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Limited Liability and Incentive Contracting with Ex-ante Action Choices*

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This article examines a principal–agent model of financial contracting in which a risk-neutral entrepreneur (agent) makes an unobservable ex-ante effort choice while employing the investment funds of a risk-neutral investor (principal). The key innovation is that the investment contract is subject to statutory liability limits. Given these liability limits, two settings are considered, one in which the investor payoff function is also constrained to be monotonically nondecreasing in firm profit, and another in which no such "monotonic contract" constraint is imposed. In the former case, a standard debt contract is shown to emerge and a "first best" effort choice is not achieved. In the latter setting, the optimum is characterized by a "live-or-die" payoff function, and a "first best" effort level may or may not be realized. *Journal of Economic Literature* Classification numbers: 026, 022, 521.

1. INTRODUCTION

A firm's liability to its security-holders is limited to firm assets and profits. Likewise, security-holders are not liable for firm losses over and above their investment. These liability rules restrict the types of securities/contracts which firms can write. Two recent papers, Sappington [19] and Demski, Sappington, and Spiller [3], have investigated the implications of these liability constraints for optimal incentive contracts. However, both papers specify models in which the agents choose their actions *after* observing the state of nature. This paper evaluates the effects of liability constraints in a different incentive contracting problem, namely, one in which an agent's "effort" choice is made *before* the state of nature is realized (as in Ross [18], Shavell [20], Holmstrom [7], and Grossman and Hart [5], among many others)[1]

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[1] Holmstrom [7] and Lewis [13], among others, have noted the prospective importance of liability limits in principal–agent models of the kind examined here. However, to my knowledge, prior research on these models has not investigated the effects of liability limits on qualitative properties of optimal incentive contracts, which is the objective of this paper.

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To focus the analysis on effects of liability rules (rather than risk-sharing issues), I assume that both the principal and the agent are risk-neutral. In the absence of liability limits, it is well known that this setting will yield a contract that gives the principal a fixed payment, thereby inducing the agent to choose his "first-best" effort level (see, for example, Shavell [20] and Harris and Raviv [6]). However, with limited liability, this contract is not feasible unless there is no possibility that ex-post firm profits will be less than the fixed payment.

Motivated by the infeasibility of a fixed payment, the following analysis develops an entrepreneur's financial contract choice problem wherein (1) investors (c.f., principals) are competitive, simply requiring that they receive a market-determined "fair" return on their investment; (2) given the financial contract, the entrepreneur (c.f., the agent) makes an effort choice that the investors cannot observe and, thus, that cannot be specified in the contract; and (3) the entrepreneur chooses the investor payoff function (c.f., fee schedule) to maximize his expected utility (of profit and effort) subject to (i) limited liability restrictions, (ii) the investor expected payoff requirement, and (iii) his own effort choice responses.[2] Given this construction, two cases are considered, one in which no constraints are placed on the form of the financial contract (other than limited liability), and another in which the investor's payoff function is constrained to be nondecreasing in firm profit. The latter "monotonic contract" constraint can be motivated either by a requirement that investors never have an incentive to sabotage the firm or by an ability of entrepreneurs to costlessly revise their profit reports upward (with hidden borrowing, for example). This restriction is deemed to be interesting not only because observed financial contract forms (including debt, equity, convertible bonds, and stock options) are all nondecreasing, but also because this constraint elicits an optimal contract that is pervasive in practice. Specifically, the optimal monotonic contract is shown to take a standard debt form. Without the nondecreasing payoff restriction, limited liability instead leads to a contract of the following "live-or-die" form: The investor takes a constant share of firm profit when this profit is less than some critical level and nothing when the profit is higher.

On an intuitive level, the debt-contracting result can be explained as follows: With any monotonic contract that is strictly increasing in some region, some of the benefits of marginal effort are shared with investors; thus, since the entrepreneur still bears the total cost of effort, he will choose

[2] The most natural specification of this contracting problem entails the agent choosing the contract rather than the principal. In contrast, most principal–agent models have the principal choosing the contract, subject to the agent receiving a prespecified reservation utility level. Since solutions to both problems characterize points on the same utility-possibility-frontier, qualitative properties of these solutions are also the same.

an effort level that is less than his "first best" choice.[3] The latter observation implies that the entrepreneur will select a contract form that implicitly commits him to the highest possible effort level, thereby permitting him to reap as much of the "first best" surplus as possible. Now note that higher effort increases probability weight placed on high-profit outcomes. Thus, with a contract that gives the entrepreneur maximal payoffs in high-profit states, the entrepreneur is induced to choose maximal effort. Among monotonic contracts subject to liability limits, the debt contract has this "maximal high-profit-state payoff" property and, hence, will be selected.

With a debt contract, the entrepreneur still works "too little" (relative to a first best). This inefficiency gives rise to a positive value for investor information on effort. However, the absence of a "monotonic contract" constraint often leads to a "first best" effort choice, implying no value to information (as in principal–agent analyses without liability limits). The intuition for the latter result is roughly as follows: Without the monotonicity constraint, the entrepreneur must still share profits with the investor in some states. However, by giving the entrepreneur a 100% share of profits in some high-profit states, the share of marginal-effort benefits captured by investors in lower-profit states is offset by investor losses from a higher probability of zero payoff. By appropriate choice of the critical profit level above which investors get nothing, the entrepreneur will lose none of the marginal effort benefits (at the "first best" effort choice) and "first best" efficiency will prevail.

There are many analyses of capital structure choices in the finance literature that have also noted the importance of agency issues for financial contracting.[4] By endogenizing the contract choice and allowing for almost any contract form, this analysis formalizes and generalizes much of the intuition developed in the famous Jensen and Meckling [10] paper, from which most of this literature springs.[5,6] In doing so, the analysis provides an alternative explanation for use of standard debt instruments (in a general contract setting) to those provided in recent models of costly state

[3] Note that the only monotonic contract that (i) satisfies liability constraints and (ii) is *not* strictly increasing in some region, is one with zero investor payoff in all states of nature. The latter contract cannot meet any positive investor expected return requirement and, thus, is ruled out here.

[4] See, for example, Darrough and Stoughton [1], Williams [21], and the references therein.

[5] Darrough and Stoughton [1] point out the prospective importance of the investigation undertaken in this paper by noting (p. 503) that general results in the principal–agent literature do "not extend trivially to the corporate finance environment because limited liability does impose some restrictions on the effective contract space."

[6] Another notable distinction between this paper and the specification of Jensen and Meckling [10] (as well as Williams [21]) is the latter authors' use of an *ex post* perquisite choice variable rather than the *ex-ante* effort choice variable of interest here.

verification (e.g., Gale and Hellwig [4] and Williamson [22]) and adverse selection (e.g., DeMeza and Webb [2], Myers and Majluf [16], and Innes [9]).

2. The Model

Consider the following two-date model. A risk-neutral entrepreneur is endowed with a technology that produces a stochastic time 1 dollar payoff of $\pi \in [0, \infty)$, which I will call "profit." This payoff is produced with two time 0 inputs: (1) investment funds, and (2) entrepreneurial effort.[7] Since the concern here is not with scale choice, the input of time 0 investment funds is assumed to be fixed, as is the portion of these funds that the entrepreneur must obtain from investors.[8] The latter amount, the funds needed from outsiders, will be denoted by $I > 0$. Suppressing the dependence of profits on time 0 investment, let $g(\pi \mid e)$ and $G(\pi \mid e)$ denote, respectively, the twice continuously differentiable profit density and distribution functions, where e represents the entrepreneur's effort level and $g(\pi \mid e) > 0 \ \ \forall(\pi, e) \in R_+^2$. Higher effort levels produce "better" profit distributions in the sense of the monotone likelihood ratio property (MLRP) (Milgrom [14]); formally,

$$\frac{\partial}{\partial \pi}\left(\frac{g_e(\pi \mid e)}{g(\pi \mid e)}\right) > 0 \tag{2.1}$$

for all $e > 0$ and $\pi \geqslant 0$.[9] In addition, $E\{\pi \mid e = 0\} = 0$.

Investors in the entrepreneurial firm are risk-neutral and competitive. Competitive behavior implies that investors require an expected return equal to that available on a risk-free bond. The latter risk-free return will be denoted by ρ. While investors cannot observe effort e, they can infer the entrepreneur's effort choice from his utility maximization problem. Thus, they require an expected return of ρ on their investment I, considering the inferred effort choice effects of contract terms.

With a possible caveat to be discussed shortly, investors observe the entrepreneur's ex-post profit, π. Therefore, the financial contract specifies an investor payoff function, $B(\pi)$. For convenience, admissible $B(\pi)$

[7] As defined here, the term "profit" represents gross payoffs to the firm *before* the opportunity cost of invested funds is subtracted.

[8] Effectively, this analysis assumes that the investment funds required by the entrepreneur exceed his available wealth, all of which the entrepreneur will invest in his entreprise.

[9] The MLRP condition, (2.1), implies first order stochastic dominance (FOSD), $G_e(\pi \mid e) < 0$ $\forall \pi > 0$. However, the MLRP is a stronger condition than FOSD; hence, the converse is not true.

functions are assumed to be integrable. In addition, liability limits constrain this function in two ways: (1) $B(\pi) \leqslant \pi$; the entrepreneur cannot be required to pay more than the profits available to him; and (2) $B(\pi) \geqslant 0$: the investor's liability is limited to his investment in the firm.

Subject to these constraints, the entrepreneur will choose the contract that gives him maximal utility. Formally, let $V(w, e)$ denote the entrepreneur's twice continuously differentiable utility function in his time 1 dollar payoff, $w = \pi - B(\pi)$, and effort. Due to the assumption of risk neutrality, this utility function takes the form

$$V(w, e) = a(e)w - v(e), \tag{2.2}$$

where $a(\ ) > 0$ insures a positive utility dependence on dollar payoffs.

Given (2.2) and a fixed $B(\pi)$ function, the entrepreneur will choose effort to solve the following problem:

$$\max_{e \geqslant 0} E\{V(\pi - B(\pi), e) \mid e\} = a(e) \int_0^\infty (\pi - B(\pi))\, g(\pi \mid e)\, d\pi - v(e). \tag{2.3}$$

At this point, neither existence nor uniqueness of a solution to (2.3) can be ensured. However, the following assumption (and associated corollary) resolve some of this ambiguity:

Assumption 1 (A1). There exists a finite $e_{\max}$ such that

$$\lim_{e \to e_{\max}} E\{V(\pi, e) \mid e\} < E\{V(\pi, 0) \mid 0\}.$$

Assumption 1 implicitly required that there be some entrepreneurial *dis*utility of effort and that this disutility grow large as effort approaches $e_{\max}$. Given this assumption, the entrepreneur's effort choice opportunities can be limited to the interval $[0, e_{\max}]$ without loss of generality.

COROLLARY 1. *For all $B(\pi)$ functions satisfying the liability constraints* $0 \leqslant B(\pi) \leqslant \pi\ \forall \pi$, (a) *$E\{V(\pi - B(\pi), e) \mid e\}$ is bounded above and below on the domain $e \in [0, e_{\max}]$, and therefore,* (b) *there exists at least one solution to* (2.3).

Proof. For all $B(\pi)$ satisfying liability constraints, $E\{V(\pi - B(\pi), e) \mid \} \leqslant E\{V(\pi, e) \mid e\}$ and, for $e \in [0, e_{\max}]$, $E\{V(\pi - B(\pi), e) \mid e\} \geqslant E\{V(0, e) \mid e\} \geqslant E\{V(\pi - k^*, e) \mid e\}$, where $k^* \equiv E\{\pi \mid e_{\max}\}$. Since $E\{V(\pi, e) \mid e\}$ and $E\{V(\pi - k^*, e) \mid e\}$ are continuous, they are bounded on the compact set $e \in [0, e_{\max}]$, implying result (a). Result (b) follows from result (a) and the Weierstrass Theorem. Q.E.D.

Corollary 1 implies the existence of optimal effort choices with eligible payoff functions. Given this result, the entrepreneur's contract choice problem can be stated as follows:

$$\max_{B,e} \; a(e)\int_0^\infty (\pi - B(\pi))\, g(\pi\,|\,e)\, d\pi - v(e) \tag{2.4}$$

$$\text{s.t.} \int_0^\infty B(\pi)\, g(\pi\,|\,e)\, d\pi \geqslant (1+\rho)I \tag{2.4a}$$

$$e \text{ solves } (2.3) \tag{2.4b}$$

$$0 \leqslant B(\pi) \leqslant \pi \qquad \forall \pi. \tag{2.4c}$$

Conditions (2.4a)–(2.4c) represent investor return requirement, entrepreneurial effort choice, and limited liability constraints, respectively.

As noted in the introduction, I will also consider the implications of a futher constraint on contract forms, namely, that $B(\pi)$ be nondecreasing in firm profit. This constraint can be written as follows:

$$B(\pi+\varepsilon) \geqslant B(\pi) \qquad \forall (\pi, \varepsilon) \in R_+^2. \tag{2.4d}$$

There are two possible rationales for the "monotonic contract" constraint given in (2.4d):

(1) After observing a perfect signal of firm profits, investors may be in a position to sabotage the firm, essentially burning as much of these profits as they choose. In this case, investors would choose to burn profits in any decreasing segment of their payoff function and a nonmonotonic contract would never be chosen.

(2) Alternately, the entrepreneur may observe a perfect signal of firm profits slightly before they are realized, although investors can only observe total net cash flows of the firm on the date profits are realized. In this case, the entrepreneur could not alter firm profits (except, perhaps, by sabotage), but he could supplement these inflows with costless borrowings, $M > 0$, revealing an apparent profit of $\pi^* = \pi + M$ to investors. Thus, the entrepreneur would borrow in any decreasing segment of the payoff function, implying an equivalent nondecreasing payoff function, $B^*(\pi) = \min_{\{\pi^* > \pi\}} B(\pi^*)$.[10]

Note that if the entrepreneur could sabotage the firm, then $B(\pi)$ would also be subject to the constraint that the entrepreneur's payoff, $\pi - B(\pi)$, be nondecreasing. However, the latter requirement turns out to be satisfied at

[10] Since the entrepreneur cannot be *compelled* to borrow ex-post, the limited liability constraint, $B(\pi) \leqslant \pi$, still applies here. Indeed, given a bankruptcy option, the entrepreneur would never choose to borrow to pay an investor more than his available profits.

any optimum, even when it is not imposed. Therefore, this constraint neither alters nor drives any of the results in this paper and will be ignored throughout.

The next two sections characterize solutions to (2.4) with and without the "monotonic contract" constraint (2.4d). In both cases, a useful benchmark for comparison is the solution to poblem (2.4)'s "first-best" analog, namely, (2.4) subject only to constraint (2.4a). Trivially, (2.4a) will bind at any solution to this "first-best" problem, implying the equivalent maximization

$$\max_{e \geqslant 0} E\{V(\pi - (1+\rho)I, e) \mid e\}, \tag{2.5}$$

where B is set in any way which satisfies $E\{B(\pi) \mid e\} = (1+\rho)I$. For convenience, I will impose the following regularity conditions on problem (2.5):

Assumption 2 (A2). $E\{V(\pi - (1+\rho)I, e) \mid e\}$ is strictly concave in e.[11] Further, $\exists e > 0$: $E\{V(\pi - (1+\rho)I, e) \mid e\} > E\{V(\pi, 0) \mid 0\}$.

The second condition in Assumption 2 states that the entrepreneur would choose to undertake the investment project in a "first best" world. Since $E\{V(\pi, 0) \mid 0\} > E\{V(\pi - (1+\rho)I, 0) \mid 0\}$, this condition also implies that any solution to the "first best" choice problem contains a *positive* effort level which is defined by the first order condition

$$\frac{dE\{V(\pi - (1+\rho)I, e) \mid e\}}{de} = a_e(e)(E(\pi \mid e) - (1+\rho)I) + a(e)\frac{\partial E(\pi \mid e)}{\partial e} - v_e(e) = 0. \tag{2.6}$$

The first statement in (A2) ensures that (2.6) has a unique solution, e^*, the "first best" effort level.

3. The Optimal Monotonic Contract

To prove the emergence of debt contracts in the presence of a monotonic contract constraint, this section first posits the optimality of some monotonic *nondebt* contract, $B^{ND}(\pi)$. It then shows that by moving to a

[11] The requirements of Assumption 2 are quite weak by standards of the principal agent literature. For example, the following conditions are sufficient (though not necessary) for concavity of $E\{V(\pi - (1+\rho)I, e) \mid e\}$ in e: (i) $a_e(e) = 0\ \forall e$, (ii) $d^2v(e)/de^2 > 0\ \forall e$ (i.e., increasing disutility of effort), and (iii) concavity of $E(\pi \mid e)\ \forall e$ (which is implied by, but does not require, convexity of $G(\pi \mid e)$ in e).

debt contract that yields investors the same expected return as $B^{\mathrm{ND}}(\pi)$, the entrepreneur will commit himself to a higher effort level. Finally, since the debt-contract effort level is still *less* than the "first best" choice, it is shown that the entrepreneur will prefer the debt contract to the nondebt contract, contradicting the supposition that $B^{\mathrm{ND}}(\pi)$ was optimal.[12]

Before proceeding, one further assumption and associated corollary are required:

Assumption 3. When $B(\pi)$ takes a standard debt form, $B(\pi) = B^{\mathrm{D}}(\pi, z) \equiv \min(\pi, z)$, $z > 0$, there is a unique solution to the entrepreneur's effort choice problem, (2.3). This solution will be denoted $e^{\mathrm{D}}(z)$.[13]

COROLLARY 2. *$e^{\mathrm{D}}(z)$ is a continuous function.*

Proof. The Corollary follows directly from Assumption 3, continuity of $E\{V(\pi - \min(\pi, z), e) \mid e\}$ in e and z, and the Theorem of the Maximum. Q.E.D.

Now consider a proposed non-debt solution to the monotonicity-constrained version of (2.4), namely $B^{\mathrm{ND}}(\pi)$ and $e^{\mathrm{ND}} > 0$.[14] As a nondebt contract, $B^{\mathrm{ND}}(\pi)$ must differ from any debt contract on nondegenerate sets of profit levels. Formally, $\forall z$, $\{\pi\colon B^{\mathrm{ND}}(\pi) \neq B^{\mathrm{D}}(\pi, z)\}$ is of positive measure. Further, as a proposed solution to (2.4), $\mathrm{B}^{\mathrm{ND}}(\pi)$ and e^{ND} must satisfy constraints (2.4a)–(2.4d).

To compare $B^{\mathrm{ND}}(\pi)$ to a possible debt alternative, define

$$Q \equiv E\{B^{\mathrm{ND}}(\pi) \mid e^{\mathrm{ND}}\}, \tag{3.1}$$

and construct $B^{\mathrm{D}}(\pi; z_0) = \min(\pi, z_0)$ such that

$$E\{B^{\mathrm{D}}(\pi; z_0) \mid e^{\mathrm{ND}}\} = Q. \tag{3.2}$$

In words, $B^{\mathrm{D}}(\pi; z_0)$ is a debt contract that gives investors exactly the same expected return as $B^{\mathrm{ND}}(\pi)$ when the entrepreneur chooses his *nondebt*

[12] An alternative derivation of this paper's debt contracting result employs the "first order condition" (FOC) approach to principal–agent problems. Although the FOC-based proof is somewhat simpler than the argument presented here, it is also less general. Specifically, it requires (i) that Assumption 3 above be strengthened to specify strict concavity of $E\{V(\pi - \min(\pi, z)e) \mid e\}$ in e, and (ii) that $B(\pi)$ be constrained (a priori) to be piecewise smooth with a right-hand derivative, $B'(\pi)$, no greater than one. Copies of the alternative proof are available from the author.

[13] The following conditions are sufficient for Assumption 3 to hold: (i) $a_e(e) = 0$ $\forall e$; (ii) $d^2v(e)/\partial e^2 > 0$ $\forall e$, and (iii) convexity of $\int_z^\infty G(\pi \mid e)\, d\pi$ in e $\forall (e, z)$. These conditions are slightly stronger than those given in footnote 11 as sufficient for concavity of $E\{V(\pi - (1+\rho)I, e) \mid e\}$ in e.

[14] Since $E\{\pi \mid e = 0\} = 0$, e^{ND} must be positive to permit satisfaction of constraint (2.4a).

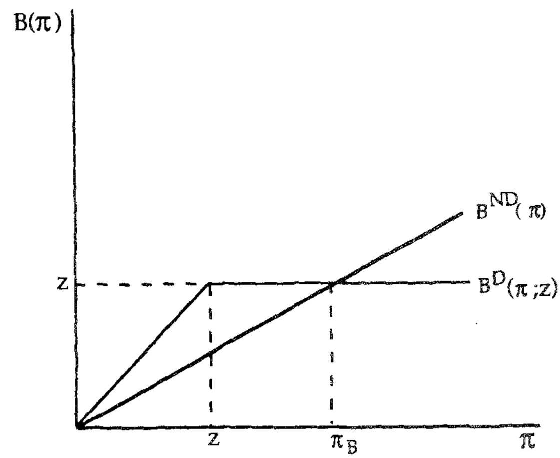

FIGURE 1

contract effort level, e^{ND}. Note that, since $B^{\mathrm{ND}}(\pi)$ is monotonic, there is a critical profit level, $\pi_B > 0$, such that $B^{\mathrm{ND}}(\pi) \leqslant B^{\mathrm{D}}(\pi; z_0)$ $\forall \pi < \pi_B$ (with strict inequality on sets of positive measure) and $B^{\mathrm{ND}}(\pi) \geqslant B^{\mathrm{D}}(\pi; z_0)$ $\forall \pi > \pi_B$ (again with strict inequality on sets of positive measure). Fig. 1 depicts this relationship. The following key lemma characterizes the effects of this property on relative effort choice incentives. This characterization leads into another lemma which relates the entrepreneur's effort choice under $B^{\mathrm{D}}(\pi; z_0)$ to that under $B^{\mathrm{ND}}(\pi) \neq B^{\mathrm{D}}(\pi; z_0)$. An intuitive discussion follows the lemmas.

LEMMA 1. *Suppose two payoff functions*, $B_0(\pi)$ *and* $B_1(\pi)$, *satisfy the following inequalities for some* $\pi_B > 0$: (i) $B_0(\pi) \leqslant B_1(\pi)$ $\forall \pi \leqslant \pi_B$ (*with strict inequality on a set of positive measure*), *and* (ii) $B_0(\pi) \geqslant B_1(\pi)$ $\forall \pi \geqslant \pi_B$ (*with strict inequality on a set of positive measure*). *Then* $\partial E\{B_1(\pi) - B_0(\pi) \mid e\}/\partial e < 0$ *if either*

(a) $E\{B_1(\pi) - B_0(\pi) \mid e\} = 0$, *or*

(b) $E\{B_1(\pi) - B_0(\pi) \mid e\} \leqslant 0$ *and* $B_1(\pi) - B_0(\pi)$ *is monotone nonincreasing in* π $\forall \pi > \pi_B$.

Proof. See Appendix.

Lemma 1 compares the extent to which marginal effort benefits are captured by investors under two different payoff function regimes: $B_0(\pi)$ and a replacement, $B_1(\pi)$, which has higher payments in low profit states and lower payments in high profit states. Under the circumstances indicated in the lemma, the replacement function, $B_1(\pi)$, tends to yield investors fewer benefits from marginal effort and, hence, correspondingly greater benefits for the entrepreneur. Since the move from a monotonic nondebt contract achieves exactly the type of shift represented by the replacement of $B_0(\pi)$ wth $B_l(\pi)$, Lemma 1 is crucial to the following derivation:

LEMMA 2. $e^{\mathrm{D}}(z_0) > e^{\mathrm{ND}}$.

Proof. Define

$$\phi(\pi) \equiv B^{\mathrm{D}}(\pi; z_0) - B^{\mathrm{ND}}(\pi). \tag{3.3}$$

From the definition of $V(w, e)$ in (2.2),

$$E\{V(\pi - B^{\mathrm{D}}(\pi; z_0), e) \mid e\} - E\{V(\pi - B^{\mathrm{ND}}(\pi), e) \mid e\} = -a(e)E\{\phi(\pi) \mid e\}. \tag{3.4}$$

(3.4) gives the difference between the entrepreneur's expected utility with the contract $B^{\mathrm{D}}(\pi; z_0)$ and that with $B^{\mathrm{ND}}(\pi)$, for a given effort level e. Since e^{ND} maximizes $E\{V(\pi - B^{\mathrm{ND}}(\pi), e) \mid e\}$ by choice of e, the following relationship (together with (3.4)) are sufficient to prove the lemma.

Property (1). $E\{\phi(\pi) \mid e\} = 0$ at $e = e^{\mathrm{ND}}$.

Property (2). $E\{\phi(\pi) \mid e\} < 0$ at $e = e^{\mathrm{ND}} + \varepsilon$, some $\varepsilon > 0$.

Property (3). $E\{\phi(\pi) \mid e\} > 0$ $\forall e < e^{\mathrm{ND}}$.

Property (1) follows directly from (3.1), (3.2), and (3.3). To establish Properties (2) and (3), let $B_1(\pi) \equiv B^{\mathrm{D}}(\pi; z_0)$ and $B_0(\pi) \equiv B^{\mathrm{ND}}(\pi)$. Now note that the prior conditions of Lemma 1 are satisfied. Thus, since $E(\phi(\pi) \mid e^{\mathrm{ND}}) = 0$ from Property (1), Lemma 1(a) implies $\partial E\{\phi(\pi) \mid e^{\mathrm{ND}}\}/\partial e < 0$ and Property (2) holds. Finally, to verify Property (3), suppose the contrary, so that $E\{\phi(\pi) \mid e_0\} \leqslant 0$ for some $e_0 < e^{\mathrm{ND}}$. Then Lemma 1(b) can be invoked since $\phi(\pi) = z_0 - B^{\mathrm{ND}}(\pi)$ $\forall \pi > \pi_B$ and, therefore, $\phi(\pi)$ is non-increasing in π $\forall \pi > \pi_B$ (due to the monotonicity of $B^{\mathrm{ND}}(\pi)$). From Lemma 1(b), $E\{\phi(\pi) \mid e_0\} \leqslant 0$ implies $E\{\phi(\pi) \mid e\} < 0$ $\forall e > e_0$ and, in particular, $E\{\phi(\pi) \mid e^{\mathrm{ND}}\} < 0$. But the last inequality violates Property (1), a contradiction. Q.E.D.

Lemma 2 indicates that the $B^{\mathrm{D}}(\pi; z_0)$ contract yields higher effort than $B^{\mathrm{ND}}(\pi)$. The intuition underlying this result is quite simple. A move from the monotonic nondebt contract, $B^{\mathrm{ND}}(\pi)$, to the debt contract, $B^{\mathrm{D}}(\pi; z_0)$, increases the entrepreneur's obligations in low profit states and decreases them in high profit states (recall Fig. 1). Loosely speaking, increased effort shifts probability weight to the high profit states, implying that the debt contract gives the entrepreneur more of the benefit from increased effort than does the nondebt contract.

In making this argument, the importance of the MLRP (condition (2.1)) should be recognized. The MLRP implies that, for any given profit level, increased effort leads to relatively greater probability weight on all higher profit levels. For example, suppose that higher effort does not alter the probability weights on profit levels $\pi > z$, but shifts probability weight from the interval $(0, z/2)$ to the interval $(z/2, z)$. In this case, $B^{\mathrm{ND}}(\pi)$ would yield the entrepreneur a greater benefit from increased effort, rather than $B^{\mathrm{D}}(\pi; z)$. However, since higher effort leads to relatively less probability weight on profits above z than on profits in the interval $(z/2, z)$, this relationship between effort and the profit distribution is inconsistent with the MLRP.Under the MLRP, if higher effort leads to greater probability weight on some profit levels below π_B (where π_B is as indicated in Fig. 1), it must also induce a proportionally greater increase in probability weight on the profit interval, $[\pi_B, \infty)$. An appropriate matching of these changes implies greater marginal effort benefits with a contract which concentrates payments as much as possible in the lowest profit states. Among monotonic limited liability contracts, debt functions have this maximal low-profit-state payment property, implying Lemma 2.

Note now (from Eq. (2.1)) that $E\{\min(\pi, z) \mid e\}$ is increasing in e, that is,

$$\frac{\partial E\{\min(\pi, z) \mid e\}}{\partial e} = -\int_0^z G_e(\pi \mid e)\, d\pi > 0, \tag{3.5}$$

where the equality is obtained with integration by parts. Given (3.5), Lemma 2 implies that investors' expected return on the $B^{\mathrm{D}}(\pi; z_0)$ contract, $E\{B^{\mathrm{D}}(\pi; z_0) \mid e^{\mathrm{D}}(z_0)\}$, is greater than Q, the expected investor payoff on $B^{\mathrm{ND}}(\pi)$. Therefore, the pair $(B^{\mathrm{D}}(\pi; z_0), e^{\mathrm{D}}(z_0))$ satisfies all of the constraints, (2.4a)–(2.4d). Moreover, since $e^{\mathrm{D}}(z_0)$ is chosen optimally, the entrepreneur prefers $(B^{\mathrm{D}}(\pi; z_0), e^{\mathrm{D}}(z_0))$ to $(B^{\mathrm{D}}(\pi; z_0), e^{\mathrm{ND}})$. Thus, since the entrepreneur is indifferent between $(B^{\mathrm{D}}(\pi; z_0), e^{\mathrm{ND}})$ and the proposed nondebt solution, $(B^{\mathrm{ND}}(\pi), e^{\mathrm{ND}})$, the following lemma holds:

LEMMA 3. *Any solution to* (2.4) (*subject to constraints* (2.4a)–(2.4d)) *contains a debt contract.*

Given Lemma 3, the following assumption is necessary (and sufficient) for the existence of a solution to (2.4), leading into the paper's main proposition:

Assumption 4 (A4). $\exists z\colon E\{B^{\mathrm{D}}(\pi; z) \mid e^{\mathrm{D}}(z)\} \geqslant (1+\rho)I.$

PROPOSITION 1. *Subject to the monotonic contract constraint* (2.4d) *a solution to problem* (2.4) *exists and has the following properties*:

(i) $B(\pi) = B^{D}(\pi; z) \equiv \min(\pi, z)$, $z > 0$ (from Lemma 3);

(ii) $E\{B(\pi) \mid e\} = (1+\rho)I$; *and*

(iii) $e < e^{*} \equiv$ *first best effort choice.*

Proof. Existence: By Lemma 3, (2.4) has a solution if and only if the following problem has a solution:

$$\max_{z \geq 0} E\{V(\pi - B^{D}(\pi; z), e^{D}(z)) \mid e^{D}(z)\} \tag{3.6}$$

$$\text{s.t. } E\{B^{D}(\pi; z) \mid e^{D}(z)\} \geq (1+\rho)I. \tag{3.6a}$$

Further, z can be bounded above without loss of generality. Given this bound, the choice set in (3.6) is compact and, by (A4), nonempty. Moreover, the objective function is continuous in z by continuity of $V(\ )$, $B^{D}(\ )$, $g(\ )$ and $e^{D}(\ )$ (Corollary 2). Thus, (3.6) has a solution by the Weierstrass Theorem.

Properties: (ii) Suppose not, so that $E\{B^{D}(\pi; z) \mid e^{D}(z)\} > (1+\rho)I$. Then, by continuity of $e^{D}(z)$, there is a sufficiently small positive ε so that $B^{D}(\pi; z-\varepsilon)$ meets the investor's return requirement (2.4a). Further, from the definitions of $V(\ )$ and $e^{D}(z)$, the following inequalities hold:

$$E\{V(\pi - B^{D}(\pi; z), e^{D}(z)) \mid e^{D}(z)\} < E\{V(\pi - B^{D}(\pi; z-\varepsilon), e^{D}(z)) \mid e^{D}(z)\}$$
$$< E\{V(\pi - B^{D}(\pi; z-\varepsilon), e^{D}(z-\varepsilon)) \mid e^{D}(z-\varepsilon)\}. \tag{3.7}$$

Inequalities (3.7) establish that the entrepeneur prefers $B^{D}(\pi; z-\varepsilon)$ to $B^{D}(\pi; z)$, implying that $B^{D}(\pi; z)$ cannot solve (2.4)

(iii) Given results (i) and (ii), the first order necessary condition for the entrepreneur's effort choice will be

$$\frac{dE\{V(\pi - (1+\rho)I, e) \mid e\}}{de} - a(e)\frac{\partial E\{\min(\pi, z) \mid e\}}{\partial e} = 0. \tag{3.8}$$

The first term in (3.6) represents the "first best" effort choice first-order-condition derivative (see Eq. (2.6)). Due to Assumption 2, this derivative is nonpositive for all $e > e^{*} \equiv$ "first best" effort choice. Thus, given the inequality in (3.5), the left side of (3.8) is strictly negative for all $e \geq e^{*}$ and any solution to (3.8) must be less than e^{*}. Q.E.D.

In summary, the entrepreneur expends too little effort when there is a "monotonic contract" constraint. A debt contract, by achieving maximal effort among available monotonic payoff functions, enables the entrepreneur to get closer to the 'first best."

4. The Optimal Nonmonotonic Contract

Without a "monotonic contract" constraint, the entrepreneur can commit himself to higher effort levels by shifting more high-profit-state payment obligations to lower-profit-states than is possible with any monotonic contract. In the extreme, the entrepreneur can maximize his incentives to exert effort by signing a "live-or-die" contract of the form

$$B^{\mathrm{LD}}(\pi; z) = \begin{cases} \pi & \forall \pi \leqslant z \\ 0 & \forall \pi > z \end{cases}. \tag{4.1}$$

If this contract yields an effort level which is still less than e^*, the first best level, then it will enable the entrepreneur to get as close to the first best as is possible given liability limits. However, if this contract yields an effort level greater than e^*, then one can surmise that a contract with less extreme effort incentives will elicit a first best.

To develop these thoughts formally, this section will analyze Rogerson's [17] "relaxed first order condition" (RFOC) analog to problem (2.4) (without constraint (2.4d)).[15] Specifically, consider the RFOC maximization

$$\max_{B,e} E\{V(\pi - B(\pi), e) \mid e\} \tag{4.2}$$

$$\text{s.t. } E\{B(\pi) \mid e\} \geqslant (1+\rho) I \tag{4.2a}$$

$$dE\{V(\pi - B(\pi), e) \mid e\}/de \geqslant 0 \tag{4.2b}$$

$$0 \leqslant B(\pi) \leqslant \pi \qquad \forall \pi. \tag{4.2c}$$

As is well known, the solution to the RFOC problem, (4.2), need not coincide with that for the underlying incentive contracting problem, (2.4). The reason is that constraint (4.2b) may permit the entrepreneur a larger opportunity set than does the true effort choice constraint, (2.4b). Therefore, in principle, the solution to (4.2) may lie outside of the true opportunity set available to the entrepreneur (e.g., see Jewitt [11], Grossman and Hart [5], and Mirrlees [15] for discussion of this point). However, if a solution to the RFOC problem, (4.2), satisfies the true effort choice constraint (2.4b) and, therefore, lies within the entrepreneur's true opportunity set, then it must also solve the original problem, (2.4).

Drawing upon this last observation, the following assumption will prove to be sufficient for the coincidence of solutions to problems (2.4) and (4.2):

Assumption 5 (A5). If $B(\pi) = \alpha B^{\mathrm{LD}}(\pi; z)$, with $\alpha \in (0, 1]$ and $z \geqslant 0$, then $d^2 E\{V(\pi - B(\pi), e) \mid e\}/de^2 < 0$ $\forall e$ and, therefore, there is a unique solution

[15] I am indebted to the referee for suggesting this approach.

to the entrepreneur's effort choice problem, (2.3). This solution will be denoted $e^{\mathrm{LD}}(z, \alpha)$.

The next corollary is a direct implication of (A5):

COROLLARY 3. *If a solution to* (4.2) *satisfies constraint* (4.2b) *with equality and takes the form* $B(\pi) = \alpha B^{\mathrm{LD}}(\pi; z)$, $\alpha \in (0\ 1]$ *and* $z > 0$, *then it also solves* (2.4).

Bearing this result in mind, consider the following Lagrangean function for problem (4.2):

$$L(B, e, \lambda, \mu, \theta, \eta) = E\{V(\pi - B(\pi), e) \mid e\} + \lambda(E(B(\pi) \mid e) - (1+\rho)I)$$
$$+ \mu(dE\{V(\pi - B(\pi), e) \mid e\}/de) + \int_0^\infty \theta(\pi) B(\pi)\, d\pi$$
$$+ \int_0^\infty \eta(\pi)(\pi - B(\pi))\, d\pi,$$

where λ, μ, $\theta(\ )$ and $\eta(\ )$ are multipliers for constraints (4.2a)–(4.2c). Using (4.3), necessary conditions for a solution to (4.2) include

$$g(\pi \mid e) \cdot \left\{\lambda - a(e) - \mu a_e(e) - \mu a(e) \frac{g_e(\pi \mid e)}{g(\pi \mid e)}\right\} + (\theta(\pi) - \eta(\pi)) = 0 \tag{4.4}$$

$$\frac{dE\{V(\pi - B(\pi), e) \mid e\}}{de} + \lambda \int_0^\infty B(\pi)\, g_e(\pi \mid e)\, d\pi$$
$$+ \mu \frac{d^2 E\{V(\pi - B(\pi), e) \mid e\}}{de^2} = 0. \tag{4.5}$$

Due to nonnegativity and complementary slackness conditions for $\theta(\pi)$ and $\eta(\pi)$, condition (4.4) yields

$$\phi(\pi, e) \equiv \lambda - a(e) - \mu a_e(e) - \mu a(e) \frac{g_e(\pi \mid e)}{g(\pi \mid e)} > 0 \quad \Rightarrow \quad B(\pi) = \pi \tag{4.6a}$$
$$\phi(\pi, e) = 0 \quad \Rightarrow \quad B(\pi) \in [0, \pi] \tag{4.6b}$$
$$\phi(\pi, e) < 0 \quad \Rightarrow \quad B(\pi) = 0. \tag{4.6c}$$

From (4.6) and nonnegativity of μ, it is evident that there are two important cases to consider: *Case* 1: $\mu > 0$ and *Case* 2: $\mu = 0$.

For Case 1, the above conditions lead to the following characterization of a solution to (4.2):

PROPOSITION 2. *If the effort choice constraint* (4.2b) *strictly binds at a solution to* (4.2) (i.e., $\mu > 0$), *then this solution has the following properties:*

(i) $B(\pi) = B^{\mathrm{LD}}(\pi; z)$, *for some* $z > 0$;

(ii) $E\{B(\pi) \mid e\} = (1+\rho)I$;

(iii) $e < e^* \equiv$ *first best effort level*; *and*

(iv) B *and* e *solve* (2.4).

Proof. (i) $\mu > 0$ and the MLRP (condition (2.1)) imply that $\phi(\pi; e)$ is decreasing in π. Thus, result (i) follows from (4.6) and constraint (4.2a).

(iv) $\mu > 0$ implies $dE\{V(\ ) \mid e\}/de = 0$ (from complementary slackness). The latter equality, together with result (i) and Corollary 3, establish result (iv).

(ii and iii) From result (i) and Assumption 5, $d^2E\{V(\ ) \mid e\}/de^2 < 0$. Thus, given $\mu > 0$ and $dE\{V(\ ) \mid e\}/de = 0$, the second term in (4.5) must be strictly positive. Therefore, since $\lambda \geqslant 0$, λ must be positive and so too must be $\int_0^\infty B(\pi)\, g_e(\pi \mid e)\, d\pi$. $\lambda > 0$ implies result (ii) (from complementary slackness). Given result (ii), constraint (4.2b) can be written

$$\frac{dE\{V(\ ) \mid e\}}{de} = \frac{dE\{V(\pi - (1+\rho)I, e) \mid e\}}{de} - a(e) \int_0^\infty B(\pi)\, g_e(\pi \mid e)\, d\pi \geqslant 0. \tag{4.7}$$

Since $\int_0^\infty B(\pi)\, g_e(\pi \mid e)\, d\pi > 0$, the first term in (4.7) is strictly positive. Given Assumption 2, this last inequality implies result (iii). Q.E.D.

For Case 2 ($\mu = 0$), similar reasoning leads to the following characterization of a solution to (4.2):

LEMMA 4. *If* $\mu = 0$ *at a solution to* (4.2), *then this solution has the following properties*:

(i) $e = e^* \equiv$ *first best effort level*; *and*

(ii) $E\{B(\pi) \mid e\} = (1+\rho)I$.

Proof. To prove the lemma, it is useful to show first that $\lambda = a(e)$ when $\mu = 0$. Suppose not. Then either $\lambda > a(e)$ or $\lambda < a(e)$. If $\lambda > a(e)$ (when $\mu = 0$), then $B(\pi) = \pi\ \forall \pi$ from (4.6) and the last two terms in (4.5) equal $\lambda \partial E\{\pi \mid e\}/\partial e > 0$; but this last inequality, together with constraint (4.2b), implies that (4.5) is violated, a contradiction. If $\lambda < a(e)$, then $B(\pi) = 0\ \forall \pi$ from (4.6) and constraint (4.2a) is violated, a contradiction.

$\lambda = a(e) > 0$ implies result (ii) from complementary slackness. Further substituting $\lambda = a(e)$ and $dE\{V(\ ) \mid e\}/de$ from (4.7) into (4.5) yields (with $\mu = 0$)

$$\frac{dE\{V(\pi - (1+\rho)I, e) \mid e\}}{de} = 0, \tag{4.8}$$

which implies result (i). Q.E.D.

Lemma 4 describes two key properties of a solution to (4.2) when the effort choice constraint, (4.2b), is slack (i.e., $\mu = 0$). But it remains to relate this solution to the original problem, (2.4), and also to determine if and when μ will be zero. The following lemma lays the groundwork necessary to address these issues:

LEMMA 5. *There is a unique* $z^* > 0$ *such that*

$$\left.\frac{\partial E\{B^{\mathrm{LD}}(\pi; z^*) \mid e\}}{\partial e}\right|_{e=e^*} = 0, \tag{4.9}$$

where e^* *is the first best effort level defined in* (2.6).

Proof. If the derivative in (4.9) is written out, the lemma is seen to be a direct consequence of the MLRP (Eq. (2.1)). Formally,

$$\left.\frac{\partial E\{B^{\mathrm{LD}}(\pi; z) \mid e\}}{\partial e}\right|_{e=e^*} = \int_0^z \pi g_e(\pi \mid e^*)\, d\pi = \int_0^{\pi_g} \pi g_e(\pi \mid e^*)\, d\pi + \int_{\pi_g}^z \pi g_e(\pi \mid e^*)\, d\pi, \tag{4.10}$$

where, due to Eq. (2.1), π_g can be defined such that $g_e(\pi \mid e^*) < 0\ \forall \pi < \pi_g$ and $g_e(\pi \mid e^*) > 0\ \forall \pi > \pi_g$. The first right-hand term in (4.10) has a fixed negative value. The second right-hand term is continuous and increasing in z, and zero at $z = \pi_g$. Further, since $\int_0^\infty g_e(\pi \mid e^*)\, d\pi = 0$, this second term is greater in absolute value than the first right-hand term at $z = \infty$. Thus, by the Intermediate Value Theorem, there is a $z^* \in (\pi_g, \infty)$ at which (4.9) is satisfied. Fixing this z^*, the derivative in (4.10) is negative for all z: $0 < z < z^*$ and positive for all $z > z^*$. Hence, the $z^* > 0$ that solves (4.9) is also unique. Q.E.D.

To appreciate the significance of Lemma 5, define α^* such that

$$\alpha^* E\{B^{\mathrm{LD}}(\pi; z^*) \mid e^*\} = (1 + \rho) I, \tag{4.11}$$

and suppose that this α^* is in the unit interval, (0, 1]. Now consider the entrepreneur's effort choice first order condition derivative, (4.7), with $B(\pi) = \alpha^* B^{\mathrm{LD}}(\pi; z^*)$. Given (4.9), the derivative vanishes at $e = e^*$. Thus, since $E\{V(\ ) \mid e\}$ is concave in e (from (A5)), this contract will support a first best effort choice, as well as satisfying investor return requirement and liability constraints. In summary,

PROPOSITION 3. *If* $\exists \alpha^* \in (0, 1]$ *such that* (4.11) *is satisfied, then* $(\alpha^* B^{\mathrm{LD}}(\pi; z^*), e^*)$ *solves problems* (4.2) *and* (2.4).

The following proposition will now close the circle between problems (2.4) and (4.2), thereby permitting a complete characterization of the optima:

PROPOSITION 4. *$\mu = 0$ at a solution to* (4.2) *if and only if* $\exists \alpha^* \in (0, 1]$ *such that* (4.11) *is satisfied.*

Proof. See Appendix.

From Propositions 2, 3, and 4 and Corollary 3, problem (2.4) has two types of possible solutions, depending upon whether or not there is an $\alpha^* \in (0, 1]$ satisfying (4.11). If there is such an α^*, the solutions to (2.4) will be first best and will include the contract $\alpha^* B^{\mathrm{LD}}(\pi; z^*)$. If not, the only possible solution to (2.4) is a pure live-or-die contract which yields an effort level that is less than first best.[16] It is clear from Eq. (4.11) that the first best outcome is more likely to be possible as the required external investment I is lower and as the value $E\{B^{\mathrm{LD}}(\pi; z^*) \mid e^*\}$ is higher.

One final corollary is a direct implication of Propositions 1–4 and concludes the analysis:

COROLLARY 4. *If (B, e) solves* (2.4) *without constraint* (2.4d), *then B is nonmonotonic.*

5. SUMMARY AND CONCLUSION

The purpose of this article was to deduce the effects of liability limits on the optimal financial contract between a risk-neutral investor and a risk-neutral entrepreneur who makes an unobservable ex-ante effort choice. Principal conclusions included the following:

[16] For the sake of brevity, the foregoing analysis does not prove the necessary existence of a solution to problem (2.4) (without constraint (2.4d)). However, an expanded version of this paper shows that a solution to problem (2.4) always exists. Paralleling the existence proof in Section 3, the expanded paper first shows that any feasible "non-live-or-die" contract is weakly dominated by a corresponding feasible "live-or-die" contract (i.e., a $B(\pi)$ of the form $\alpha B^{\mathrm{LD}}(\pi; z)$, where $\alpha \in (0, 1]$ and $z \geq 0$). Therefore, the entrepreneur's opportunity set can be restricted to "live-or-die" contracts without loss in generality, and (2.4) will have a solution if and only if the following problem has a solution:

$$\begin{aligned} &\max_{z, \alpha} E\{V(\pi - \alpha B^{\mathrm{LD}}(\pi; z), e^{\mathrm{LD}}(z, \alpha)) \mid e^{\mathrm{LD}}(z, \alpha)\} \\ &\text{s.t. } E\{\alpha B^{\mathrm{LD}}(\pi; z) \mid e^{\mathrm{LD}}(z, \alpha)\} \geq (1 + \rho) I, \quad \alpha \in (0, 1], \qquad z \geq 0. \end{aligned} \tag{2.4R}$$

From Section 3, the opportunity set defined by (2.4a)–(2.4c) contains the contract characterized in Proposition 1, $(B^{\mathrm{D}}(\pi; z), e^{\mathrm{D}}(z))$. Therefore, the restricted choice problem, (2.4R), also has a nonempty opportunity set. Given (A5) and a nonempty opportunity set, it is easily shown that (2.4R) satisfies the requirements of the Weierstrass theorem.

(1) When the investor payoff function is constrained to be monotone nondecreasing in firm profits, a standard debt contract is optimal, eliciting less effort than in a perfect information setting.

(2) Without a "monotonic contract" constraint, a "live-or-die" payoff function is optimal, giving the investor nothing in high-profit states of nature and a constant share of firm profits in low-profit states of nature. Depending upon the technology and the extent of external investment, the induced effort choice may or may not be "first best" in this case.

In closing, four limitations of the analysis merit mention: (1) Both the entrepreneur and the investor were assumed to be risk-neutral, eliminating any risk-sharing considerations from the analysis. (2) The investors and the entrepreneur were assumed to have symmetric beliefs about the profit distribution and its relationship to the effort choice. (3) The investor was assumed to have complete information about the entrepreneur's preferences, and hence his effort choice response to contract terms. (4) The entrepreneur's choices of an investment level and investment policy were assumed to be fixed and known by all agents.

With respect to the first of these limitations, entrepreneurial risk aversion will lead to nontrivial conflicts between risk-sharing and incentive objectives of the contract choice, just as in Shavell [20] and Harris and Raviv [6].[17] However, the beneficial *incentive* properties of the contract forms derived here are likely to withstand such a generalization.

With respect to the second and third limitations, preliminary research by the author (Innes [9]) indicates that asymmetric information about attributes of the entrepreneur and his investment project is unlikely to alter this paper's conclusions on equilibrium payoff functions. For example, suppose entrepreneurs can be ordered such that, given any specified investor payoff function, "higher quality" types have preferences and effort-contingent profit functions that always yield a "better" firm-level profit distribution (in the sense of the MLRP). Then any informational equilibrium (in a risk-neutral setting) can be shown to have the same payoff function properties as derived here.

Of course, the informational asymmetry may not be amenable to such a simple characterization and may even be such that the investor has no basis for probability assignment (as in Hurwicz and Shapiro [8]). Both of these possibilities suggest scope for further work.

[17] Without an incentive problem, entrepreneurial risk aversion and liability limits can be shown to lead to an optimal financial contract that gives the investor $\max(\pi - K, 0)$ for some positive constant K. In other words, the entrepreneur becomes the debt holder and the investor the residual claimant. With an incentive problem, it would be interesting to know the extent to which risk-sharing considerations lead an optimal contract to deviate from the forms derived here, as well as the parametric determinants of this deviation.

Finally, the most provocative of the limitations mentioned above is probably the fourth, which suggests a number of interesting complications to the foregoing analysis. One such complication, with potentially important implications for this paper, is as follows. Suppose that the entrepreneur could commit to investing \$$I$ of the investor's money in his firm and another \$$x$ of the investor's money in a riskless bond, with x now representing an added contractual choice variable. The right-hand side of the investor return constraint, (2.4a), would then become $(1+\rho)(I+x)$, and the limited liability restriction, (2.4c), would be as follows: $0 \leqslant B(\pi) \leqslant \pi + x(1+\rho)$. Further, the entrepreneur's objective function would become $E\{V(\pi + x(1+\rho) - B(\pi), e) \mid e\}$. Now, without loss of generality, consider replacing $B(\pi)$ with $\hat{B}(\pi) \equiv B(\pi) - (1+\rho)x$. The original choice problem, (2.4), would then be recovered, with two modifications: (i) the addition of the choice variable x, and (ii) a modified liability constraint, $-x(1+\rho) \leqslant \hat{B}(\pi) \leqslant \pi$. Thus, x would serve to relax the lower bound liability limit.

Since none of the derivations in Section 3 relied upon the lower bound constraint, $B(\pi) \geqslant 0$, all of the results from the monotonicity-constrained choice problem would persist in this altered setting. But the same cannot be said for the results in Section 4. In fact, it can be shown that, without a monotonicity constraint and with the choice variable x, a "first best" can always be achieved by setting x sufficiently high.[18,19] On an intuitive level, relaxation of the lower bound constraint permits contracts with even greater incentives for entrepreneurial effort than are provided by a "live-or-die" contract. With a positive x, these enhanced effort incentives can be obtained by making the investor's high-profit-state payoffs negative. Thus, when the best "live-or-die" contract elicits too little effort (as in Proposition 2), a positive x can be chosen to raise the effort level to the "first best," e^*.

This simple treatment of the choice variable x does have some problems, however. For example, the investor may not be able to compel the entrepreneur's adherence to his agreed-upon level of riskless investment

[18] The analysis above found that a "first best" would *not* be achieved when the contract $B^{\text{LD}}(\pi; z^*)$ (where z^* was selected to elicit the "first best" effort choice e^*) failed to meet the investor return requirement. But with $x > 0$, another contract, $B^{\text{LD}^*}(\pi; x)$, can be defined such that

$$B^{\text{LD}^*}(\pi; x) \equiv \begin{cases} \pi & \forall \pi \leqslant z^*(x) \\ -x(1+\rho) & \forall \pi > z^*(x) \end{cases}$$

with $z^*(x)$ selected to elicit the "first best" effort level e. Since the investor's expected payoff on $B^{\text{LD}}(\ )$ can be shown to be increasing in x (proof available from the author), x can always be set sufficiently high for $B^{\text{LD}^*}(\ )$ to meet the investor return requirement and thereby achieve a "first best."

[19] I am indebted to the Associate Editor for these observations.

(i.e., x), perhaps due to the investor's inability to observe this entrepreneurial choice. In this case, the entrepreneur would have an incentive to invest the x dollars in risky firm projects, so long as the expected return on these projects is not too low. The latter incentive derives from the increased probability of large high-profit-state entrepreneurial payoffs that higher risk levels elicit. Of course, these additional moral hazard concerns merit a much more complete analysis than can be given here. They implicitly raise the broader question of optimal contracting when investment policy choices, as well as effort, are made privately by the entrepreneur (e.g., see Lambert [12]). This topic, as well as the implications of incentive problems for optimal scale choices, merits more thought.

APPENDIX

Proof of Lemma 1. Define

$$\phi(\pi) = B_1(\pi) - B_0(\pi) \qquad \begin{cases} \geqslant 0 & \forall \pi \leqslant \pi_B \\ \leqslant 0 & \forall \pi \geqslant \pi_B. \end{cases} \tag{A.1}$$

As $E\{\phi(\pi) \mid e\} \leqslant 0$, there is a profit level $\pi^* > \pi_B$ such that the the following condition holds:

$$\int_0^{\pi_B} \phi(\pi)\, g(\pi \mid e)\, d\pi + \int_{\pi_B}^{\pi^*} \phi(\pi)\, g(\pi \mid e)\, d\pi = 0. \tag{A.2}$$

Now consider the derivative $\partial E\{\phi(\pi) \mid e\}/\partial e$:

$$\partial E\{\phi(\pi) \mid e\}/\partial e = \int_0^{\pi^*} \phi(\pi)\, g_e(\pi \mid e)\, d\pi + \int_{\pi^*}^{\infty} \phi(\pi)\, g_e(\pi \mid e)\, d\pi. \tag{A.3}$$

To evaluate the first term on the right-hand side of (A.3), define

$$\delta(\pi_L) \equiv \frac{\phi(\pi_L)\, g(\pi_L \mid e)}{\int_0^{\pi_B} \phi(\pi)\, g(\pi \mid e)\, d\pi} = \frac{\phi(\pi_L)\, g(\pi_L \mid e)}{-\int_{\pi_B}^{\pi^*} \phi(\pi)\, g(\pi \mid e)\, d\pi} \qquad \text{for} \quad \pi_L \in [0, \pi_B), \tag{A.4}$$

where the second equality follows from (A.2). By construction, $\int_0^{\pi_B} \delta(\pi_L)\, d\pi_L = 1$. Using this fact and substituting for $\phi(\pi_L)$ from (A.4), the first term in (A.3) can be rewritten as

$$\int_0^{\pi^*} \phi(\pi)\, g_e(\pi \mid e)\, d\pi = \left\{ \int_0^{\pi_B} \delta(\pi_{\mathrm{L}}) \frac{g_e(\pi_{\mathrm{L}} \mid e)}{g(\pi_{\mathrm{L}} \mid e)} \left[-\int_{\pi_B}^{\pi^*} \phi(\pi_{\mathrm{H}})\, g(\pi_{\mathrm{H}} \mid e)\, d\pi_{\mathrm{H}} \right] d\pi_{\mathrm{L}} \right\}$$
$$+ \left\{ \int_{\pi_B}^{\pi^*} \phi(\pi_{\mathrm{H}})\, g_e(\pi_{\mathrm{H}} \mid e)\, d\pi_{\mathrm{H}} \int_0^{\pi_B} \delta(\pi_{\mathrm{L}})\, d\pi_{\mathrm{L}} \right\}$$

$$= \int_0^{\pi_B} \int_{\pi_B}^{\pi^*} \delta(\pi_L)\phi(\pi_H)\, g(\pi_H \mid e) \left[\frac{g_e(\pi_H \mid e)}{g(\pi_H \mid e)} - \frac{g_e(\pi_L \mid e)}{g(\pi_L \mid e)} \right]$$

$$\times\, d\pi_H\, d\pi_L < 0, \tag{A.5}$$

where π_H denotes the variable of integration over the interval $[\pi_B, \pi^*]$ and π_L denotes the variable at integration over $[0, \pi_B)$. The inequality in (A.5) follows from $\delta(\pi_L) \geqslant 0\ \forall \pi_L < \pi_B$ (with strict inequality on a set of positive measure in the interval $[0, \pi_B)$), $\phi(\pi_H) \leqslant 0\ \forall \pi_H > \pi_B$ (with strict inequality on a set of positive measure in the interval (π_B, ∞)), and Eq. (2.1) (since $\pi_H > \pi_L$). Since $\pi^* = \infty$ when $E\{\phi(\pi) \mid e\} = 0$, (A.5) establishes part (a) of the lemma.

To establish part (b), the second term in (A.3) must be signed nonpositive under the indicated circumstances. To this end, note the following:

(I) $\int_0^\infty g_e(\pi \mid e)\, d\pi = 0$; and

(II) due to (I) and condition (2.1), $\exists \pi_g$ for any given e such that $g_e(\pi \mid e) > 0\ \forall \pi < \pi_g$ and $g_e(\pi \mid e) > 0\ \forall \pi > \pi_g$.

If $\pi^* \geqslant \pi_g$, then the second term in (A.3) is non-positive (due to (II) and $\phi(\pi) \leqslant 0\ \forall \pi \geqslant \pi^* > \pi_B$). If $\pi^* < \pi_g$, then $\exists \pi^{**}$ such that $\int_{\pi^*}^{\pi^{**}} g_e(\pi \mid e)\, d\pi = 0$ and, if $\phi(\pi)$ is nonincreasing in π for $\pi \in [\pi^*, \pi^{**}]$, the second term in (A.3) satisfies the inequalities

$$\int_{\pi^*}^{\infty} \phi(\pi)\, g_e(\pi \mid e)\, d\pi \leqslant \int_{\pi^*}^{\pi^{**}} \phi(\pi)\, g_e(\pi \mid e)\, d\pi \leqslant \phi(\pi_g) \int_{\pi^*}^{\pi^{**}} g_e(\pi \mid e)\, d\pi = 0. \tag{A.6}$$

The first inequality in (A.6) follows from $\phi(\pi) \leqslant 0$ and $g_e(\pi \mid e) > 0$ for $\pi > \pi^{**} > \pi_g$. The second inequality follows from a nonincreasing $\phi(\ )$ (so that $\phi(\pi_g) \leqslant \phi(\pi)$ for $\pi \in [\pi^*, \pi_g)$ and $\phi(\pi_g) \geqslant \phi(\pi)$ for $\pi \in (\pi_g, \pi^{**}]$) and the definition of π_g in (II). Q.E.D.

Proof of Proposition 4. The "if" direction follows directly from Propositions 2 and 3. To prove the "only if" direction, the following claim must first be verified:

Claim. Suppose (B^*, e^*) solves (4.2) and z_0 satisfies $E\{B^{LD}(\pi; z_0) \mid e^*\} = (1+\rho)I$. Then $(B^{LD}(\pi; z_0), e^*)$ also solves (4.2).

Proof of Claim. Since $E\{B^*(\pi) \mid e^*\} = (1+\rho)I$ (Proposition 2 and Lemma 4),

$$E\{V(\pi - B^*(\pi), e^*) \mid e^*\} = E\{V(\pi - B^{LD}(\pi; z_0), e^*) \mid e^*\}. \tag{A.7}$$

Given (A.7), it is sufficient to show that $(B^{LD}(\pi; z_0), e^*)$ satisfy (4.2a)–(4.2c). (4.2a) and (4.2c) are satisfied by construction. Further, if $B_0(\pi) \equiv B^*(\pi)$ and $B_1(\pi) \equiv B^{LD}(\pi; z_0)$, the prior conditions of Lemma 1(a) will be met at $e = e^*$, implying that $\partial E\{B^{LD}(\pi; z_0) - B^*(\pi) \mid e^*\}/\partial e \leqslant 0$ (with strict inequality when $B^*(\pi) \neq B^{LD}(\pi; z_0)$). Using this last inequality (and Eq. (4.7)),

$$\frac{\partial E\{V(\pi - B^{LD}(\pi; z_0), e^*) \mid e^*\}}{\partial e} \geqslant \frac{\partial E\{V(\pi - B^*(\pi), e^*) \mid e^*\}}{\partial e} \geqslant 0.$$

Thus $(B^{LD}(\pi; z_0), e^*)$ also satisfies (4.2b). Q.E.D. Claim

Given $\mu = 0$, Lemma 4, and the foregoing claim, $(B^{LD}(\pi; z_0), e^*)$ solves (4.2). Thus, since $E\{B^{LD}(\pi; z) \mid e^*\}$ is increasing in z, Proposition 4 will follow from the condition $z_0 \leqslant z^*$, where z^* is as defined in Lemma 5. To derive this condition, suppose the contrary, $z_0 > z^*$. Then

$$\frac{\partial E\{B^{LD}(\pi; z_0) \mid e^*\}}{\partial e} = \frac{\partial E\{B^{LD}(\pi; z^*) \mid e^*\}}{\partial e} + \int_{z^*}^{z_0} \pi g_e(\pi \mid e^*)\, d\pi > 0, \quad (A.8)$$

where the inequality follows from Lemma 5 and $g_e(\pi \mid e^*) > 0\ \forall \pi > z^* > \pi_g$. ($\pi_g$ is defined in the proof of Lemma 5, where it is shown that $z^* > \pi_g$.) But with $\mu = 0$ and $\lambda > 0$ (see the proof of Lemma 4), conditions (4.2b) and (4.5) imply that $\partial E\{B(\pi) \mid e\}/\partial e \leqslant 0$ at a solution to (4.2), contradicting (A.8). Q.E.D.

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