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Comment on: "Neutron star under homotopy perturbation method" Ann. Phys. 409 (2019) 167918

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ABSTRACT

In this comment we discuss the application of homotopy perturbation method to a nonlinear differential mass equation that solves the Tolman–Oppenheimer–Volkoff equation for an isotropic and spherically symmetric system. We show that one obtains the same results, more easily and straightforwardly, by means of a textbook power-series method.

1. Introduction

In a recent paper Aziz et al. [1] derived a mass function that, according to the authors, provides a solution to the Tolman–Oppenheimer–Volkoff [2,3] equation for an isotropic and spherically symmetric system. They solved the nonlinear differential equation for the mass by means of the homotopy perturbation method and obtained a seventh-order polynomial function of the radius. With the aid of Einstein field equations they developed three solutions for different properties of a neutron star. The purpose of this comment is the discussion of the application of the homotopy perturbation method to the mass equation. In Section 2 we derive an approximate solution to that equation and in Section 3 we summarize the main results and draw conclusions.

2. The mass equation

The core of the paper by Aziz et al. [1] is the nonlinear differential mass equation

$$m' - \frac{1}{2} r m'' + m m'' - \frac{5\omega + 1}{2\omega} \frac{m m'}{r} - \frac{\omega + 1}{2} (m')^2 = 0, \tag{1}$$

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where ω is the constant of proportionality between the pressure p and the density ρ in the linear equation of state $p = \omega\rho$. All the equations are given in units that make the speed of light c and the gravitational constant G equal to unity. According to the authors, Eq. (1) solves the Tolman–Oppenheimer–Volkoff one [2,3].

In order to solve Eq. (1) the authors applied the so called homotopy perturbation method. It consists of separating the linear and nonlinear parts of the equation, introducing a perturbation parameter ϵ and expanding the solution in a Taylor series about $\epsilon = 0 : m(r) = m_0(r)+m_1(r)\epsilon+\cdots$. In this way the authors derived an approximate solution of the form $m(r) = a_1r^3 + a_2r^5 + a_3r^7$. The coefficients a_j depend on three constants of integration C_2, C_3 and C_5 originated in the perturbation equations through second order. Further analysis shows that $a_i \propto C^i$, where C is proportional to the density ρ_c at the center of the star ($C = 4\pi\rho_c$). The conclusion is that $a_i = a_1^i f_i(\omega)$, where f_i is a rational function of ω. This result looks suspiciously like a Taylor expansion of $m(r)$ about $r = 0$; in what follows we show that this is actually the case.

For convenience we rewrite Eq. (1) as

$$D(m) = rm' - \frac{1}{2}r^2m'' + rmm'' - \omega_1 mm' - \omega_2 r(m')^2 = 0,$$
$$\omega_1 = \frac{5\omega+1}{2\omega},\ \omega_2 = \frac{\omega+1}{2}, \tag{2}$$

and look for a solution of the form $m^{[N]}(r) = a_1r^3 + a_2r^5 + \cdots + a_Nr^{2N+1}$ such that $D\left(m^{[N]}\right) = \mathcal{O}(r^{2N+3})$. The calculation of the coefficients a_j is trivial and we obtain

$$m^{[4]}(r) = a_1r^3 - \frac{3a_1^2(\omega_1+3\omega_2-2)}{5}r^5 + \frac{3a_1^3(\omega_1+3\omega_2-2)(4\omega_1+15\omega_2-13)}{35}r^7$$
$$-\frac{a_1^4(\omega_1+3\omega_2-2)\left(61\omega_1^2+2\omega_1(243\omega_2-224)+9\left(105\omega_2^2-192\omega_2+88\right)\right)}{315}r^9, \tag{3}$$

that satisfies $D(m^{[4]}) = \mathcal{O}(r^{11})$. We have obtained one coefficient more than those shown by Aziz et al. [1] and one can easily derive as many terms as desired without any effort. Note that they are of the form $a_i = a_1^i f_i(\omega)$ and agree with those obtained by Aziz et al. [1], except for the model parameter n that the authors arbitrarily introduced with the purpose of improving the accuracy of their theoretical expressions. Without doubt this approach is far simpler and more straightforward than the homotopy perturbation method. The form of the coefficients a_i shown in Eq. (3) suggests that one obtains an exact solution when $\omega_1 + 3\omega_2 - 2 = 0$. In fact

$$D\left(m^{[1]}\right) = -3a_1^2r^5(\omega_1+3\omega_2-2), \tag{4}$$

clearly shows that $m^{[1]}(r) = a_1r^3$ is an exact solution when $\omega = -1/3$ or $\omega = -1$, the latter already mentioned by the Aziz et al. [1].

3. Conclusions

The purpose of this comment is to show that the application of the homotopy perturbation method to the nonlinear equation (1) in the way proposed by Aziz et a [1] is merely a tortuous way of obtaining a power-series expansion of the solution. The straightforward application of a textbook power-series method provides much more information with much less effort. We focus our attention on the solution of Eq. (1) because it appears to be the core of Aziz et al.'s paper. It is worth noting that several applications of the homotopy perturbation method were discussed in the past [4–7] some of which lead to power-series expansions [4,5,7] and in most of those cases to extremely poor or even nonsensical results.

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