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Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering 2013 227:

309 originally published online 30 November 2012

DOI: 10.1177/0954408912466350

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Modelling laminar transport phenomena in a Casson rheological fluid from a horizontal circular cylinder with partial slip

V Ramachandra Prasad¹, A Subba Rao², N Bhaskar Reddy²,
B Vasu¹ and O Anwar Bég³

Proc IMechE Part E:
J Process Mechanical Engineering
227(4) 309–326
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DOI: 10.1177/0954408912466350
uk.sagepub.com/jpme



Abstract

The laminar boundary layer flow and heat transfer of Casson non-Newtonian fluid from a permeable horizontal cylinder in the presence of thermal and hydrodynamic slip conditions is analysed. The cylinder surface is maintained at a constant temperature. The boundary layer conservation equations, which are parabolic in nature, are normalised into *non-similar* form and then solved numerically with the well-tested, efficient, implicit, stable Keller–Box finite-difference scheme. Increasing velocity slip induces acceleration in the flow near the cylinder surface and the reverse effect further from the surface. Increasing velocity slip consistently enhances temperatures throughout the boundary layer regime. An increase in thermal slip parameter strongly decelerates the flow and also reduces temperatures in the boundary layer regime. An increase in Casson rheological parameter acts to elevate considerably the skin friction (non-dimensional wall shear stress) and this effect is pronounced at higher values of tangential coordinate. Temperatures are however very slightly decreased with increasing values of Casson rheological parameter. Increasing mass flow injection (blowing) at the cylinder surface causes a strong acceleration, whereas increasing suction is found to induce the opposite effect. The study finds applications in rheological chocolate food processing.

Keywords

Non-Newtonian fluid mechanics, Casson model, yield stress, slip condition, Keller–Box numerical method, heat transfer, skin friction, Nusselt number, boundary layers, chocolate food processing

Date received: 6 July 2012; accepted: 8 October 2012

Introduction

Non-Newtonian transport phenomena arise in many branches of process mechanical, chemical and materials engineering. Such fluids exhibit shear stress–strain relationships, which diverge significantly from the classical Newtonian (Navier–Stokes) model. Most non-Newtonian models involve some form of modification to the momentum conservation equations. These include power-law fluids,¹ viscoelastic fluids including Maxwell upper-convected models,² Walters-B short memory models,^{3,4} Oldroyd-B models,⁵ differential Reiner–Rivlin models^{6,7} and Bingham plastics.⁸ The flow of non-Newtonian fluids in the presence of heat transfer is an important research area due to its relevance to the optimised processing of chocolate,⁹ toffee and other foodstuffs.¹⁰ Of the various rheological models developed in biotechnology and food engineering, the Casson model has proved very successful. This simple, yet elegant rheological model was introduced originally¹¹ to

simulate industrial inks. This model¹² constitutes a plastic fluid model which exhibits shear thinning characteristics, yield stress and high shear viscosity. The Casson fluid model is reduced to a Newtonian fluid at a very high wall shear stress i.e. when the wall stress is much greater than yield stress. This fluid model also approximates reasonably well the rheological behaviour of other liquids including physiological suspensions, foams, cosmetics, syrups, etc.

A number of theoretical, numerical and experimental studies of transport phenomena in Casson fluids

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have been presented in a variety of areas including biomedical engineering^{13–15} and manufacturing technology.¹⁶ Neofytou¹⁷ studied computationally the flow characteristics of both power-law and Casson fluids in symmetric sudden expansions, showing that the critical generalised Reynolds number of transition from symmetry to asymmetry and subsequently the inverse dimensionless wall shear stress are linearly related to the dimensionless wall shear rate. Kandasamy et al.¹⁸ numerically studied the thermal convection in concentric annuli using a Casson model. Mass transfer in a Casson flowing through an annular geometry was examined by Nagarani et al.¹⁹ who derived analytical solutions and also considered boundary absorption effects. Hemodynamic simulations of Casson blood flow in complex arterial geometries were studied by Gorla et al.²⁰ and Murthy and Pradhan.²¹ Attia and Sayed-Ahmed²² studied the unsteady hydromagnetic Couette flow and heat transfer in a Casson fluid using the Crank–Nicolson implicit method, showing that Casson number (dimensionless yield stress parameter) controls strongly the velocity overshoot and has a significant effect on the time at which the overshoot arises. Hayat et al.²³ obtained homotopic solutions for stagnation-point flow and heat transfer of a Casson fluid along a stretching surface, also considering viscous heating effects. Mustafa et al.²⁴ very recently investigated also with a homotopy method, the transient dissipative flow and heat transfer of a Casson fluid over a moving flat plate with a parallel free stream, showing that surface shear stress and surface heat transfer are increased with the Casson fluid parameter and also Eckert number (viscous heating parameter).

The above studies invariably assumed the ‘no-slip’ condition at the boundary. Slip effects have, however, shown to be significant in certain industrial thermal problems and manufacturing fluid dynamics systems. Sparrow et al.²⁵ presented the first significant investigation of laminar slip-flow heat transfer for tubes with uniform heat flux. Inman²⁶ further described the thermal convective slip flow in a parallel plate channel or a circular tube with uniform wall temperature. These studies generally indicated that velocity slip acts to enhance heat transfer whereas thermal slip (or ‘temperature jump’) depresses heat transfer.

Many studies have appeared in recent years considering both hydrodynamic and thermal slip effects. Interesting articles of relevance to process mechanical engineering include Larrode et al.²⁷ who studied thermal/velocity slip effects in conduit thermal convection, Spillane²⁸ who examined sheet processing boundary layer flows with slip boundary conditions and Crane and McVeigh²⁹ who studied slip hydrodynamics on a micro-scale cylindrical body. Further studies in the context of materials processing include Ameal et al.,³⁰ Yu and Ameal,³¹ Crane and McVeigh.³² Studies of slip flows from curved bodies include Bég et al.³³ who examined using network numerical simulation the magneto-convective slip

flow from a rotating disk, Wang and Ng³⁴ who studied using asymptotic analysis the slip hydrodynamics from a stretching cylinder. Wang³⁵ has also examined stagnation slip flow and heat transfer from an axially moving cylinder showing that heat transfer increases with slip, Prandtl number and Reynolds number, and that in the case of large slip, the flow field decays exponentially into potential flow.

The objective of the present paper is to investigate the laminar boundary layer flow and heat transfer of a Casson rheological fluid past a horizontal cylinder. Mathematical modelling through equations of continuity and motion leads to dimensionless non-linear coupled differential boundary layer equations. The velocity and thermal slip conditions along with conservation law of mass, momentum and energy completes the problems formulation for velocity components and temperature. The considered slip conditions are important in non-Newtonian foodstuff transport processes,^{10,12} which often exhibit wall slip. It has been experimentally verified that fluid possesses non-continuum features such as slip flow when the molecular mean free path length of fluid is comparable to a representative geometrical length of the system being studied e.g. diameter of a cylindrical body. A finite difference numerical solution is presented for the transformed boundary layer equations and a parametric study is conducted the Prandtl number, Casson rheological parameter, wall suction/injection and velocity/thermal slip effects on the momentum and heat transfer characteristics conducted. The present problem has to the authors’ knowledge not appeared thus far in the scientific literature.

Mathematical flow model

Consider the steady, laminar, two-dimensional, viscous, incompressible, buoyancy-driven convection heat transfer flow from a horizontal permeable circular cylinder embedded in a Casson non-Newtonian fluid. Figure 1 shows the flow model and physical coordinate system. The x -coordinate is measured along the circumference of the horizontal cylinder from the lowest point and the y -coordinate is measured normal to the surface, with ‘ a ’ denoting the radius of the horizontal cylinder. $\Phi = x/a$ is the angle of the y -axis with respect to the vertical ($0 \leq \Phi \leq \pi$). The gravitational acceleration, g acts downwards. Both the horizontal cylinder and the fluid are maintained initially at the same temperature. Instantaneously they are raised to a temperature $T_w > T_\infty$ the ambient temperature of the fluid which remains unchanged. The rheological equation of state for an isotropic flow of Casson fluid, following Steffe¹⁰ in tensorial notation may be stated as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi \geq \pi_c \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

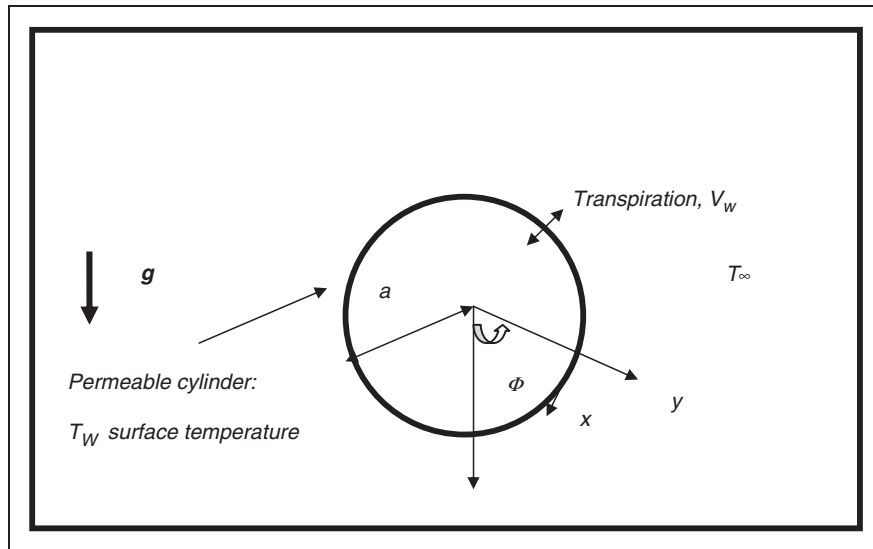


Figure 1. Physical model and coordinate system.

in which $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j) th component of deformation rate, π denotes the product of the component of deformation rate with itself, πc shows a critical value of this product based on the non-Newtonian model, μ_B the plastic dynamic viscosity of non-Newtonian fluid and p_y is the yield stress of fluid. In line with the approach of Yih³⁶ and introducing the boundary layer approximations, the governing conservation equations can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\Lambda(T - T_\infty) \sin\left(\frac{x}{a}\right) + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{4}$$

Where u and v are the velocity components in the x - and y -directions, respectively, ν , the kinematic viscosity of the fluid, β , the non-Newtonian Casson parameter, Λ , the co-efficient of thermal expansion, α , the thermal diffusivity, T , the temperature, respectively. The boundary conditions are prescribed at the cylinder surface and the edge of the boundary layer regime (free stream), respectively as follows

$$\begin{aligned} y = 0, \quad u = N_0 \left(1 + \frac{1}{\beta}\right) \frac{\partial u}{\partial y}, \quad v = -V_w, \quad T = T_w + K_0 \frac{\partial T}{\partial y} \\ y \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow T_\infty \end{aligned} \tag{5}$$

in which N_0 is the velocity slip factor and K_0 is the thermal slip factor. For $N_0=0=K_0$, one can recover the classical *no-slip* case.

The stream function ψ is defined by $u = \partial\psi/\partial y$ and $v = \partial\psi/\partial x$ and therefore, the continuity equation (2) is automatically satisfied. In order to write the governing equations and the boundary

conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} \xi = \frac{x}{a}, \quad \eta = \frac{y}{a} \sqrt[4]{Gr}, \quad f(\xi, \eta) = \frac{\psi}{\nu \xi \sqrt[4]{Gr}}, \quad Pr = \frac{\nu}{\alpha} \\ \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\Lambda(T_w - T_\infty)a^3}{\nu^2}, \\ \beta = \mu_B \frac{\sqrt{2\pi c}}{p_y} \end{aligned} \tag{6}$$

where, T_∞ is the free stream temperature, and V_w is the uniform blowing/suction velocity.

In view of equation (6), equations (2) to (4) reduce to the following coupled, non-linear, dimensionless partial differential equations for momentum and energy for the regime

$$\left(1 + \frac{1}{\beta}\right) f'' \prime + ff'' - f'^2 + \frac{\sin \xi}{\xi} \theta = \xi \left(f' \frac{\partial f''}{\partial \xi} - f'' \frac{\partial f'}{\partial \xi}\right) \tag{7}$$

$$\frac{\theta''}{Pr} + f\theta' = \xi \left(f' \frac{\partial \theta'}{\partial \xi} - \theta' \frac{\partial f'}{\partial \xi}\right) \tag{8}$$

The transformed dimensionless boundary conditions are

$$\begin{aligned} \text{At } \eta = 0, \quad f = S, \quad f' = \left(1 + \frac{1}{\beta}\right) S f''(0), \\ \theta = 1 + S_T \theta'(0) \\ \text{As } \eta \rightarrow \infty, \quad f' \rightarrow 0, \quad \theta \rightarrow 0 \end{aligned} \tag{9}$$

In the above equations, the primes denote the differentiation with respect to η , the dimensionless radial coordinate, ξ , the dimensionless tangential coordinate, Φ , the azimuthal coordinate, Gr , the Grashof (free convection) parameter, $Pr = \frac{\nu}{\alpha}$, the Prandtl number, $S_f = \frac{N_0 Gr^{1/4}}{a}$ and $S_T = \frac{K_0 Gr^{1/4} \alpha}{a}$, the

non-dimensional velocity slip and thermal slip parameters, respectively and $f_w = \frac{-V_w a}{\nu \sqrt{Gr}}$, the blowing/suction parameter ($f_w < 0$ for $V_w > 0$ (the case of blowing), and $f_w > 0$ for $V_w < 0$ (the case of suction)). Of course the special case of a *solid cylinder surface* corresponds to $f_w = 0$.

The engineering design quantities of physical interest include the skin-friction coefficient and Nusselt number, which are given by

$$\frac{1}{2} C_f Gr^{-3/4} = \left(1 + \frac{1}{\beta}\right) \xi f''(0) \tag{10}$$

$$\frac{Nu}{\sqrt{Gr}} = -\theta'(0) \tag{11}$$

Numerical solution

In this study, the efficient Keller–Box implicit difference method has been employed to solve the general flow model defined by equations (7) and (8) with boundary conditions (9). This method was originally developed for low speed aerodynamic boundary layers by Keller,³⁷ and has been employed in a diverse range of industrial multi-physical fluid dynamics problems. These include cross-diffusion boundary layer flows,³⁸ radiative-convective hydromagnetic flow from curved bodies in porous media,^{39,40} heat and mass transfer in non-linear porous media,⁴¹ micropolar convection,⁴² nanofluid dynamics,^{43,44} chemically reactive double-diffusive convection⁴⁵ and unsteady viscoelastic flow and mass diffusion.⁴⁶ The fundamental phases intrinsic to the Keller–Box scheme are:

- (a) Reduction of the N th order partial differential equation system to N first-order equations
- (b) Finite difference discretisation
- (c) Quasilinearisation of non-linear Keller algebraic equations
- (d) Block-tridiagonal elimination of linear Keller algebraic equations

Phase a: Reduction of the N th order partial differential equation system to N first-order equations

Equations (7) and (8) subject to the boundary conditions (9) are first written as a system of first-order equations. For this purpose, we reset equations (7) and (8) as a set of simultaneous equations by introducing the new variables u, v and t

$$f' = u \tag{12}$$

$$f'' = v \tag{13}$$

$$\theta' = t \tag{14}$$

$$\left(1 + \frac{1}{\beta}\right) v' + fv - u^2 + \frac{\sin \xi}{\xi} \theta = \xi \left(u \frac{\partial u}{\partial \xi} - v \frac{\partial f}{\partial \xi}\right) \tag{15}$$

$$\frac{1}{Pr} t' + ft = \xi \left(u \frac{\partial s}{\partial \xi} - t \frac{\partial f}{\partial \xi}\right) \tag{16}$$

where primes denote differentiation with respect to η . In terms of the dependent variables, the boundary conditions become

$$\text{At } \eta = 0 : \quad u = \left(1 + \frac{1}{\beta}\right) f''(0), \quad f = f_w, \quad s = 1$$

$$\text{As } \eta \rightarrow \infty : \quad u \rightarrow 0, \quad s \rightarrow 0 \tag{17}$$

Phase b: Finite difference discretisation

A two-dimensional computational grid is imposed on the ξ - η plane as sketched in Figure 2. The stepping process is defined by

$$\eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j, \quad j = 1, 2, \dots, J, \quad \eta_J \equiv \eta_\infty \tag{18}$$

$$\xi^0 = 0, \quad \xi^n = \xi^{n-1} + k_n, \quad n = 1, 2, \dots, N \tag{19}$$

where k_n and h_j denote the step distances in the ξ and η directions, respectively.

If g_j^n denotes the value of any variable at (η_j, ξ^n) , then the variables and derivatives of equations (12) to (16) at $(\eta_{j+1/2}, \xi^{n-1/2})$ are replaced by

$$g_{j-1/2}^{n-1/2} = \frac{1}{4} (g_j^n + g_{j-1}^n + g_j^{n-1} + g_{j-1}^{n-1}) \tag{20}$$

$$\left(\frac{\partial g}{\partial \eta}\right)_{j-1/2}^{n-1/2} = \frac{1}{2h_j} (g_j^n - g_{j-1}^n + g_j^{n-1} - g_{j-1}^{n-1}) \tag{21}$$

$$\left(\frac{\partial g}{\partial \xi}\right)_{j-1/2}^{n-1/2} = \frac{1}{2k_n} (g_j^n - g_{j-1}^n + g_j^{n-1} - g_{j-1}^{n-1}) \tag{22}$$

We now state the finite-difference approximation of equations (12) to (16) for the *mid-point* $(\eta_{j-1/2}, \xi^n)$

$$h_j^{-1} (f_j^n - f_{j-1}^n) = u_{j-1/2}^n, \tag{23}$$

$$h_j^{-1} (u_j^n - u_{j-1}^n) = v_{j-1/2}^n, \tag{24}$$

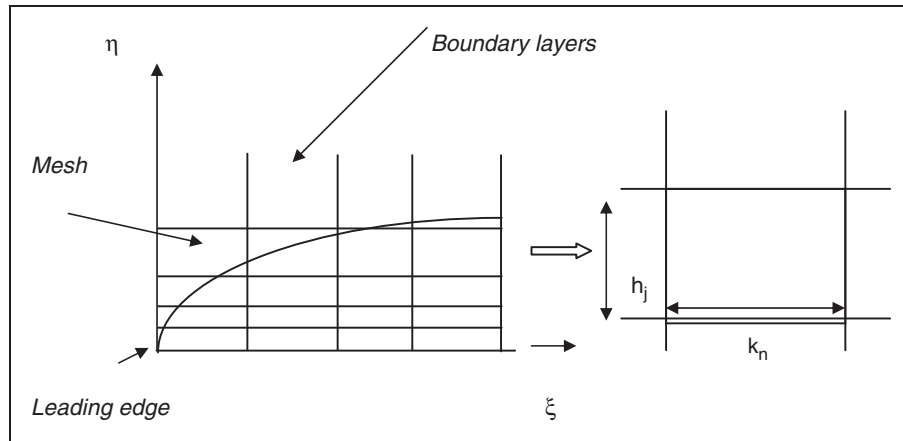


Figure 2. Grid meshing and a Keller–Box computational cell.

$$\begin{aligned} & \left(1 + \frac{1}{\beta}\right)(v_j - v_{j-1}) + \frac{(1 + \alpha)h_j}{4} \\ & \times [(f_j + f_{j-1})(v_j + v_{j-1})] - \frac{h_j}{4}(1 + \alpha)(u_j + u_{j-1})^2 \\ & + \frac{\alpha h_j}{2} v_{j-1/2}^{n-1}(f_j + f_{j-1}) - \frac{\alpha h_j}{2} f_{j-1/2}^{n-1}(v_j + v_{j-1}) \\ & + \frac{B h_j}{2}(s_j + s_{j-1}) = [R_1]_{j-1/2}^{n-1} \end{aligned} \tag{25}$$

$$h_j^{-1}(\theta_j^n - \theta_{j-1}^n) = t_{j-1/2}^n \tag{26}$$

$$\begin{aligned} & \frac{1}{Pr}(t_j - t_{j-1}) + \frac{(1 + \alpha)h_j}{4} [(f_j + f_{j-1})(t_j + t_{j-1})] \\ & - \frac{\alpha h_j}{4} [(u_j + u_{j-1})(s_j + s_{j-1})] \\ & + \frac{\alpha h_j}{2} s_{j-1/2}^{n-1}(u_j + u_{j-1}) - \frac{\alpha h_j}{2} u_{j-1/2}^{n-1}(s_j + s_{j-1}) \\ & - \frac{\alpha h_j}{2} f_{j-1/2}^{n-1}(t_j + t_{j-1}) \\ & + \frac{\alpha h_j}{2} t_{j-1/2}^{n-1}(f_j + f_{j-1}) = [R_2]_{j-1/2}^{n-1} \end{aligned} \tag{27}$$

where we have used the abbreviations

$$\alpha = \frac{\xi^{n-1/2}}{k_n}, \quad B = \frac{\sin(\xi^{n-1/2})}{\xi^{n-1/2}} \tag{28}$$

$$\begin{aligned} [R_1]_{j-1/2}^{n-1} = & -h_j \left[\left(1 + \frac{1}{\beta}\right) \left(\frac{v_j - v_{j-1}}{h_j}\right) \right. \\ & + (1 - \alpha)(f_{j-1/2} v_{j-1/2}) \\ & \left. + (\alpha - 1)(u_{j-1/2})^2 + B(s_{j-1/2}) \right] \end{aligned} \tag{29}$$

$$\begin{aligned} [R_2]_{j-1/2}^{n-1} = & -h_j \left[\frac{1}{Pr} \left(\frac{t_j - t_{j-1}}{h_j}\right) + (1 - \alpha)(f_{j-1/2} t_{j-1/2}) \right. \\ & \left. + \alpha(u_{j-1/2} s_{j-1/2}) \right] \end{aligned} \tag{30}$$

The boundary conditions are

$$f_0^n = u_0^n = 0, \quad \theta_0^n = 1, \quad v_0^n = 0, \quad \theta_j^n = 0 \tag{31}$$

Phase c: Quasilinearisation of non-linear Keller algebraic equations

If we assume $f_j^{n-1}, u_j^{n-1}, v_j^{n-1}, s_j^{n-1}, t_j^{n-1}$ to be known for $0 \leq j \leq J$, equations (23) to (27) are a system of $5J+5$ equations for the solution of $5J+5$ unknowns $f_j^n, u_j^n, v_j^n, p_j^n, s_j^n, t_j^n, j = 0, 1, 2, \dots, J$. This non-linear system of algebraic equations is *linearised* by means of Newton’s method as explained in Keller,³⁷ Prasad et al.,^{38–40} Vasu et al.⁴¹ and Anwar Bég et al.^{42,43}

Phase d: Block-tridiagonal elimination of linear Keller algebraic equations

The linear system (23) to (27) can now be solved by the *block-elimination* method, since they possess a *block-tridiagonal* structure. Commonly, the block-tridiagonal structure consists of variables or constants, but here, an interesting feature can be observed, namely that it consists of *block matrices*. The complete linearised system is formulated as a *block matrix system*, where each element in the coefficient matrix is a matrix itself. Then, this system is solved using the efficient Keller–Box method. The numerical results are affected by the number of mesh points in both directions. After some trials in the η -direction (radial coordinate) a larger number of mesh points are selected whereas in the ξ direction (tangential coordinate) significantly less mesh points are utilised. η_{max} has been set at 10 and this defines an adequately large value at which the prescribed boundary conditions are satisfied. ξ_{max} is set at 3.0 for this flow domain. *Mesh independence* is therefore achieved in the present computations. The computer program of the algorithm is executed in MATLAB running on a PC. The method demonstrates excellent stability, convergence and consistency, as elaborated by Keller.³⁷

Results and discussions

In order to verify the accuracy of our present method, we have compared our results with those of Merkin⁴⁷ and Yih.³⁶ Table 1 shows the comparisons of the values of $-\theta'(\xi, 0)$, $\beta \rightarrow \infty$, $f_w = 0$, $N_0 = 0$, $K_0 = 0$.

Comprehensive solutions have been obtained and are presented in Figures 3 to 20. The numerical problem comprises two independent variables (ξ, η), two dependent fluid dynamic variables (f, θ) and six thermo-physical and body force control parameters, namely $Pr, S_f, S_T, \beta, f_w, \xi$. In the present computations, the following default parameters are prescribed (unless otherwise stated): $Pr = 0.71$, $S_f = 0.5$, $S_T = 1.0$, $\beta = 1.0$, $f_w = 0.5$, $\xi = 1.0$. In addition we also consider the effect of *stream wise (transverse) coordinate* location on flow dynamics. The value of the parameter ξ is extremely important. For $\xi \sim 0$, the location is in the vicinity of the *lower stagnation point* on the cylinder. The governing dimensionless equations (7) to (8) in this case reduce to the following ordinary differential equations

$$\left(1 + \frac{1}{\beta}\right) f'' + ff'' - f'^2 + \theta = 0 \quad (32)$$

$$\frac{1}{Pr} \theta'' + f\theta' = 0 \quad (33)$$

since $\frac{\sin \xi}{\xi} \rightarrow 0/0$ i.e. 1, so that $\frac{\sin \xi}{\xi} \theta \rightarrow \theta$. Another special case arises at $\xi \sim \pi$, which physically corresponds to the *upper stagnation point* on the cylinder surface

Table 1. Values of the local heat transfer coefficient (Nu) for various values of ξ with $\beta \rightarrow \infty$, $f_w = 0$, $N_0 = 0$, $K_0 = 0$.

ξ	$-\theta'(\xi, 0)$		
	Merkin ⁴⁷	Yih ³⁶	Present results
0.0	0.4212	0.4214	0.4213
0.2	0.4204	0.4207	0.4206
0.4	0.4182	0.4184	0.4185
0.6	0.4145	0.4147	0.4146
0.8	0.4093	0.4096	0.4095
1.0	0.4025	0.4030	0.4031
1.2	0.3942	0.3950	0.3947
1.4	0.3843	0.3854	0.3857
1.6	0.3727	0.3740	0.3733
1.8	0.3594	0.3608	0.3602
2.0	0.3443	0.3457	0.3458
2.2	0.3270	0.3283	0.3280
2.4	0.3073	0.3086	0.3079
2.6	0.2847	0.2860	0.2858
2.8	0.2581	0.2595	0.2590
3.0	0.2252	0.2267	0.2261
π	0.1963	0.1962	0.1961

(diametrically opposite to the lower stagnation point). We note that since the Grashof-free convection parameter, Gr , is absorbed into the definitions for radial coordinate (η) and dimensionless stream function (f), it is not considered explicitly in the graphs.

In Figures 3 and 4, the influence of velocity slip parameter on velocity and temperature distributions is illustrated. Dimensionless velocity component (Figure 3) at the wall is strongly reduced with an increase in slip parameter, S_f . There will be a corresponding decrease in the momentum (velocity) boundary layer thickness. The influence of S_f is evidently more pronounced closer to the cylinder surface ($\eta = 0$). Further from the surface, there is a transition in *velocity slip effect*, and the flow is found to be accelerated markedly. Smooth decays of the velocity profiles are observed into the free stream demonstrating excellent convergence of the numerical solution. These trends in the response of velocity field in external thermal convection from a cylinder were also observed by Wang and Ang³⁴ and Wang.³⁵ Furthermore the acceleration near the wall with increasing velocity slip effect has been computed by Crane and McVeigh³² using asymptotic methods, as has the retardation in flow further from the wall. The switch in velocity slip effect on velocity evolution has also been observed for the case of a power-law rheological fluid by Ojadi et al.⁴⁸ Figure 4 indicates that an increase in velocity slip parameter significantly enhances temperature in the flow field and thereby increases thermal boundary layer thickness enhances. This will result therefore in the transport of more thermal energy from the cylinder surface to the Casson fluid and will therefore accentuate heat transfer to the fluid, as noted also by Wang.³⁵ Temperature profiles consistently decay monotonically from a maximum at the cylinder surface to the free stream. All profiles converge at large value of radial coordinate, again showing that convergence has been achieved in the numerical computations. A similar pattern of thermal response to that computed in Figure 4 for a wide range of velocity slip parameters has been noted by Aziz⁴⁹ who has indicated also that temperature is enhanced since increasing velocity slip parameter decreases shear stresses and this permits a more effective transfer of heat from the wall to the fluid regime.

In Figures 5 and 6, the variation of velocity and temperature with the transverse coordinate (η), with increasing thermal slip parameter S_T is depicted. The response of velocity is much more consistent than for the case of changing velocity slip parameter (Figure 3) – it is strongly decreased for all locations in the radial direction. The peak velocity accompanies the case of no thermal slip ($S_T = 0$). The maximum deceleration corresponds to the case of strongest thermal slip ($S_T = 3$). Temperatures (Figure 6) are also strongly depressed with increasing thermal slip. The maximum effect is observed at the wall. Further into the free stream, all temperature profiles converge smoothly

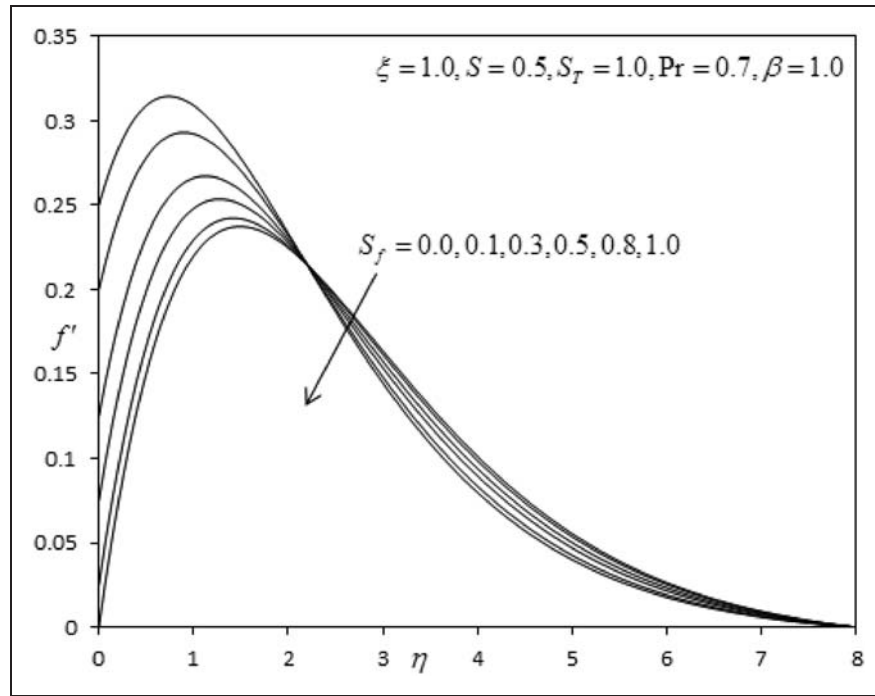


Figure 3. Influence of S_f on velocity profiles.

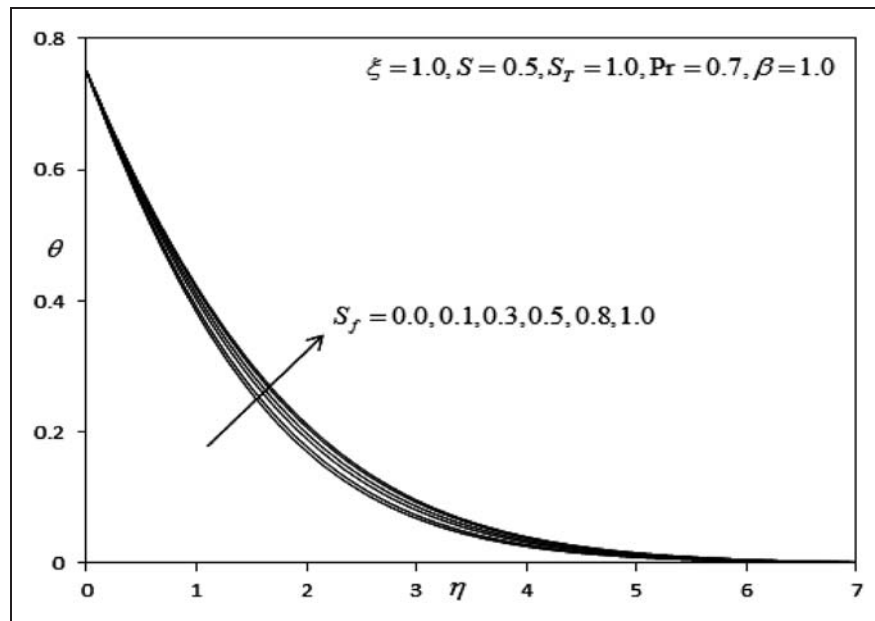


Figure 4. Influence of S_f on temperature profiles.

to the vanishing value. The numerical computations correlate well with the results of Larrode et al.²⁷ who also found that temperature is strongly lowered with increasing thermal slip and that this is attributable to the decrease in heat transfer from the wall to the fluid regime, although they considered only a Newtonian fluid.

In Figures 7 and 8, depict the influence Casson fluid parameter, β on velocity and temperature profiles. This parameter features in the shear term in the

momentum boundary layer equation (7), and also in the velocity boundary condition (9). For Newtonian flow, yield stress p_y is zero and $\beta = \mu_B \sqrt{\pi_c/p_y} \rightarrow \infty$ i.e. the appropriate term in equation (7) reduces from $(1 + 1/\beta)f''' \rightarrow 1$. Similarly the velocity boundary condition in (9) reduces from $(1 + 1/\beta) S_f f''(0) \rightarrow S_f f''(0)$. An increase in β implies a decrease therefore in yield stress of the Casson fluid. This effectively facilitates flow of the fluid i.e. accelerates the boundary layer flow close to the cylinder surface, as

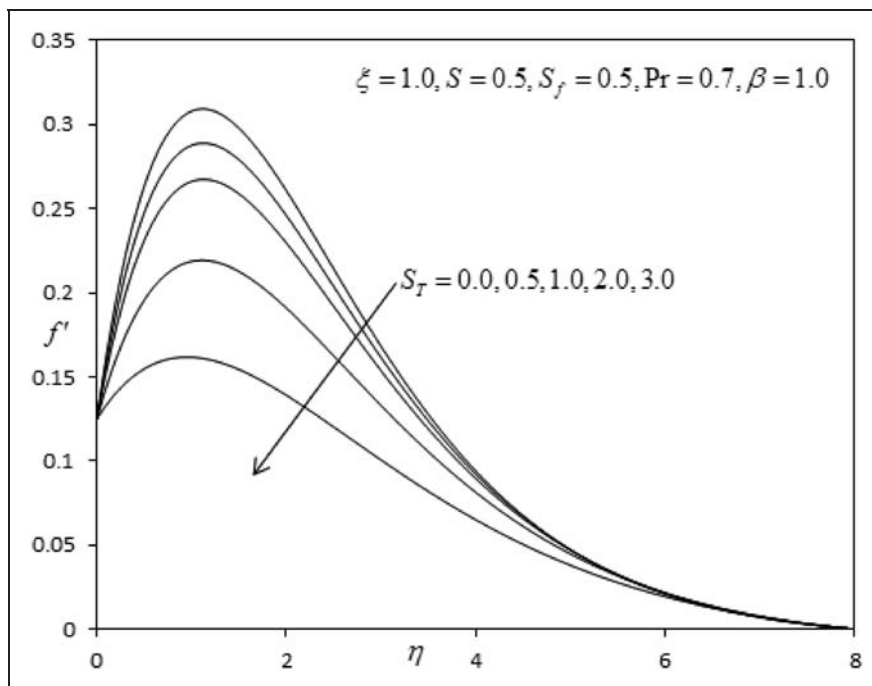


Figure 5. Influence of S_T on velocity profiles.

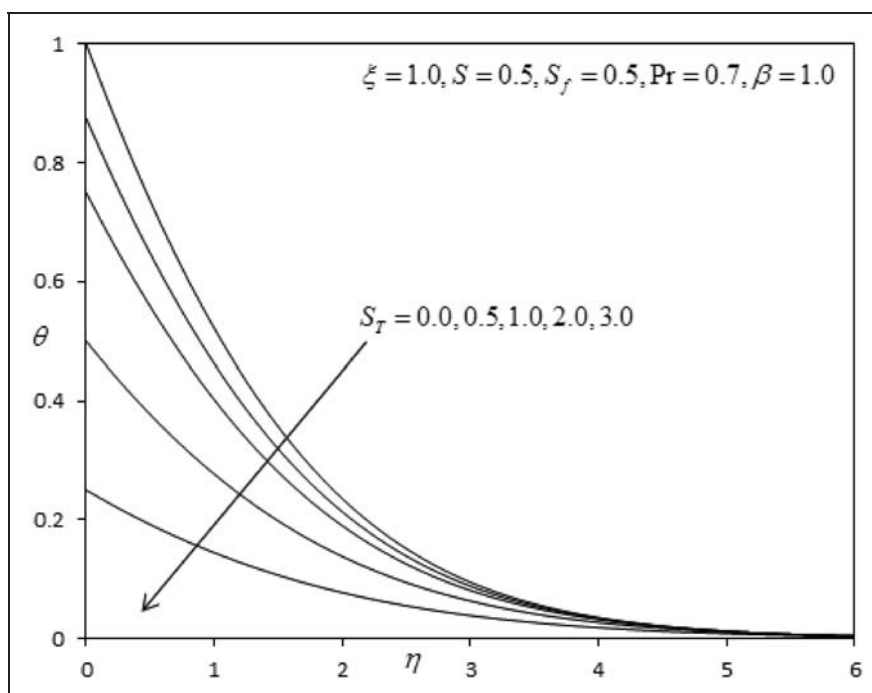


Figure 6. Influence of S_T on temperature profiles.

demonstrated by Figure 7. Since the Casson parameter is also present in the wall boundary condition, the acceleration effect is only confined to the region close to the cylinder surface. Further from this zone, the velocity slip factor, S_f will exert a progressively reduced effect and an increase in Casson parameter, β , will manifest with a deceleration in the flow. Overall, however, the dominant influence of β , is

near the wall and is found to be assistive to momentum development (with larger β values the fluid is closer in behaviour to a *Newtonian* fluid and further departs from *plastic* flow) Only a very small decrease in temperature is observed with a large enhancement in Casson fluid parameter, as shown in Figure 8. The Casson parameter does not arise in the thermal boundary layer equation (8), nor does it feature in

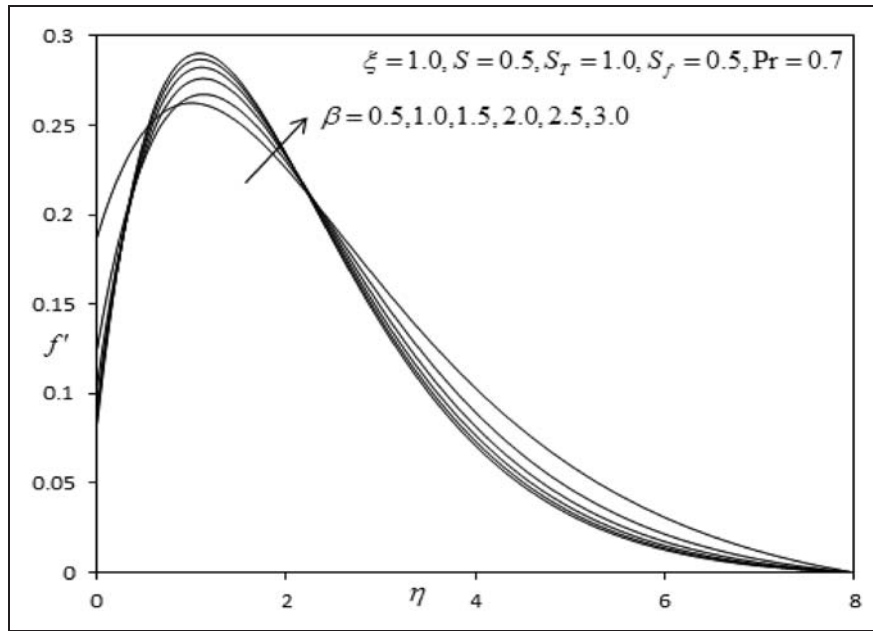


Figure 7. Influence of β on velocity profiles.

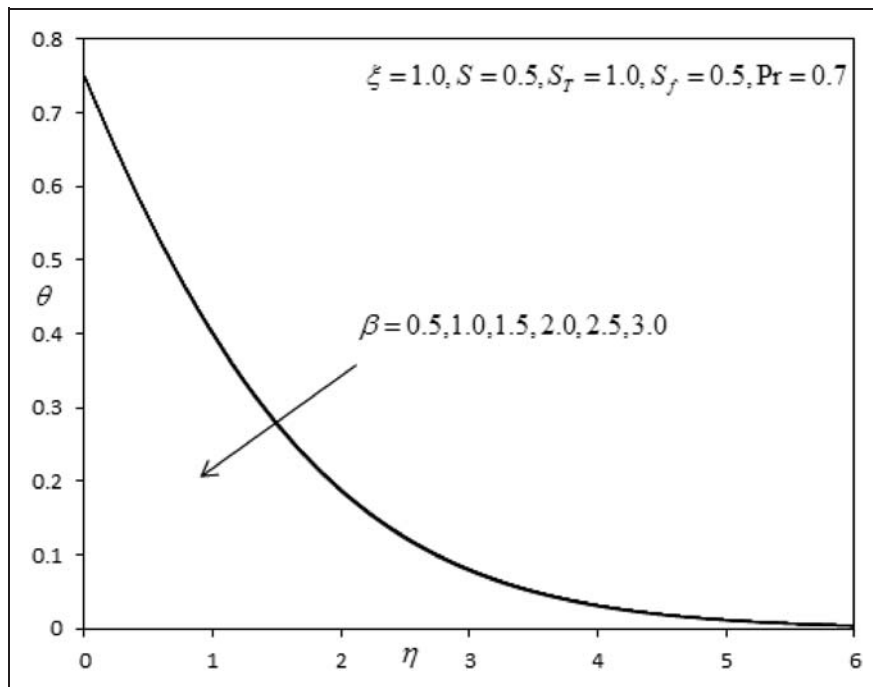


Figure 8. Influence of β on temperature profiles.

the thermal boundary conditions. The influence on temperature field is therefore experienced indirectly via coupling of the thermal equation (8) with the momentum equation (7). Similar behaviour to the computations shown in Figures 7 and 8, has been observed by Attia and Sayed-Ahmed²⁷ who also observed acceleration in Casson fluid flow near a curved surface, and additionally by Mustafa et al.²⁴ who also observed an elevation in velocities near the

wall and a slight reduction in temperatures throughout the boundary layer regime.

Figures 9 and 10 present the effect of Prandtl number (Pr) on the velocity and temperature profiles along the radial direction, normal to the cylinder surface. Prandtl number embodies the ratio of *viscous diffusion* to *thermal diffusion* in the boundary layer regime. It also expresses the ratio of the *product of specific heat capacity and dynamic viscosity*, to the

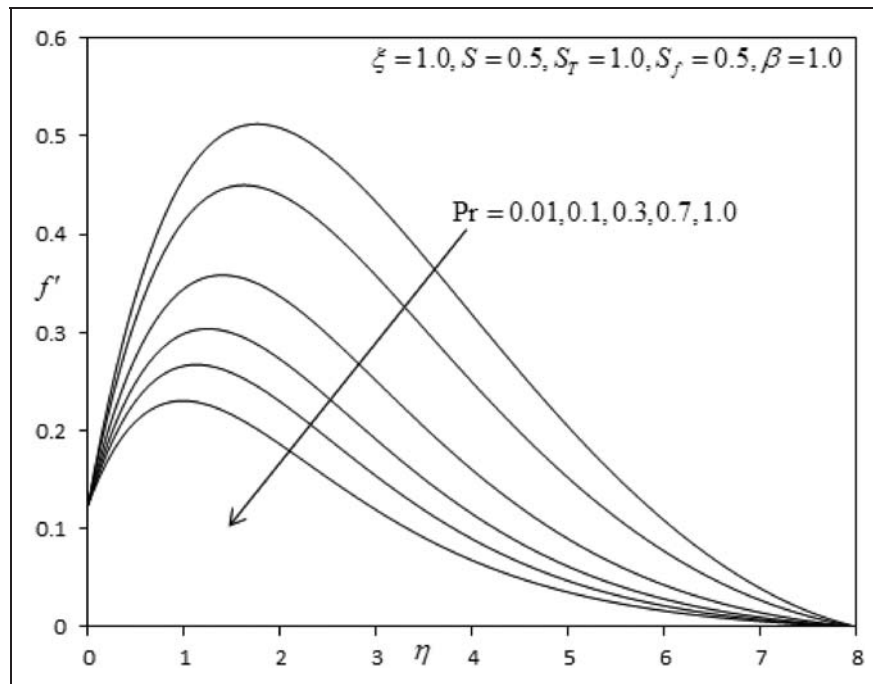


Figure 9. Influence of Pr on velocity profiles.

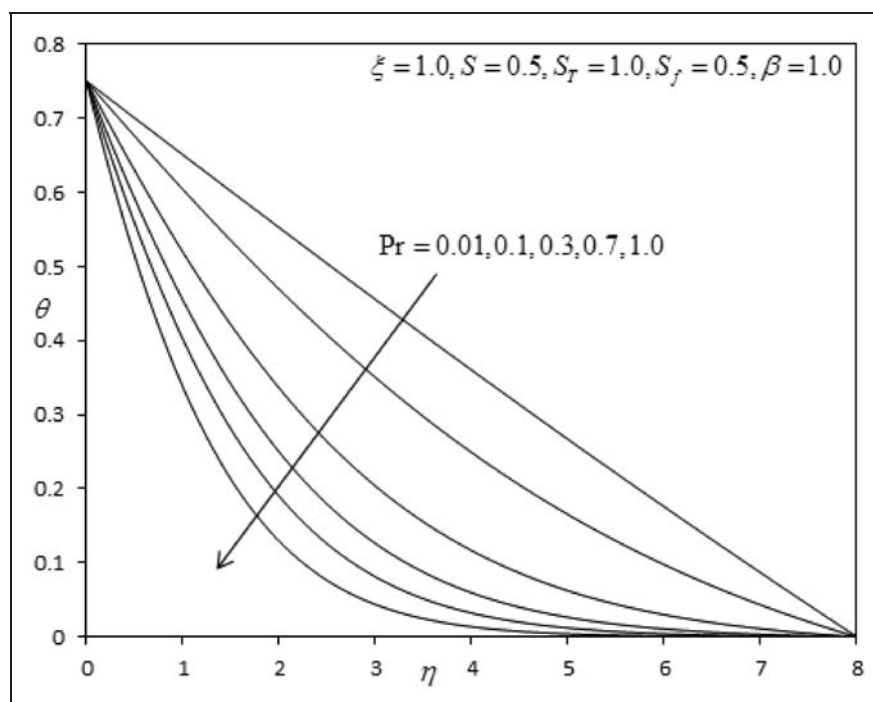


Figure 10. Influence of Pr on temperature profiles.

fluid thermal conductivity. When Pr is high, viscous diffusion rate exceeds thermal diffusion rate. An increase in Pr from 0.01 through 0.1, 0.3, 0.5, 0.7 to 1.0, is found to significantly depress velocities (Figure 9) and this trend is sustained throughout the regime i.e. for all values of the radial coordinate, η . For $Pr < 1$, thermal diffusivity exceeds momentum diffusivity i.e. heat will diffuse faster than momentum.

Therefore for lower Pr fluids (e.g. $Pr = 0.01$ which physically correspond to liquid metals), the flow will be accelerates whereas for greater Pr fluids (e.g. $Pr = 1$) it will be strongly decelerated, as observed in figure. For $Pr = 1.0$, both the viscous and energy diffusion rates will be the same as will the thermal and velocity boundary layer thicknesses. This case can be representative of food stuffs e.g. low-density

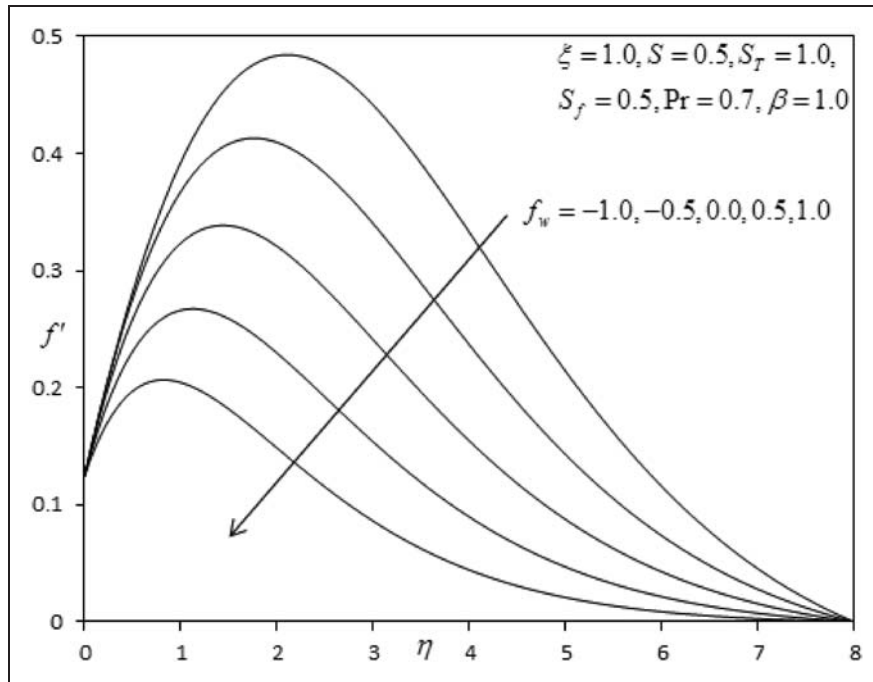


Figure 11. Influence of f_w on velocity profiles.

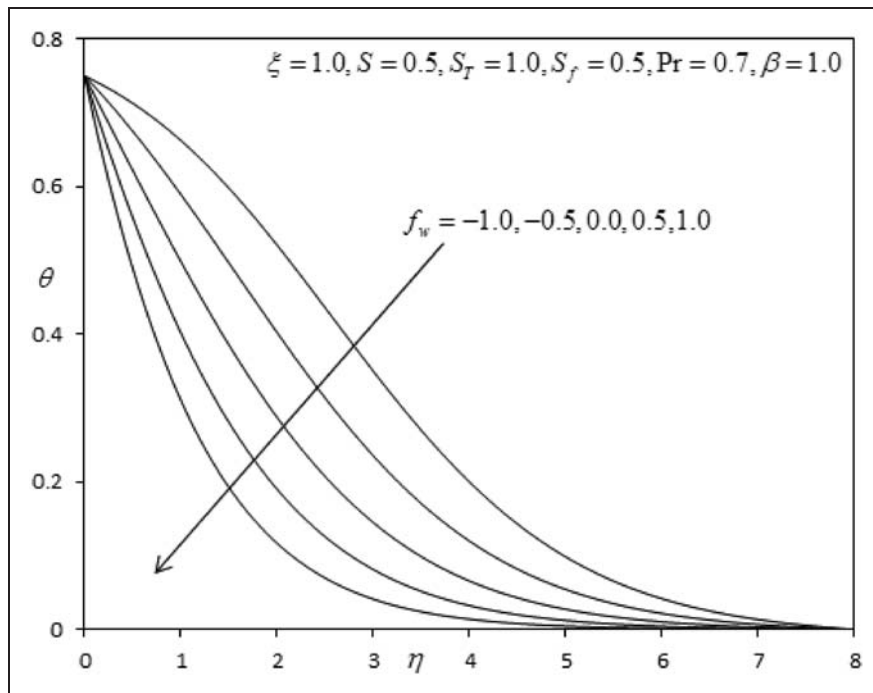


Figure 12. Influence of f_w on temperature profiles.

polymorphic forms of chocolate suspensions, as noted by Steffe¹⁰ and Debaste et al.⁵⁰ Temperature is found to be strongly reduced with increasing Prandtl number. For the case of $Pr=0.01$, the decay is almost exactly *linear*. For larger Pr values, the decay is found to be *increasingly monotonic*. Therefore for lower thermal conductivity fluids (as typified by liquid

chocolate and other foodstuffs), lower temperatures are observed throughout the boundary layer regime.

Figures 11 and 12 illustrate the influence of wall transpiration on the velocity and temperature functions with radial distance, η . With an increase in suction ($f_w > 0$) the velocity is clearly decreased i.e. the flow is decelerated. Increasing suction causes the

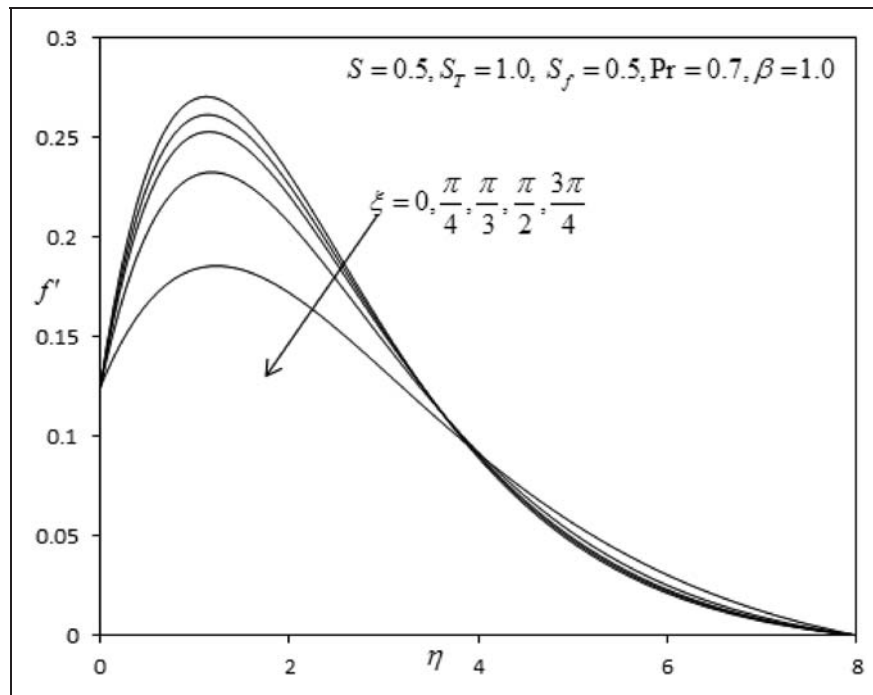


Figure 13. Influence of ξ on velocity profiles.

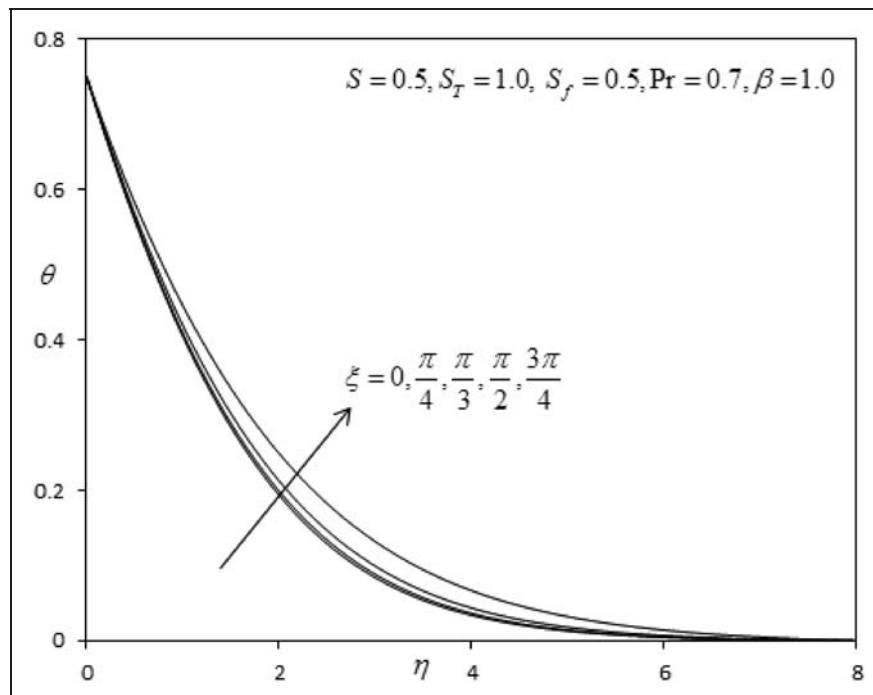


Figure 14. Influence of ξ on temperature profiles.

boundary layer to adhere closer to the flow and destroys momentum transfer; it is therefore an excellent control mechanism for stabilising the external boundary layer flow on the circular cylinder. Conversely with increased blowing i.e. injection of fluid via the cylinder surface in to the porous medium regime, ($f_w < 0$), the flow is strongly accelerated i.e. velocities

are increased. As anticipated the case of a solid cylinder ($f_w = 0$) falls between the weak suction and weak blowing cases. Peak velocity is located, as in the figures described earlier, at close proximity to cylinder surface. With a decrease in blowing and an increase in suction the peaks progressively displace closer to the cylinder surface, a distinct effect described in detail in

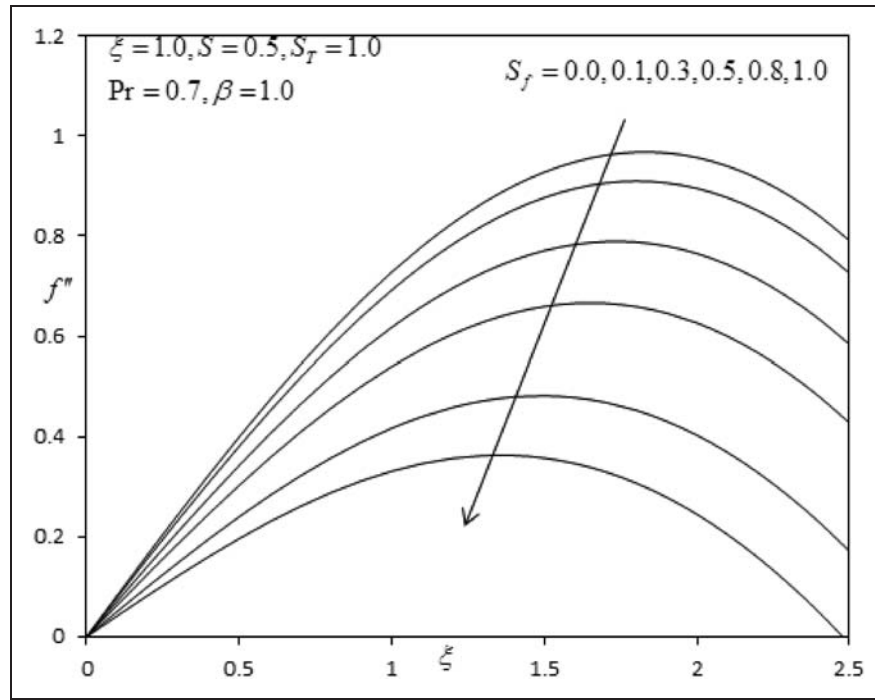


Figure 15. Effect of S_f on the skin friction coefficient results.

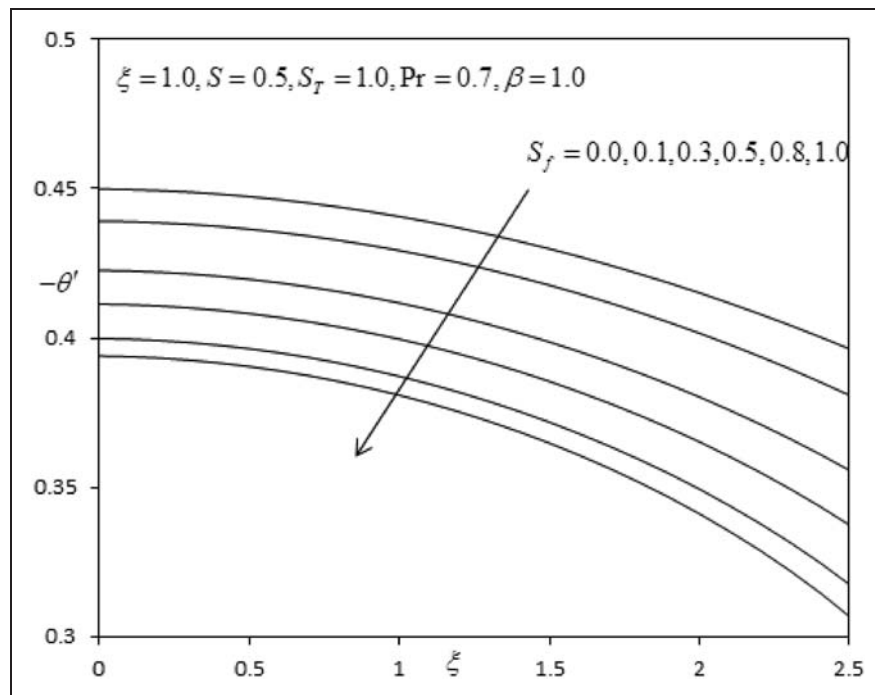


Figure 16. Effect of S_f on the local Nusselt number results.

several studies of non-Newtonian boundary layers.^{1,3,7,23,24} Temperature, θ , is also elevated considerably with increased blowing at the cylinder surface and depressed with increased suction. The temperature profiles, once again assume a continuous decay from the cylinder surface to the free stream, whereas the velocity field initially ascends, peaks

and then decays in to the free stream. The strong influence of wall transpiration (i.e. suction or injection) on boundary layer variables is clearly highlighted. Such a mechanism is greatly beneficial in achieving flow control and regulation of heat and mass transfer characteristics in food processing from a cylindrical geometry.

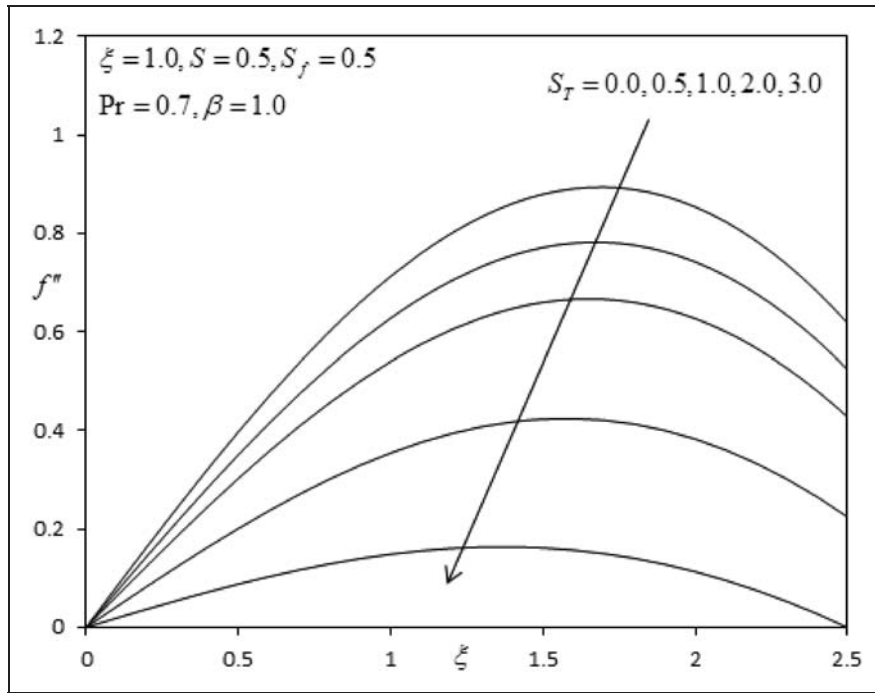


Figure 17. Effect of S_T on the skin friction coefficient results.

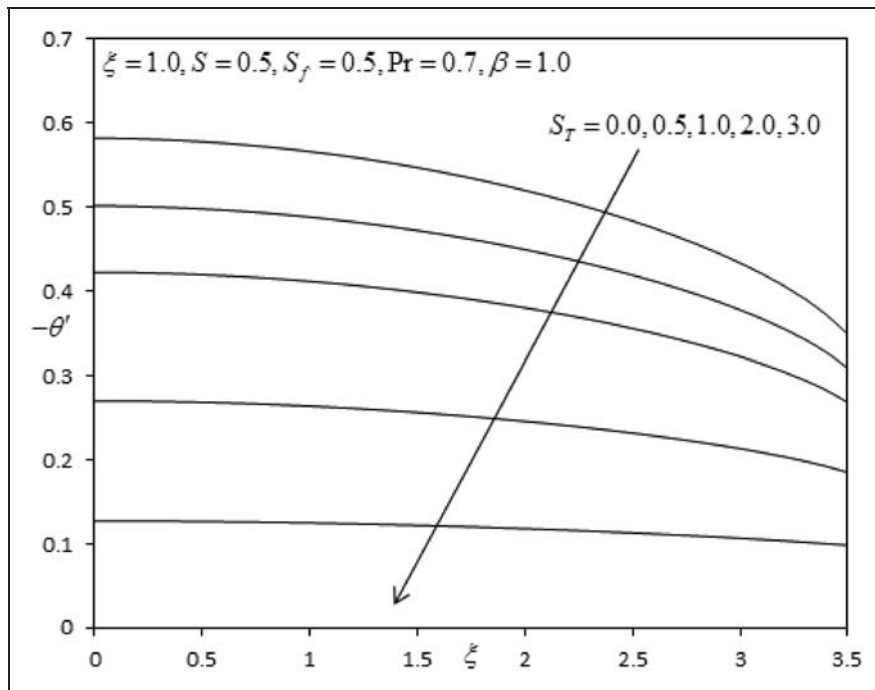


Figure 18. Effect of S_T on the local Nusselt number results.

Figures 13 and 14, the variation of velocity and temperature fields with different transverse coordinate, ξ , is shown. In the vicinity of the cylinder surface, velocity (f') is found to be maximised closer to the lower stagnation point and minimised with progressive distance away from it i.e. the flow is decelerated with increasing ξ . However, further from the wall, this trend is reversed and a slight acceleration in the flow is

generated with greater distance from the lower stagnation point i.e. velocity values are higher for greater values of ξ , as we approach the upper stagnation point temperature θ , is found to noticeably increase through the boundary layer with increasing ξ values. Evidently the fluid regime is cooled most efficiently at the lower stagnation point and heated more effectively as we progress around the cylinder periphery upwards

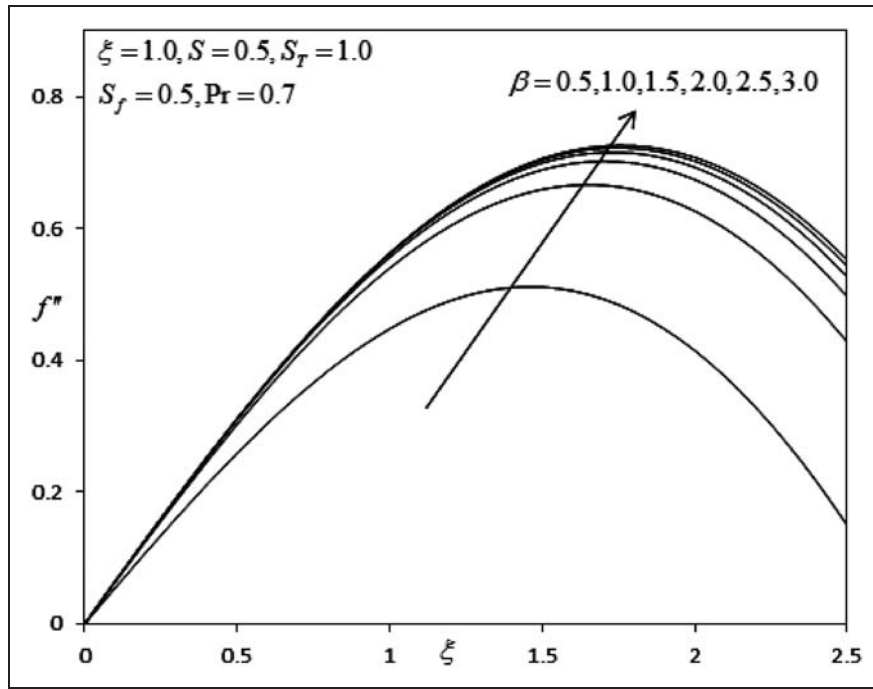


Figure 19. Effect of β on the skin friction coefficient results.

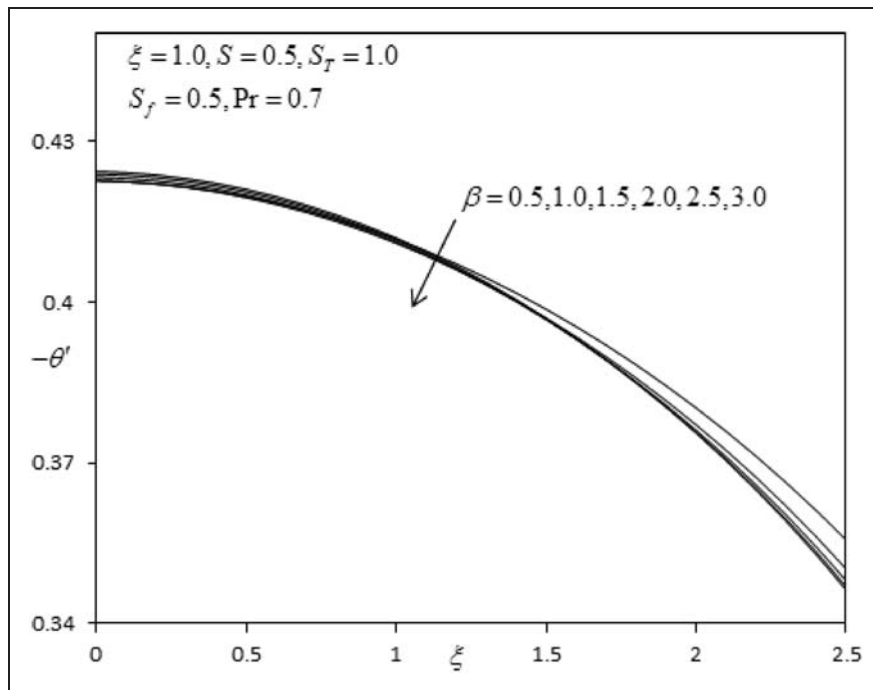


Figure 20. Effect of β on the local Nusselt number results.

towards the upper stagnation point. These patterns computed for temperature and velocity evolution around the cylinder surface are corroborated with many other studies including work on non-Newtonian Casson fluid convection by Kandasamy et al.¹⁸ and studies of Newtonian convection from a cylinder by Wang³⁵ and Prasad et al.⁴⁰

Figures 15 and 16 show the effect of velocity slip parameter S_f on cylinder surface shear stress (f'') and local Nusselt number ($-\theta'$) variation. In consistency with the earlier graphs described for velocity evolution, with an increase in S_f , wall shear stress is consistently reduced i.e. the flow is *decelerated* along the cylinder surface. Again this trend has been observed

by Wang and Ang³⁴ and Wang³⁵ using asymptotic methods. There is also a *progressive migration* in the peak shear stress locations further from the *lower stagnation point*, as wall slip parameter is increased. The impact of wall slip is therefore significant on the boundary layer characteristics of Casson flow from a cylinder. With an increasing S_f , the local Nusselt number is also considerably decreased and profiles are generally monotonic decays. Maximum local Nusselt number always arises at the cylinder surface and is minimised with proximity to the lower stagnation point i.e. greater distance from the *upper stagnation point*. This pattern of behaviour has also been observed and emphasised by Yih³⁶ for *Newtonian* flow. In both Figures 15 and 16, skin friction coefficient and local Nusselt number are maximised for the case of no-slip i.e. $S_f=0$, this result concurring with the analyses of Wang³⁵ and also Hayat et al.²³

Figures 17 and 18 show the effect of thermal slip parameter S_T on dimensionless wall shear stress function i.e. skin friction coefficient and local Nusselt number, respectively. Increasing S_T is found to decrease both skin friction coefficient and local Nusselt number. A similar set of profiles is computed as in Figure 15 for velocity distributions, and we observe that with *increasing thermal slip*, peak velocities are displaced *closer* to the lower stagnation point. For lower values of thermal slip, the plots are also similar to those in Figure 16, and have a parabolic nature; however, with S_T values greater than 1, the profiles lose their curvature and become increasingly *linear* in nature. This trend is maximised for the highest value of S_T ($=3.0$) for which local Nusselt number is found to be almost invariant with transverse coordinate, ξ

Figures 19 and 20 illustrate the effect of Casson fluid parameter, β , on skin friction coefficient and local Nusselt number, respectively. With an increase in β the skin friction coefficient increases, since as computed earlier, the flow velocity is enhanced with higher values of β . Larger β values correspond to a progressive decrease in yield stress of the Casson fluid i.e. a reduction in rheological characteristics. With higher β the flow approaches closer to *Newtonian* behaviour and the fluid is able to shear faster along the cylinder surface. Local Nusselt number is conversely found to decrease slightly as Casson fluid parameter is increased. This concurs with the earlier computation (Figure 8) on temperature distribution. With increasing β values, less heat is transferred from the cylinder surface to the fluid regime, resulting in lower temperatures in the regime external to the cylinder and lower local Nusselt numbers, as observed in Figure 20.

Conclusions

Numerical solutions have been presented for the transport phenomena i.e. combined heat and flow of

Casson rheological fluid external to a horizontal circular cylinder, with suction/injection effects and velocity/thermal slip. The model has been developed to simulate foodstuff transport processes in industrial manufacturing operations. A robust, extensively-validated, implicit finite difference numerical scheme has been implemented to solve the transformed, dimensionless velocity and thermal boundary layer equations, subject to physically realistic boundary conditions. The computations have shown that:

1. Increasing the velocity slip parameter, S_f , reduces the velocity near the cylinder surface and also skin friction coefficient and also increases temperature and decreases local Nusselt number.
2. Increasing the thermal slip parameter, S_T , decreases velocity and skin friction coefficient and also reduces temperature for all values of radial coordinate i.e. throughout the boundary layer regime, and furthermore decreases local Nusselt number.
3. Increasing the Casson fluid parameter, β , increases the velocity near the cylinder surface but decreases velocity further from the cylinder, and also fractionally lowers the temperature throughout the boundary layer regime.
4. Increasing the Casson fluid parameter, β , strongly increases the wall shear stress (skin friction coefficient) and slightly decreases the local Nusselt number, with the latter more significantly affected at large distances from the lower stagnation point i.e. higher values of transverse coordinate.
5. Increasing Prandtl number, Pr , decelerates the flow and also strongly depresses temperatures, throughout the boundary layer regime.
6. Increasing suction at the cylinder surface ($f_w > 0$) decelerates the flow whereas increasing injection ($f_w < 0$, i.e. blowing) induces a strong acceleration.
7. Increasing suction at the cylinder surface ($f_w > 0$) reduces temperature whereas increasing injection ($f_w < 0$ i.e. blowing) induces the opposite response and elevates temperature.
8. Increasing transverse coordinate, ξ , depresses velocity near the cylinder surface but enhances velocity further from the cylinder, whereas it continuously increases temperature throughout the boundary layer.

The current study has been confined to steady-state flow i.e. ignored transient effects⁴ and also neglected thermal radiation heat transfer effects.^{39,40} These aspects are also of relevance to rheological food processing simulations and will be considered in future investigations.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

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Appendix

Notation

a	radius of the cylinder
C_f	skin friction coefficient
S_f	non-dimensional velocity slip parameter
S_T	non-dimensional thermal slip parameter
f	non-dimensional steam function
g	acceleration due to gravity
Gr	Grashof number
N_0	velocity slip factor
K_0	thermal slip factor
Nu	local Nusselt number
Pr	Prandtl number
T	temperature
u, v	non-dimensional velocity components along the x - and y -directions, respectively
x, y	non-dimensional Cartesian coordinates along and transverse to cylinder surface
α	thermal diffusivity
β	non-Newtonian (rheological) Casson parameter
Λ	co-efficient of thermal expansion
Φ	azimuthal coordinate
η	dimensionless radial coordinate
μ	dynamic viscosity
ν	kinematic viscosity
θ	non-dimensional temperature
ρ	density of fluid
ξ	dimensionless tangential coordinate
ψ	dimensionless stream function

Subscripts

w	conditions on the wall
∞	free stream conditions