Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Economic Theory 114 (2004) 198–230
JOURNAL OF Economic Theory
http://www.elsevier.com/locate/jet

Optimal fiscal and monetary policy under sticky prices

Stephanie Schmitt-Grohé[a] and Martín Uribe[b,*]

[a] *Rutgers University, CEPR, and NBER, USA*
[b] *Department of Economics, University of Pennsylvania and NBER, 3718 Locust Walk, Philadelphia, PA 19104-6297, USA*

Received 10 September 2002; final version received 25 November 2002

Abstract

This paper studies optimal fiscal and monetary policy under sticky product prices. The theoretical framework is a stochastic production economy. The government finances an exogenous stream of purchases by levying distortionary income taxes, printing money, and issuing nominal non-state-contingent bonds. The main findings of the paper are: First, for a miniscule degree of price stickiness (i.e., many times below available empirical estimates) the optimal volatility of inflation is near zero. Second, small deviations from full price flexibility induce near random walk behavior in government debt and tax rates. Finally, price stickiness induces deviation from the Friedman rule.

JEL classification: E52; E61; E63

Keywords: Optimal fiscal and monetary policy; Sticky prices; Optimal inflation volatility; Tax smoothing

1. Introduction

Two distinct branches of the existing literature on optimal monetary policy deliver diametrically opposed policy recommendations concerning the long-run and cyclical behavior of prices and interest rates. One branch follows the theoretical framework laid out in Lucas and Stokey [16]. It studies the joint determination of optimal fiscal and monetary policy in flexible-price environments with perfect competition in

*Corresponding author. Fax: 1-215-573-2057.
E-mail address: uribe@econ.upenn.edu (M. Uribe).

0022-0531/03/$ - see front matter © 2003 Elsevier Science (USA). All rights reserved.
doi:10.1016/S0022-0531(03)00111-X

product and factor markets. In this group of papers, the government's problem consists in financing an exogenous stream of public spending by choosing the least disruptive combination of inflation and distortionary income taxes. The criterion under which policies are evaluated is the welfare of the representative private agent.

Calvo and Guidotti [4,5] and Chari et al. [6] characterize optimal monetary and fiscal policy in stochastic environments with nominal non-state-contingent government liabilities. A key result of these papers is that it is optimal for the government to make the inflation rate highly volatile and serially uncorrelated. Under the Ramsey policy, the government uses unanticipated inflation as a lump-sum tax on financial wealth. The government is able to do this to the extend that it has nominal, non-state-contingent liabilities outstanding. Thus, price changes play the role of a shock absorber of unexpected innovations in the fiscal deficit. This 'front-loading' of government revenues via inflationary shocks allows the fiscal authority to keep income tax rates remarkably stable over the business cycle.

On the other hand, a more recent literature focuses on characterizing optimal monetary policy in environments with nominal rigidities and imperfect competitions.[1] Besides its emphasis on the role of price rigidities and market power, this literature differs from the earlier one described above in two important ways. First, it assumes, either explicitly or implicitly, that the government has access to (endogenous) lump-sum taxes to finance its budget. An important implication of this assumption is that there is no need to use unanticipated inflation as a lump-sum tax; regular lump-sum taxes take up this role. Second, the government is assumed to be able to implement a production (or employment) subsidy so as to eliminate the distortion introduced by the presence of monopoly power in product and factor markets.

A key result of this literature is that the optimal monetary policy features an inflation rate that is zero or close to zero at all dates and all states.[2] The reason why price stability turns out to be optimal in environments of the type described here is straightforward: the government keeps the price level constant in order to minimize (or completely eliminate) the costs introduced by inflation under nominal rigidities.

Taken together, these two strands of research on optimal monetary policy leave the monetary authority without a clear policy recommendation. Should the central bank pursue policies that imply high or low inflation volatility? The goal of this paper is to contribute to the resolution of this policy dilemma. To this end, it incorporates in a unified framework the essential elements of the two approaches to optimal policy described above. Specifically, we build a model that shares three elements with the earlier literature: (a) The only source of regular taxation available

[1] See, for example, [10,12,13,20,28,29].

[2] In models where money is used exclusively as a medium of account, the optimal inflation rate is typically strictly zero [30]. Khan et al. [13] show that when a transaction role for money is introduced, the optimal inflation rate lies between zero and the one called for by the Friedman rule. However, in calibrated model economies they find that the optimal rate of inflation is in fact very close to zero and smooth. Erceg et al. [10] show that in models with sluggish price adjustment in product as well as factor markets price stability is suboptimal. Yet, for realistic calibrations of their model, the optimal inflation volatility is close to zero.

to the government is distortionary income taxes. In particular, the fiscal authority cannot adjust lumpsum taxes endogenously in financing its outlays. (b) The government cannot implement production subsidies to undo distortions created by the presence of imperfect competition. (c) The government issues only nominal, one-period, non-state-contingent bonds. At the same time, our model shares two important assumptions with the more recent body of work on optimal monetary policy: (a) Product markets are not perfectly competitive. In particular, we assume that each firm in the economy is the monopolistic producer of a differentiated intermediate good. (b) Product prices are assumed to be sticky. We introduce price stickiness à la Rotemberg [19] by assuming that firms face a convex cost of price adjustment. An assumption maintained throughout this paper that is common to all of the papers cited above (except for Lucas and Stokey [16]) is that the government has the ability to fully commit to the implementation of announced fiscal and monetary policies.

In this environment, the government faces a tradeoff in choosing the path of inflation. On the one hand, the government would like to use unexpected inflation as a non-distorting tax on nominal wealth. In this way, the fiscal authority could minimize the need to vary distortionary income taxes over the business cycle. On the other hand, changes in the rate of inflation come at a cost, for firms face nominal rigidities.[3] The main result of this paper is that under plausible calibrations of the degree of price stickiness, this trade off is overwhelmingly resolved in favor of price stability. The optimal fiscal/monetary regime features relatively low inflation volatility. Thus, the Ramsey allocation delivers an inflation process that is more in line with the predictions of the more recent body of literature on optimal monetary policy referred to above, which ignores fiscal constraints by assuming that the government can resort to lump-sum taxation. Moreover, we find that a miniscule amount of price stickiness suffices to bring the optimal degree of inflation volatility close to zero. Specifically, our results suggest that for a degree of price stickiness that is 10 times smaller than available estimates for the US economy, price stability emerges as the central feature of optimal monetary policy.

The fragility of front-loading government revenue via surprise changes in the price level reveals that the welfare gains of this way of government financing must be small. To understand why this is so, it is useful to relate price stickiness to the ability of the government to make nominally non-state-contingent debt state contingent in real terms. Under full price flexibility, the government uses unexpected variations in the price level to render the real return on nominal bonds state contingent. Under price stickiness, this practice is costly for firms are subject to price adjustment costs. It follows that as price adjustment costs become large, the Ramsey planner is less likely to use variations in the price level to create state-contingent real debt. Thus, the more sticky prices are, the more the economy will resemble one without real

[3] This point was first discussed by Woodford [27]. Christiano and Fitzgerald [7] and Sims [25], although limiting their analysis to the case of flexible prices, also recognize this point and propose as a potentially fruitful line of research the quantitative assessment of the costs and benefits of price volatility in models with sluggish price adjustment. This paper can be interpreted as following up on their suggestions.

state-contingent debt. Recent work by Aiyagari et al. [1] shows that the level of welfare in Ramsey economies with and without real state-contingent debt is virtually the same. As a consequence, in the sticky-price model studied in this paper, the Ramsey planner is willing to give up front loading all together to avoid price adjustment costs even when such costs are fairly small.[4]

Indeed, in financing the budget the Ramsey planner replaces front-loading with standard debt and tax instruments. For example, in response to an unexpected increase in government spending the planner does not generate a surprise increase in the price level. Instead, he chooses to finance the increase in government purchases partly through an increase in income tax rates and partly through an increase in public debt. The planner minimizes the tax distortion by spreading the required tax increase over many periods. This tax-smoothing behavior induces near-random walk dynamics into the tax rate and public debt. By contrast, under full price flexibility (i.e., when the government can create real-state contingent debt) tax rates and public debt inherit the stochastic process of the underlying shocks.

An important conclusion of our study is thus that the Barro [2]-Aiyagari et al. [1] result, namely, that optimal policy imposes a near random walk behavior on taxes and debt, does not require the unrealistic assumption that the government can issue only non-state-contingent real debt. This result emerges naturally in economies with nominally non-state contingent debt, clearly the case of greatest empirical relevance, and a minimum amount of price rigidity.

The remainder of the paper is organized in 8 sections. Section 2 describes the economic environment and defines a competitive equilibrium. Section 3 presents the Ramsey problem. Section 4 analyzes the business-cycle properties of Ramsey allocations. It first describes the calibration of the model. Then it presents the central result of the paper, namely, that even under very small price adjustment costs the optimal inflation volatility is near zero. Section 5 shows that when prices are sticky, public debt and tax rates are near random walk processes whereas when prices are flexible they have a strong mean reverting component. Section 6 shows that price-stickiness introduces deviations from the Friedman rule. Section 7 presents a discussion of the accuracy of the numerical solution method. Section 8 investigates whether the time series process for the nominal interest rate implied by the Ramsey policy can be represented as a Taylor-type interest rate feedback rule. Finally, Section 9 presents concluding remarks.

2. The model

In this section we develop a simple infinite-horizon production economy with imperfectly competitive product markets and sticky prices. A demand for money is motivated by assuming that money facilitates transactions. The government finances

[4] Again, by small we mean a value for the parameter determining the degree of price sluggishness that is many times smaller than available empirical estimates of that parameter based on data from the United States and other industrialized economies.

an exogenous stream of purchases by levying distortionary income taxes, printing money, and issuing one-period nominally risk-free bonds.

2.1. The private sector

Consider an economy populated by a large number of identical households. Each household has preferences defined over processes of consumption and leisure and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \tag{1}$$

where c_t denotes consumption, h_t denotes labor effort, $\beta \in (0,1)$ denotes the subjective discount factor, and E_0 denotes the mathematical expectation operator conditional on information available in period 0. The single period utility function U is assumed to be increasing in consumption, decreasing in effort, strictly concave, and twice continuously differentiable.

In each period $t \geqslant 0$, households can acquire two types of financial assets: fiat money, M_t, and one-period, state-contingent, nominal assets, D_{t+1}, that pay the random amount D_{t+1} of currency in a particular state of period $t+1$. Money facilitates consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost $s(v_t)$ that depends on the household's money- to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t}, \tag{2}$$

where P_t denotes the price of the consumption good in period t. The transaction cost function, $s(v)$, satisfies the following assumption:

Assumption 1. The function $s(v)$ satisfies: (a) $s(v)$ is non-negative and twice continuously differentiable; (b) there exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (c) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$; and (d) $2s'(v) + vs''(v) > 0$ for all $v \geqslant \underline{v}$.

Assumption 1(b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption 1(c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level. Assumption 1(d) ensures that the demand for money is decreasing in the nominal interest rate. (Note that Assumption 1(d) is weaker than the more common assumption of strict convexity of the transaction cost function.)

The consumption good c_t is assumed to be a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit–Stiglitz type. Each household is the monopolistic producer of one variety of intermediate goods. In turn, intermediate goods are produced using a linear technology, $z_t \tilde{h}_t$, that takes labor, $\tilde{h}_t$, as the sole input and is subject to an exogenous productivity shock, z_t. The household hires labor from a perfectly competitive market. The demand for the intermediate input is of the form $Y_t d(p_t)$, where Y_t denotes the level of aggregate demand and p_t denotes the relative price of the intermediate good in terms of the composite consumption good. The relative price p_t is defined as $\tilde{P}_t/P_t$, where $\tilde{P}_t$ is the nominal price of the intermediate good produced by the household and P_t is the price of the composite consumption good. The demand function $d(\cdot)$ is assumed to be decreasing and to satisfy $d(1) = 1$ and $d'(1) < -1$. The restrictions on $d(1)$ and $d'(1)$ are necessary for the existence of a symmetric equilibrium. The monopolist sets the price of the good it supplies taking the level of aggregate demand as given, and is constrained to satisfy demand at that price, that is,

$$z_t \tilde{h}_t \geqslant Y_t d(p_t). \tag{3}$$

We follow Rotemberg [19] and introduce sluggish price adjustment by assuming that the firm faces a resource cost that is quadratic in the inflation rate of the good it produces

$$\text{Price adjustment cost} = \frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2.$$

The parameter θ measures the degree of price stickiness. The higher is θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$, then prices are flexible.

The flow budget constraint of the household/firm unit in period t is then given by

$$\begin{aligned} &P_t c_t[1 + s(v_t)] + M_t + E_t r_{t+1} D_{t+1} \\ &\quad \leqslant M_{t-1} + D_t + P_t\left[\frac{\tilde{P}_t}{P_t} Y_t d\left(\frac{\tilde{P}_t}{P_t}\right) - w_t \tilde{h}_t - \frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2\right] \\ &\qquad + (1 - \tau_t) P_t w_t h_t, \end{aligned} \tag{4}$$

where w_t is the real wage rate and τ_t is the labor income tax rate. The variable r_{t+1} denotes the period-t price of a claim to one unit of currency in a particular state of period $t+1$ divided by the probability of occurrence of that state conditional on information available in period t. The left-hand side of the budget constraint represents the uses of wealth: consumption spending, including transactions costs, money holdings, and purchases of interest bearing assets. The right-hand side shows the sources of wealth: money, the payoff of contingent claims acquired in the previous period, profits from the sale of the differentiated good net of the price-adjustment cost, and after-tax labor income.

In addition, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j \to \infty} E_t q_{t+j+1}(D_{t+j+1} + M_{t+j}) \geqslant 0, \tag{5}$$

at all dates and under all contingencies. The variable q_t represents the period-zero price of one unit of currency to be delivered in a particular state of period t divided by the probability of occurrence of that state given information available at time 0 and is given by

$$q_t = r_1 r_2 \cdots r_t$$

with $q_0 \equiv 1$.

The household chooses the set of processes $\{c_t, h_t, \tilde{h}_t, \tilde{P}_t, v_t, M_t, D_{t+1}\}_{t=0}^{\infty}$, so as to maximize (1) subject to (2)–(5), taking as given the set of processes $\{Y_t, P_t, w_t, r_{t+1}, \tau_t, z_t\}_{t=0}^{\infty}$ and the initial condition $M_{-1} + D_0$.

Let the multiplier on the flow budget constraint (4) be λ_t/P_t and the one on the production constraint (3) be $mc_t\lambda_t$. Then the first-order conditions of the household's maximization problem are (2)–(5) holding with equality and

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)], \tag{6}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1 - \tau_t)w_t}{1 + s(v_t) + v_t s'(v_t)}, \tag{7}$$

$$v_t^2 s'(v_t) = 1 - E_t r_{t+1}, \tag{8}$$

$$\frac{\lambda_t}{P_t} r_{t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}}, \tag{9}$$

$$mc_t = \frac{w_t}{z_t}, \tag{10}$$

$$\begin{aligned} 0 = {} & \lambda_t[Y_t d(p_t) + p_t Y_t d'(p_t) - \theta\pi_t(\pi_t p_t/p_{t-1} - 1) - mc_t Y_t d'(p_t)] \\ & + \beta\theta E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1}p_{t+1}/p_t - 1)p_{t+1}/p_t^2, \end{aligned} \tag{11}$$

where $\pi_t \equiv P_t/P_{t-1}$ denotes the gross consumer price inflation rate. The interpretation of these optimality conditions is straightforward. First-order condition (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. The assumed form of the transaction cost function ensures that this wedge is zero at the satiation point $\underline{v}$ and increasing in money velocity for $v > \underline{v}$. Eq. (7) shows that both the labor income tax rate and the transaction cost distort the consumption/leisure margin. Given the wage rate, households will tend to work less and consume less the higher is τ or the larger is v_t. Eq. (8) implicitly defines the household's money demand function. Note that $E_t r_{t+1}$ is the period-t price of an asset that pays one unit of currency in every state in period $t+1$. Thus $E_t r_{t+1}$ represents the inverse of the gross nominal risk-free interest rate. Formally, letting R_t denote the gross risk-free nominal interest rate, we have

$$R_t = \frac{1}{E_t r_{t+1}}.$$

Assumption 1 implies that the demand for money is strictly decreasing in the nominal interest rate and unit elastic in consumption. Eq. (9) represents a standard pricing equation for one-period-ahead nominal contingent claims. Eq. (10) states that marginal cost equals the ratio of wages to the marginal product of labor.

Finally, Eq. (11) states that the presence of price-adjustment costs prevents firms in the short run from setting their prices so as to equate marginal revenue, $p_t + d(p_t)/d'(p_t)$, to marginal cost, mc_t.

2.2. The government

The government faces a stream of public consumption, denoted by g_t, that is exogenous, stochastic, and unproductive. These expenditures are financed by levying labor income taxes at the rate τ_t, by printing money, and by issuing one-period, risk-free (non-contingent), nominal obligations, which we denote by B_t. The government's sequential budget constraint is then given by

$$M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_t g_t - \tau_t P_t w_t h_t$$

for $t \geqslant 0$. The monetary/fiscal regime consists in the announcement of state-contingent plans for the nominal interest rate and the tax rate, $\{R_t, \tau_t\}$.

2.3. Equilibrium

We restrict attention to symmetric equilibria where all households charge the same price for the good they produce. As a result, we have that $p_t = 1$ for all t. It then follows from the fact that all firms face the same wage rate, the same technology shock, and the same production technology, that they all hire the same amount of labor. That is, $\tilde{h}_t = h_t$. Also, because all firms charge the same price, we have that the marginal revenue of the individual monopolist is constant and equal to $1 + 1/d'(1)$. Let

$$\eta \equiv d'(1)$$

denote the equilibrium value of the elasticity of demand faced by the individual producers of intermediate goods. Then in equilibrium equation (11) gives rise to the following expectations augmented Phillips curve:

$$\lambda_t \pi_t(\pi_t - 1) = \beta E_t \lambda_{t+1} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_t \eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - \frac{w_t}{z_t}\right]. \tag{12}$$

An aggregate supply relation of this type is a standard feature of the recent neo-Keynesian literature on optimal monetary policy.

Because all households are identical, in equilibrium there is no borrowing or lending among them. Thus, all interest-bearing asset holdings by private agents are in the form of government securities.

That is,

$$D_t = R_{t-1}B_{t-1}$$

at all dates and all contingencies. Finally, in equilibrium, it must be the case that the nominal interest rate is non-negative,

$$R_t \geqslant 1.$$

Otherwise pure arbitrage opportunities would exist and households' demand for consumption would not be well defined.

We are now ready to define an equilibrium. A competitive equilibrium is a set of plans $\{c_t, h_t, M_t, B_t, v_t, mc_t, \lambda_t, P_t, q_t, r_{t+1}\}$ satisfying the following conditions:

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)], \tag{13}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1-\tau_t)z_t mc_t}{1 + s(v_t) + v_t s'(v_t)}, \tag{14}$$

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t}, \tag{15}$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}}, \tag{16}$$

$$R_t = \frac{1}{E_t r_{t+1}} \geqslant 1, \tag{17}$$

$$\lambda_t \pi_t(\pi_t - 1) = \beta E_t \lambda_{t+1} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_t \eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - mc_t\right], \tag{18}$$

$$M_t + B_t + \tau_t P_t z_t mc_t h_t = R_{t-1}B_{t-1} + M_{t-1} + P_t g_t, \tag{19}$$

$$\lim_{j\to\infty} E_t q_{t+j+1}(R_{t+j}B_{t+j} + M_{t+j}) = 0, \tag{20}$$

$$q_t = r_1 r_2 \cdots r_t; \quad \text{with } q_0 = 1, \tag{21}$$

$$[1 + s(v_t)]c_t + g_t + \frac{\theta}{2}(\pi_t - 1)^2 = z_t h_t, \tag{22}$$

$$v_t = P_t c_t / M_t, \tag{23}$$

given policies $\{R_t, \tau_t\}$, exogenous processes $\{z_t, g_t\}$, and the initial condition $R_{-1}B_{-1} + M_{-1} > 0$.

3. The Ramsey problem

The optimal fiscal and monetary policy is the process $\{R_t, \tau_t\}$ associated with the competitive equilibrium that yields the highest level of utility to the representative household, that is, that maximizes (1). As is well known, in the absence of price stickiness, the Ramsey planner will always find it optimal to confiscate the entire initial nominal wealth of the household by choosing a policy that results in an infinite initial price level, $P_0 = \infty$. This is because such a confiscation amounts to a non-distortionary lump-sum tax. To avoid this unrealistic feature of optimal policy, it is typically assumed in the flexible-price literature that the initial price level is given. We follow this tradition here to make our results comparable to those obtained under frictionless price adjustment. However, we note that in the presence of price adjustment costs it may not be optimal for the Ramsey planner to choose $P_0 = \infty$. The reason is twofold. First, such policy would be distortionary as it would introduce a large deviation of marginal cost from marginal revenue. Second, an infinitely large initial inflation would absorb a large amount of output because the implementation of price changes requires the use of real resources.

A key difference between our model with sticky prices and non-state-contingent nominal government debt and models with flexible prices (such as [4,5,6]) or models with sticky prices but state-contingent debt (like the model considered by Correia et al. [9]) is that in our model the primal form of the competitive equilibrium can no longer be reduced to a single intertemporal implementability (budget) constraint in period 0 and a feasibility constraint holding in every period. This feature of the Ramsey problem is akin to the one identified by Aiyagari et al. [1] in their analysis of optimal policy in a real economy without state-contingent debt. The reason why under sticky prices and nominally non-state contingent debt the Ramsey constraints cannot be stated in terms of a single time-zero implementability constraint is the following. Under price flexibility, given a real allocation, the path for prices is uniquely determined in such a way that it ensures that the implied real return on nominal debt satisfies the transversality condition of the competitive equilibrium at all dates and all states. Under price stickiness, the price path is more constrained for it must also satisfy the expectations augmented Phillips curve. However, a price path that satisfies the expectations augmented Phillips curve and a time-zero implementability constraint may not result in a state-contingent real government debt path that satisfies the transversality condition of the competitive equilibrium (Eq. (20)) at all dates and under all contingencies.

The following proposition presents a simpler form of the competitive equilibrium.[5]

Proposition 1. *Plans* $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}_{t=0}^{\infty}$ *satisfying* (13), (18), (22), *and*

$$\lambda_t = \beta\rho(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{24}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t,h_t)\gamma(v_t)}{U_c(c_t,h_t)}\right)h_t = \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t, \quad t>0, \tag{25}$$

$$\frac{c_0}{v_0} + b_0 + \left(mc_0 z_0 + \frac{U_h(c_0,h_0)\gamma(v_0)}{U_c(c_0,h_0)}\right)h_0 = \frac{R_{-1}B_{-1} + M_{-1}}{P_{-1}\pi_0} + g_0,$$

$$\lim_{j\to\infty} E_t\left[\beta^j\frac{\lambda_{t+j+1}}{\pi_{t+j+1}}\left(\rho(v_{t+j})b_{t+j} + \frac{c_{t+j}}{v_{t+j}}\right)\right] = 0, \tag{26}$$

$$v_t \geqslant \underline{v} \quad \textit{and} \quad v_t^2 s'(v_t) < 1,$$

[5] This statement of the competitive equilibrium could be simplified further by using (13) and (18) to eliminate λ_t and mc_t.

for all dates and under all contingencies given $(R_{-1}B_{-1} + M_{-1})/P_{-1}$, *are the same as those satisfying* (13)–(23), *where*

$$\gamma(v_t) \equiv 1 + s(v_t) + v_t s'(v_t)$$

and

$$\rho(v_t) \equiv 1/[1 - v_t^2 s'(v_t)].$$

Proof. See Appendix A.

We assume that the government has the ability to commit to the contingent policy rules it announces at date 0. It then follows from Proposition 1 that the Ramsey problem can be stated as choosing contingent plans $c_t, h_t, v_t, \pi_t, \lambda_t, b_t$, and mc_t so as to maximize (1) subject to (13), (18), (22), (24)–(26), $v_t \geqslant \underline{v}$, and $v_t^2 s'(v_t) < 1$, taking as given $(M_{-1} + R_{-1}B_{-1})/P_0$ and the exogenous stochastic processes g_t and z_t. The Lagrangian of the Ramsey planner's problem as well as the associated first-order conditions are shown in Appendix B. □

3.1. Alternative representation of the Ramsey constraints

While it is not possible to reduce the constraints of the Ramsey problem to a single intertemporal implementability constraint in period 0 and one feasibility constraint holding at every date and at every state, as is the case under price flexibility, it is possible to express the set of constraints the Ramsey planner faces in terms of a sequence of intertemporal implementability constraints rather than in terms of the sequence of transversality conditions given in (26). The next proposition presents such a representation.

Proposition 2. *Plans* $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}_{t=0}^{\infty}$ *satisfying the feasibility constraint* (22), *the expectations augmented Phillips curve*

$$\pi_t(\pi_t - 1) = \beta E_t \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \frac{\gamma(v_t)}{\gamma(v_{t+1})} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - mc_t\right], \quad (27)$$

the sequential budget constraint of the government,

$$\frac{c_t}{v_t} + b_t + \left[mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)}\right] h_t = \frac{\rho(v_{t-1}) b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t \quad \forall t \geqslant 1, \quad (28)$$

$$\frac{c_0}{v_0} + b_0 + \left[mc_0 z_0 + \frac{U_h(c_0, h_0)\gamma(v_0)}{U_c(c_0, h_0)}\right] h_0 = \frac{R_{-1}B_{-1} + M_{t-1}}{P_{-1}\pi_0} + g_0,$$

the sequence of intertemporal budget constraints

$$E_t \sum_{j=0}^{\infty} \beta^j \left[U_c(c_{t+j}, h_{t+j}) c_{t+j} + U_h(c_{t+j}, h_{t+j}) h_{t+j} + z_{t+j} h_{t+j}(mc_{t+j} - 1) \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right.$$
$$\left. + \frac{\theta}{2}(\pi_{t+j} - 1)^2 \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right] = \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1}) b_{t-1}}{\pi_t} \right] \quad (29)$$

and the boundary conditions on v_t

$$v_t \geqslant \underline{v} \quad \textit{and} \quad v_t^2 s'(v_t) < 1,$$

for all dates and under all contingencies given $(R_{-1}B_{-1} + M_{-1})/P_{-1}$, *are the same as those satisfying the definition of a competitive equilibrium, that is*, (13)–(23).

Proof. See Appendix C.

This more compact representation of the restrictions of a competitive equilibrium facilitates comparison with those arising in real economies without state-contingent debt [1].

4. Optimal inflation volatility

In this section we characterize numerically the dynamic properties of Ramsey allocations. We compute dynamics by solving first- and second-order logarithmic approximations to the Ramsey planner's policy functions around a non-stochastic Ramsey steady state. We first describe the calibration of the model and then present the quantitative results.

4.1. Calibration

We calibrate our model to the US economy. The time unit is meant to be a year. We assume that up to period 0, the economy is in the non-stochastic steady state of a competitive equilibrium with constant paths for consumption, hours, nominal interest rates, inflation, tax rates, etc. To facilitate comparison to the case of price flexibility we adopt, where possible, the calibration of Schmitt-Grohé and Uribe [23]. Specifically, we assume that in the steady state the inflation rate is 4.2 percent per year, which is consistent with the average growth rate of the US GDP deflator over the period 1960:Q1 to 1998:Q3, that the debt-to-GDP ratio is 0.44 percent, which corresponds to the figure observed in the United States in 1995 (see the 1997 Economic Report of the President, Table B-79), and that government expenditures are equal to 20 percent of GDP, a figure that is in line with postwar US data. We follow Prescott [18] and set the subjective discount rate β to 0.96 to be consistent with a steady-state real rate of return of 4 percent per year.

We assume that the single-period utility index is of the form

$$U(c,h) = \ln(c) + \delta \ln(1-h).$$

We set the preference parameter δ so that in the flexible-price steady-state households allocate 20 percent of their time to work. The resulting parameter value is $\delta = 2.9$.[6]

[6] See [23] for a derivation of the exact relations used to identify δ.

To calibrate the price elasticity of demand η, we use the fact that in a flexible price equilibrium the value-added markup of prices over marginal costs, which we denote by μ, is related to η as $1+\mu = \eta/(1+\eta)$. Drawing from the empirical study of Basu and Fernald [3], we assign a value of 0.2 to μ. Basu and Fernald estimate gross output production functions and obtain estimates for the gross-output markup of about 0.1. They show that their estimates are consistent with values for the value-added markup of up to 0.25.

To calibrate the degree of price stickiness, we use Sbordone's [22] estimate of a linear new-Keynesian Phillips curve. Such a Phillips curve arises in our model from a log-linearization of equilibrium condition (18) around a zero-inflation steady state:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{h}{\theta\mu}\widehat{mc}_t,$$

where a circumflex denotes log-deviations from the steady state. Using quarterly postwar US data, Sbordone estimates the coefficient $\theta\mu/h$ to be 17.5. Given our calibration $h = 0.2$ and $\mu = 0.2$, we have that the price-stickiness coefficient θ is 17.5. As pointed out by Sbordone, in a Calvo–Yun staggered-price setting model, this value of θ implies that firms change their price on average every 9 months. Because in our model the time unit is a year, we set θ equal to 17.5/4.

We use the following specification for the transactions cost technology:

$$s(v) = Av + B/v - 2\sqrt{AB}. \tag{30}$$

This functional form implies a satiation point for consumption-based money velocity, $\underline{v}$, equal to $\sqrt{B/A}$. The money demand function implied by the above transaction technology is of the form

$$v_t^2 = \frac{B}{A} + \frac{1}{A}\frac{R_t - 1}{R_t}.$$

Note that money demand has a unit elasticity with respect to consumption expenditures. This feature is a consequence of the assumption that transaction costs, $cs(c/m)$, are homogenous of degree one in consumption and real balances and is independent of the particular functional form assumed for $s(.)$. Further, as the parameter B approaches zero, the transaction cost function $s(.)$ becomes linear in velocity and the demand for money adopts the Baumol–Tobin square-root form with respect to the opportunity cost of holding money, $(R-1)/R$. That is, the log–log elasticity of money demand with respect to the nominal interest rate converges to 1/2 as B vanishes. To identify the parameters A and B, we estimate the above equation using quarterly US data from 1960:1 to 1999:3. We measure v as the ratio of non-durable consumption and services expenditures to Ml. The nominal interest rate is taken to be the 3-month Treasury Bill rate. The OLS estimate implies that $A = 0.0111$ and $B = 0.07524$.[7] At the calibrated steady-state interest rate of 8.2

[7] The estimated equation is $v_t^2 = 6.77 + 90.03(R_t - 1)/R_t$. The t-statistics for the constant and slope of the regression are, respectively, 6.81 and 5.64. The $\bar{R}^2$ of the regression is 0.16. Instrumental-variable estimates using three lagged values of the dependent and independent variables yield similar estimates for A and B.

Table 1
Calibration

Symbol	Definition	Value	Description
β		0.96	Subjective discount factor
π		1.042	Gross inflation rate
h		0.2	Fraction of time allocated to work
s_g	g/y	0.2	Government consumption to GDP ratio
s_b	$B/(Py)$	0.44	Public debt to GDP ratio
$1+\mu$	$\eta/(1+\eta)$	1.2	Gross value-added markup
θ		17.5/4	Degree of price stickiness
A		0.0111	Parameter of transaction cost function $s(v) = Av + B/v - 2\sqrt{AB}$
B		0.07524	Parameter of transaction cost function
λ^g		0.9	Serial correlation of $\log g_t$
σ_{ε^g}		0.0302	Standard deviation of innovation to $\ln g_t$
λ^z		0.82	Serial correlation of technology shock
σ_{ε^z}		0.0229	Standard deviation of innovation to $\ln z_t$

Note: The time unit is a year. The variable $y \equiv zh$ denotes steady-state output.

percent per year, the implied semi-elasticity of money demand with respect to the nominal interest rate $(\partial \ln m/\partial R)$ is equal to -2.82. When the nominal interest rate is zero, our money demand specification implies a finite semi-elasticity equal to -6.6.

Government spending, g_t, and labor productivity, z_t, are assumed to follow independent AR(1) processes in their logarithms,

$$\ln g_t = (1-\lambda^g)\ln g + \lambda^g \ln g_{t-1} + \varepsilon_t^g; \quad \varepsilon_t^g \sim N(0, \sigma^2_{\varepsilon^g})$$

and

$$\ln z_t = \lambda^z \ln z_{t-1} + \varepsilon_t^z; \quad \varepsilon_t^z \sim N(0, \sigma^2_{\varepsilon^z}).$$

We assume that $(\lambda^g, \sigma_{\varepsilon^g}) = (0.9, 0.03)$ and that $(\lambda^z, \sigma_{\varepsilon^z}) = (0.82, 0.02)$. This specification is in line with the calibration of the stochastic processes for g_t and z_t given in [6]. Table 1 summarizes the calibration of the economy.

4.2. Numerical results

In [23], we show that under flexible prices (with and without imperfect competition) it is possible to find an exact numerical solution to the Ramsey problem. The reason is that in that case the constraints of the Ramsey problem reduce to a static feasibility constraint and a single intertemporal, time-separable, implementability constraint. On the other hand, when price adjustment is sluggish and the government issues only nominal state non-contingent debt, the Ramsey problem contains a sequence of intertemporal implementability constraints, one for each date and state. This complication renders impossible the task of finding an exact numerical solution. One is thus forced to resort to approximation techniques. In this section we limit attention to results based on log-linear approximations to the

Table 2
Dynamic properties of the Ramsey allocation (linear approximation)

Variable	Mean	Std. dev.	Auto. corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
Flexible prices and perfect competition ($\theta = 0$ and $\eta = -\infty$)						
τ	18.7	0.044	0.834	−0.322	0.844	−0.516
π	−3.66	6.04	−0.0393	−0.245	0.313	−0.321
R	0	0	—	—	—	—
y	0.25	0.00843	0.782	1	0.203	0.975
h	0.25	0.00217	0.834	−0.322	0.846	−0.516
c	0.21	0.00827	0.778	0.955	−0.0797	0.997
Flexible prices and imperfect competition ($\theta = 0$)						
τ	25.8	0.0447	0.616	0.236	−0.845	0.511
π	−1.82	6.8	−0.0411	−0.207	0.329	−0.321
R	1.83	0.0313	0.797	−0.237	0.845	−0.513
y	0.208	0.00675	0.783	1	0.289	0.951
h	0.208	0.0024	0.833	−0.237	0.845	−0.513
c	0.168	0.00645	0.777	0.93	−0.0624	0.998
Baseline sticky-price economy						
τ	25.1	0.998	0.743	−0.283	0.476	−0.238
π	−0.16	0.171	0.0372	−0.123	0.385	−0.289
R	3.85	0.562	0.865	−0.949	−0.0372	−0.969
y	0.209	0.00713	0.815	1	0.199	0.943
h	0.208	0.00253	0.813	−0.124	0.611	−0.424
c	0.168	0.00707	0.819	0.938	−0.131	0.958

Note: τ, π, and R are expressed in percentage points and y, h, and c in levels. Unless indicated otherwise, the parameter values are: $\beta = 1/1.04$, $\delta = 2.9$, $g = 0.04$, $b_{-1} = 0.088$, $\eta = -6$, $\theta = 17.5/4$, $A = 0.0111$, $B = 0.07524$, $T = 100$, and $J = 500$.

Ramsey planner's optimality conditions. In Section 7, we present results based on a second-order approximation to the Ramsey planner's decision rules. We show there that the results of this section are robust to higher-order approximations.

Table 2 displays a number of sample moments of key macroeconomic variables under the Ramsey policy. The moments are computed as follows. We first generate simulated time series of length T for the variables of interest and compute first and second moments. We repeat this procedure J times and then compute the average of the moments. In the table, T equals 100 years and J equals 500. In Section 7 we explain the criterion for choosing these two parameter values.

The top panel of Table 2 corresponds to a flexible-price economy with perfect competition ($\theta = 0$ and $\eta = -\infty$), the middle panel to a flexible-price economy with imperfect competition ($\theta = 0$, $\eta = -6$), and the bottom panel to an economy with sluggish price adjustment and imperfect competition ($\theta = 17.5/4$ and $\eta = -6$).

4.2.1. *Optimal inflation volatility under price flexibility*

Under flexible prices and perfect competition, the nominal interest rate is constant and equal to zero. That is, the Friedman rule is optimal. Because under perfect competition the nominal interest rate is zero at all times, the distortion introduced by the transaction cost is driven to zero in the Ramsey allocation $(s(v) = s'(v) = 0)$. On the other hand, distortionary income taxes are far from zero. The average value of the labor income tax rate is 18.7 percent. The Ramsey planner keeps this distortion smooth over the business cycle; the standard deviation of τ is 0.04 percentage points.

In the Ramsey allocation with perfect competition and flexible prices, inflation is on average negative (-3.7 percent per year). The most striking feature of the Ramsey allocation is the high volatility of inflation. A two-standard deviation band on each side of the mean features a deflation rate of 15.7 percent at the lower end and inflation of 8.3 percent at the upper end. The Ramsey planner uses the inflation rate as a state-contingent lump-sum tax/transfer on households' financial wealth. This lump-sum tax/transfer is used mainly in response to unanticipated changes in the state of the economy. This is reflected in the fact that inflation displays a near zero serial correlation. The result that in the Ramsey equilibrium inflation acts as a lump-sum tax on nominal wealth is due to Calvo and Guidotti [4,5] and Chari et al. [6] and has recently been stressed by Sims [25].

The high volatility and low persistence of the inflation rate stands in sharp contrast to the smooth and highly persistent behavior of the labor income tax rate. Our results on the dynamic properties of the Ramsey economy under perfect competition and flexible prices are consistent with those obtained by Chari et al. [6].

Under imperfect competition and flexible prices, the volatility and correlation properties of inflation, income tax rates, and other real variables are virtually unchanged. The main effect of imperfect competition is that the Friedman rule ceases to be optimal. The average nominal interest rate rises from zero to 1.8 percent. The reason for this departure from the Friedman rule is the presence of monopoly profits. As shown in [23] these profits represent pure rents for the owners of the monopoly rights, which the Ramsey planner would like to tax at a 100 percent rate. If profit taxes are either unavailable or restricted to be less than 100 percent, then the social planner uses inflation as an indirect tax on profits. Inflation acts as an indirect tax on profits because when consumers transform profits into consumption, they must hold money to perform the required transaction. The Friedman rule reemerges if (a) monopoly profits are completely confiscated; (b) profit tax rates are constrained to be equal to income tax rates; (c) monopolistically competitive firms make no profits (as could be the case in the presence of fixed costs); and (d) the Ramsey planner has access to consumption taxes.[8] Another difference between the perfectly and imperfectly competitive economies is that in the latter the average income tax rate is 7 percentage points higher than in the former, even though initial public debt and the process for government purchases are the same in both economies. The reason for this difference is that under imperfect competition, the labor income tax base is smaller due to the presence of market power.

[8] For a formal derivation of these results and a more detailed discussion, see [23].

4.2.2. Optimal inflation volatility under price stickiness

If price changes are brought about at a cost, then it is natural to expect that a benevolent government will try to implement policies consistent with a more stable behavior of prices than when price changes are costless. However, the quantitative effect of an empirically plausible degree of price rigidity on optimal inflation volatility is not clear a priori. When price adjustment is costly, the social planner faces a tradeoff. On the one hand, the planner would like to use unexpected changes in the price level as a state-contingent lump-sum tax or transfer on nominal wealth. In this way, the benevolent government avoids the need to resort to changes in distortionary taxes and interest rates over the business cycle. The use of inflation for this purpose would imply a relatively large volatility in prices. On the other hand, the Ramsey planner has incentives to stabilize the price level in order to minimize the costs associated with nominal price changes. The bottom panel of Table 2 shows that for the degree of stickiness that has been estimated for the US economy, this tradeoff is to a large extent resolved in favor of price stability. The Ramsey allocation features a dramatic drop in the standard deviation of inflation from about 7 percent per year under flexible prices to a mere 0.17 percent per year when prices adjust sluggishly.[9] This implication of the Ramsey allocation under sticky prices is more in accord with the recent neo-Keynesian literature on optimal monetary policy that ignores fiscal considerations (see the references cited in footnote 1).[10]

Indeed, the impact of price stickiness on the optimal degree of inflation volatility turns out to be much stronger than suggested by the numbers displayed in Table 2. Fig. 1 shows that a minimum amount of price stickiness suffices to make price stability the central goal of optimal policy. Specifically, when the degree of price stickiness, embodied in the parameter θ, is assumed to be 10 times smaller than the estimated value for the US economy, the optimal volatility of inflation is below 0.52 percent per year, 13 times smaller than under full price flexibility.

Therefore, the question arises as to why even a modest degree of price stickiness can turn undesirable the use of a seemingly powerful fiscal instrument, such as large re- or devaluations of private real financial wealth through surprise inflation. Our conjecture is that in the flexible-price economy, the welfare gains of surprise inflations or deflations are very small. Our intuition is as follows. Under flexible prices, it is optimal for the central bank to keep the nominal interest rate constant

[9] In independent and contemporaneous work, Siu [26] obtains similar results in a cash-credit economy where nominal rigidities are introduced by assuming that a fraction of firms must set their price one period in advance and the only source of uncertainty are government purchases shocks.

[10] An important assumption driving the result that significantly less inflation volatility is desirable in the presence of sticky prices is that government debt is state-noncontingent. When government debt is state contingent, the presence of sticky prices may introduce no difference in the Ramsey real allocation (see [9]). The reason for this result is that, as shown in [16], when government debt is state contingent and prices are fully flexible, the Ramsey allocation does not pin down the price level uniquely. In this case there is an infinite number of price level processes (and thus of money supply processes) that can be supported as Ramsey outcomes. Loosely speaking, the introduction of price stickiness simply 'uses this degree of freedom' without altering other aspects of the Ramsey solution. This is not possible under state-noncontingent debt. For in this case the price level is uniquely determined in the flexible- price economy. Thus, the presence of nominal rigidities modifies the optimal real allocation in fundamental ways.

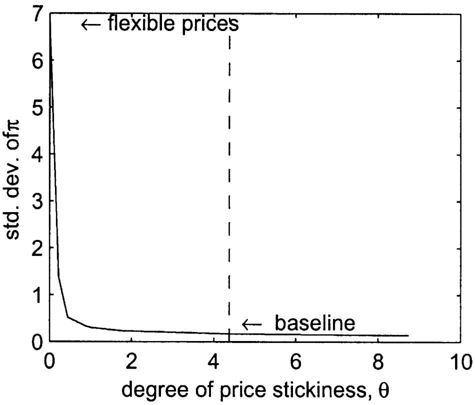

Fig. 1. Degree of price stickiness and optimal inflation volatility. The baseline value of θ is 4.4. The standard deviation of inflation is measured in percent per year.

over the business cycle. This means that large surprise inflations must be as likely as large deflations, as variations in real interest rates are small. In other words, inflation must have a near-i.i.d. behavior. As a result, high inflation volatility cannot be used by the Ramsey planner to reduce the average amount of resources to be collected via distortionary income taxes, which would be a first-order effect. The volatility of inflation serves primarily the purpose of smoothing the process of income tax distortions—a second-order source of welfare losses—without affecting their average level.

An additional way to gain intuition for the dramatic decline in optimal inflation volatility that takes place even at very modest levels of price stickiness is to interpret price volatility as a way for the government to introduce real state-contingent public debt. Under flexible prices the government uses state-contingent changes in the price level as a non-distorting tax or transfer on private holdings of government assets. In this way, non-state contingent nominal public debt becomes state-contingent in real terms. So, for example, in response to an unexpected increase in government spending the Ramsey planner does not need to increase tax rates by much because by inflating away part of the public debt he can ensure intertemporal budget balance. It is therefore clear that introducing costly price adjustment is as if the government was limited in its ability to issue real state-contingent debt. It follows that the larger is the welfare gain associated with the ability to issue real state-contingent public debt—as opposed to non-state contingent debt—the larger is the amount of price stickiness required to reduce the optimal degree of inflation volatility. Recent work by Aiyagari et al. [1] shows that indeed the level of welfare under the Ramsey policy in an economy without real state-contingent public debt is virtually the same as in an economy with state-contingent debt. Our finding that a small amount of price stickiness is all it takes to bring the optimal volatility of inflation from a very high level to near zero is thus perfectly in line with the finding of Aiyagari et al.[11]

[11] We note that the optimal degree of inflation volatility is an increasing function of the volatility of government purchases shocks.

If this intuition is correct, then the behavior of tax rates and public debt under sticky prices should resemble that implied by the Ramsey allocation in economies without real state-contingent debt. We investigate this issue in the next section.

5. Near random walk property of taxes and public debt under sticky prices

Lucas and Stokey [16] show that under state contingent government debt tax rates and public debt inherit the stochastic process of the underlying exogenous shocks. This implies, for example, that if the shocks driving business cycles are serially uncorrelated, then so are government bonds and tax rates. The work of Barro [2] and more recently Aiyagari et al. [1] suggests that the Lucas and Stokey result hinges on the assumption that the government can issue state-contingent debt. These authors show that independently of the assumed process for the shocks generating aggregate fluctuations, tax rates and public debt exhibit near random walk behavior. Calvo and Guidotti [4,5] and Chari et al. [6] show that the Ramsey allocation of a flexible price economy with nominally non-state-contingent debt behaves exactly like that of an economy with real state-contingent debt. It follows that under flexible prices and state non-contingent nominal debt, tax rates and government bonds inherit the stochastic process of the exogenous shocks.

In this section we investigate the extent to which the introduction of nominal rigidities brings the Ramsey allocation closer to the one arising in an economy without real state contingent debt. In other words, we wish to find out whether the Barro-Aiyagari et al. result can be obtained, not by ruling out complete markets for real public debt, but instead by introducing sticky prices in an economy in which the government issues only non-state-contingent nominal debt.

To this end, we consider the response of the flexible- and sticky-price economies under optimal fiscal and monetary policy to a serially uncorrelated government purchases shock. The result is displayed in Fig. 2. The response of the flexible price economy is shown with a dashed line and the response of the sticky price economy with a solid line. Government purchases are assumed to increase by 3 percent (one standard deviation) in period 1. Under flexible prices and perfect competition, taxes and bonds, like the shock itself, return to their pre-shock values after one period. By contrast, under sticky prices both variables are permanently affected by the shock. Specifically, when prices are sticky, bonds and taxes jump up on impact and then converge to values above their pre-shock levels. The difference in behavior under the two model specifications can be explained entirely by the behavior of the price level. Under flexible prices, the Ramsey planner inflates away part of the real value of outstanding nominal debt, bringing real public debt to its pre-shock level in just one period. Under sticky prices, the government finds it optimal not to increase the price level much. This is because price increases are costly. Instead, the planner finances the increase in government spending partly by increasing public debt and partly by increasing taxes. In order to avoid a large distortion at the time of the shock, the planner smoothes the tax increase over time. As a consequence, the stock of public debt displays a persistent increase.

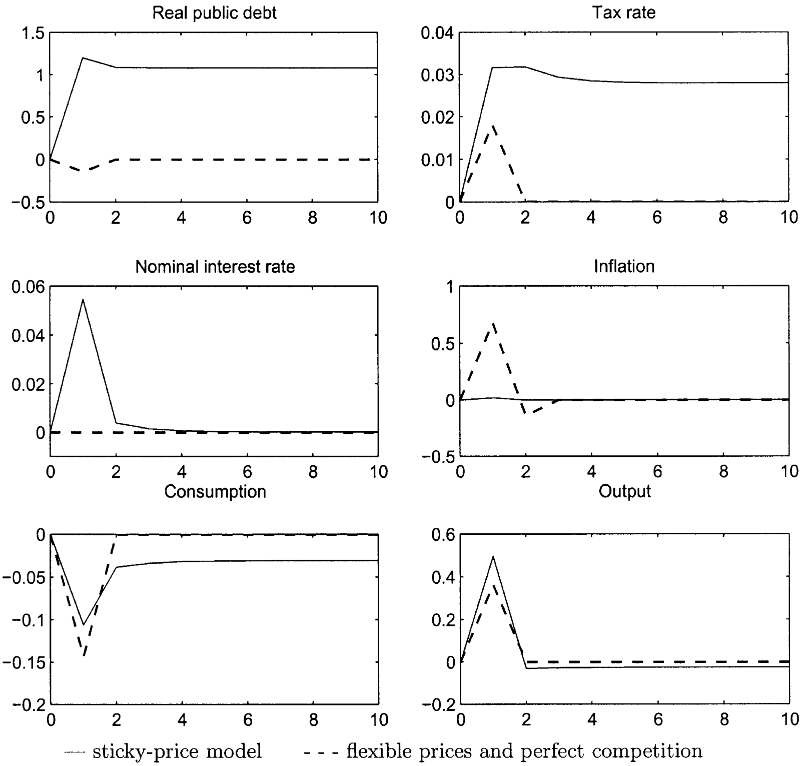

Fig. 2. Impulse response to an i.i.d. government purchases shock. *Note*: The size of the innovation in government purchases is one standard deviation (a 3 percent increase in g). The shock takes place in period 1. Public debt, consumption, and output are measured in percent deviations from their pre-shock levels. The tax rate, the nominal interest rate, and the inflation rate are measured in percentage points.

Thus, our sticky price model appears to replicate the near random walk behavior of bonds and tax rates found under the Ramsey allocation in real models without state-contingent debt, the Barro-Aiyagari et al. result. Indeed, the Barro-Aiyagari et al. result obtains not only under the baseline calibration of the degree of price stickiness (i.e., $\theta = 17.5/4$, or firms change prices once every 9 months), but for a minimal degree of nominal rigidities. Specifically, if we reduce θ by a factor of 10, bonds and tax rates maintain their near-random-walk behavior. This result is consistent with Fig. 1, which documents that a small amount of price rigidity suffices to bring the volatility of inflation close to zero.

6. Price stickiness and optimal deviations from the Friedman rule

In our baseline sticky-price economy the Friedman rule fails to hold. The average nominal interest rate is 3.8 percent per year. This significant deviation from the

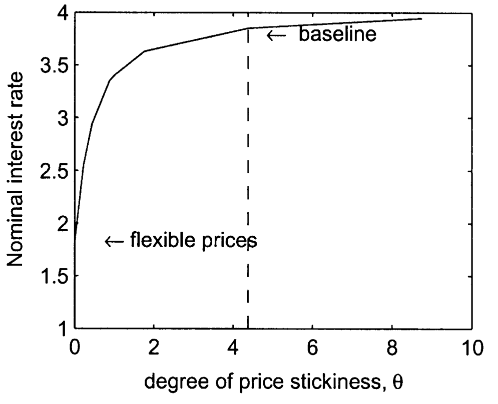

Fig. 3. Degree of price stickiness and deviations from the Friedman rule. The baseline value of θ is 4.4. The nominal interest rate is measured as percent per year.

Friedman rule can be decomposed in two parts. First, as shown by Schmitt-Grohé and Uribe [23], the presence of monopolistic competition induces the social planner to tax money balances as an indirect way to tax monopoly profits. Comparing the top and middle panels of Table 2, it follows that imperfect competition induces a deviation from the Friedman rule of 1.8 percentage points per year. Comparing the middle and bottom panels, it then follows that in our baseline economy price stickiness explains half of the 3.8 percentage points by which the nominal interest rate deviates from the Friedman rule. Indeed, as Fig. 3 illustrates, there exists a strong increasing relationship between the degree of price stickiness and the average nominal interest rate associated with the Ramsey allocation. The intuition behind this result is simple. The more costly it is for firms to alter nominal prices, the closer to zero is the inflation rate chosen by the benevolent government.

7. Accuracy of solution

The quantitative results presented thus far are based on a log-linear approximation to the first-order conditions of the Ramsey problem. In [23] we show how to compute exact numerical solutions to the Ramsey problem in the flexible-price economies (with perfectly and imperfectly competitive product markets). The availability of exact solutions allows us to evaluate the accuracy of the log-linear solution for the flexible-price economies considered above. The top and middle panels of Table 3 show that the quantitative results obtained using the exact numerical solution and a log-linear approximation are remarkably close. The most noticeable difference concerns the standard deviation of inflation. The log-linear approximation underpredicts the optimal volatility of inflation by about one percentage point.

Table 3
Accuracy of the approximate numerical solution

Variable	Mean	Std. dev.	Auto. corr.	Mean	Std. dev.	Auto. corr.
	Exact solution			Log-linear approximation		
Flexible prices and perfect competition						
τ	18.8	0.0491	0.88	18.7	0.044	0.834
π	−3.39	7.47	−0.0279	−3.66	6.04	−0.0393
R	0	0	—	0	0	—
Flexible prices and imperfect competition						
τ	26.6	0.042	0.88	25.8	0.0447	0.616
π	−1.46	7.92	−0.0239	−1.82	6.8	−0.0411
R	1.95	0.0369	0.88	1.83	0.0313	0.797
	Log-quadratic approximation			Log-linear approximation		
Baseline sticky-price economy						
τ	25.2	1.04	0.75	25.1	0.998	0.743
π	−0.16	0.18	0.03	−0.16	0.171	0.0372
R	3.83	0.56	0.86	3.85	0.562	0.865

Note: τ, π and R are expressed in percentage points.

Next, we gauge the accuracy of the log-linear approximation to the sticky-price Ramsey allocation by comparing it to results based on a log-quadratic approximation. The quadratic approximation technique we used is described in [24]. The results shown in the bottom panel of Table 3 suggest that the log-linear and log-quadratic approximations deliver similar quantitative results. In particular, the dramatic decline in inflation volatility vis-a-vis the flexible-price economy also arises under the higher-order approximation.

We close our discussion of numerical accuracy by pointing out that in both the flexible- and sticky-price economies the first-order approximation to the Ramsey allocation features a unit root. As a result, the local approximation techniques employed here become more inaccurate the longer is the simulated time series used to compute sample moments. The reason is that in the long run the log-linearized equilibrium system is bound to wander far away from the point around which the approximation is taken. We choose to restrict attention to time series of length 100 years because for this sample size the log-linear model of the flexible-price economy performs well in comparison to the exact solution. The need to keep the length of the time series relatively short also applies when a log-quadratic approximation is used. If the system deviates far from the point of approximation, then the quadratic terms might introduce large errors. These discrepancies can render the quadratic approximation even more imprecise than the lower-order one. The quadratic approximation is guaranteed to perform better than the linear one only if the system's dynamics are close enough to the point around which the model is approximated.

8. Interest-rate feedback rules

In this section we address the question of whether the time series arising from the Ramsey allocation imply a relation between the nominal interest rate, inflation, and output consistent with available estimates of such relationship for US data. In recent years there has been a revival of empirical and theoretical research aimed at understanding the macroeconomic consequences of monetary policy regimes that take the form of interest-rate feedback rules. One driving force of this renewed interest can be found in empirical studies showing that in the past two decades monetary policy in the United States is well described as following such a rule. In particular, an influential paper by Taylor [26a] characterizes the Federal Reserve as following a simple rule whereby the federal funds rate is set as a linear function of inflation and the output gap with coefficients of 1.5 and 0.5, respectively. Taylor emphasizes the stabilizing role of an inflation coefficient greater than unity, which loosely speaking implies that the central bank raises real interest rates in response to increases in the rate of inflation. After his seminal paper, interest-rate feedback rules with this feature have become known as Taylor rules. Taylor rules have also been shown to represent an adequate description of monetary policy in other industrialized economies (see, for example, [8]).

To see whether the nominal interest rate process associated with the Ramsey allocation can be well represented by a linear combination of inflation and output, we estimate the following regression using artificial time series from the sticky-price model:

$$R_t = \beta_0 + \beta_1 \pi_t + \beta_2 y_t + u_t.$$

Here the nominal interest rate, R_t, and inflation, π_t, are measured in percent per year, and output, y_t, is measured as percent deviation from its mean value. To generate time series for R_t, π_t, and y_t, we draw artificial time series of size 100 for the two shocks driving business cycles in our model, government consumption and productivity shocks. We use these realizations to compute the implied time series of the endogenous variables of interest using the baseline calibration of the sticky-price model. We then proceed to estimate the above equation. We repeat this procedure 500 times and take the median of the estimated regression coefficients. The OLS estimate of the interest rate feedback rule is

$$R_t = 0.04 - 0.14\pi_t - 0.16 y_t + u_t, \quad R^2 = 0.92.$$

Clearly, an interest rate feedback rule fits quite well the optimal interest rate process. The R^2 coefficient of the regression is above 90 percent. However, the estimated interest-rate feedback rule does not resemble a Taylor rule. First, the coefficient on inflation is less than unity, and indeed insignificantly different from zero with a negative point estimate. Second, the output coefficient is negative. The results are essentially unchanged if one estimates the feedback rule by instrumental variables using lagged values of π, y, and R as instruments. Thus, an econometrician working with a data sampled from the Ramsey economy would conclude that monetary policy is passive, in the sense that the interest rate does not seem to react to changes in the rate of inflation.

The results are also insensitive to the introduction of a smoothing term à la Sack [21] in the above interest-rate rule. Specifically, adding the nominal interest rate with one lag to the set of explanatory variables yields

$$R_t = 0.03 + 0.15\pi_t - 0.11y_t + 0.34R_{t-1} + u_t, \quad R^2 = 0.96.$$

One issue that has attracted the attention of both empirical and theoretical studies on interest-rate feedback rules is whether the central bank looks at contemporaneous or past measures of inflation. It turns out that in our Ramsey economy, a backward-looking rule also features an inflation coefficient significantly less than one. Specifically, replacing π_t with π_{t-1} in our original specification of the interest rate rule we obtain

$$R_t = 0.04 + 0.21\pi_{t-1} - 0.16y_t + u_t, \quad R^2 = 0.92.$$

We close this section by pointing out that the results should not be interpreted as suggesting that optimal monetary policy can be implemented by passive interest-rate feedback rules like the ones estimated above. In order to arrive at such conclusion, one would have, in addition, to identify the underlying fiscal regime. Then, one would have to check whether in a competitive equilibrium where the government follows the resulting monetary/fiscal regime, welfare of the representative household is close enough to that obtained under the Ramsey allocation. An obvious problem that one might encounter in performing this exercise is that the competitive equilibrium fails to be unique at the estimated policy regime. This is a matter that deserves further investigation.

9. Conclusion

The focus of this paper is the study of the implications of price stickiness for the optimal degree of price volatility. The economic environment considered features a government that does not have access to lump-sum taxation and can only issue nominally risk-free debt. The central finding is that for plausible calibrations of the degree of nominal rigidity the volatility of inflation associated with the Ramsey allocation is near zero. Indeed, a very small amount of price stickiness suffices to make the optimal inflation volatility many times lower than that arising under full price flexibility.

Our results show that when prices are sticky, the social planner abandons the use of price surprises as a shock absorber of unexpected innovations in the fiscal budget. Instead the government chooses to rely more heavily on changes in income tax rates. The benevolent government minimizes the distortions introduced by these tax changes by spreading them over time. The resulting tax smoothing behavior induces a near random walk property in tax rates and public debt. This characteristic of the Ramsey real allocation under sticky prices resembles that of economies where the government can issue only real non-state-contingent debt, like the ones studied by Barro [2] and Aiyagari et al. [1].

Our results suggest that the fragility of the use of the price level as a shock absorber is not limited to the introduction of small degrees of nominal rigidities. Any friction that causes changes in the equilibrium real allocation in response to innovations in the price level is likely to induce the Ramsey planner to refrain from using the price level as an instrument to front-load taxation. Examples of such frictions could be informational rigidities as in [14,17] and costs of adjusting the composition of financial portfolios, as in limited participation models [11,15]. We plan to explore these ideas further in future research. If this conjecture is correct, our sticky-price model is simply a metaphor to illustrate a deeper mechanism at work in the macroeconomy that leads central banks all over the world to favor price stability above any other goal of monetary policy.

Acknowledgments

We thank Paul Evans, Fabio Ghironi, Pedro Teles, Alan Viard, and seminar participants at SUNY Albany, Ohio State University, Queen's University, the March 2001 Texas Monetary Conference, the April 2001 CEPR/INSEAD Workshop in Macroeconomics, the Bank of Portugal, the 2001 NBER Economic Fluctuations Meeting, the 2001 FRB NBER Nominal Rigidities Conference, the European Central Bank, the Bank of Canada, and the CEPR/CREI Workshop on New Developments in Fiscal Policy Analysis held at Universitat Pompeu Fabra (Barcelona) for comments.

Appendix A

Proof of Proposition 1. We first show that plans $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}$ satisfying (13)–(23) also satisfy (24)–(26) $v_t \geqslant \underline{v}$, and $v_t^2 s'(v_t) < 1$. It follows from the definition of $\rho(v_t)$ and (15) that $\rho(v_t) = R_t$. It is easy to see then that (15), (17), and Assumption 1 together imply that $v_t \geqslant \underline{v}$ and $v_t^2 s'(v_t) < 1$. Taking expectations conditional on information available at time t of (16), using the definition of $\rho(v_t)$, and combining it with (17) one obtains (24). To obtain (25) divide (19) by P_t. Solve (14) for τ_t and use the resulting expression to eliminate τ_t from (19). Use (23) to replace M_t/P_t and let $b_t = B_t/P_t$. Finally, multiply and divide (20) by P_{t+j} and replace q_{t+j+1} with (21) and (16). Multiply by $\lambda_t/(q_t P_t)$ to get (26).

Next, we must show that for any plan $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}$ satisfying (13), (18), (22), (24)–(26) and $v_t \geqslant \underline{v}$, and $v_t^2 s'(v_t) < 1$ one can construct plans $\{M_t, B_t, q_t, r_{t+1}, \tau_t, R_t\}$ so that (14)–(17), (19)–(21), and (23) hold at all dates and under all contingencies. Set τ_t such that (14) holds. Set $R_t = \rho(v_t)$. It follows from the definition of $\rho(v_t)$ that (15) holds. Assumption 1, the constraints $v_t \geqslant \underline{v}$ and $v_t^2 s'(v_t) < 1$ ensure that $R_t \geqslant 1$. Let r_{t+1} be given by (16). Taking expected value and comparing the resulting expression to (24) shows that (17) is satisfied. With r_t in hand, let q_t be given by (21). Using $B_t = b_t P_t$ and (23) to write $M_t/P_t = c_t/v_t$, and the definition of τ_t we recover (19). Let $P_t = \pi_t P_{t-1}$ and recall that P_{-1} is given. Multiply (26) by $q_t P_t/\lambda_t$. Note that $q_t P_t \lambda_t \beta^j \lambda_{t+j+1}/\pi_{t+j+1}$ using (16) and (21) can be expressed as $q_{t+j+1} P_{t+j}$. Finally, replace c_{t+j}/v_{t+j} with (23) to obtain (20). □

Appendix B

B.1. The Lagrangian of the Ramsey problem

$$\begin{aligned}
\mathscr{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(c_t, h_t) + \lambda_t^f \left[z_t h_t - [1 + s(v_t)] c_t - g_t - \frac{\theta}{2}(\pi_t - 1)^2 \right] \\
& + \lambda_t^b \left[\lambda_t - \beta \rho(v_t) E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \right] \\
& + \lambda_t^s \left[\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)} \right) h_t - \frac{\rho(v_{t-1}) b_{t-1}}{\pi_t} - \frac{c_{t-1}}{v_{t-1}\pi_t} - g_t \right] \\
& + \lambda_t^p \left[\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta} \left(\frac{1+\eta}{\eta} - mc_t \right) - \pi_t(\pi_t - 1) \right] \\
& + \lambda_t^c [U_c(c_t, h_t) - \lambda_t \gamma(v_t)] \Bigg\}.
\end{aligned} \tag{B.1}$$

B.2. First-order conditions of the Ramsey problem for $t \geqslant 1$

$$z_t h_t = [1 + s(v_t)] c_t + g_t + \frac{\theta}{2}(\pi_t - 1)^2, \tag{B.2}$$

$$\lambda_t = \beta \rho(v_t) E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{B.3}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)} \right) h_t = \frac{\rho(v_{t-1}) b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t, \tag{B.4}$$

$$\pi_t(\pi_t - 1) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta} \left(\frac{1+\eta}{\eta} - mc_t \right), \tag{B.5}$$

$$U_c(c_t, h_t) = \lambda_t \gamma(v_t), \tag{B.6}$$

$$U_c(t) - \lambda_t^f [1 + s(v_t)] + \frac{\lambda_t^s}{v_t} + \lambda_t^s h_t \gamma(v_t) M_c(t) - \beta E_t \frac{\lambda_{t+1}^s}{v_t \pi_{t+1}} + \lambda_t^c U_{cc}(t) = 0, \tag{B.7}$$

$$U_h(t) + \lambda_t^f z_t + \lambda_t^s(mc_t z_t + M_t\gamma(v_t)) + \lambda_t^s h_t M_h(t)\gamma(v_t) + \frac{\lambda_t^p \eta z_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right) + \lambda_t^c U_{ch}(t) = 0, \tag{B.8}$$

$$\lambda_t^b - \frac{\rho(v_{t-1})\lambda_{t-1}^b}{\pi_t} - \beta\frac{\lambda_t^p}{\lambda_t^2}E_t\lambda_{t+1}\pi_{t+1}(\pi_{t+1}-1) + \frac{\lambda_{t-1}^p}{\lambda_{t-1}}\pi_t(\pi_t-1) - \lambda_t^c\gamma(v_t) = 0, \tag{B.9}$$

$$-\lambda_t^f s'(v_t)c_t - \beta\lambda_t^b\rho'(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t^s c_t}{v_t^2} + \lambda_t^s M_t h_t\gamma'(v_t) - \beta b_t\rho'(v_t)E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}} + \beta\frac{c_t}{v_t^2}E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}} - \lambda_t^c\lambda_t\gamma'(v_t) = 0, \tag{B.10}$$

$$-\lambda_t^f\theta(\pi_t-1) + \lambda_{t-1}^b\rho(v_t-1)\frac{\lambda_t}{\pi_t^2} + \lambda_t^s\frac{\rho(v_t-1)b_{t-1} + c_{t-1}/v_{t-1}}{\pi_t^2} + \lambda_{t-1}^p\frac{\lambda_t}{\lambda_{t-1}}(2\pi_t-1) - \lambda_t^p(2\pi_t-1) = 0, \tag{B.11}$$

$$\lambda_t^s = \beta\rho(v_t)E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}}, \tag{B.12}$$

$$\lambda_t^s = \frac{\eta}{\theta}\lambda_t^p, \tag{B.13}$$

$$\lim_{j\to\infty} E_t\left[\beta^j\frac{\lambda_{t+j+1}}{\pi_{t+j+1}}\left(\rho(v_{t+j})b_{t+j} + \frac{c_{t+j}}{v_{t+j}}\right)\right] = 0. \tag{B.14}$$

B.3. First-order conditions of the Ramsey problem at time 0

$$z_t h_t = [1+s(v_t)]c_t + g_t + \frac{\theta}{2}(\pi_t-1)^2, \tag{B.15}$$

$$\lambda_t = \beta\rho(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{B.16}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t,h_t)\gamma(v_t)}{U_c(c_t,h_t)}\right)h_t = \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t, \tag{B.17}$$

$$\pi_t(\pi_t-1) = \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}(\pi_{t+1}-1) + \frac{\eta z_t h_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right), \tag{B.18}$$

$$U_c(c_t,h_t) = \lambda_t\gamma(v_t), \tag{B.19}$$

$$U_c(t) - \lambda_t^f[1+s(v_t)] + \frac{\lambda_t^s}{v_t} + \lambda_t^s h_t\gamma(v_t)M_c(t) - \beta E_t\frac{\lambda_{t+1}^s}{v_t\pi_{t+1}} + \lambda_t^c U_{cc}(t) = 0, \tag{B.20}$$

$$U_h(t) + \lambda_t^f z_t + \lambda_t^s(mc_t z_t + M_t\gamma(v_t)) + \lambda_t^s h_t M_h(t)\gamma(v_t) + \frac{\lambda_t^p \eta z_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right) + \lambda_t^c U_{ch}(t) = 0, \tag{B.21}$$

$$\lambda_t^b - \beta\frac{\lambda_t^p}{\lambda_t^2} E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1} - 1) - \lambda_t^c\gamma(v_t) = 0, \tag{B.22}$$

$$-\lambda_t^f s'(v_t)c_t - \beta\lambda_t^b\rho'(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t^s c_t}{v_t^2} + \lambda_t^s M_t h_t\gamma'(v_t) - \beta b_t\rho'(v_t)E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}} + \beta\frac{c_t}{v_t^2}E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}} - \lambda_t^c\lambda_t\gamma'(v_t) = 0, \tag{B.23}$$

$$\lambda_t^s = \beta\rho(v_t)E_t\frac{\lambda_{t+1}^s}{\pi_{t+1}}, \tag{B.24}$$

$$\lambda_t^s = \frac{\eta}{\theta}\lambda_t^p, \tag{B.25}$$

$$\lim_{j\to\infty} E_t\left[\beta^j\frac{\lambda_{t+j+1}}{\pi_{t+j+1}}\left(\rho(v_{t+j})b_{t+j} + \frac{c_{t+j}}{v_{t+j}}\right)\right] = 0. \tag{B.26}$$

B.4. Steady state of the Ramsey economy

Assume that $b_t = b_{-1}$ for all t and that $x_t = x_{t-1} = x_{t+1} = x$ for all endogenous and exogenous variables. Also, $z = 1$. Note that the steady-state value of the marginal cost $mc_t = w_t/z_t$ is simply w.

$$h = [1 + s(v)]c + g + \frac{\theta}{2}(\pi - 1)^2, \tag{B.27}$$

$$1 = \beta\rho(v)\frac{1}{\pi}, \tag{B.28}$$

$$\frac{c}{v} + b + \left(w + \frac{U_h(c,h)\gamma(v)}{U_c(c,h)}\right)h = \frac{\rho(v)b}{\pi} + \frac{c}{v\pi} + g, \tag{B.29}$$

$$\pi(\pi - 1) = \frac{\eta h}{\theta(1-\beta)}\left(\frac{1+\eta}{\eta} - w\right), \tag{B.30}$$

$$U_c(c,h) = \lambda\gamma(v), \tag{B.31}$$

$$U_c - \lambda^f[1 + s(v)] + \frac{\lambda^s}{v} + \lambda^s h\gamma(v)M_c - \beta\frac{\lambda^s}{v\pi} + \lambda^c U_{cc} = 0, \tag{B.32}$$

$$U_h + \lambda^f + \lambda^s(w + M\gamma(v)) + \lambda^s hM_h\gamma(v) + \frac{\lambda^p\eta}{\theta}\left(\frac{1+\eta}{\eta} - w\right) + \lambda^c U_{ch} = 0, \tag{B.33}$$

$$\lambda^b - \frac{\rho(v)\lambda^b}{\pi} - \beta\frac{\lambda^p}{\lambda^2}\lambda\pi(\pi-1) + \frac{\lambda^p}{\lambda}\pi(\pi-1) - \lambda^c\gamma(v) = 0, \tag{B.34}$$

$$\begin{aligned} &-\lambda^f s'(v)c - \beta\lambda^b\rho'(v)\frac{\lambda}{\pi} - \frac{\lambda^s c}{v^2} + \lambda^s Mh\gamma'(v) - \beta b\rho'(v)\frac{\lambda^s}{\pi} \\ &\quad + \beta\frac{c}{v^2}\frac{\lambda^s}{\pi} - \lambda^c\lambda\gamma'(v) = 0, \end{aligned} \tag{B.35}$$

$$-\lambda^f\theta(\pi-1) + \lambda^b\rho(v)\frac{\lambda}{\pi^2} + \lambda^s\frac{\rho(v)b + c/v}{\pi^2} + \lambda^p(2\pi-1) - \lambda^p(2\pi-1) = 0, \tag{B.36}$$

$$\pi = \beta\rho(v), \tag{B.37}$$

$$\lambda^s = \frac{\eta}{\theta}\lambda^p. \tag{B.38}$$

Appendix C

Proof of Proposition 2. We first show that plans $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}$ satisfying (13)–(23) also satisfy (27)–(29), $v_t \geqslant \underline{v}$, and $v_t^2 s'(v_t) < 1$. It follows from the definition of $\rho(v_t)$ and (15) that $\rho(v_t) = R_t$. It is easy to see then that (15), (17), and Assumption 1 together imply that $v_t \geqslant \underline{v}$ and $v_t^2 s'(v_t) < 1$. To obtain (27) divide (18) by λ_t and then use (13) to eliminate λ_t. Next divide (19) by P_t. Solve (14) for τ_t and use the resulting expression to eliminate τ_t from (19). Use (23) to replace M_t/P_t and let $b_t = B_t/P_t$. This yields (28). For any $t, j \geqslant 0$, (19) can be written as

$$M_{t+j} + B_{t+j} + \tau_{t+j}P_{t+j}z_{t+j}mc_{t+j}h_{t+j} = R_{t+j-1}B_{t+j-1} + M_{t+j-1} + P_{t+j}g_{t+j}.$$

Let $W_{t+j+1} = R_{t+j}B_{t+j} + M_{t+j}$ and note that W_{t+j+1} is in the information set of time $t+j$. Use this expression to eliminate B_{t+j} from (19) and multiply by q_{t+j} to obtain

$$\begin{aligned} &q_{t+j}M_{t+j}(1 - R_{t+j}^{-1}) + q_{t+j}E_{t+j}r_{t+j+1}W_{t+j+1} - q_{t+j}W_{t+j} \\ &\quad = q_{t+j}[P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}mc_{t+j}z_{t+j}h_{t+j}], \end{aligned}$$

where we use (17) to write R_{t+j} in terms of r_{t+j+1}. Take expectations conditional on information available at time t and sum for $j = 0$ to J

$$\begin{aligned} &E_t\sum_{j=1}^{J}[q_{t+j}M_{t+j}(1 - R_{t+j}^{-1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}mc_{t+j}z_{t+j}h_{t+j})] \\ &\quad = -E_t q_{t+J+1}W_{t+J+1} + q_t W_t. \end{aligned}$$

Take limits for $J \to \infty$. By (20) the limit of the right-hand side is well defined and equal to $q_t W_t$. Thus, the limits of the left-hand side exists. This yields

$$E_t \sum_{j=0}^{\infty} [q_{t+j} M_{t+j}(1 - R_{t+j}^{-1}) - q_{t+j}(P_{t+j} g_{t+j} - \tau_{t+j} P_{t+j} mc_{t+j} z_{t+j} h_{t+j})] = q_t W_t.$$

By (16) we have that $P_{t+j} q_{t+j}/q_t = \beta^j \lambda_{t+j} P_t/\lambda_t$. Use (13) to eliminate λ_{t+j}, (23) to eliminate M_{t+j}/P_{t+j} to obtain

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \left[\frac{c_{t+j}}{v_{t+j}} (1 - R_{t+j}^{-1}) - (g_{t+j} - \tau_{t+j} mc_{t+j} z_{t+j} h_{t+j}) \right]$$
$$= \frac{W_t}{p_t} \frac{U_c(c_t, h_t)}{\gamma(v_t)}.$$

Solve (14) for τ_{t+j}. Then $\tau_{t+j} mc_{t+j} z_{t+j} h_{t+j} = mc_{t+j} z_{t+j} h_{t+j} + \gamma(v_{t+j})/U_c(c_{t+j}, h_{t+j}) U_h(c_{t+j}, h_{t+j}) h_{t+j}$. Use this in the above expression and replace g_{t+j} with (22). This yields

$$E_t \sum_{j=0}^{\infty} \beta^j \left[U_c(c_{t+j}, h_{t+j}) c_{t+j} \frac{1 + s(v_{t+j}) + \frac{1 - R_{t+j}^{-1}}{v_{t+j}}}{\gamma(v_{t+j})} + U_h(c_{t+j}, h_{t+j}) h_{t+j} \right.$$
$$\left. + \frac{z_{t+j} h_{t+j} U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} (mc_{t+j} - 1) + \frac{\theta}{2} (\pi_{t+j} - 1)^2 \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right]$$
$$= \frac{W_t}{P_t} \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$

Finally, use (15) to replace $(1 - R_{t+j}^{-1})/v_{t+j}$ with $v_{t+j} s'(v_{t+j})$ and use the definition of W_t to get (29).

We next show that plans $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}$ satisfying (22), (27)–(29), and $v_t \geqslant \underline{v}$, and $v_t^2 s'(v_t) < 1$ also satisfy (13)–(23). Construct λ_t so that it satisfies (13). Let τ_t be given by (14). Let R_t be given by (15). Let r_{t+1} be given by (16). Let q_t be given by (21) and M_t/P_t by (23). By the same arguments given in the proof of Proposition 2 one can show that (18) and (19) then hold. Thus, what remains to be shown is that (17) and (20) are satisfied. Note that $R_t = \rho(v_t) = 1/[1 - v_t^2 s'(v_t)]$, then the restriction $v_t \geqslant \underline{v}$ and $v_t^2 s'(v_t) < 1$ and Assumption 1 imply that $R_t \geqslant 1$. Write (29) as

$$U_c(c_t, h_t) c_t + U_h(c_t, h_t) h_t + z_t h_t (mc_t - 1) \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \frac{\theta}{2} (\pi_t - 1)^2 \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$
$$+ E_t \sum_{j=1}^{\infty} \beta^j \Big\{ U_c(c_{t+j}, h_{t+j}) c_{t+j} + U_h(c_{t+j}, h_{t+j}) h_{t+j}$$

$$+ z_{t+j}h_{t+j}(mc_{t+j}-1)\frac{U_c(c_{t+j},h_{t+j})}{\gamma(v_{t+j})}$$
$$\left. + \frac{\theta}{2}(\pi_{t+j}-1)^2\frac{U_c(c_{t+j},h_{t+j})}{\gamma(v_{t+j})}\right\} = \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[\frac{c_{t-1}/v_{t-1}+\rho(v_{t-1})b_{t-1}}{\pi_t}\right]. \quad \text{(C.1)}$$

Make a change of index $h=j-1$.

$$U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t + z_th_t(mc_t-1)\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \frac{\theta}{2}(\pi_t-1)^2\frac{U_c(c_t,h_t)}{\gamma(v_t)}$$
$$+ \beta E_t\sum_{h=0}^{\infty}\beta^h\left\{U_c(c_{t+h+1},h_{t+h+1})c_{t+h+1} + U_h(c_{t+h+1},h_{t+h+1})h_{t+h+1}\right.$$
$$+z_{t+h+1}h_{t+h+1}(mc_{t+h+1}-1)\frac{U_c(c_{t+h+1},h_{t+h+1})}{\gamma(v_{t+h+1})}$$
$$\left.+ \frac{\theta}{2}(\pi_{t+h+1}-1)^2\frac{U_2(c_{t+h+1},h_{t+h+1})}{\gamma(v_{t+h+1})}\right\}$$
$$= \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[\frac{c_{t-1}/v_{t-1}+\rho(v_{t-1})b_{t-1}}{\pi_t}\right]. \quad \text{(C.2)}$$

Using (29) this expression can be simplified to read

$$U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t + z_th_t(mc_t-1)\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \frac{\theta}{2}(\pi_t-1)^2\frac{U_c(c_t,h_t)}{\gamma(v_t)}$$
$$+ \beta E_t\left\{\frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})}\left[\frac{c_t/v_t+\rho(v_t)b_t}{\pi_{t+1}}\right]\right\}$$
$$= \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[\frac{c_{t-1}/v_{t-1}+\rho(v_{t-1})b_{t-1}}{\pi_t}\right]. \quad \text{(C.3)}$$

Take expectations of (16) and use the resulting expression to eliminate $\beta E_t\{\frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})\pi_{t+1}}\}$. This yield

$$U_c(c_t,h_t)c_t + U_h(c_t,h_t)h_t + z_th_t(mc_t-1)\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \frac{\theta}{2}(\pi_t-1)^2\frac{U_c(c_t,h_t)}{\gamma(v_t)}$$
$$+ E_tr_{t+1}\frac{U_c(c_t,h_t)}{\gamma(v_t)}[c_t/v_t+\rho(v_t)b_t] = \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[\frac{c_{t-1}/v_{t-1}+\rho(v_{t-1})b_{t-1}}{\pi_t}\right]. \quad \text{(C.4)}$$

Multiply by $P_t\gamma(v_t)/U_c(c_t,h_t)$ and replace $\theta/2(\pi_t-1)^2$ with (22). Combine (15) with (23) to express, $c_t/v_t(v_t^2s'(v_t))$ as $M_t/P_t(1-R_t^{-1})$. Finally, use (14) to replace $U_h/U_c\gamma(v_t)h_t$. The resulting expression is

$$M_t(1-R_t^{-1}) + \tau P_tmc_tz_th_t - P_tg_t + E_tr_{t+1}(M_t+R_tB_t)$$
$$= M_{t-1} + R_{t-1}B_{t-1}. \quad \text{(C.5)}$$

Subtracting (19) from this expression it follows that (17) must hold. Finally, we must show that (20) holds. Multiply (19) in period $t+j$ by q_{t+j} and take information conditional on information available at time t to get

$$\begin{aligned}&E_t[q_{t+j}M_{t+j}(1-r_{t+j+1})+q_{t+j+1}W_{t+j+1}]\\&\quad = E_t[q_{t+j}W_{t+j}+q_{t+j}(P_{t+j}g_{t+j}-\tau_{t+j}P_{t+j}w_{t+j}h_{t+j})].\end{aligned}$$

Now sum for $j=0$ to J.

$$\begin{aligned}&E_t\sum_{j=0}^{J}[q_{t+j}M_{t+j}(1-r_{t+j+1})-q_{t+j}(P_{t+j}g_{t+j}-\tau_{t+j}P_{t+j}w_{t+j}h_{t+j})]\\&\quad = -E_tq_{t+J+1}W_{t+J+1}+q_tW_t.\end{aligned}$$

Divide by q_tP_t

$$\begin{aligned}&E_t\sum_{j=0}^{J}\frac{q_{t+j}P_{t+j}}{q_tP_t}[(c_{t+j}/v_{t+j})(1-r_{t+j+1})-(g_{t+j}-\tau_{t+j}w_{t+j}h_{t+j})]\\&\quad = -E_tq_{t+J+1}W_{t+J+1}/(q_tP_t)+\frac{W_t}{P_t}.\end{aligned}$$

It follows from (29) that the limit of the left-hand side of the above expression as $J\to\infty$ is W_t/P_t. Hence the limit of the right-hand side is well defined. It then follows that

$$\lim_{J\to\infty} E_tq_{t+J+1}W_{t+J+1}=0$$

for every date t. Using the definition of W_t, one obtains immediately (20). □

References

[1] R. Aiyagari, A. Marcet, T.J. Sargent, J. Seppälä, Optimal taxation without state-contingent debt, J. Polit. Economy 110 (2002) 1220–1254.

[2] R.J. Barro, On the determination of public debt, J. Polit. Economy 87 (1979) 940–971.

[3] S. Basu, J.G. Fernald, Returns to scale in U.S. production: estimates and implications, J. Polit. Economy 105 (1997) 249–283.

[4] G.A. Calvo, P.E. Guidotti, Indexation and maturity of government bonds: an exploratory model, in: R. Dornbusch, M. Draghi (Eds.), Public Debt Management: Theory and History, Cambridge University Press, Cambridge, 1990, pp. 52–82.

[5] G.A. Calvo, P.E. Guidotti, On the flexibility of monetary policy: the case of the optimal inflation tax, Rev. Econom. Stud. 60 (1993) 667–687.

[6] V.V. Chari, L.J. Christiano, P.J. Kehoe, Optimal fiscal and monetary policy: some recent results, J. Money Credit Banking 23 (1991) 519–539.

[7] L.J. Christiano, T.J. Fitzgerald, Understanding the fiscal theory of the price level, Fed. Reserve Bank Cleveland Econom. Rev. 36 (2) (2000) 2–37.

[8] R. Clarida, J. Galí, M. Gertler, Monetary policy rules in practice: some international evidence, European Econom. Rev. 42 (1998) 1033–1067.

[9] I. Correia, J.P. Nicolini, P. Teles, Optimal fiscal and monetary policy: equivalence results, Mimeo, Banco de Portugal, April 2001.

[10] C.J. Erceg, D.W. Henderson, A.T. Levin, Optimal monetary policy with staggered wage and price contracts, J. Monet. Econom. 46 (2000) 281–313.

[11] T. Fuerst, Liquidity, loanable funds, and real activity, J. Monet. Econom 29 (1992) 3–24.

[12] J. Galí, T. Monacelli, Optimal monetary policy and exchange rate volatility in a small open economy, Mimeo, Universitat Pompeu Fabra, May 2000.

[13] A. Khan, R.G. King, A.L. Wolman, Optimal monetary policy, Mimeo, Federal Reserve Bank of Philadelphia, May 2000.

[14] R.E. Lucas Jr., Expectations and the neutrality of money, J. Econom. Theory 4 (1972) 103–123.

[15] R.E. Lucas Jr., Liquidity and interest rates, J. Econom. Theory 50 (1990) 237–264.

[16] R.E. Lucas Jr., N.L. Stokey, Optimal fiscal and monetary policy in an economy without capital, J. Monet. Econom 12 (1983) 55–93.

[17] G.N. Mankiw, R. Reis, Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve, Mimeo, Harvard University, February 2001.

[18] E.C. Prescott, Theory ahead of business cycle measurement, Fed. Reserve Bank Minneapolis Quart. Rev. 10 (1986) 9–22.

[19] J.J. Rotemberg, Sticky prices in the United States, J. Polit. Economy 90 (1982) 1187–1211.

[20] J.J. Rotemberg, M.D. Woodford, Interest rate rules in an estimated sticky price model, in: J.B. Taylor (Ed.), Monetary Policy Rules, The University of Chicago Press, Chicago, 1999, pp. 57–119.

[21] B. Sack, Uncertainty, learning, and gradual monetary policy, Board of Governors of the Federal Reserve System, July 1998, unpublished.

[22] A. Sbordone, Prices and unit labor costs: a new test of price stickiness, J. Monet. Econom. 49 (2002) 265–292.

[23] S. Schmitt-Grohé, Stephanie, M. Uribe, Optimal fiscal and monetary policy under imperfect competition, Manuscript, University of Pennsylvania, January 2001.

[24] S. Schmitt-Grohé, M. Uribe, Solving dynamic general equilibrium models using a second-order approximation to the policy function, NBER Technical Working Paper # 282, October 2002, J. Econom. Dynam. Control, forthcoming.

[25] C. Sims, Fiscal consequences for Mexico of adopting the dollar, J. Money Credit Banking 33 (2001) 597–616.

[26] H. Siu, Optimal fiscal and monetary policy with sticky prices, Mimeo, Northwestern University, March 2001.

[26a] J.B. Taylor, Discretion versus rules in practice, Carnegie–Rochester Series on Public Policy 39 (1993) 195–214.

[27] M.D. Woodford, Public Debt and the Price Level, Mimeo, Princeton University, Princeton, NJ, 1998.

[28] M.D. Woodford, Inflation stabilization and welfare, Mimeo, Princeton University, June 1999.

[29] M.D. Woodford, The Taylor rule and optimal monetary policy, Mimeo, Princeton University, December 2000.

[30] M.D. Woodford, Interest and Prices, Princeton University Press, Princeton, forthcoming.