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Bardeen-like regular black holes in 5D Einstein–Gauss–Bonnet gravity

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ABSTRACT

We find an exact spherically symmetric regular Bardeen-like solutions by considering the coupling between Einstein-Gauss–Bonnet theory and nonlinear electrodynamics (NED) in five-dimensional spacetime. These solutions, with an additional parameter g apart from the mass M, represent black holes with Cauchy and event horizons, extremal black holes with degenerate horizons or no black holes in the absence of the horizons, and encompasses as a special case Boulware-Deser black holes which can be recovered in the absence of magnetic charge ($g = 0$). Owing to the NED corrected black hole, the thermodynamic quantities have also been modified and we have obtained exact analytical expressions for the thermodynamical quantities such the Hawking temperature T_+, the entropy S_+, the specific heat C_+, and the Gibbs free energy F_+. The heat capacity diverges at a critical radius $r = r_C$, where incidentally the temperature has a maximum, and the Hawking-Page transitions even in absence of the cosmological term. The thermal evaporation process leads to eternal remnants for sufficiently small black holes and evaporates to a thermodynamic stable extremal black hole remnants with vanishing temperature. The heat capacity becomes positive $C_+ > 0$ for $r_+ < r_C$ allowing black hole to become thermodynamically stable, in addition the smaller black holes are globally stable with positive heat capacity $C_+ > 0$ and negative free energy $F_+ < 0$. The entropy S of a 5D Bardeen

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black hole is not longer a quarter of the horizon's area A, i.e., $S \neq A/4$.

1. Introduction

The cosmic censorship conjecture asserts that physically reasonable matter never creates spacetime singularities which are observable to a distant observer [1]. However, the conjecture does not rule out the possibility of regular (singularity-free) black holes. Sakharov [2] and Gliner [3] that suggested one of the ways to avoid singularities is to choose matter with the equation of state $p = -\rho$, i.e., matter having a de Sitter core. This could provide a proper discrimination at the final stage of gravitational collapse, replacing the future singularity [3]. Motivated by this idea, Bardeen [4] proposed one of the first regular black holes having horizons with no singularity, which results as an exact solution from Einstein equations coupled to nonlinear electrodynamics (NED) [5]. The spherically symmetric Bardeen black hole is a regular spacetime with a de sitter core, whose metric is given by

$$ds^2 = -\left(1 - \frac{2mr^2}{(r^2+g^2)^{3/2}}\right)dt^2 + \frac{1}{\left(1 - \frac{2mr^2}{(r^2+g^2)^{3/2}}\right)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where m and g, are respectively mass and magnetic charge. It turns out that $g^{rr} = 0$ is a coordinate singularity indicating the possibility of horizons, if they exist, tare positive roots of $g^{rr} = 0$ or

$$1 - \frac{2mr^2}{(r^2+g^2)^{3/2}} = 1 - \left(\frac{m}{g}\right)\frac{2(r/g)^2}{(1+(r/g)^2)^{3/2}} = 0, \quad \text{and} \quad r \geq 0. \tag{2}$$

Solving (2) numerically implies a critical value ζ^* such that (2) has a double root if $\zeta = \zeta^*$, two distinct roots if $\zeta < \zeta^*$ and no root if $\zeta > \zeta^*$, with $\zeta = m/g$ [6]. These cases illustrate, respectively, an extreme black hole with degenerate horizons, a black hole with Cauchy and event horizons, and no black hole. We address the regularity of black hole (1) by calculating the following scalar invariants

$$\begin{aligned} R_{ab}R^{ab} &= \frac{6mg^2(4g^2-r^2)}{(r^2+g^2)^{7/2}}, \\ R_{abcd}R^{abcd} &= \frac{12m^2}{(r^2+g^2)^{7/2}}\left[8g^8 - 4g^6r^2 + 47g^4r^4 - 12g^2r^6 + 48r^8\right], \end{aligned} \tag{3}$$

where R_{ab} and R_{abcd}, respectively are Ricci and Riemann tensors. These invariants are well behaved everywhere including at $r = 0$; hence we can infer that the Bardeen black hole (1) is a regular. The Bardeen black hole is asymptotically flat, it is according to Schwarzschild for large r, whereas near the origin it behaves as de Sitter, since

$$f(r) \approx 1 - \frac{2m}{g}r^2, \quad r \approx 0^+,$$

Thus, the Bardeen black hole, unlike the Schwarzschild black hole, does not result in a singularity but develops a de Sitter region, eventually settling with a regular center [7]. Subsequently, there has been intense activity in the investigation of regular black holes [8–10], and more recently on rotating regular black holes [11,12].

One of the natural modification of the general relativity to higher dimensions is by supplementing the Einstein–Hilbert action with second order curvature term, which is so called Einstein–Gauss–Bonnet (EGB) theory encompasses general relativity as special case. It is free from the ghosts and also equations of motion are no more than second order and also it describes a wide variety of interesting models and hence, received significant attention. In particular, EGB gravity includes string theory inspired corrections to the Einstein–Hilbert action and admits Einstein's general relativity as a particular case [13]. Boulware and Deser [14] gave an exact static spherically symmetric black holes in EGB theory and demonstrated that the only stable case has a Schwarzschild-type spacetime structure with unavoidable central singularity. The solution was also extended for the charged case [15] with similar conclusion. Several generalizations of the Boulware-Deser solution with matter source have also been obtained [16,17], and from viewpoint of gravitational collapse to a black hole [18]. The exact spherically symmetric, static, regular black hole for EGB [19]. However, the Bardeen models are still unexplored in EGB theory. Since the Bardeen black holes is the first regular black holes and also opened gate for research in the regular black hoes and it is pertinent to consider the Bardeen-like black hole in EGB theory. It the purpose of this paper to construct an exact Bardeen-like black holes in $5D$ EGB theory. We shall not only discuss the horizon of obtained solution, but also investigate the thermodynamic properties including the stability of the system.

Motivated by the development of superstring and other field theories, there is renewed interest in models with extra dimensions. Indeed there are several reasons to study higher-dimensional black holes, in particular, the gravity correspondence [20] relates the black holes in D dimensions to that of $D-1$ dimensions and the possibility of producing tiny higher-dimensional black holes at colliders in "brane-world" scenarios [21]. Further, the higher-dimensional black hole spacetimes may have several useful mathematical properties [22]. The five-dimensional ($5D$) spacetime is particularly more important as spacetime results after dimensional reduction [23]. Also, the statistical calculation of black hole entropy using string theory was first done for certain $D = 5$ black holes [24]. Hence, the study of higher-dimensional black holes is a worthwhile contribution in developing a theory of quantum gravity. In particular, Myers and Perry [25] have found solutions to Einstein's equations representing black holes in D dimensions, and this was extended to Einstein–Maxwell configurations by Dianyan [26]. Other examples with Einstein's equations are the analysis of spherically symmetric perfect fluids in higher dimensions [27], in $5D$ spacetimes [28] and the collapse of different fluids [29].

The paper is organized as follows. We present relevant equations of EGB theory coupled to NED to obtain an exact $5D$ Bardeen-like black holes EGB theory in Section 2 . The investigation of structure and location of the black holes horizons is the subject of Section 3. Section 4 is devoted to detailed investigation of the thermodynamical properties of $5D$ EGB-Bardeen black holes. The thermal stability and black hole remnant are in Section 5, and the concluding remarks are given in Section 6. We shall adopt the signature $(-, +, +, +, +)$ for the metric and use the units $8\pi G = c = 1$.

2. Einstein–Gauss–Bonnet gravity coupled to nonlinear electrodynamics

The action for a full interacting EGB theory coupled to NED in $5D$ is given by

$$S = \int d^5x\sqrt{-g}\left[R + \alpha\left(R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right) - \mathcal{L}(F)\right], \tag{4}$$

which is an extension of the Einstein-Hilbert action and $\alpha \geq 0$ is the GB coupling constant. The matter is described by NED $\mathcal{L}(F)$ with $F = F_{ab}F^{ab}/4$, $F_{ab} = \partial_a A_b - \partial_b A_a$. In $4D$ the Euler–Gauss–Bonnet term becomes invariant and it does not contribute to the equation of motion. The action (4) can be found in heterotic superstring. theory [30] in its low energy limit with α regarded as the inverse string tension [30,31]. It is easy to show that the action (4), on variation [31,32], leads to the following field equations of motion

$$G_{ab} + \alpha H_{ab} = T_{ab} \equiv 2\left[\frac{\partial L(F)}{\partial F}F_{ac}F_b^c - g_{ab}L(F)\right], \tag{5}$$

$$\nabla_a\left(\frac{\partial L(F)}{\partial F}F^{ab}\right) = 0 \qquad \text{and} \qquad \nabla_\mu(*F^{ab}) = 0, \tag{6}$$

where G_{ab} and H_{ab}, respectively, are the Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R, \tag{7}$$

and the Lanczos tensor

$$H_{ab} = 2\Big[R R_{ab} - 2 R_{ac} R^{c}_{b} - 2 R^{cd} R_{acbd} + R_{a}^{\ cde} R_{bcde} \Big] - \frac{1}{2} g_{ab} L_{GB}, \tag{8}$$

is divergence free tensor. It is remarkable that the field equations (6) have derivatives of maximum second order and the theory is free from ghosts [30,33]. The Maxwell field for NED, with suitable modifications in $5D$ spacetime, reads

$$F_{ab} = 2\delta^{\theta}_{[a}\delta^{\phi}_{b]} Z(r,\theta,\phi), \tag{9}$$

where Z is function of r, θ and ϕ in $5D$. It turns out that, in $5D$ spacetime, $F_{\theta\phi}, F_{\theta\psi}$ and $F_{\phi\psi}$ are the nonvanishing components. Integrating Eq. (5) yields

$$F_{ab} = 2\delta^{\theta}_{[a}\delta^{\phi}_{b]} g(r) \sin^2\theta \sin\phi. \tag{10}$$

Eq. (6) implies $dF = 0$, which leads to $g(r) = g =$ constant. To obtain our regular black hole solution, we suitably modify Lagrangian density [34,35]

$$L(F) = \frac{3}{s g^2}\left(\frac{(\sqrt{2g^2F})}{(1+\sqrt{2g^2F})}\right)^{7/3}. \tag{11}$$

where s is a constant which can be chosen appropriately. When NED Lagrangian $\mathcal{L}(F) \approx F$, the one obtains the $5D$ charged black hole solution [15]. Interestingly, the components of $F_{\theta\phi}$ dominates over the other components and hence can be neglected. Hence, we have

$$F_{\theta\phi} = \frac{g}{r}\sin\theta, \qquad \text{and} \qquad F = \frac{g^4}{2r^6}. \tag{12}$$

Using Eqs. (6) and (10), the energy momentum tensor (EMT) for $5D$ spherically symmetric spacetime is given by

$$T^{t}_{t} = T^{r}_{r} = \rho(r) = \frac{6g^5}{s(r^3+g^3)^{7/3}}. \tag{13}$$

Other components can be easily obtained using the Bianchi identity $T^{ab}_{;b} = 0$ [36],

$$\partial_r T^{r}_{r} + \frac{1}{2} g^{00}[T^{r}_{r} - T^{t}_{t}]\partial_r g_{tt} + \frac{1}{2}\sum g^{ii}[T^{r}_{r} - T^{i}_{i}]\partial_r g_{ii} = 0. \tag{14}$$

to obtain

$$T^{\theta}_{\theta} = T^{\phi}_{\phi} = T^{\psi}_{\psi} = \rho(r) + \frac{r}{3}\partial_r\rho(r) = -\frac{2g^5}{s}\frac{3g^3-4r^3}{(r^3+g^3)^{10/3}}. \tag{15}$$

Thus we have determined complete EMT.

3. Exact bardeen-like regular black holes

Here we are interested in a regular solution of the EGB field Eqs. (5) with EMTs (13) and (15) in a $5D$ spacetime and will investigate its properties. We are seeking $5D$ spherically symmetric solution, for which the metric has a form [12]:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_3^2, \tag{16}$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta\,(d\phi^2 + \sin^2\phi\, d\psi^2)$ denotes the metric on the 3D sphere. The Einstein field equations (5) with metric (16) take the form

$$G_t^t = G_r^r = f'(r) - \frac{2}{r}(1-f(r)) + \frac{4\alpha}{r^2}(1-f(r))f'(r) = \frac{6g^5}{s(r^3+g^3)^{7/3}}, \tag{17}$$

$$G_\theta^\theta = G_\phi^\phi = G_\psi^\psi = f''(r) + \frac{4}{r}f'(r) + \frac{2}{r^2}(1-f(r)) + \frac{4\alpha}{r^2}\left[f'(r)(1-f(r)) + f'(r)^2\right] = -\frac{2g^5}{s}\frac{3g^3-4r^3}{(r^3+g^3)^{10/3}}, \tag{18}$$

where a ($'$) means a derivative with respect to coordinate r. It turns out that Eq. (17) admits an exact solution of the form

$$f(r) = 1 + \frac{r^2}{4\alpha}\left(1 \pm \sqrt{1 + \frac{8M\alpha}{(r^3+g^3)^{4/3}}}\right), \tag{19}$$

where M is the integration constant related to the black hole mass with dimensions (length)^2 and $s = g^2/M$. Further, the metric function (19) satisfies the other field equations. It turns out that $M = 0$ corresponds to

$$f_+ = 1 + \frac{r^2}{2\alpha}, \qquad \text{and} \qquad f_- = 1. \tag{20}$$

Thus, for upper sign ($+ve$) the solution are asymptotically de Sitter and for lower sign ($-ve$) they are asymptotically flat. Further, we note that $+ve$ branch of the solution is unstable, whereas $-ve$ branch is stable and free from ghost [37]. In a realistic physical solution, (19) must become the 5D Schwarzschild solution when $\alpha, e \to 0$. However when considering the limit $\alpha, e \to 0$, it does not recover the 5D Schwarzschild solution, hence we conclude the ($+ve$) branch solution has no physical interest and hence the ($-ve$) branch solution (16) will be discussed. Thus, we have (16) with metric function (19) and the EGB field equations encompasses the Boulware–Deser solution [14], when $g = 0$. For definiteness, we shall call the solution (16) EGB–Bardeen black holes. The negative branch of the solution, in the weak limit of GB coupling ($\alpha \to 0$), becomes

$$ds^2 = -\left[1 - \frac{Mr^2}{(r^3+g^3)^{4/3}}\right]dt^2 + \frac{1}{\left[1 - \frac{Mr^2}{(r^3+g^3)^{4/3}}\right]}dr^2 + r^2\, d\Omega_3^2, \tag{21}$$

which is 5D Bardeen-like solution [34], and goes over to 5D Schwarzschild–Tangherilini solution once we switch off the charge ($g = 0$). The causal structure of the Bardeen-like solution (16) is similar to that Boulware and Deser [14] black holes, except it has no more the curvature singularity at $r = 0$ [38]. The solution (16) can also be understood as a black hole of EGB coupled to NED, henceforth we call it EGB-Bardeen black holes. For the metric (16), it is seen that the curvature invariants are well behaved and regular with

$$\lim_{r\to 0} R = -\frac{5}{\alpha} + \frac{5}{\alpha}\sqrt{1 + \frac{8M\alpha}{g^4}},$$

$$\lim_{r\to 0} R_{ab}R^{ab} = \frac{10}{\alpha^2} + \frac{40}{g^4\alpha} - \frac{10}{\alpha^2}\sqrt{1 + \frac{8M\alpha}{g^4}},$$

$$\lim_{r\to 0} R_{abcd}R^{abcd} = \frac{5}{\alpha^2} + \frac{20}{g^4\alpha} - \frac{5}{\alpha^2}\sqrt{1 + \frac{8M\alpha}{g^4}}. \tag{22}$$

Thus, for $M, \alpha \neq 0$, the invariants are finite everywhere including at the origin. The weak energy condition states that $T_{ab}t^a t^b \geq 0$ for all timelike vectors t^a, or the local energy density must be non-negative. On the other-hand, the dominant energy condition requires that $T_{ab}t^a t^b \geq 0$ for a

Table 1
Inner and outer horizons and $\delta = r_+ - r_-$ for different values of parameter g.

	$\alpha = 0.1$				$\alpha = 0.2$		
g	r_-	r_+	δ	g	r_-	r_+	δ
0	–	0.89	–	0	–	0.776	–
0.40	0.37	0.81	0.44	0.25	0.27	0.74	0.47
0.45	0.45	0.76	0.31	0.30	0.35	0.71	0.36
$g_E = 0.493$	0.624	0.624	0	$g_E = 0.373$	0.566	0.566	0
0.55		No roots		0.45		No roots	

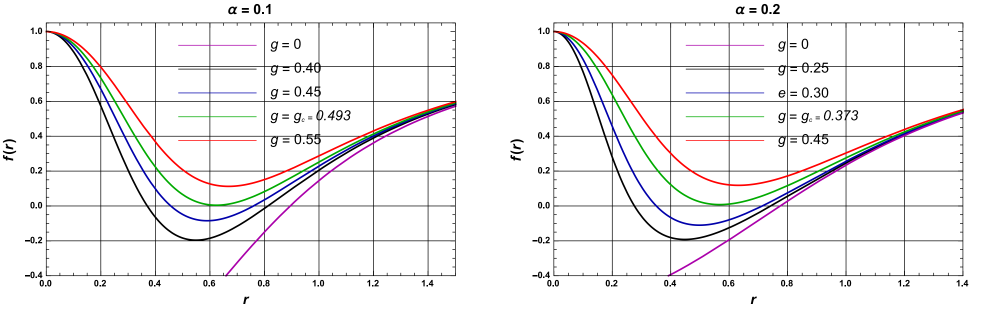

Fig. 1. Metric function $f(r)$ as a function of r for different values of parameters g and $\alpha = 0.1$ (left) and $\alpha = 0.2$ (right). The critical value of parameter $g = g_E = 0.493$ for $\alpha = 0.1$ and $g = g_E = 0.373$ for $\alpha = 0.2$ respectively.

timelike vector t^a. Hence for energy condition to be satisfied we must have $\rho \geq 0$ and $\rho + P_i \geq 0$. In the 5D case, $T^{ab}t_b$ is spacelike if

$$\rho + P_2 = \rho + P_3 = \rho + P_4 = \frac{6Mg^3}{(r^3 + g^3)^{7/3}} \geq 0. \tag{23}$$

Clearly, $\rho > P_3$ and $P_1 = -\rho$ and hence, EGB–Bardeen black holes obey the energy condition.

The event horizon is the largest root of $g^{rr} = f(r_+) = 0$, for which we seek a numerical solution. It is possible to find values of $\alpha \neq 0$ and $g \neq 0$ such that $f(r) = 0$ admits two positive roots ($r_\pm$), where r_+ corresponds to the outer event horizon and r_- to the inner Cauchy horizons.

We have plotted the horizon of the black hole in Fig. 1 and also tabulated some numerical values in Table 1. Fig. 1 suggests, for a given fixed value of α and M, there exists a critical value $g = g_E$ and critical radius r_E. Such that $f(r_E) = 0$, which corresponds to the extremal EGB-Bardeen black hole, where two horizons shrink to one, i.e., $r_\pm = r_E$ such that $f(r_E) = f'(r_E) = 0$.

4. Black hole thermodynamics

The thermodynamics of black holes is useful to throw light on the quantum properties of the gravitational field. The thermodynamics of AdS black holes have been of great interest due to the existence of a phase transition in AdS black holes [39]. Here, we are interested in the thermodynamical properties of the 5D EGB-Bardeen black holes, which is characterized by mass M, Gauss–Bonnet coupling α and parameter g. The black hole mass M using Eq. (19), in terms of the horizon radius r_+, reads

$$M_+ = r_+^2 \left(1 + \frac{2\alpha}{r_+^2}\right)\left(1 + \frac{g^3}{r_+^3}\right)^{4/3}, \tag{24}$$

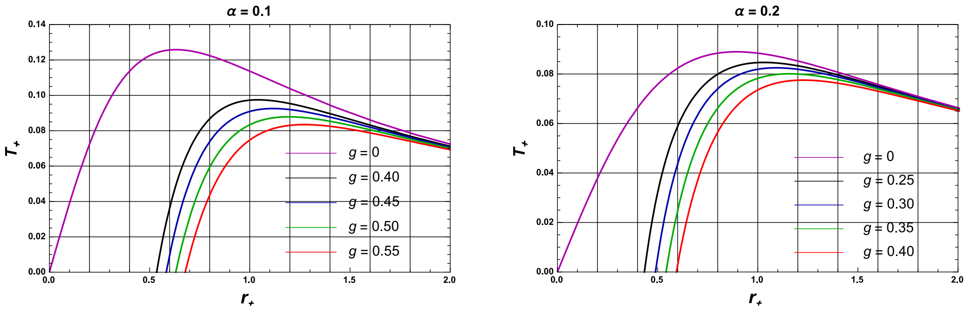

Fig. 2. Hawking temperature T_+ as a function of horizon radius r_+ for different values of parameter g.

when $g = 0$, we have $M_+ = r_+^2 + 2\alpha$ the EGB-Bardeen black hole mass [17,31,40] and further for $\alpha = 0$, we have $M_+ = r_+^2$ [31] the 5D Schwarzschild-Tangherilini black hole. The surface gravity is

$$\kappa = \frac{1}{2\pi}\left(-\frac{1}{2}\nabla_\mu \xi_\nu \nabla^\mu \xi^\nu\right)^{1/2}, \tag{25}$$

and $\xi^\mu = \partial/\partial t$ is a Killing vector. Now we are ready to analyze the thermodynamic quantities of 5D EGB-Bardeen black hole. The Hawking temperature 5D EGB-Bardeen black hole is $T = \kappa/2\pi$ with κ the surface gravity. The EGB-Bardeen black hole temperature, using (16), (19) and (25), reads

$$T_+ = \frac{f'(r_+)}{4\pi} = \frac{1}{2\pi r_+}\left[\frac{r_+^2 - \frac{g^3}{r_+^3}\left(r_+^2 + 4\alpha\right)}{(1+\frac{g^3}{r_+^3})(r_+^2 + 4\alpha)}\right], \tag{26}$$

Obviously positive temperature $T_+ > 0$ requires $g^3 > r_+^5/r_+^2 + 4\alpha$. The temperature vanishes if the black hole is extremal degenerate horizons, i.e., when $r_+ = r_-$. The Hawking temperature has a peak (cf. Fig. 2 and Table 2) that shifts to right and decreases with increasing g. The temperature (26), in the absence of the parameter ($g = 0$), becomes

$$T_+ = \frac{1}{2\pi}\left(\frac{r_+}{r_+^2 + 4\alpha}\right), \tag{27}$$

It is the temperature of the EGB black hole [17,31,40]. Further, in the limit $\alpha \to 0$, the temperature (26) reduces to the case of 5D Bardeen black hole [34]

$$T_+ = \frac{1}{2\pi r_+}\left(\frac{1 - \frac{g^3}{r_+^3}}{1 + \frac{g^3}{r_+^3}}\right). \tag{28}$$

Figs. 2 and 3 show that the temperature grows to a maximum T_{max} then drops to zero temperature. The Hawking temperature of the 5D EGB-Bardeen black hole has maximum at the critical radius shown in Table 2. When we increase the values of g and α, the local maximum of the Hawking temperature decreases (cf. Fig. 2). Further, the temperature diverges when the horizon radius shrinks to zero for 5D Schwarzschild black hole (cf. Fig. 3). In turn, the temperature yields $T_+ = 1/2\pi r_+$ when $g = 0$ for the 5D Schwarzschild–Tangherilini black holes (28) [17,31].

The entropy associated with a black hole, can be obtained by the first law of thermodynamics [17,31]

$$dM_+ = T_+\, dS_+ + \phi\, dg, \tag{29}$$

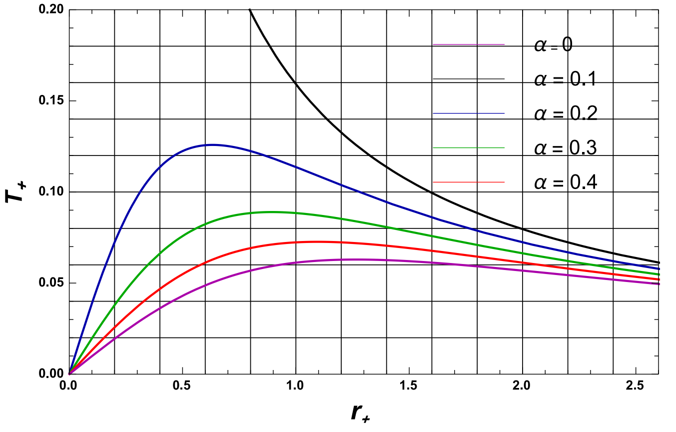

Fig. 3. Temperature T_+ as a function horizon radius r_+ for $5D$ EGB-Bardeen black holes for different values of α.

Table 2
Maximum Hawking temperature (T_+^{Max}) at critical radius (r_c^T) for different g.

	$\alpha = 0.1$					$\alpha = 0.2$				
g	0	0.40	0.45	0.50	0.55	0	0.25	0.30	0.35	0.40
r_c^T	0.63	1	1.036	1.11	1.12	0.897	0.99	1.00	1.14	1.20
T_+^{Max}	0.125	0.097	0.091	0.087	0.083	0.088	0.084	0.081	0.079	0.076

where ϕ is the potential for the constant parameter g [41]. Thus, the entropy of $5D$ EGB-Bardeen black hole

$$S_+ = \int \frac{1}{T_+}\frac{\partial M_+}{\partial r_+} dr_+, \tag{30}$$

and substituting (24) and (26) into (30), an exact expression for the entropy of the $5D$ EGB Bardeen black hole reads

$$S_+ = \frac{4\pi r_+}{3}\Bigg[\left(1+\frac{g^3}{r_+^3}\right)^{1/3}\left(r_+^2 + 12\alpha - \frac{g^3}{r_+^3}(3r_+^2+4\alpha)\right) - \frac{8\alpha g^3}{r_+^5} {}_2F_1\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{g^3}{r_+^3}\right] + \frac{2g}{r_+} {}_2F_1\left[\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{r_+^3}{g^3}\right]\Bigg]. \tag{31}$$

The expression of entropy Eq. (31) is modified due parameters g and α with the identification between entropy and area is no longer valid for $5D$ EGB-Bardeen black holes.

In the absence of NED ($g = 0$), the entropy becomes

$$S_+ = \frac{4\pi r_+^3}{3} + 16\alpha r_+, \tag{32}$$

the entropy of the EGB-Bardeen black hole [17,31,40]. Further, when $\alpha \to 0$, the entropy is exactly same as the $5D$ Bardeen black hole [34]. When both $g = 0, \alpha \to 0$, one sees $S_+ = 4\pi r_+^3/3$ as the entropy of the $5D$ Schwarzschild–Tangherilini black hole [31] and area law is valid.

5. Thermodynamical stability and black hole remnant

It is of considerable interest to study the stability of a given static field configuration, and focus on the local stability or thermal stability. A black hole is stable/unstable when heat capacity C_+ is

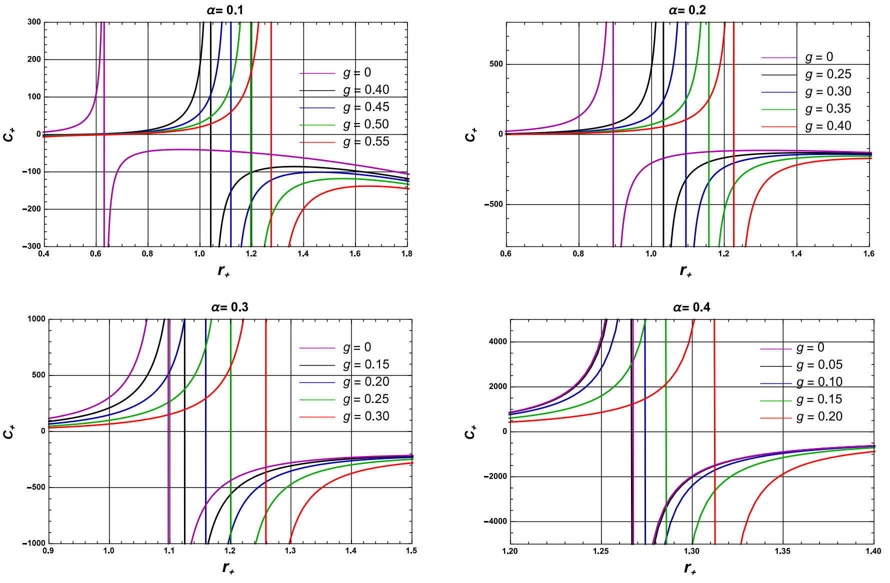

Fig. 4. The specific heat C_+ as a function of horizon radius r_+ for different values of parameters g and α.

positive/negative [42]. The thermodynamical stability of a black hole is performed by studying the behavior of its heat capacity (C_+), as the local stability of black hole is related to its heat capacity. The heat capacity of a black hole is given by [17,31]

$$C_+ = \frac{\partial M_+}{\partial T_+} = \left(\frac{\partial M_+}{\partial r_+}\right)\left(\frac{\partial r_+}{\partial T_+}\right). \tag{33}$$

Substituting (24) and (26) into (33), we obtain the heat capacity of the 5D EGB Bardeen black hole as

$$C_+ = -4\pi r_+^3 \left[\frac{(r_+^2+4\alpha)^2 (r_+^3+g^3)^{\frac{7}{3}} \left(r_+^2 - \frac{g^3}{r_+^3}(r_+^2+4\alpha)\right)}{[r_+^3(g^3(48\alpha r_+^5 + 8\alpha r_+^2 g^3 + 6r_+^7 + r_+^4 g^3 + 64\alpha^2 r_+^3 + 16\alpha^2 g^3) - r_+^8(r_+^2 - 4\alpha))]} \right]. \tag{34}$$

The heat capacity, for different values of parameter g and α, is depicted in Fig. 4 . We observe that the heat capacity is positive for $r_+ < r_C$ suggesting that the black hole is thermodynamically stable and is negative heat capacity for $r_+ > r_C$, implying the instability of black holes, with heat capacity discontinuous at critical $r_+ = r_C$, which implies second order phase transition happens [39,43]. A discontinuity of the heat capacity occur at $r_+ = 1.11$, at which the Hawking temperature maximum value $T_+ = 0.087$ for $\alpha = 0.1$ and $g = 0.493$ (Fig. 4). The critical radius r_C increases with parameter α (cf. Fig. 4 and Table 2). The EGB-Bardeen black hole has stable region $r_+ < r_C$ and heat capacity diverges at $r_+ = r_C$ when the temperature is maximum. In the limit $g \to 0$ one obtains the heat capacity for the EGB-Bardeen black holes

$$C_+ = -4\pi r_+^3 \frac{(r_+^2+4\alpha)^2}{r_+^4 - 4\alpha r_+^2}. \tag{35}$$

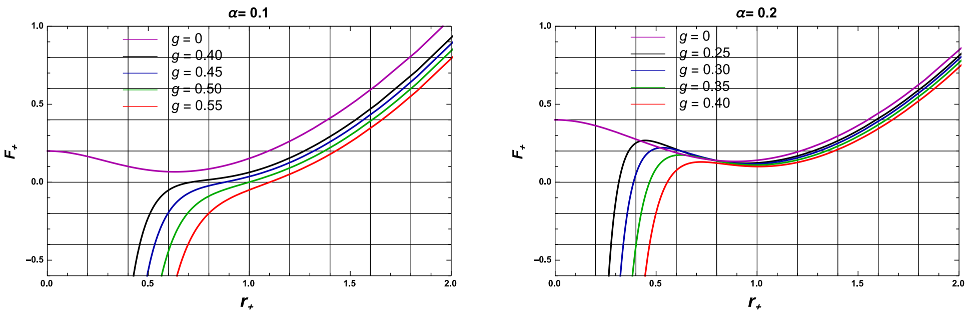

Fig. 5. Free energy F_+ as a function of horizon radius r_+ for different values of parameters g and α.

In the limit $\alpha \to 0$, the heat capacity (34) reduces to

$$C_+ = -4\pi r_+^3 \left[\frac{r_+(r_+^3+g^3)^{\frac{7}{3}}\left(r_+^2 - \frac{g^3}{r_+}\right)}{r_+^7 - g^3(6r_+^7 + r_+^4 g^3)} \right], \tag{36}$$

which is exactly same as 5D Bardeen black hole [34], and (36) with $g = 0$ corresponds to the heat capacity of the 5D Schwarzschild–Tangherilini black hole [17,31]. From Eq. (35), we observe that $C_+ \to \infty$ as $r_+^2 \to 4\alpha$ and also at $r_+^2 = 4\alpha$ the heat capacity flips the sign, representing a transition point (cf. Fig. 4).

One can also calculate the Gibbs free energy to discuss the global stability of the black holes [44] given by

$$F_+ = M_+ - T_+ S_+. \tag{37}$$

Substituting the values of Eqs. (24), (26) and (31) in Eq. (37), we get

$$\begin{aligned} F_+ = {} & (r_+^2 + 2\alpha)\left(1 + \frac{g^3}{r_+^3}\right)^{4/3} - \left(\frac{r_+^5 - g^3(r_+^2 + 4\alpha)}{(4\alpha + r_+^2)(r_+^3 + g^3)}\right)\left[\frac{4\pi r_+}{3}\left[\left(1 + \frac{g^3}{r_+^3}\right)^{1/3}\right.\right. \\ & \left(r_+^2 + 12\alpha - \frac{g^3}{r_+^3}(3r_+^2 + 4\alpha)\right) - \frac{8\alpha g^3}{r_+^5} 2F_1\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{g^3}{r_+^3}\right] \\ & \left.\left. + \frac{2g}{r_+} 2F_1\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{r_+^3}{g^3}\right]\right]\right]. \end{aligned} \tag{38}$$

In the limit $\alpha \to 0$, gives the Gibbs free energy for the 5D Bardeen black hole as

$$\begin{aligned} F_+ = {} & r_+^2\left(1 + \frac{g^3}{r_+^3}\right)^{4/3} - \frac{4\pi r_+}{3}\left(\frac{r_+^3 - g^3}{r_+^3 + g^3}\right)\left[\left(1 + \frac{g^3}{r_+^3}\right)^{1/3}\left(r_+^3 - \frac{3g^3}{r_+^3}\right)\right. \\ & \left. + \frac{2g}{r_+} 2F_1\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{r_+^3}{g^3}\right]\right]. \end{aligned} \tag{39}$$

The Gibbs free energy for various value of parameter g is shown in Fig. 5 which shows that $F_+ < 0$ for smaller r_+ where the heat capacity $C_+ > 0$, and thus indicating that EGB-Bardeen black holes are stable for smaller radius r_+.

The black hole remnant is a well-merited apprehension in theoretical astrophysics, which is one of the candidates to resolve the information loss puzzle [45] and also a source for the dark energy [46]. As pointed above, The double root $r_- = r_E$ of $f(r) = 0$ corresponds to the extremal black hole with degenerate horizon and hence $f'(r_E) = 0$.

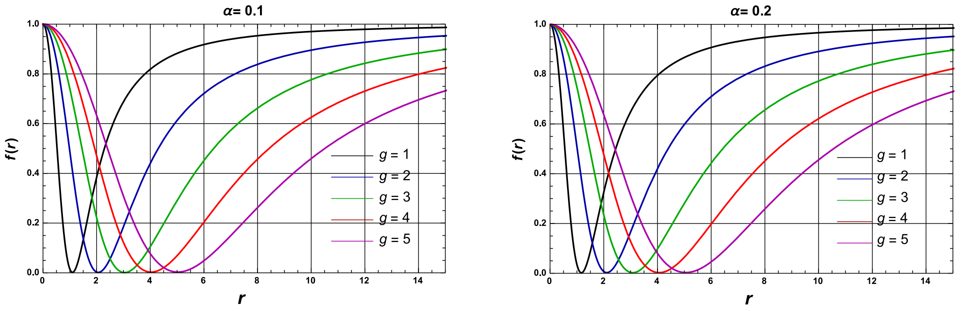

Fig. 6. The plot of metric function $f(r)$ as the function horizon radius r_+ for different values of parameters g and α.

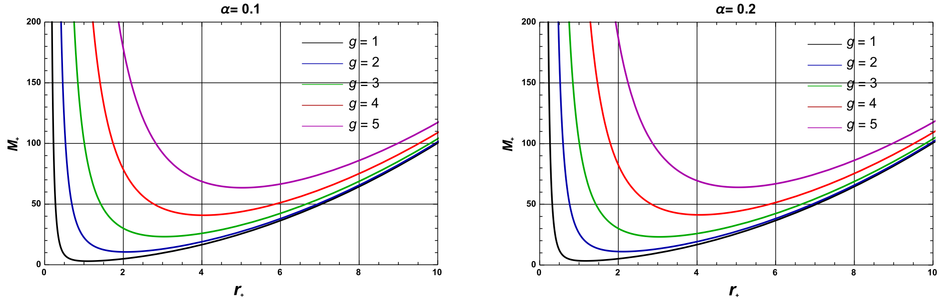

Fig. 7. The plot of mass M_+ as the function horizon radius r_+ for different values of parameters g and α.

Table 3
The critical radius r_C, remnant mass M_0, and maximum temperature T_{max} for different values of parameter g.

GB coupling	$\alpha = 0.1$			$\alpha = 0.2$		
Parameter (g)	r_C	M_0	T_{max}	r_C	M_0	T_{max}
$g = 1$	1.16	2.97	0.055	1.18	3.37	0.050
$g = 2$	2.30	10.55	0.030	2.10	11.2	0.029
$g = 3$	3.43	23.15	0.020	3.10	23.6	0.020
$g = 4$	4.59	40.70	0.015	4.06	41.2	0.015
$g = 5$	5.72	63.30	0.012	5.07	63.8	0.012

To get black hole remnant we solve $f'(r_E) = r_E^5 - g^3(r_E^2 + 4\alpha) = 0$ for r_E [47] and the results of remnants size, minimum mass and maximum temperature for different value of parameters g and α shown in Table 3.

The emitted feature of such a regular black hole can now be simply analyzed by displaying the temporal component of the metric as a function of radius for an extremal EGB-Bardeen black hole with different values of g. This has been presented in Fig. 6. This figure exhibits the possibility of having an extremal configuration with one degenerate event horizon at a minimal nonzero mass M_0 corresponding to zero temperature. In fact, the condition for having a degenerate horizon is that $M = M_0$ which means for $M < M_0$ there are no horizons (cf. Fig. 6 and Fig. 7). In the near extremal region the temperature increases from zero to local maximum T_{max} corresponding to $r_+ = r_{max}$. As r_+ increase further T_+ drops to a local minimum corresponding to $r_+ = r_{min}$ and then start increasing linearly.

The temperature decreases with increasing value of the critical r_C and becomes zero when the two horizons coincide, i.e., $T \to 0$ as $r_- \to r_+$. Hence, the 5D EGB Bardeen black hole has a stable remnant with $M = M_0$ and admits phase transition with divergence of a specific heat at the critical radius. The black hole cools down with regular double horizon remnant.

6. Conclusion

We find 5D Bardeen-like black holes as an exact solution of EGB gravity minimally coupled to NED and the famous Boulware-Deser black holes is encompassed as a special case in the absence of NED ($g = 0$). In turn, we characterized the solution by analyzing horizons which are maximum two, viz. inner Cauchy (r_-) and outer event horizons (r_-). The regularity of EGB-Bardeen black hole spacetime is confirmed by calculating curvature invariant and they are shown to be well behaved everywhere including at the origin. We have thoroughly analyzed thermodynamics to analytically compute thermodynamical quantities like the Hawking temperature, entropy, specific heat and free energy associated with EGB-Bardeen black holes with a focus on the stability of system. It turns out that the heat capacity blows at horizon radius r_+^C which is a double horizon and incidentally local maxima of the Hawking temperature also occur at r_+^C. It is shown that the heat capacity is positive for $r < r_+^C$ suggesting the stability of small black holes against perturbations in the region, and the phase transition exists at r_+^C. While the black hole is unstable for $r > r_+^C$ with negative heat capacity. The global analysis of stability of black holes is also analyzed by calculating free energy F_+. It turns out that the smaller black hole are globally stable with positive heat capacity $C_+ > 0$ and negative free energy $F_+ < 0$. Finally, we have also shown that the black hole evaporation results into a stable black hole remnant with zero temperature $T_+ = 0$ and positive specific heat $C_+ > 0$.

Further, generalization of such a regular black hole configuration to Lovelock gravity is an important direction for future which is being under active consideration. In addition, it would be also interesting to generalize this solution to include dS/AdS background.

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