



# Conditional diagnosability of optical multi-mesh hypercube networks under the comparison diagnosis model



Xianyong Li<sup>a,\*</sup>, Xiaofan Yang<sup>a,b</sup>, Li He<sup>a,c</sup>, Jing Zhang<sup>a</sup>, Cui Yu<sup>a</sup>

<sup>a</sup> College of Computer Science, Chongqing University, Chongqing 400044, PR China

<sup>b</sup> School of Electronic and Information Engineering, Southwest University, Chongqing, 400715, PR China

<sup>c</sup> College of Computer Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, PR China

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## ABSTRACT

Due to integrated positive features of both hypercubes and tori, optical multi-mesh hypercube (OMMH) networks are regarded as a class of promising optical interconnection topologies. The notion of conditional diagnosability helps enhance the self-diagnosing capability of multicomputers. This paper determines the conditional diagnosabilities of OMMH networks under the Maeng–Malek comparison model.

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## 1. Introduction

With the ever increasing size of multicomputers, the possibility that faulty processors (nodes) are present in such systems is becoming increasingly large. The so-called *system-level diagnosis*, which aims at automatically identifying the faulty nodes present in a system, is widely regarded as an effective means of maintaining its high availability [28]. See Ref. [6] for a comprehensive review about this subject. Recently, system-level diagnosis technique has been applied successfully to mobile ad hoc networks [5,7] and optical networks [33,34].

The conventional fault diagnosis is conducted provided that any set of nodes may fail simultaneously, leading to the relatively conservative result that the diagnosability of such a system cannot exceed the minimum vertex degree of its underlying network. In practical situations, however, the probability that all neighbors of some vertex in the network fail simultaneously is vanishingly small and, hence, can be ignored. Taking this fact into account, Lai et al. [20] introduced the notion of *conditional diagnosability*, significantly enhancing the self-diagnosing capability of multicomputers. In this context, a crop of interesting results have been obtained. Roughly speaking, under the comparison diagnosis model [26,27,29], the conditional diagnosabilities of an  $n$ -dimensional cube, an  $n$ -dimensional augmented cube, and an  $n$ -dimensional BC network are  $3n - 5$ ,  $6n - 17$ , and  $3n - 5$ , respectively [15,9,16], whereas their counterparts under the classical PMC model are  $4n - 7$ ,  $8n - 27$ , and  $4n - 7$ , respectively [20,3,38]. For more information on this issue, see Refs. [13,14,21,32,35–37]. Finally, it is worth mentioning that Hsieh et al. have explored other kinds of diagnosabilities [4,10–12,18,19]. Due to appealing properties, ranging from extremely high bandwidth to extremely low power consumption and extremely low latency, optical interconnection networks have emerged as a promising alternative to electrical interconnection networks [8,30]. Recently,

\* Corresponding author.

E-mail addresses: xian-yong@163.com (X. Li), xfyang1964@gmail.com (X. Yang).

the conditional diagnosabilities of the optical hypermesh networks were derived under the PMC and comparison models, respectively [33,34].

In 1994, Louri and Sung [24,25] suggested a class of promising optical interconnection topologies, known as the *optical multi-mesh hypercube* (OMMH) networks, which possess a two-level structure: a local connection level representing a collection of cube modules and a global connection level representing a torus network connecting the hypercube modules. Typically, an OMMH network can be characterized by a triplet  $(l, m, n)$ , where  $l$  and  $m$  represent the numbers of rows and columns of the torus, respectively,  $n$  the dimension of the cube module. OMMH networks integrate positive features of both hypercubes (smaller diameter, larger connectivity, excellent symmetry and fault tolerance, simpler routing strategies) and tori (constant node degree and excellent scalability). Indeed, OMMH networks have been physically demonstrated using a combination of free-space and optical fiber technologies, showing good performance characteristics [22,23]. To our knowledge, however, the conditional diagnosability of OMMHs under the Maeng–Malek comparison model has yet to be determined.

This paper investigates the conditional diagnosabilities of OMMH networks under the Maeng–Malek comparison model. For that purpose, some interesting properties of OMMH networks are revealed. On this basis, the conditional diagnosability of an  $(l, m, n)$ -OMMH network is shown to be  $3n + 7$  if either (a)  $n \geq 1, l, m \geq 4$ , or (b)  $n = 0, l, m \geq 4, (l, m) \neq (4, 4), (4, 5), (5, 4), (4, 6), (6, 4)$ .

This paper is organized as follows: Section 2 introduces preliminary knowledge. Section 3 gives some interesting properties of OMMH networks. Section 4 is devoted to the proof of the main result. This work is closed by Section 5.

## 2. Preliminaries

For a vertex subset  $S$  of graph  $G$ , let  $N_G(S)$  denote its *neighborhood*, i.e.,

$$N_G(S) = \{u \in V(G) \setminus S : u \text{ is adjacent to some vertex in } S\}.$$

For a graph  $G$ , let  $\kappa(G), \delta(G), \alpha(G)$  and  $\beta(G)$  denote its connectivity, its minimum vertex degree, its independence number and its matching number, respectively. For other fundamental graph-theoretic notations and terminologies, see Ref. [2].

For our purpose, a multicomputer system shall be represented by its underlying interconnection network, an undirected graph  $G = (V, E)$  whose vertices and edges stand for processors and communication links between processors, respectively. A *fault set* of a graph is a vertex subset that may possibly be the set of all faulty vertices present in the graph. A fault set is *t-fault* if it contains no more than  $t$  vertices. A fault set is *conditional* if it does not include the neighborhood of any vertex. A fault set is *conditional t-fault* if it is both conditional and  $t$ -fault.

Under the Maeng–Malek comparison model (MM model, for short), the *comparison graph* for a graph  $G = (V, E)$  is defined as an edge-labeled multigraph  $M^* = (V, C)$ , where  $C$  contains an edge  $(u, v)$  labeled with  $w$ , denoted  $(u, v)_w$ , if and only if vertices  $u$  and  $v$  are both adjacent to vertex  $w$ . Let  $\sigma((u, v)_w) = 0$  denote that  $w$  judges that the outputs produced by  $u$  and  $v$  are identical, and let  $\sigma((u, v)_w) = 1$  denote that  $w$  judges that the outputs produced by  $u$  and  $v$  are different. Suppose vertex  $w$  is fault-free, then  $\sigma((u, v)_w) = 0$  implies that  $u$  and  $v$  are both fault-free, whereas  $\sigma((u, v)_w) = 1$  implies that at least one of the three vertices in question is faulty. All comparison results, denoted  $\sigma : C \rightarrow \{0, 1\}$ , form a *syndrome*.

A syndrome  $\sigma$  is *consistent* with a fault set  $F$  if  $\sigma$  can be produced by  $F$ . Let

$$\sigma(F) = \{\sigma : \sigma \text{ is consistent with } F\}.$$

Two distinct fault sets  $F_1, F_2 \subseteq V$  are *distinguishable* if  $\sigma(F_1) \cap \sigma(F_2) = \emptyset$ , otherwise they are *indistinguishable*.

**Definition 1.** (See [20].) A graph is *conditionally t-diagnosable* if any two distinct conditional  $t$ -fault sets in the graph are distinguishable. The *conditional diagnosability* of a graph  $G$ , denoted  $t_c(G)$ , is defined as the maximum integer  $t$  such that  $G$  is conditionally  $t$ -diagnosable.

For a pair of sets,  $S_1$  and  $S_2$ , let  $S_1 \Delta S_2 = (S_1 \setminus S_2) \cup (S_2 \setminus S_1)$ .

**Lemma 1.** (See [15,17,29].) Let  $F_1, F_2$  be two distinct vertex subsets of graph  $G$ . Then,  $F_1$  and  $F_2$  are distinguishable if and only if either

- (C1)  $G \setminus (F_1 \cup F_2)$  has a vertex  $w$  that has a neighbor  $u$  in  $G \setminus (F_1 \cup F_2)$  and has a neighbor  $v$  in  $F_1 \Delta F_2$ , or
- (C2) there exist two vertices  $u, v$  both in  $F_1 \setminus F_2$ , or both in  $F_2 \setminus F_1$ , and there exists a vertex  $w$  in  $V(G) \setminus (F_1 \cup F_2)$  such that  $(u, v)_w \in C$ .

See Fig. 1 for an explanation of this lemma.

**Definition 2.** (See [24,25].) An  $(l, m, n)$ -OMMH network, denoted  $MH_n^{l \times m}$ , is defined as the Cartesian product graph  $C_l \times C_m \times Q_n$ , where  $C_r$  denotes an  $r$ -cycle whose vertices are labeled sequentially as  $0, 1, \dots, r - 1$ ,  $Q_n$  denotes an  $n$ -cube.

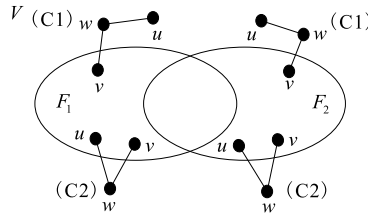


Fig. 1. Explanation of Lemma 1.

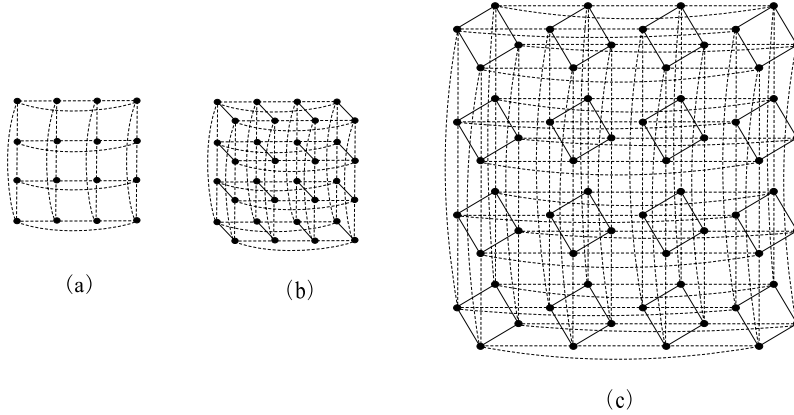


Fig. 2. OMMH networks: (a)  $MH_0^{4 \times 4}$ , (b)  $MH_1^{4 \times 4}$  and (c)  $MH_2^{4 \times 4}$ .

It is obvious from this definition that  $l, m \geq 3$ . Fig. 2 presents three small-sized OMMH networks.

**Remark 1.**  $MH_0^{l \times m} = T_{l \times m}$ , a torus with  $l$  rows and  $m$  columns.

**Remark 2.**  $MH_n^{l \times m} = T_{l \times m} \times Q_n$ .

**Remark 3.** By Definition 2, we have

$$V(MH_n^{l \times m}) = \{(i, j, k) : 0 \leq i < l, 0 \leq j < m, k \text{ is an } n\text{-length binary string}\}.$$

Two nodes of  $MH_n^{l \times m}$ ,  $(i, j, k)$  and  $(i', j', k')$ , are adjacent if and only if either

- (a)  $i' = (i \pm 1) \bmod l$ ,  $j' = j$ , and  $k' = k$ , or
- (b)  $i' = i$ ,  $j' = (j \pm 1) \bmod m$ , and  $k' = k$ , or
- (c)  $i' = i$ ,  $j' = j$ , and  $k'$  differs from  $k$  in exactly one bit position.

For brevity, the symbols for modulo operations shall be abbreviated.

For a node  $(i, j, k)$  of  $MH_n^{l \times m}$ , we shall call  $(i, j)$  and  $k$  as its *torus address* and its *cube address*, respectively. A *torus edge* of  $MH_n^{l \times m}$  is one that is not contained in any  $n$ -cube, and a *cube edge* of  $MH_n^{l \times m}$  is one that is contained in an  $n$ -cube.

**Definition 3.** Suppose  $n \geq 1$ . Let

$$S_0 = \{(i, j, k) \in V(MH_n^{l \times m}) : \text{the first bit of } k \text{ is } 0\},$$

$$S_1 = \{(i, j, k) \in V(MH_n^{l \times m}) : \text{the first bit of } k \text{ is } 1\}.$$

Then,  $MH_n^{l \times m}$  is composed of two disjoint graphs,  $0MH_{n-1}^{l \times m}$  and  $1MH_{n-1}^{l \times m}$ , and a perfect matching connecting them, where

$$0MH_{n-1}^{l \times m} = MH_n^{l \times m}[S_0], \quad 1MH_{n-1}^{l \times m} = MH_n^{l \times m}[S_1].$$

$0MH_{n-1}^{l \times m}$  and  $1MH_{n-1}^{l \times m}$  are both isomorphic to  $MH_{n-1}^{l \times m}$ .

### 3. Some properties of OMMH networks

**Lemma 2.** (See [1].)  $\kappa(Q_n) = n$ .

**Lemma 3.** (See [31].) Let  $G_1, G_2, \dots, G_n$  be connected graphs. Then

$$\kappa(G_1 \times G_2 \times \dots \times G_n) \geq \sum_{i=1}^n \delta(G_i) - \max_{1 \leq i \leq n} \{\delta(G_i) - \kappa(G_i)\}.$$

By these two lemmas, we immediately have

**Theorem 4.**  $\kappa(MH_n^{l \times m}) = n + 4$ .

**Lemma 5.** Any two distinct vertices of  $MH_n^{l \times m}$  have at most two common neighbors.

**Proof.** The assertion can be verified, respectively, when the two vertices belong to a same  $n$ -cube and when they belong to different  $n$ -cubes.  $\square$

**Lemma 6.** Let  $S$  be a vertex cutset of  $MH_n^{l \times m}$ ,  $|S| \leq 2n + 5$ . Then, the removal of  $S$  from  $MH_n^{l \times m}$  leaves exactly two components, one of which is trivial.

**Proof.** By induction on  $n$ . The assertion is easily verified for  $n = 0$ . Suppose the assertion holds for  $n = r \geq 0$ . Now, let  $S$  be a vertex cutset of graph  $G = MH_{r+1}^{l \times m}$ ,  $|S| \leq 2r + 7$ . Let  $G_0 = 0MH_r^{l \times m}$ ,  $G_1 = 1MH_r^{l \times m}$ ,  $S_0 = V(G_0) \cap S$  and  $S_1 = V(G_1) \cap S$ . Without loss of generality, assume  $|S_0| \leq |S_1|$ . Then,  $|S_0| \leq r + 3$  and, hence, **Theorem 4** ensures that  $G_0 \setminus S_0$  is connected. At this point, we claim that  $G_1 \setminus S_1$  is disconnected, because otherwise it follows from the observation that  $|V(G_0)| \geq 9 \times 2^r > 2r + 7 \geq |S|$  that there is an edge connecting  $G_0 \setminus S_0$  to  $G_1 \setminus S_1$ , implying that  $G \setminus S$  is connected, a contradiction. Next, let us distinguish among three possibilities.

**Case 1.**  $S_0$  is empty. As every vertex of  $G_1 \setminus S_1$  is connected to a vertex of  $G_0 \setminus S_0$ , it follows that  $G \setminus S$  is connected, a contradiction.

**Case 2.**  $|S_1| \leq 2r + 5$ . It follows from the inductive hypothesis that  $G_1 \setminus S_1$  has exactly two components, one of which is trivial with vertex  $u$ , say. As  $|V(G_0)| \geq 9 \times 2^r > 2r + 8 \geq |S| + 1$ , there is an edge connecting  $G_0 \setminus S_0$  to  $G_1 \setminus (S_1 \cup \{u\})$ . So, it follows from the disconnectedness of  $G \setminus S$  that the matching vertex of  $u$  in  $G_0$  must belong to  $S_0$ . Thus,  $G \setminus S$  has exactly two components, one of which is trivial with vertex  $u$ , and the assertion holds.

**Case 3.**  $S_0$  is not empty,  $|S_1| \geq 2r + 6$ . Then,  $|S_0| = 1$ ,  $|S_1| = 2r + 6$ . Let  $S_0 = \{u\}$ , and let  $v$  denote the matching vertex of  $u$  in  $G_1$ . As every vertex of  $G_1 \setminus (S_1 \cup \{v\})$  is connected to a vertex of  $G_0 \setminus S_0$ , it follows from the disconnectedness of  $G \setminus S$  that  $G \setminus S$  has exactly two components, one of which is trivial with vertex  $v$ , and the assertion holds.

By combining the above discussions, we get that the assertion holds for  $n = r + 1$ . Thus, the assertion holds for all  $n \geq 0$ .  $\square$

**Lemma 7.** Suppose  $(l, m) \neq (3, 3)$ , and let  $S$  be a vertex subset of  $MH_n^{l \times m}$ ,  $|S| \leq 3n + 6$ . Then,  $MH_n^{l \times m} \setminus S$  has a component with at least  $l \times m \times 2^n - |S| - 2$  vertices.

**Proof.** By induction on  $n$ . The assertion is easily verified for  $n = 0$ . Suppose the assertion holds for  $n = r \geq 0$ . Now, let  $S$  be a vertex subset of graph  $G = MH_{r+1}^{l \times m}$ ,  $|S| \leq 3r + 9$ . Let  $G_0 = 0MH_r^{l \times m}$ ,  $G_1 = 1MH_r^{l \times m}$ ,  $S_0 = V(G_0) \cap S$  and  $S_1 = V(G_1) \cap S$ . Without loss of generality, assume  $|S_0| \leq |S_1|$ . Next, let us examine three possibilities.

**Case 1.**  $0 \leq |S_0| \leq 2$ . It follows from **Theorem 4** that  $G_0 \setminus S_0$  is connected. Let  $T_1 \subset V(G_1)$  be the set of vertices with matching vertices falling in  $S_0$ . As every vertex of  $G_1 \setminus T_1$  is connected to  $G_0 \setminus S_0$ , it follows that  $G \setminus S$  has a component with at least  $l \times m \times 2^{r+1} - |S| - 2$  vertices, and the assertion holds.

**Case 2.**  $3 \leq |S_0| \leq r + 3$ . It follows from **Theorem 4** that  $G_0 \setminus S_0$  is connected. As  $|S_1| \leq 3r + 6$ , it follows from the inductive principle that  $G_1 \setminus S_1$  has a component  $H$  with at least  $l \times m \times 2^r - |S_1| - 2$  vertices. As  $|V(G_0)| \geq 12 \times 2^r \geq 3r + 12 > |S| + 2$ , there is an edge connecting  $G_0 \setminus S_0$  to  $H$ . Thus,  $G \setminus S$  has a component with at least  $l \times m \times 2^{r+1} - |S| - 2$  vertices, and the assertion holds.

**Case 3.**  $|S_0| \geq r + 4$ . Then,  $|S_1| \leq 2r + 5$ . By **Lemma 6**,  $G_0 \setminus S_0$  has a component  $H_0$  with at least  $l \times m \times 2^r - |S_0| - 1$  vertices, and  $G_1 \setminus S_1$  has a component  $H_1$  with at least  $l \times m \times 2^r - |S_1| - 1$  vertices. As  $|V(G_0)| \geq 12 \times 2^r \geq 3r + 12 > |S| + 2$ , there is an edge connecting  $H_0$  to  $H_1$ . Thus,  $G \setminus S$  has a component with at least  $l \times m \times 2^{r+1} - |S| - 2$  vertices, and the assertion holds.

Combining the above discussions, we conclude that the assertion holds for  $n = r + 1$ . Thus, the assertion holds for all  $n \geq 0$ .  $\square$

As a corollary of this lemma, we have

**Theorem 8.** Suppose  $(l, m) \neq (3, 3)$ , and let  $S$  be a vertex subset of  $MH_n^{l \times m}$ ,  $|S| \leq 3n + 6$ . Then, the removal of  $S$  from  $MH_n^{l \times m}$  leaves either

- (1) a connected graph, or
- (2) exactly two components, one of which is trivial, or
- (3) exactly three components, two of which are trivial, or
- (4) exactly two components, one of which has exactly two vertices.

**4. Conditional diagnosability of OMMH networks**

**Lemma 9.** Suppose  $l, m \geq 4$  and  $l \times m \geq \frac{10n+25}{2^n}$ . Then,  $MH_n^{l \times m}$  is conditionally  $(3n + 7)$ -diagnosable.

**Proof.** Let  $F_1, F_2$  be two distinct conditional  $(3n + 7)$ -fault sets in  $G = MH_n^{l \times m}$ . Let  $S = F_1 \cap F_2$ . Then,  $|S| \leq 3n + 6$ . It follows from Theorem 8 that there are three possibilities, which are treated, respectively, as follows.

**Case 1.**  $G \setminus S$  has a trivial component with node  $u$ , say. Then,  $N_G(u) \subseteq S = F_1 \cap F_2 \subseteq F_1$ . This, however, contradicts that  $F_1$  is conditionally faulty.

**Case 2.**  $G \setminus S$  is connected. We proceed by considering five subcases.

**Case 2.1.** The vertex subset  $V(G) \setminus (F_1 \cup F_2)$  is not independent. Then, it is easily verified that  $G \setminus (F_1 \cup F_2)$  has a vertex that has a neighbor in  $G \setminus (F_1 \cup F_2)$  and has a neighbor in  $F_1 \Delta F_2$ . It follows from Lemma 1 that  $F_1$  and  $F_2$  are distinguishable.

**Case 2.2.**  $V(G) \setminus (F_1 \cup F_2)$  is independent,  $|S| \leq n + 1$ . As

$$|V(G)| = l \times m \times 2^n > 6n + 15 \geq |F_1 \cup F_2| + 1,$$

$V(G) \setminus (F_1 \cup F_2)$  has a vertex  $u$ , say. Note that  $u$  has no neighbors in  $V(G) \setminus (F_1 \cup F_2)$ , and  $u$  has at most  $|S| \leq n + 1$  neighbors in  $S$ . Thus,  $u$  has greater than or equal to three neighbors in  $F_1 \Delta F_2$ . By the pigeonhole principle, there either exist two vertices  $v, w$  in  $F_1 \setminus F_2$  or exist two vertices  $v', w'$  in  $F_2 \setminus F_1$  such that either  $(v, w)_u \in C$  or  $(v', w')_u \in C$ . From Lemma 1(C2),  $F_1$  and  $F_2$  are distinguishable.

**Case 2.3.**  $V(G) \setminus (F_1 \cup F_2)$  is independent,  $|S| \geq n + 2$ . If  $|S| \geq n + 2$ , then,

$$|V(G) \setminus (F_1 \cup F_2)| \leq \alpha(G) \leq \beta(G) \leq l \times m \times 2^{n-1}.$$

As

$$|V(G) \setminus (F_1 \cup F_2)| = |V(G)| - |F_1| - |F_2| + |F_1 \cap F_2| \geq l \times m \times 2^n - (5n + 12),$$

it follows that  $l \times m \times 2^n - (5n + 12) \leq l \times m \times 2^{n-1}$  or, equivalently,  $l \times m \times 2^n \leq 10n + 24$ . A contradiction occurs.

**Case 3.**  $G \setminus S$  has a component with two vertices  $u$  and  $v$ , say, and a component with  $|V(G)| - |S| - 2$  vertices. Then,  $u, v \in V(G) \setminus (F_1 \cup F_2)$ . Otherwise, we either have  $N_G(u) \subseteq F_1$  or have  $N_G(u) \subseteq F_2$ , a contradiction. Now, let us consider two possibilities.

**Case 3.1.**  $V(G) \setminus (F_1 \cup F_2 \cup \{u, v\})$  is not independent. Then, it is easily verified that  $G \setminus (F_1 \cup F_2)$  has a vertex that has a neighbor in  $G \setminus (F_1 \cup F_2)$  and has a neighbor in  $F_1 \Delta F_2$ . It follows from Lemma 1 that  $F_1$  and  $F_2$  are distinguishable.

**Case 3.2.**  $V(G) \setminus (F_1 \cup F_2 \cup \{u, v\})$  is independent. Then,

$$|V(G) \setminus (F_1 \cup F_2 \cup \{u, v\})| \leq \alpha(G) \leq \beta(G) \leq l \times m \times 2^{n-1}.$$

As  $N_G(\{u, v\}) \subseteq F_1 \cap F_2$ , it follows from Theorem 4 and Lemma 5 that

$$|F_1 \cup F_2| = |F_1| + |F_2| - |F_1 \cap F_2| \leq (3n + 7) + (3n + 7) - (2n + 6) = 4n + 8.$$

Thus,

$$|V(G) \setminus (F_1 \cup F_2 \cup \{u, v\})| \geq l \times m \times 2^n - (4n + 10),$$

it follows that  $l \times m \times 2^n - (4n + 10) \leq l \times m \times 2^{n-1}$  or, equivalently,  $l \times m \times 2^n \leq 8n + 20$ . A contradiction occurs.

The lemma follows by combining the above discussions.  $\square$

**Lemma 10.** Suppose  $l, m \geq 4$ . Then,  $MH_n^{l \times m}$  is not conditionally  $(3n + 8)$ -diagnosable.

**Proof.** Take three vertices,  $v_1 = (0, 0, 0^n)$ ,  $v_2 = (0, 1, 0^n)$  and  $v_3 = (1, 0, 0^n)$ . Let

$$F_1 = N(\{v_1, v_2, v_3\}) \cup \{v_2\}, \quad F_2 = N(\{v_1, v_2, v_3\}) \cup \{v_3\}.$$

Clearly,  $|F_1| = |F_2| = 3n + 8$ . Furthermore, it is easily verified that neither one of the two conditions in Lemma 1 holds. It follows that  $F_1$  and  $F_2$  are indistinguishable. Hence, the lemma is true.  $\square$

From Lemmas 9–10, we get the following

**Theorem 11.** Suppose  $l, m \geq 4$  and  $l \times m \geq \frac{10n+25}{2^n}$ . Then,  $t_c(MH_n^{l \times m}) = 3n + 7$ .

For  $(l, m, n) = (4, 4, 1)$ , there is an isomorphism between  $MH_1^{4 \times 4}$  and  $Q_5$ . By [15,17], we have the following

**Theorem 12.**  $t_c(MH_1^{4 \times 4}) = 10$ .

From Theorems 11–12, we get the main result of this paper, which is formulated in the following form.

**Theorem 13.** Suppose  $l, m \geq 4$ . Then,  $t_c(MH_n^{l \times m}) = 3n + 7$  if either

- (C1)  $n \geq 1$ , or  
 (C2)  $n = 0$  and  $(l, m) \neq (4, 4), (4, 5), (5, 4), (4, 6), (6, 4)$ .

## 5. Conclusion

We have determined the conditional diagnosabilities of OMMH networks. Specifically, we have shown that the conditional diagnosability of an  $(l, m, n)$ -OMMH network is  $3n + 7$  if either (a)  $n \geq 1$ ,  $l, m \geq 4$ , or (b)  $n = 0$ ,  $l, m \geq 4$ ,  $(l, m) \neq (4, 4), (4, 5), (5, 4), (4, 6), (6, 4)$ .

Our next work is to develop efficient algorithms for the conditional diagnosis of OMMH networks as well as other kinds of optical interconnection networks under the Maeng–Malek comparison model.

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