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Coalition-Proof Nash Equilibria
I. Concepts*

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In an important class of "noncooperative" environments, it is natural to assume that players can freely discuss their strategies, but cannot make binding commitments. In such cases, any meaningful agreement between the players must be self-enforcing. Although the Nash best-response property is a necessary condition for self-enforceability, it is not sufficient—it is in general possible for coalitions arrange plausible, mutually beneficial deviations from Nash agreements. We provide a stronger definition of self-enforceability, and label the class of efficient self-enforcing agreements "coalition-proof." *Journal of Economic Literature* Classification Numbers: 022, 025.

1. INTRODUCTION

In an important class of "noncooperative" environments, it is natural to assume that players can freely discuss their strategies, but cannot make binding commitments. In such circumstances, agreements among the players are meaningless unless they are self-enforcing. Clearly, then, any

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meaningful agreement must, for each player, prescribe a strategy which is a best response to those indicated for other players; that is, agreed upon strategies must constitute a Nash equilibrium. However, while the Nash best-response property is certainly a requirement for self-enforceability, it is not generally sufficient: it is frequently possible for coalitions of players to arrange plausible, mutually beneficial deviations from Nash agreements. Here we provide a stronger notion of self-enforceability that accounts for coalitional deviations, and label the class of efficient self-enforcing agreements "coalition-proof."

To see the potential importance of such non-binding communication, consider the following pure coordination game. The game involves two players, each of whom names either "heads" or "tails." If they "match" each receives a payoff of one; otherwise, they receive zero. As is well known, there are three Nash equilibria: (heads, heads), (tails, tails), and a mixed strategy equilibrium in which each player places equal probability on the two possibilities. In a one-shot game with no preplay communication, the mixed strategy equilibrium is descriptively appealing (presumably, each player is indifferent between his two choices, and matching is observed only half of the time in one-shot trials with distinct players). Suppose, though, that non-binding preplay communication between the players is possible. In that case, we would expect the players to match in all trials, since agreeing to one of the pure strategy equilibria is both self-enforcing and Pareto superior to the mixed strategy equilibrium.

Considerations of this sort have motivated the common practice of refining the Nash equilibrium set by restricting attention to its efficient frontier (we will refer to this practice as the Pareto dominance refinement). While appealing in two player games, this procedure is, more generally, unsatisfactory. A difficulty arises because this practice implicitly assumes that no proper subset of players can privately communicate, so that in reaching a meaningful agreement, the whole set of players need only eliminate incentives for unilateral deviations. For environments in which players can freely discuss their strategies, however, it seems far more natural to assume that any group of players can agree privately upon a joint deviation. In that case, any meaningful agreement by the whole set of players must be stable against deviations by all possible coalitions of players. Generally, then, one should apply a Pareto refinement not to the Nash set, but rather to the set of coalitionally self-enforcing agreements (a set which is typically strictly smaller).

Another commonly used refinement, the notion of Strong Nash equilibrium (Aumann [1]), does require stability against deviations by every conceivable coalition. An equilibrium is strong if no coalition, taking the actions of its complement as given, can cooperatively deviate in a way that benefits all of its members. More formally, for the n-player game with strategy sets $\{S^j\}_{j=1}^n$ and payoff functions $\{g^j: \prod_{i=1}^n S^i \to R\}_{j=1}^n$, Strong Nash equilibrium is defined by,

DEFINITION. $s^* \in \prod_{j=1}^n S^j$ is a *Strong Nash equilibrium* if and only if for all $J \subseteq \{1,..., n\}$ and for all $s_J \in \prod_{j \in J} S^j$ there exists an agent $i \in J$ such that $g^i(s^*) \geqslant g^i(s_J, s^*_{-J})$ [where $s^*_{-J} \equiv \{s^*_j\}_{j \notin J}$].

Thus while the Nash concept defines equilibrium only in terms of unilateral deviations, Strong Nash equilibrium allows for deviations by every conceivable coalition. We believe, however, that the Strong Nash concept is actually "too strong." In particular, coalitions are allowed too much freedom (in fact, complete freedom) in choosing their joint deviations: while the whole set of players must originally be concerned with arriving at an agreement that is immune to deviations by any coalition, no deviating group of players (including the coalition of the whole) faces a similar restriction. In environments with unlimited private communication, however, any meaningful agreement to deviate must also be self-enforcing (i.e., immune to deviations by subcoalitions). This inconsistency in the Strong Nash concept most clearly manifests itself in the stringent requirement that a Strong Nash equilibrium must be Pareto efficient (within the entire feasible payoff space of the game). As a result of this requirement, Strong Nash equilibria almost never exist.[1]

In this paper, we introduce a new refinement of the Nash set, the concept of Coalition-Proof Nash equilibrium, that is designed to capture the notion of an efficient self-enforcing agreement for environments with unlimited, but nonbinding, pre-play communication. An agreement is coalition-proof if and only if it is Pareto efficient within the class of self-enforcing agreements. In turn, an agreement is self-enforcing if and only if no proper subset (coalition) of players, taking the actions of its complement as fixed, can agree to deviate in a way that makes all of its members better off. However, in contrast to the strong equilibrium concept, we do not entertain all possible deviations by such coalitions. Internal consistency requires us to judge the validity of deviations by the same criteria which we use to judge the original agreement—a valid deviation must be self-enforcing, in the sense that no proper sub-coalition can reach a mutually beneficial agreement to deviate from the deviation. Likewise, any potential deviation by a sub-coalition must be judged by the same criterion, and so on.[2]

[1] However, certain important classes of voting games do possess strong Nash equilibria (see, e.g., Peleg [9]).

[2] The reader should note that our notion of self-enforcability is restrictive in one potentially important respect: when a deviation occurs, only members of the deviating coalition may contemplate deviations from the deviation. This rules out the possibility that some member of the deviating coalition might form a pact to deviate further with someone not included in this coalition. We return to this issue in Section 2.

As is evident, the consistent application of the notion of self-enforceability involves a recursion. This recursion makes the formal definition of Coalition-Proof Nash equilibrium which we present below somewhat tricky. However, by consistently applying the requirement of self-enforceability, the Coalition-Proof Nash equilibrium concept avoids the inconsistencies of the Strong Nash concept. In addition, by requiring agreements to be self-enforcing against all possible coalitional deviations, the coalition-proof concept corrects the difficulty involved in the use of the Pareto dominance refinement.

In order to highlight the distinction between the Pareto dominance refinement, Strong Nash equilibria, and Coalition-Proof Nash equilibria, we consider a simple example. In the following three player game, player A chooses rows, player B chooses columns, and player C chooses boxes[3]:

TABLE I

A Three Player Game

	C_1			C_2	
	B_1	B_2		B_1	B_2
A_1	1, 1, −5	−5, −5, 0	A_1	−1, −1, 5	−5, −5, 0
A_2	−5, −5, 0	0, 0, 10	A_2	−5, −5, 0	−2, −2, 0

Suppose that the three players wish to come to an agreement regarding the strategies that they will each play. As we argued above, any meaningful agreement must be a Nash equilibrium. In this game there are two Nash equilibria, (A_2, B_2, C_1) and (A_1, B_1, C_2). Note, also, that the first of these equilibria Pareto dominates the second. Should we therefore expect (A_2, B_2, C_1) to be the chosen agreement? We believe not. If players have unlimited opportunities to communicate, (A_2, B_2, C_1) seems an implausible outcome—player C should recognize that players A and B (whose interests are completely coincident throughout the game) would have the opportunity and the incentive to jointly renege on the agreement by playing (A_1, B_1) [note that this is a Nash equilibrium for A and B, holding C's action as fixed]. Here the only meaningful (i.e., self-enforcing) agreement is the Coalition-Proof Nash equilibrium (A_1, B_1, C_2). Finally, what can be said about the set of Strong Nash equilibria? As is easy to see, no Strong Nash equilibria exist for this game (since (A_1, B_1, C_2) is not Pareto efficient).

[3] We would like to thank Andreu Mas-Colell for suggesting the use of this example.

The paper is organized as follows. A formal definition of Coalition-Proof Nash equilibrium is presented in Section II and its relation to the two more familiar concepts mentioned above is discussed. Section III considers the question of existence. Unfortunately, games can lack a Coalition-Proof Nash equilibrium. This should not be surprising given the conceptual similarities between coalition-proofness and other coalition-oriented notions such as Strong equilibria and the Core. The concept is, nevertheless, useful in a variety of circumstances; several economic applications are discussed in a companion piece (Bernheim and Whinston [4]). We generalize the analysis to extensive form games in Section IV, where we define Perfectly Coalition-Proof Nash Equilibria. The paper closes with a brief conclusion.

II. Definitions

Our objective is to define a notion of equilibrium which represents the efficient frontier of meaningful agreements for environments in which communication is possible, but binding commitment is not. By "meaningful" we mean that the agreement is self-enforcing, in the sense that no coalition can (taking the strategies of its complement as fixed) make a mutually beneficial, self-enforcing joint deviation from it. Note that the intuitive definition is naturally recursive—we require a notion of self-enforceability (of deviations for a coalition) to define self-enforceability (of original agreements). To avoid the confusion which can easily arise from this apparent circularity, it is helpful to keep the following scenario (intended only to aid understanding of our definition) in mind.

A group of agents is to play a game, as follows. All players meet in a room, where free discussion of strategies is permitted. Any player may leave the room at any time, but upon leaving must cast a secret "ballot," upon which he indicates his choice of strategy. An agreement among the players is meaningful only if, by consenting, casting his ballot, and leaving the room, each player indicates his intent to cooperate. But then the following problem arises: if any player leaves the room first, those remaining may take his action as fixed, and reach a new agreement among themselves. The first to leave may, however, be comforted by the following thought: any agreement among the remainder will also be suspect, since every other player may be reluctant to be the *next* to leave. We wish to find an agreement such that, regardless of the order of exit, the remainder will never wish to deviate.

We can find such an agreement by backward induction. The last player left in the room clearly has no incentive to deviate if his last agreement had the best-response property. The last two players in the room may therefore deviate to any Nash equilibrium in the game induced on them by others' choices; in a self-enforcing agreement, each pair of players must therefore be playing a Pareto undominated Nash equilibrium in the component game induced on the pair by the other players' actions. The induction argument continues through the total number of players.

Formally, consider an n-player game $\Gamma = [\{g^i\}_{i=1}^n, \{S^i\}_{i=1}^n]$, where S^i is player i's strategy set and $g^i: \prod_{j=1}^n S^j \to R$ is player i's payoff function. Let $\mathbf{J}$ be the set of *proper* subsets of $\{1,..., n\}$, and denote an element of $\mathbf{J}$ (a "coalition") as $J \in \mathbf{J}$. Let $S^J \equiv \prod_{i \in J} S^i$; for the case of $\{1,..., n\}$ we will simply write S. Also let $-J$ denote the complement of J in $\{1,..., n\}$. Finally, for each $s^0_{-J} \in S^{-J}$, let Γ/s^0_{-J} denote the game induced on subgroup J by the actions s^0_{-J} for coalition $-J$, i.e.,

$$\Gamma/s^0_{-J} \equiv [\{\bar{g}^i\}_{i \in J}, \{S^i\}_{i \in J}],$$

where $\bar{g}^i: S^J \to R$ is given by $\bar{g}^i(s_J) \equiv g^i(s_J, s^0_{-J})$ for all $i \in J$ and $s_J \in S^J$.

We are now prepared to define self-enforceability and coalition-proofness recursively.

DEFINITION. (i) In a single player game Γ, $s^* \in S$ is a *Coalition-Proof Nash equilibrium* if and only if s^* maximizes $g^1(s)$.

(ii) Let $n > 1$ and assume that Coalition-Proof Nash equilibrium has been defined for games with fewer than n players. Then,

(a) For any game Γ with n players, $s^* \in S$ is *self-enforcing* if, for all $J \in \mathbf{J}$, s^*_J is a Coalition-Proof Nash equilibrium in the game Γ/s^*_{-J}.

(b) For any game Γ with n players, $s^* \in S$ is a *Coalition-Proof Nash equilibrium* if it is self-enforcing and if there does not exist another self-enforcing strategy vector $s \in S$ such that $g^i(s) > g^i(s^*)$ for all $i = 1,..., n$.

The logic is simple: an agreement is coalition-proof if it is efficient within the class of self-enforcing agreements, where self-enforceability requires that no coalition can benefit by deviating in a self-enforcing way. Observe that this notion of equilibrium has an appealing internal consistency: in any coalition-proof equilibrium, every subgroup plays a Coalition-Proof Nash equilibrium strategy vector in its component game. Not all refinements have this same internal consistency. For example, Pareto undominated Nash equilibria may entail various subgroups playing Pareto dominated equilibria in the component game induced upon them by others' prescribed actions.

The reader should note that our notion of self-enforcability is restrictive in one potentially important respect: when a deviation occurs, only members of the deviating coalition may contemplate deviations from the deviation. This rules out the possibility that some member of the deviating coalition might form a pact to deviate further with someone not included in this coalition. Such arrangements are clearly much more complex than those made entirely by members of the coalition itself. In particular, coalition members have observed the original deviation first hand. In contrast, nonmembers lack verifiable information on prior deviations. At times, the willingness of some party to form a coalition may reveal his agreement to some prior deviation (in the absence of a prior deviation, it would not be in his interests to join the coalition) but may not identify that deviation. Similarly, the unwillingness of some party to join a coalition may also be informative. Further, there may be times when one player wishes to convince another that a prior deviation exists, in order to secure cooperation in deviating further. Implicitly, we assume that these information problems cripple any attempt to form coalitions consisting of both members and nonmembers from some other deviating coalition. Clearly, further is required to resolve these issues in a fully satisfactory way.

We now turn to the relationships between the set of Coalition-Proof Nash equilibria (C), the set of Strong Nash equilibria (S), and the set of Nash equilibria which are not Pareto dominated by any other Nash equilibrium (P). It is easy to check that strong equilibria are both coalition-proof and Pareto undominated (the former relationship follows from the fact that the Coalition-Proof Nash equilibrium concept differs from the Strong Nash equilibrium concept precisely because it restricts the set of feasible coalitional deviations). However, for $n > 2$ it is impossible to establish an inclusion relation between C and P.

For the special case of 2 player games, it follows immediately from the above definitions that $C = P$. This is natural: for 2 player games, the only proper coalitions are single agents, and the best-response property alone guarantees that single agents cannot profitably deviate. Thus, the "best" Nash equilibria are, in this case, coalition-proof. Furthermore, as long as the set of Nash equilibrium payoffs is compact (this requirement is satisfied under mild regularity conditions), it is easy to see that C is nonempty. However, for games with three or more players, existence becomes problematic. We now turn to this issue.

III. Existence

While it would be reassuring to discover that Coalition-Proof Nash equilibria exist for some very general class of games, we have, unfortunately, not found this to be the case. This revelation should not be too surprising, given the conceptual similarly between coalition-proofness and other coalitionally oriented concepts such as Strong Nash equilibrium and the Core.

Non-existence can be established in very simple three player games. We provide the following example.[4]

EXAMPLE. Consider a three player (A, B, C) game of pie division, where the allocation is decided by majority. That is, all players simultaneously announce allocations; if two or more players propose the same allocation, then that division is implemented, while if all disagree, the pie is discarded. Players wish to maximize the expected size of their shares.

It is straightforward to establish that no Coalition-Proof Nash equilibrium exists for this game. Consider any mixed strategy equilibrium, $(\pi_A^*, \pi_B^*, \pi_C^*)$ (where π_i^* is a probability distribution over feasible allocations). Let e_i^* denote the expected size of i's share in this equilibrium. Without loss of generality, assume $1 - e_B^* - e_C^* = \eta > 0$. Now suppose that B and C form a coalition, and announce the allocation $(0, e_B^* + \eta/2, e_C^* + \eta/2)$ (where the entries indicate A, B, and C's shares, respectively). Clearly, this deviation benefits both players. Furthermore, since $(\pi_A^*, \pi_B^*, \pi_C^*)$ was an equilibrium, no other choice can yield an expected payoff for i exceeding e_i^* $(i = B, C)$. Also note that the deviation is Pareto efficient for B and C. Combining these observations, we conclude that the deviation is self-enforcing. Thus, the original equilibrium is not coalition-proof.

Thus far, we have been unable to find sufficient conditions which guarantee the existence of Coalition-Proof Nash equilibria for a reasonably large set of games. It is, of course, easy to see that coalition-proof equilibria exist in any model for which, given any set of actions by any proper subset of players, the game induced on the remaining players has a unique Nash equilibrium. In this case the set of coalition-proof equilibria coincides with the Pareto efficient frontier of the set of Nash equilibria.

Nevertheless, Coalition-Proof Nash equilibria certainly exist in a larger number of games than do Strong Nash equilibria. Further, examination of a number of examples indicates that coalition-proof equilibria do exist quite frequently, and in such cases provide a valuable tool for refining the Nash equilibrium set. Bernheim and Whinston [4] consider several such examples.

IV. Games in Extensive Form

For games in extensive form, it is well known that the Nash equilibrium behavior of individual agents may be dynamically inconsistent. This

[4] In the example, strategy spaces are continuous. We have also constructed examples of three person games with *finite* sets of pure strategies which do not have Coalition-Proof Nash equilibrium in mixed strategies—see Bernheim and Whinston [3] and Peleg [8].

TABLE II

A Two Player Game

	B_1	B_2	B_3
A_1	5, 5	0, 6	0, 0
A_2	6, 0	4, 4	0, 0
A_3	0, 0	0, 0	2, 2

problem has led to a variety of refinements, such as perfect equilibria (Selten [11, 12]) and sequential equilibria (Kreps and Wilson [7]).

In our context, it is natural to explore the implications of requiring dynamic consistency on the part of coalitions.[5] If our objective is to identify the class of self-enforcing agreements in environments where players have unlimited ability to communicate, but no recourse to binding contracts, such a requirement is essential. To clarify this point, we consider a simple example. Table II displays a static, two player game, where each player has three choices. There are two (static) Nash equilibria: (A_2, B_2) and (A_3, B_3).

Suppose this game is played twice, with no discounting. The repeated game has a unique coalition-proof equilibrium, which is constructed as follows. In period 1, the players choose (A_1, B_1). If anyone deviates in period 1, second period play is (A_3, B_3); otherwise, players choose (A_2, B_2). Equilibrium payoffs are (9, 9).

In addition to being coalition-proof, this equilibrium is also perfect. However, when there are opportunities to communicate throughout the game, it requires the group to behave in a dynamically inconsistent fashion. In particular, suppose A deviates in period one. The agreement (equilibrium) specifies that A and B will play the static equilibrium (A_3, B_3) in period 2. However, prior to this second round of play, they clearly have an incentive to arrange a joint deviation to (A_2, B_2). In other words, (A_3, B_3) is *not* a coalition-proof equilibrium for this proper subgame. Thus, the group cannot use it to enforce first period agreements if ongoing communication is possible. By this argument, the only possible outcome in period 2 is (A_2, B_2)—this is the unique coalition-proof equilibrium in the static game. Since this leaves no room for punishing deviations, players must choose (A_2, B_2) in period one as well.

We wish to generalize this argument, thereby producing a notion of "perfectly coalition-proof equilibria." Intuitively, we would like such

[5] Bernheim and Ray [5] have explored this question for two player games with both finite and infinite horizons. This section extends their analysis of the finite horizon to cases involving more than two players.

equilibria to satisfy several conditions. First, in order to be self-enforcing, an intertemporal agreement reached by the whole group should be dynamically consistent, in the sense that it should not specify actions in *any proper subgame* that are Pareto dominated by another vector of actions that is self-enforcing in that subgame. Second, no proper subgroup of players should be able to make a mutually advantageous deviation from the agreement in *any subgame*. Now, however, not only must any such deviation itself be self-enforcing, but it must also be a dynamically consistent agreement for the deviating set of players. Similar criteria apply to deviations from deviations (and so on). Finally, the agreement reached by the coalition of the whole should be Pareto efficient within the set of such (dynamically) self-enforcing agreements.

Formally, define the number of "stages" (t) in an extensive form game to be the maximum number of nested proper subgames. We restrict attention to games with a finite number of stages. We define our equilibrium concept inductively on both the number of players and stages (note that the following definition is consistent with the definition previously provided for one stage games).[6]

DEFINITION. (i) In a single player, single stage game Γ, $s^* \in S$ is a *Perfectly Coalition-Proof Nash equilibrium* if and only if s^* maximizes $g^1(s)$.

(ii) Let $(n, t) \neq (1, 1)$. Assume that Perfectly Coalition-Proof Nash equilibrium has been defined for all games with m players and s stages, where $(m, s) \leqslant (n, t)$, and $(m, s) \neq (n, t)$.

(a) For any game Γ with n players and t stages, $s^* \in S$ is *perfectly self-enforcing* if, for all $J \in \mathbf{J}$, s_J^* is a Perfectly Coalition-Proof Nash equilibrium in the game Γ/s^*_{-J}, and if the restriction of s^* to any proper subgame forms a Perfectly Coalition-Proof Nash equilibrium in that subgame.

(b) For any game Γ with n players and t stages, $s^* \in S$ is a *Perfectly Coalition-Proof Nash equilibrium* if it is perfectly self-enforcing, and if there does not exist another perfectly self-enforcing strategy vector $s \in S$ such that $g^i(s) > g^i(s^*)$ for all $i = 1,..., n$.

It is helpful to conceptualize perfectly coalition-proof Nash equilibria in the two player case. Consider any multi-stage two player game. First, we restrict agents to play Pareto undominated equilibria in all of the terminal stages. These allow us to support various equilibria in subgames consisting

[6] Note that this definition is *not* equivalent to the statement that "a perfectly coalition-proof Nash equilibrium is coalition-proof in every proper subgame." This alternative definition is, in our view, too strong—note that our simple example possesses no equilibrium which satisfies this condition.

of the terminal two-stage games; again, we restrict agents to play the equilibria which are Pareto undominated within this set. The recursion continues in this manner. Thus, our concept isolates the outcome $[(A_2, B_2), (A_2, B_2)]$ for the game, as desired.

This refinement bears some relationship to Rubinstein's notion of "strong perfect equilibria" (see Rubinstein [10]). In fact, it is easy to verify that every strong perfect equilibrium is perfectly coalition-proof. However, for the same reasons as before, we feel that the strong equilibrium concept is "too strong."

As with our original refinement, the existence of perfectly coalition-proof equilibria is problematic. Clearly, one cannot hope to obtain an existence theorem without placing additional structure on the extensive form. One natural and important restriction is perfect information. It is relatively easy to verify that every two-stage game of perfect information (such as the direct kingmaker game of Hurwicz and Schmeidler [6]) has a perfectly coalition-proof equilibrium. We have yet to examine games with three or more stages.

V. Conclusion

When players have unlimited ability to communicate and reach non-binding agreements regarding their strategy choices, a meaningful agreement requires more than the Nash best-response property. This is true because coalitions of players can typically arrange mutually beneficial agreements to deviate from a Nash equilibrium. Here we have introduced a stronger concept of self-enforcability that accounts for coalitional deviations, and have labeled the set of efficient self-enforcing agreements "Coalition-Proof Nash equilibria." As we have argued above, we feel that the coalition-proof concept captures the notion of an efficient self-enforcing agreement for such environments in a more satisfactory way than two other, frequently used, communication-based refinements: strong Nash equilibria and the Pareto dominance refinement.

It would be interesting to extend the analysis developed here in at least two directions. First, when pre-play communication is permitted, it may sometimes be natural to assume that players can correlate their strategies. Presumably, one could refine the notion of correlated equilibria (Aumann [2]) in the spirit of our analysis.

Second, it would be desirable to extend our analysis of Perfectly Coaliton-Proof Nash equilibrium to infinite stage games. However, this case poses additional problems. Consider, for example, the infinitely repeated prisoner's dilemma game. For certain ranges of discount rates, there are only two types of perfect (pure strategy) Nash equilibria: those which repeat the static (inefficient) Nash equilibrium, and those which sustain cooperation by using the static Nash equilibrium to punish unilateral deviations. Is either perfectly coalition-proof? Suppose cooperation is possible in some period. Then it is possible in all periods (all subgames are identical). Since it strictly dominates the static Nash equilibrium, group rationality implies that the latter outcome can never arise. But then there is no way to enforce cooperation, so the static Nash equilibrium becomes a possible outcome once more (it is no longer dominated by another possible outcome). Yet this, in turn, allows us to use the static Nash equilibrium to enforce cooperation, so cooperation is possible once again. Clearly, the issues are complex, and require further study.

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