2009

Rational Number Fluency: A Prerequisite for Success in High School Mathematics Courses?

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Rational Number Fluency: A Prerequisite for Success
in High School Mathematics Courses?

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Dissertation
presented in partial fulfillment of the requirements
for the degree of

Doctor of Education
in Curriculum and Instruction

The University of Montana
Missoula, MT
August 2009

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Rational Number Fluency: A Prerequisite for Success in High School Mathematics Courses?
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This dissertation examined high school students' skill with rational numbers and compared that skill level with the skill level of middle school students by testing 147 high school seniors with 16 rational number word problems released from the National Assessment of Educational Progress assessments. The questions included equal amounts of fraction and money problems. The seniors were grouped by gender and by whether they had completed only beginning algebra and geometry or had more mathematics than geometry.

The findings of this study replicated the high school seniors' low rational number skill level seen in other studies with various age groups. The seniors had higher levels of skill on problems that were more contextually based in common real world problems. Problems with percents were more difficult for the seniors unless the problem involved tipping. Fraction problems that involved operations on fractions were the most difficult.

The high school seniors performed better than the middle school students on all but one of the problems. Standardized middle school test scores predicted the seniors' rational number skills only for students with mathematics beyond geometry while students with only algebra and geometry demonstrated a higher level of rational number skills than was expected.

A gender gap was seen as female students generally performed better than male students on money problems and the male students performed better on the fraction problems, particularly those that assessed formal knowledge. A positive association was found between students' beliefs in their mathematics ability with the level of mathematics courses and with their rational number skill level.

While students must have some prior knowledge of the system of rational numbers in order to learn algebra concepts that are highly connected to that system, the level of rational number skill does not need to be at the mastery level for students to be successful and enroll in more mathematics courses.

ACKNOWLEDGEMENTS

I wish to thank Superintendent Thorne, the counselors, principals, and high schools seniors of my district for allowing me to interrupt their busy schedules to allow me to learn more about how students think mathematically.

My sincerest appreciation goes to Dr. David Erickson who, in spite of having so many other responsibilities, has given freely of his time to listen, ask questions, and give me advice. I am also grateful to Dr. Georgia Cobbs, Dr. Maurice Burke, Dr. Libby Knott, and Dr. Jim Hirstein for serving on my dissertation committee and especially for the feedback and suggestions they have offered to make my work the best it could be.

I am especially grateful to Dr. Libby Knott and Dr. Georgia Cobbs for the opportunity to have been a CLTW fellow which provided me with not only the resources and the opportunity to complete most of my course and doctoral work at a distance, but the opportunity to work with other colleagues from five universities with the goal of improving the equity of mathematics and science education. I wish to extend a special thank you to Dr. Cobbs for being a wonderful educator, constantly encouraging me to complete this work.

I have gained great inspiration from my friends, Debby, Hydee, Patty, and Betty who have consistently supported me and convinced me that I could complete this difficult task. I am so thankful for my friendship with Dr. Marty Larkin who lent me her students, provided editing advice, and also gave me encouragement. My greatest appreciation goes to Hydee for her incredible gift of time and talent in helping me organize, think about, and edit my work.

Finally, I wish to thank my family: my children, Matt, Nick and Janelle, for their belief in me; my beautiful granddaughter Alyssa whose love of life is contagious; and most of all my loving husband Gary for all the dinners cooked, dishes cleaned, and for all the times he left me alone when he really wanted to talk.

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Need for a Study

As a high school teacher of mathematics, I teach freshmen in a beginning algebra course and seniors in an AP Statistics course thus providing me the opportunity to see the full continuum of students' levels of understanding of mathematics and to watch my students' mathematical progression through their high school years. I am very concerned that each of my beginning algebra students is afforded the same opportunity of learning as much mathematics as possible as those students who have advanced to calculus and statistics. However, students who are struggling with algebra concepts are more likely to be non-white and of lower economic status (Moses & Cobb, 2001) resulting in inequity. Addressing these issues is a high priority of The No Child Left Behind Act (NCLB) (United States Department of Education, 2001).

A survey of beginning algebra teachers conducted by the National Mathematics Advisory Panel (2008) reported that the majority of these teachers believe their students are unprepared in rational number operations as they begin algebra. The benchmarks for fractions itemized by the panel indicate that "by the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality" (p. 20), and yet those skills are not being demonstrated as mastered in national testing. The panel reiterated teachers' concern that fluency with fractions is essential before students begin algebra coursework, especially since proportional reasoning is the basis of the major topics of linear functions and slope in beginning algebra (Wu, 2001).

Teaching rational number fluency is not a direct curriculum topic of high school mathematics, even though applying rational number skills and understanding the system of rational numbers is a component of many algebra topics. High school students with low rational number skills can not expect to receive additional instruction to improve their skills. While well-intentioned districts provide students with life application courses, remediation courses, or tutoring opportunities, these programs tend to keep high school students on the track of missed opportunity for advanced mathematics.

To ensure equity for my freshmen beginning algebra students who did not demonstrate the skills necessary to be advanced to algebra in the eighth grade, I added work with rational number concepts that my beginning algebra curriculum assumes has already been mastered. Although I attempt to connect these concepts with the algebra concepts that are required, this additional knowledge development slows the year's curriculum. Not only am I seeing lack of rational number skills in my freshmen student, even my advanced students display a wide range of rational number understanding with many on the same level as my freshmen thus prompting me to question how much mastery is necessary or exists for students who are taking additional mathematics courses.

Students across the nation lack rational number reasoning. Only 79% of the sophomores who participated in the NAEP Education Longitudinal Study of 2002 were able to perform simple operations with decimals, fractions, roots, and powers (National Center for Education Statistics, 2005). Fifty-five percent of eighth-graders responded incorrectly to the multiple choice addition question, "Estimate the sum of 12/13 + 7/8" by selecting either 19 or 21 instead of 2. Fifty-nine percent of seventeen year olds were able to correctly find the product of 12 and 3/4 (Rutledge, Kloosterman, & Kenney, 2009). For the 1992 National Assessment of Educational Progress (NAEP) only seven percent of the twelfth grade students demonstrated mastery of rational numbers (Byrnes, 2003). Even many teachers were not able to solve problems involving rational numbers and of those that were able to solve the problems, a large majority was not able to explain the processes correctly (L. Ma, 1999; Post, Harel, Behr, & Lesh, 1991).

With the increased emphasis on high-stakes testing and standards-based instruction, proficiency is becoming the driving force in classrooms and is pushing aside the conceptual understanding of the complex domain of rational numbers (Balfanz, McPartland, & Shaw, 2002). Too often teachers make sure their students have the best method to get the correct answers rather than engaging students to determine why a particular method works or allowing students to find the method that will work best for them (Charles, 2008). Students may have passed state exit exams and be taking additional higher mathematics courses, but may still not have the conceptual understanding of rational numbers to apply in their daily lives.

The educational community is responding in a number of ways. Students' lack of prerequisite mathematical reasoning skills has moved mathematics reform towards more conceptual understanding, which has increased reasoning skills (Schoenfeld, 2002). Spielhagen (2006) found that enrolling more students in an eighth grade algebra course that contains rigorous beginning algebra concepts helps close the achievement gap related to socioeconomic status (SES), and that many states and districts have increased the number of required mathematics courses for high school graduation in hopes of increasing mathematics achievement (Hagedorn, Siadat, Fogel, Nora, & Pascarella, 1999). Remediation is another response — particularly in higher education. Community colleges and public institutions are providing more remediation courses for high school graduates who are not fluent in basic skills in order to compensate for inequities in higher educational opportunities (Balfanz, McPartland, & Shaw, 2002). Besides attempts at remediation, ability grouping has also been shown to close the gap (Bol & Berry, 2005).

Perhaps one of the biggest obstacles to improvement is the fact that some of the most difficult issues are not easily addressed by educational reform or remediation at any level. Student attitude, motivation, and family economics play a significant role in student achievement (Berkas & Pattison, 2006; McCoy, 2005; McLeod, 1991; Thomas & Higbee, 2000). The component of effort was addressed in the National Mathematics Advisory Panel (2008) recommendations: "The Panel recommends that teachers and other educational leaders use research-based interventions to help students and parents understand the vital importance of effort in learning mathematics" (p. 31).

Research has identified factors that contribute to the achievement gap and while there is considerable research regarding what elementary and middle school students know and understand about rational numbers, there is little research regarding high school students' knowledge and understanding of rational numbers or how that knowledge or lack of that knowledge contributes to the achievement gap. Ideally all students will leave middle school with good rational number understanding; the reality is that many students do not yet reach that goal.

Secondary students are also entering the world of work and becoming consumers where, by necessity, they develop, some form of understanding of rational numbers such as decimal representations of money. Financial literacy classes outside of the mathematics department in high schools provide students with practical information about the use of rational numbers in their lives. By better understanding what knowledge of and skills in rational numbers high school students have, as well as how that knowledge may have changed for them as they have matured or taken more mathematics courses, educational researchers can determine the role that rational number knowledge plays in the achievement gap.

Purpose of the Study

The primary purpose of this study was to determine high school seniors' skill in rational number operations and to explore the possible effects of those skills on students' learning of advanced mathematics since those courses may affect the achievement gap. The study examined how maturation or informal knowledge improved rational number skills, and how the use of money decimals as a component of the informal knowledge contributes to a student's rational number knowledge. The study examined the effect of additional mathematics courses on students' rational number operational skills and students' attitudes towards taking additional mathematics courses as it relates to their perceived abilities with rational numbers. A final purpose was to compare rational number operation skills of male and female students to determine if there is an achievement gap associated with gender.

Statement of the Problem

Existence of an achievement gap is demonstrated by the low number of students of diverse ethnicity or gender in advanced high school mathematics courses and the differences in testing scores between these students and white students. Beyond socio-economic factors, the reasons for this gap may include a lack of basic mathematics skills in these students, particularly in rational number operations (National Center for Education Statistics, 2006), the belief that students must demonstrate mastery of basic mathematics before progressing to algebra and abstract thinking (Akos, Shoffner, & Ellis, 2007; Wu, 2001), a poor attitude toward math among these students, and a lack of motivation to put forth the effort to understand and communicate mathematics (Balfanz, McPartland, & Shaw, 2002). Colleges are faced with increased remediation in areas of basic mathematics (Attewell, Lavin, Domina, & Levey, 2006) and remediation efforts at all levels often apply the deficit model creating a lower level of self-esteem for students with regard to their mathematics abilities (Balfanz, McPartland, & Shaw, 2002; Byrnes, 2003; Roderick & Camburn, 1999). Students are not taking additional mathematics courses either because of their own belief or the belief of others about their ability to do mathematics (Bol & Berry, 2005). Research on rational number achievement level is conducted with middle school students but is not conducted with high school students, since basic mathematics concepts are not part of the high school mathematics curriculum (Balfanz, McPartland, & Shaw, 2002).

Research Questions

1. What is the skill level for rational number problem solving of high school senior students and how does that skill differ from the skills of middle school students?
2. How do high school seniors' skills compare when using money and fraction forms of rational numbers?
3. How do the rational number problem solving skills of high school seniors with advanced mathematics courses compare with the rational number problem solving skills of high school seniors with minimal mathematics courses?
4. What attitude and belief differences are apparent between high school seniors with more advanced mathematics courses and high school senior students with minimal mathematics courses?

Significance of the Study

In general, high school students improve their mathematics skills as they take additional mathematics courses. However, the question of whether there are basic skills that should be mastered as a pre-requisite to taking advanced classes remains to be answered. If the goal of American education is to provide opportunities for all students (Resnick, 1987), then students' inability to successfully do computations with rational numbers can be an obstacle to that goal. Mastery of students' rational number skills can grow along with the students' own maturation and the formal learning of the mathematics that connects rational number computations with the algebraic concepts upon which advanced mathematics and science are built. Students who learn to compensate for their inabilities in formal mathematic skills and are successful in informal mathematical situations demonstrate how knowledge can be transferred. Educators can build on these informal skills to help students continue to grow in their conceptual understanding of the rational number system and proportional reasoning.

CHAPTER II

REVIEW OF THE LITERATURE

This chapter describes the current literature on secondary students' knowledge of rational numbers, how students learn to understand rational numbers, and how that knowledge affects the achievement gap and issues of equity. This review begins by examining how students' low skill level with rational numbers is part of the achievement gap. It includes the role of the National Assessment of Educational Progress in reporting achievement data. The next section describes the literature on prerequisite knowledge for algebra, particularly rational number knowledge, and the role of algebra courses as an equity issue. The third section reviews the role of student attitude and belief in their mathematical abilities as a background for student decisions about what mathematics course to take. The final section describes the literature on the complex domain of rational numbers and what is known about how students learn rational numbers.

Rational Number Knowledge and the Achievement Gap

The achievement gap is part of the political agenda as the basis of the No Child Left Behind Act (NCLB) (United States Department of Education, 2001). The short title of NCLB simply states: "An Act to close the achievement gap with accountability, flexibility, and choice, so that no child is left behind." The purpose of the act is to "ensure" for all children not only the "opportunity to obtain a high-quality education," but also that all children meet the minimal level of proficiency on state assessments. The methods which the act will use to meet this purpose is by having quality educators, informed parental choices, "high-quality academic assessments" that align with "challenging" state standards, by "meeting the educational needs of low-achieving children in our Nation's highest-poverty schools, limited English proficient children, migratory children, children with disabilities, Indian children, neglected or delinquent children, and young children in need of reading assistance," and by "closing the achievement gap between high- and low-performing children, especially…between minority and non-minority students" (United States Department of Education, 2001).

Evidence of this gap and students' achievement level is reported by The National Center for Educational Statistics (NCES), an agency under The United States Department of Education with the responsibility for obtaining and analyzing data concerning education in the United States. The National Assessment of Educational Progress (NAEP) is the assessment used by the NCES to gather data about education in the United States. The data not only provides snapshots of what students know, but also provides data concerning the trends and changes in education. Primary data for identifying and quantifying the achievement gap comes from two ongoing NAEP assessments, the main NAEP and the long-term trend assessment (LTT).

The main NAEP assesses samples of students across the nation in fourth, eighth, and twelfth grade in mathematics every two years and provides data about achievement gaps at a certain point in time, while the long-term trend assessment (LTT) is administered to nine-, thirteen-, and seventeen-year olds every four years and provides data about changes in achievement levels over time. Both provide summary data for populations, i.e., gender, states, large urban district; but neither provides data for specific students or schools. The main mathematics NAEP is based on the five content areas: number properties and operations, measurement, geometry, data analysis and probability, and algebra. Each question is also assigned a level of mathematical complexity. The low level involves questions that may only require recall knowledge. The moderate level requires a student to make more connections. The high level might require the student to analyze a mathematical model. To assess achievement at the higher levels, the main NAEP includes many free-response questions that request students show or explain their work. In some years the High School Transcript Study brings additional data to the main NAEP assessment including mathematics course enrollment. In contrast, LTT primarily assesses students' recall and skills with most questions formatted as multiple-choice. Another difference is that the main NAEP changes about every ten years to more closely match the current curriculum of the nation's schools, while the LTT remains fairly constant.

The LTT documents that the achievement level for all groups was below 350, which is defined as proficiency with fractions and percents (Byrnes, 2003). The 2008 LTT found that while the mean score for 17 year old students (306) was higher than that of the 13 year old students (281), neither group recorded average scores that indicated proficiency with rational numbers (National Center for Education Statistics, 2008). The same holds true for both genders of 17 year olds with male students scoring 309 and female students scoring 303. The 2008 LTT also documents that middle school students improved their achievement score over previous years, while high school students did not. While there has been considerable concern about students' low skill level with rational numbers, of even more concern was the fact that students were not able to communicate effectively, or chose not to communicate, justifications for solutions on the free-response items (Kouba, Zawojewski, & Strutchens, 1997).

The most recent main NAEP administered in 2007 gave data about the achievement gap at the time for both eighth and twelfth grade students (National Center for Education Statistics, 2008). For eighth grade students in 2007, male students performed better than the female students overall as well as in the individual content areas of number properties and operations, measurement, and algebra. For twelve-grade students while there has been progress in closing the gap between the number of female and male students enrolled in advanced mathematics classes, the gender gap in achievement has not been reduced. Over the last decade a higher percentage of female students are taking a more rigorous course of study. In 2005 more female students on average took more mathematics courses than male students did; the reverse of enrollment figures in 1991. However, female students are still scoring lower than the male students when comparing students enrolled in more mathematics courses. For students taking fewer mathematics courses, the average NAEP score is the same between males and females (National Center for Education Statistics, 2006).

Looking at specific content area, female students' scores were lower specifically in the areas of number properties and operations (National Center for Education Statistics, 2006; McGraw, Lubienski, & Strutchens, 2006). In contrast, for eighth graders in the 2007 Trends in International Mathematics and Science Study (TIMSS), only in the content domain of numbers was the average score for female students (506) in the US statistically lower than the average score of male students (515) in the US (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008).

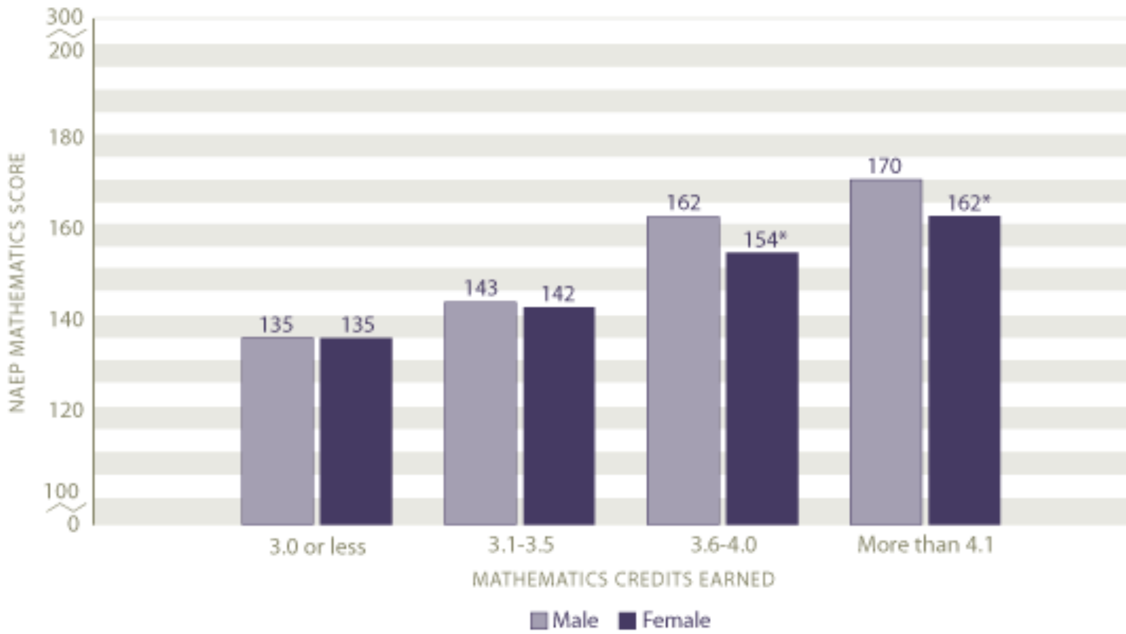

Figure 1. NAEP scores by gender and number of mathematics credits earned (National Council of Educational Statistics, 2006).

Besides giving data about overall mathematics achievement levels, NAEP allows us to tie achievement directly to specific rational number knowledge. Because there is a bank of released questions, we can identify the mathematical concept assessed with each question and can tie specific results to performance on that concept. In 2004 only 59% of seventeen year olds were able to correctly find the product of 12 with ¾ while more than 70% of the students in 1982 calculated the correct product. The decrease skill level in this type of problem could be related to the change in curriculum over 20 years with the current curriculum not including operations on numeric fractions. Another problem that showed a decrease in the percent of students answering correctly over the past two decades is changing the decimal 0.029 to a fraction for which only 35% of seventeen year olds answered correctly in 2004 but 60% answered correctly in 1982. However, there was an 18 point increase in the percentage of students converting a repeating decimal 0.333 to a fraction problem from 1982 to 2004. An explanation here is that students with calculators often see these repeating decimals and are familiar with their fraction equivalents, whereas a decimal such as 0.029 is rarely changed to a fraction using technology. However, even with the increase, only 42% of seventeen-year-olds correctly recognized 1/3 as the repeating decimal $0.\overline{3}$. While more students answered the percentage problems correctly from 1982 to 2004, again, only 56% of seventeen-year-old students could determine “what number 9 is 12% of.” However, this problem is more complex than asking the question “12% of 9 is what?” (Rutledge, Kloosterman, & Kenney, 2009).

A study of the results of the 1992 NAEP assessment provided additional evidence of the low rational number skills across fourth, eighth, and twelfth grades (Kouba, Zawojewski, & Strutchens, 1997). Students across the grades had an easier time with geometric representations to identify a rational number, but a more difficult time with identifying a rational number on a number line. When asked to locate a decimal point on a number line that was divided into fractional parts, 58% of the eighth graders answered correctly and 77% of the twelfth graders answered correctly. A problem where only the twelfth-graders were asked to locate the point 1.75 on a scale divided into thirds from zero to two, had only 50% of the students answering correctly. However, the combination of fractions and decimals, a number line longer than one unit, and the fractional divisions of thirds with an understood denominator of hundreds made this task a difficult problem (Kouba, Zawojewski, & Strutchens, 1997). Difficulty understanding that the size of a fraction is dependent on the whole was seen even by twelfth-graders on a problem that asked which candidate received more votes when the only information given was the percent of votes of different genders each candidate received. Only 26% were able to explain correctly why there was not enough information, with another 17% knowing that you couldn't tell, but these students were unable or unwilling to give a correct explanation. While 76% of twelfth graders were able to correctly recognize that you can multiply six by a number that will result in a number less than six, many did not recognize that the number could be a fraction between zero and one. Only 20% of the twelfth graders were able to find a fraction of a fraction. On a relatively simple money problem calculating the number of 85-cent rulers that can be purchased with $7.00, 75% of the twelfth grade students answered correctly (Kouba, Zawojewski, & Strutchens, 1997). This data demonstrates again students' lack of proficiency with rational numbers that has been an ongoing problem for several decades. Even in early administrations of the NAEP assessment of mathematics, concerns over students' missing knowledge were apparent. Carpenter, Lindquist, Brown, Kouba, Silver, and Swafford (1988) voiced their conclusion that "older students' difficulties with fractions, decimals, and percents reflect serious gaps in their knowledge of basic concepts involving fractions, decimals, and percents" (p. 40). These gaps were compounded by non-routine questions where only 47% of seventh grade and 44% of eleventh grade students knew that 5 ¼ is the same as 5 + ¼ (Carpenter et al., 1988).

The National Center for Educational Statistics provides data that confirms that students' skill in rational numbers continues to be low. Assessments measuring achievement over time indicates improved achievement from eighth grade to twelfth grade assessments. There continues to be evidence of a gender achievement gap between genders with males performing better. In particular, females' achievement in the area of number computations and measurement records the largest gender gap.

Algebra's Role in Educational Equity

Algebraic knowledge is an equity issue in that students with low rational number skills are often excluded from mathematics courses such as algebra that would provide students equitable access to knowledge and power in society. Our economy has a need for workers who have not only good basic skills, but the ability to apply those skills to think critically and solve problems (McLester & McIntire, 2006; National Research Council, 1989). The process of algebraic thinking includes the ability to represent quantitative relationships and analyze change in both symbolic and numeric representations providing those skills to workers (Burke, Erickson, Lott, & Obert, 2001). Additionally, algebra is the symbolic language that controls computers (Moses & Cobb, 2001) – a source of power for our society. However, students, particularly low-achieving students, have difficulty symbolically representing quantities (Resnick, 1987).

The purpose of secondary mathematics education is to move students from the numeric to the symbolic, from the concrete to the abstract, "to develop symbol sense" (National Research Council, 1989, p. 49). As preparation for college mathematics, especially calculus, algebra is considered a gateway course (Smith, 1996). Unfortunately, mathematics education too often acts as a filter removing students from the opportunities for careers requiring mathematics. Each year from high school through graduate school, half the students in mathematics classes do not take mathematics the following year. There is a prevailing attitude that mathematics is a natural ability, which you either have or don't have, rather than being a product of effort and opportunity (National Mathematics Advisory Panel, 2008; National Research Council, 1989). Yet all students benefit from taking algebra, as shown in their increased mathematics achievement levels, although there is not as significant an effect for low-achieving students as for high-achieving students (Gamoran & Hannigan, 2000).

Students who are struggling with algebra concepts are more likely to be non-white and of lower economic status (Moses & Cobb, 2001). Because of the differences in students' cultural values and goals, testing may not reflect what students know (Secada, 1991). Some cultures place more emphasis on the child's own authority to determine goals and tasks, or push children to develop their maturity in accomplishing adult tasks. Children from such cultures may have a difficult time with activities developed according to a Western European tradition. Activities based on the prior knowledge shared by students in one culture may not be successful with students from a different culture who have different prior knowledge. Thus treating all students equally may not equate with treating all students justly (Secada, 1991). While eighth grade algebra is a way to close the achievement gap, the course must not only be available to students across all socio-economic levels, but must contain rich meaningful mathematics (Spielhagen, 2006).

Not being able to do algebra not only means a person will lack the qualifications for higher paying, high-tech jobs, but also puts a person at a disadvantage as a citizen who is not and cannot be in charge. While this is true for all people, minorities are additionally disadvantaged when society is determining who is in charge. “So algebra . . . is the gatekeeper for citizenship, and people who don’t have it are like the people who couldn’t read or write in the industrial age” (Moses & Cobb, 2001, p. 14). These are compelling reasons to ensure that all students have successfully completed beginning algebra by the ninth grade so that all students are prepared for advanced and college preparatory mathematics courses in high school (Balfanz, McPartland, & Shaw, 2002).

In response to this need, mathematics education reform has made secondary education available to more than an elite few (Resnick, 1987), so that algebra is no longer reserved only for those with educational goals of advanced mathematics (Burke, Erickson, Lott, & Obert, 2001). Taking algebra before or in the first year of high school does provide good results in terms of better grades and additional mathematics courses taken, particularly for white students and for female students of color (Riegle-Crumb, 2006). Schools can encourage more mathematics by requiring more mathematics credits for graduation (Hagedorn, Siadat, Fogel, Nora, & Pascarella, 1999) since NAEP data consistently show that students with more mathematics courses have higher scores. Another policy, adopted by the state of California, is to enroll all eighth grade students in a beginning algebra course.

However, enrolling all students in algebra courses or increasing mathematics requirements does not solve the problem of some students’ inability to succeed in mathematics courses. Hoffer (1997) compared the change in the NAEP scores of high school students who attended schools with three mathematics courses as a graduation requirement to those of students attending schools with two required mathematics courses. Overall there was no difference in achievement levels between the two groups. Loveless (2008) found the lowest scoring ten percent of the eighth grade students on the NAEP fit all the characteristics of low achieving students (black or Hispanic, from homes where the mother has not attended college, attending urban public schools that are predominantly low-income), and yet the 29% of those students who were enrolled in either a beginning algebra or higher mathematics course didn't fare any better than the students who were not in advanced mathematics. Hoffer and Loveless suggest that the lack of gain from more mathematics courses may be due to a watered down curriculum, a possible effect of all levels of students being placed in the class, some of which Loveless defines as misplaced.

The importance of algebra for all students and the need to avoid misplacing students demands different methods of teaching to ensure students of various cultures are successful (Loveless, 2008). The use of technology for low-achieving students, high expectations from teachers, intervention rather than remediation courses, and student-centered classrooms can improve students' motivation and the effort they are willing to expend, particularly when their rational number skills are low. The use of computer software to obtain graphs and tables of problem situations motivated students to spend large amounts of time on a problem, contradicting the belief that low-achieving students quit when they can not produce a quick answer (Yerushalmy, 2006). Students who were involved in an intervention class that previewed concepts to be discussed in their regular algebra class, demonstrated increased mathematics achievement and reduced failures (Johnson, 2008). The use of reformed curricula that allows students to construct meaning, work in groups, and reason—as opposed to traditional classroom and curricula where the teacher provides to the students the mathematics to be learned—provides results in areas that improve students' rational number skills (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Saxe, Taylor, McIntosh, & Gearhart, 2005). Understanding is built, not only with contextual problems, but also with problems that provide the use of different representations. Making connections provides deeper understanding of both concepts (Johanning, 2008). Primary students who worked in groups to construct meaning together had better problem solving techniques, particularly on non-typical fraction problems, than the students in the control group who worked either individually or with the teacher (Keijzer & Terwel, 2003). Saxe et al (2005) also found that students' prior low mastery level of facts and procedures was not a disadvantage for those students in inquiry instruction classes compared to students in traditional instruction classes. The use of metacognitive teaching strategies for low-achieving students provided structure, a necessary component for low-achieving students, and gave those students a higher achievement level and better attitude than other low-achieving students not taught with the same strategies (Cardelle-Elawar, 1995).

While cultural values and goals, family background, ethnicity, motivation, and opportunity all contribute to the students' low-achievement, either with or without beginning algebra, academic factors also matter. For many algebraic concepts such as slope, understanding rational numbers provides the foundation (Wu, 2001). Fluency with both whole numbers and fractions along with the geometric concepts of similar triangles, perimeter, area, surface area, and volume are the three clusters defined as the Critical Foundation of Algebra by the National Mathematics Advisory Panel (2008). The panel further clarifies fluency of fractions to include the ability to locate positive and negative fractions on a number line, compare and represent rational numbers in the forms of fractions, percents, and decimals, efficiently perform operations on fractions, and to also understand the properties and meaning of the different forms of rational numbers (National Mathematics Advisory Panel, 2008). Not only should students be fluent with fractions, but the use of symbolic representation in early explanations of rational numbers can not only contribute to students' preparedness to learn algebra, but also to students' understanding of rational numbers (National Mathematics Advisory Panel, 2008; Wu, 2001).

The Effect of Attitudes on Rational Number Knowledge

The role of affect in students' learning of rational numbers cannot be overstated. Thomas and Higbee (2000) found that of all the variables that can affect students' mathematics achievement, attendance in class and the students' attitude toward their learning are factors that always correlate to mathematics achievement. McCoy (2005) found that a positive attitude combined with socio-economic status and ethnicity contributed significantly to mathematics achievement. Attitude is shaped by both positive and negative emotions as the students engage in mathematical activities. The negative emotions can be expressed as frustration, often leading a student to make guesses, educated or not, to get past the frustration and ultimately resulting in a negative attitude towards that type of mathematics (McLeod, 1991).

Beginning algebra generates both negative and positive attitudes for students. Not understanding the symbolic nature of algebra, especially for students with low rational numbers skills, can produce negative attitudes after taking beginning algebra (McCoy, 2005). While a positive attitude develops for high-achieving and average-achieving minority eighth grade students placed in algebra; a negative attitude often develops for white students of average skills ability who take algebra as eighth graders (X. Ma, 2002). The positive attitude for students who take advanced courses is because they are grouped with other high-achieving students with high levels of success as a group, contributing to favorable attitudes toward taking more mathematics and resulting in higher mathematics achievement (Smith, 1996). Schools and teachers can provide motivation for students by providing high expectations for students to be successful in mathematics rich courses. Students of color particularly benefit from having teachers and other adults who recognize their abilities and push them to succeed (Berkas & Pattison, 2006).

Students' beliefs about their own mathematical abilities, the rules, the difficulty and importance of mathematics are often related to gender (McLeod, 1991). While a higher percentage of male twelfth-grade students responded that they like mathematics and are good at mathematics, the students' perception of how well they understand the material in their mathematics was not different between the male and female students (McGraw, Lubienski, & Strutchens, 2006). The rates at which students look forward to mathematics problems, believe that mathematics class is a quiet place, and believe they are good at math are all lower for female students than male students. Yet the female students have higher grades in mathematics courses (Jacobson, 1999). In a study that looked at beliefs of college students taking remedial mathematics (Stage & Kloosterman, 1995), the number of high school mathematics courses the students had taken was not related to their beliefs about the importance of mathematics. For the female students the study found a negative association between their math skills and the students' perception of the level of difficulty of their mathematics.

Students need to have positive beliefs about their own mathematical abilities to not only be successful in high school mathematics courses, but to learn the concepts that are taught in those courses. These beliefs develop throughout elementary school and affect students' abilities in rational number skills. Negative beliefs that develop from beginning algebra and from students' perception of their own ability with rational numbers can impact students' future success in higher mathematics.

Student Acquisition of Rational Number Knowledge

It is essential we understand how students acquire rational number skills because factors such as teacher beliefs in students' inabilities and students' beliefs about their own abilities can determine their future mathematics course selection and ultimately their opportunities in society. It should come as no surprise that beginning algebra teachers find their students under prepared in fluency and understanding of rational numbers (National Mathematics Advisory Panel, 2008) given that the concept of rational numbers takes the most time to develop, is the most difficult to teach, and the most difficult to understand Lamon (2007). The high level of complexity comes from the many constructs or different uses that together build the entire domain of rational numbers; these constructs include partitioning or part-whole, quotient, measure or linear coordinate, ratio or rate, and operator or stretcher/shrinker (Behr, Lesh, Post, & Silver, 1983; Kieren, 1988; Marshall, 1993).

Deep knowledge is the result of connections, and for rational numbers the connections involve movement among the different constructs, the different representations, between constructs and representations or between procedures and the constructs (Behr, Lesh, Post & Silver, 1983; Carpenter & Fennema, 1991). Making connections requires building on students' prior knowledge (Carpenter & Fennema, 1991). The prior knowledge may include formal knowledge acquired through classroom instruction, intuitive knowledge a student just knows, and informal knowledge acquired through activities and experiences outside the classroom. Intuitive knowledge is socially derived knowledge and comes from children's reactions to the world (Cobb, Yackel, & Wood, 1991, pp. 95-96) and is the connection that teachers want to tap to help students construct their own learning.

Piaget categorized knowledge as physical, social, and logico-mathematical. It is this final category that becomes what students know about the relationships in mathematics. The lack of logico-mathematical knowledge is apparent when a four year old does not accept that equal cakes are indeed equal when one is cut into pieces. So while it may be possible to teach children what an equivalent fraction is and how to find equivalent fractions, students must construct for themselves those meanings and algorithms. Curricula that allow students to construct meanings and algorithms by presenting problems rather than algorithms allow students to build on their intuitive knowledge (Kamii & Warrington, 1999).

Young children appreciate relative quantities, how quantities change, and how parts make up a whole even before they have good number skills and protoquantitative knowledge. The concept of “fittingness” in early child development is an external ratio, comparing elements from different spaces, which evolves into the concept of direct covariation in preschoolers as they understand that a bigger object matches a bigger object (Resnick & Singer, 1993). The curricula of primary grades taps into this intuitive knowledge of fair-sharing by utilizing the part-whole or partitioning construct as a good introduction to fractions. But as the students mature, their understanding of rational numbers is limited unless their teachers include the other constructs in activities and problem situations and help students to identify differences and similarities. Unfortunately, middle grades curricula often look at each construct separately and in isolation (Lamon, 2007; Murray, Olivier, & Human, 1996).

Another reason the domain of rational numbers is very complex is because for most constructs a rational number is seen as two numbers which are usually considered only as whole numbers with a specific relationship. For the part-whole construct, one number is the whole, the other number is the part. For the operator construct, one number is the dividend and the other is the divisor. For the ratio construct, each number represents a different type or quantity being compared. For the quotient construct, the two numbers are being divided. Only for the measure construct must the student consider the rational number as one number which is a location on a number line. The complexity comes in determining the relationship between those two numbers and how that relationship affects the problem and procedure for solving the problem.

When the construct is a ratio, the procedure for finding the total of two ratios is to add corresponding parts: There are two boys to three girls in one room and five boys to four girls in another room, what is the total ratio of boys to girls between the two rooms? 2+5 to 3+4 or 7 to 7 or 1 to 1. When the construct is a measurement, the procedure for totaling two measurements requires common denominators: How long is a row of beads if a string of beads 2/3 of a foot long is combined with another string 5/6 of a foot long? 2/3 + 5/6 = 4/6 + 5/6 = 9/6 = 1 ½ feet (Marshall, 1993). Students who interpret a fraction as two whole numbers develop misconceptions due to their prior knowledge of the properties of whole numbers, i.e., multiplication of two numbers gives a larger number. Without understanding the role of the two whole numbers, students do not understand the inverse property of rational numbers, i.e., multiplying by a fraction to give a smaller number (Hecht, Vagi, & Torgesen, 2007).

Added to the complexity of the many different constructs of rational numbers are the different symbolic notations, i.e., fractions, decimals, and percents; and the different representations for rational numbers (Hecht, Vagi, & Torgesen, 2007). Children naturally use some sort of visual model to represent problems related to their intuitive knowledge (Schliemann, Carraher, & Brizuela, 2007) and since students' perceptions of rational numbers representations are very different, contexts and representations must be real and comfortable for each student (Ball, 1993; Saxe, Taylor, McIntosh, & Gearhart, 2005; Streefland, 1993). Different perceptions will affect one student's inability to see fractions in pictures or with manipulative aides in the same way other students are likely to see them (Post, Wachsmuth, Lesh, & Behr, 1985; Chick, Tierney, & Storeygard, 2007). Students may not use the manipulatives as a tool, may not represent the problem correctly with the manipulatives, or may not be able to move within different representations; all of these strategies can create misconceptions (Behr, Lesh, Post, & Silver, 1983; McNeil, Uttal, Jarvin, & Sternberg, 2009; Taube, 1997). Even for teachers, incorrect representations lead to incorrect calculations (L. Ma, 1999). For community college students enrolled in a basic math class and elementary education majors, their ability to picture a rational number was highly dependent on the use of a circular region, but students were not able to use those mental pictures to help them with the algorithms for addition of fractions (Behr, Lesh, Post, & Silver, 1983).

Studies highlight students' inability to correctly use a representation to compare rational numbers or to identify a rational number often because of an inability to move between different representations (Izsak, Tillema, & Tunc-Pekkan, 2008; Martinie & Bay-Williams, 2003; Mewborn, 1998; Ni, 2000). However, students who were comfortable with different representations were able to switch representations to place the fraction successfully even with inconsistent cues—students asked to locate 2/4 when the number line was divided into thirds (Behr, Lesh, Post, & Silver, 1983).

The use of round or rectangular food, particularly in the partitioning construct, is common among teachers and a way to tap into student's intuitive knowledge. While circle representations are often considered simple representations; they are actually quite complicated given that the use of geometric area with equivalent fractions requires both multiplicative thinking and conservation of the whole and the parts. In one study, only 44% of the fifth grade students and 51% of the sixth grade students agreed that ½ cuts from two equivalent rectangles were the same, when one was cut into triangles and the other was cut into rectangles (Kamii & Clark, 1995).

The measure construct is not as intuitive as the other constructs, since children's only prior experience with measurement is with whole number number-lines in the form of rulers and yardsticks (Marshall, 1993). Using their whole number knowledge when determining which of two rational numbers is greater forces students to consider each number as a part-whole construct. Students often confuse the ideas of size and amount as they consider which numbers to use to determine order (Post, Wachsmuth, Lesh, & Behr, 1985). Strategies students used to compare rational numbers included: comparing numerators and denominators, in some cases one was ignored in favor of the other; using a third reference point; using ratios, a strategy that was often used incorrectly; and whole number comparison, also used incorrectly (Behr, Wachsmuth, Post, & Lesh, 1984).

However, the use of a number line greater than one not only allows students to connect the measure construct to the part-whole construct (Larson, 1980) but provides additional connections to other constructs. In non-typical fraction problems as well as in fraction proficiency, nine and ten-year old students who were taught fractions with a number line representation demonstrated a higher level of skill than a control group that was taught using the circle model and fair sharing although type of instruction was different in the two groups (Keijzer & Terwel, 2003). The use of number lines and fraction bars at the secondary level gave ninth grade students the ability to recognize fractions as numbers while also building their symbolic presentation in a beginning algebra course (Darley, 2005). However, students who did not connect the part-whole construct requirement of equal intervals to a number line chose 2/6 on the problem in Figure 2 or chose 2/7 by miscounting the number of parts in the whole, even after including marks to make equal parts (Saxe, Shaughnessy, Shannon, Langer-Osuna, Chinn, & Gearhart, 2007).

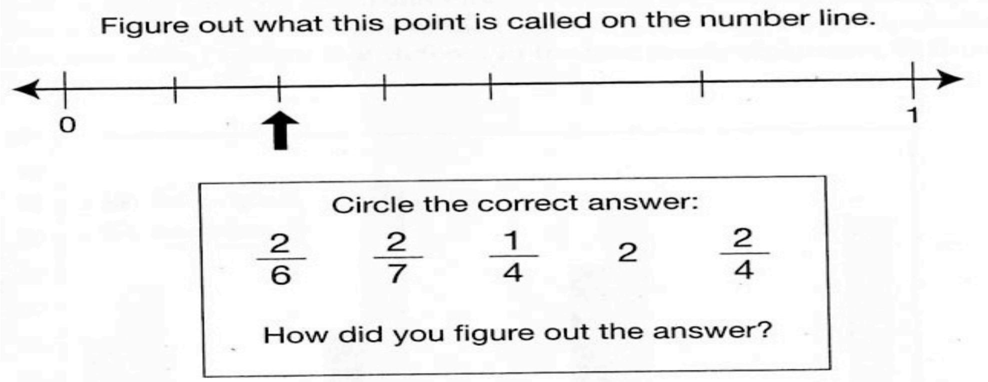

Fig. 13.8. Problem of the Day

Figure 2. Number line problem testing the part-whole construct.

The importance of different representational contexts and the ability of students to move among them are not limited to rational or whole number concept knowledge. Students benefit with different contexts for algebra topics such as slope. The usual context for rate of change in algebra is often motion and speed problems. Other contexts such as banking problems allow students to make additional connections in understanding and visualizing rate of change (Wilhelm & Confrey, 2003). Darley (2005) reported that freshmen students in a beginning algebra class connected numeric rational numbers with algebraic rational expressions. In the pretest, even her students who had good procedural knowledge of numeric fraction computation were unable to transfer that knowledge to the algebraic rational expression computations. By providing structured lessons with number lines and fraction bars, her students increased their knowledge of fractions and their ability to solve problems involving fractions. At the same time they also showed improvement at simplifying and performing computations on algebraic rational expressions.

Students' informal knowledge builds as they become consumers using money when comparing numbers. While normally this strategy is successful, it can let students down when they are comparing decimals, such as 4.4502 and 4.45. Steinle and Stacey (2004) found it was much more common with tenth grade students to use the money contexts, as well as for them to be confused about what to do when the two numbers looked the same in that context. Moloney and Stacey (1997) found that even with students increasing knowledge of money as they mature, students became more consistent in their errors instead of becoming more expert. When students do not have a clear understanding of the different constructs, even with their informal and intuitive knowledge, they often select incorrect procedures (Hecht, Vagi, & Torgesen, 2007).

Kalchman, Moss, and Case (2001) found that building on students informal knowledge of percents and a halving and doubling strategy, a curricular sequence moved students in fourth through eighth grade through levels identifying volumes in terms of percents, doing compositions of those volumes and percents, identifying the percents as decimal numbers, and on to level four where they worked with all rational number representations including fractions. These students had greater gains over the students in the control group. Students' ability to move between the different representations of rational numbers, the fluency in which they used the benchmarks, i.e., ½, ¼, to move between these representations, and their ability to quantify the rational number was developed by beginning with students' knowledge of the common percents of 50%, 100%, or 25%, and allowing students to develop their own strategies for doing computations using their understanding of the rational number as a quantity based on a hundred (Moss, 2003). Even though students do well when asked to compare shaded areas of continuous regions using percents, these students do not perform as well when given problems to compare shaded areas of discrete circles, or when asked to compare percentages without a pictorial (Gay & Aichele, 1997) demonstrating again the necessity of moving between constructs and representations for students to build knowledge that can be transferred.

Based on their intuitive knowledge of partitioning, children have been seen to use a standard ratio to compare pizzas per person or to create different groups for comparison purposes (Lamon, 2007). Lamon (1993) found that sixth graders considered a ratio as a unit and counted up by doubling, tripling, and such to solve proportional problems. Few students used the proportional algorithm of cross products. Bay-Williams and Martinie (2003) observed that fifth and sixth grade students ordered fractions by comparing the fractions to a benchmark fraction such as ¼, ½, and ¾ and changing the fractions to decimals to compare them, especially when the context of problems provided the opportunity for students to consider strategies that would arise from the problem. D'Ambrosio and Mewborn (1994) reported that students defined fractions as iterations of unit fractions using their whole number knowledge and a number line model as opposed to the traditional area model. However, these same students were not able to move between representations, from area partitioning to a quantity representation like M&M's. When asked what 1/3 is of 24, the students' response of 8/24 indicated a clear understanding of equivalency but no concept of the unit involved.

Similar to the part-whole and ratio constructs, students often see the operator construct intuitively as two whole numbers that affect a third number in the normal way whole number multiplication and division affects other whole numbers. Using scaling problems where students were asked to enlarge a figure, about a third of thirteen to fifteen year old students correctly added repetitively rather than multiplying on easy problems when the scale factor was two or three times as large. However, when the scale factor required multiplying by a fraction, these students resorted to inaccurate adding strategies. Even though they could see that their method produced a distorted result, they did not know why or how to fix the distortion. While these students had all been taught the concepts of ratio, proportion, and operations on fractions, starting with easier problems that could be correctly solved by addition may have convinced these students that their additive strategy was correct (Hart, 1988).

Riddle and Rodzwell (2000) speak to the loss of intuitive knowledge in adults. As an elementary school teacher who also does remedial work with military personnel, Rodzwell sees the loss in adults of the intuitive and fun sense of fractions demonstrated by kindergarten students. When students from kindergarten to sixth grade were allowed to demonstrate their own strategy for solving the problem 2 ½ + ¾ , the older students became dependent on algorithms that they were unable to perform successfully. Students who were successful usually completed either ½ or ¾ to make a whole, and then determined the total parts and the one piece (Riddle & Rodzwell, 2000). For community college students enrolled in a basic math class and elementary education majors, students' ability to picture a rational number was highly dependent on the use of a circular region, but the students were not able to use those mental pictures to help them with the algorithms for addition of fractions (Behr, Lesh, Post, & Silver, 1983).

Students do not build rational number knowledge sequentially, but rather in a movement back and forth starting with the fractions the student sees utilizing images and thinking tools (intuitive and informal knowledge that makes sense with the student's world), moving into symbolic representations (formal knowledge) and finally developing the logic of the system of rational numbers. Students move from doing to image-making, image-having, property-noticing, formalizing, observing, structuring, and inventing, but not sequentially, often returning to earlier levels (Cobb, Yackel, Wood, 1991; Kieren, 1993). The action of sharing or partitioning is an example of the doing and seeing levels of early understanding that can allow unit fractions to be more comprehensible at an earlier age and allow for building algorithms at a later age (Kieren, 1993). Unfortunately, students often decide that they must rely on their formal knowledge acquired at school, whether that knowledge has clear meaning for them or not, with the result that they ignore their intuitive and informal knowledge gained from experiences (Resnick, 1987). "Rather than thinking intuitively about what it might mean to add one fifth and another fifth, Cassandra switched over to thinking about adding the numbers in the symbolic form" (Ball, 1993, p. 181).

When there is a choice between using informal strategies (strategies learned outside of school) and formal strategies, the informal strategies will take precedence depending on the importance of the situation and context of a problem. Problems situated in meaningful contexts for students resulted in manipulating amounts rather than using the algorithms taught the children in school. Problems not situated in meaningful contexts resulted in the students solving through learned algorithms, which were often solved incorrectly without much concern by the student (Carraher, Carraher, & Schliemann, 1987). Brazilian fishermen with little formal education preferred informal strategies when given proportional problems involving the ratio of processed to unprocessed fish. The fishermen first used a trial and error strategy comparing to common ratios and then determined the correct relationship between the known quantities to find the unknown amount. None of the fishermen used an incorrect additive strategy. When the same problems were given to students of the same community, only one student used the proportion algorithm which had been formally taught and one student used the incorrect additive strategy, while the rest found the relation in much the same manner as the fishermen and used that quantity to solve the problem. The fishermen's informal knowledge was deep enough that they were able to reverse direction in the problems regarding the processed fish and to transfer their knowledge to other types of contextual problems that were outside their own expertise (Nunes, Schliemann, & Carraher, 1993); both are skills which are difficult for students.

There is a difficulty in connecting children's intuitive knowledge of mathematics which they use to solve problems even before they start school, with the mathematics that they are taught (Resnick, 1987). In the classroom, probably due to the lack of importance to obtain correct solutions as compared to situations connected to a family's livelihood, students will disconnect their informal knowledge from their formal knowledge. Students may need to be brought back to their informal knowledge in order to bridge to more complex ideas and operations (Mack, 1993). As they progress students have a difficult time separating their natural understandings from the procedures that have been taught to them, making constructing the required knowledge a difficult task (Lampert, 1991). While students from families with a lower socio-economic status were able to solve open-ended problems using informal strategies and practical considerations, they were more likely to not apply mathematics to their solutions (Lubienski, 2000). Since the goal for conceptual understanding must be equitably attainable for all students, teachers with open-ended lessons should use class discussion to make sure students understand what is said in the problem, what tasks are to be accomplished, and how the contextual problems relate to students' real world contexts. Students then need to be taught how to explain their solutions and to justify the mathematics they are doing (Boaler, 2002). Students able to complete patterns in tables must be formally directed to naming or formalizing the pattern and determine the mathematical rule (Schliemann, Carraher, & Brizuela, 2007).

Computer software provides opportunities for teachers to build on students' prior knowledge; intuitive, informal, and formal. In one study third grade students worked with a software program that provided multiples of fractions as outputs, based on the input and the multiplier given by the student. These students were able to draw upon their knowledge of whole number operations and facts to determine the correct multiplier to enter using the program as a tool to provide the correct outputs to solve the problem. When asked to find a number between two fractions such as two-fifths and one-half, one student turned to the complex fraction of two and one-fourth-fifths as a number slightly larger than two-fifths which he could then simplify by doubling the numerator and denominator twice with the use of the software to obtain 9/20 (Hunting, Davis, & Pearn, 1996). A different program that allows students to partition and iterate bars to determine fractional values provides students with opportunities to explore with fractions while increasing their learning and understanding of fractions (Norton, 2008).

The domain of rational number is very complex with the different constructs and representations. Conceptual understanding of rational numbers is built as students move back and forth between their intuitive or informal knowledge and their formal knowledge among the different constructs and representations until they can analyze and identify properties. From this understanding, algorithms and strategies for problem solving with rational numbers can be constructed if students are given opportunities to use all their prior knowledge. Students with conceptual understanding can move to symbolic representations and an understanding of the domain and properties of the system of rational numbers. The symbolic representation will give students the skills necessary to be successful algebra students and successful members of society.

CHAPTER III

METHODOLOGY

This chapter begins by enumerating the specific research questions used to answer the general question of whether mastery of rational numbers is a prerequisite for advanced mathematics. The chapter includes a description of the research design of this study, the procedures for selecting the students in the sample, and the methods for gathering the data. Next it explains the variables and their categories, the creation of the rational number instrument, and the scoring rubric of the instrument. The final section outlines the different statistical tests as each relates to the research questions.

Research Questions

1. What is the skill level for rational number problem solving of high school senior students and how does that skill differ from the skills of middle school students?
2. How do high school seniors' skills compare when using money and fraction forms of rational numbers?
3. How do the rational number problem solving skills of high school seniors with advanced mathematics courses compare with the rational number problem solving skills of high school seniors with minimal mathematics courses?
4. What attitude and belief differences are apparent between high school seniors with more advanced mathematics courses and high school senior students with minimal mathematics courses?

Research Design

This study is a mixed methods observational study. There is no treatment except for the natural treatment of a public education on the participants who are public education seniors. The research is observational to explore how well high school seniors solve rational number problems. The rational number instrument consisted of an equal number of money problems and fraction problems (see Appendix A) selected from the NAEP released problems (see Appendix B). It was administered to high school seniors in the fall of 2008 after approximately six weeks of school. The students' responses provided the quantitative data that was blocked using the variables of gender and whether the student took a minimum number of mathematics courses (identified as minimal mathematics courses) through the junior year, or more than the minimum (identified as advanced mathematics courses). The total scores for the rational number instrument were compared to the students' eighth grade standardized test scores and to the responses to the survey concerning mathematics attitudes of the participants (see Appendix C). The qualitative component involved four interviews that provided a lens into these students' thinking about rational number problems and the reasons for their decisions regarding mathematics courses taken (see Appendix D).

Population/sample

The population was high school seniors in a rural school district in the Intermountain West where approximately 91.5 % of the students are of Caucasian ethnicity. Sixty percent of the students are released for one period a day to receive religious instruction. The district has a total of approximately 350 seniors at three regular high schools which are ninth through twelfth grades and one alternative high school which serves junior and senior students who are lacking enough credits to graduate with their class on a regular schedule. Seniors have eleven to twelve years of formal instruction in rational numbers in their mathematics classes, and many have informal knowledge acquired as they have taken vocational courses and have become working adults. The district policy is that all students will pass a beginning algebra course. Special education students with an Individualized Educational Plan may be exempted from this requirement. About one-third of the students are advanced to algebra in the eighth grade.

The sample was a voluntary convenience sample from the seniors enrolled in the researcher's school district. The researcher had taught middle school mathematics in the district for four years, taught in other districts for 13 years and returned to teach high school mathematics for the past three years in the district. While all students were invited to voluntarily participate, the sample consisted of 147 high school seniors. All participants received a $10 Walmart gift card. Participants' standardized test scores, grades, and state exit exam scores were collected.

Instruments

The study used two instruments to gather data concerning students' knowledge of rational numbers and their beliefs about mathematics. The two instruments were the rational number instrument and the survey instrument.

The rational number instrument was composed of sixteen problems of which eight were money situations and eight were fractional situations. Most of the students were able to complete the problems in a 50-minute time period, but all students were given as much time as they needed. The problems were selected from the online bank of released main National Assessment of Educational Progress (NAEP) problems (U.S. Department of Education Institute of Education Sciences, 2009). The bank consisted of 45 rational number problems for the eighth grade NAEP participants, and 21 rational number problems given to only the twelfth grade NAEP participants. Out of the 66 released rational numbers problems, 27 were fraction problems, 20 were money problems, eight were decimal but not money problems, seven were percent but not money problems, and four were proportional problems involving whole numbers. Fourteen problems were selected from the eighth grade assessment, and two from the twelfth grade assessment (see Appendix B). Most of the selected problems were chosen because they were contextually based real world problems with an equal number of problems using money and fractions. However, two fraction problems were chosen because they could demonstrate students' understanding of the meaning of a fraction. Because students were asked to show their thinking as well as their work, five of the eight fraction problems and four of the eight money problems were changed from multiple-choice to free response. One fraction and one money problem were free response and did not need to be adjusted. Two money problems and three fractions problems were not changed from multiple-choice because the researcher felt the answer choices aided in the solving of the problem.

The main NAEP assessment only allows calculator use on one-third of the problems at all grade levels. When a calculator is allowed, the student is asked whether or not they used a calculator. Of the problems chosen for the rational number instrument, only two of the fraction problems, Frac3, the ordering fraction problem and Frac10, the weight on the moon problem, on the NAEP assessment allowed calculators. Four of the money problems for the NAEP assessment allowed calculators, Money3 the plywood problem, Money6 the apple problem, Money8 the sales tax problem, and Money10 the percent increase problems. Approximately thirty-eight percent of the problems used on the rational number instrument allowed the NAEP participant to use a calculator. To facilitate easier administration, to encourage students to show their work, and to allow the researcher to determine what operational skills students have independent of the calculator, the participants were not allowed to use a calculator. The problems were randomly assigned a position on the instrument.

The survey instrument asked demographic information such as gender, age, years attending the current school, what mathematics courses the student had taken, extra-curricular activities, work experience, post-secondary plans, and the students' belief in their abilities as a good student, at doing mathematics in and out of school, at doing money problems, fraction problems, and working without a calculator (see Appendix C).

Data Collection and Scoring

Working with the principals and guidance counselors of the four high schools in the district, the researcher distributed invitations to participate to all district seniors. The researcher administered the problems and survey instruments to the seniors who volunteered and obtained parental consent at each of the schools during pre-arranged school days in September and early October. Each student was assigned a number for his/her instrument that corresponded to a master list with the student names. Immediately following the administration of the instrument, the researcher used the master list and the students' permanent records to obtain overall grade point average, individual math course quarter grades, number sense scores from the state exit exam, and the Iowa Test of Basic Skills (ITBS) standardized test scores from the students' eighth grade year. This data was recorded on a form that was then identified only by the same number that corresponded to the number on the student instrument.

Students' work on the problems was separated from the survey demographic and attitude information and was analyzed for correctness and scored according to the following rubric:

3 points – the answer is correct. The procedure may be correct, incorrect, or missing. The units may be missing.

2 points – the answer is not correct, but the procedure is correct and the incorrect answer is due to an arithmetic error or, in multiple step problems, one step was not completed or not completed correctly.

1 point – the answer is not correct and there are errors in the procedure that led to the incorrect answer, but there is some portion of the procedure that is correct indicating a partial understanding.

0 points – the answer is not correct and the procedure is not correct at all or no procedure is shown.

The researcher scored all completed instruments. Students in a secondary mathematics methods course at a nearby university were each asked to score ten to fifteen different students' responses. The researcher compared all scores and any discrepancies were given to a third scorer who was a district math teacher and whose decision resolved any conflicts. Two scores were totaled for each participant, one for the fraction problems and one for the money problems.

The participants were blocked into one of four groups:

Group 1. Females with no more mathematics courses than beginning algebra and geometry.

Group 2. Males with no more mathematics courses than beginning algebra and geometry.

Group 3. Females with more mathematics courses than beginning algebra and geometry.

Group 4. Males with more mathematics courses than beginning algebra and geometry.

Four students were chosen to be interviewed. One each was chosen from Groups 1 and 2 with the highest score and who was willing to be interviewed. One each was chosen from Groups 3 and 4 with the lowest score and who was willing to be interviewed. Interviews were conducted in February 2009 and captured via both video and audio media. The interviews were transcribed and all tapes were erased at the conclusion of the study.

Institutional Review Board

This study was approved by the Institutional Review Board (IRB) at The University of Montana and by the school district administration a priori. The results of the assessment had no impact on students' grades. All participants returned assent and/or consent forms as required under The Family Educational Rights to Privacy Act to obtain grades and standardized test score. All data—assessment results, attitude surveys, and transcript information—were kept confidential through the use of numbers as identifiers. The researcher kept the master list in a locked file cabinet separate from the data. These procedures minimized any concerns for participants' safety and privacy.

Limitations

The convenience sample eliminated generalizing results to other high school seniors. To maintain the students' right to privacy with parental consent for those students under eighteen, participation was voluntary and may not be representative of all the high school seniors in the district.

The majority of residents in the area and students in the district belong to one predominant religion. Sixty percent of the students are released from school one period a day for religious studies.

Students who took algebra as eighth graders were required to have at least two additional mathematics courses in high school for graduation since eighth grade algebra did not count as a high school credit. For the purposes of this study these students were categorized as having advanced mathematics courses because they had taken an additional mathematics course, usually algebra II, but may not have taken more than two years of mathematics while in high school.

Additional variables that may contribute to a student's ability to successfully do computations may go beyond the number of mathematics courses taken and may include the quality of the teachers and the curriculum used. The scores of students in the same school, regardless of their level of mathematics achievement, may not be totally independent of each other because they were all taught with the same mathematics curriculum and from the same mathematics faculty.

The students received no incentive for high scores, for working all the problems on the instrument, or for showing their thinking. The only incentive was for returning the proper consent/assent forms. While the researcher assumed that they did the best work they were able to do, the students may have given a less than expected effort when solving the problems.

Variables

The quantitative variables included the rational number scores, the fraction problems score, the money problems score, the total attitude score for each student, and the eighth grade ITBS mathematics computation score for those participants for whom the scores were available. The categorical variables included the gender of each student—male or female—and the level of mathematical courses completed by each participant—minimal or advanced.

Definitions

The following words and phrases are defined as follows:

Students with minimal mathematics courses: Students who have taken only beginning algebra and geometry in either middle school or high school.

Students with advanced mathematics courses: Students who have taken any course or part of a course above geometry.

Formal knowledge: Knowledge gained through formal education at an institution of learning.

Informal knowledge: Students' intuitive or experiential knowledge obtained outside of an institution of learning (Carpenter, Fennema, Romberg, 1993; Mack, 2001).

Rational number scores: The total score for all the problems on the rational number instrument out of a total possible of 48.

Fraction problems score: The total score for all the fraction problems on the rational number instrument out of a total possible of 24.

Money problems score: The total score for all the money problems on the rational number instrument out of a total possible of 24.

Total attitude score: The total of the six attitude questions from the survey instrument for each student on a scale of 1 to 10.

Statistical Procedures

The statistical analysis for this study was of two different types. For most of the quantitative data, one and two sample *t*-tests were used to compare the variables among the four subgroups: males with minimal mathematics courses, males with advanced mathematics courses, females with minimal mathematics courses, and females with advanced mathematics courses. In some cases, a linear regression test was conducted to determine how well one variable was explained by another variable. Once again comparisons were made between the four subgroups. Experimental consistency was set at an alpha level of 5% ($\alpha = 0.05$).

Assumption of normalcy was not met. Although the total sample size (n=147) was large enough, three of the four subgroup sample sizes shown in Table 1 were less than 40: males with advanced mathematics courses, males with minimal mathematics courses, and females with minimal mathematics courses.

Table 1

Number of Participants by Type of Mathematics Courses Taken and by Gender

	Minimal Mathematics Courses	Advanced Mathematics Courses	Total
Males	34	27	61
Females	26	60	86
Total	62	85	147

The participants are not completely independent of each other since all participants are students in the same district with the same mathematics curriculum and students in each school are somewhat dependent having been taught by the same teachers.

Hypotheses and Statistical Tests

Research Question #1

What is the skill level for rational number problem solving of high school senior students and how does that skill differ from the skill of middle school students?

To determine a basis of the students' skill level before their high school years, the best measure of students' eighth grade rational number knowledge was the Iowa Test of Basic Skills (ITBS) given to 117 of the senior students in the study in the fall of 2004. For each student, in the area of mathematics, a standard score and a national percentile rank was reported with a percent correct score reported in the mathematical areas of concepts, estimation, problem solving, data interpretation, and computation. In addition a percent correct score was reported for critical thinking mathematics. Since one of the purposes of this research was to identify students' knowledge, the use of the standard score was not appropriate. In addition, due to discrepancies found in percentile ranking and scaled scores, a more accurate assessment of the student's mathematics ability can be found from the percent correct scores (Baglin, 1986). However, even these scores are limited because there is no breakdown of concepts being tested and as such the scores are only an indication of a student's ability at that age. The researcher decided to use the mathematics' computation score as the best predictor for how the student would do on what should have been similar computation problems (rational numbers) at the eighth grade level. The students with an eighth grade ITBS score were blocked into the four groups. Linear regression *t*-tests were used to determine if the students' eighth grade ITBS scores were a good predictor of their knowledge during their senior year. Table 2 provides the number of participants by gender and type of mathematics courses who participated in the ITBS as eighth grade students.

A comparison was made for each problem, comparing the national percent of students who answered the question correctly for each problem from the NAEP data with the percent of students from this research who earned a 2 or a 3 on each question. While this comparison cannot be statistically tested, it can demonstrate differences in knowledge that may or may not have occurred between the two age groups.

Table 2

Number of Participants with ITBS Scores by Type of Mathematics Courses Taken and by Gender

	Minimal Mathematics Courses	Advanced Mathematics Courses	Total
Males	21	23	44
Females	23	50	73
Total	44	73	117

Transcriptions from the interviews were analyzed for each student’s explanation of the work on selected problems from the rational number instrument to determine the level of skills for these students.

Research Question #2

How do high school seniors’ skills compare when using money and fraction forms of rational numbers?

A matched-pairs *t*-test was completed by using the difference in the money problems score and the fraction problems score for all students overall. These differences between the money score and the fraction score were blocked for the four subgroups, male students with advanced mathematics courses, male students with minimal mathematics courses, female students with advanced mathematics courses and female students with minimal mathematics courses. Matched-pairs *t*-tests were conducted on all four subgroups to determine if the money problem scores for any of those group were significantly higher than fraction problem scores.

Research Question #3

How do the rational number problem solving skills of high school seniors with advanced mathematics courses skills compare with rational number problems of high school seniors with minimal mathematics courses?

Two sample *t*-tests were calculated by comparing the total rational number problem scores between students with advanced mathematics courses and students with minimal mathematics courses. Subgroups were also compared, the total problem scores between male students with advanced mathematics courses and male students with minimal mathematics courses, and the total problem scores between female students with advanced mathematics courses and female students with minimal mathematics courses.

Research Question #4

What attitude and belief differences are apparent between high school seniors with more advanced mathematics courses and high school senior students with minimal mathematics courses?

On the survey instrument the students gave their rating score for each of six different attitudes on a scale from one to ten where ten is best. The six attitude questions had the students rating themselves on their ability as a good student, their ability to do math at school, their ability to do math outside of school, their ability to do problems with money, their ability to do problems with fractions, and their ability to do math without a calculator. The six ratings were summed for a total possible scale of 0 to 60 where 60 indicated the highest rating as a good student, highest ability to do mathematics in and outside of school with money and fractions and without a calculator. This attitude score was compared between the male students and the female students and between all the students with minimal mathematics courses and all the students with advanced mathematics courses using two sample *t*-tests. Attitudes of the students who were interviewed were analyzed as they compared to the results of the *t*-tests.

CHAPTER IV

RESULTS

The results of the different statistical tests and summaries of the student interviews are given in this chapter and are organized according to each of the four research questions. Summaries of different portions of the four interviews are also included in the appropriate research question section. While most students made good effort to show their thinking as part of their solution to the problems on the rational number instrument, eight percent of the problems showed no written work with only an answer or no answer at all.

Research Question 1

What is the skill level for rational number problem solving of high school senior students and how does that skill differ from the skills of middle school students?

Comparison of ITBS Scores with Rational Number Scores

The scatterplots for each of the four groups of seniors who had taken the ITBS test as eighth graders is shown in Figure 3. Each point represents the students' ITBS score on the x- scale and the students' rational number scores on the y-scale. A mildly strong positive linear relationship is seen in the scatterplot for all scores with a correlation factor of 0.52. Linear regression *t*-tests were run for all students and the students in each subgroup where *x* is the students' eighth grade IOWA score and *y* is the students' rational number score. Results of these tests can be found in Table 3. While there was evidence that there is a linear relationship between the students' ITBS score and the rational number scores for all the students, the correlation was not very strong (r=0.521), with only 27% of the variability in the rational number scores being explained by the variability in the ITBS scores.

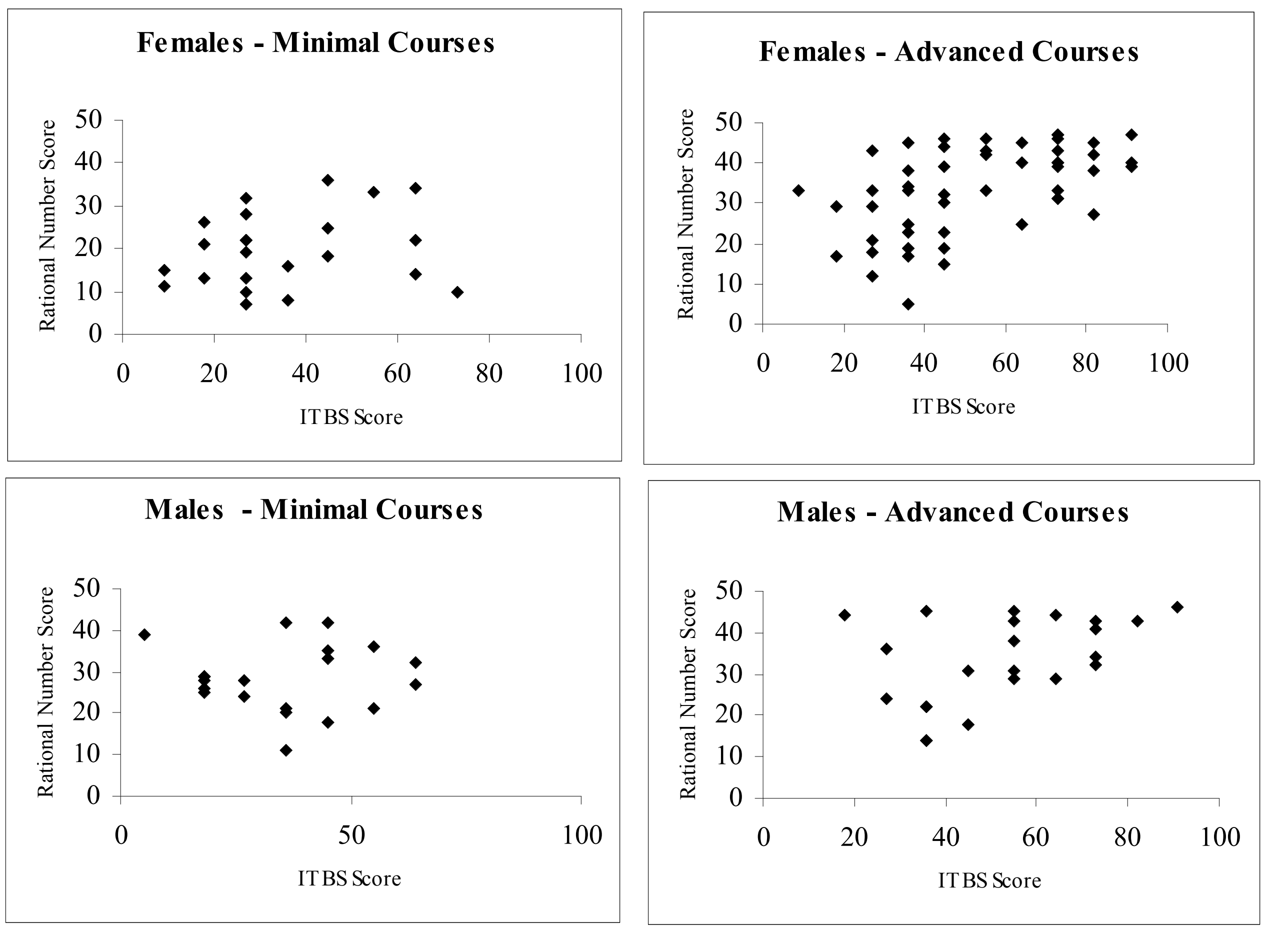

Figure 3. Scatterplots of Rational Number scores versus ITBS scores for the four subgroups.

When looking at each of the four subgroups, the linear relationship of the male and female students with minimal mathematics courses both had slopes much closer to zero, and in the case of the males, the relationship was actually negative. The linear regression *t*-test for both these groups showed that there was not a statistically significant association between the students' ITBS score and their rational number scores, indicating that for students who did not take more mathematics courses, their score on the ITBS was not a good predictor of their rational number scores four years later.

Table 3

Regression Equation, Correlation Factor, t-Score, and p-Value for all Scores and for the Scores in the Four Subgroups

	Regression equation	Correlation factor	R squared factor	*t*-score	p-value
All participants	Y=17.53+0.27X	0.52	0.27	6.54	<0.001
Females					
minimum mathematics courses	Y=16.11+0.10X	0.22	0.05	1.02	0.16
advanced mathematics courses	Y=19.94+0.26X	0.52	0.27	4.23	<0.001
Males					
minimum mathematics courses	Y=28.41-0.002X	-0.005	0.0002	-.021	0.51
advanced mathematics courses	Y=20.77+0.24X	0.45	0.198	2.28	0.017

Problem Comparisons

Using the data from the NAEP website, a comparison was made for each problem to see how the high school seniors performed compared to the students who took the NAEP exams in the various years. With the exception of two questions, the NAEP questions were all given to eighth grade students. The exceptions are the Fract4 problem involving a scale model and Frac1 about the division of a partial amount of pie among three people; both problems were given to twelfth grade students. Half of the problems for this study, which were released problems from prior NAEP exams, were changed from multiple-choice to free-response (see Appendixes A and B for the source of the problems and the changes that were made). A score of two or three on any of the research instrument problems would indicate sufficient knowledge that the problem could be considered correct, either with a correct solution or correct procedure shared.

The rational number instrument contained sixteen problems, however, only thirteen problems were given to eighth graders taking the NAEP exams. Two of the sixteen were only on the twelfth grade NAEP, and one of the problems, the dryer problem, had a typographical error on the rational number instrument. On twelve out of the thirteen problems common to both the 8th grade NAEP and rational number instrument, the percent of high school seniors with correct scores was the same or higher than the percent of eighth graders who answered correctly. The only problem on which the seniors had a lower percentage of correct responses than the eighth graders was Money8, the sales tax problem. On Frac4, the scale-drawing problem, a lower percentage of the seniors in this study scored lower than the twelfth grade NAEP participants. Results for the money problems are given in the next section followed by the results for the fraction problems.

Problem Comparisons - Money Problems

Table 4 provides the percentage of students who answered each money problem correctly for the national data base of NAEP participants, the percentage of high school seniors who answered the question correctly by scoring either a two or a three, and the average score from zero to three of the high school seniors. The two highest scoring money problems for both the eighth graders and the seniors were Money2, the class trip problem, (58% and 88% respectively) and Money 6, the apple problem, (62% and 71% respectively). Both problems had vocabulary that was common for both ages of students and applicable to the real world.

The seniors and the eighth graders had the same percentage (53%) answering correctly on Money3, the plywood profit problem. The concept of profit should be familiar for eighth grade students, but they may not be very familiar with the vocabulary. However, for the high school seniors, the vocabulary should not have been a stumbling block. This problem, however, had the added obstacle of being solved by division.

The lowest scoring money problem for both the eighth graders and the high school seniors (16% and 32% respectively) was Money10, the percent increase problem. This contextual money problem was not only a percent problem, but also a percent increase problem. Percent problems were the most difficult for the senior

Table 4

Percent of Participants Who Answered Each Money Problem Correctly from the NAEP Data and from the Rational Number Data

Problem number by type	% with correct answer from NAEP	% with score 2 or 3 from study	Average Score 0-3	Problem
Money1	38	65	2	Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?
Money2	58	88	2.7	Jill needs to earn $45 for a class trip. She earns $2 each day on Mondays, Tuesdays, and Wednesdays, and $3 each day on Thursdays, Fridays, and Saturdays. She does not work on Sundays. How many weeks will it take her to earn $45?
Money3	53	53	1.7	Peter bought 45 sheets of plywood at a total cost of $400. He plans to sell each sheet of plywood for $15. If peter has no other expenses, what is the fewest number of sheets he must sell to make a profit?
Money5	67	47	1.5	It costs $0.25 to operate a clothes dryer for 20 minutes at a Laundromat. What is the total cost to operate one clothes dryer for 30 minutes, a second for 40 minutes, and a third for 50 minutes?
Money6	62	71	2.2	What is the greatest number of 30-cent apples that can be purchased with $5.00?
Money7	30	53	1.7	Mrs. Thiery and 3 friends ate dinner at a restaurant. The bill was $67. In addition, they left a $13 tip. Approximately what percent of the total bill did they leave as a tip?
Money8	45	42	1.5	Kate bought a book for $14.95, a record for $5.85, and a tape for $9.70. If the sales tax on these items is 6% and all 3 items are taxable, what is the total amount she must pay for the 3 items including tax?
Money 10	16	32	1.3	If the price of a can of beans is increased from 50 cents to 60 cents, what is the percent increase in the price?

students to solve. Money8, the sales tax problem, also required students to calculate a percent and was the one money problem on which a smaller percent of the high school seniors (42%) answered the question correctly compared to the eighth graders (45%).

The two tip problems showed very different results even though they were also percent problems. In Money1, the problem that asked for the closest approximation of a 15% tip, the high school seniors showed a great improvement over the eighth graders (65% to 38%). The other tip problem, Money7, showed a much smaller percent correct for both the eighth graders (30%) and the high school seniors (53%). This problem gave the amount of the tip and total bill for the food and asked the students to determine the percent.

Problem Comparisons – Fraction Problems

Table 5 provides the percentage of students who answered each fraction problem correctly for the national data base of who took the tests from NAEP, the percentage of high school seniors who answered the question correctly by scoring either a two or a three, and the average score from zero to three of the high school seniors. For the fraction problems, the problem on which both eighth graders and high school seniors scored the highest percentage correct (64% and 78%) was Frac5, the recipe problem. While the high school students did better on this problem, the two age groups seemed to have equal experience with this type of problem.

Frac2, the equivalent ratio problem, and Frac3, the arranging fractions in order problem, were the only problems chosen that were more formal knowledge assessment problems as opposed to real life problems. In the case of the equivalent ratio, 69% of

Table 5

Percent of Participants Who Answered Each Fraction Problem Correctly from the NAEP Data and from the Rational Number Data

Problem number by type	% with correct answer from NAEP	% from study who scored 2 or 3	Average Score 0-3	Problem
Frac1	22	24	1.1	In a certain restaurant a whole pie has been sliced into 8 equal wedges. Only 2 slices of the pie remain. Three people would each like an equal portion from the remaining slices of pie. What fraction of the original pie should each person receive?
Frac2	60	69	2.1	Which of the following ratios is equivalent to the ratio of 6 to 4?
Frac3	49	63	2	In which of the following are the three fractions arranged from least to greatest?
Frac4	56	21	0.9	If you were to redraw the diagram below using a scale where ¾ inch represents 10 feet, what would be the length of the side that is 48 feet?
Frac5	64	78	2.4	If 1 1/3 cups of floor are needed for a batch of cookies, how many cups of flour will be need for 3 batches?
Frac6	55	55	1.8	Jim has ¾ of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?
Frac9	39	69	2	On the road shown below, the distance from Bay City to Exton is 60 miles. What is the distance from Bay City to Yardville?
Frac10	48	55	1.8	The weight of an object on the Moon is about 1/6 the weight of that object on the Earth. An object that weighs 30 pounds on earth would weigh how many pounds on the Moon?

the high school seniors answered this question correctly compared to 60% of the eighth graders – not a very large increase. However, 63% of high school seniors answered the ordering of fractions problem correctly which was a larger percentage than the 49% of the eighth graders who answered correctly. The high school seniors showed a much higher skill level than the eighth graders on Frac9, the map problem with proportional distances, where 69% of the high school seniors answered correctly compared to 39% of the eighth graders. Frac4, the scale problem, was the problem with the smallest number of high school students obtaining a correct answer (21%). This was a NAEP problem given to twelfth graders in 2005, where 56% of the national twelfth graders answered correctly. The next most difficult problem for the seniors was Frac1, the pie problem, where only 24% of the students answering correctly. This problem was also only given to twelfth grade NAEP participants where only 22% answered correctly.

Skill Level for Interviewed Students

Three of the four interviewed students had been employed.

Student #1 had worked as a cashier at a restaurant and was good at doing addition in her head, but had little knowledge with percents. She indicated that she did not waitress and so did not earn tips based on percentages. She was completing geometry during her senior year, having repeated freshman algebra I as a sophomore and taking no mathematics her junior year. Her eighth grade ITBS computation score was 27% while her score on the rational number instrument was 53%. She answered 10 problems correctly and 3 problems with some partial knowledge. She demonstrated good reasoning ability as she explained how she worked the problems and had a high skill level on money problems with the exception of the percent problems on which she scored either a zero or one.

Student #2 had been employed during his high school years as a painter and as an ice bagger. He had also taken auto mechanics courses and had fairly good informal knowledge of the meaning of fractions. His eighth grade ITBS computation score showed 36% correct while he scored 70% correct on the rational number instrument, on which he got all but two problems correct or practically correct. Of the two problems he missed, one was the apple problem. His work and solution on that problem had been scored as a one because he didn't show much work with an answer of 15. During the interview he quickly demonstrated that he had determined he could buy 3 apples per dollar and so for 5 dollars, he would have 15 apples. When asked about the left over 10 cents from each dollar, he immediately realized he could buy one more apple, thus obtaining the correct answer. The other problem he missed was the equivalent ratio problem where he erroneously took the reciprocal of the ratio and reduced to find an answer. Student #2 demonstrated the highest skill level of the four students interviewed.

Student #3 who had not worked during high school scored 27% correct on the eighth grade ITBS computation problems and scored 20% correct on the rational number instrument. She had only three problems correct which were the two tip problems and the string problem. During the interview she indicated that she would use the tip calculator on her cell phone as a tool when dining out and admitted she guessed on the tip problems since she was not allowed to use a calculator or her cell phone on the test. During the interview she was told to use her cell phone to check the problems and she was surprised to see that she had guessed correctly. This student took beginning algebra as an eighth grader, geometry as a freshman and intermediate algebra as a sophomore and was taking college algebra as a senior after having no mathematics class as a junior, but she demonstrated a very low skill level with rational numbers and, in particular, a very low skill level with fractions.

Student #4 had worked as a farmer. His work with building construction classes, had given him an informal knowledge of a tape measure and he applied this knowledge incorrectly to some of his problems. While he had taken more mathematics than the minimal mathematics, he had only taken three quarters of algebra II and had failed the last quarter. His computation score from the eighth grade ITBS was 36%. On the rational number instrument, he scored a 23% correct where he only answered three problems correctly and showed partial knowledge on five problems. His explanation of problem solutions showed a lot of confusion of what to do and a low level of belief in his ability to solve the problems. He demonstrated the lowest skill level of all the interviewed students.

Research Question #2

How do high school seniors' skills compare when using money and fraction forms of rational numbers?

The distribution of the difference of the total money problems score less the total fraction problems score for all the high school seniors is reasonably symmetric with two outliers on each tail. The median is 1 point; the mean is 0.59 points, with a standard deviation of 4.53 points. The mean is not statistically different $\left(t_{146} = 1.57, p = 0.06\right)$ from zero indicating that overall high school seniors are not better at money story problems than fraction story problems.

The distributions of difference scores between money and fractions for the four subgroups are all reasonably symmetrical; however there is an outlier on the right side for both the male students with advanced mathematics courses and the male students with minimal mathematics courses. The distribution of the difference scores for the females with advanced mathematics courses has an outlier on the left side. Table 6 contains the statistical information for the four subgroups. The only subgroups whose mean difference between the money and fraction scores is statistically greater than zero are the female students with minimal mathematics courses and the female students with advanced mathematics courses. All of the female students on average scored almost two points higher (1.59) on the money problems than on the fraction problems.

An additional comparison between money and fraction problems was made considering the percent of students of each gender who answered each problem correctly. The percent of students who answered correctly for each individual problem segregated by the high school seniors and the NAEP participants can be found in Table 7. A higher percent of male seniors than female seniors solved the problem correctly for each of the individual fraction problems with the exception of Frac4, the scale problem, and Frac9, the map problem. A higher percentage of male NAEP participants answered the problem correctly for each of the fraction problems except on Frac5, the recipe problem, where a higher percentage of female than male NAEP participants answer correctly. For the individual money problems, the gender difference switches between the seniors in this study and the NAEP participants. A higher percentage of female than male students answered correctly all of the money problems except for Money2, the trip problem, and Money10, the percent increase problem. For the NAEP participants, a higher percentage of male than female students answered correctly all but three of the money problems: Money1, the tip problem, Money2, the trip problem, and Money8, the sales tax problem.

Table 6

Difference in Money Score and Fraction Score by Subgroups

	Median difference	Mean difference	Standard deviation	*t*-score	df	p-value
Females						
Minimal courses	4	3	5.1	3.02	25	.003
Advanced courses	1	0.98	3.14	2.42	59	0.01
All	1	1.59	3.91	3.78	85	<0.01
Males						
Minimal courses	-1	-0.59	5.53	-0.62	33	0.73
Advanced courses	-.5	-0.92	4.41	-1.07	25	0.85
All	-1	-0.84	4.98	-1.31	60	0.90

Table 7

Percent of Participants with Correct Solutions per Individual Problem by Gender

Problem	% Female from NAEP with Correct Answer	% Female from RNS with Correct Answer	% Male from NAEP with Correct Answer	% Male from RNS with Correct Answer
Frac1	19	21	25	30
Frac2	58	66	62	75
Frac3	44	59	55	71
Frac4	53	23	59	20
Frac5	65	78	63	82
Frac6	53	55	56	59
Frac9	33	71	44	69
Frac10	48	54	49	61
Money1	39	71	37	61
Money2	62	87	55	93
Money3	49	59	57	46
Money5	65	55	69	39
Money6	60	78	65	66
Money7	28	55	31	54
Money8	49	44	40	41
Money10	12	28	20	39

Research Question #3

How do the rational number problem solving skills of high school seniors with advanced mathematics courses skills compare with rational number problems of high school seniors with minimal mathematics courses?

Table 8 shows the means, *t*-statistics, and p-values for three comparisons of rational number scores: all students with advanced courses with all students with minimal courses, male students with advanced courses to male students with minimal courses, female students with advanced courses to female students with minimal courses. A description of each comparison follows. The distribution of the students with minimal mathematics courses (n=61) is skewed right while the histograms for the students with advanced mathematics courses (n=86) are skewed left with no outliers in either distribution. The range for both distributions is approximately the same (40 and 42) with standard deviation of 9.9 for the students with minimal mathematics courses and a standard deviation of 10.4 for the students with advanced mathematics courses. The mean score of 22.5 for students with minimal courses is statistically less than the mean score of 33.3 for students with advanced courses $\left(t_{137.1} = !\ 6.44, p < .0001\right)$.

The distribution of the total scores for male students with minimal mathematics courses (n=35) is fairly symmetrical and the distribution of the total scores for male students with advanced mathematics courses (n=26) is skewed left with no outliers. The five number summary (minimum, quartile 1, median, quartile 3, and maximum scores) for the male students' scores with advanced mathematics courses is higher (8, 13, 17, 21, 23) than the five number summary for the students' scores with minimal mathematics courses (4, 8, 12, 15, 23) although they both have the same maximum. The means are 11.77 and 16.62 and the standard deviations are 4.93 and 4.89 for the scores of students with minimal mathematics and the scores of students with advanced mathematics respectively. A two-sample *t*-test confirms that the male students with advanced mathematics courses scored higher than the male students with minimal mathematics courses $\left(t_{54.2} = !\ 3.81, p < 0.001\right)$.

Table 8

Comparison of Means Between Course Levels

<table>
<tr><th>Group</th><th>Mean</th><th>t-statistic</th><th>p-value</th></tr>
<tr><td>All students with minimal courses</td><td>22.5</td><td rowspan="2">-6.44</td><td rowspan="2"><0.0001</td></tr>
<tr><td>All students with advanced courses</td><td>33.3</td></tr>
<tr><td>Male students with minimal courses</td><td>11.7</td><td rowspan="2">-3.81</td><td rowspan="2"><0.001</td></tr>
<tr><td>Male students with advanced courses</td><td>16.62</td></tr>
<tr><td>Female students with minimal courses</td><td>19.5</td><td rowspan="2">-6.08</td><td rowspan="2"><0.001</td></tr>
<tr><td>Female students with advanced courses</td><td>32.8</td></tr>
</table>

The distribution of the total scores for the female students with minimal mathematics courses (n=26) is not symmetrical, but there is no extreme skewness. The histogram for the female scores with advanced mathematics courses (n=60) is skewed left with no outliers. The five number summary for the female students' scores with advanced mathematics courses is higher (5, 26, 34, 41, 47) than the five number summary for the female students' scores with minimal mathematics courses (6, 13, 20, 26, 34) except that the students with advanced mathematics courses actually has a lower minimum. The means are 19.46 and 32.82 and the standard deviations are 8.58 and 10.93 for the scores of students with minimal mathematics and the scores of students with advanced mathematics respectively. A two-sample *t*-test confirms that the female students with advanced mathematics courses scored higher than the female students with minimal mathematics courses $\left(t_{59.9} = !\ 6.08, p < 0.001\right)$.

Since the interviewed students were selected because of the inverse relationship of their rational numbers scores and their level of mathematics courses an additional question of "Why these students did or did not take mathematics?" was raised. For students 3 and 4 who both had additional mathematics, the decision to take more mathematics was related to college athletics and recommendations made by their coaches, although neither took as much mathematics as was available. Student #4 followed the high school courses of beginning algebra, geometry, and started intermediate algebra his junior year, but quit after failing the third quarter which corresponded to the completion of wrestling season. The student indicated that his teacher felt it best for him to quit and his parents did not mind since he had the two necessary mathematics credits to graduate. Student #3 had taken beginning algebra as an eighth grader, had completed geometry and intermediate algebra through her sophomore year to satisfy her high school requirement, but took no math her junior year. Realizing that in order to play college volleyball she would need additional mathematics, she took college algebra and trigonometry her senior year. Students 1 and 2 who did not take additional mathematics gave different reasons. Student #1 felt her freshman beginning algebra teacher was the primary reason she and other students did not earn credit causing her to retake the course as a sophomore. Instead of taking additional mathematics her junior year, she filled her schedule with sociology courses and, as a senior, was taking geometry to complete her required mathematics credits. Student #2 had successfully completed beginning algebra and geometry by his sophomore year, but chose to not take any mathematics his junior year, mostly due to not wanting to put forth the effort he felt would be involved. As a senior he was taking intermediate algebra. For all students except student #2, teachers had either a positive or negative influence on their decisions to take additional mathematics.

Research Question #4

What attitude and belief differences are apparent between high school seniors with more advanced mathematics courses and high school senior students with minimal mathematics courses?

Using the total attitude score from the students' surveys, both the distributions of the attitude scores of the male students and of the attitude scores of the female students are fairly symmetrical, but there are outlier scores on both sides for the female attitude scores. A two-sample *t*-test for these two distributions shows that there is no difference in the attitude scores for the female students from the attitude scores for the male students $\left(t_{131.2} = 0.38, p = 0.71\right)$.

Analyzing gender by level of mathematics courses we find that there is no significant difference in the mean attitude scores for female students with minimal mathematics courses and male students with minimal mathematics courses

$(t_{52.5} = !\,.93, p = 0.35)$ or for female students with advanced mathematics course and male students with advanced mathematics courses $(t_{44.8} = !\,0.39, p = 0.70)$.

The distributions of the attitude scores in students with minimal mathematics courses and of the attitude scores in students with advanced mathematics courses are both reasonably symmetrical with n=61 and n=86. For these distributions there is a statistical difference $(t_{127.7} = !\,4.54, p < 0.001)$ in the attitude scores with a mean attitude score for students with minimal mathematics courses of 34.45 and a mean attitude score for students with advanced mathematics courses of 41.85 demonstrating that students with advanced mathematics courses have a more positive belief in themselves with regards to math abilities with money and fractions.

The four students who were chosen to be interviewed had the opposite association which, again, could be expected because of the method of selection. The male and female student with advanced mathematics courses had total attitude scores of 16 and 24 respectively out of a possible high of 60. The male and female students with minimal mathematics courses had total attitude scores of 38 and 32 respectively. These students were chosen because they scored opposite of what was expected on the rational number instrument for the type of mathematics courses they had taken. All four students still had a fairly low total attitude score with scores for ability with fractions all less than five out of a possible high of ten. The reasons these students gave for taking or not taking additional mathematics courses did not seem related to their belief or attitude about their math abilities.

Figure 4 shows a scatterplot of the students' total attitude score plotted against the students' rational number scores. A linear regression test gave the regression equation $y = !\, 0.17 + .74x$ where x is the attitude score and y is the rational number score. The correlation coefficient was r=.678. With a *t*-value of 11.09 and a p-value of 0, there is sufficient evidence to conclude that there is a positive linear relationship between the attitude score and the rational number score. In fact 45% of the variability in the rational number scores can be explained by the variability in the attitude scores.

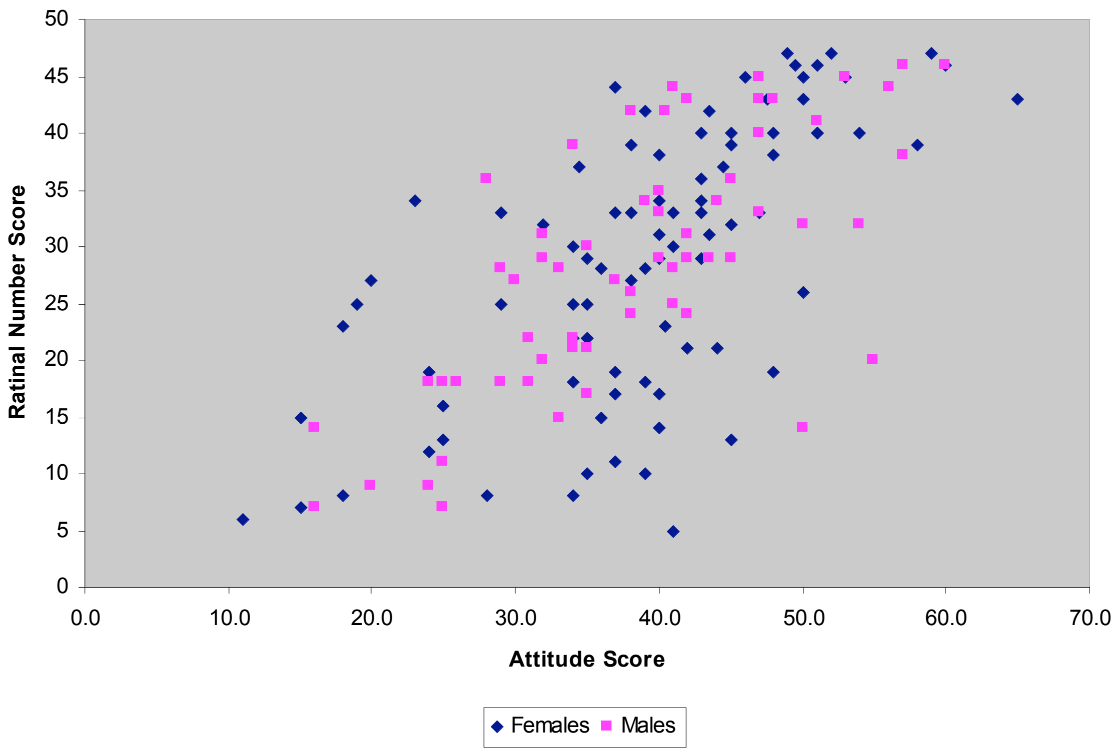

Figure 4. Scatterplot of students' attitude score versus rational number score.

CHAPTER V

CONCLUSIONS

Chapter five reviews the design of the study, the research questions, and the conclusions found for each of those questions. The chapter explores additional limitations of this study. In the final section the chapter provides recommendations for secondary mathematics educators and for future research.

Research Design

The purpose of this study was to examine the skills of high school seniors in rational numbers operations as they may impact the achievement gap. Since rational number skills are not a direct component of secondary mathematics courses, rational number skills are not tested as part of a high school students' standardized testing, and are not included in criterion reference testing for purposes of determining a schools' annual yearly progress as defined by the No Child Left Behind Act. Therefore, the researcher created an instrument for measuring these skills. The NAEP questions provided valid and reliable problems that could answer what high school seniors know about rational numbers.

With the exception of the ordering fractions and equivalent fraction problems, the researcher chose problems that were real world problems and chose an equal number of problems using money and using fractions. The fraction problems chosen did not cover sufficiently all of the operations at which students are expected to be proficient. The money problems were often tied up with percents thus creating an added variable when looking at rational numbers since rational numbers can take the form of fractions (a/b where b is not zero), decimals, or percents.

Due to time constraints for the researcher, the rational number instrument and survey were administered in the fall of the students' senior year. Administering the instruments in the spring of the students' senior year would have allowed for additional knowledge the seniors gained by mathematics classes taken that senior year and the increase of maturity from fall to spring. Also, if a long term study could be completed, the validity of this research design would have been made much stronger by administering the same instrument to eighth graders and, then, four years later to the same students as high school seniors to show/compare individual student growth.

Research Question 1

What is the skill level for rational number problem solving of high school senior students and how does that skill differ from the skills of middle school students?

High school seniors do have better skills than those demonstrated by eighth graders and those skills were better than expected for seniors without advanced mathematics courses. In spite of the higher percentage of students scoring correctly, the skill level of the high school seniors is still below what many consider mastery—at least 70% correct—with an average percent correct for the rational number scores of only 48%. The students did better on contextually based problems and worse on fraction and percent problems.

Analysis of individual problems provides specific information about what students know and from what kind of sources that knowledge is based. On only three of the rational number problems did the students indicate proficiency with more than 70% of the seniors answering correctly. These questions were all contextually based problems that seniors would be more likely to find in everyday experiences.

The seniors demonstrated a low skill level particularly with percent problems. The lowest scoring money problem for the seniors was Money10, the percent increase problem. Although the percentage of those who answered correctly for the high school seniors was double that of the eighth graders, the fact that less than a third of the high school seniors were able to correctly determine the percent increase of fairly easy numbers (50 cents to 60 cents) highlights a concern that students struggle not only with the fractional representation of rational numbers, but they also struggle with the percent representation as well; a result also seen from an analysis of the 1992 NAEP (Kouba, Zawojewski, & Strutchens, 1997). Students' struggle with percents is further evidenced in the low percentage of success for Money8, the sales tax problem. While sales tax is definitely part of all students' everyday life, sales tax is not something that ever has to be calculated thanks to the common use of technology. So whether a student works as a cashier or is just a consumer, one's real-world success depends only on one's ability to very roughly estimate how that tax will affect the total purchase, not on one's ability to calculate the exact amount.

Seniors do have better skills with percent problems that involving tipping because high school seniors have increased experience in eating out, paying for their own meals, and determining tips even though they often use the tip calculator on their cell phones. One of the interviewed students indicated, "$4.50 seemed like too much, but $3.00 seemed too small" demonstrating students' ability to estimate without any mental or physical calculation, but rather just a hunch. The senior students did not do as well on calculating the percent a tip was of the total bill as they did on calculating the actual tip. The use of division may have influenced the lower score on this problem compared to determining the tip problem. However, because most people rarely calculate the percent of a tip, the seniors' lower performance is probably due to the uncommon nature of this problem.

The seniors have fairly good skills with fraction problems that assess formal fraction knowledge. Even with requiring both vocabulary knowledge and a successful procedure to build up or reduce fractions, 69% of seniors answered the equivalent fraction problem correctly. However, it was this problem that one of the interviewed students, who had answered most of the problems correctly or nearly correctly, missed by using the reciprocal of the fraction to determine equivalency. Since he solved the string problem by changing three-fourths to six-eighths, it is more likely the vocabulary caused him difficulty rather than his misunderstanding of the equivalency of fractions.

The seniors did not do quite as well on the arranging fractions problem, but had a much higher percentage of accuracy than the eighth graders indicating that seniors have increased their understanding of the meaning of fractions even though ordering fractions is not a skill directly assessed or practiced in high school mathematics classes as it is in the middle schools.

Seniors skill level is very low when using fractions in multi-step problems. The most difficult problem for the seniors was the scale problem for which a much higher percentage of twelfth grade NAEP participants answered correctly. Part of the reason for this discrepancy could be that the NAEP problem was multiple-choice allowing NAEP participants to estimate and make a reasonable guess. The difficulty of this problem was high because it was a two-concept problem, incorporating proportional reasoning with the provided scale, and doing computations that involved a fraction as part of the scale ratio. Since 69% of the high school seniors were able to solve the map problem that also used proportional reasoning, it is likely that having the fraction in the ratio was the part of the problem that made the problem so difficult.

Another fractional problem that involved multi-steps was Frac1, the pie problem, the second most difficult problem for the seniors. The problem asked for the final fraction size of 2/8 of a pie split between three people. Even though students are often taught fractions using the circle model, in this case the model was not sufficient to help students see the solution. While a lot of students drew the pie, correctly identified ¼ of the pie remaining, and even showed this amount divided into 3 pieces; their answers demonstrated that they could not complete the final step to identify one of those pieces as a fractional part of the whole pie. This problem is another example of a problem that is not necessarily a real life problem. While it may be that three people want to divide the 2 out of 8 pieces remaining, it is not a common question to ask what fraction of the whole pie each person receives.

High school seniors' knowledge improved due to both formal and, particularly for students without additional mathematics courses, informal knowledge. The seniors performed better on problems that were more like problems they would encounter in their everyday living.

Research Question #2

How do high school seniors' skills compare when using money and fraction forms of rational numbers?

Overall the seniors were no more skilled with money problems than fraction problems except the female students did better with money problems than fraction problems and outperformed the male students on most money problems. In contrast the male students outperformed the female students on the fraction problems. The two problems where the male students outperformed the female students the most were both fraction problems designed to measure formal knowledge.

The purpose of having money problems in the rational number instrument was to provide the students with problems that could assess their informal knowledge. While the majority of the money problems were all contextual common life problems, the percent increase problem was not a common problem and was the most difficult money problem for the seniors and even more difficult for the female students. Because determining how much money is being saved is of more value than knowing the percent value of that savings, this problem was more an assessment of formal knowledge on which female students did not perform as well as they did with problems that are more common life and contextually based. The female students came closer to performing equally with the male students on fraction problems that were contextually based, but performed much worse than the male students on fraction problems that assessed formal knowledge such as equivalent fractions and ordering fractions. A possible explanation is that female students struggle to apply and remember formal knowledge because they do not acquire the understanding of that knowledge in the same manner as male students.

There was a difference on the number of problems answered correctly between genders from the high school seniors and the NAEP participants. The NAEP results for all of the money problems were from eighth grade students, with the males outperforming the females on all but three problems. Those three problems were more contextually based and two of them were percent problems. One was the sales tax problem which was one of two money problems on which the high school female seniors did not outperform the male students. Even though eighth grade students seem to be performing better on contextually based money problems than male students, the high school seniors demonstrated that result on most of the money problems. High school female students do seem to have improved skills with money problems, especially when they are common real world problems, when compared to either high school male students or middle school female students.

Research Question #3

How do the rational number problem solving skills of high school seniors with advanced mathematics courses compare with the rational number problem solving skills of high school seniors with minimal mathematics courses?

As expected, students with advanced mathematics courses demonstrated better rational number skills than the students with minimal mathematics courses. Even comparing average scores for each of the rational number instrument problems, the students with more mathematics performed better on every problem. The only indication that this conclusion may not be reliable was the spread of the scores between the two groups based on mathematics course level, especially when segregated by gender. The wide variability and similar minimum and maximum scores for both female groups indicates that there are “misplaced” students that do not fit the *more mathematics, the better skill* conclusion. These misplaced students with advanced courses may have weak skill levels similar to the students who do not take more mathematics. The misplaced students with minimal courses may have strong skill levels similar to students who do take more courses.

The interviewed students all fit the misplaced student definition. The students with more mathematics worked through their poor skills to complete additional mathematics credits, one very successfully, even though those courses did not seem to help their rational number skills. The students without more courses gained skills without the benefit of additional mathematics courses. These students are examples that mastery in rational numbers can improve without formal learning and contradict the theory that more courses improve skills leading to an inconclusive answer to the question of how students’ knowledge is different because of additional courses.

Research Question #4

What attitude and belief differences are apparent between high school seniors with more advanced mathematics courses and high school senior students with minimal mathematics courses?

Students with more mathematics course have better attitudes and beliefs about their mathematical abilities. Additionally students with better attitudes and beliefs have higher mathematics achievement. However, it is not possible to segregate the cause and effect relationship of attitude, course taking, and achievement. The interviewed students are naturally contradictory to this conclusion; they were chosen because they did not perform as expected on the rational number instrument. Student #2 was the only interviewed student whose rating of his ability to do mathematics outside of school was the lowest of all his beliefs. During the interview it became apparent that he thought the question meant how well he worked on his math school work outside of school. When the question was clarified, he replied that he did not know what his ability is, since the only math he does is the math he does in school. Yet this student demonstrated a high level of skill without the additional mathematics courses or a high ITBS score as an eighth grader. Boaler (1998) observed that students were unable to connect mathematics in school with mathematics outside of school, which often affects their ability to perform well in formal assessments.

While students who take more mathematics courses have a better belief in their ability, that belief is also associated with the students' skill level. Additional mathematics courses and improved attitudes have to go hand-in-hand because it is not clear which is the cause and which the effect. However, by improving students' beliefs and attitudes, students are likely to be motivated to take more mathematics courses, increasing the probability that their skills will improve not only from informal knowledge but formal knowledge as well.

Discussion

A basis for this study was the concern of the researcher that students' low ability with rational numbers may be an obstacle to success in higher mathematics courses, putting those students at a disadvantage because of the many opportunities those courses open. The underlying question for this research was, "What is the effect of students' low rational numbers skill on their ability to take additional mathematics courses, and is mastery of rational numbers a necessary skill to succeed in high school mathematics courses, particularly beginning algebra?"

While unable to answer the question directly, this study presented additional evidence that high school seniors, as national testing has demonstrated for middle school students, have low skills with rational numbers, particularly in the areas of fraction number sense and percents. This study also provided evidence that high school seniors' skills are better than middle school students' skills due to maturation from changes in students' informal knowledge and students' formal knowledge. While the study substantiated that student ability in rational number increases when they have taken more mathematics courses, the spread of scores indicates that there is a great deal of variability in students' knowledge regardless of the level of mathematics courses. This study also provided additional evidence of the gender gap, showing that female students perform better with rational number problems represented by money than with problems represented by fractions, and that while the number of female students in advanced courses is increasing, their abilities with formal mathematics learning is still below that of male students.

This study suggests that while exposure to rational numbers is necessary for students to learn algebra topics, demonstrating mastery of rational number is not necessary before students take additional mathematics courses. The National Mathematics Advisory Panel reported that a poor understanding of the principles such as commutativity and equality of arithmetic, especially with fractions, is a problem for student success in algebra. The Panel did not necessarily report low computational skills as being an obstacle. While this researcher understands the concern that students cannot learn algebra topics such as linearity of functions without some knowledge of rational numbers, students in this study showed that as long as students have been exposed to rational numbers, their skills and their knowledge of rational numbers will improve partly because of their algebra course and learning about algebra topics that do require application of rational numbers and partly because of other factors such as informal learning. Knowing that students' abilities will improve with maturity, allows educators to give all students every opportunity to learn as much mathematics as possible, not worrying that such an action will set the students up for failure in algebra just because of low skills at that point in time.

If one of education's goals is to increase the number of students taking additional mathematics, rather than only considering factors such as rational number skill, we must consider other factors such as student attitude towards mathematics, experiences with mathematics classes and past teachers, and family expectations. Every student is different, and state and district requirements must be established that not only provide equal opportunities for all students to take algebra courses, but that also allow the timing of the course to be adjusted according to student maturity without making it impossible for a student to progress to the advanced mathematics courses.

The essence of equity lies in our ability to acknowledge that even if our actions are in accord with a particular set of rules, or even if certain social arrangements are established and traditional, the results of those actions or of the arrangements may still be unjust (Secada, 1991, p.19).

Recommendations

Placement Decisions

Placement decisions, such as which students to advance to either seventh grade pre-algebra and/or eighth grade beginning algebra, or whether students need a remedial course before progressing, are often based on standardized tests. In this study, for students who did not take more mathematics courses, their standardized test scores were not good predictors of their rational number scores as seniors. Nearly half of the seniors with minimal mathematics courses scored higher than expected on the rational number instrument based on their ITBS score. This is evidence that placing students into mathematics courses based solely on ITBS mathematics scores could limit some students' later success in mathematics. A missed opportunity in the eighth grade can be recovered, but such a decision often contributes to other influences such as attitude and beliefs that continue to have a negative affect on students taking more mathematics.

The current district practice for the students in this study is to have approximately one-third of the eighth grade students take beginning algebra. Starting with the class of 2011 these students must have three high school mathematics credits to meet state graduation requirements. For the two-thirds of the students who do not take beginning algebra as eighth graders, the new graduation requirement will definitely assist those female students with career plans that require post-secondary education, but who do not realize early in their high school years the necessity of taking additional mathematics courses. This group includes students like the interviewed female student who struggled with beginning algebra as a freshman, retook the course as a sophomore, but did not take mathematics as a junior. If she had been required to complete three courses, this would necessitate taking mathematics her junior year in order to complete her credits, thus helping her towards her career goal in psychology.

While the new requirements appear to be helping with state concerns of equity for learning advanced mathematics, potential inequities still remain for students who are not seen as ready or able to complete high school mathematics courses through intermediate algebra. These students may be placed into courses such as algebra A and algebra B, a sequence providing the curriculum of beginning algebra over two years. This approach is intended to provide more depth and time for students who demonstrate low mathematics skills on standardized tests and might be in jeopardy of failing a regular beginning algebra course. However, the decision made for these students as freshmen will track them into a course of study that will limit the amount of mathematics they can take, even more so than the decision not to advance a student in middle school.

When making placement decisions, educators need to be very careful to ensure that their decisions do not close opportunities for students. The reasons students struggle with mathematics, whether in elementary school, middle school, or high school, are highly correlated with motivation, attendance, parental support, and social influences. Schools can have some influence on these factors by the courses that are offered, by high teacher expectations for students regardless of their abilities, and by understanding students' learning based on their cultural backgrounds.

<u>Educator's Attitudes</u>

Some of the students in this study demonstrated little mathematical knowledge, motivation, or work ethic as eighth graders and freshmen, but they set goals with better maturity as seniors. This change of ability and attitude provides justification of the importance of secondary teachers' response to these students throughout their middle school and high school years. This researcher desired to interview a female student who was attending the alternative high school who had very few successful mathematics credits, and who had the highest score on the rational number instrument of all female students with minimal mathematics courses. In fact her score was the same as the median score for the female students with advanced mathematics courses. The student had taken pre-algebra type courses in high school and had attempted beginning algebra several times attempting to obtain good grades, yet her ITBS computation score as an eighth grader was 64% with a percentile ranking of 87%. Unfortunately this student dropped out of school before an interview could be scheduled. Regardless of the reasons this student took the courses she did or the reasons she struggled earning credit and eventually dropped out, students are being denied the same opportunities as other students. Teachers need to be aware that students who do not perform well in class are still capable of succeeding. While having students with a mixed range of abilities and different prior knowledge in a classroom is a very difficult challenge for teachers, it is important to allow all students opportunities and to acknowledge and recognize what each student knows.

Future Research

High school students are changing throughout their high school years and those changes affect their knowledge, their attitudes, and their decisions. The goal of research should be to better understand those changes and their impact on students' decisions to take additional mathematics courses. The more educators understand that decision-making process, the more they will be able to influence students to take the additional mathematics courses that will broaden their future opportunities.

NAEP assessments can assist in this goal by categorizing problems to specific skills in the five content standards: number and operations, algebra, geometry, measurement, and data analysis and probability. This will allow additional analysis of skill levels that can be more specific than the general strand of number reasoning or algebra. The NAEP assessment should have more of the same problems on both the eighth grade and twelfth grade assessments to allow analysis of how students' skill in not only basic areas such as rational numbers changes over time, but also how their skill in algebra and geometry and other advanced topics changes and how that change is affected by more mathematics courses.

Long term research needs to be conducted to identify changes in students' attitudes as they progress through middle school and high school as well as studies to document the changes in their knowledge of rational numbers. Using similar rational number assessment instruments to test the same group of students at the beginning and at the conclusion of their high school years, and utilizing attitude or belief surveys, can provide better evidence of how students' thinking, beliefs, and knowledge change as they mature. Surveys should also include questions regarding the different reasons students are influenced to take certain courses, to have good attitudes, and to demonstrate a good academic work ethic.

Problems involving percent and proportion are a part of every person's everyday world. Although there are many ways for a person to function successfully without good skills in percents and proportions, students' opportunities for prosperous lives are much greater when they have a good understanding of these topics. Therefore, long term research is needed to not only document students' changing skill with fractions, but also students' changing skill with percents and proportions. Research needs to determine which areas do not improve due to student maturity independent of mathematics courses taken so that recommendations can be made to improve secondary mathematics curricula and other content area curricula to help students improve the skills necessary for not only advanced mathematics courses, but for better citizenship.

As called for by the National Mathematics Advisory Panel, additional research is needed into how students learn algebra. Of equal importance is research into what causes many students to fail beginning algebra and the negative attitudes that often result for many students even if they are able to earn credit. Questions that need answering are "How closely are the reasons for failure correlated with a students' lack of preparedness with rational numbers?" and "How closely are those reasons for failure correlated with maturity?" By better understanding students' attitude with their first high school mathematics course, educators can better understand and influence students' decisions to take additional courses.

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APPENDIX A

RATIONAL NUMBER INSTRUMENT

Rational Number Survey

Research Study

Renae Seegmiller
North Sevier High School 529-3717
University of Montana
September 2008

INSTRUCTIONS

In order to participate in this survey, you must have the appropriate consent/assent forms with you with the appropriate signatures.
Please write the number that appears in the upper right hand corner of this paper on your consent/assent form packet

DO NOT PLACE YOUR NAME ANYWHERE ON THIS PACKET.

There are 16 story problems followed by 20 personal questions.

Please do not use a calculator of any kind.

On the story problems, please show all your thinking. Even if you can do the problem in your head, please write down the calculation or thinking you did in your head in the space after each problem. Part of this study is to determine how students think about rational numbers, and the more you can write, the better the results of this study. If you need more room, you may use the back sides of any pages, but please number your work and indicate by the problem that there is more work on the back.

On the personal questions, please answer as completely and truthfully as possible.

Your answers on all parts of the survey will not be connected to your name and there will be no consequence to you due to your answers.

If you have any questions about the wording of any problem or question, please raise your hand and I will be happy to assist you.

Thank you for your participation in this study.

1. If the price of a can of beans is increased from 50 cents to 60 cents, what is the percent increase in the price? Explain or show by doing calculations why you chose the answer you did.

a. 83.3%
b. 20%
c. 18.2%
d. 16.7%
e. 10%

2. Kate bought a book for $14.95, a record for $5.85, and a tape for $9.70. If the sales tax on these items is 6 percent and all 3 items are taxable, what is the total amount she must pay for the 3 items, including tax?

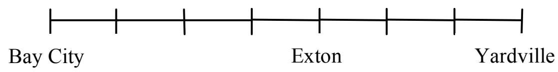
3. In which of the following are the three fractions arranged from least to greatest? Explain or show by doing calculations how you know.

a. $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$
b. $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$
c. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$
d. $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$
e. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

4. Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99? Explain or show by doing calculations how you know.

a. $2.50
b. $3.00
c. $3.75
d. $4.50
e. $5.00

5. On the road shown below, the distance from Bay City to Exton is 60 miles. What is the distance from Bay City to Yardville?

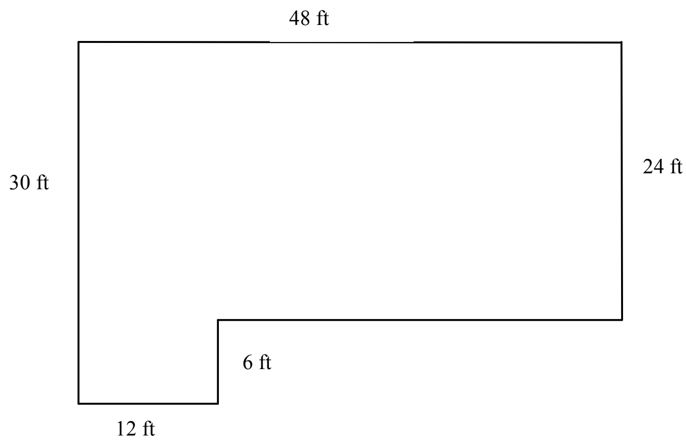


6. Mrs. Thierry and 3 friends ate dinner at a restaurant. The bill was $67. In addition, they left a $13 tip. Approximately what percent of the total bill did they leave as a tip? Explain or show by doing calculations how you know.
   a. 10%
   b. 13%
   c. 15%
   d. 20%
   e. 25%

7. In a certain restaurant a whole pie has been sliced into 8 equal wedges. Only 2 slices of the pie remain. Three people would each like an equal portion from the remaining slices of pie. What fraction of the original pie should each person receive?

8. Jim has ¾ of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?

9. If you were to redraw the diagram below using a scale where $\frac{3}{4}$ inch represents 10 feet, what would be the length of the side that is 48 feet?



10. It costs \$0.25 to operate a clothes dryer for 20 minutes at a Laundromat. What is the total cost to operate one clothes dryer for 30 minutes, a second for 40 minutes, and a third for 50 minutes?

11. Which of the following ratios is equivalent to the ratio of 6 to 4? Explain or show by doing calculations how you know.

a. 12 to 18
b. 12 to 8
c. 8 to 6
d. 4 to 6
e. 2 to 3

12. What is the greatest number of 30-cent apples that can be purchased with \$5.00?

13. If 1 1/3 cups of flour are needed for a batch of cookies, how many cups of flour will be needed for 3 batches?

14. Jill needs to earn $45.00 for a class trip. She earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays. She does not work on Sundays. How many weeks will it take her to earn $45.00?

15. Peter bought 45 sheets of plywood at a total cost of $400. He plans to sell each sheet of plywood for $15. If Peter has no other expenses, what is the fewest number of sheets he must sell to make a profit?

16. The weight of an object on the Moon is about 1/6 the weight of that object on the Earth. An object that weighs 30 pounds on earth would weigh how many pounds on the Moon?

APPENDIX B

SOURCE OF RATIONAL NUMBER PROBLEMS

For each problem used on the rational number instrument, the problem is shown exactly as given on the NAEP website along with the identifying information and difficulty level

Frac1 : Taken from the 2005 12th grade NAEP Block No. M3 Problem No. 14 difficulty level hard

M 14. In a certain restaurant a whole pie has been sliced into 8 equal wedges. Only 2 slices of the pie remain. Three people would each like an equal portion from the remaining slices of pie. What fraction of the original pie should each person receive?

Answer: __________________________

Frac2 Taken from the 2003 8th grade NAEP Block No. M10 Problem No. 10 difficulty level medium

10. Which of the following ratios is equivalent to the ratio of 6 to 4?

A) 12 to 18

B) 12 to 8

C) 8 to 6

D) 4 to 6

E) 2 to 3

Frac3 Taken from the 2007 8th grade NAEP Block No. M9 Item No. 12 Difficulty Level Medium

12. In which of the following are the three fractions arranged from least to greatest?

A) $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$

B) $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$

C) $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

D) $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$

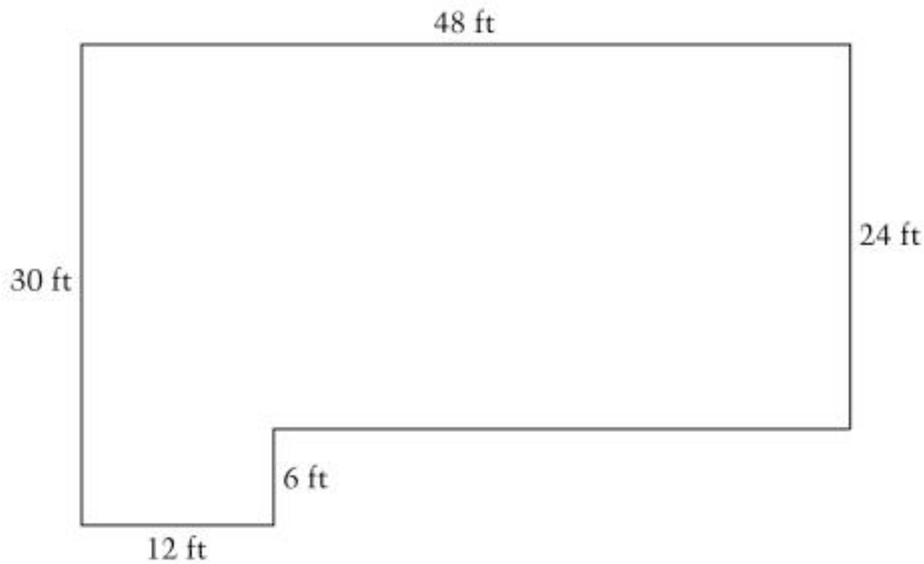
E) $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

Did you use the calculator on this question?

○ Yes ○ No

Frac4 ***Taken from 2005 12th grade NAEP Block No. M4 No. 3 Difficulty Level Medium***

Questions 1-3 refer to the following diagram.



3. If you were to redraw the diagram using a scale of $\frac{3}{4}$ inch = 10 feet, what would be the length of the side that is 48 feet?

A) 3.0 in

B) 3.6 in

C) 5.6 in

D) 7.5 in

E) 12.0 in

Frac5 ***Taken from 1992 8th grade NAEP Block No. M7 No. 6 Difficulty Level Easy***

6. If 1 1/3 cups of flour are needed for a batch of cookies, how many cups of flour will be needed for 3 batches?

A) 4 1/3

B) 4

C) 3

D) 2 2/3

Frac6 ***Taken from 2003 8th grade NAEP Block No. M6 No. 17 Difficulty Level Medium***

17. Jim has 3/4 of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?

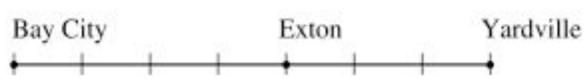
A) 3

B) 4

C) 6

D) 8

Frac9 ***Taken from 2003 8th grade NAEP Block No. M6 No. 19 Difficulty Level Hard***



19. On the road shown above, the distance from Bay City to Exton is 60 miles. What is the distance from Bay City to Yardville?

A) 45 miles

B) 75 miles

C) 90 miles

D) 105 miles

Frac10 ***Taken from 1990 8th grade NAEP Block No. M9 No. 9 Difficulty Level Hard***

9 The weight of an object on the Moon is 1/6 the weight of that object on the Earth. An object that weighs 30 pounds on Earth would weigh how many pounds on the Moon?

Answer: ____________________

Did you use the calculator on this question?

○ Yes ○ No

Money1 ***Taken from 1996 8th grade NAEP Block No. M3 No. 5 Difficulty Level Hard***

5. Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?

A) $2.50

B) $3.00

C) $3.75

D) $4.50

E) $5.00

Money2 ***Taken from 1992 8th grade NAEP Block M7 No. 7 Difficulty Level Easy***

7 Jill needs to earn $45.00 for a class trip. She earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays. She does not work on Sundays. How many weeks will it take her to earn $45.00?

Answer:____________________

Money3 ***Taken from the 2007 8th grade NAEP Block No. M7 No.8 Difficulty Level Medium***

8. Peter bought 45 sheets of plywood at a total cost of $400. He plans to sell each sheet of plywood for $15. If Peter has no other expenses, what is the fewest number of sheets he must sell to make a profit?

A) 3

B) 15

C) 16

D) 26

E) 27

Did you use the calculator on this question?

O Yes O No

Money5 ***Taken from 2007 8th grade NAEP Block No. M7 No. 3 Difficult Level Easy***

3. It costs $0.25 to operate a clothes dryer for 10 minutes at a laundromat. What is the total cost to operate one clothes dryer for 30 minutes, a second for 40 minutes, and a third for 50 minutes?

A) $3.25

B) $3.00

C) $2.75

D) $2.00

E) $1.20

Note: This problem was not included in the analysis because the researcher wrote the problem to say, "It costs $0.25 to operate clothes dryer for **20** minutes." The rubric for the problem indicated the answer was correct for either $1.50, calculated by totaling the time and determining the cost proportional to $0.25 for 20 minutes, or $1.75, calculated with the assumption that you could only put quarters in the machine making the cost for thirty minutes the same as the cost for forty minutes.

Money6 ***Taken from 2007 8th grade NAEP Block No. M9 No. 1 Difficulty Level Easy***

1. What is the greatest number of 30-cent apples that can be purchased with $5.00 ?

A) 6

B) 15

C) 16

D) 17

E) 20

Did you use the calculator on this question?

○ Yes ○ No

Money7 ***Taken from 2005 8th grade NAEP Block No. M2 No. 11 Difficulty Level Hard***

11. Ms. Thierry and 3 friends ate dinner at a restaurant. The bill was $67. In addition, they left a $13 tip. Approximately what percent of the total bill did they leave as a tip?

A) 10%

B) 13%

C) 15%

D) 20%

E) 25%

Money8 ***Taken from 1990 8th grade NAEP Block No. M9 No. 11 Difficulty Level Medium***

11. Kate bought a book for $14.95, a record for $5.85, and a tape for $9.70. If the sales tax on these items is 6 percent and all 3 items are taxable, what is the total amount she must pay for the 3 items, including tax?

A) $32.33

B) $32.06

C) $30.56

D) $30.50

E) $ 1.83

Did you use the calculator on this question?

○ Yes ○ No

Money10 ***Taken from 1990 8th grade NAEP Block No. M9 No. 18 Difficulty Level Hard***

18. If the price of a can of beans is raised from 50 cents to 60 cents, what is the percent increase in the price?

A) 83.3%

B) 20%

C) 18.2%

D) 16.7%

E) 10%

Did you use the calculator on this question?

○ Yes ○ No

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP)_

APPENDIX C

SURVEY INSTRUMENT

Age ______________________ Gender: Male Female

Number of years (including this year) you have attended your current high school?________________

Number of years (including this year) you have attended a high school in Sevier District? ______

In what extra-curricular activities (sports and clubs) have you participated during high school?

What math have you had to use in your extra-curricular or community activities?

What employment have you had during your high school years?

What math have you had to use in the jobs you have had?

What types of math have you had to use in other areas outside of school such as within your family?

Circle all the math courses you have completed in high school

Prealgebra

Algebra I

Geometry

Algebra II

Precalculus

College Algebra

Trigonometry

Calculus

Do you plan to continue your education after graduation? ____________________

If so, where do you plan on attending?

What type of job or career do you plan on having after your education is finished?

On a scale of 1 to 10, where 10 is best, how would you:

rate yourself as a good student? ___________

rate your ability to do math at school? ______________________

rate your ability to do math outside of school? _________________

rate your ability to do problems involving money? _________________

rate your ability to do problems involving fractions? _________________

rate your ability to do math **without** a calculator? _________________

Would you be willing to be interviewed concerning your responses to these questions?

____

If you respond yes and are selected, the interview will be held at your school at a time of convenience to you. You will receive a $25 gift card to Walmart for your participation in a 30 minute interview.

APPENDIX D

INTERVIEW QUESTIONS

Follow up the rating scales from the survey on the items involving each student's perceived abilities with the questions of

Why do you rate yourself at that value?

Would others (friends, family, teachers) rate you the same way? Why?

What do you think has influenced you or helped you to decide to take or not take additional mathematics courses in high school beyond the required courses?

What do you remember about your middle school and elementary classes that helped or hindered you to learn to do fraction problems?

Tell me about a particular person, place or event that made you decide to either take as many math classes as possible no matter the challenge or made you decide that it was not important to take as many math classes as possible?

What encouragement or discouragement have you received from other people about whether to take more or less math classes?

What are your plans after graduation? How did your decision to take or not take additional math courses affect those plans?

Explain how well prepared you feel you are for the math that you will need to do after graduation, either courses at post-secondary schools and/or in work requirements?

How well prepared do you feel for doing the math required for everyday living such as keeping a check book, doing your taxes, making purchases and dining out, budgeting your income and expenses? Explain your strengths and your weaknesses.

How much were you allowed to use a calculator in school math courses? Do you think that this policy helped or hindered your ability to do fraction and money problems?

Do you have any tricks that you have learned about doing math and in particular money or fraction problems that may make doing those computations easier?

For each of these problems, please explain your thinking for me.
In a certain restaurant a whole pie has been sliced into 8 equal wedges. Only 2 slices of the pie remain. Three people would each like an equal portion from the remaining slices of pie. What fraction of the original pie should each person receive?

What is the greatest number of 30-cent apples that can be purchased with $5.00 ?

Jim has 3/4 of a yard of string which he wishes to divide into pieces, each 1/8 of a yard long. How many pieces will he have?