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Duopoly Information Equilibrium: Cournot and Bertrand

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In a duopoly model where firms have private information about an uncertain linear demand, it is shown that if the goods are substitutes (not) to share information is a dominant strategy for each firm in Bertrand (Cournot) competition. If the goods are complements the result is reversed. Furthermore the following welfare results are obtained:

(i) With substitutes in Cournot competition the market outcome is never optimal with respect to information sharing but it may be optimal in Bertrand competition if the products are good substitutes. With complements the market outcome is always optimal.

(ii) Bertrand competition is more efficient than Cournot competition.

(iii) The private value of information to the firms is always positive but the social value of information is positive in Cournot and negative in Bertrand competition. *Journal of Economic Literature* Classification Numbers: 022, 026, 611.

1. INTRODUCTION

Consider a symmetric differentiated duopoly model in which firms have private market data about the uncertain demand. We analyze two types of duopoly information equilibrium, Cournot and Bertrand, which emerge, respectively, from quantity and price competition, and show that the incentives for information sharing and its welfare consequences depend crucially on the type of competition, the nature of the goods (substitutes or complements), and the degree of product differentation.

The demand structure is linear and symmetric, and allows the goods to be substitutes, independent or complements. There is uncertainty only about the

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common price intercept of the demand functions. Firm i receives a signal s_i which provides an unbiased estimate of the intercept and formulates a conjecture about the behavior of its competitor which together with its beliefs about the joint distribution of the intercept and the other firm's signal given it has received s_i determines the expected profit of any action firm i may take. We assume there is a joint Normal distribution of the intercept and the signals which is common knowledge to te firms. Firms have constant and equal marginal costs and are risk neutral. In this context a Bayesian Nash equilibrium requires that firms maximize expected profit given their conjectures, and that the conjectures be right.

We suppose that there is an agency, a trade association for example, which collects market data on behalf of each firm. Firm i may allow part of its private information to be put in a common pool available to both firms. The signal a firm receives is the best estimate of the price intercept given its private information and the information in the common pool. If there is no sharing of information the error terms of the signals are independent. Pooling of information correlates them positively. A firm, when sharing market data is, at the same time, giving more information to its rival and increasing the correlation of the signals.

Since we are interested in self-enforcing pooling agreements we consider a two-stage game where first the firms, prior to the market data collection, instruct the agency how much of their private information to put in the common pool. At the second stage market research is conducted and the agency sends the signals to the firms which choose an action (quantity or price). Therefore at the second stage a Bayesian (Cournot or Bertrand) game is played. We show that the two-stage game has a unique subgame perfect equilibrium in dominant strategies at the first stage. With substitutes it involves no pooling of information in Cournot competition and complete pooling in Bertrand competition. With complements the result is reversed.

When the goods are substitutes, in Cournot competition it turns out that increases in the precision of the rival's information and increases in the correlation of the signals have adverse effects on the expected profit of the firm and we find that not to share any information is a dominant strategy. Consequently the unique subgame perfect equilibrium of the two-stage game involves no information sharing. On the other hand, in Bertrand competition the two factors mentioned above have positive effects on the expected profit of the firm and to put everything in the common pool is a dominant strategy. This is true even when a firm's information is much better than the one of its rival. When the goods are complements the situation is reversed. Since, in any case, expected profit of firm i increases with the precision of its own information, with substitutes and in Bertrand competition the firms always obtain an efficient outcome (in profit terms). This is not the case in Cournot competition, where complete pooling of information may dominate in terms of profits the no sharing arrangement if the goods are not very good substitutes and therefore the firms are in a Prisoner's Dilemna type situation since not to share any information is a dominant strategy for each firm.

Consider now a symmetric situation where firms start with the same amount of information, neglect the resource cost of information and restrict attention to the two extreme arrangements: complete pooling of information and no pooling at all. If the goods are substitutes, then in welfare terms the market outcome (the outcome of the two-stage game) is never optimal with respect to information sharing in Cournot competition since pooling always dominates no pooling in terms of expected total surplus. In Bertrand competition it may be optimal if the goods are close enough substitutes. Then pooling dominates no pooling. Otherwise no pooling is better. This contrasts with the complements case, where the market outcome is always optimal: in either Cournot or Bertrand competition it maximizes expected total surplus with respect to information sharing.

We confirm in our incomplete information setting that Bertrand competition is more efficient (in expected total surplus terms, for example) than Cournot competition although with substitute products profits may be larger in the Bertrand case if we look at the outcome of the two-stage game.

We find that the private value of information to firm i is always positive and larger or smaller in Cournot than in Bertrand competition according to whether the goods are substitutes or complements. On the other hand the social value of information is positive in Cournot and negative in Bertrand competition.

In Section 2 we survey very briefly some related literature. Section 3 describes the model without uncertainty and states some results for this case. Section 4 extends the duopoly model to an incomplete information context. Section 5 deals with the two-stage game. Section 6 examines the welfare consequences of the two extreme information sharing arrangements. Efficiency and the value of information, private and social, are considered in Section 7. Concluding remarks including extensions and policy implications of the analysis follow in the last section.

2. Relationships with the Literature

Leland, in his paper about a monopoly facing an uncertain demand, states that "Although under certainty the choice of behavioral mode by a monopolistic firm is unimportant we show that it critically conditions performance under uncertainty" (Leland, 8, p. 278]). This paper can be seen, in part, as an extension to a duopoly context with incomplete information of this statement.

Strategic transmission of information is dealt with in an abstract setting by Crawford and Sobel [3]. The oligopoly literature on uncertain demand and incomplete information focuses on Cournot competition with homogenous product. This is the case of the work briefly surveyed below. They also assume that demand is linear with a random intercept. Normality is assumed in all of the papers considered except the first one.

Novshek and Sonnenschein [11] consider a duopoly model with constant costs and examine the incentives for the firms to acquire and release private information. Our modelling of the signals of the firms is based on theirs.

Basar and Ho [1] consider a duopoly model with quadratic cost functions. They show existence and uniqueness of affine equilibrium strategies and that, in equilibrium, expected profits of firm i increase with the precision of its information and decrease with the precision of the rival's information.

Clarke [2] considers an n-firm oligopoly model and shows that there is never a mutual incentive for all firms in the industry to share information unless they may cooperate on strategy once information has been shared.

Harris and Lewis [6] consider a duopoly model where firms in period one decide on plant capacity before market conditions are known. In period two they choose a level of production contingent on the state of demand and their plant size. They argue that observed differences in firm size and market share may be explained by producers having access to different information at the time of their investment decisions.

Gal–Or [5] considers an oligopoly model with two stages. At the first firms observe a private signal and decide whether to reveal it to other firms and how partial this revelation will be. At the second, they choose the level of output. She shows that no information sharing is the unique Nash equilibrium of the game both when private signals are completely uncorrelated and when they are perfectly correlated.

In our model Cournot competition with a homogenous product is a particular case. Our findings for this case are consistent with those of the authors who use the Normal model.

The demand structure (with no uncertainty) we consider is a symmetric version of a duopoly model proposed by Dixit [4] the duality and welfare properties of which are analyzed in Singh and Vives [13].

3. The Certainty Model

In our economy we have, on the production side, a monopolistic sector with two firms, each one producing a differentiated good, and a competitive numéraire sector, and, on the consumption side, a continuum of consumers of the same type with utility function linear and separable in the numéraire good. The representative consumer maximizes $U(q_1, q_2) - \sum_{i=1}^{2} p_i q_i$, where

$q_i \geqslant 0$, $i = 1, 2$, are the amounts of the goods and p_i, $i = 1, 2$, their prices. U is assumed to be quadratic, (strictly) concave and symmetric in q_1 and q_2. $U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}(\beta q_1^2 + 2\gamma q_1 q_2 + \beta q_2^2)$ with $\alpha > 0$, $\beta > |\gamma| \geqslant 0$. The goods are substitutes, independent, or complements according to whether $\gamma \gtreqless 0$. When $\beta = \gamma$ the goods are perfect substitutes. When $\beta = -\gamma$, "perfect complements." γ/β goes from 1 to -1. Note that the maximization problem of the consumer may not have a solution in the perfect complements case. Inverse demands are given by

$$p_1 = \alpha - \beta q_1 - \gamma q_2 \qquad \text{in the region of quantity space}$$
$$p_2 = \alpha - \gamma q_1 - \beta q_2 \qquad \text{where prices are positive.}$$

Letting $a = \alpha/(\beta + \gamma)$, $b = \beta/(\beta^2 - \gamma^2)$, and $c = \gamma/(\beta^2 - \gamma^2)$,

$$q_1 = a - bp_1 + cp_2 \qquad \text{in the region of price space where}$$
$$q_2 = a + cp_1 - bp_2 \qquad \text{quantities are positive.}$$

Firms have constant and equal marginal costs. From now on suppose prices are net of marginal cost. The Cournot equilibrium is the Nash equilibrium in quantities and the Bertrand equilibrium the one in prices. Profits of firm i are given by $\pi_i = p_i q_i$. Notice that since π_i is symmetric in p_i and q_i and the demand structure is linear, Cournot (Bertrand) competition with substitute products is the perfect dual of Bertrand (Cournot) competition with complements and they share similar strategic properties. For example, in both cases reaction functions slope downwards (upwards). A useful corollary is that we only need to compute equilibria for one type of competition and the other follows by duality. In the Cournot case there is a unique equilibrium given by $q_i = \alpha/(2\beta + \gamma)$, $i = 1, 2$, and correspondingly a unique Bertrand equilibrium given by $p_i = a/(2b - c)$, $i = 1, 2$, which equals $\alpha(\beta - \gamma)/(2\beta - \gamma)$.

In this context total surplus (TS) is just equal to $U(q_1, q_2)$. For future reference we give the equilibrium values of profits π, consumer surplus (CS), and total surplus for both types of competition.

Note that if q is the Cournot output and p the Bertrand price then $\pi_i^C = \beta q^2$ and $\pi_i^B = bp^2$, $i = 1, 2$, so that the profit formulae are perfectly dual. This is not the case for the other formulae. For example, (see Table I) the dual of $(\beta + \gamma)\,\alpha^2/(2\beta + \gamma)^2$ would be $(b - c)\,a^2/(2b - c)^2$ which equals $(\beta - \gamma)^2\,\alpha^2/(2\beta - \gamma)^2\,(\beta + \gamma)$ and not $\beta^2\alpha^2/(2\beta - \gamma)^2\,(\beta + \gamma)$. This is because the CS and TS functions do not treat prices and quantities symmetrically.

Note that when the goods are perfect substitutes ($\beta = \gamma$) the Bertrand price and profits are zero and we have the efficient outcome (price equal marginal cost). When the goods are perfect complements the Cournot consumer surplus is zero and the Bertrand magnitudes are not defined since at the Bertrand prices the consumer demands infinite quantities.

TABLE I

	π_i	CS	TS
Cournot	$\frac{\beta}{(2\beta-\gamma)^2}\alpha^2$	$\frac{\beta+\gamma}{(2\beta+\gamma)^2}\alpha^2$	$\frac{3\beta+\gamma}{(2\beta+\gamma)^2}\alpha^2$
Bertrand	$\frac{b}{(2b-c)^2}a^2$	$\frac{\beta^2}{(2\beta-\gamma)^2(\beta+\gamma)}\alpha^2$	$\frac{\beta(3\beta-2\gamma)}{(2\beta-\gamma)^2(\beta+\gamma)}\alpha^2$

The following proposition, the proof of which is in Singh and Vives [13], states that Bertrand competition is more efficient than Cournot competition. In all the propositions that follow we assume, unless otherwise stated, that $\beta > |\gamma|$, that is, we forget about the two extreme cases.

PROPOSITION 1. *Consumer surplus and total surplus are larger in Bertrand than in Cournot competition except when the goods are independent, in which case they are equal. Profits are larger, equal or smaller in Cournot than in Bertrand competition according to whether the goods are substitutes, independent or complements.*

The intuition behind the proposition is simple. Firms have less capacity to raise prices above marginal cost in Bertrand competition because the perceived elasticity of demand of a firm when taking as given the price of the rival is larger than that which the same firm perceives when taking the quantity of the rival as given. The result is that in Bertrand competition and in equilibrium firms quote lower prices than the Cournot ones. This is always good for consumers. For firms it is bad if the goods are substitutes since low prices mean low profits, if the goods are complements the situation is reversed, to increase profits firms have to lower prices to gain market.

4. THE UNCERTAINTY MODEL

Consider the model advanced in the last section but now with α, the demand intercept, being a random variable normally distributed with mean $\bar{\alpha}$ and variance $V(\alpha)$. Firm i receives a signal s_i which consists of α plus some noise ε_i, $s_i = \alpha + \varepsilon_i$, $i = 1, 2$. We assume that the error terms $(\varepsilon_1, \varepsilon_2)$ follow a bivariate normal distribution, independent of α, with zero means and covariance matrix $\left[\begin{smallmatrix} v_1 & \sigma_{12} \\ \sigma_{12} & v_2 \end{smallmatrix}\right]$, with $v_i \geqslant \sigma_{12} \geqslant 0$, $i = 1, 2$. All this is common knowledge to the firms. Given these assumptions, $E(\alpha | s_i) = (1 - t_i)\,\bar{\alpha} + t_i s_i$ and $E(s_j | s_i) = (1 - d_i)\,\bar{\alpha} + d_i s_i$, where $t_i = V(\alpha)/(V(\alpha) + v_i)$ and $d_i = (V(\alpha) + \sigma_{12})/(V(\alpha) + v_i)$, $i = 1, 2$, $i \neq j$. Note that $1 \geqslant d_i \geqslant t_i \geqslant 0$, so that both conditional expectations are convex combinations of $\bar{\alpha}$ and the received signal s_i. We say that signal s gives more precise information about α than signal s' if its mean squared prediction error is smaller, i.e., if $E\{\alpha - E(\alpha|s)\}^2 < E\{\alpha - E(\alpha|s')\}^2$. This is equivalent to saying that the variance of the error term of signal s is smaller than the one for s'. Therefore as v_i ranges from 0 to ∞ the signal goes from being perfectly informative to being not informative at all and at the same time t_i ranges from 1 to 0. When the information is perfect, $E(\alpha|s_i) = s_i$, when there is no information, $E(\alpha|s_i) = \bar{\alpha}$.

A strategy for a firm is a Borel measurable function that specifies an action, price or quantity, for each possible signal the firm may receive. Firms are assumed to be risk neutral. Each firm makes a conjecture about the opponent's strategy. A Bayesian Nash equilibrium[1] is then a pair of strategies and a pair of conjectures such that (a) each firm strategy is a best response to its conjecture about the behavior of the rival and (b) the conjectures are right.

Cournot Equilibrium

In the Cournot game, firms set quantities and a strategy for firm i specifies a quantity for each signal the firm may receive. We show that there is a unique equilibrium with linear (affine, to be precise) strategies.[2]

PROPOSITION 2. *The unique Bayesian equilibrium of the Cournot game is* $(\sigma_1^*(\cdot), \sigma_2^*(\cdot))$, *where* $\sigma_i^*(s_i) = A + B_i t_i(s_i - \bar{\alpha})$ *with* $A = \bar{\alpha}/(2\beta + \gamma)$ *and* $B_i = (2\beta - \gamma d_j)/(4\beta^2 - \gamma^2 d_1 d_2)$, $i = 1, 2, j \neq i$.

Proof. We first show that if firm 1 uses $\sigma_1^*(s_1) = A + B_1 t_1(s_1 - \bar{\alpha})$ the unique best response for firm 2 is to use $A + B_2 t_2(s_2 - \bar{\alpha})$. To see this notice that the expected profit of firm 2 choosing the quantity q_2 given the signal s_2 if firm 1 uses $\sigma_1^*(s_1)$ is

$$E(\alpha - \gamma\sigma_1^*(s_1) - \beta q_2 \mid s_2)\, q_2.$$

So the optimal choice for firm 2 is

$$q_2^* = \frac{1}{2\beta} E(\alpha - \gamma\sigma_1^*(s_1) \mid s_2) = \frac{1}{2\beta}(E(\alpha \mid s_2) - \gamma E(\sigma_1^*(s_1) \mid s_2))$$

which, after some computations, equals $A + B_2 t_2(s_2 - \bar{\alpha})$.

[1] See Harsanyi [7].

[2] Note that given our normality, assumption α and the signals may take negative values. Firms are constrained to choose positive princes and quantites. For convenience we ignore this and, given the firm's strategies that we derive, we can get negative prices and outputs for certain combinations of α and the signals. The probability of such an event can be made arbitrarily small by appropriately choosing the variances of the model.

Uniqueness follows similarly as in Basar and Ho [1] or Clarke [2], using the fact that $\beta \geqslant |\gamma|$. Q.E.D.

Remark 4.1. Suppose $v_i = v$, $i = 1, 2$, when the firms have no information at all. For $v = \infty$, the equilibrium strategy is constant and equal to $\bar{\alpha}/(2\beta + \gamma)$, the Cournot outcome when there is no uncertainty. As the information the firms receive improves, i.e., as v declines and t goes towards one, the slope of the linear strategy increases till it reaches $1/(2\beta + \gamma)$ when $t = 1$. Then $\sigma_1^*(s_1) = s_1/(2\beta + \gamma)$, which is the full information outcome.

Remark 4.2. The expected Cournot output always equals the Cournot certainty output (with $\bar{\alpha}$). Since $\sigma_i^*(s_i) = A + B_i t_i(s_i - \bar{\alpha})$ and $Es_i = \bar{\alpha}$, $E(\sigma_i^*(s_i)) = A$, which equals $\bar{\alpha}/(2\beta + \gamma)$. Note that when $s_i = \bar{\alpha}$, $\sigma_i^*(s_i) = A$ so that equilibrium strategies always go through the point $(s_i, q_i) = (\bar{\alpha}, \bar{\alpha}/(2\beta + \gamma))$.

We would like to know how expected profits in equilibrium are affected by variations in the precision and correlation of the signals the firms receive. Expected profits in equilibria are easy to compute. $E(\pi_1 \mid s_1) = E((\alpha - \gamma\sigma_2^*(s_2) - \beta\sigma_1^*(s_1)) \mid s_1)\sigma_1^*(s_1) = [E(\alpha - \gamma\sigma_2^*(s_2) \mid s_1) - \beta\sigma_1^*(s_1)]\sigma_1^*(s_1)$, but $E(\alpha - \gamma\sigma_2^*(s_2) \mid s_1) = 2\beta\sigma_1^*(s_1)$ according to the first order conditions, therefore $E(\pi_1 \mid s_1) = \beta(\sigma_1^*(s_1))^2$ and $E\pi_1 = \beta E(\sigma_1^*(s_1))^2$. Substituting in $\sigma_1^*(s_1) = A + B_1 t_1(s_1 - \bar{\alpha})$ we get $E\pi_1 = \beta(A^2 + B_1^2 t_1 V(\alpha))$. The slope of the linear strategy $\sigma_1^*(s_1)$, $B_1 t_1$, is the channel through which changes in the precision and correlation of the signals get transmitted to expected profits.

LEMMA 1. *The slope of* $\sigma_i^*(\cdot)$

(a) *increases with the precision of the information of firm i.*

(b) *decreases, is unaffected, or increases with the precision of its competitor's information and with the correlation of the signals according to whether the goods are substitutes, independent or complements.*

Proof. The slope in question is $B_1 t_1 = ((2\beta - \gamma d_2)/(4\beta^2 - \gamma^2 d_1 d_2))\, t_1$. Noting that $d_i = t_i(1 + (\sigma_{12}/V(\alpha)))$ and $t_i = V(\alpha)/(V(\alpha) + v_i)$, $i = 1, 2$, and using the fact that $d_i \leqslant 1$, $i = 1, 2$, we get by inspection that $B_1 t_1$ decreases with v_1 and, upon differentiating, that $\operatorname{sign} \partial B_1 t_1/\partial v_2 = \operatorname{sign} -\gamma = \operatorname{sign} \partial B_1 t_1/\partial \sigma_{12}$. Q.E.D.

The intuition behind (a) is clear. As firm 1 gets better information it trusts more the signal received and responds more to divergences of s_i and $\bar{\alpha}$ (see Remark 1). This is independent of the nature of the products. To understand (b) note that the covariance between the signals is $V(\alpha) + \sigma_{12}$, which is always positive and increasing in σ_{12} since $\sigma_{12} \geqslant 0$. Suppose the goods are substitutes. If firm 1 observes a high signal, $s_1 > \bar{\alpha}$ (recall that the signals are positively correlated), this means that probably firm 2 has observed a high signal too. Now, firm 2, according to (a), will produce less if v_2 is high than if it is low. The optimal thing to do for firm 1 is to produce a high output since in Cournot competition with substitutes if you expect the competitor to produce low you want to produce high. Therefore $\partial B_1 t_1/\partial v_2 > 0$ in this case. To evaluate the impact of an increased correlation in the signals we can reason similarly. If firm 1 observes a high signal, $s_1 > \bar{\alpha}$, it will produce less if σ_{12} is high than if it is low since in the former case the probability that the competitor has received a high signal too is larger and if firm 1 expects a high output of the competitor it has an incentive to reduce its own output. Therefore $\partial B_1 t_1/\partial \sigma_{12} < 0$.

We are now ready to state

PROPOSITION 3. *In equilibrium, the expected profit of firm i*

(a) *increases with the precision of its own information,*

(b) *decreases, is unaffected, or increases with the precision of the competitor's information and with the correlation of the signals according to whether the goods are substitutes, independent or complements.*

Proof. Recall that $E\pi_1 = \beta(A^2 + B_1^2 t_1 V(\alpha))$, then

(a) B_1 and t_1 decrease with v_1.

(b) Sign $\partial E\pi_1/\partial v_2 = \text{sign } \partial B_1/\partial v_2 = \text{sign } \gamma$ according to Lemma 1.

(c) Sign $\partial E\pi_1/\partial \sigma_{12} = \text{sign } \partial B_1/\partial \sigma_{12} = \text{sign } -\gamma$ according to Lemma 1.

($\gamma > 0$ for substitutes and $\gamma < 0$ for complements.) Q.E.D.

Bertrand Equilibrium

In the Bertrand game firms set prices and a strategy for firm i specifies a price for each signal the firm may receive. The duality argument gives us the Bertrand equilibrium strategies. Identifying α with a, β with b, γ with $-c$, and s_i with $\hat{s}_i$, where $\hat{s}_i = s_i/(\beta + \gamma)$, we get

PROPOSITION 2a. *The unique Bayesian equilibrium of the Bertrand game is* $(\tau_1^*(\cdot), \tau_2^*(\cdot))$, *where* $\tau_i^*(\hat{s}_i) = \hat{A} + \hat{B}_i t_i(\hat{s}_i - \bar{a})$ *with* $\hat{A} = \bar{a}/(2b - c)$ *and* $\hat{B}_i = (2b + cd_j)/(4b^2 - c^2 d_1 d_2)$, $i = 1, 2, j \neq i$.

Remarks similar to Remark 1 and 2 apply to the Bertrand case and Lemma 1 and Proposition 2 hold replacing $\sigma_i^*(\cdot)$ by $\tau_i^*(\cdot)$ and exchanging substitutes for complements.

5. The Two-Stage Game

In Section 4 we assumed firms received signals satisfying certain properties. We provide now, along the lines of Novshek and Sonnenschein, a rationale for these signals.

Suppose firm i starts with an n_i independent observation sample $(r_{i1},..., r_{in_i})$ satisfying $r_{ik} = \alpha + u_{ik}$, where the u_{ik}'s are i.i.d. normal with mean zero and variance σ_u^2 and independent of α. Firm i decides to put $\lambda_i n_i$, $0 \leqslant \lambda_i \leqslant 1$, observations in a common pool. The signal firm 1 receives, s_1, is then the best (minimum variance unbiased) estimate of α based on $n_1 + \lambda_2 n_2$ observations, its own sample plus the observations put in the common pool by firm 2. This is just the average, $s_1 = \alpha + (1/(n_1 + \lambda_2 n_2))(\sum_{k=1}^{n_1} u_{1k} + \sum_{k=1}^{\lambda_2 n_2} u_{2k})$. With this information structure the error terms of the signals $(\varepsilon_1, \varepsilon_2)$ follow a bivariate normal distribution with zero means and covariance matrix $\left[\begin{smallmatrix} v_1 & \sigma_{12} \\ \sigma_{12} & v_2 \end{smallmatrix}\right]$, where $v_i = \sigma_u^2/(n_i + \lambda_j n_j)$, $i = 1, 2, j \neq i$ and $\sigma_{12} = ((\lambda_1 n_1 + \lambda_2 n_2)/(n_1 + \lambda_2 n_2)(n_2 + \lambda_1 n_1))\,\sigma_u^2$. Note that $v_i \geqslant \sigma_{12} \geqslant 0$, $i = 1, 2$. λ_i is, thus, the proportion of observations firm i puts in the common pool. $\lambda_i \in \Lambda_i$, where $\Lambda_i = \{0, 1/n_i,..., (n_i - 1)/n_i, 1\}$, $i = 1, 2$. When $\lambda_1 = \lambda_2 = 0$ there is no pooling of information, $v_i = \sigma_u^2/n_i$, $i = 1, 2$, and $\sigma_{12} = 0$. When $\lambda_1 = \lambda_2 = 1$ there is complete pooling and $v_i = \sigma_{12} = \sigma_u^2/(n_1 + n_2)$, $i = 1, 2$. Information sharing has two effects: it decreases the variance of the error terms and it increases their correlation (and therefore the correlation of the signals).

Lemma 2. (a) *v_i decreases with λ_j, $j \neq i$, and is independent of λ_i.*

(b) *σ_{12} increases with λ_i if $\lambda_j < 1$, $i = 1, 2, j \neq i$.*

Otherwise is independent of λ_i.

Proof. (a) By inspection.

(b) $\sigma_{12} = ((\lambda_1 n_1 + \lambda_2 n_2)/(n_1 + \lambda_2 n_2)(n_2 + \lambda_1 n_1))\,\sigma_u^2$. Differentiating with respect to λ, one gets $\partial\sigma_{12}/\partial\lambda_1 = ((1 - \lambda_2\, n_1 n_2/(n_1 + \lambda_2 n_2)(n_2 + \lambda_1 n_1)^2)\,\sigma_u^2$. (In fact, λ_i is discrete but this does not matter here.) Q.E.D.

Consider now a two-stage game where first firms decide how much information are they going to put in the common pool. We suppose there is an agency, a trade association, for example, that collects an $n_1 + n_2$ observation sample and that forms the signals according to the instructions of the firms, the λ_i's. At the first stage, then, firm i picks independently $\lambda_i \in \Lambda_i$ and communicates it to the agency. At the second stage, firms, knowing the selected pair (λ_1, λ_2), play the Bayesian (Cournot or Bertrand) game. For each pair (λ_1, λ_2) we have a well-defined (proper) subgame. We are interested in subgame perfect Nash equilibria of the two-stage game, where equilibrium strategies form a (Bayesian) Nash equilibrium in every subgame (see Selten [12]).

LEMMA 3. *In Cournot competition with substitutes (or Bertrand with complements) expected profits of firm i decrease with λ_i. In Bertrand competition with substitutes (or Cournot with complements) expected profits of firm i increase with λ_i and with $\lambda_j, j \neq i$. If the goods are independent $E\pi_i$ are increasing with λ_j and unaffected by $\lambda_i, j \neq i$, $i = 1, 2$.*

Proof. Consider the Cournot case. Increases in λ_i give better information to firm j, $j \neq i$, and increase (maybe weakly) the correlation of the firm's signals. If the goods are substitutes, according to Proposition 3, both effects decrease $E\pi_i$. If the goods are complements both effects increase $E\pi_i$. Increases in λ_j, $j \neq i$, give better information to firm i and increase (maybe weakly) the correlation of the firm's signals. If the goods are complements both effects increase $E\pi_i$. Note that if they are substitutes the second decreases $E\pi_i$ so that nothing can be said a priori except if $\lambda_i = 1$. Then the covariance of the signals cannot be increased and the first effect dominates. The Bertrand case, as usual, follows by the duality argument. Q.E.D.

According to Lemma 3 in Cournot competition with substitutes to set $\lambda_i = 0$ is a dominant strategy for firm i since $E\pi_i$ decreases with λ_i whatever the value of λ_j, $j \neq i$. Symmetrically, in Bertrand competition to put all the information in the common pool is a dominant strategy for firm i. If the goods are independent $E\pi_i$ is unaffected by λ_i, $i = 1, 2$. Therefore we have established the following proposition.

PROPOSITION 4. *Suppose the goods are not independent. Then the two-stage game has a unique subgame perfect equilibrium in dominant strategies at the first stage. With substitutes it involves no pooling of information in Cournot competition and complete pooling in Bertrand competition. With complements the result is reversed.*

Remark 5.1. If the goods are independent any pair (λ_1, λ_2), $\lambda_i \in \Lambda_i$, $i = 1, 2$, is an equilibrium.

Remark 5.2. If the goods are perfect substitutes Proposition 4 holds for Cournot competition since in this case Lemma 3 holds as well. In Bertrand competition prices and expected profits are zero independently of the pooling decisions of the firms. Any pair (λ_1, λ_2) is an equilibrium in this case.

Remarks 5.3. Note that in Bertrand competition with substitute products to pool information is a dominant strategy for firm i even if the firm has much better information than its competitor, i.e., even if n_i is much larger than n_j, $j \neq i$.

TABLE II

Equilibrium Strategies for Firm i

	NP	P
C	$\frac{\bar{a}}{2\beta+\gamma}+\frac{t}{2\beta+\gamma t}(s_i-\bar{a})$	$\frac{1}{2\beta+\gamma}(\bar{a}+\tilde{t}(s_i-\bar{a}))$
B	$\frac{\bar{a}}{2b-c}+\frac{t}{2b-ct}(\hat{s}_i-\bar{a})$	$\frac{1}{2b-c}(\bar{a}+\tilde{t}(\hat{s}_i-\bar{a})$

Remark 5.4. Suppose the goods are substitutes. Note that given that at the second stage a Bayesian Bertrand equilibrium is reached the firms obtain an efficient outcome by completely pooling their information since $E\pi_i$ is increasing in λ_i and λ_j, $j \neq i$, $i = 1, 2$, in this situation. When the second stage is Cournot the firms, by choosing noncooperatively not to share any information, may not reach an efficient outcome. If the products are not very good substitutes complete pooling may dominate, in profits terms, the no pooling arrangement.

The following proposition compares the profits for the firms under the two extreme information sharing situations when each firm has a sample of size n. (For the rest of the paper we are going to restrict attention to these cases.) Let $v = \sigma_u^2/n$, $t = V(\alpha)/(V(\alpha)+v)$ and $\tilde{t} = V(\alpha)/(V(\alpha)+v/2)$. When no information is pooled $v_j = v$, $i = 1, 2$, and $\sigma_{12} = 0$ so that $t_i = d_i = t$, $i = 1, 2$. With complete pooling, $v_i = \sigma_{12} = v/2$, $i = 1, 2$, so that $d_i = 1$ and $t_i = \tilde{t}$, $i = 1, 2$.

First, we give expressions for equilibrium strategies and expected profits of the four possible combinations of Cournot C, or Bertrand B; pooling P, or not pooling NP. Using Proposition 2 and the expressions for expected profits we get Tables II and III.

TABLE III

Equilibrium Expected Profits for Firm i

	NP	P
C	$\beta\left(\frac{\bar{a}^2}{(2\beta+\gamma)^2}+\frac{t}{(2\beta+\gamma t)^2}V(\alpha)\right)$	$\frac{\beta}{(2\beta+\gamma)^2}(\bar{a}^2+\tilde{t}V(\alpha))$
B	$b\left(\frac{\bar{a}^2}{(2b-c)^2}+\frac{t}{(2b-ct)^2}V(a)\right)$	$\frac{b}{(2b-c)^2}(\bar{a}^2+\tilde{t}V(a))$

Let $E\pi_i^{\mathrm{P}}$ and $E\pi_i^{\mathrm{NP}}$ denote, respectively, expected profits of firm i with complete pooling and with no pooling of information.

PROPOSITION 5. *Let $v > 0$, then, in equilibrium,*

(a) *in Bertrand competition with substitute products (or in Cournot with complements), $E\pi_i^{\mathrm{P}} > E\pi_i^{\mathrm{NP}}$, $i = 1, 2$;*

(b) *in Cournot competition with substitutes (or in Bertrand with complements), letting $\mu = \gamma/\beta$,*

(i) *If $|\mu| \geqslant 2(\sqrt{2} - 1)$ then $E\pi_i^{\mathrm{NP}} > E\pi_i^{\mathrm{P}}$, $i = 1, 2$.*

(ii) *If $2(\sqrt{2} - 1) > |\mu| > \frac{2}{3}$ then $E\pi_i^{\mathrm{P}} \gtreqless E\pi_i^{\mathrm{NP}}$ iff*

$$\frac{v}{V(\alpha)} \gtreqless \frac{4 - 4\mu - 3\mu^2}{\mu^2 + 4\mu - 4}, \qquad i = 1, 2.$$

(iii) *If $|\mu| \leqslant \frac{2}{3}$ then $E\pi_i^{\mathrm{P}} > E\pi_i^{\mathrm{NP}}$, $i = 1, 2$.*

(See Fig. 1.)

Remark 5.5. When the goods are perfect substittes the proposition applies for the Cournot case. In the Bertrand case $E\pi_i$ are zero with pooling or no pooling.

Proof. (a) Follows from Lemma 3.

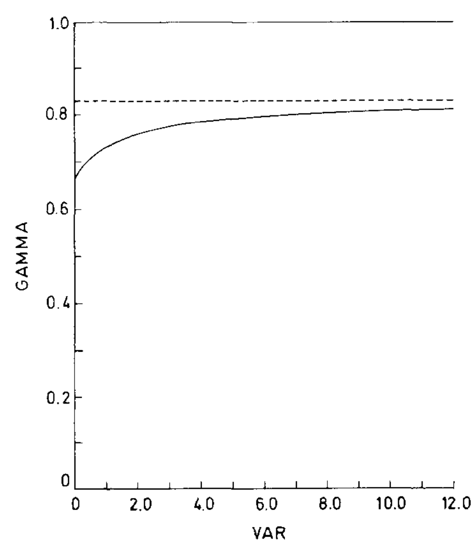

FIG. 1. Cournot with substitutes. Expected profits. $E\pi_i^{\mathrm{P}}$ greater (smaller) than $E\pi_i^{\mathrm{NP}}$ below (above) the continuous line, (GAMMA $\equiv \gamma/\beta$, VAR $\equiv v/V(\alpha)$).

(b) We only have to compare $t/(2\beta+\gamma t)^2$ with $\tilde{t}/(2\beta+\gamma)^2$ in the Cournot case and $t/(2b-ct)^2$ with $\tilde{t}/(2b-c)^2$ in the Bertrand case. The second follows from the first, noting that c is negative for complements and $\gamma/\beta = c/b$. After some computations, in the Cournot case, we have that $E\pi_i^{\mathrm{NP}} \gtreqless E\pi_i^{\mathrm{P}}$ iff $3\mu^2+4\mu-4 \gtreqless (v/V(\alpha))(4-4\mu-\mu^2)$. The values $\frac{2}{3}$ and $2(\sqrt{2}-1)$ are, respectively, the unique roots in the $[0,1]$ interval of the LHS and RHS. For $\mu < \frac{2}{3}$ the LHS is negative and the RHS positive. For $2(\sqrt{2}-1) > \mu > \frac{2}{3}$ both are positive. For $\mu > 2(\sqrt{2}-1)$ the LHS is positive and the RHS negative. Q.E.D.

The proposition has an easy intuitive explanation. Complete pooling of information cuts the variance of the error terms of the signals the firms receive by half and correlates perfectly the strategies of the firms. In Cournot competition with substitutes the second effect is bad for expected profits, the first, the joint decrease in variance, it is easily seen to be good by differentiating $E\pi_i$ with respect to v.

Which effect dominates depends on the degree of product differentiation. If the goods are close substitutes, i.e., γ/β is close to one, the correlation effect is going to prevail since it is weighted precisely by γ/β and conversely if the goods are not good substitutes. There is also an intermediate region where it pays to pool information if the precision of the firm's information is poor enough. Note that if the goods are perfect substitutes, $\beta=\gamma=1$, it never pays to share information. (See Fig. 1.) In view of Proposition 5 we see that when the goods are not very good substitutes, in Cournot competition, the firms face a Prisoner's Dilemma type situation since not to pool any information is a dominant strategy for each firm but by sharing information the firms would increase their profits.

6. Welfare

We analyze the welfare consequences, in terms of expected consumer surplus ECS and expected total surplus ETS of two extreme situations, no sharing and complete sharing of information, when the firms have the same information to start with. Tables IV and V give the equilibrum values of ECS and ETS in the four possible cases we are considering. Note that the Cournot and Bertrand expressions are not "dual." To compute them note that the expected value of any equilibrium strategy is equal to the equilibrium strategy when $\bar{\alpha}$ is known to obtain with certainty (see Remark 4.1). All the expressions decompose into two parts: one analogous to the certainty expression with $\bar{\alpha}$ (see Section 3) and another obtained by taking deviations from the mean $\bar{\alpha}$.

TABLE IV

Expected Consumer Surplus

	NP	P
C	$\frac{\beta+\gamma}{(2\beta+\gamma)^2}\bar{\alpha}^2 + \frac{\beta+\gamma t}{(2\beta+\gamma t)^2} tV(\alpha)$	$\frac{\beta+\gamma}{(2\beta+\gamma)^2}(\bar{\alpha}^2 + \tilde{t}V(\alpha))$
B	$\frac{\beta^2}{(2\beta-\gamma)^2(\beta+\gamma)}\bar{\alpha}^2 + \frac{\beta[\beta(4-3t)-\gamma t(1-t)]}{(2\beta-\gamma t)^2(\beta+\gamma)} V(\alpha)$	$\frac{\beta^2\bar{\alpha}^2 + [(2\beta-\gamma)^2 - (\beta-\gamma)(3\beta-\gamma)\tilde{t}]V(\alpha)}{(2\beta-\gamma)^2(\beta+\gamma)}$

TABLE V

Expected Total Surplus

	NP	P
C	$\frac{3\beta+\gamma}{(2\beta+\gamma)^2}\bar{\alpha}^2 + \frac{3\beta+\gamma t}{(2\beta+\gamma t)^2} tV(\alpha)$	$\frac{3\beta+\gamma}{(2\beta+\gamma)^2}(\bar{\alpha}^2 + \tilde{t}V(\alpha))$
B	$\frac{\beta(3\beta-2\gamma)}{(2\beta-\gamma)^2(\beta+\gamma)}\bar{\alpha}^2 + \frac{\beta[\beta(4-t)-t(3-t)]}{(2\beta+\gamma t)^2(\beta+\gamma)} V(\alpha)$	$\frac{\beta(3\beta-2\gamma)\bar{\alpha}^2 + [(2\beta-\gamma)^2 - (\beta-\gamma)^2\tilde{t}]V(\alpha)}{(2\beta-\gamma)^2(\beta+\gamma)}$

Before making any welfare comparisons we will see how variations in the precision of the firm's information have very different welfare effects in Cournot or Bertrand competition. Note that we consider here exogenous variations in v, i.e., variations induced not by information sharing decisions but by changes in the size of the sample firms receive (equal by assumption for both firms) or by changes in σ_u^2.

PROPOSITION 6. *If firms pool their information,* ECS *and* ETS *increase (decrease) with the precision of the information in Cournot (Bertrand) competition. If firms do not pool their information the same holds except when* $(|\gamma|/\beta)\, t$ *is greater (or equal) than* $\frac{2}{3}$, *then with complements* $\mathrm{ECS}^{\mathrm{C}}$ *decreases (weakly), and with substitutes* $\mathrm{ETS}^{\mathrm{B}}$ *increases (weakly), with the precision of the information.*

Proof. In obvious notation, $\mathrm{ECS}_{\mathrm{P}}^{\mathrm{C}}$ and $\mathrm{ETS}_{\mathrm{P}}^{\mathrm{C}}$ increase and $\mathrm{ECS}_{\mathrm{P}}^{\mathrm{B}}$ and $\mathrm{ETS}_{\mathrm{P}}^{\mathrm{B}}$ decrease with $\tilde{t}$ by inspection of the formulae in Tables IV and V. On the other hand, differentiating we get

$$\text{Sign}\left\{\frac{\partial \mathrm{ECS}_{\mathrm{NP}}^{\mathrm{C}}}{\partial t}\right\} = \text{Sign}\{2\beta + 3\gamma t\}$$

which is positive if $\dfrac{\gamma}{\beta} t > -\dfrac{2}{3}$ and nonnegative otherwise.

$$\text{Sign}\left\{\frac{\partial \mathrm{ECS}_{\mathrm{NP}}^{\mathrm{B}}}{\partial t}\right\} = \text{Sign}\{-(\beta - \gamma)(6\beta - \gamma t)\}$$

which is negative always.

$$\text{Sing}\left\{\frac{\partial \mathrm{ETS}_{\mathrm{NP}}^{\mathrm{C}}}{\partial t}\right\} = \text{Sign}\{6\beta + \gamma t\}$$

which is positive always.

$$\text{Sign}\left\{\frac{\partial \mathrm{ETS}_{\mathrm{NP}}^{\mathrm{B}}}{\partial t}\right\} = \text{Sign}\{(\beta - \gamma)(3\gamma t - 2\beta)\}$$

which is negative if $\dfrac{\gamma}{\beta} t < \dfrac{2}{3}$ and nonnegative otherwise.

Q.E.D.

Remark 6.1. If the goods are perfect substitutes the proposition applies for the Cournot case. In the Bertrand case ECS and ETS, which are equal since $E\pi_i$ are zero, are not affected by the precision of the firm's information. This is clear since firms set prices equal to marginal cost anyway.

PROPOSITION 7. *Let $\mu = \gamma/\beta$ and $\bar{\mu}$ be the unique root of $\mu^3 + \mu^2 - 8\mu + 4$ in the interval* $[0, 1]$ $(\bar{\mu} \approx 0.56)$.

In Cournot competition, pooling dominates no pooling in terms of ECS *and* ETS *except maybe when the goods are complements. In that case:*

(a) *If* $|\mu| \geqslant 2(\sqrt{2} - 1)$, *then* $\mathrm{ECS}_{\mathrm{NP}} > \mathrm{ECS}_{\mathrm{P}}$.

(b) *If* $2(\sqrt{2} - 1) > |\mu| \geqslant \bar{\mu}$ *then* $\mathrm{ECS}_{\mathrm{NP}} \gtreqless \mathrm{ECS}_{\mathrm{P}}$ *iff*

$$\frac{v}{V(\alpha)} \lesseqgtr \frac{4 + 8\mu + \mu^2 - \mu^3}{\mu^2 - 4\mu - 4}.$$

(See Fig. 2.)

In Bertrand competition, no pooling dominates pooling in terms of ECS *and* ETS *except maybe when the goods are substitutes. In that case:*

(a) *If* $\mu \geqslant 2(\sqrt{2} - 1)$ *then* $\mathrm{ETS}_{\mathrm{P}} > \mathrm{ETS}_{\mathrm{NP}}$.

(b) *If* $2(\sqrt{2} - 1) > \mu \geqslant \bar{\mu}$, *then* $\mathrm{ETS}_{\mathrm{P}} \gtreqless \mathrm{ETS}_{\mathrm{NP}}$ *iff*

$$\frac{v}{V(\alpha)} \lesseqgtr \frac{4 - 12\mu + 9\mu^2 - \mu^4}{8\mu - 3\mu^2 - \mu^3 - 4}.$$

(See Fig. 3.)

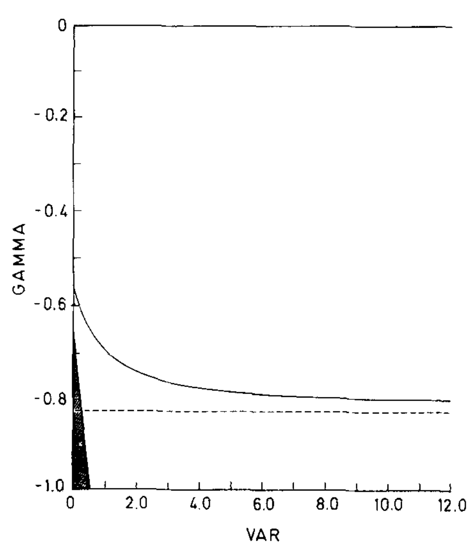

FIG. 2. Cournot with complements. Expected consumer surplus. In the interior of the shaded region $\partial \mathrm{ECS}^{\mathrm{NP}}/\partial t > 0$. $\mathrm{ECS}^{\mathrm{P}}$ greater (smaller) than $\mathrm{ECS}^{\mathrm{NP}}$ above (below) the continuous line. (GAMMA $\equiv \gamma/\beta$, VAR $\equiv v/V(\alpha)$).

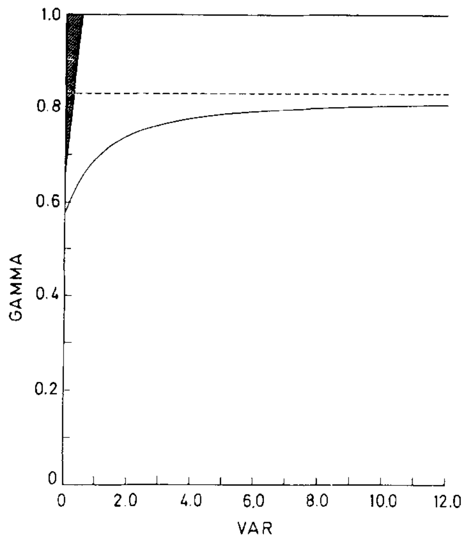

FIG. 3. Bertrand with substitutes. Expected total surplus. In the interior of the shaded region $\partial \mathrm{ETS}^{\mathrm{NP}}/\partial t > 0$. $\mathrm{ETS}^{\mathrm{P}}$ greater (smaller) than $\mathrm{ETS}^{\mathrm{NP}}$ above (below) the continuous line. (GAMMA $\equiv \gamma/\beta$, VAR $\equiv v/V(\alpha)$).

Proof. Let $f_2(\mu) = 4 - 12\mu + 9\mu^2 - \mu^4$ and $f_1(\mu) = 4 - 8\mu + \mu^2 + \mu^3$. Note that $f_2(\mu) = (1-\mu) f_1(\mu)$. It is easily seen that $\bar{\mu}$ is the unique root of f_1 in $[0, 1]$ ($\bar{\mu} \approx 0.56$), so that $f_2(\bar{\mu}) = 0$. Also the unique root of $4 + 8\mu + \mu^2 - \mu^3$ in $[-1, 0]$ is $-\bar{\mu}$ since this function is equal to $f_1(-\mu)$. Let $g_2(\mu) = 8\mu - 3\mu^2 - \mu^3 - 4$ and $g_1(\mu) = \mu^2 + 4\mu - 4$. Note that $g_2(\mu) = (1-\mu) g_1(\mu)$. The unique root of g_1 in $[0, 1]$ is $2(\sqrt{2} - 1)$ (which is 0.83 approximately), therefore $2(\sqrt{2} - 1)$ is also a root of g_2. Now, using the formulae in Tables IV and V and after some computations we obtain,

(i) $\mathrm{ECS}^{\mathrm{C}}_{\mathrm{P}} \gtreqless \mathrm{ECS}^{\mathrm{C}}_{\mathrm{NP}}$ iff $f_1(-\mu) \gtreqless (v/V(\alpha))\, g_1(-\mu)$. For $\mu \geqslant -\bar{\mu}$ the LHS is nonnegative and the RHS negative. For $-\bar{\mu} > \mu > 2(1 - \sqrt{2})$ both are negative. For $\mu \leqslant 2(1 - \sqrt{2})$ the LHS is negative and the RHS is nonnegative.

(ii) $\mathrm{ETS}^{\mathrm{C}}_{\mathrm{P}} \gtreqless \mathrm{ETS}^{\mathrm{C}}_{\mathrm{NP}}$ iff $12 - 5\mu^2 - \mu^3 \gtreqless (v/V(\alpha))(3\mu^2 + 4\mu - 12)$. The LHS is always positive and the RHS is always negative. (Recall $|\mu| < 1$.)

(iii) $\mathrm{ECS}^{\mathrm{B}}_{\mathrm{NP}} \gtreqless \mathrm{ECS}^{\mathrm{B}}_{\mathrm{P}}$ iff $(12 + 3\mu^3 - 8\mu - 7\mu^2)(v/V(\alpha)) \gtreqless \mu^4 + 5\mu^2 + 12\mu - 6\mu^3 - 12$. The LHS is always positive and the RHS always negative. (Recall $|\mu| < 1$.)

(iv) $\mathrm{ETS}^{\mathrm{B}}_{\mathrm{P}} \gtreqless \mathrm{ETS}^{\mathrm{B}}_{\mathrm{NP}}$ iff $g_2(\mu)(v/V(\alpha)) \gtreqless f_2(\mu)$. For $\mu \leqslant \bar{\mu}$, the LHS is negative and the RHS nonnegative. For $\bar{\mu} < \mu < 2(\sqrt{2} - 1)$ both are negative. Otherwise the LHS is nonnegative and the RHS negative. (Note that $\mathrm{ECS}^{\mathrm{B}}_{\mathrm{NP}} = \mathrm{ECS}^{\mathrm{B}}_{\mathrm{P}}$ and $\mathrm{ETS}^{\mathrm{B}}_{\mathrm{NP}} = \mathrm{ETS}^{\mathrm{B}}_{\mathrm{P}}$ when $\mu = 1$.) Q.E.D.

Remark 6.2. The proposition applies when the goods are perfect substitutes ($\mu = 1$) if Cournot competition prevails. In Bertrand competition pooling makes no difference in ECS or ETS. Prices equal marginal cost and consumers get the maximum surplus they can get in either case.

Remark 6.3. The exceptions in Proposition 6 and 7 are when we consider $\mathrm{ECS}^{\mathrm{C}}$ or $\mathrm{ETS}^{\mathrm{B}}$. In Cournot competition, welfare, in terms of ECS or ETS, increases with the precision of information and is greater with pooling of information with the possible exception of ECS when the goods are strong complements. In Bertrand competition, welfare decreases with the precision of information and is greater with no information sharing except possibly in terms of ETS when the products are good substitutes. (See Figs. 2 and 3.)

To keep things simple and in the spirit of the welfare comparisons we are making suppose that firms can only choose to share completely or not share at all the information they own, i.e. they instruct the testing agency $\lambda_i \in \{0, 1\}$, $i = 1, 2$. We would like to compare in welfare terms the outcome of the two-stage game, the market outcome, with the outcome an authority or planner could induce either by not allowing the agency to form or by requiring that all information be disclosed, thus enforcing no pooling or complete pooling of information respectively. The objective of the planner would be to maximize ETS. We say an outcome is optimal (with respect to information sharing) if it gives at least as much ETS as the planner can obtain.

PROPOSITION 8. *If the goods are complements the market outcome is always optimal. If the goods are substitutes, in Cournot competition the market outcome is never optimal, in Bertrand competition it is optimal if the goods are close to perfect substitutes or if they are moderately substitutes and the precision of the information is low.*

Proof. For complements. In Cournot competition the market outcome involves pooling, $\lambda_i = 1$, $i = 1, 2$, and $\mathrm{ETS}^{\mathrm{C}}_{\mathrm{NP}} < \mathrm{ETS}^{\mathrm{C}}_{\mathrm{P}}$ from Proposition 7. In Bertrand, we have $\mathrm{ETS}^{\mathrm{B}}_{\mathrm{NP}} > \mathrm{ETS}^{\mathrm{B}}_{\mathrm{P}}$ and the market outcome involves no pooling, $\lambda_i = 0$, $i = 1, 2$.

For substitutes. In Cournot competition the market outcome is NP, but $\mathrm{ETS}^{\mathrm{C}}_{\mathrm{NP}} < \mathrm{ETS}^{\mathrm{C}}_{\mathrm{P}}$. In Bertrand competition the market outcome is P and $\mathrm{ETS}^{\mathrm{B}}_{\mathrm{P}} > \mathrm{ETS}^{\mathrm{B}}_{\mathrm{NP}}$ under (a) and (b) of Proposition 7. Q.E.D.

Remark 6.4. If the goods are independent there are four equilibria in the two-stage game and therefore four possible market outcomes. Discarding the nonsymmetric ones, we will have that one of the remaining is going to be optimal in each type of competition. (The pooling one in Cournot and the no pooling one in Bertrand.)

Remark 6.5. If the goods are perfect substitutes Proposition 8 holds. In Bertrand competition any pooling arrangement is self-enforcing (i.e., it can be a market outcome) and optimal since in that case ETS is constant over arrangements.

7. Efficiency and the Value of Information

In this section we extend Proposition 1 to the incomplete information case, confirming thus that "Bertrand competition is more efficient than Cournot competition," and we compare the private and social value of information under the two types of competition.

PROPOSITION 9. *In welfare terms, either* ECS *or* ETS, *Bertrand is strictly better than Cournot. Furthermore,* $E\pi_i^C \gtreqless E\pi_i^B$ *according to whether the goods are substitutes, independent or complements. This holds comparing either the no pooling subgames or the complete pooling ones.*

Proof. First note that it is sufficient to show it when $\bar{\alpha} = 0$. If $\bar{\alpha} > 0$, then all the expressions have "certainty" terms (with $\bar{\alpha}$) which we can rank according to the certainty proposition. Let then $\bar{\alpha} = 0$. Using Tables III–V with $\bar{\alpha} = 0$, the relevant inequalities follow noting that $|\gamma|/\beta$, t, and $\tilde{t}$ are between zero and one. Q.E.D.

Remark 7.1. Notice that when the goods are independent and we have two monopolies expected profits are equal with price and quantity setting but, contrary to the certainty case, ECS qnd ETS are larger with price setting. Thus under uncertainty and incomplete information consumers and society have another reason to prefer price over quantity setting apart from the traditional one that firms have less monopoly power under Bertrand competition.

Remark 7.2. Bertrand competition is more efficient than Cournot competition (in terms of ETS even if we look at the outcomes of the two-stage game. Take the substitutes case. With Cournot the market outcome involves no pooling and with Bertrand, pooling. From Propositions 7 and 9 we know that $\mathrm{ETS}_P^B > \mathrm{ETS}_P^C > \mathrm{ETS}_{NP}^C$, so that $\mathrm{ETS}_P^B > \mathrm{ETS}_{NP}^C$. It may happen though that Bertrand profits be larger than the Cournot ones. From Table III we get $E\pi_{NP}^C = \beta((\bar{\alpha}^2/(2\beta+\gamma)^2) + (t/(2\beta+\gamma t)^2)\, V(\alpha))$ and $E\pi_P^B = (b/(2\mathrm{b}-c)^2)(\bar{a}^2 + \tilde{t}V(a))$. We know that the "certainty" term (involving $\bar{\alpha}$ or $\bar{a}$) will be larger in the Cournot case, but not very much if the products are very differentiated (γ small). The other term may be larger in the Bertrand case, and make up the difference, if the information of the firms is not very precise ($v/V(\alpha)$ not close to zero), and if there is enough basic uncertainty

TABLE VI

Private Value of Information to Firm i

	P	NP
C	$\frac{\beta}{(2\beta+\gamma t)^2} tV(\alpha)$	$\frac{\beta}{(2\beta+\gamma)^2}(\tilde{t}-t)V(\alpha)$
B	$\frac{b}{(2b-ct)^2} tV(a)$	$\frac{b}{(2b-c)^2}(\tilde{t}-t)V(a)$

($V(\alpha)$ not too small relative to $\bar{\alpha}$). For example, if $\gamma = 0.1$, $\bar{\alpha} = 10$, $V(\alpha) = v = 1$, then $E\pi_{\mathrm{P}}^{\mathrm{B}} > E\pi_{\mathrm{NP}}^{\mathrm{C}}$. Similarly, if the goods are complements one sees immediately that $\mathrm{ETS}_{\mathrm{NP}}^{\mathrm{B}} > \mathrm{ETS}_{\mathrm{P}}^{\mathrm{C}}$ but it may be the case that $E\pi_{\mathrm{P}}^{\mathrm{C}} > E\pi_{\mathrm{NP}}^{\mathrm{B}}$ for the same type of parameter configurations as above.

The Value of Information

Recall we are considering symmetric situations where both firms receive an n-sample. We define the private value of information to firm i, PVI as the difference in expected profits between receiving the n-sample and getting no information at all (the other firm gets an n-sample in either case). As before let $v = \sigma_u^2/n$ and $t = V(\alpha)/(V(\alpha)+v)$. When $n_1 = 0$ and $n_2 = n$, with no pooling $v_1 = \infty$, $v_2 = v$, and $\sigma_{12} = 0$ so that $d_1 = t_1 = 0$ and $d_2 = t_2 = t$; with pooling $v_i = \sigma_{12} = v$, $i = 1, 2$, so that $d_i = 1$, $t_i = t$, $i = 1, 2$. Now, recalling that the formulae for the Cournot expected profits is $\beta(A^2 + B_i^2 t_i V(\alpha))$ where $A = \bar{\alpha}/(2\beta+\gamma)$ and $B_i = (2\beta - \gamma d_j)/(4\beta^2 - \gamma^2 d_1 d_2)$, $j \neq i$, and using the formulae in Table III we can get the PVI in the Cournot case. The Bertrand case follows by duality. Table VI gives the results.

We define the social value of information (SVI) as the difference in ETS between the firms receiving signals of the same finite variance v and the firms receiving no information at all. Using the formulae in Table V and noting that when t or $\tilde{t}$ equal zero, $\mathrm{ETS}^{\mathrm{C}}$ is just the Cournot certainty expression (with $\bar{\alpha}$) while $\mathrm{ETS}^{\mathrm{B}}$ is the Bertrand certainty expression plus $V(\alpha)/(\beta+\gamma)$, one gets Table VII.

TABLE VII

Social Value of Information

	NP	P
C	$\frac{3\beta+\gamma t}{(2\beta+\gamma t)^2} tV(\alpha)$	$\frac{3\beta+\gamma}{(2\beta+\gamma)^2}\tilde{t}V(\alpha)$
B	$-\frac{(\beta-\gamma t)(\beta-\gamma)}{(2\beta-\gamma t)^2(\beta+\gamma)} tV(\alpha)$	$-\frac{(\beta-\gamma)^2}{(2\beta-\gamma)^2(\beta+\gamma)}\tilde{t}V(\alpha)$

PROPOSITION 10. *The social value of informtion is positive in Cournot and negative in Bertrand competition. The private value of information to the firms is always positive and larger or smaller in Cournot than in Bertrand competition according to whether the goods are substitutes or complements. This holds comparing either the no pooling subgames or the pooling ones and also in the two-stage game where the information sharing decision is endogenous.*

Proof. From the proof of Proposition 9 it follows that $(\beta/(2\beta+\gamma t)^2)\, V(\alpha) \gtreqless (b/(2b-ct)^2)\, V(a)$ if and only if $\gamma \gtreqless 0$ and $(\beta/(2\beta+\gamma)^2)\, V(\alpha) \gtreqless (b/(2b-c)^2)\, V(a)$ if and only if $\gamma \gtreqless 0$, so that from Table VI we get that $\mathrm{PVC}^{\mathrm{C}} \gtreqless \mathrm{PVI}^{\mathrm{B}}$ if and only if $\gamma \gtreqless 0$ with either pooling or not pooling of information. Now, with substitutes $\mathrm{PVI}^{\mathrm{C}}_{\mathrm{NP}} > \mathrm{PVI}^{\mathrm{C}}_{\mathrm{P}}$ since $1/(2\beta+\gamma t)^2 > 1/(2\beta+\gamma)^2$ and $t > \tilde{t} - t$ and therefore $\mathrm{PVI}^{\mathrm{C}}_{\mathrm{NP}} > \mathrm{PVI}^{\mathrm{B}}_{\mathrm{P}}$. With complements we get similarly that $\mathrm{PVI}^{\mathrm{B}}_{\mathrm{NP}} > \mathrm{PVI}^{\mathrm{C}}_{\mathrm{P}}$. The inequalities for the SVI follow by inspection of Table VII. Q.E.D.

Remark 7.3. When the goods are perfect substitutes the private and social value of information is zero in Bertrand competition since prices equal marginal cost independently of the information received.

6. CONCLUDING REMARKS: EXTENSIONS AND POLICY IMPLICATIONS

Extensions

We have considered a symmetric duopoly model. In principle there should be no difficulty in relaxing the symmetry assumption (and deal with a nonsymmetric model as in Singh and Vives [13]) or the duopoly assumption and deal with more than two firms. Computations would be very cumbersome, particularly when trying to relax both at the same time.

Note that the Cournot model can be reconsidered to accommodate the case where firms are uncertain about their common marginal costs and receive signals giving information about them. We could imagine a situation where firms have a common technology with only one variable input, oil, for example, the price of which is uncertain. The variable cost of producing one unit of output m is a constant times the price of oil. Letting $\hat{\alpha} = \alpha - m$ we can use our model with $\hat{\alpha}$. The signals in this example could come from an energy forecasting agency. For the Bertrand case new computations need to be made.[3]

[3] A situation where two firms are bidding for a government contract can be thought as a Bertrand model. If the firms have the same (unknown) costs we are in the *common value* case of the auction literature. The incentives to gather and share information in this context (see [9, 10]) contrast sharply with the results we have obtained in the paper for Bertrand competition with substitutes. I am grateful to an anonymous referee for pointing this out.

Finally, a word about the Normality assumption. We use it for analytical convenience although in our context it would be more natural to use a distribution with compact support. In fact the property we need to get linear (affine) equilibrium strategies is that conditional expectations be linear (affine). The Normal is the most common distribution with this property. Note that if $E(\alpha \mid s_i) = T + ts_i$, for some constants T and t, and $Es_i = \bar{\alpha}$ then necessarily $T = (1-t)\,\bar{\alpha}$ since $E\{E(\alpha \mid s_i)\} = \bar{\alpha}$ so that $\bar{\alpha} = T + t\bar{\alpha}$.

Policy Implications

We have seen that the market outcomes and optimal outcomes (with respect to information sharing) depend crucially on the type of competition, the nature of the goods and the degree of product differentation. This has immediate policy implications regarding information sharing. If the goods are complements the best policy is no intervention since the market outcome is already optimal. If the goods are substitutes and Cournot competition prevails, public policy should encourage information sharing. (It could do that by requiring, e.g., trade associations or testing agencies to disclose all information to the firms.) If Bertrand competition prevails and the goods are close substitutes no intervention is needed. If they are poor substitutes pooling of information should be avoided (no trade association allowed to form). In the intermediate region where the goods are moderately good substitutes if the precision of the firms' information is good enough no intervention is required, otherwise the authority should discourage the sharing of market data. Note that in this case the authority has no incentive to improve the precision of the firms' information (by subsidizing information acquisition, e.g.) since expected total surplus is decreasing with the precision of information.

We see therefore that policy prescriptions, or inferences of firm behavior, based on the Cournot model with homogenous product could be misleading when out of context. For example, if the goods are substitutes observing the firms pool information in Cournot competition is not evidence that they are setting quantities collusively if the goods are not very good substitutes. In this case pooling of information increases expected profits and although a pooling agreement is not self-enforcing in our two-stage game it could be in a repeated situation. Firms would be colluding then in their market research but not in setting outputs.

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