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Events in context

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ABSTRACT

In this short tribute to Glynn Winskel, I recall some memories of the first time we met, and describe some recent work on contextual semantics of observational systems which can be used to model quantum non-locality and contextuality, and which has been influenced by Glynn's work on event structures and presheaf semantics for concurrency.

1. Introduction

It is a great pleasure to write this short note in honour of Glynn Winskel on his 60th birthday. Glynn and I are almost exact contemporaries (I am two months older). By the time I entered the research community in semantics of computation, *ca.* 1982/83, he was already a rising star of the field, with his seminal thesis and paper with Gordon Plotkin and Mogens Nielsen on event structures [1,2]. I believe we first met at ICALP 1983 in Barcelona, almost exactly 30 years ago. We fell immediately into an animated conversation as everyone filed out of the conference hall for lunch. At some point, we realized that we had taken a wrong turn somewhere, and were in the middle of a large, empty and rather dusty square, in the heat of the mid-day sun. Neither of us has been blessed with a great sense of direction, in spatial terms!

But Glynn has unerringly followed a consistent path throughout his scientific work. The features I would particularly like to emphasize are his concern for clarifying fundamental issues, and his scientific approach. A pervasive theme running through much of his work, starting from his thesis, and continuing to his current work on game semantics, is to articulate the mathematical structure of events in computation. His work has been a reference point for me, and he has been both a friend and an inspiring colleague over the past three decades.

I would like to give a very brief discussion of some of my own recent work in quantum information and foundations [3], which can be seen as using similar tools to those Glynn has developed in his pioneering work on presheaf semantics for concurrency [4]. Thus there is some confluence of ideas, while at the same time the distinctively quantum features of contextuality and non-locality emerge rather clearly in this setting.

2. Contextual semantics

Consider the following scenario. There are two agents or experimenters, Alice and Bob, each of whom can each select one of several different measurements to perform, and observe one of several different outcomes.

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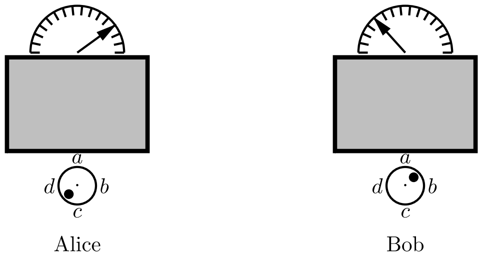

When a system is prepared in a certain fashion and measurements are selected, some corresponding outcomes will be observed. These individual occurrences or 'runs' of the system are the basic events. Repeated runs allow relative frequencies to be tabulated, which can be summarized by a probability distribution on events for each selection of measurements. We can abstract from the probabilities themselves, and consider only the *support* of the probability distributions—distinguishing those events which are *possible* (have non-zero probability) from those which can never happen.

Consider the following example of such a support table:

	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
(a,b)	1	1	1	1
(a',b)	0	1	1	1
(a,b')	0	1	1	1
(a',b')	1	1	1	0

Here Alice can select measurement settings a or a', and Bob can select b or b'. Each measurement can result in an outcome of 0 or 1. We consider the joint outcomes when Alice and Bob both select measurements, and tabulate which are possible. For example, if Alice chooses measurement setting a and Bob chooses b', then according to the above table, the joint outcome $(1,0)$, *i.e.* outcome 1 for a and 0 for b', is possible, while $(0,0)$ is not possible.

The question arises how such *correlated outcomes* can be explained, since we are assuming that Alice and Bob are spatially separated, and there is no means for the different sites at which they operate to be in communication with each other while the measurements are being performed. One mechanism which could be used to explain behaviour of this kind is to assume that there are preset values for each of the measurements which Alice and Bob can make, independently of the measurement made by the other. Such preset values are specified by functions $v : \{a, a', b, b'\} \to \{0, 1\}$. We allow for the fact that there may be several possible such preset values, which are determined by circumstances beyond our control, by saying that the behaviour of the system is 'covered' by a *set* of such functions $\{s_1, \ldots, s_p\}$. Such a set generates a support table by the rule that an entry for measurements (x, y) and outcomes (u, v) is set to 1 if and only if for some i, $s_i(x) = u$ and $s_i(y) = v$.

We can now ask the question: can the above support table be realized by a set of functions in such a fashion? In fact, it is easy to see that it *cannot*. This can be shown by using only the following information from the table: the joint outcome $(0,0)$ is possible for measurements (a,b), while the joint outcome $(0,0)$ is impossible for the pairs (a',b) and (a,b'), and the joint outcome $(1,1)$ is impossible for the pair (a',b').

What makes this fact remarkable is that a support table of the form given above can be *realized physically*. That is, we can generate a two-qubit quantum state, and local spin measurements for Alice and Bob corresponding to the measurements a, a', b, b', such that quantum mechanics predicts, and experiment confirms, the support table given above. Moreover, these predictions are verified even under conditions of spatial separation of the two subsystems corresponding to Alice and Bob. The particular construction we have described is known as *Hardy's paradox* [5], a variant of *Bell's theorem* [6].

Thus Nature realizes *non-local correlations*, which do not admit any explanation in terms of actual values possessed by the physical quantities under consideration independently of the actual combination of measurements which are performed. Such an explanation is called a *local hidden-variable model*, so Hardy's paradox is showing that no such local hidden variable model exists for the empirically observed table of possible behaviours we gave above.

As explained in [3], the language of sheaf theory can be used to give an elegant general account of these fundamental quantum phenomena of contextuality and non-locality. An "empirical model"—a table of the kind described above—can be viewed as a family of local sections of a certain presheaf, indexed by a *cover*—the empirically accessible combinations of measurements corresponding to the rows of the table. The compatibility of the cover in the usual sheaf-theoretic sense corresponds precisely to the *no-signalling* property, a fundamental physical constraint on such systems imposed by relativity. The presence of essentially contextual or non-local behaviour corresponds to *obstructions to the existence of global sections* for this family. A linear programming view of these obstructions leads to a unifying principle for *Bell inequalities* in terms of purely logical consistency conditions in [7], while they are described in terms of *sheaf cohomology* in [8].

These ideas provide the basis for ongoing work on the structure of multipartite entanglement, structural reasons for macroscopic locality, and the axiomatic characterization of quantum mechanics. There are also interesting connections with a number of topics in *classical* computation, see e.g. [9,10].

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