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INFORMAL MATHEMATICS ACTIVITIES AND THE BELIEFS OF ELEMENTARY TEACHER CANDIDATES

Matt B. Roscoe
The University of Montana

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By

Matthew Brady Roscoe

Masters of Education, University of Montana, Missoula, Montana, 2001
Bachelor of Science, University of Notre Dame, Notre Dame, Indiana, 1993

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Approved by:

Perry Brown, Associate Provost for Graduate Education
Graduate School

Dr. Bharath Sriraman, Chair
Mathematical Sciences

Dr. Albert Borgmann
Philosophy

Dr. Jon Graham
Mathematical Sciences

Dr. James Hirstein
Mathematical Sciences

Dr. Ke Norman
Mathematical Sciences

Roscoe, Matt B., Ph. D., May 2011 Mathematics Education

Informal Mathematics Activities and the Beliefs of Elementary Teacher Candidates

Chairperson: Dr. Bharath Sriraman

Researchers have identified the important role that beliefs about mathematics play in instructional decision making (i.e. Ernest, 1988; Schoenfeld, 1985). Given the central role that beliefs play in the classroom it follows that an element of preservice teacher education should concern itself with the development of beliefs that facilitate the learning of mathematics. Of particular concern are the beliefs of preservice teachers that characterize the subject as purely formal (procedural) while neglecting the informal (process-oriented) aspects of the science (Ball, 1990; Ernest, 1988; Skemp, 1978). This study sought to determine the relationship between participation in informal mathematics activities and the formal-to-informal beliefs of university teacher candidates in elementary education. Three classes of preservice teachers participated in the study through their enrollment in a content mathematics course for elementary education majors. Four informal mathematics activities were employed as part of the course requirements. Pre and post formal-to-informal beliefs about mathematics and mathematics instruction were measured using a Likert-scale beliefs assessment instrument used by Collier (1972) and Seaman et al. (2005). Changes in beliefs about mathematics and mathematics instruction were compared to a control group. Student reflection upon personal experience derived from participation in the activities was analyzed for formal and informal belief statements.

DEDICATION

This work is dedicated to Ruby.

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CHAPTER 1: INTRODUCTION

> One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential to it. (Hersh, 1986)

Mathematics teachers are routinely required to make decisions about how to proceed towards instructional goals. Educational activities are chosen by the teacher and a lesson plan is constructed which guides actions carried out in the classroom. When asked to describe the factors that influence instructional decisions, teachers often rely upon belief statements about mathematics and mathematics instruction (i.e. Lerman, 1983; Thompson, 1984; Steinberg et al, 1985; Kuhs and Ball, 1986; Raymond, 1997; Sztajn, 2003). Elements are chosen or omitted for the lesson according to teachers' beliefs about their relative importance. These elements are then taught to the class in a manner that is informed by teachers' beliefs regarding appropriate methods of educational presentation. Hersh's (1986) quote makes clear: the act of teaching mathematics is an expression of a teacher's beliefs about mathematics and mathematics instruction.

Students in mathematics classes learn from their teachers. They learn the lessons arithmetic, geometry, algebra, statistics, calculus and the like. The content and the delivery of these lessons are shaped by their teachers' beliefs. Through teachers' actions or omissions, students also learn lessons about what the teacher *believes* to be the fundamental aspects of the science. What is mathematics? How is mathematics learned? Why is mathematics important? Where is mathematics used? Who should learn mathematics? These lessons provide the student with a sense of utility, motivation, purpose and meaning in the pursuit of mathematical knowledge. And so it seems clear that what a student learns in school about mathematics, in terms of both its content and its essential qualities, is fundamentally shaped by teachers' beliefs about the subject and her beliefs about its proper instruction.

Universities prepare individuals to teach mathematics. A curriculum is developed which consists of content and methodology courses to provide the prospective teacher with the skills and knowledge which society has come to expect in those who would educate our children. Given the significant role that beliefs about mathematics and mathematics instruction play in the classroom it follows that an element of preservice teacher education should concern itself with the development of beliefs that facilitate the learning of mathematics with understanding.

Alba Thompson (1992), in her synthesis of research on teacher beliefs, points out that there is no "universal agreement on what constitutes 'good mathematics teaching'" (p. 127). Indeed, the debate over what comprises good mathematics teaching has risen to such heights to earn the label "the math wars" in the state of California (for an account see Wilson, 2003). Many have pointed out that the debate over "good mathematics teaching" rests upon opposing beliefs about the subject (i.e. Lerman, 1983; Ernest, 1988; Kuhs and Ball, 1986; Skemp, 1987; Torner, 2002). One way in which scholars (i.e. Collier, 1972; Seaman et al. 2005) have differentiated two sides in the debate over good mathematics teaching is through a formal-informal characterization of the subject.

On the one side there are those that advocate for a *formal* presentation of mathematics. Here, the distinguishing characteristics of the science are its well-known rules and procedures which empower the user in quantitative settings: the familiar algorithms of long division, the Pythagorean Theorem, the quadratic formula, and

L'Hospital's Rule. Not surprisingly, those who advocate for a formal presentation of mathematics propose a teacher-centered educational setting in which knowledge of mathematics is passed from teacher to student through traditional lecture reinforced through drill and practice. Student knowledge in mathematics is then envisioned as the possession and accurate application of these procedures.

On the other side of the debate are those that advocate for an *informal* presentation of mathematics emphasizing both creative and investigative features of the science. Here, the central characteristics of the subject are the processes through which mathematics is constructed: proof, logical reasoning, multiple representations, connections, communication and problem solving. Those who advocate for an informal presentation of the subject propose a student-centered classroom environment in which students are encouraged to explore, investigate and make conjectures about mathematical objects en route to a connected conceptual understanding of mathematical structure. Student knowledge in mathematics is then envisioned according to one's ability to actively engage in these creative and investigative processes thereby demonstrating a conceptual understanding of the topic.

From a historical standpoint, this debate is not at all a new one. In a paper presented to the Assistant Master's Society in 1830, Walter Coburn writes:

> By the old system the learner was presented with a rule, which told him how to perform certain operations on figures, and when they were done he would have the proper result. But no reason was given for a single step. . . . And when [the learner] had got through and obtained the result, he understood neither what it was nor the use of it. Neither did he know that it was the proper result, but was obliged to rely wholly on the book, or more frequently on the teacher. As he began in the dark, so he continued; and the results of his calculation seemed to be obtained by some magical operation rather than by the inductions of reason. (quoted in Wilson, 2003, p. 9)

Further evidence of the historical debate is found in the words of English mathematician and philosopher Alfred Lord Whitehead, who, in 1911, offered his critique of the school mathematics problem:

> The reason for this failure of [mathematics] to live up to its reputation is that its fundamental ideas are not explained to the student disentangled from the technical procedure which has been invented to facilitate their exact presentation in particular instances. Accordingly, the unfortunate learner finds himself struggling to acquire a knowledge of a mass of details which are not illuminated by any general conception. Without a doubt, technical facility is a first requisite for valuable mental activity: we shall fail to appreciate the rhythm of Milton, or the passion of Shelley, so long as we find it necessary to spell the words and are not quite certain of the forms of the individual letters. In this sense, there is no royal road to learning. But it is equally an error to confine attention to technical processes, excluding consideration of general ideas. (p. 8)

The debate continued in the 1920s and 1930s as progressive education advocates, most notably John Dewey, posited that “traditional education” through its strict authoritarian approach placed too much emphasis on the rote transmission of knowledge and not enough emphasis on student understanding. Highly critical of the cultural uniformity that most schools of the day promoted, the progressive platform proposed a “child-centered” (i.e. informal) agenda where students would acquire critical and socially engaged intelligences through generative and creative processes (Dewey, 1938).

The launch of the Soviet built Sputnik space module in 1957 elevated the debate to a national level and motivated large-scale curricular reforms in mathematics. The event gave rise to a fear that American educational deficiencies in science, technology, engineering and mathematics might allow for Soviet world domination. Congress responded to this fear in 1958 by passing the National Defense of Education Act which allocated funds to address the perceived shortfalls. In the same year, the School Mathematics Study Group (SMSG) was formed by the American Mathematical Society (AMS), the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) to develop a new curriculum for elementary and secondary schools in America. The new curriculum that the SMSG proposed would come to be known as the *New Math* movement.

Speaking generally, the New Math curriculum advocated for a more informal approach to the subject, placing greater emphasis on mathematical structure through the incorporation of new content, most notably axiomatic set theory but also alternate bases and functions. These "new" elements were all introduced at a young age in an effort to facilitate greater understanding of mathematical systems which would be presented later (Klein, 2003). In addition to changes in content there were also changes in approach. The movement advocated for lessons which incorporated more exploration and student-led discovery in lieu of memorization and teacher-led lecture (Wilson, 2003).

The New Math curriculum was perhaps the first to recognize the importance of *both* formal and informal elements in mathematics instruction. As Ed Begle, head of the

SMSG made clear, teaching that "emphasizes understanding without neglecting the basic skills is best for all students" (quoted in Wilson, 2003, p. 13). And, for a short ten-year period, the movement was successful. New district and state curricula were written that reflected the principles of the movement. School textbooks soon followed. Private and federal funds were also made available for the retraining of teachers. But, enthusiasm in the new approach waned towards the start of the 1970s fueled in part by the frustrations of teachers and parents as well as by characterizations of the approach as unfounded and poorly implemented, often by those in the mathematics community (i.e. Kline, 1973; Goodlad et al., 1970; Sarason, 1971). By the mid 1970s the New Math movement had lost nearly all of its original momentum and a period of "back to the basics" (i.e. formal approaches) in mathematics education was ushered in.

In 1983, The National Commission on Excellence in Education's report *A Nation at Risk* once again fueled the debate surrounding mathematics education. The report documented the decline of American educational standards and urged reforms in order to maintain "American prosperity, security, and civility" (p. 8). In the report the authors noted that:

> Some worry that schools may emphasize such rudiments as reading and computation at the expense of other essential skills such as comprehension, analysis, solving problems, and drawing conclusions. (A Nation At Risk, 1983, p. 12)

The document served as a catalyst in the mathematics education community that would ultimately lead to a national standards movement in mathematics.

In 1989, the National Council of Teachers of Mathematics (NCTM) released *Curriculum and Evaluation Standards for School Mathematics* and soon thereafter *Professional Standards for Teaching Mathematics* (1991). These documents called for restructuring reforms in mathematics content and pedagogy grounded in the learning theory of constructivism. Von Glaserfeld (1989) described the two driving tenets of constructivism as:

> 1. Knowledge is not passively received by the senses; rather, it is actively built up by cognizing the subject
>
> 2. The function of cognition is adaptive and serves to create meaning and to organize the experiential world; tending towards goodness of fit or viability (Von Glaserfeld, 1989)

Grounded in constructivist learning theory, the NCTM *Standards* (1989) called for reform in mathematical content as well as pedagogy. The movement advocated a vision of school mathematics that diverged from strictly formal notions of the subject and advocated for a more informal approach whereby students actively construct their knowledge of mathematics through inquiry-based, student-centered investigation of the subject.

This vision of restructuring reform was further refined in NCTM's *Principles and Standards* (2000) where we find the following:

> Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills…Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge…By having students talk about their informal strategies, teachers can help them become aware of, and build on, their implicit informal knowledge…Moreover, in such settings, procedural fluency and conceptual understanding can be developed through problem solving, reasoning, and argumentation. (p. 21)

NCTM's vision of school mathematics is one which advocates an informal approach to the subject where students "actively engage", "conjecture", "reason", and "evaluate their own thinking". Instruction is inherently student-centered; teachers are called to facilitate activities where students can build their own knowledge through active investigation. The authors are quick to point out that "conceptual understanding" and "procedural fluency" can *both* be achieved through informal approaches to mathematics education. This union of both formal and informal aspects of mathematics is further evidenced by the fact that, of the ten standards for school mathematics that the document proposes, five standards are allocated to "content" and another five standards are allocated to "process". NCTM's (2000) vision of mathematics education is consistent with the beliefs of the researcher and the research conducted herein.

In spite of the long historical struggle to address American school children's conceptual knowledge of mathematics through the inclusion of informal approaches to the subject, recent research has shown that little change has occurred in American classrooms (i.e. Ball, 1990; Ma, 1999; Heibert et al., 2003). This fact is much lamented by mathematics teacher educators working towards reforms in mathematics education which are consistent with NCTM's *Principles and Standards* (2000).

For example, in a study of principal importance to this research, Seaman et al. (2005) found in their replication of Collier's (1972) study that, in spite of prominent national educational reforms (i.e. NCTM (1989), NCTM (1991)), prospective elementary school teachers in 1998 held only slightly more informal views than their 1968 counterparts and continued to significantly focus on "memorized rules, formulas, and procedures" (p. 206). Recognizing that teacher candidates arrive at the university with well-formed beliefs about mathematics as a formal subject through their 12 years of previous schooling, the authors noted:

> Although teacher education can prompt students to adopt beliefs more aligned with a constructivist learning theory, they must also explicitly challenge student's existing beliefs about mathematics as an authoritarian discipline (p.206).

Seaman et al. (2005) conclude with a call for teacher education programs to provide students with sufficient opportunity to reflect upon their assumptions regarding the nature of mathematics as a subject as well as "good mathematics teaching" and to compare their beliefs against "new ideas" such as those proposed by the *Principles and Standards* (NCTM, 2000).

This call for activities in teacher education that challenge teacher candidates' formal beliefs about mathematics education is a consistent theme in the literature (i.e. Skemp, 1987; Ernest, 1988; Ball, 1990; Grant, Hiebert and Wearne, 1998; Cooney, 1999) and is the theme of this research. In particular, the following research questions are investigated:

1. What is the relationship between participation in informal mathematics activities and the formal-to-informal beliefs of university teacher candidates in elementary education?
2. Does reflection upon personal experience derived from participation in informal mathematics activities reveal any transformation of the formal-to-informal beliefs of university teacher candidates in elementary education?
3. What is the value of informal mathematics activities in elementary teacher education?

CHAPTER 2: LITERATURE REVIEW

This study focuses on the beliefs of preservice elementary school teachers with regard to mathematics and mathematics instruction. But what exactly are beliefs? Why are they important in educational settings? How have beliefs about mathematics and mathematics instruction been measured and classified? How and why do such beliefs change? These questions must be answered before any meaningful research can be conducted.

This chapter outlines the literature that informs this study. In the first section several competing definitions of the term belief are presented. The second section addresses the notion of belief and its role in the philosophical study of knowledge: epistemology. The third section gives a brief overview of the notion of belief and its position in the study of human behavior, or psychology. In the fourth section a summary of theoretical research on the various beliefs of teachers with regard to mathematics and mathematics instruction is offered. In the last section a summary of relevant empirical research on teacher beliefs is presented.

DEFINING BELIEF

Merriam-Webster's dictionary (2010) provides the following three definitions of the term belief:

1. a state or habit of mind in which trust or confidence is placed in some person or thing

2. something believed; especially a tenet or body of tenets held by a group

3. conviction of the truth of some statement or the reality of some being or phenomenon especially when based on examination of evidence (Merriam-Webster, 2010, p.1)

In Stanford's Encyclopedia of Philosophy (2010) we find another notion of the term belief:

> Contemporary analytic philosophers of mind generally use the term "belief" to refer to the attitude we have, roughly, whenever we take something to be the case or regard it as true (Schwitzgebel, 2006, Para. 1).

The psychological theorist Rokeach (1968) broadly defined the term belief as,

> Any simple proposition, conscious or unconscious, inferred from what a person says or does (p. 113)

Rokeach (1968) went on to make the distinction between those beliefs which are *descriptive*, *evaluative* and *prescriptive* in nature.

BELIEFS AND EPISTEMOLOGY

Epistemology is described as the "philosophical inquiry into the nature, conditions, and extent of human knowledge (Sosa, et al., 2009, p.i)." Epistemology asks such basic questions concerning knowledge including: What counts as knowledge? What can we say that we know? How do we know that we know? To answer such questions a philosophical account of knowledge is required. Plato is credited with proposing an enduring theory of knowledge in his dialogue with Theaetetus in which, after some debate, a hypothesis is proposed: "Knowledge is true opinion accompanied by reason (Plato, 1952, p.223)".

Chisolm (1982) and others have refined Plato's notion of knowledge as *justified true belief*. Here, one is said to have knowledge of a proposition if the following are all satisfied: the proposition is true, the proposition is believed to be true, and the belief in the proposition is justified. In more formal terms this assertion is given account analytically as follows:

> S knows that P if and only if:
>
> P is true, and,
>
> S believes that P is true, and,
>
> S is justified in believing that P is true. (Chisolm, 1982)

So, the traditional philosophical components of knowledge are truth, belief and justification. Of particular interest to this study is the important role that belief plays in the philosophical account of knowledge: knowledge entails belief. That is, in order to know, one is required to first believe.

More recently, the philosophical conception of knowledge as justified true belief has suffered significant impasse with the presentation of so-called Gettier problems (Gettier, 1963). In these scenarios the prerequisite requirements of justification, truth and belief are all met, yet, knowledge is intuitively not possessed. Zagzebski (1994) and others (Goldman, 1967; Quine, 1969) have pointed out that problems with the idea of knowledge as justified true belief are inescapable and have proposed alternative theories to the traditional justified-true-belief notion where knowledge is based on other dependencies such as virtue or natural science. In these alternate theories the concept of belief often plays a less central role.

BELIEFS AND PSYCHOLOGY

Broadly construed, human psychology is the study of human behavior. Theories of psychology, then, can be identified by the way in which they explain the causes of human behavior. *Behaviorism* or behavior analysis describes human behavior as a function of the environment. *Neuroscience* describes human behavior as a function of human biology. *Cognition* describes human behavior as a function of human mental processes. The psychological study of belief, then, is firmly rooted in the study of cognition.

Historically, beginning around the turn of the 20th century, psychologists and social scientists showed considerable interest in the study of the nature of human beliefs and their interaction with human behavior. Starting around the 1920s, interest in beliefs began to fade due in part to the rising popularity of behaviorism promoted by Pavlov, Thorndike, Watson, Skinner and others. The psychological study of beliefs experienced renewed interest in the 1960s and 1970s with the advent of the cognitive sciences led by the work of Jean Piaget and Lev Vygotsky (Thompson, 1992, pp. 128-129). Emphasis on mental processes allowed for "a place for the study of belief systems in relation to other aspects of human cognition and human affect (Abelson, 1979, p. 355)."

In the cognitive theory of Piaget, beliefs play a central role in knowledge formation. Piaget characterized intelligence as a successful adaptation of an individual to the external environment through human behavior which is controlled by schemes: representations of the world which designate particular actions. Piaget theorized that humans are born with innate schemes but quickly learn to build and modify schemes to more successfully adapt to a given environment. Piaget believed that humans altered and refined their schemes throughout their lives through a process of *accommodation* and *assimilation*. Assimilation occurs when a particular environmental stimulus is used or transformed in such a way that it can be incorporated into a preexisting cognitive scheme. Accommodation occurs when a particular environmental stimulus cannot be incorporated into a preexisting cognitive scheme and thus forces the existing cognitive scheme to change. Further, Piaget proposed that as a child matures, schemes are organized into complex systems, or structures, which are hierarchically characterized as stages of cognitive development. Specifically, Piaget proposed that human cognitive development passes through four principle stages: sensory-motor, pre-operational, concrete operational, and formal operational (Gruber, 1995).

So, for Piaget, an individual's beliefs are firmly tied to their cognitive development through the presence or absence of a scheme which allows for the successful adaptation to environmental stimuli. One's profession "I believe" is an indication that one holds a particular scheme that has proven useful, even advantageous, in making sense of environmental stimuli. Alternatively, one's profession "I do not believe" is an indication that the proposition fails to agree with an existing scheme or that the proposition is simply unintelligible to one's particular stage of cognitive development.

For Vygotsky, beliefs might best be understood as cultural artifacts. Vygotsky built a theory of cognitive development around his observation that certain cultural groups exhibited higher mental functioning which pointed to the importance of social interaction in the acquisition of knowledge. In *Thought and Language* (1962), he theorized that knowledge of the world is best described as an inner voice that directs and regulates behavior. This "self-talk" is developed through interactions with others in social settings. Thus, external communication is gradually transformed into an internalized inner voice which conducts our thoughts, actions and behaviors. Knowledge, therefore, is inherently social in its form, transmission, and function.

Vygotsky also identified what has come to be known as the "zone of proximal development". In *Mind in Society: Development of Higher Psychological Processes* (1978), he describes the zone of proximal development as follows:

> The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers (p. 86)

Simply put, the zone of proximal development is the range of possible educational goals that a learner is able to attain with capable instruction versus without capable instruction. Inherent to the theory is the important role of dialogue between learner and instructor.

Thus, for Vygotsky, a belief might best be described as a cultural artifact that is transmitted through social interaction. To say "I believe" a proposition is an admission that I have received such a proposition through social interaction and have internalized the proposition through a process of self talk which has situated the proposition as an inner voice which directs and informs my thoughts and behaviors. In educational settings, beliefs are seen as a powerful directive in student-teacher social negotiation of the zone of proximal development.

CHARACTERIZING BELIEFS

Previous research has been carried out in an attempt to characterize those aspects of a proposition that identify the proposition as a belief. Often times, characteristics of beliefs are contrasted with differing characteristics of knowledge. This body of research most often seeks to identify the salient differences between what it means when one says, "I believe" versus what it means when one says, "I know".

Rokeach (1968) theorized that any profession of belief falls into one of three categories: descriptive, evaluative and prescriptive beliefs. Descriptive beliefs are those which indicate a profession of what one takes to be the present state of being, as in, "I believe that students learn in school." Evaluative belief statements indicate a personal commitment to an uncertain proposition, as in, "I believe that mathematics is useful knowledge." Finally, prescriptive beliefs are those that indicate a personal commitment to action or treatment, as in, "I believe that every student should be taught mathematics in school."

One common identifying feature of beliefs is a varying degree of conviction (Thompson, 1992). That is, a belief is held with a level of commitment that varies on a scale from weak to strong. This feature is not a common characteristic of knowledge which is characterized as either present or absent in an individual. Ableson (1979) describes this characteristic of beliefs:

> The believer can be passionately committed to a point of view; or at the other extreme could regard a state of affairs as more probable than not, as in "I believe that micro-organisms will be found on Mars." This dimension of variation is absent from knowledge systems. One would not say that one knew a fact strongly (p. 360).

Thus beliefs are identified by a varying degree of commitment, a feature which is not commonly exhibited in knowledge claims.

Thompson (1992) and others have pointed out that another characterizing feature of beliefs is non-consensuality. That is, a belief statement can be identified by an awareness of possible disagreement. Ableson (1979) points out beliefs are offered with an awareness of disputability or an admission that “others may think differently (Ableson, 1979, p.356).” Thompson (1992) points out that this notion firmly distinguishes beliefs from knowledge due to the fact that philosophical notions of knowledge are aligned with truth and certainty. Thompson (1992) quotes Scheffler (1965) with regard to this notion:

> In general, if you think I am mistaken in my belief, you will deny that I know, no matter how sincere you judge me to be and no matter how strong you consider my conviction. For X [an individual] to be judged mistaken is sufficient basis for rejecting the claim that he knows. It follows that if X is admitted to know, he must be judged not to be mistaken, and this is the point of the truth condition…Knowing, it would appear, is incompatible with being wrong or mistaken, and when I describe someone as knowing, I commit myself to his not being mistaken…knowing, unlike believing, has independent factual reference (p. 23-24).

Scheffler points out that knowledge claims are identified by a “truth condition”. This condition serves as a division between those propositions which are non-consensual (i.e. beliefs) and those which are judged to be true (i.e. knowledge). Note that Scheffler’s truth condition is compatible with the epistemological concept of knowledge as justified true belief.

Building on the notion of truth conditions associated with knowledge claims, researchers have pointed out that there must be general consensus for the verification of such claims. As Thompson (1992) puts it, “Knowledge must meet criteria involving canons of evidence (p. 130).” Beliefs, on the other hand, are characterized by a lack of any consensual means of judgment, verification or evaluation. As Nespor (1987) writes:

> Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge systems are (p. 321).

Nespor makes clear that beliefs are characterized by their personal and often private means of justification which cannot be challenged on the basis of an agreed upon public standard of evaluation.

Some researchers have proposed the idea that beliefs are structured into systems which are organized according to predictable principles (Torner, 2002; Green, 1971; Rokeach, 1960). Thompson (1992) noted that this approach parallels the practice of characterizing conceptual domains according to cognitive structures.

Perhaps most cited of these belief systems and of particular importance to this study is that of Green (1971). Green's system attempts to explain the ways in which the beliefs of an individual are interrelated and identifies three salient dimensions of human belief systems: a quasi-logical relationship, a degree of conviction, and a clustered structure.

Green (1971) claims that no belief is held in isolation; rather, belief systems tend towards a structure in which *derivative* beliefs are linked to *primary* beliefs. This structure is seen as quasi-logical in the sense that a derivative belief is often justified on the basis of some other primary belief. If a teacher, for example, believes that mathematics is best learned through hands on activities then they are likely to hold derivative beliefs about the importance of manipulatives in the mathematics classroom. Here the derivative belief regarding manipulatives is linked to a primary belief concerning how mathematics is learned.

Green (1971) also notes that beliefs are characterized by their degree of conviction: some beliefs are *central* while others are *peripheral*. Central beliefs are those which are most strongly held. Peripheral beliefs are those which are weakly held and most likely to change. He goes on to note that the primacy of a belief is not necessarily indicative of a belief's centrality. Rather, these two characteristics act independently of one another. It is entirely possible, then, that a teacher might have stronger (central) convictions about the use of manipulatives in her classroom than her (peripheral) belief in teaching mathematics through hands-on activities even though this belief is her primary justification for the use of manipulatives in the classroom.

Finally, Green (1971) claims that beliefs are held in clusters which are "in isolation from other clusters and protected from any relationship with other sets of beliefs (p. 48)." This clustered structure allows for independence among sets of beliefs, making it possible for persons to hold seemingly conflicting beliefs. Thompson (1992) noted that this clustering feature of belief systems may explain the incongruities that many researchers have noted in studies on the beliefs of teachers (i.e. Brown, 1985; Cooney, 1985; Thompson, 1982, 1984).

BELIEFS AND MATHEMATICS EDUCATION – THEORETICAL STUDIES

Theoretical research into the role of beliefs in mathematics education began to receive significant attention in the early 1980s. Research in this area initially focused on the descriptions of theoretical frameworks to aid in the identification and characterization of beliefs that teachers hold with regard to the subject of mathematics as well as beliefs about how mathematics is taught and learned.

Lerman (1983) was one of the first to propose that one's perspective in mathematics education is a logical consequence of one's epistemological commitments to mathematics as a subject. He argued for a shift from the old "Euclidean program" towards a "problem solving" perspective as a means of advancing mathematical understanding in the classroom. Schoenfeld (1985) in his well known book *Mathematical Problem Solving* drew attention to beliefs as a necessary component to explain the activities of students when faced with mathematical problem solving tasks.

Paul Ernest's 1988 paper *The Impact of Beliefs on the Teaching of Mathematics* categorized the beliefs held by teachers regarding the nature mathematics. Ernest noted that attempts to reform mathematics education are fundamentally tied to the beliefs of teachers and their relationship to classroom behavior:

> A shift to a problem solving approach to teaching requires deeper changes. It depends fundamentally on the teacher's conception of the nature of mathematics and mental models of teaching and learning mathematics. Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change (p.1)

Ernest distinguished three conceptions of mathematics which he based on prevalent theories in the philosophy of mathematics (i.e. Lakatos, 1976; Davis & Hersh, 1980; Benacerraf & Putnam, 1964). These conceptions are the instrumentalist view, the Platonist view and the problem solving view.

Ernest (1988) first identifies the *instrumentalist* view of mathematics. In this view, mathematics is envisioned as a collection of facts, rules and skills that are to be used practically to pursue some external end. The rules of mathematics are conceived as separate entities: unstructured and unrelated. The teacher's role is then conceived of as one of *instructor* who correctly models the skills and procedures through the strict application of a curriculum. Student knowledge is demonstrated through correct performance and mastery of skills.

Next, Ernest (1988) describes the *Platonist* view of mathematics. Similar to Lerman's "Euclidean program", the Platonist view envisions mathematics as a static but unified body of knowledge which is taken to be certain. Mathematics is thought to be discovered by humans, not created. The teacher's role in this view is one of *explainer* who demonstrates mathematical objectivity and works to promote a conceptual understanding and a unified perception of the science. Student demonstration of knowledge, then, extends beyond algorithms and routines of the traditional textbook to include additional problems and activities that are conceptually linked to the curriculum.

Finally Ernest (1988) describes the *problem solving* view. Here, mathematics is characterized as a dynamic and expanding field that is the product of human creativity and invention. Mathematics is a cultural product. Most notably, the problem solving view sees mathematics as a process of inquiry whose products remain open to revision. The teacher's role in this view is one of *facilitator* who confidently models the problem-posing and problem-solving dialectic. Students demonstrate mathematical knowledge through their own active participation in the process of problem-posing and problem-solving.

Ernest (1988) concludes his piece noting that any one teacher's espoused model of teaching and learning mathematics may, or may not, match their particular enacted model of teaching mathematics (i.e. Cooney, 1985) due to the presence of external influences on the practice of teaching mathematics. Ernest identifies two such influences: social contexts and teacher awareness. Social contexts which are imposed on mathematics classrooms such as school culture, district-chosen curricula and the national system of schooling have a homogenizing effect on mathematics instruction and often impede the enactment of a teacher's espoused beliefs regarding mathematics and mathematics instruction. Teacher awareness of their own beliefs and the level of self reflection upon their practice of teaching mathematics also influence the enactment of beliefs. Here, Ernest points out that one must first be aware and able to justify one's beliefs regarding mathematics and mathematics instruction before these beliefs can be enacted and integrated into teaching practices.

Richard Skemp (1987), primarily known for describing how individual concepts in mathematics are linked together to form concept structures or *schemas*, also proposed a theoretical framework with regard to the goals of learning and the qualities of understanding in the mathematics classroom. Skemp's theory is based on the assumption that there are three competing beliefs about what counts for "understanding" in mathematics. Skemp's three types of understanding are "relational understanding" and "instrumental understanding" and "formal understanding". He summarizes these three types of understanding as follows:

> Instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works.
>
> Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships.
>
> Formal understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning. (p. 166)

Skemp points out that these three conceptions of mathematics influence the goals of mathematics education and the schemas that students construct in each setting.

According to Skemp, the goal of learning in an instrumental setting is to "give the right answers, as many as possible, to questions posed by the teacher (p.168)." The schemas acquired by students in such classrooms are a set of rules appropriate for a limited class of tasks which provide for the quick acquisition of the correct answer. Such learning is characterized by its limited adaptability. Here mathematics becomes a "degenerative schema" of isolated concept connections among groups of symbols which are disassociated from their symbol-meaning.

In contrast, the goal of learning in a relational setting is the construction of relational schemas. That is, learning in a relational classroom is evidenced by the ability of the student to connect a newly encountered mathematical object into an existing schema in such a way that the object is relationally understood. In this sense, existing schemas grow and reorganize when learning has taken place. Understanding in mathematics, then, becomes the ability to incorporate new and previously unknown mathematical objects into one's existing schema of mathematics. Here mathematics becomes a connected field of interrelated schema which is characterized by both cohesiveness and adaptable flexibility.

Finally, in formal understanding, the goals of learning are neither the provision of the correct answer nor the acquisition of new schemas but rather the demonstration of the logical necessity of a mathematical assertion through a chain of inference from a set of premises, axioms, and proven theorems. Formal understanding, then, is the construction of mathematical proof. Skemp notes that this highest stage of mathematical understanding assumes that a level of relational understanding has been previously achieved prompting the learner to shift her focus to "being sure that the schemas that have been constructed, the solutions which have been devised, are sound and accurate (p. 171)." According to Skemp, formal understanding is what makes mathematics unique among the sciences for the results of mathematics can be understood as the logical necessities of the premises, axioms and theorems of mathematics. Note that Skemp's formal understanding plays an important role in educational settings which aim to promote mathematical understanding.

Skemp comments that it is these different meanings of "understanding" which are at the heart of current debates concerning mathematics education. Skemp takes clear issue with those promoting instrumentalism:

> Instrumental understanding I would until recently not have regarded as understanding at all. It is what I have in the past described as "rules without reasons," without realizing that for many pupils *and their teachers* the possession of such a rule, and the ability to use it, was what they meant by "understanding" (p. 153)

He goes on to advocate for the relational and subsequently refined formal understandings in the classroom by playing devil's advocate in the debate, imagining the advantages of an instrumentalist approach to mathematics:

1. Instrumentalist mathematics is usually easier to understand
2. Instrumentalist mathematics offers more apparent and immediate rewards
3. Instrumentalist mathematics allows one to get the right answer more quickly and reliably (p.158)

To which Skemp offers four advantages of relational understanding in the mathematics classroom:

1. Relational understanding is more adaptable to new tasks
2. Relational understanding makes it easier to remember mathematics
3. Relational knowledge can be effective as a goal in itself and becomes self motivating
4. Relational schemas are organic in quality (p. 159)

Noting that relational classrooms offer significant advantages over their instrumentalist counterparts Skemp notes that many situational factors contribute to the difficulty of teaching mathematics for relational understanding, among them are:

1. The backwash effect of examinations
2. Over burdened syllabi
3. Difficulty of assessment
4. The great psychological difficulty for teachers of reconstructing their existing and longstanding schemas (p.161)

Skemp concludes by making a plea for a transition away from instrumental understanding in favor of relational understanding to renew the practical, cultural and intellectual value of mathematics education in the face of recent trends which indicate a popular rejection of the subject and a fear of the classrooms in which it is taught.

Taking a slightly different approach, a large number of researchers (i.e. Copes, 1979, 1982; Dougherty, 1990; Helms, 1989; Kesler, 1985; McGalliard, 1983; Meyerson, 1978; Owens, 1987; Stonewater & Oprea, 1988, cited in Thompson, 1992) have used Perry's (1970) scheme of development as a framework for characterizing teachers' beliefs regarding mathematics and mathematics instruction. In one example of this line of research, Copes (1979) proposed a framework adapted from Perry's (1970) scheme with four different conceptions of mathematics each corresponding to a distinct historical perspective prevalent in the development of the subject: absolutism, multiplism, relativism and dynamism. Here, the *absolutism* view embraces a conception of mathematics as a collection of facts verifiable in the real world. This view corresponds to the historical period up to the middle of the nineteenth century where the discovery of non-Euclidean geometry promotes a *multiplistic* view of the subject where mathematical facts regarding physically-impossible objects begin to arise in the study of mathematics. This is followed by the historical shift to *relativism* marking the abandonment of the effort to prove the logical consistency of different mathematical systems in exchange for an acceptance of the coexistence of equally valid systems which have been alternatively axiomatized. Finally *dynamism* characterizes the presently held notion of mathematics which is characterized by a commitment to one of many possible mathematical systems with an understanding of that system's relativistic status within the subject. Copes (1979) theorized that one's teaching style might indicate one's conception of the subject understood in this historical framework. Interestingly, Stonewater and Oprea (1988) found evidence for this prediction in their study of three high school teachers. In her summary of this line of research, Thompson (1992) raised the question as to whether one's beliefs about the subject might be "predicted by their level of intellectual development (p. 133)" as understood according to Copes' (1979) theoretical framework.

A number of researchers have conducted theoretical studies on the beliefs of teachers with regard to mathematics instruction. Kuhs and Ball (1986) conducted a meta-study of relevant literature in mathematics education, philosophy of mathematics, and philosophy of education as a means of identifying four theoretical models of how mathematics should be taught. They summarized these four views according to the following:

> *Learner-focused*: mathematics teaching that focuses on the learner's personal construction of mathematical knowledge.
>
> *Content-focused with an emphasis on conceptual understanding*: mathematics teaching that is driven by the content itself but emphasizes conceptual understanding.
>
> *Content-focused with an emphasis on performance*: mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures
>
> *Classroom-focused*: mathematics teaching based on knowledge about effective classrooms. (p. 2)

The reader should notice that Kuhs and Ball extend earlier theories of mathematical knowledge to theories of mathematics *teaching*.

Kuhs and Ball's (1986) *learner-focused* view of mathematics teaching is aligned with a constructivist view of mathematical learning which emphasizes the student's active involvement doing mathematics: exploring mathematical questions, making mathematical conjectures and demonstrating (i.e. actively proving) the veracity of such conjectures. The role of the teacher in such an environment is one of facilitator and stimulator of student curiosity, "posing interesting questions and situations for investigation, challenging students to think, and helping them uncover inadequacies in their own thinking (Thompson, 1992, p. 136)." Students are held responsible for determining the sufficiency of their own understanding and student knowledge is assessed according to the consistency between constructed knowledge and accepted understandings in mathematics through demonstrations where students validate mathematical conjectures, defend mathematical findings, or support mathematical conclusions.

Kuhs and Ball's (1986) *content-focused with emphasis on understanding* view of mathematics teaching places mathematics content at the center of educational activity. Here the emphasis is placed upon the structure, logic and interrelatedness of mathematical content. The role of the teacher in such a classroom is one of instructor: pointing out the connectivity of the subject matter, demonstrating the logical necessity of various results, displaying to the novice student "how things work" in mathematics. Students depend upon the instructor for knowledge in such a setting. Student knowledge is assessed according to criteria which are nearly the same as the *learner-focused* model.

Kuhs and Ball's (1986) *content-focused with an emphasis on performance* also places mathematical content at the center of educational activity in the classroom, but, deemphasizes student understanding and replaces it with an emphasis on student performance over a hierarchy of skills and procedures. Kuhs and Ball enumerate several premises of this view of mathematics instruction:

> 1. Rules are the basic building blocks of all mathematical knowledge and all mathematical behavior is rule governed.
>
> 2. Knowledge of mathematics is being able to get answers and do problems using the rules that have been learned.
>
> 3. Computational procedures should be "automatized".
>
> 4. It is not necessary to understand the source or reason for student errors; further instruction on the correct way to do things will result in appropriate learning.
>
> 5. In school, knowing mathematics means being able to demonstrate mastery of the skills described by instructional objectives. (p. 22)

The role of the teacher in this view is described as one of demonstrator: presenting mathematical processes and procedures by means of example. Student learning is then evidenced through reiteration and correct application of rules and procedures modeled by the teacher during instruction.

Kuhs and Ball's (1986) last theoretical model for the teaching of mathematics is the *classroom-focused* view. While the first three views all approach teaching from different standards for what counts as mathematical content (i.e. process, product or procedure) this view focuses on successful methods of classroom instruction as identified by studies of teaching efficacy. Teachers holding this view see "successful teaching" as a process characterized by key elements such as organization, structure and routine. So, according to Kuhs and Ball (1986), teachers who hold this view are more likely to attribute student success to elements of the classroom environment such as "maintaining high expectations" or "insuring a task-focused environment" than to an approach to the mathematical content. They note that this view, in its most extreme form, does not question mathematical content but rather views it as external to the teaching process: determined by state and local curriculum. The role of the teacher in this view is one of manager who must "skillfully explain, assign tasks, monitor student work, provide feedback to students, and manage the classroom environment, preventing, or eliminating, disruptions that might interfere with the flow of the planned activity (p. 26)." Student learning is then measured according to one's ability to listen attentively, cooperate, follow instructions and complete assigned tasks.

Noting the multitudinous definitions of mathematical beliefs in the literature, some researchers have called for more consensus and presented new theoretical frameworks that attempt to allow for more universal study of the construct. Torner (2002) suggested a four component framework consisting of a belief object, a range of mental association, activation level or strength, and a mapping of association. Further, Torner (2002) posited that mathematical beliefs be ordered hierarchically from global beliefs about mathematics teaching and learning, to domain specific beliefs about discrete areas within mathematics such as geometry or calculus, to subject matter beliefs regarding the organization of content. He suggested that beliefs in each of these hierarchical categories interact with each other, exerting either "bottom-up" or "top-down" influences. According to Torner (2002), one's beliefs about the subject of geometry exert bottom-up influence on one's beliefs about mathematics as a subject. Conversely, one's beliefs about mathematics as a subject exerts top-down influence on one's beliefs about how the subject should be organized and presented in the classroom.

Some researchers have conducted large scale statistical studies of teacher populations to identify the framework of beliefs that these populations possess. This research probes for an empirical basis which prompts a theoretical framework of beliefs. In one example of this approach, Barkatsas and Malone (2005) constructed a theoretical framework for the beliefs of teachers through a large scale statistical survey of secondary teachers in Greece. A factor analysis of the results of the survey revealed two dominant categories of teacher beliefs: a contemporary-constructivist orientation and a traditional-transmission-information-processing orientation.

Hannula et al. (2005, 2006) investigated the *structure* of mathematical beliefs of elementary teachers. They conducted a statistical survey with factor analysis of 269 beginning elementary school teachers at three Finnish school and found evidence for a "core view of mathematics" based on a cluster of three beliefs: beliefs about the difficulty of mathematics, beliefs about one's own talent with regard to mathematics, and beliefs about liking or disliking mathematics.   These researchers found evidence that beliefs differ significantly according to the factors of gender, previous grades and previous course selection. Females were found to have lower self-confidence and were more likely to hold critical images of their mathematics teachers. Elementary teachers who received higher grades in mathematics were more likely to hold positive beliefs about their talent in the subject and their perception of themselves as “hard working”. High scoring teachers also were more likely to enjoy mathematics. Finally, elementary teachers who studied more advanced courses in mathematics in high school were more likely to have higher self-confidence and a less critical view of their teachers. Other researchers who have focused their study on the structure of beliefs of teachers include Benken (2005), Archer (1999), and Hannula et al. (2009)

BELIEFS AND MATHEMATICS EDUCATION – EMPIRICAL STUDIES

Many researchers have conducted empirical studies on the role of beliefs in mathematics education. Some researchers have probed the relationship between teacher beliefs and instructional practice in mathematics education. Still others have studied how beliefs of teachers change over time. Some have “cataloged” beliefs. Finally, a handful of studies have focused on specific programs aimed at changing teachers’ beliefs.

In a heavily cited case study analysis of middle school mathematics teachers, Thompson (1984) studied the relationship between teachers’ conceptions of mathematics and their instructional behaviors through a case study analysis of three differing teachers: Jeanne, Kay and Lynn. Jeanne held beliefs about mathematics as “a coherent subject consisting of logically interrelated topics (p. 119).” In contrast, Kay’s beliefs indicated that she “regarded mathematics primarily as a challenging subject whose essential processes were discovery and verification (p.119)." Finally, Lynn's beliefs of mathematics indicated "a view of mathematics as essentially prescriptive and deterministic in nature (p. 119)." Thompson (1984) found that these beliefs played an important role in the teaching process that she observed in each of the three teachers' classrooms. Kay's classroom promoted reasoning and student discovery of mathematical concepts. Jeanne's classroom was characterized by lessons which focused on the logical derivation of mathematical concepts. Finally, Lynn saw her role in teaching mathematics "was to demonstrate the procedures that the students were to use in performing the tasks in the daily assignments (p. 120)." Thompson (1984) generally found that the teachers in her case study enacted instructional programs that were consistent with their exposed beliefs about the nature of mathematical knowledge. She summarized her findings:

> …teachers' beliefs, views and preferences about mathematics and its teaching, regardless of whether they are consciously or unconsciously held, play a significant, albeit subtle, role in shaping the teachers' characteristic patterns of instructional behavior (p. 125).

She concluded her study with a call for future research on the stability of teachers' conceptions of mathematics and mathematics instruction and the interaction of teachers' and students' beliefs regarding mathematics and mathematics instruction.

Other researchers have found evidence of the importance of teacher beliefs about mathematics in shaping instruction. In a case study analysis, Steinberg et al. (1985) found that teachers equipped with deeper content knowledge held and enacted beliefs about mathematics instruction which were conceptually orientated. Conversely, teachers with less content knowledge tended to hold and enact beliefs which were instrumental in nature: emphasizing procedural knowledge. McGalliard (1983) found a high correlation between teachers' conception of mathematics and their instructional practices in teaching high school geometry. He found that teachers holding dualistic (i.e. right versus wrong) conceptions of mathematics emphasized instrumentalist approaches in teaching their students (i.e. memorized rules and procedures). Lerman (1983) found that "absolutist" preservice secondary teachers (those holding absolute or Platonist views of the subject) were more likely to encourage teacher centered instruction while "fallibilist" pre-service secondary teachers (those holding fallible or constructivist views of the subject) were more likely to encourage student centered instruction.

Studies of teachers' beliefs regarding mathematics instruction and actual teaching practices have produced confounding results. While some researchers have reported a high degree of consistency between a teacher's espoused and enacted beliefs with regard to mathematics instruction (i.e. Grant, 1984; Shirk, 1973) others have noted inconsistencies. For example, in McGalliard's (1983) study of geometry teachers, he found that many teachers espoused a belief that the subject promotes the development of logical reasoning while contradictorily teaching the subject from an instrumental perspective: divorced from logic and reasoning and driven by rules and procedures. Cooney (1985), Shaw (1989) and Thompson (1982) have also noted similar discrepancies between espoused and enacted beliefs with regard to mathematics instruction.

The prevalence of studies documenting inconsistencies between espoused and enacted beliefs has motivated a number of researchers to probe for possible explanations of this phenomenon. The prevailing theme that arises from this line of research points to the importance of the role of contextual issues that shape instructional action in the mathematics classroom. Raymond (1997) found in her case study of *Johanna* that one's beliefs about mathematics played a more important role than one's beliefs about mathematics teaching and learning in predicting instructional behavior. Hoyles (1992) theorized that the "embodied nature" of situational constraints might explain the inconsistencies that researchers often noted between professed and enacted beliefs.

Similarly, Skott (2001) concluded that "multiple and sometimes conflicting educational priorities (p. 18)" often lead to inconsistencies between espoused and enacted beliefs in the classroom. Specifically, Skott (2001) found that the shifting priorities of learning, classroom management and developing student confidence lead to instructional practices that conflicted with a teacher's professed beliefs.

Gregg (1995), in his case study of beginning high school mathematics teachers, found that encouraging teachers to examine the discrepancies between espoused and enacted beliefs was insufficient in promoting reform in mathematics education. He documented the strong influences of the "school mathematics tradition" which acts to institutionalize and promote taken-as-shared beliefs and practices in the field. Gregg (1995) documented the view of "ability as capacity" works against reforms in mathematics education in that it explains away pedagogical errors as a source for student misunderstanding. He also noted the separation of teaching and learning in the school mathematics tradition allows teachers to meet their contractual obligations and give the appearance of competency while distancing themselves from unfavorable outcomes in student learning. Lastly, Gregg (1995) noted that the mathematics tradition often identifies an assessment as "too hard" or "unfair" when many students score poorly. Here again, the tradition provides a means for a teacher to distance himself from student failure rather than taking responsibility for a lack of student understanding. This taken-as-shared school mathematics tradition, Gregg (1995) theorized, might negate any efforts in reform in mathematics education that focus on teacher beliefs.

Finally, Sztajn (2003) in her case study of two elementary school teachers found that teacher beliefs about the *needs of their students* were more predictive of mathematics instruction than beliefs about how mathematics should be taught and learned. Troubling from a social justice standpoint, she found that teachers serving students of low socio-economic status tended to explain instrumentalist approaches to instruction on the basis of the perceived needs of their students: preparing them for rule-following roles in the workplace. Conversely, she found that teachers serving students of high socio-economic status tended to explain their problem-solving approaches to instruction on the basis of the perceived needs of their students: preparing them for complex problem solving roles in the workplace.

While some researchers have focused on the relationship between beliefs and practice, others have attempted to measure the beliefs of teachers over time in the absence of any intervention. One early longitudinal study of the beliefs of preservice elementary teachers was that of Collier (1968) conducted at the University of Wisconsin Oshkosh. He devised two Likert scale instruments to study the beliefs that preservice elementary teachers hold about mathematics and about mathematics instruction. Collier used these instruments to measure the beliefs of preservice teachers on a formal-informal scale at four stages of their undergraduate preparation. Collier characterized “formal” beliefs as those which identify mathematics as a body of rules and procedures which are largely prescriptive in nature. Collier characterized “informal” beliefs as those which identify mathematics as a creative and investigative subject.

With regard to beliefs about mathematics, Collier (1968) found that students entered the elementary education program with neutral beliefs. After two content courses, students still held neutral beliefs, but, high achieving students moved to a slightly informal view of mathematics. After taking two content courses and a teaching methods course, students moved to a slightly informal view of mathematics with high achievers holding more informal beliefs than their low achieving counterparts (p. 159).

With regard to beliefs about mathematics instruction, Collier (1968) found that students entered their program of studies with neutral beliefs. After two content courses their beliefs remained neutral. After two content courses and a teaching methods course their beliefs shifted to moderately informal with little difference between high achieving and low achieving students (p. 159).

Reflecting upon the overwhelming neutrality of teacher beliefs with regard to mathematics and mathematics instruction, Collier (1968) ended his study in discussion of two factors that may limit the range of beliefs of the population. He pointed out prospective teachers arrive at the university with beliefs about mathematics and mathematics instruction that are informed by many years of experience as students of mathematics. These well-formed beliefs may be resistant to change. Secondly, he noted that few students are exposed to courses in mathematics that included the formation of beliefs as an educational objective.

Seaman et al. (2005) replicated Collier's (1968) study in 1998. They sought to determine whether student beliefs with regard to mathematics and mathematics instruction had changed in response to 30 years of educational reform in mathematics instruction promoting the subject as both creative and investigative (i.e. NCTM, 1989;

NCTM, 1991). Seaman et al. (2005) found that students in 1998 did indeed hold significantly more informal beliefs when compared to their 1968 counterparts. And, similar to Collier's (1968) finding, students in 1998 did move towards more informal beliefs over the course of their program of study. Seaman et al. (2005) also noted that students hold seemingly contradictory beliefs both at the start and at the end of their program of studies indicating that modern students fail to develop "robust, consistent philosophies of mathematics education (p. 197)" while at the university.

Taking a similar approach as Collier (1972) and Seaman et al. (2005), Peterson, Fennema, Carpenter and Loef (1989) developed a Likert scale instrument to study teachers' beliefs about how students' thinking informs instruction, a practice known as Cognitively Guided Instruction (CGI). Their study found evidence of salient differences in instructional practice between first grade mathematics teachers that were more cognitively based (CB) versus less cognitively based (LCB). For example CB teachers were found to have a greater knowledge of different types of word problems, spent more time developing counting strategies before introducing formal symbolism, and relied more heavily on observation (rather than formal assessment) to inform their instruction (p. 36). Other researchers have used Peterson, Fennema, Carpenter and Loef's (1989) CGI beliefs instrument to study prospective teachers (Vacc & Bright, 1999) and in-service teachers (Fennema et al, 1996).

Other researchers who have taken observational approaches to the study of teacher beliefs about mathematics include Wilmott (2005), who studied the beliefs of preservice elementary teachers before and after participating in a mathematics pedagogy course. Wilmott (2005) found little evidence of spontaneous beliefs change resulting from participation in the course and called for "the need to provide opportunities for pre-service teachers to engage each others thinking in a critical and reflective manner" (p. 2). Smith et al. (2005) investigated the effect of Developing Mathematical Ideas (DMI) curricular materials in motivating beliefs change in preservice elementary teachers. They found that the variation in beliefs change across participants in the study was largely explained by the variation in the level of engagement in the educational opportunities available in the DMI course. Lloyd (2002) compared the beliefs of two populations of student teachers engaged in mathematics pedagogy courses employing different curricular materials to display how teacher beliefs are affected differentially by such experiences. Finally, Perrenet & Taconis (2009) adopted a *learning as enculturation* theoretical framework in their study of the beliefs of bachelor level mathematics education students. They found that the beliefs of the students in their study shifted towards the beliefs of their teachers although each student developed an individualized approach to mathematical problem solving. Most students explained this shift in beliefs to the shift in the nature of the mathematical tasks associated with university level work as compared to mathematics encountered in secondary school.

Some researchers have studied the beliefs of teachers and students in the absence of any theoretical framework in an attempt to identify and type the beliefs held by the population. The purpose of such empirical research is to create a list of the different beliefs that such populations hold. Cooper (2004) studied the beliefs of mathematics teachers with regard to aboriginal learning styles and found three distinct categories of beliefs: one that held that aboriginal learning difficulties cannot be solved by schooling, a second that held that aboriginal learning styles differ from styles of the non-aboriginal population, and a third that believed that aboriginal learning styles were no different than the non-aboriginal population. Furinghetti & Pehkonen (2002) surveyed 18 mathematics educators about their stances on nine characterizations of beliefs about mathematics which are found in the literature. They found consistency in some of the educators' stances and inconsistency in others. This finding led to suggestions for redefining certain characterizations of beliefs about mathematics to better align future research on the topic. Rosken and Torner (2009) examined and characterized the beliefs of university mathematics instructors using an epistemological approach. Greer et al. (2002) conducted research on student beliefs about word problems in mathematics and combined their results with other researchers' findings to arrive at a general set of beliefs that underlie the "word problem game" in the classroom. Finally, Muis (2004) conducted a critical review of 33 research articles in mathematics education literature to arrive at a general characterization of the personal epistemological beliefs held by students. She found "significant relationships between beliefs and cognition, motivation, and academic achievement" (p. 317) as well as "relationships between beliefs and learning behaviors" (p. 317).

In response to early studies which pointed towards the important role that beliefs play in the classroom (i.e. Thompson, 1984) and the largely undeveloped and often contradictory beliefs that preservice elementary teachers hold (i.e. Collier, 1972) many researchers have focused on the role of interventions aimed at the beliefs of teachers-in-training. Vacc & Bright (1999) used the Peterson et al. (1989) CGI instrument to study the changing beliefs of preservice elementary school teachers over the course of the last two years of their college preparation. They found that the beliefs of 34 teachers that they studied changed little over the course of the first two semesters of study but changed significantly (aligning with CGI principles) after the third semester of study in which they were enrolled in a mathematics pedagogy course which included CGI instructional techniques as part of the curriculum. Further, student beliefs continued to significantly change in the direction of CGI alignment during the fourth semester in which the students conducted their student teaching experience. They concluded that contrary to the previously held notion that that preservice teachers' beliefs are resistant to change "the data indicate the possibility that intensity of experience and focus on children's thinking in the mathematics methods course may be keys for helping preservice teachers change their views (p. 108)."

In another interventionist study, Fennema et al. (1996) used the Peterson et al (1989) CGI instrument in a large-scale longitudinal study of 21 in-service teachers over the course of four years as they participated in a CGI professional program while examining the growth of learning for the students in each of the participating teachers' classrooms. Fennema et al (1996) found that most of the teachers in the study displayed an increased level of sophistication in their beliefs regarding mathematics and their beliefs regarding mathematics instruction. Studying the 17 that increased in both areas, they found no evidence of a general rule that a change of one type of belief (i.e. in mathematics or mathematics instruction) precedes a change of the other. Finally, they found good evidence that "gains in students' concepts and problem solving performance appeared to be directly related to changes in teachers' instruction" (p. 430) resulting from participation in the CGI professional development. This result was replicated by Staub and Stern (2002) in their study of 496 German elementary school students in 27 classrooms. Like Fennema et al (1996), Staub and Stern (2002) found that students in the classrooms of teachers scoring higher on the CGI beliefs scale demonstrated a significantly higher aptitude in solving word problems than those students in classrooms of teachers scoring lower on the CGI beliefs scale.

More recently Cordy, et al. (2005) found promising change in elementary school teacher beliefs in response to a restructured mathematics course which incorporated nine "math therapy" sessions in which students worked collaboratively on interesting problems in mathematics. Kajander (2005) found that preservice elementary teachers' procedural and conceptual *values* changed over the course of a semester in response to a special methods course which focused on conceptual understanding of mathematics. Meel (2002) documented beliefs change in student teachers in response to exposure to research articles. Kaasila et al. (2006, 2005) found evidence of beliefs change in a mathematics methods course which included autobiographical writing. Liljedahl (2005) incorporated personal journal-writing in a problem-solving based mathematics course for elementary school teachers and found qualitative evidence of profound beliefs change. Specifically Liljedahal found that most students shifted in their perception of mathematics from a *noun* to a *verb*, that is, from something one "learns" to something one "does". Rolka et al. (2006) found a similar result in their study of 39 preservice elementary teachers enrolled in a special mathematics course which incorporated journal writing aimed at the examination of teacher beliefs. Of particular interest to this study, Rolka et al. noted in their conclusion that, "Through their own experiences with mathematics in a non-traditional setting most of the students come to see, and furthermore to believe, in the value of teaching and learning mathematics in the sense of the process aspect" (p. 447).

In another interventionist study Tillema (2000) investigated the role of *immersion* in practice and *reflection* upon practice and the interplay of these elements as agents of beliefs change in elementary teachers. Two groups of teachers were involved in the study. One group which experienced a special course in which immersion preceded reflection while another experienced reflection before immersion. Interview data and Likert questionnaires were collected upon completion of the course and were analyzed for evidence of beliefs change. Tillema summarized her findings:

> One could…assert that the greater the correspondence between practice and prior beliefs, the easier it is to accept and build up a coherent knowledge base for teaching and — conversely — that the more tenuous the correspondence is, the more relevant and supportive reflection could be. (p. 588)

She concluded that her research indeed supported the notion that beliefs change occurs primarily as a result of practice.

Some research has been conducted to investigate the relationship between the beliefs of teachers and the principles espoused in national curriculum documents. Zollman and Mason (1992) created a Likert scale instrument that was designed to study teachers' beliefs with regard 16 standards chosen from the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (1989). Using their *Standards'* Beliefs Instrument, Zollman and Mason (1992) found evidence that teachers who studied the *Standards* as part of a graduate level course tended to hold beliefs that were more consistent with those embodied in the document than a general population of teachers who had not participated in the same course.

Borrowing items from Zollman and Mason's (1992) *Standards'* Beliefs Instrument, Hart (2002) created a three part Likert scale beliefs survey that was administered to preservice elementary school teachers both before and after an innovative combined course in mathematics content and pedagogy. They found evidence that the teachers participating in the course experienced a change in beliefs "in a direction consistent with the philosophy of the program and the current reform efforts in mathematics education (p. 10)."

The use of Likert scale instruments to examine the beliefs of teachers has been criticized in the literature. Phillip (2007) points out that "one concern about self-report surveys is whether teachers' reports are accurate (p. 269)" and goes on to enumerate three weaknesses of using Likert scale instruments to measure the beliefs of teachers:

1. Inferring how a respondent interprets the meaning of words presented in Likert scale items is difficult.
2. Likert scale items provide little information for determining the centrality of certain beliefs.
3. Likert scale items provide little or no context which often shades the beliefs that teachers hold (pp. 269-270).

In response to these difficulties, Ambrose et al (2004) devised an alternative to Likert scale beliefs assessment instruments consisting of a web-based survey in which prospective and in-service teachers respond to open-ended questions about mathematics instruction after viewing a short video of a classroom in which a mathematics lesson is being taught.

Using the aforementioned instrument, Phillip et al (2007) conducted an experimental study of preservice teachers taking a mathematics course who were randomly assigned to one of three concurrent study groups: one that learned about children's thinking through direct interaction with children or by watching children on video, another that visited the classrooms of specially selected teachers, and finally a third control group received no extra instruction outside of the traditional mathematics course. Phillip et al (2007) found that those that participated in the study group that examined children's thinking through direct interaction and video displayed the most profound change in their beliefs about mathematics, mathematics learning and mathematics instruction. This group also experienced the highest gains in mathematical content knowledge. Surprisingly, the study group that visited the classrooms of specially selected teachers experienced the smallest change in beliefs even when compared to the control group.

Still other researchers have studied the changing beliefs of teachers through the analysis of interview transcript data. Grant, Hiebert and Wearne (1998) motivated by Guskey's (1986) observation that "significant change in teachers' beliefs and attitudes is likely to take place only after changes in student learning outcomes are evidenced (p. 7)" devised a program in which nine in-service elementary school teachers were asked to observe guest instructors as they taught reform-oriented lessons over the course of several weeks. Using interview data Grant, Hiebert and Wearne (1998) classified each of the participating teachers along a continuum from *skills/teacher-responsibility* to *process/student-responsibility*. They found that a teacher's particular position on their beliefs continuum influenced their interpretation of the lesson that they observed.

Teachers on the *skills/teacher-responsibility* end of the continuum tended to interpret reform lessons as confusing, even detrimental to student success. Those teachers classified in the middle of the continuum tended to correctly interpret some goals of the reform lessons, however, these goals rarely translated into reforms in their mathematics instructional beliefs which continued to emphasize skills and procedures. Lastly, those teachers on the *process/student-responsibility* end of the continuum tended to correctly interpret the goals of the reform lessons that they observed. Grant, Hiebert and Wearne (1998) concluded that "teacher development program that simply prompts teachers to observe other teachers teach will likely be of little benefit (p. 234)" without recognizing the important ways that an individual's beliefs shade interpretation. They hypothesized that one means of overcoming this obstacle might be to couple future reform efforts with structured personal reflections and collegial discussions that might confront personal beliefs more directly.

Borko et al. (1997) also found strong evidence that one's beliefs act as a filter. In a study of 14 in-service teachers participating in a staff development program, they found that teachers whose beliefs were contrary to the goals of the program tended to either ignore or improperly apply the new ideas presented. Like Grant, Hiebert and Wearne (1998), Borko et al (1997) also hypothesized about the importance of challenging beliefs through personal reflection as a prerequisite for education change.

In a rich case study of four pre-service elementary school teachers participating in a teacher education program, Cooney et al. (1998) (also in Cooney, 1999) identified four archetypal perspectives that describe how these teachers hold their beliefs: isolationist, naïve idealist, naïve connectionist, and reflective connectionist. The isolationist, as the name implies, tends to insulate their beliefs from those of others, rejecting accommodation. The naïve idealist blindly accepts and absorbs the beliefs of others without reflection upon one's own beliefs. The naïve connectionist considers the beliefs of others, but fails to resolve differences in beliefs. Finally, the reflective connectionist considers the beliefs of others and resolves differences in belief through reflective thinking. The researchers posited that students holding reflective connectionist perspectives were the most likely to become reflective practitioners. They identified the goal of teacher education as the cultivation of reflective connectionist perspectives with regard to mathematics and mathematics instruction. Their research, however, found only one in four students attained this perspective at the end of his training. They suggested

> The inculcation of doubt and the posing of perplexing situations would seem to be central to the promotion of movement from being a naïve idealist or even isolationist to becoming a connectionist. Inciting doubt and making the previously unproblematic problematic can have significant impact on a person's world and lead to varied and perhaps unsettling responses. It is not enough to make mathematics and teaching problematic for teachers. We need to understand the effect of this inculcation of doubt and also understand the kind of support that teachers need to make sense of it (Cooney, 1998, pp. 330-331).

They concluded their research with a call for research addressing the relationship between activities employed in teacher education programs and the effect of these activities on the belief systems of preservice teachers.

Mewborn (1999) studied the role of reflection on changing beliefs in her study of four preservice elementary school teachers participating in a field-based course in mathematics pedagogy. She found that teachers need support to become fully reflective in issues related to teaching mathematics, issues in children's mathematical thinking, and issues in mathematical content. She found that reflectivity in these areas was largely dependent upon these teachers' development of an internal locus of control with regard to mathematics content and pedagogy. Mewborn (1999) suggested five elements for any program aimed at producing reflective mathematics teachers through the encouragement of an internal locus of control:

1. Create and maintain an inquiry perspective
2. Allow for a community of learning
3. Maintain a focus on instruction
4. Restrict to subject-specific experiences (pp. 338-339)

Mewborn (1999) concluded her piece with a call for teacher education reform, calling on teacher educators to avoid the presentation "of knowledge, beliefs, attitudes, experiences and expectations about mathematics teacher education into some easily digestible form and expect the preservice teachers to make it their own" (p. 339-340). She instead suggests that teacher educators design programs that allow students to "interact with their own knowledge, beliefs, attitudes, experiences and expectations to develop their interpretations and understandings of mathematics teaching" (p. 340).

Addressing the challenges presented to new teachers by recent mathematics reform documents (i.e. NCTM, 1989; NCTM, 2000) some researchers have focused their attention on those beliefs that are seen as obstacles to reform. Cooney (1999) in his large study of prospective teachers found three common beliefs held by such students that are at odds with the mathematics reform movement: a dualistic (right versus wrong) conception of the subject of mathematics, a belief that a caring teacher should minimize student struggles in the subject through strict adherence to rules and procedures, and a belief that the act of teaching equates with the act of telling.

Chazan and Ball (1999) examined the traditional notion of “teaching as telling” in expert teachers. These researchers found that constructivist reform in mathematics education must go beyond the pathological identification of “teaching as telling” by providing alternatives to traditional teaching methodologies which allow for students to create, discover and generate their own mathematical knowledge. That is, the exhortations “teaching is not just telling” and “learning is not just listening” championed by the reform movement in mathematics education must be accompanied by new reform-movement-inspired examples teaching and learning in order to earn the consideration of expert teachers.

Similarly, Smith (1996) found that the tenets of the mathematics reform movement challenge traditional notions of teacher efficacy which are tied to traditional beliefs about the subject. He found that most teachers think of mathematics as a fixed set of facts and procedures and tend to place the authority for school mathematics in the textbook or curriculum. Teachers think of themselves, then, as a sort of intermediary where student procedural competency serves to reinforce a teacher’s notion of self efficacy. This phenomenon, Smith (1996) noted, promotes the notion that teaching equates with telling. Based on this finding, Smith (1996) suggested that teachers be made more aware of the principles of reform movement in mathematics both in their preparation and professional development. Additionally, Smith (1996) called for significant changes in teacher education programs to help educators to reconceptualize their notions of efficacy in light of the reforms in mathematics education.

CHAPTER 3: THEORETICAL FRAMEWORK

This chapter describes the theoretical framework that serves as the foundation for this study. The author's own conception and adopted notions of the term "belief" are presented. This is followed by a description of mathematical beliefs held on a scale that ranges from formal to informal. Next there is a description of the author's framework for accessing beliefs of prospective elementary school teachers using both quantitative and qualitative approaches. Finally, a short personal biography details the genesis of the study and points to its intended outcome.

BELIEF – A WORKING DEFINITION

While many researchers in mathematics education make use of the word "belief" few take time to define the term. In an effort to avoid such confusion this researcher presents the following working definition (adapted from Phillip, 2007):

> A belief is any psychologically held proposition about the world that is thought to be true.

Three salient characteristics of belief structures include the following (adapted from Green, 1971; Torner, 2002; Phillip, 2007):

1. Beliefs are held in clusters. Individual clusters are held independent and isolated from one another.
2. Within belief clusters, an individual belief exists on a scale from primary to derivative according to its quasi-logical relationship to other beliefs
3. Within belief clusters, an individual belief exists on a scale from central to periphery according to its degree of conviction in relationship to other beliefs

Beliefs have an affective component but are more cognitively held (i.e. reasoned) than emotions and attitudes. On the other hand, beliefs differ from knowledge in that knowledge is belief held with certainty (i.e. justified true belief) where there is "general agreement about procedures for evaluating and judging…validity" (Thompson, 1992, p. 130), whereas beliefs are held with an awareness of the non-consensuality or disputability of the proposition.

MATHEMATICAL BELIEFS

Rene Thom (1973) noted that "all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (quoted in Thompson, 1992) which is rooted in one's beliefs about the nature of the subject. Thus, a framework of beliefs in mathematics and mathematics pedagogy is necessary before any meaningful research on the subject can be conducted. This researcher adopts a general and dualistic framework for the study of beliefs in mathematics that was first used by Collier (1972) and again by Seaman (2005). The framework envisions one's beliefs regarding mathematics on a scale that ranges from *formal* beliefs on one end to *informal* beliefs on the other.

Formal beliefs about the nature of mathematics identify the subject as one of procedures. Mathematics consists of rules, algorithms, and formulas which are hierarchically organized according to various cannons (e.g. arithmetic, algebra, geometry). Knowing mathematics is then evidenced through knowledge of and proficient performance of these procedures (e.g. times tables, the quadratic formula, the Pythagorean Theorem). Teaching mathematics is then conceived as a teacher-centered activity in which the teacher provides a clear presentation of procedures and encourages students to acquire these skills through individual drill and practice.

Formal beliefs in mathematics most closely align Skemp's (1987) "rules without reasons" *instrumental* characterization of the subject in which possession of a rule is equated with knowing mathematics. This notion of mathematics is also similar to Ernest's (1988) description of *instrumentalist* beliefs, Raymond's (1997) *traditional* beliefs, Kuhs and Ball's (1986) description of the *content focused with emphasis on performance* approach to teaching the subject, Grant, Hiebert and Wearne's (1998) *skills/teacher-responsibility* perspective and Barkatsas and Malone's (2005) traditional-transmission-information-processing orientation.

Informal beliefs about the nature of mathematics identify the subject as one of creative and investigative processes. Mathematics consists of the processes of problem-solving, proof and reasoning, communication, connection and representation (NCTM, 2000) among others. Knowing mathematics is evidenced through active and successful engagement in these processes. Teaching mathematics then is conceived as a student-centered activity in which the teacher facilitates student construction of mathematical knowledge through activities that are inherently exploratory and open-ended.

Informal beliefs about the nature of mathematics are most closely aligned with Skemp's (1987) *relational* understanding, Ernest's (1988) *problem solving* view, Raymond's (1997) *nontraditional* beliefs, Kuhs and Ball's (1986) *learner focused* approach to teaching the subject, Grant, Hiebert and Wearne's (1998) *process/student-responsibility* perspective and Barkatsas and Malone's (2005) contemporary-constructivist orientation. Informal beliefs about mathematics also align with constructivist views of teaching and learning (Von Glasserfeld, 1989) which are central in the current reform movement in mathematics education and promoted by NCTM's (2000) Principles and Standards for School Mathematics.

Table 3.1: Summary of formal versus informal beliefs in mathematics.

	Formal Beliefs	Informal Beliefs
Mathematics consists of…	rote procedures: rules, algorithms, formulas	creative and investigative processes: proof and reasoning, problem solving, communication, connections, representation
Mathematics is characterized as…	absolute, fixed, certain, predictable, applicable	dynamic, expanding, surprising, relative, doubtful, aesthetic
Knowing mathematics consists of…	individual memorization, mastery of facts, rules, algorithms and formulas	the ability to actively reason, prove, problem solve, communicate, connect and represent mathematically
Teaching mathematics consists of…	a teacher-centered activity focusing on the clear transmission of rules, algorithms, formulas	a student-centered activity focusing on student construction of mathematical knowledge

It should be noted that the scale that the researcher employs makes no epistemic claims as to the certainty or fallibility of mathematical knowledge which are commonly attached to formal-informal characterizations of the subject (i.e. Lakatos, 1976; Ernest, 1988; Hersh, 1989). And while debates in foundational beliefs about the subject of mathematics are often yoked to the current reform agenda embodied by informal notions of the subject as some have indicated (i.e. Thompson, 1992), the researcher chooses not to orientate this study in the direction of such a debate.

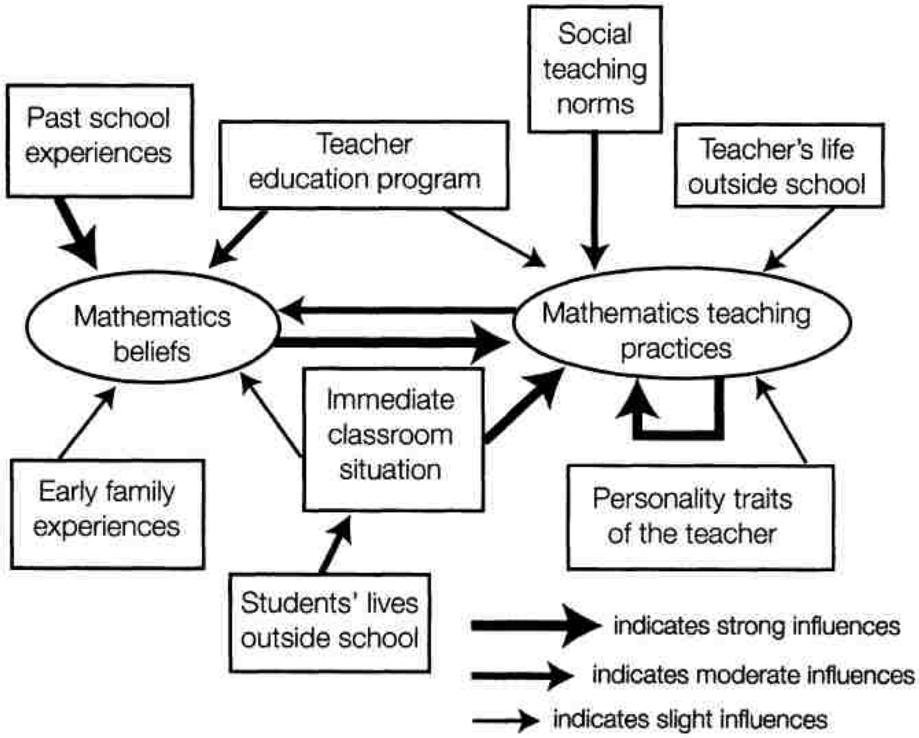
It should also be noted that although the reform movement has called for a shift towards informal beliefs as this researcher has defined them, it does not deny the importance of procedural understanding in the subject. As NCTM (1989) makes clear

> We do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity. It is clear that the fundamental concepts and procedures from some branches of mathematics should be known by all students….But instruction should persistently emphasize "doing" rather than "knowing that." (p. 7)

The researcher agrees with this notion and envisions the formal-informal continuum as one where traditional authoritarian and teacher dominated notions of the subject are being challenged by a reform movement which advocates a *shift* towards student-centered instruction which emphasizes active construction of knowledge. That such construction can result in the discovery of a rule or procedure is not inconsistent with informal beliefs concerning mathematics.

The researcher envisions the beliefs about mathematics and mathematics instruction of prospective elementary school teachers as dynamic, changing and subject to influence. Past school experience, teacher education programs, family experiences, student teaching experiences all inform these students' beliefs about mathematics and mathematics instruction. Raymond's (1997) model for the interaction of mathematics beliefs and practice is adopted as framework for this study. Raymond's (1997) framework is summarized in the following figure:

Figure 3.2: Raymond's (1997) model of the relationship between mathematical beliefs and practice



As the model indicates, student's beliefs about mathematics (Raymond includes both mathematics and mathematics pedagogy in this heading) are most strongly influenced by past school experiences in the subject. And while researchers have attempted to influence beliefs through the examination of student learning (i.e. Fennema et al., 1996; Phillip et al., 2007) the researcher has encountered no studies conducted that document the changing beliefs of teachers brought about through examining and reflecting upon their own acquisition of new mathematical knowledge. Given the larger influence that Raymond (1997) ascribes to "past school experiences" over "teacher education programs" such an investigation appears promising in terms of influencing the formal beliefs of prospective teachers.

ROLE OF RESEARCHER

The author views his role in this study as a social scientist conducting experimental research that examines the relationship between school experiences and the psychological constructs regarding the nature of mathematics and mathematics instruction in prospective elementary school teachers. In particular, the researcher seeks to identify and describe the relationship, if any, between the participation in informal mathematical activities in a classroom setting and beliefs about mathematics and mathematics instruction.

The researcher views the quantitative aspects of the study as a classical pretest-posttest study. Such studies typically divide the population into control and experimental groups and some pretest measurement is given to all participants. The control group is then given no treatment or a placebo treatment. The experimental group is given some real treatment. After the treatment has been administered, both groups are given a posttest measurement. Analysis consists of a statistical comparison of pre and posttest assessments for the two groups. Any differences observed are assumed to be the result of the action imposed on the treatment group in the study. Ideally such experiments incorporate the random assignment of participants to minimize confounding variables that might otherwise explain the response observed. (Box, 1978) For the research conducted here, this quantitative approach is expected to indicate whether participation in informal mathematical activities is linked to any change in the mathematical beliefs of prospective elementary school teachers participating in the study.

The researcher views the qualitative aspects of the study as an analysis of written student reflection data for evidence of certain concepts and themes (see Corbin and

Strauss, 2008). This method of research involves the process of open coding in which the researcher analyzes transcript data for low-level concepts pointing to high-level categories or themes. Corbin and Strauss (2008) describe this process as threefold: first the data are broken into manageable pieces, next these pieces are explored for the ideas contained within them (interpreted), finally these ideas are given conceptual names representing the ideas contained within them (p. 160). The process is one in which the researcher approaches the data without preconceived notions of expected themes or categories choosing to let the raw data "speak for themselves" and indicate those concepts that point toward higher-level themes. For the research conducted here, this qualitative approach is expected to result in a categorical description of the personal experiences of participation in informal mathematical activities incorporated in preservice elementary teacher education courses. This categorical description, in turn, is expected to provide a basis for the assessment of each activity in terms of its contribution to the transformation of formal to informal mathematical beliefs of preservice elementary school teachers. Lastly, the qualitative aspects of this research are also meant to triangulate the quantitative aspects of this research and provide a measure of reliability.

RESEARCHER BIOGRAPHY, BELIEFS AND THE RESEARCH QUESTION

The researcher holds an undergraduate degree in mechanical engineering, a master's degree in education, and has completed all coursework towards a doctorate in mathematics including preliminary exams in abstract algebra, statistics and mathematics education. The researcher's teaching experience in mathematics includes two years of tutoring the subject to university students, one year of middle school instruction, two years of high school instruction and four years of university instruction. The researcher has taught a variety of subject matter in mathematics from real number arithmetic in middle school to beginning algebra in high school to precalculus, statistics and mathematics for elementary school teachers at the university level.

The researcher's personal beliefs about mathematics and mathematics teaching are those embodied in the current reform movement in mathematics encapsulated in the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000). The researcher agrees with NCTM's (2000) call for a student-centered classroom where "effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 17). The researcher also supports the constructivist notions of learning mathematics that are embodied in the document: for students to learn mathematics with understanding requires "actively building new knowledge from experience and prior knowledge" (p. 20). These notions are incorporated in the researcher's statement of pedagogical beliefs which includes the following passage:

> I believe that the best methods of education allow for direct encounters. Educators should incorporate experiences that provide the opportunity for fuller understanding of self through reflection, society through social interaction, and the natural world through direct physical exploration and experimentation. I believe that best practices in education recognize that interaction should precede abstraction, expression come before impression, and exploration before interpretation. I believe that true education is never described by activities that are passive, receptive, or absorbing, but rather is more suitably described by activities that are inherently open-ended and self-motivated, inviting the learner to explore, conjecture and experiment. (Researcher's Statement of Pedagogical Beliefs)

In short, the researcher would characterize his beliefs about mathematics and mathematics instruction as *informal* beliefs. That is, the researcher views mathematics as a creative and investigative process involving proof and reasoning, problem solving, communication, connection and representation (NCTM, 2000). These processes give rise to the areas of algebra, geometry, number and operation, probability and statistics, and measurement. The researcher believes that mathematics is a dynamic, expanding area of study that is often surprising and relative, often incorporating a good deal of tentative doubt and uncertainty. The researcher believes that mathematics is aesthetically pleasing. The researcher believes that knowing mathematics equates with the ability to reason mathematically, prove (or disprove) mathematical statements, solve problems that are inherently mathematical in nature, communicate mathematically, make mathematical connections and construct mathematical representations. As such, the researcher believes that teaching mathematics is an inherently student-centered activity where the teacher facilitates student construction of mathematical knowledge through activities that encourage exploration and conjecture which are resolved through communication, connection, representation, or reasoning and proof. To use a well-worn colloquialism: the researcher believes that the teacher of mathematics should conceive of himself as the "guide on the side" and not the "sage on the stage".

The researcher traces the genesis of the research question to his first experience teaching the course "M135: Mathematics for Elementary School Teachers I" in the spring of 2005 and "M136: Mathematics for Elementary School Teachers II" in the fall of 2006. These courses are intended for prospective elementary school teachers and meet the nine credit-hour mathematics requirement for state certification to teach kindergarten through eighth grade. The course learning goals for M135 are

1. To identify and solve problems in elementary mathematics.
2. To model the number systems: natural numbers, integers, rationals and reals.
3. To become familiar with the use of manipulatives to enact arithmetic operations.
4. To apply basic problem-solving strategies to ratio, proportion and percent problems.
5. To use mathematical modeling and basic algebra to approach real world problems.
6. To solve problems using probability and statistics including designing simulations.
7. To communicate mathematics both in oral and written form.

The course learning goals for M136 are

1. To identify and solve problems in elementary geometry
2. To model the logic of arguments involving parallelism, congruence, and similarity
3. To use basic measurement to approach problems involving length, area, and volume.
4. To explore, conjecture, and prove mathematical ideas and theorems involving geometry.
5. To perform classical compass-straightedge constructions
6. To develop a facility with geometric theorems and proofs through hands-on and computer explorations

These goals make it clear that the department advocates an *informal* approach to mathematics. In M135 note the emphasis on *problem solving*, *communication*, *modeling*, and the use of *manipulatives*. In M136 note the emphasis on *logic*, *exploration*, *conjecture*, *proof and reasoning*, and *construction*. Notably absent from these goals are any mention of rules, procedures, algorithms which characterize purely *formal* beliefs in mathematics. And while such procedures are surely taught in the course, it is clear that they are arrived at via informal processes that are "hands on" and "exploratory" and incorporate "proof", "reasoning" and "problem solving".

It was the researcher's experience that these reform-oriented *informal* beliefs about mathematics and mathematics instruction embodied in the course goals were met with considerable resistance due to the *formal* beliefs of the students enrolled the course. In candid conversations with prospective teachers enrolled in the course, the researcher discovered that many doubted the informal perspective that the course advocated. For example, students characterized the “chip model” for concretely modeling integer operations as “confusing” or “too difficult” or “distracting” or even “making it harder to learn” and many confessed that they would not use it in their future classrooms. In another anecdote, a student recounted what she believed was a “better” device for learning the rules of integer multiplication and division:

When good things happen to good people it is a good thing	Therefore	positive times positive equals positive
When good things happen to bad people it is a bad thing	Therefore	positive times negative equals negative
When bad things happen to good people it is a bad thing	Therefore	negative times positive equals negative
When bad things happen to bad people it is a good thing	Therefore	negative times negative equals positive

While the device certainly facilitates the learning of the “rules” of integer multiplication it provides no sense of understanding of mathematical reasoning that necessitates that “negative times negative equals positive”. The device might even be characterized as providing a misunderstanding of integer multiplication, providing justification for the mathematical result through an erroneous moral code.

Still other students found the logic of classical compass and straight edge constructions, the bisection of a segment for example, “unimportant” compared to more skills-based content, such as the rules of arithmetic. One graduate student enrolled in M136 even challenged the inclusion of right triangle trigonometry in the course stating that teachers “don’t need to know” such material since they are primarily tasked with “teaching the rules of addition, subtraction, multiplication and division”.

It was this “clash of beliefs” that led the researcher to investigate the role of beliefs in mathematics education (see Literature Review). Reflecting upon the well-established beliefs of prospective elementary teachers which are informed by 12 years of school mathematics which traditionally emphasizes *formal* approaches to the subject, the researcher questioned whether the inclusion of *informal* activities in the course might have some bearing on the *formal* beliefs of these teacher candidates. Specifically, the researcher wondered whether the process of acquiring new mathematical knowledge through *informal* investigation of mathematical objects might provide a basis for self-reflection on the topic of knowing and understanding mathematics. The researcher envisioned an activity in which students would be given an assignment that required them to investigate a mathematical object, form a conjecture and then prove the conjecture. Once completed, students would be asked to *use their own experience* as a basis for personal reflection regarding the process of acquiring new mathematical knowledge to gain insight into the nature of mathematics and the optimum environment for mathematics teaching and learning.

CHAPTER 4: METHODOLOGY

The purpose of the study was an investigation of how the beliefs about mathematics and mathematics instruction of preservice elementary school teachers change in association with participation in informal mathematics activities and personal reflection upon the construction of new mathematical knowledge.

THE SETTING

The study was carried out during the summer and fall semesters of 2009 and the spring semester of 2010. The study took place at The University of Montana, located in Missoula, Montana. In 2009, the United States Census Bureau estimated that Missoula was a city of 62,982 people. Ethnically, the town was reported to be comprised of a population that was 92.3% white and non-Hispanic, 1.6% American Indian or Alaska Native, 1.6% Asian, 1.2% African American, and 2.6% Hispanic (of any race). The median income per household in the town was reported to be $35,420. The median home price was reported to be $216,800.

The University of Montana is a state funded liberal arts university. In the fall of 2009, enrollment was reported as 14,921. Nearly 62% of students are from Montana. The university offers a wide range of programs of study in colleges of Arts and Sciences, Education and Human Science, Forestry and Conservation, Health Professions and Biomedical Science, Visual and Performing Arts as well as schools of Business Administration, Journalism, Law and Technology.

At the start of fall semester, 2009, The Department of Mathematical Sciences at The University of Montana consisted of 23 tenured, tenure-track and permanent faculty.

The department is home to approximately 100 undergraduate mathematics majors and 35 graduate students. Mathematics is studied in the department at both the undergraduate and graduate levels in the areas of Algebra, Analysis, Applied Mathematics (Modeling), Mathematics Education, Combinatorics and Optimization, and Statistics.

THE PARTICIPANTS

The participants in the study were preservice elementary school teacher candidates enrolled in the course Mathematics for Elementary School Teachers (Math 136). The course is the second in a series of two semester-long courses (Math 135, Math 136) that are meant to prepare preservice elementary school teachers to teach mathematics. The courses focus on both mathematical content and mathematical pedagogy. The first course in this sequence, Math 135, presents topics that include problem–solving, sets and logic, functions, whole numbers, integers, rational numbers, real numbers, number theory, probability and statistics. The course is five credit hours and typically meets five times a week for fifty minutes. The second course in this sequence, Math 136, presents topics related to the study of geometry including geometric constructions, congruence, similarity, measurement, coordinate geometry and an introduction to computer geometry. The course is four credit hours and is offered in two formats: meeting four times a week for fifty minutes or two times a week for an hour and forty minutes.

Students enrolled in Math 136 are either elementary education majors who have been admitted to the School of Education at The University of Montana or pre-elementary education majors who have not yet been admitted to the School of Education. All elementary education majors are required to pass the course in order to fulfill the requirements of their degree and to obtain teacher licensure in the state of Montana valid for kindergarten through eighth grade.

Prerequisites for enrollment in Math 136 are the successful completion of an introductory algebra course or equivalent score of the University of Montana placement exam as well as successful completion of Math 135. Successful completion is defined as a grade of C or better.

Participants in the study were enrolled in Math 136 during the fall of 2009 or the spring of 2010. There is reason to believe that students taking the course in the fall and students taking the course in the spring are not a homogeneous group. Students taking the course in the spring are more likely to have taken and passed the course's prerequisite, Math 135, the previous fall. Since the two course sequence, Math 135 followed by Math 136, is suggested by the school of education in the first and second semesters of study, it is reasonable to assume that those students taking Math 136 in the spring semester tend to be freshmen who have not experienced any difficulty in fulfilling the suggested course sequence. Conversely, students who enroll in Math 136 in the fall are more likely to have experienced some difficulty in fulfilling the suggested course sequence intended for full-time freshmen students. It is reasonable to assume that students enrolled in the fall are more likely to have taken a remedial course in mathematics, are more likely to have failed the course's prerequisite Math 135, and are more likely to have failed Math 136. This issue is explored in the conclusion.

THE PILOT STUDY

A pilot study was carried out in the summer of 2009 to determine the feasibility of the study. Specifically, the pilot sought to investigate:

i. The compatibility of chosen reflective mathematics activities and their appropriateness to the mathematical abilities of the study population and the need to either “scale up” or “scale down” the mathematical component of each activity.

ii. The compatibility of the Collier (1972) Likert scale beliefs assessment instrument to the study population and the need to revise the instrument.

iii. The effectiveness of student reflection in revealing personal beliefs about mathematics and mathematics instruction.

The pilot was carried out in four consecutive two-hour class periods with 8 students enrolled in M136. At the start of the pilot, students were asked to complete two 20-question Likert scale beliefs assessment instruments which were first used by Collier (1972) and later by Seaman (2005). One of these instruments measured beliefs about mathematics while the other instrument measured beliefs about mathematics instruction. Students were again asked to complete the two instruments at the conclusion of the study to determine if their beliefs in the two areas had responded to their participation in the pilot study. In the four class periods between the administration of the pre and post beliefs assessment instruments, students participated in four reflective mathematics activities. Students were asked to attempt to complete a mathematical task and then provide a written reflection describing how they approached the problem and how their own learning informed them as a future teacher.

The results of the pilot study demonstrated that the four chosen mathematical activities (detailed in the following sections) were appropriate for the mathematical abilities of the study population. It was decided, however, to change the order in which the four activities would be presented in order to proceed more developmentally according to the perceptions of the pilot group from activities which were viewed as "easier" to those which were perceived as "harder".

The results of the pilot study demonstrated that the Collier (1972) beliefs assessment instruments were, in large part, compatible for measuring the beliefs of the study population. One item was identified as confusing to students. In the original Collier (1972) study it appears as:

> Many of the important functions of the mathematician are being taken over by the new computers (Collier, 1972, p. 156)

Students participating in the pilot study were perplexed by the phrase "the new computers" which reflects the relatively "new" status of computers at the time of the original study. In an effort to maintain comparability to the results obtained by both Collier (1972) and Seaman (2005) the researcher decided not to omit the item but to edit the item to read:

> Many of the important functions of the mathematician are being taken over by new computers

The pilot group found this wording to be acceptable. All other items in the original Collier (1972) beliefs assessments instruments were adopted for use in the larger study. These instruments are detailed in the following section.

The results of the pilot study demonstrated that students needed more guidance in the process of reflecting upon the acquisition of mathematical knowledge as a means of informing future practice as a mathematics teacher. While some students did generalize the methods and processes that they used in solving the four mathematical activities, many others simply focused on the problem solving aspect and made no attempt to translate their personal experience into a set of principles that might aid their future students in the acquisition of mathematical knowledge. It was decided that a more detailed prompt would be used in the collection of reflection data in the larger study. This prompt is detailed in the final section of this chapter.

THE COLLIER BAM AND BAMI INSTRUMENTS

Collier's (1972) instruments for the measurement of beliefs about mathematics were used in this research. The instruments are meant to measure an individual's beliefs about mathematics (BAM) and an individual's beliefs about mathematics instruction (BAMI) on a formal-to-informal scale. Formal beliefs in mathematics are those that emphasize a notion of mathematics and mathematics instruction that is driven by procedural knowledge of facts, rules, and algorithms. Informal beliefs in mathematics are those that emphasize a notion of mathematics and mathematics instruction that is driven by processes that are investigative, creative, and intuitive. As Seaman (2005) notes, "the survey is a reasonable measure of the constructivist philosophy and ideas about instruction that flow from this philosophy" (p. 198) which are at the heart of the current reform movement in mathematics. As such, Collier's (1972) BAM and BAMI instruments provide a means of measuring the beliefs of prospective teachers in terms of their agreement or disagreement with constructivist notions of mathematics as a subject as well as constructivist notions of teaching and learning mathematics.

Each of the two Likert-scale instruments that Collier (1972) authored consists of twenty statements: ten describe mathematics or mathematics instruction formally; ten describe mathematics or mathematics instruction informally. In his original work, Collier (1972) described the method by which these statements were chosen from an initial pool of 80 candidate statements according to the measure of their internal consistency with other like items using a procedure described by Nunnally (1967) and Winer (1962). Collier noted that the reliability of both scales exceeded the minimum standard proposed by Nunnally (1967).

Participants were asked to respond to each of the 40 items on an integer scale ranging from one to six. A response of one indicated “strongly disagree”, a response of two indicated “moderately disagree”, a response of three indicated “slightly disagree”, a response of four indicated “slightly agree”, a response of five indicated “moderately agree”, and a response of six indicated “strongly agree”. The researcher viewed this scale as continuous.

Responses were then used to form a composite score ranging from a low score of 20 to high score of 120 representing the location of the beliefs of the respondent on the formal-informal scale. Here a low score represented more formal beliefs and a high score represented more informal beliefs about mathematics and mathematics instruction. This composite was computed by summing the responses assigned by the respondent in one of two ways: if the item characterized mathematics informally, then the response value was summed in the composite score according to the number assigned by the respondent; if the item characterized mathematics formally, then the response assigned by the respondent was subtracted from seven (to reverse the scale) and then summed in the composite score.

Table 4.1: Beliefs About Mathematics (BAM) Items

Informal Items – Positively Scored	Formal Items – Negatively Scored
2. There are several different but appropriate ways to organize the basic ideas in mathematics.	1. In mathematics, perhaps more than in other fields, one can find set routines and procedures.
4. In mathematics, perhaps more than in other areas, one can display originality and ingenuity.	3. The laws and rules of mathematics severely limit the manner in which problems can be solved.
5. There are often many different ways to solve a mathematics problem.	6. Mathematicians are hired mainly to make precise measurements and calculations for scientists.
7. The field of mathematics contains many of the finest and most elegant creations of the human mind.	9. In mathematics there is usually just one proper way to do something.
8. Studying mathematics helps to develop the ability to think more creatively.	10. Mathematics is an organized body of knowledge which stresses the use of formulas to solve problems.
12. The basic ingredient for success in mathematics is an inquiring nature.	11. Solving a mathematics problem usually involves finding a rule or formula that applies.
14. Mathematics requires very much independent and original thinking.	13. Many of the important functions of the mathematician are being taken over by new computers.
15. There are several different but logically acceptable ways to define most terms in mathematics.	17. The language of mathematics is so exact that there is no room for variety of expression.
16. Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.	18. Mathematics is a rigid discipline which functions strictly according to inescapable laws.
20. Mathematics has so many applications because its models can be interpreted in so many ways.	19. The main benefit from studying mathematics is developing the ability to follow directions.

Table 4.2: Beliefs About Mathematics Instruction (BAMI) Items

Informal Items – Positively Scored	Formal Items – Negatively Scored
3. Students should be encouraged to invent their own mathematical symbolism	1. The teacher should always work sample problems for students before making an assignment
4. Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	2. Teachers should make assignments on just that which has been thoroughly discussed in class
5. Each student should feel free to use any method for solving a problem that suits him or her best	7. Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery
6. Teachers should provide class time for students to experiment with their own mathematical ideas	8. Most exercises assigned to students should be applications of a particular rule or formula
10. Teachers should frequently insist that pupils find individual methods for solving problems	9. Teachers should spend most of each class period explaining how to work specific problems
13. The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	11. Discovery methods of teaching have limited value because students often get answers without knowing where they came from
14. The teacher should consistently give assignments which require research and original thinking	12. The teacher should provide models for problem solving and expect students to imitate them
15. Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	17. Students should be expected to use only those methods that their text or teacher uses
16. Teachers must frequently give students assignments which require creative or investigative work	18. Discovery-type lessons have very limited value when you consider the time they take up
20 Students of all abilities should learn better when taught by guided discovery methods	19. All students should be required to memorize the procedures that the text uses to solve problems

The BAM and BAMI questionnaires were administered both at the start of the semester (during the first week of the course) and at the end of the semester (during the last week of the course). Composite scores for each participant in the study were then calculated. A class average of composite scores was computed and pre- and post-average composite scores were compared for statistical significance.

The researcher tested for statistical significance using a paired t-test of significance. Specifically the researcher tested the hypotheses

$$H_0 : \mu_d = 0$$
$$H_a : \mu_d \neq 0$$

where μ_d is defined as the average of the differences between post composite score (either BAM or BAMI) and pre composite score. The test statistic used was

$$t_{n-1} = \frac{\bar{d}}{SE(\bar{d})}$$

where $\bar{d}$ is defined as the mean of the pair-wise differences, *n* the number of participants, and $SE(\bar{d})$ the standard error for the mean of the pair-wise differences. The test statistic, t_{n-1}, was compared to the student's *t* distribution with *n*-1 degrees of freedom in order to obtain a p-value for the determination of statistical significance.

A control group was employed in the study in an effort to determine if students experienced any changes in beliefs about mathematics or mathematics instruction as a result of their enrollment in the course rather than any participation in the informal mathematical activities. The control group consisted of 18 students enrolled in Math 136 during the fall semester of 2009. These students completed the BAM and BAMI surveys at the start of the semester and again at the end of the semester; however, the control group did not complete the informal mathematics activities as part of their enrollment in the course. Students enrolled in the control section used the same text, followed a similar syllabus, and received the same number of exams over the course of the semester.

There were, however, significant differences between the control group and the groups participating in the informal mathematics activities that were not able to be corrected. The control group did not meet at the same hour of the day and experienced the course as a night class. The control group met from 5:10-7:30PM on Tuesday and Thursday evenings. Those participating in the informal mathematics activities met from 8:10-9:00AM on Monday, Tuesday, Wednesday, and Friday in the fall of 2009 and 1:10-2:00PM on Monday, Tuesday, Wednesday and Friday in the spring of 2010. Additionally, a control group study was only conducted during the 2009 fall semester, whereas the two informal mathematics groups participated during the 2009 fall semester and the 2010 spring semester. Finally, the control group was taught by a different instructor than the two informal mathematics groups. In an effort to identify any differences in the beliefs of the instructors, both the researcher and the control instructor completed the BAM and BAMI surveys.

It should be pointed out that the three *classes* which participated in this research can be identified as the experimental units to which the different instructional treatments were applied. That is, the fall 2009 and the spring 2010 informal groups received a significant portion of instruction which involved creative and investigative assignments as part of regular coursework while the fall 2009 control group did not experience the creative and investigative assignments. In contrast, the quantitative analysis of the BAM and BAMI instruments treats each *student* as an experimental unit which introduces potential pseudoreplication issues. Furthermore, none of the classes participating in this study can be considered a random sample of all preservice elementary school teachers enrolled in the university where the study took place. Potential pseudoreplication and the non-random nature of the sample of students participating in the study limit the results of this research. To be clear: the results presented here are strictly limited to the population participating in the study and do not generalize to the larger population of preservice elementary school teachers.

It should also be pointed out that the use of a paired *t*-test of statistical significance employed for the analysis of both BAM and BAMI scores assumes students' beliefs are independent of one another. That students' beliefs are independent of one another is questionable. Research has shown that social factors do indeed correlate with beliefs about mathematics and mathematics instruction (i.e. Benken, 2005; Archer, 1999; and Hannula et al. 2005, 2006, 2009). Furthermore, this research is carried out in a setting where social collaboration is encouraged and even required. The researcher wishes to acknowledge the unavoidability of the interaction of the beliefs of students involved in this study and the likelihood of the violation of independence that are assumed in statistical procedures employed.

THE INFORMAL MATHEMATICS ACTIVITIES

This research investigates the role of informal mathematics activities which incorporate personal reflection on the beliefs of prospective teachers. An informal mathematics activity was defined as a mathematical problem solving activity which emphasizes the creative and investigative processes in mathematics, requiring the student to communicate mathematically, to represent mathematically, to reason, to prove, and to make mathematical connections.

It was decided by the researcher that four informal activities would be incorporated into *M136: Mathematics for Elementary School Teachers II* as part of regular study in the course. As requirements in the course, the activities were chosen to address some element in the course objectives. The objectives for the course were as follows:

1. To identify and solve problems in elementary geometry
2. To model the logic of arguments involving parallelism, congruence, and similarity
3. To use basic measurement to approach problems involving length, area, and volume.
4. To explore, conjecture, and prove mathematical ideas and theorems involving geometry.
5. To perform classical compass-straightedge constructions
6. To develop a facility with geometric theorems and proofs through hands-on and computer explorations

In addition to addressing course objectives, the activities also had to be *informal* in nature, that is, the activities needed to stress the creative, investigative and constructive processes of mathematics and stand in contrast to *formal* activities which stress conformity, repetition and rule-following.

It was decided that the activities would be chosen to address each of the following four themes in the course:

Basic Notions of Geometry: points, lines, planes, angles, polygons, 3 dimensional solids

Congruence and Similarity: of lines, angles, triangles

Measurement: length, area, volume

Motion Geometry: translation, rotation, reflection, dilation, glide-reflection, tessellation

After conducting a search of textbooks and practitioner journals the following four activities were chosen for incorporation into the course:

1. Discovery and proof for the 11 nets of the cube
2. Discovery and proof of the inscribed and central angle relationship in the circle
3. Discovery and proof of the solution to the Isis problem
4. Discovery and proof of the 8 semi-regular tessellations

Each of these activities is detailed in the sections that follow.

The implementation for each of the four reflective mathematics activities was similar. Students were placed in groups of four to five and given the problem statement at the start of the class period. They were then given one 50-minute class period to investigate the problem as a small group. The instructor (who is also the researcher) circulated around the classroom during this time: answering questions, offering encouragement, clarifying statements. After this period of group-work a period of three weeks was given for the students to continue working on the activities. At the end of this three week period, each individual student was required to hand in their solution to the activity. Students were allowed to collaborate in constructing their argument but were required to submit separate solutions.   Students were asked not to access outside information resources (books, texts, internet, etc.) in their problem-solving process but rather to make an "honest attempt" at solving each of the activities relying only on knowledge gained from the course and collaboration with peers.

NETS OF THE CUBE: PROBLEM STATEMENT

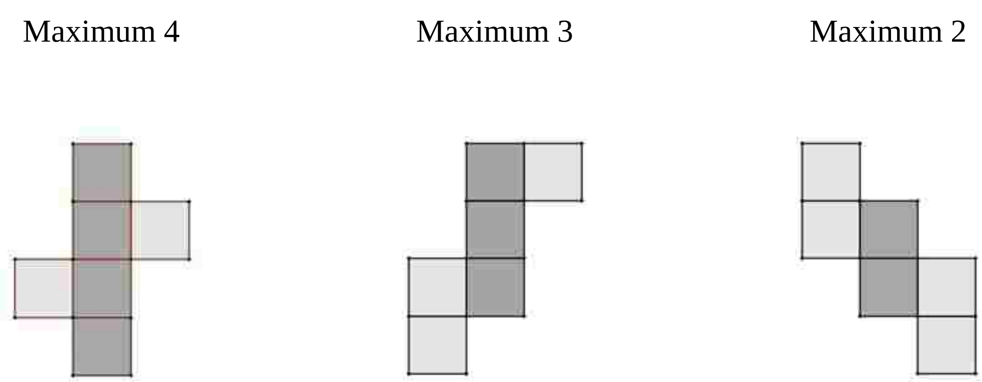
The discovery and proof of the 11 nets of the cube was the first informal mathematics activity that was presented in the course. Students were given a printed worksheet (an exact copy of the sheet appears in the appendix) that described the term "net" and gave several examples of the nets of the tetrahedron. The worksheet also made clear that any two nets that could be transformed one onto the other via rigid motion (some combination of translation, rotation or reflection) were considered equivalent. After familiarizing the student with the necessary background knowledge, the problem statement was presented:

> Your task in this reflective mathematics activity is to find all the distinct nets of the cube and then prove that no other nets of the cube exist.

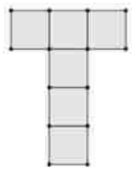
To aid in the investigation, students were provided with a set of Polydron™ manipulatives. These manipulatives consisted of snap-together regular polygons which can be used to construct any of the regular (Platonic) solids as well as many other two and three dimensional figures. In particular, the Polydron™ manipulatives can be used to create nets that can be folded and snapped together to form a cube.

NETS OF THE CUBE: SOLUTION AND PROOF

One method of proving that only 11 distinct nets of the cube exist is by method of exhaustion. It should be clear that no more than four squares can adjoin along one axis in any net of the cube or else an overlapping of faces occurs. If, for any net of the cube, we count the maximum number of faces adjoining along any axis then we can classify the nets of the cube into three subclasses where the maximum is four, three or two. Examples of members of each of these subclasses are given below.



Having identified these three subclasses, we exhaust the members in each. Examining the maximum-four subclass first, we take notice that only two squares remain to be placed. These two remaining squares cannot be placed on the axis of four else overlap occurs. Additionally, the two remaining squares cannot be placed on the same side of the axis of four or else an overlap occurs. Thus the two remaining squares must be placed on opposite sides of the axis of four. Since there are four locations that each square can be placed, there are $4 \times 4 = 16$ possible nets with a maximum of four along any axis: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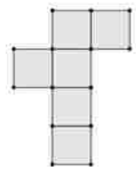
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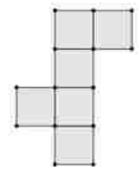
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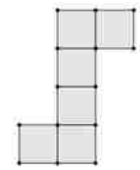
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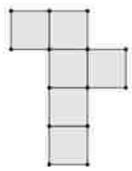
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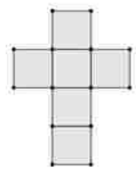
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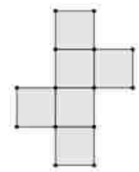
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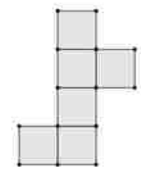
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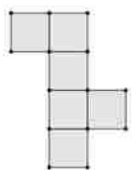
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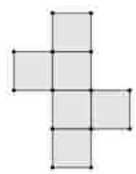
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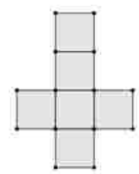
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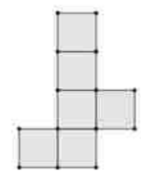
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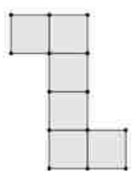
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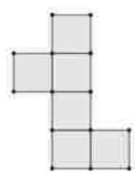
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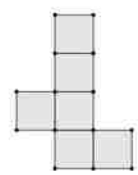
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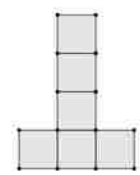
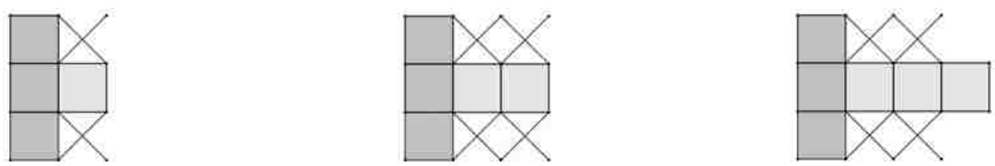
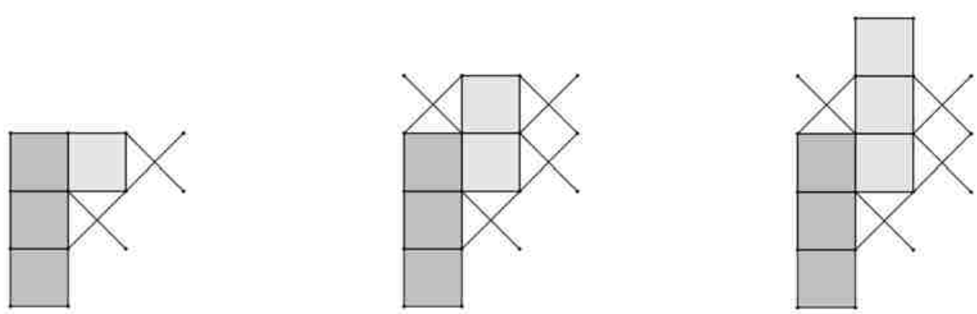

Type A

Of these 16 possibilities only 6 are unique: types A, B, C, D, E, and F.

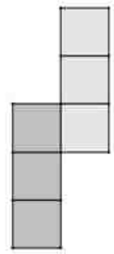
Next we examine those nets which have a maximum of three adjoining squares along any axis. We note that there are three remaining squares to be placed. As before, no square can be placed along the axis of three as this would create a maximum-four net which have already been enumerated, thus, the remaining three must be placed on one or both sides of the axis of three. There are two cases to investigate: all three remaining squares are placed on the same side of the axis of three or two are placed on one side and one square is placed on the other. We investigate the case where all three remaining squares are placed on the same side of the axis of three adjoining squares first. There are exactly three locations that the next square can be placed. If we place the square in the middle location and denote with and "X" those locations that are ruled out by overlap then this placement does not yield a viable maximum-3 net but rather a type A maximum-4 net. The inevitability of this progression is demonstrated in the following schematic:



Using a similar strategy we investigate the possible maximum-3 nets that result when all three remaining squares are placed on the same side of the axis of the three adjoining squares when the first square is placed in a location other than the middle location. A single core 3 net can be constructed in this fashion as the following schematic demonstrates:

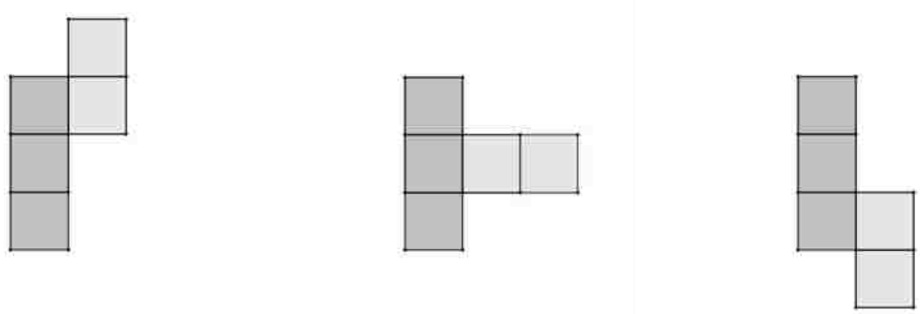


We call this first maximum-3 net type G:

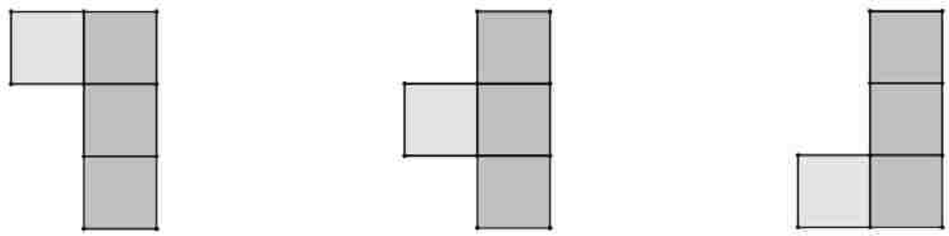


Type G

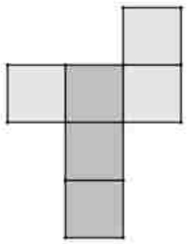
We note that starting at the lower right location (instead of upper right as above) would produce the mirror image of type G. And any placement of the three remaining squares to the left of the axis of three would produce mirror images of the placement of the three remaining squares to the right of the axis of three, thus, no other maximum-3 nets exist with all three remaining squares to one side of the axis of three. Next we investigate the possibilities if, for the three remaining squares to be placed, two are placed to one side of the axis of three and the remaining square is placed on the other side of the axis of three. Our investigation above demonstrates that if two squares are to be placed to the side of the axis of three they can be placed in only one of three ways to avoid overlap:



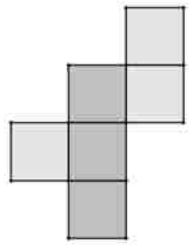
Similarly, there are three ways to place the remaining square on the opposite side of the axis of three:



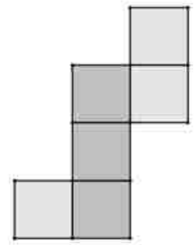
This leads to $3 \times 3 = 9$ combinations of placements in which two squares are placed to one side and the remaining square is placed on the other side of the axis of three:



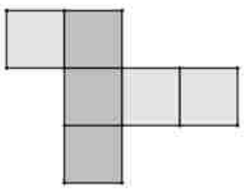
Type H



Type I



Type J



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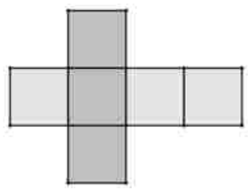
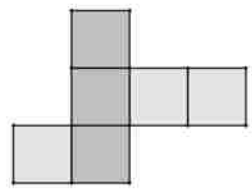


Type E



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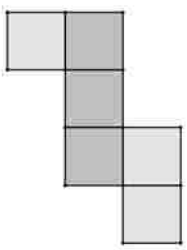

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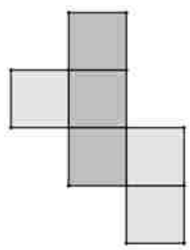
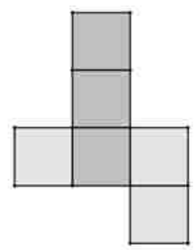


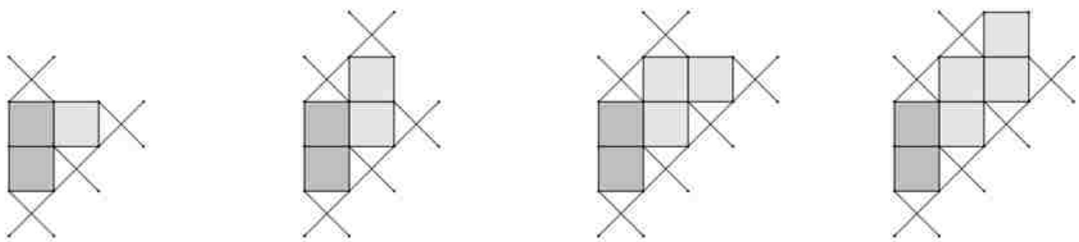
Type I



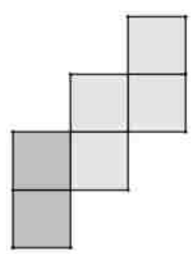
Type H

Of the nine possible nets obtained by this approach only four are unique and one (in the middle of the array) is the 4-maximum type E. Thus there are only three new nets discovered that are of the 3-maximum subclass that have two squares to one side of the axis of three and one square to the other side of the axis of three: types H, I, and J. Since there are no other ways to distribute the three remaining squares in the 3-maximum subclass, we conclude that no other 3-maximum nets exist other than types G, H, I and J.

Lastly we examine the 2-maximum subclass. We investigate the possibility of constructing a 2-maximum net that is foldable into a cube. We start with two adjoining squares and investigate which possible locations the next square can be placed while avoiding overlap as well as avoiding the creation of three or four squares in-a-row as all the 3-maximum and 4-maximum nets have already been found. These constraints lead to the creation of a single 2-maximum net seen in the following progression:



By symmetry, the choice of the placement for the first square is irrelevant, thus, all other 2-maximum nets constructed in this fashion will be equivalent. We call this last 2-maximum net type K:



Type K

Since no 1-maximum nets are possible (nets must be connected) we conclude that exactly 11 nets of the cube exist. They are six 4-maximum types A, B, C, D, E, and F, followed by four 3-maximum types G, H, I and J, and a single 2-maximum type K.

NETS OF THE CUBE: RATIONALE

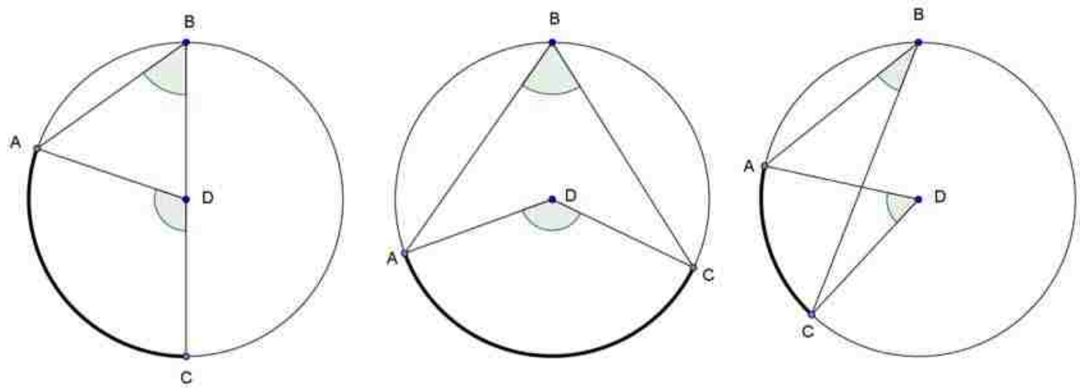
The researcher chose the discovery and proof of exactly 11 nets of the cube for this research based on a number of favorable factors. The problem nicely addressed the first theme in the course: basic notions of geometry. It incorporated notions of points, lines and planes embodied in the cube's vertices, edges and faces. It also required students to envision objects in both two and three dimensions.

The problem was judged to be an informal activity in mathematics, one whose solution requires intuition and creativity rather than the application of some rule or memorizable procedure. Indeed, the proof presented here hinges on the recognition of "classes" of nets and the intuition that these classes can be systematically exhausted. And while the division of objects into classes is a common theme in mathematics, it is rarely left up to the student to discover.

The problem could also be explored in a student-centered, hands-on fashion using the Polydron™ manipulatives. The researcher conceived that students might be able to discover the 11 nets of the cube through active investigation before attempting a mathematical proof that a *twelfth* net of the cube does not exist. This conception agreed with informal positions in mathematics education that advocate methods that are inherently exploratory and open-ended, where direct encounters provide the motivation for abstraction and mathematical structure.

INSCRIBED AND CENTRAL ANGLES: PROBLEM STATEMENT

The discovery and proof of the relationship between inscribed and central angles of the circle was chosen as the second informal mathematics activity. Students were given a printed worksheet (an exact copy of the worksheet appears in the appendix) that defined the mathematical notions of an inscribed angle and its associated central angle. Several drawings of central and inscribed angles were provided to familiarize the student with the mathematical objects that were to be investigated. The drawings provided, reproduced below, were purposely chosen as a means of demonstrating the three cases that the traditional proof (see Euclid, 2003, p. 66) of the relationship incorporates.



Those familiar with the traditional proof will recognize the three cases: the center of the circle, the point D in the figures above, lies either on the inscribed angle ABC, inside the inscribed angle ABC or outside the inscribed angle ABC. After these necessary background elements had been addressed, the following problem statement was presented:

> Your task in this reflective mathematics activity is to make a conjecture about the relationship between an inscribed angle and the central angle which subtends the same arc on any circle and then prove that conjecture.

To aid in the investigation of the mathematical relationship, students were encouraged to investigate the relationship between central and inscribed angles directly: by drawing circles with a compass, constructing central and inscribed angles with a straightedge and pencil, and measuring these angles using a protractor.

INSCRIBED AND CENTRAL ANGLES: SOLUTION AND PROOF

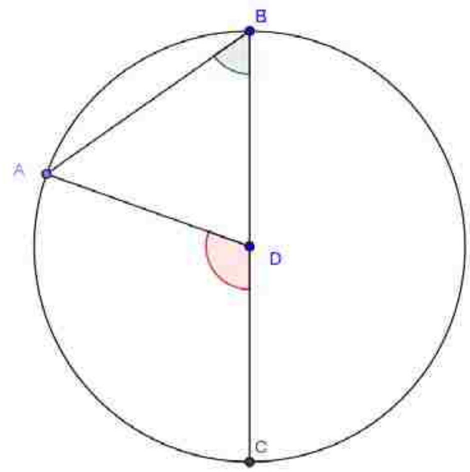
We prove that any angle inscribed in a circle is half the measure of the central angle that subtends the same arc. There are three cases to be proved:

Case 1: The center of the circle lies on a leg of the inscribed angle

Case 2: The center of the circle lies on the interior of the inscribed angle

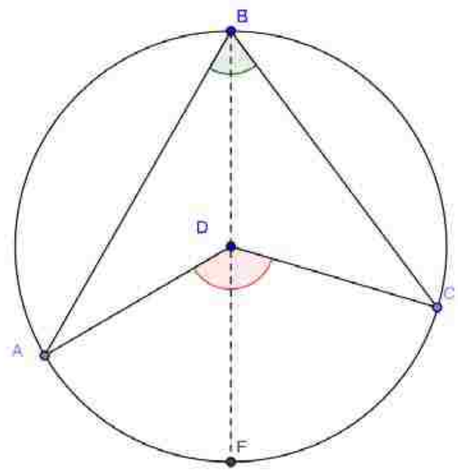
Case 3: The center of the circle lies on the exterior of the inscribed angle

Proof of Case 1: The center of the circle lies on a leg of the inscribed angle



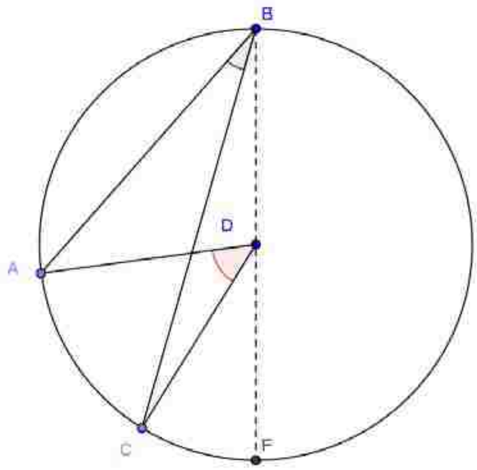
1. $\overline{AD} \cong \overline{BD}$	1. Radii of circle
2. ΔABD is isosceles	2. Definition of isosceles triangle
3. $m\angle DAB = m\angle ABD$	3. Base angles of isosceles triangles are congruent
4. $m\angle DAB + m\angle ABD + m\angle ADB = 180$	4. Triangle angle sum
5. $2m\angle ABD + m\angle ADB = 180$	5. Substitution of 3 into 4
6. $m\angle ADC + m\angle ADB = 180$	6. Straight angle
7. $2m\angle ABD - m\angle ADC = 0$	7. Subtract 6 from 5
8. $2m\angle ABD = m\angle ADC$	8. Simplification

Proof of Case 2: The center of the circle lies on the interior of the inscribed angle



1. Construct diameter passing through points B and D	1. By construction
2. $2m\angle ABF = m\angle ADF$	2. By proof of case 1
3. $2m\angle CBF = m\angle CDF$	3. By proof of case 1
4. $2(m\angle ABF + m\angle CBF) = m\angle ADF + m\angle CDF$	4. Add 2 to 3
5. $2m\angle ABC = m\angle ADC$	5. Sum of adjacent angles

Proof of Case 3: The center of the circle lies on the exterior of the inscribed angle



1. Construct diameter passing through points B and D	1. By construction
2. $2m\angle ABF = m\angle ADF$	2. By proof of case 1
3. $2m\angle CBF = m\angle CDF$	3. By proof of case 1
4. $2(m\angle ABF - m\angle CBF) = m\angle ADF - m\angle CDF$	4. Subtract 2 from 3
5. $2m\angle ABC = m\angle ADC$	5. Difference of adjacent angles

INSCRIBED AND CENTRAL ANGLES: RATIONALE

Discovery and proof of the relationship between central and inscribed angles in the circle was chosen as the second informal mathematics activity for this research based on a number of favorable factors. The investigation proved compatible with the second theme in the course, namely, the study of congruence and similarity. In particular, the activity provided an opportunity for students to put to use their knowledge of congruent angles created by intersecting lines, congruence properties of isosceles triangles, and congruence properties of radii of circles in their exploration and proof of the relationship in question.

The activity allowed for direct and student-centered investigation. Through the use of compass and straight-edge constructions, students could investigate the question first-hand in order to form a conjecture about the relationship between central and inscribed angle. This conjecture would then be followed by an attempt at proof. In this sense, the activity was one that was inherently exploratory and open-ended.

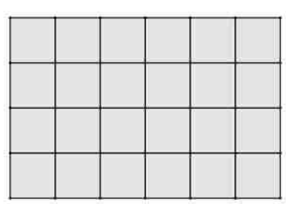
The activity also allowed for the incorporation of the various processes that characterize informal mathematics. The problem provided an opportunity for students to communicate mathematically both concerning conjectures and methods of proof. The problem provided an opportunity for students to create connections across the areas of geometry and algebra. The problem provided an opportunity for students to represent a mathematical object in various settings: geometrically, numerically, and algebraically. Finally, the problem incorporated the processes of proof and reasoning.

THE ISIS PROBLEM: PROBLEM STATEMENT

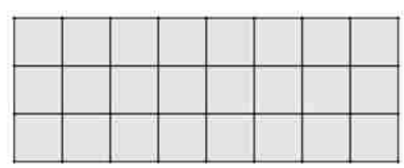
Discovery and proof of the solution to the so-called "Isis problem" was chosen as the third informal mathematics activity for this research. Students were given a printed worksheet (an exact copy of the worksheet appears in the appendix) that discussed the importance of two fundamental concepts of measurement: area and perimeter. The worksheet also introduced the notion of an "integral rectangle" as any rectangle with side-lengths which are positive integers. After this background had been provided, the worksheet introduced the problem statement:

> Your tasks in this reflective mathematics activity are to find all rectangles with sides of integral length whose area and perimeter are numerically equal and then prove that there are no others.

To aid in the investigation, each group of students was provided with a set of 40 plastic squares. These squares could be arranged in groups to form integral rectangles of various lengths and widths whose area and perimeter could then easily be visualized and calculated. For example 24 squares could be arranged to form a rectangle in the following two ways:



Area = 24 square units
Perimeter = 20 units

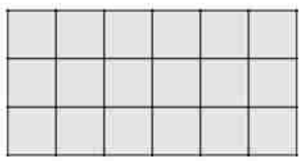


Area = 24 square units
Perimeter = 22 units

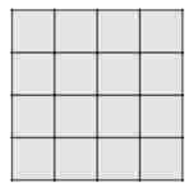
Here we can see that the rectangle on the right is "closer" to being a solution to the Isis problem than the rectangle on the left.

THE ISIS PROBLEM: SOLUTION

Through direct investigation using square tiles, it is quite easy to find two rectangles which have area and perimeter numerically equal. These two rectangles, the three-by-six and the four-by-four, are both shown below:



Area = 18 square units
Perimeter = 18 units

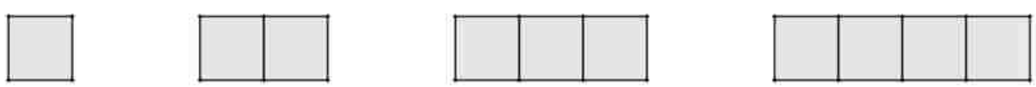


Area = 16 square units
Perimeter = 16 units

Assuming that these two solutions to the Isis problem have been discovered by direct investigation and all attempts at finding another solution prove fruitless, the task then shifts to the mathematical demonstration that no others exist. Three methods of proof that only two solutions to the Isis problem exist are explored.

Method 1 – Exhaustion

One method of proving that only two solutions to the Isis problem exist is by method of exhaustion. First we assign all rectangles to classes according to the smallest side length. These classes are then systematically exhausted. We first look at the family of rectangles possessing a smallest side length of one unit:



Which gives rise to the following table of values:

Length	1	1	1	1	…
Width	1	2	3	4	…
Area	1	2	3	4	…
Perimeter	4	6	8	10	…

Since the area increases according to an arithmetic sequence with a difference of one and the perimeter increases according to an arithmetic sequence with a difference of two and perimeter is greater than area at the onset, we can conclude that the area will never equal the perimeter for the family of smallest side length one. Looking next at the family of smallest side length of two we can construct the following table of values

Length	2	2	2	2	…
Width	2	3	4	5	…
Area	4	6	8	10	…
Perimeter	8	10	12	14	…

Again the area increases according to an arithmetic sequence, but this time with a difference of two. The perimeter also increases according to an arithmetic sequence with a difference of two, but, since perimeter is initially four greater than area this difference will be maintained for all members of the family of rectangles of smallest side length two. Next we look at the family of smallest side length three:

Length	3	3	3	3	…
Width	3	4	5	6	…
Area	9	12	15	18	…
Perimeter	12	14	16	18	…

Area increases according to an arithmetic sequence with a difference of three and perimeter again increases according to an arithmetic sequence with a difference of two. Since area is initially three less than perimeter it takes exactly three steps for the two quantities to achieve numerical equality. No more solutions exist in the family of rectangles of smallest side three as area grows faster than perimeter beyond the point of equality. Next we look at the family of smallest side length four and find:

Length	4	4	4	4	…
Width	4	5	6	7	…
Area	16	20	24	28	…
Perimeter	16	18	20	22	…

Again a solution is encountered, the four-by-four rectangle. Since area outpaces perimeter arithmetically, no other solutions will be found. Next we look at the family of smallest side length five and find:

Length	5	5	5	5	…
Width	5	6	7	8	…
Area	25	30	35	40	…
Perimeter	20	22	24	26	…

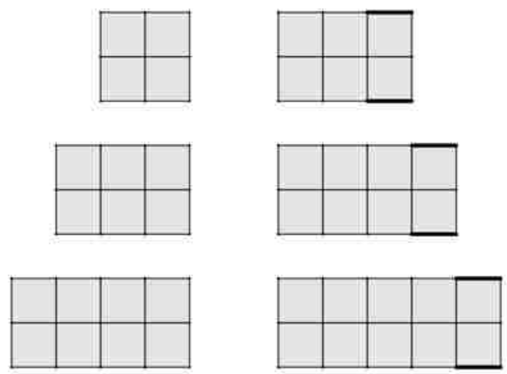
Since area is initially greater than perimeter and it outpaces perimeter arithmetically, no solution will be found in this family. It remains to be proven that no other families with a minimum side length greater than five will contain a solution. We assign n to be the minimum side length. The base case in any family is then an n by n square. The area of the base case for any family is n^2 and the perimeter for the base is $4n$. We note for the area to be greater than the perimeter:

$$n^2 > 4n$$

The following must hold:

$$n > 4$$

Thus, for all families of minimum side length greater than four the area will initially be greater than the perimeter. Now, the area for each successive member of the family will increase in an arithmetic sequence with difference of *n*. This is due to the fact that a unit increase in width corresponds to the addition of the number of units of length, *n*, to the area. Perimeter will always increase according to an arithmetic sequence with a difference of two. This is due to the fact that a unit increase in width corresponds to an increase of width on *two sides* of the figure as the following drawing makes clear:



Here we can visually see the two units of perimeter that are added to create each successive figure for the family of minimum side length two. This property is invariant for any family of any minimum side length; thus, perimeter will always increase according to an arithmetic sequence with difference two. So, we have shown that for those families whose minimum side length is $n > 4$ we have the following:

1. Initially the numerical value of the area is greater than perimeter
2. Area increases according to an arithmetic sequence whose difference is $n > 4$
3. Perimeter increases according to an arithmetic sequence whose difference is 2

These three properties logically result in the fact that the area and perimeter will never be numerically equal for any family whose minimum side length is greater than four. Therefore only two solutions to the Isis problem exist: integral rectangles measuring three-by-six and four-by-four.

Method 2 – Algebraic Approach

A second avenue of proof relies upon an algebraic representation of the problem. Assigning the value x to the width and y to the length of the integral rectangle we can then represent the area, A, as:

$$A = xy$$

And the perimeter, P, as:

$$P = 2x + 2y$$

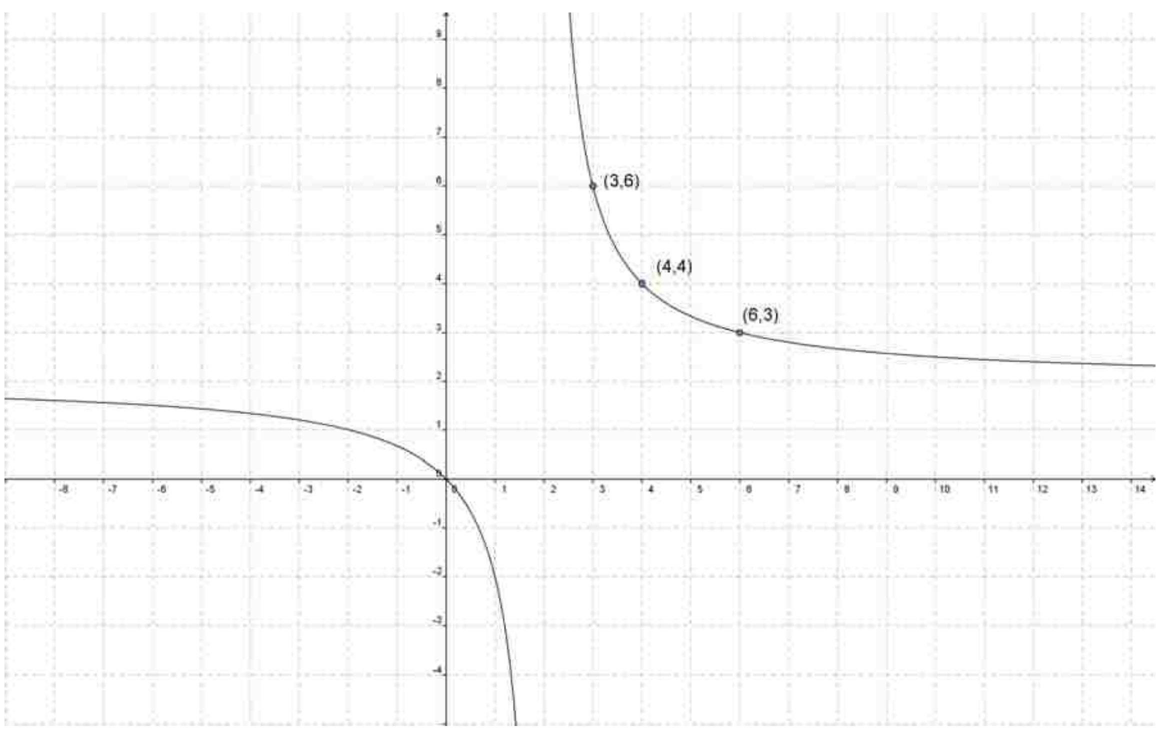
We seek the values of both x and y such that $A = P$. Setting the equations equal we find:

$$xy = 2x + 2y$$

Solving this equation for y yields:

$$y = \frac{2x}{x-2}$$

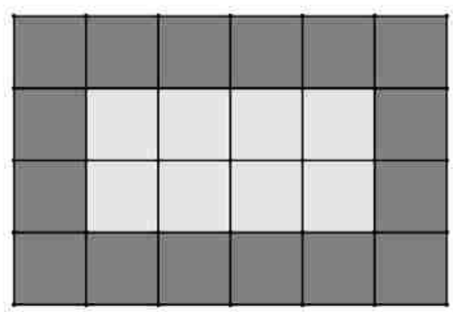
The solutions to this equation are plotted on the Cartesian plane:



Restricting our attention to the first quadrant where only positive values of *x* and *y* are found we find exactly three locations that correspond to integer solutions: (3,6), (4,4), and (6,3). The graph has a vertical asymptote at $x = 2$ and a horizontal asymptote at $y = 2$, therefore, no other integer solutions are possible. There are only two solutions: integral rectangles measuring three-by-six (taken to be the same as six-by-three) and the four-by-four.

Method 3 – Logical-Algebraic Approach

Any integral rectangle can be thought of as a tiling of squares which can be partitioned according to whether each tile is an edge tile or an interior tile:



The area, *A*, of the figure is given by the sum of the number of edge tiles, n_e, and the number of interior tiles, n_i,

$$A = n_e + n_i$$

One can then recognize that the perimeter, *P*, for any integral rectangle is given by the number of edge tiles *plus four* due to the fact that the four corner edge tiles each contribute two to the perimeter of the figure:

$$P = n_e + 4$$

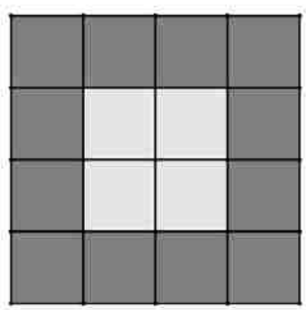
Setting area and perimeter equal we find:

$$n_e + n_i = n_e + 4$$

Which simplifies to the result:

$$n_i = 4$$

This indicates that in order for equality of area and perimeter to occur, the number of interior tiles must be four. There are exactly two ways to create an integral rectangle with four interior tiles:



Therefore, there are exactly two solutions to the problem: integral rectangles measuring three-by-six and the four-by-four.

THE ISIS PROBLEM: RATIONALE

The discovery and proof of the solution to the Isis problem was chosen as the third informal mathematics activity due to a number of favorable factors associated with the problem. The activity addressed the third objective in the course: to use basic measurement to approach problems involving length, area, and volume. The activity also addressed the sixth objective in the course: to develop a facility with geometric theorems and proofs through hands-on and computer explorations. In particular, the problem addressed the measurement topics of length, perimeter and area through hands-on exploration while seeking mathematical proof. Additionally, the problem required teacher candidates to consider the relationship between area and perimeter of rectangular figures: a theme of considerable confusion among elementary school teachers (i.e. Ma, 1999).

In addition to addressing course goals and themes, the activity was chosen based on its alignment with characteristics of informal mathematical inquiry. The activity was open-ended and allowed for direct investigation which facilitated a student-centered approach. Further, the activity was one which promoted communication and multiple representations in mathematics. The activity promoted connections between various branches of mathematics: geometry, algebra, functions, and logic. The activity also required students to apply reasoning skills to arrive at a conjecture and mathematically prove that conjecture's veracity necessitating both intuition and creativity. Finally the activity's solution could be obtained via multiple routes, dispelling notions of mathematics as algorithmically rule-bound.

SEMI REGULAR TESSELLATION: PROBLEM STATEMENT

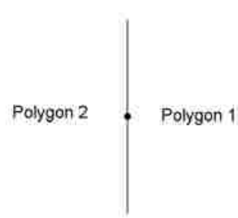
The fourth and final informal mathematics activity chosen for this research was the discovery and proof of the 8 semi-regular tessellations. Students were given a printed worksheet (an exact copy of the worksheet appears in the appendix) that discussed the term tessellation. The worksheet went on to define a *regular* tessellation as a tessellation that is made up of congruent regular polygons which meet vertex to vertex such that every vertex arrangement is identical. This definition then was amplified in defining a *semi-regular* tessellation as a tessellation that is made up of two or more congruent regular polygons which meet vertex to vertex such that every vertex arrangement is identical. Once these necessary background notions had been addressed, the following problem statement was provided:

> Your task in this reflective mathematics activity is to find all semi-regular tessellations of the plane and prove that no others exist.

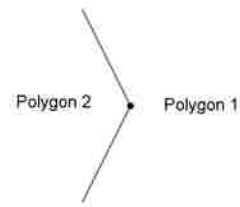
Additionally, students were provided with sets of cardboard regular polygons which could be used to investigate the question directly. These sets included multiple copies of the following regular polygons: equilateral triangles, squares, pentagons, hexagons, octagons, decagons, and dodecagons. Notably absent from the set were heptagons, nonagons and hendecagon. Students were encouraged to use the manipulatives to discover semi-regular tessellations and build intuition before attempting a proof.

SEMI REGULAR TESSELLATION: SOLUTION

The number of regular polygons meeting at a vertex in any semi-regular tessellation must be three or greater. If two regular polygons met at a vertex, then, either both would have to have an interior angle measuring 180 degrees:



Or one of the polygons would have to have an interior angle measuring greater than 180 degrees:

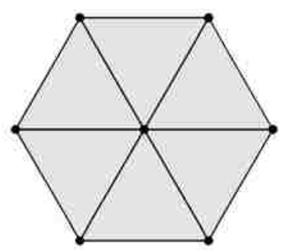


Since any n –sided regular polygon has interior angle measure given by

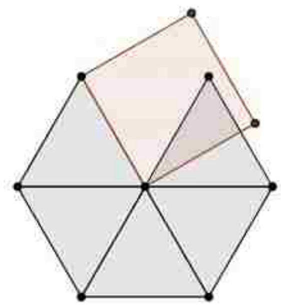
$$\frac{180(n-2)}{n}$$

the measure of the interior angle will never exceed 180 degrees, thus, three or more polygons must meet at a vertex in any semi-regular tessellation.

The number of polygons meeting at a vertex in any semi-regular tessellation must be less than six. Consider the equilateral triangle whose interior angles measure 60 degrees. Exactly six equilateral triangles can be placed around a single vertex without any gaps or overlaps (making the equilateral triangle a candidate for *regular* tessellation):



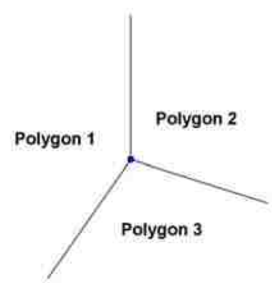
Now, since a semi-regular tessellation requires *two or more* regular polygons to meet at a vertex and every regular polygon has an interior angle greater than that of the equilateral triangle there is no way to construct a semi-regular tessellation with six polygons at a vertex without overlap of polygons. For instance, if we try to place five equilateral triangles around a vertex accompanied by an additional sixth figure, a square for instance, an overlap of polygons occurs:



Therefore, it is sufficient to investigate three cases: three polygons meeting at a vertex, four polygons meeting at a vertex, and five polygons meeting at a vertex.

Case 1: Three Polygons Meeting at a Vertex

We consider the case of three polygons meeting at a vertex. Here we have the following:



That is, three polygons meeting at a vertex in such a manner as to produce no gaps and no overlaps. It is the last requirement that necessitates that the sum of the interior angles of the three polygons must be equal to 360 degrees. If we denote the number of sides of the three polygons n_1, n_2 and n_3, and employ the use the formula for the measure of the interior angle of a regular polygon:

$$\frac{180(n-2)}{n}$$

we can construct the following sum for the interior angles of the three polygons:

$$\frac{180(n_1-2)}{n_1}+\frac{180(n_2-2)}{n_2}+\frac{180(n_3-2)}{n_3}=360$$

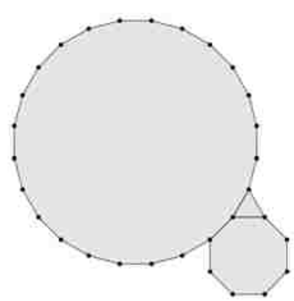
This equation simplifies to the following:

$$\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}=\frac{1}{2}$$

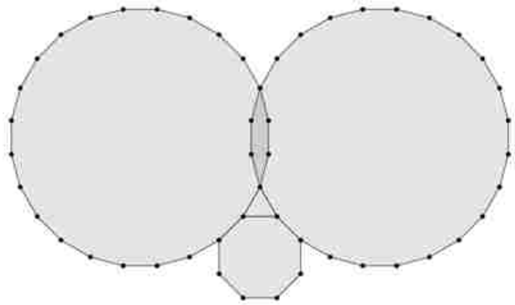
Since n_1, n_2 and n_3 represent the number of sides of a regular polygon, we need only search for positive integer solutions which are at least three (which is the minimum number of sides for a regular polygon). This leads to the following table of solutions:

n_1	6	5	4	4	4	3	3	3	3	3
n_2	6	5	8	6	5	12	10	9	8	7
n_3	6	10	8	12	20	12	15	18	24	42

This table represents the candidates for semi-regular tessellation for three polygons meeting at a vertex. These candidate polygon combinations can be placed around a single vertex, but direct attempts will demonstrate that many do not tessellate. For instance an equilateral triangle, an octagon and a 24-gon can be placed around a single vertex as required:



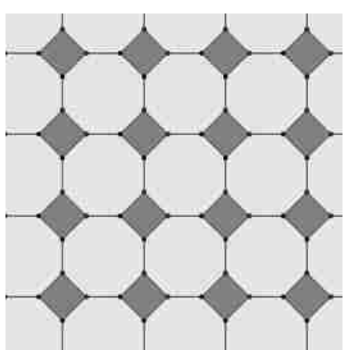
However, the vertex arrangement cannot be duplicated to produce a tessellation of the plane:



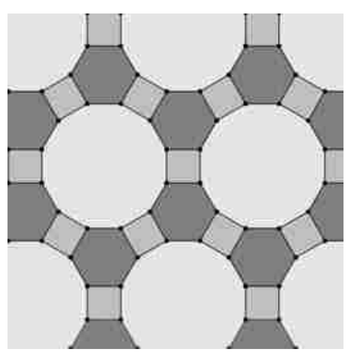
Similar investigations will disallow all but four candidates:

n_1	6	5	4	4	4	3	3	3	3	3
n_2	6	5	8	6	5	12	10	9	8	7
n_3	6	10	8	12	20	12	15	18	24	42
Tessellate	Y	N	Y	Y	N	Y	N	N	N	N

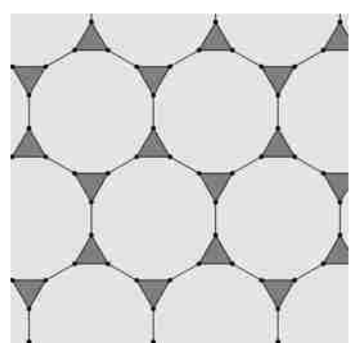
Of these candidates, we recognize that the first, 6-6-6, represents a regular tessellation of three hexagons meeting at a vertex, and as such is disallowed as a semi-regular tessellation. The rest form semi-regular tessellations which are shown below.



$n_1 = 4,\ n_2 = 8,\ n_3 = 8$



$n_1 = 4,\ n_2 = 6,\ n_3 = 12$



$n_1 = 3,\ n_2 = 12,\ n_3 = 12$

We have shown that there are exactly three semi-regular tessellations in which three polygons meet at a vertex.

Case 2: Four Polygons Meeting at a Vertex

Proceeding in a fashion similar to the three polygons to a vertex case, we can search for candidates by considering the sum of the interior angles of the four polygons that meet at a vertex. If we denote the number of sides of the four polygons n_1, n_2, n_3 and n_4 and again employ the use of the formula for the measure of the interior angle of a regular polygon, we can construct the following equation:

$$\frac{180(n_1-2)}{n_1}+\frac{180(n_2-2)}{n_2}+\frac{180(n_3-2)}{n_3}+\frac{180(n_4-2)}{n_4}=360$$

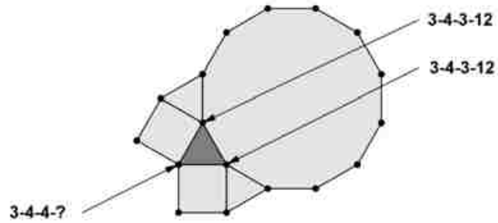
which simplifies to:

$$\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}+\frac{1}{n_4}=1$$

Again we search for semi-regular tessellation candidates by searching all positive integer solutions. Intuition would grant that the greater partition of a smaller sum should produce few tessellation candidates and indeed only four candidates are possible:

n_1	4	3	3	3
n_2	4	4	3	3
n_3	4	4	6	4
n_4	4	6	6	12

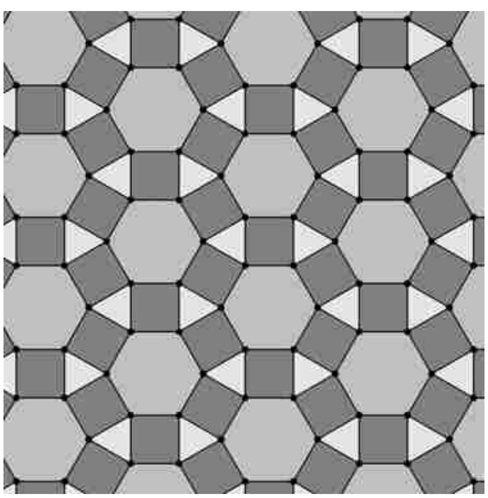
Of these four candidates, direct attempts at building will demonstrate that 3-4-3-12 does not tessellate. To demonstrate this fact we need only apply the combination 3-4-3-12 (that is equilateral triangle, square, equilateral triangle, dodecagon) at each vertex of an equilateral triangle. The attempt demonstrates that while the combination 3-4-3-12 does indeed produce a vertex arrangement with no gaps or overlaps, it fails to tessellate:



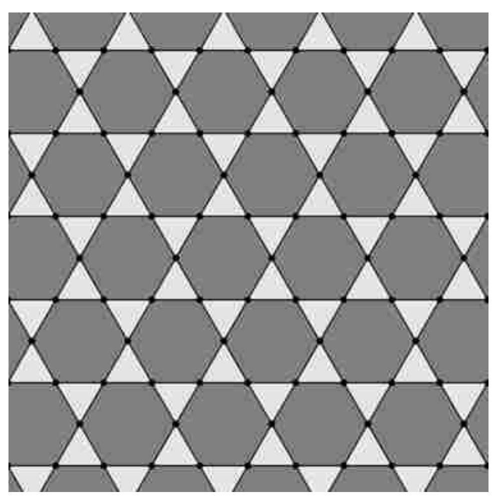
Three candidates for four polygons meeting at a vertex remain.

n_1	4	3	3	3
n_2	4	4	3	3
n_3	4	4	6	4
n_4	4	6	6	12
Tessellate	Y	Y	Y	N

Since 4-4-4-4 represents a regular tessellation it is excluded. Both 3-4-4-6 and 3-3-6-6 have two possible vertex arrangements: 3-4-4-6 and 3-4-6-4, 3-3-6-6 and 3-6-3-6. Of these four arrangements, only the following two tessellate:



$n_1 = 3,\ n_2 = 4,\ n_3 = 3,\ n_4 = 6$



$n_1 = 3,\ n_2 = 6,\ n_3 = 3,\ n_4 = 6$

We have shown that there are exactly two semi-regular tessellations in which four polygons meet at a vertex.

Case 3: Five Polygons Meeting at a Vertex

Again we search for candidates by considering the sum of the interior angles of the five polygons that meet at a vertex. If we denote the number of sides of the five polygons n_1, n_2, n_3, n_4 and n_5 and use the formula for the measure of the interior angle of a regular polygon, then we can construct the following, now familiar, equation:

$$\frac{180(n_1-2)}{n_1}+\frac{180(n_2-2)}{n_2}+\frac{180(n_3-2)}{n_3}+\frac{180(n_4-2)}{n_4}+\frac{180(n_5-2)}{n_5}=360$$

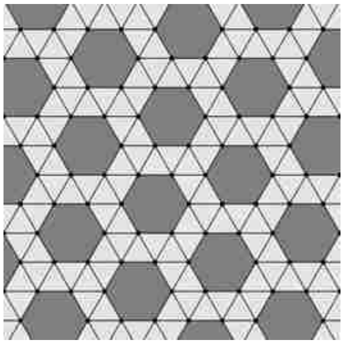
which simplifies to:

$$\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}+\frac{1}{n_4}+\frac{1}{n_5}=\frac{1}{2}$$

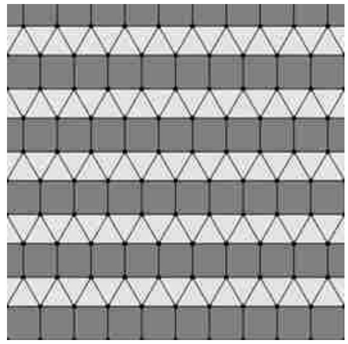
Again we search for semi-regular tessellation candidates by searching all positive integer solutions which are greater than three. Again, intuition predicts that an even greater partition of a smaller sum should produce fewer tessellation candidates than the previous cases and indeed only two candidates are possible:

n_1	3	3
n_2	3	3
n_3	3	3
n_4	3	4
n_5	6	4

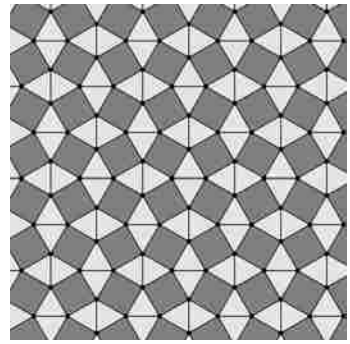
The first candidate, 3-3-3-3-6, can only be arranged in one way around a vertex, but the second candidate has two possible arrangements: 3-3-3-4-4 and 3-3-4-3-4. All three of these candidates do indeed tessellate: these tessellations are shown below.



$n_1 = 3,\ n_2 = 3,\ n_3 = 3,$
$n_4 = 3,\ n_5 = 6$



$n_1 = 3,\ n_2 = 3,\ n_3 = 3,$
$n_4 = 4,\ n_5 = 4$



$n_1 = 3,\ n_2 = 3,\ n_3 = 4,$
$n_4 = 3,\ n_5 = 4$

We have shown that there are exactly 3 semi-regular tessellations in which five polygons meet at a vertex.

Since there are no semi-regular tessellations in which six polygons meet at a vertex we have exhausted the possibilities. There are three semi-regular tessellations in which three polygons meet at a vertex. There are two semi-regular tessellations in which four polygons meet at a vertex. There are three semi-regular tessellations in which five polygons meet at a vertex. Therefore, there are exactly 8 semi-regular tessellations.

SEMI REGULAR TESSELLATION: RATIONALE

The discovery and proof of exactly eight semi-regular tessellations was chosen for the fourth and final informal mathematics activity in this research. The choice to use the activity was determined based on a number of favorable factors. The activity addressed several of the course objectives, namely objective 4: "to explore, conjecture, and prove mathematical ideas and theorems involving geometry," and objective 6: "to develop a facility with geometric theorems and proofs through hands-on and computer explorations." Additionally, the problem fit within the final theme of the course which dealt with motion geometry, transformations, and tessellation. Proof of the existence of exactly three *regular* tessellations was presented as part of the regular in-class lecture-based curriculum; thus, the proof of exactly eight *semi-regular* tessellations can be viewed as an extension to the regular curriculum.

Beyond addressing the course goals and themes, the activity was chosen based on its alignment with principles that characterize informal mathematical inquiry. The activity was open-ended and required of the student openness to exploration as well as the application of creativity and intuition. Through the use of cardboard regular polygons, the question posed could be directly investigated. The activity allowed students to physically confront the task through the construction of tessellations using different combinations of regular polygons. Proof of exactly eight semi-regular tessellations required a connected view of mathematics, incorporating arithmetic, algebra, geometry, logic, and number theory. The activity required students to reason mathematically and to ultimately prove their conjecture about the number of semi-regular tessellations. And while several algorithmic procedures were required (notably the use of the formula for the interior angle of an *n*-sided polygon), these procedures required adaptation and interpretation in this setting. All these aspects firmly placed the activity in the realm of informal mathematics.

ASSESSMENT OF INFORMAL MATHEMATICS ACTIVITIES

Two separate classes of prospective elementary school teachers participating in this research were required to complete the four informal mathematics activities described above as part of regular course requirements for Math 136 – Mathematics for Elementary School Teachers II. The first class participated in the fall of 2009. The second class participated in the spring of 2010. Assessment of student work on the four informal mathematics activities evolved from the first semester to the second in an effort to increase student motivation on the activities.

In the fall of 2009, the four informal mathematics activities were allotted 25 points each, which for the four activities, amounted to 100 points total out of a possible course total of 1000 points. The activities, therefore, comprised 10 percent of the overall grade in the course. The remainder of the course grade was allotted to mid-semester exams (300 points), homework (200 points), computer lab assignments (100 points) and a final exam (300 points). The 25 points allotted to each of the four informal mathematics activities was distributed according to the following: 10 points for completion of the activity, 10 points for completion of reflection and 5 points for responding to another student's reflection. The 10 points assigned for "completion of the activity" were awarded according to the researcher's judgment that the student in question had given the task a significant amount of effort.

Perhaps due to the vague standard imposed, a great variety of effort in student work on the informal mathematics activities was observed in the fall of 2009. In an attempt to better reward students providing exemplary work in the activities and to penalize those students whose efforts were lacking, it was decided that an assessment rubric for the informal mathematics activities be utilized for the spring 2010 participants in the study. This rubric is provided in Table 4.3.

Table 4.3: Assessment Rubric for Informal Mathematics Activities Spring 2010

INFORMAL MATHEMATICS ACTIVITIES GRADING RUBRIC	
Conjecture	
0	No conjecture is provided
1	A conjecture is provided. The conjecture is wholly incorrect and contains substantial errors.
2	A conjecture is provided. The conjecture is partially incorrect and contains several errors.
3	A conjecture is provided. The conjecture is almost correct, but contains more than one error.
4	A conjecture is provided. The conjecture is nearly correct, but contains one error.
5	A conjecture is provided. The conjecture is correct.
Proof	
0	No proof is provided.
1	Proof is provided but fails to demonstrate the certainty of the conjecture.
2	Proof is provided but only partially demonstrates the certainty of the conjecture.
3	Proof is provided. The proof makes a case for the certainty of the conjecture but several elements are left unexplained.
4	Proof is provided. The proof makes a strong case for the certainty of the conjecture but one or two elements are left unexplained.
5	Proof is provided. The proof makes a strong case for the certainty of the conjecture and no elements are left unexplained.

All other aspects of assessment for the spring 2010 group were left the same.

STUDENT REFLECTION

A common theme in the literature on mathematical beliefs is the important role of reflection in the transformation of beliefs about mathematics and mathematics instruction (i.e. Thompson, 1984; Ernest, 1988; Schram et al., 1988; Raymond, 1997; Cooney et al., 1998; Mewborn, 1999). For example, Mewborn (1999), in her study of four preservice elementary teachers, was able to identify five elements that were successful in transforming teachers' beliefs regarding mathematics instruction: (a) an inquiry perspective in coursework, (b) a student, teacher, and teacher-educator community of learners, (c) a non-evaluative community of learners, (d) a time to reflect upon coursework and practice, and (e) a subject-specific field experience. Cooney (1998) advocated for “the inculcation of doubt and the posing of perplexing situations” (p. 330) coupled with reflection as a means for transforming the beliefs of teachers away from naïve idealism and isolationist positions.

In an effort to utilize what the literature documents as a powerful means of transformation, the researcher incorporated a student reflection component in this research. The researcher questioned if reflecting upon one’s own recent acquisition of mathematical knowledge might prove transformative in promoting a more informal and constructivist view of mathematics and mathematics instruction. In this sense, this research can be seen as a variation on previous studies which employ “cognitively guided instruction” (i.e. Fennema et al, 1996; Vacc & Bright, 1999). The cognitively guided approach exposes prospective teachers to student learning through the use of vignettes, recorded videos and classroom observations. These artifacts become a means for reflection which ultimately serve to transform teacher beliefs regarding the teaching and learning of mathematics. Instead of concentrating on student learning, the researcher questioned if reflection upon one’s *own* mathematical meaning-making might prove to be an equally powerful means of transformation.

To this end, the researcher asked participants in the study to provide written reflections after engaging in each of the aforementioned mathematical activities. A reflective prompt was incorporated in the problem statement of each activity:

> At the end of this activity you will be asked to reflect on your personal experience of coming to understand this mathematical concept and what the experience "teaches you" about learning mathematics. Keep track of your strategies and procedures. Make note of your emotions and feelings. And be prepared to report your findings.

Once the activity had been completed, students were prompted again:

> You have been asked to carry out a mathematical investigation of the inscribed angle theorem to create a basis for reflection upon what it means to learn and know mathematics. Write a short reflection of 300 to 500 words about your personal process of coming to know and understand this mathematical object. Reflect upon your reactions to the problem posed: confidence, ambivalence, curiosity, or anxiety. Reflect upon your method of solution: reasoning, "dead ends", obstacles, aides, collaboration, and approaches. Reflect upon your final solution: satisfaction/dissatisfaction with solution, sense of accomplishment/frustration. Lastly, look back at the experience and reflect upon how it informs you, as a future teacher, of the process of teaching and learning mathematics.

Student reflections were posted to an internet-hosted on-line discussion forum. This forum allowed all students enrolled in the course to view one another's written reflections. In an effort to create what Mewborn (1999) describes as a "community of learners" it was required that each student read at least three reflections posted by other students and then chose one for a response. A minimum of 50 words for the response was imposed by the researcher/instructor.

These student reflections were analyzed for evidence of formal/informal beliefs according to the process described by Corbin and Strauss (2009). This process begins with a reading of the qualitative data in which the researcher takes note of common narratives which lead to the development of coarse "themes" which are defined as “higher level concepts under which analysts group lower level concepts according to their shared properties” (Corbin and Strauss, 2009, p 159). These themes serve to represent relevant phenomena emerging from the data and act as a reductive agent in qualitative analysis. Once the themes had been identified, the researcher again read each student reflection noting the presence or absence of each theme. This led to a categorical study of each activity according to the presence or absence of themes associated with dispositions associated with formal and informal approaches to mathematics and mathematics instruction.

CHAPTER 5: RESULTS AND ANALYSIS

QUANTITATIVE ANALYSIS

The results of the quantitative analysis conducted are presented in the following two sections. The results of the analysis of the beliefs for the two instructors are presented first. The second section presents the results of pre and post beliefs about mathematics for the three groups that participated in this study. The third section presents the results of the pre and post beliefs about mathematics instruction for the three groups that participated in this study.

INSTRUCTOR BELIEFS

Two instructors participated in this research. The control instructor taught the fall 2009 control group which experienced Math 136 without the four informal activities. The experimental instructor (one of the researchers) taught the fall 2009 and spring 2010 groups that experienced the four informal mathematics activities as part of the course. Both instructors completed the Collier (1972) beliefs about mathematics (BAM) questionnaire and the beliefs about mathematics instruction questionnaire (BAMI). The composite scores for the two instructors are provided in Table 5.1. The results indicate that both the control and experimental instructor hold similar beliefs about mathematics and mathematics instruction which can be characterized as highly informal; scoring near the top of the 20-120 point scale in both BAM and BAMI.

Table 5.1: Instructor BAM and BAMI Composite Scores

	BAM Composite Score	BAMI Composite Score
Control Instructor	112	113
Experimental Instructor	119	112

BELIEFS ABOUT MATHEMATICS

Beliefs about mathematics (BAM) for the fall 2009 control group, the fall 2009 informal mathematics group and the spring 2010 informal mathematics group are presented in Table 5.2, Table 5.3 and Table 5.4. Each of these tables includes the item number, whether the item was considered a formal or an informal statement about mathematics, the statement itself, the pre-course mean and standard deviation of student response to the item, and the post-course mean and standard deviation of student response to the item. At the end of each table is the pre-course and post-course mean and standard deviation of the BAM composite for each student.

Table 5.2: Fall 2009 Math 136 Control Group BAM Results (N=18)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
2	I	There are several different but appropriate ways to organize the basic ideas in mathematics.	4.56	0.98	4.72	1.23
4	I	In mathematics, perhaps more than in other areas, one can display originality and ingenuity.	5.28	0.57	3.50	1.42
5	I	There are often many different ways to solve a mathematics problem.	5.28	0.89	4.50	1.25
7	I	The field of mathematics contains many of the finest and most elegant creations of the human mind.	3.69	1.14	3.94	1.55
8	I	Studying mathematics helps to develop the ability to think more creatively.	3.83	1.15	4.17	1.54
12	I	The basic ingredient for success in mathematics is an inquiring nature.	3.94	1.26	3.89	1.02
14	I	Mathematics requires very much independent and original thinking.	3.67	1.19	3.61	1.42
15	I	There are several different but logically acceptable ways to define most terms in mathematics.	4.50	1.10	4.11	1.08
16	I	Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.	4.61	0.85	4.06	1.21
20	I	Mathematics has so many applications because its models can be interpreted in so many ways.	4.26	1.09	4.33	0.59
1	F	In mathematics, perhaps more than in other fields, one can find set routines and procedures.	5.11	1.08	4.50	1.25
3	F	The laws and rules of mathematics severely limit the manner in which problems can be solved.	3.56	1.50	3.94	1.11
6	F	Mathematicians are hired mainly to make precise measurements and calculations for scientists.	4.61	1.33	3.28	1.23
9	F	In mathematics there is usually just one proper way to do something.	3.89	1.08	2.72	1.45
10	F	Mathematics is an organized body of knowledge which stresses the use of formulas to solve problems.	3.78	1.00	4.06	0.97
11	F	Solving a mathematics problem usually involves finding a rule or formula that applies.	3.72	1.32	4.11	0.83
13	F	Many of the important functions of the mathematician are being taken over by new computers.	4.00	1.03	3.72	1.02
17	F	The language of mathematics is so exact that there is no room for variety of expression.	2.50	1.15	2.89	1.18
18	F	Mathematics is a rigid discipline which functions strictly according to inescapable laws.	2.78	1.00	3.17	1.34
19	F	The main benefit from studying mathematics is developing the ability to follow directions.	2.78	1.31	3.17	1.15
		Composite BAM	76.85	4.05	75.50	8.44

Table 5.3: Fall 2009 Math 136 Informal Group BAM Results (N=21)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
2	I	There are several different but appropriate ways to organize the basic ideas in mathematics.	5.10	0.70	4.90	0.83
4	I	In mathematics, perhaps more than in other areas, one can display originality and ingenuity.	3.95	1.16	3.35	1.27
5	I	There are often many different ways to solve a mathematics problem.	5.14	1.20	4.95	1.12
7	I	The field of mathematics contains many of the finest and most elegant creations of the human mind.	4.38	1.16	4.52	1.12
8	I	Studying mathematics helps to develop the ability to think more creatively.	4.24	1.09	4.55	0.89
12	I	The basic ingredient for success in mathematics is an inquiring nature.	3.81	1.21	4.35	0.93
14	I	Mathematics requires very much independent and original thinking.	4.10	1.14	3.86	1.28
15	I	There are several different but logically acceptable ways to define most terms in mathematics.	4.38	1.16	4.14	1.11
16	I	Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.	4.38	1.24	4.81	0.98
20	I	Mathematics has so many applications because its models can be interpreted in so many ways.	4.62	1.20	4.43	1.12
1	F	In mathematics, perhaps more than in other fields, one can find set routines and procedures.	4.90	0.77	4.95	0.86
3	F	The laws and rules of mathematics severely limit the manner in which problems can be solved.	2.81	1.29	3.24	1.22
6	F	Mathematicians are hired mainly to make precise measurements and calculations for scientists.	2.52	1.12	2.65	1.35
9	F	In mathematics there is usually just one proper way to do something.	1.86	1.06	2.24	1.00
10	F	Mathematics is an organized body of knowledge which stresses the use of formulas to solve problems.	4.19	0.93	4.36	1.04
11	F	Solving a mathematics problem usually involves finding a rule or formula that applies.	4.38	0.80	4.19	1.17
13	F	Many of the important functions of the mathematician are being taken over by new computers.	4.29	1.35	4.10	1.41
17	F	The language of mathematics is so exact that there is no room for variety of expression.	2.90	1.41	2.88	1.02
18	F	Mathematics is a rigid discipline which functions strictly according to inescapable laws.	3.38	1.07	3.35	1.35
19	F	The main benefit from studying mathematics is developing the ability to follow directions.	2.86	1.28	3.48	1.47
		Composite	80.00	8.85	79.26	11.32

Table 5.4: Spring 2010 Math 136 Informal Group BAM Results (N=25)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
2	I	There are several different but appropriate ways to organize the basic ideas in mathematics.	5.04	0.98	5.00	0.82
4	I	In mathematics, perhaps more than in other areas, one can display originality and ingenuity.	3.28	1.14	4.20	1.32
5	I	There are often many different ways to solve a mathematics problem.	5.16	0.75	5.28	1.14
7	I	The field of mathematics contains many of the finest and most elegant creations of the human mind.	4.14	0.99	4.72	0.94
8	I	Studying mathematics helps to develop the ability to think more creatively.	4.12	1.51	4.60	1.12
12	I	The basic ingredient for success in mathematics is an inquiring nature.	3.90	1.15	4.48	1.23
14	I	Mathematics requires very much independent and original thinking.	3.96	1.27	4.24	1.05
15	I	There are several different but logically acceptable ways to define most terms in mathematics.	4.68	1.11	4.60	1.08
16	I	Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.	5.00	0.87	4.88	1.33
20	I	Mathematics has so many applications because its models can be interpreted in so many ways.	4.60	0.91	4.92	0.81
1	F	In mathematics, perhaps more than in other fields, one can find set routines and procedures.	4.96	0.73	4.60	0.91
3	F	The laws and rules of mathematics severely limit the manner in which problems can be solved.	3.36	1.19	2.64	1.35
6	F	Mathematicians are hired mainly to make precise measurements and calculations for scientists.	3.29	1.16	2.92	1.22
9	F	In mathematics there is usually just one proper way to do something.	2.24	1.33	2.16	1.11
10	F	Mathematics is an organized body of knowledge which stresses the use of formulas to solve problems.	4.24	1.01	3.80	1.32
11	F	Solving a mathematics problem usually involves finding a rule or formula that applies.	4.64	0.91	4.28	1.17
13	F	Many of the important functions of the mathematician are being taken over by new computers.	3.92	1.08	3.96	1.10
17	F	The language of mathematics is so exact that there is no room for variety of expression.	3.00	1.53	2.60	1.12
18	F	Mathematics is a rigid discipline which functions strictly according to inescapable laws.	3.44	1.36	2.76	1.42
19	F	The main benefit from studying mathematics is developing the ability to follow directions.	3.22	1.39	2.48	1.42
		Composite	77.88	9.70	84.72	9.96

Subsequent analysis of the BAM data investigated the differences in pre-course versus post-course composite BAM scores. A paired *t*-test of significance was conducted for each of the three distinct groups that participated in this research. Specifically the researcher tested the hypotheses

$$H_0 : \mu_d = 0$$
$$H_a : \mu_d \neq 0$$

where μ_d is defined as the average of the differences between post BAM composite score and pre BAM composite score. The test statistic used was

$$t_{n-1} = \frac{\bar{d}}{SE(\bar{d})}$$

where $\bar{d}$ was defined as the mean of the pairwise differences, *n* the number of participants, and $SE(\bar{d})$ the standard error for the mean of the pairwise differences. The test statistic, t_{n-1}, was compared to the student's *t* distribution with *n*-1 degrees of freedom in order to obtain a *p*-value for the determination of statistical significance. The results of this analysis are presented in Table 5.5. The analysis fails to find evidence for a change in beliefs about mathematics as measured by the BAM instrument for both the fall 2009 control group ($p = 0.5410$) and the fall 2009 informal group ($p = 0.5265$). The analysis did find strong evidence indicating a change in beliefs about mathematics as measured by the BAM instrument for the spring 2010 informal group ($p = 0.0003$).

Table 5.5: Results of Paired t-Tests of Significance: BAM Post Composite versus BAM Pre Composite

Group	N	$\bar{d}$	$SE(\bar{d})$	t	df	P-Value
F2009 Control	18	-1.3491	2.1626	-0.6238	17	0.5410
F2009 Informal	21	-1.6429	2.5488	-0.6446	20	0.5265
S2010 Informal	25	6.8432	1.6281	4.2032	24	0.0003

A statistically significant change in BAM composite in the spring 2010 informal group prompted a ranking of BAM item average scores according to their contribution to a positive change in BAM composite. The results of this ranking are displayed in Table 5.6. Here, informal items were ranked according to the degree to which each item's mean score increased across the semester, and, formal items were ranked according to the degree which each item's mean score decreased across the semester. This analysis allowed for a comparison of individual beliefs, both formal and informal, according to their propensity to change in relationship to participation in informal mathematics activities. The analysis revealed that students shifted most towards agreement with the following informal statement:

> In mathematics, perhaps more than in other areas, one can display originality and ingenuity.

This statement exhibited an increase of nearly a full point towards agreement on the 6 point scale. Students also shifted, albeit at a more moderate half point, towards agreement with the following three informal statements:

> The basic ingredient for success in mathematics is an inquiring nature.

> The field of mathematics contains many of the finest and most elegant creations of the human mind.
>
> Studying mathematics helps to develop the ability to think more creatively.

The analysis also revealed that students shifted most towards disagreement with the following three formal statements about mathematics:

> The main benefit from studying mathematics is developing the ability to follow directions.
>
> The laws and rules of mathematics severely limit the manner in which problems can be solved.
>
> Mathematics is a rigid discipline which functions strictly according to inescapable laws.

Finally, to the following statements student mean response remained static, exhibited by a similar level of agreement or disagreement both before and after student participation in informal mathematics activities:

> There are often many different ways to solve a mathematics problem.
>
> There are several different but appropriate ways to organize the basic ideas in mathematics.
>
> There are several different but logically acceptable ways to define most terms in mathematics.
>
> Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.
>
> In mathematics there is usually just one proper way to do something.
>
> Many of the important functions of the mathematician are being taken over by new computers.

This ranking of items points to particular beliefs which are most susceptible to change in association with student participation in informal mathematics activities.

Table 5.6: Spring 2010 Math 136 Informal Mathematics Group BAM Results (N=25) Ranked According to Contribution to Positive BAM Change

#	F/I	Statement	Pre Mean	Post Mean	Post - Pre Mean
4	I	In mathematics, perhaps more than in other areas, one can display originality and ingenuity.	3.28	4.20	0.92
12	I	The basic ingredient for success in mathematics is an inquiring nature.	3.90	4.48	0.58
7	I	The field of mathematics contains many of the finest and most elegant creations of the human mind.	4.14	4.72	0.58
8	I	Studying mathematics helps to develop the ability to think more creatively.	4.12	4.60	0.48
20	I	Mathematics has so many applications because its models can be interpreted in so many ways.	4.60	4.92	0.32
14	I	Mathematics requires very much independent and original thinking.	3.96	4.24	0.28
5	I	There are often many different ways to solve a mathematics problem.	5.16	5.28	0.12
2	I	There are several different but appropriate ways to organize the basic ideas in mathematics.	5.04	5.00	-0.04
15	I	There are several different but logically acceptable ways to define most terms in mathematics.	4.68	4.60	-0.08
16	I	Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.	5.00	4.88	-0.12
19	F	The main benefit from studying mathematics is developing the ability to follow directions.	3.22	2.48	-0.74
3	F	The laws and rules of mathematics severely limit the manner in which problems can be solved.	3.36	2.64	-0.72
18	F	Mathematics is a rigid discipline which functions strictly according to inescapable laws.	3.44	2.76	-0.68
10	F	Mathematics is an organized body of knowledge which stresses the use of formulas to solve problems.	4.24	3.80	-0.44
17	F	The language of mathematics is so exact that there is no room for variety of expression.	3.00	2.60	-0.40
6	F	Mathematicians are hired mainly to make precise measurements and calculations for scientists.	3.29	2.92	-0.37
1	F	In mathematics, perhaps more than in other fields, one can find set routines and procedures.	4.96	4.60	-0.36
11	F	Solving a mathematics problem usually involves finding a rule or formula that applies.	4.64	4.28	-0.36
9	F	In mathematics there is usually just one proper way to do something.	2.24	2.16	-0.08
13	F	Many of the important functions of the mathematician are being taken over by new computers.	3.92	3.96	0.04

A correlative study of BAM and final course percentage was conducted for the spring 2010 group. Plots of final course percentage versus pre BAM, post BAM and BAM gain are shown in Figures 5.7, 5.8 and 5.9 and a summary of correlation factors is presented in Table 5.10. The data demonstrate that, for students in the spring 2010 group, one's degree of informality of beliefs about mathematics was not predictive of achievement in mathematics as measured by final course percentages. This finding stands in contrast to those obtained by Seaman et al. (2005) as well as Collier (1972) who both found that "high achieving" students experienced greater gains in BAM over the course of their four years of university education. Note that both researchers defined high achieving for incoming freshman as "three or more years of high school mathematics (algebra and beyond) with all grades of A or B" (Collier, 1972, p. 157).

Figure 5.7: Spring 2010 Informal Group Pre BAM versus Final Course Percentage

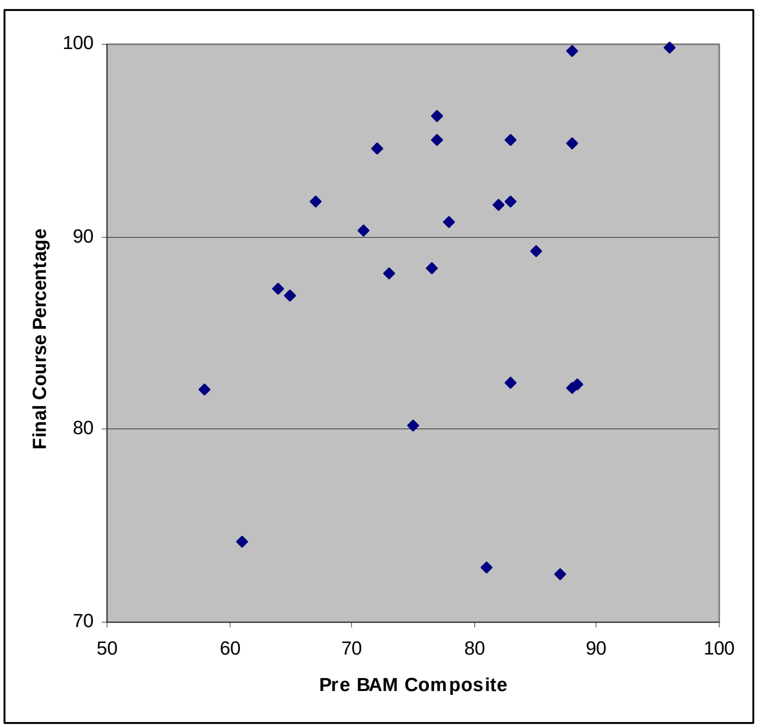


Figure 5.8: Spring 2010 Informal Group Post BAM versus Final Course Percentage

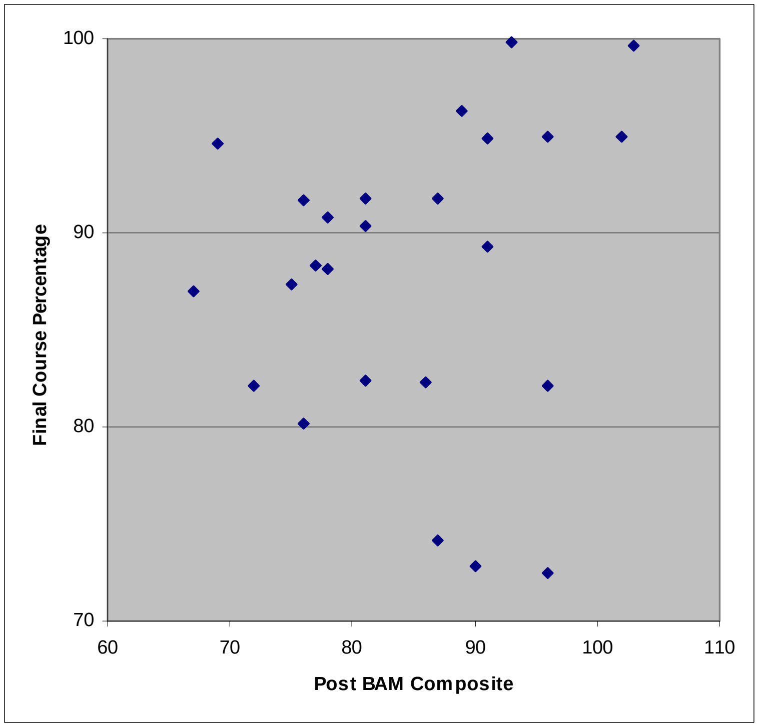


Figure 5.9: Spring 2010 Informal Group Gain in BAM versus Final Course Percentage

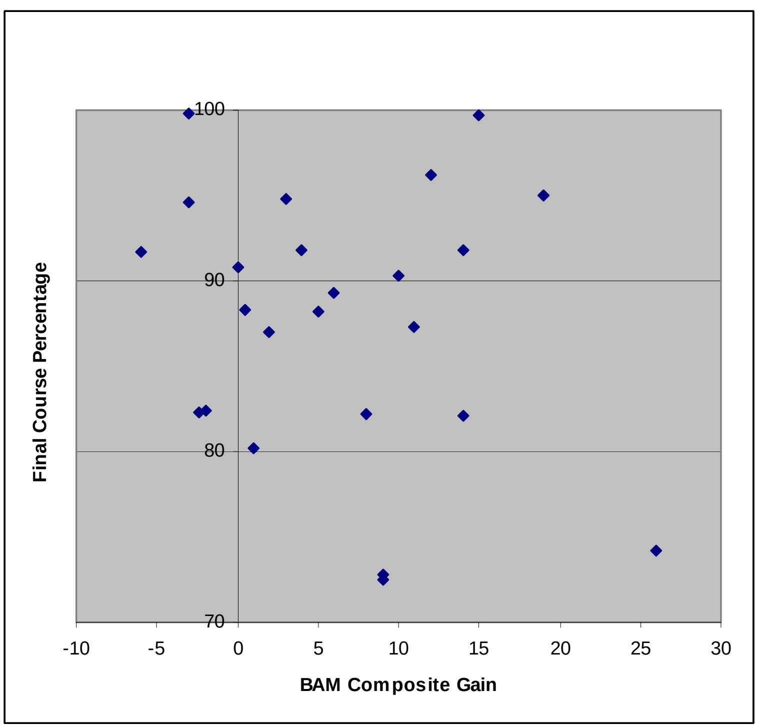


Table 5.10: Spring 2010 Informal Group Summary of Correlative Analysis of BAM Scores and Final Course Percentage

	Pre BAM	Post BAM	Gain BAM
Final Course Percentage Pearson Correlation (*P*-Value Two Tailed)	0.237 (0.254)	0.137 (0.514)	-0.115 (0.513)

BELIEFS ABOUT MATHEMATICS INSTRUCTION

Beliefs about mathematics instruction (BAMI) for the fall 2009 control group, the fall 2009 informal mathematics group and the spring 2010 informal mathematics group are presented in Table 5.11, Table 5.12 and Table 5.13. Each of these tables includes the item number, whether the item was considered a formal or an informal statement about mathematics, the statement, the pre-course mean and standard deviation of student response to the item, and the post-course mean and standard deviation of student response to the item. At the end of each table is the pre-course and post-course mean and standard deviation of the BAMI composite for each student.

Table 5.11: Fall 2009 Math 136 Control Group BAMI Results (N=18)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
3	I	Students should be encouraged to invent their own mathematical symbolism	3.56	1.50	4.33	1.37
4	I	Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	5.28	0.57	4.78	1.17
5	I	Each student should feel free to use any method for solving a problem that suits him or her best	5.28	0.89	5.11	1.23
6	I	Teachers should provide class time for students to experiment with their own mathematical ideas	4.61	1.33	4.28	1.49
10	I	Teachers should frequently insist that pupils find individual methods for solving problems	3.78	1.00	3.78	0.94
13	I	The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	4.00	1.03	3.39	1.50
14	I	The teacher should consistently give assignments which require research and original thinking	3.67	1.19	3.78	0.94
15	I	Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	4.50	1.10	4.06	1.16
16	I	Teachers must frequently give students assignments which require creative or investigative work	4.61	0.85	3.78	1.17
20	I	Students of all abilities should learn better when taught by guided discovery methods	4.26	1.09	4.35	0.93
1	F	The teacher should always work sample problems for students before making an assignment	5.11	1.08	5.11	0.90
2	F	Teachers should make assignments on just that which has been thoroughly discussed in class	4.56	0.98	4.17	1.34
7	F	Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery	3.69	1.14	3.94	1.06
8	F	Most exercises assigned to students should be applications of a particular rule or formula	3.83	1.15	3.67	1.08
9	F	Teachers should spend most of each class period explaining how to work specific problems	3.89	1.08	3.83	1.10
11	F	Discovery methods of teaching have limited value because students often get answers without knowing where they came from	3.72	1.32	2.89	1.08
12	F	The teacher should provide models for problem solving and expect students to imitate them	3.94	1.26	3.61	1.24
17	F	Students should be expected to use only those methods that their text or teacher uses	2.50	1.15	2.78	1.66
18	F	Discovery-type lessons have very limited value when you consider the time they take up	2.78	1.00	2.78	1.44
19	F	All students should be required to memorize the procedures that the text uses to solve problems	2.78	1.31	3.00	1.24
		Composite BAMI	77.61	8.19	75.82	10.37

Table 5.12: Fall 2009 Math 136 Informal Group BAMI Results (N=21)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
3	I	Students should be encouraged to invent their own mathematical symbolism	3.52	1.40	4.24	1.22
4	I	Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	5.00	0.84	5.29	0.78
5	I	Each student should feel free to use any method for solving a problem that suits him or her best	5.05	1.12	5.24	0.83
6	I	Teachers should provide class time for students to experiment with their own mathematical ideas	4.76	1.04	5.19	0.93
10	I	Teachers should frequently insist that pupils find individual methods for solving problems	3.43	1.08	3.95	0.92
13	I	The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	3.86	1.01	4.05	1.40
14	I	The teacher should consistently give assignments which require research and original thinking	4.35	0.99	4.33	1.35
15	I	Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	4.81	0.75	5.19	1.17
16	I	Teachers must frequently give students assignments which require creative or investigative work	4.86	0.79	4.65	0.93
20	I	Students of all abilities should learn better when taught by guided discovery methods	4.05	1.47	4.43	1.03
1	F	The teacher should always work sample problems for students before making an assignment	5.71	0.64	5.33	1.06
2	F	Teachers should make assignments on just that which has been thoroughly discussed in class	4.57	1.03	4.38	1.53
7	F	Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery	4.19	0.75	4.43	1.21
8	F	Most exercises assigned to students should be applications of a particular rule or formula	3.57	0.87	3.75	1.16
9	F	Teachers should spend most of each class period explaining how to work specific problems	3.21	1.21	3.57	1.21
11	F	Discovery methods of teaching have limited value because students often get answers without knowing where they came from	4.24	1.09	3.05	1.43
12	F	The teacher should provide models for problem solving and expect students to imitate them	3.86	1.24	3.43	1.12
17	F	Students should be expected to use only those methods that their text or teacher uses	2.48	1.33	2.05	1.16
18	F	Discovery-type lessons have very limited value when you consider the time they take up	2.76	1.22	2.52	1.08
19	F	All students should be required to memorize the procedures that the text uses to solve problems	2.79	1.31	2.71	1.31
		Composite BAMI	76.10	7.69	81.29	10.72

Table 5.13: Spring 2010 Math 136 Informal Group BAMI Results (N=25)

#	F/I	Statement	Pre-Course		Post-Course	
			Mean	SD	Mean	SD
3	I	Students should be encouraged to invent their own mathematical symbolism	3.58	1.32	4.08	1.71
4	I	Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	4.64	1.15	5.40	0.76
5	I	Each student should feel free to use any method for solving a problem that suits him or her best	5.29	0.69	5.40	0.91
6	I	Teachers should provide class time for students to experiment with their own mathematical ideas	4.46	0.98	5.24	0.72
10	I	Teachers should frequently insist that pupils find individual methods for solving problems	3.60	0.87	4.40	0.96
13	I	The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	3.64	1.25	3.88	1.27
14	I	The teacher should consistently give assignments which require research and original thinking	3.68	1.35	4.28	0.94
15	I	Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	4.68	1.07	5.00	0.96
16	I	Teachers must frequently give students assignments which require creative or investigative work	4.56	0.87	4.60	1.12
20	I	Students of all abilities should learn better when taught by guided discovery methods	4.38	1.05	4.64	1.08
1	F	The teacher should always work sample problems for students before making an assignment	5.28	0.68	4.92	1.38
2	F	Teachers should make assignments on just that which has been thoroughly discussed in class	4.52	1.29	4.00	1.29
7	F	Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery	3.88	0.97	3.92	1.44
8	F	Most exercises assigned to students should be applications of a particular rule or formula	3.82	1.14	3.48	1.00
9	F	Teachers should spend most of each class period explaining how to work specific problems	3.64	1.25	3.28	1.10
11	F	Discovery methods of teaching have limited value because students often get answers without knowing where they came from	3.80	1.00	3.16	1.40
12	F	The teacher should provide models for problem solving and expect students to imitate them	4.04	0.79	3.26	0.97
17	F	Students should be expected to use only those methods that their text or teacher uses	3.08	1.35	2.28	1.24
18	F	Discovery-type lessons have very limited value when you consider the time they take up	3.22	1.04	2.60	1.26
19	F	All students should be required to memorize the procedures that the text uses to solve problems	3.00	1.15	2.60	1.35
		Composite BAMI	74.35	7.52	83.42	10.97

Similar to the BAM data, the initial analysis of the BAMI data investigated pre-course versus post-course BAMI composite scores. A paired *t*-test of statistical significance was conducted to test the hypotheses

$$H_0 : \mu_d = 0$$
$$H_a : \mu_d \neq 0$$

where μ_d was defined as the average of the pair-wise differences of post BAMI composite score and pre BAMI composite score. The test statistic used was

$$t_{n-1} = \frac{\bar{d}}{SE(\bar{d})}$$

where $\bar{d}$ is defined as the mean of the pair-wise BAMI composite differences (post minus pre), *n* the number of participants, and $SE(\bar{d})$ the standard error for the mean of the pair-wise differences. The test statistic, t_{n-1}, was compared to the student's *t* distribution with $n-1$ degrees of freedom in order to obtain a *p*-value for the determination of statistical significance. The results of this analysis are shown in Table 5.14.

Table 5.14: Results of Paired *t*-Tests of Significance BAMI Post Composite versus BAMI Pre Composite

Group	N	$\bar{d}$	$SE(\bar{d})$	t	df	*P*-Value
F2009 Control	18	-1.7880	2.2457	-0.7962	17	0.4369
F2009 Informal	21	5.1905	2.5241	2.0564	20	0.0530
S2010 Informal	25	9.2400	2.1147	4.3694	24	0.0002

The analysis failed to reject the null hypothesis for the fall 2009 control group ($p = 0.4369$), indicating a lack of evidence of any change in beliefs about mathematics instruction for the 18 students enrolled in the Math 136 course which did not feature the four informal mathematics activities. The analysis did provide some evidence against the null hypothesis for the fall 2009 informal group ($p = 0.0530$) pointing towards a possible change in beliefs about mathematics instruction towards a more informal outlook for the 21 students enrolled in the fall 2009 Math 136 course that featured the four informal mathematics activities. Finally, the analysis provided strong evidence ($p = 0.0002$) against the null hypothesis for the spring 2010 informal group, strongly suggesting a change of beliefs about mathematics instruction towards a more informal outlook for the 25 students enrolled in the spring 2010 Math 136 course that featured the four informal mathematics activities.

Prompted by the evidence of a statistically significant change in beliefs about mathematics instruction for both the fall 2009 and spring 2010 informal groups a ranking of BAMI item averages according to their contribution to positive change in BAMI composite was conducted for both groups. Informal items on the survey were ranked according to the degree to which each item's mean score increased across the semester, and, formal items on the survey were ranked according to the degree which each item's mean score decreased across the semester. The results of this ranking are found in Table 5.15 (fall 2009) and 5.16 (spring 2010). This analysis allowed for a comparison of individual beliefs about mathematics instruction, both formal and informal, according to their propensity to change in relationship to participation in informal mathematics activities.

Table 5.15: Fall 2009 Math 136 Informal Group BAMI Results (N=21) Ranked According to Contribution to Positive BAMI Change

#	F/I	Statement	Pre Mean	Post Mean	Post - Pre Mean
3	I	Students should be encouraged to invent their own mathematical symbolism	3.52	4.24	0.71
10	I	Teachers should frequently insist that pupils find individual methods for solving problems	3.43	3.95	0.52
6	I	Teachers should provide class time for students to experiment with their own mathematical ideas	4.76	5.19	0.43
15	I	Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	4.81	5.19	0.38
20	I	Students of all abilities should learn better when taught by guided discovery methods	4.05	4.43	0.38
4	I	Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	5.00	5.29	0.29
5	I	Each student should feel free to use any method for solving a problem that suits him or her best	5.05	5.24	0.19
13	I	The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	3.86	4.05	0.19
14	I	The teacher should consistently give assignments which require research and original thinking	4.35	4.33	-0.02
16	I	Teachers must frequently give students assignments which require creative or investigative work	4.86	4.65	-0.21
11	F	Discovery methods of teaching have limited value because students often get answers without knowing where they came from	4.24	3.05	-1.19
12	F	The teacher should provide models for problem solving and expect students to imitate them	3.86	3.43	-0.43
17	F	Students should be expected to use only those methods that their text or teacher uses	2.48	2.05	-0.43
1	F	The teacher should always work sample problems for students before making an assignment	5.71	5.33	-0.38
18	F	Discovery-type lessons have very limited value when you consider the time they take up	2.76	2.52	-0.24
2	F	Teachers should make assignments on just that which has been thoroughly discussed in class	4.57	4.38	-0.19
19	F	All students should be required to memorize the procedures that the text uses to solve problems	2.79	2.71	-0.07
8	F	Most exercises assigned to students should be applications of a particular rule or formula	3.57	3.75	0.18
7	F	Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery	4.19	4.43	0.24
9	F	Teachers should spend most of each class period explaining how to work specific problems	3.21	3.57	0.36

For the fall 2009 informal group the data display the largest increases in agreement in the following informal statements about mathematics instruction:

> Students should be encouraged to invent their own mathematical symbolism
>
> Teachers should frequently insist that pupils find individual methods for solving problems

The data also indicates the largest decrease in agreement to the following formal statements about mathematics instruction:

> Discovery methods of teaching have limited value because students often get answers without knowing where they came from
>
> The teacher should provide models for problem solving and expect students to imitate them
>
> Students should be expected to use only those methods that their text or teacher uses

Taken together, these items which display the largest changes towards an informal outlook perhaps indicate those beliefs about mathematics instruction which are most susceptible to change as a result of participation in informal mathematics activities. The following informal statements displayed relatively little change (less than three tenths) in average level of agreement over the course of the semester:

> Each student should feel free to use any method for solving a problem that suits him or her best
>
> The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself
>
> The teacher should consistently give assignments which require research and original thinking
>
> Teachers must frequently give students assignments which require creative or investigative work

And the following formal statements also displayed little change in agreement (less than three tenths on average) over the course of the semester:

> Discovery-type lessons have very limited value when you consider the time they take up
>
> Teachers should make assignments on just that which has been thoroughly discussed in class
>
> All students should be required to memorize the procedures that the text uses to solve problems
>
> Most exercises assigned to students should be applications of a particular rule or formula
>
> Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery

These items point towards those beliefs which are more resistant to transformation associated with participation in educational activities in mathematics which can be characterized as informal.

Several statements do merit individual discussion. For instance, the statement:

> The teacher should always work sample problems for students before making an assignment

displayed a moderate shift towards disagreement (-0.38) over the course of the semester, indicating a more informal outlook on the item. However, the average level of agreement (both pre and post) is the highest among all informal statements. The post-course average score of 5.33 demonstrates a high level of agreement among students with this formal belief concerning mathematics instruction. In fact, 13 of 21 students in the post-course survey rated the item “6=highly agree”. Additionally, the formal statement:

> Teachers should spend most of each class period explaining how to work specific problems

displayed a moderate increase (indicating a shift towards a more formal outlook) in average level of student agreement (0.36) over the course of the semester perhaps indicating a strengthening of this formal belief associated with exposure to informal mathematical activities.

Table 5.16: Spring 2010 Math 136 Informal Group BAMI Results (N=25) Ranked According to Contribution to Positive BAMI Change

#	F/I	Statement	Pre Mean	Post Mean	Post - Pre Mean
10	I	Teachers should frequently insist that pupils find individual methods for solving problems	3.60	4.40	0.80
6	I	Teachers should provide class time for students to experiment with their own mathematical ideas	4.46	5.24	0.78
4	I	Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error	4.64	5.40	0.76
14	I	The teacher should consistently give assignments which require research and original thinking	3.68	4.28	0.60
3	I	Students should be encouraged to invent their own mathematical symbolism	3.58	4.08	0.50
15	I	Teachers must get students to wonder and explore even beyond usual patterns of operation in mathematics	4.68	5.00	0.32
20	I	Students of all abilities should learn better when taught by guided discovery methods	4.38	4.64	0.26
13	I	The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself	3.64	3.88	0.24
5	I	Each student should feel free to use any method for solving a problem that suits him or her best	5.29	5.40	0.11
16	I	Teachers must frequently give students assignments which require creative or investigative work	4.56	4.60	0.04
17	F	Students should be expected to use only those methods that their text or teacher uses	3.08	2.28	-0.80
12	F	The teacher should provide models for problem solving and expect students to imitate them	4.04	3.26	-0.78
11	F	Discovery methods of teaching have limited value because students often get answers without knowing where they came from	3.80	3.16	-0.64
18	F	Discovery-type lessons have very limited value when you consider the time they take up	3.22	2.60	-0.62
2	F	Teachers should make assignments on just that which has been thoroughly discussed in class	4.52	4.00	-0.52
19	F	All students should be required to memorize the procedures that the text uses to solve problems	3.00	2.60	-0.40
1	F	The teacher should always work sample problems for students before making an assignment	5.28	4.92	-0.36
9	F	Teachers should spend most of each class period explaining how to work specific problems	3.64	3.28	-0.36
8	F	Most exercises assigned to students should be applications of a particular rule or formula	3.82	3.48	-0.34
7	F	Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery	3.88	3.92	0.04

The data from the spring 2010 informal group display the highest shift towards agreement with the following informal statements concerning mathematics instruction:

> Teachers should frequently insist that pupils find individual methods for solving problems
>
> Teachers should provide class time for students to experiment with their own mathematical ideas
>
> Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error

Note that the first statement was similarly classified in the fall 2009 formal group. The data also display the largest shifts toward disagreement with the following formal statements concerning mathematics instruction:

> Students should be expected to use only those methods that their text or teacher uses
>
> The teacher should provide models for problem solving and expect students to imitate them
>
> Discovery methods of teaching have limited value because students often get answers without knowing where they came from
>
> Discovery-type lessons have very limited value when you consider the time they take up

Note that all but the second statement above were similarly identified for the fall 2009 informal group. Again, these items, taken collectively, may point towards those beliefs which are most susceptible to transformation resulting from exposure to informal mathematical activities.

The data also display relatively small shifts (less than three tenths on average) in agreement or disagreement with the following informal statements about mathematics instruction:

> Students of all abilities should learn better when taught by guided discovery methods
>
> The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for her or himself
>
> Each student should feel free to use any method for solving a problem that suits him or her best
>
> Teachers must frequently give students assignments which require creative or investigative work

and the following formal statement about mathematics:

> Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery

Note that all statements but the first, “Students of all abilities…”, are similarly characterized by the fall 2009 informal group. These statements, taken together, may point towards beliefs about mathematics instruction which are resistant to change associated with participation in informal mathematics activities.

Similar to the fall 2009 informal group, the statement:

> The teacher should always work sample problems for students before making an assignment

received the highest level of agreement among all formal characterizations of mathematics instruction in both pre and post surveys. Although the level of agreement of the spring 2010 informal group is somewhat less than that of the fall 2009 informal group (4.92 versus 5.33) the item still stands out from other formal statements which displayed considerably lower levels of agreement on the post survey (ranging from 2.28 to 4.00).

A correlative study of BAMI and final course percentage was conducted for both the fall 2009 and the spring 2010 group. Plots of final course percentage versus pre BAMI, post BAMI and BAMI gain are shown in Figures 5.17 5.18 and 5.19 and a summary of correlation factors is presented in Table 5.20 and 5.21. Similar to the earlier analysis of the BAM data, the BAMI data demonstrate that, for students in both the fall 2009 group and the spring 2010 group, one's degree of informality of beliefs about mathematics instruction is not associated with achievement in mathematics as measured by final course percentages.

Figure 5.17: Pre BAMI Composite versus Final Course Percentage Fall 2009 Informal Group and Spring 2010 Informal Group

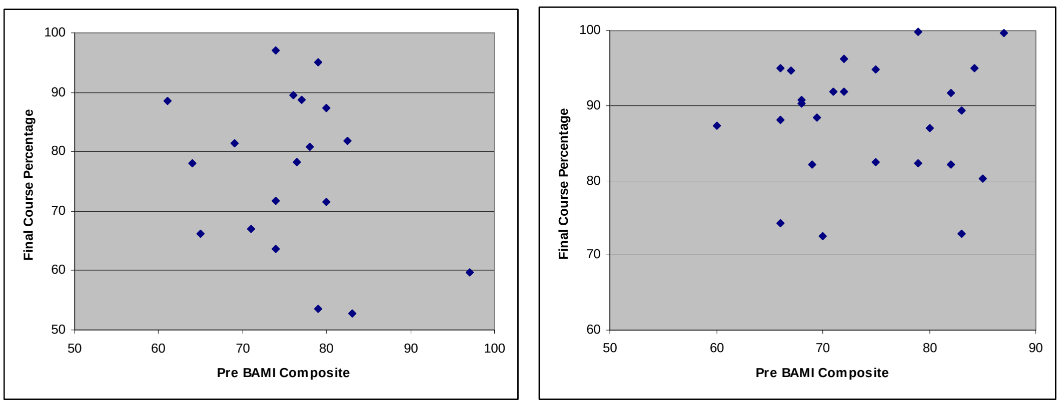


Figure 5.18: Post BAMI Composite versus Final Course Percentage Fall 2009 Informal Group and Spring 2010 Informal Group

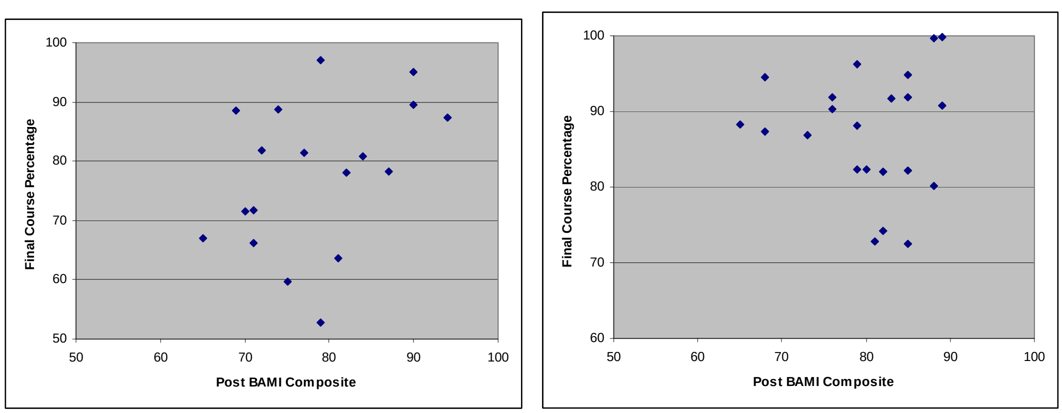


Figure 5.19: Gain in BAMI Composite versus Final Course Percentage Fall 2009 Informal Group and Spring 2010 Informal Group

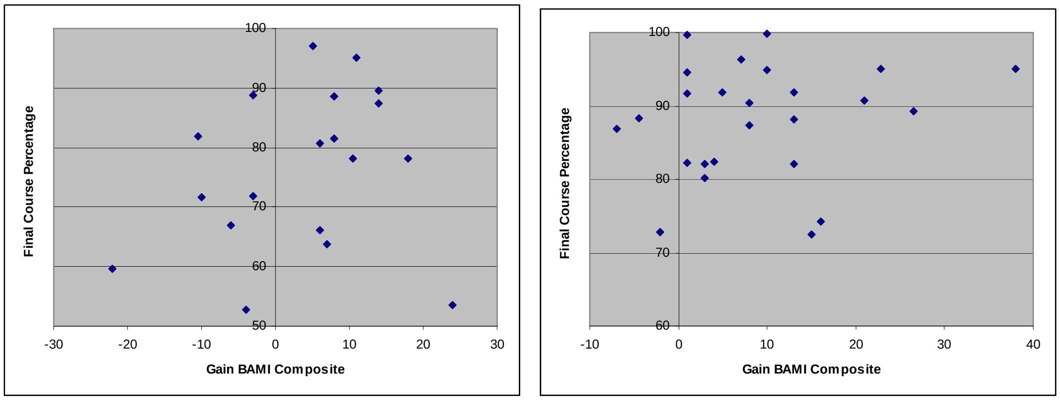


Table 5.20: Fall 2009 Informal Group Summary of Correlative Analysis of BAMI Scores and Final Course Percentage

	Pre BAMI	Post BAMI	Gain BAMI
Final Course Percentage Pearson Correlation (*P*-Value Two Tailed)	-0.208 (0.378)	0.162 (0.495)	0.293 (0.211)

Table 5.21: Spring 2010 Informal Group Summary of Correlative Analysis of BAMI Scores and Final Course Percentage

	Pre BAMI	Post BAMI	Gain BAMI
Final Course Percentage Pearson Correlation (*P*-Value Two Tailed)	0.043 (0.839)	0.178 (0.396)	0.158 (0.451)

QUALITATIVE ANALYSIS

The results of the qualitative analysis conducted in this research are presented in this section. Student-authored reflections were collected after completion of each of the four informal mathematics activities. These reflections were first read by the researcher in a quest to identify narrative concepts. These concepts were then grouped into themes according to their logical connection. Finally, the reflective data were reread and coded for the presence or absence of each theme and these results were collected into a categorical summary of themes according to activity.

Upon first reading the set of student reflections, the researcher noted 61 distinct narratives relating to beliefs about mathematics and mathematics instruction in response to each activity. These 61 narratives are provided below.

The Importance of Hands on Approach

A Foreign Experience in Comparison to Previous Mathematics Exercises

An Awareness of Problem Solving Strategies

An Awareness of Use of Tactics

An Experience of the Problem at Hand as Different

Awareness of Multiple Representations

Awareness of Multiple Solution Methods in Mathematics

A Building of Confidence in Mathematics

A Sense of Being Completely Confused

A Feeling of Mathematical Curiosity

A Difficulty in Articulating a Proof

A Difficulty Writing Up a Solution

A Positive Sense of Discovery

A Disposition to Use a Similar Activity in one's Future Classroom

A Sense of Enjoyment

A Sense of Excitement

A Growing Feeling of Competency in Mathematics

A Feeling of Certainty or Confidence in One's Solution

The Importance of Finding Patterns

The Importance of Following Mathematical Hunches

A Sense of Frustration

A Sense of Fun

A Sense of Impatience

The Importance of Discovery Methods in Building Student Confidence in Mathematics

The Importance of Giving Hints in Mathematics Education

The Importance of Monitoring Student Frustration

The Importance of Perseverance

The Importance of Positive Attitude in Problem Solving

The Importance of Problem Solving in Mathematics,

The Importance of Social Setting and Peer Collaboration in Problem Solving

The Importance of Understanding the Problem to be Solved

A Sense of Intrigue

The Invention of Mathematics Experienced as a "No-Fail" Setting

The Desire for a Traditional Teacher-Centered Classroom

Wanting or Needing More Guidance

A Sense of Never Having Done Something Like This Before,

A Sense of Newness of Experience

A Pride in One's Ability

A Questioning of One's Ability in Mathematics

A Sense of Satisfaction

Needing Confirmation from an Outside Source (internet, text, mathematician)

A Sense of Anxiety

A Sense of Being Overwhelmed

A Sense of Determination

A Sense of Fear

A Sense of Mathematical Creativity or Originality

A Sensing of Failure

An Experience of Surprise in Mathematics

Feeling Tension

The Importance of the Use of Manipulatives

The Important Role of Uncertainty in Generating Motivation in Educational Settings

Thinking Deeply About a Question in Mathematics

An Uncertainty in How to Begin

A Sense of Understanding Deeply in Mathematics

An Experience of Unexpected Outcomes

The Use of Intuition

Wanting an Immediate Answer

Wanting More Structure

Wanting to Give Up

Wanting to Know the Answer

INFORMAL THEMES

After reflection upon these narrative concepts, the researcher reflected upon the logical connectedness of each of the 61 narratives. These narratives were organized into nine themes which served as a framework for the categorical analysis of the qualitative data. Four themes were identified which align favorably with informal approaches in mathematics.   These themes are summarized in Table 5.22.

Table 5.22: Informal Themes Aligned in Favor with Informal Approaches to Mathematics

Code	Theme
I1	**Positive Affective Response to the Activity:** a sense of personal motivation, enjoyment, fun, value, creativity, curiosity, excitement; a building of personal confidence in mathematics; a sense of satisfaction and accomplishment resulting from the completion of a difficult task; a transformative experience in which a challenge is successfully overcome
I2	**Disposition in Favor of Hands-On Learning in Mathematics:** importance of the use of manipulatives, objects, self-created examples, or visual aids in generating mathematical understanding
I3	**Disposition in Favor of Social Collaboration in the Learning of Mathematics**: importance of interpersonal communication with partners, groups, tutors, and teachers as a means of *exposure* to new mathematical perspectives, ideas, and understandings; importance of social settings in the learning of mathematics as a means of *correcting* one's own misunderstandings and errors in thinking; an experience that social collaboration with peers facilitates the learning of mathematics
I4	**Disposition in Favor of Discovery Activities in the Learning of Mathematics**: importance of personal construction of knowledge through discovery; an aversion to rote memorization in mathematics education; an aversion to traditional lecture as the sole means of mathematical knowledge transfer; an awareness of the development of problem solving strategies as a legitimate educational goal in mathematics; an awareness of critical thinking, reasoning and proving as a legitimate educational goal in mathematics; a disposition to use discovery activities in one's future classroom; a disposition in favor of multiple approaches to problems in mathematics; an awareness of the importance of making connections in mathematics via creative discovery

The first theme supporting informal approaches in mathematics (I1) was a common topic in the reflective data. While not universal, many students expressed a positive affective response to certain activities. This theme was communicated in terms of "enjoyment" and "satisfaction" associated with the activity. The researcher aligns this theme with preservice elementary teacher dispositions in favor of informal approaches for the reason that teachers who enjoy and find satisfaction in such activities might be more likely to believe that such activities merit inclusion in their future classroom settings. Several examples of student reflections that were coded positively for this theme are provided below

> Overall I really liked this activity. Even though I was frustrated at points I just felt great at the end of class when I was confident with the number of nets I had come up with. This would be a great way to help kids understand the dimensions of different shapes. It will also help them figure out that they can't arrange things in any which way they want. It's a little more complicated then that. (Student F12, Activity 1)

> Starting with the same equation of perimeter and area equal, and this time solving for y, I came up with 2x/(x-2)=y. Using my graphing calculator I found out that this also worked! I was really surprised how I could come up with 3 main ways to solve this just by making a table and working on some basic equations. I personally liked the algebraic equation, just because that's the way I prefer to solve problems, but it was fun to figure out multiple ways of teaching area and perimeter. (Student F11, Activity 3)

> The third example was by far the hardest. I found myself coming to understand the two angles within this circle more and more by drawing in lines and looking for possible connections that could be useful. In the end I was not able to fully grasp this last example. Overall this activity was rewarding when a proof was discovered. It allows me to be completely confident in the conjecture and that was enjoyable, though it was not easy and sometimes quite frustrating not being able to understand it as well as I would have liked. (Student S20, Activity 2)

> This [activity] was particularly challenging, but well worth the effort that it required. It was nice to have the shapes in class so we could test our theories, although we found that having them distracted us from trying to figure out why and how tessellations are formed. (Student S22, Activity 4)

The second theme supporting informal mathematics activities (I2) was also a common aspect of student reflections. Student reflections that indicated a positive disposition for "hands-on" learning or an inclination towards the use of manipulatives in the learning of mathematics were coded positively for this theme. The researcher included this theme as an indication of alignment with informal approaches in mathematics because the use of manipulatives and "hands-on" activities are commonly associated with mathematical activities which include an element of exploration, discovery and justification. Students who believe that "hands-on" approaches aid in learning are likely to incorporate elements of discovery and exploration in their presentation of mathematics to elementary school children. Examples of student reflections that were coded positively for this theme are provided below.

> I approached the problem very hands on. I made a visual aid that literally was a cube with movable nets so that I could physically test out any ideas I had. This worked well for me; I found the majority of the solutions doing this. The rest were obtained, as I mentioned above, through collaboration with peers. I would find it very interesting to hear of anyone figuring out this problem strictly through reasoning only and no aides or collaboration. (Student F20, Activity 1)
>
> I thought that the Perimeter and Area project was easier for me to figure out than our previous project. Again I think that this has a lot to do with the fact that we had stuff to actually physically work with while in class and the visual is always very helpful to me. (Student F22, Activity 3)
>
> In terms of teaching, I can see how many students would rush to use their protractors to solve for the angles. However, although this seems like an easy short cut, this problem must be solved algebraically. I think this exercise would be a great way to emphasize that there are different ways to solve each problem, which is something they wouldn't get through direct instruction alone. This also led me to see that in many circumstances, constructivism is the winning approach when teaching a class of students especially in the field of mathematics. Students are often less engaged, and therefore take less away from a lesson when it is taught with direct instruction. Furthermore, math is simply more fun for students when it is more "hands on". (Student S19, Activity 2)

> This [activity] was the most challenging of this year. Being able to actually try out tessellations with shapes really helped. Later when we took a different approach to finding all the possibilities that added up to 360 degrees, we still needed that hands-on experimenting to test if the possibilities worked. We had a group that that could try out different ideas too, so that was very helpful in finding all the tessellations. We needed everyone in our group thinking up new ideas after we tried one that we thought was working, then we carried it out and it turned out that not all the shapes fit together. It was frustrating when you thought you found a tessellation and it turned out it wasn't one. With our group working together trying out new ideas, we figured it out though. (Student S17, Activity 4)

The third theme supporting informal approaches to mathematics (I3) centered on the role of social collaboration in mathematical meaning making. Students who commented on the utility of group settings in coming to understand and complete the informal activities were coded positively. The researcher justifies the alignment of this theme with informal approaches to mathematics based on the assumption that formal approaches which emphasize rules and procedures in mathematics are primarily concerned with the building of individual skills acquired through private and repetitive practice. In contrast, informal approaches in mathematics emphasize processes in mathematics which incorporate proof and reasoning, problem solving, making connections, communication and representation (NCTM, 2000). Many of these processes are necessarily social (i.e. communication as well as proof and reasoning). So, students who believed that social settings were important to teaching and learning in mathematics were thought to be expressing a vision of mathematics education which was committed to communication, proof and reasoning over skill-building and rote memorization. Examples of student themes that were coded positively for this theme are provided below.

> I also think working with group's helps kids learn and interact with each other. Some kids do not like talking to the teacher so talking to their peers would be easier to ask questions and help each other figure it out. When they explain it to each other they are actually helping themselves learn and remember it in the future. (Student S15, Activity 1)

> This kind of activity would be good for students because it teaches them how to work in groups and seek help when it is needed. I definitely needed help on this assignment and it was such a relief to know that I had another classmate to explain how they solved the circles. (Student S18, Activity 2)

> This problem was far more interesting to me than the second [activity]. On first look I thought there would be many rectangles with same area and perimeter in the answer, and upon finding the two early examples I was sure more would pop up. I used a system of guess and check to the point of exhaustion before looking at the problem with algebra, as my group quickly suggested. It was, for me a great experience in the group sense, due to the different viewpoints and techniques they suggested. Without the group I would not have found an algebraic proof to the problem. (Student F6, Activity 3)

> This assignment taught me a lot about how children see math. You see the first couple that super easy to find but as it gets tougher the easier it was for me to get frustrated. What helped I think was having groups because you can bounce ideas off each other and more heads are better then one. (Student S2, Activity 1)

The last theme supporting informal approaches in mathematics centered on the positive appraisal of discovery learning in mathematics. This belief took on many variations in the data: some commented on increased motivation in discovery settings while others commented on greater understanding that results from personally coming to know through discovery, still others simply found discovery learning more fun and suspenseful. The theme, by its very definition, aligns with informal approaches in the subject of mathematics. Several exemplary selections of student reflections that were coded positively for this theme are provided below.

> I learned that it is very important to let students work hard even if they are struggling because they will get it eventually. If needed, a small hint and collaboration can instill great confidence and optimism into a student. It is important for students to explore and come up with solutions on their own by problem solving, rather then being spoon fed all the answers. If someone had told us at the beginning there were 11 distinct nets, we would have lost the curiosity, satisfaction of discovery, and the excitement of coming to know and understand the answer. (Student F8, Activity 1)

> In the future when I become a teacher myself I think that doing a project similar to this would make my students really stretch their minds. The experience that I gained from doing this project was that I really tried every possible solution I could before giving up. When you do not put a limit on something then I think people work twice as hard to find the answer. So as a future teacher I will try to do many projects like this so that my students will really branch out and use their minds to their full extent. (Student F21, Activity 1)

> I also drew many of the same conclusions out of this activity about teaching and the way students learn. We should challenge them with something that is a little more than they are used to but that is still totally within their skill set. It is not only a lot more satisfying for a student to come to a conclusion or solution on their own, but they also retain more of that information because they were allowed to find their own way to the solution and have a better understanding of it because of this. This is what I experienced as a student doing the net cube project and I imagine the same holds true for all students. Student F22, Activity 1)

> I think that this sort of activity can be very useful and effective when in the classroom. I know that students will become frustrated in their attempts but sometimes that is the best way to learn something. I know that by learning constructively, students gain a better appreciation for the knowledge they gain because they have seen the struggle they have to go through in order to discover it. I will definitely be using activities such as this not only in mathematics but in many other subject areas because I see it as a very useful means of teaching and learning. (Student F9, Activity 4)

> The strategy of giving kids a vague problem to work on is very effective. A problem that they have to build their own process of solving for and a small incremental step-by-step building process that leads to a greater understanding and meaning can enlighten young minds. The strive to come up with a unique process with your group and tell the teacher about it is enough inspiration for the students to deepen their roots in math and come more in-tune with its mechanics, a good math teacher will have the [patience] to sit back and let the students struggle a bit in order to heighten that final understanding satisfaction and development. Any activity that promotes group work and collaboration with a manipulative problem or process will motivate the students to come up with creative and elaborate ways on route to a solution. (Student S8, Activity 1)

FORMAL THEMES

In addition to the four themes supporting informal approaches in mathematics, the researcher also identified five themes aligned with formal approaches in mathematics. Descriptions of these five themes which align with formal approaches to mathematics are provided in Table 5.23.

Table 5.23: Formal Themes Aligned in Favor with Formal Approaches to Mathematics

Code	Theme
F1	**Low-Level Negative Affective Response to the Activity:** A sense of frustration, tension, anxiety or fear experienced in association with the activity which is eventually resolved, lessened, or overcome.
F2	**High-Level Negative Affective Response to the Activity:** an *unresolved* and *persistent* sense of dread, confusion, anxiety, fear, or bewilderment; an *unresolved* and *persistent* sense of being lost, of not knowing what to do, of feeling stupid, dumb or ignorant
F3	**Disposition against Discovery Activities in the Learning of Mathematics:** a disposition not to use discovery activities in one's future classroom; a vision of mathematics instruction as primarily procedural; a confirming experience that mathematics should be taught in a teacher-centered environment to avoid confusion resulting from open ended discovery activities; a disposition that discovery learning is not always practical in mathematics classrooms; a disposition against multiple approaches to problems in mathematics
F4	**A Desire for More Guidance or Confirmation:** a sense of wanting more structure; a sense of needing a procedure to follow; a need for more hints; a sense that the role of the teacher is to resolve student frustration; a sense of wanting to know the right answer; looking up the answer in a book, on-line, or elsewhere;
F5	**Difficulty in the Construction of Mathematical Proof:** an uncertainty in the validity of one's proof; a sense of newness to proof-making; difficulty in articulating proof; knowing a proposition is true but not knowing how to demonstrate the veracity of the proposition

It is worth mentioning here that the researcher (who was also the teacher of the course) made a special effort to ask for students' "true reflections" upon completion of each activity. It was made clear that a student's standing in the course was in no way associated with either a positive or a negative response to the activities. Furthermore, the researcher made it clear that these activities were being "investigated" to determine their merit in training future elementary school teachers; thus, any negative response would actually be "helpful" in arriving at such a conclusion.

The first two themes supporting formal approaches in mathematics (F1 and F2) describe two levels of negative affective response to the activities. A low level negative affective response (F1) entails a sense of “frustration” or “anxiety” associated with the activity which is eventually lessened, resolved or overcome. A high level negative affective response (F2) entails a *persistent* and *unresolved* sense of anxiety, dread, fear. Many times the second theme was encountered in a metaphorical sense in the data. Here students often described “being lost” or “not knowing where they were going” as a persistent and unresolved response to the activity. Finally, in its most extreme form, some students commented that the end result of participation in the activity in question made them feel “dumb” or “ignorant” or “stupid”. The researcher aligned this theme with formal approaches to mathematics under the assumption that teachers who associate frustration and anxiety with informal activities might believe that the exclusion of such activities serves to help their students avoid such negative experiences. Examples of student reflections that were coded positively for low level negative affective response (F1) are provided below.

> It does make me a tad nervous when there is no definite answer. In this case and in many other reflective math activities we are never given the answer and we are expected to find one that we think is correct with no way of formality to go about such a solution. It makes me almost uncomfortable not knowing the exact solution. It's hard to even start a math problem not knowing how many answers there will be or a way of knowing you have the right one. I totally know the feeling of not being confident in your work. (Student F13, Activity 3)

> Drawing in lines in the wrong places was frustrating, but by process of elimination, figuring out which lines to draw, or by "luck of the draw" I was able to find the relationship between the two angles. I always felt like I was really close to figuring out the proof to my conjecture but kept falling short. It was frustrating because I am horrible at creating proofs for math. It feels like I am trying to communicate in a foreign language that I can understand, but not speak. (Student S12, Activity 2)

Several examples of student responses which were coded positively for high level negative affective response (F2) are provided below.

> I felt very frustrated and like I wanted to give up on the [activity]. I couldn’t figure it out and after staring at it for over an hour and a half I knew I wouldn’t get it. Geometry has always been a difficult subject for me and I have always had trouble with proofs. So it was like the two most daunting tasks for me all rolled into one. (Student S2, Activity 2)

> I think at this point I was looking for some form of an equation that might lead me to the answer. I tried out a few but nothing stuck to 11. Throughout this whole process both times around I didn’t/don’t really understand what I am looking for. I mean I know how to make a cube but why nessasarily do I have to know how many different ways there are. (Student F13, Activity 1)

> I am not sure what this activity had to do with chapter eleven. I wish there were a way to tie these activities together with homework activities and tests. I guess I am just frustrated. I feel like I am very slow at a lot of the math that we are doing this semester. I will get it eventually and I think that struggling in this class will make me a better teacher. I will be more empathetic with students who are struggling. If nothing else this class has been a humbling experience and I am learning to the best of my ability. (Student F14, Activity 2)

> There were many things that detracted from this problem for me, I just can’t ever really figure out what they are asking of me. Without my group members I do not think I could have figured it out. These problems are always usually very difficult for me so it makes me feel like I am not very good at math. I have always struggled with math making it hard for me to learn new things because im just trying to get through it. (Student S13, Activity 3)

The third formal theme (F3) that was encountered in the student reflective data was a disposition against discovery approaches in mathematics. This theme grouped together reflections that indicated that the informal activity had produced an aversion to student-centered discovery learning activities in mathematics classrooms. Reflections coded positively included those which critiqued the activity in question as “too open ended” or “too advanced for elementary school children”. Also coded positively were reflections which promoted teacher-centered notions of mathematics classrooms where students should be told exactly “what to do” before being asked to “do it”. The researcher included this theme as one which aligned with formal approaches to the subject because such reflections stand in opposition to informal approaches which are student-centered and incorporate a good deal of uncertainty, open-endedness and discovery. Examples of reflections that were coded positively for this theme are provided below.

> I really didn't like this problem because the one reasonable tactic is to guess and check. It gets boring and I lost interest in it almost right away. As I continued to guess and check I noticed that the area was becoming way too large for the perimeter to match up with. I forgot to add that I don't think I would use this type of an activity in my classroom. Unfortunately, I feel the lesson I'm taking away is that of "what not to do". I feel this activity was just about plugging in numbers and not a lot of logic or reasoning. (Student F16, Activity 3)
>
> That being said as soon as you think of the logistics involved in making a tessellation work, it really wasn't that difficult. The angles all need to work together and so all we needed to do was find which combinations worked together to make 360. I can see how this assignment would be very frustrating and confusing to kids. The math involved is not all that complicated…(Student F1, Activity 4)
>
> In this situation I can not say that no hints at all would have been better so that I was not looking in the wrong direction because maybe I just was taking the advise in the wrong way. But I can say that giving the first two proofs to do on your own and then having some guided direction for the third one would have been a much better approach for my personal learning style. The third proof left me more aggravated and stressed that I wasn't going to be able to

> figure it out than having a sense of success once it was completed. (Student S23, Activity 3)
>
> If there is some way to make math fun and interesting for kids than I think this would help a lot. Using tables and graphs and deeply explaining the problems to the kids will help a lot. (Student S13, Activity 3)

The fourth formal theme (F4) in the student reflective data documented the theme of "wanting more guidance" in solving the informal mathematical activity or "providing more guidance" for future students. Also grouped into this theme were those students who confessed to seeking "outside help" in solving the activity. The researcher aligned this theme with informal approaches in mathematics education theorizing that students who reflected upon the "need for guidance" or sought "outside help" in solving the informal activities were expressing a personal sense of inefficacy when faced with the prospect of mathematical discovery. This might be an indication, then, of authoritarian notions of mathematics education. Here knowledge flows from an experienced authority (i.e. teacher, book or internet) to an inexperienced pupil. Teaching is envisioned as "telling" and learning is envisioned as a mastery of "facts" or "rules" or "procedures". This model of "understanding" in mathematics stands in contrast to informal notions where mathematical knowledge created through activities which involve a good deal of investigation, experimentation and discovery. Examples of student reflections which were coded positively for this theme are provided below.

> I thought the assignment would be pretty simple but as I kept working on it I just got frustrated. It was hard for me to not have a lot of guidance for what to do and that detracted from my learning because I am use[d] to having more structure for an assignment and I didn't know how many nets I needed to be looking for. (Student S18, Activity 1)

> I think it is important for students to try constructing and proving problems on their own, but they do need some guidance. This [activity] needed a little more guidance to help us see where we were trying to go… It is important to challenge students, but it is also important to limit the amount of frustration that they feel. If a student gets too frustrated, they will cease to learn. (Student S5, Activity 2)

> I feel it is important for students to discover proofs on there own, at the same time i think it is important not set the students up for failure. I too feel that guidance is a key component when teaching math. (Student F17, Activity 2)

> I would love to let my students have the opportunity to learn something on their own, to explore and discover some of the cool things in math that lots of people don't know. Students need guidance when doing these projects, sometimes it depends on the students as well. If a student is on the right track, go ahead and let them keep plinking away. If not, it is important to provide other guidance to ensure that the student is not so far gone and frustrated that they don't care anymore. That happens more often than we think it does. (Student F8, Activity 4)

The fifth and final formal theme (F5) encountered in the reflective data centered on student difficulty with proof. Here, students who commented that they "struggled" to articulate their proof, or were "unsure of the validity" of their proof, or "did not know how to prove" their conjecture were coded positively for this theme. The researcher aligned this theme with formal approaches to the subject due to the fact that teachers who are uncertain of the process of proof in mathematics are likely to avoid activities that incorporate proof in their own classrooms. Since reasoning and understanding in mathematics is often tied to educational activities which incorporate some element of mathematical proof, difficulty in understanding and articulating proof might be seen as motivation for rule-bound and procedural approaches in the subject; approaches that this research identifies as “formal”. Examples of student reflective data coded positively for this theme are provided below.

> By using these squares I knew that I had the correct set just by folding them together, if it made a cube I knew I had another net. Even though my group had found what we felt was the total possible nets, no one could really explain why. Although I know our answer is correct I don’t feel confident with this problem because I was unable to figure it out for myself and I still don’t understand why there are only 11 possible nets. This question of “why” is what I think makes math so hard for people to handle. (Student F17, Activity 1)
>
> By simply looking at the angles I had a slight idea that the inner angle might be half the measure of the outer angle. So my group and I measured the angles using a protractor. This did in fact conclude that the inscribed angle is half the outer angle. But I still didn’t seem quite convinced. I had proof but I felt like I needed a more solid answer. (Student F7, Activity 2)
>
> The next challenged I happened across was actually proving that these were the only combinations. I was unsure of actually how to go about proving this. There are so many combinations out there, that I was afraid that I was missing some. (Student S3, Activity 3)
>
> Luckily for me in class we proved that we had all the 3 at a vertex and it helped to figure out how to prove 4 and 5 at a vertex. I’m still not sure that I was able to prove the problem correctly but I think I was able to find all the semi-regular tessellations. At times during this activity I felt like I wouldn’t reach the point to where I had proved that I had them all. (Student S21, Activity 4)

THEME CATEGORICAL ANALYSIS

Using these nine themes as a framework the verbal data was reread and categorically analyzed for the presence or absence of each of the nine themes on a per student basis for each activity. This analysis was conducted separately for the fall 2009 informal group and the spring 2010 informal group in order to identify any salient differences between reflections offered by each experimental group. The results of this analysis for the fall 2009 group are presented in Table 5.24 through 5.28. The results for the spring 2010 group are presented in Table 5.29 through 5.32. In each table, a "1" indicates that the presence of the theme in the student's reflection was detected by the researcher. In each table a "0" indicates that the presence of the theme in the student's reflection was not detected by the researcher. Students who failed to submit a reflection for the activity are indicated by an "N". Finally, results for each theme in each activity are summed to arrive at a count of the number of detected themes (C) as well as the percentage (%) of the student reflections in which the theme was detected. Tables 5.24 through 5.32 represent the results of the categorical analysis conducted by the researcher and are offered to the reader in an effort to maintain a level transparency associated with the researcher's judgment of the presence or absence of each theme in each student's reflection. Note that student reflective data is included in this publication and can be found in the appendix.

In an effort to judge the contribution of each informal activity towards the informal shift in beliefs that were detected across the semester in the quantitative analysis, the results from Tables 5.24 through 5.32 were aggregated in Table 5.33. This analysis provides the count of detected themes (C), the number of student reflections (N), and the ratio, as a percent, of the count of detected themes to the number of student reflections. Results are provided for the fall 2009 and the spring 2010 groups separately and aggregately. Finally, aggregate results for each theme are combined according to each theme's alignment with formal or informal beliefs as previously discussed.

The combined results in Table 5.32 display differential results in the proportion of student reflection themes associated with each activity. Activities 1, 3, and 4 all exhibit a higher proportion of student reflection themes which align with informal beliefs regarding mathematics and mathematics instruction. In activity 1, 64 % reflected upon themes aligning with informal beliefs compared to 24% reflecting upon themes aligning with formal beliefs. In activity 3, 52 % reflected upon themes aligning with informal beliefs compared to 15% reflecting upon themes aligning with formal beliefs. In activity 4, 58 % reflected upon themes aligning with informal beliefs compared to 25% reflecting upon themes aligning with formal beliefs. Moreover, the proportion of students reflecting on informal themes in activities 1, 3, and 4 is at least twice as high as those reflecting on formal themes. Activity 2 does not exhibit the same pattern of results. Here only 33% reflected upon themes aligning with informal beliefs while 39% reflected upon themes aligning with formal beliefs. The analysis identifies activities 1, 3, and 4 as activities which generally contribute to the transformation of beliefs of preservice teachers towards a more informal outlook. The analysis identifies activity 2 as an activity that does not generally contribute to the transformation of beliefs of preservice teachers towards a more informal outlook; but, instead, may be associated with the converse. That is, based on the results and analysis, activity 2 may have encouraged a more formal outlook towards mathematics and mathematics education as evidenced by the high proportion of students who offer reflections which are aligned with formal beliefs. Of special concern is the fact that 54% report a high-level negative affective response to the activity (F2) and 51% report a desire for more guidance in response to the activity (F4).

Table 5.24: Fall 2009 Theme Analysis Activity 1 - Nets of the Cube

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	0	1	0	1	0	1	0	1	0
F2	1	1	0	1	0	0	0	0	0
F3	0	1	0	1	1	0	0	0	0
F4	N	N	N	N	N	N	N	N	N
F5	1	1	1	1	0	0	0	0	1
F6	0	0	0	0	0	0	0	0	0
F7	1	1	1	1	1	0	0	0	0
F8	1	1	0	1	1	0	0	1	0
F9	1	1	0	1	1	0	0	1	0
F10	N	N	N	N	N	N	N	N	N
F11	1	1	1	1	1	0	0	0	0
F12	1	0	0	1	1	0	0	0	0
F13	0	0	0	0	0	1	0	0	0
F14	0	0	0	0	0	1	0	1	0
F15	0	0	0	0	1	0	0	1	1
F16	0	0	0	0	1	0	0	0	0
F17	0	0	0	0	0	1	1	1	0
F18	1	1	0	1	1	0	0	0	0
F19	0	0	1	0	0	1	0	0	0
F20	1	1	1	0	1	0	0	0	0
F21	0	0	0	1	0	1	0	0	0
F22	1	1	1	1	1	0	0	0	0
F23	1	0	0	0	0	1	0	0	1
C	**11**	**11**	**6**	**12**	**11**	**7**	**1**	**6**	**3**
%	**52**	**52**	**29**	**57**	**52**	**33**	**5**	**29**	**14**

Table 5.25: Fall 2009 Theme Analysis Activity 2 - Inscribed Angle Theorem

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	N	N	N	N	N	N	N	N	N
F2	1	0	0	0	0	1	0	0	1
F3	N	N	N	N	N	N	N	N	N
F4	N	N	N	N	N	N	N	N	N
F5	0	0	1	0	0	0	0	1	0
F6	1	1	1	0	0	1	0	1	1
F7	0	0	0	0	1	0	0	1	0
F8	1	1	1	1	0	0	0	0	0
F9	0	0	0	1	1	0	0	0	0
F10	N	N	N	N	N	N	N	N	N
F11	N	N	N	N	N	N	N	N	N
F12	0	0	0	0	0	1	0	1	1
F13	0	0	0	0	1	0	0	0	0
F14	0	0	0	0	1	0	0	1	0
F15	0	0	0	1	0	1	1	1	0
F16	0	0	0	1	0	1	1	1	1
F17	0	0	0	1	0	1	1	1	1
F18	0	0	0	1	0	1	1	0	1
F19	0	0	1	0	0	1	0	1	1
F20	0	0	0	0	0	1	0	0	1
F21	0	0	0	0	1	0	0	0	0
F22	0	0	0	1	1	0	1	0	1
F23	0	0	1	0	0	1	0	0	0
C	**3**	**2**	**5**	**7**	**6**	**10**	**5**	**9**	**9**
%	**17**	**11**	**28**	**39**	**33**	**56**	**28**	**50**	**50**

Table 5.26: Fall 2009 Theme Analysis Activity 3 - Isis Problem

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	N	N	N	N	N	N	N	N	N
F2	1	0	1	1	0	0	0	0	0
F3	1	0	0	1	0	0	0	0	0
F4	N	N	N	N	N	N	N	N	N
F5	1	0	0	0	0	0	0	0	0
F6	1	0	1	0	0	0	0	0	0
F7	1	0	1	1	0	0	0	0	0
F8	1	0	1	0	0	0	0	0	0
F9	1	0	1	1	0	0	0	0	0
F10	N	N	N	N	N	N	N	N	N
F11	1	0	1	0	0	0	0	0	0
F12	1	0	1	1	0	0	0	0	0
F13	0	0	0	0	1	0	0	1	0
F14	1	0	0	0	0	0	0	0	0
F15	1	0	0	0	0	0	0	1	0
F16	0	0	0	0	1	0	1	0	0
F17	1	0	0	0	0	0	0	1	0
F18	1	0	0	0	0	0	0	0	0
F19	1	0	0	0	0	0	0	0	0
F20	1	0	0	1	0	0	0	1	0
F21	0	0	0	1	0	0	0	0	0
F22	0	1	0	1	0	0	0	0	1
F23	1	0	1	1	1	0	0	0	0
C	**16**	**1**	**8**	**9**	**3**	**0**	**1**	**4**	**1**
%	**80**	**5**	**40**	**45**	**15**	**0**	**5**	**20**	**5**

Table 5.27: Fall 2009 Theme Analysis Activity 4 - Semi Regular Tessellation

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	0	1	0	0	0	0	1	0	0
F2	0	1	0	0	0	0	0	0	0
F3	N	N	N	N	N	N	N	N	N
F4	N	N	N	N	N	N	N	N	N
F5	N	N	N	N	N	N	N	N	N
F6	N	N	N	N	N	N	N	N	N
F7	N	N	N	N	N	N	N	N	N
F8	1	0	1	1	0	0	0	1	1
F9	0	0	1	1	1	0	0	0	0
F10	N	N	N	N	N	N	N	N	N
F11	N	N	N	N	N	N	N	N	N
F12	1	0	0	1	1	0	0	0	0
F13	0	1	0	1	1	0	0	0	0
F14	N	N	N	N	N	N	N	N	N
F15	1	0	0	1	1	0	0	1	0
F16	1	0	0	1	0	0	0	0	0
F17	1	1	1	1	0	0	0	0	0
F18	N	N	N	N	N	N	N	N	N
F19	1	1	0	0	0	0	0	0	0
F20	N	N	N	N	N	N	N	N	N
F21	1	0	0	1	0	0	0	0	1
F22	1	0	1	1	0	0	0	0	0
F23	N	N	N	N	N	N	N	N	N
C	**8**	**5**	**4**	**9**	**4**	**0**	**1**	**2**	**2**
%	**67**	**42**	**33**	**75**	**33**	**0**	**8**	**17**	**17**

Table 5.28: Spring 2010 Theme Analysis Activity 1 - Nets of the Cube

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	1	0	0	1	1	0	0	0	1
S2	0	1	1	1	0	0	0	0	0
S3	1	1	0	1	1	0	0	1	0
S4	1	1	1	1	1	0	0	0	0
S5	1	1	1	1	1	0	0	0	0
S6	0	1	1	1	0	0	0	1	0
S7	0	1	1	1	0	0	0	0	0
S8	1	1	1	1	0	0	0	0	0
S9	1	1	1	1	0	0	0	0	0
S10	1	1	1	0	1	0	0	0	0
S11	0	1	0	1	0	0	0	0	1
S12	1	1	1	1	0	1	1	0	0
S13	0	1	0	1	1	0	0	1	0
S14	1	1	0	0	0	0	0	0	0
S15	1	1	1	1	1	0	0	1	0
S16	0	1	0	1	0	0	0	0	0
S17	1	1	1	1	0	0	0	0	0
S18	0	1	1	1	1	0	0	1	0
S19	0	1	0	1	1	0	0	0	0
S20	1	1	0	1	1	0	0	0	0
S21	1	1	0	1	0	0	0	0	0
S22	1	1	0	1	1	0	0	0	0
S23	N	N	N	N	N	N	N	N	N
S24	1	1	1	1	1	0	0	0	1
S25	1	1	1	1	1	0	0	1	0
C	**16**	**23**	**14**	**22**	**13**	**1**	**1**	**6**	**3**
%	**67**	**96**	**58**	**92**	**54**	**4**	**7**	**25**	**13**

Table 5.29: Spring 2010 Theme Analysis Activity 2 - Inscribed Angle Theorem

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	1	0	1	0	1	0	0	0	0
S2	0	0	0	0	0	1	0	0	0
S3	0	0	0	1	0	1	1	1	0
S4	1	0	1	1	0	0	1	0	0
S5	0	0	0	1	0	1	0	1	1
S6	0	0	0	1	0	0	0	1	0
S7	0	0	0	1	0	1	0	1	1
S8	0	0	1	1	1	0	0	0	0
S9	1	0	0	1	0	1	0	0	0
S10	N	N	N	N	N	N	N	N	N
S11	1	0	0	1	1	0	0	1	0
S12	0	0	0	0	0	0	1	0	1
S13	0	0	1	0	0	1	0	1	0
S14	N	N	N	N	N	N	N	N	N
S15	1	0	1	0	0	1	1	1	0
S16	1	0	0	1	1	0	0	0	0
S17	0	0	1	0	0	1	0	0	0
S18	0	0	1	0	0	1	0	1	0
S19	0	1	1	1	1	0	0	1	0
S20	1	1	0	1	0	1	1	0	0
S21	0	0	0	1	0	1	0	0	0
S22	1	0	0	1	1	0	0	0	0
S23	1	0	0	0	1	0	1	1	0
S24	1	0	0	1	0	0	0	1	1
S25	1	0	1	1	0	1	0	1	0
C	**11**	**2**	**9**	**15**	**7**	**12**	**6**	**12**	**4**
%	**48**	**9**	**39**	**65**	**30**	**52**	**26**	**52**	**17**

Table 5.30: Spring 2010 Theme Analysis Activity 3 - Isis Problem

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	1	0	1	1	0	0	0	0	1
S2	1	0	1	1	0	0	0	0	0
S3	1	0	0	1	0	0	1	0	1
S4	1	0	1	1	1	0	0	0	0
S5	0	0	1	1	1	0	0	0	0
S6	0	0	1	1	1	0	0	0	0
S7	1	0	1	1	0	0	0	0	0
S8	0	0	1	1	1	0	1	0	0
S9	0	0	1	1	0	0	0	0	0
S10	N	N	N	N	N	N	N	N	N
S11	1	0	1	1	0	0	0	0	1
S12	1	0	1	0	0	0	0	0	1
S13	0	1	1	0	0	1	1	1	0
S14	0	1	1	1	0	0	0	0	1
S15	0	1	1	1	0	0	0	0	0
S16	0	0	1	1	0	0	0	0	0
S17	1	0	0	0	1	0	1	0	1
S18	1	0	1	1	0	0	0	0	1
S19	1	0	1	1	0	0	0	0	0
S20	1	0	1	1	1	0	0	0	0
S21	1	0	1	1	0	0	0	1	0
S22	0	0	0	1	1	0	0	0	0
S23	0	0	1	1	0	0	0	0	0
S24	0	0	1	1	1	0	0	0	0
S25	1	0	1	1	1	0	0	0	1
C	**13**	**3**	**21**	**21**	**9**	**1**	**4**	**2**	**8**
%	**54**	**13**	**88**	**88**	**38**	**4**	**17**	**8**	**33**

Table 5.31: Spring 2010 Theme Analysis Activity 4 - Semi Regular Tessellation

Student	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	1	0	0	1	0	1	0	0	1
S2	N	N	N	N	N	N	N	N	N
S3	1	0	0	1	0	0	0	0	1
S4	1	1	1	1	0	0	0	0	1
S5	1	1	0	0	1	0	1	0	0
S6	0	1	0	1	1	0	0	1	0
S7	0	1	1	0	0	0	0	0	0
S8	N	N	N	N	N	N	N	N	N
S9	0	1	1	1	0	0	0	0	1
S10	N	N	N	N	N	N	N	N	N
S11	1	1	1	0	1	0	0	0	0
S12	N	N	N	N	N	N	N	N	N
S13	0	0	0	0	0	1	1	1	0
S14	N	N	N	N	N	N	N	N	N
S15	N	N	N	N	N	N	N	N	N
S16	N	N	N	N	N	N	N	N	N
S17	0	1	1	1	1	1	0	0	0
S18	0	1	1	1	1	1	0	1	0
S19	1	0	1	0	1	1	0	0	0
S20	0	1	0	1	1	1	0	0	0
S21	1	0	0	1	0	0	1	1	1
S22	1	1	1	1	0	0	0	0	0
S23	N	N	N	N	N	N	N	N	N
S24	1	1	0	1	0	0	0	0	1
S25	N	N	N	N	N	N	N	N	N
C	**9**	**11**	**8**	**11**	**7**	**6**	**3**	**4**	**6**
%	**56**	**69**	**50**	**69**	**44**	**38**	**19**	**25**	**38**

Table 5.32: Aggregate Theme Analysis

		Fall 2009			Spring 2010			Total			Combined		
		C	N	%	C	N	%	C	N	%	C	N	%
Activity 1	I1	11	21	52	16	24	67	27	45	60	115	180	64
	I2	11	21	52	23	24	96	34	45	76			
	I3	6	21	29	14	24	58	20	45	44			
	I4	12	21	57	22	24	92	34	45	76			
	F1	11	21	52	13	24	54	24	45	53	53	225	24
	F2	7	21	33	1	24	4	8	45	18			
	F3	1	21	5	2	24	8	3	45	7			
	F4	6	21	29	6	24	25	12	45	27			
	F5	3	21	14	3	24	13	6	45	13			
Activity 2	I1	3	18	17	11	23	48	14	41	34	54	164	33
	I2	2	18	11	2	23	9	4	41	10			
	I3	5	18	28	9	23	39	14	41	34			
	I4	7	18	39	15	23	65	22	41	54			
	F1	6	18	33	7	23	30	13	41	32	80	205	39
	F2	10	18	56	12	23	52	22	41	54			
	F3	5	18	28	6	23	26	11	41	27			
	F4	9	18	50	12	23	52	21	41	51			
	F5	9	18	50	4	23	17	13	41	32			
Activity 3	I1	16	20	80	13	24	54	29	44	66	92	176	52
	I2	1	20	5	3	24	13	4	44	9			
	I3	8	20	40	21	24	88	29	44	66			
	I4	9	20	45	21	24	88	30	44	68			
	F1	3	20	15	9	24	38	12	44	27	33	220	15
	F2	0	20	0	1	24	4	1	44	2			
	F3	1	20	5	4	24	17	5	44	11			
	F4	4	20	20	2	24	8	6	44	14			
	F5	1	20	5	8	24	33	9	44	20			
Activity 4	I1	8	12	67	9	16	56	17	28	61	65	112	58
	I2	5	12	42	11	16	69	16	28	57			
	I3	4	12	33	8	16	50	12	28	43			
	I4	9	12	75	11	16	69	20	28	71			
	F1	4	12	33	7	16	44	11	28	39	35	140	25
	F2	0	12	0	6	16	38	6	28	21			
	F3	1	12	8	3	16	19	4	28	14			
	F4	2	12	17	4	16	25	6	28	21			
	F5	2	12	17	6	16	38	8	28	29			

RELIABILITY ANALYSIS

In an effort to determine the reliability of the categorical analysis of the qualitative reflective data a study of reliability was conducted. Two graduate students in mathematics education were asked to participate in the study of reliability. Both reliability analysts were asked to read a randomly selected collection of reflections and to code them for the presence or absence of each of the aforementioned 9 narrative themes. Results were compared to those obtained by the researcher.

One graduate student, henceforth referred to as RA1 (reliability analyst 1), was in his final year of study towards a Ph.D. in mathematics education at a large university in the Pacific Northwest in the United States of America. The other graduate student, henceforth referred to as RA2 was in her first year of study towards a Ph.D. in mathematics education at a large university in the Pacific Northwest in the United States of America. Both RA1 and RA2 hold undergraduate degrees in mathematics. RA1 is male. RA2 is female.

In order to investigate any reliability issues associated with a particular class, it was decided that each reliability analyst be assigned only one study group for analysis. By random assignment the fall 2009 informal group was assigned to RA2 and the spring 2010 informal group was assigned to RA1. Similarly, to investigate any reliability issues associated with a particular activity, it was decided that each reliability analyst be assigned to analyze two different activities that were employed in the study. By random selection, activity 2 and 3 were assigned to RA1. By random selection, activity 2 and 3 were assigned to RA2. It is worth noting that the selection of activity 2 and 3 for analysis for each of the two reliability analysts occurred independently, that is, the fact that both

RA1 and RA2 performed analysis of reliability on activity 2 and 3 is strictly the result of random assignment for each individual.

Each analyst was provided with a copy of Tables 5.22 and 5.23 which outlined the 9 themes which were encountered by the researcher in the qualitative data. Also provided were statements of each of the activities that were associated with student reflections (see Appendix A). Finally, all of the collected student reflections for each assigned study group and activities were provided. Each analyst was asked to read each student reflection and to code each reflection for the presence of absence of each of the nine themes. Compiled results were compared to those obtained by the researcher and are provided in Tables 5.33 to 5.38.

The comparison displayed moderately high reliability. Overall, the rate of agreement was about 78% for RA1 and about 76% for RA2. The two rates of agreement show no statistical difference to one another using a two proportion *z* test ($p = 0.4090$). The rate of agreement on a per activity basis did vary. In RA1's case, the agreement for activity 3 (80.89%) was higher than the agreement for activity 2 (75.36%) though the difference fails to be statistically significant using a two proportion *z* test ($p = 0.1643$). In the case of RA2, the agreement for activity 3 (85.56%) was also higher than that of activity 2 (64.56%) and here the difference does prove significant using a two proportion *z* test ($p = 7.929 \times 10^{-6}$). This discrepancy prompted further investigation. It was discovered that the majority of disagreement between the study results and RA2's results for activity 2 were encountered in those themes measuring affect (I1, F1, and F2). While discrepancies in the researcher's and RA2's interpretations were found to be simply differences of opinion for many items there was one concession which proved reassuring.

Many of the items on which the researcher and RA2 disagreed were explained on the basis of judging the *severity* of a negative affective response. That is, the researchers did not necessarily disagree on the presence of a negative affective response per se, but, did disagree on the severity of said response, either low-level or high-level. Had themes F1 and F2 been combined into a *single* measure of negative affective response, the level of agreement would have been 89% instead of 44% for F1 and 44% for F2 for activity 2.

Table 5.33: Reliability Results RA1 Spring 2010 Informal Group Activities 2 and 3 Study Results

	Study Results																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1
S2	0	0	1	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0
S3	0	0	0	1	1	0	0	1	0	0	0	0	1	1	0	0	0	1
S4	0	0	1	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0
S5	0	1	0	1	0	1	0	1	0	0	0	1	1	1	0	0	0	0
S6	0	1	1	0	1	0	0	0	0	1	0	1	1	1	0	0	0	0
S7	0	0	0	1	0	1	0	1	1	1	0	1	1	0	0	0	0	0
S8	0	0	1	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0
S9	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0
S10	N	N	N	N	N	N	N	N	N	0	1	0	0	0	0	0	0	0
S11	0	1	0	1	1	0	0	0	0	1	0	1	0	0	0	0	0	0
S12	0	0	0	1	0	1	0	0	1	0	1	1	0	0	0	0	0	0
S13	0	1	1	0	0	1	0	1	0	0	1	1	0	0	1	0	0	0
S14	N	N	N	N	N	N	N	N	N	0	1	1	0	1	0	0	0	0
S15	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0
S16	1	0	0	1	1	0	0	0	0	1	0	0	1	0	0	0	0	0
S17	0	0	1	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0
S18	0	0	0	0	0	1	1	1	1	0	0	1	1	1	0	0	0	0
S19	0	1	1	1	1	0	0	0	0	1	0	1	0	0	0	0	0	0
S20	1	1	1	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0
S21	0	0	0	1	0	1	0	1	0	0	0	1	0	1	0	0	0	0
S22	1	0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0
S23	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0
S24	0	0	0	1	0	1	0	0	0	0	0	0	1	1	0	0	0	0
S25	0	0	1	1	0	1	0	1	0	0	1	1	1	1	0	0	0	1
Count	5	6	11	14	14	9	1	9	4	5	6	21	13	13	1	0	0	3
N	23	23	23	23	23	23	23	23	23	25	25	25	25	25	25	25	25	25
Percent	22	26	48	61	61	39	4	39	17	20	24	84	52	52	4	0	0	12

Table 5.34: Reliability Results RA1 Spring 2010 Informal Group Activities 2 and 3 RA1 Results

	RA1 Results																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	1	0	1	0	1	0	0	0	0	1	0	1	1	0	0	0	0	1
S2	0	0	0	0	0	1	0	0	0	1	0	1	1	0	0	0	0	0
S3	0	0	0	1	0	1	1	1	0	1	0	0	1	0	0	1	0	1
S4	1	0	1	1	0	0	1	0	0	1	0	1	1	1	0	0	0	0
S5	0	0	0	1	0	1	0	1	1	0	0	1	1	1	0	0	0	0
S6	0	0	0	1	0	0	0	1	0	0	0	1	1	1	0	0	0	0
S7	0	0	0	1	0	1	0	1	1	1	0	1	1	0	0	0	0	0
S8	0	0	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0	0
S9	1	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0
S10	N	N	N	N	N	N	N	N	N	0	1	0	0	0	0	0	0	0
S11	1	0	0	1	1	0	0	1	0	1	0	1	1	0	0	0	0	1
S12	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0	1
S13	0	0	1	0	0	1	0	1	0	0	1	1	0	0	1	1	1	0
S14	N	N	N	N	N	N	N	N	N	0	1	1	1	0	0	0	0	1
S15	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0
S16	1	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
S17	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	1	0	1
S18	0	0	1	0	0	1	0	1	0	1	0	1	1	0	0	0	0	1
S19	0	1	1	1	1	0	0	1	0	1	0	1	1	0	0	0	0	0
S20	1	1	0	1	0	1	1	0	0	1	0	1	1	1	0	0	0	0
S21	0	0	0	1	0	1	0	0	0	1	0	1	1	0	0	0	1	0
S22	1	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0
S23	1	0	0	0	1	0	1	1	0	0	0	1	1	0	0	0	0	0
S24	1	0	0	1	0	0	0	1	1	0	0	1	1	1	0	0	0	0
S25	1	0	1	1	0	1	0	1	0	1	0	1	1	1	0	0	0	1
Count	11	2	9	15	7	12	6	12	4	13	4	21	21	9	1	4	2	8
N	23	23	23	23	23	23	23	23	23	25	25	25	25	25	25	25	25	25
Percent	48	9	39	65	30	52	26	52	17	52	16	84	84	36	4	16	8	32

Table 5.35: Reliability Results RA1 Spring 2010 Informal Group Activities 2 and Agreement Analysis

	Agreement (1=Agree, 0=Disagree)																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
S1	0	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1
S2	1	1	0	1	0	0	1	0	0	0	1	1	1	1	1	1	1	1
S3	1	1	1	1	0	0	0	1	1	0	1	1	1	0	1	0	1	1
S4	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	1	1	1
S5	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
S6	1	0	0	0	0	1	1	0	1	0	1	1	1	1	1	1	1	1
S7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
S8	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	0	1	1
S9	1	1	1	1	0	0	1	1	1	1	1	1	1	0	1	1	1	1
S10	N	N	N	N	N	N	N	N	N	1	1	1	1	1	1	1	1	1
S11	0	0	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	0
S12	1	1	1	0	1	0	0	1	1	0	0	1	1	1	1	1	1	0
S13	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
S14	N	N	N	N	N	N	N	N	N	1	1	1	0	0	1	1	1	0
S15	0	1	1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1
S16	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1
S17	1	1	1	1	0	0	1	1	1	0	1	0	1	1	1	0	1	0
S18	1	1	0	1	1	1	0	1	0	0	1	1	1	0	1	1	1	0
S19	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1
S20	1	1	0	1	0	0	0	0	1	0	1	1	0	0	1	1	1	1
S21	1	1	1	1	1	1	1	0	1	0	1	1	0	0	1	1	0	1
S22	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
S23	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
S24	0	1	1	1	1	0	1	0	0	1	1	0	1	1	1	1	1	1
S25	0	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
Count	17	19	19	20	16	16	16	14	19	13	23	21	17	19	25	21	23	20
N	23	23	23	23	23	23	23	23	23	25	25	25	25	25	25	25	25	25
Percent	74	83	83	87	70	70	70	61	83	52	92	84	68	76	100	84	92	80
Count	156									182								
N	207									225								
Percent	75.36									80.89								
Count	338																	
N	432																	
Percent	78.24																	

Table 5.36: Reliability Results RA2 Spring 2010 Informal Group Activities 2 and 3 Study Results

	Study Results																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F2	1	0	0	0	0	1	0	0	1	1	0	1	1	0	0	0	0	0
F3	N	N	N	N	N	N	N	N	N	1	0	0	1	0	0	0	0	0
F4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F5	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
F6	1	1	1	0	0	1	0	1	1	1	0	1	0	0	0	0	0	0
F7	0	0	0	0	1	0	0	1	0	1	0	1	1	0	0	0	0	0
F8	1	1	1	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0
F9	0	0	0	1	1	0	0	0	0	1	0	1	1	0	0	0	0	0
F10	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F11	N	N	N	N	N	N	N	N	N	1	0	1	0	0	0	0	0	0
F12	0	0	0	0	0	1	0	1	1	1	0	1	1	0	0	0	0	0
F13	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	0
F14	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	0	0
F15	0	0	0	1	0	1	1	1	0	1	0	0	0	0	0	0	1	0
F16	0	0	0	1	0	1	1	1	1	0	0	0	0	1	0	1	0	0
F17	0	0	0	1	0	1	1	1	1	1	0	0	0	0	0	0	1	0
F18	0	0	0	1	0	1	1	0	1	1	0	0	0	0	0	0	0	0
F19	0	0	1	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0
F20	0	0	0	0	0	1	0	0	1	1	0	0	1	0	0	0	1	0
F21	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
F22	0	0	0	1	1	0	1	0	1	0	1	0	1	0	0	0	0	1
F23	0	0	1	0	0	1	0	0	0	1	0	1	1	1	0	0	0	0
Count	3	2	5	7	6	10	5	9	9	16	1	8	9	3	0	1	4	1
N	18	18	18	18	18	18	18	18	18	20	20	20	20	20	20	20	20	20
Percent	17	11	28	39	33	56	28	50	50	80	5	40	45	15	0	5	20	5

Table 5.37: Reliability Results RA2 Spring 2010 Informal Group Activities 2 and 3 RA2 Results

	RA2 Results																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F2	1	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0
F3	N	N	N	N	N	N	N	N	N	1	0	0	1	0	0	0	0	0
F4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F5	1	0	1	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0
F6	1	0	1	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1
F7	1	0	0	0	1	0	0	1	0	1	0	1	1	0	0	0	0	0
F8	1	1	1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	1
F9	1	0	1	0	1	0	0	0	0	1	0	1	1	0	0	0	0	0
F10	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F11	N	N	N	N	N	N	N	N	N	1	0	1	1	0	0	0	0	0
F12	0	0	0	0	1	0	0	1	1	1	0	1	1	0	0	0	0	0
F13	1	1	1	0	1	0	0	0	1	0	0	1	0	1	0	0	1	0
F14	1	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0	1
F15	0	0	1	1	1	0	0	1	1	1	0	0	1	0	0	0	0	1
F16	1	0	0	1	1	0	0	1	1	0	0	0	0	0	0	1	0	0
F17	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	1	0	0
F18	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0
F19	0	0	1	1	1	0	0	1	1	1	0	1	0	1	0	0	0	0
F20	0	0	0	1	1	0	0	0	1	0	0	1	1	0	0	0	0	0
F21	1	1	0	1	1	0	0	0	1	0	0	1	1	1	0	0	0	0
F22	1	0	0	1	1	0	0	1	1	0	1	0	1	0	0	0	0	1
F23	0	0	1	1	1	0	0	0	0	1	0	1	1	0	0	0	0	1
Count	12	3	10	12	14	0	0	9	11	14	2	12	13	3	0	2	1	6
N	18	18	18	18	18	18	18	18	18	20	20	20	20	20	20	20	20	20
Percent	67	17	56	67	78	0	0	50	61	70	10	60	65	15	0	10	5	30

Table 5.38: Reliability Results RA2 Spring 2010 Informal Group Activities 2 and 3 Agreement

	Comparison																	
	Activity 2									Activity 3								
Student	I1	I2	I3	I4	F1	F2	F3	F4	F5	I1	I2	I3	I4	F1	F2	F3	F4	F5
F1	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F2	1	1	0	0	0	0	1	1	0	1	0	1	1	1	1	1	1	1
F3	N	N	N	N	N	N	N	N	N	1	1	1	1	1	1	1	1	1
F4	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F5	0	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1
F6	1	0	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	0
F7	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F8	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0
F9	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
F10	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
F11	N	N	N	N	N	N	N	N	N	1	1	1	0	1	1	1	1	1
F12	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1
F13	0	0	0	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1
F14	0	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	0
F15	1	1	0	1	0	0	0	1	0	1	1	1	0	1	1	1	0	0
F16	0	1	1	1	0	0	0	1	1	1	1	1	1	0	1	1	1	1
F17	1	1	0	1	0	0	0	1	1	1	1	1	1	1	1	0	0	1
F18	0	1	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1	1
F19	1	1	1	0	0	0	1	1	1	1	1	0	1	0	1	1	1	1
F20	1	1	1	0	0	0	1	1	1	0	1	0	1	1	1	1	0	1
F21	0	0	1	0	1	1	1	1	0	1	1	0	1	0	1	1	1	1
F22	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1
F23	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1	1	1	0
Count	9	15	13	11	8	8	13	16	12	18	19	14	16	16	20	19	17	15
N	18	18	18	18	18	18	18	18	18	20	20	20	20	20	20	20	20	20
Percent	50	83	72	61	44	44	72	89	67	90	95	70	80	80	100	95	85	75
Count	105									154								
N	162									180								
Percent	64.81									85.56								
Count	259																	
N	342																	
Percent	75.73																	

CHAPTER 6: DISCUSSION AND CONCLUSION

This study set out to address the following research questions:

1. What is the relationship between participation in informal mathematics activities and the formal-to-informal beliefs of university teacher candidates in elementary education?
2. Does reflection upon personal experience derived from participation in informal mathematics activities reveal any transformation of the formal-to-informal beliefs of university teacher candidates in elementary education?
3. What is the value of informal mathematical activity in elementary teacher education?

In the following sections a discussion of the conclusions of this research are presented. The presentation is divided into three sections which address each of the research questions in turn. Finally, limitations of the study are discussed along with suggestions for future research.

RESEARCH QUESTION 1

The first research question in this study sought to determine the relationship, if any, between participation in informal mathematics activities and change in the formal-to-informal beliefs of university teacher candidates in elementary education. The results and analysis of the data collected in this study reveal a complex and varied relationship.

The data suggest that the formal-to-informal beliefs of teacher candidates do not conform to the beliefs of mathematics content course instructors or those associated with course learning goals by virtue of enrollment. This conclusion is supported by the data collected on the fall 2009 control group. It was found that the formal-to-informal beliefs of the group did not experience a statistically significant change over the course of a semester in which they were enrolled in Math 136. At the onset of the study, beliefs about mathematics (BAM) were characterized as “slightly informal” for this group with a pre-course mean composite of 76.85 on the 20 to 120 point scale. At the conclusion of the study the post-course mean composite of 75.50 demonstrated no statistical change, on average, for beliefs about mathematics for the fall 2009 control group. Similarly, beliefs about mathematics instruction (BAMI) were characterized as slightly informal for this group with a pre-course mean composite of 77.61. Post-course mean BAMI showed no statistical change and was measured at 75.82. Taken together, the results provide strong evidence that the beliefs of the fall 2009 control group remained, on average, unchanged over the course of the semester in which they enrolled in Math 136.

These results are striking given the fact that the instructor of the control group held beliefs about mathematics and mathematics instruction which were characterized as “highly informal” when measured, with a mean composite score of 112 on the BAM scale and 113 on the BAMI scale. Both measures are near the “purely informal” end of the 20 to 120 point scale. Also significant is the fact that Math 136 takes a decidedly informal approach to mathematics. The course goals call for students to “to model”, “to explore, conjecture, and prove”, “to solve problems” and to perform “hands on” explorations. The course text also takes a problem solving approach to mathematics.

And while this result is, perhaps, a cause for concern, beliefs about mathematics and mathematics instruction which are resistant to change are certainly well documented in the literature. Both Collier (1978) and Seaman et al. (2005) noted little change in the beliefs of preservice teachers while enrolled in mathematics content courses such as Math 136. Both researchers found the most significant changes in beliefs about mathematics and mathematics instruction after students had completed a methods course in the subject. Other researchers (Brown, et al., 1999; Pajares, 1992; Borko, et al, 1997) have drawn attention to the fact that teacher candidates arrive at the university with well-formed beliefs which are the result of many years experience as students of mathematics in both elementary and high school. These well-formed, central beliefs are often noted for their resistance to change at the university where reform efforts in mathematics education are typically centered. These results are replicated here.

While the fall 2009 informal group showed no statistical change in BAM with pre and post-course mean composites of 80.00 and 79.26 respectively. The group did show some evidence of statistical change in BAMI composite. Here, mean composite rose from 76.10 to 81.29 over the course of the semester indicating a direction of change in favor of a more informal outlook with regard to mathematics instruction.

The spring 2010 informal group displayed the most evidence of beliefs change associated with participation in informal mathematics activities. Beliefs about mathematics shifted from 77.88 to 84.78 in mean composite and beliefs about mathematics instruction shifted from 74.35 to 83.42 in mean composite over the course of the semester. The statistically significant shift in BAMI was greater than that displayed in BAM, 9.1 compared to 6.84 respectively. The results for the spring 2010 informal group are particularly notable in contrast to the fall 2009 informal group which experienced the same course, instructor and informal activities but displayed no statistical change in beliefs about mathematics. Researcher hypotheses regarding this differential response to informal mathematics activity is taken up below as an area for further investigation.

Taken together, however, the results from the two informal groups demonstrate that beliefs about mathematics are more centrally held and therefore resistant to change than beliefs about mathematics instruction. This assertion is supported by the significant shifts in BAMI for both informal groups, as well as a greater shift in BAMI then BAM for the spring 2010 informal group.

This finding is consistent and can be considered a replication of the results found by both Collier (1972) and Seaman, et al., (2005) who documented more radical shifts in beliefs about mathematics instruction than beliefs about mathematics over the course of university instruction in the subject. The result may indicate that teacher candidates are actively forming notions of teaching mathematics while at the university and, therefore, hold more peripheral beliefs in this area which are susceptible to change in settings where mathematics is acquired through informal investigation. Conversely, beliefs about mathematics as a subject may be more firmly entrenched and resistant to such activities.

In contrast to Collier (1972) and Seaman, et al., (2005), the results presented here are associated with a content course in mathematics. Both the aforementioned researchers found that the most radical shifts towards informal approaches to the subject occurred after experiencing a methods course in mathematics. The difference is notable. Whereas one might expect that a methods course, through a reform orientated presentation of pedagogy, might induce beliefs transformation, the same expectation seems less plausible in a content course setting. It is the assertion of this researcher that the personal experience of learning new mathematical content in a creative and investigative setting was the catalyst for beliefs change found here. This assertion is supported by the absence of any change of beliefs in the fall 2009 control group as compared to the two informal groups which experienced a shift in beliefs in at least one of the two categories measured over the course of the semester.

Informal mathematical activities as part of regular instruction seem to hold promise in transforming both beliefs about mathematics and beliefs about mathematics instruction of teacher candidates. This conclusion is supported by the fact that the spring 2010 informal group displayed significant changes in mean composite of both BAM and BAMI measures towards a more informal disposition. The fact that significant changes were experienced in both categories for the spring 2010 informal group and only one category for the fall 2009 group provides evidence for two contrasting conclusions for the groups participating in this study: informal mathematical activity was consistently associated with a shift towards informal beliefs concerning methods of mathematical instruction, and, informal mathematical activity was variably associated with a shift towards informal beliefs concerning the nature of mathematics as a subject. Again, potential sources of the variability of the association of informal mathematical activity and beliefs about mathematics are taken up as a subject for further investigation below.

The results presented here provide for a finer analysis of the sub-types of beliefs that are most susceptible to change associated with participation in informal mathematics. Here conclusions are drawn based on those items that displayed the greatest shifts in formal-to-informal beliefs when a statistically significant shift in beliefs was noted. Conclusions are presented in two categories: beliefs about mathematics and beliefs about mathematics instruction.

Based on a significant change in beliefs about mathematics for the spring 2010 informal group, an investigation of sub-types of beliefs about mathematics supports the conclusion that those beliefs about mathematics that envision the subject as one incorporating *creativity* and *originality* were most susceptible to change. Students shifted towards agreement over the course of the semester with statements that characterize mathematics as a field of "ingenuity" and "originality" where one can develop the "ability to think creatively" employing only an "inquiring nature". Further, students shifted away from agreement that mathematics is a field where one only "follows directions" to acquire the "laws and rules" of the "rigid" science.

Similar analysis reveals that students in the spring 2010 informal group held beliefs about mathematics which were least susceptible to change that envision the subject as one which makes room for *multiplicity of methodology*. Students did not change in their beliefs about the existence of "many different ways to solve" mathematics problems, and "different but appropriate ways" to organize mathematics. Finally, there was almost universal agreement that mathematics is a field of "routines and procedures" where success is dependent upon the use of a "rule or procedure".

This analysis of sub-types of beliefs about mathematics certainly confirms the finding noted by Seaman, et al. (2005) that "the focus on memorized rules, formulas and procedures has become part of the belief structures of elementary education students" (p. 206). The analysis here also points to the seemingly contradictory beliefs that many teacher candidates hold with regard to the mathematics: asserting that the science makes room for creativity while primarily focusing on a single methodology. Collier (1972) was so concerned about this phenomenon in his initial study that he devised a measure of ambiguity that quantified this now well-know fact.

Instead of dwelling on the puzzling contradiction, this researcher chooses simply to make note of it here as evidence that teacher candidates in this study tended to hold contradictory views about the nature of mathematics. This contradiction seems to point to the conclusion that beliefs about mathematics as a creative and investigative subject are held independently of those which characterize the subject as one that is primarily concerned with routine and procedure. These beliefs, therefore, must be held in separate belief clusters. Similar findings have been noted by Green (1971) Torner (2002), and Philipp (2007).

Based on a significant change in beliefs about mathematics instruction for both the fall 2009 and the spring 2010 informal groups, an investigation of belief sub-types supports the conclusion that beliefs about mathematics instruction which are peripherally held incorporate *openness to discovery* in teaching the subject. Teacher candidates shifted towards agreement that students in mathematics classes should “invent their own symbolism”, should “find individual methods for solving problems”, should “experiment” and “build” their own ideas. Conversely, teacher candidates more strongly disagreed with the notion of teaching mathematics as an imitation of teacher or textbook. Disparaging statements concerning “discovery methods” in mathematics instruction also experienced a shift towards disagreement over the course of the semester.

An analysis of items which experienced little change over the course of the semester in this category supports the conclusion that beliefs about mathematics which are centrally held envision the teaching of the subject as an *authoritarian transmission of* *technique*. Statements which envision a mathematics in which students self-author methods for solution, learn independently the "basic ideas of mathematics" and are required to carry out "creative and investigative" work experienced little change over the semester. Finally, there is almost universal agreement (both pre and post) with the notion that the teacher should "always work sample problems" as part of instruction in mathematics.

Finally, the data support the conclusion that the association between informal-to-formal beliefs transformation and participation in informal mathematics activities is not linked to student achievement in the subject. This conclusion is asserted on the basis of an absence of any correlative relationship exhibited between beliefs in mathematics or mathematics instruction and final grade in Math 136 for each of the three groups that participated in this study. Note that this relationship was investigated at three levels: pre, post and gain composite scores. No association was found for any of the three groups on any level.

This conclusion diverges from that obtained by Collier (1972), who found significant differences between high and low achievers in both beliefs about mathematics and beliefs about mathematics instruction (curiously, Seaman, et al.'s (2005) replication of Collier's (1972) study neglects to take achievement into account). He found that high achievers held significantly more informal views of the subject than low achievers and generally experienced larger gains associated with the completion of content and methods courses in mathematics. Collier's (1972) finding points towards a possibly higher level of resistivity in the beliefs of low achievers.

The research here differs from that of Collier (1972) in its interventionalist approach and supports the assertion that informal mathematics activities may provide a means of transforming the beliefs of preservice elementary school teachers irrespective of their academic standing. That is, creative and investigative activity, incorporated as part of teacher preparation in mathematics, may provide a means of transitioning formal and authoritarian notions of the subject and its teaching towards more creative, constructive and investigative approaches (NCTM, 2000) for students at any academic level.

RESEARCH QUESTION 2

The second research question associated with this study sought to determine if reflection upon personal experience derived from participation in informal mathematics activities reveals any transformation of the formal-to-informal beliefs of university teacher candidates in elementary education. The result of the analysis of the student reflection data collected in the study are broken into conclusions based on general findings as well as conclusions based on individual findings associated with each activity.

In general, the data support the conclusion that reflection upon experience derived from participation in informal mathematics activities does reveal a transformational shift of beliefs towards informal notions of mathematics teaching. This conclusion is supported by the data which demonstrate that teacher candidates reflected upon themes that indicate a disposition in favor of informal approaches to mathematics more frequently than themes in favor of formal approaches to mathematics over the course of the semester which were identified in this study.

Taken as a whole, the data show that students were more likely to record reflections that noted a positive affective response, or a disposition in favor of discovery learning in mathematics, or favorable outlook upon hands-on learning, or a notion of the importance of social collaboration in mathematical learning. Students were less likely to record low-level or high-level negative affective response to the activities, or a disposition against discovery learning, or a desire for more guidance, or trouble in understanding proof.

Treating each opportunity to reflect on each theme as a single *student theme reflection opportunity* proves helpful here. Of the 632 student theme reflection opportunities which were associated with informal approaches to mathematics education, 326 reflections were detected; a rate of 51.58%. Of the 790 student theme reflection opportunities which were associated with formal approaches to mathematics education, 200 reflections were detected; a rate of 25.32%. Based on this analysis the researcher concludes that students participating in this study were nearly twice as likely to reflect upon themes which indicate a transition towards a more informal outlook in mathematics education than to reflect upon themes which indicate a transition towards a more formal outlook in mathematics education. These results, which add a measure of reliability to those found using the BAM and BAMI surveys, are summarized in Table 6.1 below.

Table 6.1: Reflection Theme Analysis Summary of Aggregate Results

	By Theme			Totals		
Theme	Count	N	Percent	Count	N	Percent
I1	87	158	55.06	326	632	51.58
I2	58	158	36.71			
I3	75	158	47.47			
I4	106	158	67.09			
F1	60	158	37.97	200	790	25.32
F2	37	158	23.42			
F3	22	158	13.92			
F4	45	158	28.48			
F5	36	158	22.78			

The results of the theme analysis in this research point towards differential effects in terms of formal-to-informal beliefs transformation when analyzed on a per activity basis. The proportion of students who reflect upon themes which support informal approaches in mathematics is generally greater than the proportion of students reflecting on themes opposing informal approaches in mathematics in each of activities 1, 3 and 4. The opposite result is found for activity 2.

This finding supports the conclusion that informal mathematical activities as agents of beliefs transformation in teacher preparation carry an element of unpredictability and risk associated with their use in terms of invoking formal-to-informal beliefs transformation which aligns with the current reform movement in mathematics education. The conclusion is evidenced by the fact that activity 2 invoked student reflections which are more likely to align with formal approaches to the subject in spite of the informal nature of the activity itself. Whether this result is linked to some perceived difference in nature of the activity, its level of difficulty, or some other factor is uncertain. There is some reason to believe that the sound rejection of the activity as an agent of informal beliefs transformation is linked to a sense of student failure at producing a correct and complete proof of the inscribed angle relationship. In the spring of 2010, of the 22 students who participated in the activity, 22 provided the correct conjecture, 16 successfully proved case 1, 12 successfully proved case 2 and only 8 successfully proved case 3. This possible link between student success (or lack thereof) solving open-ended and investigative mathematical activities and formal-to-informal beliefs transformation is one area in need of future investigation which is noted below.

RESEARCH QUESTION 3

The final research question explores the value of informal mathematical activity in elementary school teacher education. The results presented herein provide for a conclusion which supports the use of such activities in teacher preparation as a means of developing a quality that researchers have noted as absent in teacher candidates, namely, “robust and consistent philosophies of mathematics and mathematics education” (Seaman, et al., 2005).

While national efforts in reform of mathematics education (NCTM, 1989; NCTM, 2000) have called for more focus in the classroom on the *processes* involved in creating mathematics (i.e. problem solving, communication, multiple representations, connection, and proof and reasoning) in all K-12 classrooms, many researchers have noted that these efforts continue to produce little change to the traditional, rules-driven approaches employed by many in-service teachers in the field (i.e. Gregg, 1995; Skott, 2001; Barkatsas & Malone, 2005). In light of this fact, many researchers have called for teacher educators to develop programs that confront rule-possession notions of teaching and learning mathematics.

Debates in modern epistemology continue to employ notions of knowledge as justified-true-belief (Plato, 1952). Here, knowledge entails belief: to know one must first believe. Adopting this notion, beliefs become the natural starting place for an effort that aims to transform conceptions of knowledge held by elementary school teachers of mathematics from rule-bound notions of the science to more robust philosophies which incorporate the generative processes which are at the heart of mathematical discovery and meaning making.

Psychological theorist (i.e. Green, 1971; Rokeach, 1968) have indicated that beliefs, as a construct, are commonly derived from personal experiences. Further, beliefs are thought to exist in a quasi-logical relationship to other beliefs in a cluster-like fashion, where derivative beliefs are linked to primary beliefs which ultimately rest on personal experience.

A coupling of the traditional epistemological notion of knowledge together with the psychological theory of belief make it apparent that reforms in mathematics education such as those envisioned by NCTM (2000) depend upon the adoption of new epistemological notions of mathematical knowledge, which depend upon teachers' beliefs, which depend upon personal experiences in mathematics which contradict traditional, rule-bound notions of learning in the science. The availability of such informal mathematical experiences depends upon on the initiative of teacher-trainers.

The data presented in this study and the conclusions outlined above make it clear that open-ended and investigative activities do indeed hold promise in transitioning the rule-driven notions of teaching mathematics that have shown resistance to educational reform. With regard to the value of such activities, the conclusion drawn is this: the generation of personal experience and reflection associated with informal mathematical activities such as those employed in this study provide a valuable means of transitioning teacher beliefs towards a more informal notion of the science of mathematics. This transition can only contribute to constructivist educational reforms in K-12 classrooms (NCTM, 2000), resulting in a richer mathematical inheritance for students.

LIMITATIONS OF THE STUDY

The study presented here is certainly not without limitation. In an effort to insure against misinterpretation, the researcher presents here the limitations of the study.

None of the three groups participating in the study can be considered a random sample of the larger population of preservice elementary school teachers. All students who participated in the study did so by virtue of their own enrollment in the course. The non-random nature of the samples of preservice elementary school teachers participating in the study limits the results presented here to the three groups which took part in the study and do not generalize to the larger population of such students.

There are also issues of psuedoreplication. While *class* is the experimental unit to which the different treatments were applied, the statistical analysis carried out here treats each *student* as an experimental unit. This issue again limits the results presented here to the three groups which participated in the study.

There is concern that the assumption of independence which is required for the validity of a paired *t*-test of significance is not met. Researchers have documented that beliefs are tied to important social factors (i.e. Benken, 2005; Archer, 1999; and Hannula et al. 2005, 2006, 2009). The social nature of the setting in which this research has been conducted makes it unlikely that the transformation of one student's beliefs takes place independently of the transformation of another student's beliefs. This potential violation of the independence assumption required for the validity of the paired *t*-test of statistical significance calls into question the statistical results associated with both the BAM and BAMI measures.

There were differences between control and informal groups that were not able to be corrected. The control group had a different instructor and certainly differences in pedagogy exist between the control instructor and the researcher-instructor. The control group experienced the course as a night class. The two informal groups met during the day, at 8AM and 1PM in the fall of 2009 and the spring of 2010 respectively. The control group met twice a week for 100 minutes whereas the informal groups met four times a week for 50 minutes.

The two informal mathematics groups showed markedly different aptitudes in mathematics. The fall 2009 informal group had a mean final grade course percentage of 75.3 and a standard deviation of 19.7 at the end of the semester. The distribution of final grade course percentages was bimodal. The spring 2010 informal group had a mean final grade course percentage of 88.0 and a standard deviation of 7.76 at the end of the semester. The distribution of final course percentages was unimodal. The difference in mean final course percentage is statistically significant ($p = 0.0061$) as determined by a two sample t test.

Incentives for completing each informal mathematics activity changed from the fall of 2009 to the spring of 2010. In the fall of 2009 each of the informal mathematics activities was collected as part of course requirements but was graded as a participation grade only. That is, students were given full credit (ten points) if they made an attempt at a solution to each of the activities. This policy was adopted in an effort to make the activities a "no-fail" setting for participants who had not been instructed in a possible solution method. Due to dissatisfaction with student effort in the fall of 2009, the instructor decided to grade the spring of 2010 informal mathematics activities on a ten point scale, devoting 5 points to correctness in conjecture and 5 points to correctness in proof and reasoning. The researcher-instructor hoped that such a change might provide more student motivation towards a greater effort in completing the activities without imposing too harsh of a penalty to those students who might experience difficulty or frustration. The change in grading policy was associated with a higher degree of effort in the spring of 2010. It is unclear if this association is causal or is perhaps an expression of higher student aptitude for the spring 2010 informal group.

SUGGESTIONS FOR FUTHER STUDY

The research conducted here has prompted a need for further study to investigate phenomena encountered but not fully explained in the present study. What follows is an accounting of these phenomena which the researcher offers as suggestions for further study.

SOURCE OF DIFFERENTIAL GAINS

It is unclear the source of the differential gains in beliefs about mathematics (BAM) and beliefs about mathematics instruction (BAMI) noted in the fall 2009 informal group as compared to the spring 2010 informal group. Whereas the spring 2010 informal group showed strong evidence of beliefs change in both categories associated with informal mathematical activities, the fall 2009 group showed no evidence of beliefs change with regard to mathematics and only some evidence of change with regard to mathematics instruction. This result is especially curious given the fact that both groups showed similar patterns in the reflection theme analysis.

The researcher suggests further study to determine the sources of variation of formal-to-informal beliefs transformation associated with informal mathematics activities incorporated as part of regular instruction in elementary teacher education. Of special concern is the fact that these two classes displayed markedly different aptitudes in mathematics as measured by final course grade percentages.

The researcher hypothesizes that informal mathematical activities may follow a law of diminishing returns in settings where classes are populated by either students with lower than average mathematical ability or by a high levels of heterogeneity in mathematical ability. This hypothesis seems plausible, at least anecdotally, given the fact that open-ended and non-routine mathematical tasks are often judged on the basis of one’s ability to *make progress* towards a solution. A halting of progress would logically lead towards the experience of frustration and the personal judgment that such activities are not advisable in mathematics instruction.

It seems likely that such experiences, on a personal level, are related to one’s ability and aptitude in mathematics. That is, the higher one’s aptitude in mathematics, the more likely it is that one will experience progress towards a solution, producing a personal experience of both satisfaction and meaningfulness. Conversely, the lower one’s aptitude in mathematics the more likely outcome is one in which little progress is experienced, producing a personal experience of both frustration and aversion to such activities.

And while this study showed no relationship between individual scores in either BAM or BAMI and final course grades, it is possible that a relationship *does* exist at the class level. That is, when average class aptitude and average class BAM and BAMI composites are measured and compared a positive association is hypothesized. This hypothesis is confirmed here by the fact that the two classes had a statistically significant difference in aptitude as measured by final course percentage ($p = 0.0061$) and the higher performing class, spring of 2010, experienced a statistically higher gain in both average composite BAM ($p = 1.1838 \times 10^{-14}$) and average composite BAMI ($p = 4.3816 \times 10^{-7}$) as determined using a two sample *t* test. These results are presented in Figures 6.2 and 6.3 below. Unfortunately, only two classes were involved in the study making the reliability of any causal link between class aptitude and propensity for beliefs change questionable, at best.

Figure 6.2: Pre and Post BAM Fall 2009 and Spring 2010

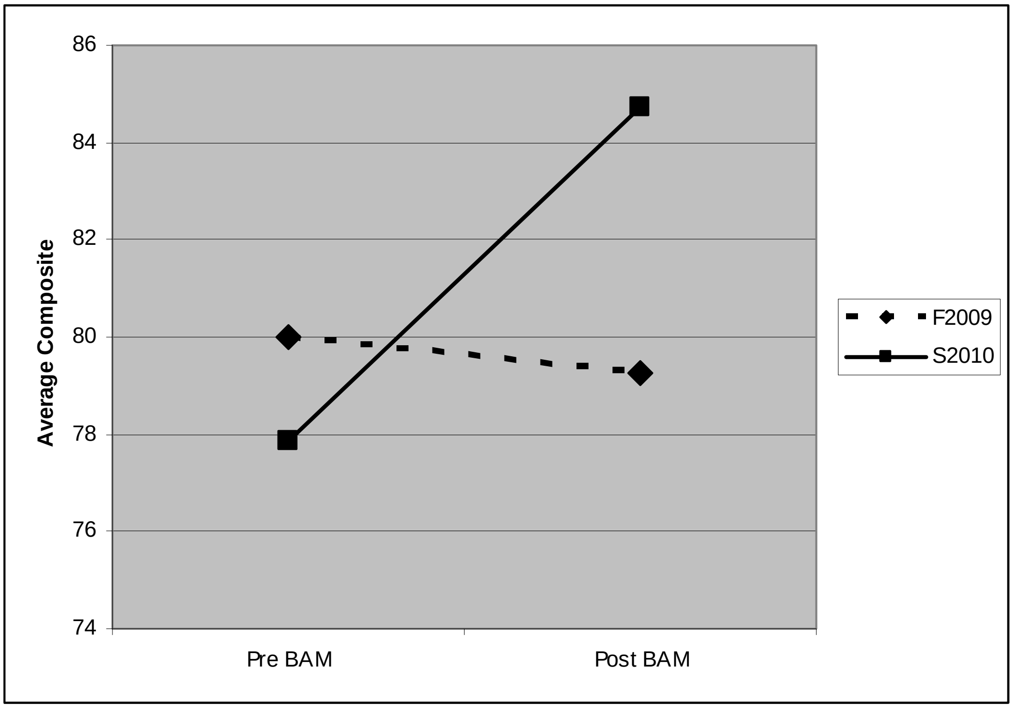
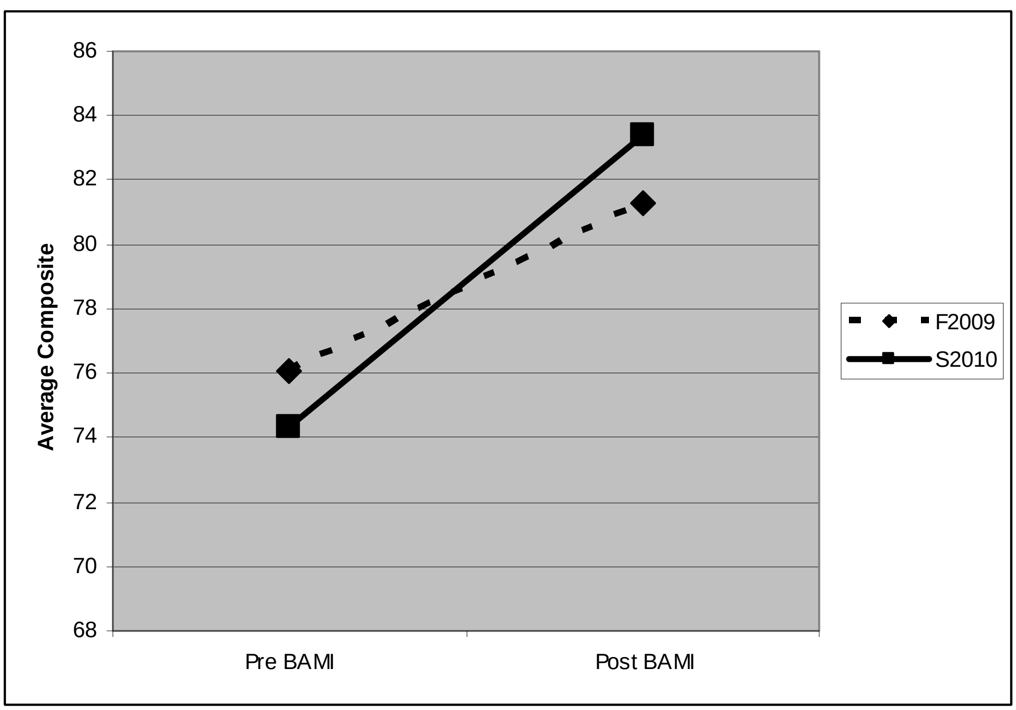


Table 6.3: Pre and Post BAMI Fall 2009 and Spring 2010



Nevertheless, the result, if proven in some future study, would certainly shed light onto the complex dynamic of classroom instruction in mathematics that focuses on creative and investigative mathematical activity. For, if the pattern discovered here holds, *individual* aptitude in mathematics may be a less important consideration than average *class* aptitude in mathematics in terms of any realignment of beliefs around the subject. That is, beliefs change, measured on a small group basis, may be inextricably bound to group dynamics which may be related to average mathematical aptitude.

Again, this hypothesis seems plausible from an anecdotal standpoint. For, students of mathematics seem to accept the fact that furthering one's education in the subject implies a measure of challenge. The data collected here, however, seem to indicate that creative and investigative mathematical experiences may be judged at a group level to be too challenging, resulting in a rejection of such activities as a model for good mathematics teaching. This hypothesis echoes other previously identified conundrums in constructivist reform of elementary mathematics education including the strong notion of "teaching as telling" (Chazan and Ball, 1999), the presence of conflicting educational priorities (Skott, 2001), the influence of the "school mathematics tradition" (Gregg, 1995), the role of teachers' perception of student needs (Sztajn, 2003) and perceived divisions between "university level" mathematics and "school level" mathematics (Perrenet & Taconis,2009). The hypothesis of an association between average mathematical aptitude and an openness to beliefs change in preservice elementary education is in need of further investigation.

SOURCE OF DIFFERENTIAL RESPONSE

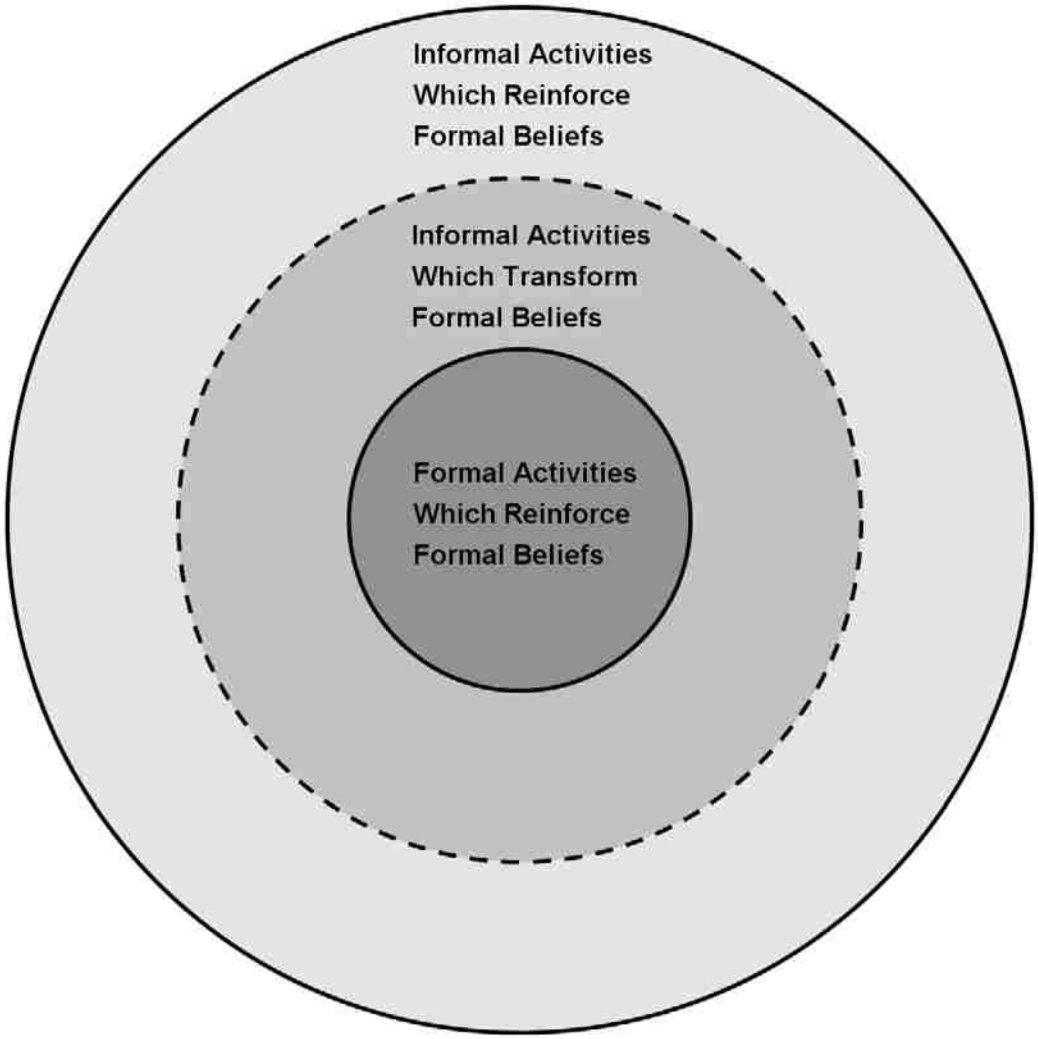
It is unclear the source of the differential response, in terms of student reflection theme analysis, to activity 2. While activities 1, 3 and 4 all prompted a higher proportion of themes associated with informal approaches to mathematics instruction, activity 2 prompted a higher proportion of themes associated with formal approaches to mathematics instruction. The researcher theorizes that there exists a “critical zone” of student perceived self-efficacy with regard to informal mathematical activity which divides such activities into two categories.

Activities within this critical zone of self-efficacy induce a shift in favor of informal beliefs about mathematics. Activities which fall outside of this critical zone of self-efficacy induce a shift in favor of formal beliefs about mathematics. The limits of this critical zone may be associated with an activity’s difficulty-level, time-to-completion requirements, content, or other factors.

The hypothesis seems plausible from an anecdotal standpoint. For, as all students of mathematics are aware, there are those mathematical investigations which are likely to produce results and then there are those that are not likely to prove fruitful. Taking this author as example, the researcher offers that an investigation into the closed form of the derivative of the cosecant is likely to produce results, whereas an attempt at a proof for, say, the Goldbach conjecture is not likely to provide similar success. This division of mathematical tasks into categories of “approachable” or “unapproachable” seems linked to a personal sense of efficacy in the subject and other complex social factors.

Interesting and worthy of further study in this area is an investigation into the characteristics of informal mathematical activity which work to transform formal notions of the subject in teacher candidates. The researcher proposes that all mathematical activity in preservice teacher education can be analyzed according to its propensity to induce formal to informal beliefs transformation on three levels: formal activity reinforcing formal beliefs (i.e. traditional formal instruction in mathematics), informal activity inducing informal beliefs transformation (i.e. activities 1,3, and 4 in this research), and, finally, informal activities which induce formal beliefs transformation (i.e. activity 2 in this research). A schematic of this hypothesized domain is provided in Figure 6.4. Here, solid lines indicate boundaries implied by the framework of the study whereas the dotted boundary and its defining characteristics are uncertain. This hypothesized domain and the nature of its uncertain boundary are in need of further study.

Figure 6.4: Hypothesized Domain of Formal and Informal Mathematical Activities and Beliefs Transformation



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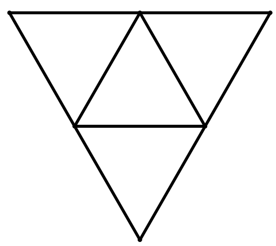
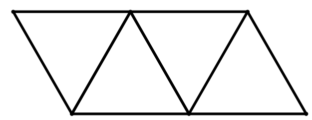
APPENDIX A: INFORMAL MATHEMATICS ACTIVITIES HANDOUTS

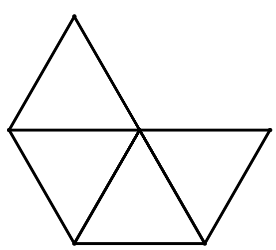
Reflective Mathematics Activity #1
The Nets of a Cube

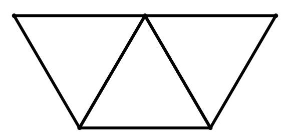
Background
In mathematics, a net is a connected two dimensional figure that can be folded into a three dimensional object. Nets are particularly powerful teaching tools because they help students extend knowledge about two dimensional objects into notions of three dimensional objects. The following examples are distinct (i.e. different) nets for the tetrahedron.





It should be noted that these two nets are called distinct because there is no way to transform one into the other by a "rigid motion," that is, a rotation (turn), translation (slide) or a reflection (flip). The following examples are not nets for the tetrahedron.

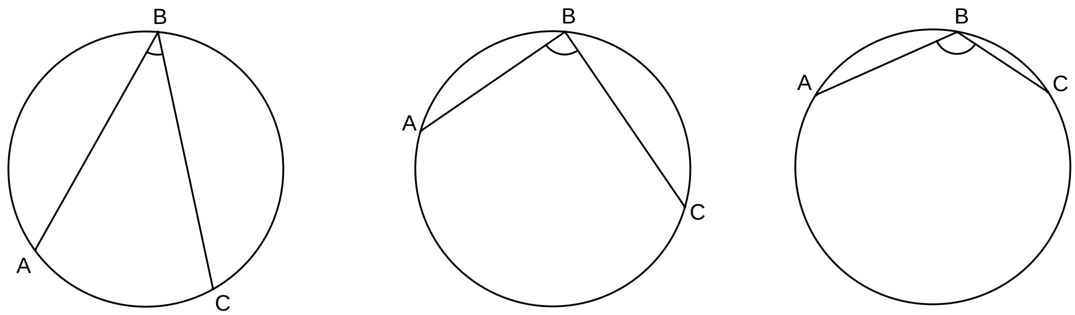




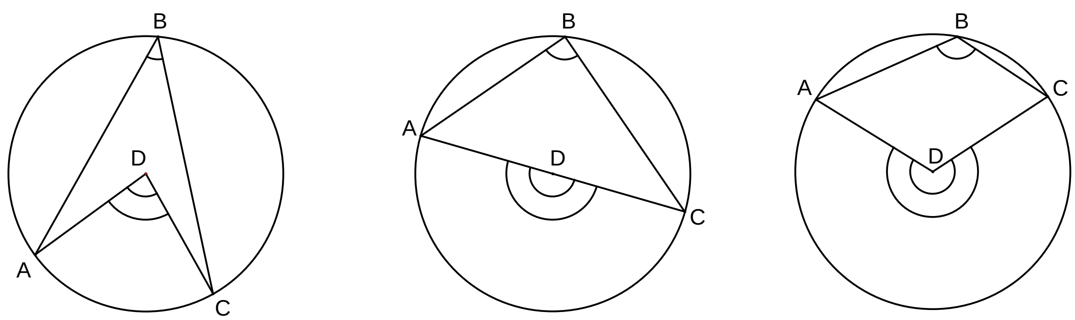
Problem Statement
Your task in this reflective mathematics activity is to find all the distinct nets of the cube and then prove that no other nets of the cube exist. At the end of this activity you will be asked to reflect on your personal experience of coming to understand this mathematical concept and what the experience "teaches you" about learning mathematics. Keep track of your strategies and procedures. Make note of your emotions and feelings. And be prepared to report your findings.

Reflective Mathematics Activity #2
Inscribed Angles of a Circle

Background
Choose any three points A, B, and C on a circle with center D. Angle ABC is then an *inscribed* angle due to the fact that the points which define it lie on the circle itself. Three examples of inscribed angles are shown below.



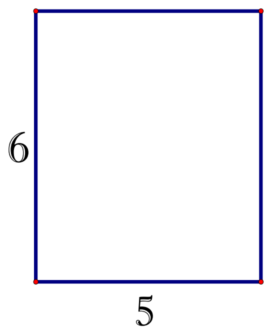
Of interest in this investigation is the relationship that exists between the inscribed angle ABC and the central angle ADC that subtends (contains) the same arc. The three examples above are again shown below each with the central angle included.



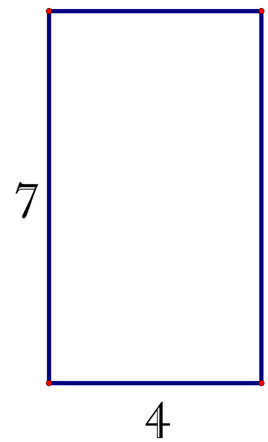
Problem Statement
Your task in this reflective mathematics activity is to make a conjecture about the relationship between an inscribed angle and the central angle which subtends the same arc on any circle and then prove that conjecture. At the end of this activity you will be asked to reflect on your personal experience of coming to understand this mathematical concept and what the experience "teaches you" about learning mathematics. Keep track of your strategies and procedures. Make note of your emotions and feelings. And be prepared to report your findings.

Reflective Mathematics Activity #3
Area and Perimeter of Integral Rectangles

Two fundamental mathematical concepts of two dimensional figures are area and perimeter. In this investigation we consider the area and perimeter of rectangles that have side lengths which are integers. Several examples of such rectangles are shown below.



Area = 30 square units
Perimeter = 22 units

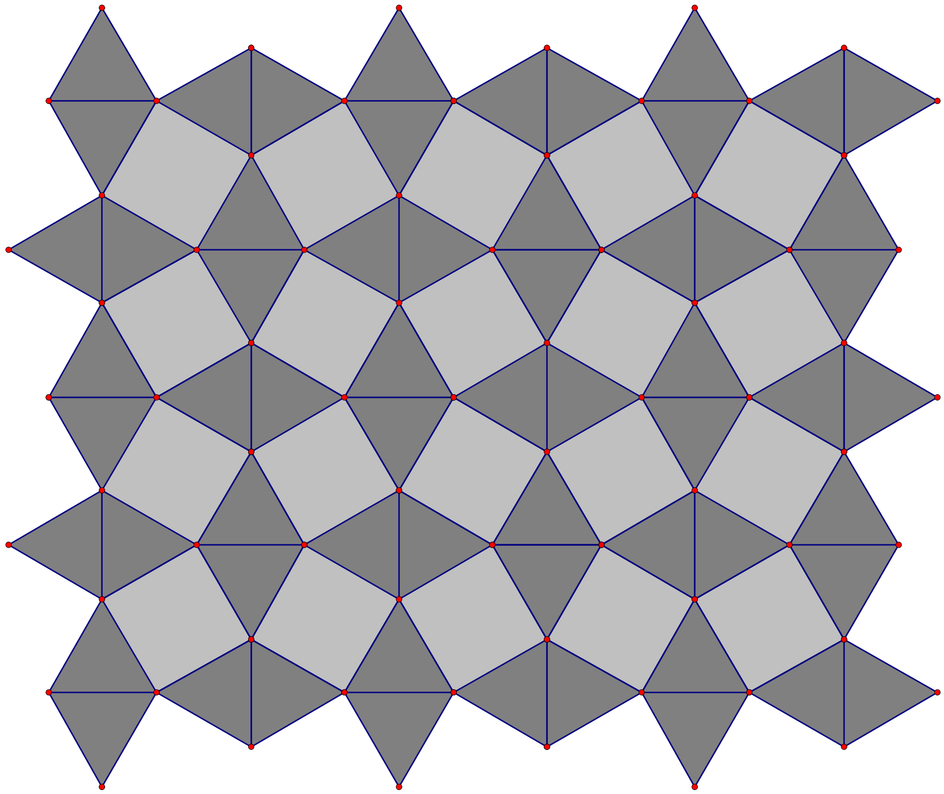


Area = 28 square units
Perimeter = 22 units

Problem Statement
Your tasks in this reflective mathematics activity are to find all rectangles with sides of integral length whose area and perimeter are numerically equal and then prove that there are no others. At the end of this activity you will be asked to reflect on your personal experience of coming to understand this mathematical concept and what the experience "teaches you" about learning mathematics. Keep track of your strategies and procedures. Make note of your emotions and feelings. And be prepared to report your findings.

Reflective Mathematics Activity #4
Tessellation

A tessellation is an arrangement of two dimensional figures that cover the entire plane without any overlaps or gaps. Tessellations are commonly found in mosaics, architectural designs, and tile-work. An example of a tessellation is given below.



A *regular* tessellation of the plane is a tessellation that is made up of congruent regular polygons which meet vertex to vertex such that every vertex arrangement is identical. We will prove in class that only three regular tessellations exist. A *semi-regular* tessellation of the plane is a tessellation that is made up of two or more congruent regular polygons which meet vertex to vertex such that every vertex arrangement is identical. Notice that the example given above is a semi-regular tessellation of the plane which is composed of squares and equilateral triangles.

Problem Statement
Your task in this reflective mathematics activity is to find all semi-regular tessellation of the plane and prove that no others exist. At the end of this activity you will be asked to reflect on your personal experience of coming to understand this mathematical concept and what the experience "teaches you" about learning mathematics. Keep track of your strategies and procedures. Make note of your emotions and feelings. And be prepared to report your findings.

APPENDIX B: STUDENT REFLECTION DATA

ACTIVITY 1 REFLECTIONS FALL 2009

F1

The assignment seemed simple – find all the nets to a cube. Once I got to sitting down and attempting to find all possible nets to a cube, however, my brain seemed to freeze. I began worrying about how I would find ALL possible nets to a cube, and how would I ensure that I had found them all?

My starting point was my knowledge of what a cube is. A Cube is a 3-D shape made up of six squares. Therefore my net must have 6 faces. Six squares in a row would not fold up into a cube, instead it left me with a hexagonal figure with no bases. From that I concluded I needed not only six sides but four lateral faces and two bases. Great, I thought, I have one net that looks like a "T" from there I began moving my bases one by one to create variations of my cube knowing that I needed to keep bases in order to create the cube. During this process I borrowed the "linkin squares" as an extra visual to aid in my discovery. I began creating the same nets I had already found but had them reversed or upside down.

So what next? I really thought that there must be more that that, so I tried putting three squares down as my base and moving others as I please around the base of three. This went on until I got down to a base of two and had completed 11 nets. No matter how much more I played around with the squares I could not come up with a new combination; this 11 nets disturbed me because I assumed the possible nets would have to be a multiple of six. I just could not come up with any other nets and was so bothered by what I thought was my inability to find further nets, I read the text book- no answer, so I looked on the internet and found a website another University had created, and they too showed that there were only 11 nets to a cube. I had a systematic way of finding out how many nets there were but when it came down to the confidence in my ability to solve this problem I did not trust my mathematical findings and searched through someone else's to build my confidence in my own answer of 11 nets.

RESPONSE

From what I understood by your reflection, we both went into the class period pretty much clueless as to how we should start figuring out ALL of the nets. I had thought through the problem and even come up with the single net I could recall from my elementary school lesson. OK- What Next? There was something very intimidating in his question to us; we were to find ALL POSSIBLE cube nets. At least for me, the combination of those words made my mind stall. No one likes being wrong and I was afraid that my grade would be unsatisfactory if I did not find them all. Being a very visual and tactile learner myself, having the ability to use the manipulative really helped. My group had come up with a way to find all the nets by using the base as a starting point. It seems you were comfortable believing the class as a whole had come up with all possible nets, whereas I took home a manipulative, played around with it, and even cross referenced our answer online! I can see how students would get very frustrated and want to give up, however, I think it is a great way to build their confidence in their own abilities to logically work through future problems.

Great Job!

F2

I had a lot of fun creating different shaped cubes. I did have the benefit of knowing for sure that there are 11 different ways to create the cube net. Basically, I just sat down and started by making a really easy small net on graph paper. Then I cut the net out, and attempted to fold it into a cube. It worked!! Wahoo! So then I drew that same net, only much larger on a separate piece of graph paper. I kind of started by making the really obvious cube nets such as the “cross” the “T” and the “S” looking shapes. I guess you could call them the base shapes. So then I just kept drawing small cube nets, cutting them out, and folding them into cubes. The process went pretty well. I had to think a teeny tiny bit, but mostly it was just drawing. Finally when after I finished my 9th cube net I ran into some trouble. I guess my creativity skills ran out …but only temporarily. My first mistake was that I created a cube net that did not fold up into an actual cube. It had a lid, but no bottom because both “bases” were on top of each other as a double lid. Then I tried to make another one. I was all excited cause I thought I made a really neat cube, but turns out I had already made that same cube, I just had to rotate it a little and they fit on top of each other when flat. So then I thought and I looked at all my shapes. I looked at different ways I could add an “arm” to the basic shapes or how I could bend the basic shape to create a new cube net. Within a few minutes I had it…in fact I had two different ideas and I had to draw as fast as possible so that my brilliant idea wouldn’t fly away. I did my usual thing of drawing, cutting, and folding. Both of my ideas worked and then I was up to 11 cube nets. I was pretty excited! It didn’t even take me an hour…and I was watching TV. too ☺ I thought this activity was a great hands on learning approach. It definitely helped to understand how a flat shape can turn into a 3-D shape. There was a point when I cut up part of my DOTS candy box to help visualize how to make the cubes. It sort of helped, but it would also be nice to try it with those connecting pieces from class (the triangles etc. to make tetrahedrons, octahedrons, icosahedrons etc.) I’ll definitely use this activity in my classroom!

RESPONSE

I thought that your approach to solving the cube nets was terrific! I also started off with no plan of action, other than to just draw and see how many I could do off the top of my head. I really liked how once you got stuck that you looked for patterns. Kids generally do the same thing. I also liked how after you struggled through it for a while you got together with group members and tried to work it out together rather than one person knowing and showing all the answers. Great Job!

F3

When we first received this assignment, I thought it was going to be effortless. When I sat down to work on the problem, I got out 6 sticky notes to represent the net. I moved these six sticky notes around until it was possible to fold up the layout into a cub. At first I thought there were going to be a mass amount of nets possible, after I started playing around with the sticky notes I found that wasn't the case at all.

I started by laying out the nets and finding the pretty easy nets. But after a while I got stuck, so I started with the cube and tried taking it apart different ways. Finding more possibilities. I took a moment to look at all the nets I had found to see if I could find a pattern. I was beginning to feel frustrated because the pattern that I did find seemed very unclear. I found that when you laid four sticky notes is a row with the two left over sticky notes there were only 6 possible ways to arrange them. Next, instead of four sticky notes in a row I now used rows of three and had three sticky notes left over to arrange. After playing around with the sticky notes I found only four possible ways to make a cube net. The next step would be to arrange the sticky notes in a row of two, and there is only one possible way to do so. Meaning that there are only 11 possible nets of a cube.

Assignments like this one have many benefits. There is clearly more then one way to solve this problem, so students can use a way that can work best for them. You can solve a problem like this visually or mathematically witch in also a perk. And it really gets your mind going!

F5

I really enjoyed the cube net project. I found that after I planned my methods out in a strategy it was easy to conclude that there are only 11 possible nets. My strategy was pretty simple; I first found a basic shape that was in a lot of nets. First I used a column of four faces, and I knew that two more needed to fit on the side of this column. I started with the basic T shape and then moved one of the side faces down one unit to form a new net. I continued this process and got several solutions including the S shaped net and the lower case t net as well. Once I exhausted all of the nets that used the column of four faces, I moved on to another shape and worked with it until I found other variations of nets.

In the end, I had a little trouble convincing myself that the 11 nets that I ended up finding were all that existed. After trying to find more options for several minutes I finally decided that I must have found them all. Once I realized that I found all the nets I was pretty satisfied and proud that I had solved the problem.

My main obstacle in this project was finding the same net twice and not realizing it. I would find a net and draw it out on my graph paper and then realize later that I had already discovered this net. It had just looked different because I drew it backwards or upside-down.

I was definitely interested in this project because it really helped me understand the different 3-D shapes. After I understood the workings of the cube and the tetrahedron, learning the remaining 3-D shapes was pretty easy. I really liked this project and I think it would be perfect in an upper elementary or middle school class. The manipulatives really helped me and I believe that they are a must in discovering the many different nets. Group discussion is also helpful because students can bounce ideas off each other and help each other find different solutions.

RESPONSE

I had a similar problem putting my methods of finding the nets into words. I used a similar method to find the solutions but I really like how you worded it as finding “cores”. I had trouble describing this same strategy and used the word column to describe the core. Now that I think about it, turning my nets on the side would turn my cores from columns to rows. So I guess my word choice in my reflection wasn’t too great. I also agree that the models really helped in solving this problem. I believe that many math problems can be solved using models, manipulatives, or pictures. This also makes the problems more realistic and relatable. Good work with the project, it seems like you really understand the basics of the nets and the 3-D shapes.

F7

When I first started to solve the cube net problem I just went about it guessing and checking. I soon realized that even though I found a few solutions, it was not the best strategy to use. I felt myself getting overwhelmed and frustrated thinking that there was no way I would know if I found all the solutions. But then I took some time away from the problem and that really seemed to help my attitude toward the activity. I then took on the challenge head on and was motivated to finding the solution. When we got together with our groups in class I found this very effective. We were able to talk through our difficulties and eventually came up with patterns that in turn lead us to the final answer of 11 cube nets. We found that each net either had a base row of 2, 3, or 4, and then the other two blocks on each side of the row. It was much easier and more enjoyable of a project once we found the pattern! After looking back on my experience I learned that when I come across a problem that seems hard and frustrating and I just want to know the answer, I just need to be patient and determined to keep working on it until I reach a point where I understand the concept and feel confident that I have the correct answer. I would definitely use a problem like this one in my classroom because it forces you to be creative, use problem solving tactics, and work well with others until you reach a final answer!

RESPONSE

I can completely relate to your experience of this activity. I also used the guess and check strategy in the beginning only to find that this was good for a while but wasn't going to bring me to the final solution. I also felt frustrated and just wanted to know what the answer was! Once I got back on my feet and accepted the challenge I seemed to enjoy it. I really liked working together with my group and found that talking about my problems made a whole lot of difference. I think using this assignment in my own classroom would challenge the students to think outside the box and deeper instead of on the surface of the problem. It forces you to work well with others and use problem solving strategies!

F8

During this activity, I learned how important problem solving and perseverance are. At the beginning, I was able to come up with 5 different nets of the cube. I didn't start with any plan of action in mind; I figured I could just experiment and easily find them all. After finding 5, I started to see a pattern. I had 4 squares in a row and the other two squares branching off of those 4. I found one more by guess and check to find a total of 6 nets. I felt stumped once I get here because I knew there must be more. Defeated, I had no idea where to go from there. Curiosity grew as I wondered how my other classmates were solving the problem, so when we started working in groups, we used reasoning and found nets with 3 squares in a row. Next came 2 in a row. We did not find any with 2 in a row, so after finding 4 with 3 in a row and 6 with 4 in a row, we had found 10 nets. We were confident that there were only 10 distinct nets of a cube because when we collaborated, we discussed how you can have no less than 2 in a row, and no more than 4 in a row. Frustrated and disappointed, we learned the other groups in the class had found 10 [11]. We went back and tried to find the last net. Just knowing there was one out there we hadn't found helped us to find it by process of elimination.

I learned that it is very important to let students work hard even if they are struggling because they will get it eventually. If needed, a small hint and collaboration can instill great confidence and optimism into a student. It is important for students to explore and come up with solutions on their own by problem solving, rather then being spoon fed all the answers. If someone had told us at the beginning there were 11 distinct nets, we would have lost the curiosity, satisfaction of discovery, and the excitement of coming to know and understand the answer.

RESPONSE

I really liked how you started out by thinking of the characteristics of a cube such as cubes having 6 faces. It seems like you really understood the problem in the beginning and you searched to find confidence at the end. I felt the same way, like there must be more. I really like the point you make about thinking the total number or nets might be a multiple of six. This went through my head as well, but I did not know how to explain that either.

You and I had similar strategies. We sat down and thought about it logically, and then we actually found a pattern to work off of. You were very smart to use the "linkin squares." I cut out paper that didn't quite help you get a true understanding of the cube structures.

I also looked on the internet because I needed an answer to ease my mind. I found 11 too, and then looked on line to make sure. Then it was much easier to prove why.

F9

At first, I felt like this problem posed by [the professor] would be relatively easy. I have had some experience with nets, although limited, in high school geometry. I had never really thought about a maximum number of nets for any polyhedron so this problem raised my curiosity about the subject. After taking a look at the problem a couple of times, I felt confident in my knowledge and confident that I could solve the problem if I really tried.

I used a method which involved “core” numbers of squares which allowed me to, in a way, categorize my findings. I used “cores” of two squares, three squares, and four squares. I used graph paper to draw each net and mentally put it together to see if it would indeed work. I found that there were six possible with a core of four squares, four possible with a core of three squares, and only one possible with a core of two squares. When we worked in class on this problem my tablemates had very similar ideas and opened my eyes to other possible routes to solving the problem. I also found it very helpful to have the physical model there to assist in solving the problem. This method proved to be effective when it came to finding the different nets of a cube. The thing that was frustrating was articulating my method in words. It made sense to me what I was doing, but I found it very difficult to describe my process so that others could understand as well. After struggling a bit I began to realize that, personally, I had a tougher time reporting my findings than actually exploring the problem and its possible solutions.

Looking at this from a teacher’s perspective is something that I feel is very important. This experience reminded me of my elementary years in school when I was just learning some of the basic ideas and concepts of math. It also allowed me to take into account the student’s perspective. I now feel that I have a better understanding of the process of problem solving in general which I think will help me as a teacher. I think that this knowledge will help me to help students when they are “discovering” new things in all subjects. I know that I will use models as much as possible to teach <u>all</u> mathematical ideas in my classroom. I strongly believe that models can make learning the material make so much more sense and make it so much easier for the students as long as the model works for the individual child. To put it simply, I feel that because I have experienced this, I can more effectively help those who will be exploring the same kinds of problems for the first time.

RESPONSE

I read your reflection and I think that we both went about the problem in similar ways. I know that when we worked together on that Friday in class, we both had very similar ideas. I agree that there were times when it got frustrating. I also agree that the problem seemed much simpler once you got to a solid answer. Good reflection overall.

F11

At the beginning, I thought this assignment was going to be a breeze because we were using our nets in class, playing around with them, and figuring it out physically by building and un-buildding the cube. However, I am extremely ADD so of course, when I got home to finish it myself I ran through it quickly at first without thinking about what I was building. The first time through I only could come up with 7 nets, when I knew because of research there was supposed to be 11. After I had frustrated myself to the max, I took a breath and remembered the method we used in class with building the nets. So I cut out 6 squares out of paper to make up the faces of the cubes and started playing around with them again. Using this hands-on approach I realized there could be 2,3, and 4 faces in the middle (going vertical on the net) as long as the 2 for the bases of the cube were left out of the middle(going horizontal on the nets.) I realized after I had found all 11 nets that there were plenty of different ways to figure out how to work out this problem I just had to find what worked for me.

RESPONSE

I had the same problem. I rushed through it, got frustrated and overwhelmed, and then once I realized I had to be patient and work it out, it was so easy to find the pattern! I agree this would be a great activity for the classroom. Not only does it teach the kids patience and problem solving skills, it is an excellent hands on group activity and it turned out to be pretty fun.

F12

One of the most enjoyable things that came out of this activity was the teamwork that my group and I put together. I had thought about the problem very little before we were able to discuss in class, made a few drawings, and hadn't even started on a proof. When we began everything was very simple, each of us showed the nets that we had got the night before and went from there. This is when things got a little more frustrating. We decided on a conclusion to our frustration by at first using trial and error but eventually checked by changing the number of blocks that made up the base. After we had thought we might have found all the nets you could tell that we still weren't very confident, but still very excited to be closing in on a answer and making progress. As the class went around saying how many nets they had found we realized we were most likely one net off. We tried one more net and we got it on the first time.

What I learned from this activity is that I need to try new things. When I didn't know the answer to the problem, I went with the simple nets that everyone uses everyday and gave up until group discussion. I did eventually get more and more involved the more I started to understand nets.

Overall I really liked this activity. Even though I was frustrated at points I just felt great at the end of class when I was confident with the number of nets I had come up with. This would be a great way to help kids understand the dimensions of different shapes. It will also help them figure out that they can't arrange things in any which way they want. It's a little more complicated then that.

RESPONSE

Overall I feel that everyone in class did a great job on the reflections and gave some really good feedback. There seemed to be a trend among a lot of people in their problem solving process. It started alright, got more frustrating, but eventually felt confident in their answer. I'm glad to know that I'm not the only one who gets frustrated over finding the nets of a cube. I also agreed with a lot of peoples reflections on teaching this activity themselves. I think it's a great way to get kids to think and raise their frustration a little but still give them a chance to feel confident in themselves.

F13

The second time around doing this activity I approached it differently, already knowing there were 11 nets I decided to get physical with this activity. I cut out all the pieces to a cube (6 ssquares) and just started piecing them together at random. I tried for about an hour to see a pattern and I got some basic shapes such as the cross, T, staircase, and the S looking shape. Then I began making these opposites which brought me closer to 11. I think at this point I was looking for some form of an equation that might lead me to the answer. I tried out a few but nothing stuck to 11. Throughout this whole process both times around I didn't/ don't really understand what I am looking for. I mean I know how to make a cube but why nessasarily do I have to know how many different ways there are. I think as a future teacher the aspect of having a physical model to show children is very important in grasping a concept. Children are very active learners and they have much to learn from an actual hands on activity. They get to use all there sences to figure out the problem. I think its interesting that past the elementary level teachers pretty much stop using manipulatives and teachers mostly resort to lectures. They believe they are "preparing you for college" when in fact there hindering an equal education. So far last semester and this semester's math courses have been all about working with manipulatives. The first time I did this activity it took me a very long time to find patterns without the help of a 3-D figure. This time around with the pieces of the cube to use as a tool it became a lot more apparent what were some easy ways to figure this out.

RESPONSE

Oh [F2], I can totally see you making those funny little noises trying to figure this whole thing out. I'm so sad we didn't work together on this project I think it would have been tons of fun. I remember my table felt the same way after we reached 9 ways. If I remember correctly it took us longer to figure out the next 2 then it did the whole 11. I spilled the beans to my table that there were indeed 11 ways so then we became determined to find the 2 left. They were the weirder looking because they didn't quite fit any particular pattern. This activity was really hard but it helped to have a group with so many different views. Even the fact that we sat on all different sides of the table we were all able to see things differently and we all came up with at least 1-2 different ways to do make a cube.

F14

I remember learning about the nets of a cube in sixth or seventh grade, so when we got this assignment, I thought it was going to be a breeze. When reading the full assignment I realized I had no recollection of how many nets there were, but also no recollection of how to figure out the answer. Now I was stumped, and frustrated. So I put the assignment away, leaving it to another day. Then on a Thursday night, I realized that we would be discussing the problem in class the next day and frantically grabbed my backpack for the paper. I stared at it for at least five minutes, not wanting to start until I knew how I was going to solve the problem. After a while, I realized that I was holding myself back. By being too meticulous, I was actually preventing myself for trying new and different ways to solve the problem. So, I finally started just writing things down. First I wrote down what I knew to begin with, and I thought, OK, this is getting me no where. My next attempt was to try and figure out how many nets there were by using the nCr button on my calculator, which told me that there was going to be 15 nets. I know that is wrong now, but it was a start, and it gave me something to go off of. So I then started drawing out all of the nets I could come up with at random. After coming up with 15, I was really excited and went to bed. In class the next day, we started looking at the drawings at our table. Not only had I drawn the same net twice, but some even three times, and when we narrowed them down, I only had 8 nets total. At this point I was just really frustrated. I had spent all that time, thinking I was right when I was very very wrong. I was also stuck on the idea that I understood all the parts of this assignmnet, I knew what a cube was, I knew what a net was, yet I couldn't get the answer. So after talking at our table, we devised a system where we would organize the nets we were making into base tiles. Starting with bases of two and working our way to bases of 4. After this, we came up with 11 nets, and realized that there could be no more. Once we realized we had the answer right, I was very satisfied, but in the end I wish it had come a little easier to me.

RESPONSE

It seems like you went through the same process that all of the rest of us went through. Random guesses, then slightly more specific guesses, and then finally accepting an answer because we give up, or just can't seem to find another answer. Which is really annoying, but seems to be a popular way to figure out an answer to a problem we don't know.

If you had been given more instruction, or more guidance, do you think that this problem would have been easier to figure out?

Are there still some concepts in math, or other subjects for that matter, that you still struggle with because you had to learn them on your own?

I know I still struggle with the concept of long division because my teacher never fully explained it, so we had to basically teach ourselves. I also wonder if we had been given a specific manipulative to work with from the beginning, if we would have been able to find the answer easier..

It is also interesting to me that you looked up the answer online to verify that you were right, because we all do that in some form or another. Usually we are told the answer by a teacher, and our results are verified, but in our case, [the teacher] wouldn't tell us if we were right, which I know was frustrating, so we had to have our answers verified elsewhere, which is very interesting to me.

I really enjoyed reading your response!

F15

When the assignment was first handed out , I was immediately anxious. I thought; "oh great, another frustrating math assignment!" Then I looked at the example of the nets of the tetrahedron and I thought; "OK, maybe I can do something with the nets of the cube by using the example to help walk me through it." The first day I found three by guess and check. I figured out that a cube had six sides and I just had to come up with combinations of the six sides that could be folded into a cube. I admit I didn't spend a lot of time dwelling on this assignment. By the end of the first week, I had come up with seven total by my self. I would first sketch them on paper and then I would scale them up to figures that were large enough to cut out and fold into cubes. I was excited every time I would come up with one that worked. When we got together in our groups it was fun to see how many everyone else had discovered. Together we decided that we had found eleven total. We were not sure how to prove that we had found them all. It was [the teacher’s] hint that we needed to classify the net cubes that we had found in order to determine if we had them all. I went home after that class period and classified mine into groups having combinations of four squares, three squares and two squares. It appeared that eleven was the magic number. I still didn't know if eleven was correct or not until class when we were told that eleven was the number of net cubes but I was much more confindent at the end of this project than I was at the beginning.

I guess what I learned from this is that students need to be given challenging math problems that may be a little beyond their ability. They need to be allowed to work the problems out at their own pace. Also, working in groups is a good way to get a idea if your on the right path to finding the answer. You can check out everyone else's ideas. Teachers can guide the students if necessary by just dropping an occasional helpful hint. Activities like this one can help students to build self confidence as well as math skills.

RESPONSE

Your right when you say that this activity took perseverance. I to went through a number of emotions from the start to the finish of this activity. Anxiety and frustration to begin with but as I started to have some success at finding the nets of a cube I started to feel excited and curious enough about the process to keep working it out. You are also correct that it does not hurt the student to struggle a little while trying to find the solution to a problem. It adds to their confidence as well as their knowledge.

F16

When I first started to work on this problem, I used a guess and check strategy. Right off the bat it is easy to discover the first few, but after that the guess and check method becomes less productive. At this point in the problem, I am enjoying this brain teaser type of puzzle. The guess and check method produced three or four nets and when the nets didn't come with ease like before, I got frustrated and soon gave up for a few days. After a while I would look at the problem and this time I tried to find some sort of pattern. As a group we noticed that some of our nets had 4 blocks across and one block on either side of the row of four. Next, we found all the nets for 3 blocks across, and lastly for 2 blocks across. Total number of nets of a cube equaled 11. After finding the pattern it was pretty easy to confidently say that 11 was the total number. Also, the problem did seem to be as difficult after you know the answer. I guess that's how I feel with a lot of math problems. When I'm working the problem the solution seems far away. In regards to this problem, it's like most math problems, I like the problem at first, then I get frustrated, then I just want to know the answer. I would use this problem in my class, to show examples of patterns and problem solving skills.

RESPONSE

[F21], I really enjoyed your comments on the net problem and I think that you and I took the same path in trying to solve this problem. You and I both used the guess and check method which yielded 4 or 5 nets rather quickly. You stated that you never found a pattern that linked all the nets, but I think that the pattern of four in a row, then 3 in a row and finally 2 in a row is the pattern. We both got frustrated after working on the net of a while, but I think that is to be expected. Most of the reviews I read, the people got frustrated at some point.

F17

As I worked on this mathematical investigation I found that I became very frustrated. My frustration came from the fact that I could not find a proof that would allow me to say that I had found all the possible ways to lay the net that would make a cube. At first I found only seven ways to lay the net but, with the help of my table mates and the internet, I found that I had missed four. Without this help, I am not certain that I would have found all the possible nets. I also found that working with the manipulative was very helpful. The manipulative allowed me to work with the six squares which make up the cube. By using these squares I knew that I had the correct set just by folding them together, if it made a cube I knew I had another net. Even though my group had found what we felt was the total possible nets, no one could really explain why. Although I know our answer is correct I don't feel confident with this problem because I was unable to figure it out for myself and I still don't understand why there are only 11 possible nets. This question of "why" is what I think makes math so hard for people to handle.

I often know that my answer is correct but if I'm ask how I know I couldn't tell you. Math is not as concrete as many people think that it is. Often there is more than one way to figure out a problem. As a future teacher this both excites and stresses me. It excites me because I know that my students will be able to figure out their own ways of working problems if I can guide them in the right direction. It stresses me because I need to be sure that I don't say something is incorrect when it isn't, rather it was just done differently.

RESPONSE

I like your almost relaxed take on this problem. I too use the guess and check method to find the first few nets. Unlike you though I found it hard to look at it like a puzzle. I would like to use your method on future math problems. I feel that I may be less stressed out by them.

I also find it interest how you felt that once you know the answer it seems easy to you. I have notice in math that I often strugle with a problem for quite sometime only to find once I know the answer how easy it really was. I wonder way our minds tend to make things more difficult then they really need to be? Thanks for your response, I think it will help a lot.

F18

When doing the cube net activity I found that there were 11 distinct nets of the cube. This mathematical concept is a good way to illustrate how 2d objects can be fit together, in several different ways, to form a 3d object. The basic way I found the nets was first by drawing out different examples on paper and then visualizing it, then later by using the physical model provided in class to run a trial and error process. My first approach was successful, but I only found six nets. Using the cube model was much more helpful, because it was getting harder to find new nets and I found that quite a few times I was repeating some, just flipped around. This exercise was very frustrating but it made me not want to stop until I had completely solved the problem. I believe it would grasp a child's interest and attention if given the cube and asked to do the same task. I think they would go about similar measures to finding all possible nets of the cube also.

RESPONSE

I read [F7], [F9], and [F23] cube net reflections and I agree with what all three of them had to say. [F7], like myself, attempted the guess and check method to begin with and found that to be very frustrating. I think math truly becomes interesting when patterns start to form, and once the pattern of the cube nets appeared the activity became a lot more fun. [F9] talked about how the cube would make a really good learning tool for future students and I completely agree with that. Hands on teaching is one of the most effective methods. [F23] just seemed to overall like the activity, which I can't say that I did at first. By the end of it, however, I did enjoy it and found it to be very interesting

F19

I went into this cube reflectoin with an open mind I thought it would be easy. I worked on the assignments for weeks. I went home and cut up even sqaures of paper and treid to make models out of them. Afte about an hour I relized that it wasnt working so well and I wished that I had borrowed a plastic cube from the class room. I thought the next best thing to do is draw patterns over and over again until I find all the possible ways. I spent about 2 hours drawing and erasing over and over again and started to get so frustrated I almost gave up. I looked at the progress I had made and saw that I only found 5 possible ways, I was overwelmed.

Finally in class we were able tog et with our small groups and discuss it, I started to feel relieved. My group pointed out that there is a pattern you ca nuse to make the process easier. So I started with 1 cube as the core and worked around it, this didnt have any results. I then did 2 cores and work around that, this had some outcomes. I moved on to 3 cores and found alot of outcomes. Next I used 4 cores and worked around that, this didnt really work so well. Afterwards my group compared notes and we realized that there is 11 possible ways to make a cube. I just wish that I would of thought about a pattern in the first place

RESPONSE

Im replying to [F18]'s response because she was in my group. I can relate to her response because we both seemed to struggle with it a bit. it helped out alot though when we were able to get together and compare our progress. Both of us put in equal work on findng the different ways of making a cube. We also worked on finding them together. I also enjoyed that we would both tell each other the do's and dont's we discovered while working on these for 3 weeks.

F20

The Cube-Net Problem was interesting and new to me. It made me very frustrated, though through that I felt some enlightenment. Several times I thought I hit dead ends and a few thought I had found my final solution. I found that working with others helped me get past these road bumps most effectively. Whether it was roommates or fellow classmates, having another person with another perspective seemed to produce the best results the fastest. Finding the final number provoked the most curiosity for me; I then wanted to know and understand why that was the solution.

I approached the problem very hands on. I made a visual aid that literally was a cube with movable nets so that I could physically test out any ideas I had. This worked well for me; I found the majority of the solutions doing this. The rest were obtained, as I mentioned above, through collaboration with peers. I would find it very interesting to hear of anyone figuring out this problem strictly through reasoning only and no aides or collaboration.

Finding the solution was still frustrating to me. Only because I had been sure I had found it already, I felt a little behind once we shared our solutions with our peers. It was helpful, and overall I felt relieved to obtain and understand the answer. I felt that I could explain not only the solution but the reasoning to a problem.

This problem makes you look at reasoning the reasoning behind it, making it great for teachers to need to understand. There are several ways one may determine the solution for this problem, it doesn’t restrict students to one routine answer and problem solving strategy. To prove you are correct in your findings this problems forces one to look at/for shapes, patterns, form criteria and other mathematical basics.

F21

In the subject of math there can be many different ways to solve one problem, so when given an open-ended problem this one it makes things very difficult to solve. When trying to solve the problem of finding all the nets for a cube there are many different methods that you could have used. I found that after discovering the more obvious nets then I used the guess and check method. The first five or six nets could be found with ease but after that they started to become more difficult I began to get more frustrated. As I kept trying and failing with the guess and check method I thought that maybe I could find a pattern within the nets that I had not already found but this turned out unsuccessful also. So then I tried to organize the nets by a common trait that linked them to other nets, I found that you could organize them by the biggest number of squares in a row which worked out well but I still could not find a pattern between all the nets. The most frustrating part of this project was that I could not find any link or pattern between all the nets. After trying and failing when attempting to find those last few nets I began to feel a bit of anxiety that I would not be able to find them all. Even when I thought that I might have discovered all of the nets I still felt nervous that I would be missing one. Also because we were not given a set number of nets that we were suppose to find I never really felt like I had accomplished the whole project but after talking with other students and comparing projects I discovered that almost everyone had the same or similar results that I did.

In the future when I become a teacher myself I think that doing a project similar to this would make my students really stretch their minds. The experience that I gained from doing this project was that I really tried every possible solution I could before giving up. When you do not put a limit on something then I think people work twice as hard to find the answer. So as a future teacher I will try to do many projects like this so that my students will really branch out and use their minds to their full extent.

F22

I found that the net cube problem was really frustrating on my own. I am typically someone who learns better when I have a visual aid and before working with groups in class we didn't have that, so it was really hard for me to figure out the net cubes. Frustration was the only real anxiety that it caused me because I knew that when I got class and had a visual aid and a group to talk it out with the problem would be easy to figure out. Working in the group with the actual squares that built up the cube made the problem a lot more manageable for me. It was better to be able to talk things out with my group members and when we were all putting our ideas on how to logically solve it together it worked a lot faster. Having the squares that we could also work with and manipulate to help us figure out the problem made things way easier to see and deal with. The reason my group came up with to solve this problem was to start with a base of 4 squares and then move around the other 2 squares. We then moved to a base of 3 squares and moved the other 3 squares around to make the cube in different ways. And finally we had a base of 2 to and moved the other 4 squares to get our final solution of 11 nets of cube. I think being forced to find our own solution to the problem without instruction or knowing exactly what we were looking for was really beneficial because it was more satisfying to find our own solution than it was to have some tell us how to do it, and I also learned more having to think on my own with no guidelines. I also think that it was beneficial because it put me in the position that many of my students are going to be in someday when I am the teacher, and now I know the frustration of not understanding, or having all the tools I need to complete the problem. I also know how I worked out the problem and seemed to work to the majority and that was having the aid of the squares the build into a cube and being able to work and talk it out with a group and knowing that will be good for when I am trying to teach my students.

RESPONSE

I had a very similar experience working in the group where everyone was coming up with ideas and trying to work the problem out logically. My group did eventually come to the base 4,3,2 way of looking at the problem, but it took some time along with trial and error. I also drew many of the same conclusions out of this acitvity about teaching and the way students learn. We should challenge them with something that is a little more than they are used to but that is still totally within their skill set. It is not only a lot more satisfying for a student to come to a conclusion or solution on their own, but they also retain more of that information because they were allowed to find their own way to the solution and have a better understanding of it because of this. This is what I experienced as a student doing the net cube project and I imagine the same holds true for all students.

F23

I have not had much experience in nets. At times, this activity became frustrating. I never really had a plan, I just started to draw. I would become irritated because I would find a new net but it was one I had just flipped or turned. When I started I thought this was easy. As I worked on figuring out the nets, I realized that it wasn't as easy as I thought. I would have to set it aside and still I found myself thinking about what net I was missing. I felt like this was a mind challenge. I enjoyed this activity and will be looking forward to the next one.

I enjoyed this activity. I haven't had a lot of experience with nets. I never really had a plan on how I was going to solve this. I just started drawing. My frustration came from running into the same nets just flipped or turned. I felt discouraged at times. I would think I was doing really well and then I would find I already had that net. I would set the homework aside and find myself still thinking about different ways of rearranging sides. It was like I felt challenged to find them all and frustrated because I couldn't seem to find them all.

RESPONSE

I agree with [F17]. I think his post was very well stated. I agree that I have no proof that eleven is all the nets. I think that working in groups was very helpful. I think the [F17] is right about different ways to figure out a problem. It is hard to see that people solve problems in a different way and that it is not always wrong to venture out to the uncommon. It can be hard to see everyone's method. I think that [F17] did a very good job with communicating thought and feelings. Way to go!

ACTIVITY 1 REFLECTIONS SPRING 2010

S1

The first part of this project was initially finding the number of nets the cube has. This part of the project interested me for the most part, but it was also quite frustrating. Our group couldn't seem to find any rhyme or reason to finding these nets. It was really easy being patient with the whole process because my group kept cool. Our strategy was mostly guess and check until we were given a hint about the bases of the nets. Once we found to start with the base of four we knew to keep getting smaller bases from there. It took us a while to figure out that there are eleven nets because we kept trying to make the same nets that didn't work. In all honesty I'm not sure what this taught me, but that could be because I'm not good with proofs, so when it came time to proving there were eleven nets I was a little confused, but thanks to helpful group members answers became clearer to me.

This experience was a very curious one. By asking us to find all the nets of the cube it made us interested in how many there really were. So it taught me that math is a lot more interesting if taught in a fun and experimental environment. It is a lot easier to learn if asked to first try and discover things for myself. It also helped me to write down all of the nets so that I could see certain patterns. One example would be that you have to start with a base of four squares. So making things interesting, making situations curious, and being pushed to learn for oneself first, I feel, are all good methods in aiding students to become motivated and willing t learn math, and also in helping them to understand.

S2

This assignment taught me a lot about how children see math. You see the first couple that super easy to find but as it gets tougher the easier it was for me to get frustrated. What helped I think was having groups because you can bounce ideas off each other and more heads are better then one. I think having the visual also helped because you may think you have one then when you tested it with the blocks you realize that it really didn't work. What distracted me was also having the visuals because as the nets got harder to find, I would vere off task and start just messing with the blocks. The groups did the same thing once we started not finding any more, we would start just making jokes or putting the block together in weird fashions.

This experience taught me that kids learning in totally different ways. Some kids learn by visuals while other learn by writing or audio. So putting the kids in groups will help get the different types of learning together hopefully giving the kids a chance to have a better understanding. I think that visuals are always a great way to get kids learning because they have something to play with, but also are learning at the same time. I think that giving examples will help kids get off on the right foot because they will know what they are looking for and will be able to find more similar to the example. But, also monitoring the kids so that they stay on task and don't start to fool around because I found me self doing that occasionally. Having the students writing how they found what they found and how they got to the conclusion, helps kids understand the whole procedure instead of just the finish product.

RESPONSE

I totally agree with the kids that are shy because i know how nerve racking it can be to talk in front of the whole class. so yeah to have smaller groups it allows for peer to peer learning which gives kinds another outlet in learning. I also agree with the learning on their own because as they come up with shapes by themselves they knew what they did and how that got their so there is [not] as much confusion and they are learning the concepts better and faster because they did it on their own.

S3

In my mathematical past, I have encountered few problems that challenged me in the way this problem did. It challenged me to go about solving a question in a more abstract way than I was use to. Most of the time, when assigned math homework, it has a simple answer that does not take such concentration or has a definite procedure to how to solve it. By being confronted with a problem that I was not use to actually, in the long run, helped me understand the working of the problem better. I came to discover the answer by my own means instead of flipping through the pages of my textbook looking for the proper procedure that needed to be carried out. It became more personal because I discovered my own procedure. In hind sight, I do like this problem, but at the time I remember being frustrated by not knowing how to get the answer in the simplest way. However, I was determined to solve the problem because I hate to accept defeat. It is that stubbornness that most children have that will help them become more motivated in their work.

Mathematics should be taught a variety of ways, in my opinion. I do believe that children should have the typical notes, homework, and test procedure for repetition so that it will stick in their brains, but I also believe that hands on problems are crucial to a well rounded learning experience. Everyone learns differently, so incorporating every style possible is important to get children interested in the mathematical world and have a desire to actually understand the concepts introduced more deeply. Being innovative in the classroom is a win-win situation because it aids children in understanding and being excited in what they are learning. Monotonous styles of teaching just drive the children into disliking mathematics and focusing their energies into other subject when in all reality mathematics could be their gift.

RESPONSE

I agree with [S25] in the aspect that soley "copying notes, memorizing rules and spitting out generic answers" is not the most effective way of teaching children. It takes innovated thinking to come [up] with hands on projects that will tease their brains and give them room to spread their wings and discover the answers by themselves. Children take pride in their accomplishment and by setting them up to understand more deeply, they will be more likely to be passionate about their learning instead of just copying problems out of a book because they are assigned.

Personally, I was frustrated with this problem, but having a group to discuss the problem made it less stressful and more fun for everyone. We got to share our input but also have others input in order to see the problem with a different set of eyes.

S4

The nets of a cube activity was an interesting learning experience in which classroom manipulatives proved to be a very useful learning tool. I discovered that for some people, including myself, certain math exercises can be difficult to visualize or wrap your head around without the aid of something tangible. In hindsight after doing this activity, I feel as though it would have been significantly more difficult to try to reach the solution of the problem without use of the manipulatives. My most successful strategies stemmed from my own experimentation and trial-and-error, as well as observing my peers and their experiments. I found that the "hands on" approach led to ownership of the task at hand as well encouraging experimentation and theories. Discussing ideas with my peers was also an effective way to brainstorm and formulate some theories and generalizations about the activity. The experiment made me feel slightly frustrated at certain points in which my group and I were failing to find new nets or when I was trying to come up with my proof. The exercise was gratifying when my group and I discovered a new net and when we concluded that we had found all nets.

The nets of a cube activity was very insightful for a future educator. I now appreciate a group math activity as a method of learning in which students feel safe to contribute ideas in a "no fail" situation. Watching each other experiment with the minipulatives helped to spark new ideas out of everyone. Discussing theories and patterns with my peers provided a form of checks and balances where we could either disprove a false theory or work to develop a solid one. This experience led me to believe that any time a math problem is taken out of its usual context (such as a problem from a book) and physically put in the hands of a student, that not only does the student demonstrate more ownership of the problem, they are also more apt to actively participate, remember the situation, and be able to more fully understand the concept. The student feels more in control of their own discovery and the group aspect encouraged everyone to participate to work towards a common goal. For those of us who have issues visualizing concepts or shapes, the tools provided greatly reduced the stress for the student. In general, the most important thing I took away from the nets of a cube activity is that if you present a problem to a student in a way they are not entirely used to and provide them the tools they need and support of their peers, they are more likely to take ownership of their own learning and also have more fun and success then they might otherwise have experienced.

RESPONSE

I agree with [S18] in that math is a great time to let students work in groups. There is no worse feeling as a student to feel alone and confused in class. In a group setting, kids discover that their questions are not "dumb" and their peers were probably wondering the same things. In a group setting, no kid is totally left behind and they have a safe environment in which to contribute ideas without judgement.

RESPONSE

I think [S13] makes a great point in her first paragraph. It is only our job as teachers to ask the tough questions and make sure we provide our students all the tools they need to reach their conclusions. It is not for us to say how they reach their answer so long as they understand the material. Nets of a cube was a good way to let everyone reach a solution in different ways.

S5

The cube nets activity was interesting. I have never really thought about all of the different combinations that could be used to make a cube. I really enjoy puzzles, so this was fun for me. It was important to remain organized and pay attention to which combinations you already used. It was frustrating to figure out the final combination because it was a combination that is unusual. It was also hard to keep with a routine because when I found one combination, it would lead me to another that maybe had a different base. The activity was fun and easy. It would be a good activity to do with kids.

This activity teaches the uses of organization and combinations. It also helps with a student's ability to work in groups. If a student couldn't figure out all the combinations, then they would have a group to help them. It is also important for a student to have a hands-on approach to activities. A teacher can include the snap together cube sides to give the students something to work with instead of just drawing and trying to visualize. It is an activity that can be used with kinesthetic or visual learners.

S6

After completing this activity it is clear to me that math is truly a process. Although one definite answer may exist to any given problem, there are certainly a number of different ways to reach this answer. It was very helpful for me to work in the group as I am not sure I could have come up with all eleven solutions on my own. I was surprised there were so many solutions. I felt as though our group came to our conclusion of the number of solutions through trial and error. None of our group members were initially aware of the concept of having cord 2, 3 or 4 nets. It was through hands on exploration with the manipulatives that we were able to discover what creates a net of a cube. I did not feel distracted during this activity because I think our group size (four people) was exactly right. We were all able to participate fully. I think a group any larger would have detracted from each members learning and individual contributions.

I think for a problem like this it was extremely helpful to work as a group. As a teacher I will need to be mindful of the size of the group and whether or not it is appropriate for the problem/task. With this particular problem I might make a list of terms on the board prior to starting the problem. I may write, cord 2, cord 3 and cord 4. I would not reveal right away what these terms mean in relation to the problem, but it would give my students some direction and basis for their exploration. I think students gain a deeper understanding when they have the opportunity to teach others and share their own solutions to problems. I would give students an opportunity to show the class the different solutions they [came] up with and explain why these solutions work. Students are motivated to learn when they feel as though their opinion is valued.

RESPONSE

I agree with [S13] in the sense that while there may be one definite answer to a math problem, there are certainly multiple ways of reaching the answer. Working in groups allows for members to see others' problem solving strategies. It is also true that students are likely to become frustrated. This is why it is important to set goals and give small clues without completely revealing the answer.

S7

The experience taught me that to understand math and complete challenges, such as this one, you need to go after it in steps. By doing it in steps it allows the problem to seem not as stressful. It makes you feel good every time you complete another step. Without using the steps strategy you will never know when you have found all of the possibilities. At the beginning of the activity my table simply put as many squares together as needed and then rearranged them, not thinking of a way to recognize if we had done that shape already. This made us a little stressed because we didn't know how to tell if we had them all at the end. We actually ended up getting 12 answers before we realized that one of them was the same shape just rotated. At the end of the activity our table was definitely excited but still a little unsure if we had all of the possibilities.

This helps me understand and see that by using models and letting kids physically touch the examples allows the student to understand the concept easier. By playing around with the objects and not simply being told what you need to do help them understand their own questions that they might have. I think being in a group definitely helps the students to be more outgoing and experiment also. Some students are even too shy to ask the teacher questions so by being in groups it allows them to ask someone they may feel more comfortable with. By doing these labs I think it makes us more interested in learning more things because we sort of taught ourselves this lesson. It makes me at least more suspicious about what other things we can figure out without real guidance of a teacher. All in all, I think these labs are a great way to teach students more than just how math works. It helps communication skills, thinking out of the box, and accomplishing a goal with a group, along with many more concepts and skills.

RESPONSE

I agree with [S25] in that learning should be more hands on rather than copying down notes, memorizing statistics and coming up with the first answer that comes to mind. I also believe that when students or people are challenged to figure out problems on their own that they are more apt to actually remember it. By lecturing with notes AND doing hands-on experiments it creates a perfect classroom for all types of students. Some may learn best by simply copying and reading what is written down while others don't understand things until it is placed in their hands. I believe that this is definitely the best way to teach any lesson.

S8

The cube net project was a great way to open up to various way of learning for me. In the process of figuring out how many nets there were total my group and I went with the strategy of simply putting the squares together in different shapes and counting them. This was a sort of guess and check process which didn't work so well and gave us lots of duplicate shapes and was very haphazard. After we caught wind of the separate net shapes we started making all the alterations for each; the four nets, three nets and the single two net. This process was much more satisfying, organized and easy. The activity definitely gave a sense of accomplishment when all was said and done.

The cube net experience really taught me that math concepts are much better understood and conceived with hands on projects in groups. This way you are allowed to collaborate with others, opening your mind to different ways of thinking and various problem solving techniques on top of having something to manipulate with your hands; not only to visualize but actually visual. The strategy of giving kids a vague problem to work on is very effective. A problem that they have to build their own process of solving for and a small incremental step-by-step building process that leads to a greater understanding and meaning can enlighten young minds. The strive to come up with a unique process with your group and tell the teacher about it is enough inspiration for the students to deepen their roots in math and come more in-tune with its mechanics, a good math teacher will have the patients to sit back and let the students struggle a bit in order to heighten that final understanding satisfaction and development. Any activity that promotes group work and collaboration with a manipulative problem or process will motivate the students to come up with creative and elaborate ways on route to a solution.

RESPONSE

I like your thinking [S2]. I completely agree with the statements you made about kids having multiple intelligences and how important it is to work in groups (especially with physical/visual problems) and have the various angles from different thinkers added into the equation. Very well said about the writing part at the end to sum up everything, it helps to really wire it into your brain. It's a great finishing move to a fun, engaging learning activity.

S9

This activity was a fun way to strategize and experience a mathematical lesson about how many shapes could be formed with a certain amount of cube-net pieces. It was helpful for our group to use the cube-net pieces to see which figures would create cubes and why others would not. Having the actual pieces in front of me helped to show the different arrangements and why they worked. This activity taught me that there are many different strategies and procedures in which a math problem can be solved. For example, besides using the pieces, my group figured out how many shapes could be formed by realizing that a certain amount of figures could be used with the different bases. We thought it out mathematically rather than just using the pieces to form the eleven different shapes. The most difficult part was picturing the eleven different forms and making sure we had not already used that shape, just in another rotation. When we found out there were eleven, it was easy to find them all and make sure none were repeated.

I love doing hands-on activities and I think it is a great method to use in a classroom. It allows the students to actually see certain items and why they work. Finding different ways in how to solve a problem would also be useful because it gives the students other ideas on how to figure out the problem. It is up to them to see which method they like best, which would be great because they would more likely understand something if they have different options upon how to solve it. Since hands-on activities have been very useful to me, I think they are a great way for students to get motivated about solving math. It allows them to experience different methods of how to find answers and lets them play around with toy pieces while still learning.

RESPONSE

I agree with [S6] completely about working in small groups. I think that for many math activities, like this cube-nets one, is much easier and understandable when there is other people around you to help solve the problem. I was in the same position as you where I do not think I could have found all eleven formations myself. It is nice to have input from others because they can come up with ideas that may not have crossed your mind and it also allows for us to expand upon a particular idea given out by someone else in the group.

S10

I enjoyed the activity because it was like a 3-D puzzle. I am a visual thinker, so I liked the activity. However, there were several times when our group got stumped. It was especially difficult to find more cube nets not knowing if there were more, and how many more. It was fun to to suddenly think of a possible new stucture, and find it works, after we thought we'd found them all. The best strategy for me was folding up the net into a cube to see where the ends meet, and folding it back down. Being able to manipulate it to be 3D so easily was helpful. Looking at the nets on paper didn't really help me at all.

Hands-on math makes more sense to students because it is more real. If math is only memorization, the student might not retain it as long because they never understood why they learned it. If it is an interactive experience, such as puting a puzzle together and discovering a pattern, the information is retained. Math should be hands-on because there is more to remember, starting with touch and sound. The student realizes their abilities when doing something hands-on. Working with a group is good too because when the student is stumped, there is someone to keep the thought process going.

RESPONSE

I agree that without the use of manipulatives I would not have been able to wrap my mind around it as well. It would have been hard to just visualize where all the edges would meet. I also agree that is was a no-fail situation because we were inventing new patterns, not knowing how many there were.

RESPONSE

I agree that it can be very fustrating to be left to solve something with guess and check. When I run out of ideas to try I feel like I can't move foreward until a different perspective is shown or an example. At the same time students need to do the problem and make discoveries. A balanced mix of discovery and guidance is probably the best way for me to learn math.

S11

For our table, the pressure was on because we were being filmed. It made the setbacks we experienced that much more embarrassing. But after a while of discussing different things that worked and comparing them to those that didn't, we were able to come up with patterns. We learned how to manipulate the excess units after setting up cores with varying numbers of units. Once we had this figured out, it was simple to find all eleven cubenets. Being able to physically manipulate the squares while working on this helped greatly, because then it was easier to see why certain setups couldn't work. This way, we were able to eliminate several different failing nets for the varying unit cores without wasting a whole lot of time on it. The hardest part was coming up with the proof, which was a little discouraging. But after some discussion on how we came up with the nets in the first place, we were able to figure out a proof.

I really like hands-on learning experiences, and this was definitely useful. My history with math has always involved listening to lectures then completing formula after formula until it's drilled into my brain. With this experience, I was able to see that when you're actually involved in figuring out the patterns (as opposed to being spoon fed them) it promotes a level of understanding and achievement that can't be found in the simple lecture style of teaching. I think that helping future students to come up with patterns can give them a better understanding of how the formulas work. Because there's more involvement than just memorization, it also would help to make the concepts stick for future use. I also think that coming up with 'real life problems' can help get kids interested in finding out the solutions in the first place, because then there's that added bit of investment.

RESPONSE

I definitely agree with [S22] that the use of a single procedure can really detract from a student's understanding of a problem. I know from my own experiences that sometimes the way shown by the teacher isn't necessarily the best way for me. Because of this, I started to struggle and just sort of gave up. It's important for us as future teachers to help kids to find the method that works best for them.

S12

This mathematical experience required us to look at a cube beyond the perspective that is has just four sides with a top and a bottom. The first strategy was to find as many solutions with a base of four as we could. We did this by moving the two free attachments around the base of four. So long as the combination was unique, we recorded our results by drawing them on our papers. The next idea was to have a base of three. We were surprised to find that this worked and our determination was lifted each time we figured out a new combination. I felt very frustrated that I couldn't figure out more successful patterns. At one point we heard another group had found 11 combinations so we knew we had two more to go. That is when we attempted to create a combination that would make a cube with a base of two. At this point we found the remaining two combinations. Overall this activity made me feel completely stupid, but once we had successfully found 11 unique combinations and knew the assignment was completed, I felt a lot better.

This experience taught me that math is better understood by completing a learning task hands-on. There is no way I would still remember those combinations if it weren't for the fact that I had to figure them out on my own. If I had merely copied them down from the book and attempted to memorize them, I would not have learned anything. I believe that the fundamental idea of the idea should be taught first, the student should try to figure out how it works on their own, and then the concept should be taught from start to finish so they can fill in any parts they may have missed or not understood. I believe if the math concept is to be learned through activity there needs to be a review to follow so no major concepts are missed. Overall, activities are more fun because they involve movement, interaction and friends.

RESPONSE

I completely agree with what you are saying, but I think it is important to remember that some students struggle with math and trial-and-error may never work for them. For some students, learning a concept by watching the teacher complete it step-by-step on the board may help them be more successful at understanding the activity. It can be very frustrating for some students and lead them to hate math instead of looking at it as an adventure. Once again, I completely support your enthusiasm for a hands-on approach to mathematical learning. I just hope we can remember that, as future teachers, all of our students are going to learn a variety of ways and it will be up to us to figure out how they learn best.

S13

In this cube net assignment I realized how many different ways that there is to go about solving a math problem. Since I was young I have always been taught one way to solve a problem and told this was the right way. It was generally straight out of the math book and no questions were to be asked. Now that I have hit the college years I realize math is a lot more abstract than many may think. If you go at it with some sort of a strategy that a teacher has taught you, you can usually figure it out in a fairly quick amount of time. You have to realize this is not the only way though usually. I think the most frustrating part for me was not knowing how many cube nets I was supposed to be finding. If I have a goal to shoot for it seems to make the problem solving easier.
Since I want to teach elementary I feel like giving the students a clue to help them strategize would help get the wheels turning. Also, like I said knowing how many you were supposed to end up with would have helped with some of my frustrations so this may be a good way to help younger students. I definitely think letting kids try to think on their own is very important opposed to lecturing out of a book all the time. Hands on experiences could be so helpful in making students realize that math can be exciting. Math does not have to be a cut and dry subject but allowing the students to work with their peers and explore on their own is so important to the learning process. I think the reason why a lot of kids don't like math is because it is not the easiest subject for them and so finding new ways to get through to them is going to be important in making sure they really do understand the subject material.

RESPONSE

I completely agree with your thoughts after completing this problem. It was frustrating at times, but it taught us to have to solve things on our own. Allowing the students to work through things on their own this will help them to accomplish a true understanding. Math does need to involve more hands on learning so that students learn to like math.

S14

I found the cube net project extremely helpful to me. I felt that having the physical "shapes" to work with, was really the only way that I was able to figure this project out. I needed to physically make the shapes and see which ways a cube could be formed. At times it was frustrating having to sort of guess and check, but at the same time guessing and checking was the one option that made sense to me. A fall back of that option is not knowing when to stop! I really actually enjoyed this activity and felt extremely satisfied when I or we would find a new way to create a cube.

Doing this activity proved to me, that ESPECIALLY with younger kids (considering it's the only thing that worked for me) you need objects in front of you that you can work with. You need to be able to "see" what you are trying to say, or figure out. I can't stress enough how helpful guessing and checking or at least just checking, and seeing with your own eyes is. I know for me just seeing those shapes we got to work with made the project seem less "math" like, and more fun. I got excited and looked forward to figuring things out and seeing them come together. This shows me that it's important to keep kids interested and wanting to learn.

RESPONSE

I completely agree with [S10] in that working with physical objects that you could see was extremely helpful. I also agree in that it was hard not knowing how many cube nets there were so we just had to keep going till we were positive we had them all, but like she said it was exceptionally satisfying when you did discover a new method of forming the cube.

S15

In the cube nets activity I had to figure out why the problem worked instead of just answering it. At first I knew what I had to do and it sounded easy. Then my table and I found all the easy and simple nets but then got stuck and did not know what to do. We thought we were finished just after about 7 nets. Then we heard other groups found more. It was frustrating not knowing how many nets we were supposed to find, but in the end it was more exciting to figure it out on my own. We figured out the rule of the base of the blocks. I really liked using the square pieces because it showed us if the net actually fit to be a cube. Then I got excited to find more and more nets.

This activity showed me that more hands on demonstrations teach better than other methods teachers use. I really can refer back to this method and remember what I actually learned because I figured it out on my own [rather] than just writing it down and having to memorize it. It helps seeing 3D objects in front of you then trying to figure out what you are looking at on paper. I think when kids get to use objects and different things with math they get more excited to learn because they get to use new and exciting equipment instead of just filling out a worksheet. I also think working with group's helps kids learn and interact with each other. Some kids do not like talking to the teacher so talking to their peers would be easier to ask questions and help each other figure it out. When they explain it to each other they are actually helping themselves learn and remember it in the future.

RESPONSE

I agree with [S5] that it does get frustrating and hard to keep a routine because one base lead to a different base so it was hard to find all one base. It is also really good for students to work together in groups. They work really well with hands-on activities. Puzzles are really fun for kids to do.

S16

This math assignment was one of the first math assignments that has ever made me really think about the math problem it was asking. This math assignment taught me that there are some things in math that can't be taught, they have to be learned by the students. After we got to seven cube nets we thought we were finished, until we heard another group was at 11. At this point we got really frustrated untill we figured it out that it was about the base of the blocks. This strategy was the most useful of all the differnent strategies we tried. Instead of guessing and checking, finding a pattern worked much better.

I think that math should be more of a hands on subject then just teach an copy. Math uses so many different tools that kids shold be able to look at and use and actually see how they work rather than just a teacher showing them. Using more things that kids can actually pick up and measure and look at would hopefully help the kids understand more clearly. Not only would it help them understand but hopefully it would also make them more motivated to learn math and new math strategies. When a child can pick up a cube and count the numbers of edges they understand more clearly then an overhaed and a teacher giving them five minutes to count the edges. So hopefully math will become more hands on in the future from technology to 3D objects.

RESPONSE

[S12], I completely agree with you that this learning expierence was a lot more useful when it was hands on rather than just memorizing something out of a book. I also agree that this method helps you remember and fully understatnd the patterns more when you have to think for yourself.

RESPONSE

[S17], my group also started out with the trial and error procedure to try and attempt the number of nets, after realizing how difficult and frustrating it was we moved on to looking for patterns. I also agree that giving kids a sense of direction without telling them the exact answer is better for their learning, rather than giving them the answer and having them memorize how they got it.

S17

The first strategy I had for finding how many nets of the cube exist was trial and error. I tried different figures with squares on both sides of the base. In this strategy I had repeated figures by doing the same thing to the base but on opposite sides. I find that the trial and error strategy can be frustrating at times because there really is no sense of direction with it. The next strategy I tried was looking at the patterns that were going on. For the base four I did all the possible positions with one square at the top and moving the square on the opposite side and kept repeating patterns like that. The base three patterns were the hardest to find because there really weren't any patterns so trial and error worked best for that. Base two only had one possibility because of the limit of two squares in a row. There were no possibilities for base one. By using the patterns strategy, it made the process seem easier and like I was actually going somewhere and doing something right. As for trial and error I never knew if I was going in the right direction or not.

This has taught me that directions you can follow might be the easiest way for students to learn. Trial and error might be necessary at times and that might work better for some students too, but I personally think it's easier when there is a pattern to follow. With direction I think students will be able to make connections to what they think is right and the correct answers. They will be able to make math "click" in their minds and understand it better. I always like it when I understand a math problem that I am doing and I can get the right answer. That is a great feeling that I want my students to have and hopefully they will came to like math.

RESPONSE

[S18], I completely agree with working in groups for math. It has always helped me to get a better understanding of a problem and to think in a different way that I might have never thought about before. I want to use this with my students because I know how much it helped me and how much I liked it. I also agree with learning with hands-on experience because it gives the kids something real and tangible that they can see in math. They are both great learning tools.

S18

In the cube-nets exercise I was asked how many nets of a cube exist. I had to think about the problem and figure out why it worked. At first, in order to figure out how many nets there were my group and I went with the strategy of putting the squares together in different shapes and counting them. I thought the assignment would be pretty simple but as I kept working on it I just got frustrated. It was hard for me to not have a lot of guidance for what to do and that detracted from my learning because I am use to having more structure for an assignment and I didn't know how many nets I needed to be looking for. However, being able to work with others helped me a lot because we were able to work together and help each other figure stuff out. Towards the end, I felt better about the assignment because I understood what I was looking for.

This experience taught me that it is extremely helpful if students have the opportunity to work in groups on math assignments. I think when students get the chance to work together they learn things they may not realize by just working alone. I think it gives students a chance to share their solutions to problems with one another and know that their ideas are valued. This experience also taught me that hands-on activities are a good idea for students because it requires problem solving skills and it gives students good visuals to learn with. I think to learn mathematics more deeply; students need to be able to see hands-on examples of why certain rules work. I think that students are more motivated to learn when they can share their ideas and when they are able to learn through hands-on experiences.

RESPONSE

I agree with [S25] that math should be taught though hands on activities. It allows students to visually see how things work. I also agree with her that it was difficult to work with little structure but later on I was able to remember what I learned because I was forced to solve the problem by myself through experimentation.

S19

No Reflection Provided

RESPONSE

I agree with [S11]’s comments about additional pressure being put on some students in the class because certain students were being filmed. I, like [S11], am used to learning mathematical skills through lecture and repetition of using formula after formula. It was nice to have the opportunity to learn in a hands-on approach. By being able to actually manipulate the cube with our hands was helpful in that we could visually see the results.

S20

In the recent mathematical activity performed in class I was able to learn firsthand how many nets make up a cube. By discovering this on my own, through trial and error and eventually through a discovered method, I was able to comprehend and understand this concept and mathematical truth far better than I could have any other way. The exploration of this mathematical question instead of being told the solution allowed me a unique understanding of the nets that make up a cube. Trial and error was the first method I used. This worked for a while, but it was not until I discovered a different method through trial and error that I was really able to get a grasp on the idea. By using the strategy of examining the core of each net I initially discovered the existence of two different cores, a core four net and a core three net. The knowledge of this allowed me to try all possible combinations on both the core three and core four net, and in the process I discovered the existence of one other net core, a core two net. Overall, both trial and error as well as using the core pattern were helpful strategies. Using the core pattern was more helpful as it truly allowed me to understand the nets of a cube. The activity became frustrating at times when many of the nets I created turned out to be repetitions of one that I had already discovered, but overall it was very rewarding. Upon trying all the possibilities for each net core I found myself fairly confident in the total number of nets that make up a cube.

Through experiencing the understanding I received from this activity, I strongly support and encourage mathematics to be taught in a manner where students can discover the answer. I believe that this would greatly increase a student's understanding of mathematics. By allowing them to try different methods and investigate all possibilities of a certain problem or question, the solution may not be reached as quickly but when it is it will not only be understood but the student will feel a greater sense of accomplishment and may even find him or herself enjoying math. The method of trial and error can be useful and even lead students to discover more effective ways of finding a solution. I think structured strategies would allow students to understand math more deeply, though I think this is more likely to be true when the student discovers the method him or herself. By teaching mathematics in a hands-on manner a student is able to make discoveries, better understand mathematics, and feel accomplished and confident.

RESPONSE

I completely agree with you. A hands-on approach to learning in the classroom will allow students to better understand mathematical concepts. Of course teaching in this method is not always practical in the classroom, but I think there needs to be a balance of both structure and freedom to explore. By teaching in this manner students are free to find their own method for solving a problem. This would not only help their understanding but may even make it enjoyable.

S21

I thought that the cube net was a question that would have been a tough question if there wouldn't have been a hands-on method to help solve it. I felt that it was best answered using the models. I was able to see the cube take shape, or not take shape, by actually putting together the squares. Then we also started with a cube and tried to destruct it into a flat surface. This was helpful because whatever shape it turned out to be had to work. Then copying the shape down to remember what ones the group had actually them figured out was very helpful. If we wouldn't have done this we probably would have recounted the ones we had already made. There wasn't anything in the experiment that detracted from my understanding. I thought it was a pretty straight forward process. During the experiment I thought that there were going to be a lot of squares, but after starting I realized that there weren't going to be as many. At the end when we thought we got them all I felt accomplished and that we truly had found all the ones that we could.

This experiment taught me that it is easier to learn with hands on tools. I feel that it is easier to justify that six squares make a cube by actually being able to see it put together then just say it makes a cube. These types of visual aids are beneficial to introduce a concept, or to prove an existing problem. By using these types of models and experiments students are able to experience hands on learning and actually conceptualize the problems. Instead of just believing just because someone said it was right. Being able to work a problem will help to teach students that math isn't just about adding and subtracting but that it can be fun and hands on. Taking this hands on and active learning approach to math will help to motivate students and help them understand the concepts.

RESPONSE

I agree with [S18] on some points and felt differently on others. I felt differently from her when she said she needed more structure. I liked not having structure I felt I was able to learn on my own terms and in my own way. I haven't done that much before and so I felt it was good for me to experience. I did agree with her when she said that it was helpful to work in groups, and also that hands on activities help students to learn. I think that it is important to include both of these aspects when teaching

S22

The reflective math activity was really helpful when it comes to figuring out how students learn mathematics. My group was a little shaky at first getting down a process but once we discovered that each functioning net had a core of two, three, or four squares it was smooth sailing as we used trial and error to discover the different nets that would work. This was a really enjoyable way to learn math particularly because it emphasized something that I have always tried to embrace; In math there is often more than one way to accomplish the same task and come to the same result. The strategy that really helped me understand was when we discovered the pattern that the nets take. There was not really any procedure or strategy that I felt detracted from my understanding.

The most significant thing this has taught me about teaching mathematics is try and give kids the ability to learn things through hands on, guided discovery. It is also important for kids to understand that there is more often more than one right way to solve a math problem. If kids are too hung up on the procedure that the teacher taught them to use when approaching a certain type of problem, it might detract from them actually having a clear understanding of why the math works the way it does. I feel that when kids have the opportunity to discover stuff for themselves through guided discovery, it means a lot more to them and they remember the mathematics better than if a teacher had taught them the entire thing in a lecture. I am not necessarily saying that teachers should not lecture for math, as there are plenty of skills that may need addressed through that manner. However, as much as the teacher can, they should try and provide opportunities to use hands on activities to further their understanding of a concept and discover new things.

RESPONSE

I really agree with [S16]'s idea that math should be more hands on. Like you, I too hope that math will have the opportunity to be more hands on future as a result of technology and 3D objects. However, I do hope that it does not become so built around technology such as computer programs that kids never get the opportunity to use tangible manipulatives which I think help kids explore because they can see the concepts they are learning come together as actual physical ideas. Call me old fashioned, but while I do believe there is great potential for the use of technology in the classroom, I would rather my kids counting the edges on a cube they can hold in their hands than a picture of a cube on a computer program.

S24

In exploring the reflective math activity our group tried to categorize the different net patterns into similar types. We found that using the number of sides aligned in the center as a “core” allowed them be to group them more easily. Subcategories were discovered within the “cores” as “T” and “L” shapes. After the patterns began to repeat themselves we realized that we had exhausted the cube net solutions. While trying to come up with proof that there were only eleven cube nets I became distracted in an attempt to apply this to a combination problem. I became frustrated with my inability to see the problem in another way in order to prove my solution although I was confident in the answer of eleven. I was satisfied after finding all the cute nets, but unsure of how I had proven the solution.

Using the manipulatives and working through the problem with their hands might be very helpful for some students and definitely aided in my ability to ‘discover’ the nets. Working in groups also allows for students to hear others perspectives on the problem and can help them work through difficulties together. Having the teacher there for ‘expert’ advice as well as clarification that they are on a correct track towards a solution would also be a positive, but as the process of learning progresses students could be given more responsibility. Students should be encouraged to ask questions of themselves and their work at each stage of the problem – Why did that just work? Can this be applied to other similar problems? – which could help students in understanding connections between math concepts. As a teacher, approaching the task at hand with enthusiasm and allowing the students to work with their hands instead of only their pencils might motivate them to participate with more excitement.

RESPONSE

I agree with your [S22] suggestion that teachers should ‘guide’ students through their discoveries. Also, importantly, realizing that there is more than one approach to problems that result in correct answers is something that we forget sometimes. As teachers it is tremendously important that we recognize and encourage creative (yet correct) means to the same end, especially in math.

RESPONSE

I agree that math should be a ‘hands on’ subject when possible. The technology that we’ve seen so far, just in geogebra alone, has so much potential for kid’s discovery learning in math. I hope we as teachers can promote that through intrinsically motivating math activities that kids can manipulate with their hands and technology.

RESPONSE

I agree that students can gain deeper understandings and perhaps differing perspectives through group work. Allowing students to explain their perspectives to others can also solidify their understanding or help them indentify gaps in their logic that group member may be able to help fill. I too was stumped at the beginning, but through interactions with my group was able to work through the problem successfully.

S25

In the cube-nets exercise, I was asked to actually think about the problem and why it worked out the way it did, rather than merely answering to get the points. It was difficult for me to grasp the task at first because I am not used to having free reign to explore why a problem works. Working with partners helped me immensely because we were able to figure it out all together and help each other to understand. The lack of direction for the assignment detracted from my learning because I thrive in stability and it took me a while to fully understand how nets worked, and how many I should be looking for. At first, the activity made me feel frustrated and lost. However, after I began to understand exactly what I was looking for I felt confident and began to become interested in what I was figuring out.

This experience has taught me that math should be taught through hands-on exercises rather than purely copying notes, memorizing rules and spitting out generic answers. With exercises that require thought and problem solving skills, comes true learning and understanding. I was able to better remember what I learned because I was forced to find the "rule" by myself through experimentation. To learn mathematics more deeply, students need to be able experiment with math and see hands-on examples of why certain rules work. When I am able to figure something out by myself, I am more likely to remember and use what I have learned. Students will also be more motivated to learn mathematics if they can see real world applications of what they are learning. I always feel more compelled to learn when I feel like what I am learning will be useful later in life. In my opinion, to make students want to learn, a teacher must be able to make the exercise interesting and useful.

RESPONSE

I agree with [S18] about how trying to work the problem out without any guidance was difficult. Without any direction to go in we both got frustrated until we found out how to use the core rule. Working in groups was also very helpful because, like [S18] mentioned, we were able to help each other to think outside the box by suggesting ideas that others may not have thought of. This whole project demonstrated the importance of group work and hands on assignments to better teach our students.

ACTIVITY 2 REFLECTIONS FALL 2009

F2

I had a little more trouble with this activity compared to the net cubes. I felt it was a little more difficult to prove it. I also thought coming to a conclusion was more ambiguous and less straight forward. At first I thought that the inscribed angle and the central angle had something to do with the shape being concave, flat, or neither. Depending on which one it was I determined that the inscribed angle was acute, right, or obtuse. However, when we got together in our groups, Matt told us to look for something different. My group ended up measuring the angles. Turns out that the central angle is twice as much as the inscribed angle. On my own I had been looking at the wrong angle for the central. I had been looking at the interior of the triangle/quadrilateral instead of I believe B, D, C...the part between the central and the circle. Then class was out of time. However, on my own I discovered something really awesome! The point D (central angle) is the center of the circle. BD and CD are the radii of the circle. When they are completely flat they are the diameter and D = 180 and A = 90. When B and C are closer to A, A and D are greater than 90 and 180 degrees respectively. When B and C are further away from A and further from creating a right angle they are nearly all the way across the circle and the measure of A and D are smaller than 90 and 180 degrees.

Response

I was also pretty lost and frustrated. I almost feel like there could have either been more than one correct answer, or that there was more than one way to come up with the same solution. I also see your frustration with how fast your group went and that you needed to go at a slightly slower pace to come to a complete understanding of the equations etc. I have that same problem sometiimes. I think as a teacher it is very crucial to have a good mix of independent work time, group work time, and then participation by the entire class. I'm glad you finally understood it so easily once you sat down with your tutor :)

F5

For this activity, I had a hunch that the relationship between the two angles in question was going to be that one was twice as big as the other. I thought this from the very beginning just by looking at the pictures on the handout. All I needed to do was prove it.

I started out with an equilateral triangle drawn inside a circle. This would represent an acute angle (60 degrees). To get started, I listed all the facts that I knew about equilateral triangles, isosceles triangles, and circles. This really helped me realize that I could label different angles and then put them equal to each other or equal to 180 degrees. I ended up with these equations: $2x+2y+2z=180$ and $180-2x=a$ where x, y, and z are all angles made by the radii of the circle and the outer triangle (x+z being the outer angle in question), and a is the inner angle in question. Since both the equations equal 180, I put them equal to each other. The 2x's cancle out leaving $a=2(y+z)$ or $1/2a = y+z$.

This proved that the acute angle drawn on the circumference of the circle was half the measure of the inner angle. My hunch was right!

Now that it worked for an acute angle, I needed to test my hunch on an obtuse angle. I did this in a similar way, using facts about circles and radii to make isosceles triangles. Then I used the facts I knew about isosceles triangles and their base angles to prove that, once again, the outer angle was half the size of the inner angle in question.

Finally, the easiest one to prove was the right angle. I already knew that my hunch should work so I just plugged in the angle measure of 90 degrees. $2(90)=180$. This is true so my hunch worked for all three triangle cases which implies that it will work for any triangle drawn.

When I first started this problem I was pretty sure I knew the relationship between the two angles. The trouble was, I needed to prove it in a mathematical way. I started with the acute angle but in hindsight, it would have been much easier to start with the right angle. The acute angle proof was the hardest one for me, but once I realized that I had isosceles triangles and therefore congruent base angles, I was able to write out several equations that linked angles together.

Once I got going, I got really interested in the problem. After the acute proof was out of the way, the other two came pretty easily. When I finished all three of them I felt satisfied that my relationship would work for any triangle drawn. This made me feel accomplished that I was able to prove this in a way that made sense. I think it would be a pretty tough problem for many grade school students, but with help from a teacher, it could be really beneficial for them to understand how and why it works.

Response

I totally agree that this problem was a lot harder than the nets project. Before when we worked on the cube nets, we all mainly used a system of organized guesses and checks. In this problem we actually had to prove the 1:2 ratio algebraically. I also thought that the time spent in class was very helpful. My group and I drew a lot of circles and triangles just like yours did. I also found the last 5 minutes of class very helpful when [the instructor] went over the equilateral example on the board. This got me going in the right direction. He used the fact that isosceles triangles have congruent base angles to his advantage when setting up a system of equations. This really helped me get started and later, I felt more confident to try a different inscribed triangle.

I also totally agree with the controversy about making a student work hard to come up with the answer to problems without leaving them high and dry and confused. I think the way we did it in class was nice because we were given group time and then met as a whole class to discuss the problem. However, it sounds like we could have maybe used a little more "whole class" time to get everyone on the same page.

F6

Upon first look at the problem in class, my first thoughts were that I had not seen a problem like it before. It struck me as more challenging than our first reflection, the nets of a cube, and I felt that more mathematical background would be necessary to solve it. Before I began the problem, I had to read it through a few times to give me a notion of how to begin and where to go from there. Even after that, if it weren't for discussing it in the classroom I don't think I would have been able to solve the problem. In class, the work with compasses and the reiteration of the ideas of inscribed circles helped tremendously when looking at this problem on my own. With that knowledge, it seemed to me that the angles in circle B would be double the angle in circle A, and likewise for circle C compared to circle B. How I would prove this was out of my range of knowledge without the boost I got from class sessions.

As with any math problem requiring multiple steps, I found myself off path often, frustrated and returning to the beginning to start again during my first attempts at solving the problem. This tends to make each attempt more sloppy as I speed through the early steps and unavoidably run into the same issues. Again, this problem proved to be beyond me when assigning a proper proof to my conjecture other than it just "seemed right" that the angles would be in a ratio of 1:2. It took a great deal of patience for me to solve the problem, which strikes me as silly given that it seems most students would handle this problem and one like it with ease. From this exercise, as with the first, I found myself enjoying the process when I put my work into the perspective of a student discovering these mathematical ideas for the first time, but when doing the work myself it was quite a struggle. Given what I know from class since approaching this problem, it seems a valuable tool to present a challenge to students at the beginning of each unit that encourages them to think outside the classroom and to find their own methods of problem solving, despite not being able to give this problem adequate proof myself.

Response

I like the idea of a "time limit" on problems such as these, mainly because when it comes to math I find myself being a student that falls behind regularly- though not for a lack of interest. I am genuinely interested in finding the answer to the problem and can relate to the frustration that you found when the answer didn't come easily in your first look at the problem. For me, the process that is used to find these answers usually needs multiple reiteration and reinforcement before taking hold. Giving the student a challenging problem to face over time is a novel idea for me, but I also like the idea of giving the means to find the answer sooner as well so that students like myself are gaining a proper method to solve so that their frustration doesn't lead them away from the task. I too found myself tossing the problem aside in favor of other work for longer than I would have liked, mainly because of a lack of base knowledge and a clear line of steps to find a solution to this problem, and for me it might have helped to have more direction earlier.

F7

When I first received this problem it took me by surprise. I looked at it for a while and just didn't know where to start. This was much more confusing and challenging than the cube nets problem. I started out by trying to remember anything I knew about inscribed angles and looked for any type of relationship between the angles. By simply looking at the angles I had a slight idea that the inner angle might be half the measure of the outer angle. So my group and I measured the angles using a protractor. This did in fact conclude that the inscribed angle is half the outer angle. But I still didn't seem quite convinced. I had proof but I felt like I needed a more solid answer. So I began to make isosceles triangle within the circle. This took some critical thinking and remembering the properties of isosceles triangles to find some more proof. Along with the information I already found I made equations from the triangles I did eventually find the final evidence that I was hoping for. This was very satisfying! Once I had finished the problem I was relieved that I was over. I would have liked to go over the problem a bit more during class to make sure that the reasoning I found matched everyone else's. But other than finding the solution this taught me a lot about what to look for and be aware of as a teacher. I can relate to those students that may be struggling with a problem or an entire concept. They may just need some extra help and a simple push in the right direction. This also made me aware that students all have strengths and weaknesses. Some students may pick up a concept with no problems at all and others may be falling behind in some area. As a teacher it is my responsibility to help this student and do whatever it takes for them to understand and in the end the student will feel a sense of accomplishment.

Response

I can relate to your process of solving this problem. I felt like I would never reach a conclusion and I began to feel discouraged. What I really learned from reading your response was that as a teacher I have to be aware and sensitive to the fact that some of my students will feel the same way that I did at times. It is my job to be a resource to them and to be able to explain it in more ways than one. When I thought I wasn't going to figure out the solution I almost wanted to give up. But I knew that eventually I would get the answer. The most important thing to learn is that it is completely ok to ask questions and get extra help if you don't understand whats going on. That is why as a teacher I think going over the solutions in class and having time set aside for questions is very important!

F8

Unlike other classmates, I felt that this assignment was easier and more enjoyable. I can only make a guess why. I believe that this was easier because we could use equations to prove why the inscribed angle was half the central angle. I really had fun doing this assignment, and that is because I went into it with a different mindset than I went into the cube net problem with. I thought, "Hey, this can't be that bad. If you let yourself get frustrated, you aren't really helping yourself. If you keep your outlook positive, you will get work faster and with less difficulty." It worked! I sat down and was happy to draw circles on both sides of a piece of paper. I started by drawing a few inscribed angles and measuring the central angles. There seemed to be the relationship that the inscribed angle was always around half the measure of the central angle. I had some measuring errors that stopped me along the way. I then decided to go about it a different way. No sense in stopping or giving up. So I drew more circles, but this time I drew them very accurately and I used different colors to draw different inscribed angles within the same circle. I found that with one central angle(for instance 60), any inscribed angle I made always measured 30 degrees. I then had my conjecture.

Next, I set out to prove it. I set out with my table mates in class to find a way to prove it. We started with an equilateral triangle inscribed in the circle. This was a great idea because it allowed us to use our knowledge of equilateral and isosceles triangles to find angles and side measures. We proved for the equilateral triangle that the inscribed angle was half the central angle by using that knowledge. Next, we inserted variables in for the angles and side lengths that we knew and explained the relationship between them. Turns out, this proved our conjecture. We found our proof during class, and it felt great. I felt I had closure, and was very confident that my proof actually did prove it for all inscribed angles.

This assignment was enjoyable and taught me the value of working with a group, but also the value of sitting down and figuring things out for yourself. It taught me that attitude really can be everything. If you think you will be frustrated, by golly, you will be! And if you think you can do it, you probably will! In the future, I will use this because I will know I must find a way to make my students excited about math, encourage them to have an open mind, and to seek help when necessary.

Response

I agree that it is important for teachers to recognize the different abilities of different students. Some may need a hint to get going, whereas some may cruise through the problems. It will be invaluable that we have realized that! I agree that the group was very helpful for the first assignment, but I think that may have been due to the availability of manipulatives to help us prove it as well. For me, I struggled with not being able to have a visual manipulative to use this time, but nonetheless, I figured it out. It is always nice to work with other people because it is amazing how different every student sees each task and the different viewpoints that you learn about.

F9

I felt very confident going into this activity. I was not positive about the relationship between the central angle and the inscribed angle with the same arc. I knew that I had seen this relationship before in high school but could not remember it. It really helped to have that past experience though as it boosted my confidence.

I began by looking at the pictures on the handout [the instructor] passed out and thought that the inscribed angle looked like about half the measure of the central angle but that was not proof. So, I began drawing circles and some inscribed and central angles. This got me to where I already was and I began to feel a little frustrated. So, what I did was pick an inscribed angle measure and drew that instead of a random one. I began with a ninety degree angle and then drew a forty five and a one hundred and ten degree angle. Each of these was in its own respective circle. In this case I knew the measure of the inscribed angle. All that I had to do was draw in the central angle with the same arc included and measure it. I did this for all three diagrams and found that the central angle was two times the measure of the inscribed angle. This made sense visually and it made sense because that is what I measured. Finally, I noticed that I could split the area of the figure formed by the two angles into triangles. These triangles turned out to be isosceles which allowed me to prove mathematically that the central angle was twice the inscribed angle.

I felt really good about my conclusion and picked up a few ideas along the way. I now see that learning the prerequisite steps is very important. If I had not known what an isosceles triangle was and the relationship between its sides and angles, I would not have gotten anywhere in the end. As a teacher, I hope to verify with my students that they know certain mathematical material before moving on to new material. Once a student has that set of basic knowledge, I feel that they will be able to apply it to new and different situations. Overall, I think this activity helped me to clarify my view on this matter.

Response

I definately felt the same way at the beginning too [F20]. I felt like it would be fairly simple seeing as how it dealt with what we were learning at the time. I too found out that it was much more difficult than the cube net problem. I struggled more with this one just like you. I wasn't able to be there for the Friday when we worked with our table mates but it sounds like it was very helpful. I know that it was for the first activity so I agree with you on that point. I also know how you feel about not being very confident in math. I feel confident in math but not in some other areas. I think you hit the nail on the head when you said that we should try very hard as teachers to learn all subjects to the best of our ability.

F12

This activity was a lot more difficult for me than the last activity. It started out kind of easy. I found the relationship between the inscribed and central angle. The inscribed angle was always half of what the central angle was. I had no idea where to go from here. I looked up inscribed angles and found out that there is a theorem about the angle being half of the central but it still didn't let me know why this was. I'm still not positive the answer I came to was right or if it was close at all but it made sense to me. I was a lot more frustrated and after a while wanted to give up. I thought to myself that I was probably just making the problem harder than it really was. I think it's interesting to see the change in emotions from the first activity. I thought I was frustrated then but this really made me think. Another thing that I noticed was that I didn't work with my group as much in this activity. It was hard to take in all the different ideas without getting confusing or most likely further away from a correct answer. What I came to conclude in the end was that because your using a circle the radius might have an effect where as the inscribed angle only consists of one radius the central angle will have two. I'm very interested to read the other reflections and see what other people came up with because I think there will be more of a variety of work ethic then the first activity and possibly a different set of emotions other than frustration alone. I'm also hoping that some will be the same in the matter of finding the answer and feeling relieved and satisfied.

Response

I read [F7]'s reflection and I was glad to see that we were on the same page. I agree with her on her comparisons to the last activity. This activity took more brain power and time then the cube nets. I also went through a variation of the same steps, however, I feel that she worked with her group a little more than me. I was thinking so hard about the answer that I wasn't really paying attention to what everyone else was saying. She also brought up the point that when you're teaching a big group of kids its very important to get everyone on the same page. It's hard after someone falls behind to get them back into the groove with the rest of the kids. So try not to leave anyone behind.

F13

When I first received the problem that very first day my group set out on a mission. They had every clue what was going on me however, not so much. When they started making equations is when I finally chimed in asking what I was supposed to be looking for. They explained that we needed to find the measure of the angle and the circle that was left over. I read and re-read the problem but still nothing was coming to mind. This was super frustrating! I watched as they made equations and solved for X and Y and this and that but I still wanted to be able to figure this out on my own. When I left he class I had many equations, number and letters written down but not anything that would lead me to an answer. At this point I knew I would have to at some point figure out how to find these measurements but I was scared I was not going to be able to. The following week when I met with my tutor I asked her how I was supposed to go about solving this. As I predetermined everything I had written down was not going to help in finding my solution. She had not a clue what I had written and nothing made legitimate sense. She started working me through the task at hand. I finally got it! It was really simple but all the equations I had written and the pace at which my group was working kept me sorta behind. It was too fast and overwhelming for me to realize just how simple this problem really was. As we worked through the three examples I began to see the pattern, which was exactly what I was supposed to be seeing. Working with my tutor I was able to write my reflection due in class. It was so simple to find the pattern and it finally made total sense. Prior to working with my tutor this problem gave me a hard time but working through it with her it made logical sense and everything just came into place.

Response

I also felt this activity was a bit harder to prove then the last one. The first one you were able to physically see how the nets formed a cube. This one you have to use the measurements and reasoning to prove the measurements of the angles and the measurement of the rest of the circle. It seems to me that you found the pattern quickly whereas it took me quite sometime to see the simplest thing in front of me. Looking at the picture of just one circle you could see the measurement of the angle and then take that number out of 360. I don’t quite no how to explain I but I feel like it’s very similar to what we are focusing on this week with the geo-boards. You can take out parts to make a whole and plug them in where needed.

F14

When I first saw this problem I was clueless. I tried to remember learning about inscribed angles, and if there was a relationship between the angles, but couldn't come up with an answer. Then, at our table, we grabbed a protractor, and started measuring the angles, to see if we could maybe get some similar angle measures. After doing this, we came to the conclusion that it seemed as though the inner angle was half of the outer angle, for all three of the circles given. This seemed like a good start, it was a reasonable conclusion, and we had used a mathematical tool to prove it. But the numbers we came up with weren't exact, so we needed something else to help prove our conclusion.

Our next attempt was to create triangles out of the inscribed angles, and use the information we knew, along with variables, to prove that the inner angle was half of the outer angle. After some tinkering, and some old algebra skills, we were able to set up a couple of equations from the triangles we had created inside the circles. Eventually, we were able to write an equation that stated exactly, that the inside angle equaled half of the outer angle.

For this problem, we were lucky to be able to guess the correct way to answer it, but usually that is not the case. For this problem, we used what we have already learned in this class, measuring angles and rues of triangles, to solve this new foreign problem. Once we had finished, we were very excited that we had solved the problem during class time, it was very successful overall.

Response

[F17], I totally understand your frustration. I went through the same process. Usually when looking at a problem like this one, you can come up with ways to maybe solve the problem, but how are you to know that what you are doing is correct without someone (like a teacher) telling you the right or wrong way to solve the problem.

Now looking back at things we learned as kids, we can understand how it was really difficult for some students to solve the problems without the teacher's assistance. I think we rely on the positive encouragement of our instructors to get us to an answer.

Hopefully, we can learn from this process and get to a place where we can have confidence in our answers without the agreement of others.

F15

I did not enjoy this reflective math assignment near as much as I enjoyed the 1st one. I think part of the problem was that one the 1st reflective assignment I had a clear idea of what the purpose of the activity was but on this 2nd assignment I was not really sure of what I was looking at or looking for. I took the assignment home and tried to duplicate it with a compass and a ruler so that I had larger pictures of the triangle to work with. I also used my sketch pad program to rebuild these triangle and their inscribed angle. This gave me the ability to measure angle ABC and angle ADC and get the most accurate measure of what the degrees of each of these angle were. It was when we worked on it in class that I realized that angle ADC was always double what angle ABC was. This was thanks to one of my table mates. Thanks [F22] ! (I think that is her name) I confess that I read a couple of posts before I did this one to see what everyone else was thinking. Thanks to [F17], I realized just today that there is a 1:2 ratio involed with all of these triangle and their inscribed angles. I just hadn't taken note of that before. Thanks [F17]! I am not sure what this activity had to do with chapter eleven. I wish there were a way to tie these activities together with homework activities and tests. I guess I am just frustrated. I feel like I am very slow at a lot of the math that we are doing this semester. I will get it eventually and I think that struggling in this class will make me a better teacher. I will be more empathetic with students who are struggling. If nothing else this class has been a humbling experience and I am learning to the best of my ability.

Response

I too thought this [activity] was a lot more difficult that the last one. If it hadn't been for our group discussions with our table mates I might not have come up with an answer at all. I was able to use my sketch pad program at home and duplicate the triangle off of the assignment. This gave me the opportunity to let the computer measure the angle for me. I discovered that the inscribe angle was double what the other angle was on a consistent basis. I realized after reading [F17]'s post that this represented a 1:2 ratio. These are the only two thing I am sure about with this assignment. I was very frustrating. I agree with you that although students need to be challenged and they do remember things better when they discover them for themselves it does not hurt for the teacher to provide guidance and maybe even an occasional prompt. The prompt would most likely be appreciated by the students who are struggling to find the answers. You are also correct that everyone has strengths and weaknesses and it is important for teachers to be aware of the weaknesses so that students do not fall through the cracks and lose large parts of their education.

F16

When I first started to solve this problem I thought it was going to be easier than the first activity. Using protractor my group and I found that the relationship between the inscribed angle's and the central angle, was that the inscribed angle was half of the central angle. I thought it was pretty fun at first, trying to come up with a proof to solve. I started by extending different line segments in hopes that it would produce some angle or triangle and I could then piece by piece find congruent angle's or sides. I mainly looked for SSS or AAS congruency. This was more difficult than I thought. I quickly got frustrated and stopped working on it after a while. When I would try again, I would run into the same problems never being able to solve it. What this exercise taught me as a future teacher is that these problems are great. They make you think many different properties of math in one solutions and if completed correctly the student will feel justly rewarded. However, I think problems like use should also have a "time limit", meaning that after the students have worked on it for a while the answer will be demonstrated to them. That way students that fell behind can see the outcome and learn the process that day.

Response

I really liked your honesty in your post. I too get frustrated with math, it's really hard for me and it can be frustrating when you can't see the answer or a solution. The more you work at math the better it will get.

Response

You had a very cool way of solving this problem. I didn't even think about inscribing a circle aroung the triangles. I too share the frustrating situation of not knowing if you have solved the problem correctly or not. I think these problem are fun and a great way to do math but I also think a shorter time limit should imply, start the problem one day then have the solution the next, or something like that.

F17

I felt that this inscribed angle problem was more difficult than our cube, net problem. If it hadn’t been for the half hour we spent in class discussing our thought, I don’t think I would have figured anything out. During that session I was able to draw a lot of circles with shapes inside of them (mostly triangles). The interesting thing was that the ratio between the angle at the top of the circle and the angle at the center of the circle was always 1:2. As a group we started looking at how this could be. My conclusion is that because one angle is in the center of the circle and the other is at a point on the circle the distance between the two is also 1:2, being that the mid-point of the circle is the radius. I felt that this ratio would carry to the angles. I also learned in class that the first triangles bottom and top angles were a 1:2 ratio, which reinforced my ideas.

What is frustrating is that I don’t know for sure that my ideas are correct. I feel like I should have learned the information in class that would allow me to make the connection to the inscribed angles, but I didn’t see it. Instead a used a common ratio idea that has no proof behind it other than it makes sense to me. I realize that it is important as a student to be given problems to struggle with; I feel that too often students are left in limbo on whether they have figured things out. When I am teaching math I want my students to come up with answers that are their own. At the same time, I want my students to have the time to discuss with their classmates and me what their thoughts are and how they could use them. This added time is not often met and I feel that it is vital if a student is going to feel success rather than confusion and frustration. I hope that we discuss this problem in the near future so that I have a better understanding of if my ideas were justified and, if not, how I could look at the problem differently.

Response

[F22] thank you for that response. I agree that this problem was more difficult than the last. I thought your idea of breaking the circles into triangles was a great one. I didn't even think to do that. I used a common ratio idea that I'm not sure is all that valid. I felt the proof was hard as well and I would have liked to have more time working on this project in class because of it. I feel it is important for students to discover proofs on there own, at the same time i think it is important not set the students up for failure. I too feel that guidance is a key component when teaching math. Allowing the student to get a little frustrated is okay, and good at times. I think that in order to give this guidance a teacher much set the time as side for clear understanding. Many times i feel like students are rushed and have to figure things out on their own because there isn't the time to discuss it. As a teacher I want that time to be available to the students. If the time is there I think all students will be able to figure out the problems in there own unique ways.

F18

When beginning this activity it appeared that the inscribed angle was half of the central angle. To check this I made several different inscribed angles (90, 80, 50, 100, 70) and measured the relation to their central angle. It was true that the central angle was twice the measure. That part was not hard and for a minute I thought that maybe this activity was much easier than the first. Then, when I sat and thought about how to proove this, I became incredibly frustrated. I couldn't really come up with any ideas as to why this would proove to be true, although it did every time. I wondered if it could be in relation to the radius and diameter of the circle. The inscribed angle expands the length of the diameter, and the central angle expands the length of the radius.

Frustration aside, I thought this was a very useful activity. It taught us a lot about something most of us never gave much thought to.

Response

I read what [F9] and [F12] had to say and I agree with them both to a great degree. [F12] actually said something similar to what I did about guessing at the relation between radius and diameter. I, too, was more in favor of the first activity than the second but I didn't seek help from my table-mates either. I responded to [F9]'s thread last time also because I think he has a lot of great things to say. He's brings up a good point about there being a lot of prerequirsites for students before they can begin learning new information. Kids need to have the basic knowledge that teachers must supply them with in order for them to progress in their learning.

F19

When I first glanced at the assignment I assumed it was going to be easy since we were learning about it but I actually really really struggled with this activity. I think the inscribed angle problem was a lot harder then the cube net problem. I really need to thank my table mates for helping me because with out them, I don't think I would have been able to do any of it on my own. Unfortunately, our table did not finish the whole thing together so I had to finish it on my own. This worries me because I'm not confident in math and I don't think my reflection activity is correct and I'm not sure if I wrote down any proof. I hope i have a better understanding of the next activity.

With this activity, I was actually really frustrated I know that we learned everything in class and yet I still struggled. Math has never been my strong point. My table mates tried to explain everything but I still did not really understand it. Because I'm not very good at math, I realize that as a teacher I should maybe focus a little more strongly on certain subjects that I realize my students are or will have a problem with. I feel that I need to have a bit more patience for upcoming activities and maybe go into them with more of an open mind and not to rely on my table so much.

Response

I can relate to [f17] with this refelction. I found that acctivity number one seemed to be easier and if it wasnt for the time we spent in class, I dont think i would of got it. I can also relate to being frustrated with not knowing if my ideas were correct.

One part in you response that I found interesting was " I realize that it is important as a student to be given problems to struggle with; I feel that too often students are left in limbo on whether they have figured things out. When I am teaching math I want my students to come up with answers that are their own. At the same time, I want my students to have the time to discuss with their classmates and me what their thoughts are and how they could use them. This added time is not often met and I feel that it is vital if a student is going to feel success rather than confusion and frustration." I have never looked at this way and I believe you make an excellent point. I think this is a great way to teach math but also help a student if they are struggling!

F20

I easily found that the inscribed angle measure was half the measure of the central angle. I drew out circles with different diameters and found the inscribed angle to be very close to half of the central angle measure so I drew new circles with more precise drawings and measuring. This showed me the conjecture that the inscribed angle on a circle will measure half that of its central angle. So, how to proof this? I started with making a triangle within the circle. I made other circles to try to different lengths and see any other patterns. I measured all different angle measures created within the circle and triangle. I made different polygons inside the circle to explore.

I definitely felt frustrated working on this problem. After finding the connection with the triangle and being able to make shapes in the circle I didn't expect such a challenge. I felt stumped and wasn't sure how to approach it after making new models and looking for connections between conjectures I had proved in the past.

I realize I should have brainstormed more mathematical solutions. I didn't very well break it down into an equation, or try to, to get a solution. I could have asked for outside thoughts on how to more forward with proving it

F21

In the beginning of this activity I was very unsure of, if any, relationship existed between the central angle and the inscribed angle. For a long period I just sat and started at the circles that I had drawn and could not see anything that related the two angles. It may have been the fact that the circles I drew were not very exact, so my angles did not directly divide each other. But after measuring the angles that were the examples on the instruction sheet I started to realize that there was a relationship that existed. I experienced a lot of frustration while working on this project because I could not find anything to draw the two angles together. After I realized the relationship it seemed so obvious and I was surprised that I had not seen this earlier.

The relationship that existed between the central and inscribed angle is the inscribed angle was half the measure of the central angle. So if you found the measure of the central angle you could just divide it by two and that would reveal its inscribed angle. I found this relationship to be so interesting because I never knew that anything like this existed. Also I had not realized that if you have a central angle of 180 degrees then the inscribed angle would always be 90 degrees, that was something now that seems so obvious but before I would not have thought twice about it.

This reflective mathematics activity seemed to be so much harder that than the first activity for me. I think that this was because you could see the solution to the problem with objects to help you reach the solution. It took a lot more actual math and not just problem solving to come to the conclusion I reached. I enjoyed this activity though after I had realized the relationship. I had never seen anything like this before so it seemed so strange but neat.

Response

I understand your frustration I felt the same way, it seemed my group knew exactly what to do and I had no idea where to even start. I also read the instruction sheet a few times before I knew what to do and where to start looking. I also had many different equations to and things labeled dividing up the triangle into two different triangles and still I saw nothing also. I had the same frustrations. Also I saw that the after so much struggle the relationship was very simple to find and I felt so much relief. I am glad someone else felt the same frustration that I did because as I was sitting in class I felt like everyone was realizing something that I was not. Like you also when I realized it I saw how simple the relationship is and I felt much relief.

F22

I found this math problem to be a lot harder to discover than the last problem. I found it frustrating because when I first started looking at the inscribed angles and the central angles I could measure with my protractor that they had a relationship of the inscribed angle being twice as much as the central angle. So my conjecture was made really early the frustration came when I had to prove it using something other than just a protractor. I tried breaking it into triangles first which is what I ended up sticking with and I could see the relationship created then by the isosceles triangles that were created by cutting the angles in half with a line that separated them into two different triangles. The next problem and the problem I still had in the end was describing the relationship as a proof. In this case a group was not as helpful to me as it was in the last problem we had because it happened that my group members were having the same problem that I was having in putting the proof that the inscribed angle was twice as large as the central angle. We did talk it out be still found it hard to proof the relationship of this angle in mathematical terms by the time the period was up. So ultimately this was a very frustrating problem for me and I am unsatisfied that I couldn't quite come up with the answer that I wanted to come up with. As a future teacher this allows me to better understand that while there is value in letting kids learn things for themselves there are always going to be students that struggle and may need just a little guidance even if they are supposed to be discovering it themselves. I still don't believe that in cases where they are supposed to be discovering it by themselves I should ever tell them the answer and rob them of the discovery and satisfaction, but that they may need guidance. Also I learned that some students will struggle with one problem but not with another, like this problem was a struggle for me, but I am sure other students did fine, and the last problem I got fairly easily, where other students may have struggled. Everyone has strengths and weaknesses sometimes within one subject area and that is something that as teachers we should if nothing else just be really aware of and pay attention to.

Response

I agree that this was a little more frustrating than the last activity. I would also say that there is value in a little bit of frustration because when you do finally figure out the problem whether it be by your own methods or the guidance of someone else you feel much more accomplished and I know that when I figure problems out this way it definitely stays with me for longer. Also I like how you feel that your struggling will make you a better teacher in the end. I think that this is not only because it will allow you to be more empathic to your students who are struggling, but also because I think that it will allow you realize that your students will all have strong areas and weak areas and you will be better equiped to help them because you have been in their shoes. I think that it is frustrating that the reflective activities don't tie directly into what we are learning in class, and are only linked by the basic concepts, however I think that more the value and even the point of these activities is to force us to struggle and learn things that we don't have a direct answer to so that we some day as teachers will be able to say I have been there, I know how you feel, and this is what I think is best to help you figure this out. Not only in math but in every subject.

F23

I really did not like this activity. I did not understand the importance of this activity. I thought it was very confusing. It took me a long time to really understand what the activity was asking. I had to do research. I felt this was a lot harder then the last reflection. I became frustrated and then irritated then confused. It was a whole big ordeal. I think that rather then learning about inscribed angles, I learned more about not understanding and how the feeling of lost can be overwhelming. I believe that this can help me as a teacher because I can better recognize and understand what it is like to feel as if things will never make sense. I thought that this activity did help me to realize that there are people out there that can help explain things in a different way so that others can understand. Thanks! This makes me think about the diversity of learning and that somethings have to be learned in a variety of ways. Although I feel like this activity did not help me, it did because it help me to better understand my own way of learning and that it is very different then others.

Response

I believe that you had very good points. I also feel like I am very slow at this math. I understand what you mean when you say that you had to look at others to get a good understanding of what was being asked. I am also thankful to the people that helped me to understand this activity. I thought that this post was very well written and very effective in communication. I think we were on the same page! I agree I did not understand the point of this activity but I felt that it gave me a better understanding on the way I learn and very possible a better understanding on the way others learn. Thank you [F15] for the very well written post!

ACTIVITY 2 REFLECTIONS SPRING 2010

S1

For me this activity was really frustrating. In fact I really just didn't understand it until I got help with it. This activity taught me that you really do need to have an open mind when it comes to math. In high school I was taught rule after rule and I think that that has not helped my learning in college. I was helped step by step through this activity which did help. This activity was just frustrating for me. Once I finally understood it I was interested in how it worked out. But I'm not sure I would've figured it out on my own.

This activity taught me that math could be taught in many different ways. This activity was explained to me in probably three different ways for just one way of solution. It also showed me that math could be taught in a very "open" way. For example, people need to understand that there can be multiple solutions for a single problem. Also students might find it fun to use the guess and check method to assist them in their learning. However the guess and check method can also be frustrating. Math is a very complex, but very interesting subject. It's hard to decide if the multiple solutions problems are easier or harder than the single solution problems.

Response

I agree with [S6] when she says that it helps to work with groups. Not only does a group give you comfort because you know they have to solve the same problem, but also people can show you easier ways of solving a problem. You might be struggling solving a problem one way and then someone can explain it to you a totally different way and you might get it right off the bat.

S2

This [activity] I felt was a lot harder than the other one. It took me it felt like forever to come up with a proof for the inscribed angle. It taught me that when teaching things like proofs, it takes more than one or two examples to come up with proofs. Children really need to understand the concepts we are teaching them. I think it would even be smart to go back to concepts over and over so we know they are getting it. The thing that distracted me the most was my own understanding. I forgot what was taught in class and couldn't figure out how to do the proofs. It also looked difficult so I had to make it look simpler so that I wouldn't get overly stressed about it. I felt very frustrated and like I wanted to give up on the [activity]. I couldn't figure it out and after staring at it for over an hour and a half I knew I wouldn't get it. Geometry has always been a difficult subject for me and I have always had trouble with proofs. So it was like the two most daunting tasks for me all rolled into one.

This taught me that patience is key, when teaching children concepts that are difficult and foreign. Sometimes taking more than one day to teach it would be a good thing and having children participate in class to make sure that they know how to do the proofs and what properties to use where. If there was some way to make proofs fun and entertaining I think that the children wouldn't fear it as much and it wouldn't be as frustrating. Even having time during class for question where the child is doing the proofs but as a teacher you are there to lend a hand if and when needed.

Response

I agree that it was difficult. But i thought even in the group it was hard because i felt that everyone in the group were haveing the same problems and i didn't understand it til [the instructor] came a show us how to do the problem.

Response

I agree completely i felt lost when i was not being helped by the teacher. I felt like i got it when i was in my group and we were going through it together but then i got home and it went out of my head and i couldn't even remember how to start to write the proof.

S3

Throughout my education, I have been told that there are numerous ways of teaching the same concept. Some are hands on, some aren't, and then there are the ones that have more to do with guided discovery. All three are viable ways to go about teaching students math, and this experience showed me that on a more personal level. The reason I say this is because many times we are told exactly what to prove and then given the exact tools prior to the problem to solve it, but this reflective mathematics activity challenged us to use all our knowledge and problem solving skills in order to come up with a conjecture and proof. That helped me learn the material on a deeper level because I had to put more thought into what I was doing instead of just looking at an example in the book that is doing the exact same problem and then moving on and forgetting what I just did five minutes later. This activity did frustrate me, however. I was able to prove the first two examples, but when it came to the third one I was basically pulling out my hair. I stared at that diagram for hours and hours and was not able to figure it out. That did deter from my willingness to learn, but it might have just been me and the answer was obvious.

This reflective mathematics activity taught me that challenging the students with problems that guide them into discovering what is being taught can be a new and refreshing way of learning. Many times classrooms are just comprised of notes and lecture and that can become boring to the students being taught. With that, I do believe that self discovery would be a advantageous thing to have within your curriculum, but I do believe that younger students should have a little more guidance than we had on this activity. Once they have that extra boost and are able to discovery ideas and equations by themselves they will be more excited about what they are learning because it will be interesting and more personal to them. Many times math is so disliked because equations and rules are just being told to student and they mean nothing to them, but if the children discover and test out their discoveries then they will understand it more deeply.

Response

I agree with you. I too was able to get the first two proofs done in class, but then when it came time to do the third one I was completely lost. I drew lines every which way and even then I could not figure it out. In the end, I wrote up a proof that probably did not make much sense.

Overall, this activity was very stressful for me, but it did challenge me and that can be a positive thing. I was dedicated to solving the problem and eventually came close, or at least I hope I did. If I had a little more guidance I am sure I would have been able to figure it out.

S4

For the past several weeks, we as students have been becoming more and more familiar with the nature of and properties of triangles. We are confident with theorems of congruency and similarity, we can correctly identify triangles based on their angle measure, we can bisect, inscribe, etc. etc. However, this reflective math activity forced us to fully concentrate on the triangle for quite a while. It seemed easy enough for everyone, including myself, to agree on a conjecture. Being able to prove it in three different scenarios required some serious knowledge regarding the properties of triangles. The most helpful strategy for me in this activity was simply "talking it out" with the members of my group. This allowed us to hear what we were thinking out loud which led to some verbal editing of our proofs before we recorded them.

As difficult as this activity was, I feel like it helped to solidify our knowledge of this important geometric shape. Even though it was obvious for most of us that the center angle was twice the measure of the inscribed angle, it was not as clear as to how we were to cite "triangle facts" in order to prove the conjecture. I like the fact that this activity occurred somewhere in the middle of our examination of triangles because we had the knowledge necessary to prove our thoughts, yet we still had time to mull it over in class while we continued to study the shape even more. An activity such as this one leads a student to understand mathematics on a deeper level because the problem asks more of you than a homework problem might. It also helps the student prepare for test situations when we are asked to organize our thoughts into proofs. As a final thought, I think if there is a way to incorporate a real-world situation into a math activity it may provide more incentive or motivation for the student uninterested in math to become more involved and take ownership of the problem.

Response

I agree with your thoughts on guided discovery. It is interesting to me that you noted that in most math classes we are told what to prove and how to prove it. In this activity we were not told what to prove. This created a "journey" for the student in that we were not told our destination or how to get there, but instead had to come up with that on our own. I appreciate that we were not just regurgetating our teacher's thougths or put under the impression that there is only one correct way to do things. Maybe the real purpose of guided discovery is to show a student that they can take their own route and still wind up at the right destination, or answer.

S5

This Reflective Math Activity taught me about how central and inscribed angles of a circle are related. It was also very difficult for me to understand. I wasn't sure how to prove that the angles were related except for showing the math. I could not figure out how to show it as an equation. It was important to do the problems in the correct order so that the first one helped with the next two problems. It was very easy to get stuck and that was what happened to me. I tried to work on the problems at home, but I kept having the same problems that I had in class. Without someone to talk to, I kept reverting back to the problems I was having originally.

This activity was important because it showed how to use trial and error. Proofs like this one are important for students because they show relationships. Activities like this one allow the students to be hands on and learn for themselves. It is a good alternative to constantly lecturing and handing out homework. It helps students with the understanding of concepts. I think it is important for students to try constructing and proving problems on their own, but they do need some guidance. This [activity] needed a little more guidance to help us see where we were trying to go.

Response

I agree with [S12] on how frustrating this [activity] was. I ran out of patience and was not able to finish the third proof. I also agree with this being a great example of trial and error. It is important to challenge students, but it is also important to limit the amount of frustration that they feel. If a student gets too frustrated, they will cease to learn.

S6

This [activity] taught me that it really helps to work as a group on certain problems. My group was selected to be video-taped and I was very grateful that someone in my group took the initiative to lead the rest of the group. At first I did not fully understand the problem. Each of the circles and their respective angles looked very different from one another. It was not until a member of my group had the idea to simply measure the angles did I start to see the relations between them. The first circle, with the inscribed angle lying on the same line as the central angle, was the easiest for our group to prove. In fact, as a group we were able to prove the first two circles in class on the first day. The third circle, with the inscribed angle lying outside the central angle, was the most difficult to prove. The third proof made me feel very frustrated at points. It was not until I drew it up on my own did I start to see some relational angles and parts of a triangle that were meaningful to me when creating a proof.

The process of forming a proof for the third inscribed angle reminded me that in order for math to truly click with a student, it has to be meaningful to them. In a way, each student needs to be guided in a way of how to make math their own. In general, people take more pride in their work when they feel a sense of ownership to it. Students are inspired to become more motivated to solve a math problem when it relates to them. This problem, and the angles, reminded me of basketball and the relationship between the ball and the goal/basket. It would fun to try and set up a similar problem on the playground or in the gym using the arcs on the court to represent the circle.

Response

I agree with [S24] that the second and third examples were indeed more challenging. It helped me that each of the examples seemed to become more complex. I too went to the Math Help Center in the library to get some extra help with formulating a proof for the third example. It was really helpful to have another person look the problem over. This helped me see it from a fresh angle. Going to the math lab was the perfect balance, for me, between guided learning and independent discovery.

S7

This activity was extremely hard and time consuming. During class I was not exactly sure what we were even assigned to do until the teacher had come over and helped us out a little bit. I was able to get the first two proofs done in class, and was told that the third was the hardest to prove. I soon found out that it was definitely harder than the others. I probably spent a total of 2-3 hours looking that that problem over the course of the couple weeks. This problem was very stressful for me and I came to the point where I was out of ideas. I tried to draw lines in every direction you could think of to make triangles that might make the problem easier. This guess and check method didn't seem to be getting me anywhere until we finally possibly found the correct position of a constructed line that helped us. I fell well accomplished when I finally had at least the proof of how I thought I could have proved it. I am not sure if it was a correct way but at least I had an answer to write down.

I found that it is very difficult to solve such a hard problem without any guidance. The first activity we did I saw that we could accomplish things without much or any guidance, but this activity was a different story. I learned that sometimes it is ok for the students to go out on a limb and try to figure things out on their own, but sometimes it would be better for a more structured setting where the students can learn from the activity rather than stress about not getting the right answer. Maybe the best idea would be to have them try it themselves for awhile but then give ideas or hints throughout so it's not as stressful on the students.

Response

I like how [S21] used the clues given to her for the previous proof to help prove the next proof that was harder. I also became very frustrated throughout the process of trying to solve the last proof. It took me forever to finally find a way to even start proving it. I also agree that you need to give the students the problem first without any help or guidance and then after they have worked on it for awhile then you can give them slight clues to help them figure out what they need to do to solve the problem. I think [S21] is right by saying I think the students will remember more of the problem and how they solved it by doing it all by themselves. This could be because they actually had to think about it by themselves and even the fact that at the beginning it made them frustrated that they couldn't solve it and later felt the feeling of accomplishing a very difficult task.

S8

The “Inscribed Angle” [activity] was really all we’ve learned about angles, triangles and rules up to this point wrapped into a few proofs. Although I didn’t fully finish the [activity] I did understand the first couple and it seemed the best way around these problems was by constructing different triangles to be able to provide evidence of the angles that we were trying to prove. It also helped to collaborate with members at our table for ideas and out of the box ways to create triangles with known angles or sides. It was really distracting to try and use a protractor or other tools to figure out angles because it was mostly algebraic and the actual numbers didn’t matter too much, but the concepts did. It was also kind of distracting working with 4-5 people because everyone had a different idea on how to go about it so it was hard to fully exploit the ideas you had on your own. Personally the activity was a bit frustrating because for me it’s hard to think around corners and construct my own objects on which to base a proof.

I feel that the activity done with one partner for 15 minutes, then switch and do with another student for 15 minutes would be a great strategy to help the student understand what is going on. That way you and your partner can really get into one way of going about it and see if it works, then move to the next and dive into another idea and share yours to help spread ideas and understanding without the riffraff of a large group. But being able to see how these simple geometric shapes can dictate algebra and really be able to visualize how the math works does help conceptualize mathematics and eventually lead to greater understanding.

Response

I shared the same experience with this [activity] that you did I think. The first two eventually revealed themselves to our group and made sense but the third required allot more ingenuity than I anticipated. It was a bit frustrating and it did take a long time. if feel in both of our cases (speaking as students) it would have been very beneficial to have some time to collaborate with different people in the classroom on ideas on how to tackle the problem and the same goes for a high school/middle school class room in order to let the kids share and help each other understand the concepts.

S9

This math activity was a challenging one. It took a lot of thought and explanation in order to complete it entirely. It showed me that in math, it is okay to make mistakes and just try, and try again. If a problem is not solved one way, it is perfectly fine to try a different route in which to solve it. For me, I found that the guess and check method is my favorite in math and one that will usually allow a person to come up with the answer, even if takes a few tries and much time. After looking at the three different samples and creating some lines and measuring the angles, my group was able to create a conjecture that showed why the inscribed angles were similar to the central angles. I created a few different lines on each problem to create triangles that would work and hopefully help to prove why our conjecture would work. My group and I came up with the first two proofs easily, but it was the third one that created a problem. We did not have time in class to finish so I had to figure it out on my own. I drew many lines to create different triangles but it was hard to prove why the conjecture worked without just assuming a certain line created an equilateral triangle, or something similar to that idea. I was frustrated since I could not figure it out. So that really showed me that sometimes in math, time and patience is needed in order to solve a problem.

I thought this activity was a perfect example to show that in math, trial and error is a common practice and one that will allow students to think in a deeper level to come up with the solution. I think it is great to challenge students, otherwise there is no way they will ever learn and think of knew ways in how to solve a problem. Even though it does get frustrating at times, it is a great way for students to use their thought processes and learn that math is a subject where mistakes can be made and many trials are needed to finish the problem. I think students will enjoy math if they get a variety of problems, some that are easy for them to find the solution, and others that are challenging and take time to solve. I think it will show them the different levels in which mathematics is based on and allow them to learn more about themselves. They will have that opportunity to practice problems and see which ones they enjoy the most.

Response

I thought your idea about guidance was an interesting one, one in which I did not even think about when writing my own reflection for this activity. I never thought about how having more guidance would have helped to solve the three problems, even though if it was available, I'm sure it would have helped greatly. Although I do agree with you somewhat on the idea, I do not necessarily think that every problem in math needs to be guided. I think having background information, clues, etc will take away from the purpose of solving certain problems that are supposed to cause a student to really think, such as this math activity. As much as I would have liked hints and clues on how to proof the problems, especially the third case, I thought it was good how much I was challenged. I believe a tough challenge every once in a while is a good thing because it really causes a person to think deep and hopefully learn something about themself and how they can find a solution. So please do not think that I am against you and your idea, I just have a different opinion and feel that guidance is not always needed.

S11

I definitely felt that this [activity] was much more challenging than the first. The physical manipulation we were able to do in the first [activity] really helped me in finding the answer, and the fact that there wasn't much physical manipulation for this one definitely took its toll. Nevertheless, we were able to figure the first two examples of the circle with inscribed angles in class. We realized that we could look at the second example by splitting it into two half circles and treating it in the same way we had treated the first. Our group was told to, "don't reinvent the wheel," in terms of figuring out the third circle, and try as I may, I had a really hard time seeing the first relationship in the third circle. I'm going to be perfectly honest and say that I did admit defeat and took the problem to my calculus TA's office hours to get his opinion, and together we came up with the idea of reflecting the inscribed angles over the diameter of the circle.

Between this and my other education classes, I'm starting to see that the idea of a constructivist approach to teaching really does create a more solid understanding of the subject being taught. In this [activity] specifically, coming up with the relationships on our own was far more effective in 'cementing' the concept into our brains than if we had been shown the relationships through direct instruction. Even though it can be incredibly frustrating at times – for both teachers and students, I'm sure – the constructivist approach definitely makes mathematics easier to deal with in the long run. Hints along the way can definitely add to the student's learning and help keep the process of learning from coming to a complete, frustrating halt. Activities such as these and the addition of manipulatives incorporate many types of learning styles and can make math more fun.

Response

I definitely agree that figuring something out yourself makes you more inclined to remember your process. It's so much more effective than simply memorizing formulas. I feel that it also helps you come up with a method of problem solving that makes the most sense to you as an individual. Because mathematics is the type of field where there are multiple ways to come to the same conclusion, it's important for students to find the method that works best of them. Sometimes the method that is simply given to students isn't the ideal solution for all individuals.

S12

The first two central and inscribed angles that we observed were much easier to understand and figure out than the third. Drawing in lines and looking at isosceles triangles helped me to understand how the inscribed angle related to the central angle. Creating a conjecture for the central angles was simple enough, but proving that conjecture for all three of the examples was much more difficult. Drawing in lines in the wrong places was frustrating, but by process of elimination, figuring out which lines to draw, or by "luck of the draw" I was able to find the relationship between the two angles. I always felt like I was really close to figuring out the proof to my conjecture but kept falling short. It was frustrating because I am horrible at creating proofs for math. It feels like I am trying to communicate in a foreign language that I can understand, but not speak.

Learning about inscribed angles and central angles will be best taught after learning about triangle and circle properties. It would be very difficult for a student to understand the properties of an angle inside a circle if they weren't previously informed. I strongly believe that for students to gain more of an appreciation for learning about inscribed and central angles, they would need to understand how this relates to a real-life problem. Understanding how math fits into their everyday lives will help them recognize its relativity and importance.

Response

I also agree that creating proofs to further understand a mathematical conjecture is important for future success on math assignments and tests. However, I believe it is crucial to remember that some students will really struggle with creating proofs (such as myself) and it may only further confuse their understanding of the conjecture. If a group of students are left to figure out the proof of a conjecture and they get it wrong but believe they are right, they will only succeed in making themselves even more lost than they would have been if the teacher showed them the proof in the first place. There are so many different learning styles that it will be difficult to cater to them all, but I think proofs are extremely difficult for students who are not mathematically inclined. Then again, it wouldn't be fair to never do proofs because students who are mathematically gifted probably enjoy them a lot

S13

To be completely honest, i felt absolutely lost during this entire assignment. I think the biggest issue was that I did not really understand exactly what I was being asked to do and in the end didn't feel like i accomplished anything. I got a few points here and there for pointing out a couple things but other than that I was lost. I think that if there would have been examples of some sort I could have followed this would have helped. Examples always seem to help me be able to better understand things. I am a very visual learner so this is a strategy that helps me along. The activity did not make me feel very good at all. I felt like it was something that I should understand and be able to do but I just couldn't get a grasp on it. Unfortunately I felt I had failed.

Obviously, since I felt so lost I wished that I could have had some examples to help me along. Even though we weren't given and of these it was nice to be able to work with our group members for a while. Trying to figure things out in teams is a great way for people to come to a conclusion. "Two heads are better than one." Sometimes others will see something that you do not see and then it gives you an opportunity to feed off of each other. Especially for young kids I think this would be a good way for them to learn and make their own discoveries. I think that math is much different than I have ever looked at it. It is not a subject that comes easy to me and so in the future I will ask for help when I am struggling with a problem like this one.

Response

I can completely relate to the problem that [S15] had with this assignment. I was also not in class when this assignment was given and this made things very hard to start off. Even after I felt like I got my bearings it proved to be very difficult to figure out. For me it was all around an extremely difficult activity. It not only proves that you need to be in class to understand things but that attendance is in fact important. Students need guidance in order to be able to figure things out that they do not know how to do. So attendance for the student as well as the teacher is important.

S15

This activity was really hard to do. I was not in class when we were assigned this assignment which was extremely difficult for me to understand without any help from my peers. I had a hard time proving the inscribed angles. When I first looked at the activity I was really confused on where to begin. There were three circles with angles in them and I had to prove them. I did not even know where to start. So I just wrote down everything I know. The first one I finally figured out and then the second one I almost got it but I could not figure out the last part of it. And then the third one I had no idea where to even begin. I couldn't finish the last one. It was really stressful for me. I would work on it then get frustrated and have to stop and work on something else and then come back to it. It was a very difficult activity.

This experience was important to show students to take risks and look at things from other perspectives. Even if it is really difficult you should always try your hardest to figure out the problem. You will soon understand it in the long run. I was not guided any where to begin this activity which made it even more difficult. Guidance for students is important for students but it also can get distracting if you give too much and the student will be unable to figure it out on their own and learn from it. Being around other students is much more useful than just working on this activity on your own.

Response

I agree that the first two we were much easier then the last one. It was very difficult for me to understand the last problem. Proofs are very hard for some people to figure out. After figuring out the first problem I was excited because I thought this activity was not going to be too hard. But the third was very hard and I couldnt it figure out. Knowing everything about triangles is extremely important before learning about inscribed angles. If you didnt know triangles this activity would very difficult for a student to comprehend and then get really confused and frustrated.

S16

This [activity] was one if the harder assignments we've had, I thought. Not only was it hard to figure out how to prove the conjecture about the interior angles, but also the fact that we needed to use the previous proofs to prove the next one. After figuring out the first proof the second one came pretty easily, but third was the most difficult. Not only did you have to use the two previous proofs but also you had to addin a line to figure it out. After finally finding the line that turned the two angles into both of the two proofs before, it became simpler to prove the conjecture.

This was so intriguing to learn by previous knowledge. This taught me a lot about teaching and how you can teach your kids to learn through wahat they have taught themselves. This makes your students teach themsleves to prove their own conjectures, along with using their own previous proofs to prove another conjecture. This was so interesting to me because i think that you can learn a lot more form yourslef and doing it yourself then having someone tell you what to do, or telling you that something is right.That's why this was my favorite proof yet, because I taught myself.

Response

Yeah [S12] I completely agree with you! Though I think this was one of the least enjoyable proofs of them all. I think that this is why it was also so hard for me to do also. I think that it was very interesting but I also felt that it was really hard to keep doing it. I'm also a kenetic thinker which is why this was so difficult. After figuring out how to do it though, it was much easier then I thought it would be.

Response S19

Yeah [S19] I completely agree with you! I think this was one of the least enjoyable proofs of them all. I think that this is why it was also so hard for me to do also. I think that it was very interesting but I also felt that it was really hard to keep doing it. I'm also a kenetic thinker which is why this was so difficult. After figuring out how to do it though, it was much easier then I thought it would be.

S17

I found this [activity] to be challenging. Our table's first strategy was to draw our own circles with angles inscribed in them. We looked at these and tried to figure out how it worked. After this process, we looked at the first problem and found out it was an isosceles triangle which gave us two equal angles which made it much easier. Then for the second problem we used the strategy from the first because it was the same type of angles with a reflection of it on the other side too. Using what helped us with the first one for the second one made it seem easy, but then we got to the third problem. The third problem was extremely frustrating and I honestly couldn't figure it out by myself. My table helped me out so much with these problems because they were difficult to do on my own. I thought this [activity] required people working together.

Math can be so much easier when people are working together. You get different points of view and perspectives on problems. It gets you thinking in ways you might not be able to do on your own. Working together helps to think creatively which is very helpful in math. Math can use different formulas and ways of solving for the same problem, and everyone thinks differently and solves problems in their own ways. I think math should be a group effort that can get those creative thoughts flowing. Discussion of how to solve problems helps get kids thinking about these problems in a deeper way, which can help kids understand the problem more and know that there isn't just one way to solve it and think about it. Finding the math behind the math will give them a deeper understanding that they can take with them for the rest of their lives. Then maybe math won't be as hated by kids as it is now.

Response

I completely agree with [S22]. He was at my table when we were working on this [activity] and I understand where he is coming from. He was a major part in our group's thinking process. We worked well as a group and I think we needed to work in a group to figure out this difficult [activity]. As [S22] said, it took more than one person to figure out this problem and math should be taught in a group process and also explain the mathematical concepts behind the problem.

S18

On the Inscribed Angles of a Circle math activity I had no idea where to start. I was actually confused throughout trying to solve the whole assignment. I had no method of how to solve this activity and I definitely feel like the assignment taught me nothing about mathematics. It was nice for me to be able to work with a group because I wouldn't have known where to start had it not been for my group members. I can honestly say the first circle only made partial sense to me. I was completely lost on the second and third circles. At the end of the assignment when we needed to prove what we solved I didn't know where to start with that either. So, I wrote what I knew and left it at that.

However, I do have to say that the good thing about this activity is that it was challenging and required me to think on a totally different level then I am used to. If anything I can say I learned that with teaching mathematics especially for students just starting to learn, there needs to be more instruction and direction. This kind of activity would be good for students because it teaches them how to work in groups and seek help when it is needed. I definitely needed help on this assignment and it was such a relief to know that I had another classmate to explain how they solved the circles. Overall, this activity was stressful and frustrating and didn't make sense to me at all. I felt like I didn't learn anything from it. I am sure I would've benefited by seeking more help on the assignment but I didn't really want to try and make sense of it. I am happy I'm done with it though and don't have to worry about it anymore.

Response S25

I agree with you, guidance is very important to have when trying to complete an assignment like this one. I didn't really think about that when I was writing my reflection, but now that I think about it guidance is the one thing that I really needed to solve the activity. If I would have had more guidance it would have helped me immensely. I think having background information, clues, is a great way to start the activity because that way students have a little to work with instead of nothing.

S19

As a kinetic learner, I had a much more difficult time with this [activity]. I found having the ability to see, hold, touch, and physically manipulate the cubes in [activity] 1 gave me a large leg up. However, being challenged mentally forced me to put more effort into this [activity] than in the past. Skills gained previously in regards to angle measures through this class helped enormously, without them, this would have seemed unsolvable. However, even with my gained knowledge in this area, I still needed hints along the way. I can see how this activity would most beneficial in a group setting where students can work off each others ideas. As we all know, every student thinks differently, therefore they each bring their own strengths to the table.

In terms of teaching, I can see how many students would rush to use their protractors to solve for the angles. However, although this seems like an easy short cut, this problem must be solved algebraically. I think this exercise would be a great way to emphasize that there are different ways to solve each problem, which is something they wouldn't get through direct instruction alone. This also led me to see that in many circumstances, constructivism is the winning approach when teaching a class of students especially in the field of mathematics. Students are often less engaged, and therefore take less away from a lesson when it is taught with direct instruction. Furthermore, math is simply more fun for students when it is more "hands on".

Personally, I did not find this activity incredibly enjoyable. I do however feel it was a great incorporation of all the material we have been covering in class. Unlike some math problems I could not rely on just one approach, and had to encompass a multitude of problem solving skills to come up with solutions.

Response

I completely agree with [S1] that this activity was frustrating to say the least. I also needed help along the way. Now that we are aware of the discrepancies of the way we learned in high school and the way we learn in college we are better prepared as teachers to help students so that when it becomes their turn we are not in the same predicament.

S20

This Reflective Mathematics Activity taught me about the central and inscribed angles of a circle, but even more valuable than this knowledge itself I discovered how to prove the conjecture I made to be true. Coming up with a conjecture was fairly simple but this was largely due to the fact that I remember learning about the relationship between inscribed and central angles of a circle in high school. Proving the conjecture is where the real work and learning process began. Taking the first of three examples that needed to be proved my group and I struggled at first to find any useful bits of evidence to support our conjecture. This lead to staring at the paper and a few feeble attempts that proved unsuccessful. After a helpful hint from [the instructor] and discussion as a group we quickly caught on how the conjecture could be proved by drawing in a line and using our knowledge of isosceles triangles to justify our proof. The next example was a bit harder and some more time was spent staring at the paper but after another helpful hint we saw that this example could be treated just as the first if broken up into two sections. The third example was by far the hardest. I found myself coming to understand the two angles within this circle more and more by drawing in lines and looking for possible connections that could be useful. In the end I was not able to fully grasp this last example. Overall this activity was rewarding when a proof was discovered. It allows me to be completely confident in the conjecture and that was enjoyable, though it was not easy and sometimes quite frustrating not being able to understand it as well as I would have liked.

Just as in the last [activity] I think that this hands-on approach to learning is essential and possibly the most beneficial manner in which a child can come to learn and better understand the concepts of mathematics. Being able to explore a mathematical idea on your own, share in that exploration with your classmates and get helpful feedback as well as hints from your teacher are techniques that combine to create a prime learning environment. Using these techniques a child is better able to understand mathematics based on his own understand rather than a given formula. Working alongside peers helps a student to offer help as well as receive it while helpful hints from a teacher guide a student yet do not subtract from the overall learning experience. I’ve certainly been more motivated to learn when I am able to come to an understanding of it by exploring it myself and with others and I believe children are not that different in this regard.

Response

I agree with you. I think that trial and error can be a great learning strategy up to a point. I experienced similar frustrations as you did with the last proof, and even though a final discovery of the solution was satisfying, the frustration that it took to get there seemed to outweigh the final result. As you said a guided direction for the third proof may have been more effective, at least for some of us.

S21

Being given the problem at first was very intimidating I wasn't quite sure what to do with it. Once we had the conjecture it was difficult to figure out the first steps to take to prove the first problem. We tried many ways, but couldn't get it. But with minimal help I was able to figure out how to prove the first one. Then trying to prove the second one was a little harder. I tried some ways but none of them seemed to work. The clue to use the first one as guidance was very helpful. I was able to then prove the second one. The third problem was the hardest, and I never did end up proving it all the way. I was able to figure out half of it but using the technique from the second one. I tried many different ways to split up the angle and none of them really worked. When I did find a way that I thought it should work I wasn't able to finish it. This made me frustrated. Trying to learn by myself with no guidance was hard. I wasn't able to have any help or hints with the third problem. After getting to a certain point I got stuck and was never able to get past it.

I think that giving a problem to a student first can be a effective way to introduce material. By getting the problem first, it made me think much harder on how I was going to approach it. I had to try many ways and failed at each one. By getting a hint after I already started became more helpful instead of in the beginning because I was able to see why that would make sense. I was able to apply it and see how it why it worked better then ideas I previously tried. I think this is a similar feeling kids would experience if they were then given a problem first without knowing exactly how to solve it. This helped me to realize that if you are able to figure something out by yourself you are more inclined to remember the process by which you got it, instead of just trying to memorize it. I think that this would help students to learn more effectively. This is a good way to help the ideas and concepts resonate.

Response

When working the math problem I had a lot in common with [S20]. My group and I went through the same processes by first coming up with the right conjecture and then being able to prove it. Like [S20] I was able to prove the first two, but was never able to fully prove the third. I also agree with her that this technique is essential to learning mathematics, and will help children to grasp the concept. Also that if these kinds of techniques are used it will help children to learn and absorb math with a better understanding of their own.

S22

This [activity] was definitely the most challenging [activity] we have done thus far but it was also very interesting. When it came to the conjecture, we drew a couple of examples and then measured the difference between the inscribed angle and the central angle. Then we set out to work. The first case was quite simple to prove once we realized that there was an isosceles triangle we could work with. Just like most everyone else, we really struggled with the third case at first. However, the third case is also the one that I found most enjoyable to work on. The way I ended up solving it required a lot of symbolic notation that was hard to keep in order but it was really interesting to see how it worked out in the end. What I found the most unique about this [activity] was that we needed to apply knowledge that we gained from solving previous cases to help us with the cases that followed.

As I noticed from the ways other people approached the [activity], there was definitely more than one way to solve this problem. This further solidifies my belief that in mathematics there is generally more than one right way to go about solving the same problem. I think this is an important concept that children need to know and I believe working through guided discoveries is a great way to demonstrate this. I think that one of the benefits of teaching through guided discoveries is that it helps ensure that the students really have a solid grasp of the mathematical concepts behind what they are doing. Not acknowledging that there are multiple ways to solve something and just teaching formula after formula or process after process may lead to the kids losing a grasp for the actual mathematical concepts behind how the math works.

Response

I agree with [S11] 100%. By taking a constructivist approach to learning concepts in math I think that students will be able gain a deeper understanding and retain the knowledge better and longer. By learning concepts through guided activities such as this, I believe it helps the students to understand and manipulate the actual mathematical properties behind why things work in math, rather than just learning a process and applying it.

S23

This activity was taught me that trial and error can be an effective strategy, though not always the fastest approach. This activity for the first two parts was fairly challenging but enjoyable. These were proofs that came easy and were very visible after looking at them for a brief amount of time. The last proof on the other hand provided a great deal of frustration. This one was much less apparent on how to approach the proof. This at times had me frustrated to the point of defeat; so i put it down and came back to it later time and time again. After two weeks of frustration and trying everything that i could think of, the proof finally was sitting there right in front of me like i had just missed it the whole time. Once I got the proof i felt very proud that i had solved the puzzle, though during the process I was not satisfied with my work.

At the beginning of the project the hint was given that iscoceles triangle could be used to prove all of the proofs. To me this was more of a hindrance than a help. It sent me looking in the wrong direction for the third proof. In this situation I can not say that no hints at all would have been better so that I was not looking in the wrong direction because maybe I just was taking the advise in the wrong way. But I can say that giving the first two proofs to do on your own and then having some guided direction for the third one would have been a much better approach for my personal learning style. The third proof left me more aggravated and stressed that I wasn't going to be able to figure it out than having a sense of success once it was completed.

S24

After learning about properties of parallel lines with a transversal and congruency of triangles it seemed natural to move into the first exercise of the [activity]. It wasn't difficult for me to see the relationship between the two angles, this proved more challenging for the second and third exercises. Our group tried applying same conjecture to the second exercise, but we couldn't come up how this was feasible until [the instructor] showed us where the line we could draw in went. As soon as we saw that it was just a doubled version of the first one, it became very clear and was only a matter of solving it algebraically. The third one stumped me for a while. I tried the same strategy of drawing in a line in lots of places, but it was extremely hard for me to wrap my brain around. It took me going to the Math Help Center in the library and working it through (with a couple dead ends) with someone else to come up with a solution. Even then it was hard for me to follow my own logic. I can see that it works, but looking at the diagram it is still difficult for me to see the proof clearly mentally. I am confident that my answer is correct step by step, but the complexity of that third one clouds the process for me.

This experience really solidified the necessity for guided discovery in some forms of mathematics. Just giving students the start down a different path or the nudge to see things out of their schemas is really important. This is important in encouraging students to take risks and try to see things from other perspectives. Too much guidance can distract from learning, so seeing that fine line between independent discovery and just directing students is crucial for teachers. The satisfaction that comes from discovering a solution independently, in a group or with limited guidance from teachers could motivate students to study mathematics with more enthusiasm.

Response

I agree that the previous knowledge of triangles, etc. was extremely helpful in solving these exercises. However, even with all the knowledge hints from [the instructor] (guide on the side) really made the difference for me in the solving them. I think you are totally right about preparing us for the test and further proofs - it was challenging! After working on this [activity] I feel like I have a better understanding of what is necessary for a proof and how the arguments have to be sequential.

S25

The Reflective Math Activity we did recently taught me that to learn math, one must be guided slightly. This was one of the most difficult math assignments that I have had to do without any guidance and I feel like it detracted from my own understanding because I wasn't able to figure it out. During class my fellow group members were able to catch on quite quickly and I was slower on the uptake, which left me farther behind. I think that more guidance (I am not detracting from the Professor's teaching skills by no means – by guidance I mean hints, background information, clues, an exercise to lead me in the right direction, a small explanation… etc.) would have helped me because I wasn't even sure how to start. Working with a friend outside of class helped me to understand somewhat what the exercise was teaching us; however, I didn't grasp the concept fully. The activity truly made me feel very frustrated with myself and my own learning capabilities because I was not able to understand let alone prove what we were learning.

I feel like math activities like the ones we are doing are very important to learning math because they make students truly think about what they are doing rather than copying and memorizing formulas. Students need to discover in order to learn, but on the other hand I also feel like students need a bit more of guidance in more difficult problems such as the activity we have just done. I feel like if I had come in to ask for a full explanation, or even just a one-on-one learning experience, I would have been better off. It is not solely the teacher/professors responsibility to find the students who are struggling, and for that I take responsibility, however, I also think that if this was my activity to instruct I might have had a day where everyone would have a discussion on their findings to teach other students. I feel that teaching is one of the greatest forms of learning and having students teach other students would not only reinforce the concepts in the students who are teaching but also pass the knowledge on to the students who are struggling. To make students more motivated to learn mathematics I would try to apply real life situations to the concepts I am teaching to illustrate the importance and to make the lessons more interesting.

Response

I agree with what you said about how challenging students with problems that guide them into discovering what is being taught can be a new and refreshing way of learning for them. It is always refreshing for me to be able to figure out something on my own. As difficult as this Reflective Math Activity was for me, I had a greater understanding of what we were learning, more so than if we had seen the proofs in lecture and had to regurgitate them. It makes for a more interesting lesson that is much more memorable and interesting.

ACTIVITY 3 REFLECTIONS FALL 2009

F2

I liked this activity tons more than the last one. I felt like the group work that we did was highly beneficial. Before coming to class I looked at the problem and noticed that right away that there couldn't be too many possible answers since the majority of the time perimeter and area just aren't equal.

[F6] came to class with like 2 pages of possible squares, the perimeter, the areas, and the differences between the two. He did a crazy amount of work. It was very helpful though. He had tons of possibilities, but the only two he had found was 3x6 and 4x4. As our group worked together more we discovered that the square could never be 1x__ because if it is 1 then the perimeter is always greater than the area by 2*(other side length) +2. The square cannot be 2x__ either because it is always 4 units more perimeter wise than area.

Then we started fiddling around with the actual unit cubes with all the different colors. I arranged the two that we for sure knew into their physical rectangles. I used blue for the whole thing. Then I took out the interiors of the rectangles and made them green. This is where I figured it out! The interior of each rectangle is 4 units. I knew that every corner had an exposed perimeter length of 2 units. All the other exposed units had perimeter of 1 unit. In order to make perimeter equal to the area you needed to have 4 units hidden away to make up for the extra 4 corner perimeter lengths. The only way to arrange 4 units in the middle is 1x4 and 2x2 so the only rectangles you can get with equal surface area and equal perimeter are 3x6 and 4x4.

I would say that I am definitely going to use this in my classroom. I felt like it has a good level of challenge to it. I think that it definitely encourages group and team work and sharing ideas, and when everyone works together a lot can be accomplished.

Response

I definitely agree that this problem was lots more fun and easier to arrive at a solution. I really like how you said that it could be a potential confidence booster for those kids struggling with math. A huge problem with math is that the students struggle a lot, don't understand it, and give up. This reflective math activity is one where all levels can work through the problem together at a good pace and stay engaged. Your group did a nice job coming up with the solutions, as well as spliting up the work for the 1-10 squares, and then getting a conclusion. Good Job!

F3

Before I actually started to work on this activity, I thought I was going to be able to find many more then two rectangles that have the same perimeter and area. I started working on this activity be drawing out rectangles, and changing the bases and heights. Increasing either the height or the base each time. After going through a lot of paper and a ton of rectangles that did not meet the requirements of this activity, I decide to go with a different approach. Instead of drawing out every rectangle I decided to make lists instead. I thought this would be a good way to see if any patterns existed. Sticking with a consistent base and increasing the height of the rectangle by one, I found the area and perimeter of each along with the difference between area and perimeter. After trying this with many different bases I found that the only two rectangles that share the same area and perimeter are a three – by – six, and a 4 – by – 4.

After I made my lists I discovered that area increases or decreases inconsistently each time an extra length unit was added, and perimeter increased consistently by two every time a length unit was added. It is very uncomment for a rectangle to share a common area and perimeter because of this.

I think this is a very important less to teach, many people have the conception that area and perimeter are some how related and both increase and decrease together. I think making a list is a good way to solve this problem because you are able to see how area and perimeter change. It's one thing to tell students that area and perimeter are not related. If they have the chance, like we did to figure out why there are only two rectangles with the same are and perimeter, they can not only see but better understand why this is the way it is.

Response

I felt the same way when I started solving this activity. It sounds like we used similar methods, and had the same beliefs at the beginning of this activity. At first after I found only two rectangles that followed these requirements I thought there were going to be many more as well. I agree that this is a good activity to share with children. It does play a mind game on you, which is fun! This is a good way for children to do something and see why there are only two rectangles that have to same area and perimeter

F5

When solving this math activity, my group and I started out making an organized table of guesses and checks. However, we soon learned that we could solve this problem much faster using algebra. We took the equation for the perimeter of any rectangle to be P=2x+2y (where length is x and width is y). Then the formula for the area of that rectangle would be A=xy. We knew that the problem asked us to find the solution for when area equaled perimeter so we set the two equations equal to each other: 2x+2y=xy. We then solved this equation by subtracting 2y from both sides (getting 2x=xy-2y) and then factoring out a y on the right side (getting 2x=(x-2)y. We finished solving this equation by dividing by (x-2) to get y=(2x)/(x-2).

The next step is to graph this equation. The resulting graph gave us two curved lines with a vertical asymptote at x=2 and a horizontal asymptote at y=2. Then we simply plugged in integers and found their corresponding y values. Whenever we got another integer for y we knew we had one of the answers. Once we started getting smaller and smaller y values we knew we had found all the possible solutions. We ended up with (3,6) (4,4) (6,3).

When first asked this question, I thought we would find many more answers. It is interesting to me why there are so few solutions. I really liked how my group and I changed our methods of solving the problem from guessing to algebra. For some reason when I solve these types of problems using algebra I understand and support my findings more than when I simply guess and check. I know that people learn in many different ways and that as a teacher I will need to prove mathematical concepts using a number of different methods.

Response

I agree that this activity was much easier than the others we have done in class. Most people in class made a table similar to yours. However, I found it much more efficient to solve it algebraically. For some reason I really understand these types of problems better when I work through then algebraically. This made me realize all the different ways my future students will learn and how I will need to adapt my teaching styles to accommodate all of them. Like you, I was confident that I found all the possible solutions, but I expected more. I am not really sure why there are so few solutions. It is very interesting.

F6

This problem was far more interesting to me than the second reflective mathematics activity. On first look I thought there would be many rectangles with same area and perimeter in the answer, and upon finding the two early examples I was sure more would pop up. I used a system of guess and check to the point of exhaustion before looking at the problem with algebra, as my group quickly suggested. It was, for me a great experience in the group sense, due to the different viewpoints and techniques they suggested. Without the group I would not have found an algebraic proof to the problem.

Response

Likewise, for me algebra was the last choice for solving this problem, although seeing how quickly using algebra found a working answer made me see how useful algebra would be. It seems writing a working equation first would have helped me to cut down on time spent guessing and checking to the nth degree. Thanks for writing it up so succinctly, it was easier for me to see how my answer worked when put in the context you suggested.

F7

When I was given the third mathematic problem in class I read through it right away and for the first time I actually knew what was going on, and I had a lot of confidence that I would turn in my paper with the correct answer, no question. So my first way of going about solving this problem was to take an educated guess. Fortunately I was right on my first try of a 4x4 rectangle which has the same perimeter and area. My next method was to try a 3x3 thinking that maybe the solution had something to do with the same lengths on all four sides. This wasn't the case. So then I tried a 5x5 and so on. But I came to the conclusion that there was no pattern to this solution. Then it was suggested that I try making a table of different rectangular dimensions and get my work organized. So within my table group we each took 2 numbers 1-10 and made a table of the perimeter, area, and the difference of each as they dimensions increased. We found that the other dimension was a 3x6. We had used all the numbers 1-10 and found that the perimeter and area increased in equal amounts and that they would never be equal and if anything just get farther and farther apart. So this led us to our conclusion that these two dimensions are the only two that their perimeter and area equal each other. I am so glad that I was able to turn this assignment in with confidence and a good sense of accomplishment! It felt much better to do it that way than it did in the previous problems where I was mostly just hoping I had the right idea. As a future teacher this activity teaches me that I would really hope that my students have the same feelings that I had when I turned in my solution. It makes homework and the entire class more enjoyable and fun when you know that you are finding the correct solutions and are able to do so through critical thinking and teamwork.

Response

When I read through your reflection it was very similar to my experience. I felt confident right away and knew that I would be able to find the solution. I also used a table to organize my data. That made it an easy way to compare area and perimeter of different dimensions. Once I found the two dimensions that were equal the tough part at first was figuring out a way to prove that these were in fact the only two. But after exhausting all possible situations of dimensions 1-10 my tablemates and I concluded that these were the only possible solutions! I feel like this problem was helpful to see relationships between area and perimeter! As a future teacher I would definitely like to use this type of activity within my classroom!

F8

This activity was different than past activities for me. It really made me realize how important it is to work as a group. I was not able to make it to class on that Friday, and felt stumped when trying to work on it alone. I found one of the solutions by guess and check the first time I sat down and tried it, and I felt like I could share this with the group and see how we could find others. Since I was unable to be in class, I was unable to collaborate with my group. I tried many different things, but never came up with anything that could be an answer, just a bunch of algebra, or a bunch of answers. I ended up asking someone in my group and when they told me to make a table, I was baffled. Why didn't I think of that?! It was a great idea and in the end, that is how I found my solution. I made a table with columns of length, width, area, perimeter, and one for perimeter-area. Then I stratigically number 1-9 in the length and width columns. I filled in the rest of the table and found that the only integer values that created area and perimeter equal were 4 and 4, 3 and 6. There was also 6 and 3, but clearly that is the same rectangle as 3 and 6. I knew these were the only answers because my group told me so.

I ended up asking [the instructor] for help because I didn't know why those were the only ones. I could not prove that there were no other solution. I just knew that these were the solutions for integer values from 1-9. He showed me some really cool ways to go about the problem and showed me that if you draw pictures of the rectangles, you will find a pattern. The patterns were sometimes that for every width you increased by one, area and perimeter would increase by 2 and 4, respectively. There are lots of patterns like these showing you that there is no way that area and perimeter can ever be the same for that length. You end up doing that for all the numbers and you will see why it works.

Response

I agree with everything you said. It is true that this can be a huge confidence booster. Reading other people's reflection, I can tell everyone felt really good about this one and they were confident in their abilities. That is an awesome, and very important, opportunity to have. I agree that we need to give our students this opportunity to succeed and feel self assured that they understand.

I went about solving this problem just about the same as you did. I feel the table was a great tool as well, and it made it all easier to understand for most students when you can see all the numbers computed and put in a table.

F9

I felt that this reflective math activity went very well. I felt pretty confident going into the problem and started off using a guess and check method. Finding the rectangles with sides of integral length whose area and perimeter are numerically equal was the easy part of this activity. The hard part was proving that these two rectangles were the only two. I actually found the two rectangles I listed by trial and error. I did that by myself but the group work was a little more structured. Working with my tablemates went very well. We started off by making a table that included length, width, area, and perimeter. We then began with the simplest rectangle with length and width of one. We soon found that the possibility of finding a rectangle with a side length of one that met the requirement was zero. We then moved on to a side length of two, three, and four and so on. We found that the 3x6 and the 4x4 worked but as we increased the width value, the difference between areas kept constant and that the difference between perimeters kept constant. This showed us that there would not be another rectangle that would meet the criteria because the difference would always be constant. The method we used to solve this was by exhaustion. Like I said, it was fairly easy to find the rectangles but harder to show why they were the only two. I can definitely see how some structure (using a table) would be helpful for any student when solving this sort of mathematical problem. I really think that working with my tablemates was very helpful in justifying our answer to the problem. I was a bit confused when we were trying to prove that there were only two rectangles of this sort. After clarifying how we would justify our answer using a table I felt better. I think that this is a great way to have students learn about this sort of math and also a great way to practice and show their skills in area and perimeter as well. They may struggle but if we as teachers give them enough guidance, this sort of activity can be very powerful.

Response

[F7], youre right, our ideas are a lot alike. I think that what you said about making a table was very well put. It also helped me to put all the information into a table in order to organize my data and to better understand what I was seeing. I too felt that I understood the problem from the beginning which always makes things easier. I think that like you, I will use activities like this in my classroom in the future. I see it as a great way to learn and discover new math principles in an interesting and meaningful way.

F11

For this activity, I started out like most of you did probably, by making a table. First with the 1 x's then the 2 x's and so on. I didn't really see a pattern at all until I had finished my table. So I had to go all the way through the numbers till I found the four numbers where the difference between the area and perimeter was the same. After I finished the table, my group and I tried to find easier ways to come up with the four numbers without having to make out a table. First, we tried to find an algebraic equation. I made area and perimeter into equations and set them equal to each other. It looked like $2x + 2y = xy$. I solved the equation till the variables were all on the same side and came up with $xy/x+y=2$. If you put in your width and length for x and y you'll see that this equations works! Next, I tried to come up with a graph method where y has a value equal to a solution. Starting with the same equation of perimeter and area equal, and this time solving for y, I came up with $2x/(x-2)=y$. Using my graphing calculator I found out that this also worked! I was really surprised how I could come up with 3 main ways to solve this just by making a table and working on some basic equations. I personally liked the algebraic equation, just because that's the way I prefer to solve problems, but it was fun to figure out multiple ways of teaching area and perimeter.

Response

I agree, working with the group on this one was essential! And the table was a great way to start it off. I really like [the instructor]'s idea also, I had never thought about it like that.

F12

I really enjoyed working through this activity. I had a good feel for figuring out a logical response to the question presented. Coming upon the right answer began with a short period of guess and check. I realized right off the bat that 4x4 would work and tried to figure out what direction if either would give me my second answer. My group was then given the idea of working out a table that went from 1-10. The table helped lots but it didn't take us all the way to ten to figure out the answer. When we got to the number three we found out that a 3x6 would also work. After we got the two solutions presented above we came to the conclusion that it was not possible for any other number to have the same perimeter and area. We came upon this assumption when I realized after working out the 4x4 that the perimeter and area were just getting further and further apart. I could tell that me and my group felt fairly confident that there were no other possible outcomes that would match the question presented and It felt good to get it done right there on our first day of looking over it. At the same time we made good conversation over the reasons that it was impossible to get any other answer and it felt good to have that knowledge and be able to discuss the way we did. Doing a project like this with children would be fun because even though it may seem really complex at first the answer is easy to come upon and very rewarding after all the work is done. This is a great confidence booster and lets you know that you understand the workings of area and perimeter.

Response

It sounds like this activity was easy for some people and harder for others. Working in groups seems to have an advantage when it comes to these reflective activities. It is way more work and frustration if you are working by yourself and not there the day of the activity. Not only do you have other brains to question certain things, but [the instructor] also is a great amount of help if you have any questions. It seems like those who were there in class had a better idea of what was going on and had a positive reflection overall.

F13

When we sat down with our groups to discuss the activity I wasn't sure what we were looking for but they helped to explain what it was we needed to find. Also a group member had start the activity prior to us all meeting and she showed me how to start the activity. She showed me "proof of exhaustion" which looked like it was going to take forever! I'm not going to lie I got a little nervous with all of her calculations but I knew I would have to do it so I went along. Then we began to go farther than she had and went up to 16 and saw no results so we decided to go up bigger. I tried 30 and someone tried 300 but still nothing. We went up by the 3's because it seemed like a number that had worked in 2 instances previously. We began to realize that there was no pattern and no equation that would get us any more answers so our quest sort of died. We then came to the conclusion that there would only ever be 3 rectangles with integral lengths. After we finalized our decision we began to create our own charts using proof of exhaustion just as our other member had started. One thing I realized going through the process is that although there wasn't a pattern to find our answer there was a pattern showing us that there would be no more solutions. The length, perimeter and area were all correlated with one another. The chart is just a concrete and orderly way to show what we were trying to prove. That because after 3x6, 6x3 and 4x4 there would be no more rectangles with integral lengths because Perimeter-Area would never again equal zero. This experience (going into being a teacher) would benefit me in how I need to teach and express my results. They need to be in some sort of easy to read way like the chart I made, it's easy to believe and grasp something when it's laid out in front of you.

Response

The last part of your response really sounded interesting. Although I was confident with my answers because that is how everyone else in the class was solving the activity I too like to use algebra to solve math problems. It does make me a tad nervous when there is no definite answer. In this case and in many other reflective math activities we are never given the answer and we are expected to find one that we think is correct with no way of formality to go about such a solution. It make me almost uncomfortable not knowing the exact solution. It's hard to even start a math problem not knowing how many answers there will be or a way of knowing you have the right one. I totally know the feeling of not being confident in your work.

F14

This activity was my favorite yet. When I first saw the assignment, I was excited because I happened to understand how to find possible answers, finding area and perimeter of a rectangle, which wasn't necessarily the case with past reflective math activities where I was slightly clueless how to start.

What made this activity interesting was the fact that when we started working on it, I wasn't sure how many possible answers there could be. I had an idea of how to go about solving this problem, and at our table, we decided to make a table with all the possible areas and perimeters starting from a 1x1 rectangle and working our way up. After going though some possible answers, we were able to find patterns in the difference between the area and perimeter of each rectangle. If the difference was increasing from zero, or decreasing from zero, as well as staying constant, we were able to determine whether we should keep trying possibilities within that grouping of rectangles. After a while, we realized that there couldn't be any possible answers over a 4x4 rectangle because every possible answer after that had a difference between the area and perimeter only got larger from there.

Throughout this process, I was confident in my answers and work, and the only problem I had with this problem was my insecurity in using a table to find the answer because I usually like to use algebra to solve an answer. But once we had finished and gotten the answers, I became confident in what answers we had found.

Response

I really liked reading how you came about solving the problem, I think using algebra was a smart way to do it. I am the same way in that I like to use algebra to solve problems like this because it seems to be the easiest and most systematic way to solve problems. I used a table, like you did in the beginning, and I believe to have gotten the answers correct, but would have liked to use something like your equations to make sure that my answers are correct, and to make sure that I found all of the answers possible. In the end, it is interesting to see that there can be so many ways to solve the same problem, and every person probably prefers do do it their own way, the way that makes most sense to them.

F15

The reflective math assignment for area and perimeter was a pretty easy assignment.The first day I had it figured out and done. I took graph paper and drew out each of the possibilities from ones to tens. example: 1 by 1, 1 by 2, ... 1 by 9, and then on to the 2's, 3's, ... 9's. I discover that there are three cases when the area and the perimeter are equals. These are 3 by 6, 6 by 3, and 4 by 4. I am not sure why these are the only three times that this happens but they are the only ones. I found these answers by exhausting the information. There were no more possibilities. In each graph the perimeter goes up by two each time and the area goes up exponentially according to what ever number you are graphing. example: on the number five the area goes up by five each time: 1 by 5 is 5, 2 by 5 is 10, and so on and so on. This was an easy reflective math to do and to understand. When we met in class to work as a group we discovered the exact same thing. I am certain that we have found all the possibilities for perimeter and area to be equal.

Response

I totally agree. This was the first reflective math assignment that I understood. I knew exactly how to proceed to find all of the cases where the perimeter might equal the area. i used graph paper and drew examples and made charts to go with my drawings. This was actually fun at least for the first five numbers. I used up all of the examples for number one through nine. I finally realized that I had exhausted all of the possibilities. I found 3 by 6 and of course 6 by 3 and 4 by 4. Working with the group in class proved these to be the only cases. This activity was fun because there was a feeling of success. I think that this was a good solo project as well as a good group project.

I thank you are right in that it is important for students to have success often enough that they gain confidence in their selves and their abilities. If kids leave class feeling frustrated and confused to often they may become discouraged and lose their willingness to apply themselves. Math is a class where kids to often feel inadequate so they cease to apply themselves. Reflective math activities might be a good way to build their knowlegde and strengthen their skills. I agree that a little more time in class to look at the results and make sure that everyone understands what is going on is a good idea

F16

I thought the over all math in this reflective problem was easy, the true test was to identify if there were any patterns that would allow you to see a set of answers. When I started this problem I took a systematic approach to identifying at least one matching set. When I found the first set 3X6, I thought that there might be a ratio connection, and so I just started trying different combinations of numbers. The next set of numbers 4X4 produced the only other correct combination if found and I believe exist. I really didn't like this problem because the one reasonable tactic is to guess and check. It gets boring and I lost interest in it almost right away. As I continued to guess and check I noticed that the area was becoming way too large for the perimeter to match up with.

I forgot to add that I don't think I would use this type of an activity in my classroom. Unfortunately, I feel the lesson I'm taking away is that of "what not to do". I feel this activity was just about plugging in numbers and not a lot of logic or reasoning.

Response

I think the strategies you used are pretty much the same as most people in our class. My group and I used the steps to identify 3X6 and 4X4 as the only correct answers. I also agree with you about fully understanding the problem before you introduce it to your class. The only big difference that I can tell is that you seemed to really enjoy this activity and I thought it was a bit boring and redundant.

Response

Our methods to solve this problem sound like they are the same. Guess and check seems to be the best and reasonable tactic when trying to solve this problem. The only difference is that you seemed to enjoy this activity, and I thought it was a little boring and redundant.

F17

Finally I feel like I turned in a reflective math assignment with the correct answer. Of the three we have had so far, this has been my most enjoyable experience. I stated this assignment working with the idea that the area and perimeter of squares would be the same. This was because I guessed on a 4 by 4 square and both the perimeter and area are 16. I quickly realized that this was only true for a 4 by 4 square as all my other attempts to find squares would not work. It was then suggested to our table that we look at the data as sets of tables. Each of my table members took 2 numbers (1-10) and made a table starting at _ by 1 and ending at _ by 10. What we found was that the only other rectangle that worked was a 6 by 3. The tables also explained by there are only two that work. As the numbers increase for perimeter then equally increase at a constant increase for area. Therefore, the numbers will never equal one another, so we had exhausted our options. This was confirmed by other tables and I left feeling good about the work we had done.

As a teacher I hope that my students leave class with this feeling more often than not. In the last two assignment I turned in my paper not sure that I had the correct answer and, even worse, feeling I didn’t know how to get the correct answer. Math is a subject that I feel need to have constant reassurance for student or they will end up feeling that they just can’t do it. If I was to teach these reflective math assignments I would not have them turned in until students at least understood why they had found the answer they found. This may simply be by giving them more time in class to work on the assignment, or having some of the students explain their answers to the rest of the class. That way the students can explore on their own, but turn in a paper that the feel confident about.

Response

I agree with you [F15], this was easier than the previous assignments. We too used exhaustion to discover the correct answers. I think that what made this assignment easier is that we were able to come to a conclusion by the end of class time. On the past two assignments I have been quite frustrated when I leave not know much more then when I got into class. I felt we were given the clues necessary to discover, for ourselves, what the answer was. The fact that I left class feeling confident with my answer gave me great joy. I hope we can all teach are students in a way that they feel this way rather than how I felt from the last two assignments. I think all it takes is time. Thanks for the response.

F18

This was definitely my favorite activity done this semester. It was a lot less frustrating and much easier to see that you had found the correct answer. All I did was set up tables with W (width) L (length) P (perimeter) and A (area). I then started at the lowest length and width; 1. From there I worked my way up and tried to see if the area and perimeter ever matched up. They didn't. Then I did a 2x2 rectangle, and repeated the same process. Eventually I found that the only rectangles that had the same perimeter and area were the 4x4, and the 3x6. I knew that this was true because after the 5x5 rectangle the difference between the perimeter and area began increasing at a higher rate. I felt a lot more confident in this activity than the others and found it a lot more enjoyable to do.

F19

I really enjoyed this reflective activity, it was a lot easier than the one before. I think I also enjoyed this one because I understood it and did not need to rely on my table mates. Of course I worked with my group for a little bit but for the rest of it I brought it home and worked on it myself :) I'm pretty proud of myself and I feel confident with my work.

I only found one solution though so I was pretty bummed but I spent a few good hours on it! I did get some what frustrated because I would notice a pattern and then it would it stop. So I don't think there is a pattern to the solution.

This activity was fun and it kept my attention, I didnt actually think of it as a dreadful homework assignment, I actually enjoyed working on it.

Response

I agree with you, this activity was better then the one before. I also agree that Jamie helped a lot, he worked pretty hard it and brought a lot of things to my attention like how it could not be a 1x anything.

But you also helped a lot too, you had a very open mind and you explains things very well. I felt very comfortable working with you.

This activity was not half as frustrating as the one before and I actually really enjoyed doing it.

F20

I felt much mroe capable of getting a handle on this activity. The conept as a whole was not as confusing or out of reach feeling as the other activities. I knew for sure there would not be too many solutions because area and perimeter are jsut not that often equal. I missed the day in class to work on this as a group so I sat down at home and started. I immediately thought of the 4X4 once I really looked at what the problem was asking and looked at the example shapes given. As it turns out, that is the only square tht has a nequal perimeter and area. The next, and only other, solution would end up being a 3X6 rectangle. I made an equation to help find this solution, and I made a table but that didn't porve as helpful for this one. Although an equation can be successfully used I found trial and error to work best here. I quickly found the 3X6 solution and next needed to prove it. It became very clear those were going to be the only two solutions once I started plugging in different combinations of numbers. Each set iof numbers, starting from 1's and going up, the perimeter was bigger than the area and then as you incresed one of the numbers the solution would hit a point where the area would then become bigger than the perimeter. If the numbers did not equal eachother (for the area and perimeter) at this turning point, they were not going to with more manipulation...the area just got larger and larger than the perimeter.

I liked this activity, I felt like I really got a solution out of it and was able to figure it out on my own. I would use this in a classroom as I think it's a good teaching activity for area and perimeter.

F21

This project was not very difficult but just very time consuming. Before we worked in class I had not really much of an idea where to start and I was very confused on what I was actually suppose to be doing. But after talking with my table I discovered that the task was just quite simple you had to find the area and perimeter then take the difference between the two. The value of the difference would then tell you if the area and perimeter were numerically equal. If the value of the difference was zero then that means the area and perimeter were numerically equal. The difficult part was finding all the different scenarios where this occurred; the only way to do this was the proof through exhaustion. This is when you must show all the ways that it does not work to prove that there are no other solutions. I showed all the possible area and perimeters for rectangles with area widths one through twelve and heights one through nine. After trying all these possible dimensions the only three that worked was the 3x6, 4x4, and 6x3 rectangles. The 3x6 rectangle and 6x3 rectangle are the same rectangles, so there are only two dimensions of a rectangle where the area and perimeter are numerically equal.

As a future educator I felt this was a great project for discovering your own solutions to problems. Having to try a bunch of different problems before finding the solution would help students realize the different ways that you may come to a solution and that it may not always be as easy as they expect. This problem really showed me that I am not always looking for a solution to a problem but sometimes you are looking for a way to prove that your solution is the correct solution.

F22

I thought that the Perimeter and Area project was easier for me to figure out than our previous project. Again I think that this has a lot to do with the fact that we had stuff to actually physically work with while in class and the visual is always very helpful to me. The frustrating part of this was that I wasn't entirely sure how to find out if I had found all the possibilities or not. The only way that I could think to do it was by using proof by exhaustion, which is when you test a bunch of perimeters and areas. I used a table to set this up, which I later lost and still can't find and had to redo, but I did a lot of examples. Within in the table I also had a column which gave me the differences in the area and the perimeter, when it was 0 obviously the area and the perimeter were equal and that was a solution. I found three solutions and decided to stop my proof by exhaustion when I felt that the differences were getting larger because the area began getting larger than the perimeter. What I found useful in this project as I have found with the previous two is that I was allowed to try to discover it by myself, but there was some prompting and guidance from [the instructor]. This was helpful because I was allowed to struggle and get a little frustrated but I wasn't left completely floundering by myself. As a teacher it is more our job to help guide the kids into making their own discoverys than just force feeding them information that they can't relate to and may not even care about.

Response

Algebra is something I hadn't even thought of using to solve this equation, but it makes a lot of sense. I personally used proof by exhaustion which is what our group started out with, and the only for sure way that I knew to be done trying to find the answers were just looking and seeing that the differences in area and perimeter getting larger. I like that the algebraic method is more definitive than the method I used, even though I came up with the same answers. I agree with you on having to know different methods as a teacher in order to get across concepts to students because the way that one person may understand something might be entirely different than another student.

F23

I thought this activity was good. There was a lot of confusion on my part at first. As I begain to think about how I would approach this, I started with just trial and error thinking that it nver hurts to see what will happen if I just start playing around with numbers. So I pretty much found that a higher number the more confusing. It took ma a little while but soon I relized that there could only be a few answers. I didn't feel very confident about saying that there were only a few but after i went over it for a while I think I just convinced myself that there were only a few. This was a fun activity. I enjoyed the mind play. I thought it created lots of thoughts in my mind that caused me to want to know the answer. Nice job [instructor]!

Response

I think that [F2] is right on! I think that this would be very useful to present in class. Team work is required and it does make the job easier. I like the way she explains it. Very simple. I also like the amount of credit that she gives to her group. It is nice to hear that team work was really helpful. I like her approach to the problem and wished I would have thought of that. Great job [F2]!

ACTIVITY 3 REFLECTIONS SPRING 2010

S1

This [activity] was, again, very challenging for me. My group and I started with making a table and then we moved onto finding equations. We mostly just used the guess and check method. This activity once again reminded me how nice it was to work in a group. Group work really does help me to learn. I think what didn't help my understanding of this was the equations however. In this [activity] I understood it best with a table and a short paragraph explaining the table. It also is always really challenging for me to prove things. Again it was nice to have group effort in creating the proofs. All [activity]'s that we have had have taught me that learning math can be very challenging. But there is no greater feeling than that of accomplishment when you solve a difficult problem that you struggled with.

These [activity]'s have taught me the importance of group work. It can be very frustrating to try and solve a problem that seems unsolvable by oneself. But throw in even one other person and it is instant relief because you know that together, which each person's different understanding, the problem will be solved. I have also realized that going into a difficult problem with an open mind makes things a lot more bearable. If you think you will never solve it then you are probably right. But if you think that you can figure it out than you probably will. It's nice to know that there is a person to help if you need it and it's also nice to know that your classmates have to figure this out also so you can assist them and they can assist you. Going into a problem with an open mind really does make a big difference.

Response

I agree with [S23] when she says that a student should not be told that their idea was a bad one. Math can be frustrating enough as it is so encouragement is always the best way to go, in my opinion. I know that I understand things more when shown other ways to do them so I think that guidance is nothing that should be looked down on.

Response

I agree that geometry is definitely not something that comes naturally to me and I always love finding algebraic ways to solve a geometry problem! I also love how willing [S23] is to learn, and I think that that is going to become very useful when she is a teacher. In my opinion every teacher should have a want to learn.

S2

This [activity] was actually one of the easier ones for me. First, we went by trial and error until we found the two that had equal areas and perimeter then once we found those we set off to find an equation that would work to find both. I like the fact that we worked on groups because like I have said in previous [activity]s more head are better than one. For once I felt that I really had a handle on the concept being taught and I could figure it out with or without the teacher in the room.

Through this [activity] I learned that even concepts that seem larger than can actually be easily handled. Children learn in many different ways and this [activity] allowed for a couple different types of learning. There was a part for the children that think more algebraically and the ones who think more by trial and error. It also allowed students to make discoveries on their own. I know when I discover something on my own it boost my confidence so if anything the students learn to be more gutsy and take more risks even if it means being wrong. I think it also helped having the groups because when you took chances there is other people around to bounce ideas off of and if it is completely wrong you have people telling you, so you not going off of a wrong Idea the whole time. But in that token, it can become a distraction with having groups because it means working together and learning together which has it perks but children gets distracted easily and if they do they will probably get distracted doing this as well. But it means distracting other children as well. But, other than that I thought it was a great [activity] of self discovery.

Response

I agree having a small group helps children that are confused by the assignment find their way, which will boost their confidence because they did not have to go to the teacher for help they could in list the help of peers that is really important to have later in life.

S3

In this [activity] we were asked to find all rectangles whose area and perimeter are numerically equal to each other. It seemed like a daunting task, but when I actually starting working on this particular [activity], I came to find that it was the easiest of solve of the three we worked on this semester. I did, however, have trouble proving my conjecture. I started by making a table that had that had the rectangles length, width, perimeter and area. By methodically placing all the information in an easy to see diagram, I was able to figure out the relationship between area and perimeter. I quickly figured out that there were only three combinations that allowed the perimeter and area to be equal. They were: 6x3, 3x6, and 4x4.

The next challenged I happened across was actually proving that these were the only combinations. I was unsure of actually how to go about proving this. There are so many combinations out there, that I was afraid that I was missing some. After looking at my table for awhile, I actually began to notice something. The farther down my table went the larger the perimeter and area became, however I realized that the area numerically grew at a faster rate. There was no way that the perimeter would be able to catch up in order to ever be equal to the area again.

Problem solving plays a large role in classrooms across our nation. This does not happen only in math but in other subjects as well. It is important to challenge the students with a problem that is not laid out step by step for them. They need to be able to look at a problem and think of a creative way to solve it. There are always different ways to solve the same problem, and this [activity] showed me that. I noticed that other students in the classroom were going about the same problem in a way that I never would have thought of. It was interesting to see the thought processes of others within my class. Refreshing even.

Response

I agree with [S2]. This [activity] had to do a lot with self discovery. There were no boundaries on how to solve the problem, so the students were able to use their previously learned knowledge to come up with a plan on how to solve the problem at hand. There are always multiple ways to solve the same problem, and it is alway intriging to see how others within your classroom came to the same conclusion as you.

S4

In this [activity] we were asked to find all rectangles whose area and perimeter are numerically equal. I've noticed that I become slightly intimidated when these activities ask us to find all solutions to the problem. However, as soon as my group and I start experimenting and diving into the problem it doesn't seem so daunting. What helped my group and I to start to narrow down the task was to group rectangles in families based on side lengths such as 4 x 1, 4 x 2, 4 x 3, etc. By looking at patterns, we found that rectangles with side lengths of 1 or 2 did not work and once side lengths exceed 6 units the area started to grow much faster than perimeter. Therefore, we could limit our search to only include rectangles with side lengths of 3, 4, 5, and 6. It was nice to be in a group at this point so that we could bounce ideas off of each other and make sure we were on the right track.

This activity brings to mind several points about how mathematics should be taught. Children need a variety of approach strategies in their "toolbox" in order to get started on word problems rather than disengaging or becoming frustrated. Word problems should be a part of teaching math at any level to familiarize students with situations in which the correct answer can be found in many different ways. Also, I am in favor of the small group setting for this type of activity because of strength in numbers. It was very beneficial in this activity to work with my group members to formulate strategies, discuss ideas, and prove our theories. Group work also teaches students other things that can be applied outside of math class such as teamwork and problem solving in group situations. This activity was quite insightful and beneficial as it brought to light the importance of several teaching strategies useful for a student to be able to complete a similar assignment.

Response

You make good points about some of the responsibilities of the teacher. It is our job not only to make sure students have the skills they need to succeed, but also that they know how to apply them. Some direction may be necessary in certain situation. It is also very important that we do what we can to create a safe learning environment where students are comfortable asking each other questions. Quality learning situations can aid the student in numerous ways and the teacher can certainly play a role in creating this type of environment.

S5

In this [activity], we had to find rectangles that had the same area and perimeter. It seemed like a very daunting task until we got started. It was a lot of trial and error until we figured out a formula to use. When the measures of the sides were over ten, then the difference in the area and perimeter was too great to even consider side lengths above ten. We found that there were only three combinations that would work: 4x4, 3x6, and 6x3. Once we figured out the three possibilities, we needed to prove that they were they only three combinations. Finding a formula was complicated and it was very important to remember all the rules associated with equations.

Math should be taught in a variety of ways which is what the [activity]s have been teaching us. A student needs to be able to think in a variety of different ways in order to solve different problems. Word problems such as the [activity]s seem to be the hardest for students to understand. To help students succeed, a teacher needs to teach them many different skills through things such as trial and error. Group work is an interesting way to teach math. Math is usually taught as an individual subject, so incorporating groups is an interesting and useful way to teach math. When a student works in a group for math, it exposes the student to new ways of approaching a problem based on the ideas of their group mates.

Response

[S25], I agree with you that this [activity] was an intimidating task because of the infinite possibilities. I also really like your idea about students remembering more about a subject when they are making the discoveries themselves. I like the idea of group work as long as it is truly group work and not one student doing all the work alone.

S6

This activity proved to be the most challenging for me so far. Yet, I feel as though I said that about [activity] #2 as well! My group started the task by making a chart that had columns as follows- dimensions of figure, number of tiles used, area and perimeter of figure. This activity taught me that when one is unclear about how to begin a problem, it is important to stay organized. My group also physically made each of the figures, which helped us see a visually representation of the information we were charting. However, after completing several rows on the chart, we were still not able to see a pattern. This activity also taught me how important it is to be flexible and willing to try a new strategy. When we reached the 4x4 square figure we knew we had found one of the solutions as 4+4+4+4=16 (perimeter) and 4x4=16 (area). With this information we were able to devise a formula. This formula is as follows 4x=x squared. After finding this first answer I felt relieved and successful! It was nice to know we were on the right track. We soon found one other solution using the formula 2x+2y=xy. With the group I was able to understand why these are the only two solutions that can exist.

Difficult problems such as this remind me of what a young student may feel when learning math concepts for the first time. This sort of problem truly shows that students may have the skills; they just need to be guided in how to use them. Group work, as well as teacher facilitation aid in the process of applying math tools to complex problems. Group work as well as popcorn techniques may be helpful in math exploration activities such as this. For example, if a member of another group understands the activity, they can move (popcorn) to another table and explain how they came to the solution. Team work and support from the entire class to one another is vital in aiding in students learning. If a member of the class does not feel comfortable with classmates, they are less likely to speak up when they are struggling.

Response

I had a similar experience when I approached this problem. At first I felt overwhelmed; it seemed as though there could be an infinite number of solutions to the problem. Yet, as I am learning, math typically has an exact answer. You are correct that there are multiple ways to get to the solution. I think it is important to empower our future students to know that although a problem may seem complex at first, they have the tools they need to solve it, simply by trying strategies that they already may know.

S7

During this [activity] I found it easiest at the beginning by simply guessing and checking for perimeters and areas that would equal each other. After coming up with the two answers (3,6;4,4)I wasn't sure how to find out if there were any more. Then our table figured out that it would be easier to make a graph. This way we could see if there was any that worked. After noticing that the bigger the numbers the farther the area and perimeter got away from each other we figured we might have the entire answer. Another group member was able to set the equations of area and perimeter and set them equal to each other. That way we could graph the answer. We found that there was a horizontal asymptote at 2 which showed us that there would be no other times that it crossed each other on whole numbers. This teaches me that maybe guessing and checking is a good way to start, but in the end you will still probably have to come up with another way to actually prove your answer. Guess and check is definitely a good way to get started though.

Again this [activity] shows me that to teach some aspects of math, or any other subject for that matter might take more than just an explanation. Since we were able to work with groups and figure out different ways to think about the problem we needed to solve really helped. In this case we found that creating a graph was an easy way, so maybe when teaching things like this the students would understand what was going on more and it would be easier to see. Letting students go off on their own is a great idea sometimes. It makes them feel like they actually need to find the answer rather than just waiting for the teacher to tell them how things work or what the equation is. It will ultimately make students more independent. I definitely again felt accomplished and excited we had figured out the answer. It almost makes you and other students feel smarter than if they had just learned the concept from a lecture.

Response

I agree with [S15]'s last paragraph completely. By having the teacher walk around the room you are able to ask questions more personally instead of in front of the entire class. This creates a more safe environment for the child where they don't need to feel embarrassed by not knowing the answer. I also like working in groups, because there might be half of the students understanding it completely while the other students have no idea what's going on. It not only allows those who don't understand have the chance to get it figured out but the students who are explaining the answer to them will now understand the problem and solution better also.

S8

This [activity] seemed much more simple than the last ones at first, but after we started i started looking at it more closely i could see it was about as challenging as the rest. i missed the group work in class unfortunately so i lacked the insight that my classmates had to offer at the time which did not do me justice. i did although have a chance to sit down and take my time and my own procedure figuring this one out which was a change. another student in class and i combined our ideas that both of us agreed on and tried to come up with a more simple way of showing the options of the shapes dimensions. in the end we just used the long route but it worked, and because of this the project was a bit frustrating but we finished and learned a few things.

with this rma as with others has a way of making the students really think outside the box to come up with unique and hopefully more efficient ways of completing the task. with this rma though vs. others we have done i think the students would have learned a lot easier/faster if they would have just been given the info and explained in class. i dont resort to that often but with this one i believe that the confusion and time could have been used more efficiently haven just been told. the other rma's though were very helpful and insightful on how to better our math skills!

Response

[S13] said in her reflection on the [activity] that kids need to learn with different strategies and i completely agree. this rma was a definate expample of how different people can solve a problem in different ways. there were various creative ways around the problem and as she said all kids have different learning patterns. i also agree that it should be made fun and exciting and if possible, executed in small groups so the kids can colaborate and work together to find a way to the solution.

S9

At first when we started this [activity], I thought it was going to be impossible to find all the rectangles that have the same area and perimeter, since there is an infinite amount of numbers to try. Once we started creating rectangles and choosing random side numbers, I then realized how there would be a lot less solutions that I at first expected. After experimenting for a few minutes, it seemed my group and I found the two possibilities quickly. Once we found 4x4 and 3x6, we then started a new technique to try to find solutions. We started with a certain number, and then went up a number each side on the opposite side. Ex- 3x1, 3x2, 3x3, etc. This idea is what allowed us to come up with the equation and see there were no other easily solvable solutions because we found the number difference between the area and perimeter of a figure with the side length a certain number. For example, 4x5=20 area, 18 perimeter (difference of two); 4x6=24 area, 20 perimeter (difference of four), so the 4's increase by two each time.

This [activity], as well as the other ones we have tested throughout the semester, has helped me realized that in math, there are always a variety of ways to solve one problem. I think this is a great idea to explain to children because it will let them experiment with their own problems and experience many different ways of how to solve the problem best. I think if they understand that there is more than one way to solve something, then they may not get as frustrated because if one way does not work, they have other options to try. Having students work in groups and come up with solutions together is also a strategy that I think works wonderfully in math. It allows the different ideas to get around the table and be built upon because there is input from person to person, rather than just one person. It will let everyone think together.

Response

[S4], I also like how you mentioned students need a variety of approach strategies in their "toolbox". I think that is a great way of putting it. Having the option to solve a problem in a different way is great because it shows different thinking skills and that math is a broad learning subject since there are many ways to solve one problem. I also think it is good to have word problems every once in awhile because it allows people to think in a different level than they would when they have a problem with a bunch of numbers on a page. It lets students experience a variety of problem types.

S10

This problem was easier for me than the last. We were encouraged to find patterns that led to a rule. Seeing the patterns helped.

I found one proof while at the same time others in my group worked on another. I found a proof because I had the list of different perimeters and areas that did and didn't coincide, and so I was able to play around with those numbers. I realized then a function with the perimeters and areas that were the same.

S11

I truly lucked out with this [activity], because I was able to see the algebra hidden in the geometry. Algebra has always been my strong suit. My first instinct was to set the area and perimeter equations equal to each other, which we (myself and group members) then used to see if we could find two different values to satisfy the resulting equation. As a group, we discovered that there were only two solutions: a rectangle with sides 3 and 6 and a square with sides 4. The proof ended up being a little bit trickier. After some thought, I realized that you could solve the equation we had created for a single variable and then graph it using the calculator. This was effective for me because it showed both a vertical and horizontal asymptote, and I was able to draw a more effective conclusion as to why the two solutions were the only ones that exist.

Because I was able to use algebra to prove this [activity], it was far less frustrating for me than the previous [activity]s. If I hadn't realized this in the beginning, I'm sure I would have been tearing my hair out. I haven't had a geometry class in eight years, and even back then I had a hard time grasping the material. In terms of learning about math, it was interesting for me to show the relationship between the area and perimeter of rectangles. I'm working with two third graders on this very subject for my [education class] observation, so I was able to use some of my insights to help them to remember the formulas and learn about how they are connected. It was a lot of fun for me, and this activity really helped increase my ability to teach them. After doing this [activity] for rectangles, I wonder if the relationship between area and perimeter is the same for other polygons. It would be interesting to see if this holds.

Response

I completely understand where [S1] is coming from when she mentioned how the [activity]s taught her the importance of group work. Being able to talk the problems out and bounce ideas off of one another is incredibly beneficial. This is an important thing to remember as we think about our own future classrooms. By allowing our students to come up with solutions as a group, we are encouraging cooperation, understanding, and communication, all of which are important in mathematics.

S12

This [activity] was the easiest to figure out the solution, but one of the hardest when trying to prove how I knew what I had figured out. Our group started by figuring out that no rectangle with a perimeter including a side length of either 1 or 2 could ever have the same area value. Then we started in on the possibilty of a rectangle witha side length of 3. We made a chart that included "3" on one side and "by __" on the other. We entered 1-9 and then began calculating the perimeter and area. When we got to "6" we had found a solution! We then continued on to our "4" chart and did the same thing, finding that a 4x4 rectangle would have the same area and perimeter. This part was quite fun and didn't take us very long. No one in my group seemed frustrated at this point.

This [activity] taught me about how instrumental students can be in each other's learning experience. For example, we each bring our own unique understanding to the table when we have to work on problems together. Doing homework or studying for an exam with peers is likely to be frequently successful in the overall amount a student will comprehend from the assigned work, compared to working alone where a student must wait for help if they cannot solve a problem. In a group setting, it is likely that someone within the group will be able to explain different steps in finding the solution. In a group setting students may also correct one another if their understanding is incorrect and that student is saved from completing their work with flawed thinking.

Response

I really like how you touched on the comfort of students in their groups. Classroom activities and projects can bring students, who normally wouldn't intereact, together. They get to know each other's learning styles and, many times, come to appreciate new ways of solving problems.

You also made a good point that if students aren't comfortable in their groups they won't participate as much. Observing our students interactions and placing them in appropriate groups in the beginning of the school year will be important to encourage interaction with each other.

It is also interesting that even in college we still interact differently in each of our new groups. I have found that in one group I flourished and in another I didn't benefit at all. Watching for things like this in our students could be very important to their success in the class and their peer interactions.

S13

This math experience was an interesting one as always. For me this time my group was a lot of help to me. Math is not easy for me to understand and with help from others and their outside ideas it really helps me to better understand. Also using tables to figure out our numbers really helped me to stay organized and better understand what information we had accumulated. There were many things that detracted from this problem for me, I just can t ever really figure out what they are asking of me. Without my group members I do not think I could have figured it out. These problems are always usually very difficult for me so it makes me feel like I am not very good at math. I have always struggled with math making it hard for me to learn new things because im just trying to get through it.

I think that mathematics should definitely be taught with visual aids and group help. There is no way that I could handle not having help like this. Mathematics is not easy for me at all and I think it is like this for a lot of kids. A lot of kids do not understand math and so different ways of teaching things needs to be explored. If there is some way to make math fun and interesting for kids than I think this would help a lot. Using tables and graphs and deeply explaining the problems to the kids will help a lot. If they are doing things and they don’t know why then it will never make sense to them and they will never be able to appreciate math. I know that when I am a teacher I am going to try to figure out ways to keep the kids involved and excited about math. If they don’t have any interest then they will never want to learn new things.

Response

I completely agree with [S14], this was a really important issue that students should learn and understand. At first, just like she said I thought this would be impossible. After working through it with others you see that it really isn't. It is important to find a pattern in order to find a solution and this is exactly what her group as well as mine did. You can't just go shooting in the dark, there has to be some logic behind what you are trying to do.

[S14]

When I started this [activity] I honestly didn't think it was possible. I thought there were wayyyyyyyyy to many witht the same area and permiter cuz there are so many numbers you would have to try. I really didn't have to much faith in this one :) We used the tools to help us see that it was possible to find some because there really wasnt that many that worked, once i physically saw that it becam easier. At first we found two solid solutions but were a little stumped as how to make sure we had them all! We decide that we had to start with a number and multiply it by 1, 2, 3, 4, etc, and hopefully we'd find a pattern. We found an equation using this theory, after a lot of hard work and a lot of confusion on my part! . We found that for instance 3*4=12 for an area and then 14 for perimeter, then continuing up we saw that 3*5=15 for area and 16 for perimiter, going up by two for every 3 then a higher number. this pattern continued. and we found our solution.

I definitely think this is something i want to teach my future students. I want to emphasize how much SEEING the shapes helped at first, just to realize that this was actually possible. Then it was significant that we realized we need a formula to be certain that we had all the possibilities. This [activity] really mad me think, and strettttch my brain a bit, and I think that's super important, especially for new learners!

Response

I totally agree with S15 on the partner and group aspect. it's so so important to feed off of other people. I also like how the teacher will give you "hints" with out giving it away. It keeps you from getting frustrated with out making you feel like you can't accomplish the task on your own.

S15

In this [activity], my group began picking out numbers and just testing to see if the area and perimeter were numerically equal. We then decided to just go in a pattern type process and do the same number of rows with different number of columns. We started to make a table and then after awhile we noticed a pattern form with the numbers. Our table included the width and height of the square and the area and perimeter. We only made the chart up to width 6 because any further we wouldn't find numerically equal area and perimeter. We examined the relationship between the area and perimeter. We didn't need to use visuals such as the little blocks to help us figure out the area and perimeter. We found the 4X4 and 3X6 and couldn't find any more. We assumed there were more and we just had to keep investigating.

When students work in groups it is easier for them to get ideas from each other and come up with different tactics. I really like when the teacher walks around the room and gives little clues on how to figure it out, or helps us get into a certain direction. When student discover the problem some of their peers will not get it entirely and the other students will have to explain it which is another great way to learn. The has taught me that working with other students is a great way to figure out the problem even if you do not totally figure it all out together. Going home and working on it by yourself after you already have ideas from other classmates is a great way to figure out problems.

Response

This [activity] did have a lot of solutions. We used a chart to figure it out but there were many ways to solve the problem. I agree using other classmates for feedback and suggestions was really helpful. Writing out the chart and using the blocks as visuals were great too. That is very true that all different kinds of math can solve the same problem. There were many different ways to approach this problem.

S16

This [activity] was one of my favorite yet. In this [activity] we needed to find out how many rectangles had equal perimenter and areas. My first thought was that this was going to a lot of trial and error, my second was that it was going to be a lot of time. After starting this project I realized how much learing I was actualy going to do.

Going through all the steps and moving from trial and error to making a chart of all possibilities to finding a pattern really taught me how a student works and thinks as they do these types of projects. I learned that you can't teach a student to learn, they have to teach themselves first. This was a really good way to understand and really put yourself in their shoes. I enjoyed this activity and thought that it was something that was really helpful.

Response

I completely agree with everyhting you talked about. This [activity] was really good for helping us learn how to learn and it really taught my group to work together. I thought that it was also helpful with my own homework and thinking about how I can stratagize this more. The [activity] was one to really put you in the students shoes. It wasn't too hard or too simple but it really made me realize that I won't always have someone to tell me how to do things, sometimes I have to learn them on my own.

S17

As always, I found this [activity] to be challenging. I seem to always think the easiest strategy at first is to guess and check. I tried this strategy for [activity] #3 as well. We think we find all the possible outcomes of equal perimeters and areas at our table, then we hear the total we are suppose to have and realize we are one short. This seems to be a reoccurring pattern with all my groups with [activity]s. Our first instinct is to guess and check, but formula or tables always seem to be the better approach. With this [activity] we made a table for all the lengths until we found the matching areas and perimeters. This helped to organize our data and figure out the missing one that we couldn't figure out before with the guess and check strategy. This activity was a little frustrating at first when we couldn't figure out that last one, but once we got the strategy down it was much easier and it made me feel accomplished when we figured it out.

Personally, I feel like I learn math better when it is organized and I know where I am going with it. I feel like formulas and tables help me understand math problems and know how they are done. Guessing and checking are okay to do for some problems, but it is definitely not my strategy of choice. It makes me feel like I could be going nowhere and I don't know if I could be right or not. I liked to be organized and know I am going in the right direction. That is how I will teach my students, with formulas. I like to show how the formulas were formed too though, like we do in class. It helps me understand the formula so I can understand the problem. I want to help my students understand the math behind the math so that they won't get lost later on when they get older studying math.

Response

I completely agree with [S15]. We started out by just picking numbers and testing them too. We then used tables like she did, but we didn't get that idea until the teacher walking around the room gave us little hints and clues. Working in groups helped us to work together and figure out new and different ideas, but we needed the teacher to help us to figure out the way to get the whole answer.

S18

For our third in class [activity] we were asked to find all rectangles whose area and perimeter are numerically equal. When I started the activity, I was able to find two rectangles right away. What my group and I had to start with was drawing out tables that showed a rectangles length, width and its corresponding area and perimeter. By doing this we quickly found all the numbers we were looking for. I came to the conclusion that there were only two combinations that worked, 3x6 and 4x4. It got challenging when I had to prove that there were no other combinations. I decided that in order to know for sure that these were the only combinations I had to go back through my tables and examine the relationships. I realized that as both perimeter and area grew in value, area increased faster than perimeter. With the area increasing in value faster than perimeter, perimeter and area could never meet. I only made tables up to six because anything higher would never meet an equal numerical value.

This activity taught me that having students experiment in the classroom is an important thing. When students can experiment on their own they are more likely to remember the information learned. I felt that this activity wasn't too challenging and was manageable to complete. I felt like I accomplished something by completely the assignment. It was good for me to be able to work with other people at my table on this activity. I wouldn't have known to draw out the tables if someone at my table hadn't suggested it. Small groups are good for young students so they can brainstorm and work together and even help other group members if they are confused and don't know what to do.

Response

I agree with you that it is important for students to work in small groups. Sometimes a student may not know where to start with an assignment so having other people to work with can get them started in the right direction. If the teacher does not give students much to work with they could get the help they need from peers.

S19

I found the last [activity] fun. I really enjoyed using my brain in different ways. Growing up I was often teased for finding pleasure in solving mathematical equations and or long answer questions. This question actually gave me the opportunity to use those "fun" ways to solve an equation. However, the way I turned in my answer is much different than how my brain initially solved the problem. As soon as I looked at the problem at hand I immediately thought of solving via linear equation. The equation in question however, was much more of trial and error type. I came up with a pair and solve equation based on numbers scaled on the positive end of a X Y correlation. I actually found joy in "beating the system." Instead of having to just insert different numbers until I had met all matches, my design (with the help of a statistician friend [the student's father]), seemed to beat the system in quickness. If I would have had to work solely in a group I never would have come up with such a system, as I was already pretty nervous to just show this to my dad. Fortunately he loved it, as did I. This activity taught me to be more creative in my answering, as my answering was pair and match- an activity that young children often take part in. I found the activity fun and also helpful, as now I can use the same system for larger points; which, dorkily, I find exciting. I won't down play the use of groups, as they are wonderful in confidence building as they are alo helpful in learning how other students brains' work- which I find incredibly intriguing. I also think this is a great activity for groups as it does help with team camaraderie. Excellent all around ☺

S20

In this [activity] our class sought to find all possible solutions where the numerical value of a rectangle’s perimeter and area are the same. My group and I began investigating this problem by simply picking and choosing different rectangles to see if they fit this criterion. It was not long before we discovered the 4x4 and the 3x6 (or 6x3) rectangles. So far these were the only solutions we found. At this point we assumed it was likely there were more solutions but our search soon became tiring and frustrating as we were left unsuccessful in our endeavor. The process became less tiring as we discovered a pattern by which to search for solutions, investigating the possibilities of each side length starting at 1 unit. We soon found that a side length of 1 or 2 units were both impossible to have the perimeter and area equal to each other. Also by setting the perimeter and area equations equal to each other and solving for one variable it became much easier to see which rectangles worked.

Having students work in groups, allowing them to bounce ideas off one another and to help teach each other, and also having a teacher supervise the activity, offering useful hints and techniques to keep the students on track were both extremely useful in teaching this mathematics activity but can also be applied to teaching mathematics in general. By allowing this type of interactive learning, students are found not only discovering the material but also explaining it students who may not be able to understand as quickly. This benefits the students who do not catch on as quickly since students and teachers can explain ideas in a different manner and different methods can be explored until the concept is grasped. For those students ahead of the game, they will not be left sitting bored waiting for their classmates to catch up because they can help their peers better understand. Through this will come to know the subject material more deeply by teaching it themselves.

Response

I like the benefits of working in a group that you point out. Like you said, we all do think in different manners and due to such fact we can all contribute a different aspect to the overall understanding, leading one another to the solution and helping steer one another back on track when we stray from the right direction. I think this is a crucial aspect of a learning environment that rewards not only understanding but even satisfaction and confidence in one's ability.

S21

I was gone on the day that we did [activity] #3, and this meant that I had to work on it by myself. After reading it though I understood what I was looking for, but I wasn't sure how to approach it. I started by taking two numbers and plugging them into the equation to see if they worked. This lead me to problems, I wasn't sure how I would be able to make sure that I found all the ones that work. After asking someone else how they approached the problem I was able to find a strategy that worked for me. With this strategy I was able to find a way to effectively solve the problem. Being able to ask some people for advice helped me to have a further understanding of how to solve the given problem. It gave me ideas that allowed me to come up with a way to solve the problem.

This activity allowed me to think what it's like for students to be learning something for the first time. This allowed me to realize that when receiving problems for the first time it can be frustrating when they are not easily solvable. But once you figure out the solution to the problem it is much more rewarding. When giving the problem first before teaching how to solve it, it allows the student to analyze the problem better. Even though they may not figure it out or may even get frustrated, once they figure out how to solve it, they will be rewarded. They will be able to understand why a formula works, or how certain things are derived from other things. They will have a better understanding of mathematics. Also by working with each other and in groups children can work off of each other and learn for each other.

Response

I used the same methods to solve the problem as [S20]. I first started by trial and error, and then found a way to show this work more effectively. I also agree with her that working in groups can be a very helpful tool to solving problems. Also I feel that students can learn from one another, collaborate their ideas, and can also have fun.

S22

Initially our group thought this [activity] was going to be really hard. We decided to do an organized method by doing a chart that measured the possible volumes and areas for a certain number of blocks. This method did not really help us discover a pattern that was good enough to prove all of the possible dimensions. We did however discover a pattern that helped us eliminate dimensions that we knew would not work. We discovered that any rectangle that had a side length of one would not work because the resulting are would always be two units greater than the perimeter. We were a little bit lost at how to mathematically prove all of the possible dimensions until we got a little guidance from [the instructor], who helped us on a path to look at the problem algebraically. We set up an equation to show that the area was equal to the perimeter for a square which was $4X=X^2$ and then solved to discover that a 4*4 is the only square that worked. We then generalized that equation for any rectangle by saying that (X*Y)=(2X+2Y). by solving that equation for Y we plugged it into our calculator and were able to find the rest of the solutions.

Doing this activity reinforced for me the idea that is important as a teacher to instill in your students that there are multiple correct ways to solve a problem in math and the more ways you can solve a problem, the deeper your understanding for the concepts involved. In addition, I really like the idea of guided discoveries as a method of teaching some concepts in mathematics because if the children are able to unravel the concept for themselves then they might come to understand it better. I believe it is best to ensure that the students truly understand the math behind why things work rather than only knowing the process of solving certain problems.

Response

I really agree with [S18] that when students get to experiment with the ideas themselves, then they are more likely to remember the information learned. Moreover, I agree with her observation of the importance of students being able to work with others, allowing them to brainstorm and communicate their ideas when performing a guided discovery.

Our groups had very similar approaches by creating a table which was very helpful and allowed us to discover an equation that related areas and perimiters.

S23

For this particular activity, going through and finding a pattern aided me in solving the end question. Though this was the long way to go about solving the problem it aided me in seeing what was really going on instead of just having a bunch of formula's. Since I knew where these formulas came from it proved to be much easier to explain my answer; as well as how I arrived upon this answer. The part that was the hardest was to attempt and find a second way to come about the same answer that I have previously found. This was difficult because I wanted to go back to the original solution because I already had a way to solve the problem. To me this was a little bit easier of an [activity] than previous ones other than finding a second explanation.

First students should be allowed to just explore the problem and see what ideas they can come up with working on their own, or in groups. Then once they have explored their own ideas getting together as a class and bouncing the ideas they thought of as a class may spark ideas that are more fully thought out than the starts they had began with. As students generate ideas and build off of others ideas there is a need for praise, encouragement and guidance. A students idea should never be cast out as a bad idea, but rather one that might not be the best idea for the problem at hand and more brainstorming is needed.

S24

In working through this activity, I thought having the group setting was extremely helpful. Since the strategy of drawing out the figures worked in previous [activity]’s, we went that direction first. It was hard to come up with a tactful charting system to put down our ideas. This was especially true after we had come up with the two solutions, but were trying to find a pattern to explore more choices and rule things out. After coming to a dead end, we tried to look at it individually to see if we had missed something. We eventually stumbled upon an algebraic strategy which led us to the function which we graphed. Only after finding the graph and plotting the points were we able to justify the solutions as the only ones. Still, after this proof I found it difficult to try to express this in terms of the lengths of the sides and the chart we had attempted previously. Turned out we had started along a couple different pathways to solutions, but were only able to see one of them through in its entirety. Even so, it was nice to see that all of our attempts could have been fruitful if pushed further.

This proof more than the previous ones exemplified to me the different approaches that can be taken to solve mathematical problems. In this case there were many solutions, all utilizing different mathematical strategies (algebra, geometry, graphs, charts, etc). I hope it’s encouraging to students successfully problem solve and having a barrage of strategies to throw at an activity would be a confidence builder in this sense. It also shows students that all different types of math can be used to solve the same problem. Math IS relevant!

Response

It is interesting that your group as well as ours attempted to use strategies, like looking for patterns, that had applied to previous [activity]'s but eventually used the algebraic strategy to solve this one. Without the little "hint" from [the instructor], I don't think we would have stumbled upon that route for a while and after much frustration. I also think its important to convey to students the depth of understanding that can come from solving a problem in multiple ways. I was a little frustrated after coming up with multiple solutions and not being able to see them through. But after successfully working through one problem it was easier to see the other ones through. Keeping this process in mind when teaching is important, as is giving students the time and tools to discover the solutions and concepts. In other words - I agree with you!

S25

For this [activity] we had to find all rectangles who's area and perimeter are numerically equal. At first I was very confused because there is an infinite number of numbers. But when I figured out the relationship of area and perimeter it became very easy. I started by making tables that showed a rectangle's length, width and its corresponding area and perimeter. In doing this I quickly found all of the numbers I was looking for. I soon realized that there were only 3 combinations that worked, 6 x 3, 3 x 6 and 4 x 4. But my predicament came when I had to prove that there were no others. At this point I felt a little lost because I wondered, "How could I possibly show that these are the only combinations of numbers that work when there are an infinite number of numbers to make combinations?" After examination and contemplation of my tables I realized that as both perimeter and area grew in value, area increased faster than perimeter. Because area increases in value faster than perimeter, perimeter can never again reach the same value as area. Experimenting with numbers higher than 6 showed that the values would never reach an equal numerical value, thus there are no other combinations of length and width that will produce an area and perimeter that are equal.

This teaches me that experimentation in the classroom is a powerful tool. When students can discover properties on their own, they are more likely to take value from the lesson and remember the information later in life. I feel like I have accomplished something personally when I can problem solve and succeed. It was also very beneficial to small groups like we did. I came in late and was very confused but I was quickly briefed by my group about the purpose of the assignment. They taught me what they had already figured out and it was very beneficial for me to hear it as well as for them to teach it. Small groups are a powerful tool for students because they can discover and learn together as they explain, brainstorm and instruct others.

Response

[S4], I like how you mentioned that children need a variety of strategies in their "toolbox" to help them to figure out problems. I completely agree and I feel like when children do get the variety they need, that they will become much more well-rounded and less stressed students. When I don't understand how to go about a problem, I get stressed because it makes me feel unintelligent and frustrated. When I know a variety of problem solving strategies I feel much more comfortable tackling the problem.

ACTIVITY 4 REFLECTIONS FALL 2009

F1

This [activity] activity was one of easier activities for me to put together. I certainly did not find it easy, however, the use of visual tools was very helpful. If I had not been given the various polygons to work with I would have gone nuts. That being said as soon as you think of the logistics involved in making a tessellation work, it really wasn't that difficult. The angles all need to work together and so all we needed to do was find which combinations worked together to make 360. I can see how this assignment would be very frustrating and confusing to kids. The math involved is not all that complicated and creating a chart - as we did in class- to discover the interior angles, etc. would really help affirm the concepts. The use of visual aids as another guiding tool is very helpful in this activity, and can act as an aid/proof to test their math/shape combinations.

F2

Well, I thought this [activity] was kinda in the middle in terms of difficulty. I briefly looked over it before class and didn't really know what to do about it. Once given the shapes in class it was easy to discover how the relationship worked-having 360 meet up at the interior of the shapes. I thought the class did a great job finding all sorts that worked! [the instructor] then asked us to find if there were more. I thought that there would be an easy way of making a chart kind of like "handshakes" to make sure we didn't miss any. But I had no clue how to do it easily without handwriting them all and taking an eternity. Later on in class when [the instructor] showed the special trick on the calculator with y = (180 * (n-2) / n...it made much better sense. Also we determined it had to be between 3 and 6 shapes because of triangles and hexagons making 360 degrees regular style. So then I just used my calculator to find all the combos that would work. I found a total of 7. then i fiddled around with the shapes that had more than one of the same number of sides and more than 3 sides to see if you could arrange them in other orders. the only one that seemed to double was the 33334 or 33434. so i believe there is a total of 8 shapes and 7 angle combos. i would say #1 it helps to have the physical shapes that you can manipulate becuase visualization without them can be nearly impossible and can appear to work but then when you put the shape together or repeat it you see it doesn't work #2 a systematic way of double checking or even listing possible solutions makes your brain less tired.

F8

This assignment for me was more difficult than all the rest. For some reason I could not wrap my head around the concept of the proof. I made my best effort, and even though we were given much more guidance this time, I was not able to understand fully the reason there were only the number that we found. I learned in this activity that even though you understand the conept fairly well, proving it is what it hard. Other we have done have been more clear in the answer, whereas this one is a bit harder for me. Maybe I am spatially challenged! Either way, I think that this was a good experience. We learned more about working with a team, the importance of perseverance, problem solving, self-discovery in math, and most of all about tessellations! When I am a teacher, I would use these types of activities, maybe not this exact one. I would love to let my students have the opportunity to learn something on their own, to explore and discover some of the cool things in math that lots of people don't know. Students need guidance when doing these projects, sometimes it depends on the students as well. If a student is on the right track, go ahead and let them keep plinking away. If not, it is important to provide other guidance to ensure that the student is not so far gone and frustrated that they don't care anymore. That happens more often than we think it does. Overall, I think we can see clearly that these assignments have huge benefits when teaching math because it really allows a student to evaluate their knowledge, apply it, and use it prove something tangible. This whole idea of proof as we have seen can be very difficult for students. I believe, however, it is very important. I always remember kids asking, "But why?" Now, we know why, and we can use the skills we have learned so we won't have to keep asking, "But why?" for the rest of our lives.

F9

Initially, I thought that this activity would be fairly easy to complete. I think that the tablework was very helpful, in that it gave us all a chance to see what others were thinking. I thought that the models provided were very helpful. It was really easy to find semi-regular tessellations using these. My frustration started when we figured out that some of the candidates we had found did not work. They worked around a single vertex but did not tessellate. This made the process a little bit harder but it was made easier by [the instructor]'s method on the calculator. You did need to find a couple of rules though before beginning. You had to find out that there are 360 degrees around a single vertex and that the maximum was 6 and the minimum was 3. After that though, it was a breeze.

I think that this sort of activity can be very useful and effective when in the classroom. I know that students will become frustrated in their attempts but sometimes that is the best way to learn something. I know that by learning constructively, students gain a better appreciation for the knowledge they gain because they have seen the struggle they have to go through in order to discover it. I will definitely be using activities such as this not only in mathematics but in many other subject areas because I see it as a very useful means of teaching and learning.

F12

As I worked on activity 4 I felt that I had an idea of how to minimize the possibilities. I had an idea because of what I've learned from the past activities in the class. That is, if you want to be positive about your answer, make a chart, or draw a picture, basically come up with a method that makes it easier for you to understand. For some reason I like to start out with guess and check and make sure I'm not going through a whole chart when it's not necessary. Yes, sounds pretty lazy, but this way I can try and think about the problem in many different ways with out putting a direct method on the problem. After thinking about it my group and the rest of the class agreed that the angles meeting at an intersection must add up to 360 degrees. This is where the chart came into play. We tried every angle from a triangle to a twelve-a-gon. This was the part where everything started coming together and I really started to feel confident in the semi-regular tessellations I had found so far. During the process of coming up with an answer I felt a little frustrated but more determined than in the past. And as most would say the ending result is a great feeling, especially when you a fairly confident in your answer. These activities have been of great help and I believe that if given enough instruction many people could come up with the answers to the problems presented in the activities. The key is to have patients and not rush into an answer, think logically about the questions and a method that will work for the situation. I hope to teach my students that it's not how fast you solve a problem but how much you learn from the problem solved no matter how long it takes.

F13

As with every other activity (I don't know which one was the hardest) I was lost from the start. Then we got the shape blocks and things started to make more sense. Although I found patterns that worked with the regular shapes I wanted to stick some shapes in there that weren't regular and then I was told I couldn't do that. I seem to always think there's another theory when there's really not. I also had a hard time checking them. I'm a terrible drawer so it was hard for me to see which ones would work and which arrangements wouldn't tessellate. Many of the arrangements we found in class actually worked and they showed me something else. As working through which worked and didn't I saw a pattern of sorts. The tessellations that did work had shapes with a number of sides that were multiples of each other. That seems confusing so the arrangement 6.3.3.3.3. has 3 which is a multiple of 6 the same with the 6 other arrangements that work but 4.3.4.6. doesn't work because although it adds up to 360 degrees 4 is not a multiple of 6 and when you put it together they do not fit correctly. Back to the actual MRA, it wasn't easy however it's a great experience to work hands on with a mathematical theory. My final solution to this whole activity was many arrangements that made 360 degrees however only 7 of them can actually tessellate the plane without overlapping or leaving a gap between them. It's good for kids to work with the shape blocks so they can see how many sides a shape has, its angle measurements and what shapes can be out together and create patterns with no spaces in them. This experience is a great time to show kids that math isn't all about definite equations or particular solutions but it's about experimentation. That's the one problem I have in math because I am such a concrete person that experimenting with new methods is really hard for me and I never believe that I have the right answer. Starting these activities in class started me out on the right track and I was then able to guide my own way to the solution. I wouldn't suggest dumping activities such as these on a young student but with a little push in the right direction they can see what I have learned. I have learned that math is what you make of it. There's not always one solution, equation or way to go about solving everything. It's up to you as an individual and a learner to reflect in your mind the way you learn and they way you perceive things to be able to find your own way.

F15

I enjoyed working on the tesselations with my group in class. I thought it was interesting to see how many semi regular tesselations we could find. We found a couple that we were sure would work until we built them out one more and discovered that they only worked once so they actually didn't tesselate. Our group was able to find eight semi regular tesselations. I admit that I peeked on google.com to see how many semi regular tesselations are possible. I think that the web site that I ended up on was called mathfun.com or maybe funwithmath.com. Anyway, the web site said there are only eight semi regular tesselations. Our group had found all eight just by working with the manipualtives (shapes) in class the first day.

I did plot the formula (X-2)180/X into my calculator. I tried to see if there were any more by using the formula and the hints that [the instructor] gave us in class. I have to admit this was frustrating to me so I gave up. [the instructor] had mentioned that there were only a few possibilities so I decided after using the formula to try to find others and having no luck finding any more than what we had found in class that I would look on the internet for the answers. I think that the reason I was getting frustrated was that we had already found all eight so naturally I was not finding any more.

I think that this was a useful [activity]. I also enjoyed the pictures that we made for the bulletin board. I can see using both of these ideas in a class room in the future. I think that students would get a great deal out of both of these projects. These assignments are challenging enough to get the kids to think and participate but they are not so hard that the kids would get frustrated and stressed out and quit.

F16

Overall I really liked this [activity]. I thought it was challenging and required a lot of thought. At first, I had a hard time seeing a way to solve this problem. If I was to guess and check it would take for ever. My group in class had tried to find some sort of math equation that would allow you to just plug in shapes and the answer will come about.
I first thought that there was a total of 8 tessellation but after taking a look at all eight I found that one of them didn't work as I first thought it would, so I concluded that there is only 7. I think this is a good activity to have a math class tackle. It will help the students become better problem solvers and teaches them to think "out side the box" when looking for patterns and possible solutions. As a teacher I would be able to see the different strategies the students are using, thus allowing me as a teacher to better connect with my students.

F17

Over the last four [activities]. I have learned a lot about how I learn and how I want to teach math. Every assignment started out with me misunderstanding and often, particularly on number 2, quite frustrated. I couldn't understand how giving a student a problem to figure out without instruction could be helpful. I would go home and work on the question only to become disheartened as a math student. As the assignments moved through the semester I began to realize their value and, when give a little help, started really enjoying them. This fourth [activity] is the crowning of the progression. Rather than felling lost and dumb, I felt like I was always on a good track and knew I would find the correct answer.

This is because of the way the tessellation question was set up. I felt that when I left the classroom I had a better understand of what I was looking for; I knew what a semi-regular tessellation was and how I could start finding them. Even better was the use of manipulative on our work day. By using the manipulative my table was able to discover there are only three regular tessellations and, with guidance from the professor, deduct that we could only have vertices of 3, 4, 5, or 6. From there the class began testing possibilities and as a group made a list of every one we found. Once we felt they were all found we worked as groups to find out how we know which sets worked. When I left class that day I know I had too many, but I know how to figure out which worked and which didn't. To add to my enjoyment of this question we discussed how we could make a table to be sure we had found all possible semi-regular tessellations. I had no doubt that I had found everyone when I turned in my paper.

As a teacher this is how I want my students to feel about math. I want them to discover things on their own and be okay with failing at first. In order for that to happen I feel I need to give them the proper scaffolding (I'm a Vygotsky fan) to attain their goals. If I can give the proper guidance the students not only learn to think critically, but do so in an environment that supports them the whole way through.

F19

This activity was my favorite one we did all semester, its like [the instructor] saved the best for last :) When we got the hand out I was confused as I was reading it and then you did some examples and explained it to us and I got super excited! I loved that we were able to work with the all the shape blocks! I love being creative so I really enjoyed moving all the shapes around and mixing them up with other shapes to see if it would make a tesselation. When we did this in class, the class went by so fast! My table found a lot of tesselations and then we had to bring it home adn work on it ourselves. I worked on it but I couldnt find any more tesselations so I figured my table had them all. Then we went back to class and showed us another way to look for them and I found a few more.

This was the only reflective activity that I didnt get frustrated with and I really enjoyed it, it was a great way to end the semester!

F21

[Activity] #4 was by far the most difficult activity we have had so far. I stretched my brain to the max and eventually ended up with a general idea not the specific answer I feel though. I found that there were only eight times that this could occur but trying to explain why was the hardest part. So even though I could understand the basic concept I still do not fully understand. The other activities were fairly easy to explain once you figured out the problem of the proof but this one took more time and effort to explain. This was a good activity though because you do not realize how much work goes into those everyday tiles that you see in your house or in other buildings. It is amazing to think that so much math is used in the process of tiling your floor. That is why these types of activities are great you have to solve a problem plus apply your knowledge rather than just solve an equation. It was nice to have the guidance in class that set me on the right track for what I was actually looking for otherwise I would have been completely lost. To solve this problem you had to actually lie out all of the possibilities to see if they would work. After trying out a few and realizing that the answers made sense the activity began to come to light. These activities were a great way for students to discover things on their own and come to a conclusion why.

F22

I personally liked this problem better than any of the other ones that we have done this semester. I think that it presented the right amount of difficulty and initial understanding for me personally. I also really liked that we worked on this problem not only in groups but also more as a class than we have on other problems. I think that having groups ideas put into it along with knowing what everyone else in the class was doing was beneficial in a couple of ways. First of all I think that having input from the entire class eased some of the minor anxiety I get from not knowing if I or my group are on the right track. Also when working with the class it felt like we were coming to conclusions faster and ideas of how to make sure that all the solutions were found were be generated faster. I can use what I experienced in this exercise as a teacher by realizing that even though it is a good idea for kids to come to conclusions and discover things on their own it is also a good thing to let them work not only in groups but also a class to discover the answer from time to time. Because some kids might see things different than others, just like in our class some people are able to solve things by putting together the tessellations and others were better at solving things using algebra.

ACTIVITY 4 REFLECTIONS SPRING 2010

S1

This [activity] was, once again, very frustrating for me. This one was different however because the only way for me to really figure it out was proof by exhaustion. It became really annoying, especially when I wasn't sure if I was doing it correctly. Without assistance I really don't think I would've figured it out. I think it was the most frustrating for me. It really helped to find a pattern in class, but then it felt really repetitive after that. It was really fun right at first, but in all honesty I was glad to be done with it. I guess over the course of the class [these activities] were my least favorite part. But reflecting back I must say they were a very good way to learn how curious math can be. Over the semester these [activities] proved that math problems can be solved a number of different ways. Also it showed us how frustrating annoying and repetitive math can be. But really one can say that that is life in general!

This [activity] was a good example of showing students how sometimes there is only one way to solve a mathematical problem, and that was proof by exhaustion. Frustrating as it can be, it is very rewarding to discover it for yourself, by yourself. I feel that math isn't something that can be drilled into a student's mind with different equations, rules and proofs, but basic concepts can be taught that will lead to learning and understanding. In a way math is like art. Some people are natural at it while others need a little more assistance, and the beauty of it is there is more than one way to do it. It is also, in a sense like history. History doesn't change and I don't believe math really does either. There is always time to understand math more deeply, and it is a subject with great patience for the different people trying to learn it.

S3

When I received the fourth and final [activity], I was excited to see what was next. When looking at the title “Tessellations” I was extremely optimistic on how fun it would be. I had remembered how much I enjoyed making tessellations in elementary/middle school, and was glad to have to opportunity and challenged to discover more about tessellations than I had previously known.

Once it came to time actually start coming up with a proof about the tessellations, I was stumped. I had no clue on where to start. With no plan set, I messed around with the combinations I could make at one vertex. I knew that the sum of all the angles at the vertex had to equal three hundred and sixty degrees. After fiddling around I realized that a square has ninety degrees in one corner and a triangle has sixty degrees in a corner. After that I just fiddled around figuring out all the different combination I would make with those numbers so that they would add up to three hundred and sixty degrees. I am not sure if I actually came up with a solid proof, but I did learn something about tessellations, and I believe that that was the objective of the [activity].

This [activity] was interesting for me. I did not think it was overly hard to come to a conclusion, but I did have troubles proving my conclusion. Many times, students can come to a conclusion and not know exactly how they got there or how to explain what happened. I recall having that issue a lot growing up. I would come with a correct answer, but the teacher would mark me partially wrong because I had not shown all of my work. With practice I was able to break down what was going on in my head to show my work, and I believe by giving a child a common known concept like tessellations, it will challenge them to discover more in depth about things they already know, but have not put onto paper yet.

S4

I found this [activity] to be the most fun to experiment with. When we were trying out different combinations it was interesting to see what my group members came up with that I hadn't thought of. I think it helps me to better understand a problem that I really have no previous conceptions towards by starting to physically experiment with manipulatives. After we discovered what "should" tesselate, I became slightly confused when we found out that some of those combinations didn't tesselate after all. It was helpful to try that out in the classroom versus outside of class when we didn't have access to the manipulatives.

I thought this [activity] was one of the hardest for me to prove. I felt like my proof was believable, but not entirely convincing. It sort of reminded me of the beginning of the semester when I was first being exposed to proofs and I was unsure of myself when first starting to prove things on my own. I finally decided on using a proof by exhaustion but I didn't really elaborate on why the answer is what it is. This activity reminds me that making connections is an incredibly important ability of students. Using what we know in many areas of mathmatics including algebra, geometry, measurement, etc. and applying them to activities like these is essential part of math education. Many students feel the various areas of math are totally distinct and can never be applied into the other areas. Exposure to problems that include several math disciplines can lead to higher levels of thinking.

S5

In this [activity], it was very important to be patient. It took a lot of trial and error to find how many semi-regular tessellations there were. It showed me that mathematics is sometimes best explained by hands-on activities like building the tessellations and finding out which ones do not work by seeing the combinations. The trial and error would be difficult for some students that prefer to have a formula instead of creating their own way of solving a problem. I thought that the building part was very fun, but after trying about 20 combinations, it was frustrating. To know if you have found them all is simply by testing and building.

It was a very good activity to use with students who need to touch something and be able to manipulate it in order to understand how it works. It was a new way to look at polygons and how they interact with each other.

S6

This task reminded me that in math it is important to not give up when one strategy does not seem to be working. I started the task and before I knew it I was disorganized. It took patience to start over and try a different approach. It was also helpful for me to attempt the task in a variety of settings and work for short amounts of time. For example, I worked in the math lab and at home. I also went to get extra help during the instructor's office hours. I was absent the first day of the assignment; therefore I was somewhat confused about the task. My absence distracted from my learning. It is important to have good attendance! I felt frustrated trying to catch up on the assignment. It was a time consuming task.

When I become a teacher I hope to offer students time when they can review the information one-on-one with me. I also hope to inspire students to have good attendance and motivate them to come to class every day. I would also offer a variety of learning tools for my students to use. For example, websites may aid in deeper understanding of the topic. For this particular tessellations problem, Geogebra may have been helpful. With a complex task I may allow a couple days for hands on exploration. If the students seem to be struggling, I would also give guidance and direction for staying organized during the problem. (A chart may be helpful). Although I would allow for use of certain websites, I would encourage students to avoid Googling the answer to the problem; at least until after they have reached a conclusion and full understanding of the solution. Students may also understand tessellations better and be more motivated if they have an artistic outlet, such as creating their own tessellation.

S7

During this [activity] my table worked really well together. First we pretty much started guessing and checking to see how many different combinations we could come up with. We ended up actually finding them all this way but then our job was to prove that we had them all. A group member came up with the idea just to create a graph with all possibly options on it. Then even though they were possible by the numbers it didn't mean they were possible to create. This then crossed off all of the ones that couldn't work and we were left with only the ones that would. This was a good way to learn this concept. The graph was definitely the easiest way for our group to do it. Even though we had already found the correct answer we still needed to prove it and this helps us understand the fact that you need a way to backup your ideas and a way to show others that your answer is correct.

If I were trying to teach this lesson to my students I don't think it would have been at all successful without the figures [manipulatives] to workout with first hand. It would have had to require them to draw all of the figures and that would have been very frustrating for many. As teachers I think you need to learn and recognize that not all students will think the same way you will. Some will be visual learners while others will simply need to just take down notes. By leaving these figures out on the table for the students to use if and when they need them is a great idea because some students may have to create each figure as they go and others may simply create a graph to figure out which ones will and will not work. The key to teaching is recognizing and accepting the fact that everyone is unique, especially in learning styles.

S9

For this activity, it was very helpful to start out by creating tessellations with the provided polygons. It gave me a sense of what the assignment was asking for and was a great visual for me to see why or why not certain polygons would not tessellate. It was very helpful to have the actual polygons in front of me because then I was able to just start choosing different polygons and putting them together to see if they would tessellate or not. With the use of the visuals, the entire class was able to come up with many of the semi-regular tessellations. Now came the hard part, proving why only those certain tessellations worked. I had realized the polygons that could tessellate had something to do with the vertex, and that each vertex must add up to only 360 degrees. Even though this helped to narrow down how I could prove why certain polygons could tessellate, I still was not entirely sure how to prove it without taking up loads of time. After we did the table in class about having three polygons at a vertex, I had a much better sense of how to go about solving the problem.

I think this activity is a great one to prove that using visuals helps to solve certain problems and allows for the students to get at least the basic concept of what the problem is asking for and how they can go about solving it. Some students may be more visual learners so having the opportunity to use tools will allow them to understand the problem and create a solution. This activity also made me realized how sometimes if we are given time in groups, and then come together as class to discuss what we have, and then finish the problems by ourselves, we are able to get a wide variety of input. It gives us the chance to do some work by ourselves, but also create some answers with others, something I believe is very helpful when solving mathematics.

S11

When we first started manipulating the polygons at our table, I was pretty excited about this [activity]. My family's always been big on puzzles, and I hoped that this tessellation would be easier for me than some of our other projects. However, as other classmates started calling out polygon combinations, I started to realize that this project was going to require a bit more than making flowers out of triangles and hexagons. In fact, I was unable to come up with any combinations during class.

With the reminder that at any meeting place of vertices for the polygons there would be a total of 360 degrees, as well as the removal of the physical polygon pieces, I had an easier time. From there it was simple addition of the interior angles to add up to 360 degrees. I kept getting overwhelmed though, thinking of the possible combinations of internal angles, so this was an [activity] that I had to take many breaks during. Working in a group with [S19] and [S1] helped, not only because it seemed we were all equally overwhelmed by the proof by exhaustion aspect, but also because we were able to bounce ideas off of each other and create patterns to make the process less confusing.

Because I'm not a fan of proof by exhaustion, and that seemed to be the only option for this [activity], it was definitely not my favorite. It was also the first [activity] where not having manipulatives in front of me made the process easier. As a future teacher, I have to say that having the polygon tiles available for students is important. While they weren't helpful to me, I'm sure they would have been to someone else. Not only that, but they served to generate interest in the [activity] that may not have been present otherwise.

S13

For this particular [activity] when [the instructor] walked us through it in class I completely understood what you were talking about and followed the procedure we were supposed to follow. It made me realize that there are a lot of interesting ways to figure out math problems. For the most part the concept of tesselations makes sense to me. The regular tessellation and how there is only three makes complete sense but then when I was given the task to find the semi-regular tesselations I got completely lost. I had no idea where to begin and couldn't get anywhere on my own. This activity didn't make me feel very good as many of them didn't. I felt most of them were very abstract and I am just not used to learning math in this way. If I am given the tools to figure it out then I can do it but if not im not very good at coming up with things on my own.

I don't think that all these [activities] were a waste of time, they were just hard. I think it is a good idea for kids to have to figure things out on their own. I think something that would be helpful in the future would be to have the kids try it on their own but then within the next few class times walk them through things step by step so they are sure to understand and maybe even teach each other something. I think that by explaining a technique step by step, although it may be time consuming, could be helpful for children. Especially if they are more visual learners. Sometimes things will just fall in to place for certain kids if different techniques are used. There are different kinds of understanding that every kid has and you have to be willing to help them find their particular learning technique.

S17

This [activity] was the most challenging of this year. Being able to actually try out tessellations with shapes really helped. Later when we took a different approach to finding all the possibilities that added up to 360 degrees, we still needed that hands on experimenting to test if the possibilities worked. We had a group that that could try out different ideas too, so that was very helpful in finding all the tessellations. We needed everyone in our group thinking up new ideas after we tried one that we thought was working, then we carried it out and it turned out that not all the shapes fit together. It was frustrating when you thought you found a tessellation and it turned out it wasn't one. With our group working together trying out new ideas, we figured it out though.

All the [discovery activities] this year, and especially this last one, has taught me that one of the best ways to learn math and understand it is to work in groups and have hands on experiments. Groups bring out new and interesting ideas that I might not be able to come up with on my own. It gets everyone working together and that might spark a new idea from me too. It can be inspiring to work with others in groups. Also, hands on experiments give you a tangible activity that can really enhance the learning process. When you can see and experiment with actual shapes, it is easier to understand instead of just talking about it. I strongly believe in learning and teaching with groups, especially in math. It has helped me with my learning experiences and I plan on using the same strategy for when I teach. I feel the same way with hands on learning. It's much easier to learn something when you can see it rather than just lecturing about it.

S18

This [activity] was very challenging for me. Being able to actually try out tessellations with shapes really helped as well as when we took a different approach to finding all the possibilities that added up to 360 degrees. It really helped having that that hands on experimenting to test if the possibilities worked. We had a group that that could try out different ideas too, so that was very helpful in finding all the tessellations. We needed everyone in our group thinking up new ideas after we tried one that we thought was working, then we carried it out and it turned out that not all the shapes fit together. It was frustrating when you thought you found a tessellation and it turned out it wasn't one. With our group working together trying out new ideas, we figured it out though.

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S19

In our final [activity] I found more frustration than I found logic. I am not a fan of exhaustion techniques, I am always searching for a way to “beat the system”. It is possible that there is a way to pattern the problem out into a way that would be quicker, but I didn’t look into it. Instead I just used a check and guess pattern as a way to try all possible outcomes. I worked with [S11] and [S1], and working in a group was helpful. We worked together to organize the possible variations. It helped me to work through it as a groups because I often find my brain working ahead of itself; I will see in my head what the answer is, or what I think a good candidate will be, way in advance. Then when I do get it on paper I have often lost track of where I was when the idea struck me.

It did however exemplify how difficult it must be to be a tessellation guru. I mean, sure you know what vertexes need to pair up, but once I got to really visualizing the possibilities it seemed pretty boggling. I have a hard time with regular tessellations, and this was with semi regular. It would however be a fun area to study. I look at math as a puzzle, and tessellations literally are a puzzle, so to me that was neat.

I do understand the thinking behind the assignment despite my frustration. It did reiterate matching vertexes and did also drive home which n-gon has which angle measures. If one did not know the angles that matched n sides this would have taken even longer. It was in that aspect a good review.

[These activities] have been helpful and fun for the most part, but this particular one was not my favorite.

S20

In this mathematics activity my group and I sought to find all the semi-regular tessellations that exist. We were given an array of cardboard polygons that served as great visuals to help explore this problem. By using trial and error my group successfully discovered several semi-regular tessellations but became too caught up in using the hands-on supplies in front of us. We soon became frustrated with not knowing how many more semi-regular tessellations exist, or how to make new discoveries when there appeared so many possible combinations of the polygons in front of us. This part was difficult but with the help of our instructor we were able to focus our attention on a more beneficial and efficient way in which to make new discoveries. We began investigating the interior angles of each polygon and mathematically determined the possibilities that could create semi-regular tessellation. Then after finding a possible solution the polygons in front of us were used to check and see if the solution was indeed a solution.

I believe that giving students a problem and materials that may aid in the discovery of a solution is key when exploring mathematical concepts. However with neat objects and tools in front of them (and us) it can be easy to get overly focused on the tools and forget to explore different ways in which a problem can be solved. This is where the guidance on part of the instructor and fellow students comes in. This is also key when exploring mathematical problems because it keeps the student on track. It allows the student to explore and gain their own understanding and at the same time ensures that they are not left struggling and frustrated. It is a beneficial use of their time, allowing a further and deeper understanding to be grasped on their own with helpful guidance when needed from each other and an instructor.

S21

When the [activity] #4 was handed out I thought that this could be fun. To start we were given the regular polygon models up to twelve. It was fun putting them together in random combinations, as a class we were able to find some that worked. After we heard that we realized that we needed a proof to prove that we had found them all we knew that we were going to need some kind of method to make sure we did. I didn't know how to even start to prove this. Luckily for me in class we proved that we had all the 3 at a vertex and it helped to figure out how to prove 4 and 5 at a vertex. I'm still not sure that I was able to prove the problem correctly but I think I was able to find all the semi-regular tessellations. At times during this activity I felt like I wouldn't reach the point to where I had proved that I had them all.

This [activity] was the hardest for me. I tried to figure out how I would go about proving this by myself, but I wasn't able to. I needed help. This helps me to realize that not all students will be able to figure out a problem even after given a chance to discover the method for them self. After receiving the help and the start I needed I was able to realize how things worked and it gave me a better understanding of why this way would work. I think allowing students to have a chance to figure out a problem for themselves, even if they aren't able to, helps them realize when they get help why that might make sense. Understanding how students learn and what methods are helpful is a very important lesson for me to learn becoming a teacher.

S22

This [activity] was particularly challenging, but well worth the effort that it required. It was nice to have the shapes in class so we could test our theories, although we found that having them distracted us from trying to figure out why and how tessellations are formed. We basically just started grabbing tiles and tried to stick them together until we found a set that tessellated. Once we figured out that for a candidate to work, all of the angles meeting at a vertex needed to sum to 360, the polygons proved useful in helping us verify whether or not a candidate worked.

I feel that the most important lesson that this [activity] teaches, beyond the value of using guided discoveries, is the value of allowing students to discover through the use of manipulatives. There is only so much learning that can be done from a description or a picture. I think that having something tangible that the students can manipulate, can lead to a clearer understanding of what is being taught. In addition, I find it very valuable to conduct guided activities such as this in a group setting to allow a variety of different ideas and techniques to come together and contribute to the group’s understanding as a whole.

S24

This last [activity] was the hardest to prove, I thought. At first I was having a lot of fun with the colorful manipulatives and I thought that there would be lots of solutions to come up with. The farther we got along in the process however, it became clear that just a guess and check, fooling around with the tiles wasn't going to get us the answers that we needed to solve the problem. The tiles were great for the initial visualization of the problem, but the formula for the interior angles of regular polygons proved most helpful in the trying to answer the question of how many semi-regular tessellations exist. The whole "proof through exhaustion" was very intimidating and I wasn't sure where to start to most efficiently systematically find solutions and prove they were the only ones. Having the discussion in class of how to get started on the three at a vertex solutions helped me to follow a similar pattern when taking on the four and five at a vertex.

This is experience was a nice reminder that even though we as teachers think we might have everything laid out in front of the students to help them solve the problem, sometimes is takes an extra little push to make it "click." I think I could have figured it out, but the complexity and time-involvement made me hesitate to start down a path that I wasn't sure was going to be the most effective. I think it is important to give the students tools to work with that give them confidence that they can solve the problem, but still making them work it through so they are getting an understanding of the math behind the solution and not just copying an example. It was great to work with the colorful tiles at the beginning, that really got me excited about the activity and I think that was a great introduction – if we had just started with the numbers I would have been way less interested.