

Robust Overlapping Decentralised Control of Linear Uncertain Time-Delay Systems with Application to Power Systems

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Submitted in total fulfilment of the requirements of the degree of
Doctor of Philosophy

Department of Electrical and Electronic Engineering
THE UNIVERSITY OF MELBOURNE

April 2016

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Abstract

There are many real-life or man-made large scale systems comprise subsystems with overlapping parts. In other words, the subsystems are strongly coupled in the overlapping (shared) parts but weakly interconnected otherwise. Such systems are called to have overlapping structure. Feedback controller with the overlapping structure, which is consistent with structure of the system, has been used widely to control such systems. The controller with overlapping structure comprises of local controllers which are fed by local information in addition to shared (overlapping) information. The overlapping controller design is based on the mathematical framework called the inclusion principle. The design procedure has three steps. First, the expanded system (includes the original system) comprising interconnected subsystems is generated. Then, local controllers are designed for interconnected subsystems of the expanded system. The local controllers form a decentralised controller. Finally, the decentralised controller is contracted (transformed) to an overlapping controller for implementation on the original system.

In this thesis, we will present a comprehensive study of the inclusion principle for linear systems. First, a necessary and sufficient condition for stabilisability of linear time-invariant systems with overlapping structure is presented. The conditions are related to minimality and the concept of quotient fixed modes (QFMs). Meanwhile, an iterative algorithm is presented to guarantee the contractibility of decentralised output feedback control law designed for the expanded system. Also, stabilisability of overlapping uncertain linear systems by overlapping static output feedback controllers is studied.

Furthermore, stabilisability of linear state-delay systems with overlapping structure is investigated. An extension of the inclusion principle is presented to design robust overlapping output feedback controllers for linear state-delay systems. Non-commensurate communication delays are assumed to be constant and unknown but bounded by given values. First, the expanded system is gener-

ated using the extended inclusion principle. Then, an iterative algorithm is suggested to design robust local controllers for the interconnected subsystems. Finally, the designed decentralised controller which is formed by local controllers, is contracted to an overlapping controller. It is proven that stability and performance are preserved through the contraction process. From application viewpoint, a two-area interconnected power system experiencing communication delay and model uncertainties is studied. The power system is decomposed into two overlapping subsystems with tie-lines being the overlapping parts. Afterwards, the proposed overlapping design approach is used to design an overlapping Load Frequency Controller (LFC) for the case study. Simulation results and a quantitative criterion clearly demonstrate the improved performance obtained by the proposed overlapping LFC compared with existing ones under different scenarios for communication delays and uncertain parameters.

Motivated by the load frequency control problem in the presence of network delays, the inclusion principle is used to design an overlapping output feedback controller for linear uncertain input-delay system. The network delays are unknown and time-varying but with bounded size and rate of change. Similar to before, the expanded system is generated at the first step. Then, robust local controllers are designed for the interconnected subsystems using a proposed iterative algorithm. The obtained decentralised control design is then contracted to an overlapping one for implementation on the original system. The preservation of stability and performance through contraction is proven. As an application, the inclusion principle is used to design a robust overlapping LFC for a three-area interconnected power system experiencing time-varying input delay. The simulation results and quantitative criteria with the proposed overlapping LFC are compared with those of existing decentralised LFCs.

Declaration

This is to certify that

1. the thesis comprises only my original work towards the PhD,
2. due acknowledgement has been made in the text to all other material used,
3. the thesis is less than 100,000 words in length, exclusive of tables, maps, bibliographies and appendices.

Adel Ahmadi, April 2016

Publications

The outcomes of this thesis are published or under review for publication in the following journals and conferences.

- Chapter 2
 - **A. Ahmadi**, M. Aldeen and M. Abdolmaleki “Decentralized stabilisability condition for expanded and contracted LTI systems and quotient fixed mode” In *Proceedings of 13th European Control Conference (ECC 14)*, pp. 406-411, Strasbourg, France, June 2014.
 - **A. Ahmadi**, M. Aldeen and M. Abdolmaleki “Robust overlapping output feedback control design in uncertain systems with unknown uncertainty bounds” In *Proceedings of the 2015 Australian Control Conference (AUCC 15)*, pp. 69-74, Gold Coast, Australia, November 2015.
- Chapter 3
 - **A. Ahmadi** and M. Aldeen “An LMI Approach to the Design of Robust Delay-Dependent Overlapping Load Frequency Control of Uncertain Power Systems” *International Journal of Electrical Power & Energy Systems*, vol. 81, pp. 48-63, 2016.
 - **A. Ahmadi** and M. Aldeen “Stabilizability of Multiple Time-Delay Overlapping Systems: The Inclusion Principle Approach” Accepted for publication in *Proceedings of the 2016 American Control Conference (ACC 16)*, Boston, USA, July 2016.
- Chapter 4
 - **A. Ahmadi** and M. Aldeen “Design of Robust Overlapping Load Frequency Controller for Power Systems with Communication Delays: An LMI Approach” Submitted to *International Journal of Electrical Power & Energy Systems*.
 - **A. Ahmadi** and M. Aldeen ‘Inclusion principle approach to the design of robust overlapping decentralised output feedback controller for uncertain time-delayed systems’ Submitted to *International Journal of Control*

Other Publications

The following papers were published during my study, however they're not included in the thesis.

- **A. Ahmadi** and M. Aldeen “New input-output pairing based on eigenvalue contribution measures” In *Proceedings of the 9th Asian Control Conference (ASCC 13)*, pp. 1-6, Istanbul, Turkey, June 2013.
- M. Abdolmaleki, M. Aldeen, and **A. Ahmadi** “System graph approach to decomposition and decentralised control of LTI interconnected systems”, In *Proceedings of the 13th International Conference on Control Automation Robotics & Vision (ICARCV 14)*, pp. 1587-1592, December 2014.

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Acknowledgements

This thesis owes its existence to the help, support and inspiration by many people. First and foremost, I would like to express my gratitude to my supervisor A/Professor Mohammad Aldeen for his continuous support and patience during my Ph.D journey. I am thankful for his aspiring guidance, invaluable constructive criticism and friendly advice during my study. I would also like to thank my committee members Professor Girish Nair, Dr. Iman shames and Dr. Sajeeb Saha for their insightful comments and encouragement.

I gratefully acknowledge National ICT Australia (NICTA) for supporting my research financially. Also, I wish to thank Professor Erik Weyer and A/Professor Peter Dower who provided me an opportunity to experience lab demonstration. A sincere thank goes to Dr. Ehsan Nekouei, who as a good friend, is always willing to help and give his best suggestions.

I would like to thank my office mates Sasan, Amin, Ahvand, Majid, Saeed, Shabnam, Roghieh, and Sahar. Also, I am thankful to my friends Mohammad, Amir, and Laven foor their support and advice. My research would have not been fun without my amazing friends Jack and Xiaoxi.

Most importantly, none of this would have been possible without the love and patience of my family. My deepest gratitude goes to my family for their love and unconditional support throughout my life.

To everybody else who accompanied me in this Ph.D. journey: THANK YOU!

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To my beloved parents, Hossein and Parvin, and my caring brother, Ali.

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ABBREVIATIONS

LFC	Load Frequency Controller
QFM	Quotient Fixed Mode
LMI	Linear Matrix Inequality
ACE	Area Control Error
PI	Proportional-Integral
Blkdiag	Block-diagonal
ILMI	Iterative Linear Matrix Inequality
SDFM	Structured Decentralised Fixed Mode
UDFM	Unstructured Decentralised Fixed Mode
DFM	Decentralised Fixed Mode
GEVP	Generalized Eigenvalue Minimization Problem
QMI	Quadratic Matrix Inequality
PID	Proportional-Integral-Derivative
GRC	Generation Rate Constraint

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Chapter 1

Introduction

1.1 Motivations

Control of large-scale systems plays a vital role in different engineering disciplines, [1–3]. These systems are comprised of interconnected subsystems, which are geographically distributed and the information is exchanged among them through communication channels. Robotic vehicles [1], Multi-area interconnected power systems [4], and network control systems [5] are examples of such systems, to name a few.

For large scale systems, centralised control, where information is sent to a central station for control design purposes, may not be physically feasible, due to many reasons such as information accessibility, highly cost of implementation and considerable computational effort as all information has to be sent to a central controller. To overcome difficulties arising from centralised controllers, structurally constrained controllers have been proposed. These types of controllers are more feasible, easier to implement, require no information transfer to a central controller and demand much less computational effort. A common type of structurally constrained controller is the one with block-diagonal structure which is usually called a *decentralised controller*. The decentralised controller is formed by local controllers, which are designed for local subsystems. Each controller uses only information of a single subsystem to generate the control input for the same subsystem. In other words, each subsystem is controlled separately. The problem of decentralised control design has been studied for decades due to its significant applications in many real-life systems [4, 6, 7].

However, an extensive number of natural or man-made large scale systems comprise of subsystems with *overlapping parts* (see for example, power systems [8], platoon of vehicles [9, 10],

mechanical systems [11], and large segmented telescope [12], to name a few). In these systems, the subsystems are strongly coupled in the overlapping (shared) parts but weakly interconnected otherwise. In other words, such systems are called to have *overlapping structure*. Recognizing the overlapping structure is advantageous as a feedback controller with overlapping structure, which is consistent with the structure of the system, can be implemented on it. A common approach to the design of a controller with an overlapping structure is based on the expansion-contraction process. In this process, a mathematical framework, called *the inclusion principle*, which provides the conditions for a system to include another lower dimensional system is used [13]. The expansion-contraction process to design an overlapping controller comprises three steps (i) the original interconnected overlapping system is expanded into a higher dimensional system (expanded system) with disjointed subsystems. The expansion is carried out in such a way that the properties of the original system are transferred to the expanded one [14], (ii) local controllers are designed for each of the disjointed subsystems of the expanded system by using standard controller design approaches [4, 6, 15], and (iii) the local controllers form a decentralised controller which is then contracted (transformed) to an overlapping controller for implementation on the original system. The main advantage of this approach is that the design of overlapping controller for the original high-dimensional overlapping system is converted into the design of local controllers for lower-dimensional subsystems, thus reducing the complexity of the design and associated computational cost.

Furthermore, in many real-life large scale systems, behaviour of system state depends on both present and past states. This property of the systems is called a delay. Systems with this feature are called time-delay systems, and they have been extensively studied since 1963 due to their importance and widespread observation. So, there have been many survey papers and books on time-delay systems [16–18]. There are many motivations to study time-delay systems which can be explained as follows to some extent.

- It is beneficial to model real-life systems with a high accuracy for engineering purposes such that the the system's behaviour can be obtained accurately through simulation. On the other hand, internal dynamics of many real-life processes experience after-effect event. There are chemical, biological, and engineering systems demonstrating this behaviour [19, 20]. So, it is essential to consider this behaviour in the modelling stage to achieve high performance in

feedback loops.

- There exist network delays which are originated from actuators, sensors and other essential components of a feedback control system. Since these components are commonly used in various areas such as communication and control domains [21] or robotics [22], taking communication delays into consideration is crucial.
- On the other hand, it has been shown that injection of delays in many cases can be useful for control purposes [23]. Many case studies, such as delayed resonators [24], time delay controllers [25, 26], and limit cycle control in nonlinear systems [27], have been studied in the open literature to investigate the advantages obtained through injection of delays.
- It is worth mentioning that delays are often source of poor performance and even instability when classical controllers are implemented on the dynamical system. This happens due to resilience of time-delays to existing control design approaches. It has to be noted that although using finite-dimensional approximations instead of delays seems to be the simplest approach to deal with time delay systems, it may lead to high degree of complexity and disastrous behaviour in terms of oscillation and even stability.

A very fundamental issue in time-delay systems is stability which has been widely studied due to its importance in dynamical systems [28, 29]. The study of stability was started by frequency domain approaches [30] which is then followed by time-domain based approaches [31]. Time domain approaches are more common due to appearance of Matlab toolboxes such as linear matrix inequalities (LMIs) [32] which can be used to construct appropriate Lyapunov-Krasovski function. Consequently, various stability criteria have been proposed in the literature [18]. They can be categorized into two groups (i) delay-independent stability criteria: The stability analysis is done disregarding nature and characteristics of delay. (ii) delay-dependent stability criteria: characteristics of delay such as bounds on its size or rate of change, are considered in the stability analysis. It is evident that since delay-independent approaches do not use any information on delay in stability analysis, they lead to conservative results, especially in the presence of small delays. On the other hand, the delay-dependent approaches provide more relaxed results as they consider characteristics of delay in the stability analysis.

Finally, it is crucial to design a controller such that obtained closed loop system is robust to model's

uncertainties. The drawback of not accounting for uncertainties is that the designed controller may not function according to the design specification when used in practice, as almost all industrial models contain such uncertainties due to modelling error, approximations, and equipment aging. Therefore, considerable attention has been given to design a robust controller such that performance of the feedback control system is preserved regardless of changes in the plant's parameters [33–35].

Based on the above discussions, in the thesis, we are dealing with a system with overlapping decomposition, which is experiencing disturbance (unknown input), network delays, and model uncertainties. For such systems, this thesis designs a robust delay-dependent overlapping output feedback controller based on the inclusion principle. The resultant closed loop system is robust to both constant and time-varying communication delays and unknown energy bounded disturbance. Finally, the load frequency controller (LFC) with overlapping structure is designed for multi-area interconnected power systems to confirm the applicability of the thesis's results.

1.1.1 Application

Load frequency control is essential for the successful operation of power systems, especially power systems comprise of interconnected areas. Without it the frequency of power networks may not be able to be controlled within the required limit band. The task of frequency control is primarily achieved by the primary control loops through the governor droop control mechanism. However, through governor control only, it is not possible to achieve zero steady state frequency deviation and zero tie-line power transfer after changes in the loading condition. It is for this reason that secondary LFC is required with aims of (i) achieving zero steady state errors in the frequency deviations and zero tie-line power transfer, and (ii) damping out, as soon as possible, the transient oscillations in the frequency and tie-line power deviations after changes in the load demand anywhere in the system.

Due to the major roles LFC plays, it has been a subject of much research over many decades. The reader is suggested to read survey papers on LFC [36, 37]. In some research works, centralised LFC design based on state feedback theory has been addressed [38–40]. The centralised LFC refers to a central (global) controller which receives measurements from all states of a power system to generate control inputs for all areas. In other words, there exists a central control station

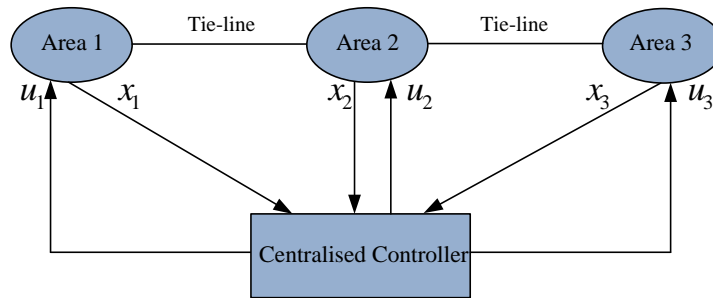


Figure 1.1: A centralised LFC scheme

where all calculations are done with access to all states of power system. The structure of a centralised LFC for a three-area longitudinal interconnected power system is shown in Fig. 1.1. Based on Fig. 1.1, measured state of all areas $x_i; i = 1;2;3$ are transferred to the centralised controller to generate the control inputs $u_i; i = 1;2;3$ for the areas.

However, there are three main practical drawbacks with the centralised LFC scheme of Fig. 1.1 as follows (i) a single fault in the central control station affects the control inputs of all areas, and consequently, performances of all areas might be degraded. (ii) communication channels have to be installed for information transfer from all areas to the central controller. Since power systems consist of interconnected areas, which are geographically distributed and separated by large distances, extra communication channels add more complexity, delays, and probability of faults occurring in the communication channels. (iii) measuring all states of a power system is costly and complicated even if it is practically possible.

To overcome the above mentioned problems, decentralised LFC has been proposed, as an alternative and has been studied widely [41–46]. The decentralised LFC scheme consists of local controllers such that each controller is responsible to generate the control input for each area using the area control error (ACE) measurement, which includes the frequency and tie-line power exchanges, from the same area. In other words, each area is controlled separately in this LFC scheme. Figure. 1.2 demonstrates the decentralised LFC scheme for a longitudinal three-area interconnected power system. As observed in Fig. 1.2, control input for each area is generated using local ACE signal, which includes frequency deviation and tie-lines.

On the other hand, decentralised LFC which is obtained by elimination of information from other areas, leads to loss of performance compared with centralised LFC [47]. In other words, there

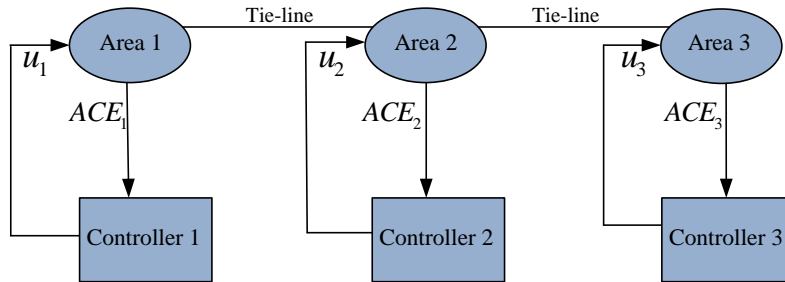


Figure 1.2: A decentralised LFC scheme

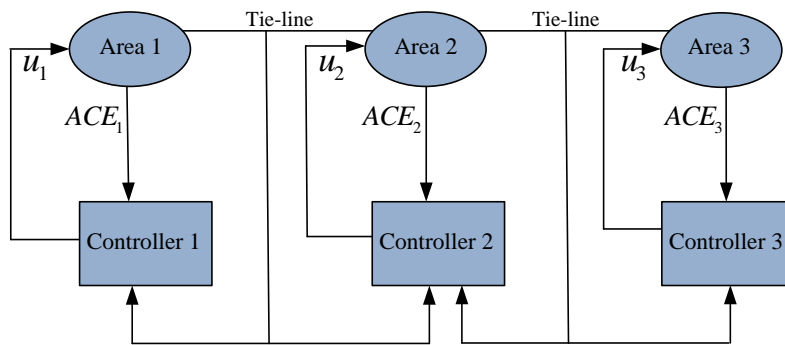


Figure 1.3: An overlapping LFC scheme

exists a trade-off between performance and simplicity of LFC scheme. In order to improve performance without a complex LFC scheme, overlapping LFC has been suggested [4, 8, 48]. In this context, the multi-area interconnected power system is decomposed into overlapping areas such that a tie-line between two neighbouring areas is the overlapping (shared) part between them. Once the overlapping decomposition is determined, the overlapping LFC has to be designed. Based on the overlapping decomposition where tie-lines are the overlapping parts, it is natural to add the locally available overlapping (shared) tie-lines to the local controllers, in addition to local ACE signals, to obtain enhanced performance. The scheme of overlapping decentralised control for a longitudinal three-area interconnected power system is demonstrated in Fig. 1.3. It can be clearly seen from Fig. 1.3 that overlapping LFC consists of local LFCs which are fed by local information in addition to overlapping parts (tie-lines).

On the other hand, there exist time-delays in transferring measurements through communica-

tion networks to governors and secondary controllers in real life [49], but have been neglected in above cited approaches. It is worth recalling that time-delay can degrade LFC performance, and may even cause instability in acute circumstances. This rather important theoretical and practical issue in the load frequency control problem has been addressed in some recent publications [26, 50–52]. Centralised state feedback LFC considering constant communication delays are designed in [26, 50]. However, It is usually not possible to directly measure the entire state vector. So, decentralised output feedback LFC, which is robust to constant communication delays, is suggested in [51] as alternative, which is easier to implement and more cost-effective than state feedback LFC. A decentralised output feedback LFC robust to time-varying communication delays is proposed in [52].

However, the above cited LFC design methods, though deal effectively with time-delays, they cannot guarantee performance and stability against model's uncertainties. So, this thesis aims at designing an overlapping LFC, which is robust to communication delays (constant and time-varying) and model's uncertainties, to achieve (i) zero steady state deviation in frequency and tie-line power deviations of each area (ii) better transient performance compared with existing LFCs. To this end, an overlapping output feedback Proportional Integral (PI)-type LFC design approach based on the inclusion principle is considered in this thesis. Two and three area interconnected power systems experiencing constant and time-varying communication delays are considered in Sections 3.5 and 4.5 respectively. In both case studied, extensive simulation results under different scenarios demonstrate the advantage of robust overlapping LFC compared with existing LFCs.

1.2 Literature Review

Most of real-life systems are called large-scale either due do (i) their high dimensions which lead to failure of traditional approaches for modelling or control in providing reasonable solutions, (ii) they can be decomposed into lower-dimensional interconnected subsystems [53]. Control of large-scale systems has become of a great interest over the past decades due to their applications in various research areas, such as power systems [54], urban traffic network [55], and large space structures [56]. To control such systems, centralised controller has been the first solution [57]. In this context, all calculations are done in one central centre with access to all information of system.

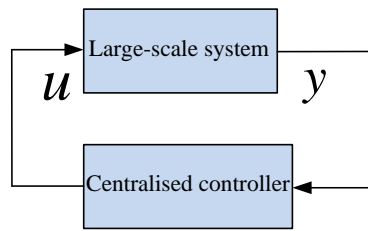


Figure 1.4: Centralised feedback control system

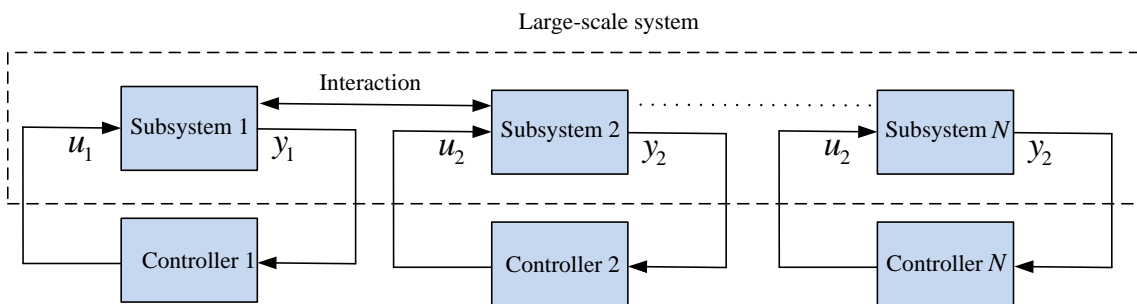


Figure 1.5: Decentralised feedback control system

The centralised control system is shown in Fig. 1.4, where u and y are control inputs and outputs of large-scale system respectively.

However, as discussed in Section 1.1.1, there are some drawbacks, such as reliability or complexity, with the centralised feedback structure. In order to overcome problems arising from the centralised feedback structure, decentralised controller has been proposed. In this context, first, large-scale systems are decomposed into lower-dimensional subsystems, which are interacting with each other through interaction signals. Examples of such systems are electrical power system which has several local stations, or highway system comprises of local traffic stations. Then, a local controller is designed to control each area using measurements from the same area. In other words, each area is controlled separately. A general structure of decentralised feedback control system is shown in Fig. 1.5. As shown in Fig. 1.5, local control input u_i for the i^{th} subsystem is generated using information y_i . It is evident to see that as the large-scale system is divided into lower-dimensional interconnected subsystems, computational cost decreases significantly compared with centralised control. Finally, it has to be noted that the reliability of the

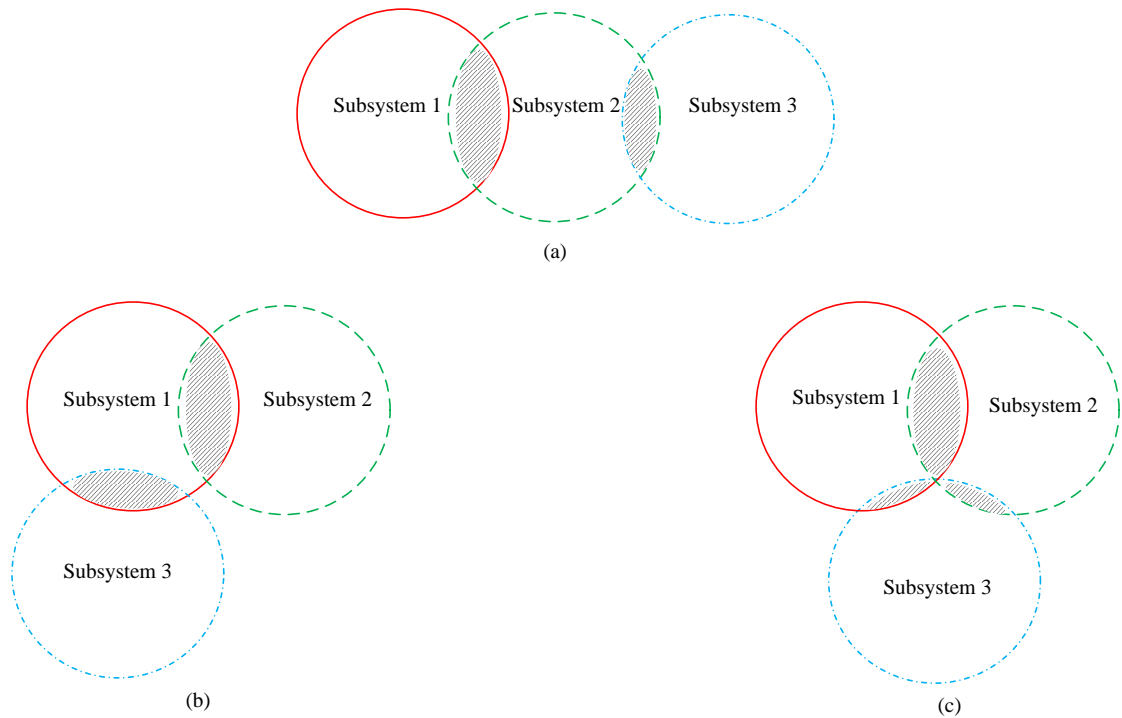


Figure 1.6: Decomposition of three-area interconnected system with (a) longitudinal (b) loop (c) radial

feedback control system increases considerably as the failure in one feedback loop does not affect the overall closed loop system.

However, the simplicity of decentralised controller is obtained at the cost of performance loss. In other words, there exists a trade-off between simplicity of feedback control system and the overall performance. In order to improve the performance with preservation of controller's simplicity, an overlapping decentralised controller has been proposed [13, 58]. In this scheme, the large-scale system is first decomposed into overlapping subsystems i.e. the subsystems with overlapping (shared) parts. There are three main overlapping structures [59]: (i) longitudinal (chain) overlapping structure (ii) loop (circle) overlapping structure (iii) radial (star) overlapping structure. These overlapping structures for a system comprises of three-subsystems are shown in Fig. 1.6. In Fig. 1.6, three subsystems, where each one is determined by the coloured circle, have shared (overlapping) parts. The shared parts have been coloured.

It is worth mentioning that comparing Fig. 1.6 with Fig. 1.5 illustrates the difference between

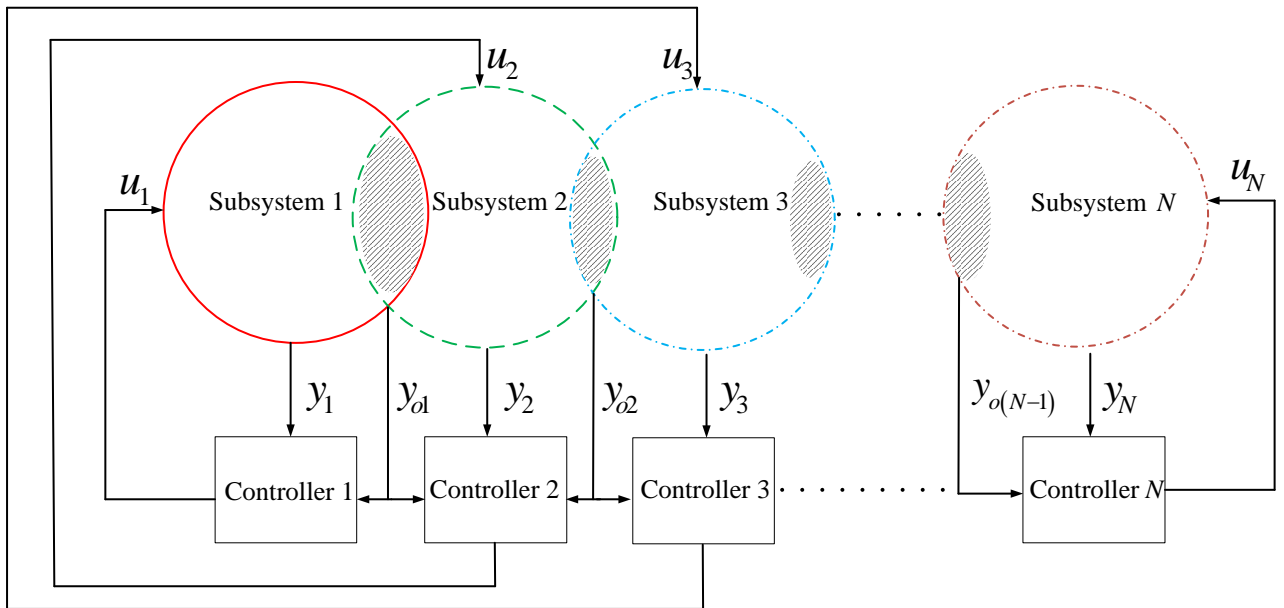


Figure 1.7: Overlapping feedback control system

decentralised and overlapping decompositions. In the decentralised decomposition of Fig. 1.5, subsystems are interconnected to each other. However, in overlapping decomposition of Fig. 1.6, subsystems have overlapping (shared) parts.

Once the overlapping decomposition is determined, the overlapping output feedback controller has to be designed, and implemented on the overlapping system shown in Fig. 1.6. The overlapping controller comprises of local controllers, however, due to overlapping structure, it is advantageous to add overlapping parts to local controllers to enhance the overall performance. Without loss of generality but for simplification, the structure of overlapping controller for a system comprising N subsystems in a longitudinal way is demonstrated in Fig. 1.7. In Fig. 1.7, $y_i; i = 1, 2, 3, \dots, N$ are the control outputs which can be measured from the i^{th} subsystem. The variable y_{oi} denotes overlapping measured outputs between subsystems i and $i + 1$ i.e. these outputs are available in both areas i and $i + 1$ to be measured. It is evident from Fig. 1.7 that overlapping controller comprises of local controllers where each controller is fed by local measurements as well as all overlapping parts.

The main idea to design an overlapping feedback control system is proposed by [13] based on the inclusion principle, which is thoroughly explained in Section 1.3.1. In this context, overlapped

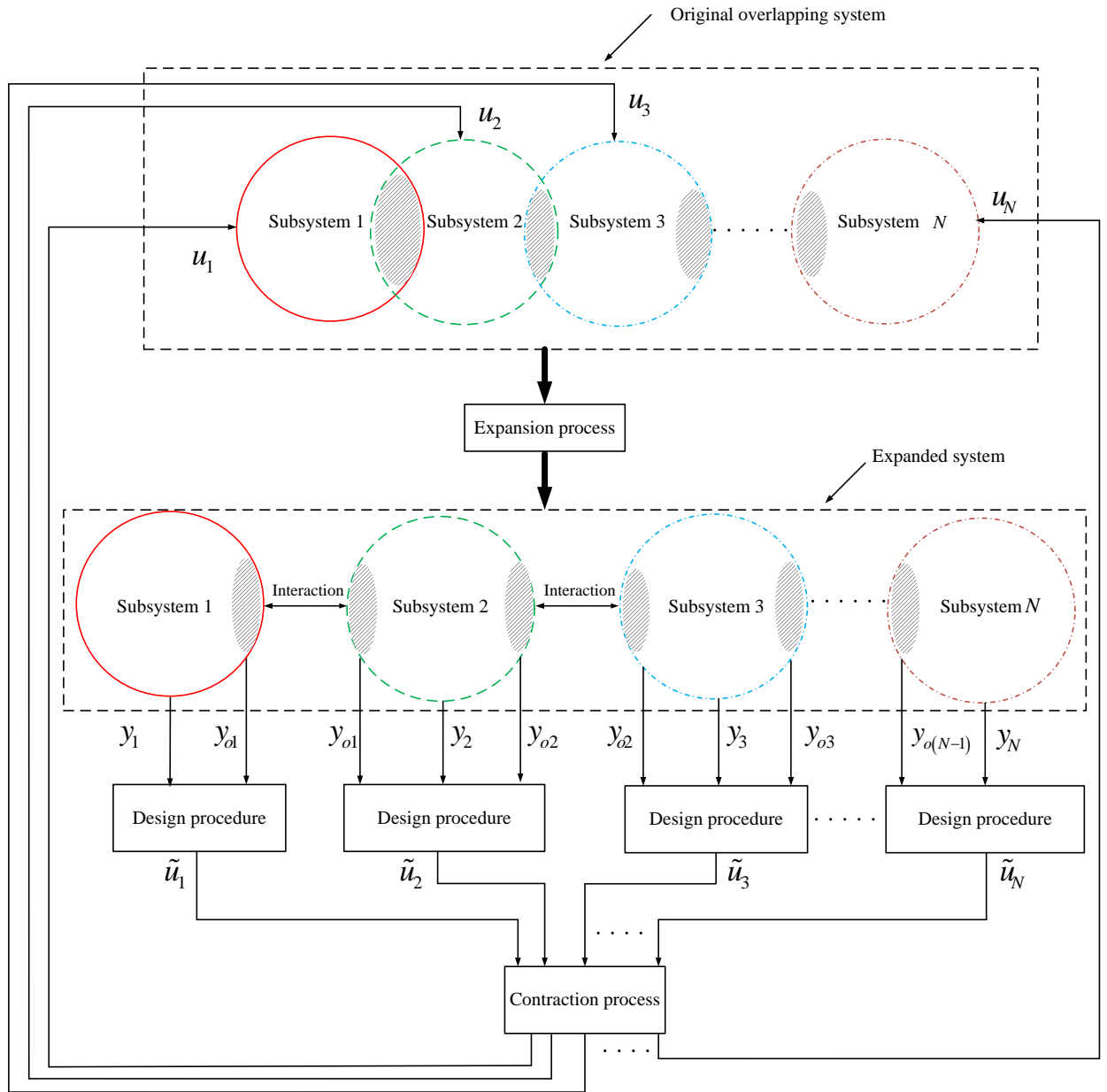


Figure 1.8: The expansion-contraction process to design overlapping control system

spaces are first expanded to isolate the overlapped subsystems. Then, local control laws are designed for disjoint subsystems. Finally, designed control laws are contracted (transformed) for implementation on the original system. These steps have been illustrated in Figure. 1.8. As shown in Fig. 1.8, first, the expanded system, which is determined by black dashed box, is generated

by applying the expansion process to the original overlapping system determined by black dashed box. In other words, the overlapping system is expanded such that overlapped parts between subsystems are duplicated, which leads to interconnected subsystems of the expanded system. Then, local stabilising control laws $\tilde{u}_i; i = 1, 2, 3, \dots, N$ are obtained for interconnected subsystems of the expanded system through local design procedures. The aggregation of the designed control laws leads to stabilising decentralised control law for the expanded system. Finally, through the contraction process, the control laws $u_i; i = 1, 2, 3, \dots, N$ are obtained and applied to the original overlapping system. Under certain conditions, stability and performance of the overlapping system with obtained control laws $u_i; i = 1, 2, 3, \dots, N$ can be evaluated. The advantage of this idea is that design of an overlapping controller for a large-scale system is formulated as design of local controllers for interconnected subsystems of an expanded system and thus reducing complexity and computational effort.

The inclusion principle, which provides mathematical framework for the expansion-contraction process for linear time-invariant systems, is initially suggested in [13, 60]. It provides conditions such that one large dynamical system (expanded system) includes another smaller dynamical system (original system). Thus, all the information about the smaller one can be extracted from the larger one. In the preliminary version of the inclusion principle presented in [13, 60], the state space is only expanded i.e. it is assumed that subsystems have overlapping states while the subsystems have their own inputs and outputs. The extension of the inclusion principle to overlapping states, inputs, and outputs is presented in [58]. In other words, subsystems have overlapping inputs, states, and outputs in results of [58]. From application viewpoint, an overlapping state feedback controller for a string of four moving vehicles has been designed based on the inclusion principle [58]. Also, the expansion-contraction process, shown in Fig. 1.7, requires a selection of sets of matrices, called complementary matrices. Set of necessary and sufficient conditions for complementary matrices to satisfy is given in Theorem 2.17 of [13]. However, satisfying these conditions is not practical especially for large-scale systems. Thus, two special cases of the inclusion principle, called aggregation and restriction, have been introduced in [13]. Using these two concepts, sufficient conditions which complementary matrices should satisfy, are provided. These conditions are more practical to satisfy and requiring less computation effort. However, the results by [61] show that simultaneous transmission of controllability and observability from

the original system to the expanded one is not possible through restriction and aggregation frameworks. In other words, although the original overlapping system is minimal, the expanded system has uncontrollable (with the restriction concept) and unobservable modes (with the aggregation concept), which may lead to major issues in decentralised control design and achieving acceptable performance. The transmission of controllability/observability properties from the original minimal system to the expanded one is the essential requirement as the decentralised control design is done for the expanded system and then contracted for implementation on the original system. To ensure simultaneous transmission of controllability/observability from the original system to the expanded one, general structures of complementary matrices have been introduced in [62, 63]. The proposed structures of complementary matrices provide more freedom in selection of complementary matrices than standard restriction and aggregation, and thus, minimal expanded system can be obtained from the minimal original system [14, 64].

Of particular interest has been the problem of contractibility of stabilising decentralised control laws designed for the linear expanded system to stabilising overlapping decentralised control laws for the original systems. It has been demonstrated in [58] that any decentralised control law designed for the expanded system cannot be contracted (transformed) to stabilising overlapping control for implementation on the original system, and the designed decentralised control law has to be modified to be contractible. However, the performance and even stability is not guaranteed once the designed decentralised control law is modified, and these properties have to be inspected again. To overcome this problem, the extension principle has been proposed in [65–67]. It has been shown that if the extension principle is used to generate the expanded system, then any designed decentralised static controller for the expanded system is contractible to overlapping controller for implementation on the original system. However, the expanded system obtained by the extension principle is not minimal, and consequently the expanded system may not be stabilisable. The problem of contractibility has been extensively studied in [68] where it has been discussed that observer (estimator) and feedback gains are contractible in the case of aggregation and restriction respectively. Furthermore, many real-life systems such as chemical systems [69], electrical circuits [70], and mechanical systems [71] have been represented by descriptor models (generalized state space models). So, due to wide appearance of descriptor systems, [72] has presented the inclusion principle framework for descriptor systems. The main objective is to determine conditions under

which the solutions of a descriptor system can be reproduced with solutions of a larger descriptor system. It has been shown that the inclusion principle framework for descriptor systems is more sophisticated than those of linear time invariant systems. Finally, it is widely accepted that many physical large-scale systems can be modelled as interaction of continuous and discrete systems [73]. Such systems are called hybrid systems. The extension of the inclusion principle has been given for hybrid systems [74], and its application has been illustrated in control of vehicle flight formation control with hybrid model [75].

All the above cited approaches deal with linear time invariant systems. However, since large scale systems with overlapping decomposition are likely to experience network delays and parametric uncertainties, the inclusion principle should be extended to deal with control of overlapping uncertain time-delay systems due to the importance of communication delays and uncertainties as mentioned in Sections 1.1. The inclusion principle has been extended in [76] to design an overlapping state feedback controller for uncertain continuous-time state delay systems. Measuring all states of large-scale systems, however, is costly and requires high computational cost even if it is possible. Thus, an overlapping robust, output feedback controllers for uncertain discrete time systems with constant communication delays is presented in [77]. However, there are two major drawbacks with [76,77]. First, delay-independent design procedures of [76,77] are conservative as no information on delay nature and its characteristics are involved in design procedures. Second, the decentralised control design is done based on the bigger-dimensional of the expanded system, thus, no benefits are taken from the decentralised structure of the expanded system.

The inclusion principle in all above mentioned research works have been applied to linear time-invariant systems. The inclusion principle, however, for linear time-varying systems was first introduced in [78, 79] where time-invariant transformations are used in the inclusion principle concept. The conditions have been determined such that a time-varying linear system includes smaller time-varying linear system. These results are then extended by [80] where time-varying transformations are used in the inclusion principle definition. Finally, the extension of the inclusion principle to non-linear systems without inputs and outputs is presented in [81, 82]. The sufficient conditions have been derived such that a non-linear system includes another smaller non-linear system. Stability of Lotka-Volterra equations as a class of non-linear systems which are used to model ecosystems [83], are investigated in [81, 82] The inclusion principle for heredi-

tary dynamical systems, as another class of non-linear systems, has been introduced in [84].

1.3 Preliminaries

In this section, all preliminaries required to derive main results of this thesis are given.

1.3.1 The Inclusion Principle

The inclusion principle has been used extensively in the domain of large scale systems since the early eighties. This principle provides the mathematical scheme such that a larger system contains all the essential information about the smaller system. In other words, solutions of the larger system include solutions of the smaller one. In this section, the summary of the inclusion principle is given.

Consider pair of linear time invariant systems as:

$$\begin{aligned}
 \Sigma : \quad & \dot{x}(t) = Ax(t) + Bu(t), \\
 & y(t) = Cx(t), \\
 \tilde{\Sigma} : \quad & \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\tilde{u}(t), \\
 & \tilde{y}(t) = \tilde{C}\tilde{x}(t)
 \end{aligned} \tag{1.1}$$

with $u(t)$, $x(t)$, and $y(t)$ are m , n , and l dimensional vectors of input, state, and output respectively. Also, $\tilde{u}(t) \in R^{\tilde{m}}$, $\tilde{x}(t) \in R^{\tilde{n}}$, and $\tilde{y}(t) \in R^{\tilde{l}}$ are input, state, and output of system $\tilde{\Sigma}$ respectively where $\tilde{m} > m$, $\tilde{n} > n$, $\tilde{l} > l$. Let V , R , and T be full column rank matrices, and U , Q , and S are full row rank matrices such that $UV = I_m$, $QR = I_n$, $ST = I_l$. Then, the following definition describes the inclusion principle [58].

Definition 1.1. *The System $\tilde{\Sigma}$ includes system Σ if there exists quadruple (U, V, R, S) such that for any initial state x_0 and any input $u(t)$, choices $\tilde{x}_0 = Vx_0$ and $\tilde{u}(t) = Ru(t)$ result in $x(t; x_0, u) = U\tilde{x}(t; \tilde{x}_0, \tilde{u})$ and $y(t) = S\tilde{y}(t)$ for all $t \geq 0$.*

The inclusion principle implies that all the essential information about the smaller system Σ , e.g. stability or performance, can be extracted from the behaviour of larger system $\tilde{\Sigma}$. This is the

prime feature of the inclusion principle.

Remark: The choice of basis does not have any effect on the inclusion principle. To show this, let $\tilde{\Sigma}$ includes Σ based on definition 1.1. Let change of basis in both systems Σ and $\tilde{\Sigma}$ be $\bar{x} = Ex$ and $\tilde{x} = \tilde{E}^{-1}\bar{x}$ respectively. Then, it can be easily seen that $\bar{x}(t; \bar{x}_0, u) = \bar{U}\tilde{x}(t; \bar{V}\bar{x}_0, \tilde{u})$ and $y(t) = S\tilde{y}(t)$, where $\bar{V} = \tilde{E}^{-1}VE^{-1}$ and $\bar{U} = EU\tilde{E}$.

The next step is to obtain straightforward relations between matrices of Σ and $\tilde{\Sigma}$. First, the following expressions have been introduced [58].

$$\tilde{A} = VAU + M, \quad \tilde{B} = VBQ + N, \quad \tilde{C} = TCU + L \quad (1.2)$$

where M, N , and L are complementary matrices with appropriate dimensions. A proper selection of the complementary matrices M, N, L is required for $\tilde{\Sigma}$ to include Σ . This is addressed in the following theorem based on definition 1.1 [58].

Theorem 1.1. *The system $\tilde{\Sigma}$ is an expansion of Σ in (1.1) if and only if*

$$UM^iV = 0, \quad UM^{i-1}NR = 0 \quad SLM^{i-1}V = 0, \quad SLM^{i-1}NR = 0 \quad (1.3)$$

for $i = 1, 2, \dots, \tilde{n}$.

However, choosing the complementary matrices based on the Theorem 1.1 is not trivial for large \tilde{n} . To solve this problem, special cases of the inclusion principle called restriction and aggregation have been presented in [58].

Definition 1.2. *The dynamical system $\tilde{\Sigma}$ is restriction of Σ if there exist transformations V, R, T such that for arbitrary x_0 and $u(t)$, choices $\tilde{x}_0 = Vx_0$ and $\tilde{u}(t) = Ru(t)$ lead to $\tilde{x}(t) = Vx(t)$ and $\tilde{y}(t) = Ty(t)$ for all $t \geq 0$.*

Definition 1.3. *The dynamical system $\tilde{\Sigma}$ is aggregation of Σ if there exist transformations U, Q, S such that for arbitrary x_0 and $u(t)$, choices $x_0 = U\tilde{x}_0$ and $u(t) = Q\tilde{u}(t)$ lead to $x(t) = U\tilde{x}(t)$ and $y(t) = S\tilde{y}(t)$ for all $t \geq 0$.*

Based on definition 1.1, it can be clearly seen that since $UV = I$, $ST = I$, and $QT = I$, we can say that the system $\tilde{\Sigma}$ includes Σ if definition 1.2 or 1.3 holds.

Next, based on definitions 1.2-1.3, certain conditions which complementary conditions should satisfy are given as follows [58].

Theorem 1.2. *The system $\tilde{\Sigma}$ is restriction of Σ if and only if:*

$$MV = 0, \quad NR = 0, \quad LV = 0 \quad (1.4)$$

It is evident that conditions (1.3) hold if conditions of (1.4) hold. This is consistent with the fact that if the system $\tilde{\Sigma}$ is restriction of Σ , then $\tilde{\Sigma}$ includes Σ , however, the vice-versa is not true.

Theorem 1.3. *The system $\tilde{\Sigma}$ is aggregation of Σ if and only if:*

$$UM = 0, \quad UN = 0, \quad SL = 0 \quad (1.5)$$

Similarly, we can say that the system $\tilde{\Sigma}$ includes Σ if set of conditions (1.5) holds. It is worth mentioning that advantages of the restriction and aggregation definitions are introducing more straightforward conditions, (1.4)-(1.5), for selection of complementary matrices.

1.3.2 Overlapping Decentralised Control

In this section, application of the inclusion principle in overlapping decentralised control design is introduced.

1.3.3 Overlapping Decomposition

In order to clarify the overlapping decomposition, consider the following overlapping structure for the system Σ given in (1.1):

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \\ \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \end{aligned} \quad (1.6)$$

where $A_{ij} \in R^{n_i \times n_j}$, $B_{ij} \in R^{n_i \times m_j}$, and $C_{ij} \in R^{l_i \times n_j}$ with $i, j = 1, 2, 3$. The dashed lines denote that this system is composed of two overlapping subsystems, Σ_1 and Σ_2 which are shown in (1.7). Based on the overlapping decomposition, u_2 , x_2 , and y_2 correspond to overlapping parts of the input, state, and output spaces, respectively. This overlapping structure can be generalized for any number of interconnected overlapping subsystems, but the structure with two overlapped subsystems has been extensively used as a prototype in the literature [14, 62, 76, 85].

$$\begin{aligned}
\Sigma_1 : \quad & \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} A_{13} \\ A_{23} \end{bmatrix} x_3(t) + \\
& \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + \begin{bmatrix} B_{13} \\ B_{23} \end{bmatrix} u_3(t), \\
& \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} C_{13} \\ C_{23} \end{bmatrix} x_3(t) \\
\\
\Sigma_2 : \quad & \begin{bmatrix} \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} A_{21} \\ A_{31} \end{bmatrix} x_1(t) + \\
& \begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix} \begin{bmatrix} u_2(t) \\ u_3(t) \end{bmatrix} + \begin{bmatrix} B_{21} \\ B_{31} \end{bmatrix} u_1(t), \\
& \begin{bmatrix} y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} C_{22} & C_{23} \\ C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} C_{21} \\ C_{31} \end{bmatrix} x_1(t) \tag{1.7}
\end{aligned}$$

The overlapping decentralised static output feedback controller, with the structure consistent with the structure of the original system Σ , consists of two static controllers K_1 and K_2 given in (1.8).

$$\begin{aligned}
K_1 : \quad & \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} \\ k_{21}^{(1)} & k_{22}^{(1)} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \\
K_2 : \quad & \begin{bmatrix} u_2(t) \\ u_3(t) \end{bmatrix} = \begin{bmatrix} k_{11}^{(2)} & k_{12}^{(2)} \\ k_{21}^{(2)} & k_{22}^{(2)} \end{bmatrix} \begin{bmatrix} y_2(t) \\ y_3(t) \end{bmatrix} \tag{1.8}
\end{aligned}$$

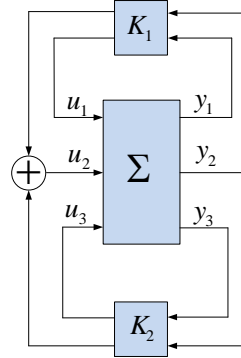


Figure 1.9: Closed loop system with overlapping output feedback controller

The implementation of overlapping decentralised output feedback controller is shown in Fig.1.9. Using (1.8), the overlapping output feedback controller K can be shown as follows:

$$\begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} k_{11}^{(1)} & k_{12}^{(1)} & 0 \\ k_{21}^{(1)} & k_{22}^{(1)} + k_{11}^{(2)} & k_{12}^{(2)} \\ 0 & k_{21}^{(2)} & k_{22}^{(2)} \end{bmatrix}}_K \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} \quad (1.9)$$

In order to use the inclusion principle to design the overlapping controller K , first, the expanded system which includes the original system Σ based on definition 1.1 has to be generated. To this end, the singular transformations V, R, T have to be chosen appropriately. Based on the overlapping decomposition of (1.6), the full column rank transformations are chosen as:

$$V = \begin{bmatrix} I_{n_1} & 0 & 0 \\ 0 & I_{n_2} & 0 \\ 0 & I_{n_2} & 0 \\ 0 & 0 & I_{n_3} \end{bmatrix}, \quad R = \begin{bmatrix} I_{m_1} & 0 & 0 \\ 0 & I_{m_2} & 0 \\ 0 & I_{m_2} & 0 \\ 0 & 0 & I_{m_3} \end{bmatrix}, \quad T = \begin{bmatrix} I_{l_1} & 0 & 0 \\ 0 & I_{l_2} & 0 \\ 0 & I_{l_2} & 0 \\ 0 & 0 & I_{l_3} \end{bmatrix} \quad (1.10)$$

It has to be noted that the singular transformations are chosen such that the overlapping dynamics are duplicated in the expanded space through $\tilde{u}(t) = Ru(t)$, $\tilde{x}(t) = Vx(t)$, and $\tilde{y}(t) = Ty(t)$.

According to the singular transformations V, R, T , the full row rank transformations U, Q, S are:

$$\begin{aligned}
 U &= \begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{n_2} & \frac{1}{2}I_{n_2} & 0 \\ 0 & 0 & 0 & I_{n_3} \end{bmatrix}, \quad Q = \begin{bmatrix} I_{m_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{m_2} & \frac{1}{2}I_{m_2} & 0 \\ 0 & 0 & 0 & I_{m_3} \end{bmatrix}, \\
 S &= \begin{bmatrix} I_{l_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}I_{l_2} & \frac{1}{2}I_{l_2} & 0 \\ 0 & 0 & 0 & I_{l_3} \end{bmatrix} \tag{1.11}
 \end{aligned}$$

Now, the complementary matrices can be chosen using (1.4) or (1.5). In the sequel and without loss of generality, structures of the complementary matrices using the set of conditions (1.4) are shown.

$$\begin{aligned}
 M &= \begin{bmatrix} 0 & M_{12} & -M_{12} & 0 \\ 0 & M_{22} & -M_{22} & 0 \\ 0 & -M_{33} & M_{33} & 0 \\ 0 & M_{42} & -M_{42} & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & N_{12} & -N_{12} & 0 \\ 0 & N_{22} & -N_{22} & 0 \\ 0 & -N_{33} & N_{33} & 0 \\ 0 & N_{42} & -N_{42} & 0 \end{bmatrix}, \\
 L &= \begin{bmatrix} 0 & L_{12} & -L_{12} & 0 \\ 0 & L_{22} & -L_{22} & 0 \\ 0 & -L_{33} & L_{33} & 0 \\ 0 & L_{42} & -L_{42} & 0 \end{bmatrix}, \tag{1.12}
 \end{aligned}$$

where M_{ij}, N_{ij}, L_{ij} are arbitrary entries.

Since the idea of the inclusion principle is expansion of the original system Σ to comprise two weakly interconnected subsystems, it is desired to provide maximum zero off-diagonal blocks in the expanded system's matrices [58]. To this end, the complementary matrices M, N , and L in

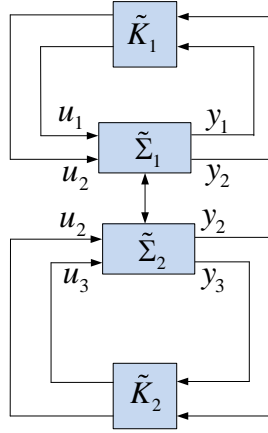


Figure 1.10: Closed loop system with decentralised output feedback controller

$$\begin{aligned} \tilde{\Sigma}_2 : \quad \dot{\tilde{x}}_2(t) &= \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix} \tilde{x}_2(t) + \begin{bmatrix} A_{21} \\ A_{31} \end{bmatrix} \tilde{x}_1(t) + \begin{bmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{bmatrix} \tilde{u}_2(t) + \begin{bmatrix} B_{21} \\ B_{31} \end{bmatrix} \tilde{u}_1(t), \\ \tilde{y}_2(t) &= \begin{bmatrix} C_{22} & C_{23} \\ C_{32} & C_{33} \end{bmatrix} \tilde{y}_2(t) + \begin{bmatrix} C_{21} \\ C_{31} \end{bmatrix} \tilde{y}_1(t) \end{aligned}$$

where $\tilde{x}_1^T = (x_1^T, x_2^T)$, $\tilde{x}_2^T = (x_2^T, x_3^T)$, $\tilde{y}_1^T = (y_1^T, y_2^T)$, $\tilde{y}_2^T = (y_2^T, y_3^T)$, $\tilde{u}_1^T = (u_1^T, u_2^T)$, $\tilde{u}_2^T = (u_2^T, u_3^T)$. The description of states show that the overlapping parts $u_2(t)$, $x_2(t)$, $y_2(t)$ have been duplicated to disjoint the overlapping subsystems.

Afterwards, local static output feedback controllers \tilde{K}_1 and \tilde{K}_2 are designed for the interconnected subsystems of the expanded system, as given in (1.15):

$$\begin{aligned} \tilde{K}_1 : \quad \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} &= \begin{bmatrix} \tilde{k}_{11}^{(1)} & \tilde{k}_{12}^{(1)} \\ \tilde{k}_{21}^{(1)} & \tilde{k}_{22}^{(1)} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \\ \tilde{K}_2 : \quad \begin{bmatrix} u_2(t) \\ u_3(t) \end{bmatrix} &= \begin{bmatrix} \tilde{k}_{11}^{(2)} & \tilde{k}_{12}^{(2)} \\ \tilde{k}_{21}^{(2)} & \tilde{k}_{22}^{(2)} \end{bmatrix} \begin{bmatrix} y_2(t) \\ y_3(t) \end{bmatrix} \end{aligned} \quad (1.15)$$

The implementation of local feedback gains on the expanded system is demonstrated in Fig. 1.10. The designed local controllers are then used to form a decentralised controller $\tilde{K}_D = \text{Blkdiag}\{\tilde{K}_1, \tilde{K}_2\}$, where *Blkdiag* denotes Block-diagonal matrix. The decentralised gain \tilde{K}_D is then contracted to an overlapping controller for implementation on the original system as shown

in Fig. 1.9. The definition of contractibility is given next [58].

Definition 1.4. *Let the system $\tilde{\Sigma}$ includes the system Σ by definition 1.1. Then, the designed decentralised control law $\tilde{u}(t) = \tilde{K}_D \tilde{y}(t)$ for the expanded system $\tilde{\Sigma}$ is contractible to an overlapping control law $u(t) = Ky(t)$ to be implemented on the original system Σ if for any initial state x_0 , the choice $\tilde{x}_0 = Vx_0$ leads to $RKy(t) = \tilde{K}_D \tilde{y}(t)$.*

Definition 1.4 implies that if the decentralised controller \tilde{K}_D is contractible to the overlapping controller K , then the closed loop:

$$\dot{\tilde{x}}(t) = (\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\tilde{x}(t) \quad (1.16)$$

includes (or is expansion of) the closed loop:

$$\dot{x}(t) = (A + BKC)x(t) \quad (1.17)$$

Thus, all the essential information such as stability or performance corresponding to the original closed loop system (1.17) can be extracted from the expanded closed loop system (1.16).

In the sequel, the necessary and sufficient condition for contractibility is given [58].

Theorem 1.4. *Let the system $\tilde{\Sigma}$ includes the system Σ based on definition 1.1. Then, the controller \tilde{K}_D is contractible to the overlapping controller K if and only if*

$$\begin{aligned} \tilde{K}_D \tilde{C} \tilde{A}^i V &= RKCA^i, \\ \tilde{K}_D \tilde{C} \tilde{A}^i \tilde{B} R &= RKCA^i B \end{aligned} \quad (1.18)$$

for $i = 0, 1, 2, \dots, \tilde{n} - 1$.

However, the contractibility conditions (1.18) are reduced to one simple condition when the expansion is done through the restriction concept given in definition 1.2. [58]

Theorem 1.5. *Let the system $\tilde{\Sigma}$ includes the system Σ by definition 1.2. Then, the controller \tilde{K}_D is contractible to the overlapping controller K if and only if $\tilde{K}_D T = RK$.*

If the contractibility condition of theorem 1.5 holds, then the overlapping controller K can be obtained by $K = Q\tilde{K}_D T$. However, based on structures of K and \tilde{K}_D and transformations R and T

(1.10), it can be clearly seen that in order to satisfy the contractibility condition, the decentralised controller \tilde{K}_D has to be modified as follows [58]:

$$\tilde{K}_D = \begin{bmatrix} \tilde{k}_{11}^{(1)} & \tilde{k}_{12}^{(1)} & 0 & 0 \\ \tilde{k}_{21}^{(1)} & \frac{1}{2}(\tilde{k}_{22}^{(1)} + \tilde{k}_{11}^{(2)}) & 0 & \tilde{k}_{12}^{(2)} \\ \tilde{k}_{21}^{(1)} & 0 & \frac{1}{2}(\tilde{k}_{22}^{(1)} + \tilde{k}_{11}^{(2)}) & \tilde{k}_{12}^{(2)} \\ 0 & 0 & \tilde{k}_{21}^{(2)} & \tilde{k}_{22}^{(2)} \end{bmatrix} \quad (1.19)$$

Once this modification is done, the overlapping controller K can be calculated by $K = Q\tilde{K}_DT$:

$$K = \begin{bmatrix} \tilde{k}_{11}^{(1)} & \tilde{k}_{12}^{(1)} & 0 \\ \tilde{k}_{21}^{(1)} & \frac{1}{2}(\tilde{k}_{22}^{(1)} + \tilde{k}_{11}^{(2)}) & \tilde{k}_{12}^{(2)} \\ 0 & \tilde{k}_{21}^{(2)} & \tilde{k}_{22}^{(2)} \end{bmatrix} \quad (1.20)$$

On the other hand, if the system $\tilde{\Sigma}$ includes the system Σ by aggregation definition 1.3, then the contractibility definition has become:

Definition 1.5. *Let the system $\tilde{\Sigma}$ includes the system Σ by definition 1.3. Then, the designed decentralised control law $\tilde{u}(t) = \tilde{K}_D\tilde{y}(t)$ for the expanded system $\tilde{\Sigma}$ is contractible to an overlapping control law $u(t) = Ky(t)$ to be implemented on the original system Σ if for any initial state x_0 , the choice $\tilde{x}_0 = Vx_0$ leads to $Ky(t) = Q\tilde{K}_D\tilde{y}(t)$.*

The following theorem determines the contractibility condition based on definition 1.5:

Theorem 1.6. *Let the system $\tilde{\Sigma}$ be expansion of Σ by definition 1.5. Then, the decentralised control law $\tilde{u}(t) = \tilde{K}_D\tilde{y}(t)$ is contractible to overlapping control law if and only if $Q\tilde{K}_D = KS$.*

Similar to theorem 1.5, it can be shown that the decentralised control \tilde{K}_D has to be modified in order to satisfy the contractibility condition.

Once the overlapping controller K is obtained through the contraction process, it has to be implemented on the original system Σ . Steps of designing the overlapping controller based on the inclusion principle are demonstrated in Fig. 1.11.

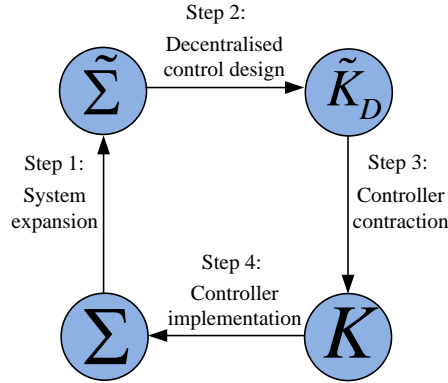


Figure 1.11: Scheme of overlapping controller design

1.3.4 Controllability/Observability in The Inclusion Principle

One fundamental question in the inclusion principle is whether the properties of the original system such as controllability/observability are transmitted to the expanded system through the expansion process. To ensure simultaneous transmission of controllability/observability from the original system to the expanded one, first, general structures of complementary matrices introduced in [62] are presented through the following theorem.

Theorem 1.7. *The system $\tilde{\Sigma}$ includes Σ based on definition 1.1 if and only if:*

$$\begin{bmatrix} M_{12} \\ M_{23} + M_{33} \\ M_{42} \end{bmatrix} [M_{22} + M_{33}]^{i-2} \times \begin{bmatrix} M_{21} & M_{22} + M_{23} & M_{24} \end{bmatrix} = 0,$$

$$\begin{bmatrix} M_{12} \\ M_{23} + M_{33} \\ M_{42} \end{bmatrix} [M_{22} + M_{33}]^{i-2} \times \begin{bmatrix} N_{21} & N_{22} + N_{23} & N_{24} \end{bmatrix} = 0,$$

$$\begin{bmatrix} L_{12} \\ L_{23} + L_{33} \\ L_{42} \end{bmatrix} [M_{22} + M_{33}]^{i-2} \times \begin{bmatrix} M_{21} & M_{22} + M_{23} & M_{24} \end{bmatrix} = 0,$$

$$\begin{bmatrix} L_{12} \\ L_{23} + L_{33} \\ L_{42} \end{bmatrix} [M_{22} + M_{33}]^{j-2} \times \begin{bmatrix} N_{21} & N_{22} + N_{23} & N_{24} \end{bmatrix} = 0 \quad (1.21)$$

for all $i = 2, \dots, \tilde{n}$ and $j = 2, \dots, \tilde{n} + 1$, where:

$$M = \begin{bmatrix} 0 & M_{12} & -M_{12} & 0 \\ M_{21} & M_{22} & M_{23} & M_{24} \\ -M_{21} & -(M_{22} + M_{23} + M_{33}) & M_{33} & -M_{24} \\ 0 & M_{42} & -M_{42} & 0 \end{bmatrix} \quad (1.22)$$

and N and L have the same structure as the matrix M when substituting M_{ij} by N_{ij} and L_{ij} respectively.

It has to be mentioned that conditions (1.21) are more straightforward than those of theorem 1.1. The conditions of (1.21), however, can still be simplified by taking into consideration that they can be satisfied in two ways. Either the right brackets in (1.21) are zero, i.e.

$$\begin{aligned} \begin{bmatrix} M_{21} & M_{22} + M_{23} & M_{24} \end{bmatrix} &= 0, \\ \begin{bmatrix} N_{21} & N_{22} + N_{23} & N_{24} \end{bmatrix} &= 0 \end{aligned} \quad (1.23)$$

Or the left brackets in (1.21) are zero, i.e.

$$\begin{bmatrix} M_{12} \\ M_{23} + M_{33} \\ M_{42} \end{bmatrix} = 0, \quad \begin{bmatrix} L_{12} \\ L_{23} + L_{33} \\ L_{42} \end{bmatrix} = 0 \quad (1.24)$$

Using the new conditions on complementary matrices, the following theorem has been given in [14] regarding minimality of the expanded system.

Theorem 1.8. *Let the expanded system $\tilde{\Sigma}$ be constructed from the minimal system Σ such that the complementary matrices M, N, L satisfy either conditions (1.23) or (1.24). Then, the expanded system $\tilde{\Sigma}$ is both controllable and observable.*

1.3.5 Overlapping Time-Delay Systems

As mentioned in Section 1.2, the inclusion principle has been extended for continuous [76] and discrete time [77] large-scale systems, experiencing communication delays and model uncertainties. In the sequel, some of the relevant results of [76] are given.

The following uncertain state-delayed system \mathbf{S} is borrowed from [76]:

$$\begin{aligned} \mathbf{S}: \quad \dot{x}(t) &= (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t-d) + (B + \Delta B)u(t) + B_1w(t), \\ z(t) &= Cx(t) + Du(t), \\ x(t_0) &= \varphi(t_0), \quad -d \leq t_0 \leq 0 \end{aligned} \quad (1.25)$$

where $\varphi(t_0)$ is the initial condition. $w(t) \in \mathbb{R}^q$ is an unknown input (disturbance), and $z(t) \in \mathbb{R}^p$ is the controlled output. Definition of the other variables are the same as those of (1.1).

The norm bounded uncertainties $\Delta A, \Delta A_d, \Delta B$ are assumed to be expressed as follows:

$$\begin{bmatrix} \Delta A & \Delta A_d & \Delta B \end{bmatrix} = \begin{bmatrix} H_A & 0 & 0 \\ 0 & H_d & 0 \\ 0 & 0 & H_B \end{bmatrix} F \begin{bmatrix} E_A \\ E_d \\ E_B \end{bmatrix} \quad (1.26)$$

where $H_A, H_d, H_B, E_A, E_d, E_B$ are known matrices with appropriate dimensions, and F is an unknown matrix with Lebesgue measurable entries such that $F^T F \leq I$.

Similar to (1.25), consider larger system $\tilde{\mathbf{S}}$ as follows:

$$\begin{aligned} \tilde{\mathbf{S}}: \quad \dot{\tilde{x}}(t) &= (\tilde{A} + \Delta \tilde{A})\tilde{x}(t) + (\tilde{A}_d + \Delta \tilde{A}_d)\tilde{x}(t-d) + (\tilde{B} + \Delta \tilde{B})u(t) + \tilde{B}_1w(t), \\ \tilde{z}(t) &= \tilde{C}\tilde{x}(t) + Du(t), \\ \tilde{x}(t_0) &= \tilde{\varphi}(t_0), \quad -d \leq t_0 \leq 0 \end{aligned} \quad (1.27)$$

The uncertainties $\Delta \tilde{A}, \Delta \tilde{A}_d, \Delta \tilde{B}$ are assumed to be norm-bounded with the following structures:

$$\begin{bmatrix} \Delta \tilde{A} & \Delta \tilde{A}_d & \Delta \tilde{B} \end{bmatrix} = \begin{bmatrix} \tilde{H}_A & 0 & 0 \\ 0 & \tilde{H}_d & 0 \\ 0 & 0 & \tilde{H}_B \end{bmatrix} F \begin{bmatrix} \tilde{E}_A \\ \tilde{E}_d \\ \tilde{E}_B \end{bmatrix} \quad (1.28)$$

Now, let the following inequalities related to systems \mathbf{S} and $\tilde{\mathbf{S}}$ hold for any square integrable signal $w(t)$:

$$\begin{aligned}\|z(t)\|_2 &\leq \gamma \|w(t)\|_2, \\ \|\tilde{z}(t)\|_2 &\leq \tilde{\gamma} \|w(t)\|_2\end{aligned}\tag{1.29}$$

It can be clearly seen that smaller values of H_∞ performance indexes γ and $\tilde{\gamma}$ denote better performance as they determine the effect of unknown input (disturbance) on the controller output vector. Thus, the primary objective in robust control design is minimizing H_∞ performance index.

In the sequel, a new definition of the inclusion principle considering H_∞ performance index is provided [76]:

Definition 1.6. *A pair $(\tilde{\mathbf{S}}, \tilde{\gamma})$ includes (or is expansion of) the pair (\mathbf{S}, γ) if there exists a pair of transformations (U, V) such that for any initial condition $\varphi(t_0)$, any input $u(t)$ and any disturbance $w(t)$, the choice $\tilde{\varphi}(t_0) = V\varphi(t_0)$ leads to $x(t) = U\tilde{x}(t)$ and $\tilde{\gamma} = \gamma$.*

In order to derive relations between the system \mathbf{S} and larger system $\tilde{\mathbf{S}}$, consider the following relations similar to (1.2):

$$\begin{aligned}\tilde{A} &= VAU + M, \quad \Delta\tilde{A} = V\Delta AU, \quad \tilde{C} = TCU + L, \\ \tilde{B} &= VB + N, \quad \Delta\tilde{B} = V\Delta B, \quad \tilde{A}_d = VA_dU + M_d, \\ \tilde{B}_1 &= VB_1 + M_1, \quad \Delta\tilde{C} = CU + L,\end{aligned}\tag{1.30}$$

Based on the relations (1.30), necessary and sufficient conditions which complementary matrices should satisfy are given by the following theorem:

Theorem 1.9. *The pair $(\tilde{\mathbf{S}}, \tilde{\gamma})$ includes the pair (\mathbf{S}, γ) if and only if:*

$$\begin{aligned}UM^iV &= 0, \quad UM^{i-1}M_dV = 0, \quad UM^{i-1}N = 0, \\ UM^iM_1 &= 0, \quad LV = 0,\end{aligned}\tag{1.31}$$

for all $i = 1, 2, \dots, \tilde{n}$.

Once the expanded system is generated, the decentralised state feedback controller $u(t) =$

$\tilde{K}_D \tilde{x}(t)$ has to be designed for the expanded system such that the closed loop $\tilde{\mathbf{S}}_c$:

$$\begin{aligned} \tilde{\mathbf{S}}_c : \quad \dot{\tilde{x}}(t) &= (\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})\tilde{x}(t) + (\tilde{A}_d + \Delta\tilde{A}_d)\tilde{x}(t-d) + \tilde{B}_1 w(t), \\ \tilde{z}(t) &= (\tilde{C} + D\tilde{K}_D)\tilde{x}(t) \\ \tilde{x}(t_0) &= \tilde{\varphi}(t_0), \quad -d \leq t_0 \leq 0 \end{aligned} \quad (1.32)$$

is robustly quadratically stable with H_∞ index $\tilde{\gamma}$ for all admissible uncertainties i.e. the closed loop system $\tilde{\mathbf{S}}_c$ is quadratically stable and the inequality $\|\tilde{z}(t)\|_2 \leq \tilde{\gamma} \|w(t)\|_2$ holds for zero initial condition.

The following theorem can be used to design \tilde{K}_D such that the closed loop (1.32) is robustly quadratically stable [76].

Theorem 1.10. *Let the expanded system be generated using (1.30)-(1.31). If for some positive scalars ϵ_1, ϵ_2 , there exist positive-definite matrices Q and Y such that the following matrix inequality holds:*

$$\begin{bmatrix} W_1 & QW_2^T & Y^T \tilde{E}_B^T \\ * & -I & 0 \\ * & * & -\epsilon^{-1}I \end{bmatrix} < 0 \quad (1.33)$$

where

$$\begin{aligned} W_1 &= \tilde{A}Q + Q\tilde{A}^T + \tilde{B}Y + Y\tilde{B}^T + Z + \tilde{\gamma}^{-2}\tilde{B}_1\tilde{B}_1^T, \\ W_2 &= \begin{bmatrix} \tilde{E}_A^T & \tilde{E}_d^T & I \end{bmatrix}, \\ Z &= \tilde{H}_A\tilde{H}_A^T + \tilde{H}_d\tilde{H}_d^T + \left(1 + \frac{1}{\epsilon_1}\right)\tilde{H}_B\tilde{H}_B^T + \tilde{A}_d\tilde{A}_d^T \end{aligned} \quad (1.34)$$

then, there exists static state feedback controller $\tilde{K}_D = YQ^{-1}$ such that the resultant expanded closed loop system (1.32) is robustly quadratically stable with H_∞ performance index $\tilde{\gamma}$.

Remark: It is evident from theorem 1.10 that design procedure is in category of delay-independent approaches as no information on delay is used in the design procedure. So, conservative results may be obtained by solving (1.33).

Finally, the designed robust decentralised controller \tilde{K}_D is contracted to an overlapping controller

K to be implemented on the original system. The contraction is done as follows [76]:

$$K = \tilde{K}_D V \quad (1.35)$$

It has been proven in [76] that the closed loop system:

$$\begin{aligned} \mathbf{S}_c : \quad & \dot{x}(t) = (A + \Delta A + (B + \Delta B)KC)x(t) + (A_d + \Delta A_d)x(t - d) + B_1 w(t), \\ & z(t) = (C + DK), \\ & x(t_0) = \varphi(t_0), -d \leq t_0 \leq 0 \end{aligned} \quad (1.36)$$

is quadratically stable with H_∞ performance γ , where $\tilde{\gamma} = \gamma$.

1.4 Summary

In this chapter, first, we provide the introduction to overlapping feedback control systems. To motivate the application of such systems, a load frequency control problem is discussed. The inclusion principle as the mathematical framework is introduced to deal with overlapping feedback control systems. This is then followed by surveying the existing research works on the inclusion principle. The inclusion principle and its properties for linear systems are presented thoroughly. Finally, designing an overlapping feedback controller for time-delay systems based on the inclusion principle is investigated.

1.5 Thesis Outline

Chapter 2: Stabilisability of Overlapping Linear Systems. In the first part of this chapter, a necessary and sufficient condition for stabilisation of linear time invariant systems with overlapping parts is presented. To this end, these systems are first expanded to decouple the overlapping parts and then a necessary and sufficient condition, which relates to their algebraic properties, is derived. The algebraic properties are characterised in terms of minimality (full controllability and observability) and the concept of quotient fixed modes (QFMs) of the expanded systems. In the second part of this chapter, stabilisability of overlapping uncertain linear systems with fixed

modes by means of overlapping static output feedback controllers is studied. The expanded system, where the overlapping parts appear as interconnected ones, is generated. An iterative linear matrix inequality (ILMI) algorithm is proposed to find (i) maximum upper bounds on the induced 2-norm of the uncertainties such that the uncertain expanded system is robustly stabilisable with a decentralised static output feedback control, and (ii) a robust decentralised guaranteed static output feedback cost controller for the expanded system. Finally, the returned results of ILMI are contracted (transformed) to a robust overlapping guaranteed cost controller and upper bounds on the uncertainties of the original system. Illustrative examples are used to confirm the results of this chapter.

Chapter 3: Stabilisation of Overlapping Time-Delay Systems. Stabilisability of overlapping linear continuous-time uncertain systems with constant communication delays by robust overlapping output feedback controllers is studied in this chapter. To this end, an extension of the inclusion principle is presented, where the overlapping system is first expanded into a higher dimensional system (expanded system). Then, robust local delay-dependent output feedback controllers, for disjoint subsystems, are designed, using a proposed LMI based iterative algorithm. Finally, the designed local controllers are contracted to an overlapping controller to be implemented on the original system. The preservation of stability and performance through contraction is proven. The robust, overlapping, LFC for an uncertain two-area interconnected power system, experiencing communication delays and parametric uncertainties is designed to verify the results of this chapter.

Chapter 4: Robust Controller Design for Overlapping Uncertain Systems with Time-varying Measurement Delay. The inclusion principle is used to design a robust, overlapping, H_∞ static, output feedback controller for continuous input-delayed uncertain systems with overlapping decomposition. The input delay is unknown but assumed to be time-varying with given upper bounds on the size and derivative of delay. The system considered is comprising a number of overlapping subsystems with structured, time-varying, and norm-bounded uncertainties. In this approach, the original overlapping system is first expanded into a higher dimensional system. Then, an LMI based delay-dependent iterative algorithm is proposed for the design of robust H_∞ local output feedback controllers for the each of the decoupled subsystems of the expanded system. Finally, the designed decentralised controller is contracted to a robust overlapping controller for implemen-

tation on the original system. The preservation of stability and performance through contraction is proven. The validity of the proposed design approach is demonstrated by designing an overlapping LFC for an uncertain 3-area interconnected power system experiencing measurement delay. Extensive simulation results provided under different scenarios verify the accuracy of the proposed design approach.

Chapter 5: Conclusion. This chapter concludes the thesis and provides future research directions.

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Chapter 2

Stabilisability of Overlapping Linear Systems

In this chapter, first, a necessary and sufficient condition for stabilisation of linear time invariant systems with overlapping parts is presented. These systems are first expanded to decouple the overlapping parts and then a necessary and sufficient condition, which relates to their algebraic properties, in terms of minimality, is derived. An iterative algorithm is suggested to design stabilising decentralised controller such that contractibility is guaranteed. A numerical example is used to verify the results of this section.

Then, a sufficient condition for stabilisability of uncertain linear systems by overlapping static output feedback controllers is presented. The overlapping system is first expanded based on the inclusion principle. Then, an ILMI algorithm is proposed to find (i) the maximum upper bounds on the induced 2-norm of the uncertainties such that the uncertain expanded system is robustly stabilisable with a decentralised static output feedback control, and (ii) a robust decentralised guaranteed static output feedback cost controller for the expanded system. Finally, the obtained results by ILMI are contracted to the original space. An illustrative example is provided to confirm the accuracy of results.

2.1 Introduction

A discussed in the preceding section, expansion-contraction process based on the inclusion principle is the useful tool in dealing with large scale systems with overlapping decomposition. The expansion-contraction process requires a judicial selection of matrices, referred to as complementary matrices. On the other hand, a key consideration in the expansion process is the desirable requirement for the expanded system to preserve the controllability-observability (minimality) properties of the original system. However, the transmission of minimality from the original system to the expanded one is not guaranteed through the standard selection of complementary ma-

trices, as shown in [61]. Therefore, standard complementary matrices may render the expanded system to be centralised unstabilisable. On the other hand, as shown in [14], the choice of general structure of complementary matrices proposed in [62] can ensure preservation of minimality in the expanded system. However, centralised stabilisability by static output feedback control does not directly translate to: (i) decentralised stabilisability of the expanded system, which is the key theme of this section, or (ii) overlapping decentralised stabilisability of the original system by the contracted overlapping decentralised controller.

On the other hand, as mentioned in Section 1.2, of particular interest has been the problem of contractibility of stabilising decentralised control laws designed for the expanded system to stabilising overlapping decentralised control laws for the original systems. To overcome this problem, the extension principle, as the special case of inclusion principle, has been used to guarantee contractibility of decentralised output controllers [66]. This approach, however, leads to non-minimal expanded systems, and consequently the expanded system may not be stabilisable the contractibility. It is worth recalling that decentralised stabilisability of expanded systems can be assessed in terms of existence of QFMs [86], which are fixed modes that cannot be removed by any type of decentralised control. If any of the QFMs is unstable, then decentralised stabilisability will not be possible by any type of controller.

In the first part of this chapter, a necessary and sufficient condition for stabilisability of linear time invariant systems with overlapping parts by overlapping decentralised static output feedback controllers is presented. The conditions relate to properties of expanded system matrices and the concept of QFMs. Then, in order to overcome the problems arising from the contractibility conditions, we introduce a procedure where stabilising decentralised static output feedback controllers designed for the expanded systems can always be contracted to stabilising overlapping decentralised controllers for the original systems.

2.2 Preliminaries

Consider linear time invariant system Σ given in (1.1) with the overlapping decomposition determined in (1.6). In order to generate the expanded system $\tilde{\Sigma}$, first, the complementary matrices M, N, L are chosen by theorem 1.7 and relations 1.23, which ensure simultaneous transmission

of controllability/observability from the original system to the expanded one as discussed in theorem 1.8. Then, based on conditions (1.23) and matrix L as per theorem 1.7, we obtain following structures for complementary matrices:

$$\begin{aligned}
 M &= \begin{bmatrix} 0 & M_{12} & -M_{12} & 0 \\ 0 & M_{22} & -M_{22} & 0 \\ 0 & -M_{33} & M_{33} & 0 \\ 0 & -M_{42} & M_{42} & 0 \end{bmatrix}, & N &= \begin{bmatrix} 0 & N_{12} & -N_{12} & 0 \\ 0 & N_{22} & -N_{22} & 0 \\ 0 & -N_{33} & N_{33} & 0 \\ 0 & -N_{42} & N_{42} & 0 \end{bmatrix}, \\
 L &= \begin{bmatrix} 0 & L_{12} & -L_{12} & 0 \\ L_{21} & L_{22} & L_{23} & L_{24} \\ -L_{21} & -(L_{22} + L_{23} + L_{33}) & L_{33} & -L_{24} \\ 0 & L_{42} & -L_{42} & 0 \end{bmatrix} \tag{2.1}
 \end{aligned}$$

In order to provide the expanded system's matrices with maximum zero off-diagonal blocks, the standard complementary matrices given in [62] are chosen. Then, matrices \tilde{A} , \tilde{B} , and \tilde{C} of the expanded system $\tilde{\Sigma}$ given in (1.1) through relations 1.2 are obtained as follows:

$$\begin{aligned}
 \tilde{A} &= \begin{bmatrix} A_{11} & A_{12} & 0 & A_{13} \\ A_{21} & A_{22} & 0 & A_{23} \\ \hline A_{21} & 0 & A_{22} & A_{23} \\ A_{31} & 0 & A_{32} & A_{33} \end{bmatrix}, & \tilde{B} &= \begin{bmatrix} B_{11} & B_{12} & 0 & B_{13} \\ B_{21} & B_{22} & 0 & B_{23} \\ \hline B_{21} & 0 & B_{22} & B_{23} \\ B_{31} & 0 & B_{32} & B_{33} \end{bmatrix}, \\
 \tilde{C} &= \begin{bmatrix} C_{11} & C_{12} & 0 & C_{13} \\ 2C_{21} & C_{22} & 0 & 0 \\ \hline 0 & 0 & C_{22} & 2C_{23} \\ C_{31} & 0 & C_{32} & C_{33} \end{bmatrix} \tag{2.2}
 \end{aligned}$$

Next, a decentralised static output feedback controller $\tilde{K}_D = \text{Blkdiag}\{\tilde{K}_1, \tilde{K}_2\}$ is designed where \tilde{K}_1 and \tilde{K}_2 are shown in (1.15). Once the design is complete, \tilde{K}_D should be contracted to an overlapping decentralised static output feedback control K (1.20) to be implemented on the original system Σ .

Now, suppose conventional partition for \tilde{K}_D as $\tilde{K}_D = RKS + J_K$ [87], where J_K denotes complementary matrix. Then, the following theorem determines equivalent conditions as those of theorem 1.4 based on the complementary matrix J_k [87].

Theorem 2.1. *A controller \tilde{K}_D for $\tilde{\Sigma}$ is contractible to K for Σ if and only if:*

$$\begin{aligned} (I) \quad & J_K L M^{i+1} V = 0, \\ (II) \quad & J_K L M^i N R = 0, \\ (III) \quad & J_K (T C + L V) = 0 \end{aligned} \tag{2.3}$$

for all $i = 0, 1, \dots, \tilde{n}$.

2.3 Problem Statement

Consider systems Σ and its expansion $\tilde{\Sigma}$ described in (1.1) such that the latter system includes the former one by definition 1.1. The aims of this section are as follows:

- Deriving necessary and sufficient condition for stabilisability of Σ with overlapping decentralised static output feedback K (1.20) through the inclusion principle.
- Designing stabilisable and contractible decentralised static output feedback for the expanded system.

2.4 Overlapping Decentralised Stabilisability

With the above preliminary materials, we now introduce main results of this section.

Theorem 2.2. *Consider minimal system Σ with overlapping parts as shown in (1.6). Let the minimal expanded system be constructed per theorem 1.7 with the complementary submatrices satisfying (1.23). Then, original system Σ is stabilisable by an overlapping decentralised static output feedback controller K if and only if the expanded system does not have any unstable QFM.*

Proof. The sufficiency of the theorem is straightforward. If the minimal expanded system obtained by choice of complementary matrices as per theorem 1.7, does not have any unstable QFM

(stabilisable via decentralised static output feedback control), then the original system is stabilisable through contracted overlapping decentralised static output feedback controller as shown in [4]. Therefore, we now need to prove the necessity of the theorem, that is the minimal expanded system does not have any unstable QFM. In order to prove necessity of the theorem, we use contradiction approach, where we assume that there exists an unstable $\tilde{\lambda} \in \text{eig}(\tilde{A})$ in the minimal expanded system which is a QFM i.e. as defined in [86]:

$$\cap_{\tilde{K}_D} \text{eig}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C}) \neq \emptyset \rightarrow \det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C}) = 0, \quad (2.4)$$

where $\det(\cdot)$ stands for determinant, and $\cap_{\tilde{K}} \text{eig}(\cdot)$ denotes any eigenvalue of (\cdot) which is independent of \tilde{K}_D . Since expanded closed loop system $\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C}$ includes original closed loop system $A + BKC$ [4], we have:

$$\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C} = V(A + BKC)U + M_{cl}, \quad (2.5)$$

with M_{cl} is the complementary matrix for closed loop expansion satisfying the condition $UM_{cl}^i V = 0$ ($i = 1, 2, \dots, \tilde{n}$), based on theorem 1.1. Let $\hat{V} \in \mathbb{R}^{\tilde{n} \times (\tilde{n}-n)}$ be any basis matrix formed from the null space of U , i.e. \hat{V} for matrix $U = (V^T V)^{-1} V^T$ can be:

$$\hat{V} = \begin{bmatrix} 0 & I_{n_2} & -I_{n_2} & 0 \end{bmatrix}^T \quad (2.6)$$

Then by constructing nonsingular matrix $\zeta = [V \quad \hat{V}]$ and using (2.5), we have:

$$\zeta^{-1} (\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C})\zeta = \begin{pmatrix} \tilde{\lambda}I - A - BKC & UM_{cl}\hat{V} \\ \hat{U}M_{cl}V & \tilde{\lambda}I - \hat{U}M_{cl}\hat{V} \end{pmatrix}, \quad (2.7)$$

where $\zeta^{-1} = \begin{pmatrix} U \\ \hat{U} \end{pmatrix}$ and $\hat{U} \in \mathbb{R}^{(\tilde{n}-n) \times \tilde{n}}$ is left inverse of \hat{V} , can be denoted as:

$$\hat{U} = \frac{1}{2} \begin{bmatrix} 0 & I_{n_2} & -I_{n_2} & 0 \end{bmatrix} \quad (2.8)$$

According to right hand side of (2.4) and using (2.7), if expanded system has unstable QFM, then

for an arbitrary \tilde{K}_D , we have:

$$\det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C}) = 0 \rightarrow \det((\tilde{\lambda}I - A - BKC)(\tilde{\lambda}I - \hat{U}M_{cl}\hat{V}) - UM_{cl}\hat{V}\hat{U}M_{cl}V) = 0 \quad (2.9)$$

It is easy to see that $UM_{cl}\hat{V}\hat{U}M_{cl}V = 0$ as (i) $\hat{V}\hat{U} = I_{\tilde{n}} - VU$ concluded from $\zeta\zeta^{-1} = I_{\tilde{n}}$ and (ii) $UM_{cl}^iV = 0$ ($i = 1, 2, \dots, \tilde{n}$) concluded from theorem 1.1. So, (2.9) can be re-written as:

$$\det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C}) = \det(\tilde{\lambda}I - A - BKC)\det(\tilde{\lambda}I - \hat{U}M_{cl}\hat{V}) = 0 \quad (2.10)$$

On the other hand, pre- and post-multiplication of (2.5) to \hat{U} and \hat{V} respectively leads to:

$$\hat{U}M_{cl}\hat{V} = \hat{U}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\hat{V} \quad (2.11)$$

Therefore, (2.10) becomes:

$$\det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C}) = \det(\tilde{\lambda}I - A - BKC)\det(\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\hat{V}) = 0 \quad (2.12)$$

for arbitrary \tilde{K}_D . In other words, (2.12) implies

$$\det(\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\hat{V}) = 0 \quad (2.13)$$

According to the structure of \tilde{A} , \tilde{B} , \tilde{C} (2.2), decentralised structure of $\tilde{K}_D = \text{Blkdiag}\{\tilde{K}_1, \tilde{K}_2\}$ (1.15) and also using (2.6) and (2.8), we have:

$$\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\hat{V} = \tilde{\lambda}I - A_{22} - \frac{1}{2}B_{22} \begin{bmatrix} (\tilde{k}_{21}^{(1)})^T \\ (\tilde{k}_{22}^{(1)} + \tilde{k}_{11}^{(2)})^T \\ (\tilde{k}_{12}^{(2)})^T \end{bmatrix}^T \begin{bmatrix} C_{12} \\ C_{22} \\ C_{32} \end{bmatrix} \quad (2.14)$$

Thus, if unstable $\tilde{\lambda} \in \text{eig}(\tilde{A})$ is a QFM, then determinant of (2.14) should be zero for arbitrary \tilde{K}_D .

On the other hand, minimality of the expanded system implies that:

$$\cap_{\tilde{K}} \text{eig}(\tilde{A} + \tilde{B}\tilde{K}\tilde{C}) = \emptyset \rightarrow \det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}\tilde{C}) \neq 0 \quad (2.15)$$

where \tilde{K} has full centralized structure:

$$\tilde{K} = \begin{bmatrix} \tilde{k}_{11} & \tilde{k}_{12} & \tilde{k}_{13} & \tilde{k}_{14} \\ \tilde{k}_{21} & \tilde{k}_{22} & \tilde{k}_{23} & \tilde{k}_{24} \\ \tilde{k}_{31} & \tilde{k}_{32} & \tilde{k}_{33} & \tilde{k}_{34} \\ \tilde{k}_{41} & \tilde{k}_{42} & \tilde{k}_{43} & \tilde{k}_{44} \end{bmatrix} \quad (2.16)$$

Through same non-singular transformation ζ and same procedure as outlined above, (2.15) results in:

$$\det(\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}\tilde{C})\hat{V}) \neq 0 \quad (2.17)$$

Similarly, based on the structures of \tilde{A} , \tilde{B} , \tilde{C} and \tilde{K} in (2.2) and (2.16) respectively, we have:

$$\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}\tilde{C})\hat{V} = \tilde{\lambda}I - A_{22} - \frac{1}{2}B_{22} \begin{bmatrix} (\tilde{k}_{21} - \tilde{k}_{31})^T \\ (\tilde{k}_{22} - \tilde{k}_{23} + \tilde{k}_{33} - \tilde{k}_{32})^T \\ (\tilde{k}_{34} - \tilde{k}_{24})^T \end{bmatrix}^T \begin{bmatrix} C_{12} \\ C_{22} \\ C_{32} \end{bmatrix} \quad (2.18)$$

So, minimality of the expanded system implies that determinant of (2.18) should be non-zero for arbitrary \tilde{K} . However, comparing (2.14) with (2.18) reveals that $\det(\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})\hat{V}) = 0$ for arbitrary \tilde{K}_D is in contradiction with $\det(\tilde{\lambda}I - \hat{U}(\tilde{A} + \tilde{B}\tilde{K}\tilde{C})\hat{V}) = 0$ for arbitrary \tilde{K} . So, minimal expanded system cannot have any unstable QFM. This completes proof of the Theorem. \square

2.4.1 Contractible and Decentralised Output Feedback Design

As shown in (1.19), the decentralised controller \tilde{K}_D has to be modified for satisfaction of the contractibility conditions. However, it is worth recalling that performance and even stability is not guaranteed with the modified controller. Thus, it is essential to design a decentralised controller which does not need any modification for contractibility. To overcome the contractibility issue,

this section presents a design procedure such that any decentralised controller \tilde{K}_D :

$$\tilde{K}_D = \begin{bmatrix} \tilde{k}_{11}^{(1)} & \tilde{k}_{12}^{(1)} & 0 & 0 \\ \tilde{k}_{21}^{(1)} & \tilde{k}_{22}^{(1)} & 0 & 0 \\ 0 & 0 & \tilde{k}_{22}^{(1)} & \tilde{k}_{12}^{(2)} \\ 0 & 0 & \tilde{k}_{21}^{(2)} & \tilde{k}_{22}^{(2)} \end{bmatrix} \quad (2.19)$$

is contractible to an overlapping controller K , shown below:

$$K = \begin{bmatrix} k_1 & k_2 & 0 \\ k_3 & k_4 & k_5 \\ 0 & k_6 & k_7 \end{bmatrix} \quad (2.20)$$

To this end, we propose an LMI based iterative algorithm similar to [15]. The result of proposed algorithm in this section is a stabilisable and contractible decentralised \tilde{K}_D . First, the contractibility conditions stated in theorem 2.1 are examined. Since selection of complementary matrix M with (2.1) results in $MV = 0$, condition (I) of theorem 2.1 holds i.e. $J_K L M^{i+1} V = J_K L M^i M V = 0$. Similarly, condition (II) of theorem 2.1 holds because selection of complementary matrix N as (2.1) results in $NR = 0$. Thus, the following conditions only remain to be satisfied for an arbitrary \tilde{K}_D :

$$J_K(TC + LV) = 0, \quad (2.21)$$

$$\tilde{K}_D = RKS + J_K \quad (2.22)$$

where complementary matrix J_K has a general structure:

$$J_K = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} \\ J_{21} & J_{22} & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix} \quad (2.23)$$

The relation (2.22) with transformations R and S given in (1.10) and (1.11) respectively, and also the overlapping controller K (2.20) leads to:

$$\tilde{K}_D = \left[\begin{array}{cc|cc} k_1 + J_{11} & \frac{k_2}{2} + J_{12} & \frac{k_2}{2} + J_{13} & J_{14} \\ k_3 + J_{21} & \frac{k_4}{2} + J_{22} & \frac{k_4}{2} + J_{23} & k_5 + J_{24} \\ \hline k_3 + J_{31} & \frac{k_4}{2} + J_{32} & \frac{k_4}{2} + J_{33} & k_5 + J_{34} \\ J_{41} & \frac{k_6}{2} + J_{42} & \frac{k_6}{2} + J_{43} & k_7 + J_{44} \end{array} \right] \quad (2.24)$$

Comparing (2.24) with block diagonal \tilde{K}_D (2.19) results in:

$$\begin{aligned} J_{12} &= J_{13} + \tilde{k}_{12}^{(1)}, & J_{21} &= J_{31} + \tilde{k}_{21}^{(1)}, & J_{22} &= J_{23} + \tilde{k}_{22}^{(1)}, & J_{33} &= J_{23} + \tilde{k}_{22}^{(2)}, \\ J_{34} &= J_{24} + \tilde{k}_{12}^{(2)}, & J_{43} &= J_{42} + \tilde{k}_{21}^{(2)}, & J_{14} &= 0, & J_{41} &= 0 \end{aligned} \quad (2.25)$$

Finally, by (2.25), matrix T (1.10) and standard complementary matrix L given in [62], contractibility condition (2.21) becomes:

$$\bar{J}_K \bar{C} = 0 \quad (2.26)$$

where

$$\bar{J}_K = \left[\begin{array}{cccc} J_{11} & J_{13} + \tilde{k}_{12}^{(1)} & J_{13} & 0 \\ J_{31} + \tilde{k}_{21}^{(1)} & J_{23} + \tilde{k}_{22}^{(2)} & J_{23} & J_{24} \\ J_{31} & J_{23} & J_{23} + \tilde{k}_{11}^{(2)} & J_{24} + \tilde{k}_{12}^{(2)} \\ 0 & J_{42} & J_{42} + \tilde{k}_{21}^{(2)} & J_{44} \end{array} \right], \quad \bar{C} = \left[\begin{array}{ccc} C_{11} & C_{12} & C_{13} \\ 2C_{21} & C_{22} & 0 \\ 0 & C_{22} & 2C_{23} \\ C_{31} & C_{32} & C_{33} \end{array} \right] \quad (2.27)$$

With the above development, we now introduce an algorithm based on LMI to provide a contractible and stabilisable decentralised static output feedback control for the expanded system as following (see [15] for details of ILMI algorithm). The algorithm comprises the following 4 steps.

Step 1. Select $Z > 0$ and solve the Riccati equation for P :

$$\tilde{A}^T P + P \tilde{A} - P \tilde{B} \tilde{B}^T P + Z = 0 \quad (2.28)$$

Set $i = 1$ and $X_1 = P$.

Step 2. Solve the following optimization problem (OP) for P_i , \tilde{K}_D , α_i and \bar{J}_K .

OP1: Minimize α_i subject to the following LMI and equality constraint.

$$P_i = P_i^T > 0, \quad (2.29)$$

$$\begin{bmatrix} \tilde{A}^T P_i + P_i \tilde{A} - X_i \tilde{B} \tilde{B}^T P_i - & (\tilde{B}^T P_i + \tilde{K}_D \tilde{C})^T \\ P_i \tilde{B} \tilde{B}^T X_i + X_i \tilde{B} \tilde{B}^T X_i - \alpha_i P_i & \\ & (\tilde{B}^T P_i + \tilde{K}_D \tilde{C}) & -I_{\tilde{n}} \end{bmatrix} < 0, \quad (2.30)$$

$$\bar{J}_k \bar{C} = 0, \quad (2.31)$$

Step 3. Let α_i^* be the optimal value returned by the above optimization problem [88]. If $\alpha_i^* \leq 0$, \tilde{K}_D is the decentralised stabilising and contractible static output feedback controller and the algorithm stops. However, if $\alpha_i^* > 0$, then proceed to step 4.

Step 4. Solve the following optimization problem for \tilde{K}_D and P_i .

OP2. Minimize $\text{trace}(P_i)$ subject to LMI constraints (2.29)-(2.31) after setting $\alpha_i = \alpha_i^*$.

Denotes P_i^* as the solution of OP2. Set $i = i + 1$ and $X_i = P_{i-1}^*$, then go to Step 2.

Remark: Similar to [15], it can be shown that value of $\tilde{\alpha}$ at each iteration is not larger than its value at the preceding iteration. In other words, $\tilde{\alpha}$ does not have increasing trend in the iterative algorithm.

2.5 Numerical Example

Consider system Σ with the following minimal (fully controllable and observable) representation:

$$\Sigma : \begin{aligned} \dot{x}(t) &= \begin{bmatrix} 1 & 3 & 1 \\ 2 & -2 & 2 \\ -2 & 2 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & 4 \\ -3 & -2 & 2 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} -2 & -1 & -3 \\ 1 & -2 & -1 \\ 0 & 1 & 1 \end{bmatrix} x(t) \end{aligned} \quad (2.32)$$

where the overlapping decomposition is determined by dashed lines.

The eigenvalues of the original system are $\{-4.7, 0, 1.7\}$, which show that the original system is unstable. In the ensuing, we expand this system as per theorem 1.7 and show that the necessary and sufficient parts of theorem 2.2 must hold to be able to stabilise the original system (2.32) by an overlapping decentralised static output feedback control. To check the necessity condition, we compute complementary matrices M , N , and L as follows:

$$\begin{aligned} M &= \begin{bmatrix} 0 & 1.5 & -1.5 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}, & N &= \begin{bmatrix} 0 & 1.5 & -1.5 & 0 \\ 0 & 0.5 & -0.5 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \\ L &= \begin{bmatrix} 0 & -0.5 & 0.5 & 0 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & -1 \\ 0 & -0.5 & 0.5 & 0 \end{bmatrix} \end{aligned} \quad (2.33)$$

Then, via (1.2) and transformations (1.10), we obtain the following expanded system

$$\begin{aligned} \tilde{\Sigma} : \quad \dot{\tilde{x}}(t) &= \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & -2 & 0 & 2 \\ 2 & 0 & -2 & 2 \\ -2 & 0 & 2 & -2 \end{bmatrix} \tilde{x}(t) + \begin{bmatrix} 2 & 3 & 0 & 5 \\ 3 & 1 & 0 & 4 \\ 3 & 0 & 1 & 4 \\ -3 & 0 & -2 & 2 \end{bmatrix} \tilde{u}(t), \\ \tilde{y}(t) &= \begin{bmatrix} -2 & -1 & 0 & -3 \\ 2 & -2 & 0 & 0 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tilde{x}(t) \end{aligned} \quad (2.34)$$

Note that the resulting expanded system is fully controllable and observable, however, it is still unstable with eigenvalues $\{-4.7, -2, 0, 1.7\}$. Now we design a decentralised static output feedback control \tilde{K}_D (2.19). But, first, note that the expanded system $\tilde{\Sigma}$ does not have any QFMs, since:

$$\det(\tilde{\lambda}I - \tilde{A} - \tilde{B}\tilde{K}_D\tilde{C}) \neq 0 \quad \text{for } \forall \tilde{\lambda} \in \text{eig}(\tilde{A}), \quad (2.35)$$

To find decentralised stabilizing and contractible static output feedback controller, the proposed iterative algorithm is used as outlined next. In Step 1, matrix $Z = 20I_4$ is found to satisfy the feasibility of the Riccati equation (2.28) and also provides fast convergence. After the first iteration, the algorithm returns $\alpha^* = -0.35$. Then the following decentralised static output feedback controller is obtained for the expanded system:

$$\tilde{K}_D = \begin{bmatrix} 0.0053 & -0.8177 & 0 & 0 \\ 0.1725 & 0.1725 & 0 & 0 \\ 0 & 0 & 0.2071 & -0.1035 \\ 0 & 0 & 0.0378 & -0.0757 \end{bmatrix} \quad (2.36)$$

The controller (2.36) stabilises the closed loop system, as $\text{eig}(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C}) = \{-2.51, -0.67 \pm j3.6, -0.39\}$.

Then, through \bar{J}_k obtained by algorithm and \tilde{K}_D in (2.36), the complementary matrix J_K is com-

puted as:

$$J_K = \begin{bmatrix} -0.32 & -0.32 & 0.49 & 0 \\ 2.73 & 2.73 & 2.56 & 13.34 \\ 2.56 & 2.56 & 2.77 & 13.23 \\ 0 & 0 & 0.0378 & 0.0756 \end{bmatrix} \quad (2.37)$$

Finally, by (2.37) and $\tilde{K}_D = RKS + J_K$, we obtain the following overlapping decentralised static output control:

$$K = \begin{bmatrix} 0.33 & -0.98 & 0 \\ -2.56 & -5.13 & -13.34 \\ 0 & 0 & -0.15 \end{bmatrix} \quad (2.38)$$

where (2.38) results in $eig(A + BKC) = \{-0.67 \pm j3.6, -0.39\}$. Stabilisability of the original system with (2.38) is expected as expanded closed loop $(\tilde{A} + \tilde{B}\tilde{K}_D\tilde{C})$ includes closed loop $(A + BKC)$. So, according to Theorem 2.2, it has been shown that necessary and sufficient condition for stabilisability of the original system by the contracted overlappinng decentralised static output feedback control K is that the minimal expanded system is stabilisable via decentralised static output feedback control.

2.6 Robust Overlapping Guaranteed Cost Control

The results reported in Section 2.1 deal with the linear systems which exhibit no parametric uncertainties, whereas such uncertainties are inherent in most of real world engineering systems. The feedback control system may not function properly if these uncertainties are not considered in the design procedure. Now, let the uncertain expanded system be generated using the expansion of the original overlapping uncertain linear system. To ascertain stabilisability of the expanded system, which can be translated to stabilisability of the original overlapping system, the notion of Structured Decentralised Fixed Modes (SDFMs) was introduced in [89] to identify those modes that always remain fixed no matter what the values of the nonzero entries of the system matrices are. This was then followed by the introduction of Unstructured Decentralised Fixed Modes (UDFMs), which are defined as those decentralised fixed modes (DFMs) that vanish when the nonzero parameters are changed arbitrarily [90].

This section deals with overlapping linear systems with parameter uncertainties arising from changes to its physical components due to operating conditions. We consider the case where such uncertainties render the system unstabilisable by structurally constrained controllers, i.e. by controllers with overlapping structure. We then expand the system and formulate an LMI problem to be solved iteratively to return (i) an upper bounds on the induced 2-norm of the uncertainties of the expanded system such that the expanded system does not have any unstable fixed mode for admissible uncertainties and (ii) a robust decentralised guaranteed cost controller for the expanded system. Finally, the decentralised guaranteed cost controller and the bounds on the induced 2-norm of the uncertainties are contracted (transformed) to the original space.

Consider uncertain continuous time systems Σ and $\tilde{\Sigma}$ with quadratic performance functions J and \tilde{J} respectively:

$$\begin{aligned} \Sigma : \quad \dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t), \\ y(t) &= Cx(t), \\ J(x_0, u) &= \int_0^{\infty} (x^T(t)Q_w x(t) + u^T(t)R_w u(t))dt \end{aligned} \quad (2.39)$$

and

$$\begin{aligned}\tilde{\Sigma} : \quad \dot{\tilde{x}}(t) &= (\tilde{A} + \Delta\tilde{A}(t))\tilde{x}(t) + (\tilde{B} + \Delta\tilde{B}(t))\tilde{u}(t), \\ \tilde{y}(t) &= \tilde{C}\tilde{x}(t), \\ \tilde{J}(\tilde{x}_0, \tilde{u}) &= \int_0^\infty (\tilde{x}^T(t)\tilde{Q}_w\tilde{x}(t) + \tilde{u}^T(t)\tilde{R}_w\tilde{u}(t))dt\end{aligned}\quad (2.40)$$

where $Q_w = Q_w^T \in \mathbb{R}^{n \times n} > 0$, $\tilde{Q}_w = \tilde{Q}_w^T \in \mathbb{R}^{\tilde{n} \times \tilde{n}} > 0$, $R_w = R_w^T \in \mathbb{R}^{m \times m} > 0$ and $\tilde{R}_w = \tilde{R}_w^T \in \mathbb{R}^{\tilde{m} \times \tilde{m}} > 0$ are state and control weighting matrices, respectively. Then, an extension of definition 1.2 by considering quadratic cost functions J and \tilde{J} is presented next.

Definition 2.1. *The pair $(\tilde{\Sigma}, \tilde{J})$ includes the pair (Σ, J) (The pair $(\tilde{\Sigma}, \tilde{J})$ is an expansion of pair (Σ, J)) if there exists triple (V, R, T) such that for any initial state x_0 and any input $u(t)$, choices $\tilde{x}_0 = Vx_0$ and $\tilde{u}(t) = Ru(t)$ result in $\tilde{x}(t; \tilde{x}_0, \tilde{u}) = Vx(t; x_0, u)$, $\tilde{y}(t) = Ty(t)$, and $\tilde{J}(\tilde{x}_0, \tilde{u}) = J(x_0, u)$ for all $t \geq 0$.*

Similar to relations (1.2), the following expressions can be used to determine relations between matrices of $\tilde{\Sigma}$ and Σ .

$$\begin{aligned}\tilde{A} &= VAU + M, \quad \tilde{B} = VBQ + N, \quad \tilde{C} = TCU + L, \\ \tilde{R}_w &= Q^T R_w Q + H, \quad \tilde{Q}_w = U^T Q_w U + F, \\ \Delta\tilde{A}(t) &= V\Delta A(t)U, \quad \Delta\tilde{B}(t) = V\Delta B(t)Q\end{aligned}\quad (2.41)$$

where M, N, L, H , and F are called complementary matrices with appropriate dimensions.

Based on definition 2.1, the conditions which complementary matrices should satisfy are given as follows:

$$MV = 0, \quad NR = 0, \quad LV = 0, \quad R^T H R = 0, \quad V^T F V = 0, \quad (2.42)$$

After generating an expanded system by the set of relationships and conditions given in (2.41) and (2.42) respectively, a decentralised static output robust guaranteed cost controller \tilde{K}_D has to be designed for the expanded system. Robust static output feedback guaranteed cost control is defined in [91] as follows:

Definition 2.2. Consider the uncertain linear system $\tilde{\Sigma}$ given in (2.40). A static decentralised output feedback controller \tilde{K}_D is a robust guaranteed cost controller for the expanded system if the closed loop:

$$\dot{\tilde{x}}(t) = (\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})\tilde{x}(t) \quad (2.43)$$

is stable, and its cost function satisfies the following criterion:

$$\tilde{J} = \int_0^\infty (\tilde{x}^T(t)\tilde{Q}_w\tilde{x}(t) + (\tilde{K}_D\tilde{C})^T\tilde{R}_w\tilde{K}_D\tilde{C})dt \leq \tilde{x}_0\tilde{P}\tilde{x}_0 \quad (2.44)$$

for all admissible uncertainties, with \tilde{x}_0 being the initial condition of the expanded system and $\tilde{P} = \tilde{P}^T > 0$ is an associated cost matrix.

Once the design is complete, the control law $\tilde{u}(t) = \tilde{K}_D\tilde{y}(t)$ is contracted to a robust overlapping guaranteed static output cost feedback control law $u = Ky$. Based on theorem 1.5, the contraction process is done as follows:

$$\tilde{K}_D T = RK \quad (2.45)$$

Note that the contractibility condition (2.45) leads to $K = Q\tilde{K}_D T$ only when $\tilde{K}_D T = RQ\tilde{K}_D T$. In other words, the decentralised controller \tilde{K}_D is contractible to overlapping controller K when $(I - RQ)\tilde{K}_D T = 0$.

2.7 Problem Formulation

Given uncertain linear systems (2.39)-(2.40) such that $(\tilde{\Sigma}, \tilde{J})$ includes (Σ, J) by definition 2.1. This section looks into the stabilisability of dynamical system (2.39) by overlapping static output feedback controllers. In this respect, the following contributions are made:

- An ILMI algorithm is proposed to find the maximum positive scalars $\tilde{\mu}_A, \tilde{\mu}_B$, and associated static output feedback controller \tilde{K}_D (2.19) such that for all admissible uncertainties, i.e. $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and $\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$, the closed loop:

$$\dot{\tilde{x}}(t) = (\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})\tilde{x}(t) \quad (2.46)$$

is stable, and its cost function satisfies:

$$\tilde{J} = \int_0^{\infty} (\tilde{x}^T(t) \tilde{Q}_w \tilde{x}(t) + (\tilde{K}_D \tilde{C})^T \tilde{R}_w \tilde{K}_D \tilde{C}) dt \leq \tilde{J}_0 \quad (2.47)$$

for a given \tilde{J}_0 .

- The inclusion principle and proper contraction are used to obtain (i) maximum upper bounds on the induced 2-norm of uncertainties in the original system Σ (2.39), and (ii) a corresponding robust guaranteed static output cost feedback control with overlapping structure.

2.8 The Inclusion Principle and Unstructured Decentralised Fixed Modes

Theorem 2.3. *Consider an uncertain linear system Σ with overlapping parts and its expansion $\tilde{\Sigma}$ constructed by (2.41) through the complementary matrices satisfying (2.42). Then, the expanded system $\tilde{\Sigma}$ has UDFM(s) if the original system Σ is unstabilisable w.r.t overlapping controller K for some admissible uncertainties.*

Proof. Let Σ be unstabilisable w.r.t overlapping K for some admissible uncertainty ΔA and ΔB i.e. for arbitrary K , there exists an unstable fixed mode $\lambda \in \text{eig}(A)$ such that:

$$\lambda \in \cap_K \text{eig}(A + \Delta A + (B + \Delta B)KC) \rightarrow \det(\lambda I_n - A - \Delta A - (B + \Delta B)KC) = 0 \quad (2.48)$$

Now, consider expanded system $\tilde{\Sigma}$ (2.40) with contractible decentralised controller \tilde{K}_D . Since expanded closed loop $\tilde{A} + \Delta \tilde{A} + (\tilde{B} + \Delta \tilde{B})\tilde{K}_D \tilde{C}$ includes original closed loop $A + \Delta A + (B + \Delta B)KC$ [4], we have:

$$\tilde{A} + \Delta \tilde{A} + (\tilde{B} + \Delta \tilde{B})\tilde{K}_D \tilde{C} = V(A + \Delta A + (B + \Delta B)KC)U + M_{cl} \quad (2.49)$$

where M_{cl} is the complementary matrix for the closed loop expansion with $M_{cl}V = 0$ based on (2.42).

Pre-post multiply (2.49) by ζ^{-1} and ζ where $\zeta = [V \quad \hat{V}]$ with \hat{V} given in (2.6), we get:

$$\begin{aligned} & \zeta^{-1}(\lambda I_{\tilde{n}} - \tilde{A} - \Delta\tilde{A} - (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})\zeta = \\ & \begin{bmatrix} \lambda I_n - A - \Delta A - (B + \Delta B)KC & -UM_{cl}\hat{V} \\ 0 & \lambda I_{n_2} - \hat{U}M_{cl}\hat{V} \end{bmatrix} \end{aligned} \quad (2.50)$$

Substitute the right side of (2.48) in determinant of (2.50) to obtain:

$$\begin{aligned} & \det(\lambda I_{\tilde{n}} - \tilde{A} - \Delta\tilde{A} - (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C}) = \\ & \det(\lambda I_n - A - \Delta A - (B + \Delta B)KC)\det(\lambda I_{n_2} - UM_{cl}\hat{V}) = 0 \end{aligned} \quad (2.51)$$

Equation (2.51) implies that $\lambda \in \cap_{\tilde{K}_D} \text{eig}(\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})$ for arbitrary \tilde{K}_D for some admissible uncertainties $\Delta\tilde{A}$ and $\Delta\tilde{B}$. That is the unstable mode λ in the expanded system is a DFM for some admissible uncertainties and hence λ is a UDFM in the expanded system. This completes the proof. \square

2.8.1 Decentralised Control Design with LMIs

In the following, the results in [15] will be generalized to include parametric uncertainties in the system matrices. An iterative algorithm based on LMI is proposed to find upper bounds $\tilde{\mu}_A, \tilde{\mu}_B$ on the induced 2-norm of the uncertainties and an associated decentralised guaranteed cost controller \tilde{K}_D for the expanded system. Then, an overlapping guaranteed cost control is obtained through proper contraction. But, first we introduce a well-known lemma to be used in the LMI problem formulation.

Lemma 2.1. *For any matrices X and Y of appropriate dimensions, the following inequality holds for an arbitrary $\epsilon > 0$:*

$$X^T Y + Y^T X \leq \epsilon X^T X + \frac{1}{\epsilon} Y^T Y \quad (2.52)$$

Now, consider the expanded system $\tilde{\Sigma}$ and the corresponding cost function (2.40). If there exist $\tilde{P} = \tilde{P}^T > 0$, a decentralised controller \tilde{K}_D , and a scalar $\tilde{\alpha} < 0$ such that for all admissible

uncertainties $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and $\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$, the following inequality holds:

$$\begin{aligned} & (\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})^T\tilde{P} + \tilde{P}(\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C}) \\ & + \tilde{Q}_w + \tilde{C}^T\tilde{K}_D^T\tilde{R}_w\tilde{K}_D\tilde{C} - \tilde{\alpha}\tilde{P} < 0 \end{aligned} \quad (2.53)$$

then, the closed loop $\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C}$ is stable with all of its eigenvalues placed to the left of the line $\frac{\tilde{\alpha}}{2}$ in the complex plane. In addition, the closed loop cost function will satisfy the condition $\tilde{J}(\tilde{x}_0, \tilde{K}_D\tilde{y}) \leq \tilde{x}_0^T\tilde{P}\tilde{x}_0$.

Now, using Lemma 2.1, we have:

$$\begin{aligned} (i) \quad & \tilde{P}\Delta\tilde{B}\tilde{K}_D\tilde{C} + \tilde{C}^T\tilde{K}_D^T(\Delta\tilde{B})^T\tilde{P} \leq \tilde{P}\Delta\tilde{B}(\Delta\tilde{B})^T\tilde{P} + \tilde{C}^T\tilde{K}_D^T\tilde{K}_D\tilde{C}, \\ (ii) \quad & (\Delta\tilde{A})^T\tilde{P} + \tilde{P}(\Delta\tilde{A}) \leq \epsilon(\Delta\tilde{A})^T\Delta\tilde{A} + \tilde{P}\epsilon^{-1}\tilde{P} \end{aligned} \quad (2.54)$$

Then, based on (2.54), the matrix inequality (2.53) holds if the following is true:

$$\begin{aligned} & \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} + \tilde{P}\tilde{B}\tilde{K}_D\tilde{C} + \tilde{C}^T\tilde{K}_D^T\tilde{B}^T\tilde{P} + \tilde{P}\Delta\tilde{B}(\Delta\tilde{B})^T\tilde{P} + \tilde{C}^T\tilde{K}_D^T\tilde{K}_D\tilde{C} + \epsilon(\Delta\tilde{A})^T\Delta\tilde{A} + \tilde{P}\epsilon^{-1}\tilde{P} \\ & + \tilde{Q}_w + \tilde{C}^T\tilde{K}_D^T\tilde{R}_w\tilde{K}_D\tilde{C} - \tilde{\alpha}\tilde{P} < 0 \end{aligned} \quad (2.55)$$

Using the induced 2-norm matrix definition, the inequalities $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and $\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$ lead to $\Delta\tilde{A}(\Delta\tilde{A})^T \leq \tilde{\mu}_A I_{\tilde{n}}$ and $\Delta\tilde{B}(\Delta\tilde{B})^T \leq \tilde{\mu}_B I_{\tilde{n}}$. Using these latter inequalities, it is easy to see that (2.55) is satisfied if the following matrix inequality holds:

$$\begin{aligned} & \tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} + \tilde{P}\tilde{B}\tilde{K}_D\tilde{C} + \tilde{C}^T\tilde{K}_D^T\tilde{B}^T\tilde{P} + \tilde{P}\tilde{\mu}_B I_{\tilde{n}}\tilde{P} + \tilde{C}^T\tilde{K}_D^T\tilde{K}_D\tilde{C} + \epsilon\tilde{\mu}_A I_{\tilde{n}} + \tilde{P}\epsilon^{-1}\tilde{P} \\ & + \tilde{Q}_w + \tilde{C}^T\tilde{K}_D^T\tilde{R}_w\tilde{K}_D\tilde{C} - \tilde{\alpha}\tilde{P} < 0 \end{aligned} \quad (2.56)$$

Using (2.56), the following corollary, which provides a new sufficient condition for \tilde{K}_D to be a robust guaranteed cost controller is obtained.

Corollary 2.1. *The decentralised static output feedback controller \tilde{K}_D is a robust guaranteed cost controller for the uncertain expanded system $\tilde{\Sigma}$ (2.40) if there exist a matrix $\tilde{P} = \tilde{P}^T > 0$, a scalar $\tilde{\alpha} < 0$ and scalar $\epsilon > 0$ such that for all admissible uncertainties $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and*

$\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$, the following symmetric matrix inequality holds:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & 0 & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 \\ * & * & * & \Phi_{44} & 0 \\ * & * & * & * & \Phi_{55} \end{bmatrix} < 0 \quad (2.57)$$

where $*$ denotes the symmetric part, and

$$\begin{aligned} \Phi_{11} &= \tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} - \tilde{P} \tilde{B} \tilde{B}^T \tilde{P} + \tilde{Q}_w - \tilde{\alpha} \tilde{P} + \epsilon \tilde{\mu}_A I_{\tilde{n}}, \\ \Phi_{12} &= \tilde{P}, \quad \Phi_{13} = \tilde{P}, \quad \Phi_{14} = \tilde{C}^T \tilde{K}_D^T, \\ \Phi_{15} &= (\tilde{B}^T \tilde{P} + \tilde{K}_D \tilde{C})^T, \quad \Phi_{22} = -\epsilon I_{\tilde{n}}, \\ \Phi_{33} &= -\tilde{\mu}_B^{-1} I_{\tilde{n}}, \quad \Phi_{44} = -\tilde{R}_w^{-1}, \quad \Phi_{55} = -I_{\tilde{m}} \end{aligned} \quad (2.58)$$

Proof. It is easy to see that (2.56) can be written as follows:

$$\begin{aligned} &\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} - \tilde{P} \tilde{B} \tilde{B}^T \tilde{P} + (\tilde{B}^T \tilde{P} + \tilde{K}_D \tilde{C})^T (\tilde{B}^T \tilde{P} + \tilde{K}_D \tilde{C}) \\ &+ \tilde{P} \tilde{\mu}_B I_{\tilde{n}} \tilde{P} + \tilde{Q}_w + \epsilon \tilde{\mu}_A I_{\tilde{n}} + \tilde{P} \epsilon^{-1} \tilde{P} + \tilde{C}^T \tilde{K}_D^T \tilde{R}_w \tilde{K}_D \tilde{C} - \tilde{\alpha} \tilde{P} < 0 \end{aligned} \quad (2.59)$$

Then, by applying Schur complement on (2.59), we obtain (2.57) with entries shown in (2.58). \square

However, due to the existence of the term $-\tilde{P} \tilde{B} \tilde{B}^T \tilde{P}$, (2.57) is not a standard LMI in \tilde{P} . In order to transform (2.57) into a standard LMI form, which can be solved efficiently, a procedure similar to [15] is used. To achieve this, we introduce an arbitrary matrix $\tilde{X} = \tilde{X}^T > 0$ for which the following inequality always holds:

$$\tilde{X} \tilde{B} \tilde{B}^T \tilde{X} - \tilde{X} \tilde{B} \tilde{B}^T \tilde{P} - \tilde{P} \tilde{B} \tilde{B}^T \tilde{X} \geq -\tilde{P} \tilde{B} \tilde{B}^T \tilde{P} \quad (2.60)$$

Then, through (2.60) and schur complement, we can say that (2.59) holds if there exists a negative

scalar $\tilde{\alpha}$ such that:

$$\tilde{\Psi} = \begin{bmatrix} \tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} + \tilde{X} \tilde{B} \tilde{B}^T \tilde{X} - \tilde{X} \tilde{B} \tilde{B}^T \tilde{P} - & \tilde{P} & \tilde{P} & \tilde{C}^T \tilde{K}_D^T & (\tilde{B}^T \tilde{P} + \tilde{K}_D \tilde{C})^T \\ \tilde{P} \tilde{B} \tilde{B}^T \tilde{X} + \tilde{Q}_w - \tilde{\alpha} \tilde{P} + \epsilon \tilde{\mu}_A I_{\tilde{n}} & & & & \\ * & -\epsilon I_{\tilde{n}} & 0 & 0 & 0 \\ * & * & -\tilde{\mu}_B^{-1} I_{\tilde{n}} & 0 & 0 \\ * & * & * & -\tilde{R}_w^{-1} & 0 \\ * & * & * & * & -I_{\tilde{n}} \end{bmatrix} < 0 \quad (2.61)$$

The above matrix inequality is linear in \tilde{P} , \tilde{K}_D , and ϵ for fixed $\tilde{\mu}_A$, $\tilde{\mu}_B$, and \tilde{X} . The following iterative algorithm summarises the LMI formulation and design problem.

Step 1) Calculate \tilde{P} by solving the Riccati equation for nominal part of $\tilde{\Sigma}$ (2.40):

$$\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} - \tilde{P} \tilde{B} \tilde{R}_w^{-1} \tilde{B}^T \tilde{P} + \tilde{Q}_w = 0 \quad (2.62)$$

Set $\tilde{X} = \tilde{P}$ and choose the bounds $\tilde{\mu}_A$ and $\tilde{\mu}_B$.

Step 2) Solve the following generalized eigenvalue minimization problem (GEVP) for \tilde{P} , \tilde{K}_D , ϵ , $\tilde{\alpha}$:

OP1: Minimize $\tilde{\alpha}$ subject to the following LMI and equality constraints:

$$\begin{aligned} \tilde{\Psi} < 0, \quad \tilde{P} = \tilde{P}^T > 0, \\ \tilde{x}_0^T \tilde{P} \tilde{x}_0 < \tilde{J}_0, \quad (I - RQ) \tilde{K}_D^T = 0 \end{aligned} \quad (2.63)$$

Based on Definition 2, where $\tilde{J}(\tilde{x}_0, \tilde{K}_D \tilde{y}) \leq \tilde{x}_0^T \tilde{P} \tilde{x}_0$, the constraint $\tilde{x}_0^T \tilde{P} \tilde{x}_0 < \tilde{J}_0$ guarantees $\tilde{J}(\tilde{x}_0, \tilde{K}_D \tilde{y}) \leq \tilde{J}_0$ for a given \tilde{J}_0 .

Step 3) Let $\tilde{\alpha}^*$ be the outcome of OP1. If $\tilde{\alpha}^* < 0$, then the decentralised static output feedback controller \tilde{K}_D is a contractible robust guaranteed cost controller with the upper bounds on induced 2-norm of uncertainties being $\tilde{\mu}_A$ and $\tilde{\mu}_B$. If so, then stop. Else, go to step 4.

Step 4) Solve the following optimization problem for \tilde{P} , \tilde{K}_D and ϵ with $\tilde{\alpha} = \tilde{\alpha}^*$.

OP2: Minimize $\text{trace}(\tilde{P})$ subject to constraints (2.63). Denote \tilde{P}^* as the solution of OP2.

Step 5) If a predetermined tolerance is achieved $\|\tilde{X} - \tilde{P}^*\| < \delta$, then go to step 6. Else, set $\tilde{X} = \tilde{P}^*$ and go to step 2.

Step 6) The expanded system may not be robustly stabilisable for the chosen bounds $\tilde{\mu}_A$ and $\tilde{\mu}_B$. If so, then decrease $\tilde{\mu}_A$ and $\tilde{\mu}_B$ and go to step 2.

Remark: The convergence of the algorithm is similar to [15].

Remark: The largest chosen values for $\tilde{\mu}_A$ and $\tilde{\mu}_B$ such that the proposed iterative algorithm returns negative $\tilde{\alpha}_j^*$ in step 3, are considered as the maximum robustness bounds of the expanded system.

After obtaining the upper bounds and the associated robust guaranteed cost controller for the expanded system, they have to be contracted for implementation on the original system.

Theorem 2.4. Consider overlapping linear system Σ , and its expanded system $\tilde{\Sigma}$. Let a robust decentralised guaranteed static output cost feedback controller \tilde{K}_D for the expanded system $\tilde{\Sigma}$ (2.40) be obtained through the proposed algorithm with admissible uncertainties $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and $\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$. Then, contraction of the robust decentralised guaranteed cost control \tilde{K}_D will lead to a robust guaranteed cost control with overlapping structure K for Σ with the admissible uncertainties satisfying $\|\Delta A\|_2^2 \leq 2\tilde{\mu}_A$ and $\|\Delta B\|_2^2 \leq 2\tilde{\mu}_B$.

Proof. If the decentralised control \tilde{K}_D is a robust guaranteed cost control for the expanded system $\tilde{\Sigma}$ (2.40) with $\|\Delta\tilde{A}\|_2^2 \leq \tilde{\mu}_A$ and $\|\Delta\tilde{B}\|_2^2 \leq \tilde{\mu}_B$, then there exists $\tilde{P} = \tilde{P}^T > 0$ such that for all admissible uncertainties, we have:

$$\begin{aligned} & \tilde{P}(\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C}) + (\tilde{A} + \Delta\tilde{A} + (\tilde{B} + \Delta\tilde{B})\tilde{K}_D\tilde{C})^T\tilde{P} \\ & + \tilde{Q}_w + \tilde{C}^T\tilde{K}_D^T\tilde{R}_w\tilde{K}_D\tilde{C} < 0 \end{aligned} \quad (2.64)$$

Now, pre-post multiply (2.64) by V^T and V , and using (2.41), (2.42) and (2.45) we obtain:

$$\begin{aligned} & P(A + \Delta A + (B + \Delta B)KC) + (A + \Delta A + (B + \Delta B)KC)^T P \\ & + Q_w + C^T K^T R_w K C < 0 \end{aligned} \quad (2.65)$$

with $P = V^T \tilde{P} V > 0$ as V is the full column rank matrix. So, (2.65) is implying that an overlap-

ping static output feedback control K obtained by contraction of \tilde{K}_D is the robust guaranteed cost control for uncertain original system Σ with admissible uncertainties $\Delta A(t)$ and $\Delta B(t)$.

In order to get upper bounds on 2-norm of uncertainties $\Delta A(t)$ and $\Delta B(t)$, first of all using (2.41) and the facts that $UV = I_n$ and $QR = I_m$, it is easy to see that:

$$\begin{aligned}\Delta A(t) &= U\Delta\tilde{A}(t)V, \\ \Delta B(t) &= U\Delta\tilde{B}(t)R\end{aligned}\quad (2.66)$$

Then, from induced 2-norm definition i.e. $\|\Delta\tilde{B}(t)\|_2^2 \leq \tilde{\mu}_B$ implying $\Delta\tilde{B}(t)(\Delta\tilde{B}(t))^T \leq \tilde{\mu}_B I_{\tilde{n}}$, and structures of R (1.10) and U which lead to $RR^T \leq 2I_{\tilde{m}}$ and $UU^T \leq I_n$ respectively, we have:

$$\begin{aligned}\Delta B(\Delta B)^T &= U\Delta\tilde{B}RR^T(\Delta\tilde{B})^T U^T \leq (U\Delta\tilde{B})2I_{\tilde{m}}(\Delta\tilde{B})^T U^T \\ &\leq 2\tilde{\mu}_B UU^T \leq 2\tilde{\mu}_B I_n\end{aligned}\quad (2.67)$$

Similarly, we have:

$$\Delta A(\Delta A)^T = U\Delta\tilde{A}VV^T(\Delta\tilde{A})^T U^T \leq (U\Delta\tilde{A})2I_{\tilde{n}}(\Delta\tilde{A})^T U^T \leq 2\tilde{\mu}_A UU^T \leq 2\tilde{\mu}_A I_n \quad (2.68)$$

Thus, the obtained overlapping output feedback control K through contraction is a robust overlapping guaranteed cost control for the original system (2.39) with admissible uncertainties satisfying $\|\Delta A\|_2^2 \leq 2\tilde{\mu}_A$ and $\|\Delta B\|_2^2 \leq 2\tilde{\mu}_B$. This completes the theorem's proof. \square

2.9 Numerical Example

Consider an uncertain linear system Σ with the unstable nominal system described by the following state and output equations:

$$\dot{x}(t) = \begin{bmatrix} 0.9 + q_1(t) & 0 & 0.01 + q_2(t) & 0 \\ 0 & 0.9 + q_3(t) & 0 & 0.1 + q_4(t) \\ 0.1 + q_5(t) & 0 & -0.9 + q_6(t) & 0 \\ -0.1 + q_7(t) & 0 & 0.1 + q_8(t) & -3 + q_9(t) \end{bmatrix} x(t) +$$

the ILMI algorithm returned $\alpha = -0.0022$ and the following robust guaranteed cost decentralised static output feedback controller:

$$\tilde{K}_D = \begin{bmatrix} 12.8 & -20.1 & 0 & 0 \\ 0 & -4.1 & 0 & 0 \\ 0 & 0 & -4.1 & 0 \\ 0 & 0 & 2.5 & -0.99 \end{bmatrix} \quad (2.70)$$

Contraction of (2.70) leads to the following robust overlapping guaranteed static output cost controller:

$$K = \begin{bmatrix} 12.8 & -20.1 & 0 \\ 0 & -4.1 & 0 \\ 0 & 2.5 & -0.99 \end{bmatrix} \quad (2.71)$$

Theorem 2.4 will confirm that the controller (2.71) is a robust guaranteed cost controller for the original uncertain system (2.69) for all of the uncertainties within the induced 2-norm bounds $\|\Delta A\|_2 \leq \sqrt{2 \times 0.01} = 0.14$ and $\|\Delta B\|_2 \leq \sqrt{2 \times 0.01} = 0.14$. This can be easily verified through the following simulation:

Let the norm bounded uncertainties $\Delta A(t)$ and $\Delta B(t)$ in the original system satisfying $\|\Delta A(t)\|_2 = 0.13 < 0.14$ and $\|\Delta B(t)\|_2 = 0.11 < 0.14$ for $t \geq 0$. Using the robust overlapping guaranteed cost control (2.71) on the original uncertain system (2.69) leads to a stable closed loop system with the the state responses shown in Fig. 2.1, where the original system (2.69) is assumed to have the initial condition $\tilde{x}_0^T = [1 \ 1 \ 1 \ 1]$. The closed loop quadratic cost function is computed as $J \cong 60 < \tilde{J}_0 = 200$. The uncertain system (2.69), however, is not robustly stabilisable with respect to K (2.71) for uncertainties outside of the regions $\|\Delta A\| \leq 0.14$ and $\|\Delta B\| \leq 0.14$. Let the following system parametric uncertainties be:

$$\Delta A = \begin{bmatrix} 0.1 & 0 & -0.01 & 0 \\ 0 & 0.1 & 0 & -0.1 \\ -0.1 & 0 & -0.1 & 0 \\ 0.1 & 0 & -0.1 & 0 \end{bmatrix}, \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.4 & 0 & 0.25 \\ -0.5 & 0 & 0 \end{bmatrix} \quad (2.72)$$

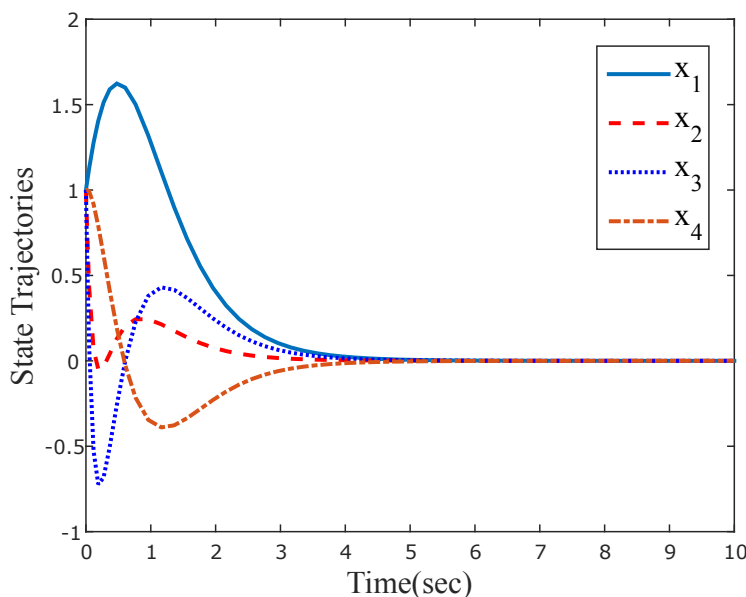


Figure 2.1: Closed loop responses with robust overlapping guaranteed cost control (2.71).

The induced 2-norms of the above uncertainties are $\|\Delta A\| = 0.17$ and $\|\Delta B\| = 0.66$, which are outside the stability bounds determined above, i.e. $\|\Delta A\| = 0.17 > 0.14$ and $\|\Delta B\| = 0.66 > 0.14$. Incorporating (2.72) into the open-loop system Σ (2.69) yields:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 0.5 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} u(t), \\ y(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0.5 & 2 & 0 \end{bmatrix} x(t) \end{aligned} \quad (2.73)$$

The system (2.73) is not stabilisable by any overlapping static output controller, due to the presence of a fixed mode at $\lambda = 1$. For example, the controller (2.71) results in the closed loop system eigenvalues being $\{-4.1 \pm 0.4j, 1, 10.9\}$.

2.10 Summary

In this chapter, first, necessary and sufficient condition for stabilisation of linear time invariant systems with overlapping parts has been provided via the expansion-contraction process. It is proven that there exists an overlapping decentralised static output feedback control for the original system if and only if the minimal expanded system is stabilisable by a decentralised output feedback control (there exists no decentralised QFMs in the expanded system). In this case, the contracted controller stabilises the original system. Also, an iterative algorithm is proposed to design decentralised static output feedback controller which is simultaneously stabilisable and contractible. A numerical example is used to confirm results of this section.

In the second section, new results on robust stabilisability of uncertain linear systems with overlapping static output feedback control have been presented. The results show that if the induced 2-norms of the uncertainties are inside a bounded region, then it is possible to design a robust guaranteed static output cost controller with overlapping structure. A new iterative algorithm based on LMI is applied to the expanded system to find a stabilising robust decentralised static output controller and the maximum upper bounds on the induced 2-norm of the uncertainties. Finally, by using proper contraction, the robust overlapping output controller and the upper bounds on the induced 2-norm of the uncertainties are both contracted to the original uncertain system with an overlapping decomposition. A numerical example has been presented to verify main results of this section.

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Chapter 3

Stabilisation of Overlapping Time-Delay Systems

In this chapter, a sufficient delay-dependent condition for the stabilisation of overlapping linear continuous-time uncertain systems with multiple constant delays by robust overlapping output feedback controllers is given. The overlapping system is first expanded into a higher dimensional system (expanded system) where the overlapping subsystems appear as disjointed from each other. Then, robust local delay-dependent output feedback controllers, for disjoint subsystems, are designed, using an LMI based iterative algorithm. Finally, the designed local controllers are contracted to an overlapping controller to be implemented on the original system. The preservation of stability and performance through contraction is proven. The design approach is used to design an overlapping LFC of an uncertain, state-delayed, 2-area interconnected power system with overlapping parts.

3.1 Introduction

In Chapter 2, stabilisability of overlapping linear systems has been studied, where the inclusion principle is used to design an overlapping static output feedback controller for the system. Although the results of Chapter 2 can be used for both certain and uncertain linear systems, their application in the presence of communication delays is of practical concern. The importance of network delays in stability analysis of complex systems has been discussed in Section 1.1. However, so far, with the exception of [76, 77], communication delays and inherit parametric uncertainty have not received sufficient level of consideration in the design of overlapping controllers. This is despite the fact that communication delays can degrade the system performance, and may even cause instability. In [77], the authors designed a delay-independent, output-feedback controller with overlapping structure for uncertain, single state-delayed, discrete-time systems. An exten-

sion of the inclusion principle is used in [76] to design robust, delay-independent, state-feedback controllers with overlapping structures for uncertain, single state-delayed, continuous-time systems. However, the above results can be improved significantly as explained next: (i) in [76, 77], the decentralised controller design is carried out using the whole higher-dimensional, expanded system, and therefore the decentralised structure of the expanded system is not taken advantage of; (ii) it is well known that state-feedback implementation may only be realizable, from physical viewpoint, if state accessibility is possible and the cost of implementation is not prohibitively high; and (iii) delay-independent approaches are, in general, more conservative than delay-dependent approaches, for the fact that the characteristics of the delay are not used in the design procedure. In order to address the above mentioned issues, this chapter presents an extension of the inclusion principle, such that it is suitable for overlapping, linear, continuous-time, uncertain systems experiencing constant, but unknown, non-commensurate delays. We establish a sufficient condition for the expansion process and use LMI approach to design local delay-dependent, uncertain, multiple state-delayed controllers for each subsystem. Then, we contract the local controllers to an overlapping controller to be implemented on the original system such that stability and performance are preserved. The effectiveness of the proposed approach is demonstrated through the design of an overlapping, robust LFC for an uncertain 2-area interconnected power system with constant communication delays. Simulation results and quantitative comparison criterion clearly show that an improved performance is obtained by our controller in compared with others.

3.2 Preliminaries

Let the uncertain, continuous-time system, Σ , comprising N overlapping subsystems with m non-commensurate delays, be presented as:

$$\begin{aligned}
 \dot{x}(t) &= (A + \Delta A(t))x(t) + \sum_{k=1}^m A_d^{(k)}x(t - d_k) + Bu(t) + B_w w(t), \\
 y(t) &= Cx(t), \\
 z(t) &= C_z x(t), \\
 x(t_0) &= \varphi(t_0), \quad -\max(d_k) \leq t_0 \leq 0
 \end{aligned} \tag{3.1}$$

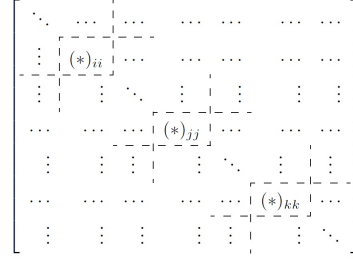


Figure 3.1: The overlapping decomposition of dynamical system (3.1)

where $u(t) \in \mathbb{R}^r$, $x(t) \in \mathbb{R}^n$, and $y(t) \in \mathbb{R}^l$ are input, state, and measured output vectors respectively. $w(t) \in \mathbb{R}^p$ is the norm bounded disturbance, $z(t) \in \mathbb{R}^q$ is the controlled output vector, and $\phi(t_0)$ is the state initial condition. The pointwise delays are assumed to be unknown, but bounded by given constants h_k , i.e. $0 < d_k \leq h_k$. Matrices $A, B, A_d^{(k)}, B_w, C, C_z$ are known with appropriate dimensions. The structured time-varying, norm bounded uncertainty matrix, denoted by $\Delta A(t)$, is assumed to satisfy the following matching condition:

$$\Delta A(t) = GF_A(t)E \quad (3.2)$$

with G, E are known matrices. Matrix $F_A(t)$ is an unknown satisfying the inequality condition $F_A^T F_A \leq I$.

The system (3.1) has N overlapping subsystems, where every two adjacent subsystems share overlapping states and outputs. This structure has been demonstrated in Fig. 1.7. Thus, each of the matrices $A + \Delta A(t)$, $A_d^{(k)}$, and C can be decomposed into overlapping blocks as illustrated in Fig. 3.1. In Fig. 3.1, the state and output variables defined by $x_i \in \mathbb{R}^{n_i}$, $x_j \in \mathbb{R}^{n_j}$, $x_k \in \mathbb{R}^{n_k}$ and $y_i \in \mathbb{R}^{l_i}$, $y_j \in \mathbb{R}^{l_j}$, $y_k \in \mathbb{R}^{l_k}$, respectively, are the overlapping parts of two neighboring subsystems. The decomposition of Fig. 3.1 reflects many real-life, large scale plants, such as interconnected power systems, as detailed in Section 3.5.

Let U and S be full row rank matrices satisfying the conditions $UV = I_n$ and $ST = I_l$. Then, using these conditions and the transformation matrices in (3.5), the expanded system $\tilde{\Sigma}$ can be described as:

$$\begin{aligned}\dot{\tilde{x}}(t) &= (\tilde{A} + \Delta\tilde{A}(t))\tilde{x}(t) + \sum_{i=1}^m \tilde{A}_d^{(i)}\tilde{x}(t - d_k) + \tilde{B}u(t) + \tilde{B}_w w(t), \\ \tilde{y}(t) &= \tilde{C}\tilde{x}(t), \\ z(t) &= \tilde{C}_z\tilde{x}(t), \\ \tilde{x}(t_0) &= \tilde{\varphi}(t_0), -\max(d_k) \leq t_0 \leq 0\end{aligned}\quad (3.6)$$

In (3.6), the system matrices can be expressed as:

$$\begin{aligned}\tilde{A} &= VAU + M, \quad \tilde{A}_d^{(k)} = VA_d^{(k)}U + M_{dk}, \quad \Delta\tilde{A}(t) = V\Delta A(t)U, \\ \tilde{B} &= VB + N, \quad \tilde{B}_w = VB_w + N_w, \quad \tilde{C} = TCU + H, \\ \tilde{C}_z &= C_zU + L\end{aligned}\quad (3.7)$$

where M, M_{dk}, N, N_w, H, L for $k = 1, \dots, m$ are complementary matrices with appropriate dimensions.

The solution and transition matrix corresponding to the expanded system (3.6) are similar to (3.3) and (3.4), respectively, but with the symbol (\sim) added.

To state the problem addressed in this chapter, we need to introduce an extension of the classic definition of the inclusion principle. To do so, let us consider performance levels γ and $\tilde{\gamma}$ for the original and expanded systems Σ and $\tilde{\Sigma}$ respectively such that:

$$\|z(t)\|_2 \leq \gamma \|w(t)\|_2, \quad \|\tilde{z}(t)\|_2 \leq \tilde{\gamma} \|\tilde{w}(t)\|_2 \quad (3.8)$$

Then, an extension of the inclusion principle definition 1.6 is presented next.

Definition 3.1. *The pair $(\tilde{\Sigma}, \tilde{\gamma})$ expressed in (3.6) and (3.8) includes, or is an expansion of, the pair (Σ, γ) , expressed in (3.1) and (3.8), if there exist transformations (U, V, S) , such that for any initial state $\phi(t_0)$, any given input $u(t)$, and any disturbance input $w(t)$, the state transformation $\tilde{\phi}(t_0) = V\phi(t_0)$ results in $x(t; \phi(t_0), u(t), w(t)) = U\tilde{x}(t; \tilde{\phi}(t_0), u(t), w(t)), y(t) =$*

3.4 Main Results

3.4.1 An Extension of The Inclusion Principle

In order to generate the expanded system, given in (3.6), the complementary matrices of (3.7) have to be chosen. Theorem 3.1 below states the conditions to be satisfied by the complementary matrices.

Theorem 3.1. *The pair $(\tilde{\Sigma}, \tilde{\gamma})$ given in (3.6) and (3.8) is an expansion of the pair (Σ, γ) described in (3.1) and (3.8) if and only if:*

$$UM^iV = 0, \quad UM^{i-1}M_{dk}V = 0, \quad UM^{i-1}N = 0, \quad UM^{i-1}N_w = 0, \quad (3.11)$$

$$SHM^{i-1}V = 0, \quad SHM^{i-1}M_{dk}V = 0, \quad SHM^{i-1}N = 0, \quad SHM^{i-1}N_w = 0, \quad (3.12)$$

$$LM^{i-1}V = 0, \quad LM^{i-1}M_{dk}V = 0, \quad LM^{i-1}N = 0, \quad LM^{i-1}N_w = 0, \quad (3.13)$$

for $i = 1, \dots, \tilde{n}$ and $k = 1, \dots, m$.

Proof. The proof of conditions (3.11) is given in [76]. In order to prove (3.12), we use (3.7) into descriptions of $x(t)$ and $\tilde{x}(t)$. Then, we substitute the resultants into the relation $y(t) = S\tilde{y}(t)$ of definition 3.1. It can then be clearly seen that the relation $y(t) = S\tilde{y}(t)$ holds if and only if we have $SH\tilde{\Phi}(t,0)V = 0$, $SH\tilde{\Phi}(t,s)M_{dk}V = 0$, $SH\tilde{\Phi}(t,s)N = 0$, and $SH\tilde{\Phi}(t,s)N_d = 0$ for $t, s \geq 0$ and $k = 1, \dots, m$. Then, using expression of transition matrix $\tilde{\Phi}(t,s)$ by Peano-Baker series and (3.7), it can be shown that the relation $SH\tilde{\Phi}(t,0)V = 0$ is equivalent to $SHM^{i-1}V = 0$ for $i = 1, \dots, \tilde{n}$. Similarly, it can be demonstrated that conditions $SH\tilde{\Phi}(t,s)M_{dk}V = 0$, $SH\tilde{\Phi}(t,s)N = 0$, and $SH\tilde{\Phi}(t,s)N_w = 0$ are equivalent to $SHM^{i-1}M_{dk}V = 0$, $SHM^{i-1}N = 0$, and $SHM^{i-1}N_w = 0$, respectively. Analogously, the set of conditions (3.13) can be proven by the relation $\tilde{C}_z\tilde{x}(t) = C_zx(t)$ originated from the same controlled output vector $z(t)$ in (3.1) and (3.6). \square

Remark: It is worth recalling that conditions (3.11) are the same as those given in [76]. However, the extension of the inclusion principle to an overlapping output-feedback controller requires, in addition to those in [76], conditions (3.12)-(3.13) of theorem 3.1 to be satisfied simultaneously. Choosing the complementary matrices based on the theorem 3.1 is not trivial for large \tilde{n} . However, it is easy to see that conditions of (3.11)-(3.13) hold if the complementary matrices satisfy

the following conditions:

$$MV = 0, M_{dk}V = 0, N = 0, N_w = 0, HV = 0, LV = 0 \quad (3.14)$$

Substituting the singular transformations (3.5), and the complementary matrices satisfying (3.14) into the relations (3.7) yield the expanded system (3.6), comprising the disjoint subsystems given in (3.15):

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= (\tilde{A}_{ii} + \Delta\tilde{A}_{ii}(t)) \tilde{x}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N (\tilde{A}_{ij} + \Delta\tilde{A}_{ij}(t)) \tilde{x}_j(t) + \sum_{k=1}^m \tilde{A}_{dii}^{(k)} \tilde{x}_i(t - d_k) + \\ &\quad \tilde{B}_{ii}u_i(t) + \tilde{B}_{wi}w_i(t), \\ \tilde{y}_i(t) &= \tilde{C}_{ii}\tilde{x}_i(t), \\ z_i(t) &= \tilde{C}_{zi}\tilde{x}_i(t), \\ \tilde{x}_i(t_0) &= \tilde{\varphi}_i(t_0), \quad -\max(d_k) \leq t_0 \leq 0, \end{aligned} \quad (3.15)$$

Consider the local, output-feedback control laws:

$$u_i(t) = \tilde{k}_i\tilde{y}_i(t), \quad i = 1, \dots, N \quad (3.16)$$

Applying these local controllers to subsystems of (3.15) leads to the following closed-loop subsystems:

$$\begin{aligned} \dot{\tilde{x}}_i(t) &= (\tilde{A}_{ii} + \Delta\tilde{A}_{ii}(t) + \tilde{B}_{ii}\tilde{k}_i\tilde{C}_{ii}) \tilde{x}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N (\tilde{A}_{ij} + \Delta\tilde{A}_{ij}(t)) \tilde{x}_j(t) + \sum_{k=1}^m \tilde{A}_{dii}^{(k)} \tilde{x}_i(t - d_k) + \\ &\quad \tilde{B}_{wi}w_i(t), \\ z_i(t) &= \tilde{C}_{zi}\tilde{x}_i(t) \end{aligned} \quad (3.17)$$

The problem then is to design local robust controllers \tilde{k}_i such that closed-loop subsystems (3.17) are stable with disturbance rejection level $\tilde{\gamma}$ i.e. $\sum_{i=1}^N \|z_i(t)\|_2^2 \leq \tilde{\gamma}^2 \sum_{i=1}^N \|w_i(t)\|_2^2$ for all admissible delays and uncertainties. The design procedure is given next.

3.4.2 Robust Decentralised Control Design and LMI

Before addressing the design problem, note that the following relationship is always true:

$$\sum_{i=1}^N \left[(N-1) \tilde{x}_i^T(t) \tilde{x}_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{x}_j^T(t) \tilde{x}_j(t) \right] = 0 \quad (3.18)$$

Also, the uncertainties of the original system ΔA can be translated to uncertainties $\Delta \tilde{A}_{ii}$ and $\Delta \tilde{A}_{ij}$ of (3.15). To this end, substitution of (3.2) into (3.7) leads to:

$$\Delta \tilde{A}(t) = \tilde{G} F_A(t) \tilde{E} \quad (3.19)$$

where

$$\tilde{G} = VG, \quad \tilde{E} = EU \quad (3.20)$$

The uncertainty (3.20) can then be decomposed into lower dimensional sub-matrices as follows:

$$\Delta \tilde{A}_{ii}(t) = \tilde{G}_{ii} F_A(t) \tilde{E}_{ii}, \quad \Delta \tilde{A}_{ij}(t) = \tilde{G}_{ij} F_A(t) \tilde{E}_{ij}, \quad (3.21)$$

Now, we can proceed to the design problem by constructing the Lyapunov-Krasovskii function as follows:

$$\begin{aligned} V(x, t) = & \sum_{i=1}^N \tilde{x}_i^T(t) \tilde{P}_i \tilde{x}_i(t) + \sum_{k=1}^m \int_{t-d_k}^t \tilde{x}_i^T(s) \tilde{J}_{ik} \tilde{x}_i(s) ds + \\ & \int_{-d_k}^0 \int_{t+\theta}^t \tilde{x}_i^T(s) \tilde{R}_{ik} \tilde{x}_i(s) ds d\theta \end{aligned} \quad (3.22)$$

where $\tilde{P}_i, \tilde{R}_{ik}, \tilde{J}_{ik}$ are symmetric positive-definite matrices. Let the derivative of $V(x, t)$ in (3.22) be taken with respect to the closed loop description (3.17). Adding the term (3.18) to the resultant $\dot{V}(x, t)$ and using the delay upper bounds, h_k , we obtain the following upper bound on $\dot{V}(x, t)$.

$$\dot{V}(x, t) \leq \sum_{i=1}^N \tilde{\gamma}^2 w_i^T(t) w_i(t) - \tilde{x}_i^T(t) \tilde{C}_{zi}^T \tilde{C}_{zi} \tilde{x}_i(t) + \tilde{\zeta}_i^T(t) \tilde{\Xi}_i \tilde{\zeta}_i(t) - \sum_{k=1}^m \int_{t-d_k}^t \tilde{x}_i^T(s) \tilde{R}_{ik} \tilde{x}_i(s) ds \quad (3.23)$$

where

$$\begin{aligned} \tilde{\zeta}_i(t) = & \left[\tilde{x}_i^T(t) \quad \tilde{x}_i^T(t-d_1) \quad \dots \quad \tilde{x}_i^T(t-d_m) \right. \\ & \left. \tilde{x}_1^T(t) \quad \dots \quad \tilde{x}_{i-1}^T(t) \quad \tilde{x}_{i+1}^T(t) \quad \tilde{x}_N^T(t) \quad w_i^T(t) \right]^T \end{aligned} \quad (3.24)$$

$$\tilde{\Xi}_i = \begin{bmatrix} \tilde{\phi}_i & \tilde{P}_i \tilde{A}_{dHi} & \tilde{P}_i (\tilde{A}_{Hi} + \Delta \tilde{A}_{Hi}) & \tilde{P}_i \tilde{B}_{wi} \\ * & -Blkdiag \{ \tilde{J}_{ik} \} & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\tilde{\gamma}^2 I \end{bmatrix} \quad (3.25)$$

In (3.25), (*) denotes symmetric part, and

$$\begin{aligned} \tilde{\phi}_i = & (\tilde{A}_{ii} + \Delta \tilde{A}_{ii})^T \tilde{P}_i + \tilde{P}_i (\tilde{A}_{ii} + \Delta \tilde{A}_{ii}) - \tilde{P}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{P}_i + \tilde{C}_{zi}^T \tilde{C}_{zi} + \\ & \left(\tilde{B}_{ii}^T \tilde{P}_i + \tilde{k}_i \tilde{C}_{ii} \right)^T \left(\tilde{B}_{ii}^T \tilde{P}_i + \tilde{k}_i \tilde{C}_{ii} \right) + (N-1)I + \sum_{k=1}^m \tilde{J}_{ik} + h_k \tilde{R}_{ik} \\ \tilde{A}_{Hi} = & \left[\tilde{A}_{i1} \quad \dots \quad \tilde{A}_{i(i-1)} \quad \tilde{A}_{i(i+1)} \quad \dots \quad \tilde{A}_{iN} \right] \\ \Delta \tilde{A}_{Hi} = & \left[\Delta \tilde{A}_{i1} \quad \dots \quad \Delta \tilde{A}_{i(i-1)} \quad \Delta \tilde{A}_{i(i+1)} \quad \dots \quad \Delta \tilde{A}_{iN} \right] \\ \tilde{A}_{dHi} = & \left[\tilde{A}_{dii}^{(1)} \quad \dots \quad \tilde{A}_{dii}^{(m)} \right], \end{aligned} \quad (3.26)$$

Now, let the matrix inequality $\tilde{\Xi}_i < 0$ holds for all admissible delays and uncertainties. Then, we obtain from (3.23):

$$\dot{V}(x, t) < \sum_{i=1}^N \tilde{\gamma}^2 w_i^T(t) w_i(t) - \tilde{x}_i^T(t) \tilde{C}_{zi}^T \tilde{C}_{zi} \tilde{x}_i(t) \quad (3.27)$$

where (3.27) leads to the following conclusions:

- (i) $\dot{V}(x, t) < 0$ when $w_i(t) = 0$, i.e., the closed loop system (3.17) is stable.
- (ii) For zero initial condition, $V(0) = 0$, taking integrals of both sides of (3.27) yields:

$$\int_0^\infty \sum_{i=1}^N -\tilde{\gamma}^2 w_i^T(t) w_i(t) + z_i^T(t) z_i(t) dt < -V(\infty) < 0 \quad (3.28)$$

From (3.28), it is easy to show that the H_∞ performance level $\tilde{\gamma}$ can be obtained through the inequality $\sum_{i=1}^N \|z_i(t)\|_2^2 \leq \tilde{\gamma}^2 \sum_{i=1}^N \|w_i(t)\|_2^2$.

In the sequel, we will show how to formulate the condition $\tilde{\Xi}_i < 0$ into an LMI, which can then be used for control design purposes. To this end, a new variable $\tilde{X}_i = \tilde{X}_i^T > 0$ is introduced to deal with the negative term $-\tilde{P}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{P}_i$ in (3.26). It follows that the inequality $(\tilde{X}_i - \tilde{P}) \tilde{B}_{ii} \tilde{B}_{ii}^T (\tilde{X}_i - \tilde{P}) \geq 0$, which always holds, can be expressed as:

$$\tilde{X}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{X}_i - \tilde{X}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{P}_i - \tilde{P}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{X}_i \geq -\tilde{P}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{P}_i \quad (3.29)$$

Through Lemma 2 [92], expression of uncertainties (3.21), Schur complement, and the inequality (3.29), it can be shown that $\tilde{\Xi}_i < 0$ holds if there exist symmetric, positive-definite matrices $\tilde{P}_i, \tilde{X}_i, \tilde{R}_{ik}, \tilde{J}_{ik}$, and positive scalars $\varepsilon_{1i}, \varepsilon_{2i}, h_k, \tilde{\gamma}$ such that the matrix inequality $\tilde{Y}_i < 0$ shown in (3.31) holds.

$$\tilde{Y}_i = \begin{bmatrix} \tilde{\Psi}_i & \tilde{P}_i \tilde{A}_{dHi} & \tilde{P}_i \tilde{A}_{Hi} & \tilde{P}_i \tilde{B}_{wi} & \tilde{P}_i \tilde{G}_{ii} & \tilde{P}_i \tilde{G}_{Hi} & (\tilde{B}_{ii}^T \tilde{P}_i + \tilde{k}_i \tilde{C}_{ii})^T \\ * & \text{Blkdiag} \{ -\tilde{J}_{ki} \} & 0 & 0 & 0 & 0 & 0 \\ * & * & -I + \tilde{E}_{Hi}^T \varepsilon_{2i} \tilde{E}_{Hi} & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{\gamma}^2 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{1i} I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_{2i} I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0$$

$$\tilde{\Psi}_i = \tilde{A}_{ii}^T \tilde{P}_i + \tilde{P}_i \tilde{A}_{ii} + \tilde{X}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{X}_i - \tilde{X}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{P}_i - \tilde{P}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{X}_i + \sum_{k=1}^m \tilde{J}_{ik} + h_k \tilde{R}_{ik} + (N-1)I + \tilde{C}_{zi}^T \tilde{C}_{zi} + \tilde{E}_{ii}^T \varepsilon_{1i} \tilde{E}_{ii}, \quad (3.30)$$

$$\begin{aligned} \tilde{E}_{Hi} &= \text{Blkdiag} \{ \tilde{E}_{i1}, \dots, \tilde{E}_{i(i-1)}, \tilde{E}_{i(i+1)}, \dots, \tilde{E}_{iN} \}, \\ \tilde{G}_{Hi} &= \begin{bmatrix} \tilde{G}_{i1} & \dots & \tilde{G}_{i(i-1)} & \tilde{G}_{i(i+1)} & \dots & \tilde{G}_{iN} \end{bmatrix} \end{aligned} \quad (3.31)$$

The matrix inequality (3.31) is a quadratic matrix inequality (QMI), which can be converted to a standard LMI with specified \tilde{X}_i, h_k , and $\tilde{\gamma}$. However, the LMI with fixed parameters is known to be conservative, and it may even be infeasible. A more relaxed LMI formulation can be obtained

by solving (3.32), where $\tilde{\alpha}_i$ is a new scalar variable.

$$\tilde{Y}_i < \begin{bmatrix} \tilde{\alpha}_i \tilde{P}_i & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \quad (3.32)$$

Since $\tilde{P}_i > 0$, (3.31) holds if there exists $\tilde{\alpha}_i \leq 0$ such that (3.32) holds. So, we solve the robust, output-feedback control problem by using the iterative algorithm shown in Fig. 3.2 to achieve $\tilde{\alpha}_i \leq 0$. In addition, the binary search (half-interval search) technique is used to minimize the parameter $\tilde{\gamma}$ in (3.32) in order to obtain adequate control performance. The iterative algorithm of Fig. 3.2 comprises three major steps. The first step, identified by green dotted box, is the initialization of the algorithm. The second step, identified by red dashed box, is to find the robust gain \tilde{k}_i with an H_∞ performance level $\tilde{\gamma}$. Finally, the third step, identified by the blue dash-dotted box, performs binary search to minimize the parameter $\tilde{\gamma}$ of (3.32).

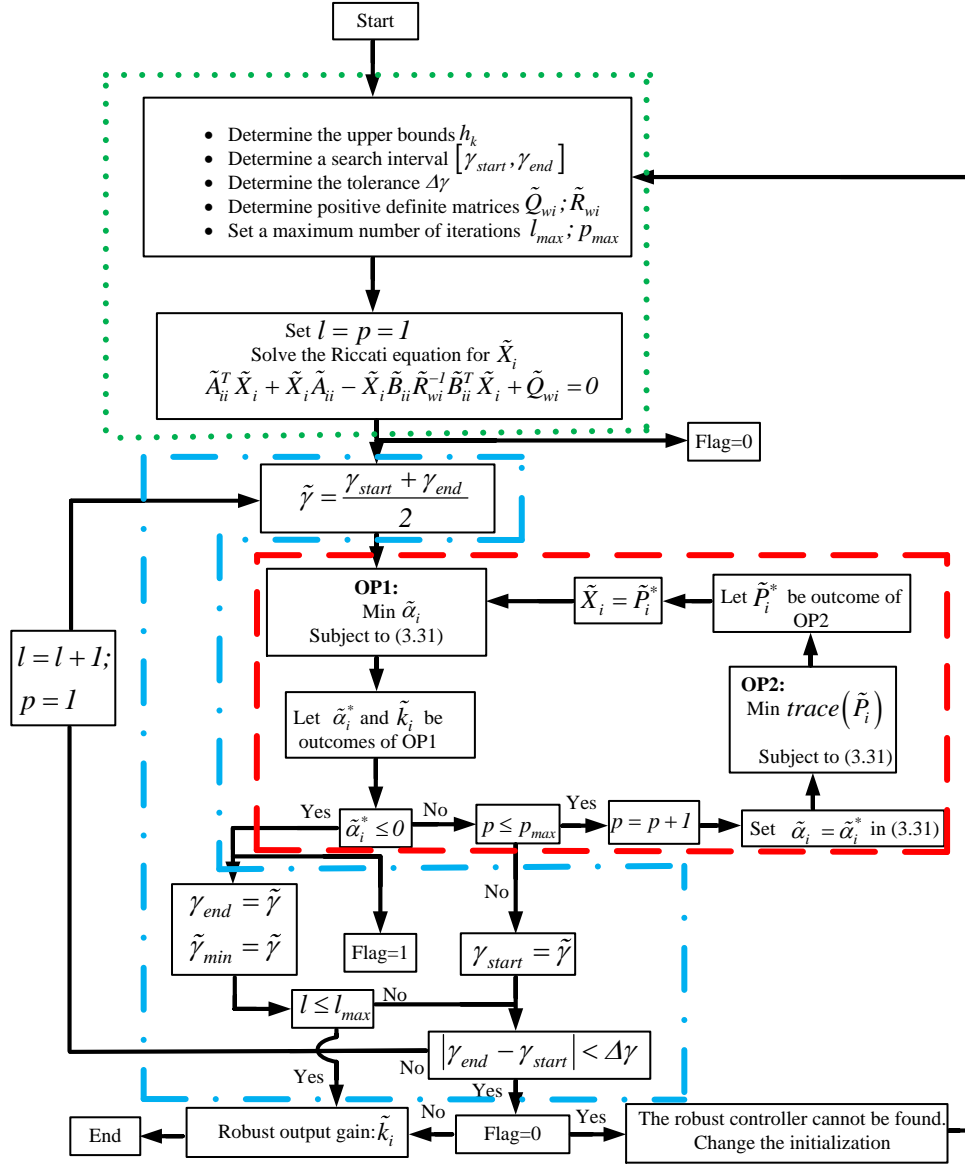
Remark: In the red dashed box, the Optimization Problem 1 (OP1) is the GEVP. Similar to [15], it can be shown that at any two consecutive steps, p and $p + 1$, the OP1 ensures that the values of $\tilde{\alpha}_i$ are such that $\tilde{\alpha}_i^{(p+1)} \leq \tilde{\alpha}_i^{(p)}$. This implies that the iterative algorithm ensures convergence as long as the LMI problem posed in (3.32) is feasible at the first step $p = 1$.

Remark: Note that the feasibility of the problem depends on the initialisation step, where the delay margins, h_k , and the H_∞ performance level search interval $[\gamma_{start}, \gamma_{end}]$ are set. It might be necessary to adjust these margins to ensure feasibility.

Once a decentralised controller $\tilde{K}_D = \text{Blkdiag}\{\tilde{k}_i\}$ is obtained through the iterative algorithm of Fig. 3.2, it is then contracted (transformed) to an overlapping controller K (3.9) for implementation on the original overlapping system (3.1). Due to (3.14), and similar to [68], the transformation back to the original system is done by using T of (3.5):

$$K = \tilde{K}_D T \quad (3.33)$$

With the above material presented, we can now state the main results of this chapter, stated in theorem 3.2 below, provides a sufficient condition for robust stabilisability of the original overlapping

Figure 3.2: The iterative algorithm to design robust \tilde{k}_i

system (3.1).

Theorem 3.2. *Contraction of a decentralised controller $\tilde{K}_D = \text{Blkdiag}\{\tilde{k}_i\}$ formed out of the robust, local, output-feedback controllers \tilde{k}_i with disturbance rejection level $\tilde{\gamma}$ leads to a stabilizing, robust, overlapping, output-feedback controller K for the original system (3.1) with a disturbance rejection level γ , where $\gamma = \tilde{\gamma}$.*

Proof. From the constructed Lyapunov-Krasovskii function (3.22) and design procedure of robust gains \tilde{k}_i , we know:

$$\sum_{i=1}^N \tilde{\zeta}_i^T(t) \tilde{\Xi}_i \tilde{\zeta}_i(t) < 0 \quad (3.34)$$

where $\tilde{\zeta}_i(t)$ and $\tilde{\Xi}_i$ are given in (3.24) and (3.25) respectively.

Using, the inequality $\tilde{C}_{ii}^T \tilde{k}_i^T \tilde{k}_i \tilde{C}_{ii} \geq 0$, equality (3.18), and descriptions of $\tilde{\zeta}_i$ and $\tilde{\Xi}_i$, the matrix inequality (3.34) results in $\tilde{\zeta}^T(t) \tilde{\Omega} \tilde{\zeta}(t) < 0$ where:

$$\tilde{\zeta}(t) = \begin{bmatrix} \tilde{x}^T(t) & \tilde{x}^T(t-d_1) & \dots & \tilde{x}^T(t-d_m) & w^T(t) \end{bmatrix}^T \quad (3.35)$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Theta} & \tilde{P}_D \tilde{A}_d^{(1)} & \dots & \tilde{P}_D \tilde{A}_d^{(m)} & \tilde{P}_D \tilde{B}_w \\ * & -\tilde{J}_{D1} & 0 & 0 & 0 \\ * & * & \ddots & 0 & \vdots \\ * & * & * & -\tilde{J}_{Dm} & 0 \\ * & * & * & * & -\tilde{\gamma}^2 I \end{bmatrix}$$

$$\tilde{\Theta} = \tilde{P}_D (\tilde{A} + \Delta \tilde{A} + \tilde{B} \tilde{K}_D \tilde{C}) + (\tilde{A} + \Delta \tilde{A} + \tilde{B} \tilde{K}_D \tilde{C})^T \tilde{P}_D + \tilde{C}_z^T \tilde{C}_z + \sum_{k=1}^m \tilde{J}_{Dk} + h_k \tilde{R}_{Dk} \quad (3.36)$$

In (3.36), $\tilde{P}_D, \tilde{J}_{Dk}, \tilde{R}_{Dk}$ are symmetric-positive definite matrices defined as below:

$$\tilde{P}_D = \text{Blkdiag} \{ \tilde{P}_i \}, \quad \tilde{J}_{Dk} = \text{Blkdiag} \{ \tilde{J}_{ik} \}, \quad \tilde{R}_{Dk} = \text{Blkdiag} \{ \tilde{R}_{ik} \} \quad (3.37)$$

for $i = 1, \dots, N$.

Now, construct the full column rank matrix $\Pi = \text{Blkdiag} \left\{ V, \underbrace{V, \dots, V}_m, I \right\}$ using matrix V given in (3.5). Then, by pre-post multiplying $\tilde{\Omega}$ (3.36) to Π^T and Π , it can be clearly seen through relations (3.7), conditions on complementary matrices (3.14), and contraction expression (3.33),

the matrix inequality $\Pi^T \tilde{\Omega} \Pi < 0$ is equivalent to (3.38) where $\gamma = \tilde{\gamma}$.

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\psi} & V^T \tilde{P}_D V A_d^{(1)} & \dots & V^T \tilde{P}_D V A_d^{(m)} & V^T \tilde{P}_D V B_w \\ * & -V^T \tilde{J}_{D1} V & 0 & 0 & 0 \\ * & * & \ddots & \vdots & \vdots \\ * & * & * & -V^T \tilde{J}_{Dm} V & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

$$\tilde{\psi} = V^T \tilde{P}_D V (A + \Delta A + BKC) + (A + \Delta A + BKC) V^T \tilde{P}_D V +$$

$$V^T \left(\sum_{k=1}^m \tilde{J}_{Dk} + h_k \tilde{R}_{Dk} \right) V + C_z^T C_z \quad (3.38)$$

In the following, it will be shown that the matrix inequality (3.38) is implying the robust stability of the closed loop (3.10) for all admissible delays and uncertainties. To this end, construct following positive definite Lyapunov-Krasovskii function using \tilde{P}_D , \tilde{J}_{Dk} and \tilde{R}_{Dk} of (3.37):

$$V_L(x, t) = x^T(t) V^T \tilde{P}_D V x(t) + \sum_{k=1}^m \int_{t-d_k}^t x^T(s) V^T (\tilde{J}_{Dk}) V x(s) ds$$

$$+ \int_{-d_k}^0 \int_{t+\theta}^t x^T(s) V^T \tilde{R}_{Dk} V x(s) ds d\theta \quad (3.39)$$

Let the derivative of $V_L(x, t)$ (3.39) be taken respect to (3.10). Then, the following upper bound on $\dot{V}_L(x, t)$ can be obtained:

$$\dot{V}_L(x, t) < \zeta^T(t) \tilde{\Phi} \zeta(t) + \gamma^2 w^T(t) w(t) - x^T(t) C_z^T C_z x(t) -$$

$$\sum_{k=1}^m \int_{t-d_k}^t x^T(s) V^T \tilde{R}_{Dk} V x(s) ds \quad (3.40)$$

with $\tilde{\Phi}$ given in (3.38), and

$$\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-d_1) & \dots & x^T(t-d_m) & w^T(t) \end{bmatrix} \quad (3.41)$$

Similar to (3.23), the inequality (3.40) through $\tilde{\Phi} < 0$ is implying the robust stability of the closed loop system (3.10) with disturbance rejection γ for all admissible delays and uncertainties. So, the theorem is proven. \square

Table 3.1: Parameters of 2-area interconnected power system

$\beta_i(p.u.MW/Hz)$	Frequency bias	K_i	Integral gain
$K_{pi}(Hz/p.u.MW)$	Power system gain	$T_{pi}(sec)$	Power system time constant
$T_{gi}(sec)$	Governor time constant	$T_{chi}(sec)$	Turbine time constant
$R_i(Hz/p.u.MW)$	Speed droop	$T(p.u.MW/Hz)$	Tie-line coefficient

3.5 Application: Load Frequency Control

Load frequency control is crucial in the operation of power systems to maintain the frequency and the power exchange between the areas as close as possible to the scheduled values when load demand exceeds generation. Due to the central roles LFC plays, it has been a subject of much research over many decades as mentioned in Section 1.1.1.

In this section, the proposed robust overlapping delay-dependent design procedure is used for LFC design for a two-area interconnected power system, and the simulation results are compared with those of [26, 50]. The simulation results clearly demonstrate that proposed robust overlapping LFC leads to enhanced performance compared with LFCs of [26, 50] despite the fact that less information is used for feedback with the proposed robust overlapping controller. The reason behind the improved performance is using delay information in the proposed design procedure, while LFCs of [26, 50] are independent of delay's characteristics.

3.5.1 Power System Description

A model of a 2-area interconnected power system is shown in Fig. 3.3 [26]. Table 3.1 shows the system physical parameters, which in real life are not fixed, i.e. they are uncertain, as they may undergo changes due to variation in the operating conditions and also due to age related factors, such wear and tear. The LFC design problem of this section deals with the presence of communication time-delays and parametric uncertainties simultaneously, captured by the following state space model:

$$\begin{aligned}
 \dot{x}(t) &= (A + \Delta A)x(t) + A_d^{(1)}x(t - d_1) + A_d^{(2)}x(t - d_2) + Bu(t) + B_d w(t), \\
 y(t) &= Cx(t), \\
 z(t) &= C_z x(t)
 \end{aligned} \tag{3.42}$$

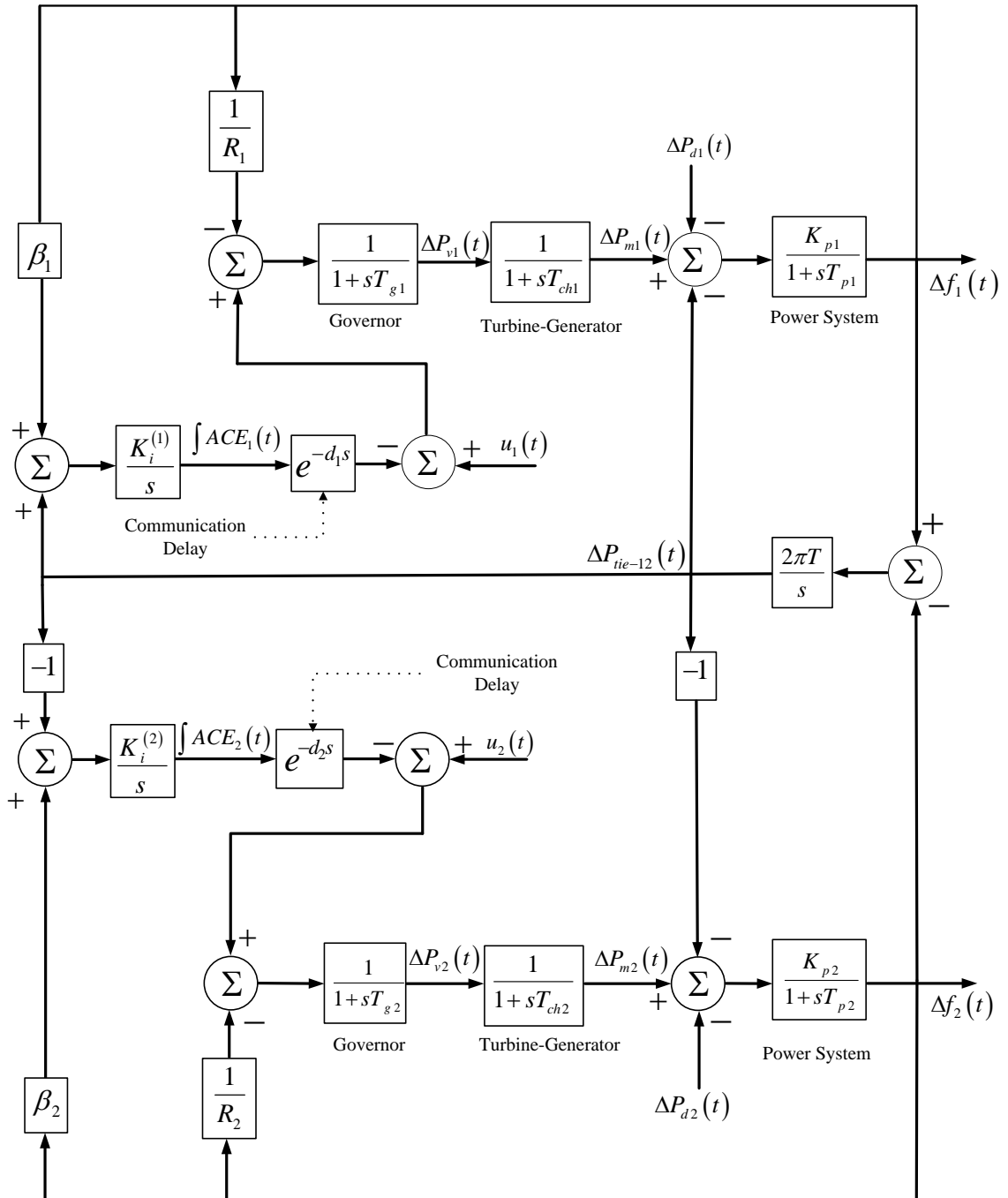


Figure 3.3: A time-delayed 2-area interconnected power system

where the state, input, measured output, controlled output, disturbance variables and system parameter matrices are defined as follows:

The state vector:

$$x(t) = [\Delta f_1(t), \Delta p_{m1}(t), \Delta p_{v1}(t), \int ACE_1, \Delta p_{tie-12}(t), \Delta f_2(t), \Delta p_{m2}(t), \Delta p_{v2}(t), \int ACE_2];$$

The input vector: $u(t) = [u_1(t), u_2(t)]$; The disturbance vector: $w(t) = [\Delta P_{d1}(t), \Delta P_{d2}(t)]$;

The measured output vector: $y(t) = [\Delta p_{m1}(t), ACE_1, \int ACE_1, \Delta p_{tie-12}(t), \Delta p_{m2}(t), ACE_2, \int ACE_2]$;

The controlled output vector: $z(t) = [\Delta f_1(t), \Delta p_{tie-12}(t), \Delta f_2(t)]$ with Δf_i being the frequency deviation in Hz, ΔP_{tie-12} is the tie-line power deviation in p.u.MW, ACE_i is the area control error in p.u.MW, Δp_{mi} is the mechanical power output of the generator in p.u.MW, Δp_{vi} is the governor valve position in p.u.MW, and Δp_{di} is the load demand change in p.u.MW for the i^{th} area. The system matrices $A, A_d^{(1)}, A_d^{(2)}, B, B_d, C, C_z$ are given below where the overlapping decomposition is determined by dashed lines.

$$A = \begin{bmatrix} -\frac{1}{T_{p1}} & \frac{K_{p1}}{T_{p1}} & 0 & 0 & -\frac{K_{p1}}{T_{p1}} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{ch1}} & \frac{1}{T_{ch1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1 T_{g1}} & 0 & -\frac{1}{T_{g1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ K_1 \beta_1 & 0 & 0 & 0 & K_1 & 0 & 0 & 0 & 0 \\ 2\pi T & 0 & 0 & 0 & 0 & -2\pi T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_{p2}}{T_{p2}} & -\frac{1}{T_{p2}} & \frac{K_{p2}}{T_{p2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{ch2}} & \frac{1}{T_{ch2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_2 T_{g2}} & 0 & -\frac{1}{T_{g2}} & 0 \\ 0 & 0 & 0 & 0 & -K_2 & K_2 \beta_2 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
A_d^{(1)} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{g1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & A_d^{(2)} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{g2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
B &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{g1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{T_{g2}} \\ 0 & 0 \end{bmatrix}, & B_d &= \begin{bmatrix} -\frac{K_{p1}}{T_{p1}} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{K_{p2}}{T_{p2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, & C &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ K_1\beta_1 & 0 & 0 & 0 & K_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_2 & K_2\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
C_z &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}
\end{aligned} \tag{3.43}$$

and ΔA is the norm bounded uncertainty with structure shown in (3.2).

Statement of the problem: For the case where (a) the communication time delays d_1 and d_2 are assumed to be constant, unknown, but upper bounded by τ_i , i.e. $0 < d_i \leq \tau_i$, $i = 1, 2$, and (b) the system parameters are uncertain, design an overlapping PI-type LFC that would achieve the following performance objectives in response to changes in the load demands: (i) zero steady state deviation in the frequency of each area; (ii) zero tie-line power exchange in the steady state; and (iii) acceptable transient performance. This PI-design problem can be reformatted as a static output feedback control problem by defining the ACE signal and its integral as state variables [41].

As a result, the feedback control law with an overlapping structure becomes

$$\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} k_1 & k_2 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_5 & k_6 & k_7 & k_8 \end{bmatrix}}_K \begin{bmatrix} \Delta P_{m1}(t) dt \\ ACE_1(t) \\ \int ACE_1(t) \\ \Delta p_{tie-12}(t) \\ \Delta P_{m2}(t) dt \\ ACE_2(t) \\ \int ACE_2(t) \end{bmatrix} \quad (3.44)$$

Applying (3.44) to (3.42) results in the following closed loop system:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A + BKC) x(t) + A_d^{(1)} x(t - d_1) + A_d^{(2)} x(t - d_2) + B_d w(t), \\ z(t) &= C_z x(t) \end{aligned} \quad (3.45)$$

Now, let induced 2-norm of the controlled outputs $z(t)$ be bounded by the following function:

$$\|z(t)\|_2^2 \leq \gamma^2 \|w(t)\|_2^2 \quad (3.46)$$

where $w(t)$ is the load variation and γ is the H_∞ performance index. It is well-known that since γ determines the effect of load variation on controlled outputs, smaller value of γ indicates better performance.

The problem then is to design a static output feedback controller K of the structure shown in (3.44) such that the closed loop (3.45) is stable with minimal H_∞ performance index γ for all admissible uncertainties and delays.

3.5.2 Case Study

In order to show the effectiveness of the proposed fixed structure LFC design, the 2-area interconnected power system shown in Fig. 3.3 is studied. As reported in [26], area 1 is represented by a generator which is equivalent to two generators and area 2 is represented by a generator which is equivalent to four generators. The aim of this exercise is to design an LFC based on the overlapping decomposition principle to (i) provide robustness to parametric uncertainties, (ii) account for the

Table 3.2: Nominal Values of Parameters

	T_{ch} (sec)	T_g (sec)	R (Hz/p.u.MW)	T_p (sec)	K_p (Hz/p.u.MW)	β (p.u.MW/Hz)	T (p.u.MW/Hz)	K_i
Area 1	0.3	0.1	0.05	10	1	41	3	0.5
Area 2	0.17	0.4	0.05	8	0.66	81.5	3	0.5

Table 3.3: Uncertain Parameters

Physical Quantity	Minimum	Maximum
T (p.u.MW/Hz)	1	5
R_1 (Hz/p.u.MW)	0.03	0.09
R_2 (Hz/p.u.MW)	0.03	0.09

communication time-delays, and (iii) achieve superior performance compared to the traditional PI-controller acting on ACE signals. In order to compare the results of our approach with that of [26], the parameters in the primary control loop are chosen to be the same as used in [26] and given in Table 3.2. As the approach in [26] does not consider parameteric uncertainties, we will use the nominal parameter values for the comparison. We will also demonstrate the robustness of the design method to changes in the system parameters by using both upper and lower bounds. The bounded uncertainty in the example is due to tie-line synchronising coefficient, T , and the droop characteristic, R_i , in both areas as given in the Table 3.3.

3.5.3 Robust Delay Dependent LFC Design

Using the expansion process explained in Section 3.4.1, the expanded system, containing the two completely disjoint subsystems $\tilde{\Sigma}_1$ and $\tilde{\Sigma}_2$, is obtained where the first subsystem $\tilde{\Sigma}_1$ is:

$$\begin{aligned}
 \dot{\tilde{x}}_1(t) = & \left(\begin{bmatrix} -0.1 & 0.1 & 0 & 0 & -0.1 \\ 0 & -3.3 & 3.3 & 0 & 0 \\ -200 & 0 & -10 & 0 & 0 \\ 20.5 & 0 & 0 & 0 & 0.5 \\ 18.84 & 0 & 0 & 0 & 0 \end{bmatrix} + \Delta\tilde{A}_{11} \right) \tilde{x}_1(t) \\
 & + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tilde{x}_1(t-d_1) + \left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -18.84 & 0 & 0 & 0 \end{bmatrix} + \Delta\tilde{A}_{12} \right) \tilde{x}_2(t) + \\
 \tilde{\Sigma}_1 : & \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix} u_1(t) + \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} w_1(t), \\
 \tilde{y}_1(t) = & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 20.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tilde{x}_1(t), \quad z_1(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tilde{x}_1(t)
 \end{aligned}$$

and the second subsystem $\tilde{\Sigma}_2$ is:

$$\begin{aligned} \dot{\tilde{x}}_2(t) &= \left(\begin{bmatrix} -0.5 & -18.84 & 0 & 0 & 0 \\ 0.083 & -0.083 & 0.083 & 0 & 0 \\ 0 & 0 & -5.88 & 5.88 & 0 \\ 0 & -200 & 0 & -10 & 0 \\ -0.5 & 40.5 & 0 & 0 & 0 \end{bmatrix} + \Delta\tilde{A}_{22} \right) \tilde{x}_2(t) + \\ \tilde{\Sigma}_2 : & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tilde{x}_2(t - d_2) + \left(\begin{bmatrix} 18.84 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \Delta\tilde{A}_{21} \right) \tilde{x}_1(t) + \\ & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \\ 0 \end{bmatrix} u_2(t) + \begin{bmatrix} 0 \\ -0.083 \\ 0 \\ 0 \\ 0 \end{bmatrix} w_2(t), \\ \tilde{y}_2(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -0.5 & 40.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tilde{x}_2(t), \quad z_2(t) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tilde{x}_2(t) \end{aligned}$$

Step 2) Setting $\gamma_{start} = 1$, $\gamma_{end} = 10$, $\Delta\gamma = 0.5$, $\tilde{Q}_{w1} = \tilde{R}_{w1} = 0.1I_5$, $\tilde{Q}_{w1} = \tilde{R}_{w1} = 0.01I_5$ and $\tau_1 = \tau_2 = 1s$ (delay bounds in both areas) in the iterative algorithm given in Fig. 3.2, the following local output controllers are obtained for the interconnected areas $\tilde{\Sigma}_1$ and $\tilde{\Sigma}_2$ with H_∞ performance $\tilde{\gamma} = 3$:

$$\begin{aligned} \tilde{k}_1 &= \begin{bmatrix} -1.63 & -2.54 & -5.78 & 3.39 \end{bmatrix} \\ \tilde{k}_2 &= \begin{bmatrix} -4.39 & -1.68 & -2.42 & -5.9 \end{bmatrix} \end{aligned} \quad (3.47)$$

Therefore, from (3.47) the decentralised LFC for the expanded system, denoted by \tilde{K}_D , is expressed as:

$$\tilde{K}_D = \begin{bmatrix} -1.63 & -2.54 & -5.78 & 3.39 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4.39 & -1.68 & -2.42 & -5.9 \end{bmatrix} \quad (3.48)$$

Contraction of (3.48) by (3.33), yields the following overlapping robust H_∞ static output feedback LFC:

$$K = \begin{bmatrix} -1.63 & -2.54 & -5.78 & 3.39 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4.39 & -1.68 & -2.42 & -5.9 \end{bmatrix} \quad (3.49)$$

The output feedback LFC has the same overlapping structure as described in (3.44). Based on theorem 3.2, the overlapping controller (3.49) is a robust LFC with $\gamma = 3$ for the original two-area interconnected power system.

3.5.4 Simulation Study

In the following, the LFC given in (3.49) is tested, through computer simulation, on the 2-area system of Fig. 3.3, and the simulation results are compared with one-term full state feedback LFC ([26], Section 4.1.1) and two-terms state feedback LFC ([26], Section 4.1.3). In the first study, let load demand change be $0.01 p.u$ step change in load demand in area 1 at $t = 0 \text{ sec}$ followed by $0.01 p.u$ step change in load demand in area 2 at $t = 20 \text{ sec}$. In the second study, the random load demand change as shown in Fig. 3.4 is considered. In each study, three scenarios are considered. In scenario 1, we use the nominal values of the parameters given in Table 3.2. In scenario 2, we use the lower limits of the uncertainties as given in Table 3.3. Finally, in scenario 3, we use the upper limits on the uncertainties as given in Table 3.3. In all the simulation studies, the communication delays are set (as in [26]) to $d_1 = 0.1 \text{ sec}$ and $d_2 = 0.6 \text{ sec}$.

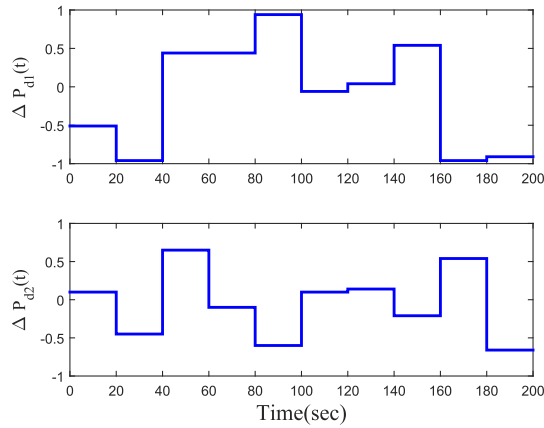


Figure 3.4: Random load demand changes applied to areas 1 and 2

Study 1. Step Load Demand Change:

Let the load demand change be $0.01p.u$ in area 1 at $t = 0$ sec followed by $0.01p.u$ step change in load demand in area 2 at $t = 20$ sec. In the sequel, the simulation results for different values of uncertain parameters are provided.

Scenario 1: In this scenario, the Matlab Toolbox DDE-BIFTOOL [93] shows that the closed loop system is unstable with the right-most pair being $\{0.06 \pm 1.53j\}$. It has been confirmed by simulation study ([26], Fig. 3). However, the closed loop system with the proposed overlapping LFC is stable, as expected, with the rightmost pole at $\{-0.86\}$. The responses of the closed-loop control system for the nominal values are shown in Fig. 3.5a-3.5c, where the responses reported in [26] for the same conditions have been superimposed for clarity of comparison.

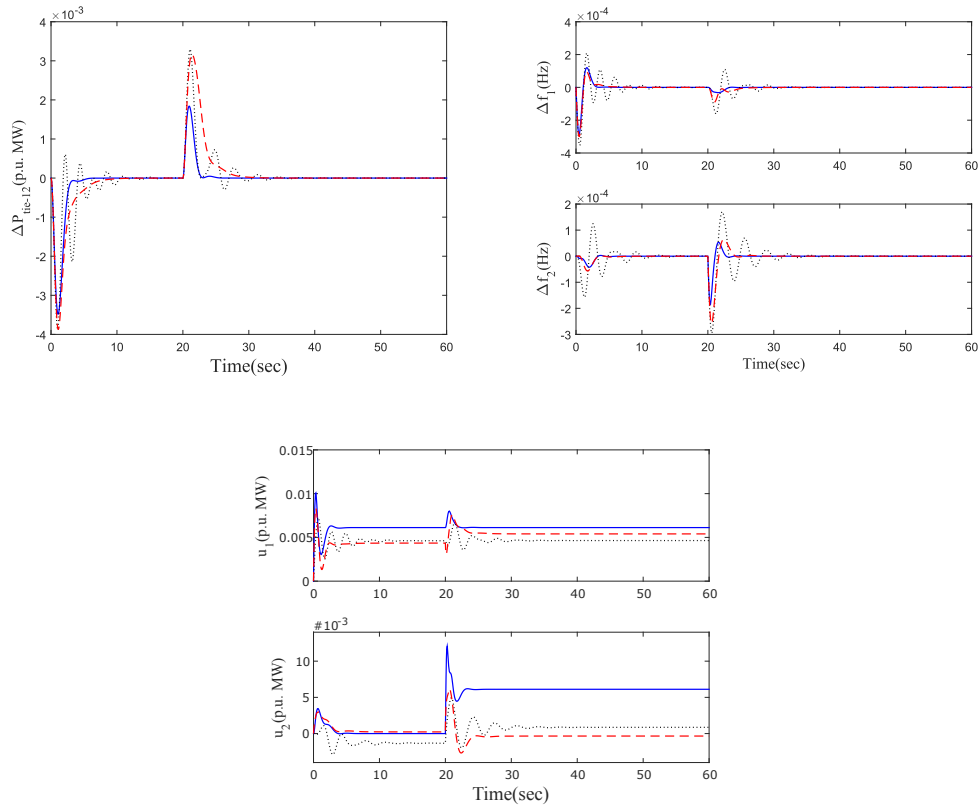


Figure 3.5: Scenario 1 (nominal parameters). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

Figure. 3.5a shows the tie-line power exchange between the two areas. Initially, during transient, the part of the load increase in area 1 is met by importing power from area 2, which then will reduce to zero in the steady state. This signifies the fact that in the steady state all of the local power demand is met locally. A careful examination of the responses shown in the Fig. 3.5a, however, demonstrates clearly the much improved response of the proposed robust output feedback controller designed over that designed in [26]. This is despite the fact that the controller in [26] is state feedback. The improved performance is reflected by the overshoot, settling time and the virtually nonexistence of oscillations.

The responses of the frequency deviations in areas 1 and 2 are shown in Fig. 3.5b. Here again our controller performs significantly better than the one term controller and slightly better than the

two terms controller, especially to responses in load demand changes in area 2, which contains 4 rather than the 2 generators in area 1. Figure. 3.5b shows clearly the improved overshoot and settling time response to area 2 load demand change compared with the two terms state feedback controller of [26].

Figure 3.5c shows the control signals, u_1, u_2 generated by the three controllers. It is clear that much less control effort (energy injection) is needed to damp out the oscillations and bring the system to a steady state than that required by the other two controllers. This is in addition to much less settling time and overshoot.

Scenario 2: Here we repeat the simulation study carried out in scenario 1 but with the nominal values of the system parameters replaced by the lower limits shown in Table 3.3. In this scenario, the open loop system is stable, with the rightmost pole placed at $\{-0.2 \pm 1.5j\}$. The overlapping LFC leads to stable closed loop system with the rightmost pole at $\{-0.25\}$. The closed-loop system performances under the three controllers are shown in Fig. 3.6a-3.6c.

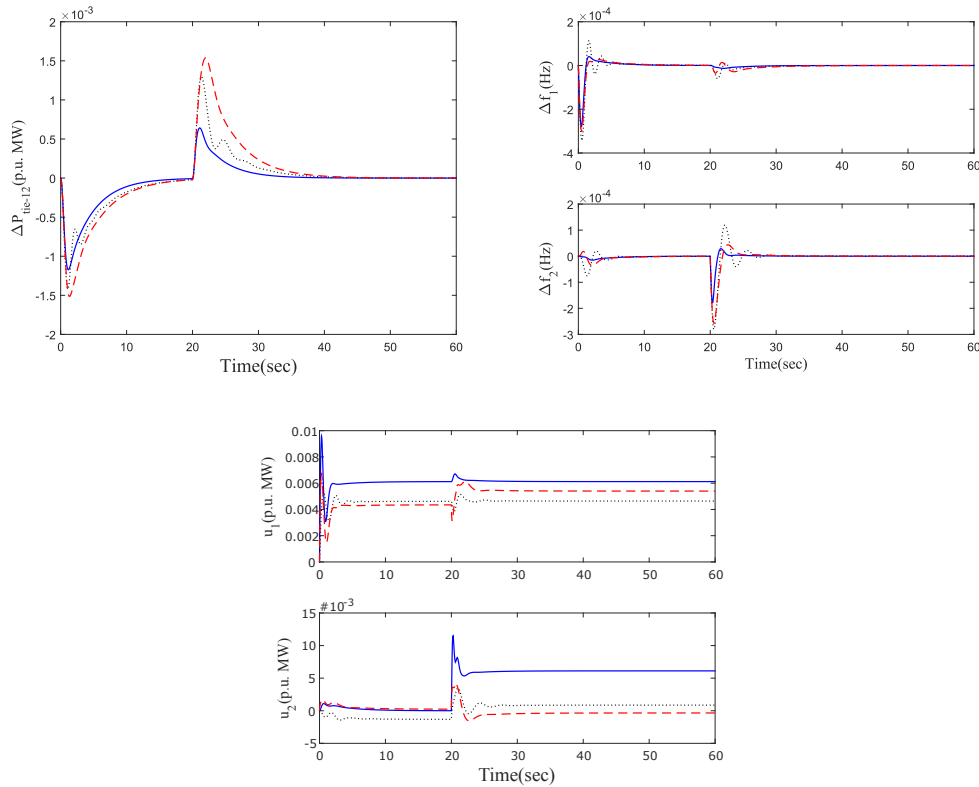


Figure 3.6: Scenario 2 (lower bound). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

The tie-line power responses to initial change in the area 1 load demand then area 2 load demand after 20 seconds are shown in Fig. 3.6a. It is clear that the response of the tie-line power exchange under our controller exhibits fewer oscillations and settles quicker than the other two controllers reported in [26].

Fig. 3.6b illustrates the responses of the frequency deviations in the two areas to the same load demand changes. Here again the same superior performance is exhibited by our controller in terms of overshoot and settling time.

The same analysis can be made with respect to the control signals generated by the three controllers shown in Fig. 3.6c. It is evident from the graphs of the control signals that our controller requires less effort (movement) to accomplish the task of achieving zero steady state error in the tie-line power and frequency and at the same time damping out oscillations in response to load demand

changes.

Scenario 3: Here again we repeat the simulation study carried out in scenario 1 but with the nominal values of the system parameters replaced by the upper limits shown in Table 3.3. The open-loop system is unstable with two complex-conjugate pairs placed in the right-hand s-plane at $\{0.26 \pm 1.4j\}$ and $\{0.01 \pm 2.52j\}$ in this case. The right-most pole, however, in the closed loop system with the proposed overlapping LFC is placed at $\{-0.67 \pm 2.24j\}$. The closed-loop system performances under the three controllers are shown in Fig. 3.7a-3.7c. The tie-line power responses to initial change in the area 1 load demand then area 2 load demand after 20 seconds are shown in Fig. 3.7a. It is clear that the response of the tie-line power exchange under our controller exhibits far superior response than the one term controller and better performance than the two terms controller reported in [26], in terms of fewer oscillations and quicker settling time. The same conclusions hold with respect to the responses of the frequency deviations in the two areas and the fact that our controller requires less effort to eliminate the steady state error in the tie-line power and frequency in both areas more quickly and efficiently than the other two controllers.

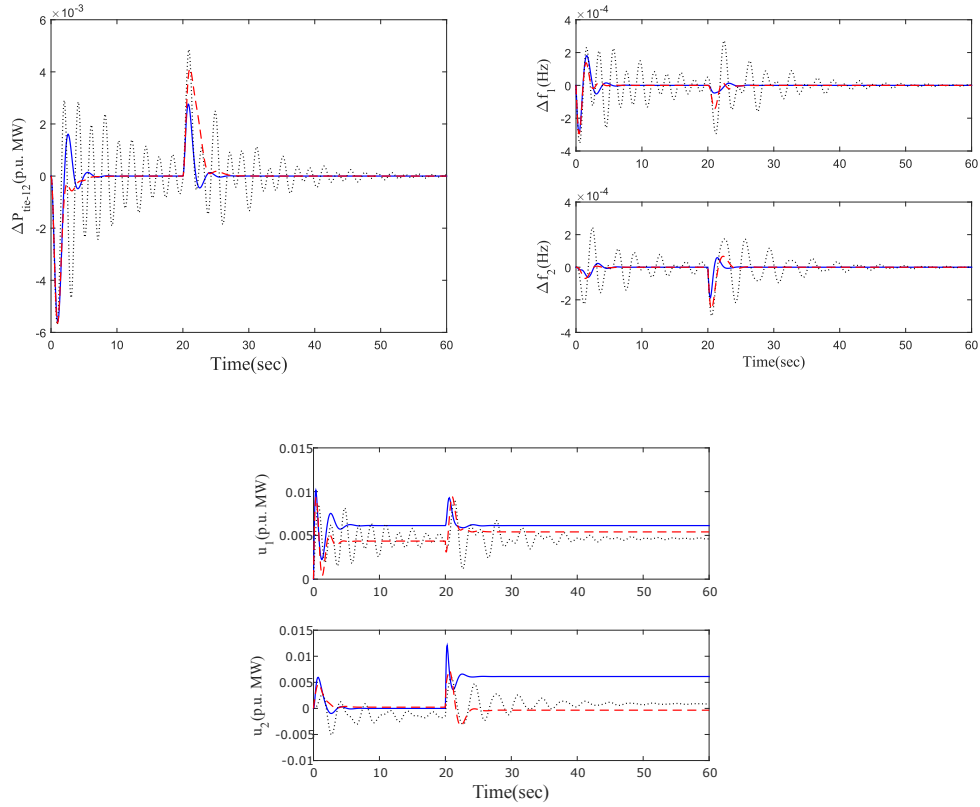


Figure 3.7: Scenario 3 (upper bound). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

Study 2. Random Load Demand Change

Let the random load demand change shown in Fig. 3.4 be applied to areas 1 and 2. In the sequel, the simulation results compared with those of [26] for various values of uncertain parameters, which are given in Table 3.3, are provided.

Scenario 1: The closed loop responses with the nominal parameters of uncertain parameters are given in Fig. 3.8a-3.8c.:

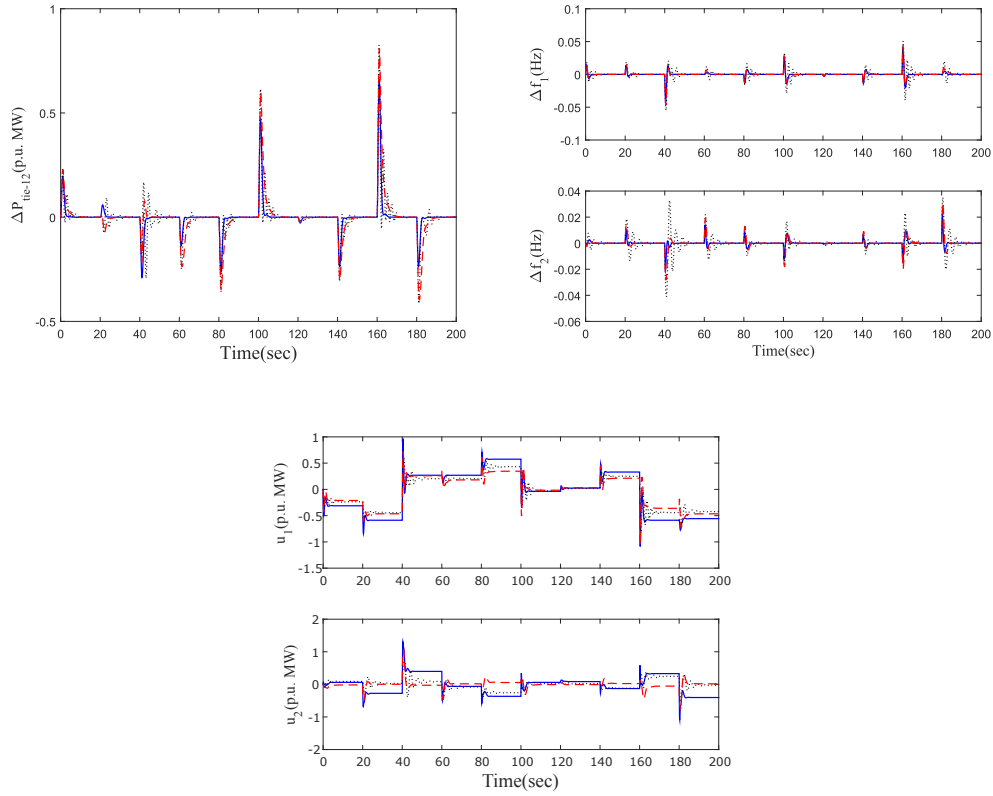


Figure 3.8: Scenario 1 (Nominal parameters). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

Scenario 2: Here, the lower bound of uncertain parameters given in Table 3.3 are considered in the simulation studies. The closed loop responses are demonstrated in Fig. 3.9a-3.9c, where the responses of [26] have been superimposed for the sake of comparison.

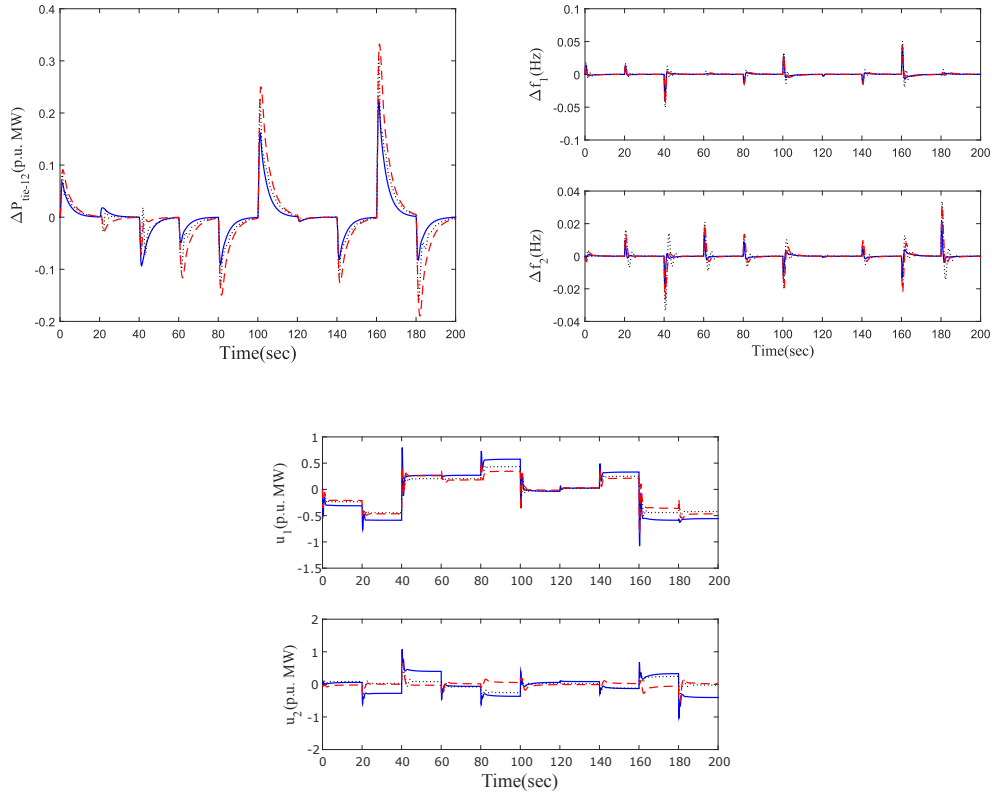


Figure 3.9: Scenario 2 (lower bound). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

Scenario 3: Finally, let the nominal values of uncertain parameters be replaced with the maximum values of Table 3.3. The closed loop responses to random load demand changes are demonstrated in Fig. 3.10a-3.10c:

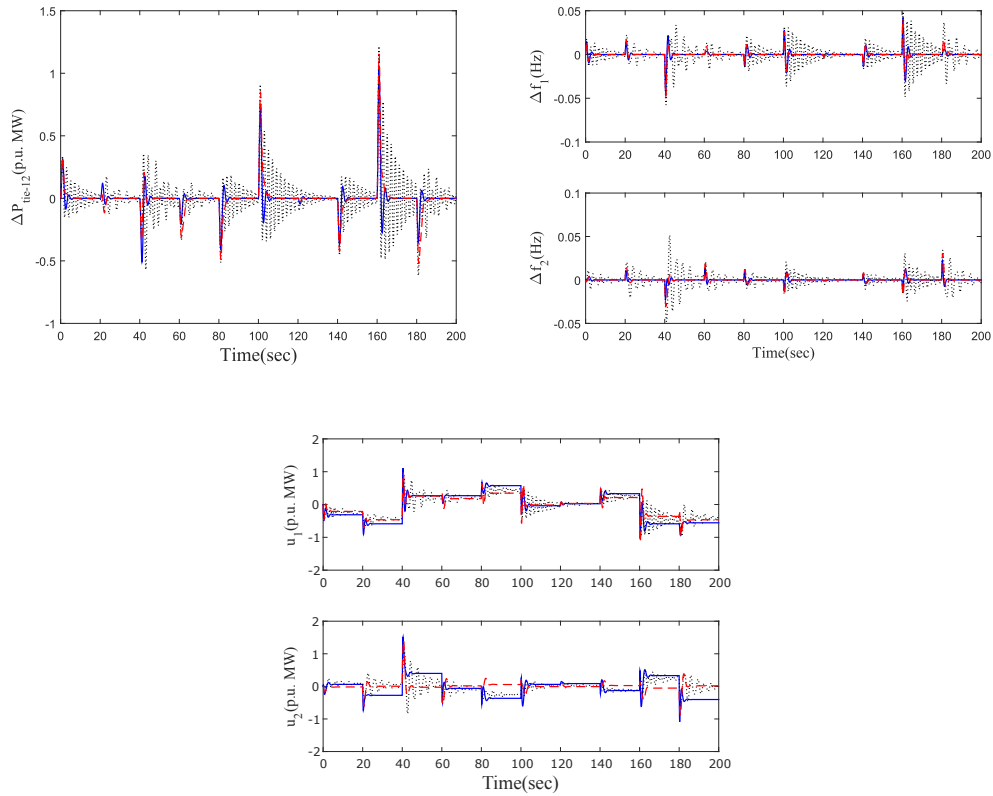


Figure 3.10: Scenario 3 (lower bound). Solid blue line (Proposed overlapping PI-type LFC), dotted black line (One term state feedback Controller [26]), dashed red line (Two terms state feedback Controller [26])

The discussions on simulation results are the same as before. But, it is still worth mentioning that based on the simulation results, performance of proposed overlapping output feedback LFC is better than state feedback LFCs of [26]. It has been confirmed through quantitative criterion in Section 3.5.5.

3.5.5 Performance Comparison

In order to provide a quantitative comparison of the performances of our controller design approach and those reported in [26], the Integral Absolute Error (IAE) criterion of form expressed in (3.50) below is used. The index J reflects the amount of energy required to bring the transients to steady-state, measured by the total absolute value of area under the ACE curve. In this application,

the energy is the amount of steam input to the turbine regulated by the governor action.

$$J = \int_0^{t_f} \{|ACE_1(t)| + |ACE_2(t)|\} dt \quad (3.50)$$

where t_f in study 1 (step load demand change) is 60 sec and study 2 (random load demand change) is 200 sec.

The result of the performance index J of each of the three designs is given in Tables. 3.4-3.5.

Table 3.4: Values of assessment criterion (unit step load demand change)

Type of Controller	Nominal Parameters	Lower bound of uncertain parameters	Upper bound of uncertain parameters
Proposed overlapping PI-type LFC	0.01	0.01	0.02
One term state feedback LFC [26]	0.06	0.03	0.17
Two terms state feedback LFC [26]	0.03	0.03	0.03

Table 3.5: Values of assessment criterion (random step load demand change)

Type of Controller	Nominal Parameters	Lower bound of uncertain parameters	Upper bound of uncertain parameters
Proposed overlapping PI-type LFC	9.79	9.02	12.8
One term state feedback LFC [26]	30.24	18.31	76.03
Two terms state feedback LFC [26]	15.08	16.72	15.02

It is clear from Table 3.4-3.5 that the proposed overlapping robust LFC provides better performance index than the full state feedback LFCs in [26]. This outcome vindicates the results of the simulation studies carried out in the three scenarios in Section 3.5.4. As well as the superior performance, our controller uses local output measurements only, as opposed to the state feedback LFC of [26] that required availability of the entire state variables in both areas to generate the local control signals. The practical implication is that implementation of the LFCs in [26] requires extra communication channels to be installed for information transfer and thus adds more complexity, delays, computational requirements and probability of faults occurring in the communication channels.

3.6 Summary

This chapter provides new results on stabilisability of overlapping, time-delay, continuous-time, uncertain systems, by overlapping static output-feedback controllers. The overlapping subsys-

tem is first expanded such that the overlapping subsystems have become disjointed. Sufficient and necessary conditions for the expansion process are given. Then, local stabilising controllers are designed for disjoint subsystems, using a LMI based iterative algorithm. A stabilising decentralised controller is then formed out of the local controllers, and contracted to a stabilising overlapping controller for implementation on the original system. Preservation of stability and disturbance rejection level through contraction process is proven. The two-area interconnected power system experiencing communication delays and parametric uncertainties is used as the case study. The simulation results, under various scenarios, show that the proposed overlapping LFC provides a better robust performance than the full state feedback LFCs of [26], although it uses local output measurements only. This is vindicated by a quantitative assessment of the performance made in terms of the amount of energy required to damp out the oscillation after a change in the load demand.

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Chapter 4

Robust Controller Design for Overlapping Uncertain Systems With Time-varying Measurement Delay

An extension of the inclusion principle is presented in this chapter to design a robust, overlapping, H_∞ static, output feedback controller for continuous input-delayed uncertain systems with overlapping decomposition. The input delay is unknown but assumed to be time-varying with given upper bounds on the size and derivative of delay. The system considered is linear comprising a number of overlapping subsystems with structured, time-varying, and norm-bounded uncertainties. In this approach, the original overlapping system is first expanded into a higher dimensional system where the overlapping subsystems are completely decoupled. Then, a delay-dependent iterative algorithm based on LMI is proposed to design robust H_∞ local output feedback controllers for the each of the decoupled subsystems. Finally, the local controllers form a decentralised controller which is then contracted (transformed) to a robust overlapping controller for implementation on the original system. The preservation of stability and performance through contraction is proven. The validity of the proposed design approach is demonstrated by designing an LFC for an uncertain 3-area interconnected power system experiencing time-varying measurement delays.

4.1 Introduction

In Chapter 3, uncertain linear systems experiencing constant state-delays are studied. The inclusion principle and design approach based on LMI are proposed to design robust overlapping output feedback controllers. However, the assumption on network delays being constant might be conservative. They may appear due to temporary data unavailability because of physical failure of a communication channel or cyber-attack on data packet. So, it is more relaxed to consider random

or time-varying communication delays in the stability analysis. Furthermore, communication delays are likely to rise in sending measurements from remote terminal units (RTUs) to control centre and from the control centre to the system. These delay can be combined as a single delay and be presented as input delay. Thus, to consider more relaxed and practical conditions, this chapter studies overlapping systems experiencing time-varying input-delay. To this end, first the inclusion principle presented in Chapter 3 is used to expand the overlapping system. Then, a relaxed LMI based iterative algorithm is proposed to design local controllers, which are aggregated to form a decentralised controller. It is then contracted to overlapping controller to be implemented on the original overlapping system. This chapter shows that stability and performance are preserved through the contraction process.

4.2 Preliminaries

In this chapter, we are dealing with uncertain input-delayed continuous system Σ described as:

$$\begin{aligned} \Sigma : \quad & \dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t - d(t)) + B_w w(t), \\ & y(t) = Cx(t), \\ & z(t) = C_z x(t), \\ & x(0) = x_0 \end{aligned} \tag{4.1}$$

Matrix $\Delta A(t)$ represents time-varying norm-bounded uncertainty with the following structure:

$$\Delta A(t) = GF_A(t)E \tag{4.2}$$

with G, E are known matrices with appropriate dimensions. $F_A(t)$ is unknown time-varying matrix satisfying the condition $F_A^T(t)F_A(t) \leq I$.

The unknown time-varying input-delay $d(t)$ in (4.1) is assumed to have the following characteristics:

$$\begin{aligned} 0 < d(t) &\leq \tau, \\ 0 < \dot{d}(t) &\leq \mu \end{aligned} \tag{4.3}$$

where τ (delay margin) and μ (maximum rate of delay change) are determined from the physical system.

Consider another larger dynamical system $\tilde{\Sigma}$ as follows:

$$\begin{aligned} \dot{\tilde{x}}(t) &= (\tilde{A} + \Delta\tilde{A}(t)) \tilde{x}(t) + \tilde{B}u(t - d(t)) + \tilde{B}_w w(t), \\ \tilde{\Sigma} : \quad \tilde{y}(t) &= \tilde{C}\tilde{x}(t), \\ \tilde{z}(t) &= \tilde{C}_z\tilde{x}(t), \\ x(0) &= x_0 \end{aligned} \quad (4.4)$$

where $\Delta\tilde{A}(t)$ represents norm-bounded uncertainty with the following structure:

$$\Delta\tilde{A}(t) = \tilde{G}\tilde{F}_A(t)\tilde{E} \quad (4.5)$$

Let the controlled output of system Σ (4.1) and its expanded form $\tilde{\Sigma}$ (4.4) be bounded by the following inequalities:

$$\begin{aligned} \|z(t)\|_2 &\leq \gamma \|w(t)\|_2 \\ \|\tilde{z}(t)\|_2 &\leq \tilde{\gamma} \|w(t)\|_2 \end{aligned} \quad (4.6)$$

It can be shown similar to Chapter 3 that the larger dynamical system $\tilde{\Sigma}$ includes (or is expansion) of the system Σ based on definition 3.1 if there exist complementary matrices M, N, N_w, H, L satisfying conditions (3.14). Then, the matrices of the expanded system is related to those of the original system Σ as given in (3.7).

4.3 Problem Setup

Let system (4.1) be comprised of N overlapping subsystems where each of the matrices $A + \Delta A(t)$ and C has overlapping blocks on its diagonal as illustrated in Fig. 3.1. The aim of this chapter is to use the inclusion principle and the expanded system $\tilde{\Sigma}$ in order to design a robust, structurally constrained controller, K (3.9), for system (4.1) with the overlapping decomposition of Fig. (3.1), such that for all admissible uncertainties and delays:

1. The closed loop:

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(t))x(t) + BKCx(t - d(t)) + B_w w(t), \\ z(t) &= C_z x(t), \\ x(0) &= x_0\end{aligned}\tag{4.7}$$

is asymptotically stable when $w(t) = 0$.

2. The H_∞ inequality $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ is satisfied by the closed loop system (4.7) with a minimal value of γ .

4.4 Main Results

Similar to Chapter 3, the generated expanded system $\tilde{\Sigma}$ (4.4) comprises of interconnected subsystems can be represented as follows:

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= (\tilde{A}_{ii} + \Delta\tilde{A}_{ii}(t))\tilde{x}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N (\tilde{A}_{ij} + \Delta\tilde{A}_{ij}(t))\tilde{x}_j(t) + \tilde{B}_{ii}u_i(t - d(t)) + \tilde{B}_{wi}w_i(t), \\ \tilde{y}_i(t) &= \tilde{C}_{ii}\tilde{x}_i(t), \\ \tilde{z}_i(t) &= \tilde{C}_{zi}\tilde{x}_i(t), \\ x(0) &= x_0; \quad i = 1, \dots, N\end{aligned}\tag{4.8}$$

Applying the local static output feedback gains $u_i(t) = \tilde{k}_i \tilde{y}_i(t)$; $i = 1, 2, \dots, N$ to interconnected subsystems of the expanded system (4.8) leads to the following closed loop system:

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= (\tilde{A}_{ii} + \Delta\tilde{A}_{ii}(t))\tilde{x}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N (\tilde{A}_{ij} + \Delta\tilde{A}_{ij}(t))\tilde{x}_j(t) + \tilde{B}_{ii}\tilde{k}_i\tilde{C}_{ii}\tilde{x}_i(t - d(t)) + \tilde{B}_{wi}w_i(t), \\ \tilde{z}_i(t) &= \tilde{C}_{zi}\tilde{x}_i(t), \\ x(0) &= x_0; \quad i = 1, \dots, N\end{aligned}\tag{4.9}$$

The problem is then to design local static gains \tilde{k}_i such that closed system (4.9) is robustly stable with H_∞ disturbance attenuation $\tilde{\gamma}_i$, i.e. $\sum_{i=1}^N \|\tilde{z}_i(t)\| \leq \sum_{i=1}^N \tilde{\gamma}_i \|\tilde{w}_i(t)\|$ for all admissible uncertainties and delays. In the sequel, this problem is addressed by adapting the design approach of [15].

But, first the following relations required in the design procedure, are presented

(i) It is easy to show that the following equality always holds:

$$\sum_{i=1}^N \left((N-1) \tilde{x}_i^T(t) \tilde{x}_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{x}_j^T(t) \tilde{x}_j(t) \right) = 0 \quad (4.10)$$

(ii) Based on well-known Newton-Leibinz formula [94], the below equality holds for appropriate dimensional matrices \tilde{Y}_i and \tilde{T}_i :

$$\sum_{i=1}^N 2 \left(\tilde{x}_i^T(t) \tilde{Y}_i + \tilde{x}_i^T(t-d(t)) \tilde{T}_i \right) \left(\tilde{x}_i(t) - \tilde{x}_i(t-d(t)) - \int_{t-d(t)}^t \dot{\tilde{x}}_i(s) ds \right) = 0 \quad (4.11)$$

(iii) For any semi-positive definite matrix $\tilde{X}_i = \begin{bmatrix} \tilde{Z}_i & \tilde{U}_i \\ \tilde{U}_i^T & \tilde{L}_i \end{bmatrix}$, we have [94]:

$$\begin{aligned} & \sum_{i=1}^N \tau \begin{bmatrix} \tilde{x}_i^T(t) & \tilde{x}_i^T(t-d(t)) \end{bmatrix} \begin{bmatrix} \tilde{Z}_i & \tilde{U}_i \\ \tilde{U}_i^T & \tilde{L}_i \end{bmatrix} \begin{bmatrix} \tilde{x}_i(t) \\ \tilde{x}_i(t-d(t)) \end{bmatrix} - \\ & \int_{t-d(t)}^t \begin{bmatrix} \tilde{x}_i^T(t) & \tilde{x}_i^T(t-d(t)) \end{bmatrix} \begin{bmatrix} \tilde{Z}_i & \tilde{U}_i \\ \tilde{U}_i^T & \tilde{L}_i \end{bmatrix} \begin{bmatrix} \tilde{x}_i(t) \\ \tilde{x}_i(t-d(t)) \end{bmatrix} ds \geq 0 \end{aligned} \quad (4.12)$$

(iv) Based on the closed loop description (4.9), the following relation holds for any appropriate dimensional matrix \tilde{G}_i, \tilde{H}_i and $\tilde{J}_i = \tilde{J}_i^T > 0$:

$$\begin{aligned} & \sum_{i=1}^N 2 \left(\tilde{x}_i^T(t) \tilde{G}_i + \tilde{x}_i^T(t) \tilde{H}_i + \tilde{x}_i^T(t-d(t)) \tilde{J}_i \right) \times \\ & \left(-\dot{\tilde{x}}_i(t) + (\tilde{A}_{ii} + \Delta \tilde{A}_{ii}) \tilde{x}_i(t) + \tilde{B}_{ii} \tilde{k}_i \tilde{C}_{ii} \tilde{x}_i(t-d(t)) + \sum_{\substack{j=1 \\ j \neq i}}^N (\tilde{A}_{ij} + \Delta \tilde{A}_{ij}) \tilde{x}_j(t) + \tilde{B}_{wi} w_i(t) \right) = 0 \end{aligned} \quad (4.13)$$

Now, we can proceed to the design problem by considering the well-known Lyapunov-Krasovskii function as follows:

$$V(x, t) = \sum_{i=1}^N \tilde{x}_i^T(t) \tilde{P}_i \tilde{x}_i(t) + \int_{-d(t)}^0 \int_{t+\theta}^t \dot{\tilde{x}}_i^T(s) \tilde{R}_i \dot{\tilde{x}}_i(s) ds d\theta + \int_{t-d(t)}^t \tilde{x}_i^T(\sigma) \tilde{S}_i \tilde{x}_i(\sigma) d\sigma \quad (4.14)$$

where $\tilde{P}_i, \tilde{R}_i, \tilde{S}_i$ are symmetric positive-definite matrices. Let the derivative of $V(x, t)$ be taken respect to the closed loop description (4.9). Using (4.10)-(4.13), the following upper bound on $\dot{V}(x, t)$ is obtained:

$$\dot{V}(x, t) \leq \sum_{i=1}^N \tilde{\gamma}_i^2 w_i^T(t) w_i(t) - \tilde{x}_i^T(t) \tilde{C}_{zi}^T \tilde{C}_{zi} \tilde{x}_i(t) + \tilde{\zeta}_i^T(t) \tilde{\Xi}_i \tilde{\zeta}_i(t) - \int_{t-d(t)}^t \tilde{\rho}_i^T(t, s) \tilde{\Gamma}_i \tilde{\rho}_i(t, s) ds \quad (4.15)$$

In (4.15), for $i, j = 1, 2, \dots, N$ and $j \neq i$, we have:

$$\begin{aligned} \tilde{\zeta}_i(t) &= \left[\tilde{x}_i^T(t) \quad \tilde{x}_i^T(t-d(t)) \quad \dot{\tilde{x}}_i^T(t) \quad w_i^T(t) \quad \tilde{x}_1(t) \quad \dots \quad \tilde{x}_{i-1}(t) \quad \tilde{x}_{i+1}(t) \quad \dots \quad \tilde{x}_N(t) \right]^T, \\ \tilde{\rho}_i(t, s) &= \left[\tilde{x}_i^T(t) \quad \tilde{x}_i^T(t-d(t)) \quad \dot{\tilde{x}}_i^T(s) \right]^T, \end{aligned} \quad (4.16)$$

$$\tilde{\Gamma}_i = \begin{bmatrix} \tilde{Z}_i & \tilde{U}_i & \tilde{Y}_i \\ * & \tilde{L}_i & \tilde{T}_i \\ * & * & \tilde{R}_i \end{bmatrix} \quad (4.17)$$

$$\tilde{\Xi}_i = \begin{bmatrix} \tilde{\Omega}_i & \tilde{\Phi}_i & -\tilde{G}_i + (\tilde{A}_{ii} + \Delta \tilde{A}_{ii})^T \tilde{H}_i^T & (\tilde{P}_i + \tilde{G}_i) \tilde{B}_{wi} & (\tilde{P}_i + \tilde{G}_i) (\tilde{A}_{Hi} + \Delta \tilde{A}_{Hi}) \\ * & \tilde{\Lambda}_i & \tilde{C}_{ii}^T \tilde{k}_i^T \tilde{B}_{ii}^T \tilde{H}_i^T - \tilde{J}_i & \tilde{J}_i \tilde{B}_{wi} & \tilde{J}_i (\tilde{A}_{Hi} + \Delta \tilde{A}_{Hi}) \\ * & * & d \tilde{R}_i - \tilde{H}_i^T - \tilde{H}_i & \tilde{H}_i \tilde{B}_{wi} & \tilde{H}_i (\tilde{A}_{Hi} + \Delta \tilde{A}_{Hi}) \\ * & * & * & -\tilde{\gamma}_i^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} \quad (4.18)$$

In (4.16)-(4.18), $(*)$ denotes symmetric parts and:

$$\begin{aligned}
\tilde{\Omega}_i &= (N-1)I + \tilde{\mathbf{S}}_i + (\tilde{\mathbf{P}}_i + \tilde{\mathbf{G}}_i) (\tilde{\mathbf{A}}_{ii} + \Delta\tilde{\mathbf{A}}_{ii}) + (\tilde{\mathbf{A}}_{ii} + \Delta\tilde{\mathbf{A}}_{ii})^T (\tilde{\mathbf{P}}_i + \tilde{\mathbf{G}}_i)^T + \\
&\quad \tilde{\mathbf{Y}}_i + \tilde{\mathbf{Y}}_i^T + \tau\tilde{\mathbf{Z}}_i + \tilde{\mathbf{C}}_{zi}^T \tilde{\mathbf{C}}_{zi}, \\
\tilde{\Phi}_i &= (\tilde{\mathbf{P}}_i + \tilde{\mathbf{G}}_i) \tilde{\mathbf{B}}_{ii} \tilde{\mathbf{k}}_i \tilde{\mathbf{C}}_{ii} - \tilde{\mathbf{Y}}_i + \tau\tilde{\mathbf{U}}_i + \tilde{\mathbf{T}}_i^T + (\tilde{\mathbf{A}}_{ii} + \Delta\tilde{\mathbf{A}}_{ii})^T \tilde{\mathbf{J}}_i, \\
\tilde{\Lambda}_i &= -(1-\mu) \tilde{\mathbf{S}}_i - \tilde{\mathbf{T}}_i - \tilde{\mathbf{T}}_i^T + \tau\tilde{\mathbf{L}}_i - \tilde{\mathbf{J}}_i \tilde{\mathbf{B}}_{ii} \tilde{\mathbf{B}}_{ii}^T \tilde{\mathbf{J}}_i + \left(\tilde{\mathbf{B}}_{ii}^T \tilde{\mathbf{J}}_i + \tilde{\mathbf{k}}_i \tilde{\mathbf{C}}_{ii} \right)^T \left(\tilde{\mathbf{B}}_{ii}^T \tilde{\mathbf{J}}_i + \tilde{\mathbf{k}}_i \tilde{\mathbf{C}}_{ii} \right), \\
\tilde{\mathbf{A}}_{Hi} &= [\tilde{\mathbf{A}}_{ij}], \\
\Delta\tilde{\mathbf{A}}_{Hi} &= [\Delta\tilde{\mathbf{A}}_{ij}]
\end{aligned} \tag{4.19}$$

The variables in (4.17)-(4.19) have been denoted by bold letters.

Now, let the matrix inequalities $\tilde{\Xi}_i < 0$ and $\tilde{\Gamma}_i \geq 0$ hold for all admissible delays and uncertainties.

Then, we obtain from (4.15):

$$\dot{V}(x, t) < \sum_{i=1}^N \tilde{\gamma}_i^2 w_i^T(t) w_i(t) - \tilde{x}_i^T(t) \tilde{\mathbf{C}}_{zi}^T \tilde{\mathbf{C}}_{zi} \tilde{x}_i(t) \tag{4.20}$$

Expression (4.20) leads to the following conclusions:

- (i) $\dot{V}(x, t) < 0$ when $w_i(t) = 0$ i.e. the closed loop system is stable.
- (ii) By taking integrals from both sides of (4.20) and with zero initial condition, we get:

$$\int_0^\infty \sum_{i=1}^N -\tilde{\gamma}_i^2 w_i^T(t) w_i(t) + \tilde{z}_i^T(t) \tilde{z}_i(t) dt < -V(\infty) < 0 \tag{4.21}$$

From (4.21), it can be clearly seen the inequality $\sum_{i=1}^N \|\tilde{z}_i(t)\|_2^2 \leq \sum_{i=1}^N \tilde{\gamma}_i^2 \|w_i(t)\|_2^2$ holds.

Therefore, it has been shown so far that satisfaction of the matrix inequalities $\tilde{\Xi}_i < 0$ and $\tilde{\Gamma}_i \geq 0$ imply robust stability of the closed-loop system (4.9). However, $\tilde{\Xi}_i < 0$ in (4.18) is not expressed in a standard LMI. To express it as LMI, Similar to Chapter 3, it can be shown that $\tilde{\Xi}_i < 0$ holds if there exist positive matrices $\tilde{\mathbf{Q}}_i, \tilde{\mathbf{P}}_i, \tilde{\mathbf{R}}_i, \tilde{\mathbf{S}}_i, \tilde{\mathbf{L}}_i, \tilde{\mathbf{Z}}_i, \tilde{\mathbf{J}}_i$ and appropriate dimensionally matrices $\tilde{\mathbf{G}}_i, \tilde{\mathbf{H}}_i, \tilde{\mathbf{Y}}_i, \tilde{\mathbf{T}}_i$ feedback gain $\tilde{\mathbf{k}}_i$ and positive scalars $\epsilon_{i1}, \epsilon_{i2}$ such that the matrix inequality $\tilde{\mathbf{Y}}_i < 0$,

shown below, holds:

$$\tilde{\psi}_i = \begin{bmatrix}
 \tilde{\Omega}_i + \tilde{E}_{ii}^T \epsilon_{i1} \tilde{E}_{ii} & -\tilde{Y}_i + \tau \tilde{U}_i + \tilde{T}_i^T + \tilde{A}_{ii}^T \tilde{J}_i & -\tilde{G}_i + \tilde{A}_{ii}^T \tilde{H}_i^T & (\tilde{P}_i + \tilde{G}_i) \tilde{B}_{wi} \\
 * & -(1 - \mu_i) \tilde{S}_i - \tilde{T}_i - \tilde{T}_i^T + \tau \tilde{L}_i + \tilde{Q}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{Q}_i - \tilde{Q}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{J}_i - \tilde{J}_i \tilde{B}_{ii} \tilde{B}_{ii}^T \tilde{Q}_i & -\tilde{J}_i & \tilde{J}_i \tilde{B}_{wi} \\
 * & * & \tau \tilde{R}_i - \tilde{H}_i^T - \tilde{H}_i & \tilde{H}_i \tilde{B}_{wi} \\
 * & * & * & -\gamma_i^2 I \\
 * & * & * & * \\
 * & * & * & * \\
 * & * & * & * \\
 * & * & * & * \\
 * & * & * & * \\
 * & * & * & * \\
 (\tilde{P}_i + \tilde{G}_i) \tilde{A}_{Hi} & (\tilde{P}_i + \tilde{G}_i) \tilde{B}_{ii} & 0 & 0 & (\tilde{P}_i + \tilde{G}_i) \tilde{G}_{ii} & (\tilde{P}_i + \tilde{G}_i) \tilde{G}_{Hi} \\
 \tilde{J}_i \tilde{A}_{Hi} & 0 & \tilde{C}_{ii}^T \tilde{k}_i^T & (\tilde{B}_{ii}^T \tilde{J}_i + \tilde{k}_i \tilde{C}_{ii})^T & \tilde{J}_i \tilde{G}_{ii} & \tilde{J}_i \tilde{G}_{Hi} \\
 \tilde{H}_i \tilde{A}_{Hi} & \tilde{H}_i \tilde{B}_{ii} & 0 & 0 & \tilde{H}_i \tilde{G}_{ii} & \tilde{H}_i \tilde{G}_{Hi} \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 -I + \tilde{E}_{Hi}^T \epsilon_{i2} \tilde{E}_{Hi} & 0 & 0 & 0 & 0 & 0 \\
 * & -I & 0 & 0 & 0 & 0 \\
 * & * & -I & 0 & 0 & 0 \\
 * & * & * & -I & 0 & 0 \\
 * & * & * & * & -\epsilon_{i1} & 0 \\
 * & * & * & * & * & -\epsilon_{i2}
 \end{bmatrix} < 0 \tag{4.22}$$

where $\tilde{G}_{Hi} = [\tilde{G}_{ij}]$ with \tilde{G}_{ij} of (3.21).

The matrix inequality (4.22) is QMI in \tilde{Q}_i which can be converted to a standard LMI with fixed \tilde{Q}_i . However, the LMI with fixed matrix variable is known to be conservative, and it may be infeasible. To provide a more relaxed condition and similar to Chapter 3, a new scalar variable $\tilde{\alpha}_i$

is introduced. Since $\tilde{Q}_i > 0$, it is evident that (4.22) holds, if there exists $\tilde{\alpha}_i \leq 0$ such that:

$$\tilde{\psi}_i < \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & \tilde{\alpha}_i \tilde{Q}_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \quad (4.23)$$

Thus, we solve the robust, output feedback control problem by using the iterative algorithm shown in Fig. 4.1 to achieve $\tilde{\alpha}_i \leq 0$, in which feasibility of (4.23) with $\tilde{\alpha}_i \leq 0$ implies feasibility of (4.22). Moreover, the binary search (half-interval search) technique is used to minimize the parameter $\tilde{\gamma}$ to obtain adequate control performance.

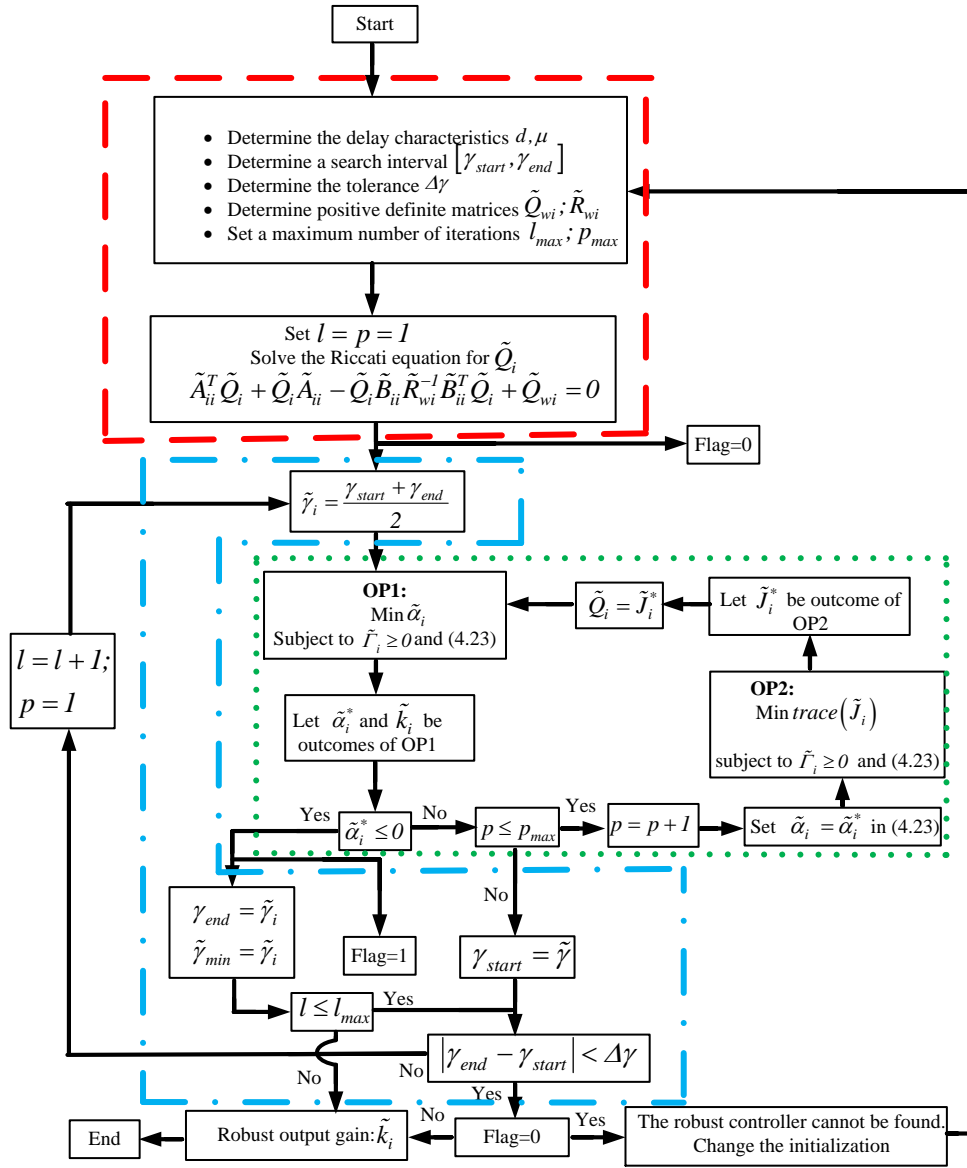


Figure 4.1: The iterative LMI algorithm to design robust local gains \tilde{k}_i

The discussions on iterative algorithm of Fig. 4.1 is similar to those of Fig. 3.2 given in Chapter 3.

4.4.1 Contraction and Stability of The Original Overlapping System

Once robust local controllers \tilde{k}_i are designed by the iterative algorithm of Fig. 4.1, the decentralised (block-diagonal) controller $\tilde{K}_D = \text{Blkdiag}\{\tilde{k}_i\}$ is formed. It is then contracted (trans-

formed) to an overlapping controller K of (3.9) for implementation on the original system. The contraction is done using (3.33).

Next, we prove that the overlapping controller K is also a robust stabilising controller for the original system with H_∞ performance γ , where $\gamma = \max\{\tilde{\gamma}_i\}$.

Theorem 4.1. *Contraction of a decentralised controller \tilde{K}_D formed out of the robust, local, output feedback controllers \tilde{k}_i leads to a stabilizing, robust, overlapping, output feedback controller K for the original system (4.1) with a disturbance rejection level γ , where $\gamma = \max\{\tilde{\gamma}_i\}$.*

Proof. From the constructed Lyapunov-Krasovskii function (4.14) and design procedure of robust gains \tilde{k}_i , we know

$$\sum_{i=1}^N \tilde{\zeta}_i^T(t) \tilde{\Xi}_i \tilde{\zeta}_i(t) < 0, \quad (4.24)$$

$$\sum_{i=1}^N \int_{t-d(t)}^t \tilde{\rho}_i^T(t,s) \tilde{\Gamma}_i \tilde{\rho}_i(t,s) \geq 0 \quad (4.25)$$

where $\tilde{\zeta}_i$, $\tilde{\Xi}_i$, $\tilde{\rho}_i$, and $\tilde{\Gamma}_i$ are given in (4.16)-(4.18).

The inequalities (4.24) and (4.25) using (4.16)-(4.18), lead to (4.26) and (4.27) shown below respectively:

$$\tilde{\xi}^T \tilde{\Omega} \tilde{\xi} < 0, \quad (4.26)$$

$$\int_{t-d(t)}^t \tilde{\eta}^T(t,s) \tilde{\Psi} \tilde{\eta}(t,s) \geq 0 \quad (4.27)$$

where:

$$\tilde{\xi}(t) = \left[\tilde{x}_1^T(t) \quad \dots \quad \tilde{x}_N^T(t) \quad \tilde{x}_1^T(t-d(t)) \quad \dots \quad \tilde{x}_N^T(t-d(t)) \quad \dot{\tilde{x}}_1^T(t) \quad \dots \quad \dot{\tilde{x}}_N^T(t) \quad w_1^T(t) \quad \dots \quad w_N^T(t) \right]^T,$$

$$\tilde{\eta}(t) = \left[\tilde{x}_1^T(t) \quad \dots \quad \tilde{x}_N^T(t) \quad \tilde{x}_1^T(t-d(t)) \quad \dots \quad \tilde{x}_N^T(t-d(t)) \quad \dot{\tilde{x}}_1^T(t) \quad \dots \quad \dot{\tilde{x}}_N^T(t) \right]^T \quad (4.28)$$

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Phi} & \tilde{\Theta} & -\tilde{G}_D + \tilde{A}^T \tilde{H}_D & (\tilde{P}_D + \tilde{G}_D) \tilde{B}_d \\ * & \tilde{\Lambda} & \tilde{C}^T \tilde{K}_D^T \tilde{B}^T \tilde{H}_D^T - \tilde{J}_D & \tilde{J}_D \tilde{B}_d \\ * & * & \tau \tilde{R}_D - \tilde{H}_D^T - \tilde{H}_D & \tilde{H}_D \tilde{B}_d \\ * & * & * & -\tilde{\gamma}^2 I \end{bmatrix} < 0, \quad (4.29)$$

$$\tilde{\Psi} = \begin{bmatrix} \tilde{Z}_D & \tilde{U}_D & \tilde{Y}_D \\ * & \tilde{L}_D & \tilde{T}_D \\ * & * & \tilde{R}_D \end{bmatrix} \geq 0 \quad (4.30)$$

and:

$$\tilde{\Phi} = \tilde{S}_D + (\tilde{P}_D + \tilde{G}_D)(\tilde{A} + \Delta \tilde{A}) + (\tilde{A} + \Delta \tilde{A})^T (\tilde{P}_D + \tilde{G}_D) + \tilde{Y}_D + \tilde{Y}_D^T + \tau \tilde{Z}_D + \tilde{C}_z^T \tilde{C}_z,$$

$$\tilde{\Theta} = (\tilde{P}_D + \tilde{G}_D) \tilde{B} \tilde{K}_D - \tilde{Y}_D + \tau \tilde{U}_D + \tilde{T}_D^T + (\tilde{A} + \Delta \tilde{A})^T \tilde{J}_D,$$

$$\tilde{\Lambda} = -(1 - \mu) \tilde{S}_D - \tilde{T}_D - \tilde{T}_D^T + \tau \tilde{L}_D + \tilde{J}_D \tilde{B} \tilde{K}_D \tilde{C} + (\tilde{J}_D \tilde{B} \tilde{K}_D \tilde{C})^T,$$

$$\tilde{P}_D = \text{Blkdiag}\{\tilde{P}_i\}, \quad \tilde{S}_D = \text{Blkdiag}\{\tilde{S}_i\}, \quad \tilde{G}_D = \text{Blkdiag}\{\tilde{G}_i\}, \quad \tilde{Y}_D = \text{Blkdiag}\{\tilde{Y}_i\}, \quad \tilde{Z}_D = \text{Blkdiag}\{\tilde{Z}_i\},$$

$$\tilde{U}_D = \text{Blkdiag}\{\tilde{U}_i\}, \quad \tilde{T}_D = \text{Blkdiag}\{\tilde{T}_i\}, \quad \tilde{L}_D = \text{Blkdiag}\{\tilde{L}_i\}, \quad \tilde{J}_D = \text{Blkdiag}\{\tilde{J}_i\}$$

(4.31)

Then, from (4.29) and (4.30), we obtain:

$$\text{Blkdiag}\{V^T, V^T, V^T, I\} \tilde{\Omega} \text{Blkdiag}\{V, V, V, I\} < 0,$$

$$\text{Blkdiag}\{V^T, V^T, V^T\} \tilde{\Psi} \text{Blkdiag}\{V, V, V\} > 0 \quad (4.32)$$

where V is the full column rank matrix given in (3.5). Afterwards, it can be clearly seen through (3.43), (3.14) and (3.33), the following inequalities can be concluded from (4.32):

$$\Psi = \begin{bmatrix} \Phi & \Theta & -V^T \tilde{G}_D V + (A + \Delta A)^T V^T \tilde{H}_D V & V^T (\tilde{P}_D + \tilde{G}_D) V B_d \\ * & \Lambda & C^T K_D^T B^T V^T \tilde{H}_D^T V - V^T \tilde{J}_D V & V^T \tilde{J}_D V B_d \\ * & * & \tau V^T \tilde{R}_D V - V^T \tilde{H}_D^T V - V^T \tilde{H}_D V & V^T \tilde{H}_D V B_d \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0,$$

$$\Pi = \begin{bmatrix} V^T \tilde{Z}_D V & V^T \tilde{U}_D V & V^T \tilde{Y}_D V \\ * & V^T \tilde{L}_D V & V^T \tilde{T}_D V \\ * & * & V^T \tilde{R}_D V \end{bmatrix} \geq 0,$$

$$\begin{aligned} \Phi &= V^T \tilde{S}_D V + V^T (\tilde{P}_D + \tilde{G}_D) V (A + \Delta A) + (A + \Delta A)^T V^T (\tilde{P}_D + \tilde{G}_D) V + V^T \tilde{Y}_D V \\ &\quad + V^T \tilde{Y}_D^T V + \tau V^T \tilde{Z}_D V + C_z^T C_z, \\ \Theta &= V^T (\tilde{P}_D + \tilde{G}_D) V B K - V^T \tilde{Y}_D V + \tau V^T \tilde{U}_D V + V^T \tilde{T}_D^T V + (A + \Delta A)^T V^T \tilde{J}_D V, \\ \Lambda &= -(1 - \mu) V^T \tilde{S}_D V - V^T \tilde{T}_D V - V^T \tilde{T}_D^T V + \tau V^T \tilde{L}_D V + V^T \tilde{J}_D V B K C + C^T K^T B^T V^T \tilde{J}_D V, \\ \gamma &= \max\{\tilde{\gamma}_i\}; \quad i = 1, 2, \dots, N \end{aligned} \quad (4.33)$$

In the sequel, it will be proven that the matrix inequalities (4.33) are implying the robust stability of (4.7). To this end, construct the following positive definite Lyapunov-Krasovskii function:

$$V_L(x, t) = \sum_{i=1}^N x^T(t) V^T \tilde{P}_D V x(t) + \int_{-d(t)}^0 \int_{t+\theta}^t x^T(s) V^T \tilde{R}_D V \dot{x}(s) ds d\theta + \int_{t-d(t)}^t x^T(\sigma) V^T \tilde{S}_D V x(\sigma) d\sigma \quad (4.34)$$

Let the derivative of $V_L(x, t)$ be taken respect to the closed loop (4.7). Then, the following upper bound on derivative of $V_L(x, t)$ through the similar relations as (4.10)-(4.13) can be obtained:

$$\dot{V}_L(x, t) \leq \gamma^2 w^T(t) w(t) - x^T(t) C_z^T C_z x(t) + \xi^T(t) \Psi \zeta(t) - \int_{t-d(t)}^t \rho^T(t, s) \Pi \rho(t, s) ds \quad (4.35)$$

where Ψ and Π are given in (4.33). Now, similar to (4.20), it can be clearly seen than the matrix inequalities $\Psi < 0$ and $\Pi \geq 0$ result in the robust stability of the closed loop system (4.7) with disturbance rejection level $\gamma = \max\{\tilde{\gamma}_i\}$ for all admissible delays and uncertainties. \square

4.5 Case Study: Three-Area Power System

In this section, load frequency problem is considered for a three-area interconnected power system experiencing network delays and model's uncertainties. First, a dynamical model of the i^{th} ; $i = 1, 2, 3$ area, comprising 3 generating units, is given in Fig. 4.2. Nominal parameters of generating

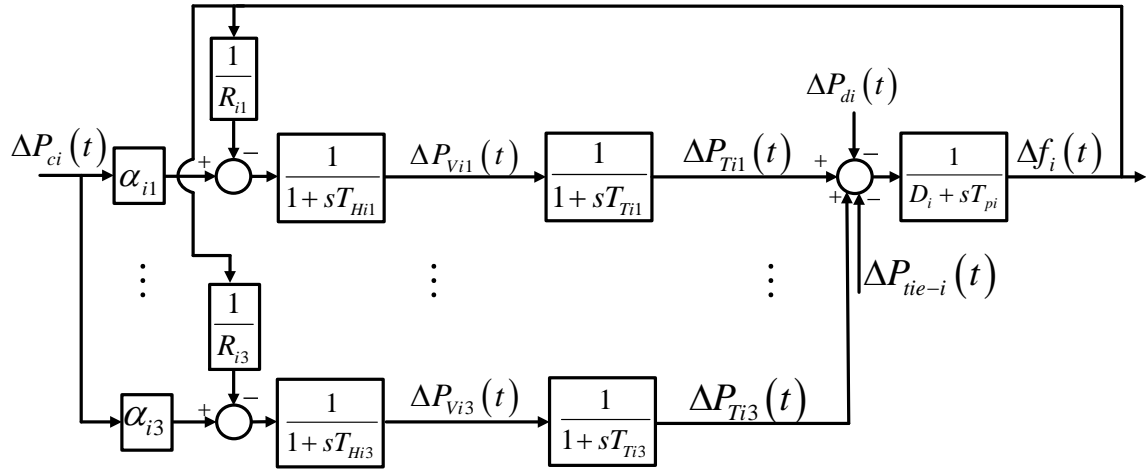
Figure 4.2: Dynamic model of i^{th} area

Table 4.1: Nominal parameters of three area interconnected power system

Generating unit number	1	2	3	4	5	6	7	8	9
$D(pu/Hz)$	0.015	0.014	0.015	0.016	0.014	0.014	0.015	0.016	0.015
$T_P(sec)$	0.1667	0.12	0.2	0.2017	0.15	0.196	0.1247	0.1667	0.187
$T_T(sec)$	0.4	0.36	0.42	0.44	0.32	0.4	0.3	0.4	0.41
$T_H(sec)$	0.08	0.06	0.07	0.06	0.06	0.08	0.07	0.07	0.08
$R(Hz/pu)$	3	3	3.3	2.7273	2.6667	2.5	2.8235	3	2.9412
$\beta(pu/Hz)$	0.3483	0.3473	0.318	0.3827	0.389	0.414	0.3692	0.3493	0.355
α	0.4	0.4	0.2	0.6	0	0.4	0	0.5	0.5

units $\{1, 2, 3\}$, $\{4, 5, 6\}$, and $\{7, 8, 9\}$ which are corresponding to areas 1, 2, and 3 respectively, are given in Table. 4.1 [51,52]. However, the droop characteristics (R) and turbine time-constant (T_T) are assumed to be uncertain with percentage uncertainty of $\pm 10\%$ around their nominal values as given in Table. 4.2. Then, an uncertain state space model of the three-area interconnected power system, where each area is depicted in Fig. 4.2, can be expressed as:

$$\begin{aligned}
 \dot{x}(t) &= (A + \Delta A)x(t) + Bu(t) + B_d w(t), \\
 y(t) &= Cx(t), \\
 z(t) &= C_z x(t)
 \end{aligned} \tag{4.36}$$

Table 4.2: Uncertain Parameters

Physical Quantity	Minimum	Maximum	Physical Quantity	Minimum	Maximum
R_{11}	2.7	3.3	T_{T11}	0.36	0.44
R_{12}	2.7	3.3	T_{T12}	0.32	0.39
R_{13}	2.97	3.63	T_{T13}	0.37	0.46
R_{21}	2.43	2.97	T_{T21}	0.39	0.48
R_{22}	2.34	3.86	T_{T22}	0.28	0.35
R_{23}	2.25	2.75	T_{T23}	0.36	0.44
R_{31}	2.52	3.08	T_{T31}	0.27	0.33
R_{32}	2.7	3.3	T_{T32}	0.36	0.44
R_{33}	2.61	3.19	T_{T33}	0.36	0.45

where

$$\begin{aligned}
x^T(t) &= \left[x_1^T(t) \quad \Delta P_{tie-12}(t) \quad x_2^T(t) \quad \Delta P_{tie-23}(t) \quad x_3^T(t) \right] \\
u^T(t) &= \left[\Delta P_{c1}(t) \quad \Delta P_{c2}(t) \quad \Delta P_{c3}(t) \right] \\
w^T(t) &= \left[\Delta P_{d1}(t) \quad \Delta P_{d2}(t) \quad \Delta P_{d3}(t) \right] \\
y^T(t) &= \left[y_1^T(t) \quad \Delta P_{tie-12}(t) \quad y_2^T(t) \quad \Delta P_{tie-23}(t) \quad y_3^T(t) \right] \\
z^T(t) &= \left[\Delta f_1(t) \quad \Delta f_2(t) \quad \Delta f_3(t) \right]
\end{aligned} \tag{4.37}$$

and

$$\begin{aligned}
x_i^T(t) &= \left[\Delta f_i(t) \quad \int ACE_i(t) \quad \Delta P_{Ti1} \quad \Delta P_{Vi1} \quad \dots \quad \Delta P_{Tin} \quad \Delta P_{Vin} \right] \\
y_i^T(t) &= \left[ACE_i(t) \quad \int ACE_i(t) \right] \quad i = 1, 2, 3
\end{aligned} \tag{4.38}$$

The matrices of state space representation (4.36) are given below where the overlapping decomposition has been determined by dashed lines.:

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & A_{14} & 0 \\ A_{21} & 0 & A_{23} & 0 & 0 \\ 0 & A_{32} & A_{33} & A_{34} & 0 \\ 0 & 0 & A_{43} & 0 & A_{45} \\ 0 & A_{52} & 0 & A_{54} & A_{55} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_3 \end{bmatrix}, \quad B_d = \begin{bmatrix} B_{d1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_{d2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_{d3} \end{bmatrix},$$

$$C_z = \begin{bmatrix} C_{z1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{z2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{z3} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_{11} & C_{12} & 0 & C_{14} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & C_{32} & C_{33} & C_{34} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & C_{52} & 0 & C_{54} & C_{55} \end{bmatrix}, \quad (4.39)$$

where

$$A_{ii} = \begin{bmatrix} AREA_i & MP_i \\ DROOP_i & TG_i \end{bmatrix}, \quad MP_i = \underbrace{\begin{bmatrix} \left[\begin{array}{cc} 1 & 0 \\ T_{pi} & 0 \end{array} \right] & \dots & \left[\begin{array}{cc} 1 & 0 \\ T_{pi} & 0 \end{array} \right] \\ 0 & 0 & 0 \end{bmatrix}}_{n \text{ blocks}}, \quad AREA_i = \begin{bmatrix} -\frac{D_i}{T_{pi}} & 0 \\ \beta_i & 0 \end{bmatrix},$$

$$DROOP_i = \begin{bmatrix} \left[\begin{array}{cc} 0 & 0 \\ -\frac{1}{R_{i1}T_{Hi1}} & 0 \end{array} \right] \\ \vdots \\ \left[\begin{array}{cc} 0 & 0 \\ -\frac{1}{R_{in}T_{Hin}} & 0 \end{array} \right] \end{bmatrix}, \quad TG_i = \text{Blkdiag} \left\{ \left[\begin{array}{cc} -\frac{1}{T_{Ti1}} & \frac{1}{T_{Ti1}} \\ 0 & -\frac{1}{T_{Hi1}} \end{array} \right], \dots, \left[\begin{array}{cc} -\frac{1}{T_{Tin}} & \frac{1}{T_{Tin}} \\ 0 & -\frac{1}{T_{Hin}} \end{array} \right] \right\}$$

(4.40)

The overlapping parts are ΔP_{tie-12} and ΔP_{tie-23} . The variable ΔP_{tie-12} is shared between areas 1 and 2, which is consistent with the fact it is accessible by both areas. Likewise, ΔP_{tie-23} is shared by areas 2 and 3, and thus is considered as the overlapping part between them. Note that since ΔP_{tie-13} is a linear combination of ΔP_{tie-12} and ΔP_{tie-23} as shown in (4.41), it has been omitted from the list of state variables defined in (4.37)-(4.38).

$$\Delta P_{tie-13}(t) = \frac{T_{13}}{T_{12}} \Delta P_{tie-12}(t) + \frac{T_{13}}{T_{23}} \Delta P_{tie-23}(t) \quad (4.41)$$

4.5.1 Problem Statement

It is well-known that a change in load demand results in frequency and tie-line power deviations. The aim of this section is to design an LFC to achieve the following performance objectives (i)

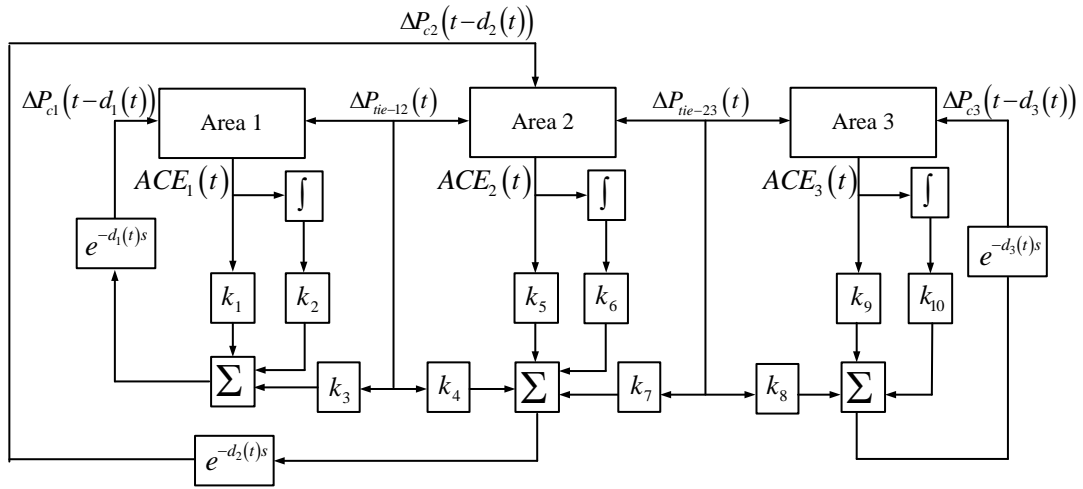


Figure 4.3: Overlapping PI-type LFC in a time-delayed three-area power system

zero steady state frequency deviation in each area (ii) zero steady state tie-line power exchange among the areas, and (iii) acceptable transient performance. In [51] and [52], LFC with decentralised structures have been considered. In other words, each area is controlled separately using a local LFC which uses the ACE signal from the same area (Fig. 1.2). In this chapter, however, the overlapping decomposition shown in (4.39) is taken advantage of. Based on the overlapping decomposition, the overlapping parts (tie-lines), which are locally available, can be added to local controllers in addition to ACE signals to improve the overall performance. This will lead to overlapping LFC. By taking time-varying measurement delays into account, the overlapping LFC is shown in Fig. 4.3, where $k_i; i = 1, 2, \dots, 10$ are feedback gains which have to be designed.

Based on Fig. 4.3, the control law can be expressed as:

$$\begin{bmatrix} \Delta P_{C1}(t) \\ \Delta P_{C2}(t) \\ \Delta P_{C3}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} k_1 & k_2 & k_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_4 & k_5 & k_6 & k_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_8 & k_9 & k_{10} \end{bmatrix}}_K \begin{bmatrix} ACE_1(t) \\ \int ACE_1(t) \\ \Delta P_{tie-12}(t) \\ ACE_2(t) \\ \int ACE_2(t) \\ \Delta P_{tie-23}(t) \\ ACE_3(t) \\ \int ACE_3(t) \end{bmatrix} \quad (4.42)$$

It is worth mentioning that considering ACE and its integral as state variables allows us to formulate the PI control problem as a static output feedback problem [41, 46, 51].

Applying the control law (4.42) to the state space representation (4.36) results in the time-delayed closed loop system being expressed by:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + BKCx(t - \tau(t)) + B_d w(t), \\ z(t) &= C_z x(t) \end{aligned} \quad (4.43)$$

The aim of this section is designing an overlapping static output feedback controller K of the structure given in (4.42) such that the closed loop system (4.43) is asymptotically stable with a minimal H_∞ performance level γ i.e. $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ for all admissible delays and uncertainties. The simulation results are also compared with decentralised PI and Proportional-Integral-Derivative (PID) type LFCs proposed in [51] and [52] respectively. These two controllers are given in Table. 4.3:

Table 4.3: Reported decentralised controllers in [51] and [52]

Area	K_1 [51]		K_2 [52]		
	K_p	K_I	K_p	K_I	K_D
1	-0.2728	-0.2296	0.0669	-0.0615	-0.0311
2	-0.1475	-0.1773	0.0305	-0.0885	-0.0325
3	-0.2142	-0.2397	0.0704	-0.0688	-0.0302

4.5.2 Robust Overlapping LFC Design

In order to design a robust, overlapping LFC based on the results of this chapter, first, based on the overlapping decomposition of under study power system, the singular transformations V and T are chosen as:

$$V \in \mathbb{R}^{28 \times 26} = \begin{bmatrix} I_8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & I_8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I_8 \end{bmatrix}, \quad T \in \mathbb{R}^{10 \times 8} = \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I_2 \end{bmatrix} \quad (4.44)$$

Afterwards, the complementary matrices satisfying (3.14) are selected, and consequently, the expanded system can be generated using (3.7). Then, the iterative algorithm of Fig. 4.1 is used to design local robust LFCs.

In order to provide a fair comparison with the reported controllers of [51, 52] given in Table. 4.3, three studies are considered. In the first study, the delay margin d is chosen to be the same as that used in [51] to obtain K_1 , i.e. $d = 3 \text{ sec}$. In the second study, the delay margin d is chosen to be 10 sec , which is the same delay margin used in [52] to obtain K_2 . In both studies 1 and 2, the nominal values of parameters given in Table 4.1 are used. However, in the third study, the designed controller of study 2 is used to provide simulation results based on different values of

uncertain parameters given in Table. 4.2. In all studies, to start the iterative algorithm, the maximum number of iterations are chosen as $l_{max} = 4$ and $p_{max} = 6$. The search interval for H_∞ criterion with $\Delta\gamma = 0.5$ is chosen to be $\gamma_{start} = 2$ and $\gamma_{end} = 11$ for area 1 and $\gamma_{start} = 1$ and $\gamma_{end} = 10$ for areas 2 and 3. The intervals, for the sake of comparison, are chosen such that they include the minimal H_∞ performance levels obtained in [51,52]. The upper bound on rate of delay change μ is set 1. Also, setting $\tilde{Q}_{wi} = I, \tilde{R}_{wi} = I, \epsilon_i = 400; i = 1, 2, 3$ has shown to provide fast convergence in the iterative algorithm.

Study 1:

Let the delay margin be $d = 3$ sec, then using the design algorithm of Fig. 4.1, the following robust output feedback gains \tilde{k}_1, \tilde{k}_2 , and \tilde{k}_3 with H_∞ performance levels $\tilde{\gamma}_1 = 2.56, \tilde{\gamma}_2 = 1.56$, and $\tilde{\gamma}_3 = 1.56$ are obtained after 4 iterations.

$$\tilde{k}_1 = \begin{bmatrix} -0.14 & -0.24 & 0.83 \end{bmatrix}, \tilde{k}_2 = \begin{bmatrix} -0.13 & -0.46 & -0.24 & 0.25 \end{bmatrix}, \tilde{k}_3 = \begin{bmatrix} -1.18 & -0.2 & -0.24 \end{bmatrix} \quad (4.45)$$

Using (4.45), the decentralised controller is obtained as:

$$\tilde{K}_D = \begin{bmatrix} -0.14 & -0.24 & 0.83 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.13 & -0.46 & -0.24 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.18 & -0.2 & -0.24 \end{bmatrix} \quad (4.46)$$

Contraction of (4.46) using $K = \tilde{K}_D T$ leads to the following robust overlapping LFC gain K (4.47):

$$K = \begin{bmatrix} -0.14 & -0.24 & 0.83 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.13 & -0.46 & -0.24 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.18 & -0.2 & -0.24 \end{bmatrix} \quad (4.47)$$

It has to be mentioned that in order to consider the physical constraints in simulation studies, each generator unit is modelled by the nonlinear model, as shown in Fig. 4.4, by taking generation rate constraint (GRC) into account. The lower and upper limits of the saturation in Fig. 4.4 are chosen

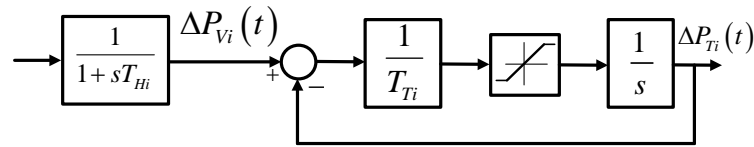


Figure 4.4: Nonlinear model of a generating unit

as $0.05pu/min$ and $-0.2pu/min$ respectively, [51,52]

Simulation study 1: The designed controller (4.47) is tested, by computer simulations, on the three-area interconnected power system. For a constant communication delay $\tau = 2 sec$, step load demand changes $0.1p.u$, $0.08p.u$, and $0.05p.u$ are applied to areas 1, 2 and 3 at $t = 0 sec$, respectively. The responses of three areas are shown in Fig. 4.5-4.7, where the responses of K_1 [51] have been superimposed for comparison. Based on Fig. 4.5b, the frequency of area 1 experiences a transient decrease immediately after load exceeds generation. After the transitional period, the local PI control generates the control signal which allows more power to be generated to meet the load demand change. As a result, the frequency deviation gets back to zero steady state gain. With respect to power exchange of area 1, Fig. 4.5a demonstrates a transient dip showing the import of power from the other two areas. This is expected as area 1 experiences the highest load demand change compared with the other two areas. After a while, the governor responds to the load demand change, and the generated power of area 1 increases. When the generated power meets the demanded one, the tie-line power exchange of area 1 with other areas reaches zero steady state gain. The improved performance respect to settling time (within 3% band of total load demand change) and frequency of oscillation obtained through robust overlapping load frequency controller (4.47) compared with robust decentralised K_1 of [51] can be seen through examination of Fig. 4.5. With respect to frequency response of area 2, Fig. 4.6b demonstrates the transient undergo after load demand changes occur. After a short period of time, however, local PI controller sends the control signal to allow the steam valve open more, and consequently more power is generated. This provides zero frequency deviation after a while. Also, there is a slight rise in tie-line power exchange suggesting the export of power from area 2, but, the power exchange reaches zero steady state when load demand is met locally. Figure 4.6 clearly demonstrates less oscillations with proposed overlapping controller compared with decentralised K_1 of

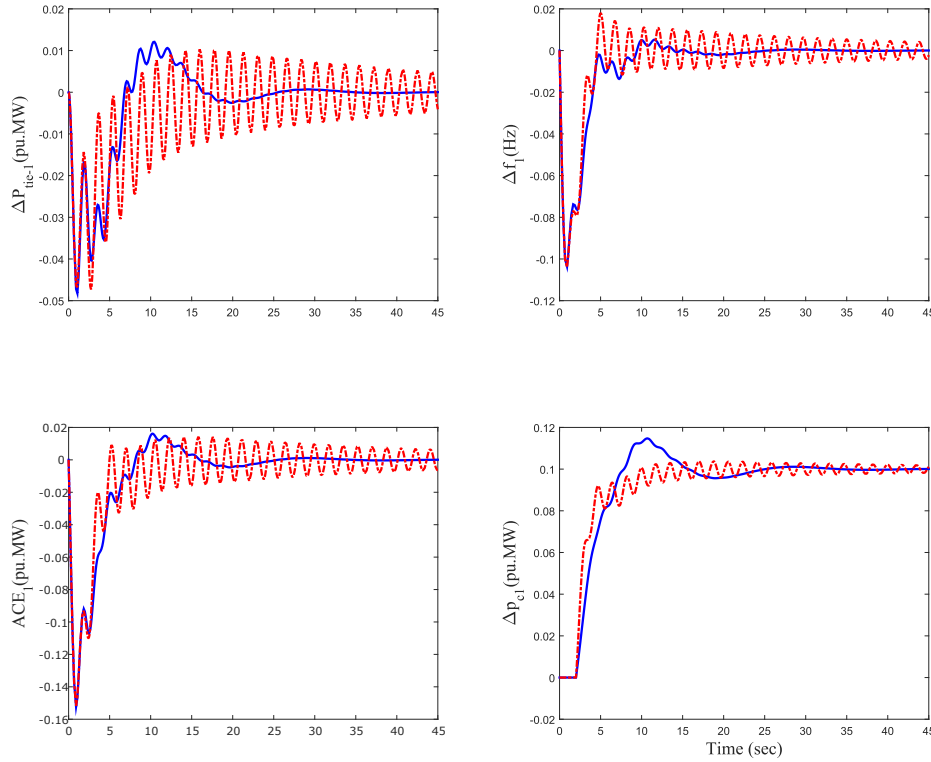


Figure 4.5: Study 1. Step change responses of area 1 (a) tie-line power deviation (b) frequency deviation. (c) ACE signal (d) control input. Solid blue line (Proposed overlapping PI-type LFC), dash-dotted red line (decentralised PI-type LFC [51])

[51]. The improved performance obtained with the proposed overlapping controller has been verified through quantitative criteria of (4.50).

Analogously, the frequency of area 3 undergoes a transient dip which disappeared after a short time due to existence of PI controllers. However, there exists a rise in tie-line power deviation after load demand increases. This means that area 3 is exporting power to other areas. It is noteworthy that the rise in tie-line power exchange is more than that of area 2 given in Fig. 4.6a. This is expected as the smallest load demand change occurs in area 3.

Careful examination of Fig. 4.5a-4.7d reveals that the proposed controller provides better responses in respect of settling time and oscillation compared with K_1 [51] in all areas.

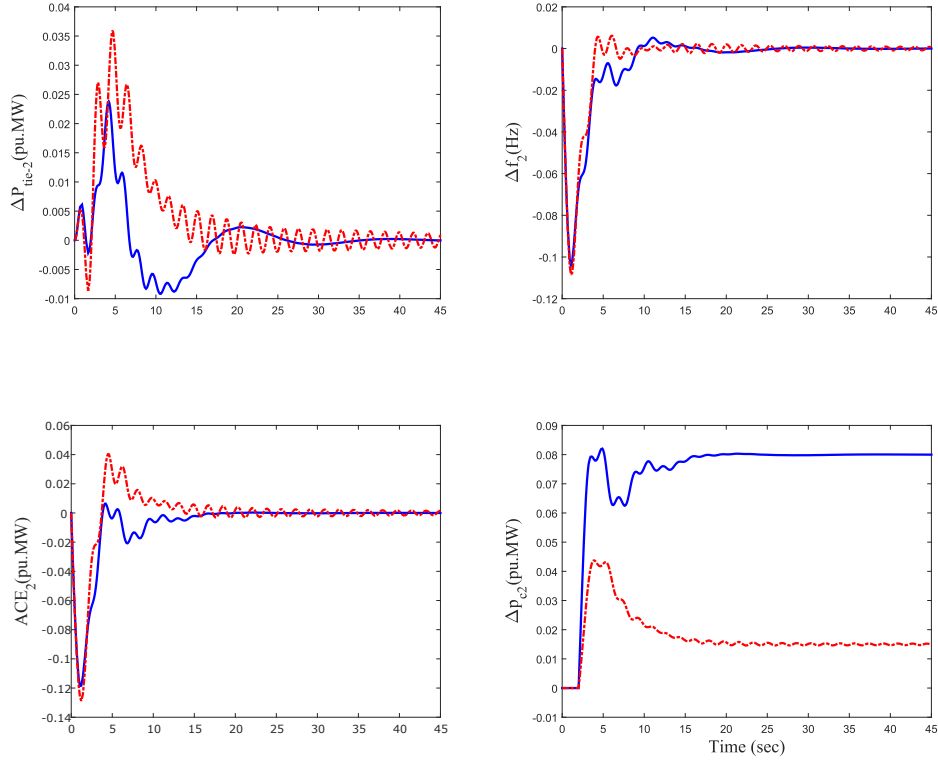


Figure 4.6: Study 1. Step change responses of area 2 (a) tie-line power deviation (b) frequency deviation. (c) ACE signal (d) control input. Solid blue line (Proposed overlapping PI-type LFC), dash-dotted red line (decentralised PI-type LFC [51])

Study 2:

In this case, the delay margin d increases to 10 sec, which is the same as that used in [52] to obtain K_2 . The following robust local output feedback controllers with H_∞ performance levels $\tilde{\gamma}_1 = 2.56$, $\tilde{\gamma}_2 = 1.56$, and $\tilde{\gamma}_3 = 1.56$ are obtained:

$$\tilde{k}_1 = \begin{bmatrix} -0.05 & -0.09 & 0.31 \end{bmatrix}, \quad \tilde{k}_2 = \begin{bmatrix} -0.02 & -0.17 & -0.09 & 0.06 \end{bmatrix}, \quad \tilde{k}_3 = \begin{bmatrix} -0.47 & -0.06 & -0.09 \end{bmatrix} \quad (4.48)$$

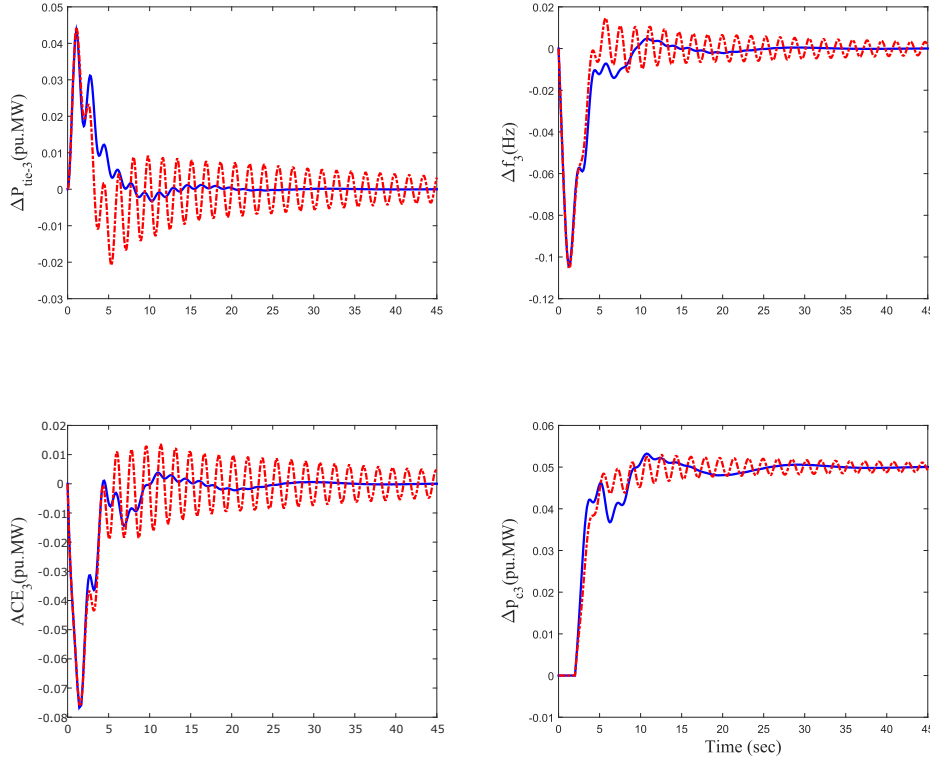


Figure 4.7: Study 1. Step change responses of area 3 (a) tie-line power deviation (b) frequency deviation. (c) ACE signal (d) control input. Solid blue line (Proposed overlapping PI-type LFC), dash-dotted red line (decentralised PI-type LFC [51]).

The decentralised controller $\tilde{K}_D = \text{Blkdiag} \{ \tilde{k}_1, \tilde{k}_2, \tilde{k}_3 \}$ is then contracted to the following overlapping structured controller:

$$K = \begin{bmatrix} -0.05 & -0.09 & 0.31 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.02 & -0.17 & -0.09 & 0.06 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.47 & -0.06 & -0.09 \end{bmatrix} \quad (4.49)$$

Simulation study 2: The controller given in (4.49) is then applied to the same three-area interconnected power system under the same conditions as described in Scenario 1 except that the network delay increases to 5 sec. The results are shown in Fig. 4.8-4.10, where the responses of controller K_2 [52] are superimposed. Based on Fig. 4.8b, the frequency drops down after load demand increases. Due to communication delay exists in sending control input signals, the fre-

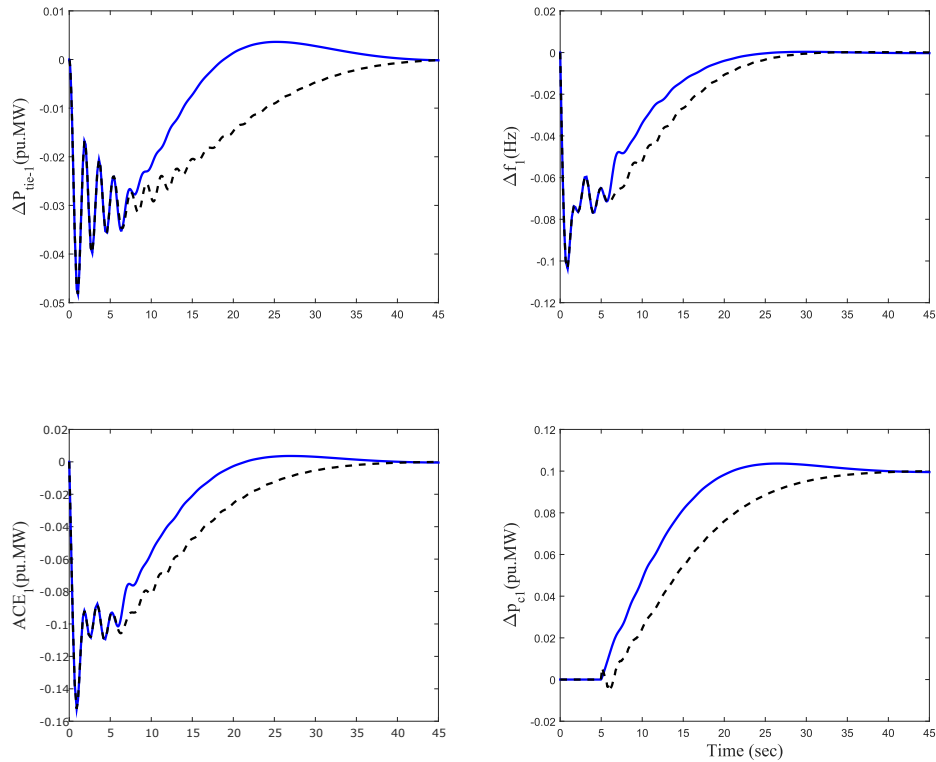


Figure 4.8: Study 2 (constant communication delay 5 sec). Step change responses of area 1. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

quency deviation oscillates around non-zero value until $t = 5$ sec after which the PI controller's command signal is received. Then, more power is generated, and the frequency deviation settles to zero steady state. Moreover, since area 1 has the highest load demand change compared with other areas, it starts importing power from other areas initially. It has been shown in ΔP_{tie-1} response of Fig. 4.8a where there is an initial dip in the response. Similar to frequency deviation, the tie-line power deviation starts moving towards zero steady state after 5 sec when the control signal is received. As it is evident from Fig. 4.8, the responses of closed loop systems with overlapping and decentralised controllers are the same until $t = 5$ sec. This is expected due to existence of communication delays in sending control command signals from PI controllers. However, when the command signals of PI controllers are received, the closed loop responses with overlapping controller settle faster than decentralised PID of [52]. This enhanced performance has been con-

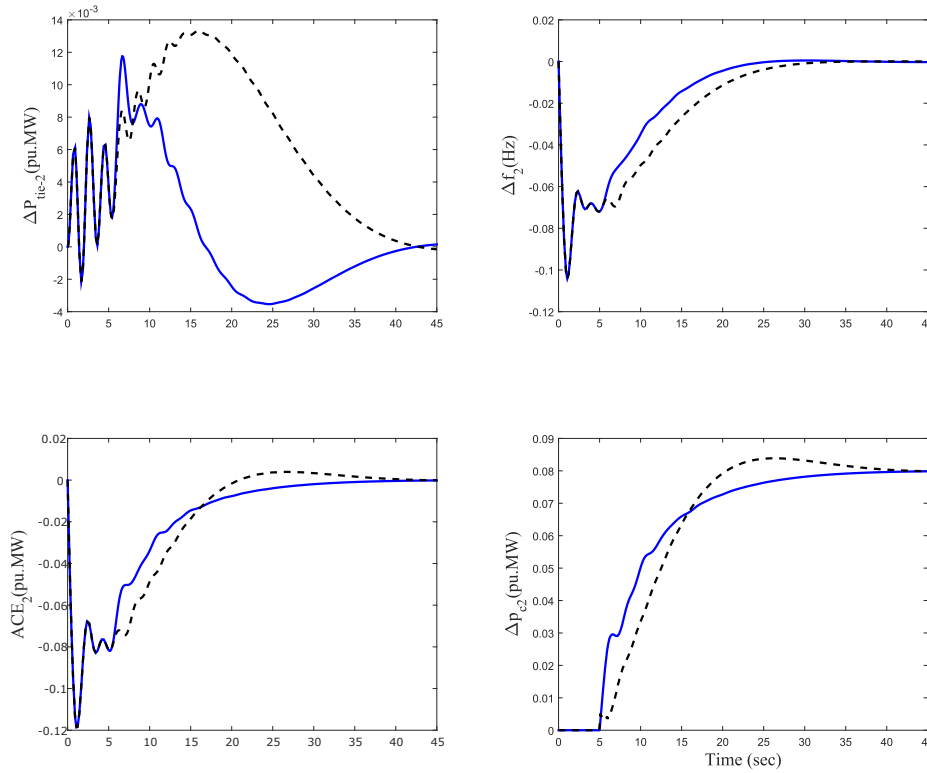


Figure 4.9: Study 2 (constant communication delay 5 sec). Step change responses of area 2. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52])

firmed by quantitative criteria of (4.50). The improved performance with overlapping controller is expected as the accessible tie-line power exchange (overlapping information) has been used in addition to local ACE signals to generate control inputs. The discussion on frequency response of Fig. 4.9b is the same as that of area 1. The frequency falls down initially. Then, due to communication delay, it takes 5 sec for the PI controller to send the command signal for more power generation. Then, the frequency moves towards zero steady state such that it settles down after about 30 sec. On the other hand, area 2 exports power initially as shown through a small rise in ΔP_{tie-2} . When PI controllers are taken into the action after 5 sec, power exchange gets back to zero steady state suggesting the local power demand is met locally. Fig. 4.9 suggests that overlapping controller provides slightly better responses than decentralised PID controller K_2 as it has been shown through quantitative criterion (4.50). The frequency response of area 3 has the similar

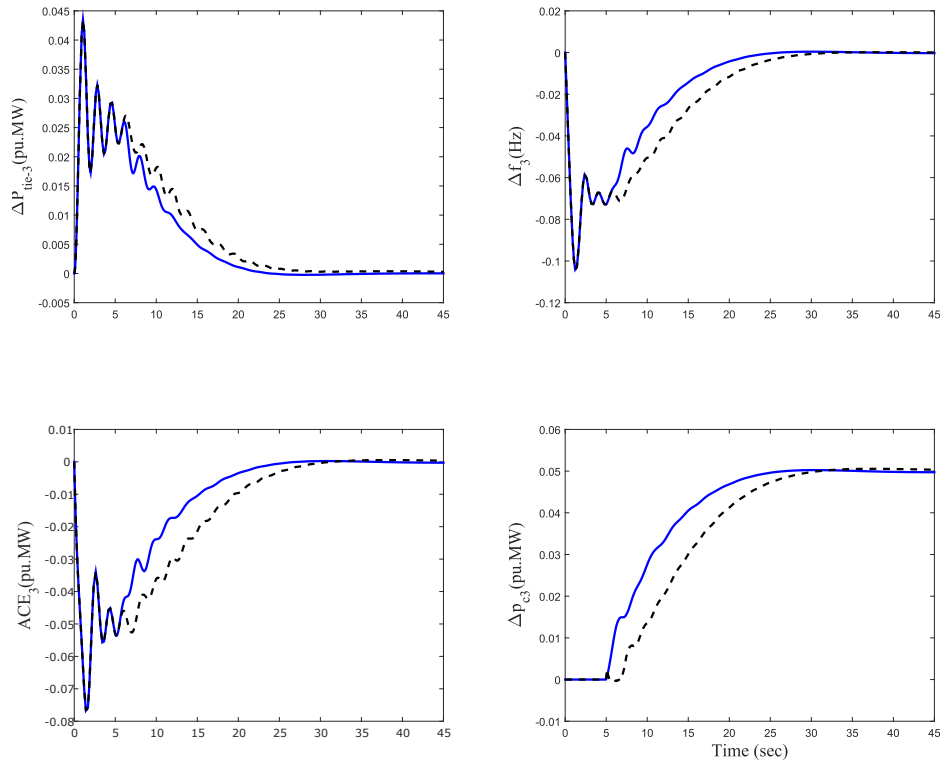


Figure 4.10: Study 2 (constant communication delay 5 sec). Step change responses of area 3. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

behaviour as the other areas. But, there is a considerable rise in tie-line power deviation of area 3 compared with that of area 2 given in Fig. 4.10a. This is expected as area 3 experiences the smallest load demand change. From the viewpoint of comparison, it is clear from Fig. 4.10 that the proposed overlapping PI controller provides enhanced performance reflected by the reduction in settling time compared with decentralised PID controller K_2 [52].

Simulation study 3: Using the designed LFC in scenario 2, the overlapping frequency controller K (4.49) is tested on the same three-area power system but with time-varying communication delay $\tau(t) = 5 + 3\sin(0.3t)$ which is in the range $2 < \tau(t) < 8$ sec with change rate $\dot{\tau}(t) < 1$. The resultant closed loop responses compared with K_2 [52] are given in Fig. 4.11-4.13. Discussions on simulation results are the same as before.

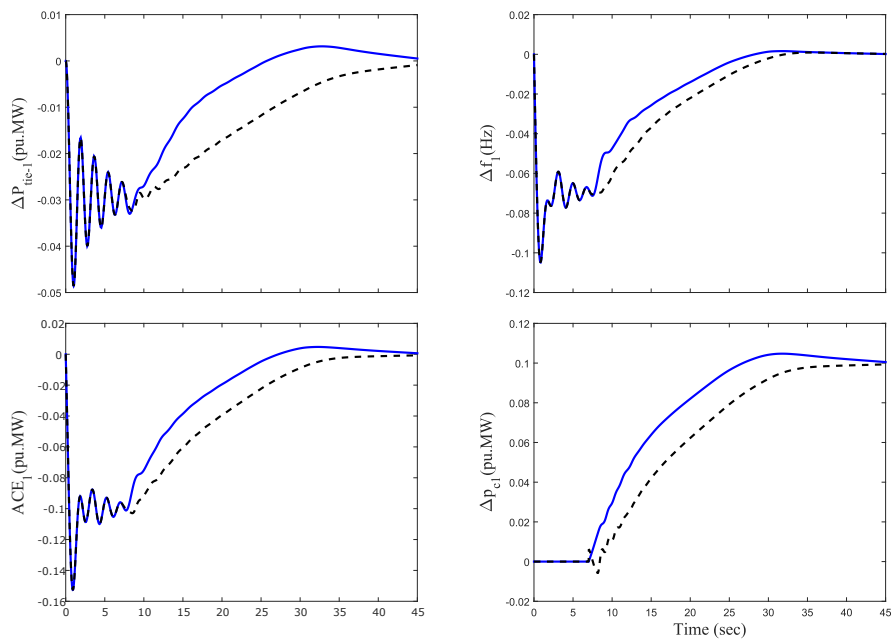


Figure 4.11: Study 2 ($\tau(t) \in [2, 8]$ sec). Step change responses of area 1. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

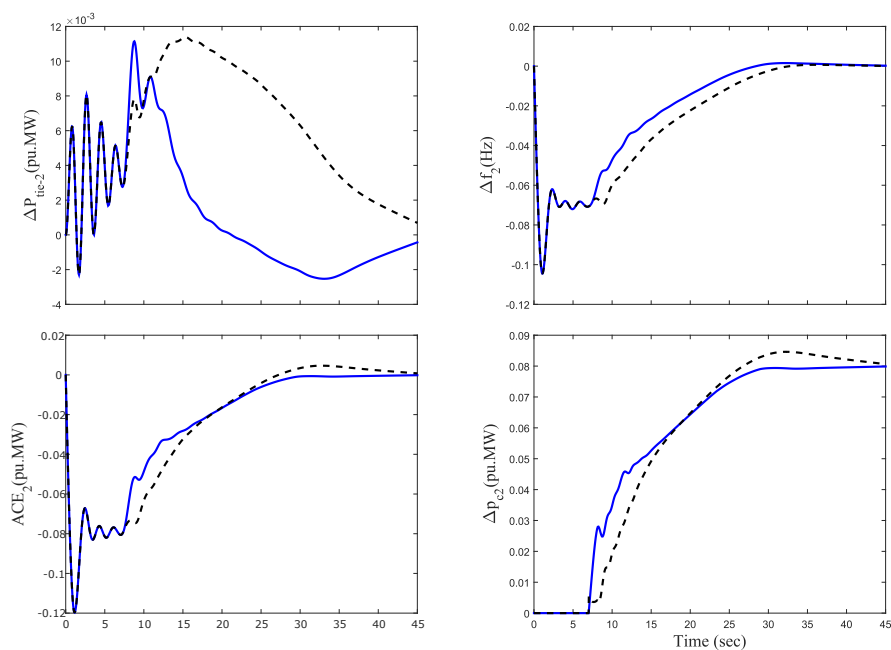


Figure 4.12: Study 2 ($\tau(t) \in [2, 8]$ sec). Step change responses of area 2. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

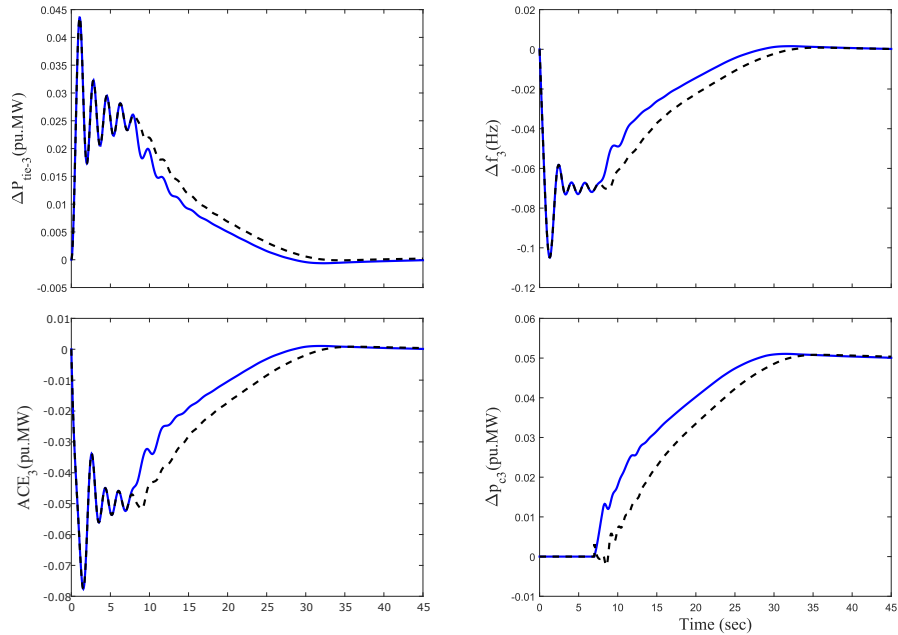


Figure 4.13: Study 2 ($\tau(t) \in [2, 8]$ sec). Step change responses of area 3. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

Study 3:

In this study, the designed overlapping output feedback gain (4.49) is used to provide simulation results for different values of uncertain parameters given in Table. 4.2. to this end, two scenarios are examined. In the first scenario, the lower bound of uncertain parameters, given in Table. 4.2 are used. The upper bounds of uncertain parameters of Table. 4.2 are used in scenario 2. In all scenarios, the time-varying communication delay is set as $d(t) = 2 + 5(1 - e^{-0.2t})$, and step load demand changes (disturbances) $0.1pu$, $0.08pu$, and $0.05pu$ are applied to areas 1,2, and 3, at $t = 0$ sec respectively. The closed loop responses are demonstrated in Fig. 4.14-4.19.

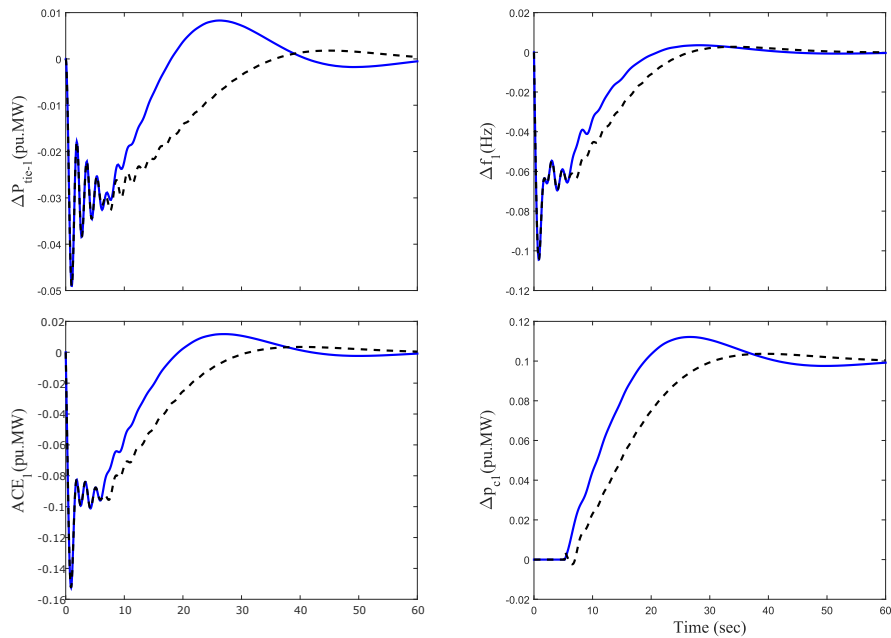


Figure 4.14: Study 3 (lower bound). Step change responses of area 1. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

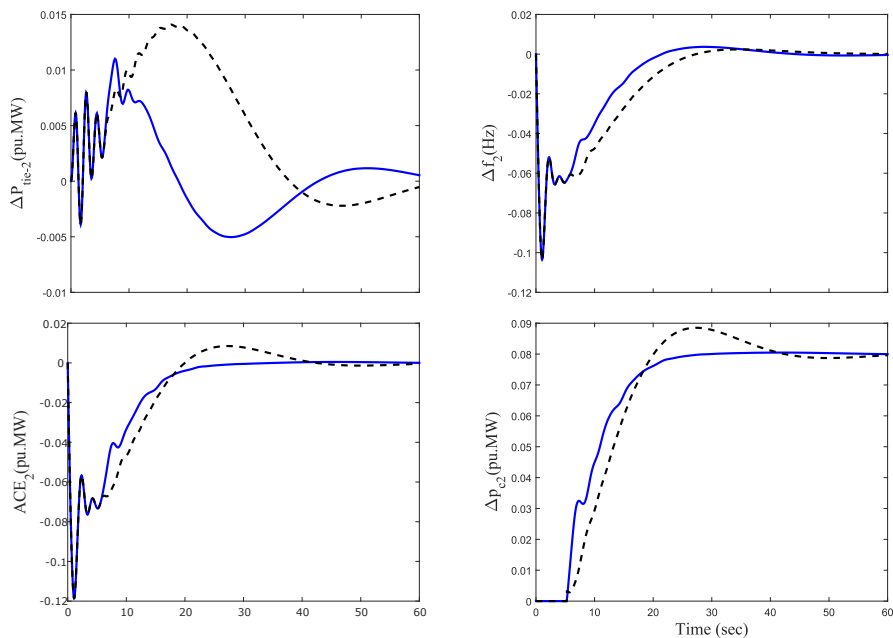


Figure 4.15: Study 3 (lower bound). Step change responses of area 2. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

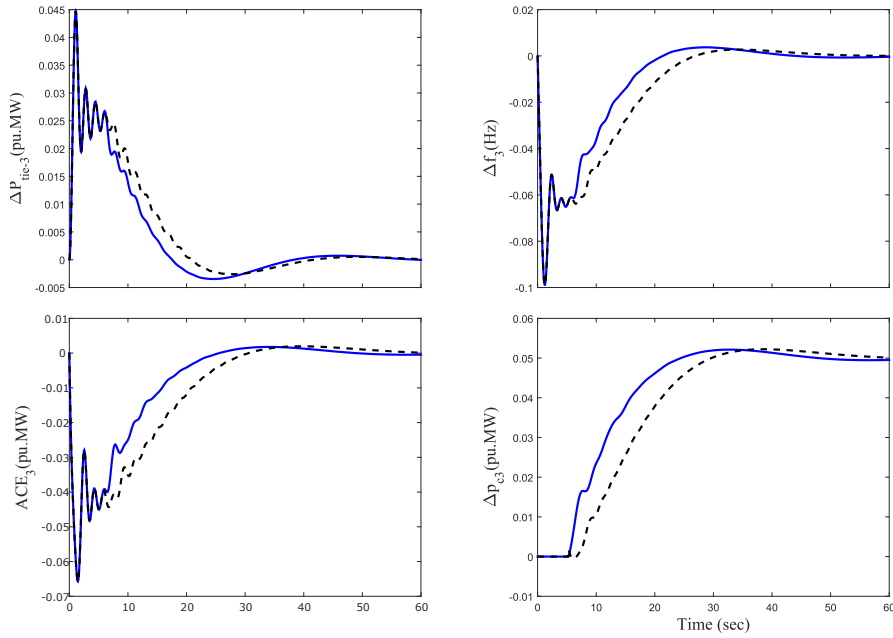


Figure 4.16: Study 3 (lower bound). Step change responses of area 3. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

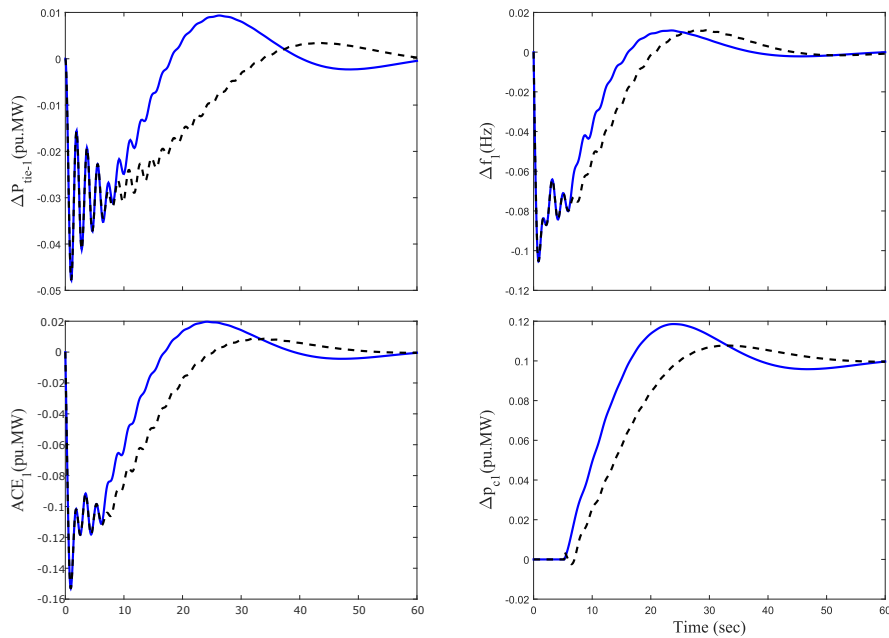


Figure 4.17: Study 3 (Upper bound). Step change responses of area 1. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

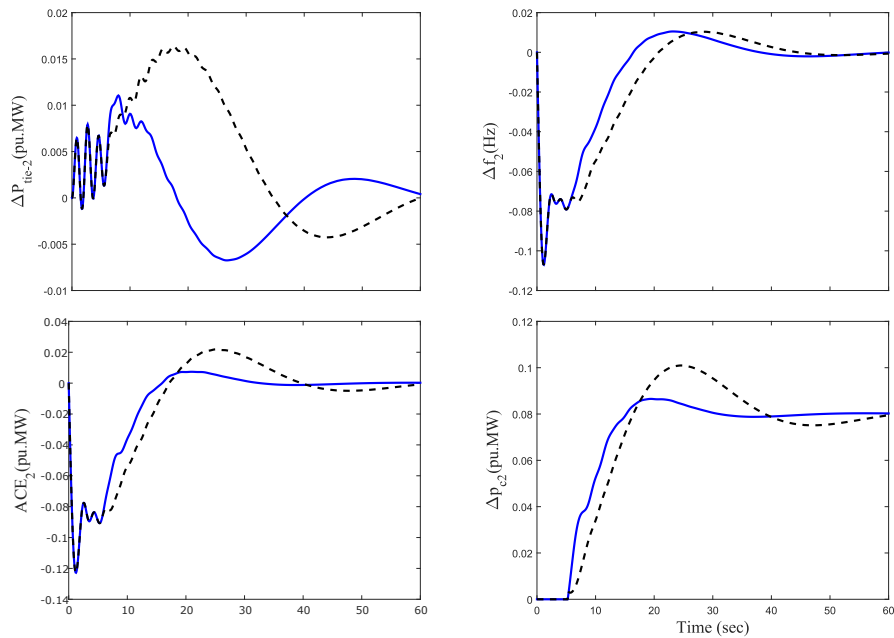


Figure 4.18: Study 3 (Upper bound). Step change responses of area 2. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

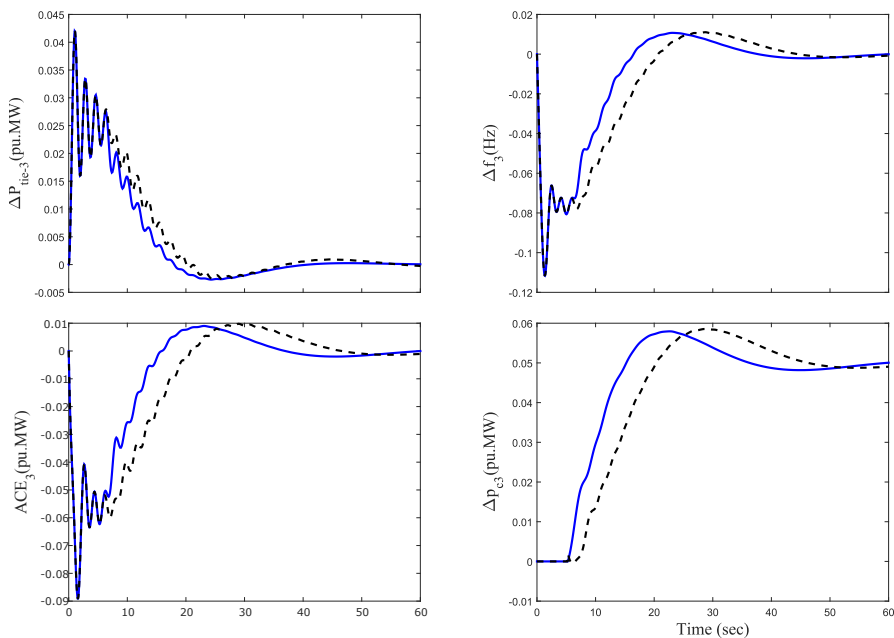


Figure 4.19: Study 3 (Upper bound). Step change responses of area 3. Solid blue line (Proposed overlapping PI-type LFC), dashed black line (decentralised PID-type LFC [52]).

4.5.3 Performance Comparison

In order to compare performance of the proposed overlapping LFC with decentralised LFCs of [51, 52] quantitatively, the integral of time multiplied absolute value of the error (ITAE) in each area, sum of the ITAE denoted by J , and the figure of demerit (FD) of forms expressed in (4.50) below are used [52]:

$$\begin{aligned}
 ITAE_i &= \int_0^{t_f} t |ACE_i(t)| dt, \\
 J &= \sum_{i=1}^3 ITAE_i, \\
 FD &= \sum_{i=1}^3 (OS_i \times 10)^2 + (FU_i \times 4)^2 + (TS_i \times 0.3)^2 \quad (4.50)
 \end{aligned}$$

where OS_i denotes overshoot, FU_i denotes first undershoot, and TS_i denotes settling time (within 3% band of total load demand change) in the frequency deviation response of the i^{th} area. Also, t_f is 45 sec in studies 1 and 2 and 60 sec in study 3.

The performance index $ITAE$ is used to penalize the long duration errors exist in ACE responses. The performance index FD is used to penalize overshoot, undershoot, and long duration error in frequency response. The values of TS , performance indexes and percentage performance improvements compared with [51, 52] in all studies are given in Tables. 4.4-4.9, for $\tau_a = 5$ sec and $\tau_b(t) \in [2, 8]$ sec.

Table 4.4: Study 1. Settling time (Sec) in frequency responses

Type of controller	TS_1	TS_2	TS_3
Proposed overlapping PI-type LFC	7.75	8.52	8.38
Decentralised PI-type LFC [51]	25.51	9.01	17.45
Percentage decrease (%)	69	5	51

Table 4.5: Study 1. Performance indexes

Type of controller	$ITAE_1$	$ITAE_2$	$ITAE_3$	J	FD
Proposed overlapping PI-type LFC	2.66	1.32	1.09	4.98	18.69
Decentralised PI-type LFC [51]	7.2	2.68	5.27	15.15	93.59
Percentage decrease (%)	63	50	79	67	80

Table. 4.4 confirms the simulation results obtained for frequency responses of three areas. The frequency responses with the proposed overlapping LFC, especially in areas 1 and 3, settle faster than those with decentralised PI-type LFC of [51]. Also, Table. 4.5 shows that performance indexes with the proposed overlapping controller are smaller than those with decentralised controller K_1 of [51]. This vindicates the closed loop responses of ACE signals.

Table 4.6: Study 2. Settling time (Sec) in frequency responses

Type of controller	$TS_1(\tau_a)$	$TS_2(\tau_a)$	$TS_3(\tau_a)$	$TS_1(\tau_b)$	$TS_2(\tau_b)$	$TS_3(\tau_b)$
Proposed overlapping PI-type LFC	17.2	17.57	17.38	23.66	23.66	23.66
Decentralised PID-type LFC [52]	22.39	22.68	22.53	26.91	27.15	27.15
Percentage decrease (%)	23	22	22	13	13	13

Table 4.7: Study 2. Performance indexes ($\tau_a = 5 \text{ sec}$)

Type of controller	$ITAE_1(\tau_a)$	$ITAE_2(\tau_a)$	$ITAE_3(\tau_a)$	$J(\tau_a)$	$FD(\tau_a)$
Proposed overlapping PI-type LFC	8.47	6.59	3.81	18.87	82.01
Decentralised PID-type LFC [52]	15.68	7.55	6.48	29.71	137.51
Percentage decrease (%)	46	13	41	36	40

Table 4.8: Study 2. Performance indexes ($\tau_b(t) \in [2, 8] \text{ sec}$)

Type of controller	$ITAE_1(\tau_b)$	$ITAE_2(\tau_b)$	$ITAE_3(\tau_b)$	$J(\tau_b)$	$FD(\tau_b)$
Proposed overlapping PI-type LFC	13.64	9.5	6.27	29.42	151.62
Decentralised PID-type LFC [52]	20.69	11.37	9.09	41.51	197.86
Percentage decrease (%)	34	16	31	29	23

Table 4.9: Study 3 (Uncertain Parameters): Values of performance index

Type of controller	Lower bound of uncertain parameters				Upper bound of uncertain parameters			
	$ITAE_1$	$ITAE_2$	$ITAE_3$	J	$ITAE_1$	$ITAE_2$	$ITAE_3$	J
Proposed overlapping PI-type LFC	13.01	5.34	4.71	23.07	16.39	6.51	7.1	30.02
Decentralised PID-type LFC [52]	16.69	9.92	7.82	34.44	17.71	17.38	10.79	45.9
Percentage decrease (%)	22	46	39	33	7.45	62	34	34

4.6 Summary

In this chapter, the inclusion principle is used to design a robust, overlapping, static, output feedback controller for uncertain continuous input-delayed system with an overlapping decomposition. The input delay is assumed to be unknown, time-varying, however, upper bounds on its magnitude and rate of change are available. The proposed overlapping, delay-dependent control design approach comprises three steps (i) The original system with overlapping subsystems is expanded into the one including disjoint subsystems (ii) robust delay-dependent decentralised controllers with H_∞ performance level $\tilde{\gamma}_i$ are designed for low-dimensional disjoint subsystems of the expanded system using the proposed iterative LMI based algorithm, and (iii) the decentralised controllers are transformed (contracted) to robust, overlapping, static output, feedback controller for implementation on the original system. It is proven that overlapping controller obtained by contraction, is a robust, stabilising controller for the original system with H_∞ performance level γ , where $\gamma = \max\{\tilde{\gamma}_i\}$. The proposed LFC design approach is applied to the three-area interconnected power system experiencing various communication delays and model's uncertainties. The simulation results and quantitative comparison criteria show that the proposed overlapping LFC provides better performance than decentralised LFC of [51] and [52] under different scenarios.

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Chapter 5

Conclusion

5.1 Conclusions

Stabilisability of overlapping linear systems is considered in this thesis. For linear time-invariant systems, Chapter 2 presents a necessary and sufficient condition for stabilisability of linear certain systems with overlapping static output feedback controllers. Furthermore, it has been shown that any decentralised control law designed for the expanded system is contractible to an overlapping controller provided a contractibility condition is incorporated in the decentralised design procedure. Stabilisability of uncertain linear systems is then studied. It has been discussed that there exists an overlapping static output feedback gain provided that the induced 2-norm of uncertainties are inside pre-determined bounds.

On the other hand, it has been widely well-known that time-delays have played a major role in stability and performance. Thus, Chapter 3 investigates stabilisability of linear state-delay systems. The inclusion principle, as the mathematical tool to design overlapping control design, is extended at first. Based on this extension, the expanded system is generated. Then, an iterative algorithm based on LMIs is proposed to design local robust controllers for interconnected subsystems of the expanded system. Once the decentralised controller is designed, it is then contracted (transformed) to an overlapping controller to be implemented on the original system. Chapter 3 shows that both stability and performance are preserved through the contraction process. As an application, state-delay two-area interconnected power system is considered. The system is decomposed to two overlapping subsystems, where tie-lines are the overlapping parts. The overlapping LFC is designed using the proposed approach, and extensive simulation results under different conditions are provided.

Finally, Chapter 4 presents the overlapping output feedback design for linear uncertain system with time-varying input delay. First, the inclusion principle is used to generate the expanded system. Then, an iterative algorithm is suggested to design local controllers for interconnected subsystems of the expanded system. The decentralised controller is formed from which an overlapping controller is obtained through the contraction process. The preservation of stability and performance is proven through the contraction process. In this chapter, a three-area interconnected power system experiencing time-varying input delay and model uncertainties is studied. The robust overlapping LFC is designed for the power system, and the extensive simulation results under different scenarios compared with existing ones are provided.

5.2 Future work

In the research carried out by this thesis, the inclusion principle and design procedures are based on linear time-invariant models. These models may not be accurate for some physical systems. Many real-life systems may be time-varying or non-linear experiencing communication delays. So, a future research direction can be extending the inclusion principle and all related design procedures to time-delay, time-varying systems.

Another direction can be related to the application used in the thesis. The frequency control in conventional power systems are studied. Recently, Microgrids (MGs) with renewable energy resources have been suggested as a solution for economical harvesting of electrical energy with attention to the environmental issues. Thus, great of interest has been devoted to control of MGs. A future work can aim at controlling the frequency of MGs based on the design procedures of the thesis.

Delay characteristics (upper bounds on its size and rate of change) are essential parameters used in the design procedure. However, these parameters may not be available in real networks. In this situation, delay estimators such as Smith predictor can be used for estimation. Upon estimation of upper bounds, they can be used in the design algorithms. Another future direction related to communication delays is associate to their types. In this thesis, network delays are assumed to constant or time-varying. On the other hand, network delays can be stochastic in real network environments. In this case, it is difficult to apply stability analysis of the thesis. An interesting

direction for future work can be developing results of the thesis for linear systems experiencing stochastic delays.

Finally, there has been a growing interest in type of systems called system of systems (SOS). These systems are complex ones whose components are also complex. Mathematical models of SOS are non-linear and high dimensional with communication delays. Thus, a future research direction can be extending the inclusion principle for SOS.

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