DEVELOPMENT OF A FRAMEWORK FOR TEACHING MATHEMATICS IN DEPTH

By

JOANNE JENSEN LAFRAMENTA

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2011

To Charmi and John
Who invited me into academia and
inspired me when I faltered
To Scott
Who set me free
To Shreve, Candice, River, and Levy
Who illuminated my purpose

ACKNOWLEDGMENTS

This work would not have been possible without the support and mentoring of my committee. My chair, Dr. Thomasenia Lott Adams, gave unstintingly of her time and energy to mentor me in the Academy, to cultivate my scholarship, to nourish my quest for knowledge, and to envelope me with friendship. Dr. Stephen Pape and Dr. Rose Pringle nurtured me as a scholar, a professional, and a friend. I thank Dr. Nancy Waldron for joining my committee as an external member, giving her vote of confidence to my future as a researcher.

Three mathematics scholars preceded me in the pursuit of this degree. I am proud to follow in their footsteps. These ladies, Dr. Fatma Aslan-Tutak, Dr. Emily Bonner, and Dr. Kristin Spencer, not only accepted me as a colleague, but also allowed me to be their mentor, their confidante, and a surrogate grandmother. Into the gap of their absence came the Friday Support Group who encouraged my scholarship by letting me offer advice and contribute to their mastering of the English language. These dear friends welcomed me into the celebrations of their lives at the same time they offered support to my data collection and analysis. I acknowledge Shih-fen Yeh, Chu-Chuan Chiu, Ji Young Kim, Dr. Zhuo Li, Dr. Feng Liu, Qing Liu, He Huang, and Dr. Vasa Buraphadeja.

This study came together with the assistance of the Qualitative Study Group, organized by Dr. Mirka Koro-Ljungberg, for qualitative scholars to discuss methods of analysis as they pertained to our initiatory studies. As a fledgling researcher, I participated in sessions called to brainstorm solutions and induce analytic thought for scholars struggling with their presentations and dissertations. Later, the group provided this support for me as I struggled with analysis of the data from my study. I am particularly grateful for the insight and wisdom of Patricia Jacobs, Aliya Zafar, Dr. Jennifer Arnold, Elizabeth Filippi, Emma Humphries, Dr. Elliot Douglas, and Dr. Mirka.

Gratefully, I acknowledge the superior educational opportunity offered me by the College of Education to associate with the exceptional professors, graduate students, and staff that are integral to the achievements of this college. My personal and professional life was enriched by the course work and the presence of visiting scholars. As a University Fellow, I had outstanding opportunities to join research studies, to teach courses, and to serve on college committees. I was always afforded respect for my work, my thoughts, and my concerns. The experience of earning a doctorate is one of the richest of my life, and I am grateful to all who played a role in its development and completion.

Finally, I gratefully appreciate my family and friends, who have sustained me on this tumultuous journey. Without Sue and Darrell Hartman, this study would not have occurred. Thanks to Dr. Katie Brkich for her coffee conversation with me that became pivotal in my analysis. I thank Dr. Elizabeth Bondy, who first empowered me to write academically, and whose friendship has sustained me. My friend and neighbor Dr. Jean Crockett interrupted her evening walk many times to offer me assistance. I remember Dr. Vicki Vescio for her encouragement at the beginning and Dr. Darby Delane for her encouragement at the end. Above all, I recognize the contributions of my participants, especially Sandra, James, and Kelly, who opened their practice for my examination and comments.

TABLE OF CONTENTS

APPENDIX

LIST OF TABLES

LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

DEVELOPMENT OF A FRAMEWORK FOR TEACHING MATHEMATICS IN DEPTH

By

Joanne Jensen LaFramenta

August 2011

Chair: Thomasenia Lott Adams
Major: Curriculum and Instruction

This study illuminates the practice of teaching mathematics in depth by developing a framework to serve practicing teachers and those who educate teachers. A thorough reading of the literature that began with all of the volumes in the decades since the publication of the *Standards* (1989) identified six elements that were profitable for effective instruction in mathematics. These elements formed the basis of a tentative framework for teaching mathematics in depth that was elaborated by the results of this study. The experience of a fifth-grade level team in a southeastern state as they implemented a mathematics curriculum and set of standards that demanded teaching mathematics in depth identified elements of the framework that were previously missing from the literature. The perceptions of other mathematics teacher educators from the university and the school district were also incorporated into the framework.

Two findings emerged from the study with implication for teaching mathematics in elementary school. The first is that the conceptualization of teaching mathematics in depth is strongly influenced by the teacher's orientation toward a learning perspective. The teachers use those practices which they believe will contribute most to an increase in student understanding of a particular topic. The second finding to emerge is that the actualization of teaching mathematics in depth is contingent upon balancing the dual forces of the pacing guide (an external scheduling

mechanism) and the teachers' desire to teach for mastery. Teachers create their instructional plans according to their assessment of existing student comprehension and understanding. They are seeking mastery of the topic, but the school administration's pacing guide pressures the tempo of their plans.

This manuscript reveals that each of these findings holds practical implication for improvement of instruction of mathematics in elementary school--addressing pacing, the definition and timing of mastery, learning theory, and assessment efficiency. Each contributes to mitigating gaps in the research literature, and each contributes to the development of the framework. Moreover, there are implications for research due to the significant overlap of this study with the recent adoption of Common Core State Standards (CCSS) and plans for national assessment instruments.

CHAPTER 1
INTRODUCTION

An Allegory

In the land of *Terre Ecole*, there was a teacher, Inez *Instructeur*, who had a formidable task. She was responsible for leading her students across the *Cinquième Vallée*, meaning she was the fifth teacher for the children. On the other side they would meet their new teacher who would direct them across the next valley. The valley floor was covered with a great number of plants that created a comprehensive maze. The foliage obscured one's vision of the horizon. There were many paths across the valley. Sometimes progress was rapid, since the path chosen moved forward without a blockade. Sometimes the path chosen was short, an immediate dead end. There were junctions when Inez had three choices or more. Over time, she learned which of these paths led where she needed to go. Generally, however, Inez groped forward almost blindly. In some years, progress was swift and purposeful, but during some years it seemed almost miraculous that she gathered her flock together on the far side.

This year there was a new *directive* from *L'Etat* that created a marvelous change on the floor of the valley. The maze was cut down, and the roots were pulled from the ground. It was possible to see clearly across the expanse to the other side. The exit was precisely marked. But the other side was far away, and there was not a well-marked highway. Although six or seven trees were growing across the valley floor, Inez missed the comforting structure of the maze that had slowed the group's progress in prior years. At places the terrain was wide and the students scattered. In other places there was a narrow and difficult path, where the students needed each other's help. During the dark of night, the group could lose its way and even begin to go backwards. Sometimes an *assistant* came to light the path and bring the group back to the best *avenue*. Frequently, over the course of the *voyage*, Inez wondered if she was going in the right direction. She worried that some of her students lagged behind while some surged ahead. At times she lost sight of her destination and pondered many thoughts. Would this *traversal* be accomplished in the time allowed? Would she deliver her students to the right spot? What would happen in the end? Would the path *directeur* be satisfied with her efforts? She was not convinced that she liked this uncharted path, because she was unsure about what would happen on a daily basis. The new landscape assured that she had more control, more choices, and more decisions, but she also carried more responsibility. She was accustomed to moving with the flow. But this new topography was the setting for the task she was given, and now she must persist. The future of her students depended on it.

Relation of Inez' Story to Florida's Mathematics Standards

The valley crossed by Inez is the fifth grade mathematics curriculum. The "maze" planted in the valley is composed of the seventeen standards, thirty-three benchmarks, and the accompanying ninety-six grade level expectations described by the 1996 Sunshine State Standards (SSS) in the mathematics standards for fifth grade (Clark & Wright, 2006; Florida Department of Education (FLDOE), 1996). There were so many topics to be taught that it was difficult for each teacher to thoroughly teach every topic (Schmidt, McKnight, & Raizen, 1997). Hence, since each teacher might lead their students on different paths through the maze, there were many permutations based on this one curriculum. It could not be guaranteed that in any two classrooms in the state, or perhaps even in the same city or school, the students were taught the same curriculum over the course of the year. And if all the topics were taught, the instruction might be sketchy. Frequently, the material would be re-taught the next year (College Board, 2005). The students arriving in August at the beginning of the school year may have taken many roads through their mathematics curriculum the year before (Schmidt, Houang, & Cogan, 2002).

The Components of the Florida Mathematics Program

There are three components of the Florida mathematics program: the standards, the state assessment, and the state-approved textbooks. Recent changes made to these elements will be briefly reported here, but are discussed in more detail in Chapter 2. In 2007 the state of Florida adopted the Next Generation Sunshine State Standards (NGSSS) for mathematics, which dramatically altered the appearance of the list of academic expectations for each grade level, from kindergarten to grade 8. Fewer topics are proscribed for each grade level, but each will be taught in great detail. Currently the standards proscribed for each grade delineates three Big Ideas and three to five Supporting Ideas, including selected benchmarks. Related Access Points for students with significant cognitive disabilities have been developed as well, to assure access for all to the mathematics curriculum. Access points reflect the core intent of the standards with reduced levels of complexity. The three levels of complexity include participatory, supported, and independent, with the participatory level being the least complex (FLDOE, 2007a).

In addition, the new set of standards anticipates that the students in a particular grade will master the mathematics taught that year. The goal of achieving articulation has the added effect of installing rigor. Not only are the Big Ideas distributed between each of the grades in a sequence built in accordance with the structure of mathematics, but also mastery of those Big Ideas is expected within that particular year. For example, third grades learn the third grade curriculum during the third grade, second graders learn the second-grade curriculum that year, etc. In any grade, the Big Ideas must be mastered, because the work of the succeeding course assumes such mastery. NGSSS expects mastery of multiplication in fourth grade; fifth grade work is built upon that assumption.

The second component of the mathematics program is the revised Florida Comprehensive Achievement Test (FCAT 2.0) that assesses the achievement of students using the changed mathematics curriculum in grades 3 through 8. Student achievement on the Access Points is measured with the new Florida Alternate Assessment. Students in high school or middle school that are taking Algebra 1 will take an end of the year examination (EOC) that will account for 30% of the grade for the course (FLDOE, 2011). Results of the examinations (FCAT 2.0 and EOC) influence the status of the school, the teachers, the principal, and the students.

The students are affected in two areas: graduation from high school and program placement. Students cannot graduate from high school without passing Algebra 1 and geometry or a year of more advanced mathematics. Since the EOC results contribute 30% of the class grade, the examinations in mathematics could impact the student's successfully completing these courses (FLDOE, 2011). Under the Enhanced New Needed Opportunity for Better Life and Education for Students with Disabilities (ENNOBLES) Act, a student's individual educational plan (IEP) team may waive this requirement (FLDOE, 2007a).

Placement in particular academic programs or mathematics courses depends on a student's score on the FCAT 2.0 taken in the prior school year, or perhaps in the year before that—middle school magnet programs refer to the scores from fourth grade, since enrollment takes place before the occasion of the fifth grade tests. However, at the writing of this document the committees that set standards are still conducting meetings to solidify those rankings (FLDOE, 2010d). Under FCAT, students in grades 3-10 who scored lower than Level 3 (the middle level of 5) in mathematics were required to have a Progress Monitoring Plan. Students in grades 6-10 whose scores were on Levels 1 or 2 were required to receive intensive remediation. This could have been given during the regular mathematics class, but schools could require that a student take two mathematics courses at the same time (FLDOE, 2007a). In practice, students are enrolled in advanced sections of mathematics based on FCAT 2.0 results (College Board, 2005; Alachua County Public Schools, 2009).

Results from the FCAT 2.0 will also impact the life of the individual teacher and school principal by 2014. The legislature recently adopted the Student Success Act and the governor signed it into law. Among its provisions is the incorporation of student learning gains on FCAT 2.0 and the EOC into teacher evaluation scales. Half of the evaluation will be based on student learning gains for classroom teachers, 30% for non-classroom personnel, and 40% on administrators. Teachers in hard-to-staff areas, such as mathematics and science, may be eligible for a higher rate of pay than other teachers. Finally, the continuing contract for teachers has been eliminated (FLDOE, 2011b; Student Success Act Factsheet, 2011). The school districts are now working with the teachers' unions to consider the implications of this act for salary, tenure, and staffing. The state department of education is in the process of holding meetings to determine the state policy on Learning Gains Calculation and Adequate Progress Requirement of the Bottom 25% (FLDOE, 2010d). Clearly, FCAT 2.0 will have a significant influence on school personnel, and since FCAT 2.0 measures the success students had in learning the curriculum, the impact of a new curriculum is tremendous.

In 2010 Florida placed among the sixteen finalists for Race to the Top Grants RTTT). This federal program, a component of the American Recovery and Reinvestment Act of 2009, offers funding for states to improve educational outcomes for all students (U.S. Department of Education, 2009). The $4.3 billion dollar appropriation can mean significant funding for school districts. Part of the application for the Local Educational Agencies refers to teacher evaluation and part is dependent on adoption of the Common Core State Standards. In compliance with the RTTT, the legislature has adopted the Common Core Standards and made changes regarding teacher evaluation as were described in the Student Success Act. In August 2010, Florida was one of several states to be named a winner of the Race to the Top Phase 2 competition, and received an award of $700 million. Over the next four years the FLDOE will be working with individual school districts to create plans to utilize these funds within standards of reform as submitted in the application (FLDOE, 2010a).

The FCAT was instrumental in determining school grades in Florida's A+ Plan and the annual yearly progress (AYP) for purposes of reports in the No Child Left Behind (NCLB) program. In an effort to address the needs of each child, the FCAT scores were considered as individual results, group results, and school results. The achievement of the child was measured as an individual score, but the child was also part of a group and a school. The child is White, Black, Hispanic, Asian, American Indian, or multiracial. He or she may also be disabled, economically disadvantaged, an English language learner, or a migrant. The desired level of individual competence is Level 3, and goals for the AYP are set as part of the state and federal legislation. The goals are being revised by state committees, meeting to consider the deadline of 2013 – 2014, when all students should meet that goal. There had been financial rewards for schools with a grade of "A" and those that had improved by one letter grade from the year prior. However, assessment and accountability are being revised continually as the state department of education modifies policy and procedures in implementation of the RTTT and the CCSS (FLDOE, 2010b).

The third component is the curriculum materials. The FLDOE has authorized a cycle of textbook adoptions. Generally, every six years school districts adopt new instructional materials for mathematics. The school year 2009-2010 was designated by the Legislature as an adoption year for mathematics instructional materials. In 2008 textbook publishers were invited to develop mathematics instructional materials to be considered by the State Instructional Materials Committee and the Commissioner of Education (Vaccari, 2009). In 2009 the school districts of Florida had the opportunity to examine the offered curricular materials. The selection process was completed in December 2009, and districts were allowed to purchase materials after April 1, 2010. Districts must adopt one of the state-approved textbook series within two years of the adoption year, but waivers are allowed when funding is problematic (Tappen & Clark, 2009).

The Problem

Changing outcomes in mathematics achievement is a goal of altering state curricular standards. States use standards to make changes in classroom practice, anticipating that teachers will change instructional practice and thus, improve educational outcomes (Cohen & Hill, 2000). With the NGSSS, the FLDOE has "cut down the maze" of grade level expectations as part of an effort to "ensure adequate rigor, relevance, logical student progression, and integration of reading, writing, and mathematics across all subject areas" (FLDOE, 2007b). The Big Ideas and Supporting Ideas structure coherence and minimize redundancy by identifying particular topics with particular grade levels. Benchmarks support this structure. Responsibility for clarity lies with the teacher, and rigor will be measured by the FCAT 2.0.

The problem for teachers like Inez is that the landscape under the NGSSS looks very different from how it looked under the SSS. Although teachers and school districts had been laboring under a complex curriculum, that curriculum was familiar and comfortable to them. This change in the mathematics standards presents a new format for instruction of this critical subject in the elementary school curriculum, as well as new benchmarks of skills and concepts (Dixon & Kersaint, 2008). As the 2010-2011 school year began, the teachers would have new mathematics standards, a new textbook series, and a new standardized state assessment—all of which would greatly influence what happened in the teacher's classroom on a daily basis.

Recommended improvements of coherence and lack of repetition have been built into the mathematics standards by legislation at the state level. However, it is the individual educator who will be enacting the rigor and depth that the new curriculum promotes and requires.

As will be more fully discussed in Chapter 2, there are compelling reasons for these curricular changes. One contributing criticism of the prior curriculum is that it was " a mile wide and an inch deep" (FLDOE, 2007b). The structure of Big Ideas and Supporting Ideas is an effort to create a context that requires "teaching in depth." As with Inez, the imaginary teacher, many teachers will be wondering how to structure their mathematics lessons and to implement the goals of the FLDOE. Many topics formerly taught are no longer in the province of their grade level (Dixon & Kersaint, 2008). With fewer imperatives, how will the teachers respond? What changes will they make? How will their work be supported? When there is a broad space with few markers, how will the teacher proceed?   How does the teacher perceive the process?

Statement of the Purpose and Guiding Research Questions

The purpose of this study was to develop a framework for "teaching mathematics in depth" by examining the practices and perceptions of teachers of mathematics in elementary schools and soliciting the considered descriptions of the practice from other professional mathematics educators. Semi-structured interviews were held with the classroom teachers to explore their experiences as they began work with the new standards. Other interviews were held with those who prepare the teacher—mathematics teacher educators (MTEs)—and those who assist the development of curriculum in the school—curriculum resource teachers (CRTs) and instructional mathematics coaches (IMCs).

The specific questions that guided this study are these:

1. How is teaching mathematics in depth understood and enacted by elementary teachers of mathematics?

A. What are the classroom teachers' perceptions of what it means to "teach mathematics in depth"?

B. What guides the teachers' enactment of "teaching mathematics in depth"?

2. How is teaching mathematics in depth defined by the MTEs?

3. How is teaching mathematics in depth defined by the CRTs?

4. How is teaching mathematics in depth defined by the IMCs?

The results of the data analysis will be merged with the review of the literature to develop a framework for "teaching mathematics in depth." This framework can be used to assist teachers of mathematics, mathematics teacher educators, and instructional mathematics coaches to make "teaching mathematics in depth" a real practice in the elementary mathematics classroom.

Dissertation Overview

The design of this study is outlined carefully in the Chapter 4, but an overview is presented here. A grade-level team of elementary school teachers was recruited to participate as they implemented the NGSSS for mathematics. The researcher served as a volunteer on a daily basis during the teachers' mathematics lesson sessions and collected primary data from the teachers in the first four months of the school year. There were formal interviews and a systematic collection of teacher reflections. The researcher's field notes were part of the secondary data sources, together with the researcher's journal, lesson plans, the corresponding curriculum materials, informal interviews that occurred both before and after lesson implementation, and notes from conversations between team teachers. MTEs and IMCs were interviewed. CRTs participated in two ways—selecting a teacher for observation and discussing that observed mathematics lesson in a recorded interview. This is a qualitative study, and grounded theory methods derived from the work of Charmaz (2006) and Clarke (2009) were used to analyze the data.

Chapters 2 and 3 anchor the study in the research literature. Chapter 2 provides background material for the legislation. The history of concerns about the SSS is traced as the impetus for the creation of the NGSSS. A brief history of mathematics reform in the national education community is outlined, culminating in publication by the National Council of Teachers of Mathematics (NCTM) of *Curriculum Focal Points*, a document that provided the structure for NGSSS. In Chapter 3, the researcher reviews research on teaching in depth, teaching for understanding, and effective instructional practices in mathematics. The publication of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) marks the starting point for this examination of instructional studies. Implementation of these *Standards* uncovered dilemmas about what it means to teach and to learn mathematics. (From this point forward, the italicized *Standards* refers to the standards included in the 1989 document and its successor, *Principles and Standards* (2000)). This research is summarized, as is the research of the implementation of reform curriculum and instruction. In these studies is found ample evidence of valuable components for effective teaching practices. These are compiled into a tentative framework for teaching mathematics in depth. The data collected in this study is used to validate the utility and accuracy of the elements of this tentative framework.

Importance of the Study

The initiation of NGSSS at this particular time situates the experience of Florida teachers on the national scene because of the mathematics standards and the textbook adoption. In 2010 the Council of Chief State School Officers and the National Governors Association Center for Best Practices (NGA Center) released the Common Core State Standards (CCSS) for Mathematics. Its publication has been endorsed by four professional associations: the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the

Association of Mathematics Teacher Educators (AMTE). These endorsements acknowledge that such common standards would offer "support for national improvement in mathematics achievement," providing a "foundation for the development of more focused and coherent instructional materials and assessments that measure students' understanding of mathematical concepts and acquisition of fundamental reasoning habits, in addition to their fluency with skills" (AMTE, NCTM, NCSM, & ASSM, 2010). The letter of support for adoption of these common standards explains that the CCSS will assist teachers and educators to focus on improving teaching and learning, looking to ensure that all students have access to a high-quality mathematics program and the support that enables them to be successful.

The CCSS are designed to make the curriculum focused and coherent. As with the NGSSS, these were written as a response to the criticism that mathematics curriculum in the United States is "a mile wide and an inch deep." The CCSS stress conceptual understanding of key mathematical ideas considered within the organizing principles of mathematics. Additionally, the writers considered how children learn as they defined what students should understand and be able to do in their study of mathematics (Common Core State Standards Initiative, 2010). The resulting document looks remarkably like the standards adopted by the Florida Legislature to specify standards for elementary school mathematics in the state (NGSSS) in that there are a small number of critical areas and their supporting ideas for each grade level, but the CCSS describe a coherent, focused curriculum that has realistically high expectations and supports an equitable mathematics education for all students (AMTE et al., 2010). The framers note explicitly that the CCSS do not define the teaching methods or curriculum but suggest only that the widest possible range of students should participate from the outset. Educators are urged to make every effort to meet the needs of individual students based on their current understanding. “These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step” (Common Core State Standards Initiative, 2010, p.4).

Secondly, this study will lead to development of a framework that will make implementation of a curriculum that is taught in depth—one that is *not* “a mile wide and an inch deep”—more feasible for classroom teachers. Although the context for this study occurs in one location, the implications for its results are pertinent to a national audience. As the CCSS are implemented in school districts across the country, the need for clear direction of the appropriate pedagogy will be essential. This study contributes to that effort, since a framework that describes the practice of teaching mathematics in depth will be helpful for mathematics educators, mathematicians, curriculum writers, textbook publishers, and policy planners across the United States, those with interest in enabling elementary school teachers to truly teach mathematics in depth.

CHAPTER 2
HISTORICAL BACKGROUND OF NEXT GENERATION SUNSHINE STATE STANDARDS

In the United States, school governance has traditionally been assigned to local governing boards. It was only during the twentieth century that states created boards of education to oversee the public school system. During that period, standards were adopted to promote uniform educational programs within each state. The idea of standardization took root more easily than the development of institutions to centralize authority in the state government (Timar, 1997). This chapter briefly describes the development of mathematics standards for public school students' education by the professional organization of mathematics teachers (NCTM) and the specific actions that led to the NGSSS in Florida. Table 2-1 summarizes the significant events described in the chapter in a timeline.

NCTM *Standards*-Based Reform

In the last half of the twentieth century, mathematicians joined mathematics educators in several efforts to make changes to school mathematics. The mathematics curriculum was so solidly anchored in tradition that "It has been said that an eighteenth century mathematician could have stepped into almost any 1950 classroom and taught any mathematics course with full confidence" (Robison, 1960). A post-World War II reform movement was popularly referred to as "New Math." Mathematics textbooks for children were rewritten to be more rigorous and more reflective of the mathematics practiced outside schools. For instance, at this time, middle school children were taught neither about inequalities nor how to solve equations with inequalities. The Cambridge Conference on School Mathematics (1963) recommended that several advanced mathematical topics could be taught to children if the material was presented properly. Among these ideas were set, function, transformation group, and isomorphism (Educational Services Incorporated, 1963). The Conference also agreed that language, notation, and symbolism were equally important and should be included in the mathematics curriculum for young children. However, significant change in instructional practice did not materialize, frequently because the teachers, particularly in grades K-8, were neither sufficiently prepared nor trained to teach the new mathematics content (Herrera & Owens, 2001; Payne, 2003).

One solution to the difficulty of finding and training sufficient numbers of mathematics teachers for the modern discipline resulted from the development of programmed learning. Many mathematics courses were created to teach complex procedures step by step, which meant that teachers could rely on the curriculum to instruct advanced topics in mathematics. As course topics multiplied, many more students enrolled in mathematics. By 1972, American high schools offered one thousand different courses in mathematics; many of these were a semester long, but some were created to be a six-week long module (Angus & Mirel, 2003; Smith, 2004b). In this same time frame, professional mathematics educators grew concerned that American students were learning procedural competency at the expense of knowing how to solve unique and contextual problems (Herrera & Owens, 2001; Porter, 1989).

In 1980, NCTM published *An Agenda for Action*. This document made several recommendations for K-12 programs in mathematics. These are (1) the focus of the mathematics program should be problem solving; (2) basic skills ought to be defined as more than computational facility; (3) mathematics courses should take full advantage of the technology of calculators and computers; (4) the teaching of mathematics should be gauged by stringent standards of efficiency and effectiveness; (5) assessment tools ought to encompass more than conventional tests; (6) more mathematics courses should be required for high school graduation, but the curriculum should be flexible; (7) mathematics teachers ought to demand of themselves and their colleagues a high level of professionalism, and, (8) commensurate with the importance of mathematical understanding to individuals and society, ask that public support for mathematics instruction be raised (NCTM, 1980).

President Ronald Reagan established The Committee on Excellence in Education that in 1983 released *A Nation at Risk.* Drawing on declining scores in both national and international test results, the report of this committee drew the attention of the educational community. To meet a growing need for a competitive workforce, the group suggested ending the menu of mathematics electives, strengthening the mathematics curriculum, and requiring three years of mathematics for graduation from high school. Concerns voiced earlier by the mathematics education community were echoed in the statements of this prestigious group (Smith, 2004a).

California was one of the first states to change their mathematics curriculum to reflect uniform standards for every child in the public schools by adopting the *Mathematics Framework for California Public Schools* in 1985. The traditional and continuing policy in California was that all textbooks and supporting materials that are purchased with state funds must be on a list of curricula that had been approved by the state board of education. Textbook companies wrote their mathematics materials to conform to these new California standards. Compliance was further secured with a statewide system of comprehensive standardized examinations, the California Learning Assessment System (CLAS), to measure student achievement of the standards. Other states have followed suit, using textbook materials and statewide standardized tests to secure changes in instructional practice (Cohen & Hill, 2001).

The work of the NCTM to improve mathematics education in the United States built on these foundations and culminated with publication of the *Curriculum and Evaluation Standards* (1989). Expanding the recommendations of the *Agenda for Action*, this document intended to ensure quality, identify explicit goals, and promote change towards the end of developing mathematically literate workers, encouraging lifelong learning, providing opportunities for all, and supporting an informed electorate (Confrey, 2007). The principles of equity, curriculum, teaching, learning, assessment, and technology describe "critical issues that, although not unique to school mathematics, are deeply intertwined with school mathematics programs" (NCTM, 2000). There were content standards for topics such as number and algebra as well as process standards, which were included to "highlight ways of acquiring and using content knowledge" (NCTM, 2000, p. 29). The latter standards relate to problem solving, reasoning and proof, communication, connections, and representation. Curricula that developed from the recommendations of this document are known as *Standards*-based curricula. One example of a specific standard relating to division as it is taught in the fifth grade is this:

> Children should be fluent in computation and practiced in estimation, use conventional and invented algorithms, and explain use of procedures and strategies. Students in these grade levels should develop understanding of multiplicative reasoning (NCTM, 2000, p. 143).

In summary, concerns for the improvement of mathematics education in the nation's schools culminated in the publication by NCTM in 1989 of *Curriculum and Evaluation Standards.* These standards were refined and expanded in NCTM's publication of *Principles and Standards for School Mathematics* (2000). This document provides an outline of expectations for reform in mathematics education, reform that has challenged the mathematics education community in the decades that have followed. A description of the impact of the *Standards* on studies about effective mathematics teaching practices will follow in Chapter 3.

Development of Curriculum Focal Points

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (CFP) (NCTM, 2006) furthers the quest for more effective elementary school mathematics education in the United States. This publication lists key mathematical concepts that ought to be taught at each grade level, in an order designed to be coherent and articulate between the grades. A few critical events had great impact in the creation of this document. One began in the late 1980s at the University of Michigan's Center for the Learning and Teaching of Elementary Subjects. The Center published results of a study called "Experts' Views on the Elementary Mathematics Curriculum: Visions of the Ideal and Critique of Current Practice" (Prawat, 1991). The study identified four mathematics educators and mathematicians of international standing in curriculum research to ask for their opinions about what makes a good elementary school curriculum. These research experts suggested to the study team three additional participants, classroom teachers who were skilled at promoting mathematical understanding, as well as higher-level thinking and problem-solving, in their students. Although the seven experts differed in some of their conclusions, there was a remarkable consensus that it was important to identify key ideas of mathematics that were important to teach in elementary school. The group also agreed that this limited number of key understandings should be taught in greater depth, that the active involvement of students was important, and that it would be possible for experts to agree on what those key mathematical concepts were (Prawat, 1991).

Another event is the Third International Mathematics and Science Study (TIMSS) of 1994/1995. TIMSS is a large-scale, cross-national comparative study of the national educational systems and their outputs in about 50 countries. Researchers examined mathematics and the sciences curricula, instructional practices, and school and social factors, as well as conducted achievement testing of students (Schmidt et al., 1997). When the results were released, Americans were dismayed to see that the participating fourth and eighth graders from the United States were not at the top of the achievement scale. Rather, students from Singapore, Korea, and Japan scored higher than the other participating countries. Other nations with high scoring students were Hong Kong, the Czech Republic, the Netherlands, and Austria. At age 9, students in the United States were the lowest of nine countries that scored significantly *higher* than the mean score for all. At age 13, the U.S. students were near the top of the group that scored significantly l*ower* than the mean of all groups (Mullis, et al., 1998; Gonzalez, Kelly, & Smith, 1998). Researchers videotaped lessons in the classrooms in the six countries and examined the curriculum to help assess mathematics and science instruction in the US. The consensus of these studies was that (1) the American mathematics curriculum was too fragmented and lacked coherence, (2) the mathematics curriculum covered too many topics and lacked depth, and (3) mathematics concentrated too much on skills and too little on problem-solving (Macnab, 2000).

A third event impacting development of the CFP occurred in July 2004, when the Park City Mathematics Institute hosted two workshops on states' K-12 mathematics standards. These workshops were supported by the National Science Foundation (NSF). One was organized by Johnny Lott and included members of the Association of State Supervisors of Mathematics (ASSM) and NCTM as well as some research mathematicians interested in K-12 curriculum. The second workshop with the Mathematics Standards Study Group (MSSG) was organized by Roger Howe. Many of the twelve assembled mathematicians had been at the first workshop. The second group asked the question, "What is important?" The answers were five principles for school mathematics: (1) whole number arithmetic and the place value system are the foundation for school mathematics—most instruction in early grades should focus here; (2) in every grade, focus on a small number of topics—devote instruction to developing deepening mastery of core topics through computation, problem-solving, and logical reasoning; (3) make instruction mathematically rigorous in a grade-appropriate fashion—use accurate language and prove key theorems and formulas whenever possible; (4) disciplined, mathematical reasoning is one of the most important goals of a school education—it must permeate all mathematical instruction; (5) most students should be taught the mathematical knowledge and reasoning skills needed to succeed in college—those interested in quantitative careers should be ready to start calculus in college (MAA Online, 2004).

From these beginnings, NCTM created CFP and executed this document to be a starting point for a discussion of the shape of educational standards for mathematics. Notably, the CFP lists three primary focal points for each grade level. Mathematicians and mathematics educators met to identify these Big Ideas, using three criteria. First, the concepts are crucial foundations of mathematics, both for further study of mathematics and for mathematical purposes outside of school. Second, the concepts must "fit" with what is known about learning mathematics, and third, there exists a logical connection between the mathematics of both earlier and later grade levels (Confrey, 2007). These primary focal points and additional supporting ideas should be addressed within the context of the process standards (NCTM, 2006). CFP iterates the central points of the earlier *Principles and Standards* that both the content and the processes of mathematics are important to the program of school mathematics. An earlier example of a standard relating to division was presented from the *Principles and Standards.* The following example from the CFP contains part of the Big Idea that relates to division in the fifth grade:

> Students apply their understanding of models for division, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends (Mirra, 2008, p. 29).

Development of Next Generation Sunshine State Standards

The NGSSS for mathematics was legislated as part of an effort to reform mathematics education in the state of Florida. There were compelling reasons to consider reformation. Florida schools have long participated in the National Assessment of Educational Progress

(NAEP). An examination of test results since 1990 reveals some interesting information. In 1990, 1992, and 1996 average scores for all students, and the 25th and 75th percentiles were below the national average, from five to eight points below. After the 1996 adoption of the SSS, the fourth grade averages rose to a level above the national averages (Kersaint & Dogbeu, 2006). These scores remained above the national average until 2009, when they were the same (National Center for Educational Statistics, 2009). However, the implementation of the SSS appeared to have no effect on the mathematics achievement of middle school and high school students. To further investigate the relationship of the state standards to mathematics achievement, the FLDOE asked the College Board organization to conduct a review of the standards in 2005 in terms of their rigor and preparation for college entrance (College Board, 2005).

The College Board conducted an investigation of the SSS in reading, mathematics, and writing. The SSS were compared not only to the *College Board Standards for College Success* but also to the mathematics standards in the states of Louisiana and Washington. The College Board examined the state standards for rigor, focus, balance, progression, specificity, clarity, and equity. Central concerns that are mentioned frequently in the 391-page report can be summarized by these remarks, “Considerable work focusing on prerequisite knowledge mastered in prior years, foci in given years, and elimination of repeated content will greatly assist in moving to a clearer vision of progression and level of expectation at each grade” (College Board, 2005, p. 25). An example of one of the benchmarks for fifth grade division is this:

> Benchmark MA.A.3.2.3: The student adds, subtracts, and multiplies whole numbers, decimals, and fractions, including mixed numbers, and divides whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator (FLDOE, 1996).
>
> Grade level expectation: Student solves real-world problems involving addition, subtraction, multiplication, and division of whole numbers, and addition,

> subtraction, and multiplication of decimals, fractions, and mixed numbers using an appropriate method (for example, mental mathematics, pencil and paper, calculator).

The Board's recommendations for the new set of mathematics standards were that the standards ought to be coherent, articulated, and limited to few topics in each grade level K-8 and for courses grades 9-12.

Most of the College Board's recommendations for mathematics were later adopted into the NGSSS. For example, one recommendation was to eliminate repetition of the same topic in several grade levels. Another was to limit the number of topics covered during a school year. Third, the comparatively low levels of cognitive expectations ought to be raised relative to reasoning, problem solving, or communicating with the mathematics outlined by the documents (College Board, 2005). Organization of the standards and benchmarks for grades K-5 was applauded, but a general revision was recommended. An example of the Big Idea for Division is identified for an example:

> Big Idea 1: Develop an understanding of and fluency with division of whole numbers.
>
> MA.5.A.1.2: Estimate quotients or calculate them mentally depending on the context and numbers involved (FLDOE, 2006).

This revision began in 2005 when the Florida legislature passed HB 7087 "to ensure adequate rigor, relevance, logical student progression, and integration of reading, writing, and mathematics across all subject areas" (FLDOE, 2007b). Consequently, the Office of Math and Science convened a committee to consider revision of the SSS for mathematics. Many adult stakeholders were represented in this committee. These were professors of education, mathematics education, and mathematics, representatives of the state and national departments of education, parents, and teachers, as well as mathematics coaches and doctoral students. This group of stakeholders were identified as the Framers (FLDOE, 2007b).

When the Framers met, they heard from national and international experts who presented their analyses of the 1996 SSS, as well as mathematics standards from other states and countries. The Framers used this information to develop a structure for the guiding principles that writers of the state standards would follow. As the work was written, the Framers reviewed and commented. Drafts were then presented to the public; over 1300 persons completed the visitor profile—teachers, administrators, district staff, other interested persons, and parents. Their comments were considered in the revision process that took place between April and June of 2007. The NGSSS were formally adopted in September 2007 (FLDOE, 2007b).

Summary of the Historical Background of NGSSS

When student achievement in Florida appeared to be at odds with comparable national achievement, the Florida Legislature investigated. The results of the investigation were recommendations that have been incorporated into NGSSS. Framers and writers of the Florida mathematics curriculum began with the CFP as a starting point for discussion. The final product has been described in both Chapters 1 and 2, and it borrows heavily from CFP. Now that the mathematics standards have been adopted, the classroom instruction will begin. Chapter 3 examines more completely what instruction is most beneficial according to educational research and how one might regard teaching mathematics in depth.

Table 2-1. Significant events in the historic background of NGSSS

Year	Significant event	Text page
1963	Cambridge Conference on school Mathematics	21
1980	An Agenda for Action published by NCTM	22
1983	A Nation at Risk published by Committee on Excellence in Education	23
1985	Adoption of Mathematics Framework for California Public Schools	23
1989	Curriculum and Evaluation Standards published by NCTM	23
1989	“Experts’ Views on the Elementary Mathematics Curriculum”	25
1994-1995	Third International Mathematics and Science Study	25
1996	FL adopts SSS	28
1996	FL students surpass national average for fourth grade	28
2004	Park City Mathematics Institute Workshops	26
2005	FLDOE asks College Board to review the SSS	28
2005	Legislature passes HB7087; a committee is convened to revise the SSS	29
2006	Curriculum Focal Points published by NCTM	27
2007	NGSSS adopted	29

CHAPTER 3
REVIEW OF THE LITERATURE

In Chapter 1 the story of Inez was told. This study proposes to develop a framework for teaching mathematics in depth. To do so, the researcher will investigate the experience, the emotions, the thoughts, and the reflections of elementary classroom teachers who are figuratively in Inez' position, embarking on a new school year of teaching mathematics with a new curriculum, new standards, and a new statewide assessment. Additionally, the thoughts and reflections of mathematics professionals—teacher educators and support personnel in the schools—will be researched in this study. Chapter 2 described the background of the NGSSS for mathematics, which was taught and assessed during the 2010-2011 school year. A uniform criticism of the SSS was that it addressed too many topics too briefly, and that it was a curriculum that lacked depth. The Framers and the Writers of NGSSS sought to alter those dimensions, and they created a set of mathematics standards that is narrow in scope and offers opportunity to study in depth. For the students, there are opportunities to explore the mathematics in depth, and the teachers are encouraged to teach mathematics in depth. Chapter 3 examines the educational research about teaching mathematics in depth.

Searching for Teaching in Depth

The literature refers to instructional practices as teaching *in* depth, teaching *for* depth, and teaching *with* depth. For purposes of this study, the term used will be teaching *in* depth. This manuscript is written in the context of NGSSS for mathematics, a set of standards that is concerned particularly that teachers go into depth when each mathematics topic is taught. Teaching with depth and teaching for depth convey other meanings—the former referring more to the subjective position of the teacher and the latter to the subjective position of the learner. Teaching in depth describes the instructional practice. This review of the literature begins with studies that specifically connect the words “teaching” and “depth,” and studies outside the discipline of mathematics will be included, should these words describe them.

Although teaching in depth has long been recommended as an instructional practice to effectively teach students mathematics, the specific characteristics of this method are rarely described in the literature. Books and articles have been written to help teachers “teach in depth,” but even then the specifics are not named. One example that described the term most closely is a book entitled *Teaching for Depth: Where Math Meets the Humanities* (Worsley, 2002). In no portion of the several chapters describing instructional activities is the meaning of the phase “teaching for depth” defined or explained. Yet from these chapters written by classroom mathematics teachers, the authors infer results of increased depth of student learning when the educators integrated mathematics, English, and social studies. As students expressed their mathematics learning by speaking and writing, greater student understanding resulted (Hauser, 2002; Worsley, 2002).

More mention directly about teaching in depth can be found in content areas outside mathematics. Advice given to teachers of English-language learners suggested that teachers teach in depth, since this practice would produce better results in the time allowed (Crandall, Jaramillo, Olsen, Peyton, & Young, 2008 Peyton, and Young, 2008). An Australian study addressing the improvement of the attitudes of preservice teachers about a career in a rural school quoted the positive comments of one of the participants:

> It was very interesting and refreshing to observe the quality educational experience teachers are able to provide their students both in terms of resources and being able to focus on teaching in depth, as opposed to behaviour management, as seen in many other classrooms (Hudson & Hudson, 2008).

The international science journal *Nature* published an essay entitled “Teaching in Depth” wherein a doctoral student praised his mentor. The mentor’s fine qualities were extolled, but teaching in depth was not defined in the piece (Stoltzfus-Dueck, 2006).

The practice of teaching in depth may be mentioned as a desirable instructional tool but yet not be described, as noted in a study of consumer science teachers in training to improve critical thinking skills (Mimbs, 2005). Teachers soon realized the need to “create more thematic-based units instead of a few real-life type assignments” in order to expand their “teaching in-depth”, but the research does not expand on the meaning of the term except to say its use would improve competency in higher order thinking skills. Teaching “for depth” of understanding facilitated retention of facts in two related studies that examined the advantages of teaching science and history with problem-based learning (van Loggerenberg-Hattingh, 2003), but the researchers reveal few characteristics of “depth” beyond the surface discussion that students taught with problem-based learning were not inferior to their counterparts who were taught through direct, lecture-based strategies.

Teaching for conceptual understanding that was focused in science yet supported by reading and language arts was the intention of In-Depth Expanded Applications of Science (IDEAS). This program replaced traditional reading and language arts instruction with interdisciplinary in-depth science concept instruction in elementary schools. As examples of teaching science in depth, Romance and Vitale (2001) offer these suggestions: concept-focused teaching, hands-on activities, extensive utilization of science process skills, enhanced reading of trade science materials, concept map construction, and journal writing (p. 374). The results after five years of the program’s implementation reported effect sizes ranging from 0.93 to 1.6 grade equivalents on the Metropolitan Achievement Test-Science and 0.3 to 0.5 grade equivalents on the Stanford Achievement Tests-Reading. Fifty-one teachers and 1200 average, above average, and at-risk students in grades 2-5 participated. Interpreting the findings, the IDEAS model was considered to provide clear evidence for the importance of focusing the teaching-learning process on the conceptual structure of the curricular knowledge to be learned (Romance & Vitale, 2001; Romance & Vitale, 2007)

Although outside the discipline of mathematics, one educational research project investigated the difference between teaching for breadth versus depth (Coyne, McCoach, Loftus, Zipoli, & Kapp, 2009). In this study that addressed extended vocabulary instruction, *breadth* refers to the number of word meanings in a student's lexicon, and *depth* addresses how well the student knows those word meanings. Each provides useful knowledge to support reading comprehension. The research team explored three methods for teaching vocabulary meaning within the classroom activity of storybook read-alouds. Teachers taught their 42 students vocabulary in three ways: incidental mention, embedded instruction, and extended instruction. Embedded instruction introduced the meaning target words during storybook readings, which used time more efficiently. Extended instruction provided multiple opportunities to interact with target words outside the context of the story instruction, but it took more time. Findings indicated that extended instruction resulted in a more full and refined word knowledge, while embedded instruction resulted in only partial knowledge of the target vocabulary. The authors point out that the teacher should take into consideration their instructional goals as they select their teaching methods (Coyne et al., 2009).

The topic of teaching in depth is beginning to appear in the mathematics education literature since the adoption of NGSSS for mathematics. The Florida Council of Teachers of Mathematics is addressing concerns about the new standards by publishing theoretical articles that focus on teaching mathematics in depth in *Dimensions in Mathematics,* the official journal. Dixon's (2008) discussion of fractions for elementary school teachers illuminates three important points. Depth does not necessarily equate with difficulty. Depth in one area of mathematics is often built on another prerequisite topic. Depth of content knowledge often results in the ability to explore problems that were previously inaccessible. Using comparison of fractional sizes, she offers an example of how this lesson—or lessons—might be taught in depth. This is one of the rare instances of specific description of this instructional practice.

In summary, the articles and studies that have been discussed heretofore mention "teaching" and "depth," but a precise definition of "teaching in depth" evades the researcher. The phrase has been cast in a positive light by critics of current mathematics education (CCSSI, 2010; FLDOE, 2007b; Schmidt et al., 2002; Schmidt et al., 1997) but the reader is unenlightened as to specifics, being left to wonder why the model studied is successful. But teaching in depth is a fundamental characteristic of the NGSSS for mathematics, a response to the criticism of the SSS for mathematics as being "shallow" or "superficial", "covering" too many topics, and teaching few of them well. However, its opposite, teaching few subjects in depth, is only beginning to be defined. Since the acknowledged intention of NGSSS in mathematics is to increase understanding of that subject, the search of educational literature now looks to elements of effective instruction, looking for practices that will help students understand deeply, actively search for meaning, and identify principles, structures, and concepts (Darling-Hammond & Bransford, 2005).

Effective Instruction that Promotes Understanding

It is the intention of the NGSSS for mathematics that students have an opportunity to learn with understanding. To facilitate this, the Framers and Writers created a set of mathematics standards with fewer topics of study during the school year. In its mandate to publishers and authors of instructional materials, the state board reiterated that it is committed to *"adopting comprehensive, focused mathematics education instructional materials"* (FLDOE, 2008). The instructions assume that teachers need more classroom time for students to learn each concept or benchmark and that there ought to be new emphasis on teaching a smaller number of mathematical concepts with a greater depth of understanding. Teaching comprehensive and focused mathematics begins with knowing what is meant by "understanding."

Understanding as a component of learning is an economical way of creating order from sensory data (Newton, 2000). Incoming information is processed in the short-term memory center, a far smaller location than the long-term memory center. Understanding enables a child to learn better because when the knowledge makes sense to them, they can create the connections, structures, and relations that move the sensory data into appropriate locations in long-term memory bank. From this bank the information is accessed easily for new applications and new problems. However, only the learner can construct understanding (Newton, 2000). In his discussion of Paideia, an educational program in the tradition of Dewey, philosopher Mortimer Adler (M. J. Adler) points out that "the truth is that the primary, though not sole, cause of learning, whenever and where ever it occurs, is the activity of the learner's own mind" (p. 5).

Teaching well creates understanding (Wiggins & McTighe, 2005). Studies document that the teacher has significant impact on the student (Peske & Haycock, 2006). A vast study undertaken by the Tennessee Value-Added Assessment System (TVAAS) using test results from the 1994 and 1995 school years validated this assertion. When the subject results of third, fourth, and fifth graders in 54 school districts were examined with factor analysis, the most influential factor was found to be the effectiveness of the teacher (Wright, Horn, & Sanders, 1997). Although many factors influence student achievement, three years of work with highly effective teachers were seen to raise the position of the Tennessee student as much as fifty percentile points (Peske & Haycock, 2006). Studies of the learning gains of students of those teachers who have successfully completed the rigorous process of National Board Certification note that these students learned 205 days' worth in 180 days—a month more than the gains made by students of non-Board certified peer teachers (Vandevoort, Amrein-Beardsley, & Berliner, 2004).

What then are the teaching practices that allow students the opportunity to understand deeply, to actively search for meaning, and to identify underlying principles, structures, and concepts (Darling-Hammond & Bransford, 2005)? Educational literature offers results of multiple studies as well as theoretical viewpoints. To group these, the researcher borrowed an organizational structure from an outline of designs for learning environments coined by Bransford, Brown, and Cocking (1999), attaching the review of additional research to their framework. Opportunities for learning are facilitated when: (1) the focus of this instruction is the student; (2) instruction is grounded in content knowledge; (3) instruction takes place in a community where the teacher and students are in dialogue; and (4) assessment is an integral part of daily instruction. This manuscript presents literature that substantiates the categories of learner-centered, knowledge-centered, community-centered, and assessment-centered instruction as presented in research studies. The instructional practices described here are applicable to all subject areas, but this review is not intended to be a comprehensive catalog of the existing literature on effective teaching. Instead it is a synthesis of learning designs that promote a student's deep understanding. This section lays the foundation for the review of studies in mathematics education later in the chapter.

Learner-Centered Instruction

Being actively involved

Children differ from adults considerably in their learning style with regard to activity. They actively engage to make sense of their worlds, particularly in mathematics and biology, and misconceptions arise from limited experiences (Bransford et al., 1999). When school actions result in success, the learning is retained (Schunk, 2001). Learning situations that offer students active participation are malleable by the teacher. The learning opportunities that are presented in the classroom are only as beneficial to the student as the student participates in these activities. Internal activities such as watching, thinking, and listening are difficult to study. External actions of volunteering thoughts and ideas, answering questions, writing, demonstrating to classmates, and participating in small group discussions have been observed by researchers and positively associated with learning gains. Not only do these actions offer students the opportunity to instantiate their learning and critique their thinking, but they are actions that teachers can reinforce with instructional practices (Turner & Patrick, 2004).

Involvement in classroom activities is associated with focused concentration, attention, and deep comprehension. The teaching behaviors that develop involvement also develop understanding. Among these are transferring responsibility for learning to the student by requiring them to explain and justify their work, balancing challenge and support, evoking students' interest and curiosity, commenting on progress, and advocating risk taking. A study team led by Turner (1998) measured the students' quality of experience in high-involvement classes where these behaviors were practiced. Students in the classrooms described as high-involvement were more likely than the others to report more experiences of flow of challenge and skill, less experiences of boredom and anxiety. They were more likely to rate their classroom experience as involving and intrinsically interesting (Turner et al., 1998).

Another effort to involve students more actively in the learning process was designed by a research team led by Black and Wiliam (2006), who worked with teachers who were interested in these alterations to change their pedagogy. As the changes were instituted the team noted the beginning of an interesting cycle—as students became more active in their own learning, they learned more. As they learned more, they became more active in the instructional tasks. (This study is further described in the Community-Centered section that follows in this chapter.) Another opportunity to increase student engagement occurs when the number of students in the classroom is reduced. The increased learning gains are attributed to greater interaction between teacher and student (Evertson & Randolph, 1989). Student activity is a valuable learning activity and one that teachers can profoundly influence.

Engaging with authentic instructional tasks

The subject of instructional tasks is an important one that will be addressed from many vantage points in this chapter. This section considers the task as it relates to the student. The task must make sense to the learner. Cooper (1998) interviewed sixth grade students from a district that had boycotted the national mathematics test to ask their opinion about potential test items. He found that there was a difference between the reactions of the children that largely conformed to the social class of their parents. For instance, the children were asked to determine the height of a tree in millimeters. The middle-class child viewed this as a school task and calculated an answer. The working-class child might not answer the question because it appeared to be ridiculous. One does not measure tree heights in millimeters! (Cooper, 1998).

Frequent checks for understanding between teacher and student facilitate student action on a task. This is more likely in smaller classes because there students can approach the teacher more frequently and gain assurance that they are doing what is intended by the task (Evertson & Randolph, 1989). Furthermore, teachers can monitor student progress in a timely manner, assuring that the students are completing the task as it was designed. Students are always learning something; the teacher needs to know that what they learn is what was intended (Butler & Cartier, 2004).

Student power is engaged when the tasks offer flexible approaches and choice. When student attention is riveted, learning is more likely to occur (Butler & Cartier, 2004; Ormrod, 2008). Active and meaningful work, such as apprentices do, creates better learning results (James, 2006; Ladson-Billings, 1994; Ormrod, 2008). Challenging work sends the student a message that school is important. Learner-centered instruction acknowledges that all students can learn challenging curriculum and should be given the opportunity to do so (Bennett et al., 2004; Irvine, 2003; Kilpatrick, Swafford, & Findell, 2001; Ladson-Billings, 1994; Nieto, 2000). This approach respects the learner at the same time it encourages their motivation, recognizing them as a unique and capable individual. Delpit, in her summary of the characteristics of teachers who have successfully taught the urban poor, notes that these professionals teach "more, not less, content" and "ensure all children gain access to conventions/strategies essential to success in American society" (Delpit, 2006).

Connecting with the world of the student

A learner-centered education recognizes and respects the world of the student. Children learn by trying to make sense of their world (Bransford et al., 1999). Schools facilitate the process when the teachers are knowledgeable about that student's world. Teachers need to be well acquainted with the culture and background within those worlds. Channels of communication can be hindered when this knowledge is missing, Miller (1995) describes four areas where differences between minority cultures and that of the majority make a difference in schools: (1) social organization, (2) sociolinguistics, (3) cognition, and (4) motivation. For example, a teacher sees the downcast eyes of a Native American student and assumes that they are not listening. An African-American student is continually dismissed as rude because the student interrupts the adult conversation, as is customary in his/her home environment. The Hawaiian student is accustomed to completing tasks with a group of peers. The Mexican-American student whose parents taught them to learn by observation is considered to be unresponsive by their Anglo teacher. Teachers do treat students differently because of differences in their expectations. These expectations can make a difference of 5-10% in academic performance (Miller, 1995). It is imperative that the teacher examines those expectations and adjusts them critically.

Moreover, success is not identified similarly in all cultures (Nieto, 2000). Schools ought not to be in the position of forcing the student to make a choice between their home culture and the culture of the school. When learning outside of school is disregarded there are several results. One is that student learning suffers. A second is the denial of one or the other world. A third is student resistance. Fourth is the student dropping out of school. Nieto's study of the literature indicates that the more successful students are those who resisted assimilation, who maintained their own culture and language. She reports that when teachers used students' resources, they helped their students learn. After all, the goal is comprehension; all languages, including the home language, can contribute to that goal.

Culture cannot be denied as a part of the student's mechanism for learning, "because students' ways of knowing and perceiving are influenced by culture, culture is a critical variable in how students learn and how teachers teach" (Irvine, 2003). Diverse student populations are a fact of public education in the United States. Successful education of a diverse student body takes place under cultural responsive teachers (CRT), where teachers wait longer for students to respond; they probe, prompt, praise, and encourage. CRTs understand and appreciate students' personal cultural knowledge. They use the students' prior knowledge to link new learning to the world the student brings to the classroom.

One truth discovered by Ladson-Billings (1994) is that effective teachers of African-American children communicate with the students in such a way that the culture of their students is validated even as the teacher is instructing within the context of the school culture. Effective teaching involves knowledge of both the students and the subject matter. She notes that good teachers demand excellence at every task students undertake, but believe that all students can succeed. Excellence is seen as a complex standard that takes student diversity and individual differences into account. The teachers are passionate about knowledge, and they help their students develop the necessary skills to pursue that knowledge.

Summary of learner-centered instruction

Learner-centered instruction insures that the students are engaged in the learning activities. These activities promote important learning of the subject matter and challenge the students. The home culture of the child is an essential component of the learning process. Learning proceeds more effectively when the child's home culture is both valued and incorporated into the classroom to better connect the students' knowledge from home with the knowledge from school. Effective teaching practices incorporate student-centered instruction.

Knowledge-Centered Instruction

A helpful distinction for considering the knowledge that constitutes instruction in U.S. schools is between content knowledge and learning knowledge. Each has a place in the curriculum because each enhances learning with understanding. Instructional practices in knowledge-centered instruction have the goal that students use their current knowledge and skills and learn with understanding rather than acquiring disconnected sets of facts and skills (Bransford et al., 1999). Since the internal organization of a body of information facilitates its retrieval, the context in which a student learns and uses information becomes part of the storage of that information and influences its retrieval (Ormrod, 2008). Effective teaching structures that internal organization.

Focus on foundational concepts

Effective teachers know what is important about the subject matter and they are certain that they teach it (Crandall et al., 2008; James, 2006; Ladson-Billings, 1994). A report of the National Academy's Panel into the Science of Learning (1999) explains that expert learning is different from novice learning. Experts notice features that novices do not; they organize their content knowledge differently; and their knowledge is embedded in contexts of applicability. Research shows that learning endures longer and transfers better if learning is guided by generalized principles and students have conceptual knowledge (Bransford, et al., 1999).

The teacher who is able to present these complex ideas is a teacher who is secure in the content knowledge. Teachers are in the position of evaluating students' conceptions and adjusting instruction to assist students' transformation of preliminary knowledge into something more useful and comprehensive (Otero, 2006). As Otero observed from observations of her preservice teachers, frequently the teacher is confronted with a student idea that is totally unexpected. She uses for a model one of her preservice teachers, Daniel, who had planned an interesting lesson about lunar phases, but abandoned it when most of his students could not explain why the moon glows. When asked why he changed his interactive lesson into a thirty-minute chalkboard demonstration, his response was, "They just don't get it." Daniel had insufficient knowledge of light and reflection to redirect the lesson from the children's existing knowledge to his plan for lunar phases.

Shulman wrote extensively about the types of knowledge teachers need to be effective. They need (a) subject matter content knowledge. In Daniel's case, the necessary knowledge from science was that the moon did not create light, but reflected it. Teachers need (b) pedagogical content knowledge, which would help Daniel realize that the students held a partially correct concept, that the moon did not create its own light. He needed to have knowledge of what content to teach them next, content that would encourage the students to alter the partially correct knowledge into a more accurate conception. Finally, teachers need (c) curricular knowledge, the knowledge of the programs and materials that would help him teach the students what they needed to know and what they might be able to learn, given the knowledge they now held (Shulman, 2004, p. 203). Teachers need to be secure in their confidence that they can make those judgments.

Offer challenge and support

The teacher's key role is negotiating through the curriculum and the textbook to create lessons that encourage students to learn the content (Bransford et al., 1999; Wiggins & McTighe, 2005). The tasks cannot be too trivial or too difficult; teachers must know what the students know and guess what they need to know next (Kilpatrick, Swafford, & Findell, 2001). Sometimes the students are not fully prepared for challenging material and they need the support of scaffolding, instructional support that enables students to move from what they know to what they need to know (Bransford et al., 1999; Ladson-Billings, 1994; Shulman, 2004). Some of these supports are familiar metaphors, analogies, and experiences from the children's world (Delpit, 2006). It is important to identify those educational needs that the classroom can particularly address, with the help of the teacher (James, 2006). Teachers then help students develop the necessary skills so their education can progress (Ladson-Billings, 1994).

The National Study Group for the Affirmative Development of Academic Ability met in 2004 in an effort to bring clarity to the discussion of academic achievement gaps and stated that all children have a right to experience high expectations and rigorous challenge (Bennett et al., 2004). After all, one can only learned what one is taught (Kilpatrick et al., 2001). When children are viewed as being competent, they are likely to demonstrate competence (Ladson-Billings, 1994). The success of expecting high quality work and offering rigorous challenge to all students regardless of their academic background was documented in two studies investigating tracking practices in U.S. high schools. Tracking is a system of enrolling students in classes according to their perceived academic ability or their previous achievement. The objective is to create groups of students, homogeneous in ability, so that instruction can be better tailored to meet the instructional needs of the group. Many research studies and court cases have reported the problems existent in the practice, but reference is made here to expectation of success with challenging content in these particular studies.

The first was conducted by Boaler (2008), who compared the mathematics instruction in three high schools in a western state with different approaches to mathematics instruction. Two of the high schools practiced tracking students in mathematics according to ability; one practiced heterogeneous grouping. Instruction in the tracked classes was conventional; students sat in rows, listened to the teacher presentation for half of the period and practiced short, closed problems for the rest of the session. In the school using heterogeneous groupings, most of the instruction was centered on student solutions to open, conceptual problems. There were very few lectures, but complex material was mastered by the students in small groups. Findings revealed greater gains in mathematics achievement of students at the school with heterogeneous classes on a number of measures. Moreover, achievement gaps between ethnic groups at this school, which were present on incoming exams, disappeared in almost all cases by the end of students' second year of study (Boaler & Staples, 2008).

The second study takes place in a school district in New York that eliminated tracking in all classes in the middle school and the high school over a period of years. Initially, only the sixth grade mathematics classes were untracked, but during every year following other grades and subjects were added to the untracked category. Today, all students, even those in special education, are in detracked classes. The sole exception is foreign language. The records of achievement on the state examinations and the Regents' tests indicated that all students were capable of mastering challenging curricula. After a high school program within detracked classes, the odds of a student achieving a Regents' diploma, as compared to their tracked counterparts with corresponding aptitude and demographic characteristics, had risen dramatically. These odds were three times greater for non-minority, whether eligible for free and reduced lunch (FRPL) or not, five times greater for minority and FRPL-eligible, and twenty-six times greater for minority, not eligible for FRPL. The dropout rate in the high school was lower after detracking high school classes, and successful completion of the International Baccalaureate program also increased in all groups of students. All these gains were statistically significant as compared to the other districts in the state (Burris, Wiley, Welner, & Murphy, 2008). Challenging and rigorous content is part of effective instructional practice.

View the curriculum critically

As well as identifying foundational knowledge, effective teachers view knowledge critically. Schools are one of the centers for cultural transmission of a society's view of knowledge. The body of knowledge is vast, and selection of particular segments for a given curriculum is made by groups and individuals. Schools are educational institutions for teaching lessons not only about content, but also about behavior, attitudes, and cultural images. High school curricula at mid-century included domestic science courses for girls (sewing and cooking) and industrial science courses for boys (auto mechanics and woodshop). The fact that these courses are seldom taught today and, if they are offered, are certainly not limited to one gender demonstrates changes in society's expectations of educational institutions. Critical theory maintains that these expectations are appropriate topics for discussion by all stakeholders in the educational process (Au, Bigelow, & Karp, 2007; Cochran-Smith, 2004; Hinchey, 1998).

The National Study Group, introduced in the prior section, also recommended that attention be given in schools and classrooms to reconciling the possible tensions between the several purposes of education—development of the intellect, skills development, and moral development—and the political agendas of diverse learners, so that academic learning might be seen as compatible with the purposes of those expected to do the learning (Bennett et al., 2004). Researchers in urban secondary schools found that teachers working specifically to create "intentional climates" that were defined by a transformational or liberating practice were effective engaging the students in the educational process. When the students were given permission and encouragement to view their education critically, to examine the structures of power and authority, work resulted in greater student achievement (Shindler, Jones, Taylor, & Cadenas, 2004).

Education is about extending the students' thinking abilities, not simply accumulating a body of factual information (Ladson-Billings, 1994). To that end, teachers lead their students to examine the content and their own thinking about that content. Successful urban teachers interviewed by Delpit (2006) demanded critical thinking of their students, whatever methodology or instructional program that was used. She also notes that these teachers provided the emotional ego strength to challenge racist societal views of the competence and worthiness of children and their families, then to recognize and build on children's strengths.

Nieto (2000) suggests incorporating praxis into the students' education. Praxis is education for social justice, teaching children to use their knowledge for thoughtful action. She proposes that talking about power and inequality is preparation for students to be citizens in a democracy. Additionally, teachers should consider their own biases because children quickly pick up the message that talking about or acknowledging differences is a negative thing. Instead, schools should make differences and similarities an explicit part of the curriculum, confronting racism and discrimination at the same time these institutions develop critical thinking and leadership skills. Since content will be taught through a lens of intention, questioning the view through the lens is an effective teaching practice.

Teaching for continuous learning

Valuable education for the twenty-first century is one that enables students to continue learning throughout their lives. It is accepted that the political, technological and social problems that face society this century will draw upon knowledge that is only now being created. For students to flourish in this world, they will need to be learners and problem solvers for the rest of their lives. Effective educators can facilitate this process by teaching students to think about their own thinking and to monitor their own learning and cognitive processes. This body of knowledge is defined as metacognition (Ormrod, 2008). Balancing metacognition and content supports the learning of struggling students (Bennett et al., 2004).

With this knowledge, students can be taught to monitor and evaluate their own learning progress. This mechanism is known as self-regulated learning (SRL), "the self-directive process through which learners transform their mental abilities into task-related academic skills" (Zimmerman, 2001, p. 1). Many of the components of SRL are successfully woven into effective teaching practices, such as goal setting, planning, application of learning strategies, self-monitoring, appropriate help-seeking, self-evaluation and self-reflection. A complete discussion of the development of SRL within effective teaching practices is beyond the scope of this manuscript, but mention here will be made of two particular areas.

Research distinguishes between mastery goals and performance goals. It has been documented that students who are working toward mastery (working to learn) are more effective learners than those who have performance goals (working to finish). Students who are completing tasks because they want to learn accomplish more than those who are looking for recognition from their parents, teachers, and peers or simply looking for the answer that completes the assignment (Hong, Chiu, Dweck, Lin, & Wan, 1999; Turner & Patrick, 2004). Goals influence learning since students are more motivated when they experience success, and greater success results from a mastery orientation (Schunk & Zimmerman, 1997). Teachers can assist students to change their goals and motivation because the teachers are in charge of the learning environment (Corno, 2004; Montalvo, Mansfield, & Miller, 2007). They can use their role of discussant to encourage class participation that supports mastery goals (Turner & Patrick, 2004). Through the instructional program, teachers can minimize public comparisons of performance and competitive instructional activities (Paris & Paris, 2001). Mastery goals are developed over performance goals when the instructional tasks utilize group goals and cooperative learning structures (Oakes, 1985).

Additionally, teachers can teach students how to learn. When students are confronted with a learning assignment, they can be taught to organize their learning—to identify the specific tasks, to develop solution strategies, to arrange the information, and to evaluate the solutions (Butler & Cartier, 2004; Paris & Paris, 2001; Randi & Corno, 2000). Effective task interpretation focuses students' attention on learning processes and promotes engagement and focus (Butler & Cartier, 2004). Teachers who design open-ended instructional activities have the opportunity to use these to scaffold assistance for student inquiry (Paris & Paris, 2001). As students experience success in the process of self-regulation and self-efficacy, they are likely to practice this more frequently. The skills and knowledge that are necessary can be taught. A student can learn that outcome expectations are influential (Schunk & Zimmerman, 1997).

Summary of knowledge-centered instruction

Knowledge-centered instruction teaches the critical and essential information of the subject area even as it challenges students. The tasks are demanding and respect the ability of all children to learn. All students are offered a challenging curriculum. This instruction is critical, willing to examine the assumptions of the curriculum and allowing students to ask questions. Students are taught to be critical thinkers. Metacognition is incorporated with content to encourage students to develop the independence required to be life-long learners.

Community-Centered Instruction

Make a community

Effective teachers create a sense of family and caring in the service of academic achievement. They foster a sense of children's connection to community (Delpit, 2006). The science of learning reminds us that community-centered cultural experience and community-centered environments promote shared norms that can increase opportunities and motivation to interact, receive feedback, and learn (Bransford et al., 1999). Education that affirms the learning capacity of the child is developed when children are socialized about the academic expectations, when they are schooled in schooling. Education that strengthens communities and families supports their child's learning and understanding (Bennett et al., 2004).

Being a learning community means that teachers and students jointly solve problems to develop their skill and understanding. Researchers in Great Britain set up partnerships with 48 high school teachers in six schools to develop changes in pedagogy in mathematics, science, and

English that would forge new relationships between the teacher and the students. Black and Wiliam led the King's-Medway-Oxfordshire Formative Assessment Project that began in 1999. The goals were to actively involve students, use learning results to change instruction, and pay attention to the attitudes and beliefs of the students. Parallel control classes, sometimes taught by the participating teachers, were selected to be as similar as possible to the experimental classes. Among the practices the group developed were changes in classroom dialogue, feedback through written comments, peer- and self-assessment, and the use of summative tests in a formative manner. As a result, the role of the teacher changed from presenter to facilitator (Black & Wiliam, 2006).

Since the students were taught to be active learners, the learning environment changed as well. The focus on student learning created a richer community of learners. The students evidenced more engagement, more motivation and confidence, and better self-regulation. Using statistical measures of effect size to measure growth across all teacher pairs (control and intervention), the results were impressive. For the 19 teachers for which they had complete data, the average effect size was about 0.3 standard deviations. Such improvements, produced across a school, would raise a school in the lower quartile of the national performance tables to well above average. By changing the "classroom contract" so that all expected that teacher and students work together for the same end, everyone's learning improved (Black, Harrison, Lee, Marshall, & Wiliam, 2004).

Build trust

Deep understanding and the active search for meaning are strengthened in an atmosphere of trust. The affirmative development of academic ability is facilitated when educators promote trust at school (Bennett et al., 2004). Students remember teachers who affirm them, because relationships are at the core of teaching and learning (Nieto, 2000). When second graders and preservice teachers were asked about their beliefs of what makes a good teacher, the results characterized good teachers as being caring, patient, not boring, polite, and organized (Murphy, Delli, & Edwards, 2004).

In a study of high school students, the team led by Montalvo (2007) found that students work harder for a “caring” teacher. This study began with 172 initial participants of diverse population and both genders from grades 10-12 in the south central United States. Complete data sets were collected from 125 students, all from the middle class socio-economic status. The students were asked to evaluate their assigned teachers with two versions of the Survey on High School Student Motivation, a measure designed for this study. The students were randomly placed into two groups. One group’s survey discussed the teacher students liked first, and the other group evaluated the teacher disliked first. The second part of the survey addressed the opposite condition. The authors wrote subscales for learning goals, performance goals, perceived instrumentality for college admissions and school recognition goals, as well as measures of perceived ability, effort, persistence, and prior interest. Achievement was measured by semester grades.

Results indicated that learning goals and performance goals were not significantly related. The overall test of differences between variables in the liking and disliking data sets was significant ($F(9,116) = 15.96$, $p > 0.001$), considering the subscales listed above. Students had higher levels of effort and persistence in classes in which they liked the teacher. This study found evidence of correlation between liking the teacher and academic achievement in that class. The areas in the subscales were lower in classes where the students disliked the teacher (Montalvo, Mansfield, & Miller, 2007). As will be made clearer in the following section, trust between the student and the teacher facilitates authentic assessment and subsequent learning gains.

Summary of community-centered instruction

Community-centered instruction acknowledges the benefits of learning in a community, one where teachers and students jointly solve problems to develop their skill and understanding. Such a learning situation stimulates active learning concomitant with greater engagement, motivation, confidence and self-regulation. When the groups are nourished with respect, students respond by exerting more energy in the learning process, and learning gains increase.

Assessment-Centered Instruction

Research indicates that formative assessment practices can be an effective support for learning gains depending upon many variables. Assessment must be related to instruction, meaning that the results of assessment influence subsequent instruction. Teachers need to inform the student of both good performance and relative performance. The feedback must be timely and be part of an open communication between the students and teachers. The ultimate goal is student self-evaluation of learning achievement.

Provide feedback

The high school study led by Black & Wiliam (2006) discussed above particularly considered changing assessment processes. One change was to use written comments on student work rather than a number or a letter grade. Another change was to teach the students to use both self- and peer-assessment. Third, summative assessments were used for formative assessment purposes. The teaching community was extensively engaged in creating a cycle of information between the students and the teachers to identify what was being learned. As the teachers and students became aware of learning deficiencies or misconceptions, instruction changed to alter those positions. Results of the study included more engagement with learning on the part of the students, increased student motivation and confidence, some improvements in behavior, improvement in metacognition, and greater teacher awareness of individual needs, as well as academic gains (Black & Wiliam, 2006).

This study points attention to the connection between assessment and instructional goals. Students need assessments that reflect learning goals (Bransford et al., 1999). This process is termed a cycle, because it requires response by the teacher. Formative assessment can lead to further learning if it is tied to changes in instruction. Assessments may be formative in intention but are not so in practice because further learning is not generated. Formative assessment is about consequences: does the teacher change instructional practice after the student is assessed (Stobart, 2006)? Teaching and assessment are blended towards the goals of learning, particularly the goal of closing the gap between current understanding and the new understandings sought (James, 2006, p. 56).

Open communication channels

Helpful formative assessment depends on two things: (1) accurate information of the student's current understanding and (2) timely and accurate feedback about that understanding. The learners need to have not only a definition of a good performance but also an evaluation of their current performance (Sadler, 1983). Additionally, teachers must know what the student now understands, and this knowledge is a result of effective communication. The teacher needs to know what the learner knows so that they can adjust instruction (Bransford et al., 1999; Wiggins & McTighe, 2005). The learners must be assured that they can openly admit that they do not understand. A study of high school students examined the connection between relationships and the students' level of comfort for sharing. When students trust the audience, they express their learning problems more openly. Embarrassment is a real fear. If teachers are to be able to connect the new knowledge with existing knowledge, they need true responses from the students. There must be sufficient trust between student and teacher for open communication to take place (Raider-Roth, 2005).

Best practices then dictate that the teacher knows what the child already believes or comes to understand about the topic under study. This is expedited when the teacher shifts responsibility to the students for employing interpretative strategies, asking questions, clarifying meanings, as well as justifying and evaluating answers. There is better response when the teacher uses the students' words (Berry & Englert, 2005). Altering the classroom discourse by increasing wait time and accepting all answers, which may be a radical change for some teachers, makes a significant difference in student engagement (Black & Wiliam, 2006). "Active learning in which teachers and students are communicating ideas does seem foundational to quality learning," (Murphy et al., 2004, p. 87).

Engage students in the process

Students are more likely to improve if they are able to monitor the quality of their own work. Self-evaluation and peer evaluation are tools that have proven to be effective in improving learning gains (Black & Wiliam, 2006; Butler & Cartier, 2004). Learning goals are effective to the extent that students understand what they yet need to do to meet the goal. Since students must be able to assess their own performance to meet learning goals, a useful teaching practice is to embed self-assessment into classroom routines and activities. Teachers can encourage students to assess their work in terms of "complete" or "initial" or at places along the continuum between the two. The students can be taught to use a rubric to examine their progress (Black & Wiliam, 2004).

Peer-assessment is an effective learning tool as well since students learn in multiple ways. They acquire a better understanding of the task as they evaluate another's success. Furthermore, students express criticism in a language that may be more readily understood than by their classmates and the student may take the advice of a peer more readily than they accept the advice of the teacher. Students are more likely to interrupt a fellow student if they do not understand the explanation. Additionally, the practice of pairing students for the critiquing activity frees the teacher to reflect before framing an intervention (Black & Wiliam, 2004).

Requiring students to actively interpret teacher feedback and evaluation is also recommended as an effective teaching practice. Teachers should match evaluation carefully to task purposes (Butler & Cartier, 2004). Student learning and motivation increase with self-esteem. Nieto (2000) notes that teachers affirm diverse students when they maintain and affirm students' pride in their culture. For example, this can be done by supporting native language approaches and letting the students tutor each other in the native tongue, thereby validating their languages. Teachers ought to support additive acculturalism and bilingualism, since students profit even if they are already fluent. Monolingual English students can be included, and the teachers can ask the students to teach the new language to them.

Summary of assessment-centered instruction

Students learn with understanding when there is a clear connection between learning goals, assessment, and instruction. Formative assessment contributes to the achievement of learning gains when feedback is immediate and accurate. Feedback is facilitated by open communication between students and teachers, but performance evaluation by the student himself is a desirable goal.

Summary of Effective Instruction that Promotes Understanding

The Framers and Writers of the NGSSS desired for the state's students a mathematics education that was focused and comprehensive. The expectation is that teachers who follow the framework will teach for proficiency in the Big Ideas and Supporting Ideas. Proficiency in mathematics includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition (Kilpatrick et al., 2001). Understanding is a result when the learning makes sense to the child. Multiple instructional practices have been identified as encouraging this understanding, wherein children identify the underlying principles, structures, and concepts of content, and then make connections between the new knowledge and their existing knowledge. Teaching for understanding incorporates instructional practices that are student-centered, content-centered, community-centered, and assessment-centered.

***Standards*-Based Instruction in Mathematics**

To this point, the discussion has been about effective instructional practices that promote student understanding generally. The summary of the research that follows is explicitly related to instruction of mathematics. Successful teachers of mathematics use the techniques that have already been discussed, but such use is guided by the structure of the discipline itself. Seen through the lens of mathematics, emphases are changed, and some new issues emerge. This review of the research about events in the mathematics classroom is organized into three sections. The first is a discussion of the theoretical framework of mathematics instruction. The second describes three projects whose significance is still central to mathematics education research, as will be demonstrated in the section that follows. The third is an overview of studies that have been conducted to investigate teaching, learning, and mathematics instruction in schools. To insure a comprehensive review of the literature in the search for information about teaching mathematics in depth, the researcher combed every volume of the hallmark periodical *Journal for Research in Mathematics Education* since the publication of the *Standards* in 1989. This overview is organized by the researcher in a manner conducive to creation of a framework for teaching mathematics in depth.

As was narrated in Chapter 2, concerns and suggestions about the improvement of mathematics education in the nation's schools resulted in the publication by NCTM in 1989 of the *Curriculum and Evaluation Standards.* The decades following have seen two major areas of discussion, focusing on *Standards*-based curricula. The first area of discussion is about the epistemology—what is knowledge in mathematics? Is it empirical and real, something that teachers deliver to their students, particularly with direct instruction, as positivists would describe it? Or is it knowledge that is completely created by the individual, as radical constructivists would define it? The *Standards* opened this debate when it identified the goals of mathematics education as creating "empowerment" and "teaching for meaning."

The other area of study is the broad category that defines what it means to teach in a reform-centered mathematics classroom. Instruction is the heart of the teacher's role. To examine this role within the context of the *Standards,* researchers have observed and analyzed, curriculum developers have composed and evaluated, and teachers have experimented—all in an effort to structure the classroom where the *Standards* are implemented with integrity. The recommendations of these voices frequently overlap, as will be explained, always keeping in mind the quest of this study to define what it means to "teach in depth" in ways that will benefit students as they learn mathematics.

Theoretical Perspective of Mathematics Education

What is mathematics knowledge? Is it part of a discipline that is independent of human invention? Is it a human structure developed from a reality bound to the physical world? The answers to these questions profoundly affect the activity of instruction. If mathematics exists as a body of knowledge intimately connected to the environment and codified by scholars over the centuries, then the role of the teacher is framed as the one who organizes and delivers this knowledge to the students. The pressure of time is always present, since there is so much information to pass on to the learner and so little time in which to pass it. This philosophical stance acknowledges that it would be beneficial if the students understood it, but the delivery of instruction is not focused on this goal as much as it is on presentation. Scholars who adopt this epistemological view are known as positivists.

In mathematics literature, this view of teaching has been called Instructional Representation (Cobb, Wood, Yackel, & McNeal, 1992). The overall goal of instruction is to help students construct mental representations that correctly or accurately mirror mathematical relationships located outside the mind in instructional representations. The method for achieving this instructional goal is to develop transparent instructional representations that make it possible for students to construct correct internal representations. External instructional materials presented to students are the primary basis from which they build their mathematical knowledge.

However, researchers in psychology and behavioral science have demonstrated that learning for understanding develops differently, as will be explained later in this section. Information comes to the individual through their sensory apparatus and lodges in their short-term memory until it is reorganized into long-term memory. From long-term memory, it *transfers* to use in a similar situation. Much of what is perceived is lost for all practical purposes during this restructuring. This information is, in effect, forgotten or never learned. Educators facilitate the movement of new data into long-term memory when the instructional tasks demand active responses from the student. The brain actually changes physically as it incorporates new information (Zambo & Zambo, 2007). Active responses include explaining, describing, reflecting, and writing (Zull, 2002). In fact, these same active responses are essential components of learning mathematics with understanding.

Hiebert and Carpenter (1992) discussed these activities in their chapter "Learning and Teaching with Understanding" in the *1992 Handbook of Research on Mathematics Teaching.* Research about learning mathematics at that time was directed into two areas: cognitive understanding and social interaction**.** These mathematics educators explain that children learn with understanding through communication and reflection. Reflection is essential for cognition because it assists the brain to connect new learning to established understanding. Connections in the brain form networks, which are the building blocks of knowledge. Strong blocks of knowledge are webs of connections, and larger webs are useful in more situations. Mathematical input from sensory experience is organized into connections and larger webs through reflection, and from there to understanding. People understand something if they see how it is related or connected to other things that they know.

The other essential area of learning mathematics for understanding is social interaction, which depends on communication. As the place of communication in instruction will be developed more completely in the sections to follow, it is briefly outlined here, again relying heavily on the team of Hiebert et al. (1997). Those same active practices of talking, listening, writing, and demonstrating that facilitate cognition are the fabric of social interaction. Sharing mathematical ideas with others and listening to them share their thoughts builds common understanding of mathematics at the same time it assists individuals to formulate their own comprehension and often leads to better thinking than might occur individually (Hiebert et al., 1997). Mathematics is taught for understanding by working with both skills and concepts while using reflection and communication.

Some philosophers take this vision of learning further and say that knowledge is existent only to the individuals when they have constructed it themselves. The creator of meaning is the person organizing the data, and there are multiple realities, as there are multiple individuals (Davis, 2004; Hinchey, 1998). Human action toward mathematical ideas and concepts is based on the meanings those ideas and concepts have for them (Crotty, 1998). In mathematics, this philosophical construct could be said to form a continuum. On one end is radical constructivism, where each person has their own mathematics, created themselves from information obtained in the physical world, which could include the abstract knowledge of mathematicians. At the other end is social constructivism, a school of thought that defines mathematics as a cultural construct, created with language and social interaction. At any place along this continuum, the role of the teacher is seen to be that of facilitator, one who offers context and tasks wherein the student can consider the information and construct mathematics from it (Davis, 2004).

Mathematics educators are still grappling with this philosophical problem. If mathematics is fixed, then the teacher presents it to the students as efficiently as possible. If mathematics is created, then the teacher offers a context from which the student can construct it. Sometimes that knowledge is constructed as a community, and sometimes it is constructed by the individual. Is there an accepted body of knowledge that students are supposed to construct? Always, the teacher has to present important mathematics. What guides selection of such mathematics? One factor is one's beliefs about the nature of learning (Prawat, 1991; Putnam, Lampert, & Peterson, 1989; Remillard, 1992), since the practice of instruction is intimately connected to the event of student learning.

Vygotsky and Piaget

Consideration of how the student learns mathematics has been clarified by the influential work of two theorists, Lev Vygotsky and Jean Piaget. Although these psychologists were not mathematicians, their theories have strong influence on theories of learning mathematics. Each wrote in the earlier part of the twentieth century, but Piaget was translated from the French much earlier than Vygotsky was translated from the Russian (Blanck, 1990). Their impact on the constructivist theory of learning is profound and continues into the twenty-first century. Each theorist regards group work as a valuable learning tool, and each emphasizes the importance of knowing the child (Confrey, 1991; Greer, 1996; Rix, 2004).

Piaget's view of child development has been very influential in the process of identifying appropriate mathematics instruction. Piaget demonstrated that children think differently than do adults, and that much of their mental development begins with interactions with physical objects (Confrey, 1991). Additionally, Piaget developed the idea of developmental stages, which postulates the concept that ideas are built on prior constructions, and children cannot advance beyond a point before they are psychologically ready to do so (Zambo & Zambo, 2007). For an example from mathematics, consider place value. Children cannot construct place value schemes before they have a grasp of one-on-one correspondence and conservation of matter. In order to properly teach mathematics the teacher has to know these stages well and to also know how to determine if the stages have been met or not. Furthermore, Piaget postulated that the child progresses when he or she perceives cognitive conflict (Tudge, 1990). When something that they believe to be true seems not to be true, the children will examine the discrepancy and learn.

Vygotsky's view is that mathematics is part of a body of knowledge that is invented by the cultural community through language. As children learn this knowledge from the adults and other more knowledgeable peers, they internalize it (Lerman, 1996). Each person has a Zone of Proximal Development (ZPD), a space where they are ready for learning, where instruction is most efficiently applied (McCaslin & Hickey, 2001; Norton & D'Ambrosio, 2008). The teacher then must know what the student already understands so that they may create appropriate learning tasks that enable the child to integrate the new knowledge and refine misunderstandings.

The process of scaffolding was thus named, being the work teachers do to bring the student's learning to greater and clearer complexity.

Vygotsky contributed to the understanding of the impact of humanity's social and cultural heritage on learning. In an interview, contemporary researchers Hargreaves, Light, Perret-Clermont, and Grossen (2004) described experiments they conducted with children doing two unrelated tasks. The success of the child doing each task was related to the directions given that child. In the first experiment, children were told it was a dexterity task, related to either sewing or engineering. Participants responded to the gender references. Boys did especially badly if they were informed that the task was related to sewing. In the other experiment, the task was to copy a complex pattern that the children had viewed only briefly. Those who were told it was an art project drew it better than those who were told it was a mathematics project. Indeed, culture positions children as learners (Rix, 2004).

The following review of the literature identifies social constructivism as the guiding theoretical framework for teaching mathematics for understanding. It is based on the foundation that instructors of mathematics teaching the *Standards* assume the philosophical perspective that mathematics is a cultural construct, created with language and social interaction. The implications for instruction under this epistemological view are addressed next.

Implications of Social Constructivism

Knowledge is a cultural invention dependent on language. The first implication of social constructivism is that knowledge is dependent on language. The cultural discoveries and traditions of any society are explained, recorded, and shared largely due to the human capacity for communication through language. Language helps makes sense of the physical world even as it creates culture and knowledge (McCaslin & Hickey, 2001). Language also assists cognitive development as people internalize the cultural legacy, since psychological development and instruction are socially embedded; (Kinard & Kozulin, 2005; O'Connor, 1998). With language, the child learns to coordinate multiple social worlds, expectations, and goals, and thus creates personal meaning for the knowledge in those worlds (McCaslin & Hickey, 2001). Mathematics, too, is created through language, for although certain numeric faculties, such as counting and basic numerosities, are part of human physiology (Butterworth, 1999), the preponderance of the subject matter is defined by the society in which the learner lives (Blanck, 1990; Cobb, Boufi, McClain, & Whitenack, 1997; Forman, 2003; Hedegaard, 1990). Classroom discourse is the vehicle for acquisition of mathematical language. At school children are taught the signs, symbols, and syntax of this language.

Knowledge is created by the community of learners. The second implication is that a community of learners is the context for development of this knowledge, knowledge that depends on a conversation between the learners and those who teach them within the school community (Kinard & Kozulin, 2005; Meyer & Turner, 2002; Schunk & Zimmerman, 1997). Some identify thought as the internal dialogue begun between the learner and the more knowing other, internalized by the learner as he/she matures (O'Connor, 1998). The teachers may be adults or peers whose knowledge is more complete than the learner. Through dialogue, the participants reformulate a problem and create a solution. The learner internalizes and transforms the help they receive from others, using it to solve new problems (Putney & Floriani, 1999).

The community of learners as created in the mathematics classroom serves a particular function. To a great extent, the content of the mathematics curriculum has been created by the greater society, but within each classroom some of the mathematics is constructed by the participating individuals acting in concert. Together they establish criteria for acceptable solutions and justification. Although there is a foundational body of knowledge, authority over mathematical truth is shared between the teacher and the learners.

Teachers have to know what learners understand. The third implication is that teachers have to know what it is that learners understand. Children bring experience-based concepts to school where that knowledge is modified by academic concepts and experiences. Vygotsky defined a ZPD where maximum learning takes place if the teacher scaffolds the learners and the learners share their existing knowledge (McCaslin & Hickey, 2001; Schunk & Zimmerman, 1997; Tudge, 1990; Zimmerman & Schunk, 2003). Both teacher and students are learners in a domain of overlapping ZPDs (Allal & Ducrey, 2000). In school, the *experience-based concepts (EBC)* interact with *academic concepts (AC)* and learning takes place (Otero, 2006). At school, the learner becomes aware of their EBCs and they try out the ACs. As the learners' EBCs are abstracted, they become more useful. Additionally, the teacher must become aware of the students' EBCs, which serve as the foundation for instruction within the ZPD. "Determining the actual level of development is the most essential and indispensable task in resolving every practical problem of teaching and educating the child" (Zimmerman & Schunk, 2003, p. 200).

Implementation of the *Standards*

This second section addresses the design of curricular projects created to incorporate students' social construction of knowledge within the guidelines recommended by the *Principles and Standards for School Mathematics*. Since its inception, many educators have developed programs to implement the *Standards* as curricula for the schools. These projects purposefully incorporated reform elements into the programs, and the research teams studied the results. Among them are (1) Cognitively Guided Instruction (CGI) developed by Carpenter, Fennema, and Franke; (2) Conceptually Based Instruction (CBI): Hiebert and Wearne; (3) The Mathematics and Teaching through Hypermedia Project (The M.A.T.H. Project) by Lampert and

Ball; (4) The Middle Grades Mathematics Project (MGMP) through Michigan State University; (5) Problem Centered Learning in South Africa: Human, Murray, and Olivier; (6) Reality in Mathematics Education Project in Australia; (7) Schoenfeld's course for problem solving at the university level; (8) The Second-Grade Mathematics Project: Cobb, Wood, and Yackel; (9) Supporting Ten-Structured Thinking: Fuson; and (10) Whitnall High School Experiment using a translation of a Dutch curriculum: deLange, van Reeuwijk, Burrill, and Romberg (D. Clarke, 1997; Hiebert et al., 1997). This is not an exhaustive list, and from it three were chosen to be highlighted. They were selected for three reasons. First, articles about each project have been published extensively in the United States. Second, each project was large in scope, considering length of time, number of participants, and large quantities of data collected. Third, each can be discussed as an important example of the issues that were raised in the prior section about the implications of social constructivism.

Knowledge is a cultural invention dependent on language: The Second-Grade Project

Paul Cobb led a team of researchers in a year-long project where second-grade classrooms experienced instruction that was characterized as socioconstructivist and reform. The principal at each school assigned second graders to heterogeneous classrooms on the basis of the students' reading skills. Ten of the teachers at the three schools agreed to use the reform curriculum. Eight other teachers at these schools taught mathematics with traditional materials. Complete sets of data were collected for 187 project students and 151 nonproject students (Cobb, Wood, & Yackel, 1991). The basic premise of the study was that it is the individual child who has to do the reflecting and reorganizing while participating in and contributing to the development of the discourse (Cobb et al., 1997, p. 266). The instructional tasks were specifically designed to assist construction of mathematical concepts. The daily lesson began with a problem to be solved. As an entire group and in pairs, students discussed the problem and offered solutions. Students justified their answers by explaining the reasoning behind the solution. Games and other instructional tasks extended their understanding of concepts of ten. Every mathematics lesson during the year was videotaped. These classes were compared with eight non-project classes at the end of the year on a standardized achievement test. While the curriculum in the project classes was derived from problem solving, the non-project classes were taught with the conventional textbook program. The comparison of the project and the non-project schools indicated that scores on computational performance were comparable, but project students had higher levels of conceptual understanding, held stronger beliefs about the importance of understanding and collaborating, and attributed less importance to conforming to the solution methods of others, competitiveness, and task-extrinsic reasons for success (Cobb, et al., 1991).

Many aspects of instruction proved to be valuable to the realization of these results. The content was organized around problem solving. Both small and large groups were used. The teacher shared authority over knowledge with the students. The students were responsible for their own learning (Cobb, Wood, & Yackel, 1991). Instruction included a wide range of games and activities. Researchers and teachers worked together to create the instructional materials. The teachers attempted to facilitate a dialogue in which interpretations and solutions were accepted because they could be explained and justified (Cobb et al., 1991). Students knew that to be successful, one had to be able to explain their thinking to others (Wood & Sellers, 1997).

This study was extended and successfully integrated into other projects in elementary schools. The Second-Grade Mathematics Project explored the question of what it means to be a reform mathematics classroom by focusing on problem solving. As the instruction pivoted upon this concern, the research team was obliged to consider what exactly is the nature of the conversation about mathematical knowledge. When students share their knowledge, does the teacher really understand what they are saying? Are teachers hearing what the students say or do they hear what fits into the teacher's conceptual knowledge? Cobb's team thus came to identify the importance of classroom norms, of determining what is agreed upon within the classroom. After the terms in the discussion are defined, the knowledge based on those norms is clearer. For instance, when students were asked if anyone solved a problem in a different way, the answer may not be clear. If the question is 27 + 50 and a child describes the process as "adding 20 to 50 and then adding 7" is that a different process than " adding 7 to 50 and then adding 20"? What, exactly, is meant by the word "different"? Discussions arising from this foundational work continue in the literature. What are acceptable solutions (Cobb, Yackel, & Wood, 1991; Cobb, Wood, Yackel, & McNeal, 1992; Yackel & Cobb, 1996)? What is the difference between procedural and conceptual understanding (Cobb, Yackel, Wood, & McNeal, 1993)? What behaviors of the teacher promote this discussion (Cobb et al., 1997)? What does it mean to teach arithmetic through problem solving (Cobb & Merkel, 1989)? The influence of this body of work is great and continuing.

Knowledge is created by the community of learners: Mathematics and Teaching through Hypermedia Project

The elementary school classroom is the forum for the learning community that was studied by Lampert and Ball as they considered what it meant to teach mathematics to children. The process of instructing the children was one that included the students in the process of identifying important mathematics, of learning to do mathematics and to think mathematically. Together the teacher and students gathered information, organized it strategically, generated and tested hypotheses, produced and evaluated solutions. The community talked, listened, appreciated, and invented mathematics (Lampert, 1986, p.340).

For several years, Magdalene Lampert was a teacher in an elementary classroom at the same time she was a professor at Michigan State University. Deborah Ball, who had taught elementary school, joined Lampert as a mathematics professor who also taught a class at an elementary school. Together they created the Mathematics and Teaching Through Hypermedia Project (The M.A.T.H. Project). This extensive study created a multimedia, computer-supported learning environment to make instances of practice available for study by prospective teachers (Lampert & Ball, 1998).

The study was conducted in the elementary schools where they each taught a class that was videotaped. The researchers were participant observers assisted by six graduate students and a project director. Additional participants were the students and teachers of other subjects in each classroom. (1991). The data included three categories of records: (1) those produced as a matter of course in everyday teaching and learning: journals, lesson plans, student journals and notebooks; (2) those collected specifically for this project: structured field notes from observers; video and audio documentation—minutes before and after class, as well as class; and (3) annotation on events and documents. Analysis of the data incorporated triangulation among the different data sources and constant comparison within and between lessons (Lampert, 1991; Lampert & Ball, 1998).

The collaboration of Ball and Lampert is impressive on two counts: one is the clear articulation of dilemmas in the work of the teacher in a school mathematics classroom; the other is their profound impact on research in mathematics education. As they researched and taught, they reflected on what it meant to be a teacher, what it meant to be a student of mathematics, and how the two worked together in relationship (Franke, Kazemi, & Battey, 2007). Technology allowed them to access the information of the videotapes completely and make that information part of their classes instructing future teachers about the pedagogy of mathematics education.

Ball and Lampert reflect on the difficulty of being totally prepared for the job of teaching.

> It is difficult to be fully prepared for a day of teaching. From moment to moment, the teacher must observe, infer, interpret, and make conjectures. Her conclusions, although tentative, are knowledge claims to herself. The assertions she makes to herself function as knowledge. She knows them the best she can in the moment and must act, treating what she knows as both reasonably reliable and also provisional (Lampert & Ball, 1998).

The teacher is always caught in the dilemma of whether to steer the class toward the planned mathematical goal or to follow a students' novel idea (Ball & Lampert, 1999). Teachers need to make connections between what they think the students know and what they want them to learn, finding language and symbols that students and teachers can use to enable them to talk about the same mathematical content (Lampert, 1991). "We used children's ideas and respected them" (Ball & Lampert, 1999, p.372).

Preparing a student to work in serious mathematical inquiry means teaching them how to be part of the discussion, how to explain their ideas and discuss if the ideas are reasonable. Lampert developed such a classroom climate where students were expected to justify their ideas, to respond to the ideas of others, and to rely on one another as the mathematical authority rather than on the teacher or a textbook "Teaching is not only about *content.* It is also teaching students what a lesson is and how to participate in it" (Lampert, 2004).

> Lampert created a new kind of classroom community, one in which students were expected to justify their ideas, to respond to the ideas of others, and to rely on one another as the mathematical authority rather than on the teacher or a textbook. (Sherin, 2004).

Thus, Lampert's work presents some of the earliest evidence that elementary school students could in fact engage in mathematical sense making and argumentation, both by identifying elements of her teaching practice and by specifically highlighting her varied role in facilitating classroom discourse and selecting appropriate tasks for students to explore.

“We sought to engage our young students in serious mathematical inquiry” (Ball & Lampert, 1999). Such serious mathematical inquiry requires that both teacher and students define what counts as mathematical knowledge. They come to agree on shared assumptions and reasoning about the consequences. In mathematics, new knowledge is produced by testing assertions in a reasoned argument. In a community of discourse, people agree upon a set of assumptions, make generalizations about a given domain, and then explore the boundaries of the domain to which the generalizations apply (Lampert, 1991).

Teachers have to know what learners understand: Cognitively Guided Instruction

Cognitively Guided Instruction (CGI) grew out of a research project at the University of Wisconsin, Madison, under a team led by Carpenter and Fennema (1991). The team acknowledged that teachers make decisions constantly about their teaching practice. CGI hypothesized that these decisions would have more effective results if teachers were aware of their students’ understandings. The study began when twenty first-grade teachers were randomly assigned to a month-long summer workshop where they studied a research-based analysis of children's development of problem-solving skills in addition and subtraction. Meanwhile, twenty other first-grade teachers in a control group attended a different workshop. The program taught to the experimental teachers emphasized problem solving and deemphasized number facts relative to the program offered the control group. The experimental group was presented with research that identified student strategies for solving arithmetic problems using addition and subtraction. At the end of the following school year, students of the treatment group teachers performed better than control groups on written and interview measures of both complex problem solving and number fact knowledge (Knapp & Peterson, 1995).

Key features of CGI instructional practice are that instructional decisions should be based on careful analyses of students' knowledge and the goals of instruction. Teachers should have a thorough knowledge of the content domain, and they must be able to effectively assess their students' knowledge in this domain (Carpenter & Fennema, 1991). Although the summer study did not focus on instructional practices, results of the study indicated that the experimental teachers taught problem solving significantly more and number facts significantly less than did the control teachers. The experimental teachers encouraged students to use a variety of problem-solving strategies, and they listened to processes their students used significantly more than did control teachers (Carpenter & Fennema, 1992; Levi, Jacobs, & Empson, 1996; Carpenter, Fennema, Peterson, Chiang, & Franke, 2004).

A companion study by Franke & Carey (1997) of the students in two areas where the teachers had been trained to use CGI techniques showed that the students regarded mathematics as a problem-solving activity and saw communication as essential. The students were not motivated by speed and accuracy, but saw their purpose as finding the answer and being able to explain how that was done. The students were quite articulate about their views. They understood that they shared authority over knowledge with the teacher. They knew whether or not they were right, and such certainty did not come from the teacher's validation (Franke & Carey, 1997). The study began by CGI indicates that it is important that the students be allowed to explain their thinking and that solving problems is a vehicle for this thinking that can be used on a regular basis (Villasenor Jr. & Kepner Jr., 1993).

The legacy of CGI is its emphasis on teacher knowledge of student thinking. The specific contributions of CGI were in these three areas: (1) knowledge of specific problem types in addition and subtraction; (2) knowledge of the ways students solve problems; and (3) knowledge of how individuals work on particular problems (Carpenter et al., 2004). Teachers who are trained to use CGI learn that students' thinking is important. A follow-up study with a participant four years later indicated that she listened to her students more and that the students learned from each other. Over time, she organized the mathematics lessons around appropriate problems, asked her students to explain their thinking, and listened to what they said. When she did not understand, she continued questioning. Using student thinking as her guide instead of the textbook, the teacher's practice depended less upon direct teaching. The mathematics taught changed from procedural to problem solving. Believing that children were both capable of solving problems and profited from sharing their thought processes with each other and herself, she taught from what the children knew and integrated mathematics with other subjects (Fennema, Carpenter, Franke, & Carey, 1993).

Commonalities of *Standards*-based programs

Although the projects and work discussed above were oriented in different directions, their programs of reform mathematics share common components. One component is that each is grounded in a philosophy of shared authority over knowledge. Authority for determining the truth is not only in the hands of the teacher or the textbook, but truth is determined by consensus of the members of the learning community. Another is that each of the projects structured the instruction around problem solving, and these problems were relevant to the student's life. In each project, students were taught to accept the problem as a starting point for discussion, and they learned how to discuss. The students justified their assumptions, listened to their classmates, and became participants in a mathematics community. The teacher learned how to know what the students were thinking, to better be able to craft the tasks and problems that would allow them to interact with the mathematics content in increasingly complex and more useful, accurate understanding.

Mathematics achievement gains from reform programs

The literature of effective instructional practices in mathematics education primarily concerns programs trying to implement the reforms suggested in the *Standards* documents. There is good reason for this focus—achievement has improved. As previously mentioned, The Second-Grade Project noted superior achievement results by the students in the problem-solving classes over the control classes in tests measuring conceptual understanding. That group did no worse on tests of procedural understanding (Cobb, Wood, Yackel, et al., 1991). Boaler's studies of contrasting high school mathematics programs in both Great Britain and California showed that achievement in mathematics was superior in the schools that used reform curriculum (Boaler, 2002). As California implemented its reform framework, the state test scores were higher in classrooms where the teachers had participated in professional development workshops where they learned to effectively implement the reform curriculum (Cohen & Hill, 2000). Teaching mathematics conceptually using flexible grouping in conjunction with formative and summative assessments and open-ended problems proved to be advantageous for reducing the achievement gap between rich and poor as well as different ethnic groups in a longitudinal study of an elementary school (Beecher, 2008).

Several of the programs undertaken to teach mathematics in accordance with the Standards were described in a report of the federal government, *Exemplary and Promising Mathematics Programs* (U.S. Department of Education Mathematics and Science Expert Panel, 1999). Researchers pursued achievement scores in mathematics to better compare results for groups of children who were taught with specific reform curricula and those who were taught more traditionally. One example is a research study conducted by Reys, Reys, Lapan, Holliday, & Wasman (2003). Notable achievement in middle school occurred after using *Connected Mathematics Project (CMP)* or *MATH Thematics* curriculum materials in Missouri, where three middle schools using the programs for two years were compared to three middle schools that were not using these materials. The study results were replicated five years later (Post et al., 2008).

Schoen, Cebulla, Finn, & Fi (2003) noted that teaching behaviors consistent with the *Standards*' recommendation as well as reflective of high mathematical expectations were positively related to growth in student achievement at the twenty-six high schools using *Core-Plus Mathematics Project (CPMP)*. In the elementary school, Riordan & Noyce (2001) compared statewide standardized test scores of fourth-grade students using *Everyday Mathematics* and eighth-grade students *Connected Mathematics* to test scores of demographically similar students using a mix of traditional curricula. Their results indicated that students in schools using either of these *Standards*-based programs as their primary mathematics curriculum performed significantly better on the 1999 statewide mathematics test than did students in traditional programs attending matched comparison schools. In high poverty middle schools materials from the University of Chicago School Mathematics Project were used successfully when supported by sustained professional development and in-class coaching. The average effect size by the end of the middle school was .24 (Balfanz, Mac Iver, & Byrnes, 2006). The curriculum alone did not catch the students up with their peers, but the significant gains acknowledge that these materials will work with every student group when the teachers are sufficiently prepared to instruct with them.

Summary of *Standards*-based Instruction in Mathematics

Since the publication of the *Standards* the theoretical perspective of the epistemology of mathematics education has been a subject of debate and discussion. Frequent reference is made to either Vygotsky or Piaget, or to both. Whether one sees the two psychologists as opposites in a continuum or as complementary views of the same phenomena, their position continues to be a point of reference in scholarly articles about mathematics education (Sfard, 2003). The *Standards* directed attention to the meaning-making process students undertake when they are learning mathematics. "Indeed, not *inspite* of students' need for meaning, but rather *because* of it, students tend to construct their own conceptions" (Sfard, 2003, p. 357). This fact has opened the conversation about what it means to be a teacher in the mathematics classroom.

Three prominent research teams investigated these questions in projects that were large and influential. The work of these scholars continues to impact the research literature. The Second-Grade Mathematics Project (Cobb et al., 1991), the M.A.T.H. Project (Lampert & Ball, 1998), and Cognitively Guided Instruction (Carpenter & Fennema, 1991) were featured as examples of programs implementing the content and process *Standards.* Among the implications these groups suggested to mathematics education research were three: (1) mathematical knowledge is a cultural invention dependent on language; (2) this knowledge is created by the community of learners; and (3) teachers have to know what learners understand if they are to teach them. The commonalities shared by the three projects were a willingness to share authority over mathematical truth between students and teacher, a focus on problem solving within contexts relevant to the student, and integration of mathematical discourse into the lessons. Students justified their assumptions, listened to their classmates, and became participants in a mathematics community, even as the teachers became aware of student understandings. Moreover, the students in these projects achieved noticeable learning gains.

Alone, the three projects described in this section would serve as sufficient examples of effective instructional practices, but multiple studies have been conducted in the decades since these were first published. The next section of this manuscript considers many of these, looking to identify specific instructional practices used to achieve mathematically effective results,

results of understanding mathematics with fluency and facility. The review of these studies is organized by the researcher into a structure that will later contribute to a framework of teaching mathematics in depth.

Effective Teaching Practices in Mathematics

Theory of Authority over Content

As was pointed out earlier, fundamental to effective instruction under the *Standards* is one's theoretical perspective: where does authority for meaningful mathematical knowledge in the classroom lie? Is it in the teacher, the textbook, or the individual? Or do all share? Reform mathematics views the content as a dynamic body of knowledge that requires justification and proof as well as consensus, and it is built on active learning. All members of the community are constructing knowledge from shared expertise (Davis & Simmt, 2003). Students who share the power know when they have the right answer; they do not need the teacher to tell them (Franke & Carey, 1997).

To be certain, when the authority is shared, there is a measured amount of uncertainty. Davis and Simmt's study (2003) of a teacher learning group working on a university mathematics course pointed out the potential of a complex, organic organizational development. As the mathematical community evolved, the path to consensus was not straight, but more of a zigzag. The members found it necessary to draw on different areas of expertise, to work with several ideas at once, and to accept ambiguity, at least temporarily. Students who were taught in an environment where risk taking was acceptable, however, became more confident learners who enjoyed mathematics (Stipek et al., 1998).

This view of shared authority over mathematical truth is problematic for many teachers. Fundamental change is unsettling, especially when one's career may have been governed by a traditional perspective where the textbook was the authority, and the teacher's role was to transmit the information from that authority. This structure has worked well for them, because they have been effective (Smith III, 1996). In fact, one of the struggles within the reform movement occurs when traditional teachers are given reform curricula to teach. The practice of transmission can be so strong that teachers implement reform curriculum in that way. Remillard (1992) studied an experienced teacher of twenty-five years as he implemented a reform state framework for mathematics. He selectively incorporated elements of the state standards into his classroom practice. Since he believed that students learn mathematics by listening, watching, and practicing, his instruction remained more procedural than conceptual (Remillard, 1992). Teachers have been found to take from the new curriculum those sections that agree with their current practice but not incorporate the entire work, unless they were inexperienced. Beginning teachers tended to treat the curriculum more like a pilot and implemented it more fully (Remillard & Bryans, 2004).

Research suggests that sharing authority over knowledge profits everyone in the class. A British study contrasting traditional and reform programs showed that students who shared the authority over mathematical knowledge were better able to transfer mathematical skills to new situations (Boaler, 2002). Reform mathematics classrooms, with an emphasis on mathematical empowerment, have opened the doors of achievement and reduced the achievement gap in schools of diverse socio-economic status (SES) (Boaler, 1998; Franke & Carey, 1997) and ethnic diversity (Gutstein, 2003; Hufferd-Ackles, Fuson, & Sherin, 2004). Boaler‘s study of Railside High School observed a program that incorporated *complex instruction*, a teaching approach that emphasizes that all learners are smart, all have strengths in mathematics, and everyone has something important to offer when working on mathematics (Boaler & Staples, 2008). Showing students that they could contribute to the community pool of knowledge heightened the students' sense of agency and learning improvement followed. Riordan and Noyce (2001) found positive gains that were consistent across students of differing gender, race, and economic status in all schools that used the reform curriculum, whether the implementation was 100% true to the curriculum or not. Middle and lower-level students in heterogeneous classes scored significantly higher than did their counterparts in homogeneous classes on achievement tests in a study undertaken by (Linchevski & Kutscher, 1998).

Instructional Tasks

Mathematical knowledge can only be constructed from interactions with mathematics. The resources are vast and cannot be grasped immediately. The teacher's role is to mediate this content into a form with which the students can interact. Both students and curriculum can be moved to a position where interaction is possible, and the teacher plays the role of mediator in this process. The selection of the appropriate tasks is of paramount importance (Boaler, 2008). The team led by Cobb, Wood, and Yackel (1991) found that beginning each class with a non-routine problem opened the doors for fruitful discussion of meaningful mathematics. The students needed opportunities to work with information to find out what made sense to them. While selecting the problems, these researchers were careful to intertwine the activities in multiple domains, as well as use both everyday and fantasy scenarios. They found that the teacher, by using legitimate notations and symbolisms to record student work, was able to support the students' growing constructions of abstract concepts and efficient procedures (Wood & Sellers, 1996).

Part of the purpose of including these rich problems is to create a classroom context where students experience a shared authority over the construction of mathematical knowledge. Revisiting the videotapes of the Second-Grade Project from a different angle, Wood, Williams, and McNeal (2006) determined that only in certain classrooms were there occasions for all students to be involved in meaning making and to develop a common ground on which to build shared understanding. In classes where argument and justification were expected and expanded, these opportunities evolved. In classrooms where the students only shared their strategies, but did not question the strategies of others, the individuals expressed quality thinking, but did not become collaborators. Students' thinking about reasonableness and identifying flaws helped them grow into a community of learners to create mathematical meaning.

Students who can identify with the problems presented and who can become more involved with the context are more successful at understanding the concepts and transferring the knowledge to new situations (Boaler, 1998; Gee, 2004; Noble, Nemirovsky, Wright, & Tierney, 2001; Verschaffel & De Corte, 1997). Noble led a team that examined pairs of fifth graders as they worked in an *Investigations* unit about rate, the mathematics of change. Following a pair of boys through several related activities, they determined that the mathematical concepts were found not in the physical materials, the computer software, or prescribed classroom activities, but in what students do and experience. The authors suggest that the educator plan a range of games, where students can experience these lived-in spaces through discussion, symbolizing, and comparing games (Noble et al., 2001).

Using more realistic and less stereotypical problem situations in a teaching experiment, Verschaffel and De Corte (1997) were able to detect that students developed more realistic mathematical models than did a control group. During the pretest, the students did not use their real world knowledge to solve mathematical problems in school. The teacher then tried to create a new classroom culture where the teaching methods and the learning materials aimed at connecting mathematical problem solving to the experiential worlds of students. After an intervention that included systematic coaching and scaffolding for the students, the post-test indicated that students were able to construct more realistic mathematics models to solve these problems where the process of addition or subtraction was not immediately discernable.

Scaffolding is an important teaching practice when students are adjusting to non-routine open-ended problems as the basis for their mathematics class work. Lubienski (2000) observed in her own teaching that students reacted in different ways to these instructional tasks. Applying research methods to her own classes for her dissertation, she determined that the differences lay not along lines of gender or intellectual ability, but along lines that followed socioeconomic status (SES) groupings. Her work has generated interest as well as criticism, but she defends the reality of this “SES gap.” The students in the lower SES group were upset and confused when the teacher did not tell them how to solve the problem. The context of the problems often diverted the students’ attention from the mathematics in question. Lubienski (2000) recommended use of abstract as well as contextualized problems and greater teacher support when transitioning to this type of problem.

Movement to the high-level type of problems that most profit development of a learning community in mathematics incorporates practices that not all teachers may employ. The tasks assigned to a class of students may be appropriate, but the mathematics does not get developed. As part of an effectiveness study of the project Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR), directed by Silver at the University of Pittsburgh, Henningsen and Stein (1997) noted that not all teachers were implementing the curriculum faithfully, despite their willingness to do so. Beyond selecting the appropriate tasks, teachers needed to scaffold their students to make meaningful connections and meaningful explanations (which is the next topic in this literature review). Some teachers had removed the complexity of the task by giving the students too many hints or changing the assignment. Effective tasks are appropriate for the students and the allotted time, and the teacher guides the class into systematic exploration of the mathematics inherent in the task.

Relying on tasks at lower-levels of thinking may not assist students in construction of important mathematical knowledge. Henry's study of first graders' learning to be fluent with the basic addition and subtraction facts for numbers up to twenty demonstrated that tasks with lower-level demands, such as memorization and procedures without connection, proved to be unsuccessful. Memory-focused instructional events did not significantly predict memorization of these facts. Students taught with the textbook and traditional practices such as flash cards and timed tests were less successful at deriving the answers. Additionally, they had a less-developed number sense and greater difficulty with word problems (Henry & Brown, 2008).

Taxonomies. There are many taxonomies that may assist teachers as they determine which tasks are high-level and which are low-level. Bloom was the most famous of many to arrange a table of cognitive activities. He ranked them in order from simple to complex (de Landsheere, 1977). See Table 3-1 for these levels and their descriptors. As Bloom studied classroom instruction, he noted that 90% of educational time stayed at the lowest level of cognitive activities. He created the taxonomy to assist teachers as they created educational objectives to match instructional tasks. Some researchers agree with his list; others criticize it for neglecting basic skills or blurring demarcations at the complex levels (Booker, 2007). Others have refined this taxonomy and some have created new ones (de Landsheere, 1977; Forehand, 2005; Krathwohl, 2002).

Taxonomies have been created for the affective and psychomotor domains, but it is the cognitive domain that is most considered in mathematics. Shulman (2002) created a taxonomy of learning that ranged from engaged to committed, a classifying table that he hoped people would use more as a story and less as a guide (Shulman, 2002). He opined that taxonomies are a potential tool for examination of the curriculum, but not as a descriptive mandate, because all types of knowledge are valuable. One taxonomy in common use today considers instructional tasks as being low or high, concrete or abstract. Tasks such as recalling, remembering, implementing, applying facts and procedures are considered to be of low cognitive demand. At the level of high cognitive demand are activities such as explain, justify, assess, decide, plan, ask questions, create, or use more than one form of representation (Silver, Mesa, Morris, Star, & Benken, 2009). Concrete activities are embedded in objects, whereas abstract or formal activities are reasoned and hypothetical rather than actual.

To better evaluate test items in the state's end-of-the-year examinations for NGSSS, the FLDOE in 2002 adopted depth-of-knowledge criteria as a tool. Depth-of-knowledge is one of many criteria developed by researchers led by Webb at the Wisconsin Center for Educational Research to align state standards and assessments (Webb, 1999). It has been modified and adapted for use in alignment of mathematics assessments and NGSSS. This modified program altered the original four levels of cognitive complexity (recall, skill or concept, strategic thinking, and extended thinking) into three levels (low, moderate, and high complexity). Further definition of these levels is found in Tables 3-2 and 3-3 (FLDOE, 2006).

Communicating Mathematics Effectively

Earlier discussion of *Standards*-based curricula established that effective teaching of mathematics depends on communication and language. Through language, the members of the learning community relate to each other what they know and what they do not know. Theory and tasks are the beginning, and dialogue keeps the process going (Lo, Wheatley, & Smith, 1994). The language for and of mathematics has particular vocabulary, symbolization, and syntax. Teachers teach the subject effectively as they teach their students how to speak and/or write this language (Adler, 1999). Students engaged in writing or talking mathematics are actively moving data from short-term memory to long-term memory (Zull, 2002). Expectations for the learning task must be clear. A study of preservice teacher understandings of division of whole numbers indicated to Ball (1990) that the understanding of the word "explain" meant, "tell a rule." They were unprepared to explain or justify their understandings (Ball, 1990). What "explain" really means in the mathematics class has to be taught to the children.

Yackel and Cobb (1996) identify the creation of sociomathematical norms, a classroom consensus about mathematical expressions and understandings. One example is that the class comes to an understanding of what statements count as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant. Another example is what counts as an acceptable mathematical explanation and justification (p. 461). While conducting Second-Grade Mathematics Project, the teachers and research team discovered that it was important for the class members to agree upon certain concepts and terms. Until all agreed upon them, speakers had to justify their thinking to the satisfaction of the listeners (Yackel & Cobb, 1996). Students had to learn what constituted a contribution to the discussion. They also had to learn how to listen to each other. The students knew that they were responsible not only for their own learning, but also to assist fellow students in their learning. The teacher used the pronoun "we" instead of "I" to train the students to think of the class as a community making meaning. The students grew accustomed to "challenging" the thinking of their classmates (McClain & Cobb, 2001; Wood, 1999).

In another study, Cobb's team distinguished between the instruction of two teachers in leading classroom discourse. Teacher One spoke as the sole validator of mathematical meaning, and the students viewed her comments as directions—treating the situation as a set of procedures, not a problem for them to solve. Teacher Two taught the children that they would have to justify their thinking and everyone ought to listen to that justification with a view to criticizing it (Cobb et al., 1992). One teacher was teaching inquiry mathematics and one was teaching school mathematics. Both were the classroom authority, but one taught the students that they could follow procedures, and the other taught them ways to create and manipulate mathematical objects in ways they could explain and, when necessary, justify.

There is general agreement that specific behaviors are fundamental to the language of mathematical discourse. Mathematicians look for patterns to conjecture a rule, test the rule, and either use it for further exploration, reject it, or modify it. Therefore, classroom expectations are to examine patterns, note regularity, and expect statements to be supported with logical reasons (Blanton & Kaput, 2005; Jacobs et al., 2006; Reid, 2002; Wood, 1999). Consider the situation of $7 + 5 = 12$, $8 + 5 = 13$, and $9 + 5 = 14$. One can count to get the sums or one can memorize to get the sums, activities normally assumed to be classroom arithmetic, but these calculations are only part of mathematics. In the classroom, teachers can offer opportunities for the students to become mathematicians when they ask the students to look for patterns, to describe the patterns, to anticipate other examples of the patterns, and to offer means for verifying the truth of their conjectures. Cobb labels this particular form of reflection mathematizing discourse—the mathematics of the set of problems is the object for discussion, not simply the answer to a calculation (Cobb et al., 1997).

Communication can be facilitated if a student can present solutions and strategies with multiple representations. If a person really understands a concept, they are able to represent it in various modes. The situation in the previous paragraph could be expressed in numerals, as above, or with physical objects (manipulatives or realia). The numbers could be placed into a graph or the context of a story. The pattern can be described as a logical argument. Brenner's team (1997) determined that moving between representations is learnable. Additionally, being able to represent the problem was related to solving it. Pyke's (2003) work with eighth grade students in algebra and geometry studies showed that the students' use of symbols, words, and diagrams to communicate about their ideas each contributed in different ways to solving tasks that reflected different kinds of cognitive processes involved in problem solving. Communication about mathematics was fostered by the bar method used prominently in Singapore to solve word problems, a method shown to be effective with language learners as well as mainstream students in a study by Ng and Lee (2009).

Teachers of mathematics must be aware of how their instruction positions the students to become owners of this language. Setati (2005) studied South African elementary schools where both the official language and the language of the home were used to educate children. It was clear that more than the words were heard by the students. Directions were given in English and concepts were taught in the home language. However, the students attached authority to the procedural language, and its mathematical impact in their home life was minimized. Teachers of bilingual students can bridge this gap by being aware of who the students are.

Instructors in two different schools were studied by Atweh, Bleicher, and Cooper (1998) to examine how teachers perceived the needs of students for mathematics instruction. The teacher in the high-SES boys school treated the students as if they were headed for the university. They were taught with the formal discourse of mathematics and learned to investigate the subject with argumentation and sarcasm. The teacher in a low-SES girls' school took a more didactic approach, using everyday language in an effort to make the subject approachable. The authors argue that the teachers were constructing the mathematical identity of the students even as they were constructing mathematical knowledge (Atweh et al.,1998). The mathematics classroom is the social context where mathematics knowledge is negotiated and constructed, and this construction depends a great deal on the teacher.

Through scaffolding, peer collaboration, and the interweaving of spontaneous and theoretical concepts, a teacher creates a culture of inquiry (Goos, 2004). Lo, Wheatley, and Smith (1994) use the example of the experience of Brad, a third-grade student, to point out several conclusions. Students profit from whole class discussions because such discussions provide opportunities for individual students to connect with classmates even as they mentally integrate their mathematical knowledge. They learn from presenting the work of their small group to the entire class, and being challenged reinforces their learning. Classroom norms that require them to communicate their mathematical ideas can promote meaningful learning. By providing many opportunities for students to work out social norms and to negotiate mathematical meaning, teachers facilitate learning.

Although research has pointed out multiple benefits of this culture of inquiry, even recognized expert teachers are not practicing this type of instruction. Truxaw and DeFranco (2008) studied the classroom instruction of three teachers chosen as participants because of their credentials as a National Board Certified Teacher or the winner of a Presidential Award for Excellence in Math and Science Teaching (PAEMST). Results fell into three types of instructional practice: inductive, deductive, and mixed. Mr. Larson’s dialogic model incorporated both exploratory and accountable talk that helped students discover the principle and construct new meaning. Periodically, the teacher infused what the research team considered to be generative assessment, to promote students’ active monitoring and regulation of thinking about the mathematics being taught. Ms. Reardon’s univocal discourse characterized the deductive model, in which she conveyed the rules and procedures and ended the discourse when students supplied the correct answers to her questions. The mixed model was attributed to Mr. Townsend whose conversation tended toward the univocal, but he did use dialogue periodically. Although his purpose was student discovery, he did not follow up on the initial conjectures of the students, but instead shifted the classroom discourse from exploratory to leading, where the exchanges are controlled by the teacher, leading to a particular point. The model with the most success in situating problems into the larger context of mathematical concepts was the inductive model (Truxaw & DeFranco, 2008).

Communication refers to many aspects of mathematics education. Language is one medium of communication; it is also specific to what it means to be doing mathematics. The language of mathematics is privileged in this society (Atweh, 1998; Brenner et al., 1997; Paul, 2005; Sfard, 2003). By teaching all students this privileged language, schools open doors to college and scientific and technical careers. Additionally, discussion is a medium for learning. Talking to others offers students opportunities to express their thoughts, to defend their position, and to activate the processing of data from their short-term memory into long-term positions.

Students' Thinking about Mathematics

Cognitively Guided Instruction.

In the preceding section of this chapter, CGI was introduced as a project that examined students' thinking about addition and subtraction. Since several studies are grounded in the work of teachers who have had CGI training, the project is presented more thoroughly here. Fennema's team (1996) enlisted teachers for a four-year period. During this time, there were regular teacher workshops throughout the school year and each school was assigned a mentor teacher and a research team member to provide teacher support. Data collected from the teachers and their classes over the four-year period proved that teachers' instructional practice had changed. As they saw their students using the strategies identified in CGI, they increasingly made problem solving a greater part of their instruction. The students' achievement in concepts and problem solving was higher at the end of the period than at the beginning. Their study provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction (Fennema et al., 1996, p. 432).

Other research has had similar results. Knapp and Peterson (1995) contacted the original CGI contingent four years later. Most were located, and half agreed to be interviewed about using CGI in their practice. They found that CGI had changed their practice, but change had been gradual. Most change occurred in the rooms where the teacher developed conceptual and flexible meanings about CGI for themselves. Teachers who considered CGI as a set of procedures grafted some changes on their existing practice. Vacc and Bright (1999) found that beliefs about the nature of teaching and learning mathematics changed for preservice teachers (PST) when CGI was part of the methods course in their program. The PSTs became more constructivist in their understanding of student learning. The researchers note, as well, that the process of changing instruction needs support that could be provided by mentors, discussions, or meetings with peers.

Villasenor and Hepner (1993) conducted a study with urban first grade classrooms. Twelve classroom teachers were trained in CGI techniques and another twelve were given professional development about the importance of problem solving. At the end of the year, the CGI classes had better achievement results in word problems and completing number facts. The CGI teachers used word problems from contexts close to their students' lives, asked students to explain strategies, taught in a variety of formats, and seldom used work sheets. They did not teach specific strategies. Results indicate that the students need not have learned all the basic facts before they could solve problems. Superior results came from the mostly-minority urban population in the treatment group.

The principles of CGI are not so much a set of procedures as they are a philosophical orientation to the instruction of mathematics. Children are given contextual problems, offered the opportunity to solve them, and then to discuss their strategies and solutions. In CGI training, the teachers are treated similarly. They are offered the opportunity to view children solving mathematics problems and explaining their thoughts about the mathematics. Teachers then are given the contextual problem—for them—which is how to use this student thinking in their classrooms as they taught mathematics. Together teachers share ideas and solutions. The sharing continues throughout the school year with professional development meetings and interviews with support personnel.

These principles about teaching addition and subtraction were successfully extended to algebraic thinking during a study led by Jacobs (2007). Teachers were introduced to student strategies with algebra, using the lens of student thinking. Nineteen elementary schools in this low-performing urban school district participated. The results were that the students of these teachers showed significantly better understanding of the equal sign and used significantly more strategies reflecting relational thinking. The participating teachers were able to generate a wider variety of student strategies, particularly those that reflected the use of relational thinking, than did nonparticipating teachers (Jacobs et al., 2007).

Cultural, ethnic, and economic background

One important reason to consider children's thinking is that not all children think alike about mathematics. Sometimes there are differences between children that are based in the cultural or ethnic life of their families. Guberman (2004) found that there were differences between children of Latin American and Korean American communities when they were engaged in solving problems regarding money. The Latin American children responded more accurately if the representation in the problem was money (currency and coins); the Korean Americans' accuracy was superior in problems represented by denomination chips. The authors trace this difference to the children's after-school activities. While in the Latin American community there were more instrumental activities with money, activities for the Korean Americans were more likely to be intended to support their school learning.

Cultural characteristics were found to influence student choice of strategy while solving problems in a study by Malloy and Jones (1998) of African American eighth graders. Although this group of participants was able to use the analytic processes preferred by students in the mainstream culture, they opted to use holistic reasoning more frequently. The group was also distinguished by confidence in their mathematical abilities, whether or not they were able to solve the problem. In another study, Berry and Englert (2008) examined the experiences of eight African American middle school boys who were successful in mathematics. Among the common themes that emerged were adult support, recognition of their individual abilities, and a positive mathematical and academic identity. A key implication that emerged from his study was that teachers ought to know their students' academic abilities well and not be riveted on aspects of behavior as indicators of future achievement.

Students who perceive themselves as outside the cultural mainstream of education and mathematics are especially needful of instruction that listens to them, to their authentic self. A team led by Gutstein (1997) worked with an elementary/middle school in a Mexican American community to improve mathematics education. The teachers were trained in culturally responsive teaching and critical pedagogy. These orientations helped the teachers elicit and listen to the students' thinking, make use of that knowledge to inform instructional decisions, and integrate it into their curriculum.

The distinction between mathematics from family life and mathematics from school life was also analyzed in a study of Brazilian street sellers by Schliemann's team (1998). The students' extensive workplace computation skills were not necessarily transferred to the school. Although the students successfully calculated complex multiplication problems during transactions, they had never adopted the commutative property of multiplication, and future work with ratio was compromised. The team concluded that demands of the workplace imposed limits on the development of mathematical ideas that are not relevant to the task at hand. For maximum effectiveness, teacher plans must be guided by what the students already understand to be true.

Advantage to expressions of thinking

This section has enumerated some of the benefits to instructional practice when teachers access student thinking and also considered some of the difficulties acquiring such access. Additionally, there are benefits to the student, since learning can be improved by these practices. Irwin (2001) studied the interaction between pairs of students who were matched according to their mathematics achievement. This school was in a lower economic area of New Zealand, and the students represented many ethnic and cultural groups. The students' thinking about contextual problems was different, separating along lines of whether or not they used the knowledge from outside school to solve mathematics problems. The questions were about decimal numbers and were embedded in situations that were common in the community. While the higher achiever worried about the decimal placement, the lower achiever viewed the logic of the problem. The latter knew that if you bought a liter of gasoline and a burger, $5 would be sufficient. The former got caught up in his calculations and thought the answer would be $50 (Irwin, 2001).

Several studies have found that students self-corrected when they voiced aloud their strategies for solving problems (Hollebrands, 2007; Johanning, 2008; Lamon, 1996). Giving students the opportunities to think aloud was described by Empson (2003) when she returned to an earlier comprehensive study to consider the work of the two boys who consistently achieved at the bottom of the class. Since she had learned in private conversation that their learning was substantial, she reviewed the video record to better understand what was happening. She discovered that the teacher made significant impact when she worked with the boys' small group. If the pupil was uncertain or incorrect, she redirected their attention in various ways, like asking them to evaluate the answer of another member of the group. Each child understood that they were contributing to the conversation. Since the classroom routines had been structured to support group work with specific roles, this validation continued even as the students worked independently. Empson concluded that the sheer frequency of productive contributions gave these boys multiple opportunities to learn the value of their ideas and the practices entailed in developing and articulating them (Empson, 2003).

Evidence of misconceptions revealed by student thinking

Teachers often wonder why the students have come up with a particular answer. Although a particular mathematical point had been taught, the students were not using the concept correctly. Chandler & Kamii (2009) used a play-store game with children from grades K-4 and discovered that they added a dime easily, but did not subtract from a dime easily. Through student interviews, the researchers were able to isolate the source of the class's misunderstanding. Although they had had many experiences with models of place value of tens and ones, the students had not firmly established equivalency of a dime and ten pennies. Student interviews were used in Hart's study (1984) of ratio, which established that some of them were using multiplicative thinking, but many were still governed by additive thinking as they approached these problems. Through discussion with the students, teachers were able to construct learning activities that reached the exact misconception and clarified the students' understanding. It profits instruction when the teacher discovers exactly what students are thinking as they approach mathematical tasks.

What students are thinking is also an international concern. A group of researchers led by Ben-Yahuda (2005) interviewed two young women, age 18, in Israel. These students had never experienced success in learning mathematics. Using discourse analysis, the researchers concluded that there were particular learning problems that could have been solved earlier in the student's educational experience. If one of the women had been given permission to use a calculator at an earlier junction of frustration, before she had written a narrative of "poor student" to describe herself, she might have advanced further in studying mathematics. The other had also enveloped herself in a negative narrative, partly because she had a limited vision of what counted as legitimate discourse in a mathematics class, never connecting her work with money and the mathematical operators she learned about in school. The authors conclude that the diagnostic tools in the files of the school have too little differential power to be useful in discovering these deficiencies (Ben-Yehuda, Lavy, Linchevski, & Sfard, 2005).

Correct answers on student examinations are not always indicators of number sense. In a study of sixth and eighth graders in Taiwan, Reys and Yang (1998) conducted two examinations in four schools with heterogeneous groups. Several students were selected for interviews after the results of the tests were determined. Parallel instruments were set up, one for number sense and one for computation. It could be seen, then, whether the student understood the sense of the problem that they had computed on the other examination. What became apparent was that high skill in computing is not necessarily number sense; correct answers do not necessarily mean good thinking. Teachers must examine more than answers and must demand from students more than answers (Reys & Yang, 1998).

When teachers are not fully certain of their students' understanding, they can unconsciously limit the students' opportunities to learn, as was discovered in a study by Izsak, Tilema, and Tunc-Pekkan (2008). The experienced teacher was using a reform curriculum to teach addition and subtraction of fractions to her sixth grade class. Both the teacher and two of the students were interviewed as to what their understanding was about this lesson of adding and subtracting fractions along a number line. The outcome of the interviews indicated that the girls had not understood that the teacher was considering the fraction as a fixed length. The girls incorporated the teacher's explanations into their own understanding of fractions as a pair of whole numbers. As the teacher had drawn the partitions and intervals on the number line, she often moved the location of "1" to make the intervals evenly spaced. This habit confused the girls' understanding, and opens to question the utility of casual drawings as a helpful tool for representation. The discrepancy of understanding was determined only through the interview process.

Using a framework developed from an analysis of the work of a first grade teacher Fraivillig, Murphy, and Fuson's (1999) team guided a cross-teacher analysis to assist teachers to support student thinking. This framework identified important elements of a teacher's dialogue with the students. Results indicated that teachers often supported children's mathematical thinking but less often elicited or extended their thinking. It is important to elicit the children's thinking, but it is also important to support the thinking of both the describer and the listener during classroom discussion. Teachers must extend the children's mathematical thinking as well, evincing high standards for all, encouraging reflection on the mathematics and cultivating a love of challenge.

Classroom Organization

Flexibility in grouping supports the other program changes that have just been outlined. Sometimes the class is taught as one group, sometimes as pairs, sometimes as individuals, and sometimes as groups of other sizes. The groups may be changed several times during the instruction period, or they may remain in one format for several weeks. The Second-Grade Mathematics Project regularly began the class with pair work and then processed that work with the whole class (Cobb, Wood, Yackel, et al., 1991; Wood & Sellers, 1996). If students are to be able to construct mathematics with authority they need to have the opportunity to discuss that work and share the results with the greater community (Yackel & Cobb, 1996). Altering the arrangement of the furniture provides visual and kinesthetic evidence to the students that agency is shared. Students working together are acting as mathematicians do, working together and sharing their ideas (Smith III, 1996; Verschaffel & De Corte, 1997).

Lampert and Ball regularly created and recreated groups for their classroom instruction to take full advantage of the non-routine problems they had to solve. Non-routine problems are beneficial because many strategies can be used to solve them. When students are placed in small groups or pairs, it is more likely that they will generate multiple strategies and solutions (Lampert & Ball, 1998). Working in small groups, students are more likely to consider if their answers make sense than when they work alone (Hart, 1993; Irwin, 2001).

Students can talk to each other, test out their ideas, find difficulties, and brainstorm in the smaller group before they present to the entire class assembly (Boaler, 2002; Empson, 2003). In a smaller group, the students have a greater option of engagement (Henningsen & Stein, 1997).

Learning is improved when students need to explain and justify their work to the entire class (Lo, Wheatley, & Smith, 1994). When the room arrangement never varies from rows of chairs in columns facing the front, a subliminal message is sent that mathematical conversation is not valued here (Jacobs et al., 2006).

Teachers who used CGI with urban school children in the study by Villasenor and Kepner (1993) incorporated flexible grouping strategies into their practice as part of the other changes mentioned earlier. In another setting, flexible grouping strategies were combined judiciously with formative and summative assessment to build student learning gains in one northeastern school documented by Beecher (2008). There the reduction of the achievement gap between rich and poor and among different ethnic groups was effectively reduced, having all groups less represented in the remedial zone.

Time

More than twenty years ago, NCTM examined *New Directions for Elementary School Mathematics* in the 1989 yearbook. The introduction said, "In general, our computational strand needs to be slowed down so that ample time can be spent developing number sense and meanings of operations, as well as applying learned computational skills and integration of topics" (Lindquist, 1989). Many of the good practices that contribute to teaching mathematics effectively require more time than has been generally allocated (Clarke, 1997; Kilpatrick et al., 2001). Lecture, review, and computational practice require less time than do cooperative group learning, mastery learning, and manipulatives (Keiser & Lambdin, 1996). With the implementation of NGSSS, it appears that there will be more time within the mathematics classroom. Formerly some activities were not incorporated into the enacted curriculum because they took more time to follow to completion. Tasks for high-level thinking are often among these, because they are complex and take longer to finish (Henningsen & Stein, 1997).

Suggestions for the use this new abundance of time in mathematics can be found in results from Jacobs and Morita (2002). In their study, 40 American and 40 Japanese teachers independently evaluated either an American or Japanese mathematics lesson captured on videotape. Their comments were classified into over 1600 *idea units*, which were then sorted into a hierarchy of categories derived from the data. Results noted that the Americans approved of a greater range of lesson scripts than did the Japanese. The Americans thought that all the lessons they saw, traditional or nontraditional, were good. However, the Japanese teachers compared the lessons they observed to a standard of an ideal lesson. The Japanese educators did not approve of the traditional American mathematics class script; teachers had not allowed sufficient time for the students to investigate the concept before they began to practice individually. In Japan, the students generate the formula. During traditional American mathematics lessons the teacher presents the formula and the students practice using it.

Results of the Trends in International Mathematics and Science Study 2007 (TIMSS) comparative study found the mathematics achievement of fourth and eighth graders in the United States to not be among the leaders. Best in the group in 2007 were Singapore and Hong Kong, whose students scored best in the fourth grade. At the eighth grade were Taipei, Korea, and Singapore. (International Association for the Evaluation of Educational Achievement, 2011). Further study of the curricula and the schools pointed out that the structure of the mathematics lesson varied considerably. In Japan, the emphasis on understanding is evident in the steps of an eighth grade lesson: (1) teacher poses a complex, thought-provoking problem; (2) students struggle with the problem; (3) various students present ideas or solutions to the class; (4) the teacher summarizes the class' conclusions; (5) students practice similar problems.

U.S. mathematics teachers can also compare the pacing of their lessons with Japanese lessons. Japanese educators spend 44% of the class period inducting the answer from the students, while American teachers do that in 1% of the session. Japanese teachers use 40% of the class time practicing a procedure; American teachers use 95% (Wiggins & McTighe, 2005). Textbooks and curriculum were analyzed as well. Researchers found that mathematics instruction in the United States contains many more topics during the course of the school year. Whereas Japanese teachers address five topics for 75% of the school year, those same topics in American schools are taught in less than half the year (Schmidt et al., 1997). Teaching for understanding and teaching in depth are more likely under the extended timetable for the mathematics lesson.

Summary of Effective Teaching Practices in Mathematics

Instructional practices that effectively teach mathematics for understanding have been measured by multiple studies published in the professional literature. These studies have been grouped together according to categories that define areas of practice particular to mathematics. These are theory of authority over content, instructional tasks, communication, student thinking and assessment, classroom organization, and time. The categories were arbitrarily selected for the utility they provide when one considers the development of a framework to assist and train teachers of elementary mathematics to be effective in their practice. The specifics of this framework are introduced in the following section.

A Tentative Framework for Teaching Mathematics in Depth

A catalogue of the beneficial practices discussed in this review of the literature describing effective instructional practices that promote deep understanding would create a very long list. Teaching is complex work, and the goals for teaching mathematics in depth will complicate it further. The desired product for this study is a framework for teaching mathematics in depth.

This framework should focus the multiple possibilities into a few elements that describe the backbone of the practice. The resulting structure should be helpful for teachers as they are challenged to teach mathematics in depth. The following section describes the elements in a tentative framework. The word tentative is appropriate until the research data are collected and analyzed. This temporary framework is the expression of the practice that can be derived from the literature. Data from the teachers, CRTs, MTEs, and IMCs may alter it, but the elements selected at this time for discussion are *theory, tasks, classroom discourse, thinking, grouping, and time.* These elements exist not in isolation, but are part of the intricate network of actions that comprise teaching mathematics in depth.

Theory

When the epistemology changes from positivist to constructivist, the paradigm governing the classroom also changes (Crawford, 2008). The holder of mathematics knowledge is neither the teacher nor the textbook. Children are constructing meaning for themselves. That meaning is not “right” or “wrong,” but is “possible.” When teaching mathematics in depth, the teacher will allow shared authority over truth. There will be alternative ways of viewing knowledge. These ways will be subject to validation, but the teacher is no longer the sole proprietor of validity. The teacher and the children work together to determine meaning that members of the learning community can agree upon. The teacher draws from the thinking of the children and the body of knowledge that comprises mathematics as a content area.

Communities of learners value the ideas and methods of the students; students have autonomy in choosing and sharing their strategies and solutions; mistakes are appreciated as a site of learning for everyone; and authority lies in the logic and structure of the subject rather than the status of the teacher or the popularity of the person making the argument (Kilpatrick et al., 2001, p. 344). Moreover, children's learning of mathematics is richest when it is self-generated rather than when it is imposed by a teacher or textbook, because it is tied to what the children already know. Children learn the concepts, facts, and skills and also how to manage and regulate the application of this new knowledge (Schroeder & Lester, Jr., 1989). This approach to mathematics may seem risky to some teachers, especially those who have been teaching conventionally, but this philosophical orientation will ground the practice of teaching mathematics in depth.

Tasks

The skeleton of the mathematics lesson is created by the instructional task drawn from an assortment of problems using diverse materials, multiple modalities, and designed to be engaging. When these activities engage student interest and intellect, more learning is possible. The problems chosen should be both abstract yet related to the context of the students' lives, since student engagement is increased when instructional activities build on their prior knowledge. As assessment activities indicate that students retain misconceptions or partial conceptions, teachers should respond to those indicators and adjust instruction. When the majority of these tasks are non-routine and open-ended, they offer occasion for the presentation of various strategies and several solutions and invite opportunities for agency and sharing. Such communication provides an opportunity for clarification and elaboration of mathematical concepts and ideas (Wood et al., 2006).

Students learn best when they are presented with academically challenging work that focuses on sense making and problem solving as well as skill building (Kilpatrick et al., 2001, p. 335). This work can lead to increased student understanding, the development of problem solving and reasoning, and greater student achievement (Silver, Mesa, et al., 2009). The cognitive demands of the tasks should be conceptual and procedural, as well as high-level and low-level. Teachers support learning within these activities by scaffolding, modeling a high level of performance, and allocating the appropriate length of time (Kilpatrick et al., 2001). Teachers should adhere to the instructional design of the task, because cognitive demand is shaped by how the students use the task. Teachers ought not give too much help nor explain too much (Silver, Mesa, et al., 2009). It is true that students can only learn what they are taught and reinforced. If the tasks support more analytic and reflective thought, then reflection will be encouraged when the students face new types of problems (Hiebert & Wearne, 1993; Stein, Grover, & Henningsen 1996).

Classroom Discourse

A central focus of teaching mathematics in depth is to teach students to speak mathematics (J. Adler, 1999). Mathematics is a language with vocabulary and syntax, and it is also a privileged language. Historically, mathematics classes have been used to stratify students (Atweh, 1998; Brenner et al., 1997; Gee, 2004; Paul, 2005). Since privileged children have greater access to this rigorous academic language outside of school, the second-language learners and other ethnic groups often experience poor instruction at school and, as a result, have limited admittance to college prep courses, four-year university entrance, and technical and scientific careers (Gee, 2004; Paul, 2005). This language is better taught when teachers build into instruction opportunities for oral or written discussion. Not only will they be teaching the appropriate vocabulary and its proper use, but additionally they will be teaching the methods of mathematicians, reasoning and justification (Ball, 1990; Yackel & Cobb, 1996; Cobb, 1992). Students need to learn that an explanation consists of a mathematical argument, not a procedural description (Ball, 1990).

Since the object of understanding cannot be defined independently of the way in which it is experienced or understood, teachers will be modeling the concept as it is understood in mathematics (Marton & Neuman, 1996). Therefore, content knowledge for teachers is essential.

Teaching mathematics in depth treats errors as opportunities to reconceptualize, to explore contradictions, and to pursue alternative strategies (Kazemi & Stipek, 2001). Managing discourse during teaching mathematics in depth is an important task, since teachers have to judge when to tell, when to ask, when to correct, when to prompt, and when to grapple. Calling on someone who is right may be the easiest course of action, but one may miss opportunities for discussion if they take that course. Recognizing that the point of discourse is to develop understanding of key ideas, the teacher needs to have those ideas clearly determined in their own mind (Kilpatrick et al., 2001).

Discussion is a path to active learning. Preparing children to be adults in the twenty-first century is to prepare them to be creators of ideas. When children are able to find things out for themselves, they have power over the subject matter. Communication can offer children this power in a mathematics class if the children are asked to describe what they think is going on in a mathematical situation, why they think they are correct, or what their answer means in the context of the original problem situation. Thus a routine lesson becomes a metacognitive lesson as well as a mathematics lesson (Lappan & Schram, 1989). Students who were in classrooms where they talked about the mathematics had more had opportunity to clarify their own thinking and support problem solving (Hiebert & Wearne, 1993). Everyone carries understandings of the topics being considered, and discussion offers the opportunities to discover what these individual understandings are (Marton & Neuman, 1996).

However, discourse depends on trust. The most effective conversation is candid conversation. Students should be able to share their confusion openly and explore their understandings without being defensive. Perhaps this means that teachers encourage conversations in the students' native languages, as effective teachers of Latino/a students discovered when they used cooperative groups, encouraged Spanish conversations, found Spanish materials, and made particular efforts to develop shared meanings for mathematical vocabulary among students in a class (Gutiérrez, 2002). Teaching mathematics in depth requires the development of a classroom climate wherein all the students feel respected and valued. They are willing to participate in dialogue because they know their understandings are respected and valued.

Formative Assessment

Teachers who teach in depth use frequent and informal assessment procedures to determine what the students are thinking and understanding about mathematics. Since the students are constructing meaningful knowledge from classroom instruction, it is imperative that the teacher periodically determines what, indeed, the students are learning. Assessment functions best when it is coordinated with instruction. These assessment tasks can be neither too trivial nor too difficult, because the teacher must be able to discover what the student now knows. Effective assessment depends on enactment of instruction to improve understanding (Kilpatrick et al., 2001).

For maximized benefit of assessment, occasions for students to formalize their thoughts should be frequent and plentiful. Although it may seem that the entire class shares a particular understanding, one cannot be assured of that unless there have been opportunities to hear from the individuals. Teachers need to make an effort to know their students well so they can understand what they are communicating, but they must also know how the diversity of the student community affects instruction (Kilpatrick, et al., 2001). Each student is creating knowledge that fits into the world of the knower’s experience. Although the class may be focused on one object or problem, each learner is looking through a different lens. The teacher must, then, determine if the meaning for the class is truly shared (von Glasersfeld, 1996). Frequent assessments offer the students' knowledge to the teacher to make this determination.

These assessment activities also offer the students opportunity for authority and ownership of their own learning. Active learning tasks not only contribute to effective knowledge organization within the brain, but also have the potential to point out to the learner areas of confusion and misunderstanding, areas of cognitive dissonance. Student abilities to be critical are increased when peer assessment is also incorporated into the classroom routine, since students use their critical capacity to evaluate another as well as themselves (von Glasersfeld, 1996). All these possibilities for assessment of the students' understanding will be included in the practice of teaching mathematics in depth.

Grouping

The instructional period uses flexible grouping organizations. Instruction may be addressed to the whole class, small groups, pairs, or individuals. These arrangements are purposefully chosen to enhance the elements discussed above. As teachers who are teaching mathematics in depth change the configuration of the classroom, they attend to the expression of a variety of teaching methods and supporting tools as well. Suitable activities for whole-group instruction are not the same as those suitable for pairs or small-group instruction. Some arrangements foster communication better than others; evaluation for different purposes can be achieved with different grouping designs. Grouping in rows facing in one direction conveys a view that the center of authority is whoever is at the front of the room. An arrangement of tables, each encircled by a few chairs, presents the view that the group at each table will be the authority for the members of those groups.

Groups in the classroom where the teacher teaches in depth are flexible. They are not arranged by achievement level or ability, but according to the need of the current subject matter.

There is evidence that benefit to high-achievers by ability grouping is offset by perpetuation of social class, racial, and ethnic inequities in schooling. The research on ability grouping is weak and mixed (Kilpatrick et al., 2001). On any day in the mathematics class some students will know something that the others do not know. This will vary from day to day. The teacher may want to organize the groups so that each has “experts” and “learners” or so that all who need support on a specific topic can work with the teacher while the rest of the class is practicing individually. The primary criteria for this flexible grouping is instructional need.

One of those instructional needs is practice. Students can practice a wide range of helping behaviors while they are working in groups. These behaviors can and should be taught. A study by Webb and Farivar (1994) showed that particular helping behaviors in small groups aided achievement, while poor behavior diminished it. The protocol they used was designed to initiate extended help, not simply give answers. The program helped students ask clear and precise questions. Results showed that simply getting the answer did not assist learning, but learning to ask the right questions and give answers in a helpful manner did raise achievement. Additionally, participation rates of minority students were increased by this training (Webb & Farivar, 1994).

Since one goal of NGSSS for mathematics is fluency, practice is an important activity in the mathematics classroom. Problem solving and new content can be used to create practice opportunities, but at times there will be individual practice sessions in the classroom. Homework offers practice opportunity, both to increase procedural fluency and to maintain skill. However, it should be realistically created so students can do it independently. Homework can also serve to launch the next day's lesson and communicate with parents (Kilpatrick et al., 2001).

Whole-class and direct instruction will also be in the toolbox of the teacher who teaches mathematics in depth. The skill and knowledge of the teacher will shared as a directed lesson at times. There is economy of effort to be considered if several children have the same question or misconception. Moreover, the entire class should be the audience for individual sharing and group reports. The community of learners will meet periodically, if not daily, to assure that meaning is commonly understood and the norms of understanding are held by all class members.

Cooperative groups in the mathematics classroom where the teacher teaches mathematics in depth are used regularly and membership varies with the instructional purpose. Students are taught to work together, offering greater opportunities for students to explain their thinking about a mathematical topic, share strategies, justify solutions, and build community. Using cooperative learning groups is a practice likely to have positive effects on achievement and other social and psychological characteristics, but it takes knowledge and skill to implement it.

Time

The message from the FLDOE to the teachers is this: there is TIME, time to teach students in-depth for long term learning and understanding. Teachers can use the extra time "to incorporate modeling, problem-solving, student justification, cooperative learning, and appropriate uses of technology to encourage students to think critically about mathematics problems, explore connections between mathematical concepts and representations, and explain their mathematical reasoning and actions" (Schoen & Clark, 2007).

Teachers will have a relative abundance of time to teach mathematics in depth. The pacing of a mathematics lesson taught in depth will be very different from conventional instruction. The teacher will have time to let all the students contribute their ideas. Each can be discussed and evaluated. If a student's explanation is not clear, it can be questioned until it is clear. Instead of asking "Does anyone have a question?' to judge for understanding, the teacher can solicit the assistance of several students about what was learned today. Assessment need not be multiple choice or bubble tests, but can assume the form of projects and demonstrations. The mathematics topic can be expressed in several modalities and with several incarnations, offering more chances to reach every student. There will be time to look for mastery and proficiency. When an observer looks at the classroom as directed by NGSSS for mathematics, one will see a more leisurely pace, with more wait time, more time spent on a single activity, and more time for students to complete their thoughts.

Summary of a Tentative Framework for Teaching Mathematics in Depth

The framework for teaching mathematics in depth will focus on theory, tasks, talk, thinking, grouping, and time. These elements overlap, since they are part of the intricate practice that is teaching mathematics. No one element stands alone, but is supported and sustained by the others. From the research on effective teaching practices and teaching for understanding, these elements surface as governing organizational topics. As used to teach in depth, the characteristics may vary from the results of the literature. This study proposes to determine that variation.

Summary of the Review of the Literature

This chapter has presented the research relevant to teaching mathematics in depth after a thorough reading of the research as presented by the *Journal for Research in Mathematics Education* over the last two decades. It was determined that, although this is a recommended practice for instruction in the United States, it has not been precisely described in the education research literature. Since teaching mathematics in depth is a fundamental characteristic of the CCSS for mathematics, a set of standards recently adopted by most states, and the acknowledged intention of the CCSS in mathematics is increased understanding of the subject, the literature was reviewed to note practices of effective instruction, particularly those that help students understand deeply, actively search for meaning, and identify principles, structures, and concepts (Darling-Hammond & Bransford, 2005).

Borrowing from the organizational structure of the outline of designs for learning environments coined by Bransford, Brown, and Cocking (1999), the literature on effective instruction was discussed in these categories: (1) learner-centered instruction, (2) knowledge-centered instruction, (3) community-centered instruction, and (4) assessment-centered instruction. Instruction that centers on the learner establishes that students should be actively involved in authentic instructional tasks that connect with their prior knowledge if they are to have the opportunity to construct meaningful knowledge. Knowledge-centered instruction focuses on foundational concepts of the discipline that challenges students. Yet, the students must be supported by the teacher as they face those challenges. Students are encouraged to view the curriculum critically, asking important questions and identifying problems that they are assisted to solve. Knowledge-centered instruction identifies the need for continuous learning, mastery goals, and metacognitive reflection. The value of community-centered instruction has been noted in several studies that researched the learning environment. Students learned better when they were part of a community built on trust. The supportive context allowed the students to more fully express their tentative understandings and comprehensions, making assessment of the learning more accurate. Assessment-centered instruction that depends on honest and immediate feedback, open channels of communication, and student participation in the process supports student understanding and concomitant adjustments to instruction. These practices encourage learning with understanding.

Reviewing the literature that is particular to studies of mathematics education research begins with a discussion of the impact of the *Standards* on mathematics instruction. Since its publication in 1989, research about its implications has dominated the literature. One discussion centers on the theoretical perspective of mathematics education, what constitutes meaningful knowledge and how students learn that knowledge. Most educators agree that meaningful mathematics knowledge is constructed by the student with understanding through communication and reflection (Hiebert & Carpenter, 1992). Current theory is informed by the work of Jean Piaget and Lev Vygotsky, who greatly influenced mathematics education although their work was within the field of the psychology of learning. The implications of social constructivism are that knowledge is a cultural invention dependent on language, knowledge is created by the community of learners, and teachers have to know what the learners understand to assist this construction of knowledge.

Many curricular programs were designed to implement the *Standards* in mathematics education. Three were highlighted as examples. The first, The Second-Grade Project directed by a team led by Paul Cobb (1991) illustrates knowledge as a cultural invention dependent on language. The second, Mathematics and Teaching through Hypermedia Project, led by Lampert and Ball (1998) shows how knowledge is created by the community of learners. The third, Cognitively Guided Instruction (CGI), a project led by Carpenter and Fennema (1991), aimed at the need for teachers to examine children's thinking about mathematics. All of these programs realized student learning gains in mathematics as they implemented the *Standards.*

The review of the literature then describes specific studies that have identified effective teaching practices. These are grouped according to six elements that distinguish reform mathematics instruction from traditional instruction. The first is sharing authority over meaningful mathematics content between the teacher and the learner. Mathematical truth is not something that can be delivered to the student, but it is something that is created by the student.

The teacher's role is to facilitate that creation. The second is using authentic instructional tasks to open opportunities for students to explore the structure of mathematics and make connections with their prior knowledge. The tasks should be relevant to the world of the student and sufficiently open-ended to encourage critical thought and discussion. The third is communicating mathematics effectively. As students discuss the tasks and their strategies, they learn to do the work of mathematicians, to justify their reasoning, to use logic, and to explain their work to others. The fourth element is student thinking about mathematics. This is a necessary and desired consequence for offering students the opportunity to communicate. Teachers must have access to student thinking and mathematics understanding in order to properly direct the instructional process. Student progress will proceed more efficiently when they have feedback about their performance and its relationship to mathematical goals. Teachers' access to student thinking will be more effective if they are well acquainted with their students, particularly those whose background is culturally or ethnically divergent from their own. Effective mathematics instruction is facilitated in a classroom where grouping is varied and flexible. Small groups, large groups, individuals, or pairs—each grouping procedure has been successful in instruction. However, as the needs of the individuals and the instructional tasks vary, so also should classroom organizational practices. The final element of effective mathematics instruction is time. The initial criticism of earlier mathematics instruction is that it was shallow and insubstantial. Student thought and understanding requires reflection and time. Classroom lessons that encourage student communication are lessons that take longer. If children are to make the mathematics their own, if they are to have power over its use, and if they are to understand it deeply, they will need to be taught with these elements in the program: shared authority over mathematical truth, authentic instructional tasks, effective communication about mathematics, teacher access to student thinking, flexible classroom structures, and time for deliberation. These elements are also the ones that are essential components of a tentative framework for teaching mathematics in depth.

Table 3-1. Bloom’s Taxonomy of Educational Objectives, Cognitive Domain

Cognitive level	Descriptors
Knowledge	Recall
Comprehension	Translation, interpretation, extrapolation
Application	Use of abstractions in particular and concrete situations
Analysis	Breakdown into elements such that the ideas are made clear and/or the relations made specific.
Synthesis	Arranging, combining to make a structure that was not clearly there before. There is no correct answer defined in advance.
Evaluation	Making judgments rationally.

Note: adapted from "On defining educational objectives," by V. de Landsheere, 1977, *Evaluation in Education: International Progress*, p. 101-104.

Table 3-2. Depth of Knowledge Levels as Classified by Webb (1999)

Level	Cognitive activity	Particulars
1	Recall	Fact, term, or procedure
2	Skill or Concept	Students must make a decision about what to do
3	Strategic thinking	Requires reasoning, planning, using evidence
4	Extended thinking	Complex reasoning, planning development, thinking

Note: adapted from “Alignment of Science and Mathematics Standards and Assessments in Four States” by N. L. Webb, 1999, p. 11. Copyright 1999 by the National Institute for Science Education, University of Wisconsin-Madison.

Table 3-3. Cognitive Complexity/Depth of Knowledge Rating for Mathematics

Level	Cognitive Complexity	Particulars
1	Low complexity	Solving or computing a one-step problem, recalling a fact, reading a graph, converting single units
2/3	Moderate complexity	Students must make a decision about what to do and to bring together skill and knowledge across domains.
4	High complexity	Requires reasoning, planning, using evidence; multiple steps and multiple decision points; justification; analyzing an argument, generalizing a pattern.

Note: adapted from “Cognitive complexity/depth of knowledge rating for mathematics” by Florida Department of Education, 2006, *Cognitive Complexity Classification of FCAT SSS Test Items*. Retrieved from http://www.floridastandards.org.

CHAPTER 4
METHODS

Preface: Qualitative Methods

The field of research methods collectively known as qualitative methods is uniquely suited to serve the needs of this study, since the topic of concern, teaching mathematics in depth, is not an empirical object that can be measured, quantified, or treated with statistical methods (Strauss & Corbin, 1990). Although the practice of teaching mathematics in depth is strongly suggested by the NGSSS and supporting instructional materials (FLDOE, 2008), classroom teachers have the option to choose from a complex array of pedagogies and tasks to determine how they will instruct their students in school mathematics. A teacher may not employ each pedagogy equally; some pedagogies are given priority. For example, a fifth grade lesson centered about "estimate with 1-digit divisors" designed to be taught in one day recommends these activities: (1) game between partners reviewing multiples of the numbers 2 through 9, (2) the real-world problem about a horse's heart beats per minute, (3) asking spontaneous questions about compatible numbers; (4) an explanation of a student's reasoning about estimating; (5) twenty-three practice problems in the textbook; (6) five story problems; and (7) a test prep question. There are also suggestions of connections that can be made between this mathematics and the science, social studies, and reading. Finally, the students can write a short essay or draw a picture to answer the essential question for the day. Additional activities are suggested for centers (Adams, Dixon, Larson, McLeod, & Leiva, 2011). From this menu of choices, the teacher will select the learning tasks that they believe are best able to improve their students' learning of mathematics.

As teachers facilitate instruction, they experiment with newly recommended teaching directives at the same time they retain practices from their repertoire. For reasons known only to the participants, some of the practices suggested in the textbook series will be adopted and some will be discarded. To access this information, the researcher must enter the lives of the teachers and create tools of data collection that draw upon the participants' knowledge (Hatch, 2002). This researcher entered the world of the participant and used qualitative research methods to collect answers to the research questions for this study.

Within this chapter the reader will encounter terms with meanings specific to qualitative research methods. Since the expression may be used previously in a different context, they are listed here to avoid confusion. The first two terms were explained in the section on the Theoretical Perspective of Mathematics Education in Chapter 3.

- Radical constructivism: each person has their own mathematics, created themselves from information obtained in the physical world, which could include mathematics created by others.
- Social constructivism: mathematics is defined as a cultural construct, created with language and social interaction.

The next three terms are defined with the meanings attributed by the qualitative research community.

- Constructivism: truth and meaning are created by human intelligence as it processes incoming data. In this study, it refers to the paradigm.
- Constructionism: meaning is embedded in a cultural context, affected by language and symbols. This is the epistemology for this study.
- Constructivist: the researcher is creating knowledge from the data collected

As trustworthiness in qualitative research derives from the paradigm and the context for knowledge, truth, epistemic conditions and justifications within the research project, the use of the most appropriate word is particularly important. ((Koro-Ljungberg, Yendol-Hoppey, Smith, & Hayes, 2009).

Paradigm: Theoretical Perspective of the Study

The set of beliefs that guide the research action of this study, defined as the paradigm, are abstract principles that all fit together (Denzin & Lincoln, 2000). The paradigm is the skeleton for the construction of the research theory that gives validity to the study. Once the paradigm is identified, other components follow. In this study, the paradigm is constructivism, which holds that truth and meaning are created by human intelligence as it processes incoming data. By processing the sensory perceptions that flood the individual with multiple impressions, the brain creates meaningful knowledge, which is uniquely significant for each individual. The tools of data collection and analysis are selected to make this access possible, and they are described below.

Epistemology

The knowledge of interest in this study, the epistemology, is constructionism (Koro-Ljungberg et al., 2009). For qualitative researchers, the difference between the two terms, constructivism and constructionism, are subtle yet important. Constructionism describes meaning as embedded in a cultural context, and language and symbols are the media between society and the individual (Grbich, 2007). Constructivism focuses more completely on "the meaning-making activity of the individual mind" (Crotty, 1998, p. 58). The paradigm acknowledges this activity, but respects the cultural context for the knowledge under investigation. Another way to express it is to say that there is not ultimate criteria, but only those that can be agreed upon at a certain time and under certain conditions in so far as truth that can be universally known (Lincoln & Guba, 2000, p. 171). Tools, participants, and the study site were all selected for the purpose of exploring how informants interpreted and made sense of the experience of teaching mathematics in depth (Grbich, 2007).

In this study, the primary informants who hold the answers to the questions are the elementary school mathematics teachers. Those who are experiencing the process of teaching the mathematics curriculum in depth are creating the knowledge about it, and the act of creation takes place in a specific context. From the input of their experience, they create meaningful knowledge. The object of the study is the meaning brought to teaching mathematics in depth by the teacher. However, even as the researcher is seeking knowledge from the participants, the researcher is looking through a lens of her experience. The data collected in the study has been interpreted by a person with a point of view based on her own history. This history must be identified and explained so its influence can be confronted (Charmaz, 2006; Hatch, 2002).

It is imperative that the researcher reflect critically on this history, and make it part of the research record. Lincoln & Guba (2000) name the multiple selves that we bring to the study: (1) the research-based self, (2) the brought selves—our historic social and personal self, and (3) situationally created selves. Each has a voice. These voices enable researchers to speak with authority of what they have seen. These selves also offer connections to the participants that allow communication between them to flow more freely (Grbich, 2007). Identification of my historical selves that impact this study follows.

Subjectivity

My historic self has participated in the adoption and enactment of mathematics reform curriculum. In 1985, the California Legislature adopted sweeping reform legislation with the passage of the *Framework for Mathematics Standards* (Cohen & Hill, 2000). The new framework called for a substantial shift in teachers' and students' views of knowledge and learning toward views that most Americans would see as unfamiliar and unconventional. It was at this point that I began a career as a teacher of middle school mathematics. As we began to implement the state standards, the textbooks were revised and the California Learning

Assessment System (CLAS) was created to measure student progress. The state encouraged professional development activities within the school district and provided financial support for workshops at external sites. I know what it means to experience changes in curriculum as a teacher. I shared these experiences with those who changed their practice, those who did not, those who opposed the implementation, and with those who ignored the new curriculum.

Another significant aspect of my experience is that of teacher leader for College Preparatory Mathematics (CPM), an exemplary program for mathematics in high school (USDOE Mathematics and Science Expert Panel, 1999). This group of teachers and mathematicians wrote middle school courses in pre-Algebra as well. I taught these courses and helped other teachers implement the curriculum in their classrooms. Working with other teachers as a teacher-leader indicated to me the problems and challenges teachers faced as they learned to use a new curriculum. The response of these instructors ranged from delighted acceptance to reluctant acquiescence. Although the materials were fully explained and designed to educate the teacher as well as the student, none of these teachers taught exactly the same curriculum as another taught.

The third historic self stems from my position as a National Board Certified Teacher of Early Adolescence/Mathematics. As such, I am prepared to propose research-based professional development programs. This rigorous and competitive process of certification both depends upon and trains one in the process of reflection of and writing about instructional practice. Board-certified teachers "excel in characteristics of expert classroom performance such as use of knowledge, the depth of their representations of knowledge, their expressed passion, and their problem-solving skills" (Vanderfoot, 2004, p. 11). Additionally, these teachers evidence higher understanding of student work, including assessment and are defined by their coaching and mentoring of each other's improvement process (Pyke & Lynch, 2005).

The personal self that I bring to this study is that of a mature woman, a wife, mother and grandmother. In my role as a non-traditional doctoral student, I have been mentor to many younger professionals. I have strong impulses to help and to advise. These life experiences gave me knowledge I urge to share and turned me from researcher into teacher. Those same life experiences allowed me to master tools of communication, such as knowing when to speak, how to affirm, to elicit confidences, and to generate trust. My situational self was a researcher, one who was aware that I had many roles contending for attention during the data collection and analysis. This ambiguity will be further discussed in the section of the manuscript relating to methods.

Study Purpose and Research Questions

As was stated in the discussion early in this chapter, this study's paradigm is constructivism. The purpose of a constructivist study is understanding and reconstruction (Lincoln & Guba, 2000). This study was designed to understand what it means for a teacher of mathematics to "teach in depth," and to reconstruct the factors that enable such instruction. Constructionism names the knowledge of interest, the knowledge of the mathematics teacher, since it is embedded in a cultural context and mediated by language and symbols. The study itself as conducted is constructivist, but it is studying knowledge that is constructionist. (Please refer to the introduction to this chapter for specific definition.) The ultimate product of the research is twofold: (1) to mitigate the gap in the literature defining this practice, and (2) to use the data from the study to develop a framework that will support mathematics teacher educators, mathematics coaches, curriculum resource teachers, and others, as they educate teachers of mathematics to teach in depth.

The action of interest placed the work in an elementary school as the teachers employed a curriculum that expects the teacher to teach mathematics in depth on a daily basis. These are the research questions.

- What are the elementary teachers' perceptions of what it means to "teach mathematics in depth"?
- What guides the elementary teachers' enactment of "teaching mathematics in depth"?
- How is "teaching mathematics in depth" described by peripheral participants: mathematics teacher educators (MTEs), curriculum resource teachers (CRTs), and instructional mathematics coaches (IMCs)?

The first and second questions relate to the first purpose of the study, defining the practice of teaching mathematics in depth. The methodology for these questions is described in the section that follows. The answers to the third research question come from a secondary data set, which will be described in a separate section of this chapter. Analysis of all the data collected was integrated with the data provided by the review of the literature to develop a framework for teaching mathematics in depth.

Methods for Primary Participants

Sampling Strategies

The study was served by a purposeful sampling, a set of teachers in a public elementary school that serves the general population of the community (Kuzel, 1999). Since the sample was small (three teachers), the importance of representativeness weighed more heavily. It was important to find a school site that was neither engaged in state sanctions nor composed primarily of a skewed student population. The site with maximum desirability was one that was facing few general problems beyond adjusting to the newly adopted state standards and textbook series. Similarly, the research would be ill-served if the participants were unwilling to consider NGSSS with an open mind, since it began with the teachers' perceptions and enactment of this curriculum. The selection process sought willing participants at such a school. At a neighborhood school, the researcher identified a fifth grade level team that not only perceived the state mathematics standards as an interesting challenge but was also enthusiastic about accepting the researcher as a quasi-member of the team. The selected teachers expressed interest in participating in the study and agreed to involve the researcher as a volunteer in their mathematics class for the first term the NGSSS was implemented.

Knowing that knowledge is by definition personal, the researcher sought to create a working relationship that encouraged the participants to come to trust and value her assistance. She wanted the participants to feel safe enough to freely communicate their perceptions and feelings. To this purpose, she offered her experience and scholarship to these professionals by becoming a volunteer in their mathematics class for the first term of the school year, from August to December of 2010. She followed their direction when students were present and positioned herself as an experienced volunteer rather than a university researcher. The teachers were enthusiastic about having the services of an experienced teacher in their classroom during mathematics. The researcher expressed her appreciation by providing each teacher with a classroom set of erasable white boards, pens, and eraser cloths.

Choosing a grade level team instead of a single individual broadened the scope of the study. Since the paradigm acknowledges multiple realities, the inclusion of multiple participants offered greater generalizability (Bogdan & Biklen, 1998; Kuzel, 1999). The four individuals in the team brought to the school year varying degrees of experience and confidence, thereby increasing the range of reaction to use of the curriculum. Different participants attributed different meanings to the questions, talked about the experience in different ways, and emphasized or omitted different components (Charmaz, 2006). One teacher was not included since she taught the gifted students mathematics and science as a pullout program, and the study looked to consider the instructional program of the general population. Nonetheless, participation of three teachers broadened the scope of experience being studied without diminishing the quality and depth of the data (Frankel, 1999). Further discussion of the limitations of this research design is in Chapter 7.

Setting

The school that housed the participating fifth grade team has been given the pseudonym, Forest Glen School. It is located in a mid-sized city in a southeastern state in the United States. As it is one of the earliest elementary schools erected in this university town, its genesis was a two-story school building built in 1939. Many additional structures have been added to create this modern plant on a wooded campus. Recess and physical education classes take place on lovely lawns surrounded by trees and a creek. This Title I school is home not only to children who live in this neighborhood near the local university, but it serves as well as the magnet school for English-language learners (ELLs) in the district.

The fifth grade classrooms are upstairs in the main building, where the students are divided into three homerooms and one combination fourth/fifth grade. For mathematics and science, fifteen students identified as gifted from the three fifth grade rooms move to the team leader's room for instruction in these subjects. Three general education teachers teach the other students, including those who are identified as language learners or as recipients of special education services. Several auxiliary teachers and paraprofessionals teach individuals at times during the week, either in the homeroom or as part of a pullout program, such as speech therapy and occupational therapy. One of the participating teachers has the services of a Title I aide for mathematics, and an aide providing special education visits one teacher's room daily. At this school are a curriculum resource teacher (CRT), a behavioral resource teacher (BRT), and a guidance counselor, in addition to the principal.

As a Title I school, the school budget is supplemented by Title I funds. Each of the rooms in the study has a permanently mounted Smart™ board and four iMacs with large screens. Each teacher has a Mac Book. Additional funding for student enrichment comes from using the school grounds as a parking lot for university football games. The school day begins at 7:45 AM daily and continues until 1:45 PM on Monday, Tuesday, Thursday, and Friday. Wednesday is early release day (12:45 PM), so teachers have collaborative time. There is an afterschool program run by the school district that charges families based on a sliding scale. The program closes at 5:30 PM. The mathematics and science classes were taught between 8:45 and 11:00 AM. (10:00 AM on Wednesday.)

Participants

The three participants have each been given pseudonyms to protect their privacy. Students are not mentioned by name. Moving from south to north in the classroom building, the first teacher is James, a man in his thirties who is solidly built and of medium height. He admits to weighing 250 pounds. When he sits at the students' table, his energy almost seems too great for the space, and it does seem that he cannot contain it, as his body is in constant motion. Although he is an imposing figure to the students, his kindness is evident in all he says and does. He is bilingual, speaking both English and Spanish. He dresses casually, wears a gold cross, and uses glasses. He carries himself confidently. James is a Master of Education in Elementary Education with eleven years' teaching experience, all at Forest Glen School and in the fifth grade. He especially enjoys teaching mathematics and science. During his work at this school, he has been team leader and advisor for the Safety Patrol. He tutors in the after-school program.

James was the teacher who first agreed to participate in the study, and he enlisted his colleagues. He co-teaches language arts to the ELL students with the ELL teacher. He views volunteers as a gift to his classroom that allows him to work with smaller groups of students at a time. During this study, he divided the class into three groups while the researcher was there. He and the intern teacher instructed two groups in mathematics, and the researcher worked with the third group, primarily on their science lesson. These groups of students rotated among the three tables during the two hours of mathematics and science. James had constructed the groups by collecting students who work well together. These groups were essentially unchanged over the course of the study.

The second participant, Kelly, was beginning her third year of teaching and indicated her happiness to have the opportunity to teach here for a second year. She holds the degree of Master of Education in Elementary Education. She is a tall young woman with a commanding figure and a friendly, direct manner. She wears her long hair in a ponytail and dresses in either skirts or slacks. Many of her students are immigrants and are learning English. She indicated that many volunteers had already signed up to assist her, and she would not know how incorporate another in her classroom routine. Therefore, she participated only in the journal writing and the interviews, and the researcher-volunteer was not part of her mathematics practice. Since the researcher was included in grade level team meetings before school and during a special release day for mathematics, there were informal meetings between the two persons.

Sandra, the third participant, is a blonde and petite young woman in her early thirties. She dresses in summer slacks, a tailored blouse, and tank top. She uses make-up and wears her hair casually in a ponytail. She uses her lunchtime and prep period to maintain her neat and organized classroom. Her degree is a Master of Arts in Science for Early Childhood Education and has been teaching fifth graders for eight years. For the last two years she was the fifth grade mathematics and science teacher at a middle school, where she taught those subjects twice in a day. This is her first year at this school. Although her appearance is youthful and her stature is tiny, she is the dominant force wherever her students are gathered. Her routines are organized and her lessons are thoroughly prepared.

Her instructional practices are directed toward the entire group, but she incorporates pair work into the classroom activities. The lessons are carefully constructed and flow smoothly. During the period the study was conducted, Sandra used many grouping strategies, always designed to increase student time-on-task. Sandra participated in a two-week summer training program focused on teaching mathematics through inquiry, and she was a member of the district textbook adoption committee. During the study, she primarily taught the entire class as a unit while the researcher was her assistant, lesson critic, and tutor to those in need during that time in her classroom, doing what the teacher asked her to do at any particular point in the lesson.

The grade level team was selected as the sample for the study to fill the criteria of the study design. More than one individual was to be studied to offer a broader range of response to the situation presented by the new standards and curriculum. However, the criteria also included a representative school population that would be learning the mathematics without major program modifications. Hence, the teacher of the gifted students was not included, so three teachers remained. Each was willing to be part of the study, but they participated in varying degrees. Initially, Kelly's reluctance for the researcher's volunteer services was viewed as a problem for the study. However, the researcher reframed this reluctance as a result that is not unexpected: some teachers are disinclined to open their practice to the view of an experienced mathematics teacher. The researcher volunteered to assist the teacher in any way that would be helpful in the teacher's view. For Sandra, this meant assisting the teacher directly. For James, this meant teaching science lessons to small groups. For Kelly, this meant responding thoughtfully to interview questions and journal prompts.

Data Collection

Primary data

The action of the teachers constructs the processes under study, and their thoughts create the knowledge of interest. The researcher's goal was to become a trusted actor in the teachers' world. Hence, she spent an average of four days a week in two of the classrooms, assisting them as they directed. She was not in the mathematics class as an observer. Her purpose was to become a safe, trusted assistant so the teachers would freely respond to the questions in the interviews. This perspective is discussed more completely in the section on secondary data that follows. Since the teachers were the fundamental informants, the primary data sources are the statements of the participating teachers (Charmaz, 2006). These were accessed through formal semi-structured interviews, journal entries, lesson plans, and informal personal contributions.

Over the course of three months, three formal semi-structured interviews were conducted at the beginning, the middle, and the end of the time period. The same questions guided each interview, although additional questions were added as needed to collect missing data (Appendix A). Digital and analog auditory processes recorded each interview. The interviews took place in the teacher's classroom, either during the planning period, lunchtime, or after school. During the school day, a teacher's consciousness is preoccupied with the classroom agenda or events that previously transpired. Interviews during the school day were more rushed than those that took place after school. Kelly chose meet after school, and these interviews are rich with data, as she was happy to elaborate on her ideas and had more leisure to do so. However, there were many more opportunities to speak informally with James and Sandra than with Kelly.

Additionally, the researcher asked the teachers for six journaling contributions. At the onset of the study, she gave to each teacher a schedule of the desired interview and journal dates. Although these deadlines were designed to be flexible, the teachers responded to her requests seriously, treating them as assignments. For one of these journal entries, she asked the general question, “Tell me about your day,” but the responses were not very reflective. For the remaining weeks, she provided the teachers with a more complete prompt (Appendix B) that resulted in more thoughtful narratives. The teachers submitted these journals electronically by writing in a Word document or directly into an e-mail letter. Other electronic data are teacher lesson plans designed for the Smart™ Board and saved on their computer.

The researcher’s daily work at the school began during the time of “specials,” while the children were with an elective teacher and the fifth grade team members had planning time. Upon her arrival, she would inquire about the day’s planned activities. This was the impetus for several interesting informal conversations. Over the course of the semester, the researcher became an intimate of the two teachers she worked with most closely. There was an obvious contrast in conversation topics between these two. Sandra came to confide in the researcher, sharing concerns about her students, her lessons, and her colleagues. James spoke mostly about the students. His curricular concerns were that the researcher would be conducting the group lesson with confidence and accuracy. Conversations between Kelly and the researcher remained impersonal and formal.

Secondary data

Secondary data sources illuminate the context of the teaching arena for analysis of the primary data sources. This teaching does not take place in a vacuum, but in an arena acted upon by many forces. Since the arena is a site of contestation and controversy, it is appropriate to analyze perspectives and power (Clarke, 2009). This study collected data to illuminate the arena from two vantage points. The first vantage point is that of the researcher, a witness to the action. Although the researcher was not officially an observer, she nonetheless did observe and record events of the classroom in the field notes. The researcher's presence in the mathematics class offers a source of verification for the report of the participant. Equally important, the contrast between what teachers say and what they do is more evident. The researcher is more likely to identify her own effects on the subjects if she is intimately knowledgeable of the setting (Bogdan & Biklen, 1998). The second vantage point will be described fully after the researcher's data is outlined.

During fifteen weeks, the researcher made forty-seven visitations to the school. These were primarily in James's and Sandra's classrooms where she followed teacher instructions as a classroom volunteer. Upon leaving the classroom, she wrote field notes for each session. Occasionally, she was able to take brief notes, but primarily she was working with students and note taking was an inappropriate activity. Field notes were written to describe the participants and their classrooms, as well as the other activities that the researcher attended, which included a fifth grade teacher planning day and a parent visiting night. Her presence was noticeable to the students, who came to view her not only as a regular visitor to their classrooms, but as a part of the fifth grade community.

The question arises as to the extent of the researcher's role as a participant observer in the classroom. Over fifteen weeks as a volunteer, the researcher did participate in the action of the classroom, in the arena of teaching mathematics in depth. She did not sit on the side and take notes, but engaged in conversations with the students and the teachers. This can be identified as the strategy of active participation (DeWalt & DeWalt, 2002). However, participating, observing, and questioning are each actions taken by the researcher, and in this study the knowledge under study was being created by the teachers, and the researcher did not participate in that creation, except incidentally. The reflexivity of the researcher was marked with careful recording of her actions and observations in field notes. Her thoughts and speculations were acknowledged in reflective notes and memos.

Since the researcher and Sandra developed a close working relationship, it is possible that Sandra considered the answers of the researcher to questions she posed about the mathematics she was teaching that day. These conversations took place before the lesson, while the students were at the Specials class. The researcher avoided comments that conveyed judgment, but she responded honestly to questions Sandra asked. The researcher's subjectivity revealed itself in James's classroom during small group instruction on the rare occasions that she was asked to direct a mathematics lesson. She did conduct the lesson according to her understanding of its objectives. There is little evidence to report that James interpreted her instruction in these small groups as an evaluation of his practice. Reports of Kelly's thoughts and practice of teaching mathematics in depth are not impacted by the presence of the researcher.

Reflective notes were constructed regularly through the semester. These are primarily written in the research field notebook. The field notes are interspersed with questions and memos. The research process is a formidable one of building categories and distinctions. As she wrote her notes, the researcher was continually asking herself, "Is this what I observed or what I felt?" "Is this comment a statement of reflection or analysis?" "Is this pithy piece of advice something that belongs in my reflection notes or in my book proposal?" A tool that proved to be very helpful was writing in different color pens. As the mode of thought changed, she changed the pen color. During analysis, she was able to organize her meandering thoughts more easily.

Additional sources of secondary data include the textbook and other instructional materials included by the publisher, such as the online supplemental activities and videos. The teachers kindly shared not only copies of the textbook and the teachers' book for both science and mathematics, but also a visitor password for the web site. It was helpful to use both at home, not only to confirm the memories of the day's events but also to prepare for future lessons. The classroom space is limited and the volume of student materials is considerable. It was less practical to use personal copies during the school time than it was to share materials with a student.

The data collection includes transcripts of nine semi-structured auditory-recorded interviews, eighteen journal entries, lesson plans, researcher field notes and reflective notes, and instructional materials. The data were analyzed by the methods associated with grounded theory.

Data Analysis

Constructivist grounded theory

Grounded theory emerges from the data collected in the study. Researchers begin with an area of the study and what is relevant to that area is allowed to emerge. The goal of analysis is to build a theory that fits the data, allows the reader to understand the phenomenon, is sufficiently general to apply to a variety of contexts, and offers control over action regarding the phenomenon (Strauss & Corbin, 1990). Data is combed for meaning as it is so designated by the participants who provide the data. In a series of cycles of reading and studying, the researcher names (codes) the units of meaning for action and relationships. This process begins with the initial data collection, and it is repeated with each additional data set. As the successive interviews and journals are coded, some of the existing codes gain more strength, and others drop away in what is similar to a spiral process of collecting data, selecting emerging ideas, and testing those ideas (Charmaz, 2006).

In the stage of initial coding, the researcher begins to create categories, note trends, and identify questions that should be addressed. The process repeats again and again, until no new data appears. This is known as saturation (Grbich, 2007). Coding offers opportunity to examine the words of the participants with fresh eyes (Hatch, 2002; Strauss & Corbin, 1990), beginning the analytic process. Focused coding, the next level, combines initial codes into categories and ideas that are tested further in the next wave of data collection. The third stage of analysis is identification of axial coding, categories that provide an axis for large amounts of information. Following this procedure, theoretical codes may emerge. These codes specify possible relationships between categories, which become the grounded theory (Charmaz, 2006).

The process of analysis introduces the subjectivity of the researcher. While coding is in process, many thoughts enter the researcher's mind. These need to be acknowledged and recorded, since they are part of the analysis process. These are known as memos in grounded theory. This researcher, too, took memos as she transcribed interviews, coded primary data, and made my field notes. At this point in the study, she fully appreciated her multiple identities and how they shifted and conflicted. She had collected data, far more than could be fully shared, and she then held the task of writing that data into a coherent story. How she perceived the action, how she translated the language, how she inferred what a participant really *meant* by the words they used—these actions were directly related to the identities she brought to the research study.

Coding of primary data

Initial coding began after the first set of interviews was transcribed. The interview text was broken into lines according to units of meaning, which could be a group of a few words or a sentence or two. Since the research questions examined the act of teaching, she was looking for actions that related to teaching mathematics in depth, in the eyes of the participants. These units were coded using gerunds that identified the action in the unit. Whenever possible, the words of the participants are the words used for codes (**bold** in this example). This is an excerpt from Sandra's first interview, after initial coding was completed. The *italicized* words are words the speaker emphasized.

Sandra: It's a struggle for the *kids* because they just want to go to what they *know*, **clinging to the familiar**

which is to divide a number, ***dividing* the number**

which today I'll actually *teach* it, finally. On Lesson 7 I'll actually *teach* **teaching *dividing* algorithm**

division, with *dirty monkeys smell bad.* [heh heh]. Divide, subtract, multiply, bring down. **teaching the algorithm**

But that's what they've been *wanting* to do this whole time. **wanting to use the algorithm**

But when you just *learn* it that way, you have no concept for what that actually *means.* **knowing what it means**

You're following steps in a process, but what does it *mean?* **knowing what it means**

The codes were copied onto sticky notes that were color-coded for teacher identification. Overlapping comments drew a check on the original. There were more than 170 different codes (example is Figure 4-1).

On large wall charts, the researcher grouped the sticky notes into rough categories. The categories were described in a special memo written to explain her thinking. At this time, these titles named the categories: (1) Reacting to legislation; (2) Managing students and materials; (3) Finding their way; (4) Juggling content; (5) Hoping to find. This process was repeated after the second set of interviews. The sticky notes were used to note new codes and the pre-existing wall charts were modified to incorporate new material. After incorporation, it was determined that the legislative category was no longer useful. Category #5 became “Seeing the benefits” and the a new category, “Isolating the practice,” emerged. Again, summaries were written to memo her thoughts about the categories at this time.

A recursive process was used to analyze the written responses to the journal prompts. On each teacher’s response, new lines of units of meaning were created, as were codes written in different color inks for each teacher. Here is an excerpt from James’s second reflection, after it was coded.

James: Today math went very well. **Going well**

> I was pleased with the engagement my kids showed. **Kids engaged**
>
> Our lesson concentrated on finding patterns in division and using base-10 block drawings to divide larger dividends with single-digit divisors. **Finding patterns**
>
> First, the kids saw this as an extension of a similar activity we did during chapter one. **Seeing extensions**
>
> They understood the math for the most part because of the spiral nature of the math content in Go Math. **Understanding mathematics**
>
> Also, most of my kids finished the related homework assignment with good accuracy. **Finishing homework accurately**

Although these reflections were not lengthy, there were a great variety of codes, so an Excel spreadsheet was used to help group the codes. Copy and paste functions helped create groupings, and often a more comprehensive code could be identified, a focused code in the teacher’s words. Going back to the interview data, the codes were grouped into a Word document, using teachers’ words for names of categories. Always the researcher was looking for actions that influenced the work of teaching—teacher actions or student actions, but clearly some of the codes could create a category that included several other initial codes.

At this point, there were two series of summary statements written into focused coding. The challenge now was to integrate these four documents—focused codes for two sets of interviews and two sets of reflections (weeks 1 to 3 and weeks 4 to 6). It became apparent that the original categories were inappropriate for the entire data set. The initial categories were generalizations and at this early point in the analysis they were not sufficiently grounded in the data. A new approach helped. By cutting the pages into pieces, one group of codes to a section, they could be arranged into fewer groups and recorded on two Word documents, one for each aspect of the research question. These fewer groups appeared to be axial codes, major categories on which large amounts of data could be assembled. Figure 4-2 is a sample from the reorganized list.

Looking at initial codes from the second reflection submission, one can see how they were followed through the process. James's remarks about "finding patterns" and "seeing extensions" were grouped under "familiar learning," while "understanding mathematics" and "finishing homework accurately" were grouped under the code, "saying it's easy." After the manual redistribution, the latter codes became organized under "assessing the mathematics," while the former joined others in an answer to the question, "What does it mean to do mathematics?" This is the stage of focused coding. Figure 4-3 illustrates this trail of coding more clearly.

The number of focused codes was still too great to organize all the data into a clear picture. Then again, codes could be grouped into broad categories to relate to each of the research questions, but there was a sea of information in each category. The task then was, "How does one write this story?" There were data to answer each question, but the narrative would be long and rambling. How could it be organized into coherent units of findings, interpretations, and implications? Constructivist grounded theory was becoming too linear of an approach for the themes that were emerging from the data. Clearly, the findings revealed more than one social process. The analysis methods needed to deal with greater difference and complexity than had been anticipated. Situational analysis proved to be the grounded theory method of analysis that was needed for this data set.

Situational Analysis

The goal is to bring clarity to the arena of teaching mathematics in depth. This arena is broader than the actual classroom and the mathematics period. Situational analysis allows the researcher to analyze several aspects of the arena, to specify, represent, and examine the most salient elements in that situation and their relations (Clarke, 2009). This method goes beyond "the knowing subject" as centered knower and decision-maker to also address and analyze salient discourses dwelling within the situation of inquiry (p.201), since the participant is within a situation real to them and real in its consequences. The unit of analysis has shifted from the individual to the situation.

Situational analysis is a method that posits constructionist grounded theory in symbolic interactionist sociology (Clarke, 2009). What we can know and how we can know are inseparable. "Symbolic interaction presumes that reality is a constructed and shifting identity and that social processes can be changed by interactions among people. Meaning is constructed through the use of symbols, signs and language," (Grbich, 2007, p.71). The researcher cannot separate the methods and the theory. This research takes place in a particular arena, a social world that is a universe of discourse, bounded by how far it reaches in terms of space, time, and meaning-making, a method that takes into account nonhuman and implicated actors as well. Situational analysis acknowledges that there is conflict and controversy, and the outcome may be anything but simple.

Rooted as it is in the Chicago School ethnographies and pragmatism, situational analysis as described by Clarke (2005) offers the researcher significant tools, ecological maps, to bring order and representation to the data analysis. The maps disrupt one's expectations and open possibilities. They force the researcher to consider impact and influence more carefully. The social maps help one understand what happens in a space and who is encountered there. The positional maps confront the researcher with the multiple positions one may take along a line of tension between opposing forces. The situational maps identify all the elements that impact the arena being studied.

The arena of teaching mathematics in depth takes place in a classroom in a school, but the elements that influence that instruction are manifold. Figure 4-4 is a situational matrix centered in the arena of the teachers' work. Around the arena are listed the elements that influence this work as noted in the data, clarifying the presence of multiple forces of potential impact. Some of these forces are human: colleagues, parents, volunteers, and students. Some are nonhuman: Smart™ board and world-wide-web. Many are discursive products: NGSSS, instructional materials, FCAT 2.0, and the pacing guide. Everything in the situation both constitutes and affects everything else in the situation. If situations are perceived as real, they are real in their consequences (Clarke, 2009). Tools such as the situational matrix guide the researcher to analysis of this arena, facilitating the representation of difference and complexity.

Situational analysis offers space to actants (a term borrowed from analysis of fiction), human or nonhuman elements with influence in the arena of study although they lack a voice. Nonetheless, actants have agency and adequate analyses of situations being researched must include the nonhuman explicitly and in considerable detail. For example, in the arena of teaching mathematics in depth, consider the Smart™ board. Its presence in the classroom and its intersection with the instructional materials identify its role as an element to configure the teacher even as the teacher programs and uses it. This was found to be true for the textbook as well. As the material world is given meaning by the participants, the material world needs to be integrated into the analysis.

Within the social world, the discourses are multiple and layered as well: teacher to student, teacher to teacher, teacher to administrator, teacher to self, and teacher to researcher. Equally powerful discourses are those that have been codified by law or published: textbook, pacing guide, standardized tests, and the curricular standards. All these players are acting and talking, informing the teachers as they teach mathematics. At the midpoint of data collection, there were 29 axial codes from primary data, and more data were to be collected. The messy situational map, found in Figure 4-5, was created to help clarify this dynamic world. Here were written the focused codes from primary data sources, unfolding possible connections between elements that were encountered in the responses of the participants. These elements were placed into an ordered situational map (Figure 4-6) that lent credence to the complexity of the discursive constructions relative to other categories.

Results began to emerge for the first two research questions, and a set of propositions to answer the questions was conceived. These are the tentative lists.

1. How is teaching mathematics in depth perceived?

- Teaching well means students understand
- Teachers want the students to do the mathematics correctly
- Engagement indicates success
- Teachers have means to gauge understanding
- Teachers direct all activities and they should be correct

2. How is teaching mathematics in depth enacted?

- Legislation: FCAT
- People with power: volunteers, colleagues

- Institution: district office
- Non-human elements: textbooks, time, computer & Smart™ board
- Experience
- Silent participants: student power

The final data were collected, coded, and checked against these codes. The codes were repeating; it appeared that saturation was reached, but a clear analysis had not appeared.

Messy situational maps (Figure 4-5) were extremely helpful tools for isolating particular relationships between the elements as the researcher explored potential thematic codes: learning (Figure 4-7), assessment, instruction, and the pacing guide/textbook. Codes from the final data collection were analyzed in a situational map found in Figure 4-8. The entire coding process is summarized in a flow chart (Figure 4-9). This iterative process of reading data, categorizing data, drawing relationships, and re-reading data accompanied the writing process, continuing until significant findings regarding enactment were clear. Answers to the conceptualization of teaching mathematics in depth emerged only during the process of writing the findings. Describing the teachers individually as they taught for understanding and mastery, as they used instructional activities and formative assessment practices, patterns of similarity and difference emerged. The findings are presented in Chapter 5 and examined more closely in Chapter 6.

Two other tools served to highlight the underlying conflict and controversy noted by the researcher. The teachers spoke and wrote with emotion about the influence of particular forces on their instruction. The story surrounding these forces is detailed in Chapter 6, but the maps will be explained here. The first is Figure 4-10, a social worlds/arenas map that was drawn several times. Composing the map necessitates a consideration of the impact of each of the elements—how large ought the shape be? Should it overlap other elements? How near and how far to the action of teaching mathematics ought the shape be? Are some of these elements in opposition? These are good questions for the researcher to answer during data analysis. A more accurate map is discussed in Chapter 6. Another tool is the positional map of the issues facing the teachers as they teach mathematics. On each axis is one of two opposing forces that together create tension for the teacher. The tool commands the researcher to consider all possible positions for the teacher along these two axes, again demanding the critical thinking needed for accurate analysis (Figure 4-11).

In summary, the analysis methods and tools described by Clarke (2005, 2009) have been used to analyze the data collected in this study. After initial grounded theory procedures were completed, situational analysis tools facilitated organization and interpretation of the primary data. Additionally, these methods facilitated compilation of the secondary data collection. The secondary data relating to the observations, reflections, and conversations of the researcher with the participants were coded by incident. This material has been used to substantiate and contextualize findings from the primary data. The conception of teaching mathematics in depth as an arena of social action offers structure to answer the second research question, how do teachers enact this practice, more completely. Moreover, utilization of this conception offers promise of advancing the researcher's purpose of constructing a framework for developing this practice fully.

Trustworthiness

This study has investigated the experience—the attitudes, opinions, and reflections—of three fifth grade teachers who opened their working world to a researcher as they taught mathematics. She has been careful to respect these participants who have so generously shared their classrooms. To protect their privacy, she has given to each of them a pseudonym. Audio recordings and transcriptions are held in secure locked areas. The school and individual students have remained anonymous. Participants had the opportunity to verify the accuracy of the transcripts. These findings have been shared with them.

To the extent possible when researching social conditions, the reliability and validity of this study can be confirmed. The primary data gathered from the teachers are supported by secondary data collected in the field notes of the researcher and in observations and interviews with peripheral mathematics education professionals. This chapter documents the methodological choices that were made, choices that could be replicated for further confirmation. There is an audit trail complete with associated raw data. Questions used for the interviews were submitted for peer review with teachers from other schools and were tested in pilot interviews with mathematics teacher educators who were not part of the study. The findings that follow are thick, rich descriptions expressed in the words of the participants as appropriate. During the extended period of classroom interaction, the participants became comfortable with the researcher and spoke freely in their journals and interviews.

Results of this study are written in sufficiently accurate descriptions so that they will be useful for programs of planned change (DeWalt & DeWalt, 2002). The sample selected is representative of a grade-level team of teachers in an inclusive elementary school. This study gives a picture of what teaching mathematics in depth means in the context of one grade level at one elementary school. This particular situation is richly described; others can determine if the study applies to other school contexts.

Methods for Peripheral Participants

Curriculum Resource Teachers (CRTs)

In this school district, the CRT works with teachers in all subject areas. He/she responds to teacher requests for support and resources. Depending on the school, the CRT acts as assistant to the principal. If the grade level teams function independently and score well on the tests, the CRT operates as a channel of communication between the district offices and the teachers. A school with a large Title I budget and more auxiliary personnel sees the CRT interacting more closely with the grade level teams, but this could be in response to issues perceived as needs by the principal and others on the leadership team.

In terms of mathematics instruction, interaction with the district office revolves around issues of testing and data management. Tests administered through the district office include On Track tests, Big Idea tests, and the FCAT 2.0. The CRT arranges for the testing, gets copies for the teachers, and discusses student results with them. Specific training in mathematics instruction might be delivered from the district to the CRT who is asked to share this information with teachers at their schools. The CRT at Hickory School reported that one recent training session was about test item specification on the FCAT 2.0 to be administered in 2011.

Sampling strategies

Considering the CRT as one of a school's administrators whose primary responsibility is the curriculum in the elementary school, the researcher sought their participation in the study. The district coordinator for elementary school mathematics suggested a list of five schools where at least one teacher had been noted as a superior instructor of mathematics. Those schools were solicited to participate in the study. The first two respondents were chosen, Hickory School and Twin Meadows School, neither of which was the school of the primary participants. Coincidentally, the two schools proved to be very different in terms of students, staff, and CRTs.

Setting

Hickory School has a student population of at least 800, most of whom are White and economically advantaged (FLDOE, 2010c). The school consistently earns As for their state grade, with more than 90% meeting high standards in mathematics. Compared to the district average, this school has fewer students with a disability. Although it has an ELL population twice that of the district, its share of this population is one-sixth that of Forest Glen School. About 10% of the students are categorized as disabled, compared to 17% in the district. The school sits on a large woodsy campus in the county, at the edge of the city. Not so long ago, it was a very small school with a very rural atmosphere. Now, newer subdivisions have been built on the surrounding acres. Most children require transportation to come to school. There are at least seven teachers in each of the six grades here. Despite the large student body, at ten o'clock in the morning Hickory School appears to be a very quiet and organized school.

Twin Meadows School is almost as large, with over 700 students, half of whom are Black and one-fourth of whom are White. This is a Title I school and as such has a large materials budget and many supplemental personnel. During the last five years, the school grades have alternated between A and C. Generally, two-thirds of the students are considered to be meeting high standards in mathematics achievement. The campus was once primarily a neighborhood school, but now most of the students are bussed to school. Parent programs are often held off campus to accommodate the families. The economically disadvantaged comprise about 78% of the school population. The ELLs make up five percent, compared to 24% of Forest Glen School, the district's ELL magnet school. Sixteen percent of the student body has a disability, close to the ratio for the district. The rambling campus fronts a busy street by the town's mall, but its buildings cover a sloping hill that backs on to a wooded area. The researcher's visit overlapped lunch hour, and the enthusiasm of the children was evident, even as they marched in orderly lines to and from the lunchroom.

Participants

The principals of Hickory School and Twin Meadows School agreed to participate in the study. These administrators enlisted the cooperation of the school CRTs, who in turn set up a classroom visitation for the researcher with a teacher perceived to be a good instructor in mathematics. Although the CRT at Hickory School shared little of her personal background, she appears to have been a classroom teacher with many years experience, most of it here at Hickory

School. The fifth-grade teacher she asked the researcher to visit is an experienced man in his fifties, with a tall and medium-sized build. He is affable and confident. The children know the classroom routines and follow them comfortably. The tables are arranged in a modified U, and all the children interact with the teacher throughout the lesson.

The CRT at Twin Meadows School is an energetic man in his thirties who expressed his interest in being an administrator. He is very active in many aspects of administration as part of his work here. The young woman the CRT selected for observation as she taught fourth grade is exceptional for her skill as a teacher at the beginning of her career. Her skill is notable because she was allowed to be a mentor for intern teachers, despite her novice status. Her mathematics classroom ran smoothly under her calm and quiet direction. The children were engaged with a lesson that incorporated the lesson from the text, the computer and Smart™ board, and manipulatives. The children’s desks were arranged in pairs facing the Smart™ board at the front of the room.

Data collection

The researcher made one visit to each of the two schools. The visit began with the classroom observation, where she took field notes. There was a brief conversation with the teacher at Hickory School, but most of the record is composed of notes on the classroom interaction and the mathematics lesson, at each school. Afterwards, the researcher conducted semi-structured interviews with each CRT that were recorded with auditory equipment. These each lasted about one hour, and the questions are found in Appendix C. The interviews were transcribed and coded by incident, as were the field notes.

Instructional Mathematics Coaches (IMCs)

The position of mathematics coaches in this district is generally created as part of an elementary school improvement plan directed toward improvement of student achievement in mathematics. The specifics of the job description would vary with the school assignment, but the coach would work with teachers to improve instruction and students to improve learning.

Sampling strategies

There are full-time IMC positions at two schools (Forest Glen School is not one of them) in this school district, and the IMC at each school agreed to participate in this study. Each of the schools is a Title I school with a large budget for supplies, resources, and supplementary personnel.

Setting

Delta School is home to 445 students, primarily Black and economically disadvantaged. About 17% of the students have a disability, which is the same ratio for the district as a whole. None of the students are considered to be ELLs. The school campus is located on a pleasant site in an historic Black community within the city limits. At the beginning of this century, the school became a magnet school for the arts and was very successful as measured by six years of As and Bs for the school's grades. However, due to a variety of factors, recently the school has received a grade of F, then D. Last year 43% of the students were identified as meeting high standards in mathematics (FLDOE, 2010c). Hiring a mathematics coach was part of the state-funded plan to improve the school.

At another high-poverty school in the city, the state had provided funds for a school improvement plan that included the hiring of an IMC. Redwood School has a history of getting Cs and Ds over the last ten years. However, two years ago the school received an F, and last year they earned a D, with 58% meeting high standards in mathematics. The student body is mostly Black (91.4%). There are no ELL students, but 12% of the students have a disability. The campus is large, with extensive grounds and trees. The buildings follow a hexagonal pod plan, concentrating the grade classrooms in their own pods. There are about 400 students.

Participants

The IMC at Delta School is Harriet, a retired classroom teacher who had developed a repertoire of mathematics activities based on the use of manipulatives to cultivate numeracy in elementary students. After a successful year as a part-time mathematics coach, she agreed to temporarily fill the full time position created this year. The principal instituted a math lab where Harriet taught rotating groups of students and their teachers the lessons Harriet thought were important, modified by teacher input. "I decided last year…that whatever the teachers needed I would do, then I would make activities *for* them…I would show them how to do it, and then I would give *them* the same pattern that I followed."

At Redwood School, Randi is the full time mathematics coach. Last year she was a fifth grade teacher, with excellent skills. She agreed to this three-year contract if her teaching position would be available to her afterwards. Her responsibilities include helping the teachers with the new standards, looking at the new test item specifications, helping the teachers with textbook, and being sure instruction is given in the best way. She meets with the grade level teams, sees individual teachers as needed, visits classrooms, and teaches mathematics to students in the intervention program. "I'm a kind of a resource for them, both on the curriculum, and the actual instruction." Managing test data with the teachers to assist them in preferred models of instruction—the gradual release model and differentiating instruction—is one of her responsibilities. The principal and she meet to examine the data, and the principal offers additional tasks as she notes them in her classroom walk-throughs.

Data collection

Each of the IMCs was interviewed for about an hour with questions found in Appendix D. The interviews were recorded by audio recording, transcribed by the researcher, and coded by incident.

Mathematics Teacher Educators (MTEs)

Sampling strategies

The researcher sought a sample of mathematics educators who were not in the immediate vicinity of the researcher's university. From a list of potential participants, two were solicited to participate, and they agreed to be interviewed.

Setting

The setting is a state university in another county in this state. This mid-sized comprehensive institution offers degrees in elementary education, middle school mathematics, and high school mathematics, with an emphasis on teaching urban education. Additionally, there are programs to certify teachers from alternate career paths. For two years in a row, the college received an award for being an exemplary professional development school (National Association for Professional Development Schools, 2011). About half the members of the student body are residents of the local county. Five percent of the student body is from out of state.

Participants

Two professors of mathematics education were participants in this study, one a former elementary school teacher and one a former high school teacher. Both share responsibility for teaching mathematics methods at the elementary level. Each recently completed the advanced degree that enables them to assume the position of assistant professor. Each retains family and social ties with their former community, one to the extent that she lives in the southern part of the state, five hours' drive away, half of each week.

Data collection

Each of the MTEs was interviewed for about an hour in their office with questions found in Appendix E. The interviews were recorded by audio recording equipment, transcribed by the researcher, and coded by incident.

Data Analysis of Peripheral Participants

The data from these participants was analyzed through situational analysis. The findings will be outlined in Chapter 6, and the voices of these participants will be integrated with the voices of the fifth grade teachers and the review of the literature to define teaching mathematics in depth and answer research questions one and three.

Summary of Methods

This qualitative study was undertaken to develop a framework for teaching mathematics in depth. Data was collected from a fifth grade level team of teachers regarding the educational practice of teaching mathematics in depth as they implemented a curriculum that expected its use. Additionally, the input of mathematics education professionals was sought to support this knowledge toward the goal of creating this framework, a framework that would support the education of teachers of mathematics. This chapter describes the participants and the context in which they taught. The constructivist grounded theory methods and situational analysis which were used to collect and analyze the data are elaborated as well. At the conclusion of the process, answers to the research questions were drawn from the statements and observations collected in this study regarding the conceptualization and actualization of teaching mathematics in depth. Moreover, a framework to describe this practice was established and is presented in Chapter 6 of this manuscript.

Table 4-1. Demographics of students at Forest Glen

Racial/Ethnic Group	Number of Students Enrolled in October		School %		District %		State %	
	Female	Male	2009-10	2008-09	2009-10	2008-09	2009-10	2008-09
WHITE	101	69	38.7	38.2	47.7	48.5	44.4	45.3
BLACK	51	52	23.5	27.6	36.1	35.9	23.0	23.0
HISPANIC	38	45	18.9	17.5	6.0	5.9	26.2	25.0
ASIAN	23	28	11.6	9.9	4.2	4.1	2.6	2.5
AM.INDIAN	1	1	0.5	0.6	0.2	0.2	0.3	0.3
MULTIRACIAL	13	17	6.8	6.0	5.7	5.4	3.4	3.9
DISABLED	10	28	8.7	11.4	17.2	18.5	14.1	14.3
ECONOMICALLY DISADVANTAGED	103	119	50.6	53.3	48.6	46.2	53.5	49.6
ELL	54	50	23.7	19.4	1.9	1.7	11.6	11.8
MIGRANT					0.1		0.5	0.5
FEMALE	227		51.7	51.2	49.1	48.9	48.7	48.7
MALE		212	48.3	48.8	50.9	51.1	51.3	51.3
TOTAL	439		100.0	100.0	100.0	100.0	100.0	100.0

Note: Adapted from "School District and State Public Accountability Report," Florida Department of Education, 2010. Retrieved 1/27/11, 2011, from http://doeweb-prd.doe.state.fl.us/eds/nclbspar/year0910/nclb0910.cfm?dist_schl=1_31

Figure 4-1. Initial codes written on sticky notes

- **A. Classroom activity directed by outer influences**
  - **1.** organizing around the NGSSS
    - racing the pacing guide
    - planning around the essential questions in the classroom
    - aligning instruction with the standards
    - worrying about the new standards
    - comparing to SSS
    - how will we implement?
  - **2.** Trusting the pacing guide
    - learning which problems can be skipped for their own classes
    - knowing there is less to teach, so we can take more time
    - limiting the scope
    - Realizing the time placement

Figure 4-2. Axial code organization

Coding trail

Initial codes	Focused codes	Tentative axial codes
-Class activities	-Teacher ideas	
-Going well -Kids engaged	-Going well when kids engaged	-Let the children explain
-Finding patterns -Seeing extensions -Understanding mathematics -Finishing homework accurately	-Familiar learning -Saying "It's easy."	-Confidence denotes understanding

Figure 4-3. Sample coding trail

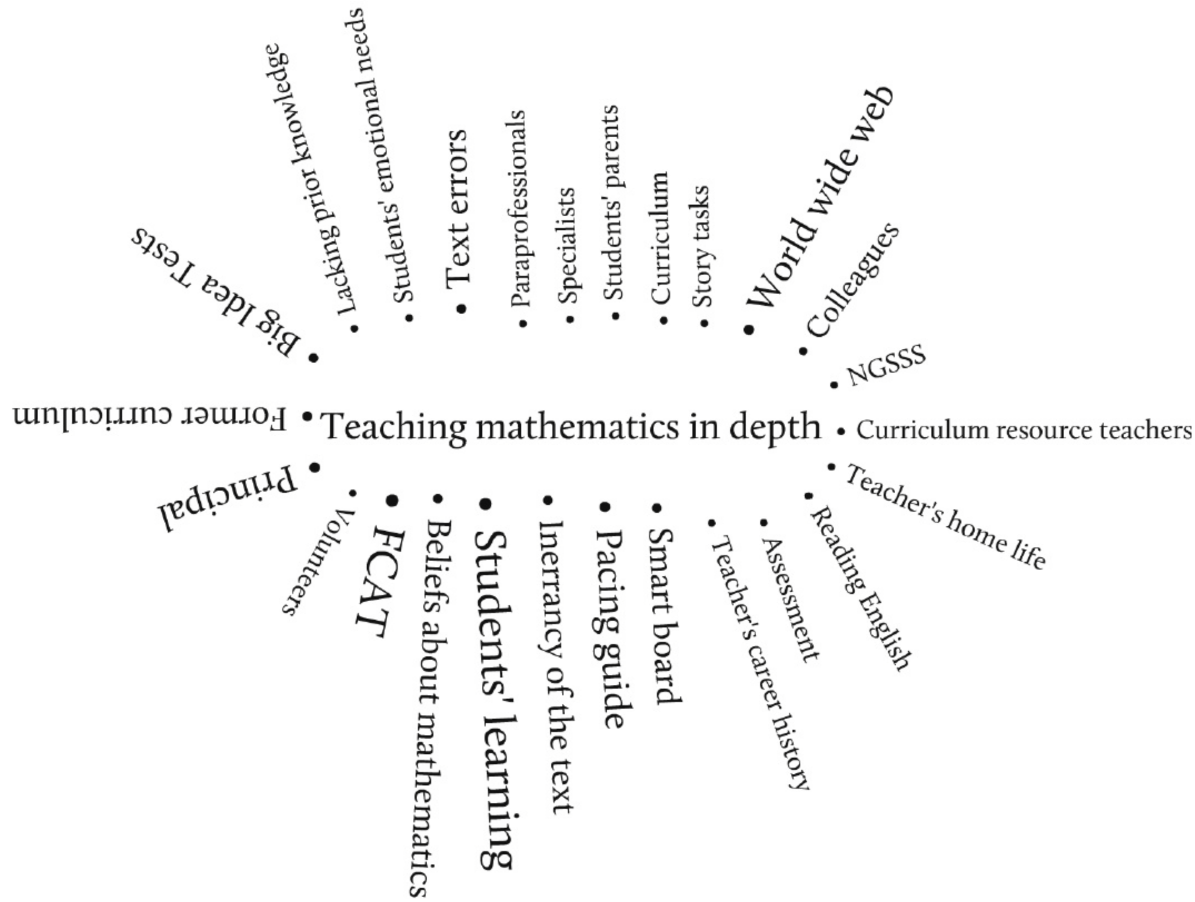

Figure 4-4. Situational matrix of 11/14/10, modified 1/11, from data 3/23/11

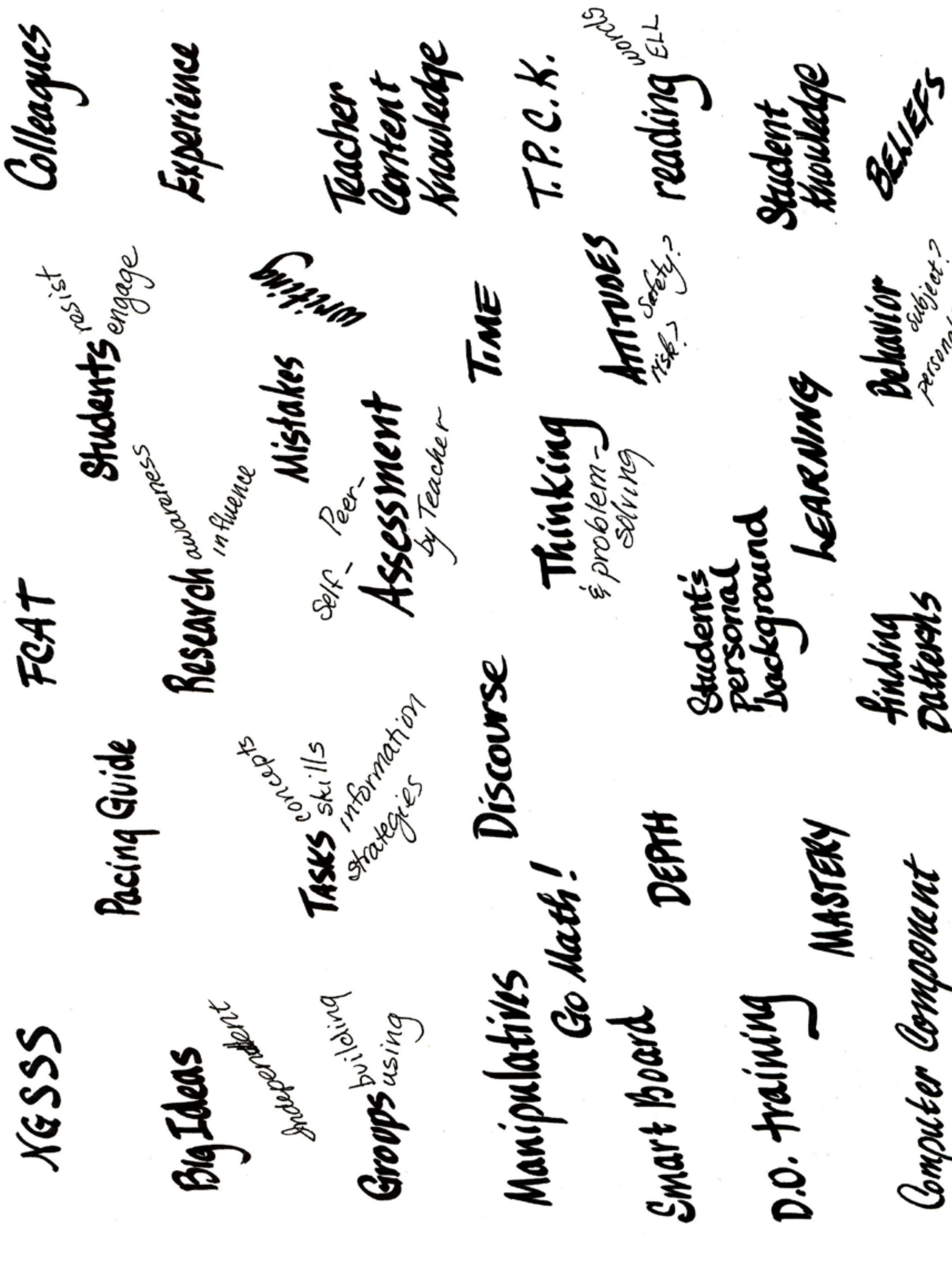

Figure 4-5. Messy situational map 11/28/10

Individual human actors: 5th grade teachers

Collective human actors: District office training-4, colleagues-8,

Discursive constructions of individual and collective human actors:

Research (influence & awareness)-10, Building groups-11, Student beliefs (resist & engage)-12, Teacher content knowledge-14, Tasks (concepts, skills, information, strategies)-15, Thinking, problem-solving-16, Talking-19, Mistakes-20, Reading-21, Big Ideas-22, Assessment (self-, peer-, teacher)- 25, Student knowledge-27, Student behavior (subject & personal)-28,

Political elements: NGSSS

Temporal elements: Time-18

Major issues debates: Mastery-23, Depth-24, Learning-26

Other kinds of elements: Past experience-13, Teacher attitudes—risk or safety?-17,

Nonhuman elements/actants: Manipulatives-9, computer component-6, Smart™ board-7,

Implicated actors: Professors, IMCs, CRTs, specialists

Silent actors: Students, parents, volunteers, paraprofessionals

Discursive construction of nonhuman actants: NGSSS-1, Pacing guide-2, FCAT-3, Textbook-5,

Sociocultural/symbolic elements: Race, ethnic background, gender, nationality-29

Spatial elements: School building, classrooms, furniture, school district

Related discourses: Ethical purposes of teaching,

Figure 4-6. Ordered version: abstract situational map of 11/28/10

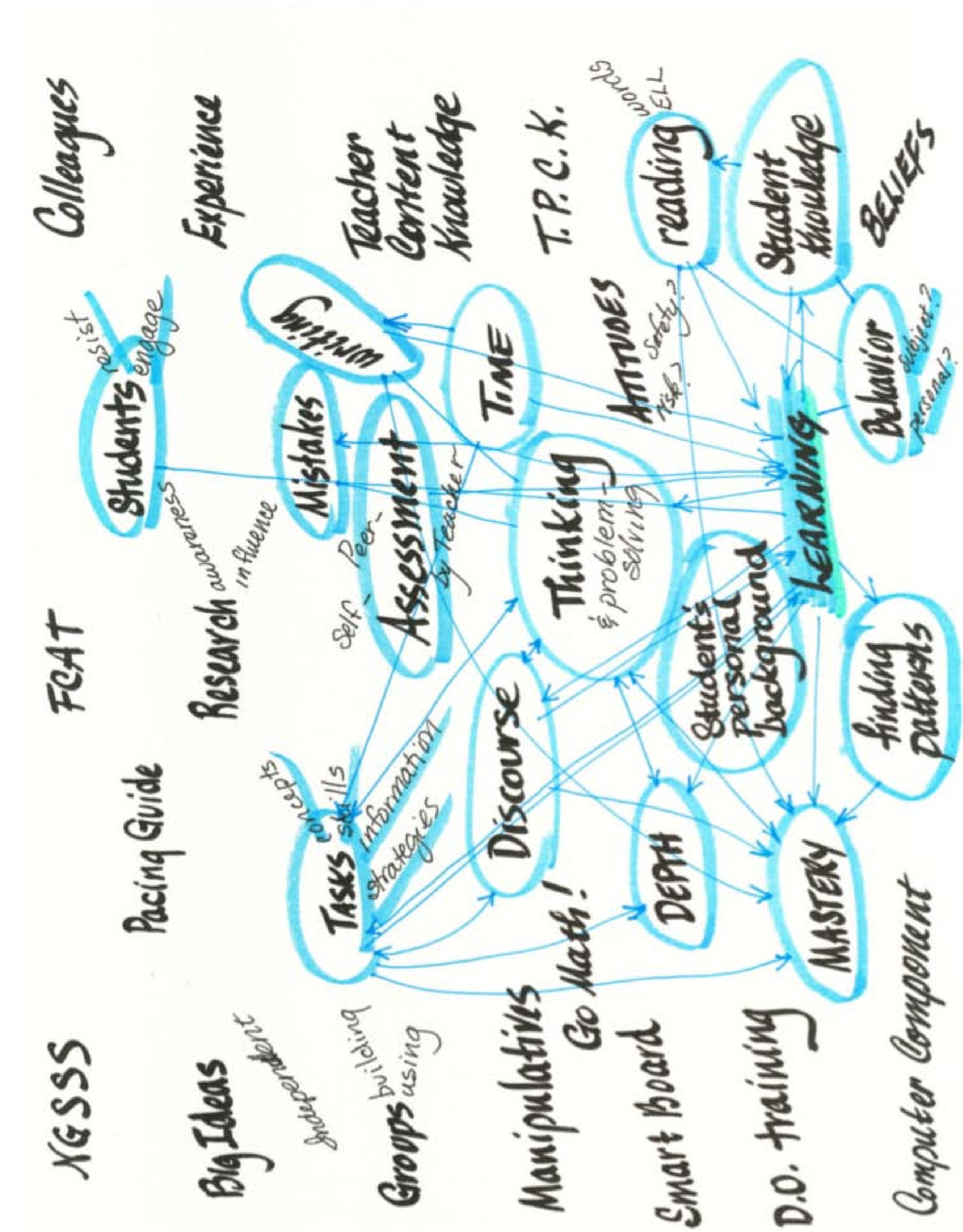

Figure 4-7. Relational analysis using situational map

Figure 4-8. Final coding situational map.

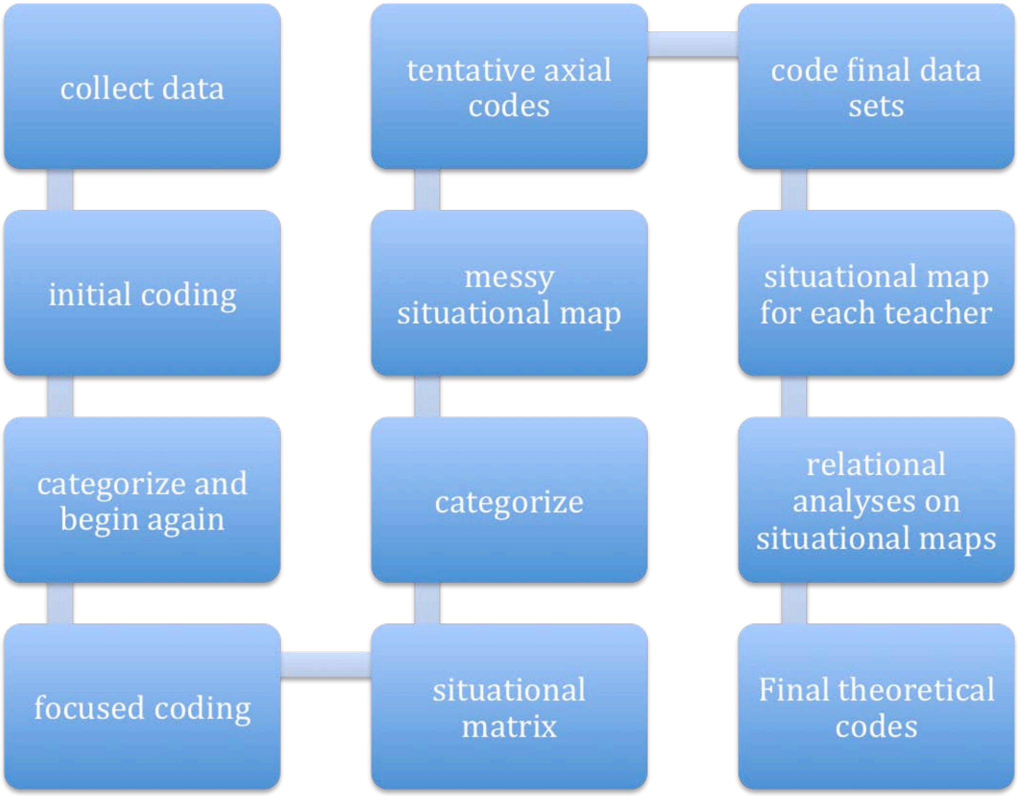

Figure 4-9. Flow chart of process

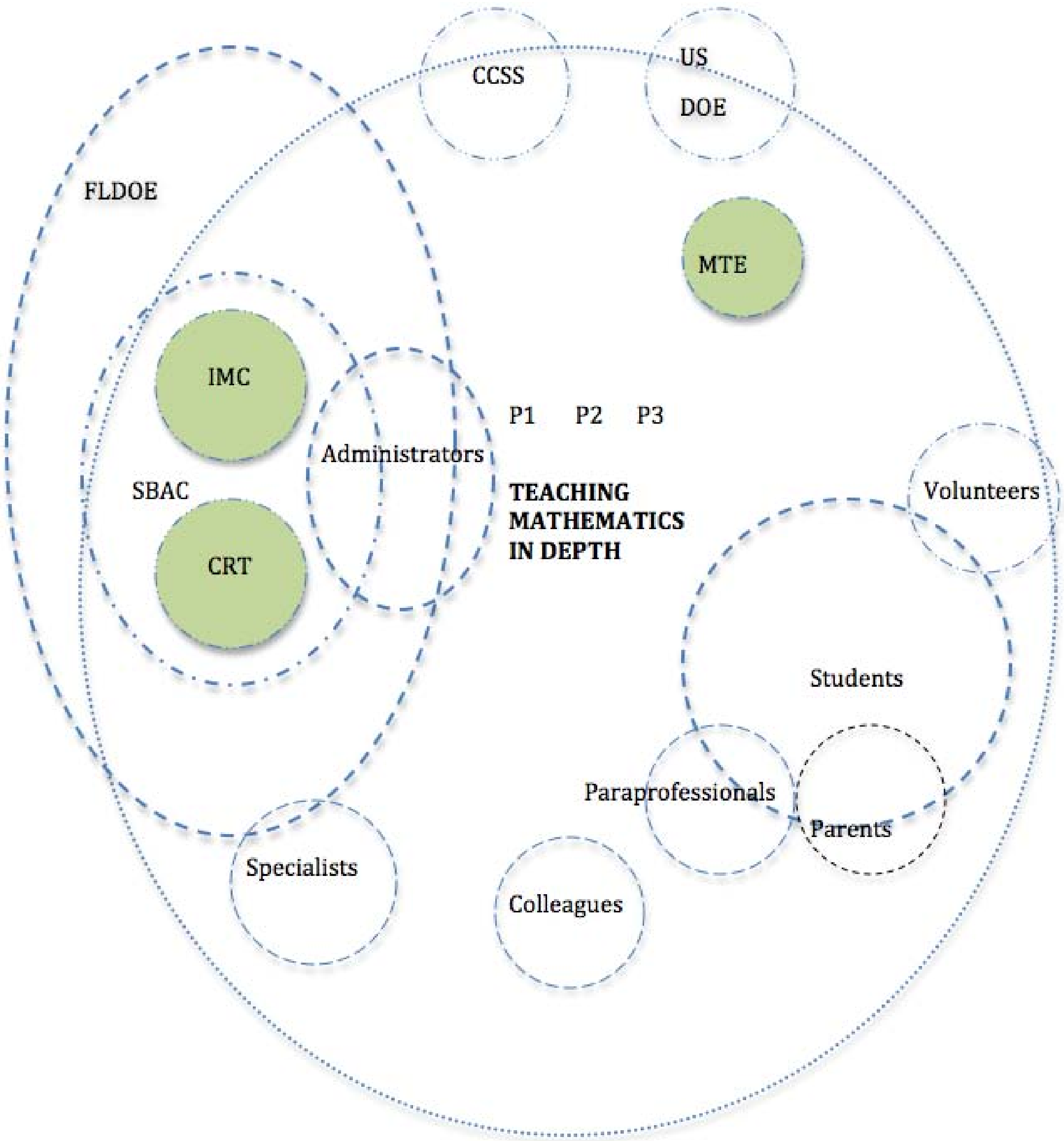

Figure 4-10. Social worlds/arenas map of 1/18/2011.

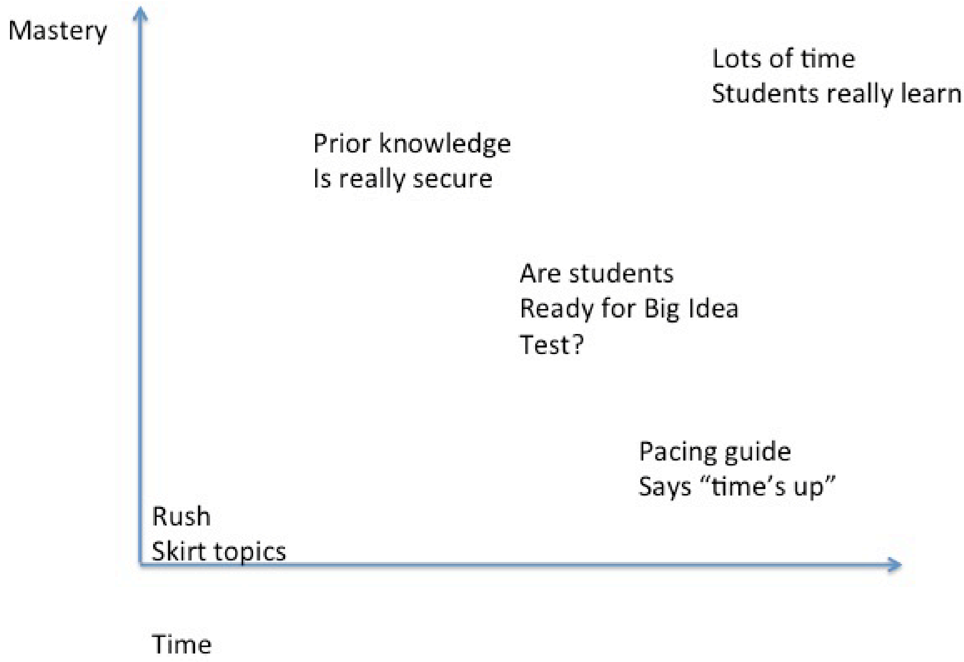

Figure 4-11. Positional map of 3/15/2011

CHAPTER 5
FINDINGS

Part of the purpose of this study was to develop a framework for teaching mathematics in depth by examining how fifth grade teachers conceptualize and actualize those practices. The context for exploring this synthesis and enactment was in an elementary school in a southeastern state where new mathematics standards, a new mathematics curriculum, and a revised high-stakes achievement examination were being implemented. The legislative mandate for these components was that mathematics be taught in depth. As the teachers taught their classes in this initial experience with this pedagogy, the researcher participated in the mathematics classes according to the direction of the teacher, being a leader of a small group, a tutor, a researcher, or a monitor. The teachers were interviewed individually using a protocol that probed both their understanding of teaching mathematics in depth and their enactment of it. Additional information was provided by the teachers in journal responses to prompts. The journal entries and the interview transcripts were analyzed by methods associated with grounded theory. This chapter is a record of the results of that analysis supplemented by analysis of the researcher's field notes.

Analysis revealed that the individual beliefs held by each teacher about learning and teaching were an important influence on how he/she defined and implemented teaching mathematics in depth. The teachers' views varied considerably about what is important to teach, how it is best taught, when they know that it has been learned, as well as what obstacles they face as they teach mathematics in depth. One finding that emerged from the data analysis is that the teachers' orientation toward learning influences their conceptualization of teaching mathematics in depth and affects their actualization of the practice. Each of the teachers proved to have a different orientation, although this result was not the purpose behind the sampling process. Teachers were recruited who were not only eager to implement the new mathematics curriculum, but who were also willing to be part of a study of this program. Because the differences between the teachers' outlook became such a significant variable in the results, the findings are presented as case studies, as portraits of three teachers implementing teaching mathematics in depth.

In the sections that follow, the teachers are discussed individually, but it is important to note that the diversity represented in each classroom by the students was similar and mirrored the demographics of the school. For mathematics and science, each fifth grade teacher in the study taught a class that included students learning English, students who were bilingual, and some who received special education services. The gifted students went to another classroom for science and mathematics instruction. The Big Idea for the term of this study was this: "Develop an understanding of and fluency with division of whole numbers" (Adams et al., 2011).

The quoted information in this section comes from interview transcripts, journal entries, and conversations recorded in the field notes of the researcher. Background information and description of classroom episodes generally arises from the field notes. The researcher has tried to be clear about the sources. Each teacher's practice is discussed in terms of strengths and areas of mutually perceived difficulties. In Chapter 6, the work of all three teachers will be integrated with the aim of answering the research questions as the answers have emerged from the data.

Sandra's Mathematics Lessons

As one entered Sandra's classroom (Figure 5-1), one knew that the teacher demanded the attention of the students so that they could learn. All desks faced the front of the room, where there was a Smart™ board, the computer stand, and a white board. Anchor charts lined the walls, "to anchor learning…so they can always refer back to it." She made the instructional decisions based on her experience as a mathematics teacher and the new textbook. The summer prior to this school year, she participated in a training program specifically designed to foster learning by inquiry methods. She was a representative on the committee that selected the textbook series.

> Sandra: That [textbook] was my choice. And I liked it initially, because I felt like it was really aligned with the standards. . . . I was seeing how it would go in depth and use manipulatives, you know, um, teach more of the *why* behind the math.

Strengths of Sandra's Instructional Practice

In the interviews, Sandra reported an advantage of the new curriculum was that the students were learning many ways to solve problems and many strategies to get answers.

> Sandra: They're really going in depth in the first four lessons of the chapter, and we're getting the different *methods* to find the answer and then applying it in the last half of the chapter.

Her instructional hope for the students was that they would be able to read a problem, determine the best method to solve the problem, and then get the correct answer to the problem. When students were telling her their favorite strategy, she responded,

> Sandra: OK. Go for it. It's totally *your* choice. It's up to you. When it comes right down to it, you just have to arrive at the correct answer. However you choose to *find* that answer is up to you, because we all work different.

Her self-confidence enabled her to pick and choose from the learning materials presented in each lesson. Sandra appreciated the pacing guide, because it took the guesswork out of her planning. Sometimes she found that her class was ahead of the guide, and she could focus on other subjects a little more. She recognized the work that this planning requires, and she was grateful that "they have *tried* in the county to take some of that legwork *out* for us. . . . I feel like we should be using it." If the textbook materials seemed overwhelming or redundant, she offered this advice to other teachers, "I recommend to other teachers to look at the benchmark. Look at the essential question and pick out what is going to help the kids *answer* and understand that benchmark." She was accustomed to checking her lesson plans with the standards, and then adjusting the instructional materials to fit the needs of her students. "Everyone's going to have their own rhythm for teaching math. They're going to have their own *flow*. I have to have problems to refer to but *I* am actively driving instruction." She knew the abundance of materials could be confusing to a new teacher, or to any teacher, so she grounded her work in the state mathematics standards.

Sandra's lessons demonstrated her concern for presenting the mathematics correctly. She prepared her lessons carefully by working the problems herself the night before and was uncomfortable making mistakes, as she wrote in a journal entry, "If I were making many mistakes, then it would lessen the student's confidence in my ability to teach them and undermine my position in the classroom." Later in the school year, when one had "a positive classroom environment, then making mistakes is okay. It is comfortable and non-threatening to everyone if handled the right way." It is important that the students trusted her, because Sandra knew that trust opened the students to risk the confusion that comes with learning new and difficult material.

She considered the mathematics in the lesson and organized her plans so the students would experience success at the beginning of a new topic. She would begin a new topic with only a few problems that the students got right, and then she would stop presenting and offered them practice. This was part of Sandra's plan, "I did not want to do a whole lesson that they would have gotten frustrated. They need that, like, seed to be planted." Her plan for gradual release was to give the students a positive experience and they would be eager to work on the task on the morrow. After an initial lesson on solving equations, she said in an interview, "I was proud of myself, through the lesson, because I feel like I was releasing responsibility in a good place."

Sandra respected the differences in learning orientations that students brought to mathematics. She was attentive to offering lessons in different modalities and recently experimented with practices to address the auditory learner. She realized one of her students had a learning disability that inhibited her understanding of the story problem when reading was required, but the same student could solve the problem when she is only listening to the narrative. As a result, she was giving all the students practice in solving problems when they heard, not read, the information. This practice proved to be beneficial for the English language learners (ELLs) as well.

Although most of the lessons taught in the presence of the researcher were direct instruction, Sandra valued the instructional practice of inquiry in mathematics. She was hopeful that this curriculum had built into it sufficient time to pursue an inquiry until the students were satisfied that they understood the concept. With excitement, she related a classroom experience that happened only because she had had time. Students were dividing three digit numbers and using base-ten blocks to solve the problem. These problems presented some difficulties, because the students needed to differentiate between the number of groups and the number of members in the group. Until this question was settled, the students were confused about working out the entire problem. Using white boards and base-ten blocks, the students puzzled over this problem until it made sense.

Sandra: Like I took that twenty minutes to let them play with the blocks and count them out, and even though it took us twenty minutes to get that *one* problem—it didn't *matter* . . . because they got it correct and they were so *engaged* and so excited to get the correct answer. . . . It took twenty minutes on one problem. It didn't matter because it was worth it.

Sandra believed that her job was to require the best that the students could offer, to make of them capable students of rigorous mathematics. On days that students did not do their homework, they walked laps during their recess or finished their homework instead of having the time outdoors. She was weaning them from reliance on the chart of basic multiplication facts with a series of activities designed to develop fluency in this subject. She had many classroom games that rewarded accuracy, such as the beloved "stand-up" game. Everyone stood until they missed an answer on their homework, then they sat down. Those with 100% correct got a piece of candy. She was proud that she was "teaching the text the way it was intended to be taught. If I skip the hard stuff then they will be doomed," noted Sandra in an interview.

Sandra paced her instruction to the level of understanding she read from her students. Frequently she circled the room, looking at their work. She may have asked for raised hands of agreement or she read faces. She was looking at the answers to computation problems. When Sandra checked, she learned that "Everybody else either missed one or none. That tells me right there that they understood. I mean in their reading and dividing and finding the answers. Unbelievable." If she saw most of the students were correct, she moved on with her instruction. She wondered, "Do I go up there and rework . . . step back a bit? Can I push them forward?"

Student eagerness to do the assignment was another indicator of student understanding to Sandra. She felt that a lesson was successful because, "They're more challenged and excited to *try* to find the answer than they were before." She continued, "I want [them] to be excited for each *other*, and excited about their *own* learning." When students were articulate and they did not struggle, Sandra said, "They *really* understood. Students are able to explain the math . . . the *math* way of doing this, the reason why."

When the assignment was given to a small group, she also circulated and listened to the group conversation. She stopped and assisted as she saw the need. When Sandra gave the students a small-group inquiry project, she moved from group to group, listening to the students, probing their responses, and suggesting further investigation. She was satisfied that her students comprehended when they not only found the numerical answer to a division problem, but they knew what that answer and that remainder meant, and they could explain it.

Another indicator of student comprehension was the willingness to write sentences voluntarily. Initially, Sandra demanded a student sentence to summarize the result of their calculations. Later, she reported that the students were writing voluntarily. She said, "I was most impressed with the kids who wrote sentences. They really understood, and wanted to explain it. I didn't tell them to do that. They actually understood it enough to *feel* that they wanted to write it down."

Lastly, Sandra wanted to know that her students would do well on standardized examinations. She saw the student scores as a reflection on her teaching ability. She continued, "On your district [On Track] scores, *if* other schools are meeting the pacing guide, my scores won't be as good…as those kids who *are* at the pacing guide." Consequently, there was pressure to be on the page suggested by the pacing guide. Yet Sandra also needed to know that her students were ready to be tested. Speaking of the pacing guide, she said, "Now there's a *kink* in that. *If* we move too fast and don't cover things in depth *enough* so they *master* it, then they won't do well enough, anyway." The summative test scores mattered to Sandra, because to her the scores connoted understanding. She took heart when one ELL student, "got *all* of the [six] problems correct and she did it on *her* own. . . . But that was an *English* issue. It wasn't a *math* issue."

Pedagogical Areas for Potential Improvement

There are three areas to discuss in this section. Both Sandra and the researcher were aware of these concerns, since Sandra's voice was heard in interviews and journal entries. The researcher noticed manifestations of the concerns in events that took place in the classroom on the days of the researcher's visits.

One day Sandra asked for a visit from the district expert on cooperative learning. All the faculty members had been trained in use of this instructional strategy at the beginning of the school year, and the specialist was available for classroom intervention. During the interview that followed this session, Sandra talked about the (1) importance of building a learning community in her classroom, as well as the (2) difficulties that inhibited that construction with this group of students. With prior groups of students, she felt that she could (3) transfer more responsibility for learning to the students themselves. Each of these topics will be discussed as pedagogical areas for improvement, as noted by both the researcher and the participant.

The model from the past that Sandra would like to implement looks like this. She grouped students in pairs or quartets and assigned them the task of self-correction and peer assistance for questions. She would check the work of one member from the group, and the group would self-correct based on that person's work. She told the student, "You're responsible. *You* are the first one to finish. *You* got the problem right. You make sure everybody else has it correct." This was a successful practice for her in the past. Moreover, her recent professional development experience motivated a desire to incorporate more lessons centered on inquiry methods, but she believed that her group of students could not handle it in a mature manner.

Sandra: I want the issue to be the *math*, not *who* touched the blocks when they shouldn't have, or who took *that* many, or *who* took my board or da-de-dah. That's not *math*…that's *stuff* I don't want to have to *worry* about.

Sandra wanted her students to be more independent, but seemed at a loss how to enhance those skills. When she was calling on student respondents, she used methods of behavior modification that were obscure to the students.

Sandra: When I make a choice to choose someone who's doing the right thing, that's not *reason* enough, not for them. Like if I say, 'I'm going to call on *this* person, because they're not asking me, they're not jumping out of their seat'. . . that's not *reason* enough to them.

She would like to see them check their work, but she has not demanded it of them, even when it was part of the text assignment, telling the researcher, "I'm not big on that." Occasionally, Sandra's emphasis on accuracy contradicted her desire for students to assess their own work. One example was from an incident during a discussion of the homework assignment. The researcher was sitting with a group of students at the end of the second month. Sandra asked for questions, but none were presented. A student asked the researcher why her answer was wrong, and the researcher brought it up as a discussion question, which Sandra readily addressed. In this instance, it appeared that there could be more than one correct answer and a profitable class discussion about checking followed.

One area of difficulty observed by the researcher related to student personality. In a class of eighteen students, it was evident to the researcher that there were as many reasons for student misbehavior as there were students. The behaviors that were most distressing for Sandra were speaking out without permission and refusing to participate in instruction, perhaps as a gesture of defiance or resistance to the content or the teacher. She expressed to the researcher her reluctance to contact parents when she was not working well with a child. However, she did contact the parents of these students and was happy to report great improvement in her relationship with each of them. There was a story behind each person's misbehavior, and each was different. Her response to the problem also varied with the individual, but she ruefully acknowledged that working with the parents reaped positive results for learning.

Sandra wanted her students to respect each other as learners and to work together on learning mathematics. She would also have liked to see them policing themselves, instead of being satisfied that there was someone whose behavior was worse than theirs. She had goals of teaching all her students to get the right answers, but also understand why math works. Sandra had a clear idea of the mathematics in the standards and how it could best be taught. She was attentive to the learning needs of her students, and she knew many methods to promote cognition. Although many of her assessment practices were traditional, student understanding was her primary objective.

James's Mathematics Lessons

The initial observer of James's classroom (Figure 5-2) might say it was smaller than the other fifth grade rooms. The tables were clustered close together, in the center of the room. Students' backpacks were stored on hooks under the windows. On the opposite wall were all the student textbooks, arranged by subject on the shelves. There were crates to store student journals for mathematics and science. The front board listed the agenda and the rotation of the groups through the agenda. There were always five chairs at a table, not because there were thirty students in the class, but so there was always a seat for an adult and an empty table for one of the visiting teachers to work with a student needing special services. International flags hung on a string cross the ceiling. In the afternoon James and the ELL teacher instructed the fifth grade ELL students in language arts. James speaks both Spanish and English and taught in each language, depending on the students addressed.

During the school term when the researcher was part of James's class, the class was always organized for small group instruction. An intern was serving as James's student teacher and several university volunteers attended class on Mondays and Fridays. A student's mother came to help every Friday. James assigned these additional adults to a table with a group of children. Usually the children rotated to the different tables where each volunteer led a lesson that they repeated three or four times during the two-hour mathematics and science period. The researcher was usually assigned to oversee a science lesson from the textbook. What the researcher learned about James's mathematics instruction originates from the interviews and the journal entries, casual conversation, and short episodes of instruction overheard from a distance.

Strengths of James's Instructional Practice

James trusted the state standards for mathematics to provide the direction for his instruction. Although he did not follow the debate prior to adoption, he was assured that the newly implemented textbook was aligned with the standards and the modified end-of-the-year examinations. His anticipation for the school year was that he would follow the textbook as written, thereby getting acquainted with its scope and sequence, taking the time he needed to fully incorporate its suggestions into his instructional practice. He was hopeful that the emphasis on the Big Ideas and its limited scope of mathematics would offer students more time to learn and greater opportunity for mastery.

Over the semester, James came to realize that the scope of the subject was limited to fewer topics, but within each topic was a depth that had not been in the previous standards. "Just the depth can be a little bit daunting for now . . . because I've never gone into this much detail…about division. I mean, here we are, still wrapping up division, and I'm itching to go into graphing and data." In the past he had two essential practices for division: how to adjust for high and low quotients and estimation. The three chapters in the new series were exploring a number of methods for division he had not addressed in the past, like repeated subtraction or partial quotients. The students were taught estimation and were asked to use it repeatedly in a number of contexts. Nonetheless, James trusted the textbook to assist him to teach the curriculum.

He was pleased that the in-depth approach was helping his students become independent thinkers. "The students have a *very* good understanding of the process now, and by understanding the process I feel they're going to have a deeper, better understanding of what to look for in word problems," since in the past he thought "it was *so* hard getting through word problems." The students have solved so many story problems requiring division that he observed; "The kids are able to see a lot more examples of problems that they've done prior so they are able to actually access their own prior knowledge from just a few weeks ago." James continued, "Whereas before we were training them how to do some of the math, now they're able to actually do it without needing any extra training necessarily."

When asked about his stance on shortcuts, he replied that he was "OK with them taking shortcuts, just as long as they understand the *process.* If they understand *how* they got to the product…then it's cool with me." James taught his students how to do a variety of problems. The knowledge of the process and the approach was emphasized far more strongly than getting the correct answer. Once the researcher was assigned supervision of a student project to create a "foldable" pamphlet describing various types of triangles. To the researcher, a ruler seemed an essential tool, but it was not offered as part of the directions. Identifying the norms for mathematical rigor was not part of his planning process for this activity.

What was generally important to James was the student's attitude and approach to a mathematical problem.

James: When I hear and see a kid participating through discussion of a problem, I think to myself, 'that kid is trying to understand how this works'. . . . I feel that students who are engaged in discussion about their math problems are, for the most part, learning.

Conversation about the mathematics was the mainstay of James's instructional approach. In small groups, he introduced new material, offered students practice opportunities, and quizzed them on their conceptual understanding. By using the rotation system, he taught the same lesson at least three times, and, more importantly to James, "Working with those smaller groups . . . gives me the luxury to tailor the instruction specifically to the needs of those kids. . . . I'm adapting what I'm doing from group to group to group."

Students' needs for understanding drove the pace of instruction in mathematics. During the final interview he was beginning to worry that he was off pace.

James: I'm feeling a little rushed right now, because I'm not where the pacing guide says I need to be. I'm probably taking, for every one day that the pacing guide says I need to be, I'm probably taking one and a half days or one point two days.

On the rare occasions the researcher observed the small group instruction, she noted that James moved the group through the lesson as a unit; students were all working on the same problem at the same time.

Groups offer James an opportunity to teach his students to help each other. He looked to his students to scaffold those who needed more direction to stay on task. He continually supported engagement of each child. He did part of the process and let the students do the rest. "If they need help, then you jump in." He believed that this assistance is beneficial to both the helper and the one who is helped; there is value and respect taught in this process. In mathematics specifically, the students interacted to improve learning. He involves the students in the process of teaching each other. "T, will you bring D up to speed?" When possible, James assigned a helper to each of the five ELL students to translate instruction into the home language. His anticipation for his Taiwanese student was that he would help his classmates with their mathematics even as they assisted him with learning English.

As he taught, sometimes James made intentional mistakes. This is a tool that informed his instruction. "The kids are seeing different ways of *fixing* those mistakes that I make." He tried to make the mistakes at different places in the procedure to offer students an opportunity to fix the mistakes that he made. "That, for me, is a way to see, like, 'Are they paying attention? Are they following along?' And, are they noticing the *process* that I'm going through?" One type of problem that presented a challenge to the students was the one that asked the student "What's the error?" where the child in the book worked the problem incorrectly. The student was asked to find the mistake. When James's students did not respond, he divided the small group into pairs to discuss it, and then report back. He used mistakes as an instructional tool and was not upset if he makes one.

As he worked with a small group, he enabled good student responses by encouraging sharing. He created an intimate group and riveted the attention of each child. "Is she right?" "Watch. This is where the other group got tripped up." "Is it easy or is it hard?" Although he sat at one table, he was constantly monitoring the room and calling the time for rotation of the groups. On some days, he did not teach a group at all, but drew individuals to the side to confer with them as the volunteers and intern taught the groups. James worked closely with his students when they made errors. He held personal interviews about important assignments, particularly summative examinations, to review with the student their strengths and weaknesses. In this session, he often retaught a misunderstood concept. When two students finished a problem, James said, " Check each other's work because one of you is right." The students worked the problem twice!

For James, the summative assessments were an indicator of how well his students were learning, and also, how well he was teaching. James would be happy to see his students score well on the Big Idea tests, results that were reported to the district.

James: If 80% of my kids do 75% or better on the [Big Idea] test, then I'll feel pretty good about it. And then, I'll actually go over the test after my kids have taken it, and do a little item analysis and see what are some of the problems. I'll definitely go back and do any kind of remediation . . . and just tailor it for those kids that need it.

Pedagogical Areas for Potential Improvement

In the course of the interviews and the journal entries, James evidenced several frustrations. The first was the challenge represented by students who are unwilling to explore the world of mathematics as revealed in class, due to personality, attitude, language, or personal history. James fretted over students who were not open to his instruction. He hoped to inspire a love of learning and a confidence to pursue it, even in the face of difficulty. He would also have enjoyed a more collegial approach to teaching mathematics. As he anticipated a teacher workday, he reported,

James: It would be nice to see . . . how they're using the components here and I want to be able to compare notes. [I want to compare] here's what was working for me . . . how did they handle this . . . so that I can verify, hey I need to change this or I need to do this better or I'm doing this right.

A third frustration for James was the knowledge gap created by the change in curricula. Many of this year's fifth graders had not mastered multiplication in the fourth grade, but that mastery was assumed by the new curriculum. James recognized the need in the students for instruction, but he was uncertain how to fit it into the schedule. He urged, "Just give me four days, to do long . . . large multiplication problems, and just process, process, process. . . . That would hopefully lead to more success with their division." He struggled to decide which problems to do and which to omit, knowing that he lacked the familiarity with the new standards that would assure him he was making the best decision.

James had taught fifth grade for many years, and he struggled with the changes between the former curriculum and the new. Watching him teach Calendar Math (Great Source, 2011), one could see the confidence he had in his ability to represent mathematics in many ways. This program that initiates the study of essential mathematical concepts with the numbers on the calendar has been part of his practice for many years. He accepted all student answers, asked the students to elaborate on their reasoning, and facilely created new questions to explore. He did not use the new curriculum with such flexibility. He was still feeling his way through its chapters. James saw the standards and the textbook as the authority over mathematical truth, mediated by the needs of the students. His priority was teaching in small groups and individual conferencing. To accomplish this, he developed student skills of working independently. He used his awareness of student knowledge, expressed in discussion and conversation, as well as the results of standardized tests to influence instructional decisions.

Kelly's Mathematics Lessons

Kelly's participation in the study was unique in that she did not agree to the presence of the researcher in the classroom during mathematics class. The researcher was in her classroom on only one day (Figure 5-3), and that was to help a substitute teacher conduct the lesson. She participated in the interviews and submitted journal entries. She was an eager respondent while she was interviewed, and she took care to write complete answers to the journal prompts. What could be reported about Kelly is from these sources.

Strengths of Kelly's Instructional Practice

Teaching mathematics was

Kelly: [to] teach them a lot of the things that they *need* to have, and eventually *will* have, like as a few years down the line, be coming in, ready to go, hopefully, for this stuff.

Kelly thought mathematics was important and valuable for future mathematics courses. She also believed that there were prerequisites that facilitated the teaching process, which this group of students lacked, being in the transition between programs. She was hopeful that the adopted curriculum would allow her to teach that mathematics to her fifth graders. She said, "We're doing pretty good. We're pushing it. But, it's like pulling teeth, these lessons." Her understanding of mathematics was that it is both numbers and processes, and it was especially important to get the right answer.

As Kelly discussed teaching her students, she noted, "They're all doing their steps pretty well. So…I'm pleased with how they're moving along right now." If a student missed a step, Kelly took that as an opportunity to reteach. She used groups as a vehicle to talk with individuals. This was an advantage because she could look at their work and show them.

Kelly: OK, you keep skipping this step. I see *why* you're skipping that. You know I can *address* it. If I sit down at least two times a week with a group of *three* kids, I can *see* what they're doing, and see, kinda, their process and which process might work a little bit better for them.

Since many of her students were learning English, she worried that too much reading is required to do the mathematics. After the first test results were recorded, she wrote in a journal entry,

Kelly: I took into account that this test was just as much of a reading test as a math test. There was one problem that I believe was way too wordy and many students got lost inside the information and got frustrated.

Choosing the best chapter test was problematic for Kelly. "The chapter tests are mostly *all* word problems. Paragraphs that they have to read and answer." She depended on the textbook to guide her review of the material before the test.

Kelly: On the chapter review within the actual textbook . . . there's a few of those, but there's also a lot of just number ones . . . they don't really align up. When you think of a math test you think of numbers, generally. That's [the test] mostly words.

Students in the fifth grade this year were caught in the transition between the former mathematics standards and the new standards. Although the current curriculum was adopted in 2007, teachers did not necessarily address the new standards prior to 2010, the year the standardized test of achievement changed. Therefore, since the change in standards rearranged some of the mathematics topics, there were knowledge gaps for some students. For instance, the SSS anticipated that students could review multiplication at the beginning of fifth grade and division would be taught after that recapitulation. The NGSSS assumes that multiplication is mastered in the fourth grade and cursory review is the expectation for fifth grade, which begins the first lesson with division. Hence, the fifth grader students were not necessarily masters of multiplication nor were they fluent with the basic number facts associated with multiplication and division. This deficiency was a very present challenge for Kelly. In the first interview, she said, "Not everybody has their multiplication facts down, and so if they don't know how to multiply, they're never going to get the concept of division." Kelly's understanding of teaching division was to introduce it after the students had mastered multiplication. She viewed the curriculum as asking the students to "jump right in," but her experience indicated that this was not possible.

Having the correct answer to computation was a major goal of Kelly's mathematics instruction, but she was also concerned that the students move confidently through the assignment. She knew that her students understood,

Kelly: If they're able to get it pretty quickly, and *walk* me through it, and tell me what I am supposed to do, then I feel like they're confident and move on to the next one.

When students said, “I don’t get it,” she quizzed them, “Have you tried it? Have you written it down?” Student answers to these questions provided the information she needed to help them understand it better. As a further check, she might have grouped her students into pairs to complete a set of problems, justifying this,

> Kelly: If a student is able to explain the process and steps of a problem to someone else, they just took their understanding of that lesson to a deeper level and it will stick with them longer.

She met student resistance to the curricular insistence that they learn several strategies to divide. At the beginning, her students asked her, “Why don’t you just divide?” Referring to the past,

> Kelly: The kids have been clearly taught, this is the quickest, this is the easiest, this is how you do it, not really told like *why* they do it.

Now it was the purpose of instruction to teach the students many ways to solve a problem and why those methods worked mathematically.

> Kelly: They see that there’s many different ways to do something, and I think that’s good, just in their life, to understand that there might not be just one way to do it (first interview). It’s much more like why do I divide…I think [it] is beneficial for them to understand that there’s that process but sometimes that is hard for them to understand (third interview).

She still wondered if the students could accomplish more complex mathematics, like solving algebra equations, if they do not get the basic one. “They have to get the basic one.” “They have to follow the steps.”

Kelly prepared for her lessons carefully. She felt more comfortable if she could teach just a few students the lesson first, as in a small group. The practice benefitted, as she detailed her lesson, “By already discussing estimation with larger numbers, I was able to see where a few problems would arise when I taught the actual lesson.” At home, she created lessons on her computer that would interact with the Smart™ board in the classroom. She made digital copies rather than rely on the Internet being available.

Kelly: One day I came in and it wasn't there, and so . . . after that, I made sure I had it there each day, just in case. I'd rather be over prepared than underprepared.

She encouraged her students to use the online text supplemental materials at their homes. But Kelly liked to be sure that she already knew the answers to each of the problems in the lesson. She struggled, weighing whether to include each textbook problem in the lesson. "Are there sometimes problems that there's just no answers to?" She also worried that there were not enough practice problems in the instructional materials, so she made copies from the old curriculum to supplement.

Student understanding was Kelly's goal. When once the class worked on a problem together, they copied a number incorrectly. They went to select an answer and it was not there!

Kelly: We spent a few minutes checking our work before we realized our silly mistake. We had a discussion right there about how even teachers could make mistakes and how important it is to go back and check our work.

She was delighted when students cried out, "Oh, that's real easy." She would have liked the students to say that about every topic in the curriculum. The curriculum has tried to relate the mathematics to the real world of the students, but it was not easy. There was one problem about shirts the school band sold to raise money. The students in the fifth grade were supposed to determine how many of the total were sweatshirts and how many were tee shirts, given the number of dollars received, and the price of each type of shirt. One of Kelly's students was fixated on the person in charge of the sale.

Kelly: He did not understand why they did not keep better records of what they were selling at concerts. He thought that they should fire who ever was in charge of knowing.

An interesting class discussion ensued, but Kelly regretted "that diverted our time away from actually figuring out how to solve the problem."

Pedagogical Areas for Potential Improvement

Kelly's areas of concern were identified in a journal response where she answered the question about three students that challenged her. She was asked to write her hopes for those students at the end of the year. Her concerns spoke to engaging student interest, developing student facility with word problems, and being confident about one's self-knowledge. These three concerns were addressed in the interviews and journal responses collected in the study.

Making mathematics fun and inviting has been a goal for Kelly this year. She has spent many hours investigating the peripheral components of the instructional program to utilize activities that engage her students. Although not substantiated by classroom visits, Kelly's conversation did not refute the conjecture that she instructed in a traditional format, with a daily routine of homework discussion, teacher explanation of new material, and student practice. She described the lesson,

Kelly: So we always, you know, I do, we do some together, and they do some, by themselves, and I circulate and see how they're doing, and answer their questions.

Her questions about the new materials pertained to the online interface, the answer key, and practice problems. She did not discuss the games, the problem of the day, or the literature in the centers kit. Kelly saw the current textbook as a tool she used as she did the former series.

Her second challenge has been working with the word problems and her students. An example is the story about band shirts that was described at the end of the previous section. One intention of the NGSSS is to make mathematics more contextual and more relevant to the students' lives. Kelly's attention has been directed toward the problems students have reading the words as individuals.

Kelly: For instance, one question talked about a baseball team and instead of just saying a baseball team they used an elaborate name for the team that many of my students could not read.

She did not acknowledge the importance of integrating mathematics with real life situations. In other words, the students were not taught to consider the mathematizing of life that is presented in the story, but only to look for the underlying calculations to be made. She journaled, “I have found myself wishing that they added/taken away some of the detail within the word problem to make it a little easier to understand.” For Kelly, story problems were important because of the tests to come. She added, “I looked at Chapter One’s test. They’re all reading problems. They have to be able to *read* to do the math.”

Kelly worried about a student who “can get a negative attitude about her ability in math very quickly. If she does not understand something the first time it is explained, she often will shut down.” This challenge was confounded because of Kelly’s own lack of confidence in her ability to teach mathematics. Initially, she was happy because the new series would be aligned with the new standards. She said in the first interview, “Actually, *using it*, I see the potential of how well it could possibly be used later on, but right now it’s hard. It’s hard.” The students were having a hard time, and so was the teacher. The publisher’s training workshop disappointed her, describing the day as, “’Look at it, it’s pretty! It’s new!’...And, so, that was pretty much the gist of it.” She said she needed a longer training. “I need more time just to kinda play with it, I know the kids are gonna have trouble here, you know, they didn't’ get this last year.” Teachers like Kelly would profit from a substantive training in the program.

Kelly is an earnest and enthusiastic young teacher at the beginning of her career. She truly wanted the students to learn mathematics well and to enjoy it. During her two years of teaching fifth grade, she had been asked to teach two different curricula with a minimum of support within the content area. Although many of her instructional and assessment practices evidenced good pedagogy, her opportunities for guided reflection on her practice were not apparent to the researcher. Some of her techniques could have been used to better advantage, and some of the tools she used too much. There was little opportunity for the fifth grade level team to collaborate about the mathematics lessons they were teaching, but such a measure would have been welcome to Kelly, James, and Sandra.

Summary of Findings

This chapter has presented the teaching story of the three teachers as they have told it and the researcher has witnessed it. These three individuals have very different styles, strengths, and challenges. Chapter 6 examines the mathematics instruction of the three teachers, identifying the common threads and differences with the purpose of describing the findings generated by the data. All voices from the study and the review of the literature will be joined to inform the creation of a framework for teaching mathematics in depth.

Figure 5-1. Sandra's classroom Photo courtesy of author

Figure 5-2. James's classroom Photo courtesy of author

Figure 5-3. Kelly's classroom Photo courtesy of author

CHAPTER 6
EXPLORING THE FINDINGS

Part of the purpose of this study was to develop a framework for teaching mathematics in depth by examining how fifth grade teachers conceptualize and actualize those practices. Chapter 5 presented the teaching story of the three teachers as they have told it and the researcher has witnessed it. These three individuals have very different styles, strengths, and challenges. This chapter examines the mathematics instruction of the three teachers, identifying the common threads and differences for the purpose of describing the findings generated by the data. All voices from the study and the review of the literature are joined to inform the creation of a framework for teaching mathematics in depth.

Chapter 4 described the methodology as grounded theory interpreted by situational analysis. These methods recognize that multiple social processes may characterize a particular phenomenon, and these may be even paradoxical or contradictory (Clarke, 2005). This appears to be the situation in the data collected for this study. The process of teaching mathematics in depth as experienced by a fifth-grade level team is characterized by two emerging findings. The task of this chapter is this: (1) to examine how the classroom teachers characterized teaching mathematics in depth, (2) to describe the actualization of this practice in the first year of NGSSS as it was generated by the data, (3) to join all the voices from the study to describe the educational practice of teaching mathematics in depth, and (4) to advance a framework created to develop this practice as it emerged from the synthesis of the review of the research literature and the data.

Conceptualization of Teaching Mathematics in Depth (TMinD)

The stories of the three fifth-grade teachers hold many common threads, but they are not identical. Although the teachers each use the same instructional materials, follow the same standards, and prepare students for the same examinations, the daily events differ in each room. Analysis revealed that these differences could be profitably considered in relation to the teacher's orientation toward learning mathematics. This orientation influences their conceptualization of teaching mathematics in depth and affects their actualization of the practice. This section will reveal the differences in orientation, outline the effects of an orientation, and prepare the groundwork for the implications for practice that will be outlined in Chapter 7.

Sandra, James, and Kelly all organize instruction toward their goal of student understanding of mathematics. They use their training and their resources to support student knowledge of the Big Ideas mandated by the NGSSS for mathematics. All of the teachers observe the dictates of the standards in terms of the accountability expected on the state examinations. As they are instructing, they employ many means to assess the extent of student learning, knowing they need to balance the requirements of the textbook with the needs of the students to be successful on achievement tests. Each teacher wants the student to know why a mathematical truth is so, to employ many methods for finding solutions, and to find the correct answer to a problem. The teachers know that an accomplished student will be able to explain not only how the answer was found, but also what the answer means mathematically. Sandra, James, and Kelly accept student confidence and engagement as indications that the students are learning and ready to move to new challenges.

That being said, each of the fifth grade teachers chooses a different classroom organization and offers different instructional activities to pursue their instructional goals. From the bevy of suggestions in each chapter, teachers choose those that they believe are most effective to engage the learner and maximize the learning of mathematics. The differences fall into these areas: (1) content, (2) authority, (3) student role, (4) grouping, and (5) learning. Writing the case histories of each teacher revealed more clearly the orientation toward learning that was noticed by the researcher during data collection and confirmed by data analysis. The case will be made that the orientation toward learning undergirds the other categories of differences.

Content Emphasis

As was said earlier, all of the fifth grade teachers taught the lessons from the textbook, as they were presented. Initially, each teacher tried to include every exercise, but as the term continued, they began to select activities, partly due to time constraints. What each teacher selected was slightly different. Kelly chose the problems that emphasized the computation process. These exercises met the criteria drawn from her definition of the essential nature of mathematics. Kelly believes that mathematics means numbers. To her, the story problems have less value in this world of mathematics. In fact, they could be confusing to the children and limit the time the class has for completing the computation.

James was surprised by the detail of the new curriculum based on the NGSSS. He had expected that there would be more time to teach division as he had taught it in the past, which included adjusting the quotient and using estimation. He was not ready for the detail of multiple methods in addition to the standard algorithm. Later in the term James did omit some of the problems in the textbook lessons. He said, “I’m skipping over a bunch of problems here and there because I see that the training that we gave them earlier has kind of taken effect.” His standard for determination of which problems to use was the success of his students. If they understood the strategy, he limited further practice of that strategy.

Sandra evaluated the lesson exercises using criteria drawn from her training in inquiry pedagogy and her experience of many years teaching division. If the mathematics was redundant, she omitted it. If the contextual problem included unnecessary information, she might strike those words from the paragraph during class instruction. Her teaching style consisted primarily of her summarization of the mathematics rather than encouraging the students to read through the textbook explanation alone or in small groups. As she said in an interview, "I mean you have all these pages, you have like six pages…five pages per lesson, with all this little step stuff. All I want is the meat." On the other hand, when she thought the textbook was lacking, she incorporated her own exercises to instruct the students more deeply, as happened when the lesson was the order of operations. The activities in the lesson were exercises in addition and subtraction. Sandra added exercises that included parentheses and multiplication, which are in the standards for grade 5, but were not part of this lesson in the textbook. The stimulus for Sandra's choices is the mathematics of the topic. Her professional judgment was that the students were ready to explore the extension of the order of operations into these areas at this time, a time earlier than that anticipated by the curriculum developers.

Authority

The paragraphs about content give hints about the teacher's view on mathematical authority. All of the fifth-grade teachers accept the textbook as the authority on the scope and sequence of the curriculum and the subsequent state examinations. However, there are differences in practice. Kelly was anxious if she did not have an answer for each question in the textbook. She wanted each student to have the correct answers in their book before they took it home. Sandra reviewed the lesson thoroughly before beginning class to be certain that she understood each of the mathematical steps in the student text before she presented it to them. James used the small group sessions to examine the class work. Until faced with a student's lack of understanding, he taught the lesson with the assumption that the work was presented clearly in the textbook.

Student Role

As was discussed in Chapter 3, the underlying theoretical perspective for mathematics education taught as suggested by the *Standards* is generally agreed to be social constructivism, acknowledging the participation of the individual in the creation of meaningful mathematical knowledge while recognizing the cultural contribution to the discipline that is mathematics. The research that supports the new standards and instructional materials assumes that the students will be constructing their own understanding of mathematical truth. In the fifth grade at Forest Glen School, the researcher collected limited evidence that this position guided instruction. Of the three teachers, only James was observed to elicit student discussion of their ideas. When that discussion was not forthcoming, he pursued the question by asking pairs of students to discuss it and report back to the group. His students were expected to solve "Find the error" problems and to review each other's work. Asking students to check their work was an opportunity for the teacher to encourage justification of a mathematical solution. However, it was not used as such in the other two rooms. Kelly encouraged her students to check their own work to find the errors, and she did use this approach in whole-class discussions to itemize the correct steps in the procedure, but not to ascertain the rationale for the strategy. When Sandra was directly asked by the researcher why she did not demand compliance to problems that said, "Check your work," she responded, "I'm not big on that." Generally, Sandra noted student errors and used those as a reason to review the procedures of the calculation.

Divergent thinking was a random occurrence during mathematics lessons. However, two examples come from the researcher's experiences in Sandra's lessons. In the first, Sandra was asking the students to read the problem and decide which word determined the best operation to solve the problem. The problem outlined a comparison situation that could be solved using subtraction. When a female student offered the word "taller" she was told she was wrong. The textbook had suggested “difference,” and Sandra told the class that this was the only correct solution. The second incident occurred just before Thanksgiving. The students were asked to answer some questions that related to a shopping list for the turkey dinner. One of the questions asked, “What is the second cheapest item on the list?” A pair of students noted that although the price for a turkey was $0.59 a pound, it would be difficult to buy only one pound of turkey, so turkey was not the lowest-priced item on the list. Sandra accepted this reasoning, but only after the researcher defended the students’ case.

Chapter 5 related a textbook problem mentioned by Kelly about a band fund-raiser selling shirts where Kelly concluded that, although the discussion was interesting, it subtracted time from the real business of mathematics, trying to solve the problem. For another example, James used the initial opening problem for Big Idea 1 as a class project. However, instead of giving the problem to the students to brainstorm and solve, he led an initiation-response-evaluation (IRE) class lesson on the situation and changed the problem into eleven separate division problems. He explained later that he anticipated student confusion over potential fractional results.

Grouping

Flexible grouping strategies are emphasized in this school district. The principal at Forest Glen School is interested in the teachers’ use of these practices. The default grouping structure in James’s room is small groups. The membership of the groups is reexamined daily and may be changed depending on which students are absent that day, but James considers how the group members get along with each other. His goal is to maximize student socialization—do the group members support and teach each other? Instruction in mathematics and science is primarily delivered to small groups. Sandra changes the seating chart in the room frequently, and she will also rearrange the furniture. Since the classroom has desks instead of tables, there are more possibilities. Since she values the need for student attention to mathematics, her criteria for arrangement is specific to limiting distractions for the students. Instruction is delivered to the entire class at once, but practice and inquiry activities might be conducted by small groups of students. As Kelly self-reports, she uses pairs, centers, and small groups as well as whole-group instruction. She uses a multi-level grouping structure that rotates students into new groups regularly, but she is careful to place the students according to their ability level, with every group assured of the same balance between skilled and unskilled. These seats are the students for a period of about a month, and they are the same for each subject during the day.

Fundamental to the teaching process is a conception of how students learn, since teaching is directed toward the initiation of learning. This study observed a difference between the teachers as they selected instructional practices. Effective instructional practices in mathematics are not easily disaggregated into separate divisions, but the disciplines of mathematics, psychology, and sociology have contributed to the identification of different, yet successful, paths to learning mathematics with understanding. As teachers have the goal of maximizing understanding, they seek the most efficient methods to achieve that goal. Each of the teachers in this study oriented toward different instructional practices because they believed that these activities increased students' learning outcomes. Analysis of the data revealed that these differences are directly related to the teacher's orientation toward learning mathematics. This orientation influences their conceptualization of teaching mathematics in depth and affects their actualization of the practice.

Orientation to Learning

The review of the literature in Chapter 3 presented the results of many studies that examined beneficial instructional activities that increase students' understanding of mathematics. This section loosely groups them into three categories according to their orientation toward cognition, sociocultural theory, or mathematics (Figure 6-1). There is considerable overlap between the three, such as the role of discourse in learning. Many theories of learning acknowledge both that learners actively construct knowledge through interaction with their surroundings and experiences, and that learners interpret these occurrences based on their prior knowledge and their rendering of their observations and actions (Campbell, 1996). Sociocultural theory emphasizes communities of practice (Forman, 2003) and forwards peer instruction to move learners from where they are to where they can be (van Oers, 1996). Cognitive theorists value the physical action of conversation and writing as helpful learning mechanisms (Ormrod, 2008). Mathematicians acknowledge the central role of explanation and justification as one of the processes of the discipline (Clarke, 1997; Nelson, 2001; Putnam et al., 1989; Truxaw & DeFranco, 2008).

For purposes of this chapter, these are the characteristics that compose an operational definition of these orientations. Drawing on the work of Vygotsky, sociocultural learning theory posits that each child has a ZPD wherein maximized learning will take place. Part of the teacher's role is to identify the ZPD and then create appropriate instructional opportunities to move the student to greater understanding. Furthermore, individual learning is dependent on social interaction. Meaningful learning is dependent on the pupils' opportunity to evaluate their own insights and ideas in critical comparison with culturally available concepts. An effective teacher need not be an adult, but can also be a more learned peer (Forman, 2003; van Oers, 1996). Teachers with leanings toward a sociocultural learning perspective incorporate learning communities and peer- and self-assessment into their practice as an essential component of instruction.

Cognitive theorists, drawing on the work of Piaget, emphasize structure and relationships among various kinds of knowledge. Their instructional activities assist students to build internal models and representations (Putnam et al., 1989). Teachers choose challenging questions and carefully organize the information into learning tasks that are essential to the knowledge and skills that must be mastered (Bennett et al., 2004). Teachers discover what prior knowledge the student brings to the learning task and use that information to determine when and how to lead students to more complex and efficient understanding (Carpenter & Fennema, 1991). Teachers who orient toward a cognitive perspective of learning are careful to focus student attention, minimize distractions, and monitor student behaviors. They bring variety to the lesson, ask questions, and organize the information so the students can absorb it in small, achievable pieces (Ormrod, 2008).

A learning perspective that orients towards the discipline of mathematics values students performing mathematical tasks both correctly and incorrectly to analyze the mathematical content in the task. Activities that mathematicians use to do this include problem solving, mathematizing, and mathematical argument (Putnam et al., 1989). Understanding mathematics requires building strong webs of connection in the brain—making an idea part of an internal network (Hiebert & Carpenter, 1992). Teachers in these classrooms train their students to not only express their mathematical ideas, but to listen to each other explain their thinking. Teachers do not necessarily inhabit the student's mathematics world, but the teacher must know what the student has in that world (Wood, Nelson, & Warfield, 2001). Successful use of multiple representations indicates mathematical understanding, but teachers need to monitor and develop this capacity in their students (Pyke, 2003). Teachers oriented toward mathematics learning perspective employ complex tasks that encourage high-level thinking and maintain student engagement in those tasks (Henningsen & Stein, 1997).

The teachers in the fifth grade employed instructional strategies from diverse perspectives toward learning, but it will be seen that each gravitated toward particular types of interventions, especially when confronted by challenges. “The sociological, psychological, and mathematical perspectives suggest that teachers attend to quite different aspects of children's mathematical learning” (Nelson, 2001, p. 257). This section of the manuscript is not intended to anchor a teacher within a specific school of learning theory, but to suggest an orientation of the teacher as evidenced by their words and actions in the mathematics classroom.

Sandra’s practice: orientation toward a cognitive learning perspective

When she talks about learning in mathematics, it is important to Sandra that she organizes the mathematics so the students can construct a representational model in their minds. She is considering her students’ achievement as she plans to use the textbook materials to meet their needs. Summing up the process,

Sandra: Going through that first couple of lessons, there’s *so* much, and it felt like *canned* teaching to me. . . . I have to be thinking it through *my* way, and I have to find *my* rhythm with it.

Her instructional practice reflects current brain research about motivation, memorization, the use of all modalities, and the development of student independence by giving them responsibility for their own learning. Her classroom routines encourage practice, self-discipline, and accuracy in the results. She values inquiry instruction and problem solving for the potential to create understanding. Sandra felt that a lesson was successful because, “They’re more challenged and excited to *try* to find the answer than they were before.” Although the class was generally taught the same lesson at the same time, individuals did work on assignments that were specific for their needs. Sandra evaluates her practice with an orientation toward a cognitive perspective (Ormrod, 2008; Putnam et al., 1989).

James's practice: orientation toward a sociocultural perspective

James's classroom is organized around student groups. This profile is supported by the selection of furniture—tables—to the use of the bookshelves where the resources are grouped not by individual ownership, but by subject matter. As the books are needed for instruction, individuals are assigned the duty of getting the materials the group needs. Before dismissal to lunch, the responsibility of returning the equipment might fall on other individuals. However, the group waits until the tasks have been completed, encouraging the development of student roles for the room's cleanliness. James cultivates a classroom ethic of helping each other, both within the small learning groups and within the entire class as monitors are asked to "bring [the absent classmate] up to speed."

Even though James would share instructional duties with students and adult volunteers, he assumes responsibility for the learning the student has achieved. On occasion, he does not lead a group, but uses the group time to hold individual conferences. This is especially important for him after a test. He wants to be sure that the student does understand. Additionally, James uses many tools to ascertain an individual's position, like asking them, "If you agree, put your colored pencil on the end of your nose." Within the small instructional group, he goes to great lengths to assure that every student participates. These conversations provide formative assessment for him to determine if the students understand what he has been teaching or if he needs to reteach them. Student discussion is a priority for his work, and to generate such conversation, he might refer to the difficulties other students had or he may intentionally make mistakes to check for student learning. James assumes that a confident student who proceeds quickly into an assignment is one who has learned the lesson. James orients towards a sociocultural learning perspective.

Kelly's practice: ambiguous orientation

As a novice teacher, Kelly is still shaping her orientation. Since the researcher was not a volunteer in her classroom, the information about her orientation is based on self-reports. She values the importance of mathematics, especially for her students' future work, and she wants to assure that her students learn the correct procedures, understand what they are doing, and get the right answer. She defines instruction as explaining the process and assigning appropriate practice exercises. As teacher, she is vigilant over her students' work, checking that they are calculating correctly. She is always ready to help them, but is dismayed that she does not always know that they need help. She uses a variety of grouping strategies, follows the guidance of the textbook, and incorporates supplemental materials offered from the publisher's website. It is important to her that the students score well on the standardized tests. Kelly works hard to be an effective instructor of mathematics, and she is teaching the new curriculum in a traditional way. She looks to professional development and experienced colleagues to direct her when she wants more guidance.

The inevitability of the influence of such orientation

The data reveal that the primary concern of the participating teachers is that the students understand the mathematics they have taught. When assessment results are available, the teachers reteach what has not yet been learned. However, the teachers return to the same pedagogies. They believe that these pedagogies are the best suited for their purposes. This argument has further credence when one considers the challenges acknowledged by each teacher. When formative assessment indicates that reteaching is required, Sandra would reorganize the information because she leans towards a cognitive perspective. James would ask the students more closely to explain what they understand about the mathematics, since he gravitates to a sociocultural perspective. Kelly would explain the procedure another time, since she understands modeling to be the foundation of teaching mathematics.

Instructional challenges

Sandra, who orients towards a cognitive perspective, noted three problem areas when discussing her challenging students. All three areas could be addressed with pedagogies offered within sociocultural perspectives. She evinced a desire to create a learning community, to promote student independence and inquiry, and to foster a classroom where mutual respect is apparent. The new curricular materials suggest pedagogies that would contribute to these instructional goals. Some examples are rich initial real-life problems that were designed to be considered in the spirit of inquiry, pair/share activities, and classroom discussions where divergent student thinking is offered and accepted. Sandra's instructional stance directs her task selection instead to activities that are teacher-centered and convergent around accepted mathematical strategies and solutions. Although she would describe mathematics taught in depth as students understanding the "math way" of solving problems, she offers few opportunities for the students to consider and discuss that "way." She recognizes that her instruction will change, now that there is more time to offer students these opportunities for exploration and thought. She has had success making changes this term, and she speaks of doing it more frequently. She also recognizes the potential help of a district specialist in cooperative learning practices and has called upon that specialist for assistance. These findings offer implications for future practice, which is discussed in Chapter 7, when the challenges for all teachers are reviewed.

James mentioned three challenges, and each of them relates to the perspective offered by the discipline of mathematics. He would like to share more planning sessions with his teammates, because he would like to share specific experiences from teaching the mathematics content. He is disturbed because of the content knowledge gap created by this year of transition from one curriculum to the next. James would like to have more specific assistance to bridge the gap, seeing the need of teaching two years' curriculum when only one is in the textbook. For example, he needs to teach multiplication strategies that were not mastered while the students were in fourth grade the year before at the same time he is teaching the required Big Idea of division. His third area of challenge was motivating students to love learning mathematics and to feel more confident about their skill in the subject. At the same time, as the researcher observed his practice, she often wondered if James taught mathematics differently than he taught reading or science; it appeared that each subject was taught by the same methods. More than once in the interviews, he associated teaching mathematics in depth with detail, detail that surprised him because he had not taught those strategies before. Were James to have access to professional development that emphasized the contribution of mathematics to learning in the elementary classroom, he might be equipped to deal more effectively with these challenges.

Kelly's view of mathematics is not readily described, since the researcher did not have access to the mathematics lessons. Her responses to the interviews indicate that in her practice the instructor models the problem and strictly follows the textbook. The children are expected to be compliant (Ernest, 1989). Yet, Kelly would like the students to be excited and enthusiastic about mathematics. She is challenged by her ELLs' difficulties with story problems and the written word. She would like to help her students be more confident about their ability in mathematics. Kelly's orientation toward a learning perspective is undefined in this study. She uses pedagogies from each area, but in a tentative way that seems uninformed of the conclusions from learning research. The challenges she identifies could be met with practices from mathematics, to address the confidence issues and the story problem issues, as well as sociocultural practices that would help her address the problems of the ELLs and generate excitement as well.

Summary of the Conceptualization of TMinD: Answering the First Research Question

The first finding generated by the data collected for this study is that the orientation of each teacher toward learning and teaching was the largest factor influencing how he/she defined and used teaching mathematics in depth. This could be seen as the answer to the first research question. Generally, the teachers saw TMinD as teaching for understanding, using tools they believed would promote learning. When asked directly for a definition of teaching mathematics in depth, the teachers struggled to express their thoughts, but this was their first year of implementation for a very different set of mathematics standards. They expressed confidence that the textbook was providing for them instructional activities that would enable them to commence TMinD. Chapter 5 presented descriptions of the mathematics instruction occurring in the classrooms of each participant. The three teachers viewed teaching and learning from a different theoretical perspective that leaned toward pedagogies informed by cognitive theory, sociocultural theory, or mathematics. When their assessment results indicated that reteaching was necessary, they selected the pedagogies from the same categories. When faced with challenging situations, the teacher's problems may have been solved more effectively if they were to orient their choices to a different perspective.

Actualization of Teaching Mathematics in Depth

The second finding generated by the data collected for this study is that instruction is actualized in elementary school mathematics by the sometimes conflicting demands of textbook and the accompanying pacing guide on the one hand, and the desire to teach mathematics for mastery on the other. Within the review of the literature the term mastery was mentioned less frequently than "understanding" or "proficiency" in mathematics. The exception was in the reports of evaluation of the U.S. mathematics program that addressed the practice of teaching for exposure, where understanding was not expected (Lindquist, 1989; MAA Online, 2004; Porter, 1989). The SSS was criticized for placing levels of cognitive demand on the students that were too low (College Board, 2005). At the same time, NGSSS anticipates that the topics in the standards will be taught for mastery by the students (FLDOE, 2008; Schoen & Clark, 2007: Vaccari, 2009). The term mastery emerged from the data collected in the teacher interviews. Teachers want to assure student understanding and mastery of the mathematics. Both summative and formative assessment inform the teacher of students' comprehension. There is overwhelming pressure on the teachers to sustain the drive for mastery even as they are following the pacing guide. Figure 6-2 diagrams this relationship.

The Pacing Guide

Instruction in the classroom is driven by the textbook and the pacing guide. Generally, the pacing guide documents are created by school district leaders to help the teachers keep instruction on track, being cognizant of the objectives of the mathematics standards or standards as well as the expectations of the standardized examinations administered at the end of the school year. Additionally, these guides have the goal of ensuring curricular continuity across the schools in the district, a purpose previously filled by scope and sequence documents. The pacing guides establish period deadlines to help teachers teach all the expected topics in a timely manner. These are sometimes accompanied by quarterly, or more frequent, benchmark assessments. Some guides specify the number of days, minutes, or instructional hours teachers should devote to specific topics; the best guides emphasize curriculum guidance instead of prescriptive pacing (David, 2008). These guides are usually prepared according to the state department of education standards, the objectives for the standardized examinations, and the textbooks used in the district (Cleveland City Schools, 2004; Sheridan School District, 2011).

The pacing guide for the teachers in this study was written by a committee of teachers in the school district. It offered a suggested date for the completion of specific chapters in the text and a window for completion of the corresponding examination for each Big Idea. The committee created the pacing guide based on the information from the publisher about the time needed to complete each chapter, but these guidelines were then coordinated with the calendar published by the district office for district exams and the state high-stakes examination.

The pacing guide is a product with direct influence over the actions of the teachers in the classroom, but its influence is derived from many political sources (Figure 6-3). Strong forces are at work to influence teaching mathematics in depth under the banner of the pacing guide. Drawing this figure helped the researcher analyze the relationships between the groups. The size of each shape is relevant to the influence of the political body, but the diagram is not drawn to scale; it is an analysis tool. For this study, authority for the pacing guide committee derived from the county school board, under state law administered by the state department of education, which, in turn, oversees the high-stakes examination, an assessment that is strongly influenced by NCLB. Additionally, since the state has been accepted into the Race to the Top federal program, the school district is influenced by the proposed common core state standards (CCSS).

The teachers accepted this guide as a suggested time frame, but it did make them nervous.

James: I'm getting a feeling inside myself right now…I'm not panicky, but I feel *rushed*. I want to make sure I can get to the point where I feel my kids are going to do well on the Big Idea test.

Kelly: Do they consider interruptions like picture day and the Veterans' Day school assembly when the calendar was made? I understand that this is where we're supposed to be, but it's a pipedream.

Sandra appreciated the pacing guide, because it took the guesswork out of her planning. Sometimes she found that her class was ahead of the guide, and she could focus on other subjects a little more. She recognized the work of the planning committee.

Sandra: I am grateful that they have *tried* in the county to take some of that legwork *out* for us…I feel like we should be using it. However, *if* other schools are meeting the pacing guide, my [district test] scores won't be as good…as those kids who *are* at the pacing guide.

Teachers do not want to handicap their students, or the school's ranking, by skipping topics (David, 2008).

As the fifth grade level team began finishing the Big Idea 1, the discussion turned to the Big Idea test. There was great concern whether each class of students would succeed in answering the test questions correctly. Kelly was concerned because the team leader, teacher of the gifted students, had mentioned problems "with some of the wording of it." Word problems form the basic structure of the high stakes test, and the fact that the preliminary chapter tests were in the same format cheered the teachers. The teachers were hopeful that after administering six chapter tests, the students would be prepared for these test items.

Teacher concerns at the onset of the new curriculum were centered on whether the new curriculum would align well with the state high states examination. They were hopeful, since this is the promise that was made when the new textbook series was adopted, but this dilemma would be on their mind until the high stakes tests were taken and passed by the students. Test preparation has an enormous influence on instruction. Teachers have these questions in their mind. Do the mid-chapter tests properly preview the chapter test? Is the chapter review a good indicator of lessons that have not been completely learned? Should one give a test that looks like the homework or one that looks like the state examination? These thoughts hover during teacher planning and preparation, but one that overrides all the others is, "Are the students ready to take a test?"

Understanding

A profound hope for the new curriculum is that "they actually gain mastery over it [mathematics]." "Do the students understand?" is primary goal of teacher planning and lesson implementation. The question is asked because teachers are most concerned with the extent of the student's understanding. If students do not understand, would they be ready to take a test? Teachers would rather not give the test until they are assured that the students understand the mathematics.

When Kelly began to teach the series, she was frustrated using the new textbook. In the past, students had been taught to find the answer in the quickest, easiest way and now they were asked to learn multiple ways of calculation, to apply those methods to real-life situations, and to determine *why* the strategies work. Although she believed that her students were handicapped by both inadequate preparation in multiplication and English language deficiencies, she continued, "…truckin' along." She continued, "We're doing pretty good. We're pushing it. But, it's like pulling teeth, these lessons." Students did not like the lessons at first. "Why don't you just divide?" was a common question. She persevered and was pleased to hear some say, "Oh, that's real easy." Her satisfaction came when "we had a few light bulbs, a few more light bulbs come on for that [division]."

Initially, James appreciated that "the textbook really, really goes into depth with all the kids." Three months later he remarked,

James: Just the depth can be a little bit daunting for now . . . because I've never gone into this much detail . . . about division. In our old series, it was *so* hard getting through word problems, and I feel like it's a little easier going through a word problem with this, but part of that is because, like this forces me to spend so much time on division with them.

James felt like this was all time well spent, "If this concept is coming across *thoroughly* for them."

As Sandra taught her lessons, she checked her students' understanding, "It was easy to see on the white boards that everybody had the answers, and they were writing them." She acknowledged that she could have changed a few words to assist them, "But they're getting it. They have a really good understanding of these equations. I'm pretty pleased with them on that." She monitored her forward progress by checking with her students, wondering if she could "move on" or if she had "time to go back." "Do I go up there and rework . . . step back a bit? Can I push them forward?" Sandra wanted them "to be excited for each *other,* and excited about their *own* learning."

These teachers depended on student understanding as a signal that they could introduce new material. Student understanding was characterized in many ways. Sandra was satisfied that her students understood when they not only found the numerical answer to a division problem, but they also knew what that answer and what that remainder meant, and they could explain it. She assumed that if students understood, they were *eager* to solve problems and to demonstrate their skill. Instead of wondering *why* they had to solve the problems, they simply did it. Students read, strategized, and solved *independently*; they learned what resources they needed to accomplish division, and they divided. And they got the right answer.

James saw understanding as the point where students could apply their prior knowledge to a new situation and successfully transfer it. Student understanding also implied that the students would notice errors and identify faulty answers. Students understood when they "owned" a method and used it by choice. For James, understanding the word problems was significant, since in this series, "the word problems are *tough*." However, some of the methods students regularly used were extended procedures such as adding several numbers to avoid multiplying them. The problem then is, “There’s too many opportunities for an error to occur.” He would like to see students attend to the details in long multiplication and division problems.

Kelly thought that there were areas that her students would not understand until she went through every example with them. She believed that practice and knowledge of [basic] multiplication facts would increase understanding. Solving many types of problems and learning to identify patterns would benefit the students.

> Kelly: that’s good, just in their life, to understand that there might not be just *one* way to do it. I like the idea of showing them a bunch of different ways.

She anticipated this would lead to better understanding. To understand, for Kelly, means to work a problem quickly and talk about it, being able to tell the teacher not only the process, but also why each step is done. She carried grave concerns about her many English language learners (ELL) since it was difficult to determine if the lack of understanding was an issue in mathematics or in literacy. In another room, Sandra discovered that when the story problems were read aloud, one of her ELL students got each of the story problems right.

Assessment

As was detailed in Chapter 5, student understanding was observed by the teachers in many ways. They observed student body language that indicated the students knew what the teacher was saying. Teachers regarded correct answers, seen on white boards and student papers or spoken in discussion, as evidence of understanding. Students who could explain their thinking and apply it to new situations demonstrated understanding in the opinion of their teachers. When students were confident about attempting new assignments, the teacher was satisfied that they had understood the instruction. Teachers assumed that the student who responded negatively or passively to teacher directions about the next activity did not understand how to do the work. These methods are all types of formative assessment, and the fifth grade teachers used them regularly. As will be discussed in this section, research on learning and mathematics learning specifically denotes assessment processes that would prove to be efficient means of improving performances. The teachers already demonstrate use of some of them, but could rely on them more.

Classroom practices of formative assessment

Classroom discussion is an activity with a dual benefit: (1) it can reveal much to the teachers about student thinking, and (2) it is a task that demands action on the part of the learner (Nicol & Macfarlane-Dick, 2006; Wilson & Kenney, 2003). Teachers can use classroom discussion to provoke thoughtful responses to questions in several ways, such as increasing wait time after they ask, assigning pairs or small groups to discuss a question and then report to the class, or asking the class to vote for the best of several possibilities (Black & Wiliam, 1998a). Although the fifth grade level team does seek student participation in discussion, this technique could be exploited for greater value as a tool of formative assessment.

Kelly checked for understanding by using paired work. "When they are able to work with a partner and complete a set of problems. I am able to see how well they understand what we are learning." She also understood the active stance that students assume when they talked about their learning. "There is great 'math talk' going on when students are helping others that are struggling." Kelly's strategy of using pairs to help each other worked well because her instruction was largely procedural. The pairs were not so much strategizing as they were monitoring use of the accepted algorithmic steps. A coach or mentor could encourage Kelly to use this same activity with more open-ended tasks and the information gleaned in this formative assessment would benefit the students' learning at the same time it informed the teacher as the to students' level of understanding.

The formative assessment tool of writing is one that the fifth grade team was beginning to use. The students were writing more than was asked of them, but the teachers could raise their expectations of minimum response to use this tool more completely as indication of student understanding. Admittedly, the students were unused to explaining how they knew that they were right. Many had the habit of waiting until another student made the response, and then they used that response as well. After the teachers become convinced of the value of these analytic tools, they will have to instruct the pupils in their use, as well. Using the white board and dry-erase pens, seated with a small group and an adult, the students were willing to offer their written contributions. It is recognized as a beneficial activity, and it is employed, but it is currently undervalued.

One challenge for the teachers in this fifth grade was to extract from the students thoughtful answers to the questions they faced. When one student was asked why she got the wrong answer to a practice item, she said, "Oh, I didn't even read it. I just used the numbers." During the researcher's conversation with a student about the remainder of two students in a school bus problem, the boy explained, "Oh, they wouldn't be able to go on the field trip. They'd have to stay home." Looking carefully at problems and examining all the information was not yet a common practice for the students.

Feedback is a term with multiple dimensions. Describing a good performance for a student is a valuable tool for formative assessment. Good feedback is more than simply identifying a given answer as wrong. This information does not provide useful feedback since it does not inform a better result. Positive feedback is another assessment tool that was used well at times, but could become more beneficial. James worked closely with his students as they made errors. He held personal interviews about important assignments, particularly summative examinations, to review with the student their strengths and weaknesses. In this session, he often retaught a misunderstood concept. When Sandra gave the students a small-group inquiry project, she moved from group to group, listening to the students, probing their responses, and suggesting further investigation. The aforementioned teaching practices were already part of the teachers' repertoire, but as yet are underused and undervalued tools of formative assessment. The information freely shared by the student informs the teacher who responds by changing instruction to meet the needs they have discovered.

There is a potential weakness in assessment when the teachers take assurance from a numerical answer to a calculating problem. A study conducted by Reys and Yang (1998) compared computational performance and number sense among sixth- and eighth-grade mathematics students in Taiwan by administering two parallel examinations that measured each variable with a similar mathematics problem. A sample of the seventeen participating students, selected to represent high, low, and middle achievement, was interviewed to explore characteristics of number sense. The researchers discovered, "Correct answers are not a safe indicator of good thinking. Teachers must examine more than answers and must demand from students more than answers," (p. 235). The fifth grade teachers in this study rely heavily on correct answers for their formative assessment. Although this is helpful, its relative value would assign it a place of far less importance.

Self-assessment and peer-assessment are two more tools that could be beneficial forms of assessment but are currently little used in these classrooms. Professional development could inform the teachers of the value of these practices. Sandra's viewpoint on self-assessment was mentioned in the last section during a discussion of checking one's work. She does not see the greater value of building independence as the students make this action part of their routine. A similar result occurs because James asks the students to work at the same pace by staying on the same item when they are in a small group. The result is not independence, but a dependence on others to verify one's answer. Instead of being in charge of their own accuracy, the students wait for the teacher to utter the pronouncement of accuracy. The potential for these small groups to share strategies is not yet tapped.

Summative assessment

Summative assessment guides instruction of mathematics for these fifth-grade teachers in two ways (Figure 6-2). On one hand, summative assessment instruments pair with the publisher's calendar to inform the pacing guide. On the other hand, the results of these examinations influence instruction. Findings reveal the influence of the state examinations on the teacher's work of teaching mathematics. Teachers want to know that they are teaching what the standards ask the students to know and that their students will do well on the end-of-the year examinations. Student scores are seen as a personal judgment on the teacher. Scores matter to these teachers because they like to know that they are indeed teaching their pupils what they need to know.

Since the school district also has a vested interest in student scores on the state examinations, district administrators have implemented district tracking tests and chapter tests. These tests are not normed, but the teachers have access to the relative standing of their students as compared to other students in the school district. James referred to these tests when he talked about doing item analysis and remediation in Chapter 5. Although James was the only participant to explicitly mention remediation after examination of the exam, the test results influence lesson planning. The avowed purpose of the district examinations was to inform instruction. The grade level teams had been given the assignment of creating focus lessons to address the weaknesses uncovered in those examinations at the beginning of the year. These focus lessons were to be used every two weeks for the duration of the school year, after the topic was introduced in class.

James: There's a lot of time for assessments, that we have to take, whereas, four years ago, three years ago, before On Track, we would have had more time…you know, teaching.

Tests inform the tension between pacing guide and mastery (Figure 6-4). Kelly reported that the test scores saddened her.

Kelly: Many had made silly mistakes and missed ones that I knew they had the skills to find the answer correctly. They want us to stay as close to the curriculum guide as possible, but if I went any faster than the way I'm going right now there would be over half the class that would be lost, and I'm not OK with losing half the class with each step forward.

In her conclusion of a report on pacing guides, David (2008) says, "Teachers are caught between a rock and a hard place: Slow down and risk lack of coverage, or speed up and sacrifice depth of learning" (para. 4).

So the schedule of summative assessments contributes to the tension the teachers feel between following the pacing guide and teaching mathematics for understanding and mastery. The teachers' biggest concern is that their students are prepared well to take these examinations and that they score well. Although this desired goal is less often achieved, the On Track and Big Idea tests are used for formative purposes,.

Summary of the Actualization of TMinD: Answering the Second Research Question

The second finding generated by the data collected for this study is that instruction is impelled in elementary school mathematics by the sometimes conflicting demands of textbook and the accompanying pacing guide on the one hand (which could be considered a surrogate for the standards) and the desire to teach mathematics for mastery on the other. Actualization of TMinD depends on the students' mastery of the mathematics required by the standards. The teachers' work of mathematics instruction has as its goal student understanding of mathematics.

The accomplishment of this goal depends on information the teachers have received from formative and summative assessments. In response to this information, teachers organize the classroom activities and learning tasks needed to teach the standards. When they are assured that the students understand the mathematics, they comfortably move on to new topics. If that assurance is not forthcoming, they reteach in hopes of better results. The tempo of this cycle of assessment and instruction is greatly influenced by the demands of the textbook and the pacing guide. This external scheduling device serves the functional needs of the school district to prepare the students for the high-stakes examinations. The teachers are caught in the tension of following the guidelines yet instructing their students for mastery of the mathematics.

Teaching Mathematics In Depth

This section of the manuscript will synthesize the findings from the data collection, incorporating for the first time the results of the interviews with the peripheral participants. Teaching mathematics in depth is then explained with words from the classroom teachers, the CRTs, the IMCs, and the MTEs, as well as the researcher's field notes and other secondary data collections

Harriet (IMC at Delta School): Teaching in depth means that the children understand what they're doing—not just the algorithm, not just the method, but understand why they're doing something. So I just think that any good teacher in the past was [teaching in] depth.

The review of the literature forced a similar conclusion about practices that could be used to define teaching mathematics in depth. Since the terminology was not described in the literature, the information regarding effective instructional practices that promoted mathematical understanding informed the search for these teaching practices. The discussion in Chapter 3 relied on such a definition when the effective practices were summarized.

The two participating CRTs had nominated teachers they believed were exemplars of the practice of teaching mathematics in depth. The defining practices could easily be considered simply as good teaching practices. The reasons for selection were that the teachers used resources well, planned effectively, and managed the classroom skillfully. The students in these classrooms were engaged in their lessons and scored well on the achievement tests. The teachers were recommended because they followed the pacing guide and generally cooperated with the CRT. When defining teaching mathematics in depth,

> CRT at Hickory School: I think it's having a concept…and having a thorough *understanding* of it so you can apply it in many different situations, not just that I can do it *here*, but if it's presented differently, or I have to apply a little bit different format, [but] I can *do* it.

Identifying teaching mathematics in depth as teaching for understanding is a common tendency in the field.

The Significance of "In Depth"

The quantity of benchmarks is significantly reduced, according to Sandra, reducing the subjects "we need to *cover*." One MTE pointed out that not only are there fewer benchmarks to teach, they are more specific and focused. The IMC at Redwood School, Randi, saw this reduction in scope as a benefit for the teacher, especially the teachers who lacked confidence in their own knowledge of mathematics. Randi said, "One thing I've noticed is that . . . I haven't seen anybody going, 'I can't teach this.' 'I don't want to teach this.'" There are only three ideas for each grade level, and the curriculum provides many tools for the teacher to present the ideas.

> MTE: We have a focus . . . I think I can help these teachers realize that, whatever grade level you are asked to teach, you can be *good* at what you're being asked to teach.

One does not have to be a master of all the topics, but only a few. The curriculum is also coherent, evolving from particulars to deeper structures inherent in the discipline (Schmidt et al., 2002). For example, adding fractions depends partly on a conceptual understanding of the difference between 1/6 and 1/5, but also on the interconnection between multiplication and division. A strong understanding of these concepts is the appropriate foundation for the study of rational numbers.

Teaching mathematics in depth means teaching fewer ideas, but in much greater detail.

James: Just the detail . . . just the depth can be a little bit daunting for now."

Harriet: One of the teachers [at Delta School] reported, "When they test the children, they don't test them on one skill or two . . . they test them on *so* many skills, that the children can't remember everything."

Randi: I think that across the grade levels the first chapter kind of was an eye-opener for everybody, just because it's very *different* from what we have been doing, the depth that they want the students to understand.

Harriet: The fifth grade Big Idea for fractions is a huge chunk of math, and then they put it like, just one just one Big Idea.

At the same time, the students are learning more than they have ever learned about a topic.

James: I don't think that they can necessarily do it in their sleep. I think they can do it really well . . . they have a *very* good understanding of the process now.

Sandra believed that this curriculum prepared her students more completely than they were prepared in the past. "I haven't *felt* that way, like, 'Oh, I'm going too *fast*, or they're not *ready* for this.' They're *ready*." Each of the classroom teachers has expressed their satisfaction that their students have learned many strategies for approaching problems and finding the correct answers. What is different about the actualization of teaching mathematics in depth is the expectation that the teachers are teaching for mastery of the material, instead of a cursory "coverage." Many methods and strategies are being taught, but they are also being learned. There is great hope that the teachers will "help the child actually *master* the content before moving them out the door" said one MTE.

The final element in this discussion of the significance of "in depth" is the issue of time. While teaching mathematics in depth, there is time to do the job well, particularly with the reduced focus on skills. The teachers noticed that aspect immediately.

James: I see that this system, along with the pacing guide that the county has provided for us, is gonna let the kids *savor* certain aspects of mathematics a little longer, a little more…so that they actually gain mastery over it.

Kelly noted that she was teaching all the ways to divide, not only the quickest and easiest. Sandra appreciated that beginning the class year with division saved several weeks that had formerly been used to review addition, subtraction, and multiplication. Last year she often felt like she was "racing the clock"—or the pacing guide. And, indeed, it is still an issue.

Sandra: I don't feel as rushed this time. . . . I haven't had as much difficulty getting [the students] through the material, so I am moving faster, and I think they have a deeper understanding for it. I like the textbook a lot because . . . I have *time*, you know. . . . I'm not falling *behind.*

One of the MTEs anticipated that the practicing teacher could say, "I can spend several days developing a concept and developing it in depth." For example, the concept of a "thousand" can be intimidating and difficult for a second grader. However, with the bonus of time, the MTE continued, the teacher actually would have the luxury of leading the students in the action of bundling groups of tens into hundreds and then thousands. This activity would promote deeper understanding of the quantity than would reading about it or viewing pictures of a thousand blocks. However, one IMC was less sanguine about the abundance of time, noticing that teachers at Delta School felt that the curriculum demanded all their available time for mathematics. They were rushing to do all that was expected for learning the Big Idea.

Redwood School's IMC believed that pacing was particularly troublesome for two reasons: (1) there were knowledge gaps caused by the transition from one curriculum to another and (2) students had not been held to the status of mastery in the past. Students in the schools of the

IMCs are playing catch-up on many fronts of the mathematics curriculum. “There was no where in the pacing guide where it says, ‘Fix your multiplication issues.’” In many ways, the pacing guide can assist the new teacher, or the teacher new to this curriculum, because it motivates the teacher to keep going.

Randi: They come in, thinking, “I have to teach to mastery. And we will stay on this until they get it.” And the reality is, we cannot teach that way. We don’t get that luxury.

She reminds the teacher that mastery will be measured at the end of the year. The exact moment of understanding is unique to each student. The teacher must persist in instruction and assessment, and postpone some re-visitation for a later time.

Teaching Conceptually

The focus of the classroom instruction is no longer procedures and skills in the words of IMCs, MTEs, CRTs, and the teachers, as it was in many traditional programs. At the onset of using the new curriculum, Kelly commented that now they were teaching the “*why* to do it,” rather than simply, “how you do it.” After teaching the course for first three months, Kelly expanded her definition of teaching mathematics in depth.

Kelli: It’s looking below just the surface of why we do a problem. And actually understand what’s the purpose if it, how can that help me, how can I use that in life and how can I build on my previous knowledge on the further things.

Randi: Teaching in depth would be teaching in a way so that the student truly understands the concept, can explain it, and then apply it. We want them to understand what they’re doing. And if they have a different strategy, if they *invent* their own strategy, if’s a *good* one, then it works for them.

Students were learning many methods that enabled them to work with the concept and understand it, but the goal of instruction is conceptual learning.

Teaching conceptually means teaching with regular assessment, knowing what the students know, and offering open-ended ways to get that information. One MTE instructs her preservice teachers in formative assessment techniques even as she is assessing their knowledge. Rubrics offer more flexibility to provide feedback even as an evaluation is taken.

Randi: If they tell you to master this at this grade level, you need to, because you're not going to get to practice that again, and next year we're going to do something else where they expect you to use that knowledge to apply it.

As the teachers teach a concept, they expect the students know it well enough to explain it and to apply it to new situations. Also, if they understand a concept like multiplication, they can show the teacher many ways to model it.

Classroom Pedagogies

The pedagogies used by the classroom teachers have been amply described in this and in Chapter 5. These paragraphs highlight teaching practices that are recommended by the peripheral participants as supports of teaching mathematics in depth. They are organized within the categories noted in Figure 6-1, mathematical, cognitive, and sociocultural perspectives, influenced by Nelson's (2001) discussion of facilitative teaching. One of the teachers nominated by the CRT was praised because she knew how to ask good questions. This pedagogy will be considered to be oriented toward mathematics, a perspective defined by the structure of the discipline that is mathematics. Other pedagogies recommended by the IMCs are also mathematical. These are three: (1) learn to find patterns, (2) use lots of strategies, and (3) help the students master the basic facts. Many of the recommendations of the MTEs illustrate the mathematical perspective. The teachers offer opportunities for the students to consider mathematical questions and evaluate their discussion in mathematical terms. The MTEs teach their students, the preservice teachers, to consider what the students are saying about mathematics and teach their students to listen to each other, also. These pedagogies offer those opportunities—problem-solving, finding patterns, and mathematizing real world situations. Other activities are recommended because they are the activities of mathematicians—logic, justification, and making multiple representations. Students must be encouraged to make sense of the mathematics. As preservice teachers, they are taught error analysis and asked to make concept maps of mathematical topics. These pedagogies are valuable teaching tools.

A cognitive perspective considers whether children are ready to learn more efficient strategies. To answer the question, the teacher must know what the student understands and what he might be ready to explore. Cognitive learning theory also informs the teacher what actions promote effective organization and transfer of information within the brain. The CRT at Hickory School thought that teachers could help children learn mathematics in depth if:

CRT: I think you have to provide a *range* of different types of activities. It can't be *all* just lecturing you've got to have things that they're *actively* involved in, and, you know, just as many different *ways* that they can react to it. You know, some are still needing more of a concrete level, some are ready to move to a more *abstract*, and being able to *provide* for that, so that hopefully, we can all get to a more abstract level.

Redwood School's IMC, Randi, taught the teachers there to transition the student from object to icon, as they worked together to prepare the students to take the high stakes tests. The students might need to begin exploration of a concept with concrete materials, but these materials are not allowed in the examination room. The IMC told the teachers to transition the students from the holding the object to drawing the object, because drawings are allowed on the examination papers. The final goal would be for the students to reach abstract levels of reasoning, but this is a developmental process that differs for each student. Another challenge at Redwood School is altering a tradition of passive learning habits.

Randi: And what we're struggling with, too, is getting the kids to do it. I think, for so many of them, too, they're kind of used to, well, "Somebody will tell us what to do. I won't have to sit here and think." That's probably a big adjustment for our students as well. So many of them have sat in classrooms and never done that. No one's ever *forced* them to.

The MTEs value manipulatives as tools for students to move from the concrete to the representational. Teaching preservice teachers how to incorporate these objects into instruction is a priority in their mathematics methods classes. Since they recognize that each person is constructing mathematical truth, they appreciate the role of reflection in the learning process. Journaling their own reflections is a requirement for students in the MTEs' courses. Deeper learning is learning with strong webs of connection within the brain. The MTEs recommend for elementary teachers integration of mathematics with other subjects, particularly science. They advocate incorporating mathematics into social studies when possible, and one MTE models the use of literature to teach mathematics concepts. Cognition theory also acknowledges the integrative activity of the brain, incorporating new knowledge into the body of prior knowledge (Ormrod, 2008). When teachers have access to that prior knowledge, they can adjust instruction in response. Hence, MTEs suggest many assessment activities to their teaching candidates.

Less emphasized by the peripheral participants are the pedagogies of the sociocultural perspective, although accessing prior knowledge is equally important for each of the perspectives. The IMC at Redwood School recognizes the importance of cooperative learning groups and partners with teachers to implement this pedagogy into their mathematics instruction. She has been most successful with the kindergarten teachers who use centers and groups regularly. The value of this tool is acknowledged by the teachers in the intermediate grades, but implementation in these classrooms is limited by perceived problems of gaps in prior knowledge.

Randi: I think that's probably one of the *hardest* things—there's so much. The teachers tend to do too much, "I'll stand and talk and talk and talk" and there's not enough time for the students to talk.

Randi advocates cooperative learning groups because she knows the students will learn in depth if they have the opportunity to talk about mathematics with their peers.

Valued pedagogies in the college methods classroom that are generated within the sociocultural perspective are cooperative learning groups and scaffolding the students within their ZPD. The MTEs encourage their preservice teachers to “share” rather than lecture, and the students are encouraged to “talk” rather than recite. These educators value the place of “play” in mathematics. Problem solving is an important realm of mathematics, and the solutions are best constructed within an atmosphere that encourages experimentation and brainstorming with one’s peers. Invented strategies are also a valued product of mathematical thought. At Delta School, Harriet taught the students to work together, to share materials, and to help each other.

Harriet: To me, the whole idea, if you’re going to sit by someone, you *help* that person. You *assist* that person. You *share*. If you know this, you help this person with it. So you’re a *helper*.

Summary of TMinD: Uniting the Voices of the Participants

This section has presented the synthesis of the findings from the data collection, incorporating for the first time the results of the interviews with the peripheral participants. Teaching mathematics in depth was explained then with words of the participating teachers, the IMCs, the CRTs, and the MTEs, as well as the researcher’s field notes and other secondary data collections. The reference most iterated was that teaching mathematics in depth was teaching mathematics for understanding. The exemplary teachers engaged their students, planned well, used resources effectively, and managed the classroom skillfully. Teaching in depth was considered to be something that good teachers did.

The experience of the teachers using the NGSSS introduced the element of depth to instructional practice. Depth meant detail, great detail, but each teacher was responsible for leading the students through fewer ideas, fewer mathematical topics. It was almost surprising how teachers encountered the aspect of time. Although there were fewer ideas to teach, the amount of detail entailed greater preparation. The teachers were surprised that there did not seem to be sufficient time to accomplish the work before the summative assessments would be given. One teacher felt like she was still “racing the clock,” because she, as the other teachers, wanted to teach the students for mastery before she broached a new subject.

The participants generally agreed that teaching mathematics in depth was teaching conceptually, teaching the students why they are using a procedure, giving them the ability to situate that learning into a new context, and use the skill in a new situation. The pedagogies recommended by the peripheral participants were organized to support the first finding of the study, that teachers chose interventions according to their own orientation toward theories of learning. These were described according to mathematical, cognitive, and sociocultural perspectives. The data collection and review of the literature is synthesized in the next section of the manuscript, the description of the framework for teaching mathematics in depth.

` The Framework for TMinD

Using qualitative research methods associated with grounded theory and situational analysis, the data generated two findings associated with teaching mathematics in depth, which were explained earlier in this chapter. Data collected from both primary and peripheral participants have generated a definition of teaching mathematics in depth. This definition, the two findings, and the literature are now synthesized to inform a framework for teaching mathematics in depth. The researcher coined the acronym *TMinD* to refer to this framework. It is designed for use by instructional mathematics coaches, mathematics teacher educators, professional developers, and other administrators to facilitate the practice of teaching mathematics in depth.

After a thorough review of the literature, including a volume-by-volume reading of the hallmark journal of mathematics research literature, this dissertation adds substance to the definition of teaching mathematics in depth. The comprehensive review of the literature was compared with two data sets collected in this study. One was the understanding of the practice by representatives of the groups that are serving in the preparation of teachers (MTEs) and support of mathematics instruction for in-service teachers (CRTs and IMCs). The other was the conception and actualization of teaching mathematics in depth as it was experienced by three classroom teachers implementing the state mandated curriculum and standards. The literature notes the value of several elements of the TMinD framework, but the experience of those who tried to teach it revealed other important aspects little studied in mathematics education research. The statements of the peripheral participants verified this experience even as they were sometimes better able to compose a definition in pedagogical language.

The framework is best considered with the metaphor of a building structure (Figure 6-5). This structure begins with a foundation that supports the framing for the walls. The space within the walls is enclosed for the most part, but there are windows for seeing to the other side. A door accesses the interior space. Even though the walls are anchored to the foundation, a roof ensures their vertical orientation and preserves the safety of the interior space, the space where teaching mathematics in depth occurs. Each component of the structure will be described completely as it supports teaching mathematics in depth (TMinD). Moreover, the six elements identified in Chapter 3 as being essential constituents of TMinD are synthesized with the data to complete the metaphor.

The Foundation

The review of the research literature demonstrating effective mathematics instructional practices noted the primacy of three curricular projects created to incorporate students' social construction of knowledge within the guidelines recommended by the *Standards*. For the foundation of TMinD, we return to commonalities shared by these three: The Second-Grade Project (Cobb, Wood et al., 1991), Mathematics and Teaching through Hypermedia Project

(Lampert & Ball, 1998), and Cognitively Guided Instruction (Carpenter & Fennema, 1991). Each was grounded in a philosophy of shared authority over knowledge, knowledge emerging from mathematics but subject to the consensus of the members of the learning community. Each structured instruction around tasks and discourse that created opportunities for the students to encounter the mathematics, grapple with it, and make meaningful knowledge by incorporating the new data into their existing understanding.

Similarly, as was noted for these significant programs, the foundations for TMinD, the forces that hold the structure in place and allow development to occur, are two: solid grounding in the discipline of mathematics and acceptance of the theoretical framework of constructivism for mathematical knowledge (Figure 6-6). Important mathematical knowledge, the cultural invention defined by those who practice it, is one of those forces. The ideal curriculum for elementary school mathematics is built upon a limited number of powerful ideas rooted in basic understandings and principles. These ideas would include the formal symbol systems of mathematics and their underlying meanings or semantics (Prawat, 1993; Putnam et al., 1989). The foundation for TMinD incorporates important mathematics upon which students can build an edifice that will take them to more complex ideas in mathematics.

A theoretical perspective of mathematics education is constructivism, meaning that students create meaning from the sensory information they gather, both in school and outside of school. As learning actively changes the brain (Zambo et al., 2007; Zull, 2002), educators accept the philosophical orientation to constructivism that indicates the instructional advantage of teachers sharing agency with their students (Smith, 1996; Vacc & Bright, 1999; Verschaffel & DeCorte, 1997). Although knowledge development takes place in a community of learners (Kinard & Kozulin, 2005; Meyer & Turner, 2002; Schunk & Zimmerman, 1997), it is the individual child who reflects and reorganizes while participating in and contributing to the development of the discourse (Cobb et al., 1997, p. 266). This understanding is strengthened when students communicate and reflect (Hiebert & Carpenter, 1992).

These two elements then, a theoretical perspective that students will be the ultimate constructivists of their mathematical knowledge and the structural integrity of the discipline that is mathematics, form the basis for TMinD. The essential task for the teacher of mathematics is to connect what they want students to learn with what they think that students already know. On a daily basis, this presents an interesting dilemma for the classroom teacher during the course of an interactive lesson: ought the teacher pursue the planned mathematical goal or instead follow that interesting student idea (Ball & Lampert, 1999)? For TMinD to occur, the goals of instruction need to be connected to the ideas of the students (Carpenter & Fennema, 1991). The instructional tasks need to construct mathematics and students are responsible for their own learning (Cobb, Wood et al., 1991).

The Framing

Every structure rises from its foundation, and the height of the building depends on the frame that is anchored to that base (Figure 6-7). The skeleton of two by four studs or steel columns reaches vertically toward the sky. Wooden braces or steel girts, securely fixed to each other and to the vertical posts, support the grid that composes the frame. Within the research literature on mathematics education, fewer references are made to the two elements that comprise framing: one is detail and the other is mastery. Critics of the curriculum that preceded NGSSS were eloquent that these two components were essential for TMinD. One group of critical remarks stems from the Third International Mathematics and Science Study (TIMSS) results that were initially introduced in Chapter 2. After concerns were raised about the relatively poor showing of the students in the United States, researchers began to examine the mathematics lessons in the nations whose student scores were better. Smith (2004) observed both Japanese lessons and American lessons. The Japanese students gave more detail in their explanations to answer questions posed by their teachers. When this was not spontaneous, the Japanese teacher specifically probed students to find more detailed and connected explanations.

To frame the structure that is TMinD, detail is essential. The few mathematical topics that are emphasized in NGSSS at each grade level are taught in great detail. After the fifth grade concludes, the students know everything there is to know about division with whole numbers. Students have many strategies to solve division problems, they know vocabulary, they can diagnose the use of division in context, and they can interpret remainders. For four months these students investigated division as a mathematical topic, and they practiced this operation in a multitude of situations. Their initial impression of Big Idea 1 (“Develop an understanding of and fluency with division of whole numbers”) is detailed and complex, which is appropriate, because abstract concepts are always embedded in concrete experience (Zull, 2002).

Additionally, the height of the building that is the metaphor for TMinD depends on the security of the framework for each floor. A strong second story can exist only if there is a strong first story. The strength of the framing for upper floors in TMinD is mastery of the mathematics. Another category of criticism of the mathematics instruction in the United States came from the Mathematics Standards Study Group (2004) that stated, "Our goal is for students to develop an in-depth mastery of the mathematical knowledge and reasoning in core topics as they tackle increasingly challenging problems. This is the surest path to success in high school and college mathematics" (p. 4). The report continued by advising that topics should not be introduced before the students are adequately prepared, and then the students should master the topic they are taught.

A third report of critical recommendations stems from the request of the state department of education to evaluate the mathematics standards in the SSS. This review (College Board, 2005) determined that that there was too much duplication of mathematical topics between the school grades. The focus on each grade was blurred because of this overlap and repetition, lacking a sense of purpose. At the end of a course, the SSS placed comparatively low levels of cognitive demand on students relative to reasoning, problem solving, or communicating with the mathematics suggested in the standards. The report specified that insufficient recommendation had been written to indicate where mastery was expected. The NGSSS addresses this critique by anticipating that a mathematical topic will be mastered by students in the year it is taught.

The instructional goal for each topic is mastery of its use. Teachers teach the detail as discussed earlier, and the students master it. A more complete description of the supporting pedagogy is contained in the discussion of walls and windows. However, the goal of mathematics instruction under TMinD is mastery—not introduction or coverage, but mastery. The teachers of the sixth grade are expecting that their students have learned how to divide in many ways, in various contexts, and with good accuracy.

The Walls

A framed structure without wall covering can be a drafty place (Figure 6-8). The contents within the building may fly out the openings. Walls of siding and sheetrock or concrete block and plaster over wood lathing insure that the interior of the structure is a safe and cozy place. This same idea of safety extends to TMinD. Without walls, the knowledge of mathematics may fly away or never coalesce at all. The tasks, procedures, and instructional activities are represented by the walls in this model. The interior space of TMinD is maintained and developed by the choices teachers make in the area of lesson planning and implementation. The findings of this study from both primary and peripheral participants have isolated many types of instructional practices. This section highlights the most effective of these practices using the lens provided in Chapter 3, the review of the literature identifying effective teaching practices in mathematics.

Tasks identified by the teacher appropriate the greater part of the mathematics lesson sessions. Much of the students' learning for the day derives from these tasks, so they must be carefully chosen for TMinD. The tasks need to derive from the essential characteristics of mathematics, develop the details that confirm the essence, demand rigor on the part of the students, and offer the student opportunity for thoughtful consideration of the mathematical truth offered by the task. Abstract mathematics begins in a context where students can interact with it in situations that make sense in their world (Bransford et al., 1999; Cooper, 1998; Irwin, 2001; Verschaffel & De Corte, 1997). The tasks are undertaken within a spirit of inquiry and problem solving, as occurs in the realm where mathematicians work (Ball & Lampert, 1999; Boaler, 1998; Franke & Carey, 1997; Good & Grouws, 2004). Incorporating non-routine problems into mathematics classes, even for young children, opens the doors for fruitful discussion of meaningful mathematics (Cobb, Wood et al., 1991). Teachers can also integrate the mathematical processes of justification, logic, representation, and connection into the instructional activities (Cobb et al., 1991; Silver et al., 2009). The goals of TMinD tasks include the expectation of higher level thinking skills (Boaler, 2002; Henry & Brown, 2008).

Moreover, these tasks can foster interdependence with the classroom community by offering the students opportunity for discussion with their peers as well as the opportunity to teach their classmates at the same time they solidify their own understanding (Truxaw & DeFranco, 2008). Language activities such as discussion of solutions, problem solving, and classroom presentations are significant builders of tight walls (Turner & Patrick, 2004). To begin, writing and discussion are active forms of learning, learning that moves information from the short-term memory centers to the appropriate file cabinet in the part of the brain that stores learning for the long term and for transfer to future similar situations (Zull, 2002). These activities also develop mathematics vocabulary, mathematical rigor, and command of the English language. Through language the students can communicate what it is that they know to the teacher, who can then plan the next steps for instruction.

Helping students make sense of their world is one of the essential objectives of TMinD. Fostering a community of learners who discuss mathematics is a positive component for this objective. The community develops rituals and processes of discourse that enhance communication through the creation of sociomathematical norms (Cobb, Yackel et al., 1991; Cobb, Wood et al., 1992; Yackel & Cobb, 1996). Teaching students to use correct mathematical language not only develops mathematics rigor and conceptualization but also promotes the ability of the students to instruct each other (Lo et al., 1994). Encouraging students to use multiple representations also promotes their ability to convey their understanding to the teacher and their peers (Pyke, 2003). Discourse that promotes explanation and justification of one's mathematical thinking is a substantial wall to protect the development of TMinD.

Windows and Door

The idea of windows and a door is associated with vision and access (Figure 6-9). Windows in this model represent assessment. The teacher must "see" into the students' minds and infer what they are thinking if they are to facilitate the students' construction of mathematical meaning. Assessment practices that expose student thought cultivate TMinD. Some of these are formative in nature, while others are summative. The largest windows are those based on formative assessment. These practices, combining definition of good practice with relevant and immediate feedback, offer great opportunity for increase in learning gains

(Black & Wiliam, 1998b; Izsak et al., 2008). These practices may be classroom assignments that are graded with comments instead of scores or specific open-ended tasks that allow many methods of solution and require justification and explanation of the methods used (Chandler & Kamii, 2009; Hart, 1984). The purpose is to expose student thinking to the teacher and provide direction for future instruction. It is important that the teacher elicit the complete answer that the student holds (Fraivillig et al., 1999). Looking at a single number answer may not be an effective indicator of student understanding (Reys & Yang, 1998).

Flexible grouping structures can enhance formative assessment practices. Teachers can conference with individuals or small groups. Self-assessment and peer-assessment are also valuable practices for evaluation (Black & Wiliam, 1998b). Pairs can assess the learning of each other. When students explain their thinking to each other, they have an opportunity to clarify the expression of their thought and evaluate it (Boaler, 2002; Empson, 2003; Hart, 1993; Irwin, 2001). The entire class can share a discussion about the solutions created by one group of students. However, there are two cautions to be mentioned. Teacher knowledge of students' understandings is enhanced in a climate of trust between the parties of the classroom. Students may not fully express their thoughts if they fear embarrassment for giving the wrong answer (Raider-Roth, 2005). A second caution arises from the need to fully know the student. There are cultural differences that impact the communication channels between teacher and students of which teachers should be aware if they are to be a party to students' mathematical understanding (Berry, 2008; Guberman, 2004; Irvine, 2003; Malloy & Jones, 1998; Miller, 1995; Nieto, 2000).

The door is a metaphor for access to TMinD for all students. Mathematics ought not be a discipline that only a privileged few can practice, because its importance extends to all aspects of modern life (Sfard, 2003). The *Standards* (1989) addresses the importance of mathematics in our technologically oriented society when it suggests as a principal goal of the mathematics curriculum “Learning to value mathematics” (p.5). The educational community owes each student the opportunity to learn this subject to the best of their ability, to make sense of it, and to incorporate it into their daily lives (Au et al., 2007; Cochran-Smith, 2004; Gutstein et al., 1997; Hinchey, 1998). Education is about extending the students’ thinking abilities (Delpit, 2006; Ladson-Billings, 1994), and freely offering access to all learners. The teacher needs to know each student well and learn the individual password that will unlock the door and help the child learn.

The Roof

No structure is complete without a roof, because the walls will fall, and the interior contents will dissipate or be contaminated by weather conditions (Figure 6-10). The roof for TMinD is time; TMinD cannot take place with out time. One of the considered criticisms of previous mathematics education in the United States is that there were too many topics and instruction of those topics was too shallow (Lindquist, 1989; Schmidt et al., 1997). In the Jacobs and Morita study (2002) Japanese teachers believed that the U.S. teachers had not allowed sufficient time for the students to investigate the concept before they began to practice individually. The 1989 NCTM Yearbook pointed out that, “In general, our computational strand needs to be slowed down so that ample time can be spent developing number sense and meanings of operations as well as applying learned computational skills (Lindquist, 1989, p. 7).

The requirements for TMinD cannot be achieved without time. Each topic must be treated in detail. Since the students are constructing their own knowledge, they must be allowed the leisure to do that. Earlier in this chapter, James remarked, “This system…is gonna let the kids savor certain aspects of mathematics a little longer, a little more, so that they actually gain mastery over it.” It will not happen if every moment is absorbed in a directed activity. Mastery is an essential element of TMinD that cannot be measured equally for each learner. Some students will need more assistance and more learning tasks to incorporate the knowledge into their existing mathematical formation. Others will seem to master a topic almost through insight. Flexibility to deal with the range of student proficiency is required on the part of the teacher and the curriculum.

Time is a gift. As Sandra said earlier, it was OK to take twenty minutes to solve one problem. It would be desirable to investigate a subject completely on the web, to explore connections with science and literature. It is beneficial to take the time to let students explain, to let them ramble until their thinking makes sense of their meanderings. Teachers must feel that they have the wisdom and the authority to regulate the pace of instruction in their classroom so that each of their students can truly learn mathematics deeply.

Summary of the framework for TMinD

TMinD is a complex pedagogical practice that incorporates effective teaching practices and those recommended for implementation since the *Standards* were published in 1989. The research literature for mathematics education has been culled to insure that a complete representation of these practices were included in the model. The study participants have added the voices of the classroom teachers and professional mathematics teacher educators, coaches, and resource teachers. The data has been synthesized into a framework that is symbolized by the model of a structure. Its foundation is the discipline of mathematics and a constructivist theoretical perspective. The framing for the walls is detail and mastery. Wall coverings are provided by the instructional tasks and communication. Assessment is represented by windows, and access for all students is designated by the door. The capstone is the roof, signifying time, an important addition to earlier collections of effective teaching practices.

The experience of the classroom teachers as they taught mathematics in depth added elements to the framework that were little noted in the research. The teachers clearly desired that their students master the subject matter, and assessment served to assist the teachers to instruct their students toward that end. At the same time, there were political and administrative forces that pressured the teachers to measure student mastery at arbitrary moments, at points dictated by a district calendar or pacing guide, rather than at a point determined by the teachers' reading of readiness. The elements added to the model by this finding are time, mastery, and detail. Each proved to be an essential component of teaching mathematics in depth. The structure lacks strength without them. Framing of mastery and detail erects the landscape on which to attach the walls, doors, and windows. The studies in the review of the literature considered these aspects less than did the critics of the American mathematics educational system (College Board, 2005; Kilpatrick et al., 2001; Lindquist, 1989; MAA Online, 2004; Porter, 1989; Schmidt et al., 1997). The first finding from the fifth grade teachers highlights the potential difficulty facing teachers trying to teach mathematics in depth when the political arena enforces time lines and high stakes testing deadlines. The roof of TMinD is suffering from a hurricane led by legislation that demands accountability even as it denies the requirement for time, the protective covering over TMinD.

A second finding provides explanation for the walls and windows of the framework. The curriculum provided for the teachers in this study offered to them several methods of instruction and formative assessment. Research literature supports the value for learning mathematics in each of the suggestions found in this wide repertoire. Results of this study indicate that teachers have inclinations toward some over others, and that these inclinations seem to stem from the teacher's perspective on learning. The teachers use the activities and procedures that they assume will provide the greatest opportunity for their students to learn. However, there are many theories of learning within mathematics education, and this study noted that the teachers could address their challenges more effectively if they were able to broaden their perspective on effective learning practices. The walls and windows of TMinD will be anchored more firmly with a broader interpretation on the part of the classroom teacher of how students learn.

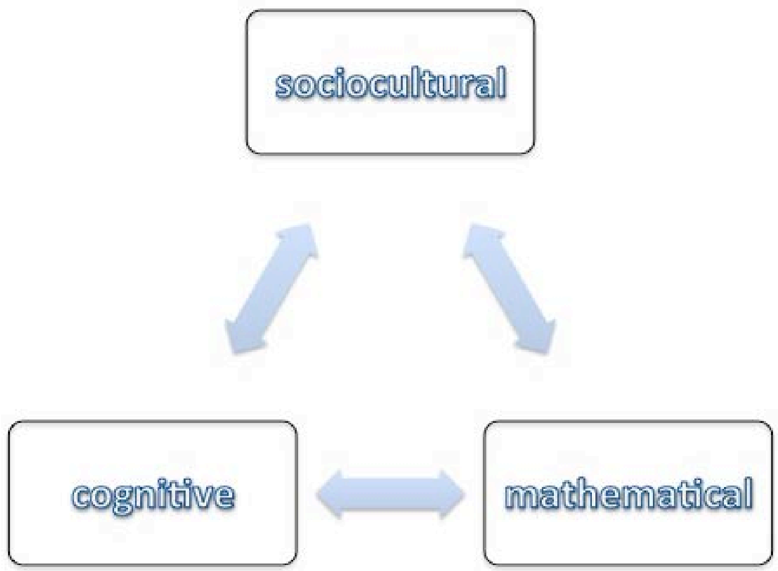

Figure 6-1. Theoretical perspectives of learning mathematics

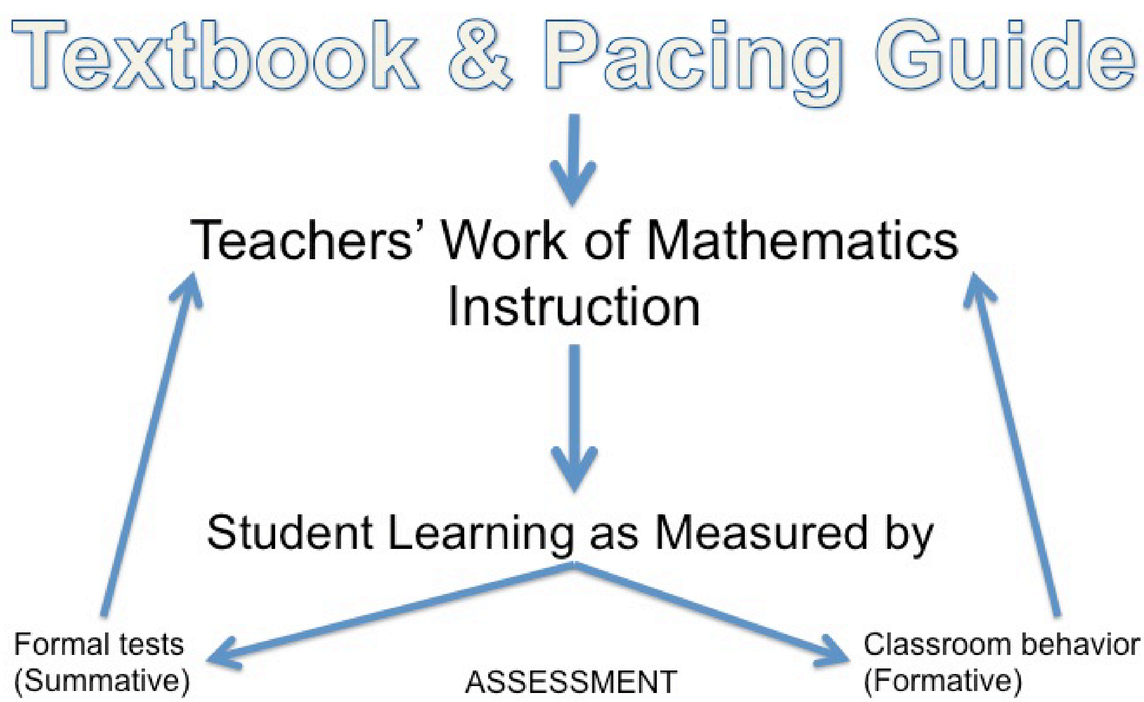

Figure 6-2. Enactment of Teaching Mathematics in Depth in the classroom

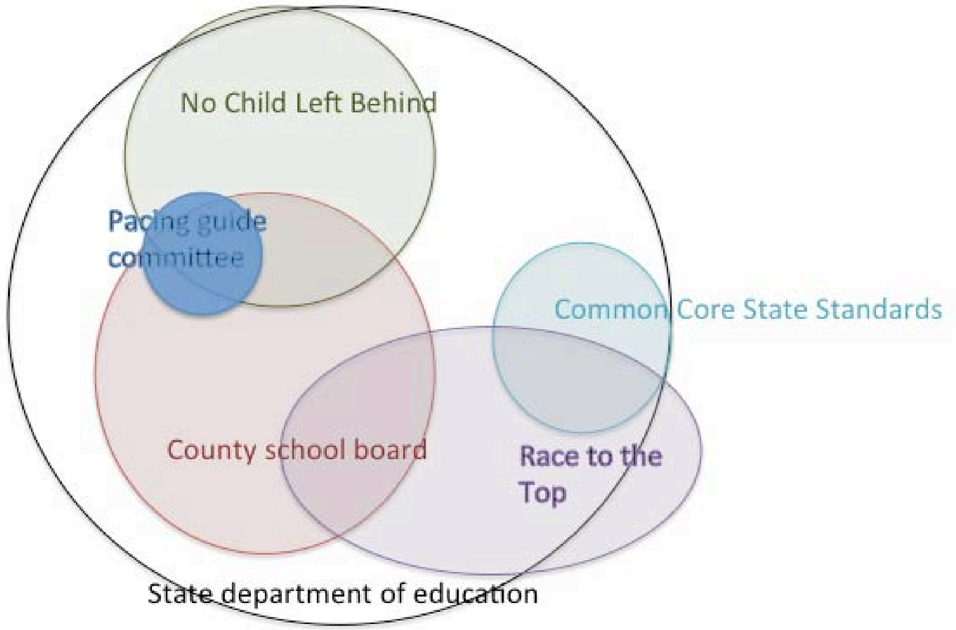

Figure 6-3. Bodies of influence on the Pacing Guide

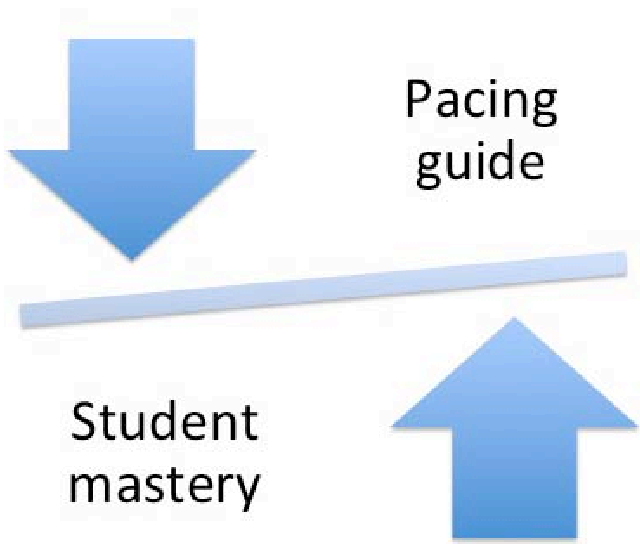

Figure 6-4. The tension teachers feel trying to balance the pacing guide and teaching for student mastery

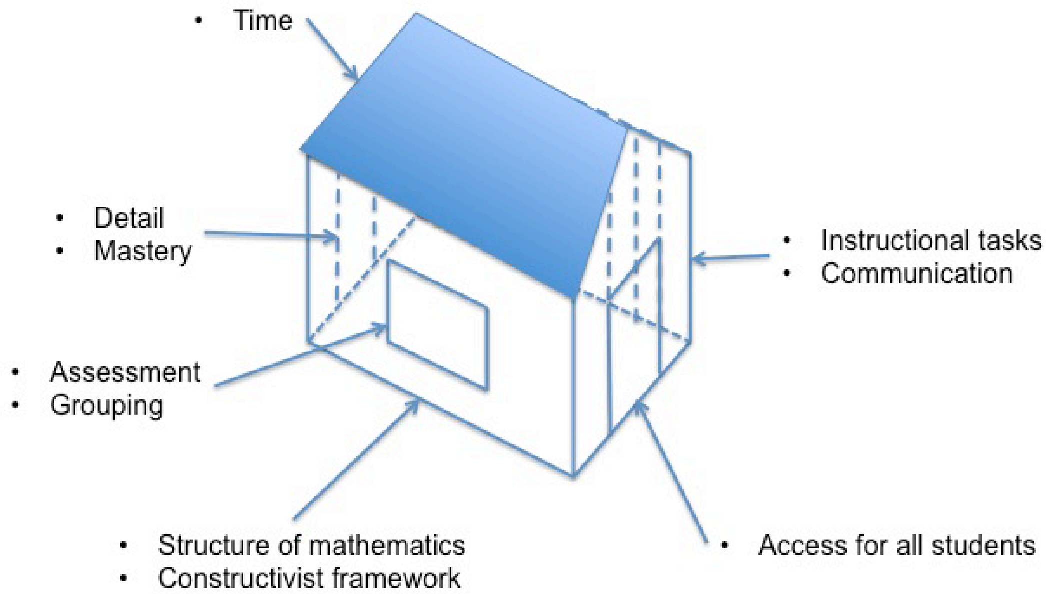

Figure 6-5. The structure: a model for Teaching Mathematics in Depth (TMinD)

Figure 6-6. The foundation: mathematics and student constructivism of meaningful knowledge

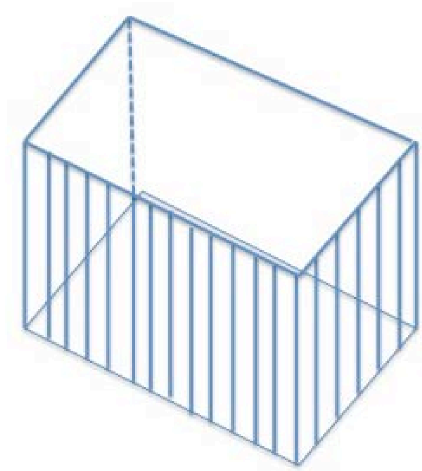

Figure 6-7. The framing: detail and mastery

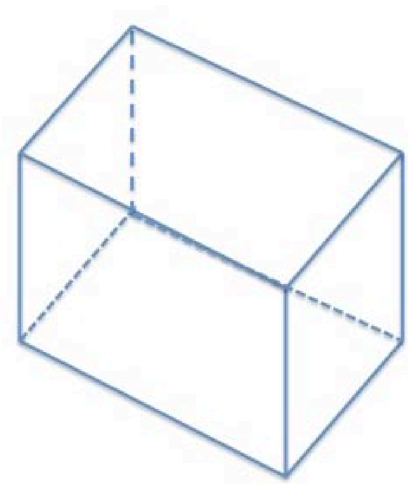

Figure 6-8. The walls: instructional tasks and communication

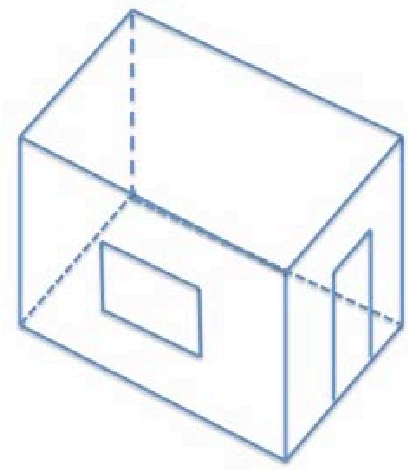

Figure 6-9. Windows: assessment, grouping and Door: access for all students

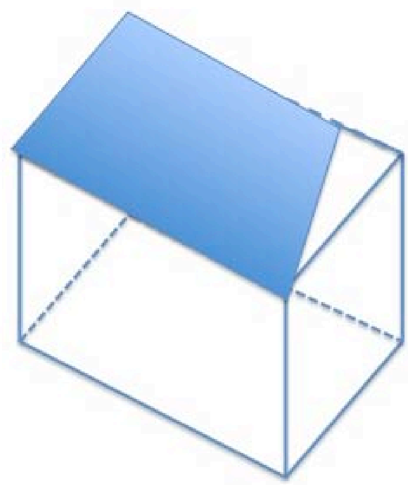

Figure 6-10. The roof: time

CHAPTER 7
CONCLUSIONS

The purpose of this research was to develop a framework for teaching mathematics in depth (TMinD) and thereby address a gap in the mathematics education literature regarding its description. This purpose has been accomplished and the description is in Chapter 6. The practice of TMinD has long been extolled as a beneficial practice, but until now few have given voice to the specifics of its structure. This study has articulated those specifics and will present a contribution to the national debate.

Implications of the Findings for Research

The movement to create national standards for mathematics has gained considerable momentum in the last year with the publication of the CCSS, which have been adopted by forty-two states, the District of Columbia, and the U.S. Virgin Islands (CCSSI, 2011). The CCSS establish what content and skills should be taught, but do not dictate how teachers should teach. States that apply for the federal grants under Race to the Top must adopt the CCSS to qualify. Additionally, the USDE recently awarded $330 million in Race to the Top funds to two consortia to help develop assessments aligned with the CCSS (Porter, McMaken, Hwang, & Yang, 2011). Methods to teach these standards are surely equally interesting, as the mathematics standards are explicit in the intention “To deliver on the promise of common standards, the standards must address the problem of a curriculum that is ‘a mile wide and an inch deep.’ These standards are a substantial answer to that challenge” (CCSSI, 2010, p. 3).

The study represented in this manuscript is an examination of TMinD to teach mathematics standards that are remarkably similar to CCSS. It is reasonable to expect that the insight gained from the teachers’ experience with NGSSS will be transferrable to the CCSS. Potential research studies could investigate the use of TMinD in different contexts. Here are some possibilities:

- TMinD at the high school or the middle school level
- Replicating this study with a different sample of teachers.
- Studying the implementation of TMinD in a longitudinal study.

As will be discussed later in this chapter, the researcher was positioned as a volunteer in the design of this project, and this choice limited the results in some ways. A similar study could take place where the researcher was an observer without any participatory role. This design would offer other benefits and present other limitations.

The subject of enacting TMinD is only in its infancy. Enactment of TMinD needs to be studied more completely than was within the scope of this work. Considering the results that emerged here, there are elements of the framework that will be more readily adopted than others by teachers in the field. For example, the *content* and *supporting details* are likely to be changed most readily, given the assistance of the curriculum and the demands of the high-stakes assessments. Teaching content is traditionally the work of mathematics teachers, and there is motivation to increase one's expertise in the structure of the key ideas. As teachers repeatedly employ the NGSSS or CCSS, their confidence and knowledge of the concepts will improve.

Other elements of the framework will be more difficult for individual teachers to implement without the support of professional development or other formal structures. Many of these elements can be derived from the Standards for Mathematical Practice (CCSS, 2010). They are listed here in correspondence to a section of the TMinD framework. (a) Make sense of problems and persevere in solving them (*foundation:* constructivist theoretical perspective; *roof:* time). (b) Reason abstractly and quantitatively (*walls:* tasks, discourse). (c) Construct viable arguments and critique the reasoning of others (*walls:* discourse; *windows:* assessment). (d) Model with mathematics (*walls:* tasks and discourse; *framing:* detail). (e) Use appropriate tools strategically (*walls:* tasks; *windows:* assessment). (f) Attend to precision (*framing:* mastery;

walls: tasks; *windows:* assessment). (g) Look for and make use of structure (*foundation*: concepts of mathematics; *walls:* tasks; *windows:* assessment). (h) Look for and express regularity in repeated reasoning (*framing:* mastery; *walls:* tasks; *windows:* assessment). Use of these mathematical practices was underrepresented in the sample of this study. One infers that these practices will not change dramatically unless there is scaffolded support for those efforts.

One finding of this study is that teachers tend to model their practice of TMinD according to their orientation toward learning perspectives. In this case, the distinction appeared to be clearly directed toward cognitive theory of learning, a sociocultural theory of learning, or a learning theory based on the discipline of mathematics. This is not the first study to consider the coaching advantage of teaching teachers according to their strengths and orientations (Kise, 2011; Nelson, 2001). The question is, can we expand the teacher's viewpoint so they attend to perspectives other than their own? Knowledge of a teacher's orientation toward one of the learning perspectives suggests an interesting line of research as well, one that may have practical application for professional development and pre-service teacher preparation. Sfard suggested in her discussion of theory for NCTM (2003) that these theories need not be contradictory, and indeed it would be beneficial for students if teachers saw these orientations as complementary. The teachers of the fifth grade at Forest Glen School could have more solutions to approach their challenges if the scope of the their orientation were broadened to include orientations beyond their initial inclination.

Another finding of this study was that the forces of following the pacing guide and teaching for mastery create tension for the teachers who are trying TMinD. Although research has noted this tension in other situations (David, 2008), finding it as a district is implementing teaching mathematics in depth is ironic. The elements of time and mastery that shape TMinD contrast greatly with the need to adhere closely to a daily schedule. There is more to be learned about the complex relationship of monitoring the rate of instruction to maximize student learning using existing time, resources, and effort with the state's imposed deadlines for high-stakes examinations (Sheridan School District, 2011). Little is yet written about the ways the pacing guides are utilized in mathematics education, and questions remain about the potential for TMinD under circumstances that are monitored in terms of time.

Implications of the Findings for the Field

The teachers represented in this study as primary participants or by peripheral participants evidenced a desire for collegial support as they implement TMinD, particularly with content knowledge and pedagogical content knowledge. These are areas that would be beneficial subjects for professional development. The teachers' concerns are also of interest to those who would be their instructional mathematics coaches. Due to the finding about the orientation toward a particular learning orientation, coaches and professional developers would be wise to discuss learning theory with their teachers. This study indicated that answers to particular challenges faced by the classroom teachers might be found within the pedagogies that were outside their personal orientation. It would appear to be beneficial that teachers consider such orientation toward other learning theories, and this would be an appropriate method for instructional coaches to work with teachers to improve instruction in mathematics.

The pacing guide created emotional unrest among the teachers in this study. These needs ought to be addressed by the school district administrators. The pacing guide should be a helpful means of allocating time and resources in preparation for the state high-stakes examination, but it often creates another layer of anxiety. Particular attention must be paid to the emotional cost of this guide over the course of the school year. Since time is such an important element in TMinD, coaches and other instructional supervisors might profitably direct the teachers into consideration of various strategies in terms of their efficiency as a tool of instruction and of assessment. For instance, incorporating writing into the curriculum is beneficial on both counts: it intensifies the learning effect and it can be a transparent means of formative assessment. As teachers use it more frequently and more purposefully, the positive results will be magnified. Similarly, the staff development sessions could focus on complex activities, such as rich problem-solving, that achieves results on multiple fronts. Other promising methods of formative assessment, such as the use of peer-assessment and self-assessment, could be incorporated into professional development for teachers.

Results of this study offer topics for consideration in preservice teacher preparation. Learning theory can be related to pedagogical techniques. The students can be led into a discussion of proclivity. Why are some pedagogies more attractive to teachers? Are there differences in the results or are the benefits primarily in the eyes of the teachers because of their orientation? Another topic that could be fruitfully addressed in the preservice program is the pacing guide. This may be a mandate in most schools; the teachers should have some practical training in its use. Balance between pacing and mastery is essential in the contemporary school culture of TMinD and testing. As the IMC at Redwood School told her teachers about working toward mastery, "The reality is, we cannot teach that way. We don't get that luxury." If teachers will not have time to teach mastery, then what will they have time for? This is an important topic for the preparing teacher.

Conclusion

Chapter 1 introduced Inez, the fifth grade teacher who was directing her students across the *Cinquième Vallée* after the landscape had been significantly altered. The alteration was a result of a new set of mathematics standards, a different curriculum, and a revised high-stakes state examination. These elements combined to create a pedagogical imperative to "teach mathematics in depth." However, the precise explanation of what the pedagogy is had not yet been defined in the research literature for mathematics education. This study was undertaken to contribute to a mitigation of that research gap and to develop a framework describing teaching mathematics in depth for the benefit of practicing teachers and those who educate teachers.

The significance of this research project is the illumination of the practice of teaching mathematics in depth by developing a framework to serve educators and coaches of mathematics teachers. A thorough reading of the literature that began with all of the JRME volumes in the decades since the publication of the *Standards* (1989) identified six elements that were profitable for effective instruction in mathematics. These elements formed the basis of a tentative framework for TMinD that was elaborated by the results of this study. The experience of a fifth-grade level team as they implemented a mathematics curriculum and set of standards that demanded teaching mathematics in depth identified elements of the framework that were previously missing from the literature. These are two–the objective of teaching for mastery and the detail required to develop the key concepts in depth. Moreover, the experience of the teachers crystallized the element of time. Time as an element was introduced by the literature, but its importance became clear as the teachers implemented these new practices.

The review of the literature in this dissertation reported the results of studies of *Standards*-based instructional programs, reports of critical committees, and suggestions of scholars and researchers about the most effective practices for teaching mathematics. This information suggested a tentative framework that was described in Chapter 3 and included the elements of theory, tasks, classroom discourse, grouping, and formative assessment. Each element could have been placed on a triangle and the triangles arranged side by side in a tent-like structure. The researcher observed through the lens of her experience that time, although rarely considered as a topic for study, would be an essential for TMinD to flourish, and added a pentagon representing time to anchor the triangles to each other more firmly, suggesting a pentagonal pyramid (Figure 7-1).

The study was undertaken, data were collected, and the results of the experience of the fifth grade teachers added information to the framework. The teachers reported in interviews and journal responses how they conceptualized and actualized what it meant to teach mathematics in depth. Results of this portion of the study were not intended to be an evaluation of the work of the teachers relative to the tentative framework, but to speak for the experience of those teachers, who were using a new set of standards and a new curriculum to teach mathematics in depth for the first time.

The peripheral participants responded to interview questions with their opinions on the practice of teaching mathematics in depth, defining it and expanding on the elements that they considered made the practice more effective. The MTEs were the most eloquent of these respondents, and they are the professionals most aware of the new standards and the preparation teachers will need to teach mathematics in depth. The IMCs whose job it is to assist the teachers at their school to instruct mathematics and increase student learning in that subject were also articulate discussants. The CRTs, the group least conversant with the new standards, held more conventional views that teaching mathematics in depth was teaching conceptually.

The Place of Grounded Theory

Ideally, a grounded theory investigation precedes the review of the literature (Strauss & Corbin, 1990). However, in a formalized program for the doctorate degree, this is not a realistic possibility. The literature review was completed, and the study was begun. The researcher was put in the position of analyzing the data without making comparisons between the perceptions of these participants and those who had participated in studies led by far more qualified scholars.

The grounded theory described in this dissertation had for its purpose the development of a framework to teach mathematics in depth. Research questions were advanced that would provide a design for a study of the perceptions of teachers and peripheral participants about the subject of teaching mathematics in depth.

The role of the researcher in this study required continuous balance between being an impartial data collector yet also a skilled interpreter of that data. In the fifth grade classrooms, the researcher primarily worked to make the teachers comfortable, helping them with their work, building trust in her so they could honestly and openly respond to her inquiries. Yes, the researcher was a participant in the mathematics classes, but she also studiously avoided judgmental comments. The object of interest in this project was the knowledge of the teachers. The teacher might have considered the words and work of the researcher as the knowledge was formed, but that was not an intention of this study.

Limitations of the Study

One limitation of this study is that Kelly did not fully participate. The researcher developed a closer relationship with the other two participants as a result of assisting them in their work during the instruction of Big Idea One. Since field notes about Kelly were limited to interactions with her and her students in the hallway or on the playground, the opportunity to confirm the description of events in Kelly's room was limited. What Kelly described is the information that the findings reveal. However, Kelly trusted the researcher as an interviewer and spoke openly during those conversations. Eliminating Kelly as a participant would not have assured more credibility in the estimation of this researcher. The number of participating teachers remained at three, and the voice of an important member of the instructional team was heard. She, too, was teaching mathematics in depth to a general population with a significant number of ELLs, and she experienced teaching mathematics in depth. Her words support the discussion of Sandra and James in many areas, particularly the effect of the pacing guide, the need for student understanding, and the concern for student mastery.

Since the researcher was not part of the work of Kelly's mathematics instructions, she could neither verify nor deny what was reported in interviews and journals, as she could with Sandra and James. Considering this distinction, the report on each teacher in Chapter 5 was designed to interpret what the teachers said about their own practice as much as possible. More evaluative statements are found in Chapter 6 where the researcher spoke of issues of concern to her, influenced by her professional experience and judgment. She was aware that the teachers were less effective at times because of what they were not doing. Also, the teachers could use formative assessment and instructional tasks more efficiently. As an outsider, the researcher noted that the teachers' partiality for learning activities from particular theoretical orientations sometimes limited their creative use of the new curriculum. As she became aware of the teachers' emotional response to the requirements of the pacing guide, she struggled to maintain a neutral position and refrain from offering advice. This good advice is reserved for professional development and preservice teacher preparation programs.

The purpose of this study was to develop a framework for teaching mathematics in depth. The findings of the study indicate that teachers who are using this practice for the first time are ill-equipped to formulate elaborate descriptions of its elements. Until this encounter with the mandate to teach mathematics in depth, the teachers had not defined it. James was unprepared to venture an opinion or description during the first interview, but during later interviews he attributed teaching in detail to teaching in depth. Sandra was no more eloquent, but she believed she had been taught to teach in depth during a professional development program geared toward problem solving. In later interviews, she enjoined the element of time to the definition, since now there was time to follow a problem to its mathematical roots. Kelly's initial impression of teaching mathematics in depth was that the students were learning why they used the procedures that they used. Later, Kelly spoke of students' being forced to think more carefully about what they were doing than before they were being taught mathematics in depth. The students were asked to look below the surface for the purpose of the mathematics used. Another limitation of the study was the use of member checking. In this particular instance, the researcher could have taken the description of the framework back to the participants for their evaluation and to explore the possibility that the teachers now could define TMinD.

Grounded Theory Informs the Model

Findings from the teachers have informed a model that is more comprehensive than the one provided by the literature alone. The fifth grade teachers observed the importance of mastery and detail for TMinD. In effect, this input significantly altered the model from a truncated pyramid (Figure 7-1) into a prism (Figure 6-5) with the potential for expansion. Mastery and detail are the framing on which rest the walls of instructional tasks and classroom discourse. The framing supports the windows of assessment and the doors of student access. The value of adding the element time into the model is pointedly apparent in the experience of the teachers. As Figure 6-2 is incorporated into the model of TMinD, the findings from the teachers inform the framework for TMinD. The teachers perceived the importance of mastery (framing), assessment (windows), instructional activities (walls), and time (roof). Because of the testimony of the teachers, one is aware of the legislative and political challenges to the tender fabric of the roof in the current climate of accountability and high-stakes testing. Time to teach mathematics in depth is assailed by forces beyond the teachers' control, forces that expect certain results at specific times.

The peripheral participants verified the elements emphasized by the teachers, and they validated the elements found in the review of the literature. Mastery, detail, and time are noted as important components by these educators. Additionally, they emphasized tasks, discourse, theory (of mathematics authority), and access for all students to an extent not addressed by the classroom teachers. The teachers were implementing the practice of TMinD for the first time, and for them some elements were more vital than others. The peripheral participants, less immediately involved in the arena of TMinD on a daily basis, reviewed the practice from a more objective stance.

Findings from the study were then added to the review of the literature to make a complete framework. All parts were needed to create a solid structure that houses TMinD. Many elements of the structure are identified in the literature, but within that body of work the specific term teaching mathematics in depth is rarely described beyond surface mention of it as a beneficial practice. This manuscript is not only the first comprehensive discussion of the practice, it is also part of a large-scale introduction of teaching mathematics in depth to all the school districts in a populous state with a diverse student body. As the experience in this state is given more study, new truths will emerge, but this initial contribution is a good starting point. Moreover, there are implications for research due to the significant overlap of this study with recent adoption of CCSS and plans for national assessment instruments. A position of relation to learning theory for both instruction and assessment has been offered here, a position that further research can verify. The research gap about teaching mathematics in depth has been mitigated, but the importance of the topic is such that this work is an introduction that can be expanded into other contexts.

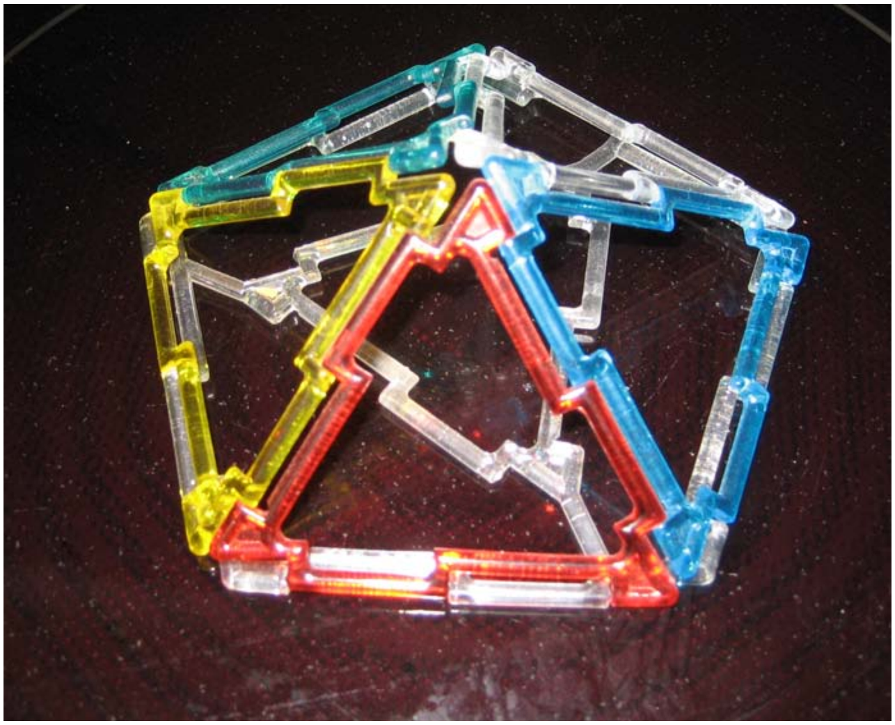

Figure 7-1. Model representing the tentative model for TMinD suggested in Chapter 3.
Photo courtesy of the author.

APPENDIX A
INTERVIEW QUESTIONS FOR PARTICIPATING CLASSROOM TEACHER

1. What is your initial impression of the Next Generation Sunshine State Standards for mathematics?
2. What interests you about the opportunities to implement this framework?
3. What do you see as its challenges?
4. What is the nature of the support you expect from the district?
5. How do you expect the students to react to the new curriculum?
6. Do you think you have ever taught in depth? Why or why not?
7. Do you have anything to add to this?

APPENDIX B
JOURNAL PROMPTS

1. How do you use groups? What did you think about the first exam you gave?

2. Tell me about your day.

3. How do you feel about making mistakes when you are working with the students? How do your students feel about making mistakes?

4. What means do you have to find out what the children understand about a particular concept?

5. Do the tasks in the math book encourage your students to use different strategies? Do the students talk about why their solutions work?

6. Please focus on three specific individuals who challenge you in mathematics—perhaps it is their behavior or their intelligence or their resistance or their lack of knowledge of English. Without identifying them beyond a number (Student #1, Student #2, Student #3), please write to me about the legacy you hope to provide for them this year in mathematics. At the end of the year, when you comment to their parents, what would you like to be able to say about their skills and dispositions in mathematics?

APPENDIX C
QUESTIONS FOR CURRICULUM RESOURCE TEACHERS

1. You recommended M____________________ as one of your best instructors of mathematics. What qualities are most impressive?
2. If you were to suggest him/her as a model, what might you expect a novice to respond to?
3. How do you interact with teachers in regards to mathematics?
4. How is the school administration preparing the teachers to teach Next Generation Sunshine State Standards for mathematics?
5. What opportunities will the new standards provide for the teachers?
6. What challenges do you anticipate for the teachers?
7. The Next Generation Sunshine State Standards for mathematics has tried to incorporate "Teaching in Depth." How would you define that practice?
8. What do you expect your role to be under Next Generation Sunshine State Standards for mathematics?
9. Is there anything you would like to add to this conversation?

APPENDIX D
QUESTIONS FOR INSTRUCTIONAL MATHEMATICS COACHES

1. What is your understanding of the Next Generation Sunshine State Standards for mathematics?
2. How are you preparing teachers under your supervision to teach this framework?
3. What opportunities do you see Next Generation Sunshine State Standards for mathematics providing teachers?
4. What challenges do you anticipate for the teachers?
5. Were you to model a Next Generation Sunshine State Standards mathematics lesson, what features would you emphasize?
6. I understand that a primary component of Next Generation Sunshine State Standards is the opportunity to "teach in depth". How would you describe what it means to "teach in depth"?
7. How might you prepare teachers to "teach in depth"?
8. Is there anything you would like to add to the remarks you already contributed?

APPENDIX E
QUESTIONS FOR MATHEMATICS TEACHER EDUCATORS

1. What is your understanding of the Next Generation Sunshine State Standards for mathematics?
2. How are you preparing your teachers-in-training to teach this framework?
3. What opportunities do you see Next Generation Sunshine State Standards for mathematics providing teachers?
4. What challenges do you anticipate for the teachers?
5. Were you to model a Next Generation Sunshine State Standards mathematics lesson, what features would you emphasize?
6. Were you to provide a PowerPoint lecture for your students, what content would you deem essential?
7. I understand that a primary component of Next Generation Sunshine State Standards is the opportunity to “teach in depth”. How would you describe what it means to “teach in depth”?
8. How might you prepare teachers to “teach in depth”?
9. Is there anything you would like to add to the remarks you already contributed?

LIST OF REFERENCES

Adams, T. L., Dixon, J. K., Larson, M., McLeod, J. C., & Leiva, M. A. (2011). *Go Math! Florida*. Orlando, FL: Houghton Mifflin Harcourt.

Adler, J. (1999). The dilemma of transparency: Seeing and seeing through talk in the mathematics classroom. *Journal for Research in Mathematics Education, 30*, 47-64.

Adler, M. J. (Ed.). (1984). *The Paideia Program: an educational syllabus*. New York, NY: Macmillan.

Alachua County Public Schools. (2009). *Middle School Course Descriptions, 2009-2010*.

Allal, L., & Ducrey, G. P. (2000). Assessment of—or in—the zone of proximal development. *Learning and Instruction, 10*(2), 137-152.

AMTE, NCTM, NCSM, & ASSM. (2010). Mathematics education organizations unite to support implementation of Common Core State Standards. Retrieved June 14, 2010, from http://www.amte.net/sites/all/themes/amte/resources/JointStatementonCCSS.pdf

Angus, D. L., & Mirel, J. E. (2003). Mathematics enrollments and the development of the high school in the United States, 1910-1994. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. I, pp. 441-489). Reston, VA: NCTM.

Atweh, B., Bleicher, R. E., & Cooper, T. J. (1998). The construction of the social context of mathematics classrooms: A sociolinguistic analysis. *Journal for Research in Mathematics Education, 29*, 63-82.

Au, W., Bigelow, W., & Karp, S. (Eds.). (2007). *Rethinking our classrooms* (New ed. Vol. 1). Milwaukee, WN: Rethinking Schools.

Balfanz, R., Mac Iver, D. J., & Byrnes, V. (2006). The implementation and impact of evidence-based mathematics reforms in high-poverty middle schools: A multi-site, multi-year study. *Journal for Research in Mathematics Education, 37*, 33-64.

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*, 132-144.

Ball, D. L., & Lampert, M. (1999). Multiples of evidence, time, and perspective: Revising the study of teaching and learning. In E. C. Lagemann & L. Shulman (Eds.), *Issues in education research: problems and possibilities* (pp. 371-398). San Francisco: Jossey-Bass.

Beecher, M. (2008). Closing the achievement gap with curriculum enrichment and differentiation: One school's story. *Journal of Advanced Academics, 19*(3), 502-530.

Ben-Yehuda, M., Lavy, I., Linchevski, L., & Sfard, A. (2005). Doing wrong with words: What bars students' access to arithmetical discourses. *Journal for Research in Mathematics Education, 36*, 176-247.

Bennett, A., Bridglall, B., Cauce, A. M., Everson, H. T., Gordon, E. W., Lee, C. D., et al. (2004). *All students reaching the top: Strategies for closing academic achievement gaps.* Paper presented at the North Central Regional Educational Laboratory.

Berry III, R. Q. (2008). Access to upper-level mathematics: The stories of successful African American middle school boys. *Journal for Research in Mathematics Education, 39,* 464-488.

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2004). Working inside the Black Box: Assessment for learning in the classroom. *Phi Delta Kappan, 86*(1), 9-21.

Black, P., & Wiliam, D. (1998a). Inside the Black Box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139-148.

Black, P., & Wiliam, D. (1998b). Assessment and classroom learning. *Assessment in Education: Principles, Policy, and Practice, 5*(1), 1-68.

Black, P., & Wiliam, D. (2006). Assessment for learning in the classroom. In J. Gardner (Ed.), *Assessment and Learning* (pp. 9-25). London, Thousand Oaks, New Dehli: SAGE.

Blanck, G. (1990). Vygotsky: The man and his cause. In L. C. Moll (Ed.), *Vygotsky and Education* (pp. 28). Cambridge, UK: Cambridge University Press.

Blanton, M. L., & Kaput, J. J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education, 36*, 412-446.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education, 29*, 41-62.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*, 239-258.

Boaler, J. (2008). *What's math got to do with it? Helping children learn to love their most hated subject—and why it's important for America*. New York: Viking.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of railside school. [study]. *Teachers College Record, 110*, 608-645.

Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research in education*. Boston, MA: Allyn and Bacon.

Booker, M. J. (2007). A roof without walls: Benjamin Bloom's taxonomy and the misdirection of American education. *Academic Quest, 20*, 347-355.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (1999). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., et al. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal, 34*, 663-689.

Burris, C. C., Wiley, E. W., Welner, K. G., & Murphy, J. (2008). Accountability, rigor, and detracking: Achievement effects of embracing a challenging curriculum as a universal good for all students. *Teachers College Record, 110*, 571-607.

Butler, D. L., & Cartier, S. C. (2004). Promoting effective task interpretation as an important work habit: A key to successful teaching and learning. *Teachers College Record, 106*, 1754-1757.

Butterworth, B. (1999). *What counts: How every brain is hardwired for math*. New York: The Free Press.

Campbell, P. F. (1996). *Transforming mathematics instruction in every elementary classroom: Using research as a basis for effective school practice*. Paper presented at the 8th International Congress on Mathematical Education.

Carpenter, T. P., & Fennema, E. (1991). Research and cognitively guided instruction. In E. Fennema, T. P. Carpenter & S. J. Lamon (Eds.), *Integrating Research on Teaching and Learning Mathematics* (pp. 1-16). Albany: State University of New York Press.

Carpenter, T. P., & Fennema, E. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research, 17*, 457-470.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Franke, M. L. (2004). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp. 18). Reston, VA: NCTM.

Chandler, C. C., & Kamii, C. (2009). Giving change when payment is made with a dime: The difficulty of tens and ones. *Journal for Research in Mathematics Education, 40*, 22.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Los Angeles: SAGE.

Clark, T., & Wright, P. (2006). Florida, math matters! Revision of the sunshine state standards for mathematics. *FCTM Dimensions, 26*(2), 9-12.

Clarke, A. E. (2009). From grounded theory to situational analysis: What's new? Why? How? In J. Morse, P. N. Stern, J. Corbin, B. Bowers, K. Charmaz & A. E. Clarke (Eds.), *Developing Grounded Theory: The Second Generation* (first ed., pp. 194-235). Walnut Creek, CA: Left Coast Press.

Clarke, D. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*, 278-308.

Cleveland City Schools. (2004). Curriculum Pacing Guide. from http://tech.clevelandschools.org/currdept/index.html

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28*, 258-277.

Cobb, P., & Merkel, G. (1989). Thinking strategies: Teaching arithmetic through problem solving. In P. R. Trafton & A. P. Shulte (Eds.), *New directions for elementary school mathematics: 1989 yearbook* (pp. 70-81). Reston, VA: National Council of Teachers of Mathematics.

Cobb, P., Wood, T., & Yackel, E. (1991). A constructivist approach to second-grade mathematics. In E. von Glasersfeld (Ed.), *Radical Constructivism in Mathematics Education* (pp. 157-176). Dordrecht, Netherlands: Kluwer.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*, 573-604.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G., Trigatti, B., et al. (1991). Assessment of a problem-centered second-grade mathematics project. *Journal for Research in Mathematics Education, 22*, 3-29.

Cobb, P., Yackel, E., Wood, T., & McNeal, B. (1993). Mathematics as procedural instructions and mathematics as meaningful activity: The reality of teaching for understanding. In R. B. Davis & C. A. Maher (Eds.), *Schools, mathematics, and the world of reality* (Vol. 4, pp. 119-133). Boston: Allyn and Bacon.

Cochran-Smith, M. (2004). Walking the road: Race, diversity, and social justice in teacher education. New York, NY: Teachers College Press.

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record, 102*, 294-343.

Cohen, D. K., & Hill, H. C. (2001). *Learning policy: When state education reform works*. New Haven and London: Yale University Press.

College Board. (2005). *Focus on success: A review of the Florida sunshine state standards for rigor and alignment to college readiness*: College Board.

Common Core State Standards Initiative, C. (2010). *Common Core State Standards for Mathematics*: The Council of Chief State School Officers of the National Governors Association Center for Best Practices (NGA Center).

Confrey, J. (1991). Steering a course between Vygotsky and Piaget. *Educational Researcher, 20*(8), 28-32.

Confrey, J. (2007). *Tracing the evolution of mathematics content standards in the United States: looking back and projecting forward*. Paper presented at the Conference on K-12 mathematics curriculum standards.

Cooper, B. (1998). Using Bernstein and Bourdieu to understand children's difficulties with "realistic" mathematics testing: An exploratory study. *Qualitative Studies in Education, 11*(4), 511-532.

Corno, L. (2004). Introduction to the special issue Work Habits and Work Styles: Volition in education. *Teachers College Record, 106*, 1669-1694.

Coyne, M. D., McCoach, D. B., Loftus, S., Zipoli Jr., R., & Kapp, S. (2009). Direct vocabulary instruction in kindergarten teaching for breadth versus depth. *The Elementary School Journal, 110*, 1-18.

Crandall, J., Jaramillo, A., Olsen, L., Peyton, J. K., & Young, S. (2008). Diverse teaching strategies for immigrant children. In R. W. Cole (Ed.), *Educating everybody's children: diverse teaching strategies for diverse learners* (pp. 64 word pages): ASCD (Association for Supervision and Curriculum Development).

Crawford, T. (2008). Winning the epistemological struggle: Consructing a cultural model of shared authority in an elementary classroom. *Teachers College Record, 110*, 1706-1736.

Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Thousand Oaks: SAGE.

Darling-Hammond, L., & Bransford, J. (Eds.). (2005). *Preparing teachers for a changing world*. San Francisco: Jossey-Bass.

David, J. L. (2008). Pacing Guides. *Educational Leadership, 66*.

Davis, B. (2004). *Inventions of teaching: A genealogy*. Mahway, NJ: Lawrench Erlbaum Associates.

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education, 34*, 137-167.

de Landsheere, V. (1977). On defining educational objectives. *Evaluation in Education: International Progress, 1*, 73-190.

Delpit, L. (2006). Lessons from teachers. *Journal of Teacher Education, 57*, 220-231.

Denzin, N. K., & Lincoln, Y. S. (2000). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of Qualitative Research* (pp. 1065). Thousand Oaks, CA: SAGE.

DeWalt, K. M., & DeWalt, B. R. (2002). *Participant observation: A guide for fieldworkers*. Walnut Creek, CA: AltaMira Press.

Dixon, J. K. (2008). An example of depth: Comparing fractions. *FCTM Dimensions, 28*(1), 5-8.

Dixon, J. K., & Kersaint, G. (2008). Unchartered territory: Using the *Curriculum Focal Points* as a basis for designing state standards. *NCSM Journal of Mathematics Education Leadership, 10*(1), 22-26.

Educational Services Incorporated. (1963). Goals for school mathematics. *Cambridge Conference on School Mathematics*, 102.

Empson, S. B. (2003). Low-performing students and teaching fractions for understanding: An interactional analysis. *Journal for Research in Mathematics Education, 34*, 305-343.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The State of the Art* (pp. 249-254). New York, NY: The Falmer Press.

Evertson, C. M., & Randolph, C. H. (1989). Teaching practices and class size: A new look at an old issue. *Peabody Journal of Education, 67*, 85-105.

Fennema, E., Carpenter, T. P., Franke, M. L., & Carey, D. A. (1993). Learning to use children's mathematics thinking: A case study. In R. B. Davis & C. A. Maher (Eds.), *Schools, mathematics, and the world of reality* (pp. 93-117). Boston: Allyn and Bacon.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 37*, 403-434.

Florida Department of Education. (2006). Cognitive complexity/depth of knowledge rating for mathematics. *Cognitive Complexity Classification of FCAT SSS Test Items*. Retrieved from http://www.floridastandards.org

Florida Department of Education. (2007). *Sunshine State Standards: Mathematics*.

Florida Department of Education. (2008). *Next Generation Sunshine State Standards*.

Florida Department of Education. (1996). *Grade level expectations for the Sunshine State Standards: Mathematics grades 3 - 5*. Retrieved from http://www.myfloridaeducation.com/.

Florida Department of Education. (2006). Cognitive complexity/depth of knowledge rating for mathematics. *Cognitive Complexity Classification of FCAT SSS Test Items*. Retrieved from http://www.floridastandards.org

Florida Department of Education. (2007a). *Assessment and accountability briefing book 2007*.

Florida Department of Education. (2007b). *Sunshine State Standards: Mathematics*.

Florida Department of Education. (2008). *2008 mathematics specifications for the 2009-2010 Florida state adoption of instructional materials*. Retrieved from thhp://www.fldoe.org/BII/Instruct_Mat/.

Florida Department of Education. (2010a). *Florida's Race to the Top memorandum of understanding for phase 2*. Retrieved from http://www.fldoe.org/ARRA/RacetotheTop.asp.

Florida Department of Education. (2010b). Home page. Retrieved July 14, 2010, from http://fcat.fldoe.org/fcatpub2.asp

Florida Department of Education. (2010c). School district and state public accountability report. Retrieved 1/27/11, 2011, from http://doeweb-prd.doe.state.fl.us/eds/nclbspar/year0910/nclb0910.cfm?dist_schl=1_31

Florida Department of Education. (2011a). *Florida begins the work of implementing Common Core State Standards and Assessments*. Tallahassee, FL: Retrieved from http://www.fldoe.org/arra/pdf/CCSSRolloutTimeline.pdf.

Florida Department of Education. (2011b). Student Success Act. Tallahassee, FL: Retrieved from http://www.fldoe.org/arra/pdf/april13tac.pdf.

Florida State Board of Education. (2010d). Workshop on Florida's System of Assessment and Accountability. Tallahassee.

Forehand, M. (2005). Bloom's taxonomy: Original and revised. In M. Orey (Eds.), Emerging perspectives on learning, teaching, and technology. Available from http://www.coe.uga.edu/epitt/bloom.htm.

Forman, E. A. (2003). A sociocultural approach to mathematics reform: Speaking, inscribing, and doing mathematics within communities of practice. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 333-352). Reston, VA: National Council of Teachers of Mathematics.

Fraivillig, J. L., Murphy, L. A., & Fuson, K. (1999). Advancing children's mathematical thinking in *Everyday Mathematics* classrooms. *Journal for Research in Mathematics Education, 30*, 148-170.

Franke, M. L., & Carey, D. A. (1997). Young children's perceptions of mathematics in problem-solving environments. *Journal for Research in Mathematics Education, 28*, 8-25.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (Vol. 1, pp. 225-256). Charlotte, NC: NCTM.

Frankel, R. M. (1999). Standards of qualitative research. In B. F. Crabtree & W. L. Miller (Eds.), *Doing qualitative research* (pp. 333-346). Thousand Oaks, CA: SAGE.

Gee, J. P. (2004). *Situated language and learning: A critique of traditional schooling*. New York: Routledge.

Good, T. L., & Grouws, D. A. (2004). The Missouri Mathematics Effectiveness Project: an experimental study in fourth-grade classrooms. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp. 10). Reston, VA: NCTM.

Goos, M. (2004). Learning mathematics in an classroom community of inquiry. *Journal for Research in Mathematics Education, 35*, 258-291.

Grbich, C. (2007). *Qualitative data analysis: an introduction*. Thousand Oaks, CA: SAGE.

Greer, B. (1996). Theories of mathematics education: The role of cognitive analyses. In L. P. Steffe & P. Nesher (Eds.), *Theories of Mathematical Learning* (pp. 179-196). Mahwah, NJ: Lawrence Erlbaum Associates.

Guberman, S. R. (2004). A comparative study of children's out-of-school activities and arithmetical achievements. *Journal for Research in Mathematics Education, 35*, 117-150.

Gutiérrez, R. (2002). Beyond essentialism: The complexity of language in teaching mathematics to Latina/o students. *American Educational Research Journal*, *39*, 1047-1088.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education, 34*, 37-73.

Gutstein, E., Lipman, P., Hernandez, P., & de los Reyes, R. (1997). Culturally relevant mathematics teaching in a Mexican American context. *Journal for Research in Mathematics Education, 28*, 709-737.

Hart, K. M. (1984). Ratio: Children's strategies and errors. A report of the strategies and errors in secondary mathematics project. Windsor, UK: The NFER-Nelson.

Hart, L. C. (1993). Some factors that impede or enhance performance in mathematical problem solving. *Journal for Research in Mathematics Education, 24,* 167-171.

Hatch, J. A. (2002). *Doing qualitative research in education settings*. Albany: State University of New York Press.

Hauser, I. (2002). Every class is an English class. In D. Worsley (Ed.), *Teaching for Depth: Where math meets the humanities* (pp. 32-46). Portsmouth, NH: Heinemann.

Hedegaard, M. (1990). The zone of proximal development as a basis for instruction. In L. Moll (Ed.), *Vygotsky and education: instructional implications and applications of sociohistorical psychology* (pp. 22). Cambridge, UK: Cambridge University Press.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. [study]. *Journal for Research in Mathematics Education, 28*, 524-549.

Henry, V. J., & Brown, R. S. (2008). First-grade basic facts: An investigation into teaching and learning of an accelerated, high-demand memorization standard. *Journal for Research in Mathematics Education, 39*, 31.

Herrera, T. A., & Owens, D. T. (2001). The "new new math"? Two reform movements in mathematics education. *Theory into Practice*, *40*, 84-93.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning: a project of the National Council of Teachers of Mathematics* (pp. 65-97). New York, NY: Macmillan

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Wearne, D., Murray, H., et al. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*, 393-425.

Hinchey, P. H. (1998). *Finding freedom in the classroom: A practical introduction to critical theory*. New York, NY: Peter Lang.

Hollebrands, K. F. (2007). The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for Research in Mathematics Education, 38*, 29.

Hong, Y.-y., Chiu, C.-y., Dweck, C. S., Lin, D. M.-S., & Wan, W. (1999). Implicit theories, attributions, and coping: A meaning system approach. *Journal of Personality and Social Psychology*, *77*, 12.

Hudson, P., & Hudson, S. (2008). Changing preservice teachers' attitudes for teaching in rural schools. *Australian Journal of Teacher Education, 33*(4), 67-77.

Hufferd-Ackles, K., Fuson, K., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*, 81-116.

International Association for the Evaluation of Educational Achievement. (2007). TIMSS 1995. from http://www.iea.nl/timss1995.html

International Association for the Evaluation of Educational Achievement. (2011). Trends in International Mathematics and Science Study 2007 Retrieved June 6, 2011, from http://www.iea.nl/timss2007.html

Irvine, J. J. (2003). *Educating teachers for diversity: Seeing with a cultural eye*. New York, NY: Teachers College Press.

Irwin, K. C. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education, 32*, 399-420.

Izsak, A., Tillema, E., & Tunc-Pekkan, Z. (2008). Teaching and learning fraction addition on number lines. *Journal for Research in Mathematics Education, 39*, 33-62.

Jacobs, J. K., Hiebert, J., Givvin, K. B., Hollingsworth, H., Garnier, H., & Wearne, D. (2006). Does eighth-grade mathematics teaching in the United States align with the NCTM *standards?* Results from the TIMSS 1995 and 1999 video studies. *Journal for Research in Mathematics Education, 37*, 5-32.

Jacobs, J. K., & Morita, E. (2002). Japanese and American teachers' evaluations of videotaped mathematics lessons. *Journal for Research in Mathematics Education, 33*, 154-175.

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education, 38*, 258-288.

James, M. (2006). Assessment, teaching and theories of learning. In J. Gardner (Ed.), *Assessment and Learning* (pp. 47-60). London: SAGE.

Johanning, D. I. (2008). Learning to use fractions: Examining middle school students' emerging fraction literacy. *Journal for Research in Mathematics Education, 39*(3), 30.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal, 102*, 59-80.

Keiser, J. M., & Lambdin, D. V. (1996). The clock is ticking: time constraint issues in mathematics teaching reform. *Journal of Educational Research, 90*, 23-30.

Kersaint, G., & Dogbeu, J. (2006). Florida's fourth grade student achievement: A comparison with the nation. *FCTM Dimensions, 26*(2), 48-54.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Kinard, J. T., & Kozulin, A. (2005). Rigorous mathematical thinking: Mediated learning and psychological tools. *Focus on learning problems in mathematics, 27*(3), 29.

Knapp, N. F., & Peterson, P. L. (1995). Teachers' interpretations of "CGI" after four years: Meanings and practices. *Journal for Research in Mathematics Education, 26*, 40-65.

Koro-Ljungberg, M., Yendol-Hoppey, D., Smith, J. J., & Hayes, S. B. (2009). (E)pistemological awareness , instantiation of methods, and uniformed methodological ambiguity in qualitative research projects. *Educational Researcher, 38*, 687-699.

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: an overview. *Theory into Practice, 41*, 212-218.

Kuzel, A. J. (1999). Sampling in qualitative inquiry. In B. F. Crabtree & W. L. Miller (Eds.), *Doing Qualitative Research* (pp. 33-45). Thousand Oaks, CA: SAGE.

Ladson-Billings, G. (1994). *The Dreamkeepers: Successful teachers of African American children*. San Francisco, CA: Jossey-Bass

Lamon, S. J. (1996). The development of unitizing: Its role in children's partitioning strategies. [study]. *Journal for Research in Mathematics Education, 27*, 170-193.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition & Instruction, 3*, 38.

Lampert, M. (1991). Connecting mathematical thinking and learning. In E. Fennema, T. P. Carpenter & S. J. Lamon (Eds.), *Integrating Research on Teaching and Learning Mathematics* (pp. 121-152). Albany: State University of New York Press.

Lampert, M. (2004). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in Mathematics Education Research* (pp. 152-171). Reston, VA: NCTM.

Lampert, M., & Ball, D. L. (1998). Teaching, multimedia, and mathematics: Investigations of real practice

Lappan, G., & Schram, P. W. (1989). Communication and reasoning: Critical dimensions of sense making in mathematics. In P. R. Trafton & A. P. Shulte (Eds.), *New Directions for Elementary School Mathematics: 1989 yearbook* (pp. 14-30). Reston, VA: National Council of Teachers of Mathematics.

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education, 27,* 133-150.

Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education, 29*, 533-554.

Lindquist, M. M. (1989). It's time to change. In P. R. Trafton & A. P. Shulte (Eds.), *New Directions for Elementary School Mathematics: 1989 Yearbook* (pp. 1-13). Reston, VA: National Council of Teachers of Mathematics.

Lo, J.-J., Wheatley, G., & Smith, A. C. (1994). The participation, beliefs, and development of arithmetic meaning of a third-grade student in mathematics class discussions. *Journal for Research in Mathematics Education, 25*, 30-49.

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31,* 454-482.

Mathematical Association of America Online. (2004). *What is important in school mathematics:*. Paper presented at the PMET, Park City, Utah.

Macnab, D. (2000). Forces for change in mathematics education: The case of TIMSS. *Education Policy Analysis Archives, 8*(15).

Malloy, C. E., & Jones, M. G. (1998). An investigation of African American students' mathematical problem solving. *Journal for Research in Mathematics Education, 29*, 143-163.

Marton, F., & Neuman, D. (1996). Phenomenography and children's experience of division. In L. P. Steffe & P. Nesher (Eds.), *Theories of Mathematical Learning* (pp. 315-334). Mahwah, NJ: Lawrence Erlbaum.

McCaslin, M., & Hickey, D. T. (2001). Self-regulated learning and academic achievement: A Vygotskian view. In B. J. Zimmerman & D. H. Schunk (Eds.), *Self-regulated learning and academic achievement: theoretical perspectives* (pp. 227-252). Mahwah, NJ: Lawrence Erlbaum.

McClain, K., & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32*(3), 31.

Meyer, D. K., & Turner, J. E. (2002). Using instructional discourse analysis to study the scaffolding of student self-regulation. *Educational Psychologist, 37*, 17-25.

Miller, L. S. (1995). *An American imperative: Accelerating minority educational advancement*. New Haven, CT: Yale University Press.

Mimbs, C. A. (2005). Teaching from the critical-thinking, problem-based curricular approach: Strategies, challenges, and recommendations. *Journal of Family and Consumer Sciences Education, 23*, 7-18.

Montalvo, G. P., Mansfield, E. A., & Miller, R. B. (2007). Liking or disliking the teacher: Student motivation, engagement and achievement. *Evaluation & Research in Education, 20*, 15.

Mullis, I. V. S., Martin, M. O., Beaton, A. E., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1998). *Mathematics achievement in the primary school years: IEA's Third International Mathematics and Science Report*. Boston, MA: International Study Center at Boston College.

Murphy, P. K., Delli, L. a. M., & Edwards, M. N. (2004). The good teacher and good teaching: Comparing beliefs of second-grade students, preservice teachers, and inservice teachers. *The Journal of Experimental Education, 72*, 69-92.

National Association for Professional Development Schools. (2011). Congratulations to the 2011 recipients of the Exemplary PDS Achievement Award. 2011, from http://www.napds.org/default.html

National Center for Educational Statistics. (2009). NAEP state comparisons. Retrieved 11/19/2009, 2009, from http://nces.ed.gov/nationsreportcard/statecomparisons/

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Retrieved June 15, 2010, 2010, from http://www.nctm.org/standards/content.aspx?id=17278

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: NCTM.

Nelson, B. S. (2001). Constructing Facilitative Teaching. In T. Wood, B. S. Nelson & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 251-274). Mahwah, NJ: Lawrency Erlbaum Associates.

Newton, D. P. (2000). *Teaching for understanding: What it is and how to do it*. London, UK: Routledge/Falmer.

Ng, S. F., & Lee, K. (2009). The model method: Singapore children's tool for representing and solving algebraic word problems. *Journal for Research in Mathematics Education, 40,* 282-313.

Nicol, D. J., & Macfarlane-Dick, D. (2006). Formative assessment and self- regulated learning: A model and seven principles of good feedback practice *Studies in Higher Education, 31,* 199-218.

Nieto, S. (2000). *Affirming diversity: The sociopolitical context of multicultural education* (third ed.). New York, NY: Longman.

Noble, T., Nemirovsky, R., Wright, T., & Tierney, C. (2001). Experiencing change: The mathematics of change in multiple environments. *Journal for Research in Mathematics Education, 32*, 85-108.

Norton, A., & D'Ambrosio, B. S. (2008). ZPD and ZPC: Zones of teaching and learning. *Journal for Research in Mathematics Education, 39*, 220-246.

Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven. CT: Yale University Press.

Ormrod, J. E. (2008). *Human learning* (fifth ed.). Upper Saddle River, NJ: Pearson, Merrill, Prentice Hall.

Otero, V. K. (2006). Moving beyond the "get it or don't" conception of formative assessment. *Journal of Teacher Education, 57*, 247-255.

Paris, S. G., & Paris, A. H. (2001). Classroom applications of research on self-regulated learning. *Educational Psychologist*, *36*, 89-101.

Paul, F. G. (2005). Grouping within Algebra I: A structural sieve with powerful effects for low-income, minority, and immigrant students. *Educational Policy, 19*, 262-282.

Payne, J. N. (2003). The New Math and its aftermath. grades K-8. In G. M. A. Stanic & J. Kilpatrick (Eds.), *A history of school mathematics* (Vol. 1, pp. 559-598). Reston, VA: NCTM.

Porter, A. (1989). A curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*, 9-15.

Post, T. R., Harwell, M. R., Davis, J. D., Maeda, Y., Cutler, A., Anderson, E., et al. (2008). *Standards*-based mathematics curricula and middle-grade students' performance on standardized achievement tests. *Journal for Research in Mathematics Education, 39*, 184-212.

Prawat, R. S. (1991). *Experts' views on the elementary mathematics curriculum: Visions of the ideal and critique of currect practice*.

Putnam, R. T., Lampert, M., & Peterson, P. L. (1989). *Alternative perspectives on knowing mathematics in elementary schools.*

Putney, L. G., & Floriani, A. (1999). Examining transformative processes and practices: A cross-case analysis of life in two bilingual classrooms. *Journal of Classroom Interaction, 34*(2), 17-29.

Pyke, C. L. (2003). The use of symbols, words, and diagrams as indicators of mathematical cognition: A causal model. *Journal for Research in Mathematics Education, 34*, 406-432.

Pyke, C. L., & Lynch, S. (2005). Mathematics and science teachers' preparation for National Board of Professional Teaching Standards Certification. *School Science and Mathematics, 105*(1), 25-35.

Raider-Roth, M. B. (2005). Trusting what you know: Negotiating the relational context of classroom life. *Teachers College Record 107*, 44.

Reid, D. A. (2002). Conjectures and refutation in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*, 5-29.

Remillard, J. T. (1992). Teaching mathematics for understanding: a fifth-grade teacher's interpretation of policy. *The Elementary School Journal, 93*, 179-193.

Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*, 352-388.

Reys, R. E., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of Standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34*, 74-95.

Reys, R. E., & Yang, D.-C. (1998). Relationship between computational performance and number sense among sixth- and eighth-grade students in Taiwan. *Journal for Research in Mathematics Education, 29*, 225-237.

Riordan, J. E., & Noyce, P. E. (2001). The impact of two Standards-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education, 32*, 368-398.

Rix, E. (2004). Learning in context: Probing the theories of Piaget and Vygotsky. On *Films for the Humanities and Sciences* [DVD Video]. Princeton, NJ: Films for the Humanities and the Sciences.

Robison, G. B. (1960). The new look in mathematics. *Teacher Education Quarterly 19*, 58-60.

Romance, N. R., & Vitale, M. R. (2001). Implementing an in-depth expanded science model in elementary schools: Multi-year findings, research issues, and policy implications *International Journal of Science Education, 23*, 373-404.

Romance, N. R., & Vitale, M. R. (2007). *Elements for bringing a research-validated intervention to scale: Implications for leadership in educational reform*. Paper presented at the Annual Meeting of the American Educational Research Association.

Sadler, D. R. (1983). Evaluation and the improvement of academic learning. *The Journal of Higher Education*, *54*, 60-79.

Schliemann, A. D., Araujo, C., Cassundé, M. A., Macedo, S., & Nicéas, L. (1998). Use of multiplicative commutativity by school children and street sellers. *Journal for Research in Mathematics Education, 29*, 422-435.

Schmidt, W. H., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator*, 1-18.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision: An investigation of U.S. science and mathematics education: Executive summary*: U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University.

Schoen, H. L., Cebulla, K. J., Finn, K. F., & Fi, C. (2003). Teacher variables that relate to student achievement when using a Standards-based curriculum. *Journal for Research in Mathematics Education, 34*, 228-259.

Schoen, R. C., & Clark, J. T. (2007). Florida's answer to "A mile wide and an inch deep:" Mathematics content standards that allow time for in-depth teaching and learning. *FCTM Dimensions, 27*(2), 6-12.

Schroeder, T. L., & Lester Jr., F. K. (1989). Developing understanding in mathematics via problem solving. In P. R. Trafton & A. P. Shulte (Eds.), *New Directions for Elementary School Mathematics* (pp. 31-42). Reston, VA: National Council of Teachers of Mathematics.

Schunk, D. H. (2001). Social cognitive theory and self-regulated learning. In B. J. Zimmerman & D. H. Schunk (Eds.), *Self-regulated learning and academic achievement: Theoretical perspectives* (pp. 125-152). Mahwah, NJ: Lawrence Erlbaum.

Schunk, D. H., & Zimmerman, B. J. (1997). Social origins of self-regulatory competence. *Educational Psychologist, 32*, 195-208.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*, 447-466.

Sfard, A. (2003). Balancing the unbalanceable: The NCTM standards in light of theories of learning mathematics. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A Research companion to principles and standards for school mathematics* (pp. 40). Reston, VA: NCTM.

Sheridan School District. (2011). Introduction to pacing guides for mathematics. Retrieved May 4, 2011, from http://www.sheridanschools.org/curriculum.cfm?subpage=59179

Sherin, M. G. (2004). Perspective on when the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. In T. P. Carpenter, J. A. Dossey & J. L. Koehler (Eds.), *Classics in Mathematics Education Research* (pp. 152). Reston, VA: NCTM.

Shindler, J., Jones, A., Taylor, C., & Cadenas, H. (2004). Does seeking to create a better classroom climate lead to student success or improved teaching? Examining the relationship between pedagogical choices and classroom climate in urban secondary schools. In L. Kim (Ed.), *2004 yearbook: urban learning, teaching, and research* (pp. 27-36): American Educational Research Association (AERA) Special Interest Group.

Shulman, L. (2002). Making differences. *Change, 34*(6), 9p.

Shulman, L. (2004). *The wisdom of practice: Essays on teaching, learning, and learning to teach*. San Francisco, CA: Jossey-Bass.

Silver, E. A., Mesa, V. M., Morris, K. A., Star, J. R., & Benken, B. M. (2009). Teaching mathematics for understanding: An analysis of lessons submitted by teachers seeking NBPTS certification. *American Educational Research Journal, 46*, 501-531.

Smith III, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*, 387-402.

Smith, T. M. (2004). Curricular reform in mathematics and science since *A nation at risk*. *Peabody Journal of Education, 79*, 25.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks use in reform classrooms. *American Educational Research Journal, 33*, 455-488.

Stipek, D., Salmon, J. M., Givvin, K. B., Kazemi, E., Saxe, G., & MacGyvers, V. L. (1998). The value (and convergence) of practices suggested by motivation research and promoted by mathematics education reformers. *Journal for Research in Mathematics Education, 29*, 465-488.

Stobart, G. (2006). The validity of formative assessment. In J. Gardner (Ed.), *Assessment and Learning* (pp. 133-145). London, UK: SAGE.

Stoltzfus-Dueck, T. (2006). Teaching in depth. *Nature, 439*, 1.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques*. Newbury Park, CA: SAGE.

Student Success Act Factsheet. (2011). Governor Scott signs Student Success Act. 2011, from http://www.flgov.com/2011/03/24/governor-scott-signs-student-success-act/

Tappen, M. J., & Clark, T. (2009). *Textbook adoption and discretionary spending*.

Timar, T. B. (1997). The institutional role of state education departments: A historical perspective. *105*, 231-260.

Truxaw, M. P., & DeFranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education, 39*, 489-525.

Tudge, J. (1990). Vygotsky, the zone of proximal development, and peer collaboration: implications for classroom practice. In L. Moll (Ed.), *Vygotsky and education: instructional implications and applications of sociohistorical psychology* (pp. 20). Cambridge, UK: Cambridge University Press.

Turner, J. C., & Patrick, H. (2004). Motivational influences on student participation in classroom learning activities. *Teachers College Record, 106*, 1759-1785.

U.S. Department of Education. (2009). *Race to the Top program executive summary*.

U.S. Department of Education Mathematics and Science Expert Panel. (1999). *Exemplary and promising mathematics programs 1999*.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*, 89-110.

Vaccari, D. (2009). Instructional materials.   Retrieved 11/18/2009, 2009, from http://www.fldoe.org/Bll/instruct_mat/

van Loggerenberg-Hattingh, A. (2003). Examining learning achievement and experiences of science learners in a problem-based learning environment. *South African Journal of Education, 23*, 52-57.

van Oers, B. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. P. Steffe & P. Nesher (Eds.), *Theories of Mathematical Learning* (pp. 91-114). Mahwah, NJ: Lawrence Erlbaum Associates.

Vandevoort, L. G., Amrein-Beardsley, A., & Berliner, D. C. (2004). National Board Certified teachers and their students' achievement. *Educational Policy Analysis Archives, 12*(46).

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education, 28*, 25.

Villasenor Jr., A., & Kepner Jr., H. S. (1993). Arithmetic from a problem-solving perspective: An urban implementation. *Journal for Research in Mathematics Education, 24,* 62-69.

von Glasersfeld, E. (1996). Aspects of radical constructivism and its educational recommendations. In L. P. Steffe & P. Nesher (Eds.), *Theories of Mathematical Learning* (pp. 307-314). Mahwah, NJ: Lawrence Erlbaum Associates.

Webb, N. L. (1999). *Alignment of science and mathematics standards and assessments in four states*. Madison, WI: National Instititute for Science Education, University of Wisconsin-Madison.

Webb, N. M., & Farivar, S. (1994). Promoting helping behavior. *American Educational Research Journal, 31*, 369-394.

Wiggins, G., & McTighe, J. (2005). *Understanding by design*. Alexandria, VA: Association for Supervision and Curriculum Development.

Wilson, L. D., & Kenney, P. A. (2003). Classroom and large-scale assessment. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 53-67). Reston, VA: National Council of Teachers of Mathematics.

Wood, T. (1999). Creating a context for argument in mathematics class. *Journal for Research in Mathematics Education, 30,* 21.

Wood, T., Nelson, B. S., & Warfield, J. (Eds.). (2001). *Beyond classical pedagogy: Teaching elementary school mathematics*. Mahwah, NJ: Lawrence Erlbaum.

Wood, T., & Sellers, P. (1996). Assessment of a problem-centered mathematics program: Third grade. *Journal for Research in Mathematics Education*, *27*, 337-353.

Wood, T., & Sellers, P. (1997). Deepening the analysis: Longitudinal assessment of a problem-centered mathematics program. *Journal for Research in Mathematics Education, 28,* 163-186.

Wood, T., Williams, G., & McNeal, B. (2006). Children's mathematical thinking in different classroom cultures. *Journal for Research in Mathematics Education, 37*, 222-255.

Worsley, D. (Ed.). (2002). *Teaching for depth: Where math meets the humanities*. Portsmouth, NH: Heinemann.

Wright, S. P., Horn, S. P., & Sanders, W. L. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluation in Education, 11*, 57-67.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27,* 458-477.

Zambo, R., & Zambo, D. (2007). Mathematics and the learning cycle: How the brain works as it learns mathematics. *Teaching Children Mathematics, 13*, 265-270.

Zimmerman, B. J., & Schunk, D. H. (Eds.). (2003). *Educational psychology: A century of contributions*. Mahway, NJ: Lawrence Erlbaum Associates.

Zull, J. (2002). The art of changing the brain: Enriching teaching by exploring the biology of learning. 263.

BIOGRAPHICAL SKETCH

Joanne LaFramenta is a retired middle school mathematics teacher with a master's degree in teaching from the University of Chicago. She taught in public schools for thirty-four years in elementary, middle, and high school, as well as in community college and the University of Florida. She is a National Board Certified Teacher of Early Adolescence/Mathematics. She graduated from the University of Florida in the summer of 2011, with a degree of Doctor of Philosophy in Curriculum and Instruction.