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{S,R}}^{(0)}}})^{L_1 - l_1} (e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{S,R}}^{(0)}}})^{l_1} (1 - e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{S,P}}^{(0)}}}) \\
&\quad + \binom{L_1}{l_1} \sum_{k=0}^{L_1 - l_1} \binom{L_1 - l_1}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{S,P}}^{(0)} (k + l_1)}{I_p \lambda_{\mathrm{S,R}}^{(0)}} + 1} e^{-[\frac{\sigma_n^2 T (k + l_1)}{I_p \lambda_{\mathrm{S,R}}^{(0)}} + \frac{1}{\lambda_{\mathrm{S,P}}^{(0)}}] \frac{I_p}{P_{\max}}}
\end{aligned} \tag{5.8}
$$

We now evaluate $P(\cup_{n=m+1}^{M+1} \mathcal{O}_n | c(\mathcal{D}_m) = l_m)$ for any $1 \leq m < M$. For $1 \leq l_m \leq L_m$, we have

$$
\begin{aligned}
P(\cup_{n=m+1}^{M+1} \mathcal{O}_n | c(\mathcal{D}_m) = l_m) &= P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) \\
&+ \sum_{l_{m+1}=1}^{L_{m+1}} P(\cup_{n=m+2}^{M+1} \mathcal{O}_n | c(\mathcal{D}_{m+1}) = l_{m+1}) \\
&\quad \times P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m)
\end{aligned} \tag{5.9}
$$

Note that in the above

$$
P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) = P(c(\mathcal{D}_{m+1}) = 0 | c(\mathcal{D}_m) = l_m)
$$

Therefore we need to evaluate $P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m)$ and $P(c(\mathcal{D}_{m+1}) = l_{m+1} | c(\mathcal{D}_m) = l_m)$.

For $m = 1, 2, \cdots, M-1$, $P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m)$ is the probability that from any relay in $\mathcal{D}_m$, the SNRs of all L_{m+1} links to the relays in RC_{m+1} are below the threshold

T. For $\mathrm{R}_i^{(m)} \in \mathcal{D}_m$, similar to (5.7), the probability that the SNRs of all L_{m+1} links from this relay to the relays in RC_{m+1} are below the threshold T, can be calculated as

$$\begin{aligned}
&P(\max_{j \in RC_{m+1}} \{\gamma_{i,j}^{(m)}\} < T) \\
&= (1 - e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{R,R}}^{(m)}}})^{L_{m+1}} (1 - e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{R,P}}^{(m)}}}) \\
&\quad + \sum_{k=0}^{L_{m+1}} \binom{L_{m+1}}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{R,P}}^{(m)} k}{I_p \lambda_{\mathrm{R,R}}^{(m)}} + 1} e^{-(\frac{\sigma_n^2 T k}{I_p \lambda_{\mathrm{R,R}}^{(m)}} + \frac{1}{\lambda_{\mathrm{R,P}}^{(m)}})\frac{I_p}{P_{\max}}}
\end{aligned} \tag{5.10}$$

Moreover, for all relays in $\mathcal{D}_m$, we have

$$\begin{aligned}
&P(\mathcal{O}_{m+1} | c(\mathcal{D}_m) = l_m) \\
&= P(\max_{i \in \mathcal{D}_m, j \in RC_{m+1}} \{\gamma_{i,j}^{(m)}\} < T) \\
&= \left[P(\max_{j \in RC_{m+1}} \{\gamma_{i,j}^{(m)}\} < T) \right]^{l_m} \\
&= \Bigg[(1 - e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{R,R}}^{(m)}}})^{L_{m+1}} (1 - e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{R,P}}^{(m)}}}) \\
&\quad + \sum_{k=0}^{L_{m+1}} \binom{L_{m+1}}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{R,P}}^{(m)} k}{I_p \lambda_{\mathrm{R,R}}^{(m)}} + 1} e^{-(\frac{\sigma_n^2 T k}{I_p \lambda_{\mathrm{R,R}}^{(m)}} + \frac{1}{\lambda_{\mathrm{R,P}}^{(m)}})\frac{I_p}{P_{\max}}} \Bigg]^{l_m}
\end{aligned} \tag{5.11}$$

For the last hop (when $m = M$), the probability that the link from $\mathrm{R}_i^{(M)} \in \mathcal{D}_M$, to destination D is in outage is given by

$$
\begin{aligned}
& P(\gamma_{i,\mathrm{D}}^{(M)} < T) \\
&= \int_{x=0}^{\frac{I_p}{P_{\max}}} P(\frac{P_{\max} g_{i,\mathrm{D}}^{(M)}}{\sigma_n^2} < T | g_{i,\mathrm{P}}^{(M)} = x) f_{g_{i,\mathrm{P}}^{(M)}}(x) dx \\
&\quad + \int_{\frac{I_p}{P_{\max}}}^{\infty} P(\frac{I_p g_{i,\mathrm{D}}^{(M)}}{g_{i,\mathrm{P}}^{(M)} \sigma_n^2} < T | g_{i,\mathrm{P}}^{(M)} = x) f_{g_{i,\mathrm{P}}^{(M)}}(x) dx \\
&= (1 - e^{-\frac{\sigma_n^2 T}{P_{\max} \lambda_{\mathrm{R,D}}^{(M)}}})(1 - e^{-\frac{I_p}{P_{\max} \lambda_{\mathrm{R,P}}^{(M)}}}) \\
&\quad + \sum_{k=0}^{1} \binom{1}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{R,P}}^{(M)} k}{I_p \lambda_{\mathrm{R,D}}^{(M)}} + 1} e^{-(\frac{\sigma_n^2 T k}{I_p \lambda_{\mathrm{R,D}}^{(M)}} + \frac{1}{\lambda_{\mathrm{R,P}}^{(M)}}) \frac{I_p}{P_{\max}}}
\end{aligned} \tag{5.12}
$$

Since in the last hop the relay with the highest SNR among all the $l_M = c(\mathcal{D}_M)$ relays in $\mathcal{D}_M$ is selected for retransmission, outage occurs when the SNR of all these l_M links are below the threshold T. Therefore we have

$$
\begin{aligned}
& P(\mathcal{O}_{M+1} | c(\mathcal{D}_m) = l_M) \\
&= \left[P(\gamma_{i,\mathrm{D}}^{(M)} < T) \right]^{l_M} \\
&= \Bigg[(1 - e^{-\frac{\sigma_n^2 T}{P_{\max} \lambda_{\mathrm{R,D}}^{(M)}}})(1 - e^{-\frac{I_p}{P_{\max} \lambda_{\mathrm{R,P}}^{(M)}}}) \\
&\quad + \sum_{k=0}^{1} \binom{1}{k} (-1)^k \frac{1}{\frac{\sigma_n^2 T \lambda_{\mathrm{R,P}}^{(M)} k}{I_p \lambda_{\mathrm{R,D}}^{(M)}} + 1} e^{-(\frac{\sigma_n^2 T k}{I_p \lambda_{\mathrm{R,D}}^{(M)}} + \frac{1}{\lambda_{\mathrm{R,P}}^{(M)}}) \frac{I_p}{P_{\max}}} \Bigg]^{l_M}
\end{aligned} \tag{5.13}
$$

Let $A_w^{(m)}$ denote the event that from a relay in $\mathcal{D}_m$, there are w channels to relays in RC_{m+1} whose instantaneous SNRs are above the threshold T, and let $B_w^{(m)} =$

$\bigcup_{w'=0}^{w-1} A_{w'}^{(m)}$. Then we have

$$P(A_w^{(m)}) = \binom{L_{m+1}}{w}(1-e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{R,R}}^{(m)}}})^{L_{m+1}-w}(e^{-\frac{\sigma_n^2 T}{P_{\max}\lambda_{\mathrm{R,R}}^{(m)}}})^{w}(1-e^{-\frac{I_p}{P_{\max}\lambda_{\mathrm{R,P}}^{(m)}}})$$
$$+\binom{L_{m+1}}{w}\sum_{k=0}^{L_{m+1}-w}\binom{L_{m+1}-w}{k}(-1)^k$$
$$\times \frac{1}{\frac{\sigma_n^2 T\lambda_{\mathrm{R,P}}^{(m)}(k+w)}{I_p\lambda_{\mathrm{R,R}}^{(m)}}+1} e^{-[\frac{\sigma_n^2 T(k+w)}{I_p\lambda_{\mathrm{R,R}}^{(m)}}+\frac{1}{\lambda_{\mathrm{R,P}}^{(m)}}]\frac{I_p}{P_{\max}}} \tag{5.14}$$

Also we have

$$P(B_w^{(m)}) = \sum_{w'=0}^{w-1} P(A_{w'}^{(m)}) \tag{5.15}$$

To evaluate $P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m)$, we note that this is the probability that l $(1 \le l \le l_m)$ relays in $\mathcal{D}_m$ have l_{m+1} "good" channels[2] to relays in RC_{m+1}, while the remaining $l_m - l$ relays in $\mathcal{D}_m$, have fewer than l_{m+1} "good" channels. Therefore we can write

$$P(c(\mathcal{D}_{m+1}) = l_{m+1}|c(\mathcal{D}_m) = l_m) = \sum_{l=1}^{l_m}\binom{l_m}{l}\left[P(A_{l_{m+1}}^{(m)})\right]^l\left[P(B_{l_{m+1}}^{(m)})\right]^{l_m-l} \tag{5.16}$$

Finally by putting (5.7), (5.8), (5.11), (5.13)-(5.16) into (5.6), we get the outage probability of the proposed strategy.

5.4 Numerical Results

In this section, we consider a four-hop relay network (M=3) for the case that $\lambda_{\mathrm{S,R}}^{(0)} = \lambda_{\mathrm{R,D}}^{(M)} = \lambda_{\mathrm{R,R}}^{(m)} = 1$, $\lambda_{\mathrm{S,P}}^{(0)} = \lambda_{\mathrm{R,P}}^{(m)} = \frac{1}{8}$, $\sigma_n^2 = 1$, and a target rate of $C = 2/(M+1)$ bps/Hz. We present our numerical results from analysis and compare to those obtained from simulation.

In Figure 5.2, all relay clusters have the same number of relays, which is 3. We can see that when $P_{\max}$ is small, the outage probability achieved for different values of the interference threshold I_p are nearly the same, and decreases as $P_{\max}$ increases. The

[2] Channels whose SNR exceed the threshold T.

reason is that for small values of $P_{\max}$, the transmit power is mainly limited by $P_{\max}$. As $P_{\max}$ increases, the transmit power becomes limited by the interference threshold I_p. Consequently the outage probabilities exhibit a floor level which is determined by and decreases with I_p.

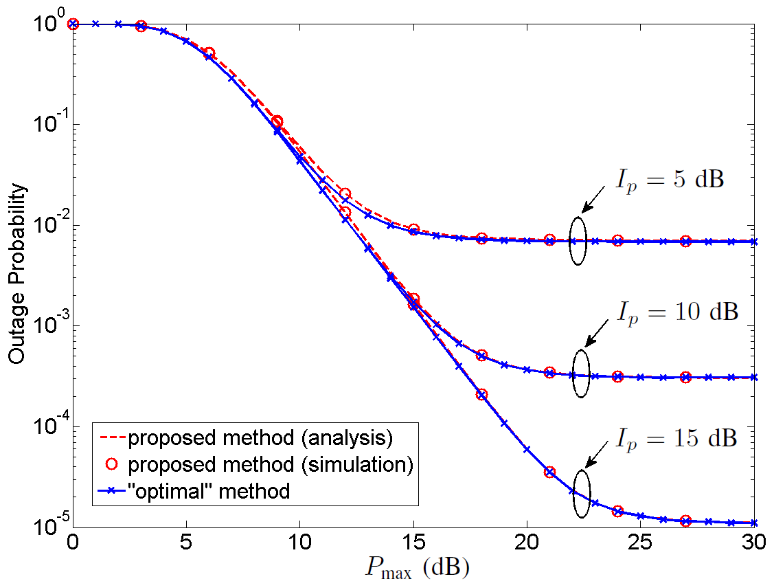

FIGURE 5.2. Outage Probability vs. $P_{\max}$ with different maximum tolerance power I_p.

As discussed in Chapter 1, the optimal relay selection strategy in [33] is a centralized method for path selection in multi-hop networks without spectrum sharing. In order to compare our proposed method with this method we have extended this strategy to underlay spectrum sharing cognitive networks as follows. In addition to the CSI of all the links in the network, the limits of the transmit power of all relays are also calculated according to their channel coefficients to PU-Rx. Then using the CSIs and the transmit power limits, the CC computes the SNR of all the links. It then selects the end-to-end path which has the highest SNR bottleneck. We have simulated this scheme and show the results of its outage probability in Figure 5.2. It can be seen that the performance of the proposed method is very close to this "optimal" method. However, the complexity of the "optimal" method is significantly higher than the proposed method. In addition, since the CSI of all the links must be collected before path selection and transmission, the collected CSI may be significantly outdated. This would not only degrade the performance of the secondary user, but more importantly, may cause interference to the primary user well beyond the specified threshold. Finally, the figure shows a close match between the results from our analysis and simulation.

In Figure 5.3, we show the outage probability vs. $P_{\max}$ for different number relays per cluster where $I_p = 10$ dB. In the case that $L_1 = L_2 = L_3 = 1$, there is a single relay in each stage which may retransmit the message. Therefore no relay selection strategy is involved. This is the same scenario studied previously by several authors including [63]. Clearly having more relays in the relay clusters substantially improves system performance in terms of outage probability. As in Figure 5.2, a floor is reached for each case as $P_{\max}$ increases, since the transmit power becomes limited by I_p.

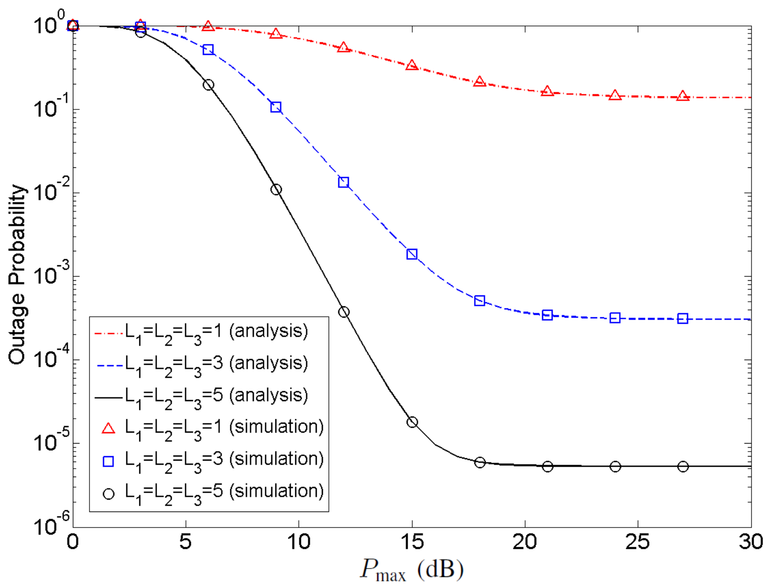

FIGURE 5.3. Outage Probability vs. $P_{\max}$ with different number of relays.

In Figure 5.4 we show the outage probability vs. interference threshold I_p. The number of relays in each cluster is 3. We can see that outage probability decreases as the interference threshold I_p increases, and again reaches a floor level for large values of I_p where the transmit power is limited by $P_{\max}$. Clearly, lower outage probability can be reached for larger values of $P_{\max}$.

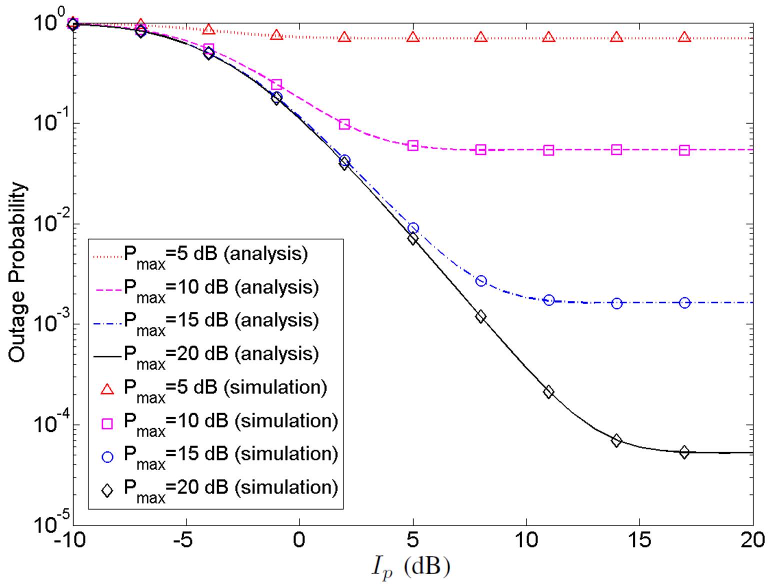

FIGURE 5.4. Outage Probability vs. I_p with different maximum transmit power $P_{\max}$.

Chapter 6
Conclusions

In this dissertation we analyze the performance of several relay selection strategies for multi-hop cooperative networks.

In Chapter 2, we analyze the performance of a hop-by-hop relay selection strategy for multi-hop DF cooperative relay networks. In each relay cluster, relays that successfully receive and decode the message from the previous hop form the candidate set for relaying, and the relay which has the highest channel gain to the next stage is selected for retransmission. Therefore in this method, a CC is not required, and relay selection of each relay cluster is only based on the CSI to the next hop. We evaluate the performance of this relay selection method in terms of end-to-end outage probability through analysis and simulation. It is shown that given the total number of relays for the entire network, unequal distribution of relays with more relays in the first and last relay clusters can significantly improve the performance. Moreover, this relay selection strategy is suitable for fast fading channels with a short coherence time, since each pair of relay selection and transmission is only based the CSI of the channels of one hop. Accurate approximations for the ergodic capacity and effective ergodic capacity of this relay selection strategy are also derived.

In Chapter 3, a novel hop-by-hop relay selection strategy for multi-hop DF cooperative relay networks is proposed where relay selection at each hop is only based on the CSI to relays in the next stage. The implementation complexity and communication overhead of our method is significantly lower than the relay selection strategies that require a CC for the entire network. We analyze the performance of the proposed method in terms of end-to-end outage probability for the cases of perfect and imperfect CSI. Numerical results from analysis closely match those obtained from simulation, and show a major improvement compared to other relay selection strategies in the literature.

In Chapter 4, we consider cognitive relay networks with imperfect CSI under interference power constraints. Reactive DF and ORS scheme for data transmission of SU is considered. We investigate the performance of DF ORS scheme for cognitive relay networks. Two power margin factors are considered for SU source and SU relays respectively in order to lower interference probability from SU to PU. We derive the outage probability and interference probability. It is shown that larger distances between SU and PU, larger values of correlation coefficient, larger number of relays lead to lower outage probabilities.

In Chapter 5, we introduce a hop-by-hop relay selection strategy for multi-hop underlay cognitive spectrum sharing systems. In each stage, relays that successfully decode the message from previous hop form a candidate set for retransmission. Each relay in this candidate set calculates its available transmit power and evaluates its instantaneous SNR to relays in the next stage. Then the relay which has the largest number of channels with an acceptable SNR level to relays in the next stage is selected for retransmission. Therefore, relay selection in each stage replies only on the CSI of the channels in that stage and does not require the CSI of any other stage. This strategy can be implemented in a distributed manner without the need for any coordination among the relays or a central controller. We analyze the performance of the introduced strategy in terms of end-to-end outage probability, and show that the performance of this method is nearly optimal.

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Appendix A: Proof of Lemma 1

$$
\begin{aligned}
&P(X_{1\sim n} < y | X_{1\sim n} \neq X_{max}) \\
&= \frac{P(X_{1\sim n} < y, X_{1\sim n} \neq X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} \neq X_{max} | X_{1\sim n} < y) P(X_{1\sim n} < y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{[1 - nP(X_1 = X_{max} | X_{1\sim n} < y)] P(X_{1\sim n} < y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{[1 - \frac{nP(X_1 = X_{max}, X_{1\sim n} < y)}{P(X_{1\sim n} < y)}] P(X_{1\sim n} < y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} < y) - nP(X_{1\sim n} < y | X_1 = X_{max}) P(X_1 = X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} < y) - \frac{n}{N} P(X_{1\sim n} < y | X_1 = X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} < y) - \frac{n}{N} P(X_{max} < y)}{P(X_{1\sim n} \neq X_{max})}
\end{aligned}
\tag{6.1}
$$

Appendix B: Proof of Lemma 3

$$
\begin{aligned}
&P(X_{1\sim n} > y|X_{1\sim n} \neq X_{max}) \\
&= \frac{P(X_{1\sim n} > y, X_{1\sim n} \neq X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} \neq X_{max}|X_{1\sim n} > y)P(X_{1\sim n} > y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{[1 - nP(X_n = X_{max}|X_{1\sim n} > y)]P(X_{1\sim n} > y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{[1 - \frac{nP(X_n = X_{max}, X_{1\sim n} > y)}{P(X_{1\sim n} > y)}]P(X_{1\sim n} > y)}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} > y) - nP(X_{1\sim n} > y|X_n = X_{max})P(X_n = X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} > y) - \frac{n}{N}P(X_{1\sim n-1} > y|X_n = X_{max})}{P(X_{1\sim n} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} > y) - \frac{n}{N}P(X_{1\sim n-1} > y|X_{1\sim n-1} \neq X_{max})}{P(X_{1\sim n} \neq X_{max})}
\end{aligned} \tag{6.2}
$$

Appendix C:
Proof of Lemma 5

$$
\begin{aligned}
&P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y | X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max}) \\
&= \frac{P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y, X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&= \frac{1}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&\quad \times [P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max} | X_{1\sim n} > y, X_{N_a+1\sim N_b} < y) \\
&\quad \times P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)] \\
&= \frac{[1 - nP(X_n = X_{max} | X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)]P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&= \frac{[1 - \frac{nP(X_n = X_{max}, X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}{P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}]P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&= \frac{1}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&\quad \times [P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y) \\
&\quad - nP(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y | X_n = X_{max})P(X_n = X_{max})] \\
&= \frac{P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y) - \frac{n}{N}P(X_{1\sim n-1} > y, X_{N_a+1\sim N_b} < y | X_n = X_{max})}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&= \frac{P(X_{1\sim n} > y, X_{N_a+1\sim N_b} < y)}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})} \\
&\quad - \frac{\frac{n}{N}P(X_{1\sim n-1} > y, X_{N_a+1\sim N_b} < y | X_{1\sim n-1} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})}{P(X_{1\sim n} \neq X_{max}, X_{N_a+1\sim N_b} \neq X_{max})}
\end{aligned} \tag{6.3}
$$

Appendix D: Proof of Lemma 6

We know that

$$P(\gamma_{i^*,j}^{(m)} < T | \tilde{\gamma}_{i^*,j}^{(m)} < T) = \frac{P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T)}{P(\tilde{\gamma}_{i^*,j}^{(m)} < T)} \tag{6.4}$$

in which

$$P(\tilde{\gamma}_{i^*,j}^{(m)} < T) = 1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}} \tag{6.5}$$

Now consider $P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T)$. According to [73], $g_{i^*,j}^{(m)}$ conditioned on its estimate, $\tilde{g}_{i^*,j}^{(m)}$, follows a non-central chi-square distribution with 2 degrees of freedom. We have

$$\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T) \\
&= P(g_{i^*,j}^{(m)} < \frac{T}{\bar{\gamma}}, \tilde{g}_{i^*,j}^{(m)} < \frac{T}{\bar{\gamma}}) \\
&= \int_{x=0}^{\frac{T}{\bar{\gamma}}} \int_{y=0}^{\frac{T}{\bar{\gamma}}} f_{g_{i^*,j}^{(m)}, \tilde{g}_{i^*,j}^{(m)}}(x,y) dx dy \\
&= \int_{x=0}^{\frac{T}{\bar{\gamma}}} \int_{y=0}^{\frac{T}{\bar{\gamma}}} \frac{1}{(1-\rho^2)\lambda_m^2} e^{\frac{-(x+y)}{(1-\rho^2)\lambda_m}} I_0(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\lambda_m}) dx dy \\
&= \frac{1}{(1-\rho^2)\lambda_m^2} \int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{(1-\rho^2)\lambda_m}} [\int_{y=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{y}{(1-\rho^2)\lambda_m}} I_0(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\lambda_m}) dy] dx
\end{aligned} \tag{6.6}$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Letting $z = \sqrt{y}$, and according to Eq. (10) in [76], we get

$$\begin{aligned}
&\int_{y=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{y}{(1-\rho^2)\lambda_m}} I_0(\frac{2\rho\sqrt{xy}}{(1-\rho^2)\lambda_m}) dy \\
&= 2\int_{z=0}^{\sqrt{\frac{T}{\bar{\gamma}}}} z e^{-\frac{z^2}{(1-\rho^2)\lambda_m}} I_0(\frac{2\rho\sqrt{x}}{(1-\rho^2)\lambda_m} z) dz \\
&= (1-\rho^2)\lambda_m e^{\frac{\rho^2 x}{(1-\rho^2)\lambda_m}} [1 - Q_1(\rho\sqrt{\frac{2x}{(1-\rho^2)\lambda_m}}, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}})]
\end{aligned} \tag{6.7}$$

where $Q_1(a,b) = \int_b^\infty x e^{-\frac{x^2+a^2}{2}} I_0(ax)dx$ is the first-order Marcum Q-function [76]. Now putting (6.7) into (6.6), we have

$$\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T) \\
&= \frac{1}{\lambda_m} \int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{\lambda_m}} [1 - Q_1(\rho \sqrt{\frac{2x}{(1-\rho^2)\lambda_m}}, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}})] dx \\
&= \frac{1}{\lambda_m} [\int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{\lambda_m}} dx - \int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{\lambda_m}} Q_1(\rho \sqrt{\frac{2x}{(1-\rho^2)\lambda_m}}, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}}) dx] \\
&= 1 - e^{\frac{T}{\bar{\gamma}\lambda_m}} - \frac{1}{\lambda_m} \int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{\lambda_m}} Q_1(\rho \sqrt{\frac{2x}{(1-\rho^2)\lambda_m}}, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}}) dx
\end{aligned} \tag{6.8}$$

By letting $y = \sqrt{x}$, and according to Eq. (37) in [76], we get

$$\begin{aligned}
&\int_{x=0}^{\frac{T}{\bar{\gamma}}} e^{-\frac{x}{\lambda_m}} Q_1(\rho \sqrt{\frac{2x}{(1-\rho^2)\lambda_m}}, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}}) dx \\
&= 2 \int_{y=0}^{\sqrt{\frac{T}{\bar{\gamma}}}} y e^{-\frac{y^2}{\lambda_m}} Q_1(\rho \sqrt{\frac{2}{(1-\rho^2)\lambda_m}} y, \sqrt{\frac{2T}{\bar{\gamma}(1-\rho^2)\lambda_m}}) dy \\
&= \lambda_m e^{-\frac{T}{\bar{\gamma}\lambda_m}} [Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) \\
&\quad - Q_1(\rho \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})]
\end{aligned} \tag{6.9}$$

Now putting (6.9) into (6.8), we have

$$\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T) \\
&= 1 - e^{\frac{T}{\bar{\gamma}\lambda_m}} \\
&\quad - e^{-\frac{T}{\bar{\gamma}\lambda_m}} [Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) \\
&\quad - Q_1(\rho \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})]
\end{aligned} \tag{6.10}$$

Here we note that when $\rho = 1$, (6.10) reduces to

$$P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T) = 1 - e^{\frac{T}{\bar{\gamma}\lambda_m}} \tag{6.11}$$

which follows the exponential distribution and agrees with the case of perfect CSI. On the other hand when $\rho = 0$, (6.10) reduces to

$$P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T) = (1 - e^{\frac{T}{\bar{\gamma}\lambda_m}})^2 \tag{6.12}$$

which corresponds to the case that $\gamma_{i^*,j}^{(m)}$ and $\tilde{\gamma}_{i^*,j}^{(m)}$ are independent. Putting (6.5) and (6.10) into (6.4), we get

$$\begin{aligned} &P(\gamma_{i^*,j}^{(m)} < T | \tilde{\gamma}_{i^*,j}^{(m)} < T) \\ &= 1 - \frac{e^{-\frac{T}{\bar{\gamma}\lambda_m}}[Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) - Q_1(\rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})]}{1 - e^{-\frac{T}{\bar{\gamma}\lambda_m}}} \end{aligned} \tag{6.13}$$

Appendix E:
Proof of Lemma 7

$$
\begin{aligned}
&P(\gamma_{i^*,j}^{(m)} < T | \tilde{\gamma}_{i^*,j}^{(m)} > T) \\
&= \frac{P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} > T)}{P(\tilde{\gamma}_{i^*,j}^{(m)} > T)} \\
&= \frac{P(\tilde{\gamma}_{i^*,j}^{(m)} > T | \gamma_{i^*,j}^{(m)} < T) P(\gamma_{i^*,j}^{(m)} < T)}{P(\tilde{\gamma}_{i^*,j}^{(m)} > T)} \\
&= \frac{[1 - P(\tilde{\gamma}_{i^*,j}^{(m)} < T | \gamma_{i^*,j}^{(m)} < T)] P(\gamma_{i^*,j}^{(m)} < T)}{P(\tilde{\gamma}_{i^*,j}^{(m)} > T)} \\
&= \frac{P(\gamma_{i^*,j}^{(m)} < T) - P(\gamma_{i^*,j}^{(m)} < T, \tilde{\gamma}_{i^*,j}^{(m)} < T)}{P(\tilde{\gamma}_{i^*,j}^{(m)} > T)} \\
&= Q_1(\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}) - Q_1(\rho\sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}}, \sqrt{\frac{2T}{(1-\rho^2)\bar{\gamma}\lambda_m}})
\end{aligned}
\tag{6.14}
$$

Appendix F: Derivation of Equation (4.18) - (4.19)

$$
\begin{aligned}
&P(outage|c(\mathcal{D})=l)\\
&= P(\gamma_{\mathrm{R}(i),\mathrm{D}} < T|A_i(l))\\
&= \int_{x=0}^{\infty}\int_{y=0}^{\infty} P[\gamma_{\mathrm{R}(i),\mathrm{D}} < T|A_i(l), \tilde{g}_{\mathrm{R}(i),\mathrm{D}} = y, \tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x]\\
&\qquad \times f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}},\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(y, x|A_i(l))\; dydx
\end{aligned} \tag{6.15}
$$

The distribution of $g_{\mathrm{R}(i),\mathrm{D}}$ conditioned on $\tilde{g}_{\mathrm{R}(i),\mathrm{D}}$, follows a non-central chi-square distribution with 2 degrees of freedom [73]. Thus

$$
\begin{aligned}
&f_{g_{\mathrm{R}(i),\mathrm{D}}|\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(x_2|x_1)\\
&= \frac{1}{(1-\rho^2)\lambda_{\mathrm{R,D}}} e^{\frac{-(x_2+\rho^2 x_1)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}} \cdot I_0\left(\frac{2\sqrt{\rho^2 x_2 x_1}}{(1-\rho^2)\lambda_{\mathrm{R,D}}}\right)
\end{aligned} \tag{6.16}
$$

Then we have

$$
\begin{aligned}
&f_{\gamma_{\mathrm{R}(i),\mathrm{D}}|\tilde{g}_{\mathrm{R}(i),\mathrm{D}},\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(z|A_i(l), \tilde{g}_{\mathrm{R}(i),\mathrm{D}} = y, \tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x)\\
&= \frac{x e^{\frac{-(\frac{xz}{\beta\bar{\gamma}_{\mathrm{R,D}}}+\rho^2 y)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}}}{\beta\bar{\gamma}_{\mathrm{R,D}}(1-\rho^2)\lambda_{\mathrm{R,D}}} I_0\left(\frac{2\sqrt{\rho^2 xyz}}{(1-\rho^2)\lambda_{\mathrm{R,D}}\sqrt{\beta\bar{\gamma}_{\mathrm{R,D}}}}\right)
\end{aligned} \tag{6.17}
$$

From (6.17) we get

$$
\begin{aligned}
&P(\gamma_{\mathrm{R}(i),\mathrm{D}} < T|A_i(l), \tilde{g}_{\mathrm{R}(i),\mathrm{D}} = y, \tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x)\\
&= \int_{z=0}^{T} \frac{x}{\beta\bar{\gamma}_{\mathrm{R,D}}(1-\rho^2)\lambda_{\mathrm{R,D}}} e^{\frac{-(\frac{xz}{\beta\bar{\gamma}_{\mathrm{R,D}}}+\rho^2 y)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}} I_0\left(\frac{2\sqrt{\rho^2 xyz}}{(1-\rho^2)\lambda_{\mathrm{R,D}}\sqrt{\beta\bar{\gamma}_{\mathrm{R,D}}}}\right)dz
\end{aligned} \tag{6.18}
$$

In order to compute of $f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}},\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(y, x|A_i(l))$ we note that

$$
\begin{aligned}
&f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}},\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(y, x|A_i(l))\\
&= \lim_{\Delta\to 0} \frac{P(\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta]|A_i(l))}{\Delta^2}\\
&= \lim_{\Delta\to 0} \frac{P(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])}{P(A_i(l))\Delta^2}\\
&\qquad \times P(\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])
\end{aligned} \tag{6.19}
$$

It is easy to see that

$$
\begin{aligned}
&P(\max_{k=1,\cdots,l \; and \; k\neq i} \tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \leq \beta\bar{\gamma}_{\mathrm{R,D}}\frac{y}{x+\Delta})\\
\leq &P(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])\\
\leq &P(\max_{k=1,\cdots,l \; and \; k\neq i} \tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \leq \beta\bar{\gamma}_{\mathrm{R,D}}\frac{y+\Delta}{x})
\end{aligned} \tag{6.20}
$$

When Δ goes to zero, we get

$$
\begin{aligned}
&\lim_{\Delta\to 0} P(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])\\
&= P(\max_{k=1,\cdots,l \; and \; k\neq i} \tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \leq \beta\bar{\gamma}_{\mathrm{R,D}}\frac{y}{x})\\
&= [F_{\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}}(\beta\bar{\gamma}_{\mathrm{R,D}}\frac{y}{x})]^{l-1}\\
&= (\frac{\lambda_{\mathrm{R,P}}y}{\lambda_{\mathrm{R,D}}x + \lambda_{\mathrm{R,P}}y})^{l-1}
\end{aligned} \tag{6.21}
$$

And,

$$
\begin{aligned}
&P(\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta], \tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])\\
&= P(\tilde{g}_{\mathrm{R}(i),\mathrm{D}} \in [y, y+\Delta])P(\tilde{g}_{\mathrm{R}(i),\mathrm{P}} \in [x, x+\Delta])\\
&\approx f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(y) f_{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(x)\Delta^2\\
&= \frac{1}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}}\Delta^2
\end{aligned} \tag{6.22}
$$

Then combining (6.21) and (6.22) into (6.19), we get

$$
\begin{aligned}
&f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}},\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(y, x|A_i(l))\\
&= l(\frac{\lambda_{\mathrm{R,P}}y}{\lambda_{\mathrm{R,D}}x + \lambda_{\mathrm{R,P}}y})^{l-1} \frac{1}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}}
\end{aligned} \tag{6.23}
$$

Finally combining (6.18) and (6.23) into (6.15), we get

$$
\begin{aligned}
&P(outage|c(\mathcal{D}) = l) \\
&= \int_{x=0}^{\infty} \int_{y=0}^{\infty} \int_{z=0}^{T} \frac{x}{\beta \bar{\gamma}_{\mathrm{R,D}} (1-\rho^2) \lambda_{\mathrm{R,D}}} e^{\frac{-(\frac{xz}{\beta \bar{\gamma}_{\mathrm{R,D}}} + \rho^2 y)}{(1-\rho^2)\lambda_{\mathrm{R,D}}}} \\
&\times I_0\left(\frac{2\sqrt{\rho^2 xyz}}{(1-\rho^2)\lambda_{\mathrm{R,D}} \sqrt{\beta \bar{\gamma}_{\mathrm{R,D}}}}\right) l \left(\frac{\lambda_{\mathrm{R,P}} y}{\lambda_{\mathrm{R,D}} x + \lambda_{\mathrm{R,P}} y}\right)^{l-1} \\
&\times \frac{1}{\lambda_{\mathrm{R,D}} \lambda_{\mathrm{R,P}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}} dz dy dx
\end{aligned}
\tag{6.24}
$$

Appendix G: Derivation of $P(\bar{\mathcal{I}}_R|c(\mathcal{D}) = l)$ when $l > 0$

Assuming that $c(\mathcal{D}) = l > 0$, let $i = \arg\max_{k\in\mathcal{D}}\{\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}\}$. The retransmit power $P_{\mathrm{R}(i)}$ is given by $P_{\mathrm{R}(i)} = \beta I_p/|\tilde{h}_{\mathrm{R}(i),\mathrm{P}}|^2$ and the interference power received at the PU from relay i is given by $I_{\mathrm{R}(i)} = P_{\mathrm{R}(i)}|h_{\mathrm{R}(i),\mathrm{P}}|^2$. Then

$$I_{\mathrm{R}(i)} = \frac{\beta I_p |h_{\mathrm{R}(i),\mathrm{P}}|^2}{|\tilde{h}_{\mathrm{R}(i),\mathrm{P}}|^2} = \frac{\beta I_p g_{\mathrm{R}(i),\mathrm{P}}}{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}} \tag{6.25}$$

Given that relay i is retransmitting, there will not be any interference to PU if $I_{\mathrm{R}(i)} \leq I_p$, or $\beta g_{\mathrm{R}(i),\mathrm{P}} \leq \tilde{g}_{\mathrm{R}(i),\mathrm{P}}$. Therefore[1],

$$\begin{aligned}
P(\bar{\mathcal{I}}_R|c(\mathcal{D}) = l) &= \sum_{i=1}^{l} P(g_{\mathrm{R}(i),\mathrm{P}} < \frac{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}{\beta}|A_i(l))P(A_i(l)) \\
&= \frac{1}{l}\sum_{i=1}^{l} P(g_{\mathrm{R}(i),\mathrm{P}} < \frac{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}{\beta}|A_i(l)) \\
&= P(g_{\mathrm{R}(i),\mathrm{P}} < \frac{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}{\beta}|A_i(l))
\end{aligned} \tag{6.26}$$

where the last equality follows from the fact that all the relay channels are *iid*. Now (6.26) is calculated as

$$\begin{aligned}
&P(g_{\mathrm{R}(i),\mathrm{P}} < \frac{\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}{\beta}|A_i(l)) \\
&= \int_{x=0}^{\infty}\int_{z=0}^{\frac{x}{\beta}} f_{g_{\mathrm{R}(i),\mathrm{P}}\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(z, x|A_i(l)) \ \ dzdx \\
&= \int_{x=0}^{\infty}\int_{z=0}^{\frac{x}{\beta}} \frac{P(A_i(l)|g_{\mathrm{R}(i),\mathrm{P}} = z, \tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x)f_{g_{\mathrm{R}(i),\mathrm{P}}\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(z, x)}{P(A_i(l))} \ dzdx \\
&= \int_{x=0}^{\infty}\int_{z=0}^{\frac{x}{\beta}} lP(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x)f_{g_{\mathrm{R}(i),\mathrm{P}}\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(z, x) \ dzdx
\end{aligned} \tag{6.27}$$

[1]Please see Remark 8.

in which $P(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x)$ is evaluated as

$$
\begin{aligned}
& P(A_i(l)|\tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x) \\
& = P(\tilde{\gamma}_{\mathrm{R}(i),\mathrm{D}} \geq \max_{k\in\mathcal{D},\, k\neq i}\{\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}\}|\tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x) \\
& = \int_0^\infty P(\tilde{\gamma}_{\mathrm{R}(i),\mathrm{D}} \geq \max_{k\in\mathcal{D},\, k\neq i}\{\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}\}|\tilde{g}_{\mathrm{R}(i),\mathrm{P}} = x, \tilde{g}_{\mathrm{R}(i),\mathrm{D}} = y) f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(y)\ dy \\
& = \int_0^\infty P(\max_{k\in\mathcal{D},\, k\neq i}\{\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}}\} \leq \frac{\beta I_p y}{\sigma_{\mathrm{D}}^2 x})\ f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(y)\ dy \\
& = \int_0^\infty P(\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \leq \frac{\beta I_p y}{\sigma_{\mathrm{D}}^2 x})^{l-1} f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(y)\ dy
\end{aligned} \tag{6.28}
$$

Using (6.27) and (6.28) we get

$$
\begin{aligned}
& P(\bar{\mathcal{I}}_R|c(\mathcal{D}) = l) \\
& = \int_{x=0}^\infty \int_{z=0}^{\frac{x}{\beta}} \int_{y=0}^\infty l P(\tilde{\gamma}_{\mathrm{R}(k),\mathrm{D}} \leq \frac{\beta I_p y}{\sigma_{\mathrm{D}}^2 x})^{l-1} f_{\tilde{g}_{\mathrm{R}(i),\mathrm{D}}}(y) \\
& \qquad\qquad\qquad \times f_{g_{\mathrm{R}(i),\mathrm{P}}\tilde{g}_{\mathrm{R}(i),\mathrm{P}}}(z,x)\ dydzdx \\
& = \int_{x=0}^\infty \int_{z=0}^{\frac{x}{\beta}} \int_{y=0}^\infty l \left[\frac{y\lambda_{\mathrm{R,P}}}{x\lambda_{\mathrm{R,D}} + y\lambda_{\mathrm{R,P}}}\right]^{l-1} \frac{1}{\lambda_{\mathrm{R,D}}} e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} \\
& \times \frac{1}{(1-\rho^2)\lambda_{\mathrm{R,P}}^2} e^{\frac{-(z+x)}{(1-\rho^2)\lambda_{\mathrm{R,P}}}} I_0(\frac{2\sqrt{\rho^2 zx}}{(1-\rho^2)\lambda_{\mathrm{R,P}}}))dydzdx
\end{aligned} \tag{6.29}
$$

Now using the first-order Marcum Q-function, we can simplify (6.29) to

$$
\begin{aligned}
& P(\bar{\mathcal{I}}_R|c(\mathcal{D}) = l) \\
& = \int_{x=0}^\infty \int_{y=0}^\infty l \left[\frac{y\lambda_{\mathrm{R,P}}}{x\lambda_{\mathrm{R,D}} + y\lambda_{\mathrm{R,P}}}\right]^{l-1} \frac{e^{\frac{-y}{\lambda_{\mathrm{R,D}}}} e^{\frac{-x}{\lambda_{\mathrm{R,P}}}}}{\lambda_{\mathrm{R,D}}\lambda_{\mathrm{R,P}}} \\
& \times \left[1 - Q_1(\sqrt{\frac{2\rho^2 x}{(1-\rho^2)\lambda_{\mathrm{R,P}}}}, \sqrt{\frac{2x}{(1-\rho^2)\beta\lambda_{\mathrm{R,P}}}})\right]\ dydx
\end{aligned} \tag{6.30}
$$

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Vita

Hui Sun was born in Nanjing, China, in 1985. He received his B.S. degree in Communication Engineering from Nanjing University of Posts & Telecommunications, Nanjing, China in 2008. He received his M.S. degree in Communication & Information System from Nanjing University of Science & Technology, Nanjing, China in 2010, and M.S. degree in Electrical Engineering from Louisiana State University (LSU), Baton Rouge, USA in 2012, respectively. He has been working as a graduate assistant in Division of Electrical & Computer Engineering at LSU since 2011. Currently, he is pursuing his Ph.D. degree in Electrical Engineering at LSU. His current research interests include cooperative relay networks and cognitive radio networks.