Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 185 (2020) 104964

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

More apples vs. better apples: Distribution and innovation-driven growth

Robert F. Kane [a,1], Pietro F. Peretto [b,*,2]

[a] *International University of Japan, Japan*
[b] *Duke University, United States of America*

Received 24 January 2018; final version received 17 October 2019; accepted 22 October 2019
Available online 25 October 2019

Abstract

We model distribution, the delivery of goods to customers, as an activity governed by its own technology and undertaken by firms subsequently to production. We then use the model to investigate how distribution shapes innovation-driven economic growth. We contrast two canonical specifications of distribution costs, iceberg vs. per-unit. The per-unit cost implies that factory-specific productivity improvements cannot sustain steady-state growth. Quality improvement, instead, raises the services that customers obtain from each unit of the good so that firms can increase the volume of services without increasing the volume of shipments. Unless technological advancements allow the distribution cost to fall to zero, quantity growth must cease and growth must be driven by quality improvement. More generally, the ratio of distribution to manufacturing unit costs must be constant in steady state. The iceberg cost delivers this property by assumption. The per-unit distribution cost, instead, yields an endogenous structure of the costs of serving the market.

JEL classification: E10; L16; O31; O40

Keywords: Endogenous growth; Distribution; Product innovation; Process innovation; Quality; Productivity

* Corresponding author.
E-mail addresses: kane@iuj.ac.jp (R.F. Kane), peretto@econ.duke.edu (P.F. Peretto).
[1] Department of Economics, International University of Japan, Minamiuonuma, Niigata 949-7248, Japan.
[2] Department of Economics, Duke University, Durham, NC 27708.

https://doi.org/10.1016/j.jet.2019.104964

1. Introduction

Distribution, defined as the delivery of goods to customers, is an essential component of economic activity. This fact supports the conventional wisdom that infrastructure—in particular the construction, maintenance and improvement of transportation and communication networks—is important for economic performance. Consider the following quote:

> "A high quality transportation network is vital to a top performing economy. Investments by previous generations of Americans—from the Erie Canal in 1807, to the Transcontinental Railroad in 1869, to the Interstate Highway System in the 1950s and 1960s—were instrumental in putting the country on a path for sustained economic growth, productivity increases, an unrivaled national market for good and services, and international competitiveness. But today, current estimates indicate that America's transportation infrastructure is not keeping pace with demands or the needs of our growing economy, for today or for future generations."
> [An Economic Analysis of Transportation Infrastructure Investment, The White House, 2014, p. 2]

Such emphasis on infrastructure is typical in policy-oriented documents and the media: it reflects the common-sense idea that the cost of moving physical objects and information from one place to the other is a crucial driver of economic performance.

The empirical evidence on the role of infrastructure is, nevertheless, far from clear. Fogel (1964), for instance, estimates that the removal of all railroads in 1890 would have only reduced the GNP of the United State in the same year by 2.7%. Even though recent work (Donaldson and Hornbeck, 2016) increases this estimate to 3.22%, the effect remains small.[3] One plausible reason for such small effects is that this literature assumes that railroads had no effect on the rate and direction of technical change. We argue that questioning this assumption and studying the relationship between distribution costs and innovation is essential to improve our understanding of the role that infrastructure, shipping, retail and so on, play in shaping economic growth.

To our knowledge, growth economics has not considered the topic. The closest literature that we could find is that on the role of public capital, pioneered by Barro (1990), which is consistent with the evidence that infrastructure raises the level of economic activity (Aschauer, 1989).[4] This literature, however, models infrastructure as public capital that enters the aggregate production function and thus fails to disentangle the production of goods (manufacturing) from the delivery of goods to the customer (distribution). We argue that to make progress we need models that disentangle the two stages. Empirically, distribution is a large part of the economy. Burstein et al. (2003), for example, show that distribution costs account for over 40% of the retail price of a typical consumer good in the United States and over 60% in Argentina. Yet, although distribution has received substantial attention in international economics, it has been neglected in growth economics.

Our goal in this paper is to shed light on the relationship between growth dynamics and distribution. We build an R&D-driven growth model that incorporates distribution and features three types of innovation: cost reduction, quality improvement, and variety expansion. The framework allows us to make progress in two complementary dimensions.

[3] Similarly, Allen and Arkolakis (2014) estimates that the introduction of the interstate highway system increased welfare between 1.1% and 1.4%.

[4] See also Turnovsky (1996) and Chatterjee and Turnovsky (2012).

First, we model distribution as a distinct economic activity and analyze its interaction with endogenous innovation. While an improvement in the distribution technology has no direct impact on the production process, it affects the incentives to engage in R&D because it reduces the cost of delivering goods to customers. We show that this disentanglement of production from distribution has important implications for how one ought to model distribution costs and for how to discipline the theory with data.

Second, in our framework distribution discriminates among the different types of innovation and thus has *qualitative* effects on how the economy grows. For example, if the distribution technology contains a constant per-unit component, cost reduction must eventually cease and is thus not an engine of steady-state growth. The intuition is that as manufacturing productivity rises, the distribution cost becomes the dominant driver of the product's price. Consequently, cost reduction becomes less effective at increasing demand and eventually the rate of return to cost reduction becomes so low that it is unprofitable to engage in it. More generally, in our framework the steady-state rate of cost reduction cannot exceed the rate of technical progress in distribution. Quality innovation, on the other hand, is not subject to this mechanism and can thus be an engine of steady-state growth. Since this is a key insight of our analysis, it is worth reviewing it in some detail.

It is commonly held that cost reduction and quality improvement are isomorphic. The classic exposition is due to Spence:

> "Suppose that products deliver services to consumers. Let s be the services and $P(s)$ be the inverse demand. Services are delivered through goods. Let x be the quantity of goods, and $c(x)$ be the cost function. Let $f(q)$ be the quantity of services per unit of the good. Then $s = f(q)x$, and the cost of delivering services s is $c(s/f(q))$. If $f'(q) > 0$, and q is raised through R&D of the product development kind, then the effect is to reduce the costs of the service. Thus formally this kind of product development is equivalent to cost reduction."
>
> [Spence, 1984, p. 101]

In our framework this isomorphism breaks down. The reason why is best illustrated with an example. The cost of shipping a Motorola DynaTAC (one of the first commercially available cellphones) is roughly the same as the cost of shipping the latest smartphone.[5] However, by every measure the latest smartphone delivers more services. Moving from the example to economic theory, quality improvement is not just a change that allows the supplier to produce a good that delivers a larger volume of services to the customer at the same manufacturing cost, it is also a change that reduces the "effective distribution cost", defined as the cost of delivering one unit of service to the consumer. Though cost reduction reduces the manufacturing cost, it fails to reduce the effective distribution cost. It is only when distribution is absent that the distinction vanishes and the Spence isomorphism applies.

The difference between cost reducing R&D and quality improving R&D is related to a policy issue that predates endogenous growth theory. The President's Commission on Industrial Competitiveness once stated: "It does us little good to design state-of-the-art products, if within a short time our foreign competitors can manufacture them more cheaply" (The White House, 1985). Japanese manufacturers, who at the time were the main competitors to American firms, did engage in more cost reduction. Mansfield (1988) found that American firms devoted two-thirds of

[5] If anything, modern cellphones are lighter and can be delivered more cheaply than their predecessors.

their R&D expenditures to product innovation while Japanese firms only devoted one-third, with the remaining R&D expenditures going to cost reduction. Our model provides an explanation for the differing R&D expenditure shares; the transitional dynamics are such that as economy's develop, they engage in relatively less cost reduction.

Our model is related to a literature, thus far focused only on international trade, that explores the differences between productivity and product quality. Sutton (2007a) shows that when there are internationally traded materials, there is a minimum level of quality that must be attained if a firm or country is to enter world export markets, but no minimum productivity. Hallak and Sivadasan (2013) also break the isomorphism. They do so by assuming that iceberg costs are a declining function of product quality. Because of this assumption, firms producing high quality goods are more likely to export than firms producing low quality goods. The purpose and mechanism of our paper is different from, though highly complementary to, the trade models of Sutton (2007a) and Hallak and Sivadasan (2013). Sutton (2007a) and Hallak and Sivadasan (2013) only allow for a "one time" increase in product quality; in both models firms cannot engage in R&D to improve manufacturing productivity. We allow firms to engage in both quality improvement and cost reduction and study the resulting long-run dynamics.

In addition to providing results on the difference between quantity and quality, our model sheds new light on the difference between iceberg and per-unit frictions. Indeed, the paper's title pays homage to the *Alchian-Allen Hypothesis*, which states that the introduction of a per-unit (tariff or shipping) cost reduces the relative price of expensive goods (Alchian and Allen, 1964). Building on that work, Sørensen (2014) and Irarrazabal et al. (2015) show that increases in per-unit frictions lead to higher welfare losses compared to increases in iceberg frictions in a heterogeneous trade model. We argue that the iceberg specification implicitly assumes that the technologies for manufacturing and distribution are identical, whereas the per-unit formulation breaks the linkage between the two.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 solves for the rates of return to all R&D activities. Section 4 solves the model. Section 5 presents the transitional dynamics and analyzes the effects of changes to the distribution technology. Section 6 extends the model to include endogenous innovation in distribution. Finally, section 7 concludes.

2. The model

We extend the framework of Peretto and Connolly (2007). We consider a closed economy with a continuum of goods. Labor is the only physical resource. Each good is produced by a single firm. Firms engage in R&D to improve labor productivity and product quality. Entrepreneurs engage in R&D to create new goods and then set up new firms to serve the market. We call this activity entry. Time is continuous. All variables are functions of time but we omit the time argument unless necessary to avoid confusion.

2.1. Households

The economy is populated by a representative household with standard Benthamite lifetime utility

$$U = \int_0^\infty e^{-\rho t} \mathrm{L}(t) \ln C(t)\, dt, \quad \rho > 0 \tag{1}$$

where ρ is the rate of time preference, $L(t) = L(0)e^{\lambda t}$ is the mass of identical household members, λ is the population growth rate, and

$$C = \left[\int_0^N \left(Q(\omega)^\theta \frac{X(\omega)}{L} \right)^{\frac{\epsilon-1}{\epsilon}} d\omega \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1, \theta \geq 0 \tag{2}$$

is an index yielding the instantaneous utility flow that each household member obtains from differentiated consumption goods. In this index, N is the mass of available goods, $X(\omega)$ is the household's purchase of good ω which has quality $Q(\omega)$, ϵ is the elasticity of substitution between goods and θ governs quality's effectiveness in increasing utility.

In this environment households supply inelastically their entire labor endowment, L, and face the flow budget constraint

$$\dot{A} = wL + rA - LE, \tag{3}$$

where A is asset holdings, w is the wage rate (it is also the numeraire and is henceforth normalized to 1), L is labor supply, r is the interest rate and $LE \equiv \int_0^N p(\omega) X(\omega) d\omega$ is household expenditure. Consumers maximize lifetime utility subject to (3). The maximization yields the saving plan

$$\frac{\dot{E}}{E} = r - \rho \tag{4}$$

and the demand curve for good ω

$$X(\omega) = LE \frac{p(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega}. \tag{5}$$

2.2. Production technology

The typical firm produces with the technology

$$L_X(\omega) = Z(\omega)^{-\sigma} X(\omega) + \phi, \quad \phi > 0, 1 > \sigma > 0, \tag{6}$$

where $L_X(\omega)$ is the total amount of labor required to produce $X(\omega)$ units of good ω. It consists of the per-unit component $Z(\omega)^{-\sigma} X(\omega)$ and the fixed component ϕ. The per-unit cost of production is a function of firm-specific knowledge $Z(\omega)$ with constant elasticity σ.

2.3. Distribution technology

In line with the representation of production, we restrict attention to a distribution technology with labor as the single input and assume in-house distribution; we can incorporate a separate competitive market for distribution services without changing the results. We posit the following distribution technology

$$L_D(\omega) = \left[\tau Z(\omega)^{-\sigma} + (1+\tau) s \right] X(\omega), \tag{7}$$

where $L_D(\omega)$ is the amount of labor required to distribute $X(\omega)$ units of good ω to the consumer. Accordingly, the distribution cost per unit sold consists of an iceberg component and a per-unit component. The iceberg component consists of the cost of producing the additional amount of the good needed to make up for the anticipated melting during delivery and is thus $\tau Z(\omega)^{-\sigma}$. Because it depends on the manufacturing knowledge of the firm, this iceberg component is *endogenous*. The per-unit component, denoted s, is instead exogenous. Overall, therefore, the cost of shipping a good is the sum of the cost of producing the anticipated melted amount, $\tau Z(\omega)^{-\sigma}$, and the per-unit cost of delivering the (unmelted) good and the anticipated melted amount, $(1+\tau)s$.

The iceberg component is well known and originally introduced by Samuelson:

> I now propose to come directly to grips with transport costs. The simplest assumption is the following: To carry each good across the ocean you must pay some of the good itself. Rather than set up elaborate models of a merchant marine, invisible items, etc., we can achieve our purpose by assuming that just as only a fraction of ice exported reaches its destination as unmelted ice, so will a_x and a_y be the fractions of exports X and Y that respectively reach the other country as imports. Of course, $a_x < 1$ and $a_y < 1$.
>
> [Samuelson (1954, p. 268)]

While this assumption captures in reduced form that shipping goods is costly, it entangles the manufacturing and distribution technologies, because it assumes that manufacturing and distribution use the same factors with identical intensities. Consequently, when a firm introduces a new production method the unit cost of distribution falls in proportion to the unit cost of production. This is a strong assumption that, perhaps not surprisingly, is solidly rejected by the data. Hummels and Skiba (2004) and Irarrazabal et al. (2015), for example, find strong evidence for the per-unit formulation in which the cost of shipping a good is independent of its cost of production.

We allow for both the iceberg and the per unit components to study their different implications. It is useful to contrast the cases $s=0$, $\tau>0$ and $s>0$, $\tau=0$. When the distribution technology consists only of the iceberg cost, an increase in manufacturing productivity reduces the labor needed to produce the extra quantity that melts in transit, thereby reducing the cost of shipping. When the distribution technology consists only of the per-unit cost, in contrast, a reduction in production cost does not entail a reduction in distribution cost.

The key to this formulation is that production and distribution are separate activities, each one with its own technology. The iceberg component allows for a "spillover" from innovation in manufacturing to distribution, but such spillover is not the whole story. Our per-unit component entails an incompressible cost of delivery that has qualitatively important effects of the composition of innovation. "Incompressible" here means that the firm's effort in improving labor productivity in the factory cannot drive to zero the price of the good for the customer — as in traditional endogenous technological change models — because such price contains the per unit distribution cost which does not fall proportionally to the manufacturing cost. To build intuition gradually, we first develop the analysis under the simple assumption that the per-unit cost is exogenous and constant. We then extend it by first letting the per-unit cost decay at a constant rate (Subsection 4.6.2) and then by modeling it as fully endogenous due to innovation (Section 6), in a manner similar to our treatment of the production cost and product quality.

2.4. Innovation technology

There are three sources of innovation: cost reduction, quality improvement and new product creation. For the first two we posit the following technologies:

$$\dot{Z}(\omega) = \alpha_Z Z L_Z(\omega), \quad Z = \int_0^N Z(\omega)\, d\omega / N; \tag{8}$$

$$\dot{Q}(\omega) = \alpha_Q Q L_Q(\omega), \quad Q = \int_0^N Q(\omega)\, d\omega / N. \tag{9}$$

Firms hire labor, $L_Z(\omega)$ and $L_Q(\omega)$ respectively, to engage in cost-reducing and quality-improving R&D. The efficiency of labor in cost reducing R&D is $\alpha_Z Z$; the efficiency of labor in quality improving R&D is $\alpha_Q Q$.[6] The R&D technologies are qualitatively identical. This is intentional: we want cost reduction and quality improvement as similar as possible so that differences in investment in each arise solely from the distribution cost channel.

Entrepreneurs hire labor to create new goods and set up firms to serve the market. The mass of firms/products evolves according to

$$\dot{N} = \left(\frac{\beta N}{LE}\right) L_N - \delta N, \quad \delta > 0, \tag{10}$$

where the exogenous death rate, δ, avoids asymmetric dynamics and hysteresis due to sunk entry costs, unnecessary complications that would distract from the main point of the paper. According to this formulation, L_N is the amount of labor devoted to gross entry and $\beta N/LE$ is the efficiency of labor in creating new products/firms. Consequently, $LE/\beta N$ is the cost of creating a new product/firm.[7]

3. Behavior of firms

In this section we characterize firm behavior and the resulting rates of return to innovation and entry. Unless of interest, we relegate all derivations to the appendix. We highlight that firms' pricing decisions capture the role of the distribution cost.

3.1. Vertical innovation

The typical firm maximizes the present discounted value of the net profit,

$$V(\omega, 0) = \int_0^\infty e^{-\int_0^t [r(s)+\delta] ds} \left[\Pi(\omega, t) - \phi - L_Z(\omega, t) - L_Q(\omega, t)\right] dt, \tag{11}$$

subject to the R&D technologies (8) and (9), where

$$\Pi(\omega) = X(\omega)\left[p(\omega) - (1+\tau)\left[Z(\omega)^{-\sigma} + s\right]\right], \tag{12}$$

[6] This specification assumes that the stock of public knowledge is the weighted sum of firm-specific knowledge stocks. Peretto and Smulders (2002) provides the micro-foundations for this spillover function.

[7] See Peretto and Connolly (2007) for a discussion of alternative specifications of entry costs that deliver the same qualitative results. More importantly, they show that if the cost of entry does not scale with market size it eventually vanishes when there is population growth. They argue that the linear scaling that we adopt here is the simplest way to write a sensible and tractable model that retains a role for entry costs as the market grows large. Recently Bollard et al. (2016) provided evidence that, as their paper's title says, "entry costs rise with development". Thus, the assumption in the text not only is analytically convenient but it is also supported by the data.

is the gross profit of the firm, defined as revenues minus variable production costs. The firm's maximization yields the following price and rates of return to innovation.

Lemma 1. *Consider the typical good ω. Let $\eta_Z(\omega)$ and $\eta_Q(\omega)$ denote, respectively, the elasticity of gross profit with respect to cost-reducing knowledge and the elasticity of gross profit with respect to quality-enhancing knowledge. Then:*

$$\frac{\partial log \Pi(\omega)}{\partial log Z(\omega)} = (\epsilon - 1)\sigma \frac{Z(\omega)^{-\sigma}}{Z(\omega)^{-\sigma} + s}; \tag{13}$$

$$\frac{\partial log \Pi(\omega)}{\partial log Q(\omega)} = (\epsilon - 1)\theta. \tag{14}$$

The prices and the rates of return to cost reduction and quality improvement are, respectively:

$$p(\omega) = \frac{\epsilon(1+\tau)}{\epsilon - 1}\left(Z(\omega)^{-\sigma} + s\right); \tag{15}$$

$$r = r_Z(\omega) \equiv \alpha_Z(\epsilon - 1)\sigma\left(\frac{Z(\omega)^{-\sigma}}{Z(\omega)^{-\sigma} + s}\right)\left(\frac{Z}{Z(\omega)}\right)\Pi(\omega) - \delta - \frac{\dot{Z}}{Z}; \tag{16}$$

$$r = r_Q(\omega) \equiv \alpha_Q(\epsilon - 1)\theta\left(\frac{Q}{Q(\omega)}\right)\Pi(\omega) - \delta - \frac{\dot{Q}}{Q}. \tag{17}$$

Note that in equilibrium the rates of return equal the interest rate, r.

Proof. See the appendix.

The pricing decision (15) and equation (5) allow us to rewrite gross profit as

$$\Pi(\omega) = \frac{LE}{\epsilon}\frac{\left(\frac{p(\omega)}{Q(\omega)^{\theta}}\right)^{1-\epsilon}}{\int_0^N \left(\frac{p(\omega)}{Q(\omega)^{\theta}}\right)^{1-\epsilon} d\omega} = \frac{LE}{\epsilon}\frac{\left(\frac{Z(\omega)^{-\sigma}+s}{Q(\omega)^{\theta}}\right)^{1-\epsilon}}{\int_0^N \left(\frac{Z(\omega)^{-\sigma}+s}{Q(\omega)^{\theta}}\right)^{1-\epsilon} d\omega}, \tag{18}$$

where LE is the size of the market, $1/\epsilon$ is the profit rate and the last term is the firm's market share. The first equality says that the firm engages in both cost reduction and quality improvement because they both reduce the quality-adjusted price of the good, $p(\omega)/Q(\omega)^{\theta}$, and thereby yield higher profit. The second equality highlights the novelty of our model: it says that manufacturing productivity and product quality affect the gross profit through different mechanisms that show up in the expressions for the elasticities in Lemma 1. The per-unit distribution cost, s, yields that as $Z(\omega)$ rises, the elasticity of profit with respect to manufacturing productivity falls. In contrast, the elasticity of profit with respect to quality is invariant with respect to quality. Remember the Spence (1984) isomorphism. In principle, we can think of cost reduction and quality improvement as effectively identical ways of delivering to the consumer a larger flow of services. However, our model says that while higher manufacturing productivity reduces the cost of *production*, it does not reduce the per-unit cost of *delivery*. Consequently, the cost of delivering the higher flow of services does not fall one for one with process innovation. An increase in quality, in contrast, does not require a larger volume of shipment to deliver the higher volume of services. Consequently, the cost of delivering the higher flow of services does fall one for one with quality innovation.

Spence (1984) is correct that cost reduction is similar to quality improvement because both reduce the cost of producing services. But only quality improvement allows the cost of delivering a given service to fall to zero. Equation (13) demonstrates the importance of modeling distribution flexibly. When $s = 0$, the distribution technology is identical to the manufacturing technology. Because of the resulting entanglement, technical progress in manufacturing automatically allows the cost of delivering services to fall to zero. When $s > 0$, instead, the distribution technology is not identical to the manufacturing technology and contains an incompressible component. As manufacturing productivity rises, the weight of the manufacturing cost in governing the price declines and in the limit vanishes, leaving only the distribution component. Quality improvement does not suffer from this limitation because it does not require the replication of physical units to deliver more services to the consumer.

3.2. Horizontal innovation

Given the entry technology (10), the free-entry condition is

$$V = LE/\beta N \tag{19}$$

where V is the lifetime value of the typical firm defined in (11). This condition says that entrants anticipate that once in operation they will run the firm efficiently and thus that the benefit from entry is the maximized value of the firm to be created. Because any agent in the economy can raise resources from the household and start a firm, in equilibrium the value created must equal the cost of creating it. Differentiating (11) with respect to time yields the rate of return to entry

$$r = r_N \equiv \frac{\pi}{V} + \frac{\dot{V}}{V} - \delta = \left(\frac{\beta N}{LE}\right)\pi + \frac{\dot{E}}{E} + \lambda - \frac{\dot{N}}{N} - \delta, \tag{20}$$

where the net profit is $\pi = \Pi - \phi - L_Z - L_Q$. As for the returns to firm-level innovation, in equilibrium the rate of return to entry also equals the interest rate, r.

4. General equilibrium

This section presents the model's general equilibrium dynamics. Having characterized households' and firms' decisions, we derive the resulting allocation of labor to production, distribution and innovation and the associated general equilibrium of the economy. The equilibrium is symmetric because all firms make identical decisions and have the same productivity and product quality. We can thus drop the firm-level argument ω. For clarity, we present the main components of the equilibrium system separately and then bring them all together to characterize the equilibrium path. Also, to isolate the role of the novel elements of the model, namely, the role of transportation costs, we first suppress population growth (we set $\lambda = 0$) and turn it on again in Subsection 4.6 where we extend the model to time-varying transportation costs.

4.1. Expenditure and interest rate

In this framework financial assets are ownership shares of firms so that in equilibrium $A = NV$. This result combined with the household budget constraint (3), the free entry condition (19) and the simplifying assumption $\lambda = 0$ yields that expenditure per capita and the interest rate are constant at all points in time:

$$E(t) = E^* = \frac{\beta}{\beta - \rho} \qquad \text{and} \qquad r(t) = \rho. \tag{21}$$

This property makes the model very tractable. Note that the restriction imposed later in the paper, (27), to guarantee existence of the equilibrium path implies $\beta > \rho$ so that $E^* > 0$.

4.2. *Innovation rates*

The main challenge in building the equilibrium system is to take into account the non-negativity constraints on L_Z, L_Q and L_N. We begin with vertical innovation and then characterize horizontal innovation. The elasticity of profit with respect to manufacturing knowledge, discussed in the previous section, plays such an important role that in the following we denote it $\eta(Z, s) = (\epsilon - 1)\sigma(1 + Z^\sigma s)^{-1}$.

4.2.1. *Vertical innovation: cost reduction and quality improvement*

In symmetric equilibrium, equations (13) and (16) combined with $r = \rho$ yields the growth rate of manufacturing productivity, $\sigma\frac{\dot{Z}}{Z}$, where

$$\frac{\dot{Z}}{Z} = \begin{cases} \frac{LE^*}{\epsilon N}\alpha_Z \eta(Z, s) - \rho - \delta & N < \frac{LE^*\alpha_Z\eta(Z,s)}{\epsilon(\rho+\delta)} \equiv \bar{N}_Z(Z) \\ 0 & N \geq \bar{N}_Z(Z) \end{cases}. \tag{22}$$

Following similar steps yields the growth rate of quality

$$\frac{\dot{Q}}{Q} = \begin{cases} \frac{LE^*}{\epsilon N}\alpha_Q(\epsilon - 1)\theta - \rho - \delta & N < \frac{LE^*\alpha_Q(\epsilon-1)\theta}{\epsilon(\rho+\delta)} \equiv \bar{N}_Q \\ 0 & N \geq \bar{N}_Q \end{cases}. \tag{23}$$

Recall that $\eta(Z, s)$ and $(\epsilon - 1)\theta$ are, respectively, the elasticities of gross profit with respect to cost-reducing and quality-enhancing knowledge; see equations (13)-(14). As is standard in this class of models, the growth rates (22) and (23) are decreasing in N.

The core of our mechanism is that as manufacturing productivity rises relative to the economy's ability to transport goods, the elasticity $\eta(Z, s)$ falls, dragging down investment in, and thus the growth rate of, manufacturing productivity. The reason is that the marginal cost of production becomes less important in the determination of the price of goods. Consequently, each reduction in the marginal cost of production results in a smaller price reduction, a smaller movement along the demand curve and a smaller gain in gross profits. Accordingly, firms engage in less and less cost-reducing R&D.

Both (22) and (23) have cutoffs of the mass of firms above which firms engage in zero R&D. These cutoffs follow from the property that both rates of return to vertical innovation (cost reduction and quality improvement) are decreasing in N and can thus fall below the household discount rate. In the case of cost reduction the presence of the distribution cost yields that the cutoff is a function of the productivity level and thus it identifies the boundary of a region in (Z, N) space. We denote this boundary $\bar{N}_Z(Z)$ and note that it is decreasing in Z. In the case of quality improvement the cutoff is a value of the mass of firms independent of Q and Z. In each case, if the mass of firms is above the cutoffs, the non-negativity constraint on R&D is binding and firms set R&D expenditure, L_Z or L_Q, to zero. The mechanism is that if the market becomes too saturated with firms, firms do not engage in R&D because the rate of return is not large enough to meet the household reservation interest rate on saving.

4.2.2. *Horizontal innovation and net entry-exit*

We now turn to horizontal innovation. Substituting the profits into (20) and using $r = \rho$ yields the net entry rate

$$\frac{\dot{N}}{N} = \left(\frac{\beta N}{LE^*}\right)\left(\frac{LE^*}{\epsilon N} - \phi - L_Z - L_Q\right) - \rho - \delta. \tag{24}$$

Substituting (8), (9), (22), (23), in (24), and taking into consideration the corner solutions $L_Z = 0$ and $L_Q = 0$, yields

$$\frac{\dot{N}}{N} = \begin{cases} \frac{\beta[1-(\epsilon-1)\theta-\eta(Z,s)]-\epsilon(\rho+\delta)}{\epsilon} - \left(\frac{\beta N}{LE^*}\right)\left(\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}\right) & N < min\left\{\bar{N}_Q, \bar{N}_Z(Z)\right\} \\ \frac{\beta[1-(\epsilon-1)\theta]-\epsilon(\rho+\delta)}{\epsilon} - \left(\frac{\beta N}{LE^*}\right)\left(\phi - \frac{\rho+\delta}{\alpha_Q}\right) & \bar{N}_Z(Z) < N < \bar{N}_Q \\ -\delta & \tilde{N}_N(Z) < N \end{cases}, \tag{25}$$

where

$$\bar{N}_N(Z) = \begin{cases} \frac{LE^*}{\epsilon}\left(\frac{1-(\epsilon-1)\theta-\eta(Z,s)-\epsilon(\rho+\delta)/\beta}{\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}}\right) & N < \bar{N}_Z(Z) \\ \frac{LE^*}{\epsilon}\left(\frac{1-(\epsilon-1)\theta-\epsilon(\rho+\delta)/\beta}{\phi - \frac{\rho+\delta}{\alpha_Q}}\right) & \bar{N}_Z(Z) \le N \end{cases} \tag{26}$$

is the $\dot{N} = 0$ locus and the inequality $\tilde{N}_N(Z) < N$ identifies the boundary of the zero gross entry region. The expression for the boundary is cumbersome and relegated to the appendix to streamline the exposition. The main benefit of the exit shock is that the steady state is in the region with positive gross entry.

We impose the following parameter restrictions

$$1 - (\epsilon - 1)\theta - (\epsilon - 1)\sigma > \epsilon(\rho + \delta)/\beta, \tag{27}$$

and

$$\phi > \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z\alpha_Q} \tag{28}$$

The first restriction ensures that when the number of firms/goods is low enough, regardless of $\eta(Z, s)$, there is positive net entry. The second restriction ensures that the rate of entry is decreasing in N.

There exists a cutoff such that when the mass of firms is too large, net entry falls to zero and such cutoff yields the boundary of a region in (Z, N) space. In the first branch of (25) all three forms of R&D are active. In the second branch quality improvement and variety expansion are both active, but cost reduction is not. In the third branch gross entry is zero and the dynamics of the mass of firms/products are determined by the exogenous exit of firms.

The rate of net entry in (24) depends on the elasticities $\eta(Z, s)$ and $(\epsilon - 1)\theta$ because R&D expenditures are an endogenous fixed sunk cost that reduces net profits and thereby the incentive for entry. This insight is well known: it was developed originally by Sutton (2007b) in partial-equilibrium IO models and incorporated in endogenous growth theory by Peretto (1996). Our model adds the novel feature that, for given mass of firms, N, the *level* of cost-reducing knowledge, Z, affects positively the rate of entry, $\frac{\dot{N}}{N}$. The mechanism is as follows. When manufacturing productivity rises *relative to* the distribution cost, the elasticity $\eta(Z, s)$ falls and firms *reduce* R&D investment in cost reduction. Consequently, the gross profit per firm rises and the rate of entry rises. The empirical prediction, therefore, is that holding constant the mass of firms the rate of entry is *positively* correlated to the average *level* of manufacturing productivity. It would be wrong, however, to interpret such positive correlation as entry driving (i.e., causing) higher productivity. Note, moreover, that throughout the above reasoning we held the mass of firms, N, constant. Therefore, the prediction is that along the transition path, for given N, the economy features a negative co-movement between average manufacturing productivity and the rate of entry. As we show below, accounting for the dynamics of N reveals that the market share effect, the term $1/N$ in (24), dominates so that the rate of entry decreases throughout the transition and the co-movement between the rate of entry and manufacturing productivity is negative. As for the prediction that holds N constant, it would be wrong to interpret such negative correlation as rising productivity deterring entry (i.e., erecting barriers to entry) since, as we argued, rising productivity reduces firms' R&D expenditure and thus encourages entry.

4.3. Interlude: the iceberg distribution cost does not matter

A glaring feature of the key components of the general equilibrium system is that the iceberg transportation cost, $\tau Z^{-\sigma}$, is absent from all equations. The reason is that, as some readers might have already noticed, the iceberg component is not present in the reduced-form profit in equation (12). It is not present because it enters the pricing decision in equation (15) through the multiplicative term $(1+\tau)$. Specifically, an increase in τ causes the prices of all goods to rise by the same proportion, leaving the firm's market share and thus its profit flow unchanged since the firm's market share is a function homogeneous of degree zero in all prices.

Due to the Alchian-Allen effect (Alchian and Allen, 1964), instead, the per-unit component, s, affects the relative price of a good based on the firm's relative productivity. Consequently an increase in s changes the firm's market share and thus its profit. In the following analysis, therefore, we discuss only the role of the per-unit component of the distribution cost because the iceberg component has no interesting effects on the economy's equilibrium dynamics.

4.4. Equilibrium dynamics

The model allows for multiple combinations of cost reduction, quality improvement and variety expansion. The main result is that cost reduction eventually shuts down while net entry goes to zero, meaning that variety expansion also ceases and gross entry simply replaces firms that exit due to the exogenous death shock.

We study the model's dynamics in the phase diagram in (Z, N) space in Fig. 1. A helpful property is that both Z and N are pre-determined state variables so that we can characterize trajectories as the solution of a partial differential equation subject to initial conditions only. Specifically, let $N = T(Z; Z_0, N_0)$ denote a trajectory starting from initial condition (Z_0, N_0). We then have:

$$\begin{aligned} &T(Z; Z_0, N_0) \\ &= argsolve \left\{ \frac{dN}{dZ} = \frac{\frac{\beta(1-\eta(Z,s)-(\epsilon-1)\theta)-\epsilon(\rho+\delta)}{\epsilon} - \left(\frac{\beta N}{LE^*}\right)\left(\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}\right)}{\frac{LE^*\alpha_Z\eta(Z,s)}{\epsilon N} - \rho - \delta} \frac{N}{Z} \right\}. \end{aligned} \tag{29}$$

Throughout the analysis we impose the following parameter restrictions

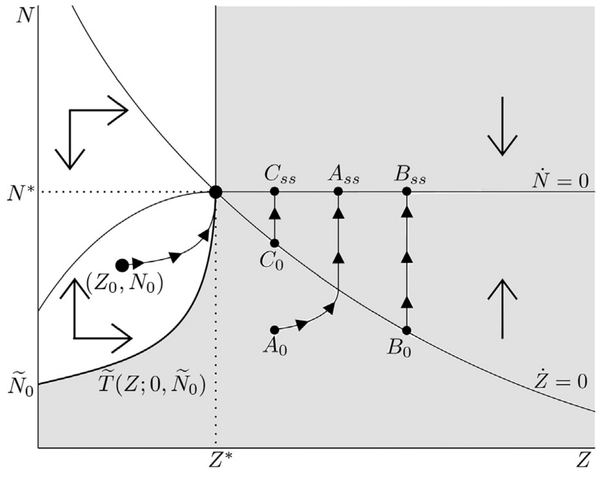


Fig. 1. Hysteresis (shaded) and non-hysteresis (unshaded) zones.

$$Min\left\{\alpha_Q, \alpha_Z\right\} \frac{\theta\left(\epsilon-1\right)\left(\phi-\frac{\rho+\delta}{\alpha_Q}\right)}{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta} > \rho+\delta, \tag{30}$$

which ensures that the cutoff value $\bar{N}_Q$ is larger than $\bar{N}_N\left(Z\right)$ and that the $\dot{Z}=0$ and $\dot{N}=0$ loci intersect.[8]

Proposition 1 *(Convergence dynamics). There exists a "watershed" trajectory $N=\widetilde{T}(Z;\widetilde{Z}_0, \widetilde{N}_0)$ that divides the state space in two regions. Such trajectory can start either from the vertical axis, i.e., $(0, \widetilde{N}_0)$ with $\widetilde{N}_0 \geq 0$, or from the horizontal axis, i.e., $(\widetilde{Z}_0, 0)$ with $\widetilde{Z}_0 \geq 0$. For initial conditions $0 \leq Z_0 \leq Z^*$ and $N_0 \geq \widetilde{T}(Z;\widetilde{Z}_0, \widetilde{N}_0)$* the economy converges to the steady state (Z^*, N^*). For initial conditions $0 \leq Z_0 \leq Z^*$ *and* $N_0 < \widetilde{T}(Z;\widetilde{Z}_0, \widetilde{N}_0)$ or $Z_0 \geq Z^*$ *the economy converges to the steady state (Z_{ss}, N^*). In both cases, the steady-state mass of firms is*

$$N^* = \frac{LE^*}{\epsilon} \frac{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta}{\phi-\frac{\rho+\delta}{\alpha_Q}} \tag{31}$$

and the steady-state dynamic of quality is

$$Q^*\left(t\right) = Q_{ss}e^{gt},\ g \equiv \frac{\theta\left(\epsilon-1\right)\left(\alpha_Q\phi-\rho-\delta\right)}{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta} - \rho - \delta,\ Q_{ss} \equiv F\left(N_0, Z_0, Q_0\right). \tag{32}$$

Trajectories that converge to (Z^*, N^*) yield the constant manufacturing knowledge level

$$Z^* = \left\{\frac{1}{s}\left[\frac{\sigma\left(\epsilon-1\right)\alpha_Z\left(\phi-\frac{\rho+\delta}{\alpha_Q}\right)}{\left(\rho+\delta\right)\left[1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta\right]}-1\right]\right\}^{1/\sigma}. \tag{33}$$

[8] Specifically, the parameter restriction ensures that $\bar{N}_Z\left(0\right) > \bar{N}_N\left(0\right)$.

Trajectories that exhibit path dependence and converge to (Z_{ss}, N^*) *yield the constant manufacturing knowledge level*

$$Z_{ss} = argsolve\left\{\frac{LE^*\alpha_Z}{\epsilon(\rho+\delta)}\eta(Z,s) = N(Z; Z_0, N_0)\right\} > Z^*. \tag{34}$$

Proof. See the appendix for a discussion.

Our first result, thus, concerns the dynamics. Fig. 1 shows two classes of trajectories: those that do not intersect the boundary $\bar{N}_Z(Z)$ in finite time to the right of the steady-state point (Z^*, N^*) and those that do. The former converge to the steady state (Z^*, N^*) and exhibit independence on initial conditions. The latter have the property that the economy shuts down cost-reducing R&D *before* reaching the steady state. This event locks in the level of manufacturing knowledge achieved at such point in time. We denote such value of cost-reducing knowledge Z_{ss}. According to the diagram and equation (34) such level of manufacturing knowledge depends on the initial conditions. These trajectories, therefore, exhibit path-dependence. The watershed trajectory $\widetilde{T}(Z; \widetilde{Z}_0, \widetilde{N}_0)$ divides the state space into the basin of attraction of the steady state (Z^*, N^*) and the basin of attraction of the hysteresis region. In all cases the economy ends up with steady-state mass of firms N^* and zero cost-reducing R&D.

The second result, therefore, is that in steady state firms do not invest in cost-reducing R&D and devote all R&D expenditure to quality improvement. The reason is that $\lim_{Z\to\infty}\eta(Z,s) = 0$: holding the distribution technology constant, as manufacturing productivity increases, the effectiveness of cost-reducing R&D decreases and in the limit cost-reducing R&D has no impact on profits because the price of goods is determined solely by the distribution technology.

Next, we consider the steady-state *level* of manufacturing productivity. By construction, trajectories that converge to the steady state (Z^*, N^*) have the property that manufacturing productivity, $(Z^*)^\sigma$, is inversely proportional to the per-unit distribution cost, s, with factor of proportionality that depends on technology and preference parameters. Interestingly, manufacturing productivity does not depend on market size, LE^*.

Fig. 1 illustrates the implications of path dependence for manufacturing productivity by contrasting three different paths. The first is path A with initial condition $A_0 = \left(Z_0^A, N_0^A\right)$. Both manufacturing knowledge, Z, and the mass of firms, N, initially increase. When the economy enters the $\dot{Z} = 0$ region and firms shut down cost-reducing R&D, the level of manufacturing knowledge is $Z_{ss}(Z_0^A, N_0^A)$. The mass of firms continues to grow until reaching the steady-state value N_{ss}, which is invariant to the initial condition. To illustrate how the level Z_{ss} depends on $\left(Z_0^A, N_0^A\right)$, we construct paths B and C. The sole difference between the initial conditions $B_0 = \left(Z_0^B, N_0^A\right)$ and $A_0 = \left(Z_0^A, N_0^A\right)$ is the level of manufacturing knowledge, which in path B is so large that, given the mass of firms, cost-reduction is not profitable. Therefore, path B locks in Z_0^B immediately, while path A exhibits a period of cost-reducing R&D and achieves $Z_{ss}(Z_0^A, N_0^A)$. Similarly, the sole difference between the initial conditions $C_0 = \left(Z_0^A, N_0^C\right)$ and $A_0 = \left(Z_0^A, N_0^A\right)$ is the initial mass of firms, which in path C is so large that, given cost-reducing knowledge, cost-reduction is not profitable. Therefore, path C locks in Z_0^C immediately, while path A achieves $Z_{ss}(Z_0^A, N_0^A)$. Clearly, the level of manufacturing knowledge that the economy locks in depends on the shape of the path in the region below the boundary $\bar{N}_Z(Z)$. We cannot solve analytically for such path but the qualitative characterization based on the phase diagram is sufficient to extract the key results.

Changes in parameters that shift up the boundary $\bar{N}_Z$, like for example larger market size LE^*, yield higher manufacturing productivity. The reason is that, holding everything else constant, such changes delay the shutting down of cost-reducing R&D.

We now turn to the relationship between initial conditions and quality. Compare paths A and B. Path A features smaller initial Z_0 and thus larger initial value of the elasticity $\eta(Z, s)$. Consequently, it features firms that devote a larger amount of resources to cost-reducing R&D. Because of such larger expenditures, the rate of entry is lower and thus the growth rate of quality decreases more slowly. This means that path A intersects the boundary $\bar{N}_Z(Z)$ with a larger value of quality than path B. Formally, the value Q_{ss} in equation (32) in the proposition is *decreasing* in Z_0. This implies that a larger initial value Z_0 shifts down the steady-state path $Q^*(t)$. Accordingly, even though the steady-state growth rate of quality is independent of initial conditions, the specific path followed by the economy in reaching the boundary $\bar{N}_Z(Z)$ has a permanent "imprinting" effect on the future evolution of the quality *level*. An analogous argument applies to the initial mass of firms, N_0. In the proposition we use the notation $Q_{ss} \equiv F(N_0, Z_0, Q_0)$ to account for this property.

4.5. *The endogenous structure of costs*

Another result worth highlighting is the following. When the economy follows a path-dependent trajectory and locks in the value of cost-reducing knowledge, Z_{ss}, it locks in the value of the elasticity $\eta(Z_{ss}, s)$. Consequently, it locks in the ratio

$$\frac{Z^{-\sigma} + \tau Z^{-\sigma} + (1+\tau)s}{Z^{-\sigma}} = (1+\tau)\left(1 + Z_{ss}^{\sigma} s\right). \tag{35}$$

This is the ratio of the per unit variable cost of serving the market (production plus distribution) relative to the per unit variable cost of production. Through the value Z_{ss}, this ratio depends on a rich set of fundamentals, including the per unit transportation cost, s, and, most importantly, parameters regulating R&D behavior and market size, LE^*. When the economy converges to the steady state (Z^*, N^*), the ratio of total cost per unit to cost of production per unit is

$$(1+\tau)\left[1 + \left(Z^*\right)^{\sigma} s\right] = (1+\tau)\frac{\sigma(\epsilon - 1)\alpha_Z\left(\phi - \frac{\rho+\delta}{\alpha_Q}\right)}{(\rho+\delta)\left[1 - \theta(\epsilon - 1) - \epsilon(\rho+\delta)/\beta\right]} \tag{36}$$

and it depends on a smaller set of fundamentals than in the case with path dependence. For example, it does not depend on the per unit cost, s, or the size of the market, LE^*.

Aside from the differences between the two classes of steady states, an interesting interpretation of the model is that the per unit cost of distribution, s, anchors the entire cost structure of the firm—production plus distribution—forcing the variable cost of production to converge to a finite value. More generally, the per-unit cost determines the overall structure of the cost of serving the customer. Although we focus on a single-industry economy for simplicity, in the sense that we represent the economy as a single monopolistically competitive sector, it is not hard to generalize our model to multiple sectors. The main result would be that the ratio of distribution cost to manufacturing cost in a given sector depends on the R&D efficiencies α_Z and α_Q and many other sector-specific parameters. Our model thus provides a structure capable of determining endogenously characteristics that are typically treated as exogenous. To further emphasize this point, note that if we set $s = 0$, and thus adopt the traditional specification with only the iceberg component of the distribution cost, we obtain the well-known property that the relative cost of delivering goods to the consumer is $(1+\tau)$. Moreover, manufacturing knowledge would grow forever at a constant exponential rate.

An interesting difference between the two classes of steady state is that the steady state with no path dependence yields a cost structure (production plus distribution) that sterilizes the effects of parameters that operate through scale. As said, although we focus on a single industry the model generalizes to multiple sectors and thus this property produces sector-specific cost structures that do not depend on sectoral size.

4.6. A convenient complementary representation of the dynamics

The core of the mechanism characterized above is the endogenous adjustment of the elasticity $\eta(Z,s)$. In both cases, the convergence to the steady state has the property that the economy stabilizes the rate of return r, which requires stabilizing $\eta(Z,s)$. Inspecting the relevant expressions suggests that we use the *ratio of the per-unit cost of delivery to the per-unit variable cost of production*,

$$\chi \equiv \frac{s}{Z^{-\sigma}}, \tag{37}$$

in place of Z as a state variable. Specifically, the elasticity $\eta(\chi) = \sigma(\epsilon-1)(1+\chi)^{-1}$ is constant when χ is constant. Moreover, as is well known, this class of models sterilizes the scale effect and thus allows for population growth (see, e.g., Peretto (1998)). We thus now turn on population growth, $\lambda > 0$, and following Peretto and Connolly (2007) work with the state variable

$$n \equiv \frac{N}{L}. \tag{38}$$

We now have $E^* = \beta/(\beta+\lambda-\rho)$, with the restriction $\beta+\lambda>\rho$, and rewrite the equilibrium system as:

$$\frac{\dot{\chi}}{\chi} = \begin{cases} \sigma\left[\frac{E^*\alpha_Z\sigma(\epsilon-1)}{\epsilon n}(1+\chi)^{-1} - \rho - \delta\right] + \frac{\dot{s}}{s} & n < \bar{n}_Z(\chi) \\ \frac{\dot{s}}{s} & n > \bar{n}_Z(\chi) \end{cases}; \tag{39}$$

$$\frac{\dot{n}}{n} = \begin{cases} \frac{\beta\left(1-\theta(\epsilon-1)-\sigma(\epsilon-1)(1+\chi)^{-1}\right)-\epsilon(\rho+\delta+\lambda)}{\epsilon} - n\beta\left(\frac{\phi-\frac{\left(\alpha_Z+\alpha_Q\right)(\rho+\delta)}{\alpha_Z\alpha_Q}}{E^*}\right) & n < \bar{n}_Z(\chi) \\ \frac{\beta(1-\theta(\epsilon-1))-\epsilon(\rho+\delta+\lambda)}{\epsilon} - n\beta\left(\frac{\phi-\frac{\rho+\delta}{\alpha_Q}}{E^*}\right) & \bar{n}_Z(\chi) < n \\ -\delta-\lambda & \tilde{n}_N(\chi) < n \end{cases} \tag{40}$$

where $\bar{n}_Z(\chi) = \bar{N}_Z(\chi)/L$ and $\tilde{n}_N(\chi) = \tilde{N}_N(\chi)/L$.

We can now easily allow for time-variation in s to obtain a flexible tool to study the effects of changes to the distribution technology. Specifically, we set $s(t) = s_0 e^{-\varsigma t}$, with $s_0 > 0$ and $\varsigma \geq 0$. We analyze separately the cases $\varsigma = 0$ and $\varsigma > 0$. The first case allows us to do comparative dynamics exercises for permanent changes in the per-unit distribution cost. The second case allows us to do comparative dynamics exercises for permanent changes in the (implied) rate of technological change in distribution and, furthermore, delivers novel properties.

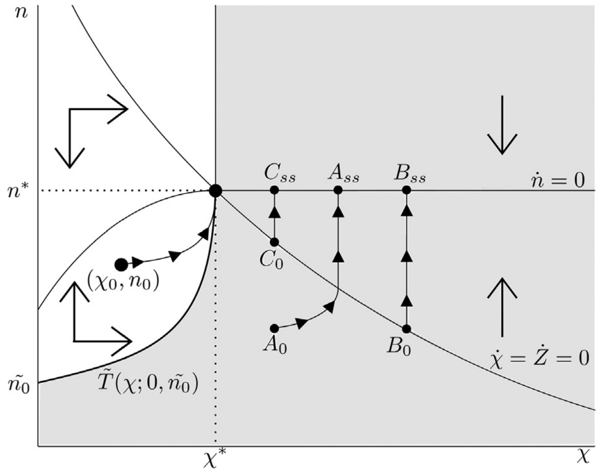


Fig. 2. Dynamics with $\dot{s}/s = 0$.

4.6.1. *Constant distribution cost*

With $\varsigma = 0$ equations (39) and (40) yield the steady-state loci:

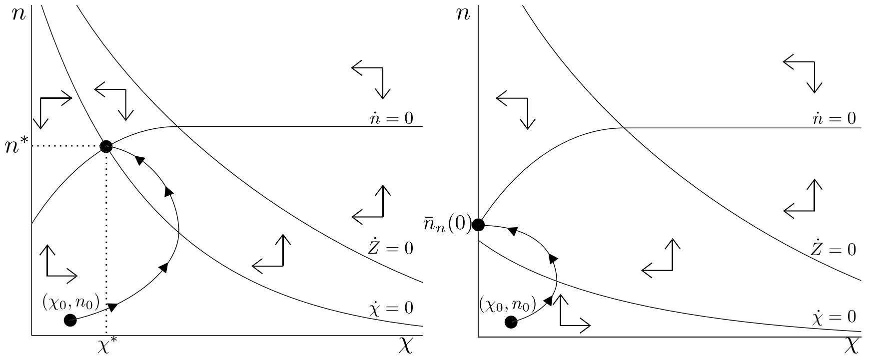
$$\dot{\chi} = 0: \ \bar{n}_\chi(\chi) = \frac{E^*}{\epsilon}\left(\frac{\alpha_Z \sigma(\epsilon - 1)(1+\chi)^{-1}}{\rho + \delta}\right); \tag{41}$$

$$\dot{n} = 0: \ \bar{n}_n(\chi) = \begin{cases} \frac{E^*}{\epsilon}\left(\frac{1-(\epsilon-1)\theta - \sigma(\epsilon-1)(1+\chi)^{-1} - \epsilon(\rho+\delta+\lambda)/\beta}{\phi - \frac{(\alpha_Z + \alpha_Q)(\rho+\delta)}{\alpha_Z \alpha_Q}}\right) & n < \bar{n}_Z(\chi) \\ \frac{E^*}{\epsilon}\left(\frac{1-(\epsilon-1)\theta - \epsilon(\rho+\delta+\lambda)/\beta}{\phi - \frac{\rho+\delta}{\alpha_Q}}\right) & \bar{n}_Z(\chi) \le n \end{cases}. \tag{42}$$

The advantage of this representation is that the relevant loci are independent of the per-unit distribution cost. Therefore a permanent fall in s_0 is a displacement of the variable χ from its steady state value along the line $n = n^*$. This property yields the first comparative-dynamics result of note: path dependence yields the possibility of zero effect of the lower distribution cost. Specifically, while an economy at steady state (χ^*, n^*) responds to a marginal fall in s_0, an economy in steady state (χ_{ss}, n^*) inside the hysteresis zone does not respond unless the change in s_0 is sufficiently large to yield an initial vale of η outside the zone. To be precise, the change in s_0 must be such that $|d\chi/ds_0| > \chi^* - \chi_{ss}$. Once the fall in the per-unit distribution cost triggers action, the dynamics replicates the properties discussed in Proposition 1. Accordingly, the dynamics in Fig. 2 replicate those in Fig. 1.

4.6.2. *Constant rate of decay of the distribution cost*

With exogenous technical progress in distribution there are two potential steady states: $\chi > 0$ and $\chi = 0$. The first occurs when the unit cost of manufacturing falls at the same rate as the distribution cost. The second occurs when the unit cost of manufacturing decreases slower than the unit cost of distribution and consequently the relative cost of distribution asymptotically goes to zero. It's useful to define



(a) Case 1: $K > \frac{\varsigma}{\sigma}$ (b) Case 2: $\frac{\varsigma}{\sigma} > K$

Fig. 3. Dynamics with $\dot{s}/s = -\varsigma$.

$$K \equiv \frac{\alpha_Z \sigma (\epsilon - 1) \left(\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q} \right)}{1 - \theta (\epsilon - 1) - \sigma (\epsilon - 1) - \epsilon (\rho + \delta + \lambda) / \beta} - \rho - \delta > 0. \tag{43}$$

Our next proposition shows that the stability of each steady state depends on the relative size of K. Fig. 3 illustrates the dynamics.

Proposition 2. *Assume constant, exponential technical progress in distribution such that $s(t) = s_0 e^{-\varsigma t}$, with $s_0, \varsigma > 0$.*

Case 1. For $K > \frac{\varsigma}{\sigma}$ the steady state $(0, \bar{n}_n(0))$ is unstable. Therefore, given initial condition (χ_0, n_0) the economy converges to (χ^*, n^*) where:

$$\chi^* = \frac{\frac{\alpha_Z (\epsilon - 1) \sigma \left(\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q} \right)}{\rho + \delta + \frac{\varsigma}{\sigma}} - (1 - \theta (\epsilon - 1) - \sigma (\epsilon - 1) - \epsilon (\rho + \delta + \lambda) / \beta)}{1 - (\epsilon - 1) \theta - \epsilon (\rho + \delta + \lambda) / \beta}; \tag{44}$$

$$n^* = \frac{E^*}{\epsilon} \left(\frac{1 - (\epsilon - 1) \theta - (\epsilon - 1) \sigma (1 + \chi^*)^{-1} - \epsilon (\rho + \delta + \lambda) / \beta}{\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q}} \right). \tag{45}$$

This steady state exhibits sustained manufacturing productivity growth at rate

$$\left(\frac{\dot{Z}}{Z} \right)^* = \frac{\varsigma}{\sigma} \tag{46}$$

and quality growth at rate

$$\left(\frac{\dot{Q}}{Q} \right)^* = \frac{\alpha_Q (\epsilon - 1) \theta \left(\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q} \right)}{1 - (\epsilon - 1) \theta - (\epsilon - 1) \sigma (1 + \chi^*)^{-1} - \epsilon (\rho + \delta + \lambda) / \beta} - \rho - \delta. \tag{47}$$

Case 2. For $\frac{\varsigma}{\sigma} > K$ the steady state $(0, \bar{n}_n(0))$ is unique and globally stable. Given initial condition (χ_0, n_0) the economy converges to $(0, \bar{n}_n(0))$, where:

$$\bar{n}_n(0) \equiv n^* = \frac{E^*}{\epsilon}\left(\frac{1-(\epsilon-1)\theta-(\epsilon-1)\sigma-\epsilon(\rho+\delta+\lambda)/\beta}{\phi-\frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}}\right). \tag{48}$$

This steady state exhibits sustained manufacturing productivity growth at rate

$$\left(\frac{\dot{Z}}{Z}\right)^* = \frac{\alpha_Z\sigma(\epsilon-1)\left(\phi-\frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}\right)}{1-(\epsilon-1)\theta-\sigma(\epsilon-1)-\epsilon(\rho+\delta+\lambda)/\beta}-\rho-\delta \tag{49}$$

and quality growth at rate

$$\left(\frac{\dot{Q}}{Q}\right)^* = \frac{\alpha_Q(\epsilon-1)\theta\left(\phi-\frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z\alpha_Q}\right)}{1-(\epsilon-1)\theta-(\epsilon-1)\sigma-\epsilon(\rho+\delta+\lambda)/\beta}-\rho-\delta. \tag{50}$$

Proof. See the appendix.

This result highlights yet another aspect of the connection between manufacturing and distribution. The rate of technical progress in distribution provides a potential upper bound (a speed limit, so to speak) to the growth rate of manufacturing productivity. To understand the differences between both cases, its useful to analyze the loci $\bar{n}_\chi(\chi)$ and $\bar{n}_n(\chi)$ again. Because of the technical progress in distribution, we have:

$$\dot{Z}=0: \ \bar{n}_Z(\chi) = \frac{E^*\alpha_Z\sigma(\epsilon-1)(1+\chi)^{-1}}{\epsilon(\rho+\delta)};$$

$$\dot{\chi}=0: \ \bar{n}_\chi(\chi) = \frac{E^*\alpha_Z\sigma(\epsilon-1)(1+\chi)^{-1}}{\epsilon\left(\rho+\delta+\frac{\varsigma}{\sigma}\right)}.$$

The restriction (43) ensures that the $\dot{Z}=0$ locus intersects the $\dot{n}=0$ locus, which in turn implies that firms engage in cost reducing R&D in steady state. For $K > \frac{\varsigma}{\sigma}$ the intersection of the $\dot{\chi}=0$ and $\dot{n}=0$ loci yields χ^* and n^*. In contrast, for $\frac{\varsigma}{\sigma} > K$, the loci $\dot{\chi}=0$ and $\dot{n}=0$ do not intersect and the economy engages in cost-reducing R&D in steady state. However the economy fails to produce *enough* cost reduction to maintain a constant χ. Consequently, the cost of distribution relative to manufacturing vanishes in the long run.

There is another feature worthy of attention. Despite our simple specification for the dynamics of s (constant exogenous decay), on the transition path the growth rate of χ changes over time and it can be non-monotonic. Consider an economy with relatively low initial conditions χ_0 and n_0. Initially χ and n grow—that is, the manufacturing cost decreases relative to the distribution cost, while the mass of firms per capita increases. Both the increases in χ and n reduce the incentive to engage in cost reduction, and eventually the economy crosses the $\dot{\chi}=0$ locus. The continued increase in the mass of firms further reduces the incentive to engage in cost reduction and consequently the growth rate of manufacturing productivity declines. But this *decreases* χ and consequently increases the incentive to engage in cost reduction. Eventually the economy must return to the $\dot{\chi}=0$ locus in which case the growth rate of manufacturing productivity equals the rate of technical progress in distribution.

5. Dynamic effects of changes in the distribution technology

In this section we use the tool just developed to study the broader effects of changes to the distribution technology. We stress that, differently from the literature on public capital, in our framework an improvement in the distribution technology—which could be driven by an increase in public capital, for example roads—has no *direct* impact on manufacturing productivity. Instead, it affects the economy through two mechanisms: it reduces the cost of delivering goods to consumers and it affects firms' innovation decisions.

5.1. Consumption

In symmetric equilibrium the consumption index reduces to $C = L^{-1} N^{\frac{\epsilon}{\epsilon-1}} Q^{\theta} X$, showing that there are three sources of utility: variety, quality, and quantity. The first two can grow without additional production *and delivery* of physical objects. Variety expansion raises utility via the now traditional love-of-variety effect. Quality improvement raises utility by delivering more services per good purchased. Productivity growth raises utility via the "classic" channel of lower prices that allow customers to purchase a larger quantity of each good. The associated steady-state consumption dynamic is

$$\left(\frac{\dot{C}}{C}\right)^{*} = \frac{1}{\epsilon - 1}\lambda + \theta \left(\frac{\dot{Q}}{Q}\right)^{*} + \sigma \left(\frac{\dot{Z}}{Z}\right)^{*}, \tag{51}$$

where the growth rates of quality and manufacturing productivity are given by Proposition 2.

To trace the effects of fundamentals, especially the pair (τ, s), it is useful to use the production technology of firms and write

$$C = L^{-1} N^{\frac{\epsilon}{\epsilon-1}} Q^{\theta} Z^{\sigma} \left(L_X - \phi\right), \tag{52}$$

where the firm's variable employment in manufacturing is

$$L_X - \phi = \frac{Z^{-\sigma}}{Z^{-\sigma} + s} \frac{L}{N} \frac{\beta\left(\epsilon - 1\right)}{\left(\beta + \lambda - \rho\right)\left(1 + \tau\right)\epsilon}. \tag{53}$$

This expression allows us to decompose the effects of changes in the fundamentals in short-run (or impact) effects and long-run dynamic effects. The former work through the jumping variable L_X, the latter through the state vector (N, Q, Z). Another advantage of this decomposition is that it allows us to trace the allocation of labor across its uses.

We characterize the main channels in the next proposition.

Proposition 3. *The elasticity of consumption with respect to Z is*

$$\frac{d \ln C}{d \ln Z} = \sigma + \frac{d \ln \left(L_X - \phi\right)}{d \ln Z} > 0, \tag{54}$$

where

$$\frac{d \ln \left(L_X - \phi\right)}{d \ln Z} = \frac{-\sigma s}{Z^{-\sigma} + s} < 0. \tag{55}$$

Therefore,

$$\frac{d \ln C}{d \ln Z} = \frac{\sigma}{1 + Z^{\sigma} s} > 0$$

The elasticity of consumption with respect to s is

$$\frac{d \ln C}{d \ln s} = \frac{d \ln \left(L_X - \phi\right)}{d \ln s} = \frac{-s}{Z^{-\sigma} + s} < 0. \tag{56}$$

Proof. Take logs and differentiate (52) and (53).

The proposition decomposes the effect of manufacturing knowledge, Z, in a direct effect and an indirect effect via employment. Specifically, for a given employment in manufacturing, L_X, higher manufacturing productivity allows the typical firm to produce more and thus, crucially, to ship more. The rise in manufacturing productivity, however, does not raise the productivity of labor in delivering goods. Consequently, to meet the larger volume of shipment employment in distribution must rise. Because of this reallocation, the overall effect of manufacturing productivity growth falls with the per-unit distribution cost. And if s is very large, manufacturing productivity has a negligible effect on consumption growth. This outcome is a hard property of our mechanism: as long as $s > 0$ the elasticity of consumption with respect to manufacturing knowledge must fall with the accumulation of manufacturing knowledge for the reasons discussed throughout the paper.

Equation (56) says that a reduction in s increases consumption. While this is not surprising, the increase in production is driven solely by the reallocation of labor towards manufacturing. Recall that in our setup distribution is separate from manufacturing and, therefore, improvements in the distribution technology have no direct effect on manufacturing productivity. The existing literature neglects this possibility because it focuses on different modeling structures, as discussed in the introduction. Our mechanism seems to be consistent with the data. For example, Fernald (1999) finds that "... changes in road growth are associated with larger changes in productivity growth in industries that are vehicle intensive." In the language of our model, this says that the marginal effect of a fall in s is larger when distribution is a larger fraction of economic activity. Recall that in the model such fraction is endogenous so that the statement entails evaluating the elasticity in equation (56) at the equilibrium value of manufacturing productivity, Z^{σ}.

Equation (55) produces an outcome that at first sight is related to Baumol's cost disease (Baumol, 1967). In that framework, faster exogenous technological change in the "progressive" sector leads to a reallocation of labor to the "stagnant" sector. Our framework yields a similar outcome but the economics behind it is quite different because the model delivers structural change within the vertical supply chain. Specifically, higher manufacturing productivity results into a larger volume of shipments, which, as we argued above, requires a rise in the amount of labor devoted to the distribution of the goods. In other words, the reallocation is *downstream*, from production to distribution, and because we model distribution as technologically stagnant, the reallocation is seemingly consistent with the cost disease. The mechanism, however, cannot be interpreted as a "disease" in that sense because reallocating labor to distribution is simply a necessary step in the process that allows the economy to take advantage of higher productivity. In this light, the model provides a new insight about structural change: only technical change that results into larger physical production of each good (quantity) requires the reallocation of labor to deliver the additional goods to the consumer; non-physical sources of growth like product variety and product quality do not.

5.2. Induced innovation for constant distribution cost

We now analyze how changes to the distribution technology affects firms' innovation decisions. For the reasons discussed in subsection 4.3 we focus on the per-unit cost, s. It is convenient to use the representation of the dynamics developed in subsection 4.6. As we showed, a permanent fall in s displaces the economy from the steady state in the sense that it yields initial

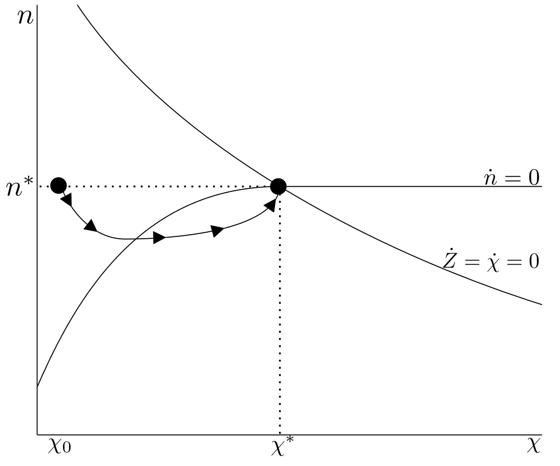


Fig. 4. Transitional dynamics due to a fall in s_0.

condition (χ', n^*) in an unchanged phase diagram. We showed that depending on whether the economy is inside the hysteresis zone there exists a critical threshold of the displacement of the elasticity η below which the fall in s has no effect.

The first result, therefore, is that the economy subject to hysteresis experiences only the short-run effects of the fall in the per-unit distribution cost, with no induced innovation effects. Our model, therefore, nests as a special case the main thrust of traditional analyses that ignore endogenous technological change. In contrast, when we consider an economy not subject to hysteresis, our model yields new insights.

Fig. 4 illustrates the adjustment process for an economy subject to a sufficiently large fall in s. The results discussed above say that the new steady state features the same mass of firms per capita n^* as the initial one. Consequently, the permanent fall in s leads to a temporary reduction to the growth rate of variety followed by a rise; see Fig. 5c. The reason is that the rise in cost-reducing R&D expenditure lowers the flow of profits which in turn reduces the rate of return to entry. Note that despite to rise of cost-reducing R&D, the temporary reduction in the mass of firms per capita drives up profitability and thus induces more quality-improving R&D. Eventually cost-reduction ceases and the mass of firms per capita starts rising, returning to the steady-state value n^*. In this phase of the process, as profitability falls the growth rate of quality returns to its steady-state value. Figs. 5a and 5b illustrate the response of manufacturing productivity and product quality. The most important feature is that the temporary deviation of the mass of firms from its steady state value locks in *permanent* gains in both productivity and quality.

How do these dynamics affect welfare? Consumption immediately rises. This is the classic channel emphasized by, e.g., Fogel (1964), Allen and Arkolakis (2014) and Donaldson and Hornbeck (2016): improvements in a nation's infrastructure allow the economy to increase production. This literature, however, is silent on the implications for technical change. As Figs. 5a, 5b and 5c demonstrate, manufacturing productivity, and hence production, is permanently higher after taking into account the induced technical progress. Product quality is likewise permanently higher. The U-shaped pattern of the mass of firms implies initial downward pressure on consumption due to the loss of product variety, but eventually this loss is made up and consumption

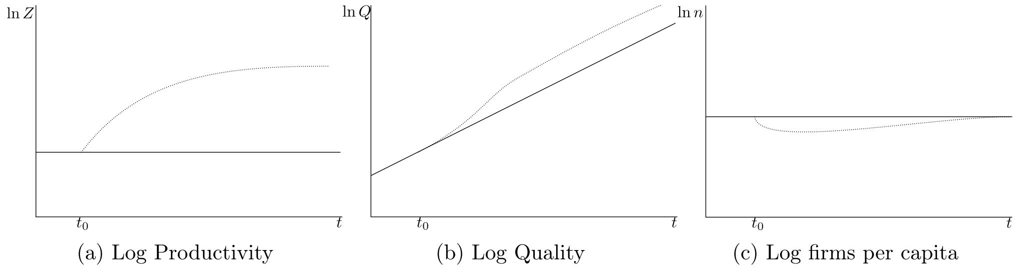


Fig. 5. Response of productivity, quality and firms per capita to a fall in s_0.

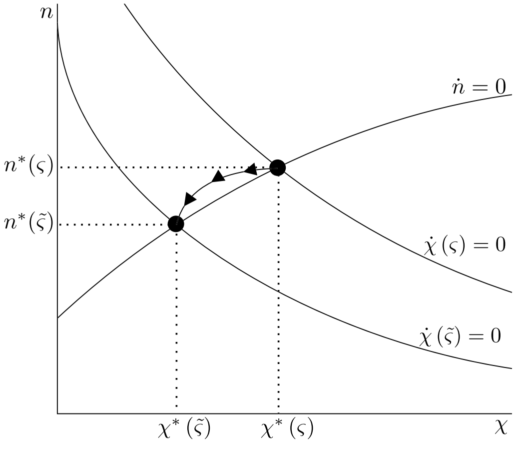


Fig. 6. The effects of an increase in ς.

rises. The net effect of the transition, therefore, is a temporary acceleration of consumption growth toward a permanently higher level. We stress that such permanently higher level is due entirely to the permanent gain in cost-reducing and quality-improving knowledge due to the transition.

5.3. Induced innovation for constantly falling distribution cost

Fig. 6 depicts the effects of an increase in the exogenous rate of technological change in distribution from ς to $\tilde{\varsigma}$. The rise in ς shifts the $\dot{\chi} = 0$ locus down from $\dot{\chi}(\varsigma) = 0$ to $\dot{\chi}(\tilde{\varsigma}) = 0$, while leaving the $\dot{n} = 0$ locus unchanged. Consequently, it yields a new steady state with fewer firms per capita, $n^*(\tilde{\varsigma})$, and lower relative cost of distribution, $\chi^*(\tilde{\varsigma})$. The logic is straightforward: with higher rate of decay of the per-unit distribution cost, firms want to do more cost-reducing R&D, which in equilibrium requires fewer firms with larger market share and associated profit flow. A by-product of this adjustment is that larger, more profitable firms also do more quality-improving R&D. Therefore, the acceleration in the exogenous rate of technological change in distribution causes an acceleration in the rate of growth of the economy.

6. Endogenous cost reduction in distribution

Thus far we have taken the distribution technology as given. We now allow firms to reduce the distribution cost through costly R&D. To do so we first modify the distribution technology to

$$L_D(\omega) = S(\omega)^{-\psi} X(\omega). \tag{57}$$

The unit cost of distribution is a function of firm-specific distribution knowledge $S(\omega)$ with elasticity ψ. Next, in line with the specifications (8) and (9), we introduce the R&D technology

$$\dot{S}(\omega) = \alpha_S S L_S(\omega), \quad S = \int_0^N S(\omega)\, d\omega / N, \tag{58}$$

where L_S is the amount of labor engaged in reducing the distribution cost and $\alpha_S S$ is its efficiency. The rest of the model is unchanged. The typical firm's problem is similar to that of Section 2. Consequently, we relegate all derivations to the appendix and proceed directly to the economy's dynamics. As before, it is useful to represent the dynamics in terms of the ratio of the unit cost of distribution to the unit cost of production, $\chi = S^{-\psi}/Z^{-\sigma}$ and the mass of firms per capita, $n = N/L$.

The growth rates of productivity in manufacturing and distribution are, respectively[9]:

$$\frac{\dot{Z}}{Z} = \begin{cases} \frac{E^*\alpha_Z\sigma(\epsilon-1)}{\epsilon n(1+\chi)} - \delta - \rho & n < \frac{E^*\alpha_Z\sigma(\epsilon-1)}{\epsilon(\rho+\delta)(1+\chi)} \equiv \bar{n}_Z(\chi); \\ 0 & \bar{n}_Z(\chi) < N \end{cases} \tag{59}$$

$$\frac{\dot{S}}{S} = \begin{cases} \frac{E^*\alpha_S\psi(\epsilon-1)\chi}{\epsilon n(1+\chi)} - \delta - \rho & n < \frac{E^*\alpha_S\psi(\epsilon-1)\chi}{\epsilon(\rho+\delta)(1+\chi)} \equiv \bar{n}_S(\chi) \\ 0 & \bar{n}_S(\chi) < n \end{cases}. \tag{60}$$

Equations (59) and (60) show that productivity growth in manufacturing (distribution) is decreasing (increasing) in χ. When the unit cost of production is relatively high, firms devote a large portion of their R&D expenditure towards reducing that cost. In contrast, when the unit cost of distribution is relatively high, firms devote more of their R&D expenditure to reducing that cost.

The growth rate of the ratio χ is

$$\frac{\dot{\chi}}{\chi} = \begin{cases} \frac{E^*(\epsilon-1)}{\epsilon n}\left(\frac{\alpha_Z\sigma^2-\alpha_S\psi^2\chi}{1+\chi}\right) - (\sigma-\psi)(\rho+\delta) & n < \bar{n}_Z(\chi), \bar{n}_S(\chi) \\ \frac{E^*(\epsilon-1)}{\epsilon n}\left(\frac{\alpha_Z\sigma^2}{1+\chi}\right) - \sigma(\rho+\delta) & \bar{n}_S(\chi) < n < \bar{n}_Z(\chi) \\ -\left[\frac{E^*(\epsilon-1)}{\epsilon n}\left(\frac{\psi^2\alpha_S\chi}{1+\chi}\right) - \psi(\rho+\delta)\right] & \bar{n}_Z(\chi) < n < \bar{n}_S(\chi) \\ 0 & \bar{n}_Z(\chi), \bar{n}_S(\chi) < n \end{cases} \tag{61}$$

[9] Quality growth is left unchanged, and hence omitted from this section.

In every branch of (61) the growth rate of χ is decreasing in its level: as the unit cost of distribution rises relative to the unit cost of production, the growth rate of productivity in manufacturing (distribution) decreases (increases). Taken to the extreme, if χ is too large (small), firms cease cost-reducing R&D in manufacturing (distribution); this is the case in the second (third) branch of (61). Finally, if the market is too saturated with firms, firms cease both forms of cost reduction.

Taking the corner solutions into account, the evolution of n is

$$\frac{\dot{n}}{n} = \begin{cases} \beta\left(\frac{B_0-(\epsilon-1)(\sigma+\psi\chi)(1+\chi)^{-1}}{\epsilon}\right) - n\beta\left(\frac{\phi-\frac{\left(\alpha_Q+\alpha_Z+\alpha_S\right)(\rho+\delta)}{\alpha_Q\alpha_Z\alpha_S}}{E^*}\right) & n < \bar{n}_Z(\chi), \bar{n}_S(\chi) \\ \beta\left(\frac{B_0-(\epsilon-1)\sigma(1+\chi)^{-1}}{\epsilon}\right) - n\beta\left(\frac{\phi-\frac{\left(\alpha_Q+\alpha_Z\right)(\rho+\delta)}{\alpha_Q\alpha_Z}}{E^*}\right) & \bar{n}_S(\chi) < n < \bar{n}_Z(\chi) \\ \beta\left(\frac{B_0-(\epsilon-1)\psi\chi(1+\chi)^{-1}}{\epsilon}\right) - n\beta\left(\frac{\phi-\frac{\left(\alpha_Q+\alpha_S\right)(\rho+\delta)}{\alpha_Q\alpha_S}}{E^*}\right) & \bar{n}_Z(\chi) < n < \bar{n}_S(\chi) \\ \beta\left(\frac{B_0}{\epsilon}\right) - n\beta\left(\frac{\phi-\frac{\rho+\delta}{\alpha_Q}}{E^*}\right) & \bar{n}_Z(\chi), \bar{n}_S(\chi) < n \\ -\delta-\lambda & \tilde{n}_N(x) < n \end{cases}, \tag{62}$$

where

$$B_0 \equiv 1-\epsilon(\rho+\delta+\lambda)/\beta-(\epsilon-1)\theta;$$
$$B_0 > (\epsilon-1)\,\mathrm{Min}\{\psi,\sigma\}. \tag{63}$$

To understand the intuition of the first branch of equation (62), it is useful to first discuss the second and third branches. When firms engage in only manufacturing cost reduction (the second branch), the entry rate is increasing in χ. An increase in χ reduces manufacturing cost-reduction R&D expenditure and hence raises the incentive to enter. When firms engage in only distribution cost reduction (the third branch), the entry rate is decreasing in χ. As χ increases, firms engage in more distribution cost-reducing R&D which lowers the incentive to enter. Returning to the first branch, when both sources of cost reduction are active, the effects of χ on the entry rate is ambiguous; an increase in χ increases (decreases), manufacturing (distribution) cost-reducing R&D expenditure. Which force dominates depends on the parameters σ and ψ: when σ is greater (less) than ψ, the first branch of (62) is increasing (decreasing) in χ. Finally, to prevent the analysis from becoming too taxonomic, we restrict parameters so that $\bar{n}_Q \equiv \bar{N}_Q/L > \bar{n}_n(\chi)$. We will return to this restriction momentarily.

Equations (61) and (62) yield the steady state loci:

$$\dot{\chi}=0\colon \bar{n}_\chi(\chi) = \begin{cases} \frac{E^*(\epsilon-1)}{(\sigma-\psi)(\rho+\delta)\epsilon}\left(\frac{\alpha_Z\sigma^2-\alpha_S\psi^2\chi}{1+\chi}\right) & n < \bar{n}_Z(\chi), \bar{n}_S(\chi) \\ Max\{\bar{n}_Z(\chi), \bar{n}_S(\chi)\} & \bar{n}_Z(\chi), \bar{n}_S(\chi) < n \end{cases}; \tag{64}$$

$$\dot{n}=0:\ \bar{n}_n(\chi)=\begin{cases}\frac{E^*}{\epsilon}\left(\frac{B_0-(\epsilon-1)(\sigma+\psi\chi)(1+\chi)^{-1}}{\phi-\frac{\left(\alpha_Q+\alpha_Z+\alpha_Q\right)(\rho+\delta)}{\alpha_Q\alpha_Z\alpha_S}}\right) & n<\bar{n}_Z(\chi),\bar{n}_S(\chi)\\ \frac{E^*}{\epsilon}\left(\frac{B_0-(\epsilon-1)\sigma(1+\chi)^{-1}}{\phi-\frac{\left(\alpha_Q+\alpha_Z\right)(\rho+\delta)}{\alpha_Q\alpha_Z}}\right) & \bar{n}_S(\chi)<n<\bar{n}_Z(\chi)\\ \frac{E^*}{\epsilon}\left(\frac{B_0-(\epsilon-1)\psi\chi(1+\chi)^{-1}}{\phi-\frac{\left(\alpha_Q+\alpha_S\right)(\rho+\delta)}{\alpha_Q\alpha_S}}\right) & \bar{n}_Z(\chi)<n<\bar{n}_S(\chi)\\ \frac{E^*}{\epsilon}\left(\frac{B_0}{\phi-\frac{\rho+\delta}{\alpha_Q}}\right) & \bar{n}_Z(\chi),\bar{n}_S(\chi)<n\end{cases}. \tag{65}$$

Before discussing the steady state we introduce the following parameter restriction which ensures $\bar{n}_Q>\bar{n}_n(\chi)$:

$$\frac{\alpha_Q(\epsilon-1)\theta}{\rho+\delta}>Max\left\{\frac{B_0}{\phi-\frac{\rho+\delta}{\alpha_Q}},\frac{B_0-(\epsilon-1)\psi}{M+\frac{(\rho+\delta)(\sigma-\psi)}{\alpha_Z\sigma}},\frac{B_0-(\epsilon-1)\sigma}{M-\frac{(\rho+\delta)(\sigma-\psi)}{\alpha_S\psi}}\right\}, \tag{66}$$

where

$$M\equiv\phi-\frac{\left(\alpha_Q+\alpha_Z+\alpha_S\right)(\rho+\delta)}{\alpha_Q\alpha_Z\alpha_S}>0. \tag{67}$$

This restriction also guarantees that firms engage in quality improvement in all possible steady states. We characterize the steady state in our next proposition.

Proposition 4. *Assume that* (66) *holds. If*

$$\frac{\alpha_S\alpha_Z M\psi\sigma(\epsilon-1)}{\rho+\delta}>\alpha_Z\sigma\left(B_0-(\epsilon-1)\psi\right)+\alpha_S\psi\left(B_0-(\epsilon-1)\sigma\right) \tag{68}$$

then there exists a unique and globally stable steady state where:

$$\chi^*=\frac{(\epsilon-1)M\alpha_Z\sigma^2-(\sigma-\psi)(\rho+\delta)\left(B_0-(\epsilon-1)\sigma\right)}{(\epsilon-1)M\alpha_S\psi^2+(\sigma-\psi)(\rho+\delta)\left(B_0-(\epsilon-1)\psi\right)}; \tag{69}$$

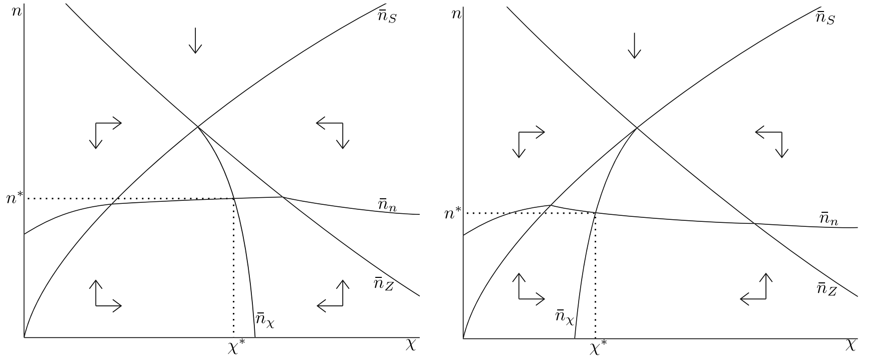
$$n^*=\frac{E^*}{\epsilon}\left(\frac{\alpha_Z\sigma^2\left(B_0-(\epsilon-1)\psi\right)+\alpha_S\psi^2\left(B_0-(\epsilon-1)\sigma\right)}{M\alpha_Z\sigma^2+M\alpha_S\psi^2+(\sigma-\psi)^2(\rho+\delta)}\right); \tag{70}$$

$$\sigma\left(\frac{\dot{Z}}{Z}\right)^*=\psi\left(\frac{\dot{S}}{S}\right)^*=\sigma\left(\frac{E^*\alpha_Z(\epsilon-1)\sigma}{\epsilon n^*(1+\chi^*)}-(\rho+\delta)\right)>0; \tag{71}$$

$$\left(\frac{\dot{Q}}{Q}\right)^*=\frac{E^*\alpha_Q(\epsilon-1)\theta}{\epsilon n^*}-(\rho+\delta)>0. \tag{72}$$

Proof. See the appendix.

Equation (68) ensures that the $\dot{\chi}=0$ and $\dot{n}=0$ loci intersect. When (68) holds, firms engage in both forms of cost reduction (distribution and manufacturing) and also in quality improvement. Similar to Proposition 2, in the long-run manufacturing and distribution unit costs must decrease at the same rate. We plot the underlying dynamics in Fig. 7.



(a) $\sigma > \psi$ (b) $\psi > \sigma$

Fig. 7. Fully endogenous cost reduction.

The intuition behind the shape of the $\dot{Z} = 0$ and $\dot{S} = 0$ loci ($\bar{n}_Z$ and $\bar{n}_S$ respectively) is straightforward. As χ increases, the incentive to engage in cost reduction in manufacturing (distribution) decreases (increases) and consequently $\bar{n}_Z$ ($\bar{n}_S$) is decreasing (increasing) in χ. The $\dot{n} = 0$ locus, $\bar{n}_n$, is a bit more complicated. The parameter restriction (63) combined with (68) ensures that $\bar{n}_n$ intersects $\bar{n}_S$ and $\bar{n}_Z$. To the left of $\bar{n}_S$, $\bar{n}_n$ is increasing in χ. Once $\bar{n}_n$ crosses $\bar{n}_S$, both sources of cost reduction are active. When σ is greater (less) than ψ, $\bar{n}_n$ is increasing (decreasing) in χ. However, after $\bar{n}_n$ crosses $\bar{n}_Z$, it is decreasing in χ. The $\dot{\chi} = 0$ locus, $\bar{n}_\chi$ lies strictly under the $\dot{Z} = 0$ and $\dot{S} = 0$ loci. When σ is greater (less) than ψ, $\bar{n}_\chi$ is decreasing (increasing) in χ.[10] The intersection of $\bar{n}_n$ and $\bar{n}_\chi$ yields the steady state mass of firms per capita n^* and ratio of manufacturing to distribution cost χ^*. When equation (68) is not satisfied, $\bar{n}_n$ and $\bar{n}_\chi$ do not intersect. In this case, firms cease both sources of cost-reduction in the steady state. Interestingly, if either R&D efficiency α_Z or α_S is too low, firms cease both forms of cost-reduction in the steady state.

7. Conclusion

In the 1980's Bill Machrone, the editor of the popular PC Magazine, observed that the price of new computer models was staying roughly constant at $5,000. *Machrone's law* soon followed: "The computer you want, always costs five thousand dollars." The failure of Machrone's law demonstrates that, in addition to increases in the quality of computers, our capability in producing the physical object known as a computer dramatically increased. But, there's more to the story: computer prices seem to have stopped falling. For example, at an investors meeting the CEO of Intel stated: "They aren't going to fall to $99" (Worthen, 2010). While the rate of decline in the price of computers has slowed, quality improvement has continued unabated—Moore's law has held for the past 30 years.[11]

In this paper we have introduced a growth model in which firms do R&D to improve product quality *and* manufacturing productivity. The key insight is that distribution makes producing

[10] Recall that σ and ψ are, respectively, the elasticity manufacturing and distribution unit costs with respect to the knowledge stocks Z and S. Consequently, when σ is greater than ψ, S must grow faster than Z—this is why $\bar{n}_\chi$ loci lies closer to $\bar{n}_Z$.

[11] Skeptics have, repeatedly, claimed that Moore's law will be over soon. However, recent innovation has again staved off the end (see Kanellos (2013) for an interesting discussion).

more objects drastically different from producing *better* objects. Unless technological advancements allow the cost of distributing goods to consumers to fall to zero, quantity growth must cease and long-run growth must be driven by quality improvement.

Gordon (2012) started a vibrant debate on the end of growth. A common rebuttal to Gordon's work is the so-called "mismeasurement problem." Simply put, because so much of our technical progress has become intangible (thinner, lighter computers, better televisions, etc.), we underestimate the growth rate of GDP in recent decades.[12] In this paper, we do not take a stand on whether quality makes up for the missing productivity growth in current measurements. Instead, we argue that the issues at the heart of the debate starkly highlight the need for models with both sources of growth. Critics of Gordon seem to rely on the idea that quality improvement has become increasingly important relative to more production of physical objects. In this paper we offered a growth-theoretic framework that allows one to study the idea systematically and rigorously.

Appendix A

To maximize readability the appendix is self contained. All necessary equations from the text are repeated with new numbering.

A.1. Lemma 1

The firms' Hamiltonian is

$$H = \Pi(\omega) - \phi - L_Z(\omega) - L_Q(\omega) + \lambda_Z(\omega)\{\alpha_Z Z L_Z(\omega)\} + \lambda_Q(\omega)\left\{\alpha_Q Q L_Q(\omega)\right\}, \tag{73}$$

where the gross profit is

$$\Pi(\omega) = \frac{LEp(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega}\left[p(\omega) - (1+\tau)\left[Z(\omega)^{-\sigma} + s\right]\right], \tag{74}$$

and $\lambda_Z(\omega)$ and $\lambda_Q(\omega)$ are the (transformed) costate variables of $Z(\omega)$ and $Q(\omega)$ respectively.

The first order conditions are:

$$p(\omega) = \frac{\epsilon(1+\tau)}{\epsilon - 1}\left(Z(\omega)^{-\sigma} + s\right); \tag{75}$$

$$\lambda_Z(\omega) = \frac{1}{\alpha_Z Z}; \tag{76}$$

$$\lambda_Q(\omega) = \frac{1}{\alpha_Q Q}; \tag{77}$$

$$-H_Z(\omega) = \dot{\lambda}_Z(\omega) - (r+\delta)\lambda_Z(\omega); \tag{78}$$

$$-H_Q(\omega) = \dot{\lambda}_Q(\omega) - (r+\delta)\lambda_Q(\omega), \tag{79}$$

where: $H_Q(\omega) \equiv \partial\Pi(\omega)/\partial Q(\omega)$ and $H_Z(\omega) \equiv \partial\Pi(\omega)/\partial Z(\omega)$.

Substituting (76) and (77) into (78) and (79) yields

$$r = \alpha_Z \frac{Z}{Z(\omega)} \frac{\partial \ln \Pi(\omega)}{\partial \ln Z(\omega)} \Pi(\omega) - \delta - \frac{\dot{Z}}{Z}; \tag{80}$$

[12] However, recent empirical work by Byrne et al. (2016) and Syverson (2016) has called this rebuttal into question.

$$r = \alpha_Q \frac{Q}{Q(\omega)} \frac{\partial \ln \Pi(\omega)}{\partial \ln Q(\omega)} \Pi(\omega) - \delta - \frac{\dot{Q}}{Q}. \tag{81}$$

Taking log of (74) and differentiating with respect to $Z(\omega)$ and $Q(\omega)$ and then using the pricing decision (75) yields

$$\frac{\partial \ln \Pi}{\partial \ln Z(\omega)} = \sigma(\epsilon - 1) \frac{Z(\omega)^{-\sigma}}{Z(\omega)^{-\sigma} + s}; \tag{82}$$

$$\frac{\partial \ln \Pi(\omega)}{\partial \ln Q(\omega)} = \theta(\epsilon - 1). \tag{83}$$

Finally substituting (82) and (83) into (80) and (81) yields

$$r = r_Z(\omega) \equiv \alpha_Z(\epsilon - 1)\sigma \left(\frac{Z(\omega)^{-\sigma}}{Z(\omega)^{-\sigma} + s} \right) \left(\frac{Z}{Z(\omega)} \right) \Pi(\omega) - \delta - \frac{\dot{Z}}{Z}; \tag{84}$$

$$r = r_Q(\omega) \equiv \alpha_Q(\epsilon - 1)\theta \left(\frac{Q}{Q(\omega)} \right) \Pi(\omega) - \delta - \frac{\dot{Q}}{Q}. \tag{85}$$

A.2. *Proposition 1*

The economy's state variables evolve according to

$$\frac{\dot{Z}}{Z} = \begin{cases} \frac{LE^*}{\epsilon N} \alpha_Z \eta(Z, s) - \rho - \delta & N < \frac{LE^* \alpha_Z \eta(Z,s)}{\epsilon(\rho+\delta)} \equiv \bar{N}_Z(Z) \\ 0 & N \geq \bar{N}_Z(Z) \end{cases}; \tag{86}$$

$$\frac{\dot{Q}}{Q} = \begin{cases} \frac{LE^*}{\epsilon N} \alpha_Q(\epsilon - 1)\theta - \rho - \delta & N < \frac{LE^* \alpha_Q(\epsilon-1)\theta}{\epsilon(\rho+\delta)} \equiv \bar{N}_Q \\ 0 & N \geq \bar{N}_Q \end{cases}; \tag{87}$$

$$\frac{\dot{N}}{N} = \begin{cases} \frac{\beta[1-(\epsilon-1)\theta-\eta(Z,s)]-\epsilon(\rho+\delta)}{\epsilon} - \left(\frac{\beta N}{LE^*} \right) \left(\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z \alpha_Q} \right) & N < min\{\bar{N}_Q, \bar{N}_Z(Z)\} \\ \frac{\beta[1-(\epsilon-1)\theta]-\epsilon(\rho+\delta)}{\epsilon} - \left(\frac{\beta N}{LE^*} \right) \left(\phi - \frac{\rho+\delta}{\alpha_Q} \right) & \bar{N}_Z(Z) < N < \bar{N}_Q \\ -\delta & \tilde{N}_N < N \end{cases}, \tag{88}$$

where $\eta(Z, s) = (\epsilon - 1)\sigma \left(\frac{Z^{-\sigma}}{Z^{-\sigma}+s} \right)$ and the zero net and gross entry loci are, respectively,

$$\bar{N}_N(Z) = \begin{cases} \frac{LE^*}{\epsilon} \left(\frac{1-(\epsilon-1)\theta-\eta(Z,s)-\epsilon(\rho+\delta)/\beta}{\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z \alpha_Q}} \right) & N < \bar{N}_Z(Z) \\ \frac{LE^*}{\epsilon} \left(\frac{1-(\epsilon-1)\theta-\epsilon(\rho+\delta)/\beta}{\phi - \frac{\rho+\delta}{\alpha_Q}} \right) & \bar{N}_Z(Z) \leq N \end{cases}; \tag{89}$$

$$\tilde{N}_N(Z) = \begin{cases} \frac{LE^*}{\epsilon} \left(\frac{1-(\epsilon-1)\theta-\eta(Z,s)-\epsilon\rho/\beta}{\phi - \frac{(\alpha_Z+\alpha_Q)(\rho+\delta)}{\alpha_Z \alpha_Q}} \right) & N < \bar{N}_Z(Z) \\ \frac{LE^*}{\epsilon} \left(\frac{1-(\epsilon-1)\theta-\epsilon\rho/\beta}{\phi - \frac{\rho+\delta}{\alpha_Q}} \right) & \bar{N}_Z(Z) \leq N \end{cases}. \tag{90}$$

In the steady state, for the reasons discussed in the paper, firms cease cost reducing R&D. Therefore there are two possible steady states: one with quality growth and the other without any growth. Restriction (30) in the main body, repeated here for convenience,

$$Min\left\{\alpha_Q,\alpha_Z\right\}\frac{\theta\left(\epsilon-1\right)\left(\phi-\frac{\rho+\delta}{\alpha_Q}\right)}{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta}>\rho+\delta \tag{91}$$

ensures that firms engage in quality improvement in the steady state. The steady state number of firms N^* is given by the second branch of (89). Substituting this value into the growth rate of product quality, yields

$$\left(\frac{\dot{Q}}{Q}\right)^*=\frac{\theta\left(\epsilon-1\right)\left(\alpha_Q\phi-\rho-\delta\right)}{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta}-\rho-\delta. \tag{92}$$

Steady-state manufacturing productivity (equation (33)) is obtained by setting $\bar{N}_Z=\bar{N}_N\left(Z\right)$, doing so yields

$$\eta\left(Z^*,s\right)=\left(\epsilon\left(\rho+\delta\right)\right)\frac{1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta}{\alpha_Z\left(\phi-\frac{\rho+\delta}{\alpha_Q}\right)}, \tag{93}$$

the definition of $\eta\left(Z,s\right)$, and simple algebraic manipulations, yields steady state manufacturing productivity.

$$Z^*=\left\{\frac{1}{s}\left[\frac{\sigma\left(\epsilon-1\right)\alpha_Z\left(\phi-\frac{\rho+\delta}{\alpha_Q}\right)}{\left(\rho+\delta\right)\left[1-\theta\left(\epsilon-1\right)-\epsilon\left(\rho+\delta\right)/\beta\right]}-1\right]\right\}^{1/\sigma}.$$

The path dependent steady state, Z_{ss}, is discussed at length in the main body of the paper.

A.3. Proposition 2

Here we discuss the steady state with exogenous decay in distribution costs. We assume constant exponential technical progress in distribution such that $s\left(t\right)=s_0e^{-\varsigma t}$, with s_0, $\varsigma>0$.

The state variables $\chi=s/Z^{-\sigma}$ and $n=N/L$ evolve according to $\dot{\chi}=F(\chi,n)$ and $\dot{n}=G(\chi,n)$, where

$$F(\chi,n)=\chi\sigma\left[\frac{E^*\alpha_Z\sigma\left(\epsilon-1\right)}{\epsilon n}\left(1+\chi\right)^{-1}-\rho-\delta-\frac{\varsigma}{\sigma}\right];$$

$$G(\chi,n)=n\left[\frac{\beta\left(1-\theta\left(\epsilon-1\right)-\sigma\left(\epsilon-1\right)\left(1+\chi\right)^{-1}\right)-\epsilon\left(\rho+\delta+\lambda\right)}{\epsilon}\right.$$
$$\left.-n\beta\left(\frac{\phi-\frac{\left(\alpha_Z+\alpha_Q\right)\left(\rho+\delta\right)}{\alpha_Z\alpha_Q}}{E^*}\right)\right].$$

There are two steady states: $0,\bar{n}_n\left(0\right)$ and χ^*,n^*. The Jacobian is

$$J=\begin{pmatrix}F_\chi & F_n\\ G_\chi & G_n\end{pmatrix}. \tag{94}$$

The Jacobian's elements evaluated at $(0, \bar{n}_n(0))$ are

$$F_\chi(0, \bar{n}_n(0)) = \sigma \left[K - \frac{\varsigma}{\sigma} \right];$$
$$F_n(0, \bar{n}_n(0)) = 0;$$
$$G_\chi(0, \bar{n}_n(0)) = \frac{E}{\epsilon} \left(\frac{1 - (\epsilon - 1)\theta - (\epsilon - 1)\sigma - \epsilon(\rho + \delta + \lambda)/\beta}{\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q}} \right) \frac{\beta(\epsilon - 1)\sigma}{\epsilon};$$
$$G_n(0, \bar{n}_n(0)) = -\beta \frac{1 - (\epsilon - 1)\theta - (\epsilon - 1)\sigma - \epsilon(\rho + \delta + \lambda)/\beta}{\epsilon},$$

where we assume $K \equiv \frac{\alpha_Z \sigma(\epsilon - 1)\left(\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q}\right)}{1 - \theta(\epsilon - 1) - \sigma(\epsilon - 1) - \epsilon(\rho + \delta + \lambda)/\beta} - \rho - \delta > 0$.

At the economy's other steady state, χ^*, n^*, the Jacobian's elements are

$$F_\chi(\chi^*, n^*) = -\chi\sigma \frac{E^*(\epsilon - 1)\alpha_Z \sigma}{\epsilon n}(1 + \chi)^{-2};$$
$$F_n(\chi^*, n^*) = -\chi\sigma \frac{E^*(\epsilon - 1)\alpha_Z \sigma}{\epsilon n^2}(1 + \chi)^{-1};$$
$$G_\chi(\chi^*, n^*) = \frac{n\beta(\epsilon - 1)\sigma}{\epsilon}(1 + \chi)^{-2};$$
$$G_n(\chi^*, n^*) = \left(\frac{-\beta n}{E^*}\right)\left(\phi - \frac{(\alpha_Z + \alpha_Q)(\rho + \delta)}{\alpha_Z \alpha_Q}\right).$$

There are two cases to analyze

Case 1. When $K > \frac{\varsigma}{\sigma}$, evaluating the Jacobian at $(0, \bar{n}_n(0))$ the trace is ambiguous, but the determinant is negative. Hence $(0, \bar{n}_n(0))$ is unstable. In contrast, at (χ^*, n^*) the trace is negative and the determinant is positive. Hence (χ^*, n^*) is the only stable steady state.

Case 2. When $\frac{\varsigma}{\sigma} > K$, the trace is negative and the determinant is positive. Hence $(0, n)$ is stable. Note that when $\frac{\varsigma}{\sigma} > K$, the $\dot{\chi} = 0$ and $\dot{n} = 0$ loci do not intersect.

A.4. Endogenous distribution innovation derivations

The firms current value Hamiltonian is

$$H = \Pi(\omega) - \phi - L_S(\omega) - L_Z(\omega) - L_Q(\omega),$$
$$\lambda_S(\omega)\{\alpha_S S L_S(\omega)\} + \lambda_Z(\omega)\{\alpha_Z Z L_Z(\omega)\} + \lambda_Q(\omega)\left\{\alpha_Q Q L_Q(\omega)\right\}$$

where the gross profit, $\Pi(\omega)$, is now

$$\Pi(\omega) = \frac{LEp(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega}\left[p(\omega) - \left(Z(\omega)^{-\sigma} + S(\omega)^{-\psi}\right)\right],$$

and $\lambda_j(\omega)$ $(j = S, Z, Q)$ are the costate variables. The firm chooses $p(\omega)$, $L_S(\omega)$, $L_Z(\omega)$, and $L_Q(\omega)$. The first order conditions are

$$p(\omega) = \frac{\epsilon\left(Z(\omega)^{-\sigma} + S(\omega)^{-\psi}\right)}{\epsilon - 1}; \quad (95)$$

$$\lambda_S(\omega) = \frac{1}{\alpha_S S}; \tag{96}$$

$$\lambda_Z(\omega) = \frac{1}{\alpha_Z Z}; \tag{97}$$

$$\lambda_Q(\omega) = \frac{1}{\alpha_Q Q}; \tag{98}$$

$$r + \delta = \frac{H_{S(\omega)}}{\lambda_S(\omega)} + \frac{\dot{\lambda_S(\omega)}}{\lambda_S(\omega)}; \tag{99}$$

$$r + \delta = \frac{H_{Z(\omega)}}{\lambda_Z(\omega)} + \frac{\dot{\lambda_Z(\omega)}}{\lambda_Z(\omega)}; \tag{100}$$

$$r + \delta = \frac{H_{Q(\omega)}}{\lambda_Q(\omega)} + \frac{\dot{\lambda_Q(\omega)}}{\lambda_Q(\omega)}; \tag{101}$$

where

$$H_{S(\omega)} = \frac{LEp(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega} \psi S(\omega)^{-\psi-1}; \tag{102}$$

$$H_{Z(\omega)} = \frac{LEp(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega} \sigma Z(\omega)^{-\sigma-1}; \tag{103}$$

$$H_{Q(\omega)} = \frac{\theta(\epsilon-1) LEp(\omega)^{-\epsilon} Q(\omega)^{\theta(\epsilon-1)-1}}{\int_0^N p(\omega)^{1-\epsilon} Q(\omega)^{\theta(\epsilon-1)} d\omega} \left[p(\omega) - \left(Z(\omega)^{-\sigma} + S(\omega)^{-\psi}\right)\right]. \tag{104}$$

In symmetric equilibrium the rates of return are

$$r + \delta = \frac{LE\alpha_S \psi (\epsilon-1)}{N\epsilon} \left(\frac{S^{-\psi}}{Z^{-\sigma} + S^{-\psi}}\right) - \frac{\dot{S}}{S}; \tag{105}$$

$$r + \delta = \frac{LE\alpha_Z \sigma (\epsilon-1)}{N\epsilon} \left(\frac{Z^{-\sigma}}{Z^{-\sigma} + S^{-\psi}}\right) - \frac{\dot{Z}}{Z}; \tag{106}$$

$$r + \delta = \frac{LE\alpha_Q \theta (\epsilon-1)}{N\epsilon} - \frac{\dot{Q}}{Q}. \tag{107}$$

A.5. Proposition 4

In the steady state with positive cost reduction we must have

$$Min\{\bar{n}_Z(\chi), \bar{n}_S(\chi)\} > n^*$$

where: $\bar{n}_Z(\chi) \equiv \frac{E^* \alpha_Z \sigma(\epsilon-1)}{\epsilon(\rho+\delta)(1+\chi)}$; $\bar{n}_S(\chi) \equiv \frac{E^* \alpha_S \psi(\epsilon-1)\chi}{\epsilon(\rho+\delta)(1+\chi)}$; $n^* = \frac{E^*}{\epsilon}\left(\frac{\alpha_Z \sigma^2 B_1 + \alpha_S \psi^2 B_2}{M\alpha_Z \sigma^2 + M\alpha_S \psi^2 + (\sigma-\psi)^2(\rho+\delta)}\right)$. Equivalently, since in the steady state $\sigma\left(\frac{\dot{Z}}{Z}\right)^* = \psi\left(\frac{\dot{S}}{S}\right)^*$, we can verify that the growth rates are positive. Tedious algebra yields

$$\frac{\dot{Z}}{Z} = \frac{(\epsilon-1) M\alpha_S \psi^2 \alpha_Z \sigma - (\rho+\delta)\psi [\alpha_Z \sigma B_1 + \alpha_S \psi B_2]}{\alpha_Z \sigma^2 B_1 + \alpha_A \psi^2 B_2} \tag{108}$$

restriction (68) in the main text ensures that (108) is positive.

We now turn to the stability of the steady state. The state variables χ and n evolve according to $\dot{\chi} = F(\chi, n)$ and $\dot{n} = G(\chi, n)$, where

$$F(\chi, n) = \chi \left[\frac{E^* (\epsilon - 1)}{\epsilon n} \left(\frac{\alpha_Z \sigma^2 - \alpha_S \psi^2 \chi}{1 + \chi} \right) - (\sigma - \psi)(\rho + \delta) \right];$$

$$G(\chi, n) = n \left[\beta \left(\frac{B_0 - (\epsilon - 1)(\sigma + \psi \chi)(1 + \chi)^{-1}}{\epsilon} \right) - n\beta \left(\frac{\phi - \frac{(\alpha_Q + \alpha_Z + \alpha_S)(\rho + \delta)}{\alpha_Q \alpha_Z \alpha_S}}{E^*} \right) \right].$$

The Jacobian is

$$J = \begin{pmatrix} F_\chi & F_n \\ G_\chi & G_n \end{pmatrix},$$

and its elements evaluated at (χ^*, n^*) are

$$F_\chi\left(\chi^*, n^*\right) = -\frac{\chi^* E (\epsilon - 1)}{\epsilon n^*} \left(\frac{\alpha_S \psi^2 + \alpha_Z \sigma^2}{(1 + \chi^*)^2} \right);$$

$$F_n\left(\chi^*, n^*\right) = -\frac{\chi^* E (\epsilon - 1)}{\epsilon (n^*)^2} \left(\frac{\alpha_Z \sigma^2 - \alpha_S \psi^2 \chi^*}{1 + \chi^*} \right);$$

$$G_\chi\left(\chi^*, n^*\right) = -\frac{n^* \beta (\epsilon - 1)}{\epsilon} \left(\frac{\psi - \sigma}{(1 + \chi^*)^2} \right);$$

$$G_n\left(\chi^*, n^*\right) = -\frac{n^* \beta}{E^*} \left(\phi - \frac{\left(\alpha_Q + \alpha_Z + \alpha_Q\right)(\rho + \delta)}{\alpha_Q \alpha_Z \alpha_S} \right).$$

Using tedious algebra we can show that $F_n(\chi^*, n^*)$ and $G_\chi(\chi^*, n^*)$ are of opposite sign. Moreover, parameter restriction (67) in the main text ensures that $G_n(\chi^*, n^*)$ is negative. Therefore

$$\det(J) = F_\chi\left(\chi^*, n^*\right) G_n\left(\chi^*, n^*\right) - F_n\left(\chi^*, n^*\right) G_\chi\left(\chi^*, n^*\right) > 0;$$
$$\operatorname{tr}(J) = F_\chi\left(\chi^*, n^*\right) + G_n\left(\chi^*, n^*\right) < 0.$$

Hence χ^*, n^* is stable.

References

Alchian, A.A., Allen, W.R., 1964. University Economics. Wadsworth Publishing Company.
Allen, T., Arkolakis, C., 2014. Trade and the topography of the spatial economy. Q. J. Econ. 129 (3), 1085–1140.
Aschauer, D., 1989. Is public expenditure productive? J. Monet. Econ. 23 (2), 177–200.
Barro, R.J., 1990. Government spending in a simple model of endogenous growth. J. Polit. Econ. 98 (5), S103–S126.
Baumol, W.J., 1967. Macroeconomics of unbalanced growth: the anatomy of urban crisis. Am. Econ. Rev. 57 (3), 415–426.
Bollard, A., Klenow, P.J., Li, H., 2016. Entry Costs Rise with Development. Discussion paper.
Burstein, A.T., Neves, J.C., Rebelo, S., 2003. Distribution costs and real exchange rate dynamics during exchange-rate-based stabilizations. J. Monet. Econ. 50 (6), 1189–1214.
Byrne, D.M., Fernald, J.G., Reinsdorf, M.B., 2016. Does the United States Have a Productivity Slowdown or a Measurement Problem? Working Paper Series 2016-3. Federal Reserve Bank of San Francisco.
Chatterjee, S., Turnovsky, S.J., 2012. Infrastructure and inequality. Eur. Econ. Rev. 56 (8), 1730–1745.
Donaldson, D., Hornbeck, R., 2016. Railroads and American economic growth: a "market access" approach. Q. J. Econ. 131 (2), 799–858.
Fernald, J.G., 1999. Roads to prosperity? Assessing the link between public capital and productivity. Am. Econ. Rev. 89 (3), 619–638.

Fogel, R., 1964. Railroads and American Economic Growth: Essays in Econometric History. Johns Hopkins University Press.
Gordon, R.J., 2012. Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds. NBER Working Papers 18315. National Bureau of Economic Research, Inc.
Hallak, J.C., Sivadasan, J., 2013. Product and process productivity: implications for quality choice and conditional exporter premia. J. Int. Econ. 91 (1), 53–67.
Hummels, D., Skiba, A., 2004. Shipping the good apples out? An empirical confirmation of the Alchian-Allen conjecture. J. Polit. Econ. 112 (6), 1384–1402.
Irarrazabal, A., Moxnes, A., Opromolla, L.D., 2015. The tip of the iceberg: a quantitative framework for estimating trade costs. Rev. Econ. Stat. 97 (4), 777–792.
Kanellos, M., 2013. With 3D chips, Samsung leaves Moore's law behind. In: Forbes Magazine.
Mansfield, E., 1988. Industrial R&D in Japan and the United States: a comparative study. Am. Econ. Rev. 78 (2), 223–228.
Peretto, P., Connolly, M., 2007. The Manhattan metaphor. J. Econ. Growth 12 (4), 329–350.
Peretto, P., Smulders, S., 2002. Technological distance, growth and scale effects. Econ. J. 112 (481), 603–624.
Peretto, P.F., 1996. Sunk costs, market structure, and growth. Int. Econ. Rev. 37 (4), 895–923.
Peretto, P.F., 1998. Technological change and population growth. J. Econ. Growth 3 (4), 283–311.
Samuelson, P.A., 1954. The transfer problem and transport costs, II: analysis of effects of trade impediments. Econ. J. 64, 264–289.
Spence, M., 1984. Cost reduction, competition, and industry performance. Econometrica 52 (1), 101–121.
Sutton, J., 2007a. Quality, trade and the moving window: the globalisation process. Econ. J. 117 (524), F469–F498.
Sutton, J., 2007b. Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration, vol. 1, 1 edn.. The MIT Press.
Syverson, C., 2016. Challenges to Mismeasurement Explanations for the U.S. Productivity Slowdown. NBER Working Papers 21974. National Bureau of Economic Research, Inc.
Sørensen, A., 2014. Additive versus multiplicative trade costs and the gains from trade liberalizations. Can. J. Econ. 47 (3), 1032–1046.
The National Economic Council, The President's Council of Economic Advisers, 2014. An Economic Analysis of Transportation Infrastructure Investment. The White House report.
The White House, 1985. Global competition: the new reality. In: President's Commission on Industrial Competitiveness. USGPO, Washington.
Turnovsky, S.J., 1996. Optimal tax, debt, and expenditure policies in a growing economy. J. Public Econ. 60 (1), 21–44.
Worthen, B., 2010. Rising PC prices buck the trend. Wall St. J.