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Differential space time modulation and demodulation for time varying multiple input multiple output channels

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DIFFERENTIAL SPACE TIME MODULATION AND DEMODULATION FOR TIME VARYING MULTIPLE INPUT MULTIPLE OUTPUT CHANNELS

A Thesis

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I would like to dedicate this work to my parents Sathyavathi Gundappa and Jaishankar Chitnis who have supported me throughout my academic endeavors, without them this may never have been possible.

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Abstract

Over the last decade there has been considerable interest in wireless communication using multiple transmit and receive antennas. Several literatures exists that show that these multiple link support very high data rates with low error probabilities when the channel state information is available at the receiver. However when multiple antennas are employed or when the mobile environments change rapidly, it is not always possible to have apriori knowledge of the channel state matrices which calls for Differential Space-Time modulation techniques.

Differential modulation is used in conjunction with Unitary Space-Time codes to evaluate their performance over time varying channels. Jakes model for frequency flat fading processes in mobile radio systems is incorporated with the differential modulation scheme to model a time-varying space-time Rayleigh fading multiple input multiple output (MIMO) radio channel. Parametric unitary codes that are known to have the largest possible diversity product for a 16-signal constellation and a 4-signal constellation with both optimal diversity sum and diversity product is used to evaluate the Block Error Rates for 2 and 5 receiver antennas that are moving at different velocities.

A fast differential demodulation for Alamouti codes is derived based on prior work by Liang and Xia and is tested using our simulations.

MATLAB R2006b V 7.1 is used to simulate the performance of M=2, N=2 and M=2 N=5 antennas over a time varying channel for velocities of 0, 50, 75, 100 and 125 kmph. We also show that the fast demodulation algorithm is almost twice as fast and also perform within 1dB of existing differential demodulation schemes.

Chapter 1 - Introduction

This chapter provides an overview of the motivation and concepts on which this thesis is based. We provide a brief summary of MIMO and Space Time Block Codes. The structure of the thesis and the material covered in the subsequent chapters are discussed.

1.1 Overview

Wireless communication has become a pervasive means of communication and with its increasing popularity the need for optimizing data rates, speed and reliability has grown exponentially. It has been shown by the information theoretic works of [1], [2], [3] and [4] that the capacity that can be achieved by using an array of transmitting and receiving antennas can be increased linearly with the number of antennas. The field of wireless communication that employs multiple antennas has been referred to as Multiple Input Multiple Output system. A considerable amount of work has been done in this area that has facilitated newer wireless technologies that support increased speeds and throughput that take advantage of the diversity in such systems.

The focus of this thesis is to design modulation and demodulation techniques for Differential Space-Time MIMO systems for a time varying channel. The performance of specific Space Time Block Codes is evaluated for various terminal speeds using simulations for the above mentioned scheme.

1.2 MIMO Fundamentals

In this section the fundamentals of MIMO systems are discussed where the channel is assumed to be flat and quasi-stationary fading.

Simply defined, a MIMO system can be thought of as a wireless communication system that has multiple (an array of) transmitting (Tx) and receiving (Rx) antennas. It was demonstrated by the works of Foschini, Marzetta, Teletar et al that the advantages of such a scheme would be an increased performance in terms of Bit Error Rate (BER) and speed (data rate in bits/sec) of the communication system.

1.2.1 MIMO Capacity

The capacity for a simple 1x1 Single Input Single Output (SISO) communication system as given by Shannon's theorem is

$$C = \log_2 \left(1 + \gamma |h|^2 \right) \qquad bits/\sec/Hz \tag{1-1}$$

Where h is the normalized complex channel state realization of a wireless channel and γ is the Signal to Noise Ratio (SNR) at any receiver antenna.

For a SIMO system, the capacity can be given by,

$$C = \log_2 \left(1 + \gamma \sum_{i=1}^{M} |h_i|^2 \right) \qquad bits/\sec/Hz \tag{1-2}$$

Where h_i is the gain for the receiver antenna i. It should be noted that increasing the number of receiving antennas leads to a logarithmic rise in average capacity.

Similarly, for a MISO system, the capacity is given by

$$C = \log_2 \left(1 + \frac{\gamma}{N} \sum_{i=1}^{N} |h_i|^2 \right) \qquad bits/\sec/Hz \tag{1-3}$$

The channel capacity for a MIMO system is given by

$$C_{Ergodic} = \log_2 \left[\det \left(\mathbf{I}_M + \frac{\gamma}{N} \mathbf{H} \mathbf{H}^* \right) \right] \qquad bits/\sec/Hz \tag{1-4}$$

Where **H** is the Channel State Information (CSI) matrix and $\mathbf{I}_M$ is an identity matrix of order M.

For M transmitting antennas and N receiving antennas, capacity increases logarithmically as M is kept constant and N is increased. Intuitively, it can be noted that if N is kept constant there would be a point when adding more number of transmitting antennas, *M*, would not make any difference to the capacity [3].

If *M* increases and $N \geq M$, then information theoretic results show that the MIMO system capacity increases at least linearly as a function of *M* [3].

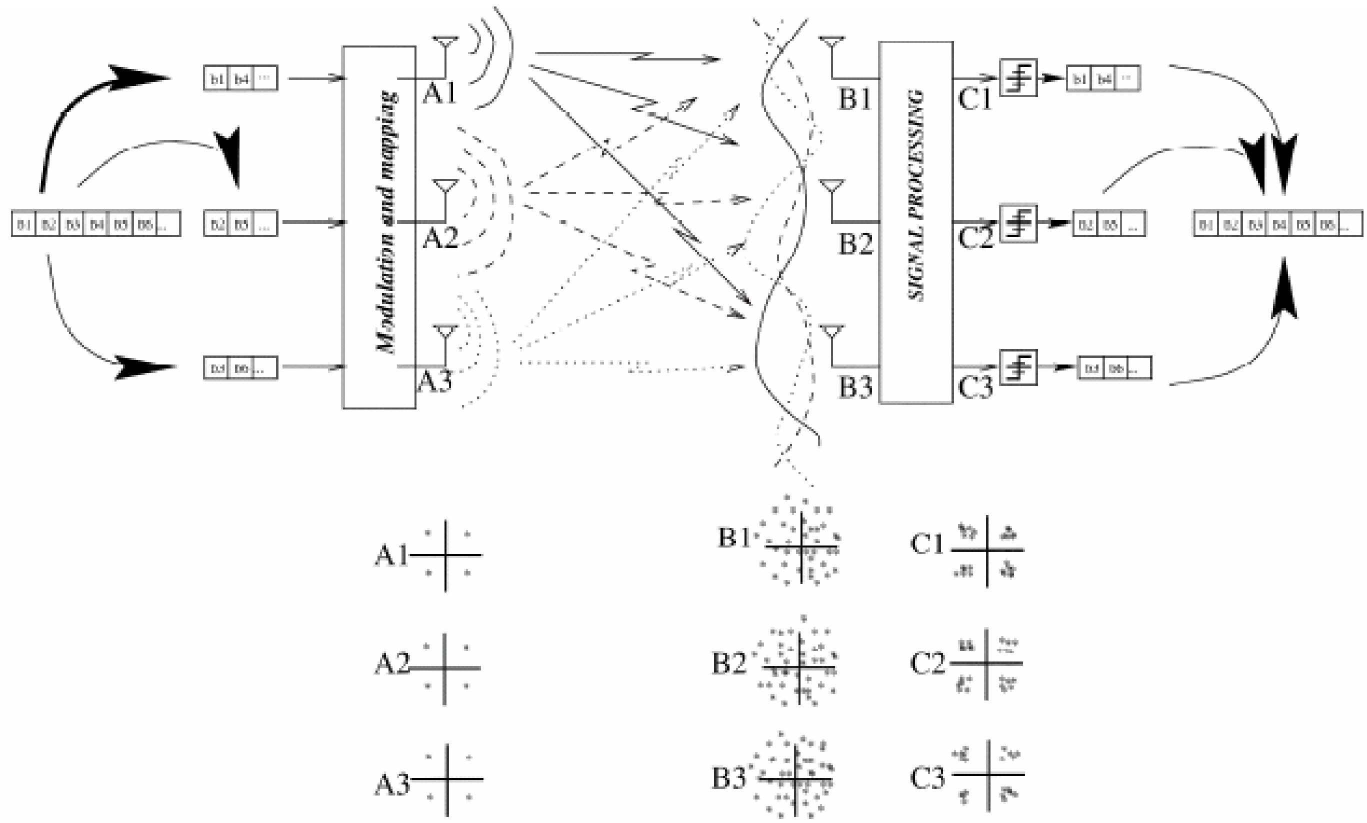

Figure1.1 Block diagram of a MIMO system

1.2.2 Diversity

Spatial multiplexing and transmit diversity are the two reasons why MIMO offers higher throughput. Multiplexing involves transmission of several independent data streams/signals over different transmitting antennas simultaneously to maximize the rate while transmit diversity sends copies of the same information signal over different transmitting antennas.

Multi-path fading causes severe degradation of signals in a wireless communication system. MIMO offers a spectacular method for countering random fading due to multi-path propagation and channel noise (scattering, reflection, refraction, interference etc by means of diversity. When we have multiple copies of the signal coming in at the receiver at the same time, the receiver has better means of extracting the most information and on deciding upon the correct transmitted signal. This technique is referred to as Diversity. Diversity mitigates the effect of fading and hence facilitates higher level modulation schemes that increase system capacity and greatly reduce Bit Error Rates (BER).

There are various types of diversity including temporal, frequency, polarization, spatial and angle diversity. In this thesis however, we shall be dealing only with spatial diversity. Spatial diversity uses multiple antennas at the receiver and the transmitter to achieve diversity. The probability of correct detection at the receiver increases exponentially with the number of decorrelated antennas being used. The number of decorrelated spatial channels available at the transmitter or the receiver is known as Diversity Order.

There are several ways to process the diverse number of signals received such that the power efficiency of the system is optimized. The most common techniques are Selection Diversity, Maximal Ratio Receive Combining (MRRC) and Equal Gain Combining (EGC). The simplest of the above techniques, of course, is Selection Diversity, wherein the receiver simply selects the one with the largest SNR as the output.

1.3 Fundamentals of Space Time Block Codes

Space Time Block Coding (STBC) is a spatial diversity techniques used to improve system performance and capacity in fading environments. They provide the natural mathematical extension and representation to diversity techniques discussed above. Initially proposed by Tarokh et al [5], these space time block codes achieve significant error rate improvement over single-antenna systems.

STBCs are represented using matrices as shown below. Each row represents a time slot while the columns represent transmitting antennas. Every element of the matrix, S_{ij} is a modulated symbol to be transmitted in time slot *i* from transmitter *j*. Generally these symbols are chosen from a Quadrature Amplitude Modulated (QAM) or an M-ary Phase Shift Keyed (M-PSK) signal constellation.

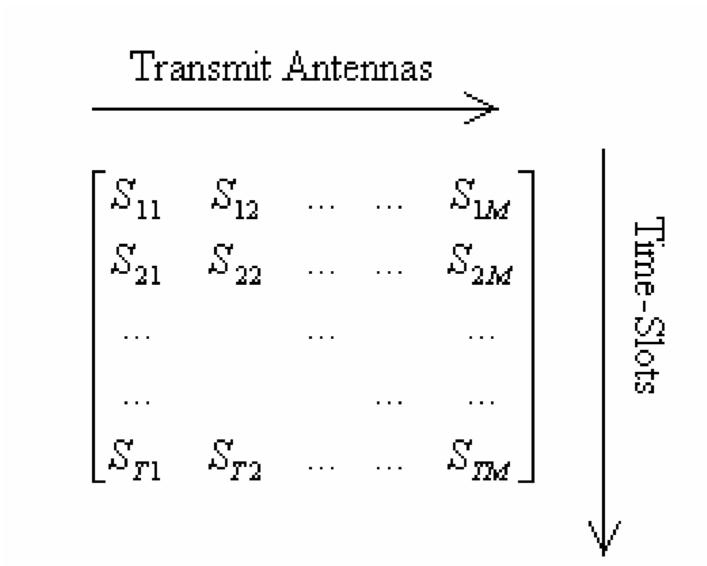

Figure 1.2 General Structure for a Space Time Block Code.

$$\mathbf{S}_{ij} \in \mathbf{V};\ 1 \le i \le T\ ;\ 1 \le j \le M; T \ge M \ge 1 \qquad (1\text{-}5)$$

Where,

T = number of time slots in one block channel use.

M= number of transmitting antennas.

V = signal constellation from which the modulated symbols are chosen for transmission.

The code rate for any given STBC is given by the number of symbols transmitted in a block length of *T* channel uses. Therefore, if our symbols are from a QPSK constellation and the block length is, say, *T*=5, the code rate, **r,** would be $4/5$.

$$\mathbf{r} = \frac{\mathbf{k}}{\mathbf{T}} \tag{1-6}$$

1.4 Orthogonal Space Time Block Codes

Orthogonal STBCs (O-STBC) are a special class of STBCs that satisfy the Diversity Criterion outlined by Tarokh et. al in [5]. Consider a codeword, **C**, and its corresponding error matrix, **E**,

$$\mathbf{C} = \begin{bmatrix} C_1^1 & C_1^2 & \ldots\ldots & C_1^M \\ C_2^1 & C_2^2 & \ldots\ldots & C_2^M \\ \ldots & & \ldots\ldots & \ldots \\ C_T^1 & C_T^2 & \ldots\ldots & C_T^M \end{bmatrix} \quad ; \quad \mathbf{E} = \begin{bmatrix} E_1^1 & E_1^2 & \ldots\ldots & E_1^M \\ E_2^1 & E_2^2 & \ldots\ldots & E_2^M \\ \ldots & & \ldots\ldots & \ldots \\ E_T^1 & E_T^2 & \ldots\ldots & E_T^M \end{bmatrix}$$

In order that a given STBC offer a full diversity of *MN*, the matrix **B**, as given below should be full-rank over any pair of distinct code words, **C** and **E**.

$$\mathbf{B}(\mathbf{C},\mathbf{E}) = \begin{bmatrix} C_1^1 - E_1^1 & C_1^2 - E_1^2 & \ldots\ldots & C_1^M - E_1^M \\ C_2^1 - E_2^1 & C_2^2 - E_2^2 & \ldots\ldots & C_2^M - E_2^M \\ \ldots & & \ldots\ldots & \ldots \\ C_T^1 - E_1^1 & C_T^2 - E_T^2 & \ldots\ldots & C_T^M - E_T^M \end{bmatrix}$$

O-STBCs follow from the theory of generalized orthogonal designs (see [6] for more details).

A generalized complex orthogonal design **G** is a $T \times M \ (T \geq M \geq 1)$ matrix with entries being complex linear combinations of the complex variables $Z_1, Z_2, Z_3, \ldots\ldots, Z_K$ over the complex field **C** and their complex conjugates $Z^*_1, Z_2^*, Z_3^*, \ldots\ldots, Z_K^*$ satisfying

$$\mathbf{G}^H \mathbf{G} = \left(|Z_1|^2 + |Z_2|^2 + \ldots\ldots + |Z_K|^2 \right) I_{M \times M} \quad (1\text{-}7)$$

The rate of this STBC is K/T. When *M*=*T*, **G** is called a complex square orthogonal design. If the entries of **G** are linear combinations of real variables instead it is called a real square orthogonal design. In this thesis we shall only consider the complex case.

A very simple O-STBC for 2 transmitters and 1 receiver was developed by Alamouti which is the *only* STBC that achieves full capacity and full diversity order while providing linear decoding at the receiver. The Alamouti scheme employs a simple 2 x 2 complex orthogonal design as shown.

$$\begin{pmatrix} Z_1 & Z_2 \\ -Z_2^* & Z_1^* \end{pmatrix} \quad (1\text{-}8)$$

The Alamouti code offers a performance improvement similar to MRRC. Since installing multiple antennas at the base station is more practical than having them at the mobile remote units, the Alamouti code can be readily extended to practical applications.

With this background on MIMO systems and STBCs, we shall now proceed to explain some of the practical concepts used in this thesis.

1.5 Doppler Effect

When we evaluate the performance of various STBCs in a wireless environment it is necessary to consider situations where the remote terminal is moving with a given velocity, either towards the terminal or away from it. In such a scenario, it is essential to consider the velocity of the mobile terminal, and the offset in carrier frequency induced because of the Doppler Effect, as this has been known to affect data rates.

Doppler Effect is a shift in the frequency of an electromagnetic or a sound wave caused by the relative movement of the source or the observer. In our thesis however, we shall consider only the change in frequency induced because of receiver's movement towards or away from the stationary transmitting unit.

The change in frequency is given by

$$\Delta f = \pm f_c \times \frac{v}{C} \tag{1-9}$$

Where $v =$ velocity of the terminal in m/sec;

C = Speed of light in vacuum = 3×10^8 m/sec.

For the purpose of this thesis *fc* is chosen to be 2 GHz as specified for the Enhanced Data rates for GSM Evolution (EDGE).The change in frequency is positive if the receiver is moving towards the terminal and negative otherwise.

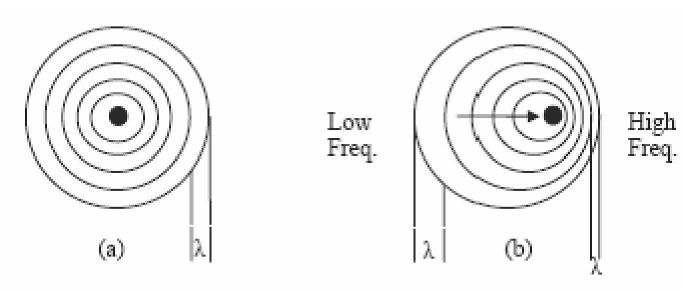

Figure 1.3. (a) Stationary Source (b) Moving Source

1.6 Thesis Overview

The remainder of this thesis is organized as follows. Chapter 2 provides the preliminaries about Differential Space Time Modulation for the time-invariant case. The motivation for this thesis is to extend this paradigm for a fast varying time selective channel. The idea behind the extension of the conventional Differential Space Time Block Codes (D-STBC) to a time varying case is thereby provided. Chapters 3 and 4 build upon this idea and demonstrate the performance of specific STBCs in terms of Block Error Rates for N number of receivers traveling at varying velocities. Finally, chapter 5 summarizes the results of our simulations and outlines the observations made thereof.

Chapter 2 - Preliminaries and Problems

2.1 Preliminaries

It is well known from information-theoretic results, [2], [3], [4] that MIMO systems offer improved channel capacity, better data rates and lower bit error rates relative to single antenna wireless systems. Extensive literature has been published proposing modulation and coding schemes that achieve these advantages in [1], [5], [6], [7]. These schemes assume the knowledge of the channel fading coefficients at the receiver. However it is not always feasible to know or to estimate the wireless channel in rapidly changing mobile environments, particularly when multiple antennas are employed.

In the absence of channel state information (CSI) at the receiver the above schemes require transmission of pilot symbols resulting in longer processing times and lower data rates.

Single antenna wireless systems that are required to operate in wireless environments with unknown channel coefficients have employed Differential Phase Shift Keying (D-PSK) [8] successfully for quite some time. Such a modulation scheme requires that the channel coefficients remain fairly constant for successive time intervals. Since most continuously fading channels change little from one time sample to the next, D-PSK is widely used in such environments. This provides the motivation for generalizing D-PSK concepts to multiple antenna systems.

Differential modulation techniques for multiple transmit antennas have been proposed in [9], [10] and [11], and are a natural extension of the standard D-PSK.

In this section we present some preliminaries about the differential unitary modulation schemes, which are a special class of differential schemes for MIMO systems as proposed by Hochwald and Sweldens in [9] and Hughes in [10].

2.1.1 Channel Model

Consider a communication link with M transmitting antennas and N receiving antennas operating in a Rayleigh flat fading environment. The channel model for such a setup is described below as defined in [9].

$$X_\tau = \sqrt{\rho}\, S_\tau\, H_\tau + W_{\tau,} \qquad \tau = 1,2,\ldots \tag{2-1}$$

Where, τ = index of the time block within which $t = \tau T, \tau T + 1, \ldots, \tau T + T - 1$ time samples are assembled in order, T = length of each time block, $S_\tau = (s_{tm})$ is the transmitted $T \times M$ matrix valued signals whose expected total power is

$$\mathrm{E} \sum_{t=1}^{T} \sum_{m=1}^{M} \left| s_{tm} \right|^2 = T \tag{2-2}$$

Where E denotes expectation, $X_\tau = (x_{tn})$ the received $T \times N$ matrix-valued signal, $H_\tau = (h_{mn}{}^{\tau})$ the $M \times N$ channel state information (CSI) matrix, the additive $T \times N$ matrix valued noise, and ρ being the expected signal-to-noise ratio (SNR) at each receiver antenna.

It is assumed that the additive noise w_{tn} at time *t* and receive antenna *n* are independent identically distributed (i.i.d.) Gaussian complex normal *CN*(0,1) with respect to both *t* and *n*. The channel fading coefficients $h_{mn}{}^{\tau}$ is constant in the τ th time block, independently of the time $t = \tau T, \tau T + 1, \ldots, \tau T + T - 1$ and also i.i.d. *CN*(0,1) distributed with respect to *m* and *n*. The CSI matrix H_τ is assumed to be nearly equal to it adjacent fading coefficient matrix, $H_\tau + 1$.

$$H_\tau \approx H_\tau + 1 \qquad \text{for } \tau = 1,2,\ldots \tag{2-3}$$

2.2 Differential Space Time Modulation and Demodulation

A general idea for differential unitary modulation is provided for any number of transmitting antennas.

Consider a MIMO system with *M* transmitting and *N* receiving antennas. We shall consider *T* to be the length of the time block in which signals are transmitted. This means that in a single time block of size *T*, the transmitting array makes use of the channel *T* times. Let $T \geq M$ .In order that this system support a data rate, *R*, we need to have $L = 2^{RT}$ different signals.

Each signal is a $T \times M$ unitary matrix V_l , from a signal constellation V consisting of $L \geq 2$ distinct unitary matrices. A $T \times M$ complex matrix V is called unitary if

$$V^H V = I_{T\times M} \tag{2-4}$$

where *I* is a $T \times M$ identity matrix and *H* is the Hermitian transpose of a complex matrix.

V is a subset of the signal constellation,

$$\Psi = \left\{ V_l \in C_{T\times M} \,\middle|\, V_l^H V_l = I_{T\times M}, l \in Z_l \right\} \tag{2-5}$$

where $Z_l = \{0,1,\ldots,L-1\}$.

The data bits to be transmitted are packed into an integer sequence $z_1, z_2, \ldots$ with $z_\tau \in Z_l \quad for\ \tau = 1,2,\ldots$.

The transmitted signal is then determined by the fundamental differential encoding or transmitter equation [9]

$$S_\tau = V_{Z_\tau} S_{\tau-1}, \quad \tau = 1,2... \tag{2-6}$$

The initial transmitted signal S_0 can be any $T \times M$ unitary matrix satisfying $S_0{}^H S_0 = I_{T\times M}$. Clearly all the transmitted matrices S_τ will be unitary.

In this thesis, we assume $S_0 = I_{T\times M}$ for the sake of simplicity. At the receiver we have,

$$X_\tau = \sqrt{\rho}\, S_\tau\, H_\tau + W_{\tau,} \quad \tau = 1,2,..... \tag{2-7}$$

$$X_\tau = \sqrt{\rho}\, \overbrace{\left(V_{Z_\tau} S_{\tau-1}\right)}^{S_\tau} H_\tau + W_\tau \tag{2-8}$$

$$X_\tau = V_{Z_\tau} \overbrace{\left(\sqrt{\rho}\, S_{\tau-1} H_\tau\right)}^{X_{\tau-1} - W_{\tau-1}} + W_\tau \tag{2-9}$$

$$X_\tau = V_{Z_\tau} \left(X_{\tau-1} - W_{\tau-1}\right) + W_\tau \tag{2-10}$$

$$X_\tau = V_{Z_\tau} X_{\tau-1} + W_\tau - V_{Z_\tau} W_{\tau-1} \tag{2-11}$$

Let $W_\tau{}' = \frac{1}{\sqrt{2}}\left(W_\tau - V_{Z_\tau} W_{\tau-1}\right)$.

Note that the noise matrices W_τ and $W_{\tau-1}$ are independent and statistically invariant to multiplication by a unitary matrices. Hence $W_\tau{}'$ is a $M \times N$ additive independent CN*(0,1)* noise. Hence, we have,

$$X_\tau = V_{Z_\tau} X_{\tau-1} + \sqrt{2}\, W_\tau{}' \tag{2-12}$$

This is the fundamental Differential receiver equation.

2.3 Problem Statement

How to extend the Differential Space Time Modulation for a time varying flat-fading channel.

In this thesis we aim to evaluate the performance evaluation of various differentially modulated STBCs for a slow-fading time varying channel. Differential Space Time Modulation for a time invariant channel however needs that the CSI matrix for successive time intervals be the same $\left(H_{\tau} = H_{\tau+1}\right)$.

In a practical wireless environment where the receiver is moving with a given velocity, the fading coefficients are bound to change because of the effects of fading and also because of the Doppler Effect on the electromagnetic waves discussed in chapter 1. In such circumstances, it is necessary to consider the change in the channel coefficients. The above Differential Space Time Modulation can be extended to a time varying case if

$$H_{\tau} \neq H_{\tau+1} \tag{2-13}$$

One of the best known channels for a slow-fading time varying channel has been the Jake's model [12]. We incorporate the Jake's model in our simulations to evaluate STBCs for differential modulation schemes over time varying channels. The model is given below,

$$H_{\tau} = \alpha_{\tau} H_{\tau-1} + W_{\tau} \tag{2-14}$$

where the coefficient α_{τ} is given by

$$\alpha_{\tau} = J_0\left(2\pi \times \tau \times f_d \times T_s\right) \tag{2-15}$$

where $J_0(.)$ is the 0^{th} order Bessel function of the first kind, f_d is the Doppler frequency and T_s is the information symbol duration, and W_τ is the i.i.d. complex Gaussian noise with zero mean and variance $\sigma_\tau^{\ 2}$.

α_τ varies with velocity of the terminal. Also, it is to be noted that the variance of W_τ is dependent on $\alpha_{\tau-1}$ and is given below.

$$\sigma_\tau^{\ 2} = 1 - |\alpha_{\tau-1}|^2 \qquad (2\text{-}16)$$

The Doppler frequency is given as

$$f_d = f_c \times \left(\frac{V}{C}\right) \qquad (2\text{-}17)$$

where, f_c is the carrier frequency, V is the velocity of the mobile terminal and c is the speed of light in vacuum.

The information symbol duration, T_s is given by

$$T_s = \frac{SF}{Chipe\ Rate} \qquad (2\text{-}18)$$

where SF is the Spreading factor. Universal Mobile Telecommunications System (UMTS), the mobile standard specifies the following values for the above variables

Chip rate $= 3.84 \times 10^6 \ \frac{chips}{\text{sec}}$ and SF =128.

2.4 Decoding

At every symbol interval τ the transmitter sends a block code, $S_\tau = V_{Z_\tau} S_{\tau-1}$, the corrupted version of which is received at the receiver. Again, we know that $Z_\tau \in \mathrm{Z}_L$. The transmitted signal is one of the L unitary signals from the constellation Ψ that is chosen apriori. Hence during decoding our aim is to find the correct Z_τ.At the receiver we have,

$$X_\tau = \sqrt{\rho}\, S_\tau\, H_\tau + \sqrt{2}\, W_\tau' \qquad \tau = 1,2,..... \tag{2-19}$$

$$X_{\tau+1} = \sqrt{\rho}\, S_{\tau+1}\, H_{\tau+1} + \sqrt{2}\, W_{\tau+1}' \tag{2-20}$$

Now this signal is a differentially modulated signal and hence the information is carried in the difference (in some parameter) between X_τ and $X_{\tau+1}$.

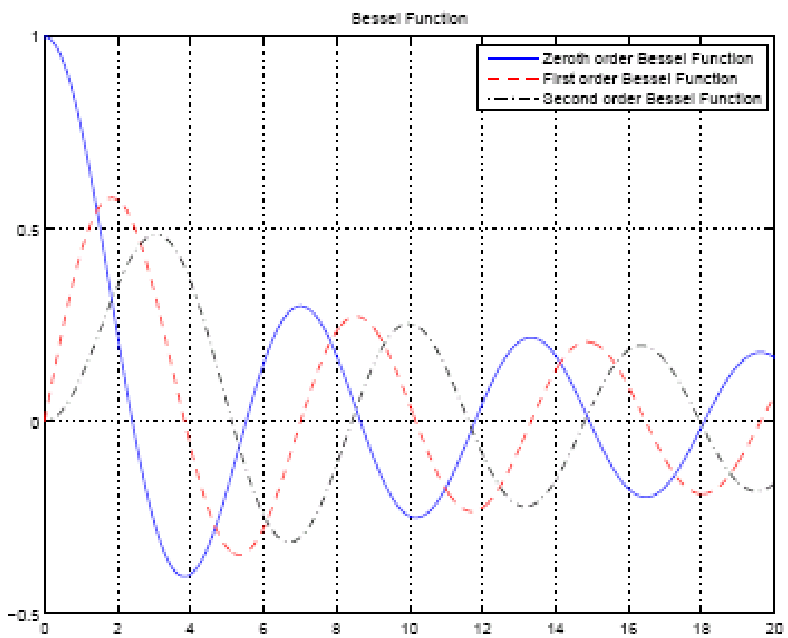

Figure 2.1. Bessel Function of 0th, 1st and 2nd order.

The Maximum Likelihood detector for the Differential modulator can be given as

$$\hat{Z}_{\tau+1}^{ML} = \arg\min_{l \in Z_L} \| X_{\tau+1} - X_\tau \|_F \tag{2-21}$$

$$\hat{Z}_{\tau+1}{}^{ML} = \arg\min_{l \in Z_L} \left\| X_{\tau+1} - \overbrace{S_\tau H_\tau}^{X_\tau} \right\|_F \tag{2-22}$$

$$\hat{Z}_{\tau+1}{}^{ML} = \arg\min_{l \in Z_L} \left\| X_{\tau+1} - V_{Z_\tau} S_\tau \alpha_\tau H_\tau \right\|_F \tag{2-23}$$

$$\hat{Z}_{\tau+1}{}^{ML} = \arg\min_{l \in Z_L} \left\| X_{\tau+1} - \alpha_\tau V_{Z_\tau} \overbrace{S_\tau H_\tau}^{X_\tau} \right\|_F \tag{2-24}$$

$$\hat{Z}_{\tau+1}{}^{ML} = \arg \min_{\substack{V \in \Psi \\ \#\Psi = L}} \left\| X_{\tau+1} - \alpha_\tau V X_\tau \right\|_F \tag{2-25}$$

This is the fundamental decoding equation used in our simulations.

For a $M \times N$ matrix $A = (a_{mn})$, its Frobenius norm or Euclidean norm is defined by

$$\begin{aligned} \|A\|_F &= \sqrt{Tr\left(A^H A\right)} = \sqrt{Tr\left(A A^H\right)} \\ &= \left(\sum_{m=1}^{M} \sum_{n=1}^{N} |a_{mn}|^2 \right)^{1/2} \end{aligned} \tag{2-26}$$

where Tr(.) denotes the trace of the augment matrix.

The Jakes model makes it possible to render Differential modulation methods to a slow-fading time varying channels and evaluate the performance of STBCs in condition where it is not possible to estimate the channel. This makes the receiver less expensive and less complex. In the forthcoming chapters we will be evaluating the performance of various STBCs using the above mentioned decoding scheme in terms of the Block Error Rate at varying terminal velocities.

Chapter 3 - Unitary Scheme

In this chapter we focus on the differential-unitary modulation scheme proposed by Hochwald and Sweldens in [9] and Hughes in [10].

Recall that a $T \times M$ complex matrix V is called unitary if

$$V^H V = I_{T \times M} \tag{3-1}$$

Design of unitary-space time constellations with large diversity products has known to be crucial for the performance (in terms of Block Error ate) of differential unitary modulation schemes. Unitary signal constellations for two transmit antennas with largest possible diversity product for a 5-signal constellation and a 16-signal constellation with the largest known diversity product has been developed by Liang and Xia in [13].We use the Parametric codes developed in [13] as the signal constellation based on which simulation results for the performance of 2 transmitting antennas over different constellation sizes and varying terminal velocities are provided.

3.1 Diversity Product and Diversity Sum

Diversity product and diversity sum are two important parameters for unitary space time constellation design.

Let the transmitted signal be denoted as $S_\tau \ for \ \tau = 1,2,3...$. The signal to be transmitted at time interval τ is chosen from a predetermined signal constellation such as the parametric unitary constellation. We strive to minimize the probability that the receiver mistakes the transmitted signal $V_l \ \ for \ V_{l'}$, where $l, l' \in Z_L \ \ where \ L = \# \Psi$ (called the pairwise probability of error, P_e) between any two signals within the given constellation.

For any two $M \times M$ unitary matrices V_1 and V_2 define M quantities that reflect the dissimilarity between the two matrices as follows:

$$D_m(V_1,V_2) = \frac{1}{2}\left(\frac{E_m(V_1 - V_2)}{\binom{M}{m}}\right)^{1/2m} \qquad ,m = 1,2,3\ldots,M \tag{3-2}$$

When m=1 and m=M, $D_m(V_1,V_2)$ are related to the Frobenius norm and determinant of the difference matrix V_1-V_2, respectively.

$$\begin{aligned} D_{euc}(V_1,V_2) = D_1(V_1,V_2) &= \frac{1}{2\sqrt{M}}\|V_1 - V_2\|_F \\ &= \frac{1}{2\sqrt{M}}\left(\sum_{m=1}^{M}\sigma_m{}^2(V_1 - V_2)\right)^{1/2} \end{aligned} \tag{3-3}$$

$$\begin{aligned} D_{\det}(V_1,V_2) = D_M(V_1,V_2) &= \frac{1}{2}\sqrt[M]{|\det(V_1 - V_2)|} \\ &= \frac{1}{2}\left(\prod_{m=1}^{M}\sigma_m(V_1 - V_2)\right)^{1/M} \end{aligned} \tag{3-4}$$

$D_{euc}(V_1,V_2)$ and $D_{\det}(V_1,V_2)$ are called the normalized Euclidean distance and normalized determinant dissimilarity between the two matrices V_1 and V_2.

For any $M \times M$ unitary signal constellation Ψ of size L,

$\Psi = \left\{V_l \in C_{T\times M} \,\middle|\, V_l{}^H V_l = I_{T\times M}, l \in Z_l\right\}$, we can define M quantities that reflect the minimum dissimilarity between any two different unitary signals in Ψ as follows:

$$\xi_m(L,\Psi) = D_m(V_l, V_{l'}), \quad m = 1,2,3,\ldots,M$$

When m=1 we have,

$$\delta(L,\Psi) \overset{def}{=} \xi_1(L,\Psi)$$
$$= \frac{1}{2\sqrt{M}} \min_{0 \le l \le l' \le L-1} \left\| V_l - V_{l'} \right\|_F \qquad (3\text{-}5)$$
$$= \frac{1}{2\sqrt{M}} \min_{0 \le l \le l' \le L-1} \left(\sum_{m=1}^{M} \sigma_m{}^2 \left(V_l - V_{l'}\right) \right)^{\frac{1}{2}}$$

where $\delta(L,\Psi)$ is called the Diversity Sum and is defined as the minimum among the sums of the squared singular values for all the difference signal matrices.

When m=M, we have,

$$\varsigma(L,\Psi) \overset{def}{=} \xi_M(L,\Psi)$$
$$= \frac{1}{2} \min_{0 \le l \le l' \le L-1} \sqrt[M]{\left|\det\left(V_1 - V_2\right)\right|} \qquad (3\text{-}6)$$
$$= \frac{1}{2} \min_{0 \le l \le l' \le L-1} \left(\prod_{m=1}^{M} \sigma_m{}^2 \left(V_l - V_{l'}\right) \right)^{\frac{1}{2M}}$$

where $\varsigma(L,\Psi)$ is called the Diversity Product and is defined as the minimum among the products of the squared singular values for all difference signal matrices.

Parametric form of 2x2 unitary matrices that have optimal diversity product and sum for L=4and a L=16 signal constellation with the largest known diversity product in addition to the optimal diversity sum has been developed in [13].

Parametric codes used for simulation in this chapter are nongroup codes, hence the transmitted symbols generated from the fundamental differential encoding equation are possibly arbitrary unitary signals [13].

MATLAB™ Release 2006b, Version 7.1 was used as the simulation tool in this thesis.

The following data has been used in our simulations according to the European Telecommunications Standard:

Carrier Frequency = 2 GHz

Transmission Rate = $144\frac{kbits}{\sec}$

Chip Rate = $3.84\times10^{6}\frac{chips}{\sec}$

Spreading Factor (SF) = 128

The components modeled using our simulation includes:

1. Complex and real square orthogonal design based on BPSK and QPSK constellations
2. Flat fading channels based on Jake's model
3. Doppler Effect
4. Additive White Gaussian Noise
5. Terminal velocity of the receiver.

Number of transmitting antennas: M =2

Number of receiving antennas: N =2, 5

Number of block channel uses: τ = 20

The transmitter is assumed to be stationary while the receiver is mobile with varying terminal velocities (0 kmph to 150 kmph).

At every instance τ, the transmitter block sends a differentially modulated signal S_τ (which in this chapter is a parametric STBC) of dimension $T \times M$. At the receiver block we have X_τ, which is corrupted by Additive White Gaussian Noise (AWGN) with mean zero and variance one. The channel model is given by

$$X_\tau = \sqrt{\rho}\, S_\tau\, H_\tau + W_{\tau,} \qquad \tau = 1,2,..... \tag{3-7}$$

Decoding is performed at the receiver in accordance with the following equation

$$\hat{Z}_{\tau+1}{}^{ML} = \arg \min_{\substack{V\in\Psi \\ \#\Psi = L}} \left\| X_{\tau+1} - \alpha_\tau V X_\tau \right\|_F \tag{3-8}$$

It is to be noted that the assumption of $H_\tau = H_{\tau+1}$ doesn't hold true in this thesis.

Since the receiver block is likely to be in transition in mobile environments, we need to consider various factors such as terminal velocity, the Doppler shift induced because of the mobile receiver and the additive noise component. Jakes model is used to rope in all of these factors in order to define a relation between consecutive CSI matrices.

Parametric unitary signal constellation for L=4 and L=16, [13] has been used in our simulations of Block Error Probability for M=2 over time varying channels

Table 3.1 Parametric Unitary Constellation with Optimal Diversity Sum and Diversity Product for L=4

$$V_0 = \begin{pmatrix} ja_1 & a_3 - ja_2 \\ -a_3 - ja_2 & -ja_1 \end{pmatrix}$$

$$V_1 = \begin{pmatrix} -ja_1 & -a_3 - ja_2 \\ a_3 - ja_2 & ja_1 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} -ja_3 & a_1 + ja_2 \\ -a_1 + ja_2 & ja_3 \end{pmatrix}$$

$$V_3 = \begin{pmatrix} ja_3 & -a_1 + ja_2 \\ a_1 + ja_2 & -ja_3 \end{pmatrix}$$

where $a_i \in R \; for \; i = 1,2,3$ and if $a_1{}^2 + a_2{}^2 = 2/3$, the above signal constellation has the optimal diversity product and optimal sum of $\sqrt{\frac{2}{3}}$.

The above constellation has been proved to have an improvement of 1 dB over cyclic code at SNR 14 Db and in the case of five receiver antennas the improvement was 1dB at SNR 8 Db.

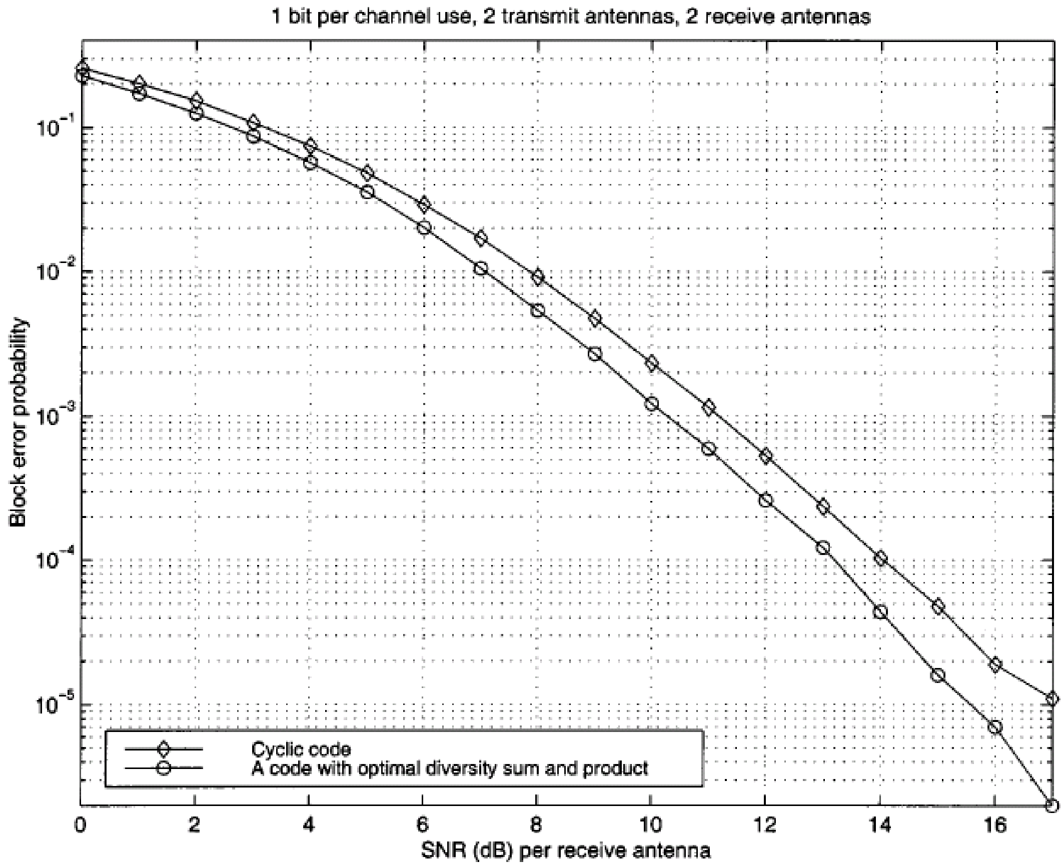

Figure 3.1 Performance evaluation of Parametric codes versus Cyclic codes for L=4 with M=2 and N=2 (Adapted from [13]).

The constellation for L=16 that has been used for simulation is given below.

Table 3.2 Signal Constellation of Parametric Code $\Psi(3,4,2)$ of Size 16 for Two Transmit Antennas

$$V_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad V_1 = \begin{pmatrix} 0 & e^{j\frac{15}{8}\pi} \\ e^{j\frac{13}{8}\pi} & 0 \end{pmatrix}$$

$$V_2 = \begin{pmatrix} e^{j\frac{7}{4}\pi} & 0 \\ 0 & e^{j\frac{5}{4}\pi} \end{pmatrix} \qquad V_3 = \begin{pmatrix} 0 & e^{j\frac{5}{8}\pi} \\ e^{j\frac{15}{8}\pi} & 0 \end{pmatrix}$$

$$V_4 = \begin{pmatrix} e^{j\frac{3}{2}\pi} & 0 \\ 0 & e^{j\frac{1}{2}\pi} \end{pmatrix} \qquad V_5 = \begin{pmatrix} 0 & e^{j\frac{11}{8}\pi} \\ e^{j\frac{1}{8}\pi} & 0 \end{pmatrix}$$

$$V_6 = \begin{pmatrix} e^{j\frac{5}{4}\pi} & 0 \\ 0 & e^{j\frac{7}{4}\pi} \end{pmatrix} \qquad V_7 = \begin{pmatrix} 0 & e^{j\frac{1}{8}\pi} \\ e^{j\frac{3}{8}\pi} & 0 \end{pmatrix}$$

$$V_8 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \qquad V_9 = \begin{pmatrix} 0 & e^{j\frac{7}{8}\pi} \\ e^{j\frac{5}{8}\pi} & 0 \end{pmatrix}$$

$$V_{10} = \begin{pmatrix} e^{j\frac{3}{4}\pi} & 0 \\ 0 & e^{j\frac{7}{4}\pi} \end{pmatrix} \qquad V_{11} = \begin{pmatrix} 0 & e^{j\frac{13}{8}\pi} \\ e^{j\frac{7}{8}\pi} & 0 \end{pmatrix}$$

$$V_{12} = \begin{pmatrix} e^{j\frac{1}{2}\pi} & 0 \\ 0 & e^{j\frac{3}{2}\pi} \end{pmatrix} \qquad V_{13} = \begin{pmatrix} 0 & e^{j\frac{3}{8}\pi} \\ e^{j\frac{9}{8}\pi} & 0 \end{pmatrix}$$

$$V_{14} = \begin{pmatrix} e^{j\frac{1}{4}\pi} & 0 \\ 0 & e^{j\frac{3}{4}\pi} \end{pmatrix} \qquad V_{15} = \begin{pmatrix} 0 & e^{j\frac{9}{8}\pi} \\ e^{j\frac{11}{8}\pi} & 0 \end{pmatrix}$$

3.2 Simulation Results

The following graphs illustrate the simulation results in terms of Block Error Probability for the Parametric Unitary constellations listed above. Results are provided for M=2, N=2 and M=2, N=5 for both the constellation sizes of 4 and 16. The receiver terminal is mobile with terminal velocities of 0, 50, 75, 100 and 125 km/hr. The number of block channel uses (τ) is limited to 20 in all our simulations. For comparison and readability purposes we plot all the Block Error Probability curves for all the above mentioned speeds, as shown in Figure 3.2.

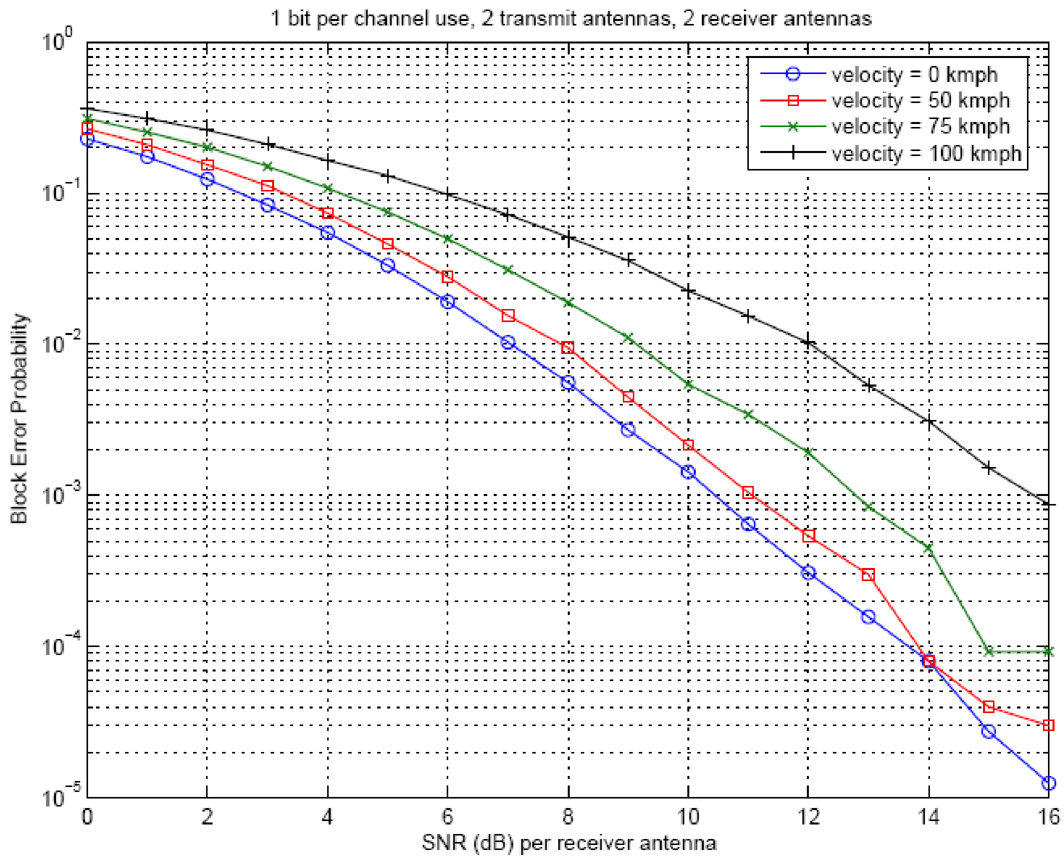

Figure 3.2 Block Error Rate for Parametric code of size 4 for M=2, N=2 for terminal velocities 0, 50, 75, 100 kmph

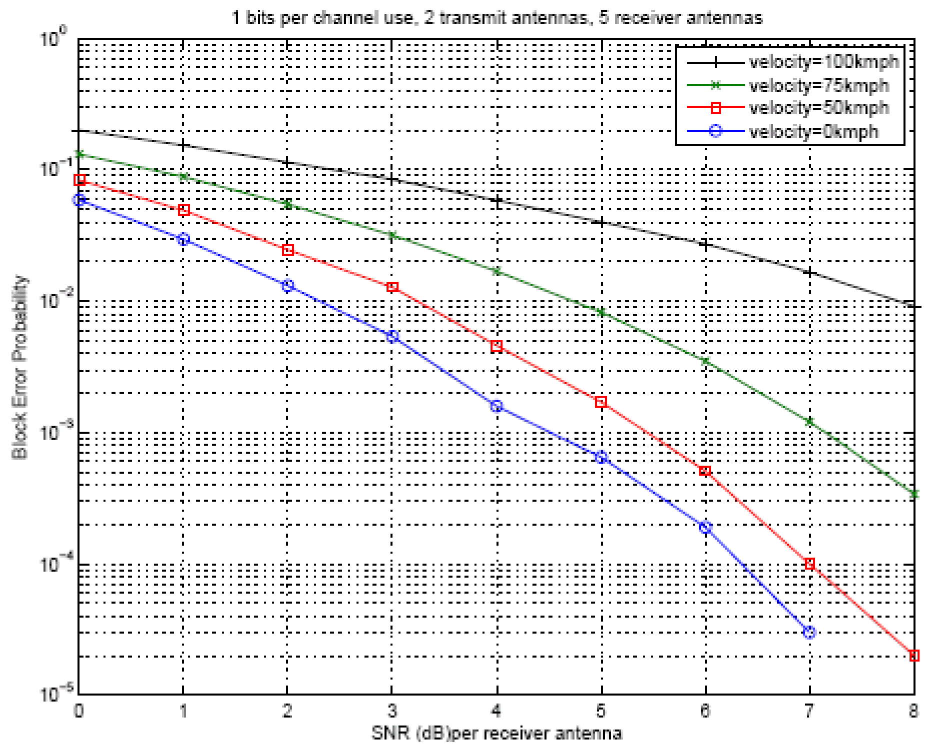

Figure 3.3 Simulation results for Parametric code of size 4 for M=2, N=5 terminal velocities 0, 75, 100 and 125 kmph.

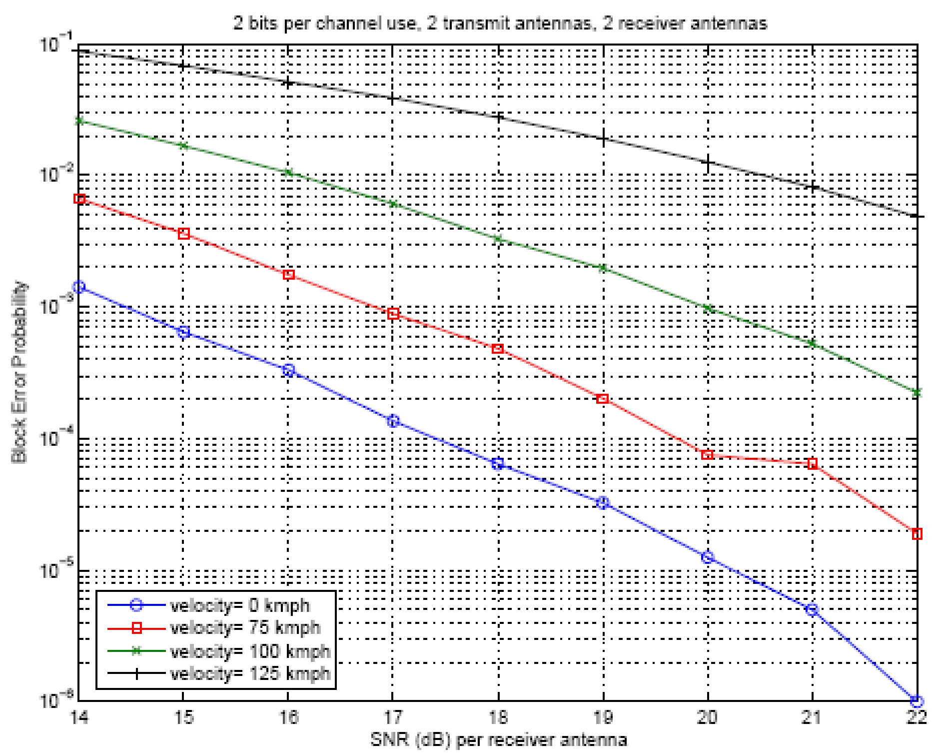

Figure 3.4 Block Error Rate for Parametric code of size 16 for M=2, N=2 Terminal velocities of 0, 75, 100, 125 kmph

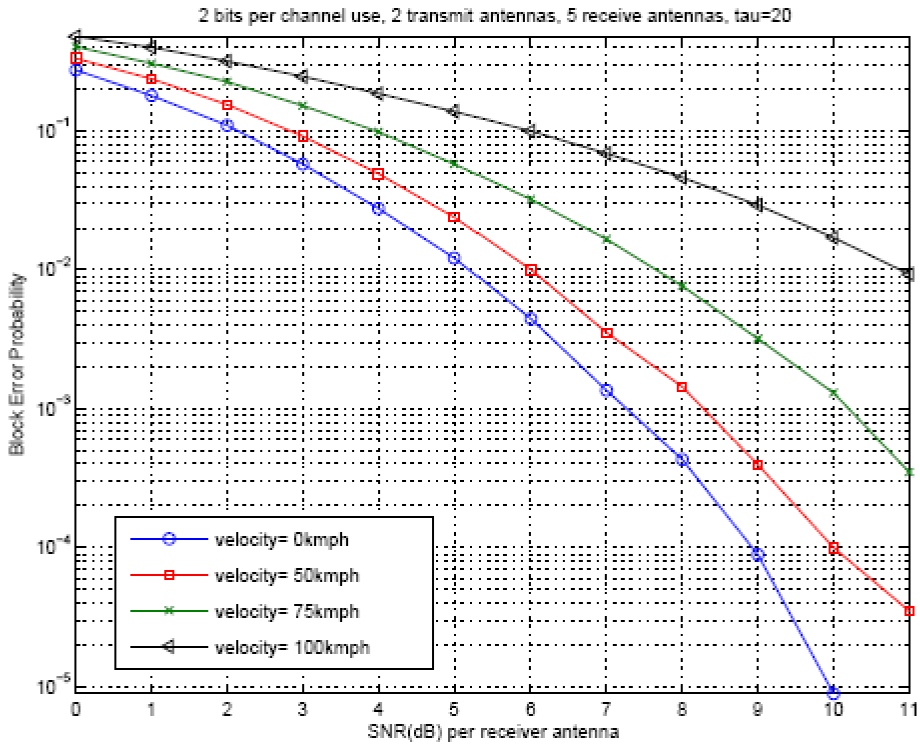

Figure 3.5 Simulation results for parametric code of size 16 for terminal velocities 0, 50, 75 and 100kmph.

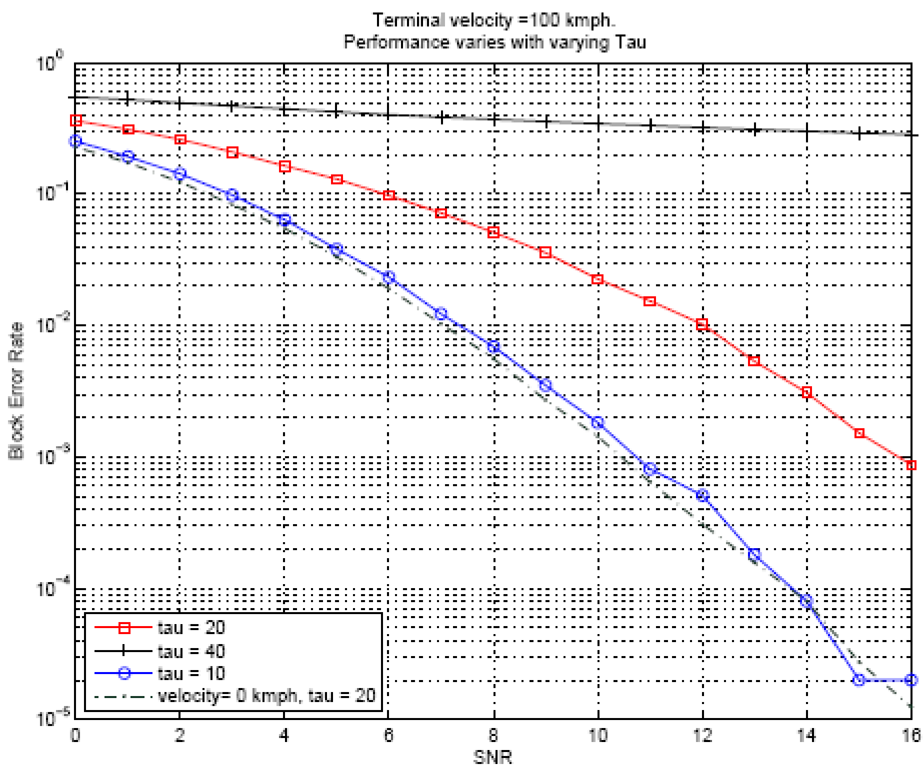

Figure 3.6 Performance of L=4 Parametric codes for a terminal velocity of 100kmph when Tau = 10, 20 and 40.

3.3 Performance Evaluation

As stated earlier the CSI matrices for successive block channel uses are not constant (as is the case for general differential modulation systems) but are correlated by the Jake's model. It can be seen that the Parametric codes perform remarkably well upto speeds of upto 75 km/hr for both cases of M=2, N=2 and M=2, N=5. Since the Jakes model degrades the CSI coefficients (for terminal velocities greater than 0 kmph) as the number of block channel uses (τ) increases, the perform degrades proportionally with the increase in τ. This phenomena is observed in Figure 3.6. for the same terminal velocity of 100 km/hr we observe an error floor for τ = 40 and 20 while it falls within less than 1dB of the curve for 0 km/hr.

The performance starts to deteriorate irreducibly as the speed increases or as τ increases since it destroys the orthogonality of the unitary matrices. By reducing τ it is however possible to obtain satisfactory performance at higher speeds, but that may in turn affect the processing overhead due to repetitive header and footer blocks.

Chapter 4 - Alamouti Scheme

4.1 System Model

Alamouti scheme for 2 transmitting antennas is used for simulation of Block Error Rates for constellation sizes of L=4 and L=16. The unique structure of Alamouti codes facilitate fast decoding at the receiver relative to other STBCs. The channel model and decoding structure is the same as described in the previous chapter.

The diversity scheme first devised by Alamouti [7] uses 2 transmitting and 1 receiving antenna. The same structure has been extended for the cases of M=2, N=2 and M=2, N=5 in our simulations for constellation sizes L=4 and L=16. Binary PSK constellation is used for L=4 and a Quarternary PSK constellation for L=16.

The Alamouti code employs the following complex square orthogonal design as shown

$$\begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix} \tag{4-1}$$

The 2x2 Alamouti code achieves full rate with full diversity and supports a maximum likelihood receiver with only linear decoding at the receiver. Since the Alamouti scheme uses transmitter diversity as opposed to receiver diversity, receiver design and cost reduces considerably as well. The simple structure and linear processing of the Alamouti code is so attractive that it is now a part of both W-CDMA and CDMA 2000 standards.

The following sections of this chapter provide simulation results obtained for a differential modulation scheme over flat-fading time varying channels where the receiver terminals are mobile at different velocities. The channel model, decoding structure, the simulation parameters and tools remain the same as used in the previous chapter for

Paramteric Unitary STBCs. A comparison of the performance of Alamouti versus Parametric Unitary codes is presented.

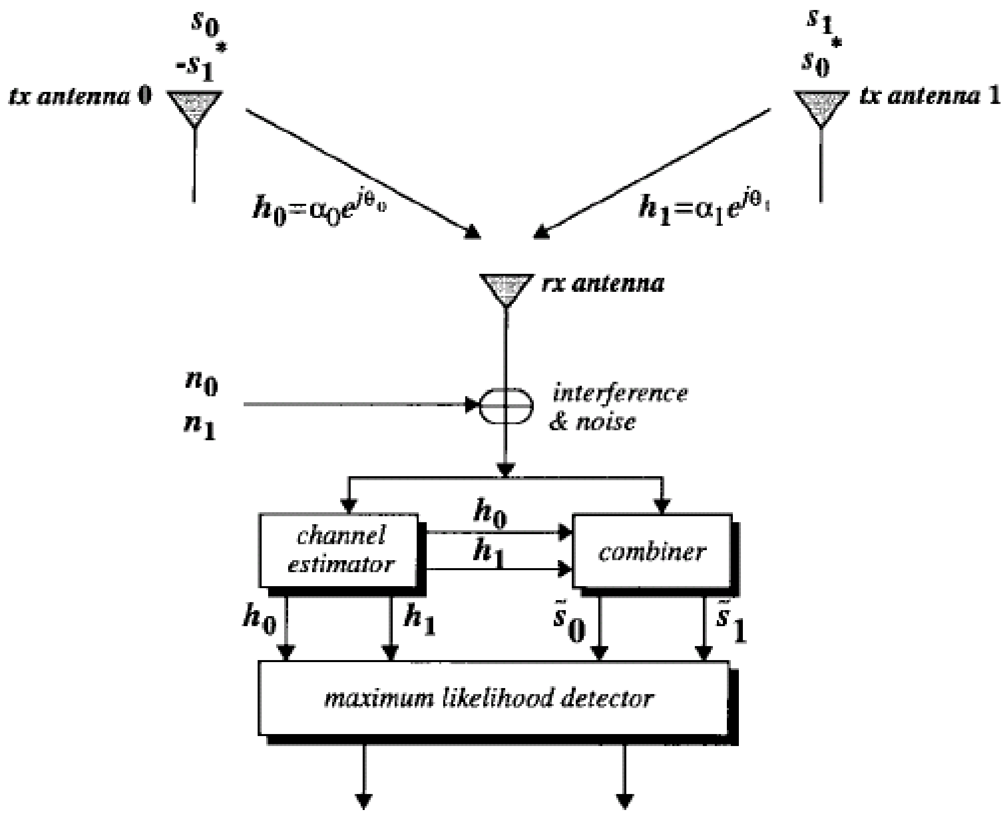

Figure 4.1 Alamouti scheme for M=2, N=1 that achieves full transmit diversity (Adapted from [7])

The constellations of size L=4 used in our simulations is drawn from BPSK and are listed below.

$$\Psi_4 = \left\{ \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \right\}$$

Similarly the QPSK signal constellation is used for the the Alamouti code of size 16.

4.2 Simulation Results

Simulation results are provided for the Alamouti constellation of size 4 an 16 for the cases of M=2, N=2 and M=2, N=5 for terminal velocities of 0, 50, 75, 100 and 125 kmph. The number of block channel uses has been restricted to 20.

For comparison and readability purposes we plot all the Block Error Probability curves for all the above mentioned speeds I the same graph as shown.

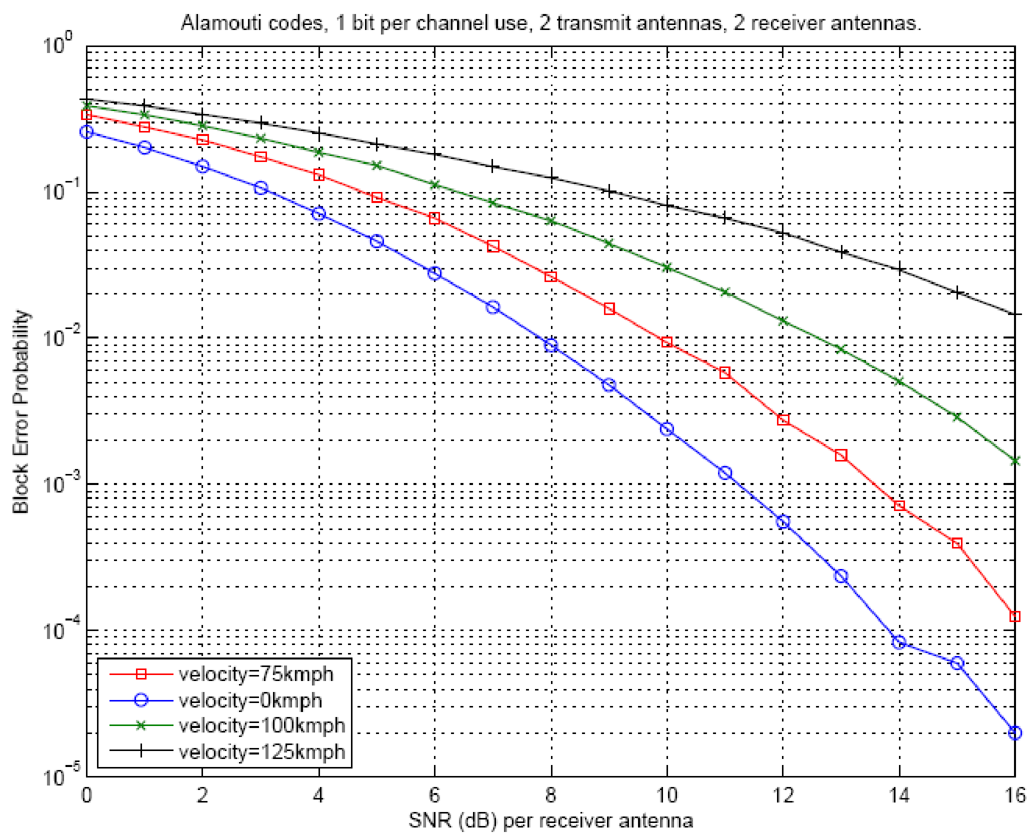

Figure 4.2 Block Error Rate for L=4, M=2, N=2 for Alamouti code.

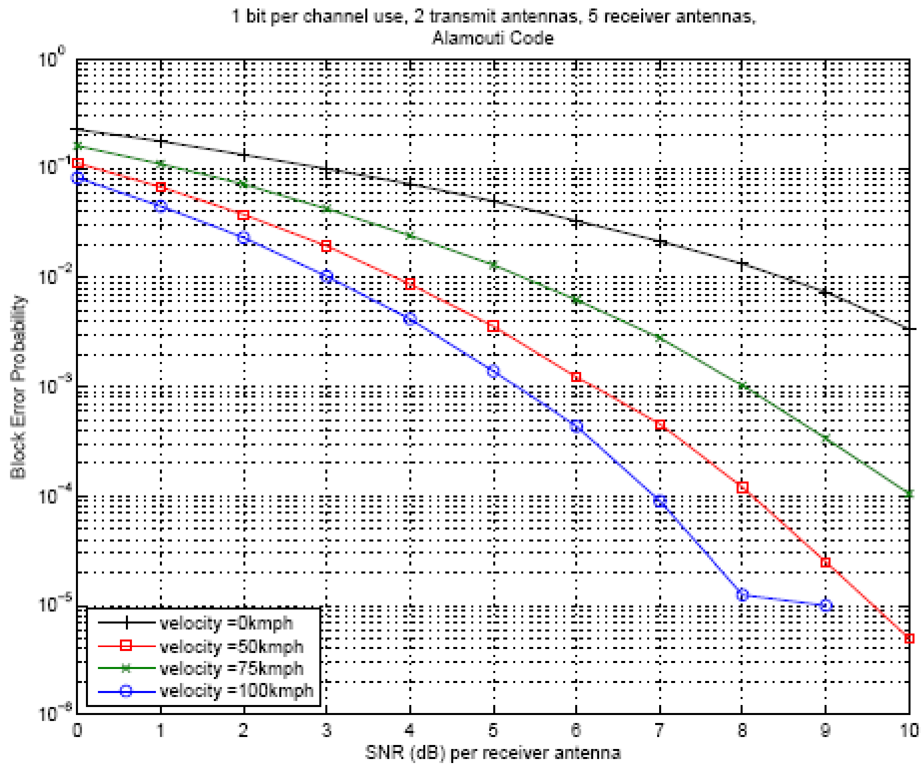

Figure 4.3 Block Error Rate for Alamouti constellation of size 4, M=2, N=5 for terminal velocities of 0, 50, 75, and 100 kmph.

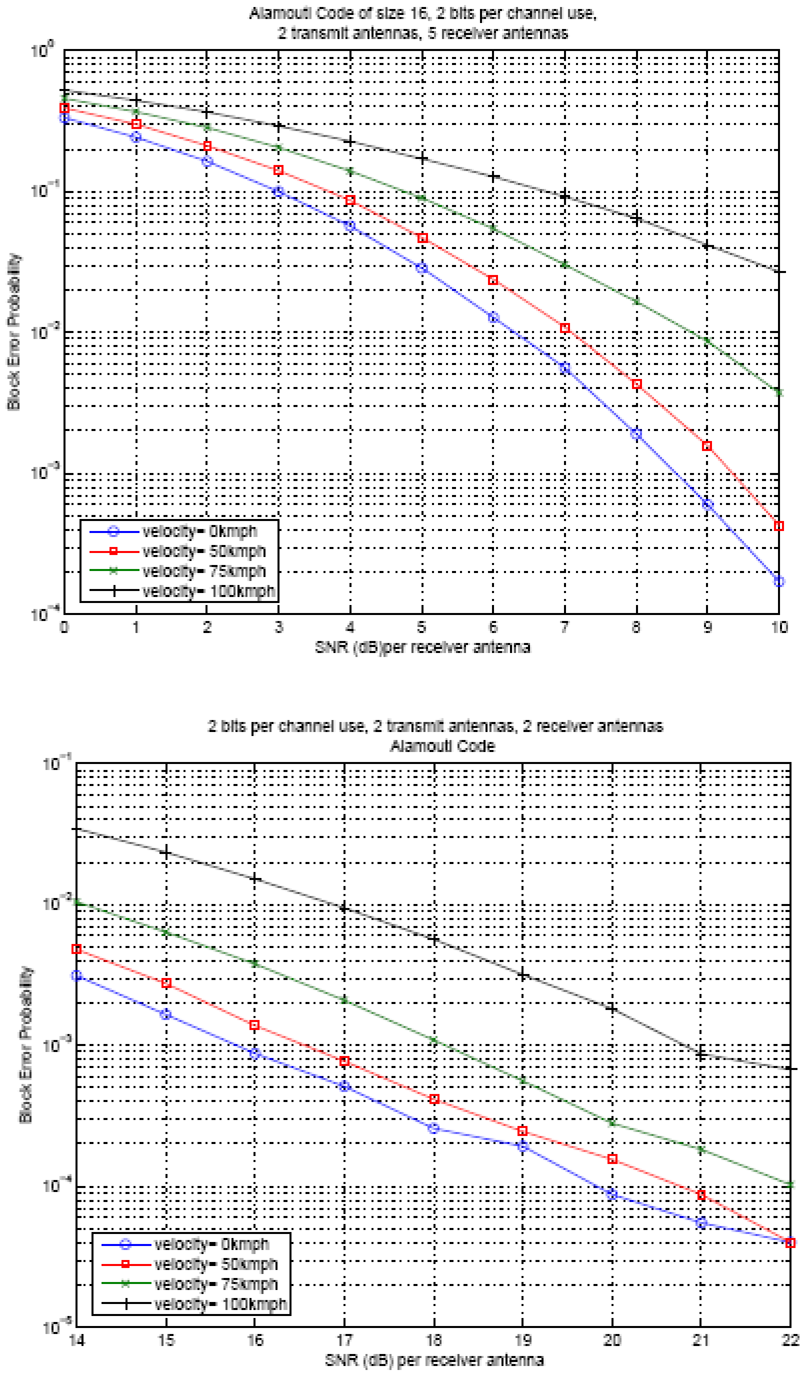

Figure 4.4 Probability of Block Error for Alamouti Code of size L=16, for (a) M=2, N=2 and (b) M=2, N=5.

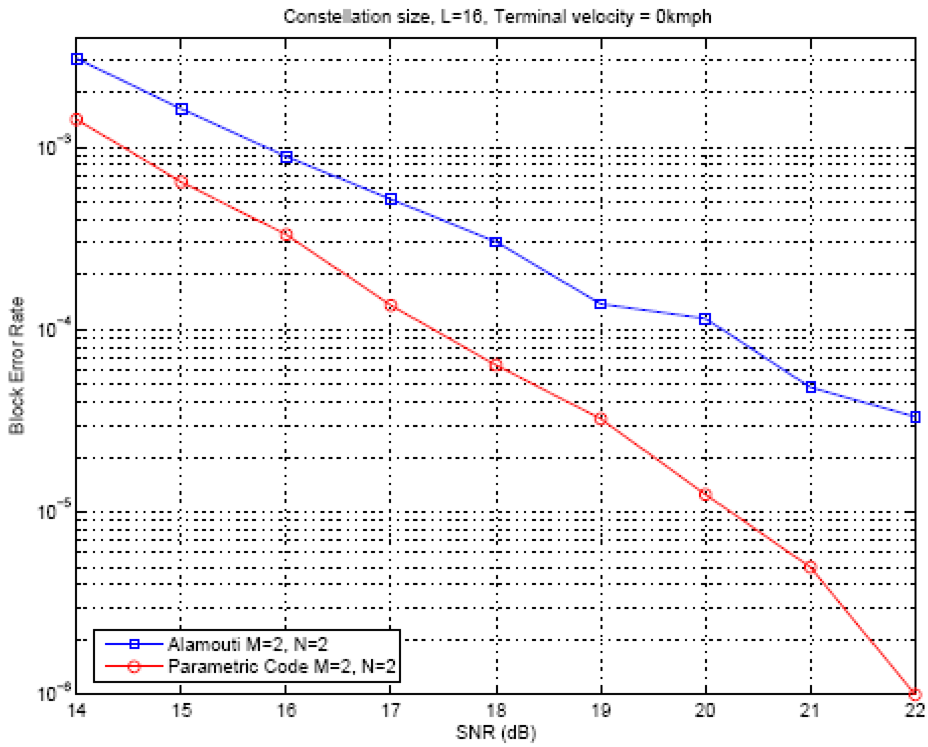

Figure 4.5 Comparison of Alamouti and Parametric codes for a constellation size of L=16, M=2, N=2 at terminal velocity = 0 kmph.

It can be seen fronm the above simulations that both Alamouti and Parametric codes perform reasonably well under 75kmph.

It can be observed from the above simulations that the performance starts degrading progressively as the terminal velocity approaches 100kmph. Figure 4.5 provides a comparison between Alamouti code and the Parametric code for a terminal velocity of 0 kmph.

4.3 Fast Differential Unitary Space Time Demodulation

Unitary space-time block codes can be primarily classified into two categories. In the first category we have the information bits are conveyed by directly mapping them onto these unitary valued matrices. Hence at the receiver the decoding algorithm's objective is to find the transmitted unitary matrix. The parametric codes of constellation sizes L=4 and L=16 used in the previous chapter fall under this category of unitary codes.

In the second category of unitary codes, the transmitted code words are still unitary in nature; however symbols from specific constellation (such as M-PSK) are used to form these unitary matrices. The information to be transmitted is mapped onto individual symbols from the (M-PSK) constellation, which in turn constitute the unitary valued matrix. The objective of the decoding algorithm at the receiver is to decode the complex scalar valued signal symbols instead. Since such square orthogonal unitary codes have a predefined structure it is relatively easier and much faster to put together the unitary matrix from the decoded symbols at the receiver. Alamouti code is a classic example of a 2x2 real and complex square orthogonal designs (unitary codes of the second kind).

The maximum-likelihood demodulators for the first category of unitary codes rely upon exhaustive searches within their constellations and generally have exponential complexity. This makes maximum likelihood demodulation impractical for real-time use in differential space-time modulation schemes that employ unitary codes of the first kind. In Real and complex square orthogonal designs the information symbols are decoupled form the unitary matrix itself and hence render themselves more easily to maximum-likelihood type decoding.

It has been shown by the works of Liang et al in [14] that there is a natural concatenation between real and complex square orthogonal designs and the general differential unitary space-time demodulation using maximum-likelihood techniques. The same has been extended to the case of Alamouti codes in the following section.

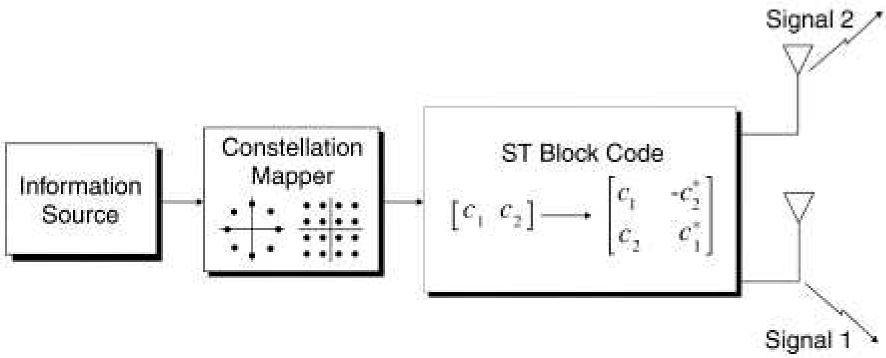

Figure 4.6 Transmitter diversity using the Alamouti space time block coding scheme (Adapted from [15])

4.4 Generalized Complex Orthogonal Designs

A generalized complex rthogonal design *G* is a $T \times M$ $(T \geq M \geq 1)$ matrix with entries being complex linear combinations of the complex variables $z_1, z_2, ..., z_k$ over the complex field C and their conjugates $z_1^*, z_2^*, ..., z_k^*$ satisfying

$$G^H G = \left(|z_1|^2 + |z_2|^2 + ... + |z_k|^2\right) I_{M \times M} \qquad (4\text{-}2)$$

The rate of *G* is defined as $\frac{K}{T}$ and when *M=T, G* is called a complex square orthogonal design.

$$\begin{aligned} G &= G(z_1, z_2, ..., z_k) \\ &= z_1 A_1 + z_2 A_2 + ... + z_k A_k + z_1^* B_1 + z_2^* B_2 + ... + z_k^* B_k \end{aligned} \qquad (4\text{-}3)$$

In the above scheme the k information symbols, $z_1, z_2, ..., z_k$ are chosen from specific constellations. For a 16 signal constellation z_1 and z_2 are chosen from a QPSK constellation while for a 4 signal constellation they are chosen from a BPSK constellation.

In order to send Kb bits in each block of size T over M transmitting antennas, we have a rate of $R = \frac{Kb}{T}$ bits per channel use. We have $L = 2^{RT} = 2^{Kb}$ different unitary matrix valued signals. The Kb bits are split into K sub-blocks of b bits, each of which are mapped onto $z_k \in A$ for $k = 1,2,....K$ using binary to decimal number mapping or gray encoding. A $T \times M$ unitary matrix V, is then generated using

$$V = z_1 A_1 + z_2 A_2 + ... + z_k A_k + z_1^* B_1 + z_2^* B_2 + ... + z_k^* B_k \qquad (4\text{-}4)$$

The above signal is them differentially modulated using the fundamental transmitter equation,

$$S_\tau = V\, S_{\tau-1} \qquad (4\text{-}5)$$

As shown in the previous chapter, at the receiver we have,

$$\begin{aligned}
\hat{Z}_\tau^{ML} &= \arg \min_{\substack{V \in \Psi \\ \#\Psi = L}} \| X_\tau - \alpha_{\tau-1} V\, X_{\tau-1} \|_F \\
&= \arg \min Tr\left[\left(X_\tau^H - \alpha_{\tau-1} X_{\tau-1}^H V^H\right)\left(X_\tau - \alpha_{\tau-1} V X_{\tau-1}\right)\right] \\
&= \arg\min_{z_k \in A, 1 \le k \le K} \sum_{k=1}^{K} \alpha_{\tau-1} \left[- z_k Tr\overbrace{\left(A_k X_{\tau-1} X_\tau^H + B_k^H X_\tau X_{\tau-1}^H\right)}^{\lambda_K^*} - z_k^* Tr\overbrace{\left(A_k^H X_\tau X_{\tau-1}^H + B_k X_{\tau-1} X_\tau^H\right)}^{\lambda_K} \right] \\
&= \arg\min_{z_k \in A, 1 \le k \le K} \alpha_{\tau-1} \sum_{k=1}^{K} \left(-\lambda_k^* z_k - \lambda_k z_k^* \right) \; for \; k = 1,2,..., K
\end{aligned}$$

Where,

$$\begin{aligned}
\lambda_k^* &= Tr\left(A_k X_{\tau-1} X_\tau^H + B_k^H X_\tau X_{\tau-1}^H\right); \; and \\
\lambda_k &= Tr\left(A_k^H X_\tau X_{\tau-1}^H + B_k X_{\tau-1} X_\tau^H\right)
\end{aligned}$$

Where $A_1, A_2, ..., A_K$ and $B_1, B_2, ..., B_K$ are constant complex matrices in $C^{T \times M}$.

For the Alamouti code we have,

$$\begin{aligned}
G &= G(z_1, z_2) \\
G &= z_1 A_1 + z_2 A_2 + z_1^* B_1 + z_2^* B_2
\end{aligned} \qquad (4\text{-}7)$$

$$z_1 A_1 + z_2 A_2 + z_1^* B_1 + z_2^* B_2 = \begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix} \tag{4-8}$$

Where $A_1, A_2, B_1, B_2 \in C_{2\times2}$

We have, $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$; $A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$; $B_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$; $B_2 = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}$

For L=4 we have A_1, A_2 *and* B_1, B_2

$$\lambda_1 = Tr\left(A_1^H X_\tau X_{\tau-1}^H + B_1 X_{\tau-1} X_\tau^H\right) \tag{4-9}$$

Let

$$X_\tau X_{\tau-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{4-10}$$

We have,

$$\begin{aligned} \lambda_1 &= a + d^*; \\ \lambda_1^* &= a^* + d; \\ \lambda_2 &= b - c^* \\ \lambda_2^* &= b^* - c \end{aligned} \tag{4-11}$$

Therefore,

$$\begin{aligned} \hat{Z}_k &= \underset{z_k \in A,\, 1\le k\le K}{\arg\min}\ \alpha_{\tau-1} \sum_{K=1}^{2} \left(-\lambda_k^* z_k - \lambda_k z_k^*\right) \\ \hat{Z}_k &= \underset{z_k \in A,\, 1\le k\le K}{\arg\min}\ \alpha_{\tau-1} \left[\left(-\lambda_1^* z_1 - \lambda_1 z_1^*\right) + \left(\lambda_2^* z_2 - \lambda_2 z_2^*\right)\right] \end{aligned} \tag{4-12}$$

4.5 Fast Decoding Algorithm

At the receiver, the decoding algorithm is summarized as follows:

After every τ^{th} block channel use of size *T*, X_τ and $X_{\tau-1}$ available at the receiver.

The complex square matrices, A_1, A_2 and B_1, B_2 are also known. The original *Kb* bits that were earlier transmitted are recovered as follows:

1) Compute the K complex numbers λ_k, for $k=1,2,\ldots,K$.

2) Round λ_k to the nearest information symbol, $\hat{z}_k \in A$ for $k=1,2,\ldots,K$.

3) Performa reverse mapping from $\hat{z}_k \in A$ onto a sub-block of *b* bits for $k=1,2,\ldots,K$

4) Put together *K* such sub-blocks of *b* bits into the original block of *Kb* bits.

4.6 Simulation Results for the Fast Decoding Algorithm

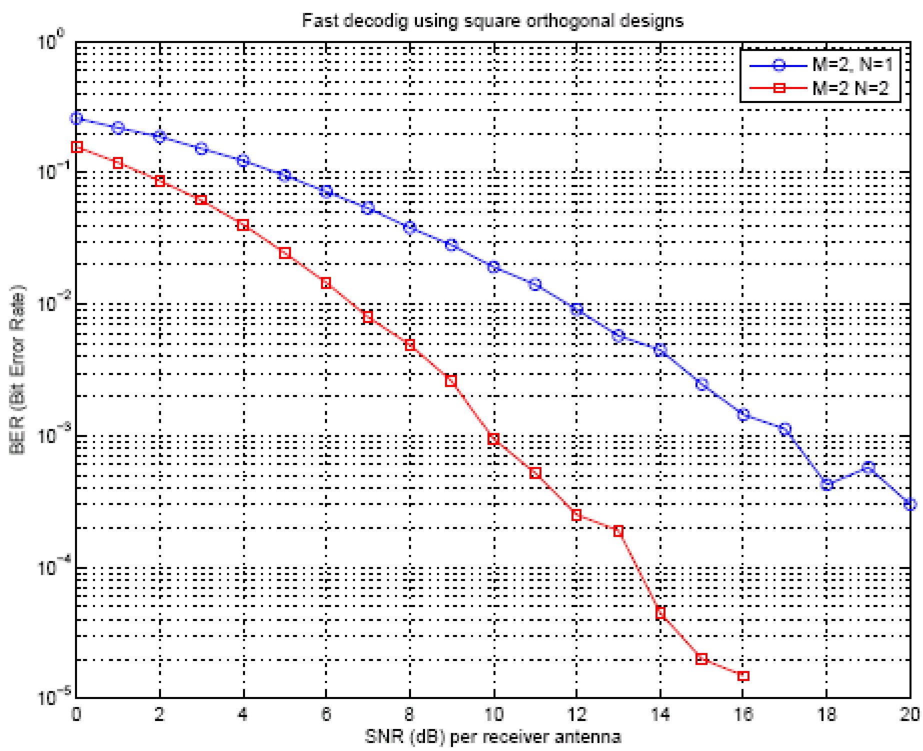

Figure 4.7 Fast decoding of Alamouti constellation of size L=16

Simulation results for Fast Maximum-Likelihood decoder for square orthogonal designs as applied to the case of a 16 constellation Alamouti code is shown above for M=2, N=1 an M=2 and N=2 antennas. It can be confirmed form the works of Alamouti in [7] that our simulation results perform very well within about 1dB of the Symbol Error Rates obtained using regular Maximum-Likelihood type demodulator. The simulation time needed for demodulation and decoding using the general differential demodulation was about 1.8 times that needed for demodulation using our fast decoding scheme.

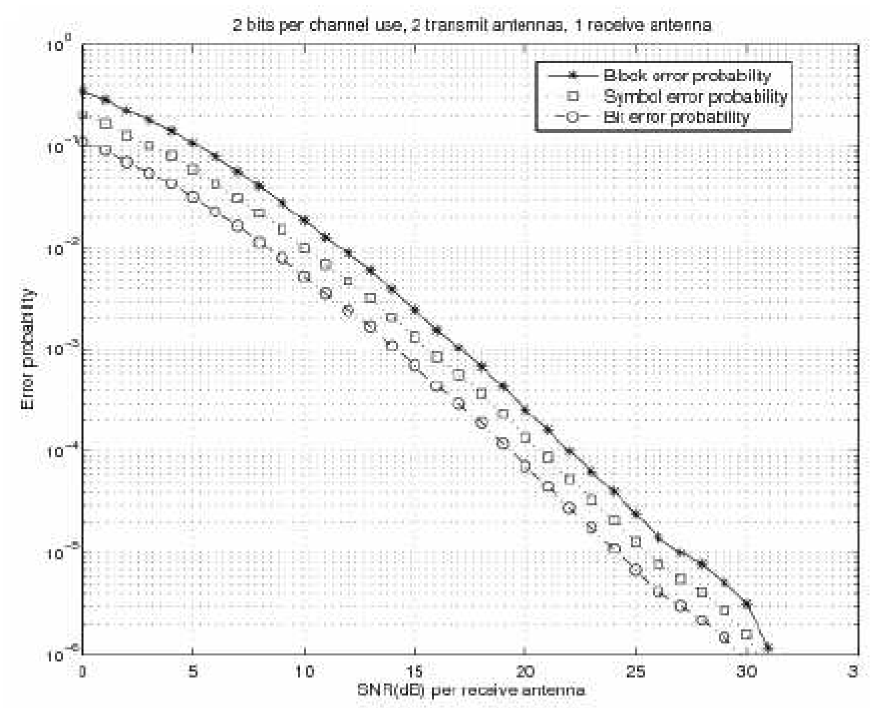

Figure 4.8 Error Probabilities for Alamouti scheme using general differential decoding
Adapted from [19]

Chapter 5 - Conclusion

Through our derivations and simulations we have shown that it is possible to extend Differential Modulation scheme for the case of a time varying flat fading Rayleigh channel. Jakes model renders it possible to incorporate velocities of the mobile wireless terminal and the Doppler shift induced thereof providing a logical correlation between successive channel coefficients. This allows us to remove the quasi static constraint on fading channels required for differential modulation schemes.

The performance of Parametric Unitary codes and Alamouti codes are evaluated based on our simulations of Block Error Rrates for two simulation models (2Tx 2Rx and 2Tx 5Rx) for constellation sizes 4 and 16. The effect of Doppler spreads introduced at the receiver due to varying terminal velocities on the channel coefficients is studied.

An important observation made was that restricting the number of block channel use τ (or frame length) was essential as the channel coefficients tend to degrade progressively with increasing number of block channel uses and is illustrated in figure 3.6.

It is seen that Parametric codes perform remarkably well upto terminal speeds of 75 kmph when a Frame Length is restricted to 20 channel uses.

It is clear from our simulation that the performance improves with the increase in the number of receiving antennas for both parametric codes and Alamouti codes.

The 16-signal constellation for the Parametric codes have the largest possible diversity product and has Block Error Rates that are better than Alamouti codes by 3dB.

The differential framework used is very flexible and allows various classes of unitary matrices to be chained together differentially.

A fast differential demodulation scheme for the Alamouti codes is developed based on the work of Liang and Xia. Our simulation results (Figure 4.7) indicate that the fast demodulation scheme developed performs almost twice (1.8 times) as fast as that of the generalized maximum likelihood differential decoding and has Symbol Error Rates within 1dB of the general differential scheme.

Future studies could involve working with more than 2 transmitting antennas for Alamouti codes to mitigate the error floor for higher terminal velocities. It could also be studied as to how well the fast differential decoding scheme performs for different terminal velocities.

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Vita

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