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Public security in Brazil: Efficiency and technological gaps

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Abstract

This article analyzes the technical efficiency, the Total Factor Productivity (TFP) and the technological gap in public security services in Brazilian States. The order—m frontier is used for results estimation. The TFP variation is built by decomposing the Malmquist productivity index into technical efficiency, scale efficiency and technological variation. More than 50% of the federative units were considered technically inefficient. Out of the 27 federative units, 12 presented a positive total productivity while all others suffered total productivity losses. Productivity gains in public security are more related to scale aspects than to efficiency improvements and technological progress.

Keywords: Public security; Criminality; Productivity; Efficiency and technology

Resumo

Este artigo analisa a eficiência técnica, a Produtividade Total dos Fatores (PTF) e a defasagem tecnológica nos serviços de segurança pública nos Estados brasileiros. A fronteira da ordem-m é usada para estimar os resultados. A variação da PTF é calculada pela decomposição do índice de produtividade de Malmquist em termos da variação da eficiência técnica, variação da eficiência de escala e da variação tecnológica. Mais de 50% das unidades federativas foram consideradas tecnicamente ineficientes. Das 27 unidades federativas, 12 apresentaram variação da produtividade total positiva, enquanto todas as outras sofreram perdas de produtividade total. Os ganhos de produtividade na segurança pública estão mais relacionados com aspectos de escala do que com a melhoria da eficiência e o progresso tecnológico.

Palavras-chave: Segurança Pública; Criminalidade; Produtividade; Eficiência e tecnologia

1. Introduction

In recent years, Brazil experienced some considerable improvements resulting from the growth in income and formal employment offer. According to IBGE (Brazilian Institute of Geography and Statistics) the employment rate of the economically active population increased from 89.5% in December 2002 to 95.0% in March 2014. The average income, according to PME/IBGE data, grew from R$ 2336.81 in December 2002 to R$ 3175.00 in March 2014. Due to these results, poverty indicators and income inequality have also experienced a decreasing tendency in recent years. According to IPEA (Institute of Applied Economic Research), the poverty index fell from 34.36% to only 15.96%.

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However, crime rates show that despite the income growth and poverty reductions, Brazil has become a more violent country. According to the Brazilian Forum of Public Security (2013) the intentional homicides rate grew 7.8% between 2011 and 2012, reaching 24.3/100,000 inhabitants. The rape rate in 2012 reached 26.1 cases per each 100,000 women. That means 50,617 rape cases nationwide. The rate of this crime increased 23% in São Paulo between 2011 and 2012.

In Brazil, internal public security services are attributions of the Federation units. Each federation unit has its own police force constituted by military and civil agents. The first one is responsible for ostensive police services and crime repression; the second one, also called judiciary police, is responsible for investigation activities.

Just like other public services, security must be guided by constitutional principles such as efficiency, which means to offer the best service possible with the smallest expense in resources. The security efficiency, besides granting economical services, implies the control of the negative effects of crime against people and assets, taking into account that crime containment results in an improvement of the social and financial environment.

There is plenty of literature aimed at the study of crime determinants. Chalfin and McCrary (2014) performed an economics research review on the effects of policing, punishment and security strategies on criminal acts. The evidence in favor of deterrence varies. While there is evidence that crime is sensitive to policing and the opportunities offered by the labor market, there is also evidence of a weak response to more severe punishment.

In Brazil, Dos Santos and Kassouf (2008) discussed economics studies on national crime. The data quality refers to the high level of official data sub-recording and highlights the inverse causality problem amongst deterrence variables and crime rates. Most studies reviewed found evidence that income inequality and crime returns seem to be factors that boost crime levels.

More recently, studies on public safety technical efficiency have become an alternative approach to the crime problem. Instead of identifying violence determinants, they search for those units that are being relatively successful in fighting criminality. This allows for the identification of reference units so that their security policies can be used in studies on crime reduction strategies. Schull et al. (2014) performed an analysis of expenses with public safety in Brazilian states applying the Data Envelopment Analysis (DEA) estimation method. Pereira Filho et al. (2010) estimated the efficiency of public safety based on the stochastic costs frontier methodology.

In this sense, this article intends to analyze the technical efficiency of public security services in Brazilian states between 2008 and 2012. Additionally, we also estimate the technological gap per state and decompose the Total Factor Productivity (TFP) of public security into technical efficiency and technological variations.

In general, the methodology applied for the estimation of technical efficiency is qualified into parametric and non-parametric methods. Among the first ones, the most widely used is the stochastic frontier, which in general is applicable to single product technologies. The main criticism and the difficulty with this method is the imposition of a functional application for production function and the distributional hypothesis on model errors. When productive processes involve multiple products, non-parametric models are more adequate. Among the main non-parametric models we should mention the Data Envelopment Analysis (DEA) and the Free Disposal Hull (FDH). The former was first proposed by Charnes et al. (1978), based on the hypothesis that productive units present constant scale returns. Later on, Banker et al. (1984) developed the DEA hypothesis of variable scale returns. The second model, developed by Deprins et al. (1984), is based on a DEA model in which the production frontier results in non- convex variable scale returns based on the free disposal assumption.

However, it is worth highlighting that estimated technical efficiency results obtained through these two last methods may be biased in the presence of measurement errors or outliers.

Seeking to solve these problems, Cazals et al. (2002) developed an alternative way to robustly estimate technical efficiency. That's how the efficiency frontier method—*m* is born.

Therefore, considering that public security offers different services, besides the possibility of the existence of measurement errors or outliers, we estimate the efficiency of public security in Brazilian states through the order—*m* production frontier method.

In order to calculate technological gap levels, according to Wongchai et al. (2012), we apply the meta-frontier theory, defined as the envelopment of production functions for decision unit subgroups, as initially proposed by Hayami (1969) and Hayami and Ruttan (1970, 1971). The production functions envelopment is defined by the most efficient aspects of each specific technology, as explained by Ruttan et al. (1978).

The productivity measurement of public security services in Brazilian states shall be equal to the TFP calculated through the Malmquist index. The advantage of its use is to allow the total productivity index to be decomposed into the technical efficiency variation, the scale variation and the technological variation. To this effect, the analysis shall permit us to discover if state productivity gains were more due to technical efficiency effects, because of scale efficiency or as a result of technological variations.

Besides this introduction, this work is organized as follows: Section 2 introduces and discusses the methodology used to calculate technical efficiency levels, technological gaps, TPF and its decomposition for Brazilian states. Section 3 discusses the construction of a database and develops a descriptive analysis. Section 4 introduces and discusses result obtained from the methods described in Section 2. Section 5 deals with the final considerations.

2. Technical efficiency methodology

The most common efficiency estimation methods in literature are classified into parametric and non-parametric. The parametric methods depend on the imposition of hypothesis on errors distribution and the production function form. As examples of this category we can name the corrected ordinary least square and the stochastic production frontier.

At the same time, non-parametric methods permit us to work with multiproduct models without the need to consider any functional form or data probability distribution.

Among non-parametric methods, it is worth mentioning the Data Envelopment Analysis (DEA). This method uses a linear programming to build production frontiers based on best practices among decision units that hypothetically, use identical production technologies that transform inputs into products. The efficiency scores for each productive unit correspond to the distance between the observed result and the optimal result projected in the efficient production frontier.

There are several DEA formulations found in literature. The method proposed by Charnes et al. (1978), also known as CCR (constant returns to scale), evaluates production technical efficiency based on the hypothesis of constant returns to scale. However, this hypothesis is quite restrictive and consequently, inefficiency values may become biased.

Alternatively, the Banker et al. model (1984) known as BCC, refers to the hypothesis of constant returns to scale assuming that these returns vary. Based on the hypothesis of variable returns to scale it is possible to separately estimate scale efficiency and technique for each decision unit. This model, although more flexible than the CCR, has been criticized for using a hypothetical value as efficiency calculation reference, which in reality is not observed, as it only existed in the constructed frontier. If the frontier value is not observed, there is no evidence that it can be reached. Therefore, a decision unit may be considered inefficient based on a reference that is impossible to reach. Besides, the BCC model is sensitive to the set of variables selected and to the existence of outliers.

Deprins et al. (1984) developed a DEA model denominated FHD (Free Disposal Hull) based on a production frontier with variable non-convex scale returns assuming a free disposal. However, the DEA and FDH models have some important disadvantages: (a) results are strongly dependent on the set of variables and they can be biased with the simple inclusion or exclusion of an input and/or output; (b) the influence of stochastic factors or measurement errors completely alters the frontier position and biased results; (c) treating inputs and/or outputs as if they were homogeneous, when in general they are heterogeneous, may distort results; (d) the presence of outliers may completely alter results.

Besides, in the FDH model, when a decision unit does not have another pair in the group to be compared to, it is considered efficient by default. To be considered efficient, it is enough not to be dominated by any other one.

Years later, Cazals et al. (2002) developed the order—m frontier approach. This approach, contrary to the DEA and FDH methods, does not include all points and it also requires much less information (data) than the previous methodologies. Its main advantage is that in the presence of significant measurement errors and a reasonable number of outliers, technical efficiency estimates are more robust if compared to the other parametric and non-parametric methods. Krüger (2012) demonstrated these properties through Monte Carlo simulations.

2.1. The Order—m Frontier method

The production process is described by a probability measure (X,Y) over $R_+^p \times R_+^q$ in which the support of (X,Y) is the set of production possibilities defined as:

$$\mathcal{P}^t(x^t) = \{(x, y) \in R_+^p \times R_+^q / x \text{ can produce } y\}.$$

The superior limit of $\mathcal{P}^t(x^t)$ represents the technology or the production frontier and shall be denoted by $\mathcal{P}^{\delta t}(x^t)$. In real terms it is defined by the intersection of $\mathcal{P}^t(x^t)$ and the closure of its compliment.

In the product space, for a point $(x, y) \in \mathcal{P}^t(x^t)$, the Farrel (1957) product technical efficiency measurement is defined as:

$$\theta(x, y) = \sup\{\theta/\theta y \in \mathcal{P}(x)\} = \sup\{\theta/(x, \theta y) \in \mathcal{P}^t(x^t)\},$$

in which $\mathcal{P}(x) = \{y \in R_+^q/(x, y) \in \mathcal{P}^t(x^t)\}$ is the required product set. Therefore, for any product level y within the Y support, the efficient frontier may be described. In the case of multiple inputs, the efficient frontier may be represented either through efficiency measurements, as the efficient frontier is defined as $\partial\mathcal{P}(x) = \{y/\theta(x, y) = 1\}$ or through the efficient level of products, which for any $y \in R_+^q$ is defined as:

$$y^\delta(x) = \theta(x, y)y \in \mathcal{P}\partial(x)$$

The econometric problem is therefore how to estimate the frontier $\mathcal{P}^t(x^t)$ through a random y sample of productive units. Then, for input data of level x_0 within the X support, consider an independent random sample identically distributed of size m and variables Y_i, $i = 1.2.\ldots\ldots,m$ generated by a distribution p-varied $F_y(y/x_0) = \mathrm{Prob}(Y \geq y/X \leq x_0)$. In this case, the empirical production technology (production frontier) can be expressed as: $\hat{\mathcal{P}}^t(x^t) = \{(x, y) \in R_+^p \times R_+^q/X \leq x_0, Y_i \geq y\}$

Consequently, for any y, the technical efficiency measure may be defined as:

$$\tilde{\theta}_m(x, y) = \max_{i=1.2.\ldots,m}\left\{\min_{j=1.2.\ldots,q}\left(\frac{y_i^j}{y^j}\right)\right\}$$

where a^j corresponds the j-th component of vector a. Observe that in the product space, based on the free disposal hypothesis, $\hat{\mathcal{P}}^t(x^t)$ is the FDH estimator of $\mathcal{P}^t(x^t)$ which shall be denoted by $\hat{\mathcal{P}}^t_{FDH}(x^t)$.

By definition, for any $y \in R_+^q$, the maximum product level expected of m order denoted by $y_m^\delta(x)$ for any x within the X support equals:

$$y_m^\delta(x) = yE\left(\tilde{\theta}_m(x, y)/X \leq x\right)$$

Thus, the existence of the expected value is assumed.

Cazals et al. (2002) showed in their article that the maximum product level of order m expected may be calculated as:

$$y_m^\delta x = y\int_0^\propto \left(1 - F_y(uy/x)\right)^m du$$

Additionally, they showed that when $m \to \infty$, the maximum product level expected converges to the efficient product level defining the frontier, such that $\lim_{m\to\infty} y_m^\delta(x) = y^\delta(x)$.

The non-parametric estimation of $\tilde{\theta}_m(x, y)$ is made by replacing the real $F_y(\cdot/x)$ by its empirical version $\hat{F}_{y,n}(\cdot/x)$ in which:

$$\hat{F}_{X,n}(x/y) = \frac{\sum_{i=1}^n \| (x_i \leq x, y_i \geq y)}{\sum_{i=1}^n \| (x_i \leq x)}$$

In these terms, the estimation of $\tilde{\theta}_m(x, y)$ denoted by $\hat{\theta}_{m,n}(x, y)$ is calculated as:

$$\hat{\theta}_{m,n}(x, y) = \hat{E}\left(\tilde{\theta}_m(x, y)/X \leq x\right) = \int_0^\infty \left(1 - \hat{F}_{y,n}(uy/x)\right)^m du$$

At the same time, the estimator of the maximum product level expected is calculated through:

$$\hat{y}_{m,n}^\delta x = y\hat{\theta}_{m,n}(x, y)$$

However, due to the multivariate nature of $\hat{F}_{y,n}(y/x)$, we notice that there is not an explicit expression for $\hat{\theta}_{m,n}(x, y)$. Therefore, we can use Monte Carlo simulations to estimate it. This procedure is performed through the following steps:

(1) For a data x we consider a sample of size $m < n$ with reposition between the y_i so that $x_i \leq x$. The sample is then described as $(Y_{1.b}, Y_{2.b}, \ldots\ldots, Y_{m,b})$. Notice that this sample forms a partial frontier;
(2) The efficiency of each y_i is calculated through DEA or FDH where $\tilde{\theta}^b_{m,n}(x, y) = \max_{i=1.2....,m}\left\{\min_{j=i,2....q}\left(\frac{y^j_{i,b}}{y^j}\right)\right\}$;
(3) The previous stage in repeated for $b = 1, 2, \ldots, B$, making B big enough.

Therefore, the efficiency order score—m is calculated as:

$$\hat{\theta}_{m,n}(x, y) \approx \frac{1}{B}\sum_{b=1}^{B}\tilde{\theta}^b_m(x, y)$$

The empirical frontier that envelops all observed data is given by the DEA solution or the FDH standard. In the product space, the technical efficiency measurement $\theta_n(x, y)$ for any (x, y) is given by:

$$\theta_n(x, y) = \max\{\theta/(x, \theta y) \in \hat{\mathcal{P}}^t_{FDH}(x^t)\}$$

The estimation is then calculated as:

$$\hat{\theta}_n(x, y) = \max_{i/x_i \leq x}\left\{\min_{j=1.2....,q}\left(\frac{y^j_i}{y^j}\right)\right\}$$

The corresponding maximum efficient product level estimated is therefore equal to:

$$\hat{y}^\delta_n(x) = y\hat{\theta}_n(x, y)$$

Cazals et al. (2002) showed that when n is fixed and $m \to \infty$, $\hat{\theta}_{m,n}(x, y) \to \hat{\theta}_n(x, y)$. This means that the maximum efficient product level of order $\hat{y}^\delta_{m,n}(x)$, converges to the efficient product level FDH $\hat{y}^\delta_n(x)$. However, for a finite m, the estimator of order m does not envelop all data and it is most robust to extreme values, noises or outliers.

2.2. Technological gap

The technical efficiency measures the relative position of states regarding any productivity element. It responds to the question over which the units, according to their technology, are using available resources efficiently. Efficiency measures also permit to evaluate the technological gap between analyzed units through the meta production frontier estimation.

As stated above, the technical efficiency is estimated considering the best practices within a set of decision units. In order to make selected units comparable, it is assumed that the adopted technology among states must be homogeneous. However, it is perfectly foreseeable that states use different technological resources according to their specificities. With regards to public security, richer states could, for example, employ more expensive material resources, training and paying their police force better. Once the technological diversity among states is verified, it is possible to calculate the technological gap between the units and correct efficiency estimates.

The technological gap shall be measured through the comparison between the specific production frontier of a group of homogeneous states with the meta-production frontier. Therefore, the formation of groups of states for the estimation of their specific frontiers must follow grouping criteria. Consequently, groups of homogeneous states shall be organized applying the Cluster Analysis technique. Then, meta-frontier results shall be compared to the frontier of each group, seeking to identify possible technological gaps.

2.3. Meta production frontier

Each subgroup of states has a specific public security technology and a corresponding production frontier. Specific production frontiers serve as reference for the calculation of efficiency scores and other productivity measurements.

The specific technologies of each subgroup may be compared between them and with the group technology. There is a specific production function that corresponds to the technology of each subgroup. Likewise, there is a production meta-frontier associated to the most efficient technologies of each group.

The meta production frontier function initially proposed by Hayami (1969) and Hayami and Ruttan (1970, 1971), is defined as the one enveloping all production functions of state subgroups. The production functions envelopment is defined by the most efficient aspects of specific technologies, as explained by Ruttan et al. (1978).

After the estimation of production functions for each subgroup and the production possibility frontier, it is possible to compare them. The comparison permits to estimate the technological gap of each subgroup, as proposed by Wongchai et al. (2012). The technological gap, $DT_{i,k}$, of the unit i belonging to the subgroup k is defined as:

$$DT_{i,k} = \frac{ET_{i,m}}{ET_{i,k}} \tag{1}$$

where $ET_{i,k}$ is the technical efficiency of the unit i based on the technology of subgroup k and, $ET_{i,m}$, is the technical efficiency of the unit i based on the meta frontier. If $DT_{i,k} > 1$ it means that the unit i of subgroup k suffers a technological gap with regards to the units that define the meta frontier. If $DT_{i,k} = 1$, this means that there is no gap. Technological gap measurements may also be used to test the technological gap of any single group with regards to the meta frontier.

2.4. Productivity variation, efficiency variation, scale variation and technological variation

The productivity measurement variation for Brazilian states shall be the PTF calculated through the Malmquist index, which is similar to the one proposed by Färe et al. (1992, 1994). The great advantage of its use is to allow the total productivity index to be decomposed into factors that explain its variation.

In order to understand the Malmquist productivity index concept, it is necessary to consider the concepts of set of production possibilities, $\mathcal{P}^t(x^t)$ and the production frontier $\mathcal{P}^{\delta t}(x^t)$ defined in Subsection 2.1. Another important concept is the distance function notion, which we will define next.

The distance function oriented by the product (product space) according to Shephard (1970) measures the distance between the observed product and the potential maximum product for a given quantity of utilized inputs, which may be defined as:

$$D^{\mathcal{P}^t}(x^t, y^t) = \inf\{\theta : (x, y^t/\theta) \in \mathcal{P}^t(x^t)\}$$

Please note that the distance function may be defined in terms of other sets different than $\mathcal{P}^t(x^t)$. For example, let's consider $\mathcal{P}^t(x^t)$), a convex cone with a vertex expanded in the origin by the set $\mathcal{P}^t(x^t)$. Thus, $\mathcal{P}^t(x^t) \subseteq \mathcal{V}(\mathcal{P}^t(x^t))$. If the production frontier $\mathcal{P}^{\delta t}(x^t)$ shows constant returns to scale, then $\mathcal{P}^t(x^t) = \mathcal{V}(\mathcal{P}^t(x^t))$ and in this case, $D^{\mathcal{P}_t}(x^t, y^t) = D^{\mathcal{V}(\mathcal{P}_t)}(x^t, y^t)$; otherwise $\mathcal{P}^t(x^t) \subset \mathcal{V}(\mathcal{P}^t(x^t))$, and $D^{\mathcal{P}_t}(x^t, y^t) \geq D^{\mathcal{V}(\mathcal{P}_t)}(x^t, y^t)$. It is evident that the production set $\mathcal{P}^t(x^t)$ and the distance function $D^{\mathcal{P}_t}(x^t, y^t)$ are not observed and therefore, they have to be estimated through observations.

Two of the most common estimators of $\mathcal{P}^t(x^t)$ and $\mathcal{V}(\mathcal{P}^t(x^t))$ and therefore of $D^{\mathcal{P}_t}(x^t, y^t)$ and $D^{\mathcal{V}(\mathcal{P}_t)}(x^t, y^t)$ are the FDH suggested by Deprins et al. (1984) and defined as:

$$\hat{\mathcal{P}}^t_{FDH}(x^t) = \left\{(x^t, y^t) \in R_+^{p+q} | y \leq y_i^t, x \geq x_i^t\right\},$$

and the convex envelopment of $\hat{\mathcal{P}}^t_{FDH}(x^t)$ given by:

$$\hat{\mathcal{P}}^t_{DEA}(x^t) = \left\{(x^t, y^t) \in R_+^{p+q} | y \leq \sum_i \lambda_i y_i^t, x \geq \sum_i \lambda_i x_i^t, \sum_i \lambda_i = 1. i = 1.2.\ldots, n\right\}$$

The convex cone $\hat{\mathcal{P}}^t_{DEA}(x^t)$ expanded by $\hat{\mathcal{P}}^t_{DEA}(x^t)$ or equivalent to $\hat{\mathcal{P}}^t_{FDH}(x^t)$ is obtained by removing the restriction $\sum_i \lambda_i = 1$ of $\hat{\mathcal{P}}^t_{DEA}(x^t)$ and therefore providing a $\mathcal{V}(\mathcal{P}^t(x^t)$ estimator.

The estimators for $D^{\mathcal{P}_t}(x^t, y^t)$ are obtained by replacing $\mathcal{P}^t(x^t)$ by $\hat{\mathcal{P}}^t_{FDH}(x^t)$ or by $\hat{\mathcal{P}}^t_{DEA}(x^t)$. Likewise, $\left(x^t, y^t\right)$ or $D^{\mathcal{V}(\hat{\mathcal{P}}^t_{FDH})}(x^t, y^t)$ shall produce an estimator of $D^{\mathcal{V}(\mathcal{P}_t)}(x^t, y^t)$. The estimators $D^{\hat{\mathcal{P}}^t_{DEA}}(x^t, y^t)$ and $D^{\mathcal{V}(\hat{\mathcal{P}}^t_{DEA})}(x^t, y^t)$ are obtained through linear programming methods whereas $D^{\hat{\mathcal{P}}^t_{FDH}}(x^t, y^t)$ is obtained through numeric algorithms.

In general, the Malmquist index is estimated replacing $\mathcal{P}^{t_1}$ and $\mathcal{P}^{t_2}$ with $\hat{\mathcal{P}}^{t_1}_{DEA}$ and $\hat{\mathcal{P}}^{t_2}_{DEA}$, respectively. In this sense, the estimation of this index has the same problems with regards to the DEA that were discussed in Section 2. Therefore, asking to solve these problems, all Malmquist index distances are estimated by applying the order—m frontier methodology.

In general, it is common to decompose the Malmquist index to identify the productivity variation sources. There are several possibilities but we shall adopt the decomposition proposed by Wheelock and Wilson (1999) as follows:

$$\mathcal{M}(x^{t_2}, y^{t_2}, x^{t_1}, y^{t_1} | \mathcal{P}^{t_1}, \mathcal{P}^{t_2}) = \frac{D^{\mathcal{P}^{t_2}}(x^{t_2}, y^{t_2})}{D^{\mathcal{P}^{t_1}}(x^{t_1}, y^{t_1})} \times \tag{2}$$

$$\frac{D^{\mathcal{V}(\mathcal{P}^{t_2})}(x^{t_2}, y^{t_2}) / D^{\mathcal{P}^{t_2}}(x^{t_2}, y^{t_2})}{D^{\mathcal{V}(\mathcal{P}^{t_1})}(x^{t_1}, y^{t_1}) / D^{\mathcal{P}^{t_1}}(x^{t_1}, y^{t_1})} \times \tag{3}$$

$$\left[\frac{D^{\mathcal{P}^{t_1}}(x^{t_1}, y^{t_1})}{D^{\mathcal{P}^{t_2}}(x^{t_1}, y^{t_1})} \times \frac{D^{\mathcal{P}^{t_1}}(x^{t_2}, y^{t_2})}{D^{\mathcal{P}^{t_2}}(x^{t_2}, y^{t_2})} \right]^{1/2} \times \tag{4}$$

$$\left[\frac{D^{\mathcal{V}(\mathcal{P}^{t_1})}(x^{t_1}, y^{t_1}) / D^{\mathcal{P}^{t_1}}(x^{t_1}, y^{t_1})}{D^{\mathcal{V}(\mathcal{P}^{t_2})}(x^{t_1}, y^{t_1}) / D^{\mathcal{P}^{t_2}}(x^{t_1}, y^{t_1})} \times \frac{D^{\mathcal{V}(\mathcal{P}^{t_1})}(x^{t_2}, y^{t_2}) / D^{\mathcal{P}^{t_1}}(x^{t_2}, y^{t_2})}{D^{\mathcal{V}(\mathcal{P}^{t_2})}(x^{t_2}, y^{t_2}) / D^{\mathcal{P}^{t_2}}(x^{t_2}, y^{t_2})} \right]^{1/2} \tag{5}$$

A Malmquist index higher than one represents an increase in total productivity whereas a value below one suggests a productivity fall and a one value shows no productivity variation. The total productivity variation shall be denoted by ΔPTF.

Note the first term on the right hand side (Eq. (2)), denominated ΔEF, measures changes in order—*m* the technical efficiency in which values higher than (equal to, less than) 1 suggest increase (constant, reduction) of efficiency. Observe that the efficiency may vary through time, because one unit (state) moves in the direction of the order—*m* frontier, as the frontier varies through time or due to a combination of these two factors.

The second term (Eq. (3)), $\Delta EESC$, measures changes in order—*m* scale efficiency. Notice that the numerator of this term compares the distances from a point in particular (*x*,*y*) to the conical hull of the order—m *m* frontier in the output direction and the distance from this same point to the order—*m* frontier in the period t_2. If these distances are identical, the order—*m* frontier presents constant returns to scale. Consequently, the point (*x*,*y*) is order—*m* scale efficient. If these distances are not the same, then the numerator shall be lower than 1, showing that the order—*m* frontier is in a region with growing or decreasing returns to scale. Since the $\Delta EESC$ denominator compares these same distances with regards to the t_1 period, then $\Delta EESC > 1$ shows an increase in the scale efficiency; $\Delta EESC = 1$ suggests no scale variation and $\Delta EESC < 1$ proves a decrease in the scale efficiency.

The third term (Eq. (4)), ΔTEC, measures frontier displacements through time. When the first ratio of this term is higher than (equal, less than) 1, then the order—*m* frontier is displaced upwards (does not change, it is displaced downwards in the place where (x^{t_1}, y^{t_1}) is projected in the direction of the product on the frontier). The second ratio is interpreted in the same way considering the point (x^{t_2}, y^{t_2}). Consequently, ΔTEC is defined as the geometric mean of these two ratios.

The fourth term (Eq. (5)), $\Delta EESCF$, measures the efficiency variation in order—*m* scale efficiency due to the variations in the order—*m* frontier through time. Notice that the first numerator on the right hand of the $\Delta EESCF$ is the same as the $\Delta EESCF$ denominator. This numerator measures the order—*m* scale efficiency at the point where (x^{t_1}, y^{t_1}) is projected in the product direction on the order—*m* frontier in the period t_1. The corresponding denominator is similar, except that $\mathcal{P}^{t_2}$ is replaced by $\mathcal{P}^{t_1}$. Therefore, this denominator measures the order—*m* scale for the order—*m* frontier in the second period (t_2), where (x^{t_1}, y^{t_1}) is projected in the product direction of $\mathcal{P}^{t_2}$.

As a result, the ratio of these two scale order—*m* efficiencies shall be lower than (equal to, higher than) the distance between $\mathcal{P}^{t_1}$ and $\mathcal{V}(\mathcal{P}^{t_1})$ through the trajectory in which (x^{t_1}, y^{t_1}) is projected in the product direction towards the frontier. Summarizing, this ratio compares the scale efficiency of (x^{t_1}, y^{t_1}) related to the technologies $\mathcal{P}^{t_1}$ and $\mathcal{V}(\mathcal{P}^{t_1})$ in the product direction; values lower than (equal to, lower than) 1 correspond to an increasing order—m scale inefficiency (constant, decreasing) for a state located at (x^{t_1}, y^{t_1}) for both periods. The second $\Delta EESCF$ ratio is interpreted likewise, however related to (x^{t_2}, y^{t_2}). Since these two situations could be different with regards to (x^{t_1}, y^{t_1}) and (x^{t_2}, y^{t_2}), $\Delta EESCF$ is defined as the geometric mean of the terms that measure the relative effect of the state localization in t_1 and t_2. In these situations, $\Delta EESCF$ measures the efficiency scale variation of order—*m* resulting of the displacement or format variation in the order—*m* frontier with $\Delta EESCF$ (<, =, >) 1 considering the order—*m* scale efficiency (increases, remains constant, decreases) throughout its fixed trajectories in the product direction.

3. Database and statistics

In the literature specialized on calculating public security efficiency, data commonly used as outputs are the crime rates or some transformation. In general, as inputs, we use human, material and financial resources used for security purposes.

Among many others, we can highlight the following studies: (a) Schull et al. (2014) use crime rates (intentional homicide, robbery, involuntary vehicular manslaughter, drug trafficking and rape) as a product and the expenses in security as an input; (b) Pereira Filho et al. (2010) estimate the public security efficiency based on a costs frontier. They use the wages of the military and civil policemen as inputs and the inverse homicide rate as product (cost); (c) Arantes et al. (2012) estimate the technical efficiency of public security in the Minas Gerais municipalities where the products are: the homicide rate, the rate of violent crimes against property, the rate of violent crimes against persons, the rate of less aggressive crimes and the rate of criminals arrested during violent crimes. The input applied was the expense per capita with public security; (d) Scalco et al. (2012) estimate the technical efficiency of the Minas Gerais military police. They used the following input variable: number of military policemen per each 1000 people. The outputs were: number of arrests registered due to violent crime acts against persons per policeman; number of arrests registered due to violent crimes against property per policemen, inverse violent crime rate against persons and inverse of violent crime rate against property.

Seeking to calculate technical efficiency scores for public security services, the following product indicators were used for each state: (a) homicide prevention rate; (b) other thefts prevention rate; (c) vehicle theft prevention rate. The inputs used were: the number of military policemen; (b) the number of civil policemen and (c) the amount of public expenses with security except for police expenses. The variable “other thefts” is the difference between total thefts minus vehicle thefts.

Prevention rates were obtained based on complementary homicide rates, other thefts and vehicle theft rates weighted by the population. If we consider $y_{\max(t)}$, as the highest homicide rate among the Federation units in a period t and σ_t as the homicide rate standard deviation, then the homicide prevention rate for unit i, in a period t, is given by;

$$z_{it} = \left(y_{\max(t)} + \sigma_t - y_{it}\right) \times N,$$

where y_{it} is the homicide rate for the Federation unit i, in a period t and N is the relation between the unit i population and the population of the unit with the highest homicide rate. The other prevention rates follow the same formula.

The expense with maintenance and remuneration of the police forces is one of the main components of public expenses on security. Seeking to avoid that the policing effect is double-counted, the variable applied in the estimations is the difference between the total expenses with security minus the expense with police services. This variable was built based on 2012 prices using the IPCA (Amplified Consumer Price Index) as published by IBGE. All variables used were extracted from the Brazilian Public Security Yearbook between 2008 and 2012.

On Table 1, the state averages for these variables related to the years 2008 and 2012 are introduced. In the Homicides, Vehicles and Thefts columns, you will find statistics related to the average number of homicides, car thefts and other thefts per 100,000 people. The column expenses introduces information related to the per capita expenses with public security, except for police-related expenses. The columns Military and Civil correspond to the number of military and civil policemen per 1000 people.

In a first approach, public security indicators statistics evidence a scarcity of inputs in order to face the high crime rates. In 2012, for example, we verified that on average, the equivalent to R$ 255.79 per capita was invested in security and that for every 1,000 people there were less than 3 military policemen and less than one civil policeman. The state of Rondônia is the one that invested the most in security, spending R$ 486.29 per capita—eight times more than the state of Amapá, which is the one with the lowest investment. As for the agents, the main characteristic is the reduced presence of police forces. Even in the Federal District, which boasts the highest relative military police, there are less than six policemen for every 1000 people.

The results obtained reveal a disturbing reality. The average homicide rate per state is above 28 per 100 thousand inhabitants, with the highest rate registered in Alagoas, with 58.2. The vehicle theft rate per state reaches 215.15 per 100 thousand people, with a peak of 737.1 in Amazonas. The other theft types have a mean of 126.04 per 100 thousand people and a maximum of 567.5 in São Paulo.

Table 1
Public security indicators in Brazilian States—2008/2012.

2008						
Statistic	Homicides	Vehicle theft	Other thefts	Expenses	Military	Civil
Mean	29.28	49.67	140.67	183.28	2.56	0.75
Standard deviation	11.91	44.11	116.46	95.67	1.14	0.54
Minimum	12.41	1.70	21.10	28.07	0.67	0.12
Maximum	60.33	175.40	514.6	370.65	5.79	2.15
2012						
Statistic	Homicide	Vehicle	Other thefts	Expenses	Military	Civil
Mean	28.26	215.15	126.04	255.79	2.44	0.75
Standard deviation	12.01	170.60	140.50	104.25	1.05	0.45
Minimum	9.9	3.50	0	55.32	1.09	0.22
Maximum	58.2	737.1	567.5	486.29	5.63	1.82

Source: developed by the authors.

The maximum and minimum values in both years show that vehicle theft, general thefts and the civil police force present a considerable dispersion. The width of all variables reveals a situation of inequality among the units, both with regards to crime rates and resources used. Considering its federative units as a reference, we can conclude that Brazil is a country with high criminality, and high crime rate levels against individuals and property. In addition to this, the availability of resources is generally insufficient.

Analyzing the trajectory of crime indicators and the resources employed in public security between 2008 and 2012, we verify important variations during this period. On average, there was an inexpressive reduction in the homicide rate (−3.4%) and a remarkable growth in the vehicle theft rate (333.15%). As for the resources applied, there was a 4.68% reduction in the number of military police while the number of civil policemen remained constant during these two years. On the other hand, there was a growth in expenses of about 39.56%.

The states that recorded improvements in their homicide rates were: Amapá (−71.23%), Espírito Santo (−51.24%), Mato Grosso do Sul (−49.63%), Roraima (−48.11%), Pernambuco (−32.39%), Rio de Janeiro (−30.86%) and Goiás (−29.69%). The worst results were found in the states of Ceará (68.93%), Paraíba (42.0%), Sergipe (32.36%), Rio Grande do Norte (27.71%), Piauí (22.42%) and Bahia (17.55%).

As for the vehicle theft rates, all states suffered significant increases. The state with the highest rates were Maranhão (1.585%), Piauí (1.240%), Amazonas (1.214%), Alagoas (1.014%), Rio Grande do Norte (800%) and Ceará (713%).

With regards to other types of thefts, the states that showed the highest reductions were Amapá (−95%), Santa Catarina (−78%), Rondônia (−71%), Paraíba (−66%), Espírito Santo (−63%), Paraná (−62%), Rio Grande do Sul (−52%) and Minas Gerais (−51%). On the other hand, the states that suffered a growth in this indicator were Amazonas (229%), Acre (214%), Sergipe (185%), Minas Gerais (131%), Piauí (96%) and Rio Grande do Norte (90%).

With regards to the per capita expenses with public security, only the states of Paraná (−45.79%) and Rio Grande do Norte (−28.83%) suffered a reduction of this input. The remaining federative units either improved or suffered insignificant reductions. The police force per one thousand people was reduced in Santa Catarina (−44.29%), Roraima (−21.7%), Piauí (−18.38%), Sergipe (−15.4%), Tocantins (−14.32%), Maranhão (−10.05%) and Mato Grosso do Sul (−8.5%). The civil force was reduced in the states of Rio Grande do Sul (−43%), Sergipe (−16%), Roraima (−15%), Mato Grosso do Sul (−14%), Federal District (−11%), São Paulo (−9%) and Ceará (−8%).

4. Analysis of results

According to Krüger (2012), when there are no outliers present or there are measurement errors in the data sampling, the DEA and FDH methods are more suitable than the order—*m* frontier. In this sense, the Super efficiency test was firstly carried out based on the Andersen and Petersen (1993) models to verify the possible presence of outliers in the sample. Table A1 in the article Appendix shows the super efficiency scores and the corrected z statistics. Super

Table 2
Efficiency scores, homicides, vehicle thefts and other thefts. Mean values from 2008 to 2012.

State	Efficiency		Homicide		Other thefts		Vehicle thefts	
	Values	Order	Rate	Order	State	Values	Order	Rate
MG	1.2655	1°	15.12	22°	MG	1.2655	1°	15.12
SP	1.2099	2°	11.40	26°	SP	1.2099	2°	11.40
BA	1.1651	3°	36.24	5°	BA	1.1651	3°	36.24
RJ	1.1648	4°	29.64	14°	RJ	1.1648	4°	29.64
RS	1.0924	5°	18.74	21°	RS	1.0924	5°	18.74
MS	1.0066	6°	19.54	20°	MS	1.0066	6°	19.54
PI	1.0043	7°	10.80	27°	PI	1.0043	7°	10.80
TO	1.0011	8°	19.84	19°	TO	1.0011	8°	19.84
SE	1.0006	9°	32.42	7°	SE	1.0006	9°	32.42
RO	1.0003	10°	30.98	11°	RO	1.0003	10°	30.98
AC	1.0000	11°	24.65	16°	AC	1.0000	11°	24.65
CE	0.9050	12°	31.96	8°	CE	0.9050	12°	31.96
PR	0.8675	13°	31.20	9°	PR	0.8675	13°	31.20
PA	0.8482	14°	37.90	3°	PA	0.8482	14°	37.90
MA	0.8310	15°	20.20	18°	MA	0.8310	15°	20.20
SC	0.7736	16°	13.14	25°	SC	0.7736	16°	13.14
GO	0.7682	17°	21.84	17°	GO	0.7682	17°	21.84
AL	0.6929	18°	65.02	1°	AL	0.6929	18°	65.02
PB	0.6077	19°	35.70	6°	PB	0.6077	19°	35.70
AM	0.5817	20°	27.24	15°	AM	0.5817	20°	27.24
ES	0.5493	21°	37.42	4°	ES	0.5493	21°	37.42
RN	0.5276	22°	30.86	12°	RN	0.5276	22°	30.86
DF	0.4676	23°	30.78	13°	DF	0.4676	23°	30.78
AP	0.4605	24°	13.44	24°	AP	0.4605	24°	13.44
MT	0.4530	25°	31.06	10°	MT	0.4530	25°	31.06
PE	0.4283	26°	41.40	2°	PE	0.4283	26°	41.40
RR	0.0406	27°	14.00	23°	RR	0.0406	27°	14.00
Mean	0.8041		27.13		Mean	0.8041		27.13

Source: authors' estimates.

efficiency results tests suggest the existence of outliers: Alagoas, in 2009; São Paulo, in 2010; Paraná, in 2009, 2010, 2011 and 2012.

Once the presence of outliers in the sample is identified, there are two possible procedures: (a) eliminating them from the sample and proceeding to estimate inefficiencies using the DEA or FDH models or (b) keeping the original sample and outlier-robust estimation methods. We chose the second alternative and efficiencies were estimated using the order—m frontier method. According to Cazals et al. (2002), the lower the *m* value in relation to the sample size, the more robust the order—*m* estimator becomes for extreme values and outliers. On the other hand, the estimated frontier gets further away from the real frontier. It is up to the researcher to adjust the m values according to his goals. In this study, we used several m values that could make order—*m* efficiency scores quite different from the scores obtained through FDH. The m value was equal to 20.

Table 2 values show the annual mean values and the public security efficiency order, as well as the Homicide, Vehicle Theft and Other Thefts rates in Brazilian states.

The technical efficiency average of Brazilian states was 0.804. Based on the best states performances, it would be possible to improve public security results in approximately 20%, considering the resources (inputs) available in the states. Among analyzed states, 11 of them were efficient, which is equivalent to 40.75% of the Federative units. The most efficient states were, in decreasing order, Minas Gerais, São Paulo, Bahia, Rio de Janeiro, Rio Grande do Sul, Mato Grosso do Sul, Piauí, Tocantins, Sergipe, Rondônia and Acre. Among the most inefficient ones we should mention Roraima, with an efficiency score lower 5%.

Apparently, there is no influence of regional factors in the determination of technical efficiency, considering that both in the efficient and inefficient groups there is a presence of states from all regions.

Table 3
Groups of homogenous states according to the cluster analysis technique.

Group	2008	2009	2010	2011	2012
Group 1	RJ, MG	RJ, MG	RJ, MG	RJ, MG	RJ, MG
Group 2	BA, RS, SP	BA, RS, SC, SP	BA, RS, SC, SP	BA, RS, SC, P,GO, PA	BA, RS, SC, SP, G0, PA,CE
Group 3	Other states	Other states	Other states	Other states	Other states

Source: elaborated by the authors.

Table 4
Annual technological gap and 2008–2012 mean value.

State	2008	2009	2010	2011	2012	Average
AC	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AL	1.0188	1.0000	1.0019	1.0000	3.2563	1.1664
AM	1.0116	1.0910	0.9989	3.0450	3.0577	1.4038
AP	2.9738	0.6388	0.3614	0.1671	0.4229	0.3841
BA	0.8217	0.8567	0.8660	0.8953	0.8641	0.8601
CE	1.0784	1.0602	1.3953	1.3807	1.2398	1.2142
DF	2.0235	2.3695	1.0023	1.0009	3.6644	1.5694
ES	1.0520	1.0025	1.7365	2.8637	2.6595	1.5388
GO	1.1592	1.1331	0.3619	0.5938	0.7833	0.6694
MA	1.0237	0.9893	1.0034	1.6070	1.5685	1.1781
MG	0.7890	0.7855	0.7758	0.8189	0.7817	0.7899
MS	0.9970	1.0000	1.0000	0.6471	0.4796	0.7538
MT	1.0251	1.2773	1.0437	4.0765	4.0501	1.5582
PA	1.2496	1.1277	1.4564	0.4691	1.1495	0.9301
PB	0.9311	1.0005	0.6721	2.7011	2.6673	1.1611
PE	1.1678	1.0687	3.1106	3.3348	3.2580	1.8380
PI	1.0000	0.9824	1.0002	1.0000	0.9989	0.9963
PR	1.0001	1.0000	0.9991	1.2305	1.7738	1.1423
RJ	0.9245	0.8438	0.8351	0.8418	0.8471	0.8573
RN	1.0065	1.0104	1.0011	3.2833	3.2576	1.3913
RO	1.0000	1.0000	1.0000	1.0000	1.0017	1.0003
RR	7.0676	10.1438	7.8839	14.4214	13.0111	9.7444
RS	0.8916	0.8735	0.9344	0.9471	0.9380	0.9160
SC	1.0012	0.5454	0.3196	0.4500	0.6579	0.5153
SE	1.0000	1.0000	1.0000	0.9990	1.0025	1.0003
SP	0.8164	0.8363	0.8162	0.8146	0.8733	0.8308
TO	0.9984	1.0005	1.0000	1.0000	0.9981	0.9994
Average	1.0683	0.9909	0.8582	0.8851	1.1467	0.9780

Source: estimated by the authors.

4.1. Meta production frontier and technological gap

The Cluster analysis applied to input and product values identified three groups with homogeneous states, as detailed on Table 3 as follows: Group 1 is constituted only the Minas Gerais and Rio de Janeiro states in all sample years; Group 2 varies its composition for every sample year. In 2008, the group was formed by the states of Bahia, Rio Grande do Sul and São Paulo. However, in 2012 the states of Santa Catarina, Goiás, Pará and Ceará were included in the group. Group three is obviously formed by all other states for each year.

Once groups were identified, the meta production frontier and the production frontier for each group were calculated through the order—*m* frontier method. Then, using the expression (Eq. (1)) from Subsection 2.3, state technological gaps were estimated. Measurements higher than 1, suggest a technological gap for that state in a given group with regards to the national frontier. Results are introduced in Table 4 below.

The estates that suffer the biggest technological gaps in decreasing order are: Roraima, Pernambuco, Distrito Federal, Espírito Santo, Mato Grosso, Amazonas, Rio Grande do Norte, Ceará, Maranhão, Alagoas, Paraíba and Paraná. Among

Table 5
Efficiency variation method. Technological variation and total factor productivity variation.

DMU	Variation				
	ΔEF	$\Delta EESC$	ΔTEC	$\Delta EESF$	ΔPTF
MG	1.002	1.377	0.987	0.939	1.279
RO	1.000	0.906	1.218	1.060	1.170
RS	0.989	1.181	1.008	0.985	1.160
SE	1.000	0.986	1.211	0.969	1.158
MS	0.997	1.030	1.089	0.996	1.114
RN	0.746	1.757	1.338	0.635	1.113
MT	0.713	1.976	1.301	0.597	1.093
PE	0.752	1.726	1.160	0.693	1.043
AC	1.000	1.070	1.000	0.958	1.025
CE	0.933	1.146	1.048	0.898	1.006
MA	0.896	1.250	1.061	0.844	1.003
ES	0.782	1.488	1.133	0.761	1.003
RJ	1.022	0.960	0.989	1.025	0.996
GO	0.911	1.189	1.050	0.872	0.992
SP	0.986	0.993	0.997	1.014	0.991
PI	1.000	0.974	0.999	1.013	0.987
AL	0.746	1.661	1.159	0.682	0.979
AP	1.366	0.538	0.862	1.543	0.978
PR	0.893	1.180	0.992	0.928	0.970
BA	0.989	0.983	1.010	0.980	0.963
PA	0.977	1.021	1.006	0.953	0.956
TO	1.000	0.912	1.000	1.047	0.955
PB	0.785	1.536	1.131	0.680	0.927
SC	0.904	0.944	1.060	0.969	0.876
AM	0.772	1.478	1.068	0.718	0.875
DF	0.924	0.760	1.165	0.955	0.781
RR	1.104	0.906	0.700	1.107	0.775

Source: estimated by the authors.

all these states, Roraima resulted the least technologically developed, as its gap is much bigger than the gap found in the other states. Therefore, the remaining states that do not suffer a technological gap are the ones that determine the efficient national frontier.

In terms of spatial distribution of states with a technological gap, we identified 7 in the Northeast, 3 in the North, 2 in the Midwest and 1 in the South.

Analyzing the composition of clusters taking 2012 as a reference, we calculated the mean sub-frontier gap. Group 1 presented an average gap of 0.822; Group 2, 0.796 and Group 3, 1.098. The evidence suggests that Group 3 adopted a differentiated technological standard lower than the other groups.

4.2. *Total factor productivity variation decomposition*

The FTP decomposition values were calculated in accordance with expressions (Eqs. (2)–(5)) of Subsection 2.4. These results are the means for the analyzed period and are detailed in Table 5.

The states with ΔPTF over 1 obtained total productivity gains. This means that results obtained in the fight against crime were proportionally superior to the amount of resources (inputs) used, or else, that they decreased more than the achieved results. Among all states, 12 experienced a total productivity growth.

In decreasing order, these states were: Minas Gerais, Rondônia, Rio Grande do Sul, Sergipe, Mato Grosso do Sul, Rio Grande do Norte, Mato Grosso, Pernambuco, Acre, Ceará, Maranhão and Espírito Santo. The remaining 15 states presented a negative total productivity variation. In other words, the crime fight results in these states were proportionally lower than the value of resources applied in these initiatives or the resources improved more than the positive results obtained.

Table 6
Identification of the most relevant productivity components.

DMU	Order (ΔPTF))	Positive contribution	Negative contribution
MG	1^+	ΔEF, $\Delta EESC$	ΔTEC, $\Delta EESCF$
RO	2^+	ΔTEC, $\Delta EESCF$	$\Delta EESC$
RS	3^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
SE	4^+	$\Delta EESC$, ΔTEC	$\Delta EESC$, $\Delta EESCF$
MS	5^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
RN	6^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
MT	7^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
PE	8^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
AC	9^+	$\Delta EESC$	$\Delta EESCF$
CE	10^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
MA	11^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
ES	12^+	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
RJ	13^-	ΔEF, $\Delta EESCF$	$\Delta EESC$, ΔTEC
GO	14^-	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
SP	15^-	$\Delta EESCF$	ΔEF, $\Delta EESC$, ΔTEC
PI	16^-	$\Delta EESCF$	$\Delta EESC$, ΔTEC
AL	17^-	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
AP	18^-	ΔEF, $\Delta EESCF$	$\Delta EESC$, ΔTEC
PR	19^-	$\Delta EESC$	ΔEF, ΔTEC, $\Delta EESCF$
BA	20^-	ΔTEC	ΔEF, $\Delta EESC$, $\Delta EESCF$
PA	21^-	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
TO	22^-	$\Delta EESCF$	$\Delta EESC$
PB	23^-	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
SC	24^-	ΔTEC	ΔEF, $\Delta EESC$, $\Delta EESCF$
AM	25^-	$\Delta EESC$, ΔTEC	ΔEF, $\Delta EESCF$
DF	26^-	ΔTEC	ΔEF, $\Delta EESC$, $\Delta EESCF$
RR	27^-	ΔEF, $\Delta EESCF$	$\Delta EESC$, ΔTEC

Source: developed by the authors.
Remarks: + indicates positive FTP variation; − suggests negative PTF variation.

Table 6 shows the PTF components for each state that contributed the most to the increase or decrease in productivity. Among the 12 states that presented a positive total productivity variation, 10 of them obtained productivity gains due to efficiency improvements and technological progress.

These components more than compensated the negative variation effects of the other components. In decreasing order, these states were Rondônia, Rio Grande do Sul, Sergipe, Mato Grosso do Sul, Rio Grande do Norte, Mato Grosso, Pernambuco, Ceará, Maranhão and Espírito Santo.

In terms of political advice, these states should prioritize their policies aimed at improving technical efficiency and scale efficiency whenever there is technical progress, seeking to obtain more productivity.

Productivity gains in the state of Minas Gerais were obtained as a result of the variation in technical efficiency and the scale variation due to the frontier displacement. The variation of these components largely compensated the negative variation of the other components. At the same time, Acre was the only state that obtained productivity gains as a result of scale returns when the frontier was displaced.

Among the 15 states that suffered total productivity losses, the components that contributed the most to these negative results were predominantly related to the scale efficiency variation and the scale variation when the frontier was displaced.

We can then verify once again that measures seeking to improve the productivity of public security services are more related to scale issues than to efficiency and technological progress. Quantitative aspects seem to be more relevant than qualitative ones.

Considering that scale adjustments have a limited implementation capacity, it is expected that technological issues, considering their unlimited benefit potential, and the efficient use of resources (technical efficiency) are given priority in public security initiatives.

5. Final considerations

Descriptive statistics of public security indicators clearly show a scarcity of resources (inputs) to face the high crime rates in Brazilian states. On average, R$ 255.79 per capita are applied to public security and for every 1000 people, there are less than three military policemen and less than one civil policeman.

Crime indicators suggest a disturbing scenario. The average homicide rate per state is above 28 per 100 thousand inhabitants, with a maximum rate of 58.2 in Alagoas. On average, the vehicle theft rate per state reaches 215.15 per 100 thousand people, with a record 737.1 in Amazonas. The other types of theft average 126.04 per 100 thousand people with a maximum of 567.5 in São Paulo.

In terms of technical efficiency, the mean value for all Brazilian states was 0.80. This result suggests that it would be possible to improve public security indicators in approximately 20%, considering the available resources (inputs) for each state. Among analyzed states, 11 of them were considered efficient. The most efficient states were, in decreasing order, Minas Gerais, São Paulo, Bahia, Rio de Janeiro, Rio Grande do Sul, Mato Grosso do Sul, Piauí, Tocantins, Sergipe, Rondônia and Acre. Among the most inefficient ones, Roraima has a technical efficiency level below 5%.

Apparently, there are no regional factors affecting technical efficiency results, considering that both the efficient and inefficient groups have states from all regions.

The states that suffer the biggest technological gaps in decreasing order are: Roraima, Pernambuco, Federal District, Espírito Santo, Mato Grosso, Amazonas, Rio Grande do Norte, Ceará, Maranhão, Alagoas, Paraíba and Paraná. Among these states, Roraima is the most technologically undeveloped, as it gap is much bigger than the other states. The states that did not suffer a technological gap are the ones that set the efficient national frontier.

Among all Brazilian states, 12 achieved a total productivity growth, being in decreasing order, Minas Gerais, Rondônia, Rio Grande do Sul, Sergipe, Mato Grosso do Sul, Rio Grande do Norte, Mato Grosso, Pernambuco, Acre, Ceará, Maranhão and Espírito Santo. The remaining 15 states had a negative total productivity variation. In other words, in states with a positive PTF variation, results of crime fight improvements were proportionally higher than the resources applied to these initiatives or either the use of such resources increased proportionally less than obtained improvements. Obviously, in states with a negative PTF, we find the opposite phenomenon.

Out of the 27 analyzed states, 12 obtained a positive total productivity variation, being that 10 of them achieved productivity gains as a result of the scale efficiency growth and technological progress. In decreasing order, these states were Rondônia, Rio Grande do Sul, Sergipe, Mato Grosso do Sul, Rio Grande do Norte, Mato Grosso, Pernambuco, Ceará, Maranhão and Espírito Santo. In terms of political advice, these states should give priority to policies aimed at improving technical efficiency and scale efficiency, seeking to reach even higher productivity levels.

As for the 15 states that suffered total productivity losses, the main components that negatively contributed to such results were the scale efficiency variation and the scale variation when there is a national frontier displacement.

We verify once again that measures aimed at improving public security productivity services are more related to scale aspects than to efficiency gains and technological progress. Quantitative issues seem to be more relevant that qualitative ones.

Considering that scale adjustments have a limited implementation capacity, it would be desirable that technological issues, because of their unlimited benefit potential, and the efficient use of resources (technical efficiency) are given priority in public security initiatives.

The most efficient units in the last sample year showed, in general terms, less scale variation, more productivity variation, more efficiency variation, less technological variation and a smaller technological gap. With fewer technological gaps, these units also presented fewer technological variations, or, in other words, as they already have a relatively more advanced technology it was not necessary or possible to include new technological upgrades in security.

The difference in efficiency is partially explained by the technological gap between unit groups. This shows that an effort in the sense of standardizing the qualification of personal and material resources should permit to obtain higher efficiency and productivity levels. However, there are intra-group efficiency gaps. This result shows the possibility of sometimes obtaining productivity gains without the need to invest in changing the technological standard. Perhaps only a better use of available material resources could promote expressive productivity gains in those units with the biggest technological gap.

Appendix A.

In small samples, we can use a test based on the Andersen and Petersen super efficiency model (1993) to identify outliers. This test shall be performed in two stages: in the first one, super efficiency estimations shall be made in order to identify DMU's with scores above the unit. In the second one, we perform a statistical test to indentify the presence of outliers.

According to Seo (2006) the z modified statistic was applied, in which for each unit i, $z_i = 0.675\,(x_i - \tilde{x})\,/Med$, where $\tilde{x}$ is the mean of efficiency scores for all units and Med is the mean of $(x_i - \tilde{x})$ absolute values. In such case, the unit shall be considered outlier if $|z_i| > 3.5$. Values obtained through these statistics are introduced in Tables A1 and A2.

Table A1
Super efficiency test results.

	2008 z-corrected	2009 z-corrected	2010 z-corrected	2011 z-corrected	2012 z-corrected
AC	1.15	0.86	0.31	0.42	0.68
AL	0.57	7.49	0.17	0.48	0.42
AP	1.90	0.68	1.21	1.47	1.22
AM	0.25	0.52	0.45	0.72	0.58
BA	1.95	0.76	0.74	0.55	0.65
CE	1.95	1.39	0.78	1.00	1.35
DF	0.03	0.89	0.85	0.93	1.01
ES	0.74	0.19	0.68	0.68	0.57
GO	0.29	0.26	0.04	0.01	0.10
MA	2.17	2.01	2.78	1.79	1.41
MT	1.25	0.45	0.98	1.05	0.92
MS	0.50	2.02	0.11	0.36	0.08
MG	2.32	2.82	1.73	1.44	1.72
PA	0.00	0.10	0.00	0.57	0.23
PB	0.37	0.00	0.68	0.00	0.36
PR	3.20	7.35	8.27	4.05	24.38
PE	0.11	1.53	0.15	0.11	0.00
PI	0.32	0.01	0.45	0.85	0.83
RJ	0.54	0.32	0.76	0.20	0.16
RN	0.68	0.18	0.39	0.10	0.00
RS	0.11	0.22	0.32	0.62	0.79
RO	0.26	0.32	0.48	0.78	0.71
RR	1.71	1.26	1.12	1.02	0.86
SC	1.43	0.12	0.15	0.22	0.64
SP	3.14	2.22	3.57	2.81	2.29
SE	0.15	0.27	0.47	0.66	0.47
TO	1.52	1.22	1.04	0.95	0.83

Source: elaborated by the authors.

Table A2
Escores de Eficiência.

Estados	2008	2009	2010	2011	2012	Média
AC	1,00000	1,00000	1,00000	1,00000	1,00000	1,00000
AL	1,00400	1,00000	1,00251	1,00000	0,31030	0,69288
AP	0,26952	0,24962	1,00000	0,93023	0,93804	0,46051
AM	1,00992	0,95614	1,00301	0,36056	0,35855	0,58170
BA	1,21701	1,16721	1,15479	1,12682	1,16352	1,16515
CE	1,06659	1,08688	0,82102	0,82686	0,80659	0,90501
DF	0,38108	0,40534	1,00079	1,00119	0,27744	0,46757

Table A2 (*Continued*)

Estados	2008	2009	2010	2011	2012	Média
ES	1,00549	1,00161	0,59204	0,36242	0,37575	0,54930
GO	0,97569	1,00856	0,66357	0,66750	0,67237	0,76819
MA	1,02668	1,01251	1,00237	0,64739	0,66096	0,83099
MT	1,00931	0,78494	0,97547	0,25548	0,26083	0,45298
MS	1,01686	1,00000	1,00000	1,01291	1,00366	1,00664
MG	1,26749	1,27304	1,28901	1,22115	1,27926	1,26554
PA	0,91041	0,95383	0,76235	0,81175	0,83048	0,84824
PB	1,00747	1,00060	1,02163	0,37821	0,38277	0,60773
PR	1,00082	1,00000	1,00091	0,83957	0,63512	0,86748
PE	0,98015	0,97317	0,31175	0,30894	0,31426	0,42828
PI	1,00000	1,01853	1,00146	1,00000	1,00186	1,00432
RJ	1,08163	1,18509	1,19741	1,18795	1,18044	1,16485
RN	1,00724	1,00308	1,00077	0,30499	0,31158	0,52758
RS	1,12153	1,14483	1,07107	1,05788	1,07161	1,09236
RO	1,00000	1,00000	1,00000	1,00000	1,00158	1,00032
RR	0,03416	0,03401	0,04161	0,04786	0,05078	0,04055
SC	1,00075	0,95514	0,67175	0,69847	0,66798	0,77356
SP	1,22489	1,19580	1,22513	1,24841	1,15911	1,20988
SE	1,00000	1,00000	1,00000	1,00257	1,00064	1,00064
TO	1,00188	1,00047	1,00000	1,00141	1,00188	1,00113

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