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Integrating lattice space-time codes of highest rank and multiplexing over rayleigh fading channels

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INTEGRATING LATTICE SPACE-TIME CODES OF HIGHEST RANK AND MULTIPLEXING OVER RAYLEIGH FADING CHANNELS

A Thesis

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DEDICATION

To my parents Samudrala Thirupathaiah and Samudrala Jyothi who have provided me consistent love, support and encouragement which guided me this far.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS, ABBREVIATIONS, AND NOMENCLATURE

3G	Third Generation
4G	Fourth Generation
A/D	Analog-to-Digital
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CBPSK	Complementary BPSK
CCI	Co Channel Interference
D/A	Digital-to-Analog
DQPSK	Differential QPSK
FFT	Fast Fourier Transform
i.i.d.	independent identically distributed
IIR	Infinite Impulse Response
ISI	Intersymbol Interference
MIMO	Multiple-Input Multiple-Output
MIMO-BC	MIMO Broadcast Channel
MIMO-MAC	MIMO Multiple-Access Channel
MIMO-MU	Multiple-Input Multiple-Output Multi User

MRC	Maximum Ratio Combining
PDF	Probability Density Function
PSK	Phase Shift Keying
QAM	Quadrature Amplitude Modulation
QPSK	Quadrature Phase Shift Keying
SER	Symbol Error Rate
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SM	Spatial Multiplexing
ST	Space-Time
STBC	Space-Time Block Code/Codes/Coding
STC	Space-Time Coding
ZF	Zero Forcing
MIMO-SU	Multiple-Input Multiple-Output Single User
MISO	Multiple-Input Single-Output
ML	Maximum Likelihood
MLSE	Maximum Likelihood Sequence Estimation
MMSE	Minimum Mean Square Error

ABSTRACT

In this thesis, we consider the problem of finding an optimal combination of multiplexing and space-time coding over a MIMO array when a transmitter has a prior knowledge about correlation. Some existing work in this area used multiplexing and space-time coding methods as a technique to achieve capacity over a MIMO wireless channel. In this work, we consider a 2x2 lattice space-time code of highest rank and multiplex it over a MIMO system to improve the bit error rate performance. The main focus of this thesis is to address the problem of switching between spatial multiplexing and space-time coding to enhance the performance of a MIMO system.

The data rate over wireless links is improved by using a MIMO system. For this type of system, the spatial dimension is exploited by using spatial multiplexing and space-time coding with diversity oriented transmission. Spatial multiplexing uses spatial degrees of freedom and space-time coding uses the antennas. To integrate them the system has to compromise to some extent in performance criterion.

This thesis investigates the need to switch between the spatial multiplexing and space-time coding over space by making the instantaneous channel feedback available at transmitter. This thesis also indicates that the bit-error rate of the combination of lattice space-time code and multiplexing can be reduced when compared with the combination of Alamouti scheme and spatial multiplexing.

CHAPTER 1

INTRODUCTION

1.1 Problem Formulation

If our principle motto is to improve the data rate of transmission over a wireless medium then MIMO system is one of the choices available. In MIMO systems multiplexing and space-time coding are two ways by which the capacity can be enhanced. Space-time coding uses antennas to fight against Rayleigh fading while the spatial multiplexing uses spatial degree of freedom to improve the data rate by simultaneously sending the independent symbol streams. There are some differences in employing these schemes, the diversity scheme with more number of antennas has less returns and the multiplexing scheme lacks in diversity using a simple linear receiver which results in poor performance. Therefore we will try to solve both these problems due to multiplexing and space-time coding by integrating these two schemes. There are proposed schemes where combining these two schemes has been carried out over space and time. There were cases where the required combination is to switch between spatial multiplexing and space-time coding over time, this exploited the fact that spatial multiplexing performance was totally dependent on eigen value spread whereas for the space-time coding depends on channel matrix energy.

In this part of our work we will concentrate on the problem of switching between the spatial multiplexing and space-time coding over space. In this case the transmitter has instantaneous channel feedback where long term characteristics are available. We will discuss about the MIMO system model and then the two schemes, spatial multiplexing and space-time coding in detail and then combining them over a Rayleigh fading channel. We will then investigate the performance gains with bit error rate simulations. Later in the second part of the work we will discuss about the lattice codes and their relation in communications and integrating with spatial multiplexing. This will be followed by studying the performance characteristics by examining the bit error rate curves and comparing it with the normal mode. We have a 2x2 lattice code of highest rank 8 as proposed by Yang [6]. We were able to rearrange the lattice in such a way that the Alamouti scheme can be applied on it and also integrate this lattice space-time code with spatial multiplexing for a MIMO system over a fading channel.

1.2 Contribution of the Thesis

The objectives of this Thesis are :

- To compare the performance of Alamouti code using 8-PSK, lattice space-time code of highest rank using BPSK, and the combined transmission scheme of lattice with spatial multiplexing for 2x1 and 2x2 MIMO systems.
- To analyze the BER and SER curves of the lattice space-time code combined with spatial multiplexing.

This thesis provides the following information:

- Preliminary information about the various MIMO systems including the channel capacities for the cases when the channel is known to the transmitter and when it is unknown to the transmitter.
- Various space-time codes, spatial multiplexing schemes and their decoding procedures.
- Formulation of the lattice space-time code of highest rank such that the Alamouti scheme for a 2x2 antenna system can be implemented for the evaluation of performance.
- Development of different procedures for simulation of SER and BER curves using matlab.
- Comparison of various space-time coding, spatial multiplexing and their combined transmission schemes.

1.3 Outline of the Thesis

This thesis is organized as follows.

Chapter 2 gives a brief information of multi-antenna systems and different types of array gain and diversity gain involved.

Chapter 3 discusses about Alamouti scheme, which is a simple full rate space-time block code scheme that has information spread across multiple antennas at the transmitter end.

Chapter 4 discusses about another diversity oriented transmission technique known as spatial multiplexing (SM or SMX). This method is also called as ”BLAST” approach. This is another approach of exploiting the spatial dimension offered by the MIMO system.

Chapter 5 focuses on combination of the Alamouti space-time code and spatial multiplexing over Rayleigh fading channel.

Chapter 6 discusses about a lattice and how they are related to communications and a 2x2 lattice space-time code of highest rank and how the Alamouti scheme is applied to this lattice code.

Chapter 7 focuses on integrating the highest rank lattice code and spatial multiplexing over space.

Chapter 8 contains the comparative study of the BER and SER of lattice space-time code, Alamouti scheme and these schemes combined with spatial multiplexing.

Chapter 9 gives conclusions and discusses about the future work.

CHAPTER 2

PRELIMINARIES

2.1 Multi-antenna Systems

The following figure Fig. 2.1 shows the different types of antenna configurations used in space-time systems. Single-input single-output (SISO) which has one antenna at the transmission end and one antenna at the receiver end and this is one of the well known configuration. Single-input Multiple-output(SIMO) which has one antenna at transmitter end and M_r receiving antennas. Multiple-input single-output (MISO) which has M_t antennas at the transmitter and one antenna at the receiver end. Multiple-input multiple-output (MIMO) which has M_t antennas at the transmitter end and M_r receiving antennas and finally MIMO-multiuser (MIMO-MU) which has a base station with multiple antennas at the transmitter end and multiple receiving antennas interacting with multiple users each with one or more antenna. Some of the terms related to MIMO systems are discussed in the next section.

2.2 Array Gain

Array gain is defined as the average increase in the signal-to-noise ratio (SNR) at the receiver due to the use of multiple antennas. Coherence combining effect of these antennas will be there either at the receiver or at the transmitter or both [17]. There are two types of array gains

- Transmitter Array gain
- Receiver Array gain

Consider MISO system where we have multiple antennas at transmitter, if the channel state information is known to the transmitter then it will weigh the transmission depending on the chan-

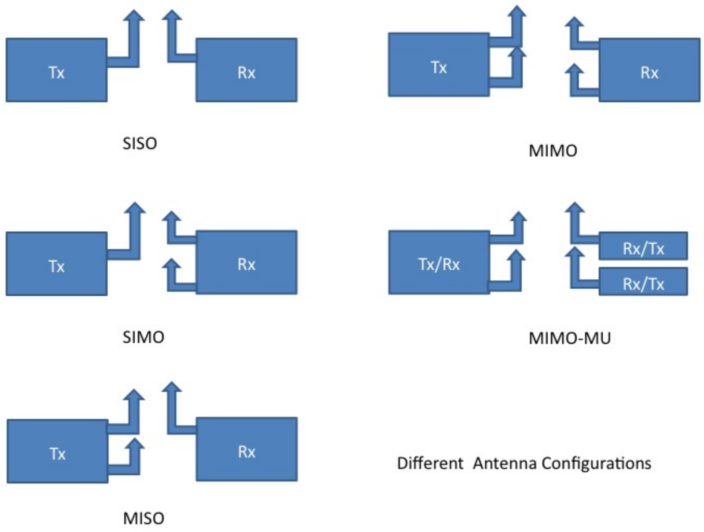

Figure 2.1 Different antenna configurations

nel coefficients. The gain at one antenna receiver which has coherent combining effect is called transmitter array gain.

Now consider the SIMO system where we have one antenna at the transmitter end which do not have any knowledge of the channel state information and multiple antennas which have complete knowledge of the channel state information can suitably weigh the received signals and coherently combine at the output thereby enhancing the signal. This is called receiver array gain.

Therefore multiple antenna system requires complete knowledge about either transmitter or receiver or both to achieve this array gain.

2.3 Diversity Gain

In a communication system one of the major problem is multi path fading due to obstacles like buildings, cars, trees etc. In a fading channel like Rayleigh channel the signal experiences fluctuations in their strength. When the power of the signal drops to an extent then the signal is said to be in a fade which results in bit error rate (BER). Diversity techniques are employed to fight against fading, in this process several replicas of the transmitted signals are used over space, time or frequency [17]. There are some traditional diversity schemes like

- Selection diversity.
- Maximal ratio diversity.
- Equal gain diversity.

These are diversity combining schemes employed for SIMO channels. The space-time coding exploits diversity across space and time. There are several types of diversity schemes in wireless communication systems,

- Temporal diversity.
- Frequency diversity.
- Spatial diversity
  - Receiver diversity.
  - Transmitter diversity.
  - Polarization diversity.
  - Angle diversity.

In *temporal diversity* [17] the replications of the transmitted signals are provided in time by combining time interleaving strategies and channel coding. The channel must provide variations in time, in some cases where the coherent time of channel is small we can guarantee that the interleaved symbol is different from the previous transmitted symbol, hence the new replication will be completely new than that of the original symbol transmitted [17].

In *frequency diversity* [17] the replications of the original transmitted signals are in frequency. This happens in the case where the bandwidth of the signal is greater than the coherence bandwidth of the channel, this guarantees that the spectrum will undergo different fades.

In *spatial diversity* the replications of the original transmitted signals are across different receiver antennas. In this case there are independent fades across different antennas as coherent distance is smaller than the antenna spacing. This is also called as antenna diversity scheme which is the best method to fight against the multi-path fading. We have to understand that, the better the independent samples of the original signal transmitted the better the diversity scheme then the probability of a signal to undergo fading in different parts of signal will be very small. There are so many constraints on which it depends like coherence time, coherence bandwidth and coherence distance. The receiver should be able to combine the different waveforms to get back the original signal with good quality. This scheme may also be categorized based on the scheme application to either transmitter or receiver [17].

Receive diversity: It is also called as maximum ratio combining, this is commonly applied receiver diversity scheme to enhance the signal quality [17]. This is costly and difficult to handle or use at the receiver side especially in cell phones, this is the reason that the other diversity scheme became popular.

Transmit diversity: This is easier to implement at the base station, where controlled redundancies are introduced at the transmitter. By employing appropriate signal processing at the receiver the signal can be restored. In this scheme the transmitter should have complete knowledge about the channel [17].

In this category of diversity we have two more types of diversity schemes.

Polarization diversity: In this type of diversity scheme there are two pairs of polarized antennas, two at the transmitter and two at the receiver. Horizontal and vertically polarized signals are sent from two differently polarized transmitter antennas and they are received by the two differently polarized antennas at receiver [17]. As they employ different polarization, there will be no correlation between the data streams.

Angle diversity: This type of scheme is applicable for carrier frequencies greater than 10GHz, at this high frequencies the transmitted signal undergoes scattering. They have two highly directional antennas facing in different directions enables receiver to collect two different samples of the same signal, which are different from each other [17].

So far in this chapter we discussed about the different multi-antenna systems and several concepts like array gain, diversity gain and how they play a role in the performance of the system. Now let switch to our main idea of the space-time codes in the next chapter.

CHAPTER 3

ALAMOUTI SPACE-TIME CODING

In this chapter we will discuss about 2x2 Alamouti code with full diversity gain and also investigate simple maximum likelihood decoding algorithm. We shall examine how these space-time codes behave in imperfect channel estimates and Rayleigh fading channels.

The Alamouti space-time block coding is a well known diversity technique which uses the information spread across multiple antennas at the transmitter end. We make some assumptions in this scheme, as the channel we are using is the Rayleigh fading channel and the modulation technique used is BPSK, the Rayleigh channel is assumed to be flat fading channel [17], [1].

Consider a transmission sequence $(x_1, x_2, x_3, x_4, x_5, ., ., ., ., .x_n)$, in normal transmission x_1 is sent in first time slot and x_2 is sent in the second time slot and x_3 is sent in the third time slot and so on x_n is sent in the n^{th} time slot, where as in this Alamouti scheme the encoder picks up the block of two modulated symbols x_1 and x_2 in one encoding operation and feed it to the transmitting antennas according to the following block code matrix [1]

$$X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}. \tag{3.1}$$

In the above matrix the columns represent the space and the rows represent time i.e., the columns tells us about the transmission periods and the rows tell us about the symbols transmitted from the antennas. Elaborating it, in the first symbol period the first and second antennas transmit x_1 and x_2 and in the second symbol period they transmit $-x_2^*$ (complex conjugate of x_2) and x_1^*

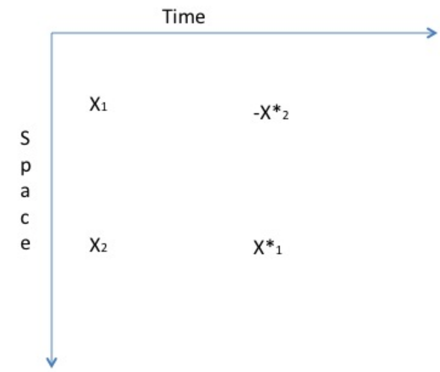

Figure 3.1 Alamouti scheme using rows and columns

(complex conjugate of x_1). In the third symbol period x_3 and x_4 and in the fourth symbol period $-x_4^*$ and x_3^* and it continues till the n^{th} symbol [1].

Actually the symbols are grouped in two and sent, by sending two symbols in two different time slots the data rate does not change. The block diagram Fig. 3.2 below gives the exact idea how the Alamouti scheme is implemented.

This is going to imply that we are transmitting both in space and time i.e., space-time coding. By examining the encoding matrix we can tell that the sequences are orthogonal, as the inner product of the x' and x'' is zero.

$$x' = [x_1, -x_2^*] \tag{3.2}$$

$$x'' = [x_2, x_1^*] \tag{3.3}$$

$$x' x''^H = x_1 x_2^* - x_2^* x_1 = 0. \tag{3.4}$$

Block diagram of Alamouti Space-time encoder

Figure 3.2 Block diagram of Alamouti coding

There will be fading and the coefficients related to fading are defined by $h_1(t)$ and $h_2(t)$, at time *t* from antennas first and second respectively. By assuming that these coefficients will be constant across two consecutive symbols gives us the following equations:

$$h_1(t) = h_1(t+T) = h_1 = |h_1|e^{j\theta_1}, \tag{3.5}$$

$$h_2(t) = h_2(t+T) = h_2 = |h_2|e^{j\theta_2}, \tag{3.6}$$

$|h_i|$ is the amplitude gain and θ_i,i=1,2 is the phase shift from transmitter *i* to the receiver antenna. This received signals after passing through the channel can be expressed as below. In 1^{st} time slot

$$y_1^1 = h_{11}x_1 + h_{12}x_2 + n_1^1. \tag{3.7}$$

$$y_2^1 = h_{21}x_1 + h_{22}x_2 + n_2^1. \tag{3.8}$$

$$\begin{pmatrix} y_1^1 \\ y_2^1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}. \tag{3.9}$$

The channel remains constant and the second time slot

$$y_1^2 = -h_{11}x_2^* + h_{12}x_1^* + n_1^2. \tag{3.10}$$

$$y_2^2 = -h_{21}x_2^* + h_{22}x_1^* + n_2^2. \tag{3.11}$$

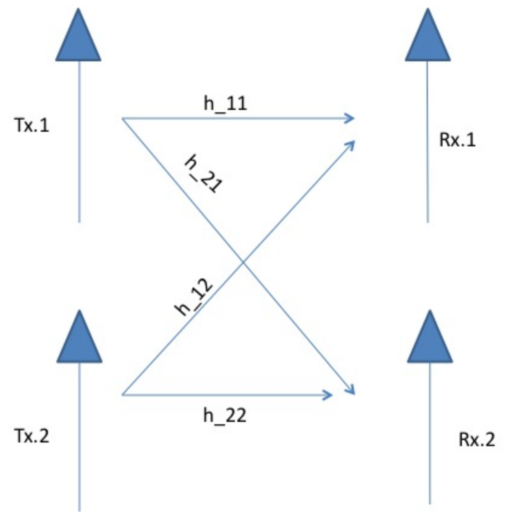

Figure 3.3 Alamouti coding in a 2x2 system

$$\begin{pmatrix} y_1^2 \\ y_2^2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} -x_2^* \\ x_1^* \end{pmatrix} + \begin{pmatrix} n_1^2 \\ n_2^2 \end{pmatrix}, \tag{3.12}$$

where h_{ij} represents i^{th} receiver to the j^{th} transmitting antenna and n_1 and n_2 are independent complex variables.

3.1 Maximum Likelihood Decoding

The channel coefficients h_1 and h_2 are assumed to be perfectly recovered at the receiver and these coefficients are used as the CSI [17]. The combiner combines the received signal as mentioned below (3.13) and sends it to the maximum likelihood detector, where it reduces to decision

$$\widetilde{x_1} = h_1^* y_1 + h_2 y_2^* = (\alpha_1^2 + \alpha_2^2) x_1 + h_1^* n_1 + h_2 n_2^*, \tag{3.13}$$

$$\widetilde{x_2} = h_2^* y_1 - h_1 y_2^* = (\alpha_1^2 + \alpha_2^2) x_2 - h_1 n_2^* + h_2^* n_1, \tag{3.14}$$

and the decision metric equation is given as

$$|y_1 - h_1 x_1 - h_2 x_2|^2 + |y_2 + h_1 x_2^* - h_2 x_1^*|^2. \tag{3.15}$$

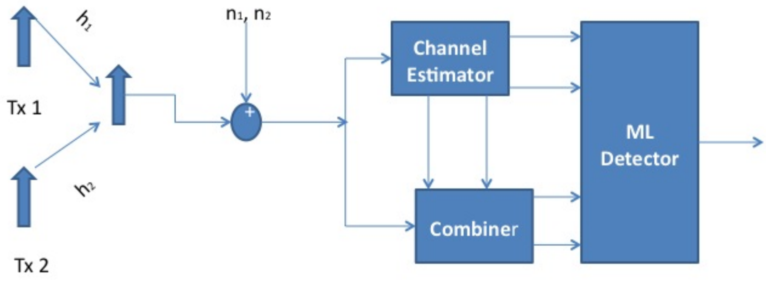

Figure 3.4 Maximum likelihood detector

Therefore enquiring all the possibilities of x_1 and x_2 and deleting the non related codewords reduce the above equation (3.15) to

$$|y_1h_1^* + y_2^*h_2 - x_1|^2 + (\alpha_1^2 + \alpha_2^2 - 1)|x_1|^2, \tag{3.16}$$

$$|y_1h_2^* + y_2^*h_1 - x_2|^2 + (\alpha_1^2 + \alpha_2^2 - 1)|x_2|^2, \tag{3.17}$$

for detecting x_1 and x_2 respectively. For the decision rule if the notation is

$$d^2(a,b) = (a-b)(a^* - b^*) = |a-b|^2. \tag{3.18}$$

The decision rule for each combined signal $\widetilde{x}_j$ behaves as

$$(\alpha_1^2 + \alpha_2^2 - 1)|x_i|^2 + d^2(\widetilde{x}_j, x_i) \leq (\alpha_1^2 + \alpha_2^2 - 1)|x_k|^2 + d^2(\widetilde{x}_j, x_k), \vee i \neq k. \tag{3.19}$$

In the case of PSK signals the above equation (3.17) reduces to

$$d^2(\widetilde{x}_j, x_i) \leq d^2(\widetilde{x}_j, x_k), \vee i \neq k. \tag{3.20}$$

3.2 Maximum Ratio Combining

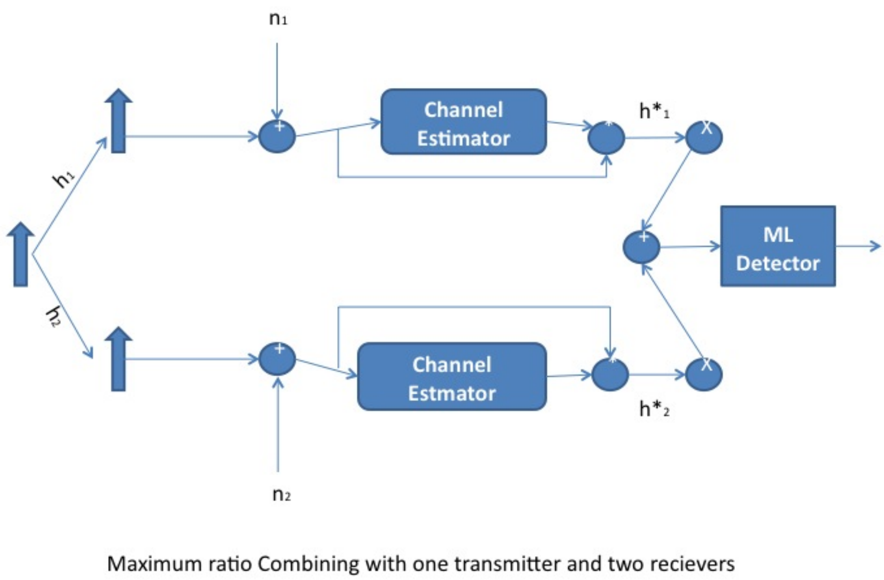

Figure 3.5 Maximum ratio combining

When we consider the case of maximum ratio combining, the received signals can be interpreted as

$$y_1 = h_1 x_0 + n_1 y_2 = -h_2 x_0 + n_2, \tag{3.21}$$

and the combined signal is

$$\widetilde{x_0} = h_1^* x_0 + h_2^* r_2 \widetilde{x_0} = (\alpha_1^2 + \alpha_2^2) x_0 + h_1^* n_1 + h_2^* n_2. \tag{3.22}$$

The maximum likelihood detector uses the same decision rule as that of PSK signals and signal x_i is detected [17]. This MRC signal is almost same as the diversity scheme signal except for small phase difference in the noise signal, which does not affect the signal-to-noise ratio. Hence we can conclude that two-branch MRC and Alamouti diversity order in this case is same.

3.3 Transmit Diversity

Consider two distinct code sequences X and $\hat{X}$ generated by the inputs (x_1, x_2) and $(\hat{x_1}, \hat{x_2})$, respectively, where

$$(x_1, x_2) \neq (\hat{x_1}, \hat{x_2}). \tag{3.23}$$

We know that the transmissions are orthogonal in Alamouti's scheme, hence the transmission diversity is of order two. The codeword difference matrix is given by the following equation:

$$B(x, \hat{x}) = \begin{pmatrix} x_1 - \hat{x_1} & -x_2^* + \hat{x_2^*} \\ x_2 - \hat{x_2} & -x_1^* + \hat{x_1^*} \end{pmatrix}. \tag{3.24}$$

The rows of this code word difference matrix are orthogonal as the rows of the code matrix are orthogonal. Code word distance matrix is given by

$$A(X, \hat{X}) = B(X, \hat{X})B^H(X, \hat{X}). \tag{3.25}$$

$$A(X, \hat{X}) = \begin{pmatrix} |x_1 - \hat{x_1}|^2 + |x_2 - \hat{x_2}|^2 & 0 \\ 0 & |x_1 - \hat{x_1}|^2 + |x_2 - \hat{x_2}|^2 \end{pmatrix}, \tag{3.26}$$

since the transmit diversity is two i.e., $M_T = 2$. The resultant determinant of $A(X, \hat{X})$ is given by

$$det(A(X, \hat{X})) = (|x_1 - \hat{x_1}|^2 + |x_2 - \hat{x_2}|^2)^2. \tag{3.27}$$

When we observe the above code word distance matrix (3.27), we can tell that it has two identical eigen values. Out of the two, the minimum eigen value is the min squared Euclidean distance in the signal constellation.

Therefore in brief according to Alamouti scheme there is no feedback to the transmitter from receiver to get full transmit diversity. The main advantage is that there is no need of complex decoders, and there is no bandwidth expansion as redundancy applied in space for these multiple antennas.

Matlab Implementation:

1. A binary sequence of +1 and -1 is generated.
2. These are grouped into pairs of two symbols.
3. These are coded as per Alamouti scheme.
4. Multiply these symbols with channel and then white Gaussian noise is added.
5. The received symbols should be equalized.
6. Hard decision decoding is being performed on the received bits and number of errors are counted.
7. This will be repeated for different values of E_b/N_0.

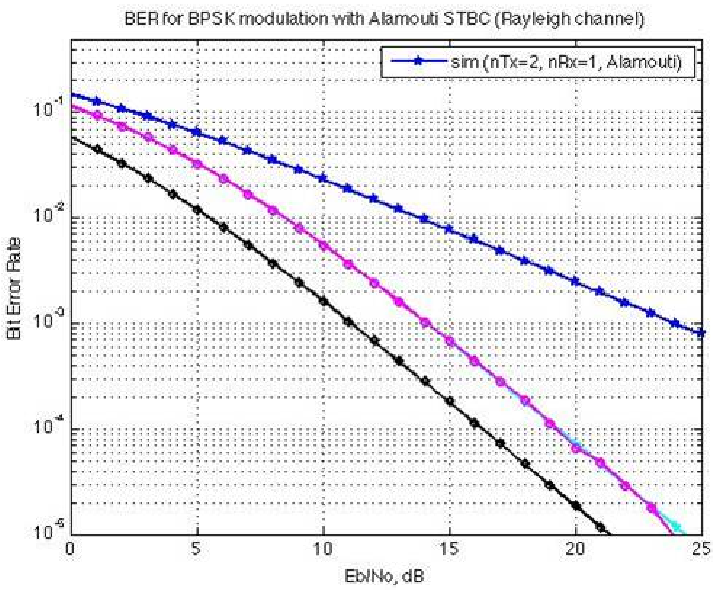

Figure 3.6 Matlab implementation of 2x1 system

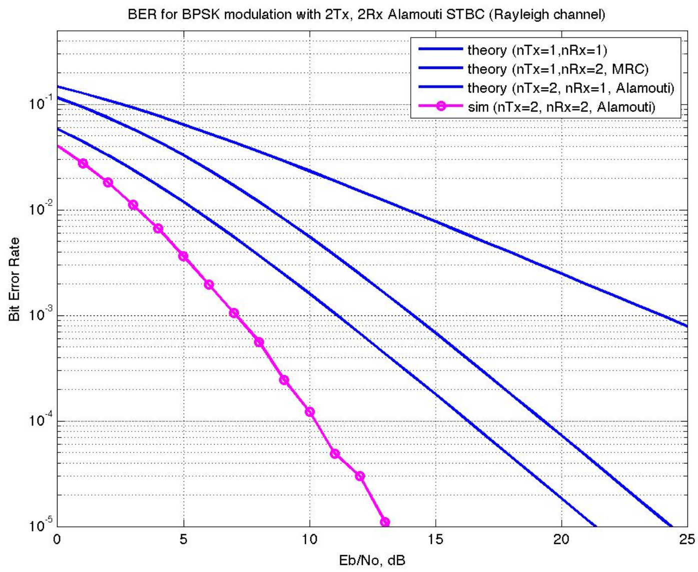

Figure 3.7 Matlab implementation of 2x2 system

CHAPTER 4

SPATIAL MULTIPLEXING

In this chapter we will discuss about another diversity oriented transmission technique known as spatial multiplexing (SM or SMX). This method is also called as ”BLAST” approach. This is another approach of exploiting the spatial dimension offered by the MIMO system. This method sends the symbol streams at a time using the spatial degree of freedom, which helps it to increase the data rate. Here the stream is referred as the independent and separately encoded data signals from the transmitting antenna. By sending the streams of symbols at a time the space dimension is being used again and again more than once i.e., reused or multiplexed [17].

In spatial multiplexing if transmitter is transmitting using M_t antennas and receiver is using M_r antennas, the maximum number of streams that can be transmitted in case of linear receiver is used is

$$M_s = min(M_t, M_r), \tag{4.1}$$

i.e., over a wireless channel M_s streams can be transmitted at a time leading to the increase in spectral efficiency by the amount of M_s without requiring any additional bandwidth or power. This capacity increment can be achieved only in MIMO channels. In this multiple antenna systems, the independent sub streams transmitted undergo scattering due to the obstacles and scattering objects like cars, walls of buildings, etc., and these transmitted signals takes different paths. This is the reason for different transmitting antennas having different spatial signature. At the receiver end the individual signal streams received undergo signal processing to decode them to get the original signal. The added advantage in employing the spatial multiplexing technique is the orthogonal signatures of the transmitted signals provided by the propagation channel, whereas in CDMA or TDMA this orthogonal signatures are obtained at the cost of frequency spreading and time spreading, hence their spectral efficiency is much decreased.

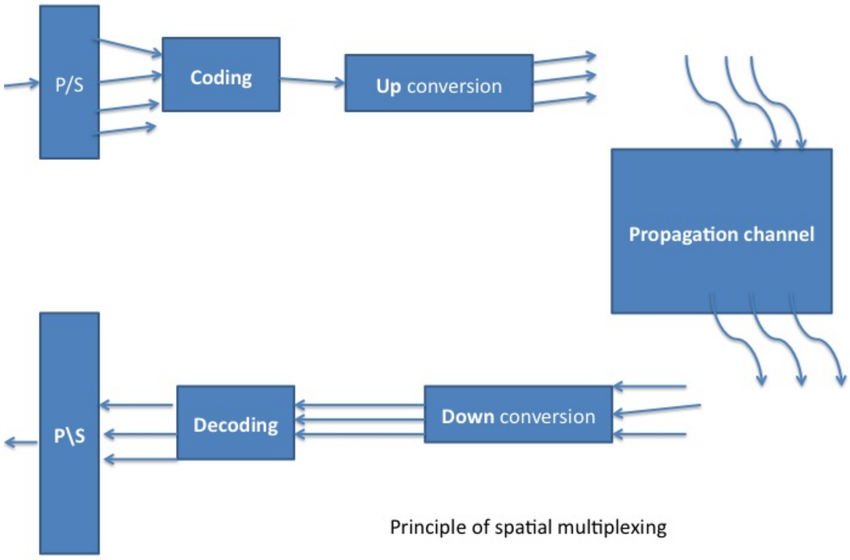

Figure 4.1 Spatial multiplexing system

Spatial multiplexing can be encoded in two different ways

1. Open-loop approach.
2. Closed-loop approach.

4.1 Open-loop Approach

This type of system uses M_t transmit antennas and M_r receiving antennas, the relation between the output y and input x is given as

$$y = Hx + n, \tag{4.2}$$

where,

- $x = [x_1, x_2, x_3, x_4, ., ., ., ., ., x_{N_t}]^T$ is the transmitted symbols M_t x 1 vector.

- y is the N_r x 1 vector of the received symbol.
- n is the noise vector.
- H is the M_r x M_t matrix of channel coefficients.

4.2 Closed-loop Approach

This type of system uses M_t transmit antennas and M_r receiving antennas, the relation between the output y and input x is given as

$$y = HWs + n, \tag{4.3}$$

where,

- $s = [s_1, s_2, s_3, s_4, ., ., ., ., ., ., s_{N_t}]^T$ are the transmitted symbols M_s x 1 vector.
- y is the M_r x 1 vector of the received symbol.
- n is the noise vector.
- H is the M_r x M_t matrix of channel coefficients.
- W is the linear pre-coding matrix.

In this case we use a pre-coding matrix W to pre-code the symbols in order to increase the performance. The columns of M_s of W are chosen in such a way that they are smaller than the columns of M_t, as in most of the systems the number of transmit antennas are greater than the receiver antennas.

For example consider a system with two transmitting antennas and two receiving antennas, if you want to extend to more antennas in number this can be applied to any general case. At the transmitter end the two antennas simultaneously transmit the modulated bit stream by splitting it into two half-rate bit streams. In this system the receiver has knowledge about channel it has receiver diversity, as the receiver knows about the channel it can recover and combine the received bits to get back the original stream of bits. Whereas this is not the case of transmitter as bit streams carry completely different data, hence transmitter diversity cannot be achieved. Therefore the number of transmission antennas pairs have an impact on the transmission rate i.e., the transmission rate increases when the number of pairs are increased, they are directly proportional to each other.

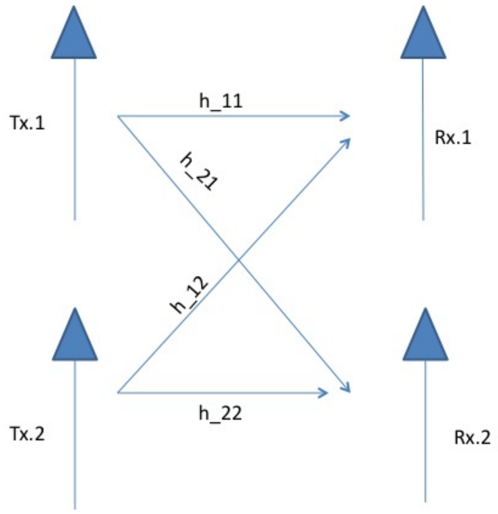

Figure 4.2 Spatial multiplexing for 2X2 system

In MIMO-MU case the two antennas simultaneously transmit the modulated bit stream by splitting it into two half-rate bit streams to the base stations (BS). This base station can separately transmit two different signals simultaneously with spatial filtering such that the receiver can decode the signal without any error. Therefore the number of transmission antennas pairs at the base station have an impact on the transmission rate, i.e., the transmission rate increases, when the number of pairs at the base station are increased and also on the number of users.

In this multiplexing system we consider the channel to be flat fading channel i.e., Rayleigh fading channel. Let $x = [x_1, x_2, x_3, x_4, ., ., ., ., ., ., x_{N_t}]$ be the sequence to be transmitted, the two transmitter antennas group the symbols in group of 2 and they send as follows

- 1^{st} time slot x_1, x_2 are sent.
- 2^{nd} time slot x_3, x_4 are sent.
- 3^{rd} time slot x_5, x_6 are sent.

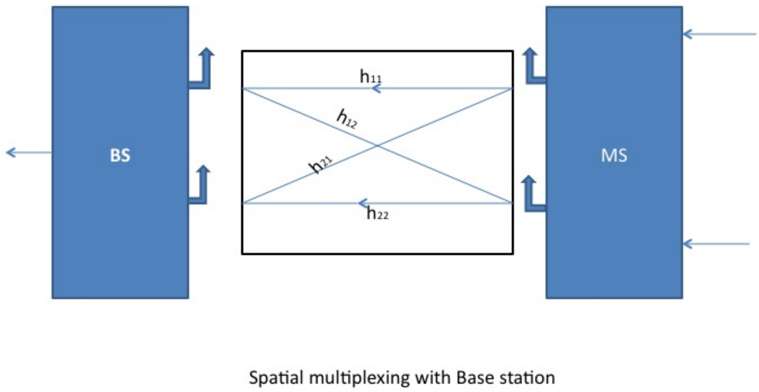

Figure 4.3 Spatial multiplexing with base station

- 4^{th} time slot x_7, x_8 are sent.
- so on.

Therefore the number of time slots required are **n/2**, thus the data rate has been doubled. Where h_{ij} is randomly varying complex number and $\mu_{h_{ij}} = 0$ and $\sigma^2_{h_{ij}} = \frac{1}{2}$. At the receiving antenna end the noise has Gaussian distribution, therefore the distribution is given as [14]

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(n-\mu)^2}{2\sigma^2}}, \tag{4.4}$$

where $\mu = 0$ and $\sigma^2 = N_0/2$.

4.3 Zero-forcing Detector

The ZF receiver is linear and it acts as a linear filter, which independently decodes each data stream. At the receiver the output can be interpreted as

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1. \tag{4.5}$$

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2. \tag{4.6}$$

Therefore the output can be expressed as

$$Y = HX + N, \tag{4.7}$$

where Y, X, H and N are the output, input, channel and noise matrices w.r.t to the above equations (4.5), (4.6). To solve for X we should be able to find a matrix *W* such that $WH = I$, where *I* is the identity matrix. Hence with this ZF linear detector we get

$$W = (H^H H)^{-1} H^H, \tag{4.8}$$

$$H^H H = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}^* h_{12} + h_{21}^* h_{22} \\ h_{12}^* h_{11} + h_{22}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}, \tag{4.9}$$

where in $H^H H$ matrix the off diagonal terms are non-zero the ZF equalizer try to nullify the interfering terms during equalization, i.e., while solving for x_1 the interference terms due to x_2 tried to be nullified and while solving for x_2 the interference terms due to x_1 are nullified. The BER for MIMO system in Rayleigh fading channel with ZF equalization is given as following referring to section 3.3 in [18]

$$p_b = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{(E_b/N_0)}{(E_b/N_0) + 1}}. \tag{4.10}$$

This shows that the diversity order of the each independent data stream is given by $M_r - M_t + 1$, i.e., M_t parallel streams from the ZF receiver has the array gain of the order of $M_r - M_t + 1$.

Matlab Implementation:

1. The symbol sequences of +1's and -1's are generated.
2. Then these pairs are grouped in pairs which has two symbols per slot.
3. These symbols are then multiplied with channel and then Gaussian noise is added.
4. Equalization using ZF is done at the receiver.
5. Hard decision decoding is done and the number of bit error are counted.
6. This is repeated for different values of E_b/N_0.

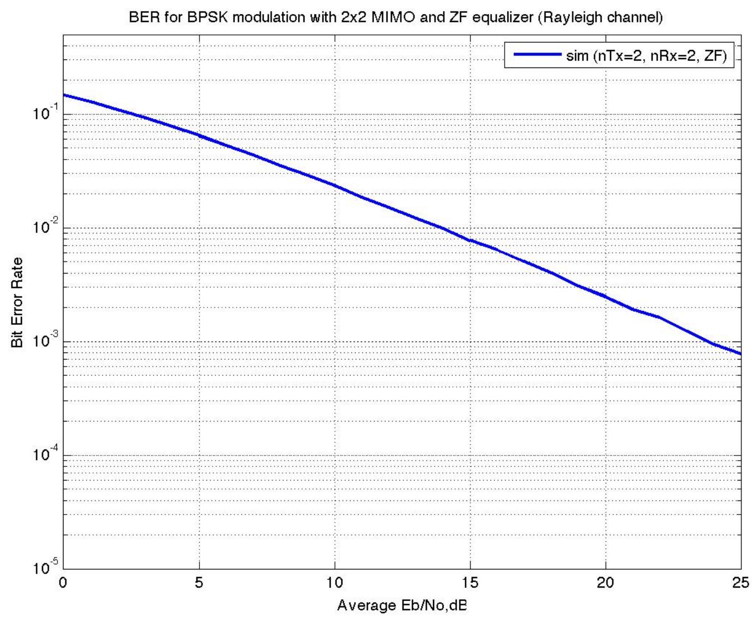

Figure 4.4 Spatial multiplexing of 2x2 system using matlab

CHAPTER 5

COMBINING ALAMOUTI'S SCHEME AND SPATIAL MULTIPLEXING

In this chapter we will discuss about combining the space-time codes and spatial multiplexing over space, the proposed scheme focuses on the problem of switching these two methods over space rather than on time. We use the simple algorithms proposed in [2] to generalize the work of [7]. For both cases the long term correlation statistics of the transmitter is known and the instantaneous channel feedback is available at transmitter. Here the problem of combining both the techniques Alamouti and multiplexing on the same array by allocating the antennas to either one scheme or the other is discussed. The combination in space inspired by [2], which in turn got inspired by the combination in time by [7].

5.1 Combining in Time

In the paper [7] these two techniques are compared by checking the instantaneous channel matrices, which uses different modulation schemes to make sure they have same bit error rate. The best method is chosen based on a criterion that the closer the symbols are in the received constellation the more likely that the decision made has been right. Therefore at any time for the received constellation the one that gives the largest minimum Euclidean distance is chosen.

The minimum squared,Euclidean distance of the received constellations denoted as $d^2_{min_{SM(H)}}$ for SM and $d^2_{min_{STC}}(H)$ for STC methods and the bounds on these are given as

$$d^2_{min,SM}(H) \geq \sigma^2_{min}(H) d^2_{min,sm}, \tag{5.1}$$

$$d^2_{(min,STC)}(H) \leq 1/N ||H||^2_F d^2_{min,stc}. \tag{5.2}$$

In the above equations (5.1), (5.2) $d^2_{min,sm}$ and $d^2_{min,stc}$ are the minimum squared Euclidean distances of the transmit constellations, and $\sigma_{min}(H)$ is the minimum singular value of H. Using conservative approach, spatial multiplexing is used only when

$$\sigma^2(H) d^2_{minsm} \geq ||H||^2_F d^2_{minstc}. \tag{5.3}$$

Therefore for a given channel matrix H, if it has a large Frobenius norm then diversity is preferred, else if there is large minimum eigen value then spatial multiplexing is preferred.

5.2 Combining in Space

In paper [2] space-time code is considered as Alamouti scheme, here all these Alamouti coded blocks have been spatially multiplexed consisting of independent group of two symbols in each one. It is assumed that $N = 2k$, $k \geq 2$,$M \geq N/2$ and the transmitted block is $X = [x_0 x_1 ....x_{N-1}]^T$ is transmitted over two symbol durations by forming matrix *X* of size Nx2 from the X_k Alamouti matrices of size Nx2. The best way of assigning the Alamouti blocks to antenna is by choosing *X* such that

$$Y = HX + N, \tag{5.4}$$

where,

$$X = \begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ . \\ . \\ . \\ X_{N/2-1} \end{pmatrix}. \tag{5.5}$$

This combination may also be expressed with respect to channel matrix

$$\tilde{y} = \tilde{H}x + n. \tag{5.6}$$

These Alamouti blocks may be assigned to any antenna combination, if there are p_N non-trivially equivalent antenna patterns, then these are figured as p_k, where $k\epsilon[1, p_N]$. When only two antenna system is used then there is only one pattern possible so there is no complexity involved but when two or more antennas are used then the pattern selection comes to light, for example when $N = 4$ and and $p_4 = 3$ then the patterns possible are shown below as in Fig. 5.2.

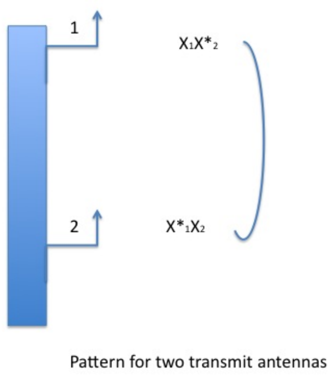

Figure 5.1 Two antenna transmit pattern

According to these different p_k patterns available the general input output equation is given as

$$\tilde{y} = \tilde{H}(p_k)x + n. \tag{5.7}$$

The main goal is to select the pattern out of possible patterns available is to reduce the feedback load and the complexity of the model.

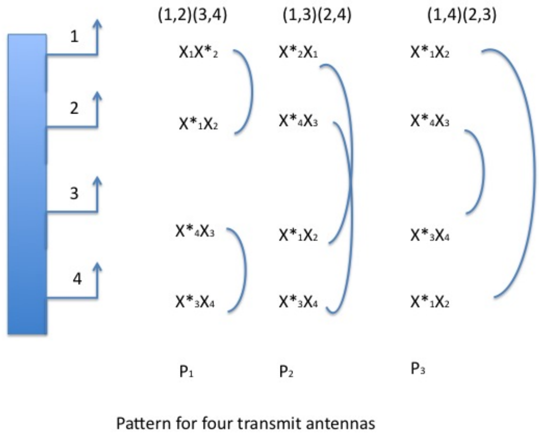

Figure 5.2 Four antenna transmit pattern

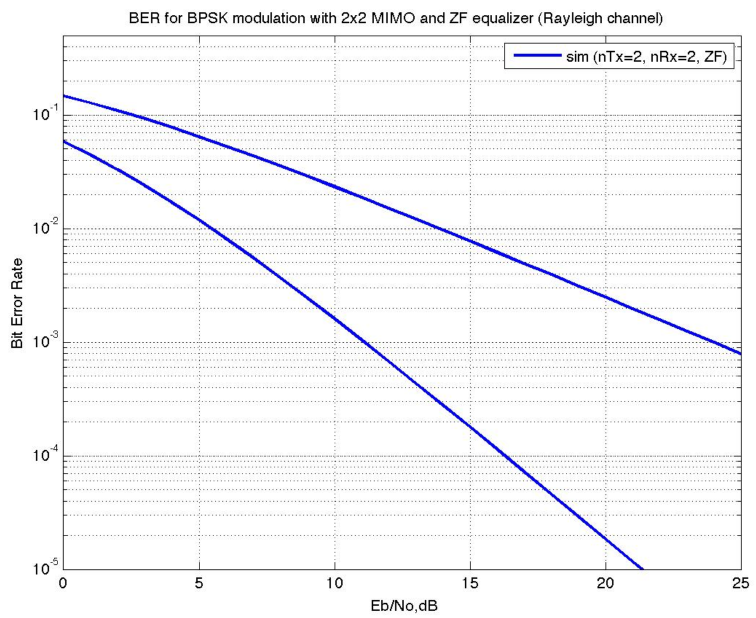

Figure 5.3 Combined spatial multiplexing with Alamouti space-time code

CHAPTER 6

LATTICE SPACE-TIME CODES

In this chapter we will discuss about lattices and how they are related to communication systems and a 2x2 lattice space-time code of highest rank. For all the pervious construction of 2x2 lattice space-time codes with a positive diversity product, the rank r(2) is such that $r(2) \leq 4$. An example of 2x2 lattice space-time code of rank 5 with a positive diversity product was given by Li, later 2x2 lattice space-time code of rank 8 with positive diversity product was given by [6] Yang, Bo and Togbe as an answer to the open problem by Xing and Li. We use this high rank lattice code and decode it such that Alamouti scheme can be applied to it and used for communication over fading channel.

Lattice in general related to number of geometric problems like sphere packing. Sphere covering and the kissing number problem and also other areas of mathematics like number theory, combinatorics. Chemistry has also lot of connections with lattices their 3-D structures has been studied in areas like crystallography and in physics the high dimensional lattices like Leech lattice and E_8 are being studied.

The main application of lattices other than the mathematics and engineering is specifically in communications. In recent years lattice codes have considerable attention because they provide high data rate constellations. The main use of lattice is in the channel coding problem, the design of codes in the bandlimited channel. Lattices are used for the construction of n-dimensional codes for n-dimensional vector quantizers.

Definition: A lattice in R^n is the set of integer linear combinations of a given linear independent set of vectors.

6.1 Lattice and Channel Codes

The theory of classical linear channel codes over finite fields is same as the theory of lattices i.e., sphere packing problem, this questions us how densely a large number of identical spheres can be packed together in n-dimensional space. One of the most pleasing consequence is the Gilbert-varshanov lower bound, which guarantees existence of channel codes with a certain rate given minimum distance and the Minkowski-Hlawka theorem of lattices, which guarantees lattice packing densities in dimensions for $n > 1000$. Following this approach the problem of achieving channel capacity on AWGN channel was solved with lattice encoding and decoding [4].

6.2 Lattice and Space-time Codes

The codes achieving the optimal multiplexing diversity tradeoff in a MIMO environment are the capacity achieving codes, involving lattice constructions. Another use of lattices for MIMO systems in detection, assuming the knowledge about the receiver and the input data vector are taken from the lattice. The data vectors are all processed by the channel matrix so that the received vector which is transformed lattice point can be applied for detection using some lattice reduction techniques [4]. Lattice sphere decoder is one of the efficient decoding algorithm, which repeatedly enumerates all the lattice points inside a sphere of a given radius with the received vector as its center.

Lattice reduction: It is a technique to get a lattice basis that is more orthogonal to original basis. In the decoding process it searches for the nearest lattice point with respect to the received vector, this reduces to simple rounding operation.

6.3 2x2 Lattice Space-time Code of Rank 8

In these recent years plenty of research on space-time codes is going on, for constructing good lattice codes mathematical subjects like number theory, algebra, combinatorics are being used.

Initially a lattice space-time code of rank 5 is proposed by Xing and Li, later the open question of any space-time code higher than the proposed one was solved by [6] Yang, Bo and Togbe, where they showed that there is a lattice space-time code of rank 8 [6].

Assume that the $\mathbb{C}$ denote complex numbers, $\mathbb{R}$ denote real numbers, $\mathbb{Z}$ denote integers, $\mathbb{N}$ denote positive integers and $M_n(\mathbb{C})$ is set of nxn matrices over $\mathbb{C}$. Lattice space-time code is defined in this case as a set **A** of matrices in $M_n(\mathbb{C})$ such that it forms a free abelian group under matrix addition. Then dimension of this group is called the rank of **A**.

6.3.1 Abelian Group

An abelian group, is a commutative group in which the order of the group has no effect if there is any group operation applied to the group. These are generally arithmetic addition of integers. They are named after Abel. This is the one of the concept under the section of abstract algebra and vector space [6].

An abelian group is a set $\mathbf{T}$, with an operation $\bigodot$ that combines *a* and *b* two elements to form a new element denoted $a \bigodot b$. The symbol $\bigodot$ is a general operation can be substituted by any operation which needs to be performed on. To make a abelian group the set and operation ($\mathbf{A}$, $\bigodot$), must satisfy the following five axioms,

1. Inverse element: For each a in **A**, there exists an element *b* in $\mathbf{T}$ such that $a \bigodot b = b \bigodot a = e$, where *e* is the identity element.
2. Identity element: There exists an element *e* in $\mathbf{T}$, such that for all elements *a* in $\mathbf{T}$, the equation $e \bigodot a = a \bigodot e = a$ holds.
3. Associativity: For all a, b and c in $\mathbf{T}$, the equation $(a \bigodot b) \bigodot c = a \bigodot (b \bigodot c)$ holds.
4. Commutativity: For all a, b in $\mathbf{T}$, $a \bigodot b = b \bigodot a$.
5. Closure : For all a, b in $\mathbf{T}$, the result of the operation $a \bigodot b$ is also in $\mathbf{T}$.

Therefore we can tell that in a group where group operation is not commutative is called as non-abelian group or non-commutative group.

The diversity product of **A** is defined as [6]

$$\delta(\mathbf{A}) = \inf\left(|\det(A - B)| : \text{ A,B} \in \mathbf{A}, \text{A} \neq \text{B}\right), \tag{6.1}$$

and the normalized diversity product of **A** is defined as [6]

$$d_g = \frac{(\delta(\mathbf{A}))}{|\det G|.|L|^{n/2}} = \frac{\delta(\mathbf{A})^2}{\sqrt{|\det g}}. \tag{6.2}$$

In the above equation (6.2) we have,

- G is defined as the generating matrix of complex lattice **A**.
- g is real generating matrix for $\bigwedge_G$.
- $|L|$ is the absolute value of determinant of generating matrix of two dimensional real base lattice L.

Criterion:

- The rank **A** should be as large as possible.
- The diversity product should be as large as possible.
- The discriminant Δ should be as small as possible.

The maximal rank r(n) of lattice space-time code is determined as [6]

$$r(n) := \max\left(rank(\mathbf{A}) : \mathbf{A} \text{ is a lattice in } M_n(\mathbb{C}), \delta(\mathbf{A}) > 0\right). \tag{6.3}$$

The upper and lower bounds are given as $2n \leq r(n) \leq 2n^2$ for n = 2 it is $4 \leq r(n) \leq 8$.

6.3.2 Linear Code

Linear code over a finite field with q elements F_q is the linear subspace $C \subset F_q^n$ and the codewords are the vectors which form the subspace. If these codewords are chosen on the criterion of maximum distance then they are called error correcting codes.

If C is a matrix then G is called the generator matrix of it, if its rows can generate all the elements of C. Let

$$G = (g_1, g_2, g_3., ., ., ., ., ., g_k). \tag{6.4}$$

Then each and every code word W of C can be given as

$$W = C_1 g_1 + C_2 g_2 + C_3 g_4 + ... + C_k g_k = CG, \tag{6.5}$$

where,

$$C = (c_1, c_2.., ., ., ., .c_K). \tag{6.6}$$

Therefore G for lattice **A**is given as below

$$G = \begin{pmatrix} G_1 & 0 \\ 0 & -G_2 \end{pmatrix}, \tag{6.7}$$

where,

$$G_1 = \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}. \tag{6.8}$$

$$G_2 = \begin{pmatrix} 1 & -i \\ -1 & -i \end{pmatrix}. \tag{6.9}$$

Consider the eight 2x2 matrices over $\mathbb{C}$

$$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, \tag{6.10}$$

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{6.11}$$

$$A_2 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \tag{6.12}$$

$$A_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{6.13}$$

$$A_4 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \tag{6.14}$$

$$A_5 = \begin{pmatrix} \sqrt{q} & 0 \\ 0 & \sqrt{q} \end{pmatrix}, \tag{6.15}$$

$$A_6 = \begin{pmatrix} \sqrt{q}i & 0 \\ 0 & \sqrt{q}i \end{pmatrix}, \tag{6.16}$$

$$A_7 = \begin{pmatrix} o & \sqrt{q} \\ \sqrt{q} & 0 \end{pmatrix}, \tag{6.17}$$

$$A_8 = \begin{pmatrix} 0 & \sqrt{q}i \\ -\sqrt{q}i & 0 \end{pmatrix}, \tag{6.18}$$

where q is the positive integer and let **A** be the lattice formed by the eight above matrices, hence the rank of **A** is 8 and the diversity product of **A** is 1. We know that the matrices

$$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, \tag{6.19}$$

are linearly independent over $\mathbb{R}$.

$$\mathbf{A} = x_1A_1 + x_2A_2 + x_3A_3 + x_4A_4 + y_1A_5 + y_2A_6 + y_3A_7 + y_4A_8, \tag{6.20}$$

where,

$$(x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4) \neq 0, \tag{6.21}$$

and

$$|det(A)| \geq 1. \tag{6.22}$$

Therefore **A** can be expressed as

$$\mathbf{A} = \begin{pmatrix} x_1 + y_1\sqrt{q} + i(-x_2 + y_2\sqrt{q}) & x_3 + y_3\sqrt{q} + i(-x_4 + y_4\sqrt{q}) \\ -x_3 + y_3\sqrt{q} + i(-x_4 - y_4\sqrt{q}) & x_1 - y_1\sqrt{q} + i(x_2 + y_2\sqrt{q}) \end{pmatrix}. \tag{6.23}$$

Let us define

$$\mathbb{Z}_1 = x_1 + iy_2\sqrt{q} \tag{6.24}$$

$$\mathbb{Z}_2 = x_2 + iy_1\sqrt{q} \tag{6.25}$$

$$\mathbb{Z}_3 = x_3 + iy_4\sqrt{q} \tag{6.26}$$

$$\mathbb{Z}_4 = x_4 - iy_3\sqrt{q}. \tag{6.27}$$

Thus **A** can be represented as below

$$\mathbf{A} = \begin{pmatrix} Z_1 - iZ_2 & Z_3 - iZ_4 \\ -Z_3 - iZ_4 & Z_1 + iZ_2 \end{pmatrix}. \tag{6.28}$$

In the above matrix (6.28) if we consider

$$|a|^2 = |Z_1 - iZ_2|^2 = Z_1^2 + Z_2^2 \tag{6.29}$$

$$|b|^2 = |Z_3 - iZ_4|^2 = Z_3^2 + Z_4^2 \tag{6.30}$$

$$|c|^2 = |-Z_3 - iZ_4|^2 = Z_3^2 + Z_4^2 \tag{6.31}$$

$$|d|^2 = |Z_1 + iZ_2|^2 = Z_1^2 + Z_2^2 \tag{6.32}$$

$$. \tag{6.33}$$

The total energy can be expressed as

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 2(Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2). \tag{6.34}$$

When we observe the lattice **A**, we can tell that it is similar to the Alamouti structure of space-time code, therefore we can apply Alamouti space-time coding scheme for the above lattice. Let us assume

$$k_1 = Z_1 - iZ_2 \tag{6.35}$$

$$k_1^* = Z_1 + iZ_2 \tag{6.36}$$

$$k_2 = Z_3 - iZ_4 \tag{6.37}$$

$$k_2^* = Z_3 + iZ_4. \tag{6.38}$$

The above lattice (6.28) can be modified to the following

$$\mathbf{A} = \begin{pmatrix} k_1 & k_2 \\ -k_2^* & k_1^* \end{pmatrix}. \tag{6.39}$$

Let us send the bits, assuming that the channel is flat fading cannel,

- k_1 and $k_2 \rightarrow$ first time slot $\rightarrow b_1$ and b_2.
- k_1^* and $-k_2^* \rightarrow$ second time slot $\rightarrow b_1^*$ and $-b_2^*$.

According to first time slot

$$a_1^1 = h_{11}b_1 + h_{12}b_2 + n_1^1. \tag{6.40}$$

$$a_2^1 = h_{21}b_1 + h_{22}b_2 + n_2^1. \tag{6.41}$$

$$\begin{pmatrix} a_1^1 \\ a_2^1 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} n_1^1 \\ n_2^1 \end{pmatrix}. \tag{6.42}$$

The channel remains constant and the second time slot

$$a_1^2 = -h_{11}b_2^* + h_{12}b_1^* + n_1^2. \tag{6.43}$$

$$a_2^2 = -h_{21}b_2^* + h_{22}b_1^* + n_2^2. \tag{6.44}$$

$$\begin{pmatrix} a_1^2 \\ a_2^2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} -b_2^* \\ b_1^* \end{pmatrix} + \begin{pmatrix} n_1^2 \\ n_2^2 \end{pmatrix}. \tag{6.45}$$

Therefore the resultant matrix can be can be given as

$$\begin{pmatrix} a_1^1 \\ a_2^1 \\ a_1^{2*} \\ a_2^{2*} \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} n_1^1 \\ n_2^1 \\ n_1^{2*} \\ n_2^{2*} \end{pmatrix}. \tag{6.46}$$

We consider the noise to be i.i.d then the expectation of the noise is given as

$$E\begin{pmatrix} n_1^1 \\ n_2^1 \\ n_1^{2*} \\ n_2^{2*} \end{pmatrix} \begin{pmatrix} n_1^1 & n_2^1 & n_1^{2*} & n_2^{2*} \end{pmatrix} = \begin{pmatrix} |n_1^2 + n_2^1| & 0 \\ 0 & |n_2^1 + n_2^2| \end{pmatrix}. \tag{6.47}$$

Let us assume that the channel matrix H as

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{pmatrix}. \tag{6.48}$$

For a MxN matrix the pseudo inverse is given as

$$H^+ = (H^H H)^{-1} H^H. \tag{6.49}$$

After solving for $(H^H H)$, we obtain the diagonal matrix and the inverse of the matrix is just inverse of the diagonal elements:

$$(H^H H)^{-1} = \begin{pmatrix} 1/|\sum h_{11}^2| + |\sum h_{21}^2| & 0 \\ 0 & 1/|\sum h_{11}^2| + |\sum h_{21}^2| \end{pmatrix}. \tag{6.50}$$

The estimate of the transmitted symbol at the receiver end is given as

$$\begin{pmatrix} \hat{b}_1 \\ b_2^* \end{pmatrix} = (H^H H)^{-1} H^H \begin{pmatrix} a_1^1 \\ a_2^1 \\ a_1^{2*} \\ a_2^{2*} \end{pmatrix}. \tag{6.51}$$

$$\begin{pmatrix} \hat{b}_1 \\ b_2^* \end{pmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + (H^H H)^{-1} H^H \begin{pmatrix} n_1^1 \\ n_2^1 \\ n_1^{2*} \\ n_2^{2*} \end{pmatrix}. \tag{6.52}$$

The estimated symbol is given by (6.52) now we will find out how to get the bit error rate. The instantaneous bit energy to noise ratio at the i^{th} receiver antenna is given as

$$\gamma_i = \frac{|h_i|^2 E_b}{N_0}. \tag{6.53}$$

If the channel is equalized with h^H in the M_r receiving antenna case, the effective bit energy to noise ratio modifies to

$$\gamma = \sum \frac{|h_i|^2 E_b}{N_0}, \tag{6.54}$$

$$\gamma = M_r \gamma_i. \tag{6.55}$$

The effective bit energy to noise ratio in M_r receive antenna case in M_r times the bit energy to noise ratio of single antenna case. If h_i is a Rayleigh distributed random variable then h_i^2 is a chi-squared random variable. The pdf of γ_i is given as

$$p(\gamma_i) = \frac{1}{(E_b/N_0)} e^{-\frac{\gamma_i}{(E_b/N_0)}}, \tag{6.56}$$

and the effective bit energy to noise ratio of γ is sum of M_r random variables, hence the pdf of γ will have M_r degrees of freedom and given by

$$p(\gamma) = \frac{1}{(M_r - 1)!(E_b/N_0)^{M_r}} \gamma^{M_r - 1} e^{-\frac{\gamma}{(E_b/N_0)}}, \gamma \geq 0. \tag{6.57}$$

The bit error rate for BPSK in AWGN, when the bit energy to noise ratio $\frac{E_b}{N_0}$ is given by [14]

$$P_b = \frac{1}{2} erfc(\sqrt{\frac{E_b}{N_0}}). \tag{6.58}$$

When the effective bit energy to noise ratio with maximal ratio combining is given as γ. The total bit error rate is the integral of the conditional bit error rate over all the possible values of γ,

$$P_e = \int \frac{1}{2} erfc(\sqrt{\gamma}) p(\gamma) d\gamma. \tag{6.59}$$

$$P_e = \int \frac{1}{2} erfc(\sqrt{\gamma}) \frac{1}{(M_r - 1)!(E_b/N_0)^{M_r}} \gamma^{M_r - 1} e^{-\frac{\gamma}{(E_b/N_0)}} d\gamma, \tag{6.60}$$

with reference to the section $11.3.1$ performance with maximal ratio combining in [14], the equation (6.60) reduces to following equation

$$P_e = p^2[1 + 2(1 - p)], \tag{6.61}$$

where,

$$p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{2}{E_b/N_0}\right)^{-\frac{1}{2}}. \tag{6.62}$$

Hence the BER for the BPSK modulation in Rayleigh channel with two transmit antenna and one receive antenna case is given by equation (6.61) P_e.

CHAPTER 7

COMBINING LATTICE SPACE-TIME CODES AND SPATIAL MULTIPLEXING

In this chapter we will try to combine the lattice space-time code using Alamouti scheme and try to multiplex over a Rayleigh fading channel, the inspiration in combining these was from the [2] Hilde, David and Nils. Although the combination can be done in time, we try to do it in space as we are trying to switch them based on the instantaneous channel information.

In the paper [7] for the narrow band MIMO channels the agreement between the diversity and multiplexing has been discussed based on the Euclidean distance of the code-book at the receiver by inspecting the union bound on the probability of error. This method is completely different form the regular schemes which examines the performance of the space-time coding schemes for the average probability of error, whereas in this the Euclidean distance approach depends on the instantaneous channel realization, because of this reason we can compare the performance of the multiplexing and diversity as a factor of channel statistics. According to this if the variations in the channel are slow then the feed back path will have low rate.

Although the comparison in the paper [7] revealed that it was for narrow band channels it is also applicable to wide-band fading channels also, at that point of time it was explicit to multivariate channels. Hence we thought of combining and comparing over space.

According to our procedure we consider our space-time code to be the lattice space-time code as discussed in the earlier chapter. We are trying to spatially multiplex various lattice coded blocks, which used the Alamouti code of two symbol groups.

We assume that $N = 2k$ and $k \geq 2$, $M \geq N/2$ and the transmitted block is $\mathbf{b} = [b_0 b_1 ....b_{N-1}]^T$ is transmitted over two symbol durations by forming matrix **B** of size Nx2 from the B_k Alamouti matrices of size Nx2. The best way of assigning the Alamouti blocks to antenna is by choosing **B** in $A = HB + N$ such that

$$\mathbf{B} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ . \\ . \\ . \\ B_{N/2-1} \end{pmatrix}. \tag{7.1}$$

This combination may also be expressed with respect to channel matrix

$$\tilde{A} = \tilde{H}\mathbf{b} + n. \tag{7.2}$$

These Lattice blocks may be assigned to any antenna combination. Now the question arises about the case, where there are multiple antennas then there will be p_N of non-trivially equivalent antennas patterns. Which can be figured as p_k, where $k\epsilon[1, p_N]$. When only two antenna system is used then there is only one pattern possible so there is no complexity involved but when two or more antennas are used then the pattern selection comes to light, for example when $N = 4$ and and $p_4 = 3$ then the patterns possible are shown Fig 5.2. According to these different p_k patterns available the general input output equation is given as

$$\tilde{A} = \tilde{H}(p_k)\mathbf{b} + n. \tag{7.3}$$

For example $H(\tilde{p_2})$ is shown below [2]

$$H(\tilde{p_2}) = \begin{bmatrix} h_{11} & h_{13} & h_{12} & h_{14} \\ -h_{13}^* & h_{11}^* & -h_{14}^* & h_{12}^* \\ h_{21} & h_{23} & h_{22} & h_{24} \\ -h_{23}^* & h_{21}^* & -h_{24}^* & h_{22}^* \\ h_{31} & h_{33} & h_{32} & h_{34} \\ -h_{33}^* & h_{31}^* & -h_{34}^* & h_{32}^* \\ h_{41} & h_{43} & h_{42} & h_{44} \\ -h_{43}^* & h_{41}^* & -h_{44}^* & h_{42}^* \end{bmatrix}. \tag{7.4}$$

7.1 Instantaneous Pattern Optimization

The problem of choosing the best pattern can be solved by finding out the optimal antenna groups over which the independent Alamouti blocks will be multiplexed. The performance of this can be improved by choosing the pattern p_{k_0} in detecting **B** according to the above mentioned equation (7.1) and the equivalent patterns will be as below [2]

$$p_{k_0} = \arg(\max_{p_k}(\lambda_{\min}^2(\tilde{H}(p_k)^H \tilde{H}(p_k)))). \tag{7.5}$$

As assumed the transmitter will have instantaneous channel information through feedback from the receiver, the best pattern can be found by using the correlation based on long term statistics. Therefore the optimization is based on average behavior of the channel. Applying the singular value decomposition to above equation (7.5) we obtain

$$p_{k_0} = \arg(\max_{p_k}[\lambda_{\min}^2(\tilde{H}(p_k)^H \tilde{H}(p_k)]). \tag{7.6}$$

By increasing the minimum eigen value of the following term

$$(\tilde{H}(p_k)^H \tilde{H}(p_k), \tag{7.7}$$

and replacing the above term (7.7) with its average the equation (7.6) changes to

$$p_{k_0} = \arg(\max_{p_k}[\lambda^2_{\min}(E(\tilde{H}(p_k)^H \tilde{H}(p_k))]). \tag{7.8}$$

$\tilde{H}_{p_k}$ will have both transmit and receive correlation, but the received correlation does not affect the transmit pattern, hence it can be ignored here. In this situation we will define $\tilde{R}_t(p_k)$, a simple function of transmit correlation matrix R_t and the transmit pattern p_k

$$\tilde{R}_t(p_k) = E(\tilde{H}^H(p_k)\tilde{H}(p_k)), \tag{7.9}$$

therefore the equation (7.8) can be rewritten to the following equation

$$p_{k_0} = \arg\max_{p_k}[\lambda_{\min}(\tilde{R}_t(p_k)]. \tag{7.10}$$

CHAPTER 8

SIMULATIONS

In this chapter we will examine the BER and SER performance of combined lattice space-time codes and spatial multiplexing and compare them with the previous cases of space-time codes, where there is no multiplexing.

For the performance criterion we assume that uniform array of antennas are used, the BER results are taken over thousands of different independent channel realizations, and zero forcing detection is used at the receiver. For the Alamouti scheme we used 8-PSK modulation scheme and for the lattice space-time code we used BPSK as we are comparing symbol error rate performance so we just want to make sure that they are compared on a same platform. In lattice space-time code the transmission matrix (6.39) has 2 bits in each symbol, by using BPSK modulation we will send in total 8 bits per symbol. Therefore for the simulation to keep up the comparison with Alamouti scheme we use 8-PSK modulation.

We can simulate the symbol error rates by calculating the bit error rate using the following equation

$$E_s = E_b \log_2 M. \tag{8.1}$$

In our simulation $MATLAB^{TM}$ release 2008b was used as simulation tool in this thesis. The components modeled using our simulation includes complex and real square orthogonal design based on BPSK and 8-PSK constellations. There are 2 transmitting antennas and a single receiving antenna. All the figures show the BER performance of all the transmission modes. There are 4 curves in each plot

- Alamouti space-time code using 8-PSK.
- Alamouti combined with spatial multiplexing scheme.
- Lattice space-time code of highest rank.
- Combined lattice space-time code and spatial multiplexing.

As observed from the simulated results, for any ratio of energy per bit to spectral noise density (E_b/N_0), the fixed rate Alamouti code is outperformed by the lattice space-time code which in turn outperformed by combination of Alamouti and spatial multiplexing scheme. The best performance is achieved by using the combined lattice and spatial multiplexing transmission.

In Fig. 8.1, Fig. 8.2, and Fig. 8.3, we have 4 different graphs showing the bit error rates. We can tell that the combined lattice and spatial multiplexing scheme outperformed the other three schemes namely Alamouti, Lattice space-time code and Alamouti combined with multiplexing. The graph was plotted with number of bits $N = 10^4$ and up to 20dB. We could not get the complete graph hence we plotted the results till 25dB. In this case the BER of combined scheme has better performance than that of Alamouti with spatial multiplexing but by what amount we are unable to conclude hence we simulated the graph till 25dB. For the simulation of second and third figures Fig. 8.2 and Fig. 8.3 we increased the number of bits for the simulation to $N = 10^5$ and $N = 10^6$ and we could observe the performance of the combined scheme has increased.

From the SER simulation curve Fig. 8.4, we can observe that the lattice combined scheme is 3dB better performance than Alamouti combined scheme. For figures Fig. 8.5 and Fig. 8.6 we have increased the number of bits and plotted the simulations up to 25dB. We observed that the SER of combined lattice and multiplexing also has performance improvement over the other schemes.

However the performance improvement of combined scheme of lattice space-time code and spatial multiplexing was at the cost of decoding time. i.e., the decoder takes more time to decode the lattice space-time code. Although the decoding time is more but it is still practical to implement, the lattice in equation (6.28) has Alamouti structure but it has complex bits hence the decoding will take some more time when compared to simple Alamouti's structure of equation (3.4). Therefore the decoding time for Alamouti's scheme, spatial multiplexing and the combined scheme of Alamouti and spatial multiplexing is less when compared to the decoding time of combined scheme of lattice space-time code and spatial multiplexing.

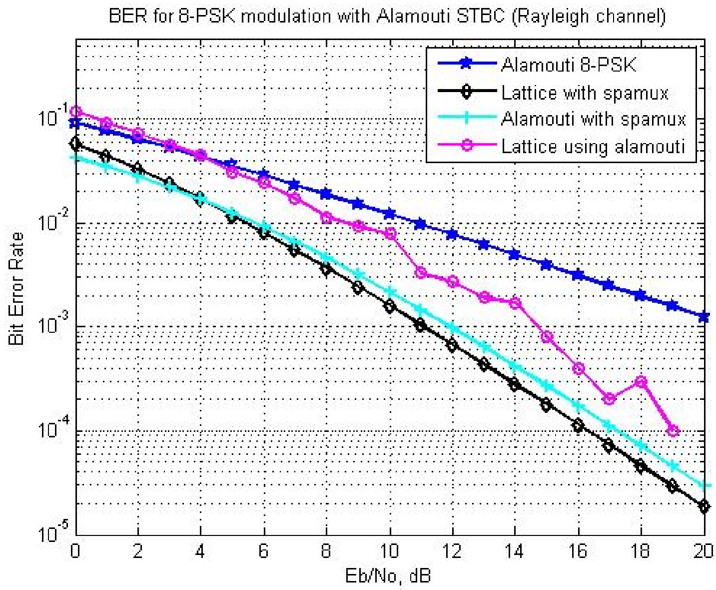

Figure 8.1 Simulation results of BER for $N = 10^4$ and SNR up to 20dB

Observing the graphs we can tell that the bit error rate and symbol error rate of the combined system of lattice space-time code and multiplexing are better than the other systems by approximately 3dB.

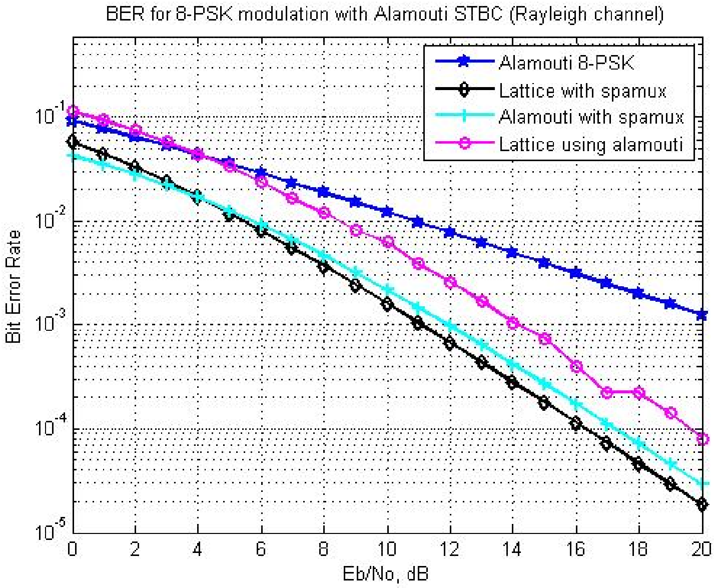

Figure 8.2 Simulation results of BER for $N = 10^5$ and SNR up to 20dB

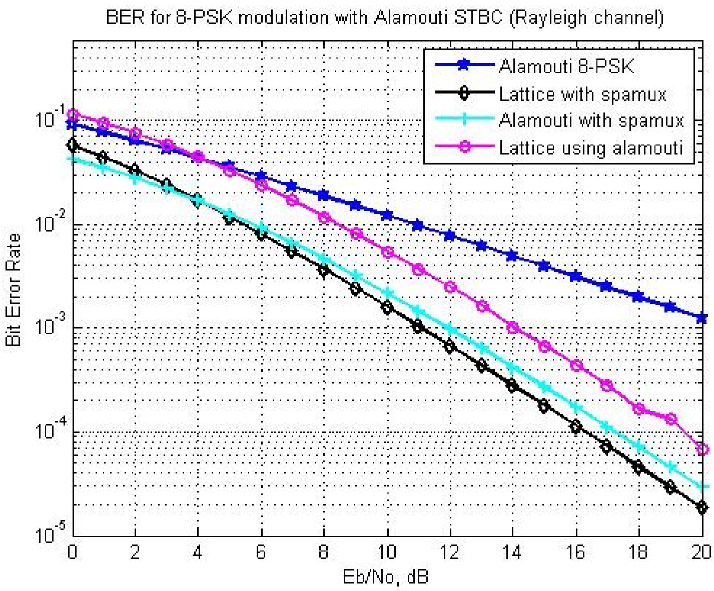

Figure 8.3 Simulation results of BER for $N = 10^6$ and SNR up to 20dB

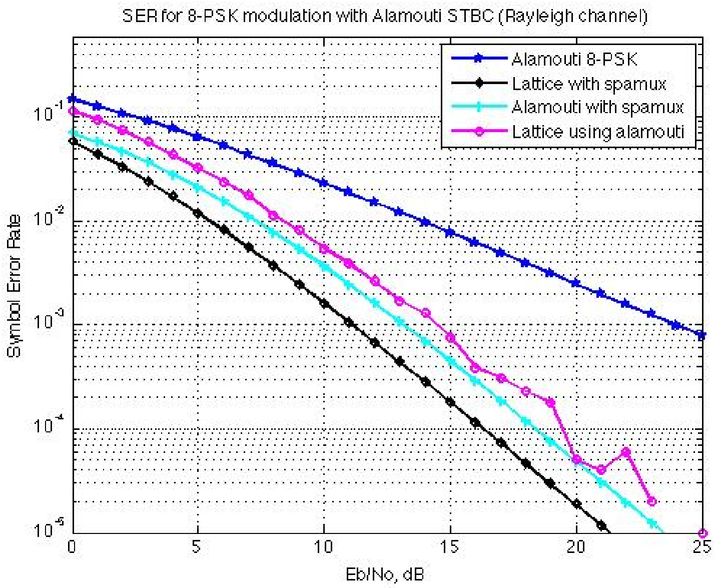

Figure 8.4 Simulation results of SER for $N = 10^4$ and SNR up to 25dB

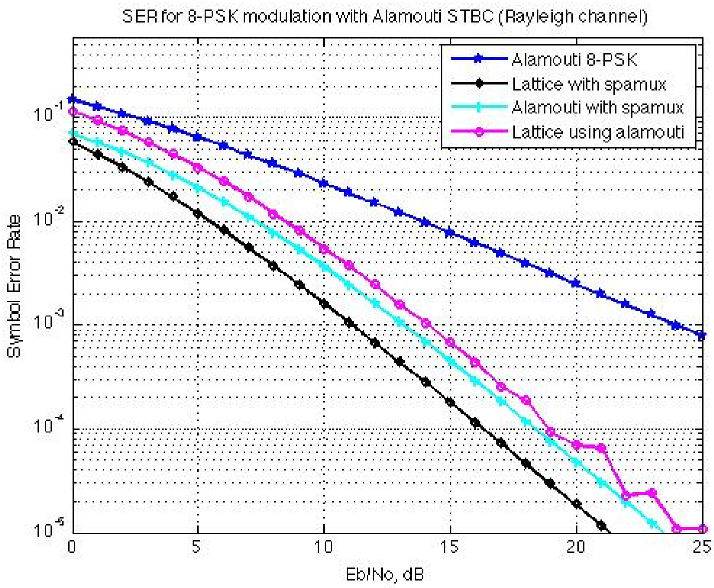

Figure 8.5 Simulation results of SER for $N = 10^5$ and SNR up to 25dB

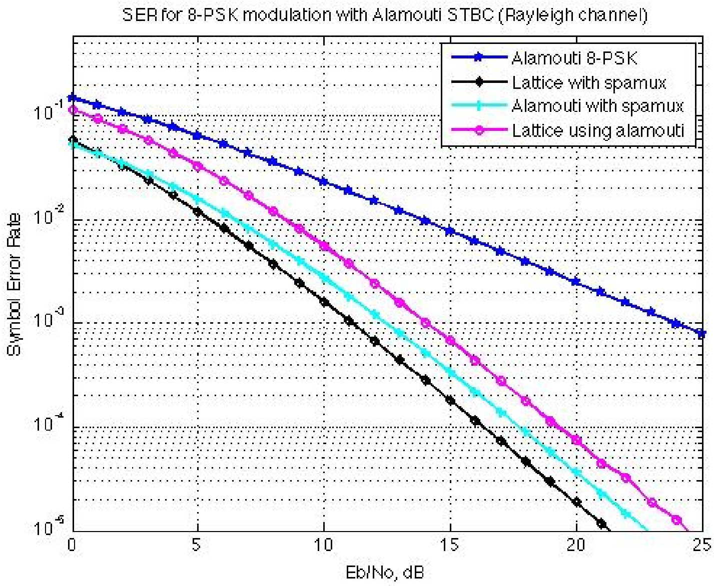

Figure 8.6 Simulation results of SER for $N = 10^6$ and SNR up to 25dB

CHAPTER 9

CONCLUSIONS

One of the main achievement of this thesis is in developing a transmission scheme that combines lattice space-time codes and spatial multiplexing through a simple block code antenna assignment scheme. The model illustrates significant potential improvement over regular space-time codes by using space dimensions of the signal. Simulation results showed an increase in capacity by an amount of 3dB for the integrated lattice space-time codes and multiplexing system over other three systems namely Alamouti, lattice space-time code and Alamouti combined with multiplexing. However, for the lattice space-time code and multiplexing combination the decoder takes more time than the combination of Alamouti's scheme and spatial multiplexing.

In this work we have implemented the proposed transmission scheme on only one receiver antenna system. This transmission scheme can be extended to multi-antenna system with multiple antennas at receiver and at transmitter for enhancing the bit error rate. In general, if the number of antennas are increased in a MIMO system, the pattern optimization has to be given more consideration. In this thesis, we have discussed the issue of choosing the best pattern available in multi-antenna system. We selected only a specific lattice code of highest rank through which this transmission scheme can be applied to any lattice space-time code which can be combined with spatial multiplexing for improving performance. Hence this transmission scheme yields much better performance on multi-antenna systems.

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