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Social norms in networks ☆

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Abstract

Although the linear-in-means model is the workhorse model in empirical work on peer effects, its theoretical properties are understudied. In this study, we develop a social-norm model that provides a micro-foundation of the linear-in-means model and investigate its properties. We show that individual outcomes may increase, decrease, or vary non-monotonically with the taste for conformity. Equilibria are usually inefficient and, to restore the first best, the planner needs to subsidize (tax) agents whose neighbors make efforts above (below) the social norms. Thus, giving more subsidies to more central agents is not necessarily efficient. We also discuss the policy implications of our model in terms of education and crime.

JEL classification: D85; J15; Z13

Keywords: Social norms; Conformism; Local-average model; Welfare; Anti-conformism; Network formation

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1. Introduction

There is substantial empirical evidence showing that peer effects matter in education (Calvó-Armengol et al., 2009; Epple and Romano, 2011; Sacerdote, 2011), crime (Ludwig et al., 2001; Patacchini and Zenou, 2012; Damm and Dustmann, 2014), risky behavior (Clark and Loheac, 2007; Hsieh and Lin, 2017), performance in the workplace (Herbst and Mas, 2015), participation in extracurricular activities (Boucher, 2016), obesity (Christakis and Fowler, 2007), environmentally friendly behavior (Brekke et al., 2010; Czajkowski et al., 2017), and tax compliance and tax evasion (Fortin et al., 2007; Alm et al., 2017), among other outcomes. The standard model used in these studies is the so-called *linear-in-means model*, which can be written as

$$x_{ig} = z_{ig}\beta + y_g\gamma + \frac{\theta}{(N_g - 1)} \sum_{j=1, j\neq i}^{N_g} x_{jg} + \epsilon_{ig} \tag{1}$$

where x_{ig} is the outcome of individual i belonging to group g,[1] z_{ig} are the observable characteristics of individual i (e.g., age, race, and gender), y_g are the observed exogenous characteristics that are common to all individuals in the same group g,[2] N_g is the number of individuals in group g, and ϵ_{ig} is an error term. Parameter θ captures the "social interaction effect" of the average outcome of the reference group on an individual's own outcome; this is the key parameter of interest that is estimated to measure peer effects.[3]

As noted by Blume et al. (2015), Boucher and Fortin (2016), and Kline and Tamer (2019), it is useful to interpret the linear-in-means model as corresponding to a perfect information game in which (1) is the best-reply function of individual i choosing action (outcome) x_i. The corresponding utility function is such that individuals have a preference to conform to the *average* action of their neighbors in a social network. For this reason, this game is often referred to as the *local-average model*. Surprisingly, the theoretical properties of this model in terms of comparative statics, welfare, and policies have not been investigated. On the contrary, the literature on games on networks[4] (Ballester et al., 2006; Bramoullé et al., 2014; Jackson and Zenou, 2015; Bramoullé and Kranton, 2016)[5] studies the properties of another model, the *local-aggregate model*, in which the sum (not the average) of actions (or outcomes) of neighbors affects own action.[6]

[1] For example, in relation to crime, x_{ig} is the criminal effort of individual i in neighborhood g and, in relation to education, it is the test score of student i in classroom g.

[2] For example, y_g are the average education or income level in a neighborhood g or the average education or income level of students' parents in a classroom g.

[3] If all agents belong to the same group g, this model is not identified, because it is difficult to distinguish between the endogenous effect θ and the exogenous effect γ. Manski (1993) referred to this as the *reflection problem*, because it is difficult to distinguish between an individual's behavior and the behavior being "reflected" back on the individual. The literature on peer effects has proposed different ways of causally interpreting θ, including field experiments that randomly allocate individuals to groups (see, e.g., Sacerdote, 2011, for an overview of peer effect studies in education).

[4] The economics of networks is a growing field. For overviews, see Jackson (2008), Ioannides (2012), and Jackson et al. (2017).

[5] One can interpret the group g in (1) in terms of networks so that group g captures all agents who individual i is connected to. In that case, the game underlying the linear-in-means model is a game on networks in which $N_g - 1$ is the number of agents who are directly connected (direct friends) to i.

[6] The key difference between the local-average and the local-aggregate model is that the former aims to capture the role of *social norms*, such as conformist behavior or peer pressure, on outcomes (Patacchini and Zenou, 2012; Liu et al., 2014; Blume et al., 2015; Topa and Zenou, 2015; Boucher, 2016), while the latter highlights the role of knowledge spillovers

Thus, there is a discrepancy between the theoretical analysis of the local-aggregate model and the empirical applications using the linear-in-means model or local-average model. In this study, we analyze the comparative statics, welfare properties, and policy implications of the local average model and show that these properties are very different from those of the local-aggregate model.[7] Indeed, we show that the differences between the local aggregate and the local average, although seemingly minor, lead to substantial divergence in both positive and normative prescriptions. In other words, the local-aggregate model fails to approximate the local-average model in each of the following key dimensions: comparative statics, welfare properties, and policy recommendations.

Our main findings are summarized as follows. First, we characterize the Nash equilibrium in the local-average model and show that individual efforts, social norms,[8] and aggregate effort are the weighted sums of productivity, whereby the weights are non-linear functions of the taste for conformity. To understand these results, we compare two extreme cases: *pure individualism* and *total conformism*. Under pure individualism, each agent's equilibrium effort is equal to her intrinsic productivity and is independent of her own social norm. By contrast, under total conformism, all agents choose the same level of effort, which is equal to the weighted mean of individual productivity, whereby the weights are proportional to the degree (numbers of links) of the agents in the network. Whether total effort is higher under pure individualism or total conformism depends on the correlation between the productivity distribution across individuals and the degree distribution of the social network.

Second, we provide comparative statics of individual and aggregate efforts with respect to the key parameters of the model. We focus especially on the taste for conformity. Endogenous social norms give rise to general-equilibrium effects. A complex interplay between these effects may result in a non-monotonic relationship between the taste for conformity and individual efforts. Whether an individual is above or below her social norm is key for understanding the shape of this relationship. Interestingly, in regular networks, aggregate effort remains neutral to changes in the taste for conformity and is always equal to aggregate productivity.

We also study the impact of adding a link on the equilibrium efforts of all agents in the network. *All* agents in the network increase their effort if and only if a link between two agents with sufficiently high productivities is added in the network. This result is driven by the following *snowball effect*. When a link is formed between two very productive agents, their social norm increases, because the effort of the newly added agent is high. The best response for the agent for whom the social norm increases is to increase her effort. This, in turn, increases the effort of her neighbor, which increases her social norm, and so forth. Note that, when a link is created between a high-productive and a low-productive agent, then the low-productive agent increases

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on outcomes (Ballester et al., 2006, 2010; Bramoullé et al., 2014; De Marti and Zenou, 2015). Bramoullé et al. (2009) provide conditions for identification in the local-average model while Liu et al. (2014) derive conditions for identification in the local-aggregate model.

[7] In this study, we are interested only in *positive* peer effects, which is why we compare the local-aggregate model with the local-average one—both are games with *strategic complementarities*; that is, an increase in the effort of a neighbor increases the marginal utility of own effort. Another well-studied model in network games is a game with strategic substitutability (Bramoullé and Kranton, 2007; Bramoullé et al., 2014; Allouch, 2015) in which there are *negative* peer effects, that is, an increase in the effort of an individual's neighbor decreases the marginal utility of making own effort. This is not the topic of analysis in this study, since we focus on the linear-in-means model in which peer effects are supposed to be positive.

[8] There are different definitions of social norms in the literature (see, e.g., Akerlof, 1997; Dutta et al., 2019). Here, we define the social norm of an agent as the average action of her neighbors.

her effort, because her social norm increases while the high-productive agent decreases her effort because her social norm is reduced. As a result, the impact of adding this link on the effort of all agents in the network is ambiguous. Using these results, we discuss the *key-link policy*, whose aim is to determine the link between two agents which, once removed, reduces total crime the most. We show that, irrespective of the network structure, the planner should remove the link between the two most productive agents in the network.

Third, we provide a complete welfare analysis of the local-average model. We derive a necessary and sufficient condition for the equilibrium to be socially optimal. However, this condition is not likely to hold in most networks. Indeed, each agent exerts externalities on her neighbors, which she does not take into account when making effort. In particular, when the effort of agent i's neighbor (say, agent j) is below her own social norm, then an increase in i's effort increases the social norm of j, which has a negative impact on j's conformist utility, because j's effort is now further away from her own social norm. In this case, agent i exerts a negative externality on her neighbor j. To restore the first best, the planner taxes agents who exert negative externalities on their neighbors. If the effort of agent i's neighbor (say, agent j) is *above* her own social norm, then the reasoning is the same in reverse, so that to restore the first best, the planner *subsidizes* agents who exert *positive* externalities on their neighbors. This is very different from the policy implications of the local-aggregate model, in which agents always exert positive externalities on their neighbors so that the planner always subsidizes agents and gives higher subsidies to more central agents. Here, if central agents have higher productivity, they are more likely to exert negative externalities on their neighbors, since the latter are more likely to have effort below their own social norms. For example, in a star-shaped network, if the central agent has, on average, higher productivity than that of the peripheral agents, in the local-aggregate model, to restore the first best, the planner gives the highest subsidy to the central agent. By contrast, in the local-average model, the planner taxes the central agent and subsidizes the peripheral agents.

We also consider different extensions of our benchmark model. First, we extend our utility function so that agents have different tastes for conformity. We show that all our results are robust to this extension. Second, we consider an *anti-conformist* model in which agents benefit from deviating from the social norm of their friends. We show that if agents are not too anti-conformist, then our results hold even if some agents provide zero effort in equilibrium. However, when agents become more anti-conformist, then either no equilibrium exists or multiple equilibria prevail. We also consider a model in which agents may want to make effort above the average effort of their friends. In this model, contrary to our benchmark model in which agents either overinvest or underinvest in efforts compared to the first best, we show that they tend to mostly overinvest, because they always want to exert efforts above the social norm of their neighbors. Finally, we extend our model to directed and weighted networks and show that all our results are robust to this extension.

Next, we study the implications of our model for network formation. Specifically, we consider a two-stage model in which, in the first stage, agents form links, and in the second stage, they exert effort. We show that, in the local-aggregate model, the unique pairwise Nash equilibrium is the complete network. On the contrary, in the local-average model, the unique pairwise Nash equilibrium is the complete *homophilous* network in which agents of the same type form a complete network but never create links with agents of the other type. In other words, the local-average model provides a simple explanation of homophilous behavior, whereas the local-aggregate model fails to do so.

Finally, we discuss the differences in policy implications of the local-average and the local-aggregate models. We show that, in the former model, group-based policies are more efficient while in the latter model, it is better to implement individual-based or key-player policies.

Contributions to the literature Other researchers have studied the local-average (conformist) model in network games.[9] Patacchini and Zenou (2012) and Liu et al. (2014) characterized Nash equilibrium and showed that it exists and is unique; Blume et al. (2015) and Golub and Morris (2017) introduced imperfect information[10]; Boucher (2016) embedded the local-average model into a network formation model, while Olcina et al. (2017) embedded it into a learning model.[11] To the best of our knowledge, ours is the first study analyzing the comparative statics properties of the local-average model as well as its welfare and policy implications. Ours is also the first study to examine how adding or removing a link changes the effort of all agents in the network.

One may argue that many peer-effect empirical studies cannot distinguish between the local-average and the local-aggregate model because, in the usual case, the size of the reference group is constant in the sample. For example, if the reference group is the neighborhood, the class, or co-workers, then the network is the same for everyone, namely, a complete graph in which all the students in a class, residents of a neighborhood, or employees of a firm are interlinked. Fortunately, because of network data availability, many recent studies have precisely described the network of agents (see, e.g., Christakis and Fowler, 2007; Bramoullé et al., 2009; Calvó-Armengol et al., 2009; Banerjee et al., 2013; for overviews, see Breza, 2016; Jackson et al., 2017) and therefore, can easily distinguish between the two models. Thus, the results of the present study can be used to derive adequate policy recommendations for each model.[12]

The rest of the paper unfolds as follows. In Section 2, we develop the local-average model and characterize the best response functions. In Section 3, we study the comparative statics properties of the model. In Section 4, we investigate the welfare properties of the local-average model. Section 5 considers different extensions of our model. In Section 6, we examine the policy implications of our results. Finally, Section 7 concludes. All proofs are in Online Appendix A. Online Appendix B provides a comparison between the local-average and the local-aggregate model. In Online Appendix C, we provide a probabilistic interpretation of our model. In Online Appendix D, we provide a simple example that shows how a mean-preserving spread of the productivity impacts own and aggregate outcome. In Online Appendix E, we provide additional results and examples on the comparative statics of the taste for conformity while in Online Appendix F, we compare equilibrium and first-best outcomes for specific networks. In Online Appendix G, we consider different extensions of our model.

[9] Some studies have introduced conformity in the utility function without an explicit network analysis but the social norm is usually assumed to be exogenous. See, among others, Akerlof (1980, 1997), Kandel and Lazear (1992), Bernheim (1994), and Fershtman and Weiss (1998).

[10] See Ghiglino and Goyal (2010), Bloch and Quérou (2013), and Chen et al. (2018), who also developed theoretical network models with average effects but focusing on different issues.

[11] Olcina et al. (2017) forms part of the wide literature on learning on networks using the DeGroot model, whereby the utility function is implicitly assumed to be equivalent to the local-average model. For an overview of this literature, see Golub and Sadler (2016).

[12] For example, Carrell et al. (2013) assigned students to peer groups so that the academic performance of the least able students was maximized. The authors showed that using average peer effects to "optimally" design these groups without taking into account the network relationships between these students could backfire, since they found a negative and significant treatment effect for the least able students.

2. The local-average model

2.1. Definitions and notation

Consider $n \geq 2$ individuals (or agents) who are embedded in a *network* $\mathbf{g}$. The *adjacency matrix* $\mathbf{G} = [g_{ij}]$ is an $(n \times n)$-matrix with $\{0, 1\}$ entries, which keeps track of the *direct connections* in the network. By definition, agents i and j are *directly connected* if and only if $g_{ij} = 1$; otherwise, $g_{ij} = 0$. We assume that the network is *undirected*, that is, $g_{ij} = g_{ji}$, and has *no self-loops*, that is, $g_{ii} = 0$.

Denote by $\widehat{\mathbf{G}} = [\widehat{g}_{ij}]$ the $(n \times n)$ row-normalized adjacency matrix defined by $\widehat{g}_{ij} := g_{ij}/d_i$, where d_i is individual i's *degree*, or the number of her direct neighbors, that is, $d_i := \sum_{j=1}^{n} g_{ij}$.

Each agent $i = 1, 2, \ldots, n$ is described by: (i) her *productivity* $\alpha_i \in \mathbb{R}_+$, which is an exogenous characteristic; (ii) her *effort* $x_i \in \mathbb{R}_+$, which is agent i's choice variable; and (iii) her position in the network $\mathbf{g}$, which defines her social norm. Following the standard notation, we set

$$\boldsymbol{\alpha} := (\alpha_1, \alpha_2, \ldots, \alpha_n)^T \in \mathbb{R}^n_+, \qquad \mathbf{x} := (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n_+,$$

while the subscript $(-i)$ means dropping a vector's ith coordinate:

$$\mathbf{x}_{-i} := (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)^T \in \mathbb{R}^{n-1}_+.$$

Finally, agent i's *social norm*, $\overline{x}_i$, is defined as the average effort across her neighbors, namely,

$$\overline{x}_i := \sum_{j=1}^{n} \widehat{g}_{ij} x_j \tag{2}$$

In equilibrium, each agent's effort x_i is represented[13] as a convex combination of her own exogenous productivity α_i and her endogenous social norm $\overline{x}_i$. This is very much in the spirit of the linear-in-means model (1).

2.2. Preferences

Agent i's utility function has a standard linear-quadratic structure and is given by

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{1}{2} x_i^2 - \frac{\theta}{2} (x_i - \overline{x}_i)^2, \tag{3}$$

where $\alpha_i > 0$ stands for agent i's *individual productivity*, while $\theta > 0$ is the *taste for conformity*.[14]

The utility function (3) has two terms. The first term, $\alpha_i x_i - x_i^2/2$, is the utility of exerting x_i units of effort when there is *no interaction* with other individuals. The second term,

[13] See equation (11) below.

[14] Note the difference between (3) and the local-aggregate model (Ballester et al., 2006), where the utility of agent $i = 1, 2, \ldots, n$ is given by

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{1}{2} x_i^2 + \theta \sum_{j=1}^{n} g_{ij} x_i x_j, \tag{4}$$

that is, it is the aggregate effort of peers, $\sum_{j=1}^{n} g_{ij} x_j$, which positively affects own utility. In Online Appendix B, we compare the local-average and the local-aggregate model.

$-\theta\left(x_i-\overline{x}_i\right)^2/2$, captures the *peer-group pressure* faced by agent i, who seeks to minimize her social distance from her reference group, and suffers a utility reduction equal to $\theta\left(x_i-\overline{x}_i\right)^2/2$ from failing to conform to others.[15]

For the sake of analytical convenience, we reparametrize the taste for conformity by setting

$$\lambda := \frac{\theta}{1+\theta}, \qquad 0 \le \lambda < 1. \tag{5}$$

By plugging (5) into (3), we obtain

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{1}{2}x_i^2 - \frac{1}{2}\left(\frac{\lambda}{1-\lambda}\right)\left(x_i-\overline{x}_i\right)^2 \tag{6}$$

The two parameterizations, (3) and (6), are clearly equivalent. Indeed, as observed from (5), λ is a monotone transformation of θ.

We now point out some important properties of the utility function (6), which provides useful intuition about our main results. First, if i and j are neighbors, we have

$$\frac{\partial U_i(x_i, \mathbf{x}_{-i}, \mathbf{g})}{\partial x_j} \gtreqless 0 \iff x_i \gtreqless \overline{x}_i. \tag{7}$$

In other words, when agent j makes effort x_j, she exerts a positive (negative) *externality* on her neighbor i if and only if the effort of i is above (below) i's social norm. This is important in the welfare section, since we observe that the equilibrium effort differs from the first-best, because agents fail to internalize externalities when choosing their effort levels. These externalities are positive or negative depending on whether the effort is above or below the social norm. This highlights the importance of having endogenous social norms.

Second, efforts are *strategic complements*. Indeed, for $\widehat{g}_{ij} > 0$,

$$\frac{\partial^2 U_i(x_i, \mathbf{x}_{-i}, \mathbf{g})}{\partial x_i x_j} > 0, \tag{8}$$

which means that the higher is the effort of an individual's peer, the higher is the individual's marginal utility of exerting effort.

Third, the cross-effect of individual i's effort x_i and the taste for conformity λ is given by:

$$\frac{\partial^2 U_i(x_i, \mathbf{x}_{-i}, \mathbf{g})}{\partial x_i \partial \lambda} \lesseqgtr 0 \iff x_i \gtreqless \overline{x}_i. \tag{9}$$

In other words, if $x_i > \overline{x}_i$ ($x_i < \overline{x}_i$), then, when agents become more conformist, an increase in x_i increases (reduces) the gap between x_i and $\overline{x}_i$, which leads to a decrease (increase) in the utility level. In other words, an increase in λ decreases (increases) the marginal utility of exerting effort for individual i if $x_i > \overline{x}_i$ ($x_i < \overline{x}_i$). We refer to this assumption when discussing the comparative statics of λ.

Finally, the cross-effects of effort and productivity are positive, as for any $i, j, k = 1, 2, \ldots, n$ we have

$$\frac{\partial^2 U_i(x_i, \mathbf{x}_{-i}, \mathbf{g})}{\partial x_j \partial \alpha_k} \geq 0. \tag{10}$$

[15] This is the standard way in which economists have modeled conformity (see, among others, Akerlof, 1980, 1997; Kandel and Lazear, 1992; Bernheim, 1994; Fershtman and Weiss, 1998; Patacchini and Zenou, 2012; Boucher, 2016).

Hence, productivities $\boldsymbol{\alpha}$ and efforts $\mathbf{x}$ satisfy the standard Milgrom–Shannon conditions, which guarantee monotone comparative statics in supermodular games (see Proposition 2 below). However, this is not the case for the comparative statics in terms of the taste for conformity λ (see (9)).

To summarize, the utility function (3)—equivalently, (6)—is the standard way economists have modeled conformity. However, the social norm $\overline{x}_i$ is usually assumed to be exogenous (see, e.g., Akerlof, 1980, 1997), which makes the problem less interesting, because it abstracts from general equilibrium effects (Dutta et al., 2019). Here, we endogenize the social norm by making it dependent on the network structure. In that case, agents create externalities for each other through the social norm that they do not take into account when exerting their effort. This leads to new policy implications that we explore in Sections 4 and 6.

2.3. Nash equilibrium

Each individual i chooses x_i to maximize (6) taking the network structure $\mathbf{g}$ and the effort choices $\mathbf{x}_{-i}$ of other agents as given. By computing agent i's first-order condition (FOC) with respect to x_i, we obtain the following best-reply function for each i:

$$x_i = (1-\lambda)\alpha_i + \lambda \overline{x}_i. \tag{11}$$

After some normalizations, it should be clear that (11) is equivalent to the standard linear-in means model (1) in which individual effort is a function of individual observable characteristics α_i, which can also depend on the characteristics of neighbors, and on the endogenous peer effect $\overline{x}_i$.

Combining (11) with the definition (2) of agent i's social norm, we find that the vector $\mathbf{x}^* := \left(x_1^*, x_2^* ..., x_n^*\right)^T$ of equilibrium efforts must be a solution to

$$\mathbf{x} = (1-\lambda)\boldsymbol{\alpha} + \lambda \widehat{\mathbf{G}}\mathbf{x}, \tag{12}$$

where $\boldsymbol{\alpha} := (\alpha_1, \ldots, \alpha_n)^T$ is the productivity vector.[16]

Proposition 1 *(Equilibrium efforts, norms, and utilities).*

(i) *There exists a unique interior Nash equilibrium* $\mathbf{x}^*$*, which is given by*

$$\mathbf{x}^* = \widehat{\mathbf{M}}\boldsymbol{\alpha}, \tag{13}$$

[16] Observe that the linear-in-means model (1) is closely related to the *spatial-autoregressive (SAR) model* in the spatial econometrics literature (LeSage and Pace, 2009) and is usually written in matrix form as

$$\mathbf{x} = \boldsymbol{\beta} + \lambda \widehat{\mathbf{G}}\mathbf{x} + \boldsymbol{\epsilon},$$

where, as in our model, $\widehat{\mathbf{G}}$ is a row-normalized matrix that captures the distance or proximity in the geographical space (or any other space, e.g., the social space) between different agents or entities, such as geographical areas. In this literature, the main reason for the matrix $\widehat{\mathbf{G}}$ to be row-normalized is to obtain an intuitive interpretation of λ as the weighted average impact of neighbors but also to avoid explosive spatial multipliers implied by λ (by analogy to time-series econometrics, in which the autoregression parameter λ is expected to be strictly less than 1 in modulus; see Hamilton, 1994). Equation (12) is clearly equivalent to the spatial-autoregressive model and it gives a microfoundation of the SAR model via the utility function (3) or (6).

where $\widehat{\mathbf{M}} = [\widehat{m}_{ij}]$ *is an* $(n \times n)$*-matrix of marginal effects defined as follows*[17]*:*

$$\widehat{\mathbf{M}} := (1-\lambda)\left(\mathbf{I} - \lambda\widehat{\mathbf{G}}\right)^{-1} = (1-\lambda)\sum_{k=0}^{\infty}\lambda^k\widehat{\mathbf{G}}^k. \tag{14}$$

(ii) *The equilibrium social norms* $\overline{\mathbf{x}}^*$ *are given by*

$$\overline{\mathbf{x}}^* = \widehat{\mathbf{G}}\widehat{\mathbf{M}}\boldsymbol{\alpha} = (1-\lambda)\sum_{k=0}^{\infty}\lambda^k\,\widehat{\mathbf{G}}^{k+1}\boldsymbol{\alpha}. \tag{15}$$

(iii) *For each* $i = 1, 2, \ldots, n$*, agent* i*'s equilibrium utility level is given by*

$$U_i^*(\boldsymbol{\alpha}, \lambda, \mathbf{g}) = \frac{1}{2}\left[\alpha_i^2 - \frac{1}{\lambda}\left(\alpha_i - \sum_{j=1}^{n}\widehat{m}_{ij}\alpha_j\right)^2\right]. \tag{16}$$

Several comments are in order. First, taking a closer look at the structure of the marginal effect $\widehat{m}_{ij}$ of agent i's productivity on agent j's effort, we obtain

$$\widehat{m}_{ij} = \sum_{k=0}^{\infty}\underbrace{(1-\lambda)\lambda^k}_{\substack{\text{geometric}\\\text{distribution}}}\,\widehat{g}_{ij}^{[k]}. \tag{17}$$

As seen from (17), $\widehat{m}_{ij}$ is decomposed into a series whose kth term is proportional to $\widehat{g}_{ij}^{[k]}$, that is, the normalized number of paths from i to j of length k in the social network. Surprisingly, the coefficients of the series are given by the standard geometric distribution with the odds ratio equal to $\theta \equiv \lambda/(1-\lambda)$. Therefore, although the game under study is fully deterministic, one may inquire whether the marginal effects $\widehat{m}_{ij}$ have some probabilistic origin. In Online Appendix C, we demonstrate that the local average model is observationally equivalent to an average outcome of a naive social learning model.[18]

Second, there is *no need to impose any conditions on* $\theta \equiv \lambda/(1-\lambda)$ (except that $\theta > 0$) to guarantee the existence of a unique and interior Nash equilibrium. This is not the case in the local aggregate model.[19]

Third, it is readily verified that, if agents are *ex ante homogeneous*, that is, if $\alpha_i = \alpha_j$ for any $i, j = 1, 2, \ldots, n$, then, regardless of the network structure, *the equilibrium effort levels are the same across agents*: $x_i^* = x_j^*$ for any $i, j = 1, 2, \ldots, n$. This result displays another significant difference with the local aggregate model, in which the outcome is represented by the Katz–Bonacich centralities of the agents. Here, the impact of the network structure on equilibrium is

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[17] Because $\widehat{\mathbf{G}}$ is row-normalized and $0 \le \lambda < 1$, the matrix $\widehat{\mathbf{M}}$ of marginal effects is well defined and can be represented by the Neumann series. This follows from Corollary 5.6.16 in Horn and Johnson (1985, Ch. 5, p. 301), in which the suitable matrix norm is the maximum row sum norm.

[18] A similar result was obtained by Golub and Morris (2017).

[19] Indeed, in the local-aggregate model for which the utility function is given by (4), one needs a condition on θ (i.e., $\theta < 1/\mu(\mathbf{G}$, where $\mu(\mathbf{G})$ is the largest eigenvalue of $\mathbf{G}$), to prove the uniqueness of equilibrium. In the local-average model, one does not need such a condition, because the matrix to be inverted is $\left(\mathbf{I} - \lambda\widehat{\mathbf{G}}\right)$, where $\widehat{\mathbf{G}}$ is the row-normalized matrix of $\mathbf{G}$. The largest eigenvalue of $\widehat{\mathbf{G}}$ equals one and thus, the condition for invertibility of $\left(\mathbf{I} - \lambda\widehat{\mathbf{G}}\right)$ is $\lambda := \theta/(1+\theta) < 1$, which is always true.

mediated by the *correlation* between the productivity distribution $\boldsymbol{\alpha}$ and the degree distribution of the network $\mathbf{g}$. We return to this property in Sections 3 and 4.

Fourth, instead of assuming (3) or (6), the following utility function can be assumed:

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{1}{2} x_i^2 - \frac{\theta}{2} \sum_{j=1}^{n} \widehat{g}_{ij}(x_i - x_j)^2, \tag{18}$$

and exactly the same first-order condition (11) can still be obtained and thus, the same equilibrium effort x_i^*. The interpretation of the utility function (18) is still in terms of conformism but now, each individual pays some cost from deviating from the action of *each* of her neighbors instead of the *average* action of her neighbors.

Even if the equilibrium effort is the same and equals x_i^*, the equilibrium utility is different.[20] As a result, the equilibrium effort x_i^* and its comparative statics results are the same but the welfare analysis and its comparative statics may differ, because the equilibrium utilities and thus, welfare are different.[21]

Finally, in part (iii) of Proposition 1, we calculate the equilibrium utility level of each agent in the network as a function of the parameters of the model. An important aspect of this model is whether individual i's effort is above or below her own social norm. The following result clarifies this relationship.

Lemma 1. *For each $i = 1, 2, \ldots, n$, we have*

$$x_i^* \gtreqless \overline{x}_i^* \iff \alpha_i \gtreqless \sum_{j=1, j\neq i}^{n} \frac{\widehat{m}_{ij}}{(1-\widehat{m}_{ii})} \alpha_j. \tag{19}$$

This lemma shows that agent i's own effort is above (below) her social norm if and only if her productivity is higher (smaller) than the weighted average of the other productivities in the network. For example, in a star network, if the central agent is more productive than the others,

[20] It is easily verified that, in our model, the equilibrium utility is given by

$$U_i(x_i^*, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i^* - \frac{1}{2}(x_i^*)^2 - \frac{\theta}{2}(x_i^*)^2 + \theta x_i^* \bar{x}_i^* - \frac{\theta}{2}\left(\sum_{j=1}^{n} \widehat{g}_{ij} x_j^*\right)^2$$

while, in this new model with preferences given by (18), we have:

$$U_i(x_i^*, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i^* - \frac{1}{2}(x_i^*)^2 - \frac{\theta}{2}(x_i^*)^2 + \theta x_i^* \bar{x}_i^* - \frac{\theta}{2}\sum_{j=1}^{n} \widehat{g}_{ij} (x_j^*)^2$$

The only difference between these two utility functions is the last term which is clearly different, since $\left(\sum_{j=1}^{n} \widehat{g}_{ij} x_j^*\right)^2 = (\bar{x}_i^*)^2 \neq \sum_{j=1}^{n} \widehat{g}_{ij} (x_j^*)^2$.

[21] As noted by Boucher and Fortin (2016), another utility function could have generated the same first-order conditions (11) and thus, the same equilibrium effort x_i^*. It is given by

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{(1+\theta)}{2} x_i^2 + \theta x_i \sum_{j=1}^{n} \widehat{g}_{ij} x_j.$$

However, in this case, the properties of the model are very different, since it is no longer a conformist model.

then her effort is always above the social norm of her neighbors (the peripheral agents), who, in turn, exert effort below that of their social norm, since the latter is the effort of the central agent. This is a useful insight that helps us to understand the main results of Sections 3 and 4.

2.4. Linear-in-means model and heterogeneity: an example

In the Introduction, we discuss how peer effects are estimated in the literature using the linear-in-means model (see (1)), which captures an *average* effect. In reality, the same average effect can have a very different impact on outcomes, depending on other moments of the distribution, in particular, the variance.[22] Contrary to the linear-in-means model, the local-average model can address this issue, since it encompasses a network approach whereby the group each individual belongs to is determined by her direct neighbors. In that case, the whole distribution matters in evaluating the impact of peers on outcomes.

To illustrate this, in Online Appendix D, we provide a simple example that shows how a mean-preserving spread of the productivity impacts own and aggregate outcome. This example shows that estimating a linear-in-means model may be misleading, because it focuses only on the average effect and does not take into account other characteristics of the distribution of efforts in the population. In this example, we show that the local-average model can have a very different prediction than the linear-in-means model, depending on the value of λ, the taste for conformity, and the value of t. Indeed, with exactly the same average characteristic (here, productivity) in the group (here, network), the individual effort level may vary a lot. In this example, these changes are driven by t, which is proportional to the standard deviation of the productivity distribution. As a result, when studying the impact of the social norm on individual effort, one should not only take into account the average social norm of the reference group but also its variance.

3. Comparative statics

We aim to understand the properties of our model by performing some comparative statics exercises of the Nash equilibrium with respect to the key parameters of the model: (i) the productivity vector $\boldsymbol{\alpha}$; (ii) the taste for conformity λ; and (iii) the density/sparsity of social network $\mathbf{g}$.

3.1. Effect of productivity

Let us start with the productivity $\boldsymbol{\alpha}$ of all agents. We have the following result:

Proposition 2 *(Comparative statics for productivity).*

(i) *For all $i, j = 1, 2, \ldots, n$, the marginal effects of a change in individual i's productivity α_i on individual j's equilibrium effort x_j^* and individual j's social norm $\overline{x}_j^*$ are positive and do not exceed 1:*

[22] For example, in a classroom of 30 students, the impact of an average test score of 50/100 is very different if all students have a test score of around 50/100 (i.e., low variance with a very homogeneous distribution of test scores) than when some students have very high test scores and others have very low test scores (i.e., high variance with a very heterogeneous distribution of test scores).

$$0 < \frac{\partial x_j^*}{\partial \alpha_i} < 1, \qquad 0 < \frac{\partial \overline{x}_j^*}{\partial \alpha_i} < 1.$$

(*ii*) *The equilibrium utility of each individual* $i = 1, 2, \ldots, n$ *is increasing with her own productivity*:

$$\frac{\partial U_i^*(\boldsymbol{\alpha}, \lambda, \mathbf{g})}{\partial \alpha_i} > 0.$$

(*iii*) *For any* $j \neq i$*, agent* i*'s equilibrium utility* $U_i^*(\boldsymbol{\alpha}, \lambda, \mathbf{g})$ *increases (decreases) in response to a small change in* α_j*, if and only if agent* i*'s equilibrium effort* x_i^* *is above (below) her equilibrium social norm* $\overline{x}_i^*$*; that is,* $\operatorname{sign}\left[\frac{\partial U_i^*}{\partial \alpha_j}\right] = \operatorname{sign}\left(x_i^* - \overline{x}_i^*\right)$*, or equivalently, using Lemma 1,*

$$\frac{\partial U_i^*}{\partial \alpha_j} \gtreqqless 0 \iff \alpha_i \gtreqqless \sum_{l=1, l \neq i}^{n} \frac{\widehat{m}_{il}}{(1 - \widehat{m}_{ii})} \alpha_l$$

The first result is straightforward because, as implied by (13), each x_i^* is a convex combination of productivity and social norms. The second result, although intuitive, is relatively difficult to show. Indeed, when own productivity α_i increases, own effort x_i^* increases, which raises U_i^*, the equilibrium utility of i, but the social norm $\overline{x}_i^*$ also increases, which can increase or decrease U_i^* depending on whether x_i^* is higher or lower than $\overline{x}_i^*$. We show in the proof that the first direct effect is stronger than the second indirect effect, so that an increase in α_i always increases U_i^*. When we analyze the effect of α_j on U_i^* for $j \neq i$, we find a similar result, that is, the impact depends on whether x_i^* is above or below $\overline{x}_i^*$.

3.2. *Effect of conformity*

We now look at the impact of taste for conformity λ on individual and social outcomes.

3.2.1. *Pure individualism versus total conformism*

To obtain some intuition, we begin by contrasting two extreme cases: *pure individualism* ($\lambda = 0$), where i's utility depends only on own productivity α_i; and *total conformism* ($\lambda \to 1$), where i's utility depends only on others' behavior. To obtain these results, we use the observational equivalence between our models and that of the Markov chain developed in Online Appendix C to compare the outcomes generated by perfect individualism ($\lambda = 0$) and total conformism ($\lambda = 1$).

It is straightforward to observe that, under pure individualism ($\lambda = 0$), we have $x_i^* = \alpha_i$. In this case, norms play no role, and there are incentives for an individual to exert neither higher nor lower effort than her intrinsically desirable level, α_i. However, the outcome when $\lambda \to 1$ is less obvious.

Proposition 3 *(Totally conformist agents). For any network structure, individual efforts in a totally conformist society are given by*

$$\lim_{\lambda \to 1} x_i^*(\lambda) = \boldsymbol{\pi}\boldsymbol{\alpha} = \sum_{j=1}^{n} \pi_j \alpha_j, \qquad \textit{for all} \qquad i = 1, \ldots, n, \tag{20}$$

where $\boldsymbol{\pi} \equiv (\pi_1, \pi_2, \ldots, \pi_n)$ *are normalized degrees of agents:*

$$\pi_i := \frac{d_i}{\sum_{j=1}^{n} d_j}, \quad \textit{for all } i = 1, 2, \ldots n, \tag{21}$$

Proposition 3 shows that, for any network structure, when agents are perfectly conformist, the equilibrium effort depends only on the weighted productivity in the network, where the weights depend on the network structure. This implies, in particular, that π_j is the probability that a *perfectly conformist* individual i exerts a level α_j of effort. This means that, when $\lambda \to 1$, the *effort of all agents in the network is the same* and that the level of these efforts depends on the network structure captured by $\boldsymbol{\pi}$ and on the productivity distribution captured by $\boldsymbol{\alpha}$. Thus, the probabilistic interpretation of the model helps us to understand the totally conformist society, which is otherwise difficult to characterize.[23]

We are now equipped to compare the purely individualist society ($\lambda \to 0$) and the totally conformist society ($\lambda \to 1$).

Proposition 4 *(Individualist versus conformist society).*

(i) *Individual effort:*

$$\lim_{\lambda \to 0} x_i^* (\lambda) \gtreqless \lim_{\lambda \to 1} x_i^* (\lambda) \iff \alpha_i \gtreqless \sum_{j=1}^{n} \pi_j \alpha_j$$

(ii) *Aggregate effort:*

$$\lim_{\lambda \to 0} \sum_i x_i^* (\lambda) \gtreqless \lim_{\lambda \to 1} \sum_i x_i^* (\lambda) \iff \sum_{j=1}^{n} \alpha_j \gtreqless n \sum_{j=1}^{n} \pi_j \alpha_j$$

Part (i) of Proposition 4 shows that the effort exerted by each agent i can be higher or lower in a pure individualist society than in a completely conformist one if the productivity of i is above or below the weighted average productivity in the network. This result depends on both own productivity and the network structure. Part (ii) of Proposition 4 shows that conformity is not necessarily good for aggregate effort. However, when α_i and π_i are positively (negatively) correlated, that is, agents with higher productivity have (less) more central positions in the network,[24] then perfect conformity increases aggregate effort.

How do individual and aggregate efforts change when the taste for conformity varies? To answer this question, we study the comparative statics with respect to the conformity parameter λ.

[23] A similar result in terms of conformity limits was shown by Golub and Morris (2017), but in the context of imperfect information.

[24] Indeed, it is straightforward to show that:

$$\sum_{j=1}^{n} \pi_j \alpha_j \gtreqless \frac{1}{n} \sum_{j=1}^{n} \alpha_j \iff \text{Corr}\,(\boldsymbol{\pi}, \boldsymbol{\alpha}) \gtreqless 0,$$

where $\text{Corr}\,(\boldsymbol{\pi}, \boldsymbol{\alpha})$ is the correlation between $\boldsymbol{\pi}$ and $\boldsymbol{\alpha}$. If $\text{Corr}\,(\boldsymbol{\pi}, \boldsymbol{\alpha}) > 0$ (< 0), then more productive agents are also more (less) central (in terms of degree centrality) in the network.

3.2.2. The impact of the taste for conformity on outcomes

Let us totally differentiate (11) with respect to λ. We obtain

$$dx_i^* = \underbrace{-\alpha_i \, d\lambda}_{\substack{\text{productivity}\\ \text{effect}}} + \underbrace{\overline{x}_i^* \, d\lambda}_{\substack{\text{direct}\\ \text{norm effect}}} + \underbrace{\lambda \left(\partial \overline{x}_i^* / \partial \lambda\right) d\lambda}_{\substack{\text{indirect}\\ \text{norm effect}}} \tag{22}$$

Indeed, when λ increases, the individual effort of individual i, x_i^*, is affected in three different ways. First, there is a negative *productivity effect*, according to which, when conformity increases, the impact of own productivity on effort decreases. Second, there is a positive *direct social-norm effect*, indicating that, when λ increases, the impact of the social norm on own effort increases. These are straightforward direct effects due to the fact that, when λ increases, agents pay more attention to their neighbors than to themselves. There is a third, more subtle effect, the *indirect social-norm effect*, which can be positive or negative. This effect shows that, when λ increases, the social norm itself changes as i changes her effort and her peers become more conformist. The effect is ambiguous as i's friends may increase or decrease their effort following an increase in λ. As a result, the total effect of λ on x_i^* is ambiguous. To understand this better, using (11), (22) can be written as

$$dx_i^* = -\left(x_i^* - \overline{x}_i^*\right) \frac{d\lambda}{1-\lambda} + \lambda \frac{\partial \overline{x}_i^*}{\partial \lambda} d\lambda$$

We now see that the total impact of a change of λ crucially depends on whether the individual effort of i is above or below her own social norm. As observed from (9), this is because the effect of λ on the marginal utility of effort is ambiguous and depends on the gap, $x_i - \overline{x}_i$, between the individual's effort and her social norm. In particular, when λ increases, agents become more conformist, and the gap between x_i and $\overline{x}_i$ matters more.

Recall that $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_n)$ is the normalized degree distribution of the network $\mathbf{g}$ (see (21)). We obtain the following result.

Proposition 5 *(Non-monotonicity of individual efforts in conformism).*

(i) *For any $\lambda \in (0, 1)$, if $\partial x_i^* / \partial \lambda > 0$ for some i, then it has to be that $\partial x_j^* / \partial \lambda < 0$ for some $j \neq i$.*

(ii) *When λ is small, we have*

$$\frac{\partial x_i^*}{\partial \lambda} \gtrless 0 \iff \alpha_i \lessgtr \sum_{j=1}^{n} \widehat{g}_{ij} \alpha_j \tag{23}$$

(iii) *Assume that the following conditions hold:*

$$\sum_{j=1}^{n} \pi_j \alpha_j \leq \alpha_i < \sum_{j=1}^{n} \widehat{g}_{ij} \alpha_j . \tag{24}$$

Then, agent i's individual effort $x_i^*(\lambda)$ has an interior global maximum in λ.

(iv) *Assume that the following conditions hold:*

$$\sum_{j=1}^{n} \pi_j \alpha_j \geq \alpha_i > \sum_{j=1}^{n} \widehat{g}_{ij} \alpha_j . \tag{25}$$

Then, agent i's individual effort $x_i^*(\lambda)$ has an interior global minimum in λ.

Part (i) of Proposition 5 provides an expression of the impact of conformity on individual i's effort. We show that it crucially depends on whether both individual i and all other agents in the network (since all agents are path-connected to each other) make efforts above or below the social norm of their friends. In particular, if we order agents by their productivity in descending order, so that $\alpha_{\max} := \alpha_1$ and $\alpha_{\min} := \alpha_n$ are the highest and lowest values of productivity among the n agents in the network, respectively, then, by Lemma 1, it has to be that $x_1^* > \overline{x}_1^*$ and $x_n^* < \overline{x}_n^*$. As a result, because some individuals exert effort above the norm and some below the norm, the total impact of λ on an individual is ambiguous, and has to increase for some individuals and decrease for others. Equation (23) shows that, for small λ, the sign of this derivative depends only on whether i's productivity is above or below that of her peers.[25]

Observe that this comparative statics result is very different to that obtained in the local-aggregate model in which an increase in λ or θ (social multiplier or social interaction effect in the local-aggregate model; see (4)) always leads to an increase in effort x_i^*. This is important for policy purposes, because, as noted by Boucher and Fortin (2016), if there is a positive policy shock on λ, and we observe that individual effort either decreases or the effect is non-monotonic, then we know that the underlying utility function is defined by the local-average model (see (3) or (6)) and not by the local-aggregate model. To know which utility function each agent has when choosing her effort is important for policy implications, as discussed in Section 6 below.

Parts (ii) and (iii) of Proposition 5 provide sufficient (but not necessary) conditions for x_i^* to vary *non-monotonically* with λ.[26] Based on these conditions, which depend only on the productivity parameters and the structure of the network, α_i cannot be neither too high nor too low for the relationship between x_i^* and λ to be non-monotonic. Clearly, if λ_i is very high (low), which implies that x_i^* is very likely to be above (below) $\overline{x}_i^*$, then $\frac{\partial x_i^*}{\partial \lambda}$ is negative (positive). Conditions (24) and (25) also guarantee a global interior maximum or minimum in λ. In particular, if α_i is above (below) the productivity in the network, there is a global interior maximum (minimum), which means that an increase in λ first has a positive (negative) impact on x_i^* and then a negative (positive) one.

In fact, the non-monotonicity expressed in parts (ii) and (iii) of Proposition 5 can be complex and not necessarily U shaped or bell shaped. In Fig. 1, we provide an example for a chain network with 13 nodes in which increasing λ yields an S shape. In this chain network, node 0 is in the middle, nodes $1, 2, 3, 4, 5$, and 6 are on the right side of node 0, while nodes $-1, -2, -3, -4, -5$, and -6 are on the left side of node 0.[27]

In Proposition 5, we show that the impact of λ on *individual effort* is very complex and difficult to sign. In Corollary E.1 in Online Appendix E, we show that the same non-monotonicity results hold for the *aggregate effort*, which is an important aspect of this model.[28] Also, in Proposition E1 in Online Appendix E, we demonstrate that, in regular networks, the aggregate effort does not vary with λ. This is because, in a regular network, there is perfect compensation between the positive impact of λ on low-productive agents and the negative impact of λ on high-productive agents. As a result, neither the average nor aggregate effort in a regular network are affected by a

25 Proposition G4 in Online Appendix G.2 generalizes Proposition 5 when the taste for conformity is individual specific and equal to λ_i for each agent i.

26 We give sharper conditions for some specific types of network structures in Section E.3 in Online Appendix E.

27 The values of productivity are assumed to be: $\alpha_0 = 0.75$, $\alpha_1 = 1 = \alpha_{-1}$, $\alpha_2 = 0.5 = \alpha_{-2}$, $\alpha_3 = \alpha_{-3} = 0.25$, $\alpha_4 = 0.5 = \alpha_{-4} =$, $\alpha_5 = 2\alpha_{-5}$, and $\alpha_6 = 0.5 = \alpha_{-6}$.

28 For example, in crime, we would be interested in analyzing how conformity affects individual crime effort but also the total crime level in the network.

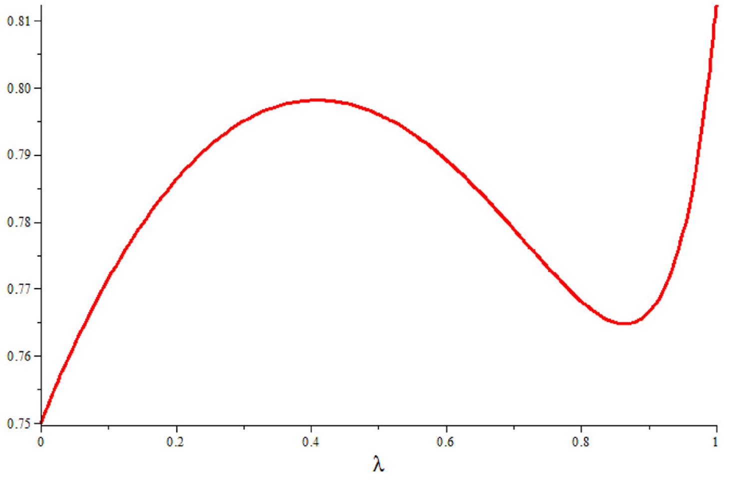

Fig. 1. Non-monotonic effect of λ on individual effort for a chain network with $n = 13$.

change in λ. In Section E.4 in Online Appendix E, we illustrate this result by means of a circular network. When we rewire the links in this network without changing the network topology, we show that the convergence of agents' efforts to the average effort can be faster or slower than in the original network depending on the rewiring.

To summarize, in this section, we show that the impact of the taste for conformity λ on i's effort depends on the productivity of each individual and the network topology, which determines the links between all agents and, thus, the peer pressure (via the social norm) that neighbors exert on own effort. Therefore, the effect of a higher taste for conformity on own effort is complex and determined by whether the individual is an "underdog" or someone who has high productivity. If we consider crime, this determination is important, since it shows how delinquents influence each other and how an individual's crime effort is affected by the degree of conformism in the peer group she belongs to.

3.3. Do agents exert more effort in denser networks?

We now consider the consequences of a change in the network structure by asking the following question: how does adding a new link to the existing network affect the equilibrium efforts? In the local-aggregate model, the answer is straightforward: because of strategic complementarities, regardless of the productivities αs, all agents always exert more effort in denser networks. However, as the following proposition shows, this is not always true in the local-average model.

Proposition 6. *Assume that agents i and j are not connected to each other ($g_{ij} = 0$). Then, adding a link between i and j leads to:*

(i) *an increase in everyone's effort, if the following two conditions hold simultaneously:*

$$\alpha_i > \frac{\sum_{l \neq i} \left(\widehat{m}_{il} - \lambda \widehat{m}_{jl} \right) \alpha_l}{1 - \lambda - \widehat{m}_{ii} + \lambda \widehat{m}_{jj}}, \tag{26}$$

$$\alpha_j > \frac{\sum_{l \neq j} \left(\widehat{m}_{jl} - \lambda \widehat{m}_{il} \right) \alpha_l}{1 - \lambda - \widehat{m}_{jj} + \lambda \widehat{m}_{ii}}; \textit{ and} \tag{27}$$

(ii) *a reduction of everyone's effort, if the inequalities are opposite in* (26) *and* (27).

Otherwise, there is an ambiguous outcome.

This proposition shows that, in any network, adding a link between two agents who have high (low) productivities not only increases (decreases) the effort of these two agents but also increases (reduces) the effort of all the other agents in the network. Indeed, if we connect agent i to a high-productivity agent j, then i's norm increases and the best response for i is to increase her effort (see (11)). This implies that the norm of i's neighbors increases, which, in turn, increases their effort, and so forth. Similarly, if we connect j to a high-productivity agent i, then j's norm increases and the best response for j is to increase her effort. We have again the same snow-ball effect. The same reasoning applies in the opposite direction if we connect two low-productive agents. Indeed, if agent i connects to low-productivity agent j, then i's norm decreases, which reduces i's effort. This, in turn, decreases the norm of i's neighbors, which reduces their effort, and so forth.

To illustrate this result, consider a star network with three agents in which agent 1 is in the center. The row-normalized adjacency matrix is then given by

$$\widehat{\mathbf{G}}^S = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Let us first assume that $\alpha_1 = 2$, $\alpha_2 = 1$ and $\alpha_3 = 0.5$, so that the star is more productive than the peripheral agents are. It is easily verified that

$$\mathbf{x}^{*S} = \frac{1}{4(1+\lambda)} \begin{pmatrix} 3\lambda + 8 \\ -\lambda^2 + 8\lambda + 4 \\ \lambda^2 + 8\lambda + 2 \end{pmatrix}, \quad \overline{\mathbf{x}}^{*S} = \frac{1}{4(1+\lambda)} \begin{pmatrix} 8\lambda + 3 \\ 3\lambda + 8 \\ 3\lambda + 8 \end{pmatrix}.$$

According to part (ii) of Proposition 6, adding the link 2–3 between the two less productive agents should decrease the efforts of all agents in the network. Let us verify this. By adding the link between agents 2 and 3, the network becomes complete and the row-normalized adjacency matrix is now given by

$$\widehat{\mathbf{G}}^C = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

In that case, we obtain

$$\mathbf{x}^{*C} = \frac{1}{2(2+\lambda)} \begin{pmatrix} 8 - \lambda \\ 4 + 3\lambda \\ 2 + 5\lambda \end{pmatrix}, \quad \overline{\mathbf{x}}^{*C} = \frac{1}{2(2+\lambda)} \begin{pmatrix} 3 + 4\lambda \\ 5 + 2\lambda \\ 6 + \lambda \end{pmatrix}.$$

It is easily verified that $x_i^{*S} > x_i^{*C}$, for all $i = 1, 2, 3$, so that adding the link 2–3, indeed, *decreases* the effort of all agents in the network. Consider first agent 2. By adding the link 2–3, her social norm decreases, that is, $\overline{x}_2^{*S} > \overline{x}_2^{*C}$, since, before adding the link 2–3, the social norm of agent 2 was equal to the effort of agent 1, a very productive agent, while, after adding the link 2–3, it becomes the average of the efforts of 1 and that of 3, a low-productive agent. Since agent 2's norm decreases, her best response is to decrease her effort. The same reasoning applies for agent 3, whose norm changes from the effort of agent 1 to the average effort of agents 2 and 3.

Thus, agent 3's norm decreases and her best response is to decrease her effort. Since both agents 2 and 3 reduce their effort, the social norm of agent 1, which is the average effort of agents 2 and 3, decreases and her best response is to decrease her effort. As a result, by adding the link 2–3, all agents reduce their effort.

Assume now that $\alpha_1 = 0.5$, $\alpha_2 = 1$, and $\alpha_3 = 2$ so that the peripheral agents are now the most productive ones in the network. Then, it is easily verified that adding the link 2–3 *increases* the effort of all agents in the network, as predicted by part (i) of Proposition 6. This is because, when the link 2–3 is added, the social norm of agent 2 increases, as it changes from being equal to the effort of agent 1, a low-productive agent, to the average effort of agents 1 and 3, where 3 is a high-productive agent. Her best response is to increase her effort. The same applies to agent 3. Since both agents 2 and 3 increase their effort, agent 1 also increases her effort, because her social norm increases.

Finally, if we assume that $\alpha_1 = 1$, $\alpha_2 = 0.5$, and $\alpha_3 = 2$, then adding the link 2–3 has no clear monotonic effect on the effort of all agents in the network. Indeed, on the one hand, it increases the social norm of agent 2, who increases her effort, but reduces the social norm of agent 3, who decreases her effort. This implies that the effect on the social norm of agent 1 (and her effort), which is the average effort of agents 2 and 3, would be ambiguous.

Observe that the results obtained in Proposition 6 can easily be extended to removing links.

Remark 1. Assume $g_{ij} = 1$. If both (26) and (27) hold, then removing the link between agents i and j decreases the effort of all agents in the network. If the inequalities are opposite in (26) and (27), then removing the link between agents i and j increases the effort of all agents in the network. Otherwise, the effect of removing the link i-j is ambiguous.

This is an important result that has interesting policy implications. Consider crime. The usual objective of the planner is to reduce total crime, which, here, amounts to reducing aggregate effort. Thus, Remark 1 helps us answer the following question: if the planner wants to reduce total crime, which *link* should she remove from the network? This is referred to as the *key-link* policy. As Remark 1 shows, the planner needs to remove the link between the two most productive agents in the network and this is independent of the network structure. Ballester et al. (2010) determined the key link in the local-aggregate model and showed that it strongly depends on the network structure, in particular, the Katz–Bonacich centrality of the two agents involved in the key link. The main advantage of our result in Remark 1 is that the planner does not need to know the network but only the crime productivity of all agents in the network, which can be determined in the data by their crime records.

What does a key-link policy mean in the real-word? A link removal would lead to a disruption of the communication between two criminals. For example, when a police officer keeps watch over a street, she disrupts the possible communication between criminals from the same neighborhood. Another example of a key-link policy is to move a delinquent teenager to another residential location where there are less delinquents.[29] By doing so, this delinquent stops her activities and communication with other delinquents in the older residential area.[30]

[29] See, for example, Ludwig et al. (2001) and Kling et al. (2005, 2007), who study the moving to opportunity experiment that relocates families from high- to low-poverty neighborhoods. The authors found that this policy reduces juvenile arrests by 30 to 50% of the arrest rate for control groups.

[30] For a general discussion of removing links and disrupting the network in criminal activities, see Lindquist and Zenou (2019).

4. Welfare and first best

We now analyze socially optimal outcomes. For that, let us first calculate the first-best outcome of this economy and then determine the taxes/subsidies that can restore the first best.

4.1. First best

Define the social welfare $\mathcal{W}$ as

$$\mathcal{W} := \sum_{i=1,2,\ldots,n} U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}). \tag{28}$$

The following proposition characterizes the first best and establishes a necessary and sufficient condition for the Nash equilibrium in efforts to be socially optimal.

Proposition 7 *(First best).*

(i) *For each $i = 1, 2, \ldots, n$, the first-best effort $\mathbf{x}^O$ is a solution to*

$$x_i = (1-\lambda)\alpha_i + \lambda \overline{x}_i + \lambda \sum_{j=1}^{n} \widehat{g}_{ji} \left(x_j - \overline{x}_j\right), \tag{29}$$

or, in matrix form,

$$\mathbf{x} = (1-\lambda)\boldsymbol{\alpha} + \lambda \widehat{\mathbf{G}}\mathbf{x} + \lambda \widehat{\mathbf{G}}^T \left(\mathbf{I} - \widehat{\mathbf{G}}\right)\mathbf{x}. \tag{30}$$

(ii) *For the Nash equilibrium to be the first best ($\mathbf{x}^* = \mathbf{x}^O$), it is necessary and sufficient that the vector $\boldsymbol{\alpha}$ of productivity satisfies the following system of linear constraints:*

$$\widehat{\mathbf{G}}^T(\mathbf{I} - \widehat{\mathbf{G}})\widehat{\mathbf{M}}\boldsymbol{\alpha} = \mathbf{0}. \tag{31}$$

(iii) *Moreover, for any network,*

$$\sum_{i=1}^{n} x_i^O = \sum_{i=1}^{n} \alpha_i. \tag{32}$$

Part (i) of Proposition 7 clearly shows the difference in effort between the Nash equilibrium (see (11)) and the first best (see (29)). In particular, compared to the Nash equilibrium, the first best has an extra term, $\lambda \sum_{j=1}^{n} \widehat{g}_{ji} \left(x_j - \overline{x}_j\right)$, which could be positive or negative. In fact, this extra term is the result of the following derivation: $\sum_{j \neq i} \frac{\partial U_j}{\partial \overline{x}_j} \frac{\partial \overline{x}_j}{\partial x_i}$ (see the proof of Proposition 7), where $\frac{\partial \overline{x}_j}{\partial x_i} > 0$, that is, an increase in i's effort increases the average effort of j's friends if i and j are friends, and $\frac{\partial U_j}{\partial \overline{x}_j} = (\frac{\lambda}{1-\lambda})\left(x_j - \overline{x}_j\right) \gtreqless 0$. This last result implies that, if $x_j > \overline{x}_j$ ($x_j < \overline{x}_j$), then an increase in $\overline{x}_j$ reduces (increases) the difference between x_j and $\overline{x}_j$, which, because of conformism, increases (decreases) utility. Thus, at the Nash equilibrium, when deciding their individual effort, agents do not take into account the effect of their effort of the social norm of their peers, which creates an externality that can be positive or negative. Indeed, if individual i has friends for whom $x_j > \overline{x}_j$ ($x_j < \overline{x}_j$), then when she exerts her effort, she does not take into account the fact that she positively affects $\overline{x}_j$, the norm of her friends, which increases (decreases) the utility of their neighbors. In that case, compared to the first best, individual i underinvests (overinvests) in effort, because she exerts positive (negative) externalities on her friends.

This result contrasts with that obtained in the local-aggregate model in which agents always underinvest in effort, because they always exert positive externalities on their neighbors. Here, even though the efforts are *strategic complements* (see (8)), agents can exert positive or negative externalities on their neighbors. This is why, in the local-aggregate model, the planner always wants to subsidize agents (Helsley and Zenou, 2014) while, in the local-average model, the planner subsidizes agents who underinvest in effort and taxes agents who overinvest in effort. We investigate these issues in detail in Section 4.3 below.

Part (ii) of Proposition 7 gives an exact condition on the productivity vector $\boldsymbol{\alpha}$ that ensures that the Nash equilibrium in efforts is always optimal. Unfortunately, this condition is very unlikely to hold in most networks, as shown in Online Appendix E.

Finally, in part (iii), we demonstrate that, for any network, the aggregate first-best effort is independent of λ, the taste for conformity, and is equal to the aggregate productivity in the network. In particular, this implies (see Proposition E1 in Online Appendix E) that, for *regular networks*, we have:

$$\sum_{i=1}^{n} x_i^* = \sum_{i=1}^{n} x_i^O = \sum_{i=1}^{n} \alpha_i .$$

In other words, for regular networks, even if the individual effort is generally not optimal, the aggregate effort in a network is always optimal. This is because, in regular networks, the positive and negative externalities imposed by agents on their neighbors exactly cancel out, so that the aggregate effect is optimal. Consequently, when the network is regular, some agents overinvest while others underinvest, and it is not possible that all agents underinvest. This result stands in sharp contrast to the local aggregate model, in which all agents exert too little effort in equilibrium, regardless of whether the network is regular or not.

Remark 2. If agents are ex ante homogeneous in productivity, that is, $\alpha_i = \alpha_j$ for all $i, j = 1, 2, \ldots, n$, then the Nash equilibrium in effort is always optimal. Furthermore, if $\det(\widehat{\mathbf{G}}) \neq 0$, the converse is also true.

Indeed, if agents are ex ante homogeneous, we know that, in equilibrium, the position in the network does not matter and all agents exert the same effort level, which is equal to the common social norm in the network. As a result, there are no more social interactions, since $x_i = \overline{x}_i$, for all i, and each utility depends only on own productivity. Thus, the equilibrium is always optimal.

In Online Appendix F, we illustrate condition (31) for specific networks. We show that for the equilibrium efforts to be optimal, there needs to be some compensation for the externalities that agents exert on others. In particular, for bipartite networks, such as the star and circular network, the average productivity of the different agents has to be the same, which is very unlikely to be the case.

4.2. Equilibrium versus first-best outcomes in a sufficiently conformist society

Let Corr $(\boldsymbol{\pi}, \boldsymbol{\alpha})$ be the correlation between the productivity distribution $\boldsymbol{\alpha}$ and the degree distribution $\boldsymbol{\pi}$.

Proposition 8 *(First best in a sufficiently conformist society). If* $\text{Corr}(\boldsymbol{\pi}, \boldsymbol{\alpha}) < 0$ ($\text{Corr}(\boldsymbol{\pi}, \boldsymbol{\alpha}) > 0$), *then there exists* $\underline{\lambda} \in (0, 1)$ *such that, in equilibrium, for any* $\lambda > \underline{\lambda}$*, all agents underinvest (overinvest) in effort compared to the first best.*

This result implies that more central agents make higher effort and exert stronger externalities on their neighbors. As a result, all agents overprovide effort. This result is true when λ is sufficiently high since, in that case, externalities to neighbors become very important. For example, in a star network with three agents, we show below that λ does not need to be very high ($\lambda > \underline{\lambda} = 1/2$) for the result in Proposition 8 to hold (see footnote 31).

Remark 3. In a perfectly conformist society,

$$\lim_{\lambda \to 1} x_i^O = \frac{1}{n} \sum_{j=1}^{n} \alpha_j, \qquad \text{for all } i = 1, 2, \ldots, n \tag{33}$$

This result shows that, when the society becomes perfectly conformist, the first-best effort is the same for all agents and does not depend on the position of each agent in the network. All agents should make an effort equal to the average productivity in the network. This implies that, unless the network is regular, the equilibrium in effort is never optimal when λ is sufficiently close to 1.

4.3. Restoring the first best

Let us return to the general case in which λ can take any value and assume that condition (31) does not hold. Then, to restore the first best, the planner can either subsidize or tax efforts. Let S_i^O denote the optimal per-effort subsidy for each agent i, where

$$S_i^O = \frac{\lambda}{(1-\lambda)} \sum_{j \neq i} \widehat{g}_{ji} \left(x_j^O - \overline{x}_j^O \right).$$

If we add one stage before the effort game is played, the planner announces the optimal per-effort subsidy S_i^O for each agent i such that,

$$U_i^{S_i^O} = \left(\alpha_i + S_i^O \right) x_i - \frac{1}{2} x_i^2 - \frac{1}{2} \left(\frac{\lambda}{1-\lambda} \right) (x_i - \overline{x}_i)^2 \tag{34}$$

Observe that, when each agent i chooses x_i that maximizes (34), she takes S_i^O as given, in particular, x_j^O and $\overline{x}_j^O$. In that case, the solution of this maximization problem for each agent i is the first-best.

Proposition 9 *(Subsidies). The first best is restored if the social planner gives to each agent* i *the following tax/subsidy per unit of effort:*

$$S_i^O = \frac{\lambda}{(1-\lambda)} \sum_{j \neq i} \widehat{g}_{ji} \left(x_j^O - \overline{x}_j^O \right) \tag{35}$$

or, in matrix form:

$$\mathbf{S}^O = \frac{\lambda}{(1-\lambda)} \widehat{\mathbf{G}}^T \left(\mathbf{I} - \widehat{\mathbf{G}} \right) \mathbf{x}^O.$$

By doing so, the planner restores the first best and subsidizes (taxes) agents *whose neighbors make efforts above (below) their social norms*. In other words, it is necessary to subsidize agents who exert effort below that of their neighbors and to tax those who exert effort above that of their neighbors.

Let us illustrate this result with an example. Assume a star network in which $n = 3$, and agent $i = 1$ is the star. Set $\alpha_1 = 2$, $\alpha_2 = \alpha_3 = 1$, so that the star is more productive than the peripheral agents are. Since $\alpha_1 = 2 > 1 = (\alpha_2 + \alpha_3)/2$, condition (31) is not satisfied, and hence, the Nash equilibrium is not optimal. We have

$$\mathbf{x}^* = \frac{1}{(1+\lambda)}\begin{pmatrix} 2+\lambda \\ 1+2\lambda \\ 1+2\lambda \end{pmatrix}, \qquad \mathbf{x}^O = \frac{1}{(1+4\lambda)}\begin{pmatrix} 2+5\lambda \\ 1+6\lambda \\ 1+6\lambda \end{pmatrix}.$$

The star agent *overinvests* compared to the first best $\left(x_1^* > x_1^O\right)$. Indeed, since $x_2^* = x_3^* < \overline{x}_2^* = \overline{x}_3^* = x_1^*$, the externality term $\lambda \sum_{j=1}^n \widehat{g}_{ji}\left(x_j - \overline{x}_j\right)$ (see (29)) is *negative* and the star, when deciding her effort level, does not take into account the *negative externalities* she exerts on agents 2 and 3. For the peripheral agents 2 or 3, we obtain $x_2^* = x_3^* \gtreqless x_3^O = x_2^O \iff \lambda \gtreqless 1/2$, so that they may overinvest or underinvest in effort, depending on the value of λ.[31] However, the externality term is always *positive*, since $x_1^* > \overline{x}_1^*$ and thus, agents 2 and 3 always exert *positive externalities* on agent 1. As a result, the planner should tax agent 1 and subsidize agents 2 and 3. Since $x_2 = x_3$, it is easily verified that the subsidies per unit of effort are equal to $S_1^O = \frac{2\lambda}{(1-\lambda)}(x_2^O - x_1^O) < 0$ and $S_2^O = S_3^O = \frac{\lambda}{(1-\lambda)}(x_1^O - x_2^O) > 0$. The subsidies or taxes exactly correct for the externalities exerted by the agents. We obtain:

$$\mathbf{S}^O = \frac{\lambda}{(1+4\lambda)}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \tag{36}$$

Clearly, this result strongly depends on the productivity values. For example, if $\alpha_1 = 0.5$ and $\alpha_2 = \alpha_3 = 1$ so that the productivity of the central agent is the lowest, then, to restore the first best, the planner now needs to subsidize agent 1 (the star) and to tax agents 2 and 3 (the peripheral agents) since, now, the former exerts *positive externalities* on agents 2 and 3 while the latter exert *negative externalities* on agent 1.

5. Extensions

In this section, we develop several extensions of the baseline local-average model. We consider weighted networks, heterogeneous tastes for conformity, anti-conformist attitudes, ambitious behavior, and network formation. These extensions show how various features of individual

[31] Observe that, for the star network with $n = 3$ and $\alpha_1 = 2$, $\alpha_2 = \alpha_3 = 1$, we have

$$\sum_{j=1}^3 \pi_j \alpha_j = \frac{\alpha_1}{2} + \frac{\alpha_2 + \alpha_3}{4} = \frac{3}{2} > \frac{4}{3} = \frac{1}{3}\sum_{j=1}^3 \alpha_j,$$

which means that $Corr(\boldsymbol{\pi}, \boldsymbol{\alpha}) > 0$, since the star has both a higher productivity and a higher degree than the peripheral agents. Thus, our results confirm Proposition 8. When $\lambda > \underline{\lambda} = 1/2$, all three agents in the star network *overinvest* in effort compared to the first best. It is also easily verified that, if we now assume for the same network that $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 2$, then, $\sum_{j=1}^3 \pi_j \alpha_j = \frac{3}{2} < \frac{5}{3} = \frac{1}{3}\sum_{j=1}^3 \alpha_j$, and thus, $Corr(\boldsymbol{\pi}, \boldsymbol{\alpha}) < 0$. In this case, if $\lambda > \underline{\lambda} = 1/2$, all three agents in the star network *underinvest* in effort compared to the first best.

behavior affect our main results and how our model can be applied to a wide range of different contexts.

5.1. Weighted networks

Consider, first, an extension of the baseline local-average model in which the network is *directed*, *weighted*, and may have *self-loops*, as in the standard DeGroot model (Golub and Jackson, 2010). Let $\mathbf{W} = [w_{ij}]$ be an arbitrary $(n \times n)$ row-normalized irreducible matrix with non-negative entries. Each cell w_{ij}, $i, j = 1, 2, \ldots, n$, gives the relative impact (weight) of agent j's effort on agent i's social norm $\overline{x}_i$, defined as follows: $\overline{x}_i \equiv \sum_{j=1}^{n} w_{ij} x_j$. In particular, we do not rule out self loops, that is, we allow for the possibility that $w_{ii} > 0$ for some i. Otherwise, agent i's utility function is the same as in the baseline model and given by (3).

In Online Appendix G.1, we study this more general model with the adjacency matrix $\mathbf{W}$ and show that most of our results (total conformism, comparative statics, and welfare) remain qualitatively the same.

In Proposition G3, we show that a slight change in the network $\mathbf{W}$ may increase everyone's effort if highly productive agents have more impact on everyone's social norms. This echoes our result established in Proposition 6 in which we demonstrate that adding a link in the network may increase everyone's effort if this link is between two highly productive agents. However, in the weighted network model, this result is much easier to prove, because the standard calculus technique can be used to study the consequences of small changes in the weights on outcomes.

5.2. Heterogeneous tastes for conformity

In Online Appendix G.2, we relax the assumption that λ is the same across all agents by allowing each agent i to have a specific taste for conformity λ_i. We first show that our existence, uniqueness, and interiority results when λ is the same for all agents (Proposition 1) are robust to this extension.

Then, Proposition G4 provides additional intuition about the non-monotonicity results of Proposition 5. We show that higher conformity of some agents—namely, those who exert efforts below their social norms—increases everyone's effort because of strategic complementarities, while higher conformity of the others has the opposite effect. Therefore, it is not surprising that the total effect is ambiguous, as Proposition 5 states.

5.3. Anti-conformism

We now consider what happens if agents are *anti-conformist*,[32] that is if the taste for conformity θ is negative. In this case, the magnitude of $|\theta|$ can be viewed as the degree to which an agent wants to be different from the others (although not necessarily better than the others). In other words, each individual obtains a benefit of $\frac{\theta}{2}(x_i - \overline{x}_i)^2$ if she does *not* conform to the norm of her neighbors. This model can still be considered a local-average model but it is now a game with strategic *substitutes* ($\theta < 0$) instead of strategic *complements* ($\theta > 0$).

In Proposition G5 of Online Appendix G.3, we derive our main results for the anti-conformist model. We show that our model can be extended to the case of anti-conformity if agents are

[32] See Bramoullé et al. (2004), Bramoullé (2007), Grabisch and Rusinowska (2010a,b), and Grabisch et al. (2017) for network models with anti-conformist agents but in very different settings.

not too non-conformist ($|\theta| < 1/2$), although it loses a good deal of tractability. In particular, because we have a game with strategic substitutes, even if the equilibrium is unique, it is not always interior.

For example, in the case of a dyad ($n = 2$), in Online Appendix G.3, we show that the two agents exert strictly positive effort only if they are not too heterogeneous in terms of productivities, not too anti-conformist, or both. When this is not the case, then some agent may exert zero effort (see (G.25), which totally characterizes the Nash equilibrium for the dyad network). Indeed, in the anti-conformist model with a dyad network, when the difference in productivity between the two agents is too high, then it becomes optimal for the low-productivity agent to exert zero effort, because she wants to differentiate herself as much as possible from the high-productivity agent (whose effort is her social norm). On the contrary, when the productivity difference is not too large, then the low-productivity agent can still differentiate herself from the high-productivity agent and exert positive effort. This never happens in the conformist model, because agents always want to be as close as possible to each other.[33]

Furthermore, we show that, in contrast to Proposition 2, the impact of α_j on x_i^* is a priori ambiguous. Finally, we demonstrate that if agents are very anti-conformist, there are either multiple equilibria or an equilibrium fails to exist.

5.4. Ambition and social norms

It seems realistic to assume that agents may benefit from choosing an effort that is higher than the average effort of their neighbors. To address this issue, let us extend our utility function (3) so that, for each individual i, it is now given by

$$U_i(x_i, \mathbf{x}_{-i}, \mathbf{g}) = \alpha_i x_i - \frac{x_i^2}{2} - \frac{\theta}{2}(x_i - \beta_i \overline{x}_i)^2,$$

where $\beta_i \geq 1$ is agent i's *ambition factor*. Since $\beta_i \geq 1$, the "reference effort" of each individual i is now higher than the social norm $\overline{x}_i$ of her neighbors.[34] Denote $\beta_{\max} \equiv \max\{\beta_1, \ldots, \beta_n\}$. Then, if $\lambda\beta_{\max} < 1$, there exists a unique interior equilibrium.

To investigate the welfare properties of this model, assume that all agents are ex ante identical, that is, $\alpha_1 = \ldots = \alpha_n = \alpha > 0$, and that they have the same ambition factor, that is, $\beta_1 = \ldots = \beta_n = \beta > 1$. In that case, we show in Appendix G.4, that, in equilibrium, each individual exerts an effort above the social norm (average effort) of their direct friends.

Moreover, for *regular networks*, we show that all agents *overinvest* in equilibrium compared to the social optimum. For *non-regular networks*, we demonstrate (see Proposition G6) that if the agents are either sufficiently conformist (high λ) or sufficiently non-conformist (low λ), they all overinvest in equilibrium compared to the first best. These results contrast with Proposition 7 for the benchmark model, in which the equilibrium is socially optimal when productivities do not vary across agents. This is because ambitious behavior creates an additional positive externality,

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33 Observe that, when all agents have the same productivity α and $|\theta|$ is low enough, then the conformist and anti-conformist models lead to the same outcome, that is, all agents make an effort equal to α. When $|\theta|$ becomes larger, then even with ex ante identical agents and a regular network, in the anti-conformist model, there may be multiple equilibria in which one agent makes a higher effort than the other. See Figure G5 in Online Appendix G.3. This never occurs in the conformist model since, with identical productivities, all agents always make the same effort for any possible network, including the regular one.

34 See Ghiglino and Goyal (2010), who also develop a model in which agents want to consume more than the average consumption (social norm) of their neighbors.

which cannot be fully internalized by individuals even in the absence of any ex ante heterogeneity.

5.5. Network formation

Thus far, we have assumed that the network is fixed and taken as given when agents decide their effort level. Consider now a two-stage game in which, in the first stage, agents create links (endogenous network formation) while, in the second stage, they exert effort.

Assume, for simplicity, that there are two types of agents: high-productivity agents for which $\alpha_i = \alpha^H$ and low-productivity agents for which $\alpha_i = \alpha^L$, with $\alpha^H > \alpha^L > 0$. Assume also that creating or severing a link is costless.

In Proposition G7 in Online Appendix G.5, we show that, in the *local-aggregate model*, the only pairwise Nash equilibrium[35] is the *complete network* in which all agents of any type are linked to each other. On the contrary, in the *local-average model*, the only pairwise Nash equilibrium is a network of two disconnected components; in each component, all agents of the same type form a complete network. This network is called the *completely homophilous network*.

This means that, in the local-aggregate model, there is complete "integration" of the two types of agents while, in the local-average model, there is complete "segregation" of the two types of agents so that *extreme homophily behavior* prevails in equilibrium. In other words, in the local-aggregate model, even if agents are heterogeneous in terms of productivities, complete homophily cannot emerge because, independently of the type, there is always a benefit of forming new links due to strong positive spillovers.

On the contrary, in the local-average model, an agent of one type never wants to form a link with an agent of the other type. Indeed, when agents have the same productivity α, independently of their position in the network, they all exert the same effort level and have the same social norm, both equal to α. As a result, they no longer bear the cost of not conforming to their social norm and their equilibrium utility equals $\alpha^2/2$. However, if an agent forms a link with someone of a different type, she suffers an extra *loss*, because a gap between her effort and her social norm emerges. For this reason, in the local-average model, agents of one type are better off not having links with agents of the other type. Using the same reasoning, one can show that, if we introduce a cost of forming and severing links, we still have the same pattern, that is, complete homophily or segregation in the local-average model, and integration and heterophily in the local-aggregate model, but there may be more than one equilibrium. Furthermore, we can easily generalize our results to more than two types of agents.

6. Policy implications: local-average versus local-aggregate model

As stated in the Introduction, there are two main models of games on networks with positive peer effects (strategic complementarities): the local-average and the local-aggregate model. In the local-average model, deviating from the average effort of one's peers negatively affects the utility of an individual (see (3)). The closer each individual's effort is to the average of her friends' efforts, the higher is her utility. By contrast, in the local aggregate model, the sum of the efforts of an individual's peers positively affects the utility of each individual (see (4)). When peers exert more effort, the utility derived from own effort increases.

[35] For a precise definition of pairwise Nash equilibrium, see Bloch and Jackson (2006).

We believe that it is important to be able to disentangle different behavioral peer-effect models because, even if they look very similar, they have different policy implications. To highlight these differences between the models, we consider in the next subsection education and crime and observe how these two models yield different policy implications.

6.1. Policy implications: education

In terms of education, if the local-aggregate model describes well the preferences of students (Calvó-Armengol et al., 2009), then any individual-based policy, such as *vouchers*, would be efficient, because if one or more "key" students (e.g., the disruptive ones) are positively affected by the policy, because of peer effects (social multiplier), many other students are also positively affected. If, on the contrary, we believe that the local-average model describes students' preferences more adequately, then we should change the social norm in the school or classroom (group-based policy) and attempt to implement the idea that it is "cool" to work hard at school. Affecting a few students will not change anything if it does not change the social norm in the school.

An example of an educational policy that has attempted to change the social norm of students is the *charter-school policy*. Charter schools are very good at screening teachers and selecting the best ones. In particular, the "No Excuses policy" (Angrist et al., 2010, 2012) is a highly standardized and widely replicated charter model that features a long school day, an extended school year, selective teacher hiring, and strict behavioral norms, while it emphasizes traditional reading and math skills. The main objective is to change the social norms of disadvantaged children by being very strict on discipline. This is a typical policy that is in accordance with the local-average model, since its aim is to change the social norms of students in terms of education. Angrist et al. (2012) focus on special needs students who may be underserved. The study's results show average achievement gains of 0.36 standard deviations in math and 0.12 standard deviations in reading for each year spent at a charter school called the Knowledge is Power Program (KIPP) Lynn, with the largest gains coming from the Limited English Proficient (LEP), Special Education (SPED), and low-achievement groups. The authors show that the average reading gains were driven almost entirely by SPED and LEP students, whose reading scores rose by roughly 0.35 standard deviations for each year spent at KIPP Lynn.[36]

In summary, an effective policy for the local-average model would be to change people's perceptions of "normal" behavior (i.e., their social norm) so that a *school-based policy* could be implemented. Meanwhile, for the local-aggregate model, this would not be necessary and an *individual-based policy* should instead be implemented.

6.2. Policy implications: crime

It is well documented that crime is, to a large extent, a group phenomenon, and the source of crime is located in the intimate social networks of individuals (see, e.g., Warr, 2002; Bayer et al., 2009; Damm and Dustmann, 2014).

[36] See also Curto and Fryer (2014), who study the SEED schools, which are boarding schools serving disadvantaged students located in Washington DC and Maryland. The SEED schools, which combine a "No Excuses" charter model with a 5-day-a-week boarding program, are the United States' only urban public boarding schools for the poor for students in grades 6–12. Using admission lotteries, Curto and Fryer (2014) show that attending a SEED school increases achievement by 0.211 standard deviation in reading and by 0.229 standard deviation in math per year.

In the local-aggregate model, a *key-player policy* (Ballester et al., 2006; Zenou, 2016; Lee et al., 2018), whose aim is to remove the criminal that reduces total crime in a network the most, would be the most effective way of reducing total crime.[37] In other words, the removal of the key player can have large effects on crime because of the feedback effects or "social multipliers" at work. Indeed, as the proportion of individuals participating in criminal behavior increases, the impact on others is multiplied through social networks. Thus, criminal behavior can be magnified, and interventions can become more effective.

On the contrary, a key-player policy would have nearly no effect in the local-average model, since it would not affect the social norm that committing crime is morally wrong. To be effective, one would have to change the norm for each of the criminals, which is clearly a more difficult objective. In that case, it is necessary to target a group or gang of criminals to reduce crime drastically. This illustrates the fact that, for the local-aggregate model, *individual-based policies* are more appropriate while, for the local-average model, *group-based policies* are more effective.[38]

6.3. *Which model is the most empirically relevant?*

Which model is relevant is clearly an empirical question. To statistically identify whether the average model or the aggregate model is more appropriate for a particular outcome, Liu et al. (2014) proposed the following methodology. It is necessary to estimate an augmented model, which includes both average and aggregate peer effects, and to determine which one is statistically significant. Using data for the National Longitudinal Study of Adolescent to Adult Health (Add Health), Liu et al. (2014) showed that, for study effort in education, the endogenous peer effect is mostly captured by a social-conformity (local average) effect rather than a social-multiplier (local aggregate) effect. This implies that a charter-school policy that aims to change the social norms of students (as in Angrist et al., 2010, 2012) would be the most effective policy to improve education in schools. On the other hand, for sport activities, Liu et al. (2014) found that both social-conformity and social-multiplier effects contribute to the endogenous peer effect. Moreover, Lee et al. (2018), who studied juvenile delinquency, showed that the local-aggregate model is at work for the AddHealth data. This implies that a key-player policy would be the most effective policy to reduce crime for adolescents in the United States.

6.4. *An illustrative example*

Let us illustrate the above discussion about individual versus group-based policy with a simple example. Consider the network **g** in Fig. 2 with $n = 11$ players. This network was considered by Ballester et al. (2006) to illustrate their formula of the key player. In this network, player 1 bridges together two fully intra-connected groups with five players each.

6.4.1. *An individual-based policy: key player*

Consider a network-crime model in which agents choose crime effort that maximizes their utility, which can be based on either the local-aggregate or the local-average model. As an illustration of an individual-based policy, we consider the key player policy, which consists of

[37] In Section 3.3, we also discuss the difference between the local-average and local-aggregate models in terms of the *key-link* policy, whose aim is to choose how to optimally remove a link between two criminals in order to minimize the total crime level in a network. See, in particular, Remark 1 and the discussion that follows.

[38] For recent overviews on place-based policies, see Kline and Moretti (2014) and Neumark and Simpson (2015).

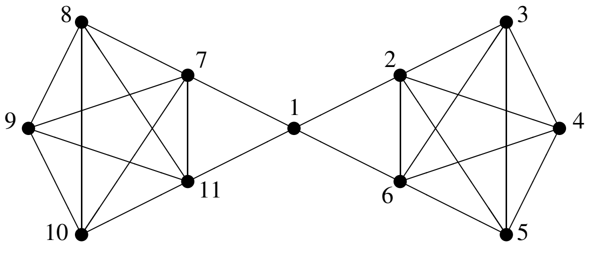

Fig. 2. A bridge network.

determining the player who, once removed from the network, reduces total crime effort the most. To make the comparison between the two models easier, we assume that agents are ex ante identical, that is, $\alpha_i = 1$, for all $i = 1, \dots n$. We also assume that $\theta = 0.2$.

The local-aggregate model Consider the local-aggregate model whereby the utility function is given by (4). Then, if $\theta < 1/\mu(\mathbf{g})$ (where $\mu(\mathbf{g})$ is the largest eigenvalue of $\mathbf{g}$),[39] then a unique Nash equilibrium in efforts exists, which is equal to:

$$\mathbf{x}^* = (\mathbf{I} - \theta \mathbf{G})^{-1}\,\mathbf{1}$$

It is easily verified that the key player is agent 1 (Ballester et al., 2006). In particular, the total crime effort in equilibrium is equal to 91.67 while, after the removal of individual 1, it is 50. Thus, the removal of player 1 leads to a decrease of total crime activity by 45.46%. This is because removing player 1 disrupts the network and leads to two different networks that are no longer connected. The change in efforts after the removal of agent 1 varies a lot depending on the position in the network. For example, agent 2, who was directly linked to 1, reduces her effort from 9.17 to 5 (45.47% reduction) while agent 3, who was two links away from 1, decreases her effort from 7.78 to 5 (35.73% reduction).

The local-average model Consider now the local-average model in which the utility function is given by (3). We have shown that the Nash equilibrium is given by

$$\mathbf{x}^* = \widehat{\mathbf{M}}\boldsymbol{\alpha} = \frac{1}{(1+\theta)}\left(\mathbf{I} - \frac{\theta}{(1+\theta)}\mathbf{G}\right)^{-1}\mathbf{1}$$

It is easily verified that all agents make the same effort level equal to 1 (which is the social norm) so that total crime effort is 11. Let us remove player 1 (or in fact any other player) and renormalize the resulting adjacency matrix. It is easily checked that nothing changes since each player still makes an effort of 1 and the social norm is exactly the same and equal to 1. Because there is one less player in the network, the total effort is now given by 10 and the reduction in total crime is then equal to 9.09%.

In summary, an individual policy, such as the key player, has a big impact on total crime when the preferences of agents are based on the local-aggregate model while it has nearly no impact when the preferences are based on the local-average model. As a result, if the planner believes that the agents have preferences according to the local-aggregate model and implements a key-player policy while, in fact, agents have local-average preferences, then this example shows that this policy will fail to reduce crime, as agents will not change their criminal behavior.

[39] This condition is verified for the network displayed in Fig. 2, since $\theta = 0.2 < 0.227 = 1/\mu(\mathbf{g})$.

6.4.2. A group-based policy: changing the norm

Consider again the network **g** in Fig. 2 and implement a group-based policy, which is common to everybody. For example, consider a reduction of α from 1 to 0.7. All agents in the network are affected in the same way.

The local-aggregate model By implementing such a policy, it is easily verified that total crime effort decreases from 91.67 (before the policy) to 64.17 (after the policy), giving a reduction in total crime of 30%.

The local-average model In this model, the effort and social norm change for all agents in the network. It is easily verified that all agents now reduce their crime effort to 0.7 and the social norm is now given by 0.7. As a result, we switch from a total crime effort of 11 (before the policy) to 7.7 (after the policy), that is, a reduction in total crime of 30%. In other words, changing the social norm from 1 to 0.7 now has a large impact on total crime in the network.

In summary, a group-based policy, such as changing the social norm by reducing the productivity of all agents in the network, has a much bigger impact on total crime when the preferences of agents are based on the local-average model. However, a group-based policy is less efficient when the agent's preferences are based on the local-aggregate model. Again, if the planner has the wrong beliefs about agents' preferences, then the impact of a group-based policy on reducing crime may be limited.

7. Concluding remarks

In this study, we analyze the linear-in-means model (also known as the local-average model in the network literature), which is the workhorse model in empirical work on peer effects. Apart from their position in the network, agents are heterogeneous in terms of productivity. We characterize the Nash equilibrium in efforts of this game in which each agent minimizes the social distance between her own effort and that of her peers (her own social norm). While individual productivity positively affects equilibrium effort, the impact of taste for conformity is non-monotone. Both the sign and the magnitude of this conformity effect depend on whether an individual is above or below her own social norm. We also study how adding or removing a link affects the aggregate effort in the network and show that it depends on the productivity of the agents involved in the link. Equilibria are usually inefficient and we provide a condition on the productivity distribution and the network structure that guarantees the efficiency of equilibrium. Because this condition often fails to hold, we show how to restore the first best. Unexpectedly, the optimal taxation/subsidy scheme is to subsidize agents whose peers would exert efforts above their social norms while taxing agents whose peers would exert efforts below their social norms. Hence, the planner does not necessarily subsidize central agents, as is the case in the local-aggregate model.

More generally, we consider our framework to be rich enough to encompass many real-world situations in which people are conformist and dislike to deviate from the social norms of their friends. We also believe that our results lead to important policy implications that can be tested empirically. In particular, we shed light on the debate on whether individual-based policies are more effective in maximizing welfare or minimizing total activity (in the case of crime) than group- or place-based policies.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2019.104969.

References

Akerlof, G.A., 1980. A theory of social custom of which unemployment may be one consequence. Q. J. Econ. 94, 749–775.

Akerlof, G., 1997. Social distance and social decisions. Econometrica 65, 1005–1027.

Allouch, N., 2015. On the private provision of public goods on networks. J. Econ. Theory 157, 527–552.

Alm, J., Bloomquist, K.M., McKee, M., 2017. When you know your neighbour pays taxes: information, peer effects, and tax compliance. Fisc. Stud. 38, 587–613.

Angrist, J.D., Dynarski, S.M., Kane, T.J., Pathak, P.A., Walters, C.R., 2010. Inputs and impacts in charter schools: KIPP Lynn. Am. Econ. Rev. Pap. Proc. 100, 239–243.

Angrist, J.D., Dynarski, S.M., Kane, T.J., Pathak, P.A., Walters, C.R., 2012. Who benefits from KIPP? J. Policy Anal. Manage. 31, 837–860.

Ballester, C., Calvó-Armengol, A., Zenou, Y., 2006. Who's who in networks. Wanted: the key player. Econometrica 74, 1403–1417.

Ballester, C., Calvó-Armengol, A., Zenou, Y., 2010. Delinquent networks. J. Eur. Econ. Assoc. 8, 34–61.

Banerjee, A.V., Chandrasekhar, A.G., Duflo, E., Jackson, M.O., 2013. The diffusion of microfinance. Science 341 (6144).

Bayer, P., Hjalmarsson, R., Pozen, D., 2009. Building criminal capital behind bars: peer effects in juvenile corrections. Q. J. Econ. 124, 105–147.

Bernheim, B.D., 1994. A theory of conformity. J. Polit. Econ. 102, 841–877.

Bloch, F., Jackson, M.O., 2006. Definitions of equilibrium in network formation games. Int. J. Game Theory 34, 305–318.

Bloch, F., Quérou, N., 2013. Pricing in social networks. Games Econ. Behav. 80, 243–261.

Blume, L.E., Brock, W.A., Durlauf, S.N., Jayaraman, R., 2015. Linear social interactions models. J. Polit. Econ. 123, 444–496.

Boucher, V., 2016. Conformism and self-selection in social networks. J. Public Econ. 136, 30–44.

Boucher, V., Fortin, B., 2016. Some challenges in the empirics of the effects of networks. In: Bramoullé, Y., Galeotti, A., Rogers, B.W. (Eds.), The Oxford Handbook on the Economics of Networks. Oxford University Press, Oxford, pp. 277–302.

Bramoullé, Y., 2007. Anti-coordination and social interactions. Games Econ. Behav. 58, 30–49.

Bramoullé, Y., Djebbari, H., Fortin, B., 2009. Identification of peer effects through social networks. J. Econom. 150, 41–55.

Bramoullé, Y., Kranton, R., 2007. Public goods in networks. J. Econ. Theory 135, 478–494.

Bramoullé, Y., Kranton, R., 2016. Games played on networks. In: Bramoullé, Y., Rogers, B.W., Galeotti, A. (Eds.), The Oxford Handbook of the Economics of Networks. Oxford University Press, Oxford.

Bramoullé, Y., Kranton, R., D'Amours, M., 2014. Strategic interaction and networks. Am. Econ. Rev. 104, 898–930.

Bramoullé, Y., López-Pintado, D., Goyal, S., Vega-Redondo, F., 2004. Network formation and anti-coordination games. Int. J. Game Theory 33, 1–19.

Brekke, K.A., Kipperberg, G., Nyborg, K., 2010. Social interaction in responsibility ascription: the case of household recycling. Land Econ. 86, 766–784.

Breza, E.L., 2016. Field experiments, social networks, and development. In: Bramoullé, Y., Rogers, B.W., Galeotti, A. (Eds.), The Oxford Handbook of the Economics of Networks. Oxford University Press, Oxford, pp. 412–439.

Calvó-Armengol, A., Patacchini, E., Zenou, Y., 2009. Peer effects and social networks in education. Rev. Econ. Stud. 76, 1239–1267.

Carrell, S.E., Sacerdote, B.I., West, J.E., 2013. From natural variation to optimal policy? The importance of endogenous peer group formation. Econometrica 81, 855–882.

Chen, Y.J., Zenou, Y., Zhou, J., 2018. Competitive pricing strategies in social networks. Rand J. Econ. 49, 672–705.

Christakis, N.A., Fowler, J.H., 2007. The spread of obesity in a large social network over 32 years. N. Engl. J. Med. 357, 370–379.

Clark, A.E., Loheac, Y., 2007. It wasn't me, it was them! Social influence in risky behaviour by adolescents. J. Health Econ. 26, 763–784.

Curto, V., Fryer, R., 2014. The potential of urban boarding schools for the poor: evidence from SEED. J. Labor Econ. 32, 65–93.

Czajkowski, M., Hanley, N., Nyborg, K., 2017. Social norms, morals and self-interest as determinants of pro-environment behaviours: the case of household recycling. Environ. Resour. Econ. 66, 647–670.

Damm, A.P., Dustmann, C., 2014. Does growing up in a high crime neighborhood affect youth criminal behavior? Am. Econ. Rev. 104, 1806–1832.

De Marti, J., Zenou, Y., 2015. Networks games under incomplete information. J. Math. Econ. 61, 221–240.

Dutta, R., Levine, D.K., Modica, S., 2019. Interventions with sticky social norms: a critique. McGill University. Unpublished manuscript.
Epple, D., Romano, R., 2011. Peer effects in education: a survey of the theory and evidence. In: Benhabib, J., Bisin, A., Jackson, M.O. (Eds.), Handbook of Social Economics, vol. 1B. Elsevier, Amsterdam, pp. 1053–1163.
Fershtman, C., Weiss, Y., 1998. Social rewards, externalities and stable preferences. J. Public Econ. 70, 53–73.
Fortin, B., Lacroix, G., Villeval, M.-C., 2007. Tax evasion and social interactions. J. Public Econ. 91, 2089–2112.
Ghiglino, C., Goyal, S., 2010. Keeping up with the neighbors: social interaction in a market economy. J. Eur. Econ. Assoc. 8, 90–119.
Golub, B., Jackson, M.O., 2010. Naïve learning in social networks and the wisdom of crowds. Am. Econ. J. Microecon. 2, 112–149.
Golub, B., Morris, S., 2017. Expectations, networks, and conventions. Harvard University. Unpublished manuscript.
Golub, B., Sadler, E., 2016. Learning in social networks. In: Bramoullé, Y., Galeotti, A., Rogers, B.W. (Eds.), The Oxford Handbook on the Economics of Networks. Oxford University Press, Oxford, pp. 504–542.
Grabisch, M., Poindron, A., Rusinowska, A., 2017. A Model of Anonymous Influence with Anti-Conformist Agents. CES Working Paper No. 2017.47.
Grabisch, M., Rusinowska, A., 2010a. A model of influence in a social network. Theory Decis. 69, 69–96.
Grabisch, M., Rusinowska, A., b. A model of influence with an ordered set of possible actions. Theory Decis. 69, 635–656.
Hamilton, J.D., 1994. Time Series Analysis. Princeton University Press, Princeton.
Helsley, R.W., Zenou, Y., 2014. Social networks and interactions in cities. J. Econ. Theory 150, 426–466.
Herbst, D., Mas, A., 2015. Peer effects in worker output in the laboratory generalize to the field. Science 350, 6260.
Horn, R.A., Johnson, C.R., 1985. Matrix Analysis. Cambridge University Press, Cambridge.
Hsieh, C.-S., Lin, X., 2017. Gender and racial peer effects with endogenous network formation. Reg. Sci. Urban Econ. 67, 135–147.
Ioannides, Y.M., 2012. From Neighborhoods to Nations: The Economics of Social Interactions. Princeton University Press, Princeton.
Jackson, M.O., 2008. Social and Economic Networks. Princeton University Press, Princeton.
Jackson, M.O., Rogers, B., Zenou, Y., 2017. The economic consequences of social network structure. J. Econ. Lit. 55, 49–95.
Jackson, M.O., Zenou, Y., 2015. Games on networks. In: Young, P., Zamir, S. (Eds.), Handbook of Game Theory, vol. 4. Elsevier Publisher, Amsterdam, pp. 91–157.
Kandel, E., Lazear, E.P., 1992. Peer pressure and partnerships. J. Polit. Econ. 100, 801–817.
Kline, B., Tamer, E., 2019. Econometric analysis of models with social interactions. In: Graham, B., De Paula, A. (Eds.), The Econometric Analysis of Network Data. Elsevier, Amsterdam. Forthcoming.
Kline, P., Moretti, E., 2014. People, places, and public policy: some simple welfare economics of local economic development programs. Annu. Rev. Econ. 6, 629–662.
Kling, J.R., Liebman, J.B., Katz, L.F., 2007. Experimental analysis of neighborhood effects. Econometrica 75, 83–119.
Kling, J.R., Ludwig, J., Katz, L.F., 2005. Neighborhood effects on crime for female and male youth: evidence from a randomized housing voucher experiment. Q. J. Econ. 120, 87–130.
Lee, L.-F., Liu, X., Patacchini, E., Zenou, Y., 2018. Who is the key player? A network analysis of juvenile delinquency. Monash University. Unpublished manuscript.
LeSage, J.P., Pace, R.K., 2009. Introduction to Spatial Econometrics. CRC Press, Boca Raton.
Lindquist, M.J., Zenou, Y., 2019. Crime and networks: 10 policy lessons. Oxf. Rev. Econ. Policy. Forthcoming.
Liu, X., Patacchini, E., Zenou, Y., 2014. Endogenous peer effects: local aggregate or local average? J. Econ. Behav. Organ. 103, 39–59.
Ludwig, J., Duncan, G.J., Hirschfield, P., 2001. Urban poverty and juvenile crime: evidence from a randomized housing-mobility experiment. Q. J. Econ. 116, 655–679.
Manski, C., 1993. Identification of endogenous social effects: The reflection problem. Rev. Econ. Stud. 60, 531–542.
Neumark, D., Simpson, H., 2015. Place-based policies. In: Duranton, G., Henderson, V., Strange, W. (Eds.), Handbook of Regional and Urban Economics, vol. 5B. Elsevier Publisher, Amsterdam, pp. 1197–1288.
Olcina, G., Panebianco, F., Zenou, Y., 2017. Conformism, Social Norms and the Dynamics of Assimilation. CEPR Discussion Paper No. 12166.
Patacchini, E., Zenou, Y., 2012. Juvenile delinquency and conformism. J. Law Econ. Organ. 28, 1–31.
Sacerdote, B.I., 2011. Peer effects in education: how might they work, how big are they and how much do we know thus far? In: Hanushek, E., Machin, S., Woessmann, L. (Eds.), Handbook of the Economics of Education. Elsevier, Dordrecht, pp. 249–277.

Topa, G., Zenou, Y., 2015. Neighborhood and network effects. In: Duranton, G., Henderson, V., Strange, W. (Eds.), Handbook of Regional and Urban Economics, vol. 5A. Elsevier, Amsterdam, pp. 561–624.

Warr, M., 2002. Companions in Crime: The Social Aspects of Criminal Conduct. Cambridge University Press, Cambridge.

Zenou, Y., 2016. Key players. In: Bramoullé, Y., Rogers, B.W., Galeotti, A. (Eds.), Oxford Handbook on the Economics of Networks. Oxford University Press, Oxford, pp. 244–274.