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ESTIMATION OF RFID TAG POPULATION SIZE BY GAUSSIAN ESTIMATOR

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science in Electrical Engineering

in

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by
Md Mahmudul Hasan
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This work was motivated by some of the tag estimation algorithms, specially the recently published tag estimation scheme proposed by Shahzad and Liu (2015).

This thesis is dedicated to all my family members, specially my parents, my younger brother and my grand -parents for their support and encouragement throughout my academic life.

Contents

Asbtract

Radio Frequency IDentification (RFID) systems are prevalent in all sorts of daily life endeavors. In this thesis we propose a new method to estimate RFID tag population size. We have named our algorithm "Gaussian Estimation of RFID Tags,"namely, GERT. We present GERT under both $\{0, 1\}$ and $\{0, 1, e\}$ channel models, and in both cases the estimator we use is a well justified Gaussian random variable for large enough frame size based on Central Limit Theorem for triangular arrays. The most prominent feature of GERT is the quality with which it estimates a tag population size. We support all the required approximations with detailed analytical work and account for all the approximation errors when we consider the overall quality of the estimation. Our simulation results agree well with analytical ones. GERT, based on standardized frame slotted Aloha protocol, can estimate any tag population size with desired level of accuracy using fewer number of frame slots than previously proposed algorithms.

Chapter 1

Introduction

RADIO frequency identification (RFID) systems are of common use particularly because of the negligible price of RFID tags (e.g., as low as 5 cents per tag [1]) compared to the products that they are attached to. Some of these applications are, object tracking [2], 3-D positioning [3], indoor localization [4], supply chain management [5], inventory control, and access control [6], [7] . The most common use of RFID is to identify individual products. Some of the world famous guitar producing companies, for example Fender, have embedded their producsts with RFID chips, which uniquely identify each guitar. If a guitar gets stolen, police can quickly know the real owner. RFID tags embedded in poker chips help the casino track how much gamblers are spending and on which tables and thus prevent theft. RFID is being used by some of the luxury product brands to let their customers scan and verify the originality of the product. A product that has RFID tag inside, can be tracked wherever there is a reader. So, factories use RFID to measure how long it takes to make product and companies track how and where their products such as vehicles and computers are used. RFID tags are very small and hence can be implanted in people and animals. One of the earliest uses of RFID in the United States and Europe was to implant pet animals with RFID chips to ensure that the pets do not go missing. RFID is used in manufacturing plants to track parts and work in process and to reduce defects, increase throughput and manage the production of different versions of the same product.

A *tag* is a microchip with an antenna that has limited computing power and communication range. A *reader*, on the other hand, is a device that has a dedicated power source and substantial amount of computational ability. Tags respond to the queries sent by the reader over a shared wireless medium. *Passive tags* do not have their own power sources and hence they are powered by the radio frequency energy from the readers. They are usually capable of communicating over a range of less than 20 ft. *Active tags* posses their own power sources and can communicate over a longer range. When an RFID reader is at a distance within the communication range of a tag, it can send and read data to identify the item the tag is embedded in.

Tag estimation is defined as the problem of designing an efficient algorithm to estimate the number of RFID tags in a deployment area without actually reading the ID of each tag. A tag estimation algorithm must ensure the following qualities [8]:

1. *Reliability* The given reliability requirement α means that atleast α fraction of the times the estimated value $\hat{t}$ of the number of tags should fall in the

allowed confidence intrval $t \pm \beta t$. This is called *required reliability.* The reliability that we actually achive running the algorithm is called the *actual reliability.* The algorithm should ensure that the *acutal reliability* is equal or greater than the *required reliability.*

2. *Scalability* The estimation for large number of tags should be scalable. There might be cases where the number of passive tags will be very high and the algorithm should take all those cases in consideration.

3. *Compliance* The estimation algorithm should be compliant with the C1G2 standard without any modifications or adjustments done to the tags

Tag estimation is useful in many everyday applications including tag identification, privacy-sensitive RFID systems and warehouse monitoring. For tag identification in frame slotted Aloha protocol [standardized in EPCGlobal Class-1 Generation-2 (C1G2) RFID standard [9] and implemented in commercial RFID systems], tag estimation often helps to decide an optimum frame size. In warehouses, it is often required to estimate the current stock of a product to plan on the future requirements or prevent theft. In a privacy sensitive RFID system, for example in a parking area where the reader does not have the permission to identify a human individual, tag estimation helps to keep track of the number of vehicles entering in a particular area.

We are proposing a very efficient method to estimate the size of a tag population. There has been a good number of estimation schemes in literature, and we present the comparative efficacy of our algorithm with respect to recently proposed Average Run-based Tag estimation(ART) [8].

Before we fomally introduce the detailed attributes of our scheme it is worth to have a quick look at the some of the preceeding works, their disticnt features and contributions. One of the first works on tag estimation was that of Kodialam and Nandagopal, titled Unified Probabilistic Estimator (UPE) [10]. UPE estimation was based on the number of empty slots in a frame or the number of collision slots in the frame. UPE has larger variance which only meant more number of rounds required. Kodialam et al. later proposed an improved framed slotted Aloha protocol-based estimation in [11] called Enhanced Zero Based (EZB) estimator. EZB makes its estimation based on the total number of empty slots in a frame. The difference between EZB and UPE is, UPE makes an estimation of the population size in each frame and at the end averages out all the estimation results, whereas EZB finds the average of the number of 0s in each frame and finally makes the estimation based on this average value.

Qian et al. in [12] proposed the scheme known as Lottery of Frame (LoF). Though it was faster than the previous schemes, it required each tag to store a big number in the scale of thousands. This issue rendered LoF's implementation impractical. LoF is also noncompliant with C1G2 because, it requires to modify both tags and communication protocol between the reader and the tags.

First Non Empty Based (FNEB) estimator, proposed by Han et al. is based on the size of the first run of 0s in a frame [13]. FNEB assumes an arbitrarily large frame size, which is impractical. Maximum Likelihood Estimator (MLE), proposed by Li et al. came with the motive to minimize power consumption by the active tags [14]. The multireader tag estimation proposed by Shah and Wong in [15] assumes that any tag covered by several readers replies to only one of them. Collision Set Estimator (CSE), proposed in [16] by Zanella uses maximum likelihood estimation to estimate the population size. CSE does not take accuracy requirements into account, hence cannot achieve required level of reliability. Shah-Mansouri and W.S. Wong presented an algorithm to estimate the cardinality or RFID tags under multiple readers with overlaping regions [17].

We compared our proposed approach GERT with relatively recent tag estimation scheme ART [8]. We owe a sincere acknowlegement to the authors of ART, for the inspiration and intuition behind writing this paper. Though our work is analytically different from their's, understanding their approach contributed to our understanding of the problem and helped us frame and organize our approach. We also credit ART for comparing their results to a good number of preceeding works, because that added significant meaning to the comparison of our results to that of ART. ART also uses standardized framed slotted Aloha protocol, in which a reader first broadcasts a value f to the tags in its vicinity, where f represents the number of time-slots present in a forthcoming frame. Each tag replies to a randomly picked slot in the frame. Thus, the reader gets a binary sequence of 0s and 1s by representing a slot with no tag replies as 0 and a slot with one or more tag replies as 1. So, it's clear that ART uses $\{0, 1\}$ channel model and does not differentiate between a *singleton* and a *collision* slot. A 1 may mean exactly 1 or any other value greater than 1. ART estimates tag population size based on the average run size of 1s in the binary sequence. They show that the average run size of 1s in a frame monotonically increases with the increase in the tag population size and hence it is invertible. ART compared their results and findings with some of the previous works in the area and claimed to be more cost effective and faster than all previous works.

Multi-category RFID Estimation was proposed in [18], which is a technique to estimate RFID tags in different categories. The first ever RFID estimation technique in presence of blocker tags was proposed in [19]. An algorithm to estimate RFID tags in composite sets was proposed in [20].

The problem of tag estimation can be formally stated as follows: for a given reliability requirement $\alpha \in [0, 1)$, a confidence interval $\beta \in [0, 1)$ a reader or a set of readers will have to estimate an unknown tag population size t in a particular area. The estimation has to maintain the minimum accuracy condition $P[|\hat{t} - t| \leq \beta t] \geq \alpha$ where $\hat{t}$ is the estimated value of the actual tag population size t.

GERT uses the framed slotted Aloha protocol specified in C1G2 as its MAC-layer communication protocol. Reader broadcasts the frame size (f) and a random seed number (S) to all the tags in its vicinity. Each of the tags participate in the forthcoming frame with probability p, where p is the *persistence probability*, the probability that decides if a tag is going to remain active to participate in the forthcoming frame. Each individual tag has an ID and uses f and S values to evaluate a hash function $h(f, S, ID)$. The value of the hash function is uniformly distributed within the range $[1, f]$. Each tag has a counter that has an initial value equal to the slot number that it has got evaluating the hash function. After each slot the reader sends out a termination signal to all the tags and each tag decreases its counter value by 1. At any given point the tags with counter value equal to 1, reply to the reader. *Empty* slots are the slots that have not been replied to by any of the tags; *singleton* slots are the ones that have been replied to by exactly one of the tags; *collision* slots are the slots that have been replied to by more than one tags. Under $\{0, 1, e\}$ channel model, each empty slot is represented by a 0, each singleton slot is represented by a 1, and each collision is represented by an e, whereas under $\{0, 1\}$ channel model, each empty slot is represented by a 0 and each non empty is represented by a 1.

At the beginning of the estimation process GERT runs a probe using the Flazolet Martin algorithm [21] to get a rough upper bound t_m on the tag population size t. The critical parameters p, f and n are calculated based on the upper bound t_m and the accuracy requirements α, β, where n is the number of rounds (i.e. frames) required to meet the accuracy requirements . Using standardized framed slotted Aloha protocol, depending on the channel model, GERT gets a reader sequence of $\{0, 1, e\}$ or $\{0, 1\}$. GERT uses $\frac{N_e - N_1}{f}$ and $\frac{N_n - N_0}{f}$ as the estimator under $\{0, 1, e\}$ and $\{0, 1\}$ channel models respectively, where N_e represents the number of e's (i.e. collision slots) and N_1 is the number of 1's, N_n is the number of non-empty slots (i.e, both 1s and e's) and N_0 represents the number 0s in the reader sequence. GERT calculates the value of the respective estimator (i.e, $\frac{N_e - N_1}{f}$ under $\{0, 1, e\}$ and $\frac{N_n - N_0}{f}$ under $\{0, 1\}$ channel model) for each round . After n rounds of these measurements we take the average of all these values. This average is finally substituted for the true mean in the expected value equation of the respective estimator to estimate the tag population size by an inverse function. We have analyzed in this paper the conditions under which $\frac{N_e - N_1}{f}$ and $\frac{N_n - N_0}{f}$ are asymptotically Gaussian distributed and their respective expected value functions are invertible, while meeting the imposed estimation accuracy requirement. Below are the prominent features of our algorithm:

1. The approximation of our estimator to Gaussian random variable has been well justified by using 'Triangular Array Central Limit Theorem'.

2. We have strictly maintained the quality of our estimation by taking the approximation error into account when calculating the overall estimation error.

3. We have presented the analysis and results of GERT under both $\{0, 1, e\}$ and $\{0, 1\}$ channel models.

4. GERT achieves same level of reliability using fewer number of slots for estimation than the recently proposed Average Run-based Tag estimation(ART) [8].

GERT uses fewer number of slots than ART to estimate any given tag population size (except for estimating very small tag population sizes) under the same accuracy requirements. Here, we define the required number of slots as $(f + l) \times n$ where f is the number of slots in a frame, n is the number or rounds and $l = 1$ms (i.e. ≈ 3.33 time slots) is the C1G2 specified mandatory time delay between the end of a frame and the start of the next one [9], [22]. Because of having more side information, it is intuitively expected that GERT under $\{0, 1, e\}$ channel model would require fewer number of slots than ART, but it incurs the extra cost of distinguishing between a *singleton* and a *collision* slot. To ensure a fair ground for comparison, we compare the results of GERT under $\{0, 1\}$ channel model with the results of ART in terms of the number of slots required for estimation and the achieved reliability.

1.1 Organization of the Thesis

Here we give some guideline about the organization of the remaining part of the thesis. The next chapter gives the detailed analysis of GERT under $\{0, 1, e\}$ channel model. Chapter 3, which refers back to chapter 2 for most parts, gives the detailed analysis of GERT under $\{0, 1\}$ channel model. Since the the analysis of GERT under $\{0, 1\}$ channel model follows similar steps as GERT under $\{0, 1, e\}$ channel model, we used the framework laid down in chapter 2 as reference for chapter 3. The comparative performance of our algorithm to the previously proposed schemes and the conclusions drawn from there are given in Chapter 4. The final part of the thesis has appendices that include all the necessary proofs to lemmas and theorems.

Chapter 2

GERT Under $\{0, 1, e\}$ Channel Model

This chapter and the next one are exclusively dedicated to analyzing the analytical properties of GERT under $\{0, 1, e\}$ channel model. We define,

$$Z_f(t) \triangleq \frac{N_e - N_1}{f} \tag{2.1}$$

$$g_f(t) \triangleq E\left[\frac{N_e - N_1}{f}\right] = E\left[Z_f(t)\right] \tag{2.2}$$

It is obvious that, Z_f can take the values only in the range $[-1, 1]$. The minimum value for Z_f is -1 considering the case where we have all the slots of a frame with exactly one reply in them and the maximum value is 1 which occurs when all the slots are strictly greater than 1.

In GERT, in each frame of the Aloha protocol, we calculate Z_f . After n rounds of these measurements we take the average of all these values. This average is finally substituted as if it were the true mean, in the expected value equation of the estimator to estimate the tag population size.

Let, $X_{ij} \sim \text{Bernoulli}(\frac{p}{f})$ be the variable that represents the probability that the ith tag replies to the j^{th} slot. So the value of X_{ij} is 1 with a probability $\frac{p}{f}$ and the value of X_{ij} is 0 with a probability $(1 - \frac{p}{f})$, i.e,

$$X_{ij} = \begin{cases} 1, & \text{with probability } \frac{p}{f} \\ 0, & \text{with probability } (1 - \frac{p}{f}) \end{cases} \tag{2.3}$$

Now we introduce Y_j to represent the random variable exactly how many replies are there to the jth slot i.e. the sum of all X_{ij} for the j^{th} slot over the total number of tags, and

$$Y_j = \sum_{i=1}^{t} X_{ij}. \tag{2.4}$$

It is straightforward to derive the following probability distribution of Y_j,

$$p_{Y_j}(y) = \begin{cases} \left(1 - \frac{p}{f}\right)^t = p_0, & y = 0 \\ t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1} = p_1, & y = 1 \\ 1 - p_0 - p_1 = p_e, & y = e \end{cases} \tag{2.5}$$

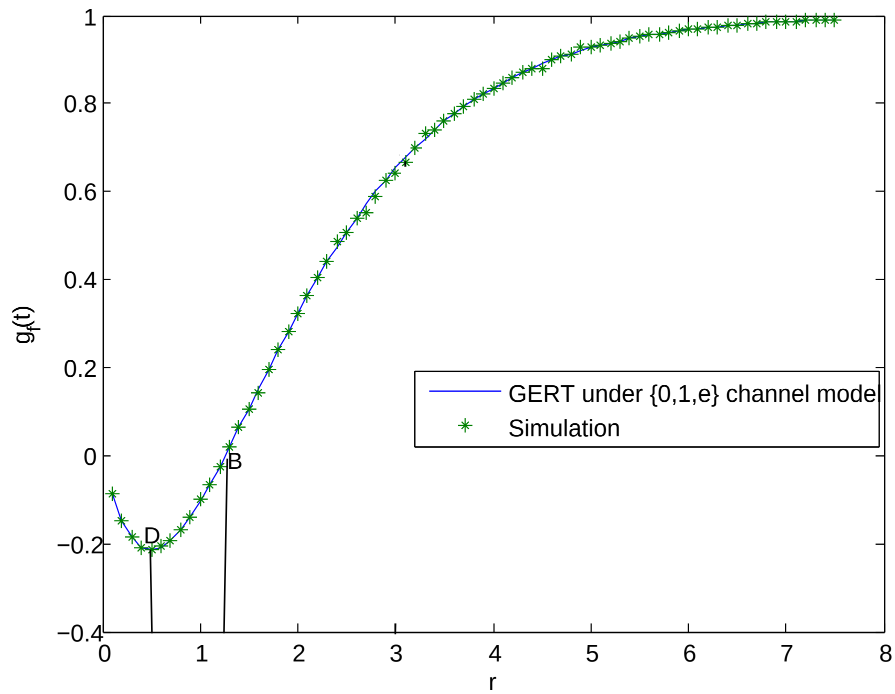

Figure 2.1: Expected value function of GERT $g_f(t)$ against $r = \frac{tp}{f}$ under $\{0, 1, e\}$ channel model . $(f = 200, p = 1)$

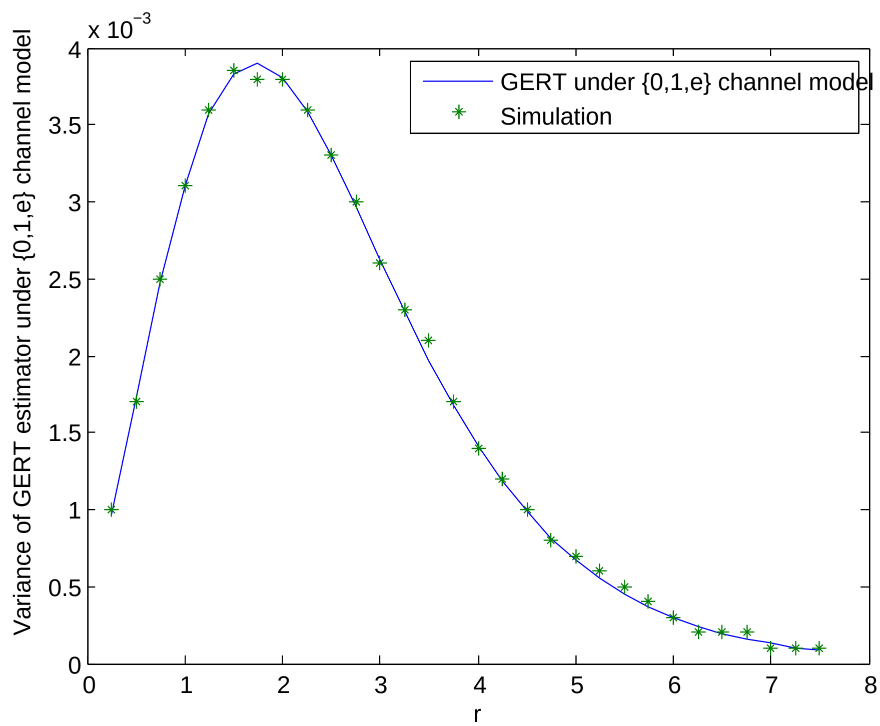

Figure 2.2: Variance of GERT estimator against $r = \frac{tp}{f}$ under $\{0, 1, e\}$ channel model. $(f = 200, p = 1)$

where p_0, p_1 and p_e are the probabilities that a slot has no reply, exactly one reply and more than one replies, respectively.

Introduction of the following two indicators $Y_j^{(1)}$ and $Y_j^{(e)}$ makes our analysis easier,

$$Y_j^{(1)} = \begin{cases} 1, & \text{when } Y_j = 1 \\ 0, & \text{when } Y_j \neq 1 \end{cases}, \quad Y_j^{(e)} = \begin{cases} 1, & \text{when } Y_j = e \\ 0, & \text{when } Y_j \neq e \end{cases} \tag{2.6}$$

So, (2.1) can be re-written as

$$\Rightarrow Z_f = \frac{1}{f}\sum_{j=1}^{f}(Y_j^{(e)} - Y_j^{(1)}) = \frac{1}{f}\sum_{j=1}^{f} Z_{j,f} \tag{2.7}$$

where

$$Z_{j,f} \triangleq Y_j^{(e)} - Y_j^{(1)}. \tag{2.8}$$

It is straightforward to show that $Z_{j,f}$ has the following PMF.

$$Z_{j,f} = \begin{cases} 0, & \text{with probability } p_0 \\ -1, & \text{with probability } p_1 \\ 1, & \text{with probability } p_e \end{cases} \tag{2.9}$$

The first and second moments of $Z_{j,f}$ can be given by,

$$E[Z_{j,f}] = \mu_{j,f} = p_e - p_1, \tag{2.10}$$
$$E[Z_{j,f}^2] = p_e + p_1. \tag{2.11}$$

Combination of (2.10) and (2.11) gives us the variance of $Z_{j,f}$,

$$\sigma_{j,f}^2 = p_e + p_1 - (p_e - p_1)^2. \tag{2.12}$$

Let, μ_f and σ_f^2 be the mean and variance of Z_f, respectively. Using (2.2), (2.7), (2.10) and (2.12) we have,

$$\mu_f = g_f(t) = p_e - p_1, \tag{2.13}$$
$$\sigma_f^2 = \frac{1}{f}[p_e + p_1 - (p_e - p_1)^2]. \tag{2.14}$$

Now using the expressions from (2.5), equation (2.13) can be rewritten as,

$$g_f(t) = 1 - \left(1 - \frac{p}{f}\right)^t - 2t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1} \tag{2.15}$$

We define,

$$r \triangleq \frac{tp}{f} \tag{2.16}$$

which essentially is the average number of active tags per slot.

Figure 2.2 presents the variance curve against different values of tag population size r. We see that for the very small values of r , the variance is low because, for given p and f if t is small we have a lot of 1s and very few e's which means p_1 is big and p_e is very small which in turn makes the term $(p_e - p_1)^2$ in (2.14) big and hence the overall variance small. For the opposite reason (i.e. p_e is big and $p_1$amall) we get small variance for bigger values of r. For the r values where $p_1 \approx p_e$ the variance curve hits its peak.

We notice in Figure 2.1 that, there is a dip at the beginning and other than that the expectation curve is monotonically increasing. The monotonically increasing part guarantees us a distinct inverse that will help us estimate the actual number of tags. But for the dip we instead have a singularity i.e. we will get more than one horizontal points for one vertical point. We either have to operate in the monotonically increasing region, or we have to find a special way out to select the actual value out of the multiple values suggested by the estimator in the singularity region. The following two lemmas shed more light on the matter.

Let the corresponding values of t and r at the point where the dip of $g_f(t)$ in Figure 2.1 occurs be t_{LM} and r_{LM}, respectively.

Lemma 2.1. *The local minimum of $g_f(t)$ curve occurs at a tag population size $t_{LM} = \frac{f}{2p}$ or equivalently at $r_{LM} = \frac{1}{2}$, given frame size f and persistence probability p.*

Lemma 2.2. *$g_f(t)$ is a convex function of t for $t < \frac{3f}{2p}$, and for the rest of the t values the function is concave, given frame size f and persistence probability p.*

The proofs to the above two lemmas are given in the appendices.

Combining Lemma 2.1 and Lemma 2.2 we can see there exists $\frac{1}{2p}f \leq t_B \leq \frac{3}{2p}f$ or equivalently $\frac{1}{2} \leq \frac{t_B p}{f} \leq \frac{3}{2}$ such that $g_f(t_B) = g_f(1)$ (considering that the minimum possible tag population size is 1). That t corresponds to the point B in the $g_f(t)$ curve and we represent the corresponding value of r by r_{min}. $g_f(t)$ will go into the singularity if we violate the following condition,

$$r \geq r_{min}. \tag{2.17}$$

We numerically got the value of $r_{min} = 1.2564$. So, the summary from this section is that, if we have an r so that (2.17) holds, $g_f(t)$ will be a monotonic function and hence invertible.

It is important to note that, we do not know the value of t rather we just have an upper-bound on t which is t_m. The frame size we are selecting corresponds to t_m not t. Since our frame size corresponds to t_m we may end up selecting a bigger f than we should. As a result we may still end up operating in the singularity region of $g_f(t)$. The next lemma helps us to solve this problem.

Lemma 2.3. *In the case of a singularity, i.e. when we have to decide between two different $\hat{t}$ corresponding to the same value of the estimator, the value to the right of the dip is always the value desired.*

The proof to lemma 2.3 is given in the appendices.

2.1 Gaussian Approximation of GERT Estimator Under $\{0, 1, e\}$ Channel Model

Our estimation of the tag population size has to maintain the accuracy requirement given by the condition,

$$P[|\hat{t} - t| \leq \beta t] \geq \alpha \tag{2.18}$$

Since we are using Z_f as our estimator to determine the value of $\hat{t}$, using (2.1) and (2.2), the condition in (2.18) can be written as,

$$\begin{aligned} & P[|g_f^{-1}\{Z_f\} - t| \leq \beta t] \geq \alpha \\ \Longleftrightarrow\ & P[(1-\beta)t \leq g_f^{-1}\{Z_f\} \leq (1+\beta)t] \geq \alpha \\ \Longleftrightarrow\ & P[g_f\{(1-\beta)t\} \leq Z_f \leq g_f\{(1+\beta)t\}] \geq \alpha \end{aligned} \tag{2.19}$$

Now to perform our estimation of the tag population size while maintaining the accuracy requirements given in (2.19), we need the following two conditions,

1. $g_f(t)$ has to be an invertible function.
2. a well approximated PDF for $Z_f(t)$.

The previous section clearly analysed the conditions under which $g_f(t)$ is monotonic function and hence invertible. This section is particularly devoted to the analysis of the conditions under which $Z_f(t)$ can be well approximated by a Gaussian random variable.

The Lindeberg-Feller Central Limit Theorem states that sums of independent random variables, properly standardized, converge in distribution to standard normal if Lindeberg Condition is satisfied. Since these random variables do not have to be identically distributed, this result generalizes the Central Limit Theorem for independent and identically distributed sequences.

It is sometimes the case that $X_1, X_2, ..., X_n$ a set of independent random variables possibly even identically distributed but their distributions depend on n. To find the asymptotic distributions for those random variables we need a special version of Central Limit Theorem, known as Triangular Array Central Limit Theorem.

In our case the probabilities for p_0, p_1, p_e given in (2.5), vary with the tag population size t for a given frame size f, or equivalently for a given tag population size t, the probabilities vary with the frame size. Hence we resort to triangular array CLT [23], to prove that $Z_f(t)$ follows Gaussian distribution. In other words, we resort to Lindeberg Feller Theorem [24].

The statement of Lindeberg Feller Theorem says, let $\{X_{n,i}\}$ be an independent array of random variables with $E[X_{n,i}] = 0$ and $E[X_{n,i}^2] = \sigma_{n,i}^2$, $Z_n = \sum_{i=1}^{n} X_{n,i}$ and $B_n^2 = Var(Z_n) = \sum_{i=1}^{n} \sigma_{n,i}^2$, then $Z_n \rightarrow N(0, B_n^2)$ distribution if the condition below holds for every $\epsilon > 0$,

$$\frac{1}{B_n^2}\sum_{i=1}^{n} E\left[X_{n,i}^2 1_{X_{n,i}}\{|X_{n,i}| > \epsilon B_n\}\right] \to 0 \tag{2.20}$$

where $1_X\{A\}$ is the indicator function of a subset A of the set X, and is defined as,

$$1_X\{A\} := \begin{cases} 1, & x \in A \\ 0 & x \notin A \end{cases}$$

In our algorithm, it is easy to see that the random variable $Z_{j,f}$ given in (2.9) are independent. From (2.10), (2.13) we see that $E[Z_{j,f}] = \mu_f$. Lindeberg Feller theorem requires the variable to have zero mean which is not the case with $Z_{j,f}$. To fulfill that requirement, we define a new variable $\widetilde{Z}_{j,f}$ such that,

$$\widetilde{Z}_{j,f} = Z_{j,f} - \mu_f \tag{2.21}$$

Now using (2.9), the probability distribution of $\widetilde{Z}_{j,f}$ can be given by,

$$\widetilde{Z}_{j,f} = \begin{cases} -\mu_f, & \text{with probability } p_0 \\ -1-\mu_f, & \text{with probability } p_1 \\ 1-\mu_f, & \text{with probability } p_e \end{cases} \tag{2.22}$$

Using (2.10),(2.12) and (2.21) we have,

$$E[\widetilde{Z}_{j,f}] = 0 \tag{2.23}$$

$$Var[\widetilde{Z}_{j,f}] = \sigma_{j,f}^2 \tag{2.24}$$

We define,

$$S_f \triangleq \sum_{j=1}^{f} \widetilde{Z}_{j,f} = \sum_{j=1}^{f}(Z_{j,f} - \mu_f) \tag{2.25}$$

Now, $\{\widetilde{Z}_{j,f}\}$ are independent random variables with $E[\widetilde{Z}_{j,f}] = 0$, $S_f = \sum_{j=1}^{f} \widetilde{Z}_{j,f}$ and $Var[S_f] = \sum_{j=1}^{f} Var(\widetilde{Z}_{j,f}) = f\sigma_{j,f}^2$. According to Lindeberg Feller Theorem, S_f will be asymptotically $N(0, f\sigma_{j,f}^2)$ if the condition below holds for every $\epsilon > 0$,

$$\frac{1}{f\sigma_{j,f}^2}\sum_{j=1}^{f} E\left[\widetilde{Z}_{j,f}^2 1_{\{\widetilde{Z}_{j,f}\}}\{|\widetilde{Z}_{j,f}| > \epsilon\sqrt{f}\sigma_{j,f}\}\right] \to 0 \tag{2.26}$$

Substituting $S_f \sim N(0, f\sigma_{j,f}^2)$ in (2.25), simple algebraic manipulations using (2.7), (2.12), (2.14) and (2.25) give us $Z_f \to N(\mu_f, \sigma_f^2)$. So, we can sum up, $Z_f \to N(\mu_f, \sigma_f^2)$ if (2.26) holds.

In the above condition given in (2.26) , the indicator function $1_{\{\widetilde{Z}_{j,f}\}}\{|\widetilde{Z}_{j,f}| > \epsilon\sqrt{f}\sigma_{j,f}\}$ plays a pivotal role. For the variable $\widetilde{Z}_{j,f}$ we have the following 3 cases of the indicator function,

$$|1-\mu_f| > \epsilon\sqrt{f}\sigma_{j,f} \tag{2.27}$$

$$|-1-\mu_f| > \epsilon\sqrt{f}\sigma_{j,f} \tag{2.28}$$

$$|-\mu_f| > \epsilon\sqrt{f}\sigma_{j,f} \tag{2.29}$$

It is easy to see that if none of (2.27), (2.28) and (2.29) holds, (2.26) not just converges to but actually becomes 0. We have proved in the appendix section that if the following condition holds, none of the (2.27), (2.28), (2.29) holds, or consequently (2.26) holds,

$$\epsilon^2 f \geq k(r) \tag{2.30}$$

where $k(r)$ is defined as

$$k(r) \triangleq max\{k_1(r), k_2(r), k_3(r)\} \tag{2.31}$$

and the values for k_1, k_2 and k_3 are given by the following equations:

$$k_1(r) = \frac{1}{\left|\frac{-e^r(1+4r)}{(1+2r)^2}\right| - 1} \tag{2.32}$$

$$k_2(r) = \left|\frac{e^{2r} + (1+2r)\left(\frac{1}{4} - e^r\right)}{\frac{1}{4}(1+2r)^2 - re^r}\right| \tag{2.33}$$

$$k_3(r) = \left|\frac{e^{2r} - 2e^r(1+2r) + (1+2r)^2}{(1+2r)^2 - e^r(1+4r)}\right| \tag{2.34}$$

Figure 2.3 demonstrates k_1, k_2, k_3 against different values of r. One noticeable thing is that only for the very small values of r, k_1 is bigger, other than that k_2 dominates the other two conditions.

We now come down to a condition, if (2.30) holds, (2.26) strictly becomes 0. So, for given r and ϵ if we select a frame size large enough so that (2.30) holds, the distribution of the estimator can be approximated as $Z_f \sim N(\mu_f, \sigma_f^2)$.

Quality Considerations of Gaussian Approximation

The quality of the approximation depends on the value of ϵ. To ensure that we satisfy the reliability requirements, we take this approximation error into account when we calculate the overall approximation error. Exactly speaking $Z_f \to N(\mu_f, \sigma_f^2)$ means, if the frame size is large enough to satisfy (2.30),

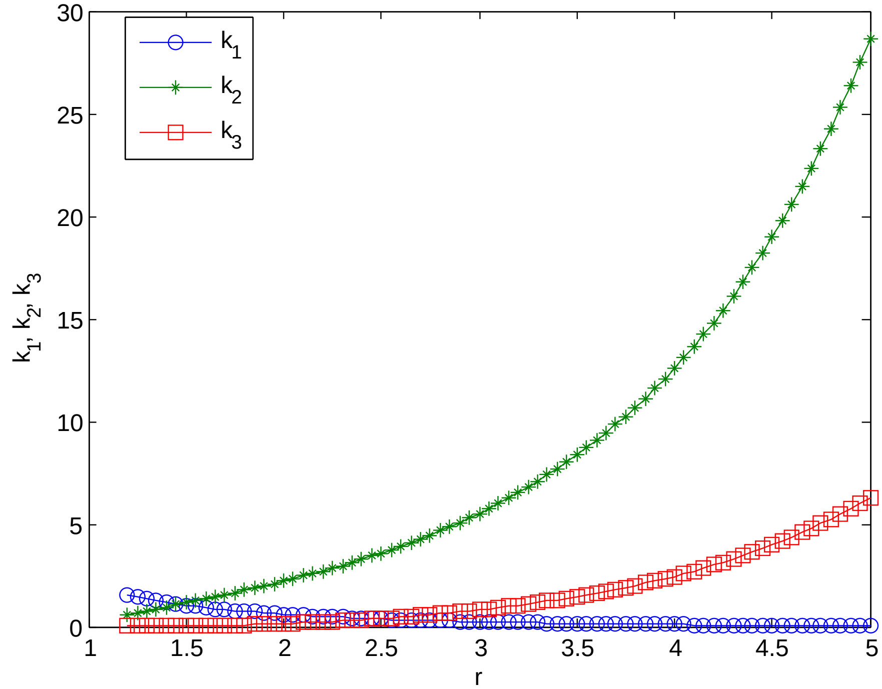

Figure 2.3: k_1, k_2, k_3 against r under {0,1,e} channel model

$$\left| P\left[l \leq \frac{Z_f - \mu_f}{\sigma_f} \leq u\right] - P\left[l \leq \theta \leq u\right] \right| \leq \epsilon \tag{2.35}$$

where, $\theta \sim N(0,1)$. Using the above equation we can write,

$$P\left[l \leq \frac{Z_f - \mu_f}{\sigma_f} \leq u\right] \geq P\left[l \leq \theta \leq u\right] - \epsilon \tag{2.36}$$

Now for the given reliability requirement α, using (2.36) we have,

$$\begin{aligned} &P\left[l \leq \theta \leq u\right] - \epsilon \geq \alpha \\ &\Rightarrow P\left[l \leq \theta \leq u\right] \geq \alpha + \epsilon \end{aligned} \tag{2.37}$$

Which means that, if we approximate $\frac{Z_f - \mu_f}{\sigma_f}$ as a standard normal, to compensate for the approximation error we will have to maintain the actual reliability $\alpha + \epsilon$ instead of the given reliability α. Using the fact that probability can not be greater than 1,

$$\alpha + \epsilon \leq 1 \Rightarrow \epsilon_{max} = 1 - \alpha \tag{2.38}$$

Equation (2.38) gives the maximum value of ϵ that we can operate on for a given reliability requirement α .

2.2 Selection of Critical Parameters

This section clarifies the steps to attain the optimum parameters for GERT under $\{0, 1, e\}$ channel model, described by Algorithm 1. Since all the parameters are functions of the tag population size t which is unknown, we have substituted t_m for t in those equations.

Frame Size f

For given r and t_m we can have different pairs of p and f satisfying (2.16). Since $p \leq 1$, (2.16) should give us the maximum possible frame size when $p = 1$ for given r and t_m. Using (2.16),

$$f_{max} = \frac{t_m}{r} \tag{2.39}$$

For a given r we will have a particular value of $k(r)$ calculated from (2.31), to satisfy the Lindeberg Feller conditions. Hence, we can get different pairs of f and ϵ satisfying (2.30). Plugging in maximum possible ϵ given by (2.38) in (2.30) should give us the minimum allowable frame size for given r. Using (2.30),

$$f_{min} = \frac{k(r)}{\epsilon_{max}^2} \tag{2.40}$$

Because of the fact that we operate on a fixed value of r, any frame size in the range $[f_{min}, f_{max}]$ will have a corresponding $\epsilon \leq \epsilon_{max}$, hence the pair will always satisfy Lindeberg Feller conditions.

We see in equation (2.16) that for given r and t_m, bigger the value of f larger is p. So, f_{max} corresponds to $p = 1$, and any f in the range $[f_{min}, f_{max}]$ will have a corresponding $p \leq 1$ for given r and t_m.

Number of Rounds n

For a given frame size f, the required number of rounds required for the estimation of the tag population follows from the accuracy requirements specified in (2.19). Since for all the frame sizes in the permissible range $[f_{min}, f_{max}]$, $g_f(t)$ is a monotonic function and $Z_f \sim N(\mu_f, \sigma_f^2)$ with an approximation error ϵ corresponding to a given f, using (2.19) and (2.36) we have,

$$P\left[\frac{g_f\{(1-\beta)t_m\} - \mu_f}{\sigma_f} \leq \frac{Z_f - \mu_f}{\sigma_f} \leq \frac{g_f\{(1+\beta)t_m\} - \mu_f}{\sigma_f}\right] \geq \alpha + \epsilon \tag{2.41}$$

Since $Z_f \sim N(\mu_f, \sigma_f^2)$, we have $\frac{Z_f - \mu_f}{\sigma_f} \sim N(0, 1)$. Now taking the value of $\alpha + \epsilon$ as the CDF of standard normal distribution we get the corresponding cut off points in the standard normal curve either side of the vertical axis. Let us the symmetric cutoff points to the right and to the left be z^* and $-z^*$ respectively.

Algorithm 1: Estimate RFID Tag Population (α, β)

Input:

1. Required reliability α
2. Required confidence interval β

Output: Estimated tag population size $\hat{t}$

Calculate $t_m :=$ **upper bound**, and substitute t by t_m.
Find the range $R = [r_{min}, r_{max}]$ specified for $\{0, 1, e\}$ channel model and discritize it. calculate, $l_R = length(R)$.
for $m := 1 : l_R$ **do**
    calculate $k(r)$ from (2.31) and use that to obtain f_{max} and f_{min} using (2.39) and (2.40) respectively for given $r = R(m)$.
    if $f_{min} \leq f_{max}$ **then**
        make the array, $f_{array} = [f_{min}, f_{max}]$
        calculate, $l_f = length(f_{array})$
        for $i := 1 : l_f$ **do**
            calculate p_i and ϵ_i from (2.16) and (2.30) respectively for given r and $f = f_{array}(i)$.
            Evaluate n_i from (2.45).
        end
        Obtain f_m, and n_m such that
        $(f_m + l) \times n_m := \min_i\{(f_{array}(i) + l) \times n_i\}$
    end
end
Obtain r_{op}, f_{op}, and n_{op} such that $(f_{op} + l) \times n_{op} := \min_m\{(f_m + l) \times n_m\}$
calculate p_{op} and ϵ_{op} from (2.16) and (2.30)
for $j := 1 : n_{op}$ **do**
    Provide the reader with frame size f_{op}, persistence probability p, and random seed S_j.
    Run Aloha on jth frame.
    Obtain $Z_f(j) = \frac{N_e - N_1}{f_{op}}$ for the jth frame
end
$\bar{Z}_f \leftarrow \frac{1}{n_{op}} \sum_j^{n_{op}} Z_f(j)$
Set $g_f(t) := \bar{Z}_f$ and solve (2.15) to get the estimated value $\hat{t}$ for tag population size t.
return $\hat{t}$

Q-function is the tail probability of the standard notmal distribution, in other words $Q(x)$ the probability that a normal random variable will obtain a value x

standard deviation above the mean. Mathematically, Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty exp\left(-\frac{u^2}{2}\right) du. \tag{2.42}$$

Now using (2.41) and (2.42) it is straightforward to find the value of z^* to be $z^* = Q^{-1}\left[\frac{1-\alpha-\epsilon}{2}\right]$. Let n_{right} and n_{left} be the number of rounds required corresponding to z^* and $-z^*$ respectively. Because of the fact that the standard deviation gets scaled down $\sqrt{n}$ times if we take n rounds of the measurements, solving the following two equations should give us the values for n_{left} and n_{right} ,

$$\frac{g_f\{(1-\beta)t_m\} - \mu_f}{\frac{\sigma_f}{\sqrt{n_{left}}}} = -z^* \tag{2.43}$$

$$\frac{g_f\{(1+\beta)t_m\} - \mu_f}{\frac{\sigma_f}{\sqrt{n_{right}}}} = z^* \tag{2.44}$$

Since the two equations are not quite symmetric, the values of n_{left} and n_{right} might differ from each other. We will go by the higer value and take the ceiling if it is not an integer to ensure that we fulfill the minimum accuracy requirements. So, using (2.43) and (2.44) the required number of rounds for given frame size f, is given by

$$n = \lceil max\{n_{left}, n_{right}\}\rceil = \left\lceil \frac{(z^*)^2\sigma_f^2}{[g_f\{(1-\beta)t_m\} - \mu_f]^2}, \frac{(z^*)^2\sigma_f^2}{[g_f\{(1-\beta)t_m\} - \mu_f]^2} \right\rceil \tag{2.45}$$

2.3 Selection of r

All other parameters of GERT were selected for a given value of r. To select a proper r, we go back to the expectation curve given in Figure 2.1 . We see that for $\{0, 1, e\}$ channel model, the value of r cannot be less than $r_{min} = 1.26$ because of the singularity considerations.

Calculation of r_{max} follows from the fact that we need to ensure that the following holds,

$$f_{max} \geq f_{min} \tag{2.46}$$

Combinig (2.40) and (2.46), we can calculate the biggest r for which (2.46) holds,

$$r_{max} = \sup_{r \geq r_{min}} \{\epsilon_{max}^2 f_{max}(r) \geq k(r)\} \tag{2.47}$$

It may happen that for a given t_m the r_{max} calculated from (2.47) is smaller than $r_{min} = 1.26$. Which implies that the range $[r_{min}, r_{max}]$ does not exist. That means the tag population size is not big enough to be estimated even at a value $r = r_{min}$ for given ϵ_{max}. Plugging in $r = r_{min}$ in (2.39) and (2.40) then solving the equations under the constraint $f_{max} \geq f_{min}$, we have

$$t_m \geq \frac{k(r_{min})r_{min}}{\epsilon_{max}^2} = t_{ml} \quad (2.48)$$

(2.48) implies that, for a given accuracy requirement we can only estimate a particular tag population size only if the upper bound is greater than t_{ml}. For example, for $\alpha = 92\%$ i.e. $\epsilon_{max} = 0.08$, we can only estimate tag population sizes that have t_m greater than 230 under $\{0, 1, e\}$ channel model .

2.4 Upper Bound on the Tag Population Size t_m

Our algorithm requires an upper-bound on the tag population size which we obtain by using Flajolet and Martin's probabilistic counting algorithm [21]. We do it before calculating the parameters p, f, and n because, to calculate all these parameters we need t_m. In Flajolet and Martin's probabilistic counting algorithm, the reader keeps issuing one slot frames till the reader gets an empty slot. The persistence probability starts with a value 1 and keeps on decreasing following a geometric distribution ($i.e., p = \frac{1}{2^{i-1}}$in the ith frame). If the empty slot occurs in the jth frame then, $t_m = 1.2897 \times 2^{j-2}$ is considered to be an upper bound on the existing tag poputation size t [21], [12]. Average of t_m values obtained in large number of rounds assymptotically approaaches t [21].

We want the upper bound to be as close to actual population size as possible. In [8] fair bit of analysis was done on how close t_m is to t. Their result shows that for 99% reliability and 1% confidence interval t_m is within $1.66 \times t$, and for 90% reliability and 10% confidence interval , which is not particularly a very tight accuracy requirement, t_m is within $1.83 \times t$. This gives us a fair idea that for reasonable accuracy requirements t_m is within $2 \times t$. This is an assumption that we made in this paper that $t_m \leq 2 \times t$ which is well supported by the findings in [8] and we take the average over 100 rounds to get that t_m.

2.5 GERT with Multiple Readers

Firstly, When there are multiple readers they are deployed after site surveys so that minimum overlapping region between readers is ensured. Secondly, our t_m is just a rough estimate with an error tolerence over $1.64 \times t$ [8]. Because of these two reasons we can let each reader calculate t_m in its own region and all the t_ms to get the overall t_m. As suggested by Kodialam et al. in [11], we will use a central controller for all the readers. Accross all the readers the GERT parameters $\alpha, \beta, t_m, p, \epsilon, n, f$ are the same. Each reader uses the seed issued to it by the central controller that means all the readers generate the same sequence of seeds. In the ith frame each reader uses the same seed S_i and the each running the hash $h(f, S_i, ID)$

will reply to the same slot number in across all frames under overlapping reader. The controller adds all the ith frames and gets a single ith frame. This bars the same tag getting counted multlple times. GERT uses total n rounds.

Chapter 3

GERT Under $\{0, 1\}$ Channel Model

To justify the fairness of the comparison between our proposed approach to previously proposed algorithms, we present the analysis of GERT under $\{0, 1\}$ channel model in this chapter. Under this model 0 represents an empty slot and 1 represents a nonempty slot. The Aloha protocol that we used for $\{0, 1, e\}$ is the same protocol that we use for $\{0, 1\}$ channel model to obtain the reader sequence. The estimator that we are using for $\{0, 1\}$ model is $\frac{N_n - N_0}{f}$ where N_n is the number of nonempty slots and N_0 is the number of empty slot in frame. The corresponding equations for (2.1) and (2.2) for this channel model can be given by, (3.1) and (3.2) respectively.

$$Z_f(t) \triangleq \frac{N_n - N_0}{f} \tag{3.1}$$

$$g_f(t) \triangleq E\left[\frac{N_n - N_0}{f}\right] \tag{3.2}$$

Lemma 3.1. *The expected value function $g_f(t)$ is a monotonically increasing function of $r = \frac{tp}{f}$ for given frame size f and persistence probability p.*

Proof to lemma 3.1 is given in the appendices. So, under $\{0, 1\}$ channel model $g_f(t)$ is invertible . As far as the requirements of GERT we only need to analyze the conditions under which Z_f can be approximated as Gaussian.

It is straightforward to see that the random variable Y_j defined in (2.4), has the following probability distribution under $\{0, 1\}$ channel model,

$$p_{Y_j}(y) = \begin{cases} \left(1 - \frac{p}{f}\right)^t = p_0, & y = 0 \\ 1 - \left(1 - \frac{p}{f}\right)^t = p_n, & y \neq 0 \end{cases} \tag{3.3}$$

where, $p_n = 1 - p_0$ is the probability that a particular slot is not empty. Corresponding to $Y_j^{(1)}$ and $Y_j^{(e)}$ in (2.6), we introduce the following two indicators for $\{0, 1\}$ channel model,

$$Y_j^{(0)} = \begin{cases} 1, & \text{when } Y_j = 0 \\ 0, & \text{when } Y_j \neq 0 \end{cases}, \quad Y_j^{(n)} = \begin{cases} 1, & \text{when } Y_j \neq 0 \\ 0, & \text{when } Y_j = 0 \end{cases} \tag{3.4}$$

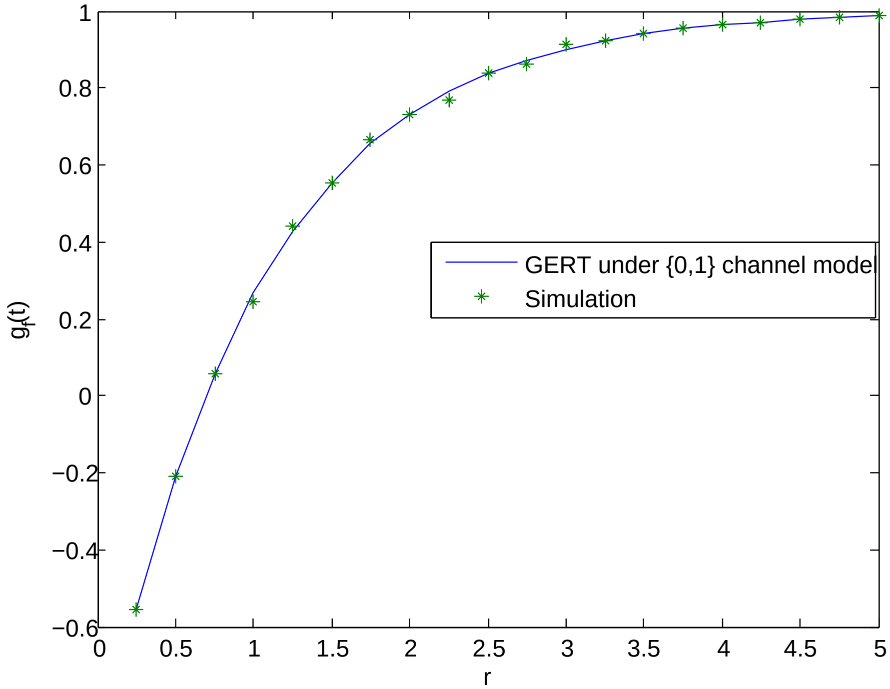

Figure 3.1: Expected value function of GERT $g_f(t)$ against $r = \frac{tp}{f}$ under $\{0, 1\}$ channel model . $(f = 200, p = 1)$

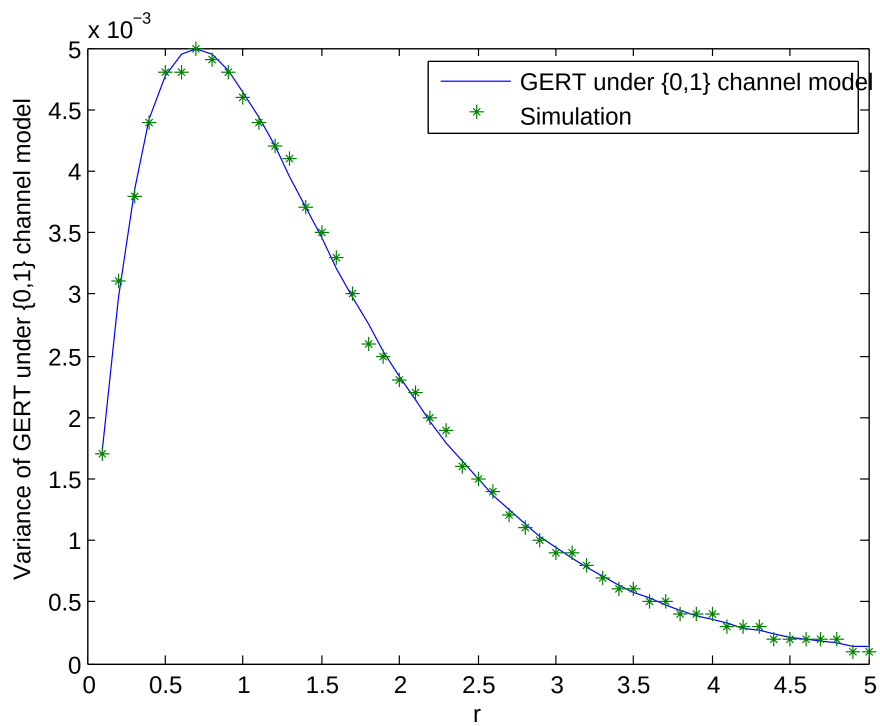

Figure 3.2: Variance of GERT estimator against $r = \frac{tp}{f}$ under $\{0, 1\}$ channel model. $(f = 200, p = 1)$

Corresponding to (2.8), we define $Z_{j,f} \triangleq Y_j^{(n)} - Y_j^{(0)}$, and its easy to see that $Z_{j,f}$ has the following PMF under $\{0,1\}$ channel model,

$$Z_{j,f} = \begin{cases} 1, & \text{with probability } p_n \\ -1, & \text{with probability } p_0 \end{cases} \tag{3.5}$$

It is important to notice that the relation $Z_f = \frac{1}{f}\sum_{j=1}^{f} Z_{j,f}$ holds for $\{0,1\}$ channel model as well. Simple algebraic manipulations should give us the mean and variance for $Z_{j,f}$ and $Z_{j,f}$ for the $\{0,1\}$ channel model. Corresponding to (2.13) and (2.14) in $\{0,1,e\}$ for $\{0,1\}$ channel model we have,

$$\mu_f = g_f(t) = p_n - p_0 = 1 - 2\left(1 - \frac{p}{f}\right)^t \tag{3.6}$$

$$\sigma_f^2 = \frac{1}{f}[p_n + p_0 - (p_n - p_0)^2] \tag{3.7}$$

Where, $p_n = 1 - p_0$ is the probability that a particular slot is not empty. Now, corresponding to (2.22), the modified version of $Z_{j,f}$, $\widetilde{Z}_{j,f}$ has the following probability distribution under $\{0,1\}$ channel model,

$$\widetilde{Z}_{j,f} = \begin{cases} 1 - \mu_f, & \text{with probability } p_n \\ -1 - \mu_f, & \text{with probability } p_0 \end{cases} \tag{3.8}$$

It is obvious that under $\{0,1\}$ channel model, the indicator function in (2.26) has only the following two cases,

$$|1 - \mu_f| > \epsilon\sqrt{f}\sigma_{j,f} \tag{3.9}$$

$$|-1 - \mu_f| > \epsilon\sqrt{f}\sigma_{j,f} \tag{3.10}$$

Consequently, the corresponding equation for k given in (2.31), under $\{0,1\}$ channel model would be,

$$k(r) \triangleq max\{k_1(r), k_2(r)\} \tag{3.11}$$

and the values for $k_1$and k_2 are given by the following equations under $\{0,1\}$ channel model.

$$k_1(r) = \frac{e^{-r}}{1 - e^r} \tag{3.12}$$

$$k_2(r) = \frac{1 - e^{-r}}{e^{-r}} \tag{3.13}$$

The proofs of the above two equations are given in the Appendix section of the paper. Based on Lindeberg Feller theorem applied in $\{0,1,e\}$ case, we can say, if we have a big enough frame size to satisfy (2.30) for given ϵ and a value of k calculated from (3.11), (2.26) holds and we have $Z_f \sim$ asymptotically $N(\mu_f, \sigma_f^2)$ under $\{0,1\}$ channel model. To ensure that we take the approximation error into account, like $\{0,1,e\}$ we have to maintain the actual reliability $\alpha + \epsilon$ instead of required reliability α for $\{0,1\}$ channel model, and we still have that upper bound on the value of ϵ given by (2.38).

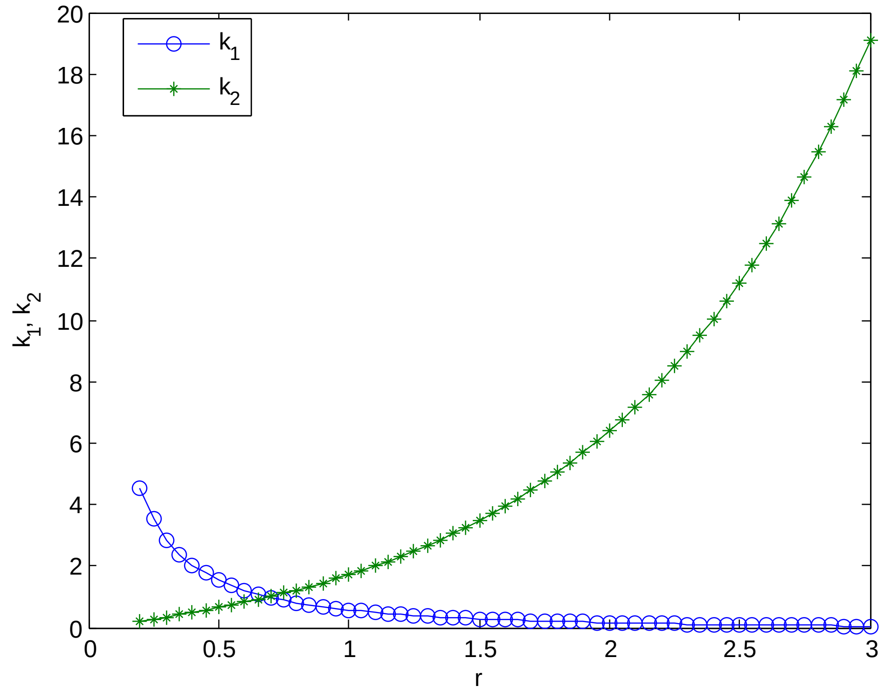

Figure 3.3: k_1, k_2 against r, under $\{0,1\}$ channel model

3.1 Critical Parameters Under $\{0, 1\}$ Channel Model

We use the same equations (2.39) and (2.40) as under $\{0, 1, e\}$ channel model, to calculate f_{max} and f_{min} for given t_m and r, except for the fact that the value of $k(r)$ we use in (2.40) is specific to $\{0, 1\}$ channel model and is calculated from (3.11). The corresponding values of p and ϵ for each f in the range $[f_{min}, f_{max}]$ follow like they did under $\{0, 1, e\}$ channel model.

Selection of the number of rounds required n to meet the accuracy requirement under $\{0, 1\}$ is no different from that of $\{0, 1, e\}$ channel model. For any f in the f_{array} mentioned in Algorithm 2, we calculate n required for the given accuracy requirements from (2.45), as we did for the $\{0, 1, e\}$ channel model.

3.2 Selection of r

For the $\{0, 1\}$ channel model we do not have any singularity issue. Hence, we do not have to maintain any lower bound on r. We still calculate r_{max} from equation (2.47) as we did uder $\{0, 1, e\}$ channel model, except for the fact that in this case we use $k(r)$ specific to $\{0, 1\}$ channel model. So, the range of the values of r that we operate for given t_m under $\{0, 1\}$ channel model is $(0, r_{max}]$. It's easy to see that, for any value of r_{max} the range $(0, r_{max}]$ exists. Since we do not have r_{min}, the maximum frame size f_{max} can be arbitrarily big and the condition $f_{max} \geq f_{min}$ is always satisfied. This implies that, unlike GERT $\{0, 1, e\}$, we can estimate arbitrarily small tag population sizes under GERT $\{0, 1\}$ channel model.

Algorithm 2: Estimate RFID Tag Population (α, β)

Input:

1. Required reliability α
2. Required confidence interval β

Output: Estimated tag population size $\hat{t}$

Calculate $t_m :=$ **upper bound**, and substitute t by t_m.
Find the range $R = (0, r_{max}]$ specified for $\{0, 1\}$ channel model and discritize it. calculate, $l_R = length(R)$.
for $m := 1 : l_R$ **do**
    Calculate $k(r)$ from (3.11) and use that to obtain f_{max} and f_{min} using (2.39) and (2.40) respectively for given $r = R(m)$.
    if $f_{min} \leq f_{max}$ **then**
        make the array, $f_{array} = [f_{min}, f_{max}]$
        calculate, $l_f = length(f_{array})$
        for $i := 1 : l_f$ **do**
            calculate p_i and ϵ_i from (2.16) and (2.30) respectively for given r and $f = f_{array}(i)$.
            Evaluate n_i from (2.45).
        end
        Obtain f_m, and n_m such that
        $(f_m + l) \times n_m := \min_i\{(f_{array}(i) + l) \times n_i\}$
    end
end
Obtain r_{op}, f_{op}, and n_{op} such that $(f_{op} + l) \times n_{op} := \min_m\{(f_m + l) \times n_m\}$
calculate p_{op} and ϵ_{op} from (2.16) and (2.30)
for $j := 1 : n_{op}$ **do**
    Provide the reader with frame size f_{op}, persistence probability p_{op}, and random seed S_j.
    Run Aloha on the jth frame.
    Obtain $Z_f(j) = \frac{N_n - N_0}{f_{op}}$ for the jth frame
end
$\bar{Z}_f \leftarrow \frac{1}{n_{op}} \sum_j^{n_{op}} Z_f(j)$
Set $g_f(t) := \bar{Z}_f$ and solve (3.6) to get the estimated value $\hat{t}$ for tag population size t.
return $\hat{t}$

3.3 Upper Bound on the Tag Population Size t_m

It's the same algorithm that we use to find the population upper bound for GERT under the $\{0, 1\}$ channel model as we used for GERT under $\{0, 1, e\}$ channel model.

3.4 GERT with Multiple Readers

Under the multiple reader arrangement GERT under $\{0, 1\}$ channel model works exactly the same way as GERT under $\{0, 1, e\}$ channel model does, except for one technical difference. The difference is that, the controller runs logical OR operation on all ith frames of different readers and gets a single ith frame, instead of adding them which we saw under $\{0, 1, e\}$ channel model.

Chapter 4

Performance Evaluation, Conclusion and Appendices

4.1 Performance Evaluation

After the completion of the analytical analysis of GERT we used MATLAB to get simulation results for GERT. Figures 4.1 and 4.2 illustrate the actual reliability of GERT for different reliability requirements under $\{0,1\}$ and $\{0,1,e\}$ channel models respectively. We see that the actual reliability of GERT is much greater than the required reliability under both channel models. This higher level of quality can be attributed to two distinct properties of GERT. Firstly, because of the restrictions imposed by the Gaussian approximation of Z_f, for a given number of tags t GERT always maintains a commensurate frame size which is highly unlikely to get saturated. Secondly, we controlled quality by taking all the approximation errors into account when we calculated the overall estimation error. This gives us the advantage of being able to target a lower reliability requirement than the required reliability. For example, under $\{0,1\}$ channel model we see that, when we have a required reliability of 91%, we can actually aim at 87%. Figure 4.3 and 4.4 present the corresponding number of slots $(f+l)\times n$ required for the achieved reliability given in Figure 4.1 and 4.2 respectively. Since $(f+l)\times n$ is random due to the randomness of t_m, we presented the mean value line along with the standard deviation over 50 samples. The gradual descent of the curves under both the models, is due to the fact that for the smaller values of t we can not shoot for the bigger values of r due to restrictions incurred by the condition $f_{max} \geq f_{man}$. As t gets bigger we can shoot for the larger values of r and the variance curves in Figures 2.2 and 3.2 suggest that under both the models variance is smaller for the bigger values of r. That saves us in terms of the number of rounds. The $(f+l)\times n$ curve eventually gets saturated due to the fact that we can not operate beyond the vaule of $r = r_{max}$. So, even if t continues increasing we cannot operate beyond $r = r_{max}$ and adjust p to account for the increment in the tag population size. The required number of slots for ART for (α, β) requirements $(91\%, 5\%)$, $(95\%, 5\%)$ and $(97\%, 5\%)$ are 1760, 2340 and 2880 respectively for any tag population size t. As we see in Figure 4.3 compared to ART,except for the very small tag population sizes GERT under $\{0,1\}$ channel model takes much less number of slots to achieve the same or higher reliability. Comparing between figures 4.3 and 4.4, we notice that GERT under $\{0,1,e\}$ requires fewer number of slots than GERT under $\{0,1\}$ to achieve similar level of accuracy. This shows the advantage of having more side

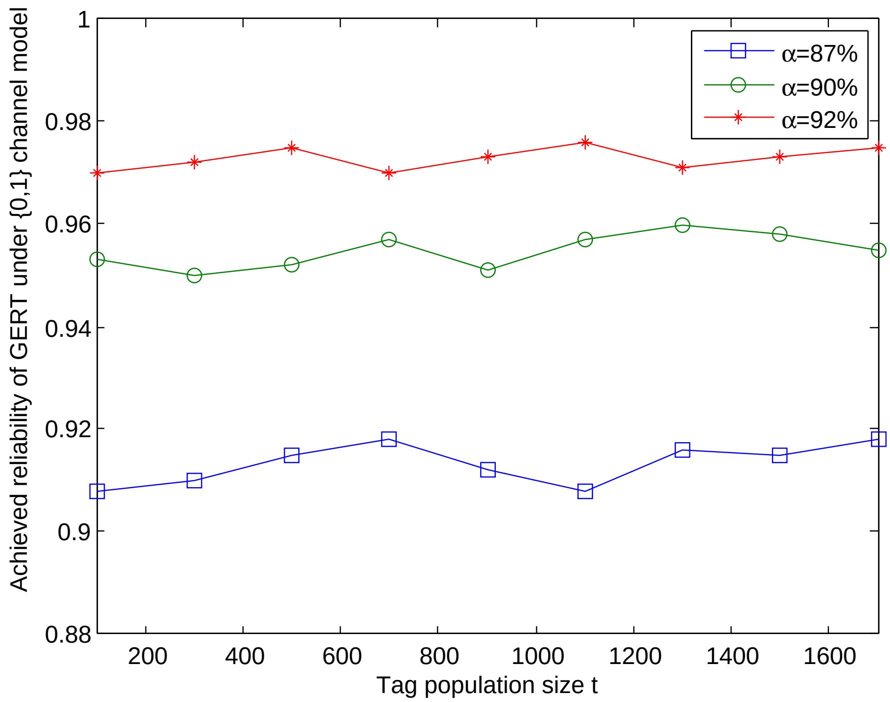

Figure 4.1: Actual reliability achieved by GERT under $\{0, 1\}$ channel model for different tag population sizes for different accuracy requirements.$\beta = 0.05$.

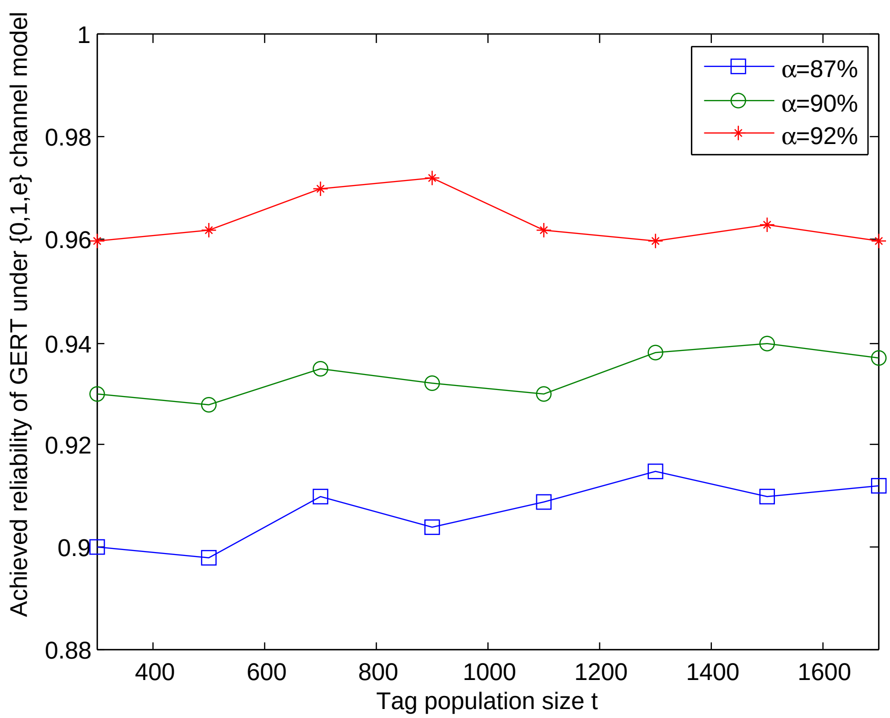

Figure 4.2: Actual reliability achieved by GERT under $\{0, 1, e\}$ channel model for different tag population sizes for different accuracy requirements. $\beta = 0.05$

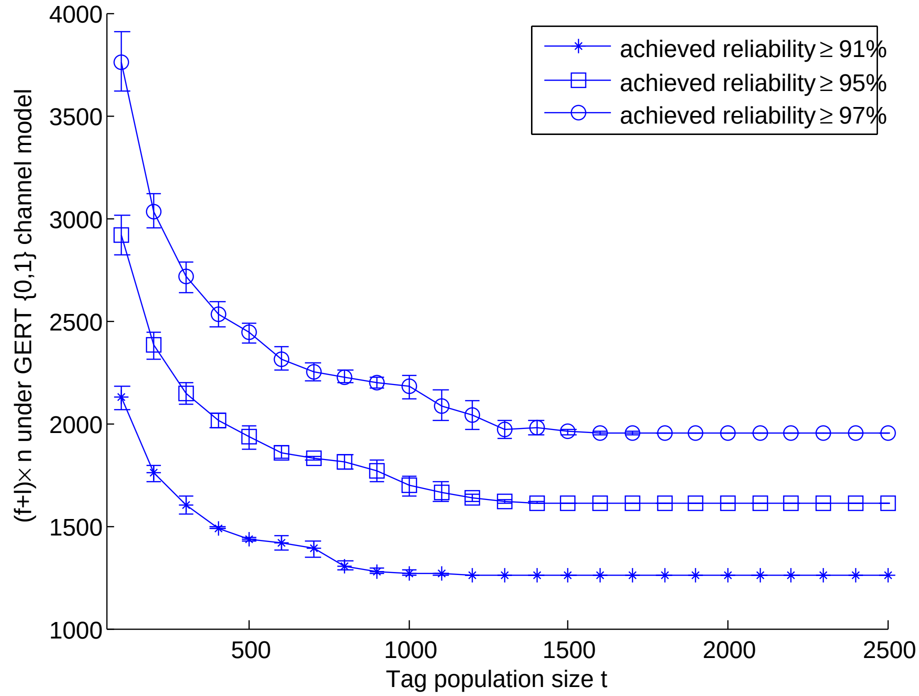

Figure 4.3: Number of slots required for estimation by GERT under $\{0, 1\}$ channel model against the tag population size t, for different levels of achieved reliability. ($\beta = 0.05$)

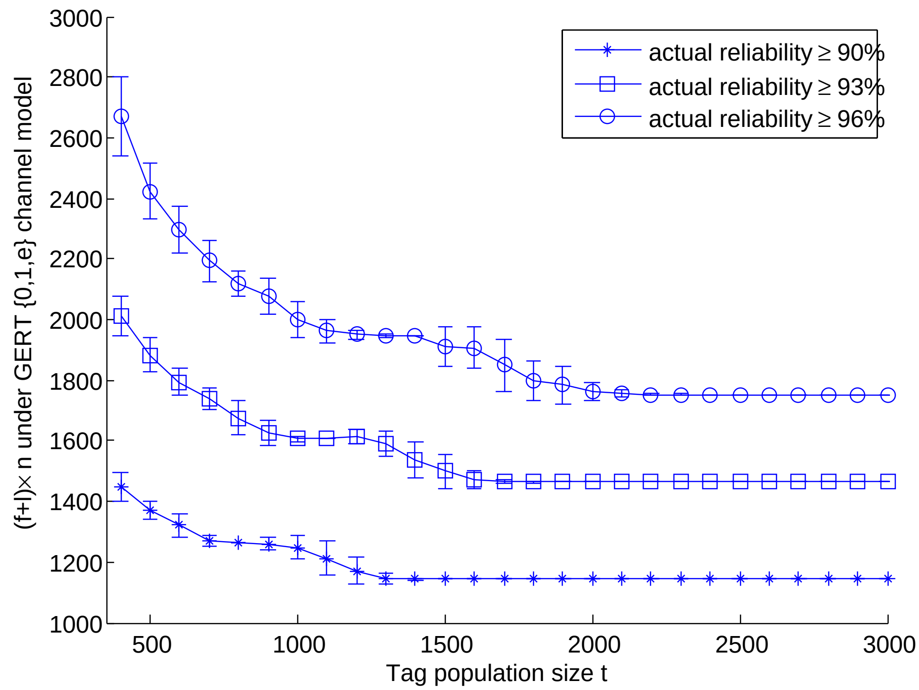

Figure 4.4: Number of slots required for estimation by GERT under $\{0, 1, e\}$ channel model against the tag population size t, for different levels of achieved reliability. ($\beta = 0.05$)

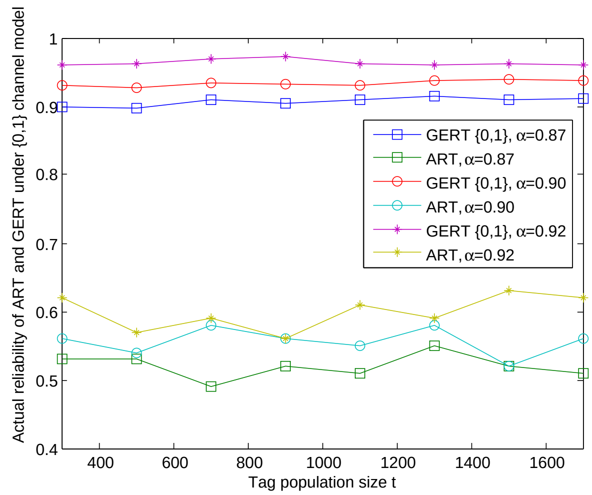

Figure 4.5: Comparison of actual reliability between ART and GERT under $\{0,1\}$ channel model for different required reliability requirements. ($\beta = 0.05$)

information. This is because under $\{0,1,e\}$ each 1 is certain. Because of this added certainty, we need fewer number of rounds to meet the accuracy requirements under $\{0,1,e\}$ than under $\{0,1\}$ channel model.

We simulated ART and compared the achieved reliability with that of GERT under $\{0,1\}$ channel model for different accuracy requirements. Figure 4.5 depicts that GERT under $\{0,1\}$ channel model performs much better that ART in terms of achieved reliability.

4.2 Conclusion

The key contribution of this thesis is that it proposes a completely new and more effective technique for the estimation of an RFID tag population size. Our estimator is Gaussian distributed and we proved that in detail with all the necessary steps. The most prominent feature of GERT is the quality of estimation coming off rigorous analysis. We supported our analytical results both under $\{0,1,e\}$ and $\{0,1\}$ channel models, by simulation results. GERT under $\{0,1\}$ channel model can estimate any arbitrary tag population size with better accuracy than other existing $\{0,1\}$ channel model schemes. Our scheme not just achieves better reliability, it generally uses less resources than other schemes in terms of the number of slots required for estimation to achieve the same reliability.

Appendix A

Proof of Lemma 2.1

Proof. To find the local minimum we need to differentiate the expectation curve and set the derivative to 0. Solving that equation we will get the local minimum of the dip. Using (2.15),

$$\frac{d}{dt} g_f(t)] = \frac{d}{dt}\left[1 - \left(1 - \frac{p}{f}\right)^t - 2t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1}\right] = 0 \tag{A.1}$$

Simple algebraic calculations give us,

$$t_{LM} = \frac{-2\left(\frac{p}{f}\right) - \left(1 - \frac{p}{f}\right)\ln\left(1 - \frac{p}{f}\right)}{2\left(\frac{p}{f}\right)\ln\left(1 - \frac{p}{f}\right)} \tag{A.2}$$

Here, t_{LM} stands for the t value where the local minimum of the dip occurs (i.e. at point D in Figure:2.1). Now, since the value of $\frac{p}{f} << 1$ we can approximate $\ln\left(1 - \frac{p}{f}\right)$ as $-\frac{p}{f}$. This will give us the following, $t_{LM} \approx \frac{f}{2p}$. Which means the local minimum for the dip occurs at a value of $t = t_{LM}$ which is supported by our simulation results. Substituting, t_{LM} in (2.16) gives us $r_{LM} = \frac{1}{2}$. □

Appendix B

Proof of Lemma 2.2

Proof. To find an inverse we need $g_f(t)$ to be a monotonic function of t . To find which part of the $g_f(t)$ demonstrates monotonic behavior we need the second derivative of $g_f(t)$ and check for it's convexity and concavity characteristics. Again using (2.15)

$$\frac{d^2}{dt^2}g_f(t) = \left(1-\frac{p}{f}\right)^{t-1}\ln\left(1-\frac{p}{f}\right)\left[-2t\left(\frac{p}{f}\right)\ln\left(1-\frac{p}{f}\right)\right. \\ \left. -4\left(\frac{p}{f}\right)-\left(1-\frac{p}{f}\right)\ln\left(1-\frac{p}{f}\right)\right] \tag{B.1}$$

If we closely follow the equation we see that, the value of the common factor $\left(1-\frac{p}{f}\right)^{t-1}\ln\left(1-\frac{p}{f}\right)$ is negative . So, for the total value to be posive the part of the equation inside the third bracket will have to be negative. After algebraic manipulations and approximating $\ln\left(1-\frac{p}{f}\right)$ as $-\frac{p}{f}$ we get for $t < \frac{3f+p}{2p}$,

$$\left[2t\left(\frac{p}{f}\right)\left(\frac{p}{f}\right)-4\left(\frac{p}{f}\right)+\left(1-\frac{p}{f}\right)\left(\frac{p}{f}\right)\right] < 0$$

Or, equivalently, for $t < \frac{3f+p}{2p} \approx \frac{3f}{2p}$, $\frac{d^2}{dt^2}g_f(t)$ is positive, indicating $g_f(t)$ is a convex function of t and for the rest of the t values the curve is concave. Substituting $t = \frac{3f}{2p}$ in (2.16) gives us $r = \frac{3}{2}$. Our simulation results strongly support that claim. □

Appendix C

Proof of Lemma 2.3

Proof. At point B in Figure 2.1, we know the corresponding frame size is f_{max}, uning (2.16) and (2.17) we have,

$$\frac{t_m p}{f_{max}} \geq r_{min} \quad \Rightarrow \frac{t_m p}{f_{max}} \geq 1.2564 \tag{C.1}$$

Now from [8], we know that the upper bound on t is always less that $2t$ for any reasonable accuracy requirements. With that knowledge, using (C.1)

$$\frac{2tp}{f_{max}} \geq 1.2567 \quad \Rightarrow \frac{2tp}{f} \geq 1.2564 \tag{C.2}$$

Using (2.16) and (C.2),

$$r \geq 0.6283 \quad \therefore r \geq \frac{1}{2} \tag{C.3}$$

From Lemma 2.1, we know that any point in $\frac{1}{2} \leq r \leq r_{min}$ corresponds to a t value to the right side of the dip in the $g_f(t)$ curve in Figure 2.1. □

Appendix D

Proof of Lemma 3.1

For GERT under $\{0, 1\}$ channel model, using equation (3.6) we get,

$$
\begin{aligned}
& g_f(t) = 1 - 2\left(1 - \frac{p}{f}\right)^t \\
\Rightarrow & \frac{d}{dt}[g_f(t)] = -2\left(1 - \frac{p}{f}\right)^t \ln\left(1 - \frac{p}{f}\right) \\
\Rightarrow & \frac{d^2}{dt^2}[g_f(t)] = -2\left(1 - \frac{p}{f}\right)^t \left[\ln\left(1 - \frac{p}{f}\right)\right]^2
\end{aligned}
$$

In our algorithm we have $p \in (0, 1]$, $f \geq 1$ and $t \geq 1$. So, the first and second derivatives of $g_f(t)$ with respect to t will always be non-negative and negative respectively. Hence, $g_f(t)$ is a monotonically increasing function of t. Equivalently, $g_f(t)$ is a monotonically increasing function of $r = \frac{tp}{f}$ for given f and p.

Appendix E

Derivation of Lindeberg Feller Conditions for GERT Under $\{0, 1, e\}$ Channel Model

First Condition

From equation (2.27) we have the following,

$$
\begin{aligned}
&\Rightarrow 1 - \mu_f > \epsilon\sqrt{f}\sigma_{j,f} \\
&\Rightarrow 1 - (p_e - p_1) > \epsilon\sqrt{f\,[p_e + p_1 - (p_e - p_1)^2]} \\
&\Rightarrow 1 - 2(p_e - p_1) + (p_e - p_1)^2 > \epsilon^2 f\left[p_e + p_1 - (p_e - p_1)^2\right] \\
&\Rightarrow 1 - \left(2 + \epsilon^2 f\right)p_e + \left(2 - \epsilon^2 f\right)p_1 + \left(1 + \epsilon^2 f\right)(p_e - p_1)^2 > 0
\end{aligned}
\tag{E.1}
$$

Letting $\epsilon^2 f$ be represented by k, and inserting the expression for p_0 , p_1 and p_e we have,

$$
\begin{aligned}
\Rightarrow 1 &- (2+k)\left[1 - \left(1 - \frac{p}{f}\right)^t - t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1}\right] + (2-k)\left[t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1}\right] + (1+k) \\
&\left[1 + \left(1 - \frac{p}{f}\right)^{2t} + 4t^2\left(\frac{p}{f}\right)^2\left(1 - \frac{p}{f}\right)^{2(t-1)} - 2\left(1 - \frac{p}{f}\right)^t + 4t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{2t-1} - \right. \\
&\left. 4t\left(\frac{p}{f}\right)\left(1 - \frac{p}{f}\right)^{t-1}\right] > 0
\end{aligned}
\tag{E.2}
$$

We know, for $x \ll 1$ and $y \gg 1$, $(1-x)^y$ can be apprriximated as $e^{y\ln[1-x]}$ and that in turn can be reduced to $e^{y[-x-\frac{1}{2}x^2]}$ applying Taylor series. Applying this we get,

$$
\begin{aligned}
\Rightarrow 1 &- (2+k)\left[1 - e^{-t\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} - t\left(\frac{p}{f}\right)e^{-(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}}\right] + (2-k)\left[t\left(\frac{p}{f}\right)e^{-(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}}\right] + \\
&(1+k)\left[1 + e^{-2t\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} + 4t^2\left(\frac{p}{f}\right)^2 e^{-2(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} - 2e^{-t\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}}\right] + \\
&(1+k)\left[4t\left(\frac{p}{f}\right)e^{-(2t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} - 4t\left(\frac{p}{f}\right)e^{-(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}}\right] > 0
\end{aligned}
\tag{E.3}
$$

Now we have a list of approximations to make. They are,

$$e^{-t\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} \approx e^{-t\frac{p}{f}} \tag{E.4}$$

$$e^{-(2t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} \approx e^{-2t\frac{p}{f}} \tag{E.5}$$

$$e^{-2(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} \approx e^{-2t\frac{p}{f}}l \tag{E.6}$$

$$e^{-(t-1)\{\frac{p}{f}+\frac{1}{2}(\frac{p}{f})^2\}} \approx e^{-t\frac{p}{f}} \tag{E.7}$$

After all these approximations and using (2.16) we have,

$$\Rightarrow 1-(2+k)\left(1-e^{-r}-re^{-r}\right)+(2-k)\,re^{-r}+(1+k)\left(1+e^{-2r}+4r^2e^{-2r}-2e^{-r}\right) + (1+k)\left(4re^{-2r}-4re^{-r}\right)>0 \tag{E.8}$$

For the equation (2.27) to not hold, (E.8) must not hold. Simple algebraic manipulations give us that for (E.8) to not hold the value of k must be,

$$k \geq \frac{1}{\left|\frac{-e^r(1+4r)}{(1+2r)^2}\right|-1} = k_1$$

So, k_1 is the minimum value of k for which (2.27) does not hold.

Second condition

From equation (2.28) we have the following,

$$\begin{aligned}
&\Rightarrow 1+\mu_f > \epsilon\sqrt{f}\sigma_{j,f}\\
&\Rightarrow 1+(p_e-p_1) > \epsilon\sqrt{f\left[p_e+p_1-(p_e-p_1)^2\right]}\\
&\Rightarrow 1+2(p_e-p_1)+(p_e-p_1)^2 > \epsilon^2 f\left[P_e+p_1-(p_e-p_1)^2\right]\\
&\Rightarrow 1+\left(2-\epsilon^2 f\right)p_e-\left(2+\epsilon^2 f\right)p_1+\left(1+\epsilon^2 f\right)(p_e-p_1)^2>0
\end{aligned} \tag{E.9}$$

letting $\epsilon^2 f$ be represented by k, and inserting the expression for p_0 , p_1 and p_e we have,

$$\Rightarrow 1+(2-k)\left[1-\left(1-\frac{p}{f}\right)^t-t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{t-1}\right]-(2+k)\left[t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{t-1}\right]+(1+k)$$
$$\left[1+\left(1-\frac{p}{f}\right)^{2t}+4t^2\left(\frac{p}{f}\right)^2\left(1-\frac{p}{f}\right)^{2(t-1)}-2\left(1-\frac{p}{f}\right)^t+4t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{2t-1}\right.$$
$$\left.-4t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{t-1}\right]>0 \tag{E.10}$$

Like the first condition $(1-x)^y$ can be apprriximated as $e^{y\ln[1-x]}$ and that in turn can be reduced to $e^{y[-x-\frac{1}{2}x^2]}$ applying Taylor series. Applying this along with approximations made in (E.4), (E.5), (E.6), (E.7) and using (2.16) we have,

$$\begin{aligned}\Rightarrow & 1+(2-k)\left(1-e^{-r}-re^{-r}\right)-(2+k)re^{-r}+(1+k)\left(1+e^{-2r}+4r^2e^{-2r}-2e^{-r}\right)\\ &+(1+k)\left(4re^{-2r}-4re^{-r}\right)>0 \end{aligned} \tag{E.11}$$

For the equation (2.28) to not hold, (E.11) must not hold. Simple algebraic manipulations give us that for (E.11) to not hold the value of k must be,

$$k \geq \left|\frac{e^{2r}+(1+2r)\left(\frac{1}{4}-e^r\right)}{\frac{1}{4}(1+2r)^2-re^r}\right| = k_2$$

So, k_2 is the minimum value of k for which (2.28) does not hold.

Third Condition

From equation (2.29) we have the following,

$$\begin{aligned}
&\Rightarrow \mu_f > \epsilon\sqrt{f}\sigma_{j,f}\\
&\Rightarrow (p_e-p_1) > \epsilon\sqrt{f\left[p_e+p_1-(p_e-p_1)^2\right]}\\
&\Rightarrow (p_e-p_1)^2 > \epsilon^2 f\left[p_e+p_1-(p_e-p_1)^2\right]\\
&\Rightarrow -\epsilon^2 f(p_e+p_1)+\left(1+\epsilon^2 f\right)(p_e-p_1)^2>0\\
&\Rightarrow -k\left[1-\left(1-\frac{p}{f}\right)^t\right]+(1+k)\left[1+\left(1-\frac{p}{f}\right)^{2t}+4t^2\left(\frac{p}{f}\right)^2\left(1-\frac{p}{f}\right)^{2(t-1)}-2\left(1-\frac{p}{f}\right)^t+\right.\\
&\quad \left.4t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{2t-1}-4t\left(\frac{p}{f}\right)\left(1-\frac{p}{f}\right)^{t-1}\right]>0
\end{aligned} \tag{E.12}$$

Like we did in the previous two conditions, $(1-x)^y$ can be apprriximated as $e^{y\ln[1-x]}$ and that in turn can be reduced to $e^{y[-x-\frac{1}{2}x^2]}$ applying Taylor series. Applying this along with approximations made in (E.4), (E.5), (E.6), (E.7) and using (2.16) we have,

$$\Rightarrow -k\left(1-e^{-r}\right)+(1+k)\left(1+e^{-2r}+4r^2e^{-2r}-2e^{-r}\right)+(1+k)\left(4re^{-2r}-4re^{-r}\right)>0 \tag{E.13}$$

For the equation (2.29) to not hold, (E.13) must not hold. Simple algebraic manipulations give us that for (E.13) to not hold the value of k must be,

$$k \geq \left|\frac{e^{2r}-2e^r(1+2r)+(1+2r)^2}{(1+2r)^2-e^r(1+4r)}\right| = k_3$$

So, k_3 is the minimum value of k for which (2.29) does not hold.

From the above three conditions we see that if we select the value of k such that $k = max\{k_1, k_2, k_3\}$ all of (2.27), (2.28) and (2.29) will not hold or equivalently (2.26) will hold. That essentially means for $k = max\{k_1, k_2, k_3\}$ the GERT estimator Z_f under $\{0, 1, e\}$ channel model is Gaussian distributed.

Appendix F

Derivation of Lindeberg Feller Conditions for GERT Under $\{0,1\}$ Channel Model

First Condition

From equation (3.9) we have the following,

$$\begin{aligned}
&|1-\mu_f| > \epsilon\sigma_{j,f} \\
\Rightarrow & 1-\mu_f > \epsilon\sigma_{j,f} \\
\Rightarrow & 1-(p_n-p_0) > \epsilon\sigma_{j,f} \\
\Rightarrow & 1-1+2p_0 > \epsilon\sigma_{j,f} \\
\Rightarrow & 2p_0 > \epsilon\sigma_{j,f} \\
\Rightarrow & 4p_0^2 > \epsilon^2 f\left[p_n+p_0-(p_n-p_0)^2\right] \\
\Rightarrow & -kp_n-kp_0+p_n^2k-2p_np_0k+p_0^2k+4p_0^2 > 0 \qquad \text{(F.1)}
\end{aligned}$$

Simple algebraic manipulations and aforementioned approximations give us, (F.1) or equivalently (3.9) does not hold if the following is satisfied,

$$k \geq \left|\frac{e^{-r}}{1-e^{-r}}\right| = k_1 = \frac{e^{-r}}{1-e^{-r}} \qquad \text{(F.2)}$$

Second Condition

From equation (3.10) we have the following,

$$\begin{aligned}
&|-1-\mu_f| > \epsilon\sigma_{j,f} \\
\Rightarrow & 1+\mu_f > \epsilon\sigma_{j,f} \\
\Rightarrow & 1+(p_n-p_0) > \epsilon\sigma_{j,f} \\
\Rightarrow & 1+1-2p_0 > \epsilon\sigma_{j,f} \\
\Rightarrow & 2-2p_0 > \epsilon\sigma_{j,f} \\
\Rightarrow & 4-8p_0+4p_0^2 > \epsilon^2 f\left[p_n+p_0-(p_n-p_0)^2\right] \\
\Rightarrow & 4-8p_0+4p_0^2-kp_n-kp_0+p_n^2k-2p_np_0k+p_0^2k > 0 \qquad \text{(F.3)}
\end{aligned}$$

Simple algebraic manipulations and aforementioned approximations give us, (F.3) or equivalently (3.10) does not hold if the following is satisfied,

$$k \geq \left| \frac{1 - e^{-r}}{e^{-r}} \right| = k_2 = \frac{1 - e^{r}}{e^{-r}} \tag{F.4}$$

From the above two conditions we see that if we select the value of k such that $k = max\{k_1, k_2\}$, (3.9) and (3.10) will not hold or equivalently (2.26) will hold. That essentially means for $k = max\{k_1, k_2\}$ the GERT estimator Z_f under $\{0, 1\}$ channel model is Gaussian distributed.

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Vita

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