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Causality in physics and computation

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ABSTRACT

Glynn Winskel has had enormous influence on the study of causal structure in computer science. In this brief note, I discuss analogous concepts in relativity where also causality plays a fundamental role. I discuss spacetime structure in a series of layers and emphasize the role of causal structure. I close with some comparisons between causality in relativity and in distributed computing systems.

1. Introduction

Several years ago, in 1982 to be exact, I decided to abandon a career in relativity and quantum mechanics and retrain myself as a theoretical computer scientist. I, like most of my physics colleagues of the time, was completely ignorant about the field. Indeed many physicists had no idea that there was such a field. I recall one of them saying to me, "Theoretical computer science! What is that about? Do you study ideal spherical computers?"

Initially, I was thinking rather unenthusiastically about job security and visa status rather than being excited about a new intellectual adventure. I found two documents that changed that dramatically. The first was an article by Lamport [1] called "Time, clocks and the ordering of events in a distributed system" and the other was Glynn Winskel's remarkable thesis [2]. Both made me realize that I could think about my new subject mathematically and grapple with the foundational questions that I loved in physics.

Since then Winskel and I become friends and have shared many exciting scientific discussions and drinks in pubs. I can think of no better way of celebrating his continued youthful vigour by offering this little note that reflects some of the ways in which he influenced (and influences) me the most. So "Happy birthday Glynn!"

2. The spacetime canvas

This section is necessarily brief; for a detailed treatment of the background mathematics I recommend the excellent book by Hawking and Ellis [3] and the equally excellent but terse monograph by Penrose [4].

The fundamental unit of physics is the *event*. This is taken as a primitive undefined concept but one can think of it as an idealization of a process as the duration and spatial extent of the process shrinks to zero. It is the spatio-temporal analogue of an idealized point. The modern presentation of classical general relativity posits the existence of a smooth 4-manifold of events on which is defined a local "metric" which specifies infinitesimal distances; this is called the *spacetime metric* and the entire structure: manifold together with this metric, is called *spacetime*.

The metric alluded to above is not like a metric that one studies in topology or analysis: it is rather the analogue of a Riemannian metric in geometry. Rather than attributing distances to pairs of points it gives lengths of infinitesimal curves; one can integrate this metric along a curve to obtain a length for a curve.

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The reason that the word "metric" appears in quotation marks is that unlike the metrics that mathematicians and computer scientists are used to, the spacetime metric takes on positive and negative values and is zero even for many curves connecting pairs of distinct points. The reason for this is the existence of independent events: events that cannot influence each other. Such pairs of events are said to be *spacelike* and the distances are said to be positive. Other pairs of events are possibly causally related and the distances between them are negative: such events are said to be *timelike* related. In order to give a coherent presentation of the structure of spacetime it is best to imagine it as a blank canvas on which more and more sophisticated mathematical structures are defined in successive layers.

As a prelude to painting the spacetime canvas I will quickly review the pre-Einstein–Minkowski picture of spacetime. Here there is a 4-dimensional manifold M of events. A manifold is a topological space so one understands what is meant by open and closed sets. Given two events A and B is it possible for A to influence B? For a fixed A there is a set of events that A can potentially influence: call it $F(A)$, the future of A. There is a set of events that can influence A: call this $P(A)$ the past of A. These two sets are *open* and share a *common* boundary: call this $N(A)$. The set $N(A)$ is "now" as far as A is concerned: it is the set of events that are *simultaneous* with A. The fact that the past and the future share a common boundary means that the points that are pairs of points to the future and past of A that are arbitrarily close to each other and arbitrarily far from A. All this testifies to the lack of any limit on the speed with which causal influences can propagate.

This structure can be neatly described by a real-valued function $t : M \to \mathbb{R}$ called *time*. For all points in $N(A)$ t takes on the same value and for all points to the past of A, t is strictly less than $t(A)$ while for all points in $F(A)$, t is strictly greater than $t(A)$. The manifold has been decomposed into a product of a 3-manifold called S (space) and $\mathbb{R}$ (time): thus $M = S \times \mathbb{R}$. The geometry of spacetime can thus be reduced to the geometry of S which is spatial and one tends to ignore time when talking about geometry. The metric on space is a positive-definite (i.e. Riemannian) metric.

The Einstein–Minkowski picture of spacetime is very different because of the *experimental fact* that the speed of light is constant in all reference frames and the concomitant *belief* that this represents an upper bound on the speed of propagation of *signals*. I now turn to the task of painting the spacetime canvas.

At the most primitive level, spacetime is just a set. At the next level it is a topological space: one has a notion of "nearly" without any metrical connotations and one understands continuity. It is at this level that one encodes the 4-dimensionality and the fact that locally every point looks like $\mathbb{R}^4$. Again the 4 is an experimental fact; perhaps more refined experiments will reveal in the future that it is really 11 dimensional or not even locally homeomorphic to any open subset of any $\mathbb{R}^n$.

The next structure that one imposes is *differential* structure. This allows one to do differential calculus and define smooth curves and tangent vectors to curves. Every point (event) p now has attached to it a 4-dimensional real vector space T_p call the *tangent space* at p. The whole assembly of all these vector spaces held together by being attached to the points of the manifold is called the *tangent bundle*.

The next structure is the crucial one for causality. First a preliminary definition.

Definition 2.1. A subset C of a real vector space V is called a *cone* if

1. $v \in C$ and $-v \in C$ implies $v = 0$,
2. $\forall r \in \mathbb{R}^+, v \in C; r \cdot v \in C$,
3. $\forall u, v \in C; u + v \in C$.

A vector $u \in C$ that can be written as $v + w$ where both v and w are in C and v and w are not scalar multiples of each other is said to be in the *interior* of the cone. A vector not in the interior of the cone is said to be on the *boundary*.

At every point p, there is a pair of subsets C_p^+ and C_p^- of the tangent space called the *future and past light cones*. Each of these sets are *cones* as defined just above. In pictures, one draws the cones as if they were on spacetime itself but they really live in the tangent spaces. That is why it is necessary to define the differential structure first. A vector in the interior of the future (past) light cone at p is said to be a future-pointing (past-pointing) timelike vector. A vector on the boundary of C^+ (C^-) is said to be a future-pointing (past-pointing) *null* vector.

In order for the subsequent discussion to get off the ground one makes a basic assumption about the light cone structure. It is assumed that it is possible to define a notion of future-pointing and past-pointing cones that vary continuously and are defined globally. Such a spacetime is said to be *time-orientable*. One can construct counter-examples to time orientability by using Möbius-strip like constructions; we will assume time orientability as a basic axiom of spacetimes henceforth.

A (smooth/continuous) curve is just a (smooth/continuous) map γ from $\mathbb{R}$ to M or $[0, 1]$ to M if one is considering a curve with end points.

Definition 2.2. A curve is said to be *timelike* if its tangent vector is everywhere timelike. A curve is said to be *causal* if its tangent vector is everywhere timelike or null.

The discussion is best couched in terms of piecewise smooth curves.

Definition 2.3. A *timelike trip* from A to B is a sequence of events $e_0 = A, e_1, \ldots, e_n = B$ together with a timelike curve γ_i from e_{i-1} to e_i for $i = 1, \ldots, n$. A causal trip is defined similarly.

Now we can actually define a causal relation between events.

Definition 2.4. We say y is in the *chronological future* of x if there is a future directed timelike trip from x to y. We write $x \ll y$. We define $I^+(x) = \ y \mid x \ll y$; similarly for chronological past and I^-.

The physical meaning of $x \ll y$ is that it is possible for a material particle to travel from x to y.

Definition 2.5. We say that x is the *causal past* of y if there is a future-directed causal trip from x to y. We write $x \leqslant y$. We define $J^+(x) = \ y \mid x \leqslant y$. Similarly for *causal future* and J^-.

The physical significance of the causal order is that one can propagate *information* from x to y if and only if $x \leqslant y$. The mechanisms used to propagate information involve sending material particles and light signals.

The two relations $\ll$ and $\leqslant$ are order structures: both are transitive. The relation $\ll$ is taken to be irreflexive while the causal order $\leqslant$ is a partial order. This last statement is an assumption about possible spacetimes. In fact, there are spacetimes that occur as solutions to Einstein's equation that violate: the most celebrated example is the Gödel universe. If one is to rule out possible pathologies like being able to revisit one's past then one can impose as an *additional condition* the fact that $\leqslant$ is a partial order.

Stronger causality conditions can also be imposed; in this paragraph we give an informal telegraphic survey of some common causality conditions. The most basic condition imposed is called causality: there are no closed causal curves. A spacetime with the property that $I^+(x) = I^+(y)$ implies $x = y$ is said to be *future distinguishing*; similarly one has *past distinguishing*. These properties say that there cannot be two distinct points with the same timelike futures. It may seem hard to believe that this is possible but there are examples in [3]. Roughly speaking, it can be thought of as an analogue of being a sober space in topology.

One of the more interesting conditions is called strong causality. A spacetime is *strongly causal* if at every point p there is a neighbourhood U in which a causal curve γ that originates at p must leave U and not reenter. This means that not only are there no closed causal curves but there cannot be curves that come "close to violating causality."

A spacetime is said to be *stably causal* if it is causal and any "small perturbation" of the light cones keeps it causal. Here, by "small" perturbation we mean that the light cones are opened out slightly. Clearly this makes more curves causal since there are now more vectors inside the light cones. In a stably causal space none of these new causal curves can be closed. Of course, the technical issue is formalizing what is meant by "slightly"; for this one has to introduce a suitable topology on the space of spacetime metrics. There is an important consequence: a spacetime is stably causal if and only if there is a real-valued function on spacetime whose gradient is everywhere timelike. One can think of such a function as a global time.

The next condition is called *causal simplicity*: it means that spacetime has no points removed. More precisely, a spacetime is said to be causally simple if it is strongly causal and for any point p, $J^+(p)$ and $J^-(p)$ are closed sets in the manifold topology. The strongest condition usually imposed is global hyperbolicity. A spacetime is said to be *globally hyperbolic* if for every pair of points p, q with $q \in I^+(p)$ the "interval" $J^+(p) \cap J^-(q)$ is compact.

There are other mathematical structures that must be defined before one has the full structure of spacetime. A very readable paper by Ehlers, Pirani and Schild [5] describes how one can set up these structures and relate them to the flow of freely falling particles and light rays. A more sophisticated treatment was subsequently given by Woodhouse [6]. My focus is on causal structure and its relation to how computer scientists view causality so I will stop here.

3. Causality and order

The most primitive mathematical structure associated with causality is a partial order. The notion of cause and effect introduces a direction between events: the fact that cause precedes effect is the essence of any kind of temporal structure that claims to capture causality.

Once one has a notion of spacetime one can define causal curves and timelike curves. These induce two orders: the causal order is typically denoted $\leqslant$ while the timelike order is written $\ll$.

Definition 3.1. Given a spacetime $\mathcal{M}$ and two points (events) x and y, we say that x **causally precedes** y, written $x \leqslant y$ if there is a piecewise smooth curve γ from x to y with the tangent to γ being everywhere future-pointing and timelike or null. If the tangent vector to γ is future-pointing and timelike we say that x **chronologically precedes** y and write $x \ll y$.

There are some subtleties that we are eliding. The points where the tangent vector is not defined could form an infinite sequence converging to a point; Penrose [4] calls these "bad trips." We will assume that γ is not a bad trip.

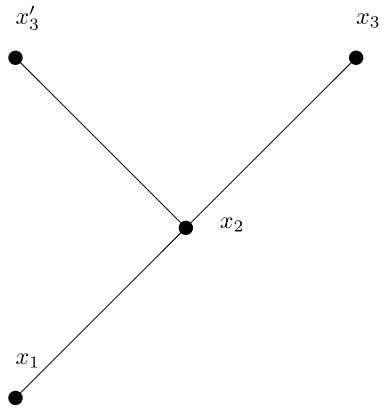

Fig. 1. The horismos in action.

The intuition that we are capturing is that if $x \leqslant y$ then a signal can propagate from x to y and if $x \ll y$ then a material object can travel from x to y.

If one abstracts away from all the various layers of structure one can define an event set equipped with two order structures. The causal orders had been studied first by Zeeman in the context of special relativity but the first axiomatic description of these orders in general spacetimes was done by Kronheimer and Penrose [7] and led to their well-known axiomatization of causal spaces. They axiomatize the properties of $\leqslant$ and $\ll$.

Definition 3.2. A **causal space** is a set X equipped with two binary relations $\leqslant$ and $\ll$ satisfying the following axioms:

1. $\leqslant$ is a partial order;
2. $\ll$ is not reflexive;
3. $\ll$ is contained in $\leqslant$, i.e. if $x \ll y$ then $x \leqslant y$;
4. if $x \leqslant y$ and $y \ll z$ then $x \ll z$;
5. if $x \ll y$ and $y \leqslant z$ then $x \ll z$.

Kronheimer and Penrose actually define a causal space in terms of *three* binary relations: the causality $\leqslant$ and chronology $\ll$ and the third is a relation derived from these two: $x \rightarrow y$ if $x \leqslant y$ but not $x \ll y$. This relation is called a *horismos*. Physically $x \rightarrow y$ means that a light signal but not a material object can travel from x to y. Thinking in terms of light cones, it means that y is on the light cone emanating from x.

We discuss some aspects of the horismos precisely because it is markedly different from anything in computer science. First we define its striking characteristic property.

Definition 3.3. A reflexive binary relation R on a set X is called **horismotic** if whenever $(x_i)_{1 \leqslant i \leqslant n}$ is a finite sequence with $x_i R x_{i+1}$ for $1 \leqslant i < n$ then for any $1 \leqslant j \leqslant k \leqslant n$, (i) $x_1 R x_n$ implies that $x_j R x_k$ and (ii) $x_n R x_1$ implies $x_j = x_k$.

Proposition 3.4. *The $\rightarrow$ relation of a causal space is horismotic.*

Proof. The second property of a horismotic relation follows immediately from the fact that $\leqslant$ is antisymmetric. From $x_1 R x_2 R x_3 \ldots R x_n$ we have that (x_i) forms a chain in the causal order; hence if $x_n \leqslant x_1$ we have $x_1 = x_2 = \cdots = x_n$.

Suppose that $x_1 R x_2 R x_3 \ldots R x_n$ and $x_1 R x_n$. Now consider x_j and x_k: we know that $x_j \leqslant x_k$ we would like to show that $x_j \ll x_k$ does **not** hold. Suppose that it does, then we have $x_1 \leqslant x_j \ll x_k \leqslant x_n$. From properties 4 and 5 of Definition 3.2 we have $x_1 \ll x_n$ which contradicts $x_1 R x_n$. □

We also have the following fact [7] which has a trivial.

Proposition 3.5. *Let $x \leqslant y \leqslant z$ in a causal space. If $x \rightarrow z$ then $x \rightarrow y \rightarrow z$.*

The fact that these are trivial propositions does not make them less interesting. The horismos relation tells us that certain events can *only* be linked by a light signal. The picture in Fig. 1 illustrates these facts. We have $x_1 \rightarrow x_2$, $x_2 \rightarrow x_3$, $x_1 \rightarrow x_3$ but note that it is not necessarily transitive since we also have $x_2 \rightarrow x_3'$ but we do not have $x_1 \rightarrow x_3'$.

In Fig. 1 one can imagine a partially silvered mirror at x_2 that bounces some of the incident light to x_3' and transmits some to x_3. The point of points 4 and 5 of Definition 3.2 is that whenever light is bounced off a mirror we can smooth the corner and construct a timelike curve. The assertion in Proposition 3.5 is that if there is a lightlike curve then anything along the way must have been lightlike as well. *Without talking about maximum speeds* these properties of the horismos relation show that light limits the region of causal influence.

Kronheimer and Penrose [7] show that from any one of the relations the other two can be defined.

4. Causal structure in computer science

Causality in computer science acquires its most mature mathematical form in the study of distributed and concurrent systems. In computation one can also take events or actions as fundamental entities. In sequential systems there is a single locus of activity and a single locus of control. The sequence of actions defines an absolute time and by the very nature of a sequence all the events are related to each other. In distributed systems actions can occur concurrently and hence independently. Thus causal independence appears as a new basic concept. Three landmark investigations appeared in computer science: Petri nets in the 1960s [8,9], Lamport's treatment of clocks in 1978 [1] and Glynn Winskel's theory of event structures [2]. Excellent reviews of all these exist so I will not attempt yet another review of these ideas. Rather, I will briefly discuss the impact of these works on the notion of event and causality.

4.1. Petri nets

I assume familiarity with the vocabulary of net theory: events, places, markings. Petri nets introduce events as a basic entity. There are many variations of the definition but the basic version (condition-event nets) illustrates the main ideas. There is a set of events E and a set of conditions C and there is a binary relation $F \subset (C \times E \cup E \times C)$: a CE net can be visualized as a bipartite graph. The most striking new idea here, relative to automata theory, is *locality*. An event is a local action and, unlike in automata theory, one has a clear notion of independent events.

A lot of the work in the field has been about exploring the structure of nets. In a paper from the 1990s Petri [10] studies continuous versions of net theory and even explored the connections with Lorentz transformations. However, the connection with causal structure via light cones in spacetime is not explored.

4.2. Time and clocks in asynchronous systems

In 1978 Lamport wrote a remarkable paper called "Time, clocks and the ordering of events in a distributed system." In this he envisages a system of computing agents each executing a sequential process, and hence viewed as a linearly ordered sequence of events. The agents communicate with each other: message passing is a convenient idiom for this kind of communication. Each agent has some receive events and some send events. A message will be associated with one send event and one receive event. The basic causality axiom is that a send event precedes the *corresponding* receive event. Thus we now have two order structures. The events located at each agent are linearly ordered and send and receive events are ordered. The causal order is the reflexive, transitive closure of the union and it is an axiom of this class of systems that the result is indeed a partial order.

Lamport goes on to define a "cut" as the downward closure of an antichain in this order and a *consistent cut* as one in which for every receive event the corresponding send event is also included. Thus a consistent cut is a possible global state of the system: it is perfect analogue of the notion of spacelike slice.

In my opinion, this paper comes closest to the spirit of relativistic causality. The fact that causality is captured by a causal order is explicit and the antichains are the possible global states. The fact that time is conventional and that different logical clocks can be defined is also explicit.

In a later paper the present author and Kim Taylor [11] looked at the whole collection of possible executions of a Lamport-style system and formulated an epistemic logic based on it. This brings in a new, from the point of view of the present paper, ingredient: the different possible executions of a system all living in the same mathematical structure. Of course, this is not due to [11], it is clearly visible in Glynn Winskel's notion of event structure [2] which is about 10 years older.

4.3. Event structures and domains

Winskel's thesis [2,12] was another landmark contribution to the subject of causality in computer science. Of course, the subject has evolved tremendously since then in large part due to Winskel himself and there are modern sheaf-theoretic incarnations of his original ideas [13] but now in a much more sophisticated form and also extensions of event structures to probabilistic [14,15] and even quantum situations. I will, however, stick to a discussion of his early work on event structures because some of the most striking ideas are already there. Winskel himself has written a wonderful reminiscence [16] of the evolution and scope of these ideas. What I find most exciting in his contributions are: (a) a clear notion of morphism of Petri nets which made the subsequent explosion of work on monoidal categories of Petri nets (and connections to linear logic), (b) a rich theory of event domains which makes it possible to treat concurrency at higher type, and (c) the notion of conflict which has been widely used in the computer science community but which needs to be looked at again from the physics point of view.

Definition 4.1. An *event structure* comprises a set E of events, a partial order $\leqslant$ on E of causality and a family Con of finite subsets of E. These are required to satisfy the following conditions:

1. $\{e\} \in \mathrm{Con}$,
2. $Y \subseteq X \in \mathrm{Con}$ implies that $Y \in \mathrm{Con}$, and
3. $X \in \mathrm{Con}$ and $e' \leqslant e \in \mathrm{Con}$ implies that $X \cup \{e'\} \in \mathrm{Con}$.

Events are again the basic entities and causality the most important relation. However, not all events can occur together, there is a new notion: *conflict* which captures inherent indeterminacy. Conflict can be axiomatized by introducing a class of consistent sets which are meant to be the conflict-free sets: they are required to satisfy some natural axioms. A special case of conflict can be axiomatized as an irreflexive, symmetric binary relation and the consistent sets can be derived from the conflict relation. The domains obtained in this case are prime event structures.

From an event structure one can construct a domain as follows. What are the possible configurations of an event structure. These have to be collections of events without conflict so every finite subset has to be in Con and every event must have its necessary predecessors so a configuration has to be downward closed in the causal order. It is easy to show that one gets an algebraic domain in this way.

The domain contains the possible executions of a system. The later presheaf models make this much clearer: essentially they describe possible dynamical trajectories of a concurrent system featuring independence and *conflict*.

5. Back to physics: causal sets

The search for a quantum theory of gravity has excited and vexed physicists for over 50 years. The standard prescriptions of quantum field theory simply did not work for gravity. Radical new ideas were sought and some of these were based on the idea that the causal order should be the fundamental structure of the theory rather than the manifold structure. The idea that the causal order should be fundamental was mainly articulated by Rafael Sorkin [17,18] and several of his collaborators.

Sorkin's work awakened interest in the mathematics of partial orders. The most active line of research has been viewing spacetime as a discrete structure obtained by randomly sprinkling points in spacetime. It is usually said that a discrete analogue of spacetime would ruin Lorentz invariance but if one imagines generating a discrete set by sprinkling points in Minkowski spacetime according to a suitably defined stochastic process [19] one can recover Lorentz invariance "on the average."

One of the interesting questions raised by Sorkin was whether one could recover the topology from the causal order alone. In the 1970s Malament [20] had shown that the class of continuous timelike curves determines the topology of spacetime. Indeed this paper was part of the inspiration for the causal set program. In Sorkin's version of the question one only has the causal order $\leqslant$ and not the chronological order $\ll$. This was shown to be possible by Keye Martin and the author [21] using ideas from domain theory. Strikingly, the $\ll$ relation turned out to be exactly the "way below" relation associated with the causal order: a remarkable notational coincidence!

6. Conclusions: comparing the concepts

The similarities between causality in physics and computer science are clear: both are based on events as the fundamental ingredient, both are partial order structures and both describe limitations on the propagation of information. There are a number of important differences however.

Spacetime in relativistic physics is the arena in which dynamics plays out. The events that make up spacetime are not "actual" occurrences as in event structures or Lamport's model, but are loci of *possible* actions. The trajectories of physical systems, say particles, are curves through spacetime. Thus one can think of spacetime as the collection of all possible trajectories. In this sense it is somewhat like an event domain. However, unlike in Petri nets or event structures conflict or choice is not very explicit in the description of dynamics.

The work of Martin and the author showed some interesting connections between the topology of domains and the topology of spacetime. It seems to me that this merits further investigation. For example, it would be fascinating to understand the proper treatment of differential structure in domain theory. The presheaf models being investigated by Winskel and others have a rich enough structure to support such a study.

To my mind, the most striking difference between the causal order in spacetime physics and in computer science is the complete absence of anything like the horismos [7,6] in the latter. The horismos is crucial in the study of relativity as it gives the trajectory of light rays that play such a major role in relativity.

In a more speculative vein, I would suggest that the way conflict is modelled in Petri nets and in event structures would be very interesting for the treatment of quantum mechanics. Sorkin has proposed a new formalism which he called anhomomorphic logic [22,23] which is based on preclusion between quantum events. This is certainly a more subtle concept that mere conflict but it is closely related and an investigation of the connections is likely to be fruitful.

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