Inattention in individual expectations ☆

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Abstract

This paper investigates the expectations formation process of economic agents about inflation rate. Using the Market Expectations System of Central Bank of Brazil, we perceive that agents do not update their forecasts every period and that even agents who update disagree in their predictions. We then focus on the two most popular types of inattention models that have been discussed in the recent literature: sticky-information and noisy-information models. Estimating a hybrid model we find that, although formally fitting the Brazilian data, it happens at the cost of a much higher degree of information rigidity than observed.

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Resumo

Este artigo investiga o processo de formação de expectativas de inflação de agentes econômicos. Utilizando o Sistema de Expectativas de Mercado do Banco Central do Brasil, percebemos que os agentes não atualizam suas projeções em todos os períodos e mesmo aqueles agentes que o fazem discordam sobre os valores previstos. Neste sentido, investigamos os dois tipos de modelos mais populares sobre inatenção discutidos na literatura recente: informação com rigidez e informação com ruído. Com base

na estimação de um modelo híbrido, concluímos que, embora formalmente o modelo seja capaz de se ajustar aos dados brasileiros, tal resultado ocorre ao custo de um grau muito maior de rigidez informacional do que o observado.

Palavras-chave: Expectativas; Inflação; Informação imperfeita; Inatenção racional

1. Introduction

The expectations formation process of economic agents about macroeconomic variables has long been one of the most debated questions in macroeconomics. Nevertheless remains an open question how expectations are formed, and how best to model it. In much classical theory, there is no room for disagreement in expectations, since it is usually assumed that all agents form expectations conditional on a common information set. However if not everyone has the same expectations and the information frictions are large and economically significant, the degree of information rigidity may have significant implications for macroeconomic dynamics and optimal policy.

What we aim to do is related to the recent empirical work trying to determine the nature of the expectations formation process. Rational expectations models with information frictions such as Mankiw and Reis (2002), Reis (2006a,b), Sims (2003) and Woodford (2003) have been associated to agents' inattention to new information, due to costs of collecting and processing information. These models have the key advantage of parsimoniously explaining some patterns of individual expectations observed in the data – such as disagreement across forecasters and predictable forecast errors – that are conflicting with the standard hypothesis of perfect information.

The sticky-information models proposed by Mankiw and Reis (2002) and Reis (2006a,b) are based on the assumption that the agents do not have access to information instantly. In Mankiw and Reis (2002), for instance, it is assumed that the acquisition of information follows a Poisson process in which, at each date, agents face a given and constant probability λ of being able to get new information. Nevertheless, once agents update their information set, they obtain perfect information and form expectations rationally. Thus we refer to λ as the attention degree for the sticky-information model and $(1 - \lambda)$ can be seen as the degree of information rigidity.

The infrequent update implies that, each period, only a fraction of the agents has access to the latest macroeconomic news and the expectations and actions of those who did not update their information sets continue to be based on their old information. As a result, agents who updated their information sets in the same period must make the same forecasts and agents who did not have access to new information should not revise their last prediction.

On the other hand, in the noisy-information models developed by Sims (2003) and Woodford (2003), although agents continuously track variables and update their information set, they only observe noisy signals about the true state. As agents know they have an imperfect access to the news they get at each period, they do not completely pass it onto their forecast. More precisely, forecasts are a weighted average of the new and the previous information received, so the weight on previous beliefs is taken as the degree of information rigidity.

Following Andrade and Le Bihan (2013), we will focus on these two most popular types of inattention models that have been discussed in the recent literature: sticky-information and noisy-information models. The model – developed by these authors – is a hybrid one: it assumes that, at each date, every forecaster faces a given probability of being able to update his information set and that, when updating, he gets a noisy perception of the state of the economy. The model is then estimated by a Minimum Distance Estimation (MDE) procedure, which allows us to test if the model is capable of quantitatively fitting the data, particularly the forecast errors and the disagreement among forecasters.

As far as we know, we are one of the first authors to use expectations data on inflation in Brazil in order to try to model it based on inattention models.[1] As the response of agents to macroeconomic dynamics is strongly impacted by the way individual expectations are formed, modeling the expectations formation process is important for better conduct of economic policy and better understanding its implications. An advantage over Andrade and Le Bihan's

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[1] Guillén (2008) investigates a set of expectations theories using Brazilian data and concludes that the median inflation forecast is more likely to conform to the sticky-information theory.

(2013) study is that forecasts on our database are observed in a daily frequency – theirs are on a quarterly frequency. This means we are able to follow the sequence of forecasts made by each agent more closely.

The remainder of this paper is structured as follows. In Section 2 we make a brief literature review and in Section 3 we present some basic facts about forecast errors and disagreement between forecasters. Section 4 presents the methodology we apply and in Section 5 we introduce the database we will use, namely the Market Expectations System of Central Bank of Brazil. Finally, Section 6 presents the results of our estimation and Section 7 concludes.

2. Literature review

The sticky-information Phillips curve proposed by Mankiw and Reis (2002) was developed in order to obtain an alternative to the new Keynesian Phillips curve – a sticky-price model – that failed in explaining some aspects of macroeconomic fluctuations.[2] In this model, in each period, a share λ of the firms update their information set[3] and compute a new path of optimal prices. The ones that did not receive new information continue to set prices based on old plans. A firm's optimal price is given by (all variables expressed in logs):

$$p_t^* = p_t + \alpha y_t$$

where p_t is overall price level and y_t output gap. A firm that updated its information set j periods ago sets the price $E_{t-j}[p_t^*]$ and aggregate price level is the average of the prices set by all firms:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j}(p_t + \alpha y_t)$$

Thus inflation rate can be represented by:

$$\pi_t = \left[\frac{\alpha\lambda}{1-\lambda}\right] y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(\pi_t + \alpha \Delta y_t).$$

Defining money supply (or aggregate demand) by $m = p + y$ and taking it to be exogenous, they examine how output and inflation respond to variation of m under the sticky-information model, comparing it to the dynamic properties of the sticky-price model and a backward-looking model. The responses of the sticky-information model seem consistent with what happens when economies are exposed to these shocks.

In the same line, Reis (2006b) and Reis (2006a) study respectively the problem of a producer and the consumption decisions of agents who face costs of acquiring and/or interpreting new information and try to understand the dynamic response of prices to shocks in order to explain inflation dynamic. In this setup, the agents rationally decide to be inattentive to new information and choose the optimal length of inattentiveness. But once they pay attention to new information, they become aware of everything that is relevant.

Sims (2003), in turn, assumes that people have limited information-processing capacity – instead of assuming that agents update their information set only sporadically. Here the limited capacity is represented by the fact that information that agents have access may be contaminated with a random noise. More precisely, information is thought of as moving through a channel in which one enters input data, and output data emerges, possibly with error. The author takes the nature of the noise agents face as exogenous and assume that noise changes systematically according to changes in the dynamic properties of the economy. Using the idea of a finite Shannon capacity,[4] he analyses the implications of including these information-processing constraints to the dynamic programming problem used to model behavior. The implications – in terms of altering part of its outcomes – seem in line with observed macroeconomic behavior.

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[2] For example: the sticky-price model cannot explain why inflation is so persistent or why shocks to monetary policy have gradual and delayed effect on inflation, and it yields that announced and credible disinflation leads to booms.

[3] Each firm has the same probability of being one of the firms that receive new information, regardless of how long it has been since its last update.

[4] In information theory, the Shannon capacity is the maximum amount of information that can be transmitted by a channel without error. It depends essentially on the channel bandwidth and on the signal-to-noise ratio.

Following the approach proposed by Sims (2003), Woodford (2003), Moscarini (2004) and Mackowiak and Wiederholt (2009) model inattentiveness by price-setters assuming that agents have limited capacity to acquire and/or process all the information in their environment. In Moscarini (2004), agents also choose not to update their information systematically. In this sense, the model is also related to Reis' (2006a,b). But here, once they update, they do not obtain a perfect signal on the state of the economy.

Woodford (2003) assumes that each price-setter acts taking into account his own subjective perception of the state of aggregate demand, which is modeled as an observation of the true value with an idiosyncratic error, and that he forms optimal estimates of the aggregate state variables given this imperfect information. He assumes that the estimates are updated in real time using a Kalman filter. The price-setter (correctly) believes that the economy's aggregate state evolves according to

$$\bar{X}_t = \bar{c} + M\bar{X}_{t-1} + mu_t$$

where $\bar{c}$ and m are vectors, M a matrix, u_t is assumed to represent a monetary policy shock and

$$\bar{X}_t = \begin{bmatrix} X_t \\ \sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} X_t^{(k)} \end{bmatrix}$$

with $X_t^{(k)}$ being higher-order expectations – others' expectations of others' expectations ... – and $\xi \in (0, 1)$ being a measure of strategic complementarity. $\sum_{k=1}^{\infty} \xi(1-\xi)^{k-1} X_t^{(k)}$ indicates that weighted average of expectations and higher-order expectations also matter for the determination of prices, output and of their future evolution.

The only information received by supplier i in period t (the observation equation) takes the form

$$z_t(i) = e_1' \bar{X}_t + v_t(i)$$

where $v_t(i)$ is a mean-zero Gaussian white noise error term, distributed independently both of the history of fundamental disturbances and of the observation errors of all other suppliers. e_j refer to jth unit vector.

Thus, i's optimal estimate of the state vector evolves according to:

$$\bar{X}_{t|t}(i) = \bar{X}_{t|t-1}(i) + k[z_t(i) - e_1' \bar{X}_{t|t-1}],$$

where k is the vector of Kalman gains. And the date t state perceived at $t-1$ is given by

$$\bar{X}_{t|t-1}(i) = \bar{c} + M\bar{X}_{t-1|t-1}(i).$$

Woodford analyses the responses of inflation and output to a monetary shock implied by his model and compares it to those implied by the sticky-price model. When the effect of changes on nominal GDP growth present considerable persistence, the noisy-information model matches the empirical evidence better.

Some authors have also exploited survey of forecasts of macroeconomic variables to produce micro-facts that characterize the formation of expectations and, then, tried to assess which model best describe the behavior of forecasters. Mankiw et al. (2004) investigate whether the amplitude of disagreement observed in the data about inflation expectations, as well as its evolution over time, can be predicted by the sticky-information model. They find that this approach seems capable of accounting for many aspects of the observed dispersion and central tendency during the period under study, but it does not consistently generate enough time variation in disagreement.

Branch (2007) extended Mankiw and Reis's (2002) sticky-information model allowing the distribution of information across agents to vary over time and found that this model provides a better fit to the distribution of the survey data used than the basic/static version does.[5] Patton and Timmermann (2010), on the other hand, focus on the relation of the source of disagreement and differences in agents' beliefs. They explore survey data cross-sectional dispersion in forecasters' predictions at several forecast horizons and find evidence that differences in opinion do not stem from differences in information sets, but from heterogeneity in priors.

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[5] In this sense, models that consider the interaction between dispersed and sticky information also provide an alternative framework to the standard approach (e.g., Morris and Shin, 2002; Areosa and Areosa, 2012; Areosa et al., 2012).

Both Coibion and Gorodnichenko (2012a) and Coibion and Gorodnichenko (2012b) find evidence against the null of full information consistent with the presence of information rigidities. Coibion and Gorodnichenko (2012b) use aggregated survey data from US professional forecasters, consumers, firms, and central bankers to explore the conditional response of the average forecast error and of the disagreement across forecasters to various structural shocks. They find evidence in favor of the two models. Coibion and Gorodnichenko (2012a) propose an approach that does not require the identification of shocks. They document evidence that the economic conditions affect how much resources are devoted to the collection and processing of information. The degree of information rigidity seems to vary across macroeconomic variables and this variation goes in the manner predicted by noisy-information models.

Andrade and Le Bihan (2013) and Andrade et al. (2013) also use survey data to analyze information rigidity models for the euro zone and the United States. They find evidence on the data in favor of imperfect information models and focus on sticky and noisy-information models. An advantage of both studies is that they rely on multivariate models, which take into account the dynamic interactions across variables when agents form their expectations. Andrade et al. (2013) use aggregated survey data to study the term structure of disagreement for US real output growth, consumer price index inflation and the federal funds rate, encountering that disagreement is time varying at all horizons, including the very long run and that the term structure of disagreement differs significantly across variables. Their results indicate that both sticky- and noisy-information models are able to characterize the data.

Andrade and Le Bihan (2013) exploit the panel dimension of the ECB Survey of Professional Forecasters. The authors develop a hybrid sticky-noisy-information model and test the empirical performance of the model. They assume that the economy can be summarized by a state vector Z_t made of p lags of 3 variables X_t – year on year inflation rate, change in unemployment rate and real GDP growth – with associated innovation ϵ_t. Its dynamics are described by:

$$Z_t = FZ_{t-1} + \eta_t,$$

where $Z_t = (X_t'\ X_{t-1}' \ldots X_{t-p+1}')'$, and $\eta_t = (\epsilon_t'\ 0 \ldots 0)'$ has a covariance matrix Σ_η.

Let i denote an individual forecaster. At each date, every i may receive new relevant information with probability λ, and when updating, agents observe a noisy signal (Y_{it}) of the true state (Z_t), that follows

$$Y_{it} = H'Z_t + v_{it}, \quad v_{it} \sim i.i.d.(0, \Sigma_v)$$

where H is assumed to be equal to the identity and with Σ_v a diagonal matrix. Besides, because every period only a share of the population updates, whenever aggregating the forecasts – or any other measure – one should weigh by $\lambda \sum_{j=0}^{\infty} (1-\lambda)^j$, as in Mankiw and Reis (2002), where j denote the forecasters that last updated in $t-j$. The authors detect that although the facts observed from the data qualitatively supports both expectation models, when estimating them, they cannot quantitatively replicate the high persistence and variance of forecast error together with the low level and time-variance of disagreement observed in the data.

3. Why inattention models?

Two particular patterns of individual expectations observed in the data that are in conflict with the assumption of perfect information and that can be reproduced by models of rational inattention are related to the disagreement among forecasters and the predictability of forecast errors. In this section, we present some basic facts focusing on both forecast errors and disagreement.

3.1. Forecast errors

The predictability of forecast errors is a property that arises from both sticky-information and noisy-information models. The infrequent update of the sticky-information model implies that, in date $t-1$, the expectations of those who did not update their information sets continue to be based on the information available at $t-2$. Accordingly, the forecast error at date t of agents who did not update their information set at $t-1$ will be predictable, based on information available at $t-1$. On the other hand, in the noisy-information model, although having access to new information every period, agents know the information they get each period is contaminated with noise. Thus they do not completely pass it onto their forecasts and forecast error will also be predictable.
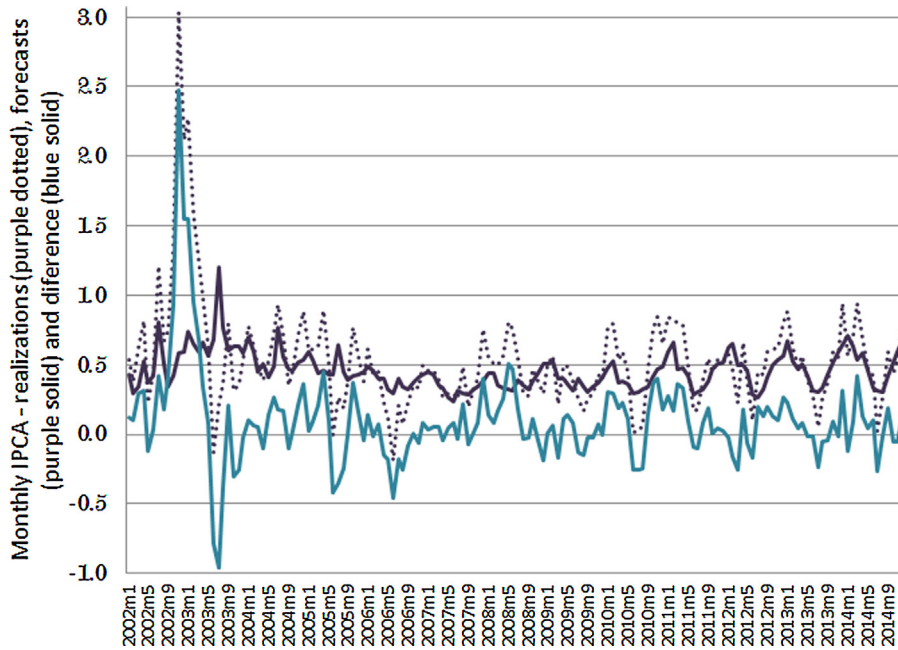
Fig. 1. Time series of realization of monthly inflation (purple dotted), average forecasts (purple solid) and difference (blue).

Individual forecast error is defined as:

$$e_{it,T} = x_T - f_{it,T}$$

where x_T denotes the realized inflation at date T event and $f_{it,T}$ indicates the forecast made by individual i at date t for the date T event.

And the average forecast error is given by:

$$e_{t,T} = x_T - f_{t,T}$$

where $f_{t,T} = (1/n_t)\sum_{i=1}^{n_t} f_{it,T}$ denote the average forecast across agents, and n_t is the number of institutions taken into account.

Fig. 1 exhibits the time series of average Brazilian monthly inflation (as measured by the IPCA – The Broad National Consumer Price Index) forecasts pooling the current and the next 11 month ahead horizons, using monthly inflation realizations from January 2002 to December 2014. One can easily see that the difference between the two series – inflation realizations minus forecasts – present considerably persistence. Periods of under- or overestimation usually last for some consecutive periods, which leads to predictable forecast errors. Also, the first-order autocorrelation of the average forecast error ($\varrho_e(1)$) is 0.6366.

All this suggest past forecast errors contain information that has not been exploited when producing present forecasts. Therefore, the predictability of forecast errors derived from both sticky and noisy-information models – and also obtained from the hybrid model featuring both types of inattention – are observed in our data.

3.2. Disagreement

Disagreement among forecasters is defined as the cross-section standard deviation of forecasts at each date:

$$\sigma_{t,T} = \sqrt{(1/n_t)\sum_{i=1}^{n_t} (f_{it,T} - f_{t,T})^2}$$

It emerges from information frictions and the fact that the agents do not all have a common information set, and can also be explained by both sticky- and noisy-information models. Fig. 2 exhibits the time series (from January 2002 to
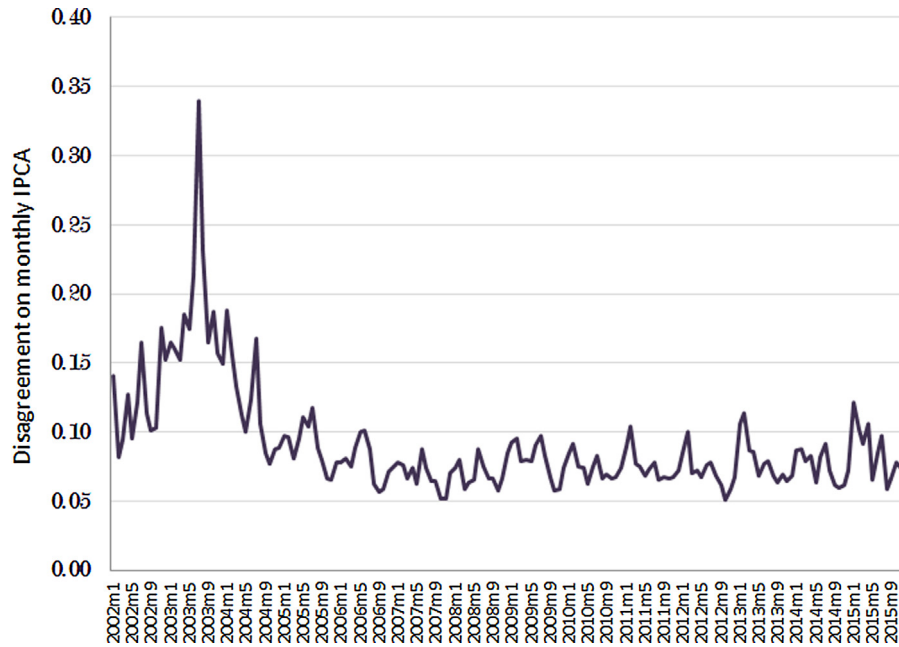
Fig. 2. Average disagreement.

November 2015) of average disagreement across forecasters pooling the current and the next 11 month ahead horizons. Noticeably, disagreement is always greater than zero, which is evidence in favor of information rigidity.

However, these two models make some different predictions about disagreement. First, sticky-information models, as opposed to the basic noisy-information – which assumes constant noise in the signal and no heterogeneity between forecasters – admit time variation in disagreement. Since in sticky-information approaches only a fraction of the agents update their information set, the size of the shocks hitting the economy will determine the variation of disagreement over time. More precisely, the difference between the forecasts produced by the agents who updated their information and the ones who did not will be much larger under large than under small shocks. On the other hand, in basic noisy-information models, disagreement is not affected by the magnitude of the shocks, because the different perceptions of reality are responsible for the difference in forecasts. It thus depends only on the variance of the noise, which is assumed to be constant over time.

Also in Fig. 2, we can see that, apart from the disagreement being always non-zero, it varies over time. Therefore, we can make an analysis of the response of disagreement to the dynamics of the economy to see if they are correlated. A possible measure for the dynamic of the economy – or the shock hitting it – is the squared variation in last period inflation, i.e., $(\Delta\pi_{t-1})^2$, or the squared change in average forecast, i.e., $(\Delta f_t)^2$. So, we regress disagreement of forecasts on these measures. The result is shown in Table 1: both are statistically significant at 1%. This indicates disagreement

Table 1
Disagreement among forecasters of monthly inflation. Mean (σ) stands for the mean of the disagreement across dates and Std-Dev (σ) for its standard error. $(\Delta\pi_{t-1})^2$ denotes the squared variation in past inflation and $(\Delta f_t)^2$ denotes the squared change in average forecast. Standard errors are in parentheses.

Disagreement, σ	Monthly IPCA
Descriptive statistics	
Mean (σ)	0.091
Std-Dev (σ)	0.039
Regressions of σ on	
$(\Delta\pi_{t-1})^2$	0.055*** (0.014)
$(\Delta f_t)^2$	0.827*** (0.078)

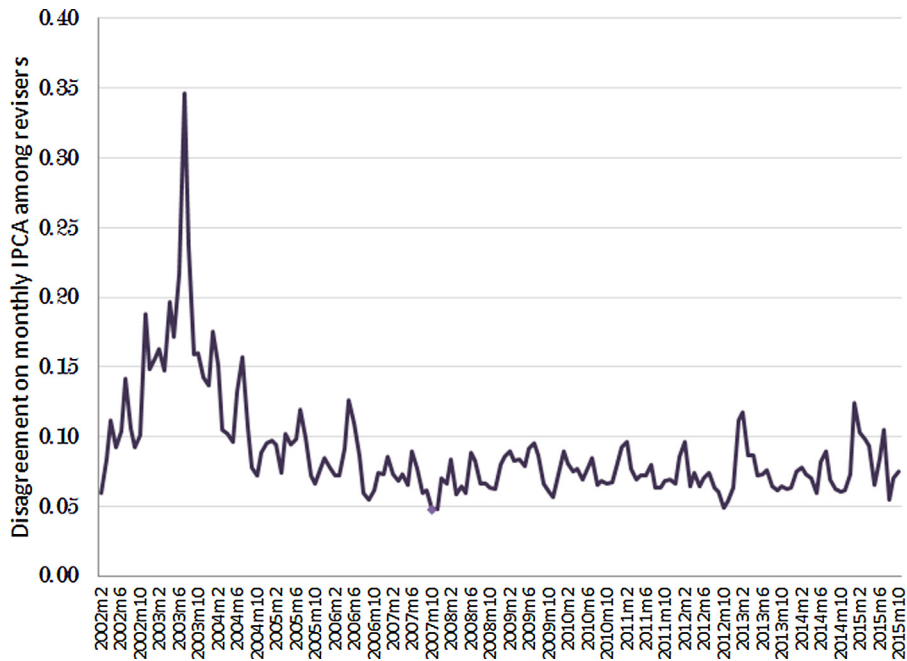*** Significant at 1%; ** Significant at 5%; * Significant at 10%.

Fig. 3. Average disagreement among revisers.

respond to shocks, which is evidence in favor of the sticky-information model. But this also goes in the manner predicted by the hybrid model. Although maintaining the constant noise in the signal, in the hybrid approach we have a share of the population that updates its information set and a share that does not. This is enough to generate time variance in disagreement.

A second point of divergency regards the analysis of the disagreement among forecasters who revise their forecasts. Non-zero disagreement in this case is evidence in favor of the noisy-information approach. While the sticky-information model assumes that agents who receive new information in the same period have the same information set – and thus make the same predictions, which should indicate zero disagreement among revisers – the noisy-information model considers that each agent receives new information contaminated with noise. Therefore, even agents who update their forecasts concomitantly may make different predictions. Fig. 3 plots disagreement among revisers for monthly IPCA forecasts. As one can see, disagreement is always non-zero between 2002 and 2015 – evidence in favour of the noisy. Still this is also predicted by the hybrid model. As agents who update receive new information contaminated with noise, it is unlikely that all forecasters that update their information set in a given date make exactly the same predictions. This leads disagreement among revisers to be non-zero.

4. Methodology

4.1. Attention/inattention degree

Working with a hybrid model implies we'll derive two measures of attention/inattention degree, which are related to the degree of information rigidity faced by the economy. As mentioned in Section 1, in the sticky-information model the probability of updating a forecast between two consecutive dates can be seen as a measure of agents' attention to new information. So, in order to obtain our first measure of the attention degree[6] – represented by λ – we want to estimate

$$P(f_{it,T} \neq f_{it-1,T})$$

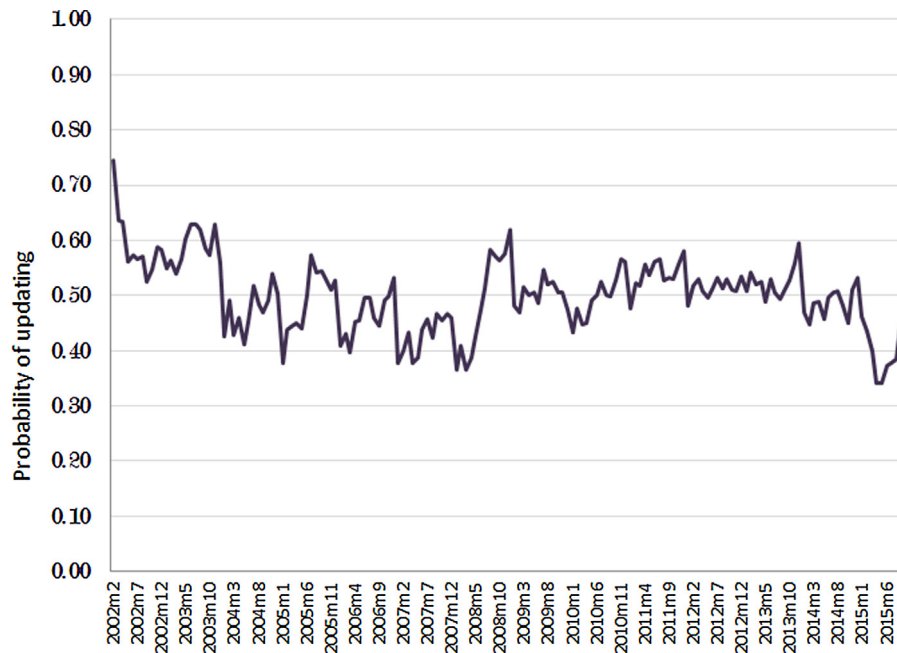[6] The corresponding "inattention degree" can be thought of as (1-λ).

Fig. 4. Average (sticky-information) degree of attention, λ.

Table 2
Degree of attention (λ) – probability of updating a forecast of monthly IPCA inflation for a date T event between two consecutive months.

Degrees of attention, λ	Monthly IPCA
Descriptive statistics	
Mean ($\hat{\lambda}$)	0.4999
Std-Dev ($\hat{\lambda}$)	0.0657

where $f_{it,T}$ denotes agent i's forecast for the IPCA inflation at date t for the date T event. Assuming that the probability is homogeneous across institutions and across horizons, we can establish some empirical counterparts to this attention degree from the survey data:

$$\hat{\lambda_{t,T}} = (1/n_t)\sum_{i=1}^{n_t} I(f_{it,T} \neq f_{it-1,T})$$

where $I(\cdot)$ is a indicator function, that assumes value 1 when the individual's forecast for date event T changes between $t-1$ and t, and 0 otherwise.

In other words, $\hat{\lambda_{t,T}}$ tells us the percentage of our population – here the institutions responding the survey – that updated their forecasts and supposedly had access to relevant new information between the dates $t-1$ and t. Thus the greater λ, the lower the degree of information rigidity faced by the economy.

Fig. 4 shows the average (across institutions and forecast horizons) attention degree for predictions made for each date T event, specified in the horizontal axis.[7] As can be seen, the attention degree never reaches 1. It in fact remains always between 0.3 and 0.8 during the period under study and its mean (over time) – denoted by $\hat{\lambda}$ and described in Table 2 – is 0.4999.

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[7] Given the structure of our database, to construct a sequence of forecasts made by the same institution for the same date T, we pool the monthly forecasts made by each institution for the current and the next 11 months.

Our second measure of inattention degree is obtained from the noisy-information model and is related to the variance of the measurement error, i.e., the error associated to the signal about the state of the economy received by the institutions each period – denoted by Σ_v. It tells us how noisy is the perception of the state of the economy – or how different from the actual inflation realization the forecast can be – and thus the greater Σ_v, the greater is the information rigidity faced by agents. This parameter will be explained in more details below.

4.2. The model

Disagreement among forecasters and predictable forecast errors observed in survey data and documented in Section 3 indicate that both sticky- and noisy-information models may describe expectation formation in a satisfactory way. So, we will implement the methodology proposed by Andrade and Le Bihan (2013). The hybrid model features both approaches and in order to assess the empirical performance of this expectations model, it is compared to some data characteristics through a Minimum Distance Estimation (MDE) procedure. This procedure allows to analyze if the model can replicate the forecast error and the disagreement observed in the survey data.

We sketch below the structure of the model. We assume that Z_t is the state that summarizes the economy. The following AR(p) describe its dynamics:

$$A(L)Z_t = w_t,$$

with $A(L) = \sum_{k=0}^{p} \rho_k L^k$, and where w_t has zero mean and variance equal to Q.

As in Mankiw and Reis (2002), at each date, every forecaster i may update his information set, with a given and constant probability λ – modeled as a Poisson. The agents who last updated their information set in $t - j$ are named the generation j. However we also consider a noisy perception of the new information when updating, which is captured by the signal Y_{it} that follows

$$Y_{it} = Z_t + v_{it}, \quad v_{it} \sim i.i.d.(0, \Sigma_v),$$

where v_{it} is a private shock and Σ_v is the inattention parameter of the noisy approach.

The estimation procedure involves three steps. First, an AR(p) model for inflation rate, with the variable previously centered, is estimated. Second, taking the AR(p) parameter as given, and for a given set of structural parameters (λ, Σ_v), four moments are simulated using the Kalman filter and the hybrid model. The selected moments – the same used in Andrade and Le Bihan (2013) – are: the mean square of forecast errors, forecast errors' first autocorrelation, disagreements' average level and disagreements' variance, respectively denoted by: $E[e^2]$, $\varrho_e(1)$, $Mean[\sigma]$ and $V[\sigma]$. These moments are related either to forecast errors or disagreement, two key features that can be reproduced by imperfect information models and that, in this hybrid model, are functions of the two measures of attention/inattention degree – λ and Σ_v.

Next, the previous model is estimated by the MDE procedure. Take the structural parameters vector $\theta_0 = (\lambda, \Sigma_v)'$, the reduced form parameter vector $\mu_0 = (E[e^2], \varrho_e(1), Mean[\sigma], V[\sigma])'$, and $h : R^2 \to R^4$ a continuously differentiable function. The MDE procedure consists of first estimating μ_0 by $\hat{\mu}$, and then choosing an estimator θ of θ_0 by minimizing the distance between $\hat{\mu}$ and $h(\theta)$. From now on, we denote $h(\theta) = \mu(\lambda, \Sigma_v)$. We thus compute the distance between the vector of data-moments $\hat{\mu}$ and the corresponding moments generated by the model, which are a function of the attention/inattention parameters $\mu(\lambda, \Sigma_v)$. The objective function is minimized over the space of parameters. I.e., one must minimize:

$$[\hat{\mu} - \mu(\lambda, \Sigma_v)]' \hat{\Omega}^{-1} [\hat{\mu} - \mu(\lambda, \Sigma_v)],$$

where $\hat{\Omega}$ is a consistent estimator of the asymptotic variance of $\hat{\mu}$, i.e., $\sqrt{T}(\hat{\mu} - \mu) \overset{a}{\to} N(0, \Omega)$ and $\mu(\lambda, \Sigma_v)$ takes the form:

$$\mu(\lambda, \Sigma_v) = \begin{bmatrix} E[e^2] \\ \varrho_e(1) \\ Mean[\sigma_t] \\ V[\sigma_t] \end{bmatrix} = \begin{bmatrix} E_t\{E_j[(e_{t,T}^j)^2]\} \\ \varrho_e(1) \\ E_t[\sigma_{t,T}] \\ V_t[\sigma_{t,T}] \end{bmatrix}$$

where[8]

$$E_j[(e_{t,T}^j)^2] = \rho^{2(T-t)}(1-G_t)^2\lambda\sum_{j=0}^{\infty}(1-\lambda)^j P_{t|t-j-1} + \rho^{2(T-t)}G_t^2\Sigma_v + \frac{\rho^{2(T-t)}-1}{\rho^2-1}Q \quad ,$$

$$\varrho_e(1) = \frac{E\{[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j-1]^2 E_i(Z_{it|t-j-1})E_i(Z_{it|t-j-2})\}}{E\{[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j-1]^2 E_i(Z_{it|t-j-1})^2\}} \quad \text{and}$$

$$\sigma_{t,T} = \sqrt{\begin{aligned}&\lambda\sum_{j=0}^{\infty}(1-\lambda)^j \ \{\rho^{2(T-t)}\{V_i(Z_{it|t-j-1}|j) + G_t^2[V_i(Z_{it|t-j-1}|j) + \Sigma_v]\}\\ &+ \ [\rho^{T-t}E_i(Z_{it|t-j-1}) + \rho^{T-t}G_t Z_t + \rho^{T-t}G_t E_i(Z_{it|t-j-1})\\ &- \left(\lambda\sum_{j=0}^{\infty}(1-\lambda)^j\right)[\rho^{T-t}E_i(Z_{it|t-j-1}) + \rho^{T-t}G_t Z_t + \rho^{T-t}G_t E_i(Z_{it|t-j-1})]]^2\}\end{aligned}}$$

5. Data

In order to ascertain the nature of the expectations formation process and to perform an empirical assessment of inattention models we will take professional forecasts on the headline rate of inflation (π), IPCA, surveyed by the Market Expectations System developed by the Central Bank of Brazil.[9] Professional forecasters are particularly interesting because these agents are some of the most informed in the economy. As a result, any evidence of information rigidity observed in their expectations should be indicative of existence of information rigidity faced by the whole economy.

Since 1999, the Central Bank of Brazil conducts a daily survey of forecasts of the main Brazilian macroeconomic variables. The so-called "Focus survey" currently covers more than 100 institutions, including commercial banks, asset management firms, consulting firms, non-financial institutions and other legal entities. Nonetheless, the dataset used in this paper includes forecasts made by 254 institutions, taking into account the whole sample period and the institutions that are currently active in the system and the ones that no longer are. We thus deal with an unbalanced panel.[10]

The Central Bank of Brazil also makes available monthly rankings including the 5 best forecasters with regard to short, medium and long run. This stimulates the institutions to update their forecasts regularly and to estimate them with accuracy, which makes this survey data more reliable. Moreover, the system only takes into account forecasts imputed in the last 30 days, which prevents the statistics to be influenced by forecasts that haven't been updated. See Marques (2013) for further details.

Another important advantage is that, while most expectations surveys conducted all over are aggregated, which implies one is not able to directly observe updates of forecasts in the micro level, in the Market Expectations System we can track the sequence of responses of a particular survey participant over time.[11] Finally, apart from being able to exploit individual data, we have access to daily responses, what means we can closely follow the sequence of forecasts made by institutions. This is an advantage over Andrade and Le Bihan's (2013) study, since the ECB SPF provides forecasts only on a quarterly frequency. For the IPCA inflation, we observe (in a daily basis) 5 annual IPCA inflation forecasts, i.e., for the current calendar year and the next 4 calendar years, besides the forecast for the twelve-month

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[8] See Appendix A for detailed calculation.

[9] The collection and manipulation of data from the Focus survey is conducted exclusively by the staff of the Central Bank of Brazil.

[10] The survey respondents are followed throughout time with a reasonable turnover. As new participants are often added to the survey, and others drop out, the panel of survey forecasts is unbalanced. Thus, from a set of 254 registered institutions in the system (in our sample), there is a smaller *active* group of around 100 institutions that frequently update their forecasts. On average, 87 institutions are daily surveyed in our dataset.

[11] The confidentiality of information is guaranteed and the anonymity of forecasters is preserved.

Table 3

Information criterion – Likelihood Ratio (LR), Akaike (AIC), Hannan-Quinn (HQIC) and Schwartz Bayesian (SBIC) information criteria.

Selection-order criteria				
Sample: May 2002 to December 2014	Number of obs = 152			
Lag	LR	AIC	HQIC	SBIC
0		0.961446	0.969527	0.98134
1	109.97*	0.251084*	0.267247*	0.290872*
2	0.14116	0.263313	0.287558	0.322995
3	0.07645	0.275968	0.308295	0.355544
4	0.05561	0.28876	0.329168	0.38823

* denotes the optimal lag in each column.

ahead cumulated inflation rate and monthly IPCA inflation forecasts for the present month and up to 17 months ahead the current date.

In Brazil, the inflation rate as measured by the IPCA is calculated and released on a monthly frequency by the Brazilian Institute of Geography and Statistics (IBGE). Thus, in this study, we use monthly IPCA inflation forecasts. The range of data goes from January 2002 to November 2014. The aggregation of the daily into monthly response is done by using the date of reference for IPCA, also known as the "critical date", which is the last business day before the IPCA-15 release date.[12] We simply take the last forecast made until the date of reference as the monthly response, because this is the forecast used by the Central Bank of Brazil to formulate the Top 5 rankings that are made publicly available. So, we understand the institutions have incentives to update their forecasts close to this date.

We assume that the probability of updating a forecast is constant across horizons and over time. Having established that, in order to construct a sequence of forecasts made for the same date T event by the same institution – so that we are able to compare revisions of consecutive forecasts for the same T and same i – we pool the monthly forecasts made by each institution for the current and the next 11 months. We thus usually have a sequence of 12 forecasts – therefore 11 revisions – for each date T.[13]

6. Estimation and results

As mentioned, first we model the inflation process. The autoregressive model that seems to best describe the inflation rate is the AR(1). The choice is made by the analysis of autocorrelation and partial autocorrelation functions and the Schwartz's Bayesian Information Criterion (SBIC) – although the result remains the same with alternative information criteria, see Table 3 – considering up to 4 lags.

Finally, the Breusch–Godofrey test of autocorrelation of the residuals does not reject the null of no autocorrelation (*p-value* = 0.6855) and the residuals analysis corroborate this choice. The graphic of the time series of the residuals can be seen in Fig. 5, as well as the original IPCA and fitted IPCA.

Having established the process that rules inflation, we proceed to the simulation of the moments, namely the mean square of forecast errors ($E[e^2]$), forecast errors' first autocorrelation ($\varrho_e(1)$), average level of disagreement ($Mean[\sigma_t]$) and variance of disagreement ($V[\sigma_t]$). The results obtained from simulation are presented in Table 4, Column (1). The respective sampled moments – that we ideally would like to match with the simulated moments – are also specified in Table 4, Column (2).

For the MDE procedure we use a diagonal matrix as estimator of Ω (asymptotic covariance matrix). The variance of each moment is computed utilizing Newey-West estimator to overcome autocorrelation in the error terms. The attention/inattention degrees simulated by the hybrid model are: $\tilde{\Sigma}_v = 0.1158$ and $\tilde{\lambda} = 0.2256$, which is very distant from the λ value of 0.4999 observed from the data – Table 4. In fact, something one can immediate draw from these

---

[12] The collecting period of IPCA is the calendar month, whereas for the IPCA-15 it ranges from the 16th of the previous month to the 15th of the reference month.

[13] Except for the first 11 and the last 11 dates T sampled – i.e., from January 2002 to November 2002 and from January 2015 to November 2015 – or when the institution does not provide a forecast for a determined date T event.
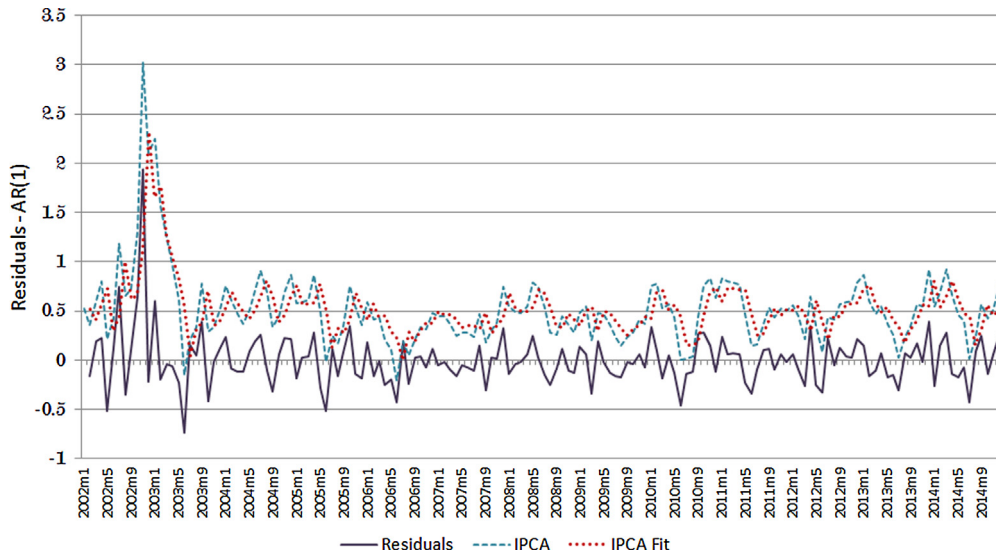
Fig. 5. Time series of the residuals after fitting an AR(1) to inflation process.

Table 4

Simulated and sampled attention/inattention degrees. λ denotes mean (over time) probability of updating a forecast for date T between two consecutive months and Σ_v stands for the variance of the noise in the signal. First column specifies the measures simulated by the hybrid model and second column the measure of λ obtained from the data, using up to 12 months ahead forecasts.

Attention/inattention degrees		
	Simulated	Sampled
λ	0.2256	0.4999
Σ_v	0.1158	
Test of overidentifying restrictions		
χ_2^2 statistic	2.3343	

*** Significant at 1%; ** Significant at 5%; * Significant at 10%.

numbers is that in order to minimize the distance between the selected moments, the model generates a much higher degree of information rigidity than the observed in the data.

Using the MDE procedure, we can also perform a test of over-identifying restrictions – which null is that the parameters to be estimated (λ and Σ_v) can accurately describe the 4 selected moments, or alternatively, that the distance between the simulated moments and the observed ones is zero. The test statistic has asymptotic Chi-square distribution with 2 degrees of freedom, χ_2^2. Our statistic equals 2.3343 (*p-value* = 0.3113), which leads us to not reject the null even under the statistical significance level of 10%. Although the over-identifying restrictions are not rejected by the data, as mentioned, that happens at the cost of a much lower frequency of updating (λ).[14]

Now we analyze how the moments respond to some alternative combinations of λ and Σ_v. This exercise is presented in Columns 3 and 4 of Table 5. Once we fix λ at the value observed from data ($\hat{\lambda} = 0.4999$) and Σ_v at the value generated by the hybrid model ($\tilde{\Sigma}_v = 0.1158$) – Column 3 – the two moments related to the forecast error and time-variance of disagreement become lower and hold off further from the data, and the mean of disagreement gets higher. If on the other hand we fix λ at 0.4999 and raise Σ_v to 0.9 for example – Column 4 – when compared to Column 3, we observe a raise in the first two moments, related to forecast error, while both moments associated to disagreement fall.

---

[14] Looking for a local minimum, we could restrict the interval for possible values of λ around the value encountered in the data. Nevertheless, independent of how fine the interval is, whenever the lower bound is greater than 0.2256, λ stays at its lower bound.

Table 5

Moments. $E[e^2]$ denotes mean square of forecast errors, $\varrho_e(1)$ forecast errors' first autocorrelation, $Mean[\sigma_t]$ the average level of disagreement and $V[\sigma_t]$ its time variance. First column specifies the moments simulated by the hybrid model and second column the moments obtained from the data, using up to 12 months ahead forecasts. Columns 3 to 5 specify the results under alternative configurations.

Moments	(1) Simulated $\lambda = \tilde{\lambda}$ $\Sigma_v = \tilde{\Sigma}_v$	(2) **Data**	(3) $\lambda = 0.4999$ $\Sigma_v = \tilde{\Sigma}_v$	(4) $\lambda = 0.4999$ $\Sigma_v = 0.9$	(5) $\lambda = 0.4999$ Σ_v *free* (=0.4596)
$E[e^2]$	0.0685	0.1267	0.0536	0.0757	0.0686
$\varrho_e(1)$	0.6271	0.6366	0.5919	0.6449	0.6280
$Mean[\sigma_t]$	0.0912	0.0912	0.1150	0.0752	0.0908
$V[\sigma_t]$	0.000414	0.0015	0.000154	0.01549×10^{-3}	0.04305×10^{-3}

These patterns are what would be expected from the model. According to the model an increase in Σ_v (i.e., an increase in the variance of the noise associated to the signal received by the agents) or a decrease in λ (i.e., a decrease in the probability of updating a forecast between two consecutive months) are both indicative of an increase in information rigidity. It should thus raise both variance and persistence of forecast errors. The effect on average disagreement, on the other hand, is ambiguous. Finally, an increase in Σ_v should decrease time variance of disagreement, while a decrease in λ should raise it – see Appendix A for more details. The opposite should happen when we face an increase in λ or a decrease in Σ_v.

Finally, in Column 5 of Table 5 we fix λ at the value observed from the data ($\hat{\lambda} = 0.4999$) and let Σ_v free to be simulated by the model. We obtain $\Sigma_v = 0.4596$. When compared to the case where all parameters are freely simulated (Column 1), the two moments related to forecast errors and the mean of disagreement do not change too much. Nevertheless the variance of disagreement gets even lower, holding off from the data value.

The results indicate that, despite of fitting the persistence of forecast errors and the level of disagreement relatively well, the model cannot account for both high variance of forecast errors and of disagreement. In fact, whenever we fit sampled variance of disagreement, for example, we must set Σ_v at a much lower level. But this in turn leads to much higher average disagreement than observed and even lower variance of forecast errors. On the other hand, to get closer to the variance of forecast errors we need a lower λ and a much higher Σ_v. Nevertheless it leads to higher persistence of forecast errors and much lower level and variance of disagreement.

7. Conclusion

In this study we used the Market Expectations System of Central Bank of Brazil to analyze how good can the expectations formation process of inflation for Brazilian data be modeled by the hybrid model featuring both sticky-information and noisy-information models. The model is based on the assumption that the agents do not have access to information instantly/every period, and that when agents update their information set, they only observe noisy signals about the true state and form expectations rationally.

Utilizing a Minimum Distance Estimation (MDE) procedure, we were able to test if the model is capable of quantitatively fitting the Brazilian data. Estimating the model we encounter that formally the model fits the data. Nevertheless, it happens at the cost of a much higher degree of information rigidity than observed in the data.

In future research, one possible way of improving the fit of the model to the data may be allowing the degree of attention of the sticky-information approach (λ) to vary across institutions – something that is observed in the data. Or one could try to include the stickiness as in Reis (2006a,b). That is, the forecaster explicitly chooses to be inattentive, solving a optimization problem to decide when to update his information set – instead of being subject to an exogenous constant probability of being able to get new information. Other possible routes are to test different models, following the approach of Coibion and Gorodnichenko (2012b), or to consider dispersed information with strategic interactions, in line with the methodology of Morris and Shin (2002).

Appendix A. The model

The state space representation is given by:

$$
\begin{cases}
Y_{it} = Z_t + v_{it} \\
Z_t = \rho Z_{t-1} + w_t
\end{cases}
\tag{A.1}
$$

where $v_{it} = \epsilon_{it} + \eta_t \sim i.i.d.(0, \Sigma_v)$ and $w_t \sim i.i.d.(0, Q)$.

The first (A.1) equation is called the observation equation and the second the state equation.

The date t state of the economy perceived by agent i at date $t-1$ is

$$
Z_{i,t|t-1} = E[Z_t | I_{i,t-1}]
\tag{A.2}
$$

where $I_{i,t-1} = (Y'_{i,t-1}, Y'_{i,t-2}, \ldots, Y'_{i,1})$ and

$$
P_{t|t-1} = E[(Z_t - Z_{i,t|t-1})(Z_t - Z_{i,t|t-1})']
\tag{A.3}
$$

Besides,

$$
Y_{i,t|t-1} = E[Y_{i,t} | I_{i,t-1}]
\tag{A.4}
$$

$$
E[Z_t] = \rho E[Z_{t-1}]
\tag{A.5}
$$

$$
E[Y_{i,t} | Z_t] = Z_t
\tag{A.6}
$$

Thus,

$$
Y_{i,t|t-1} = E[Y_{i,t} | I_{i,t-1}] = E[E(Y_{i,t} | Z_t) | I_{i,t-1}] = E[Z_t | I_{i,t-1}] \overset{A.2}{=} Z_{i,t|t-1}
\tag{A.7}
$$

The optimal forecast for date T[15] event made by generations 0 and j – i.e., the agents that updated their information set in the current period and the ones who updated j periods ago – is respectively given by:

$$
f_{it,T} = E[Z_T | Z_{i,t|t}] = \rho^{T-t} Z_{i,t|t}
\tag{A.8}
$$

$$
f_{it-j,T} = E[Z_T | Z_{i,t-j|t-j}] = \rho^{T-t} Z_{i,t|t-j}
\tag{A.9}
$$

From (A.1) and (A.7)

$$
Y_{i,t} - Y_{i,t|t-1} = Z_t + v_{it} - Z_{i,t|t-1} = (Z_t - Z_{i,t|t-1}) + v_{it}
\tag{A.10}
$$

thus

$$
\begin{aligned}
E[(Y_{i,t} &- Y_{i,t|t-1})(Y_{i,t} - Y_{i,t|t-1})'] \\
&= E[[(Z_t - Z_{i,t|t-1}) + v_{it}][(Z_t - Z_{i,t|t-1}) + v_{it}]'] \\
&= E[(Z_t - Z_{i,t|t-1})(Z_t - Z_{i,t|t-1})'] + E[v_{it}v'_{it}] = P_{t|t-1} + \Sigma_v
\end{aligned}
\tag{A.11}
$$

where the second equality comes from v_{it} and Z_t being non-correlated.

---

[15] $T = t + h$, where h is the forecast horizon.

The Kalman Filter solution to this signal extraction problem is given by:

$$Z_{i,t|t} = Z_{i,t|t-1} + G_{it}(Y_{it} - Y_{i,t|t-1}) \tag{A.12}$$

where G_{it} is the Kalman gain. We assume it to be homogeneous across agents: $G_{it} = G_t = P_{t|t-1}(P_{t|t-1} + \Sigma_v)$.

Using this solution, we can rewrite the *optimal forecast* as:

$$f_{it,T} \stackrel{A.8}{=} \rho^{T-t} Z_{i,t|t} = \rho^{T-t} Z_{i,t|t-1} + \rho^{T-t} G_t(Y_{it} - Y_{i,t|t-1}) \tag{A.13}$$

The average forecast within a generation j is:

$$E_i[f_{it-j,T}|j] = \rho^{T-t} E_i[Z_{i,t|t-j-1}] + \rho^{T-t} G_t E_i$$
$$\times [Z_t - Z_{i,t|t-j-1} + v_{it}] \stackrel{A.1}{=} \rho^{T-t+1} E_i[Z_{i,t-1|t-j-1}] + \rho^{T-t} G_t[Z_t - \rho E_i(Z_{i,t-1|t-j-1})] \tag{A.14}$$

and the forecast error within a generation j is:

$$e_{t,T}^j = Z_T - E_i[f_{it-j,T}|j] = \rho^{T-t} Z_t + \sum_{i=1}^{T-t} \rho^{T-t-i} w_{t+i} - \{\rho^{T-t} E_i[Z_{i,t|t-j-1}] + \rho^{T-t} G_t[Z_t - E_i(Z_{i,t|t-j-1})]\}$$

$$= \rho^{T-t}(1 - G_t)\{Z_t - E_i[Z_{i,t|t-j-1}]\} + \sum_{i=1}^{T-t} \rho^{T-t-i} w_{t+i} \tag{A.15}$$

The average forecast across generations follows:

$$E_j\{E_i[f_{it-j,T}|j]\} = \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_i[f_{it-j,T}] \tag{A.16}$$

and the average forecast error is given by:

$$e_{t,T} = E_j[e_{t,T}^j] = Z_T - E_j\{E_i[f_{it-j,T}|j]\} = \rho^{T-t} Z_t + \sum_{i=1}^{T-t} \rho^{T-t-i} w_{t+i} - \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_i[f_{it-j,T}]$$

$$= \rho^{T-t} Z_t + \sum_{i=1}^{T-t} \rho^{T-t-i} w_{t+i} - \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j \{\rho^{T-t} E_i[Z_{i,t|t-j-1}] + \rho^{T-t} G_t[Z_t - E_i[Z_{i,t|t-j-1}]\}$$

$$= \rho^{T-t}(1 - G_t)\lambda \sum_{j=0}^{\infty} (1 - \lambda)^j [Z_t - E_i(Z_{it|t-j-1})] + \sum_{i=1}^{T-t} \rho^{T-t-i} w_{t+i} \tag{A.17}$$

We now turn to the disagreement generated by this hybrid-model. Our measure of disagreement – presented in Section 3.2 – is the cross-section standard deviation at each date. So, we derive the corresponding formula obtained by the theoretical model. In order to make calculations simpler, so we do not have to deal with squared roots, we will here look at the cross-section variance.

Note that the total cross-section variance of point forecasts across individuals i in different generations j can be decomposed in two elements: first the differences of perceptions of the state of the economy within a given generation, weighted by its relative share in the total population and second the differences in average perception of the state of

the economy between different generations:

$$
\begin{aligned}
V_{ij}(f_{it-j,T}) &= E_j[V_i(f_{it-j,T}|j)] + V_j[E_i(f_{it-j,T}|j)] \\
&= \lambda \sum_{j=0}^{\infty}(1-\lambda)^j V_i(f_{it-j,T}|j) + \lambda \sum_{j=0}^{\infty}(1-\lambda)^j \{E_i(f_{it-j,T}|j) - E_j[E_i(f_{it-j,T}|j)]\}^2 \\
&\stackrel{A.13}{=} \lambda \sum_{j=0}^{\infty}(1-\lambda)^j \ \Big\{ V_i[(\rho^{T-t}Z_{it|t-j-1} + \rho^{T-t}G_t(Z_t - Z_{it|t-j-1} + v_{it}))|j] + \{E_i(f_{it-j,T}|j) \\
&\quad - E_j[E_i(f_{it-j,T}|j)]\}^2\Big\} \\
&= \lambda \sum_{j=0}^{\infty}(1-\lambda)^j \ \Bigg\{ \rho^{2(T-t)}\{V_i(Z_{it|t-j-1}|j) + G_t^2[V_i(Z_{it|t-j-1}|j) + \Sigma_v]\} \\
&\quad + \Bigg[\rho^{T-t}E_i(Z_{it|t-j-1}) + \rho^{T-t}G_t Z_t + \rho^{T-t}G_t E_i(Z_{it|t-j-1}) \\
&\quad - \left(\lambda \sum_{j=0}^{\infty}(1-\lambda)^j\right)\left[\rho^{T-t}E_i(Z_{it|t-j-1}) + \rho^{T-t}G_t Z_t + \rho^{T-t}G_t E_i(Z_{it|t-j-1})\right]\Bigg]^2\Bigg\}
\end{aligned}
$$

$$\tag{A.18}$$

So, disagreement generated by the model is given by:

$$
\sigma_{t,T} = \sqrt{V_{ij}(f_{it-j,T})} \tag{A.19}
$$

A.1. Selected moments

Based on the previous equation, lets derive the four moments - the mean square of forecast errors, forecast errors' first autocorrelation, disagreements' average level and disagreements' variance – used in the procedure.

First, we derive the *mean of squared forecast error*.

$$
\begin{aligned}
E_j[(e_{t,T}^j)^2] &= E_j\{E_i[(Z_T - (f_{it-j,T}|j))^2]\} \\
&= E_j\{E_i\{[(\rho^{T-t}(1-G_t)(Z_t - Z_{it|t-j-1})) - \rho^{T-t}G_t v_{it}]^2\} + E_i\left[\sum_{i=1}^{T-t}(\rho^{T-t-i}w_{t+i})^2\right]\} \\
&= E_j\{E_i[\rho^{2(T-t)}(1-G_t)^2(Z_t - Z_{it|t-j-1})^2] + \rho^{2(T-t)}G_t^2 E_i[v_{it}^2] + \left[\sum_{i=1}^{T-t}\rho^{2(T-t-i)}\right]E_i[w_{t+i}^2]\} \\
&= \rho^{2(T-t)}(1-G_t)^2 E_j\{E_i[(Z_t - Z_{it|t-j-1})^2]\} + \rho^{2(T-t)}G_t^2\Sigma_v + \frac{\rho^{2(T-t)}-1}{\rho^2-1}Q \\
&= \rho^{2(T-t)}(1-G_t)^2 E_j[P_{t|t-j-1}] + \rho^{2(T-t)}G_t^2\Sigma_v + \frac{\rho^{2(T-t)}-1}{\rho^2-1}Q \\
&= \rho^{2(T-t)}(1-G_t)^2\lambda \sum_{j=0}^{\infty}(1-\lambda)^j P_{t|t-j-1} + \rho^{2(T-t)}G_t^2\Sigma_v + \frac{\rho^{2(T-t)}-1}{\rho^2-1}Q
\end{aligned}
$$

$$\tag{A.20}$$

Our first moment is the mean (over time) of mean squared forecast error, i.e.,

$$
E[e^2] = E_t\{E_j[(e_{t,T}^j)^2]\} \tag{A.21}
$$

The *forecast errors' first autocorrelation* is detailed below.

$$\varrho_e(1) = \frac{\gamma_1^j}{V(e_{t,T}^j)} = \frac{E[(e_{t,T}^j - e_{t,T})(e_{t-1,T}^j - e_{t-1,T})]}{V(e_{t,T}^j)} \tag{A.22}$$

where

$$\gamma_1^j = E\{[\rho^{T-t}(1-G_t)(Z_t - E_i(Z_{it|t-j-1})) + \sum_{i=1}^{T-t}\rho^{T-t-i}w_{t+i} - \rho^{T-t}(1-G_t)\lambda\sum_{j=0}^{\infty}(1-\lambda)^j[Z_t - E_i(Z_{it|t-j-1})]$$

$$- \sum_{i=1}^{T-t}\rho^{T-t-i}w_{t+i}] \times [\rho^{T-t}(1-G_t)(Z_t - E_i(Z_{it|t-j-2}))$$

$$+ \sum_{i=1}^{T-t}\rho^{T-t-i}w_{t+i} - \rho^{T-t}(1-G_t)\lambda\sum_{j=0}^{\infty}(1-\lambda)^j[Z_t - E_i(Z_{it|t-j-2})] - \sum_{i=1}^{T-t}\rho^{T-t-i}w_{t+i}]\}$$

$$= \rho^{2(T-t)}(1-G_t)^2 E\{(\lambda\sum_{j=0}^{\infty}(1-\lambda)^j - 1)^2 E_i(Z_{it|t-j-1})E_i(Z_{it|t-j-2})\} \tag{A.23}$$

and

$$V(e_{t,T}^j) = E\{[\rho^{T-t}(1-G_t)(\lambda\sum_{j=0}^{\infty}(1-\lambda)^j - 1)E_i(Z_{it|t-j-1})]^2\} \tag{A.24}$$

Thus

$$\varrho_e(1) = \frac{E\{[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j - 1]^2 E_i(Z_{it|t-j-1})E_i(Z_{it|t-j-2})\}}{E\{[\lambda\sum_{j=0}^{\infty}(1-\lambda)^j - 1]^2 E_i(Z_{it|t-j-1})^2\}} \tag{A.25}$$

The *average disagreement* is:

$$Mean[\sigma_t] = E_t[\sigma_{t,T}] \tag{A.26}$$

and *disagreement's time variance*:

$$V[\sigma_t] = V_t[\sigma_{t,T}] \tag{A.27}$$

where $\sigma_{t,T}$ is given by (A.19).

A.2. Selected moments and the degree of information rigidity

We shall now take a look at how these moments relate to changes in the degrees of attention/inattention. Recall λ is the probability of updating a forecast after one period of new information. Thus λ is inversely proportional to the degree of information rigidity. Σ_v, on the other hand, describes how volatile is the noise of the perception of the state of the economy. This implies that the greater Σ_v, the higher the degree of information rigidity.

Suppose an increase in λ or a decrease in Σ_v, which are both equivalent to a decrease in information rigidity. This means agents have access to more accurate information and should consistently make better predictions. Everything else remaining constant, we will face a decrease in the variance of the forecast errors and the same happens to its persistence. It means the response of the first two moments goes in the same direction as the change in information rigidity.

With regard to the time variance of disagreement, an increase in λ decreases the difference of information between generations. Thus the arrival of a new information set that is observed only by generation $j=0$ and that can contain a lot of or just a few relevant information unnoted by the previous generations – which can be seen as the size of a shock hitting the economy on that date – will have a lower impact on the magnitude of disagreement and that leads

to lower time variance of disagreement. On the other hand, a decrease in Σ_v decreases the amount of noise about the state of the economy faced by agents. This implies new information is more informative and thus more weight is given to new information when making a forecast. Thus it can also rise time variance of disagreement, since it would lead to disagreement being higher when more relevant information (or a larger shock) arises and to less disagreement when few relevant information (or a smaller shock) arises.

Finally, with respect to the response of the average disagreement, we have two concurrent effects for both degrees of (in)attention. An increase in λ, for example, decreases the length between information updating – because the probability to receive new information faced by each agent each period is now greater -, which decreases the heterogeneity between forecasts made by different generations and should imply a decrease in disagreement. But it also decreases the share of generations basing their prediction on old information. Within the generations that haven't updated their information set for too long, the optimal forecast is simply the unconditional mean of the inflation process and disagreement tends to zero. Thus a decrease in the share of theses generations contributes to increase disagreement.

Consider now a decrease in Σ_v. On one hand, it decreases the different perceptions within each generation of forecasters (via the fall in the amount of noise faced by agents about the state of the economy) and decreases disagreement. On the other hand, a decrease in Σ_v implies agents tend to incorporate more of the new information into their forecast, because the signal is more precise. This leads to an increase in average disagreement.

Therefore, while the effect of a change in the degree of information rigidity on the moments related to forecast errors can be easily anticipated, the direction of the moments related to disagreement depend on which forces will prevail. Table A.1 summarizes it:

Table A.1
Response of the moments to changes in the attention/inattention degrees. λ stands for the probability of updating a forecast between two consecutive months and Σ_v for the variance of the noise in the signal. $E[e^2]$ denotes mean square of forecast errors, $\varrho_e(1)$ forecast errors' first autocorrelation, $Mean(\sigma_t)$ the average level of disagreement and $V(\sigma_t)$ its time variance.

Moments	Decrease in Information Rigidity	
	Increase in λ	Decrease in Σ_v
$E[e^2]$	↓	↓
$\varrho_e(1)$	↓	↓
$Mean[\sigma_t]$	ambiguous	ambiguous
$V[\sigma_t]$	↓	↑

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