

University of Montana

ScholarWorks at University of Montana

Graduate Student Theses, Dissertations, &
Professional Papers

Graduate School

2007

The Use of Mathematics Dialogues to Support Student Learning In High School Prealgebra Classes

Susann Meachelle Bradford
The University of Montana

Follow this and additional works at: <https://scholarworks.umt.edu/etd>

Let us know how access to this document benefits you.

Recommended Citation

Bradford, Susann Meachelle, "The Use of Mathematics Dialogues to Support Student Learning In High School Prealgebra Classes" (2007). *Graduate Student Theses, Dissertations, & Professional Papers*. 1098. <https://scholarworks.umt.edu/etd/1098>

This Dissertation is brought to you for free and open access by the Graduate School at ScholarWorks at University of Montana. It has been accepted for inclusion in Graduate Student Theses, Dissertations, & Professional Papers by an authorized administrator of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

**THE USE OF MATHEMATICS DIALOGUES
TO SUPPORT STUDENT LEARNING
IN HIGH SCHOOL PREALGEBRA CLASSES**

By

Susann Meachelle Bradford

B.A., St. John's College, Santa Fe, NM, 1989
M.S., University of Montana, Missoula, MT, 1992

Dissertation

Presented in partial fulfillment of the requirements
for the degree of

Doctor of Education
in Curriculum and Instruction

The University of Montana
Missoula, MT

Spring 2007

Approved by:

Dr. David A. Strobel, Dean
Graduate School

David R. Erickson, Ph.D., Chair
Curriculum and Instruction

Marian J. McKenna, Ph.D.
Curriculum and Instruction

Stephanie Wasta, Ph.D.
Curriculum and Instruction

Merle J. Farrier, Ed.D.
Educational Leadership

Libby J. Knott, Ph.D.
Mathematical Sciences

© COPYRIGHT

by

Susann Meachelle Bradford

2007

All Rights Reserved

The Use of Mathematics Dialogues to Support Student Learning In High School
Prealgebra Classes

Chairperson: David R. Erickson, Ph.D.

This study explored the use of mathematics dialogue activities as an intervention for low achieving mathematics students. These activities consisted of short scripts that portrayed mathematics students working together to solve problems like those in their lessons. These were accompanied by discussion questions and mathematics problems intended to facilitate student discourse in small groups. This intervention strategy was based on the *Professional Standards for Teaching Mathematics*, which recommends the use of teaching methods that provide opportunities for student discourse. The purpose of the study was to help teachers and schools identify whether the use of discourse could provide an effective strategy to improve student learning and meet the requirements of the No Child Left Behind Act.

A mixed methods case study design was used to provide a situated comparison of learning outcomes in two distinct instructional settings. Each of two teachers taught two prealgebra classes, one with and one without dialogue activities. Observations and classroom transcripts were used to describe the instructional settings and implementation, and to characterize classroom discourse in each setting. Quantitative methods were used to measure mathematics learning outcomes in terms of achievement and problem solving. In addition, a mathematics attitude survey and student interviews were used to address the potential influence of student attitudes and obtain feedback from students.

Results included the development of mathematics dialogue activities as a model for introducing student discourse into diverse classroom settings. Classes using the dialogue activities were found to have more opportunities for student-led questions and explanations and displayed more indicators of student learning and attitudes than control group classes. Student attitudes also emerged as an important factor influencing implementation. Quantitative results indicated that students who participated in mathematics dialogue activities had greater gains in mathematics achievement in both settings, greater gains in problem solving skill in one setting, and positive effects on student attitudes concerning self-concept in both settings. The quantitative findings were not conclusive due to small sample sizes, but indicate that mathematics dialogue activities are a promising intervention strategy for low achieving students.

Acknowledgements

This research would not have been possible without the support of Missoula County Public Schools and the two excellent teachers who collaborated on this project. I would also like to acknowledge my colleagues, reviewers, and committee members who provided critical feedback on research decisions and transcript interpretations. Special thanks go to Michael Marcinkowski, Matthew Arndt, Gail Becker, Susan Arthur, Jane Micklus, Bonnie Ferguson, George Sendon, Joe Crepeau, and Barry Adams. In addition, I would like to thank all of my committee members for their time and support, and acknowledge David Erickson, for his calm patience and the many hours spent editing drafts of this paper.

TABLE OF CONTENTS

Abstract.....	iii
Acknowledgements.....	iv
Contents	v
Tables	ix
Figures.....	x
CHAPTER ONE	
STATEMENT OF THE PROBLEM.....	1
Background.....	1
<i>Student Discourse in Mathematics Instruction</i>	7
Specific Problem Addressed by this Research.....	8
Research Questions.....	9
Importance of Study.....	9
CHAPTER TWO	
REVIEW OF LITERATURE	12
Factors Influencing Student Success in School	14
<i>General Factors on Student Success</i>	14
<i>Effective Intervention Programs</i>	19
<i>Summary: General Factors on Student Success</i>	23
Research Addressing Low Achievement in Mathematics	23
<i>Achievement Testing as a Measure of Learning and Effectiveness</i>	24
<i>Programs & Interventions to Improve Success in Mathematics</i>	30
<i>Supplemental Instruction</i>	30
<i>Parental Involvement Programs</i>	34
<i>Instructional Innovations</i>	35
<i>Technological Innovations</i>	37
<i>Summary: Research Addressing Low Achievement in Mathematics</i>	38
Discourse in Mathematics.....	39
<i>Instructional Standards</i>	40
<i>Research on Implementing Discursive Teaching</i>	43
<i>Role of Discourse in Learning</i>	46
<i>New Theoretical Frameworks</i>	49
<i>Socio-Discursive Research Tools</i>	54
<i>Summary: Discourse in Mathematics</i>	56
Chapter Summary	57
CHAPTER THREE	
METHODOLOGY	58
Research Design.....	58
<i>Concurrent Triangulation Strategy</i>	59
<i>Visual Model</i>	60
<i>Qualitative Components of the Study</i>	60
<i>Intervention: Mathematics Dialogue Activities</i>	62

<i>Data Collection</i>	66
<i>Data Analysis</i>	67
<i>Validation of Findings</i>	68
<i>Quantitative Components of the Study</i>	69
Research Questions	70
<i>Definition of Terms</i>	72
Population and Sample	75
Measures of Student Learning	76
<i>Variables and Levels of Data</i>	76
<i>Null Hypotheses</i>	77
<i>Definitions</i>	78
<i>Statistical Procedures</i>	78
Delimitations and Limitations	79
<i>Delimitations</i>	79
<i>Limitations</i>	80
Role of Researcher	80
Potential Ethical Issues	80

CHAPTER FOUR

QUALITATIVE CONTEXT	82
Development and Planning	83
<i>Collaboration</i>	83
<i>Instructional Setting</i>	85
<i>Traditional High School Setting</i>	86
<i>Alternative High School Setting</i>	89
Classroom Observations and Discourse Analysis	92
<i>Methods</i>	92
<i>Peer Review</i>	93
<i>Factors Affecting Implementation</i>	94
<i>Traditional School Implementation</i>	94
<i>Alternative School Implementation</i>	96
<i>Characteristics of Classroom Discourse in Each Setting</i>	98
<i>Teacher-Directed Whole Class Instruction</i>	99
<i>Tutoring Exchanges</i>	106
<i>Informal Student Discourse</i>	108
<i>Small Group Discourse</i>	110
<i>Silence-verification</i>	111
<i>Sharing answers</i>	112
<i>Small group tutorials</i>	114
<i>Collaboration</i>	116
<i>Overview: Small Group Discourse</i>	120
<i>Summary: Characteristics of Classroom Discourse in Each Setting</i>	121
<i>Discursive Indicators of Student Learning and Attitudes</i>	122
<i>Traditional School Control Group Classroom</i>	123
<i>Traditional School Discourse Classroom</i>	123
<i>Alternative School Control Group Classroom</i>	124

<i>Alternative School Discourse Classroom</i>	125
<i>Summary: Indicators of Student Learning and Attitudes</i>	126
Chapter Summary	127
 CHAPTER FIVE	
RESULTS	128
Achievement and Problem Solving	128
<i>Achievement</i>	129
<i>Subgroups: Achievement Comparisons by Setting</i>	130
<i>Traditional High School</i>	130
<i>Alternative High School</i>	131
<i>Problem Solving</i>	131
<i>Subgroups: Problem Solving Comparisons by Setting</i>	132
<i>Traditional High School</i>	132
<i>Alternative High School</i>	133
<i>Follow-up Assessments</i>	133
<i>Follow-up Test on Achievement</i>	134
<i>Follow-up Test on Problem Solving</i>	136
<i>Summary: Achievement and Problem Solving</i>	137
Student Attitudes towards Mathematics	138
<i>Attitude Survey Results</i>	139
<i>Overall Findings</i>	139
<i>Classroom Attitude Profiles</i>	141
<i>Traditional School Attitude Profiles</i>	141
<i>Alternative School Attitude Profiles</i>	143
<i>Student Attitudes towards Group Activities</i>	145
Student Interviews	146
<i>Traditional School Interviews</i>	146
<i>Alternative School Interviews</i>	148
<i>Summary: Student Interviews</i>	150
Chapter Summary	150
 CHAPTER SIX	
CONCLUSIONS	151
Research Questions	151
<i>Achievement</i>	152
<i>Problem Solving</i>	154
<i>Attitudes</i>	158
<i>Student Learning</i>	163
Implications	167
Recommendations for Future Research	170
 REFERENCES	 172
 APPENDICES	
A Mathematics Dialogue Activities	184

B	Interview Protocol.....	191
C	Pre-test on Achievement and Problem Solving	193
D	Post-test on Achievement and Problem Solving.....	199
E	Problem Solving Scoring Rubric	205
F	Attitude Survey	209
G	Follow-up Test on Achievement and Problem Solving.....	212

LIST OF TABLES

Table 1: Average Scores on Achievement Pre-Test and Post-Test	130
Table 2: Average Scores on Problem Solving Pre-Test and Post-Test.....	133
Table 3: Average Scores on Follow-up Tests of Achievement and Problem Solving.....	135
Table 4: Summary of Mean Gains in Achievement and Problem Solving.....	137
Table 5: Overall Distribution of Attitude Scale Scores	140
Table 6: Traditional High School, Distribution of Attitude Scale Scores	142
Table 7: Alternative High School, Distribution of Attitude Scale Scores	144
Table 8: Student Attitudes Concerning Group Activities.....	145

LIST OF FIGURES

Figure 1: Visual Model of Research Design.....	61
Figure 2: Excerpt from Mathematics Dialogue	65
Figure 3: Interactivity Flow Chart of Transcript Excerpt 9	119

CHAPTER ONE

STATEMENT OF THE PROBLEM

Low achievement in mathematics has affected schools, teachers and students throughout our nation's public school system for several decades. Public debates over school quality and accountability frequently point to statistics on low achievement as evidence of serious problems within the public schooling system. Demands for curriculum reforms and school improvement have also looked to student achievement data for evidence of program effectiveness. This longstanding issue took on new importance in 2002 when the No Child Left Behind Act was signed into law. This law requires schools and school districts to demonstrate their competence by continually improving student achievement on state mandated tests. Schools that fail to meet Annual Yearly Progress (AYP) goals may become labeled as *failing schools* and be required to adopt scientifically supported school improvement programs. This situation underscores a need for educational research to identify effective programs and instructional strategies that will help teachers, and thereby schools, improve student achievement in mathematics. This in turn will benefit students by helping them succeed in learning mathematics.

Background

A high percentage of students in our nation's public schools perform poorly in mathematics. According to the 2005 National Assessment of Educational Progress (NAEP), 71% of public school eighth graders are less than proficient in eighth grade mathematics, with 32% of these performing below the basic level (National Center for

Educational Statistics, 2005a). State level results for Montana are slightly better than the national average, with 64% of eighth graders scoring below the proficient level and just 20% below the basic level, but still a substantial majority does not appear to be obtaining proficiency in eighth grade mathematics (National Center for Educational Statistics, 2005b). It also appears from this data that one in five of Montana's public school eighth graders are headed to high school without a basic understanding of eighth grade mathematics topics. In other words, assuming these standards are reasonable, one fifth of the students entering high school are not prepared to learn high school mathematics topics.

In Montana, statewide testing mandated under the No Child Left Behind Act (NCLB) provides a somewhat brighter picture, with just 36% of public school eighth graders scoring below the proficient level on the MontCAS exam in 2004, and 37% in 2005 (Montana Office of Public Instruction, 2004, 2005). However, the most recent data from the 2006 assessment cycle indicates a substantial increase to 43% of eighth graders scoring below the proficient level, with an increase from 10% to 17% in the lowest skill bracket, the novice level (Montana Office of Public Instruction, 2007). Thus, even on statewide tests designed to reflect Montana's Content Standards for Mathematics, it still appears that a large proportion of Montana's eighth graders, about two-fifths, are not achieving proficiency in eighth grade mathematics. At the high school level, the numbers are similar and similarly increasing, with 40% of tenth graders testing below state proficiency standards in 2004, 44% in 2005, and now 46% in 2006 (Montana Office of Public Instruction, 2007). Again, at least two-fifths of Montana's public school tenth

graders appear to have insufficient mathematical knowledge and skills to demonstrate grade level proficiency in mathematics.

Low achievement in mathematics is also commonly higher among certain subgroups of students. NAEP data collected by the National Center for Education Statistics (2005a) indicates lower levels of academic achievement among students from low income family backgrounds and some ethnic minority backgrounds, such as Native American, African American, and Hispanic students. This is also the case in Montana, where available average test scores for these subgroups concur with higher percentages of students scoring below the proficient level in mathematics. In the case of Native American students, Montana's most widespread minority, 70% of eighth graders were below proficiency in 2004, 68% in 2005, and 73% in 2006. For tenth graders, this jumps to 75% below proficiency in 2004, 79% in 2005, and 76% in 2006. This is nearly double the statewide average reported for all tenth graders. In the case of low income students, as indicated by participation in free and reduced lunch programs, 53% of eighth graders were below grade level in 2004, 52% in 2005, and 60% in 2006. Among tenth graders in this category, 58% were below grade level in 2004, 63% in 2005, and 63% again 2006 (Montana Office of Public Instruction, 2007).

These statistics suggest that far too many students are leaving high school without the mathematical skills and knowledge they will need as adults, including many students who already face other substantial economic and cultural challenges. This situation may have serious consequences, both for these individuals and for society. Failure to learn mathematics and gain the confidence and skills needed to use mathematics effectively in everyday life can greatly limit the opportunities and quality of life available to

individuals. The inability to understand, apply, or interpret mathematical information not only limits access to postsecondary schooling and many career opportunities, but undercuts an individual's ability to make responsible choices in a society where mathematical data now informs a growing range of activities and issues from financial planning and healthy lifestyles to public policies and ballot issues (National Council on Education and the Disciplines, 2001). Students who experience consistent low achievement or failure may also become frustrated and lose hope or self confidence in their ability to learn mathematics (Glasser, 1988). Students who fail frequently are also more likely to drop out of school, and may end up contributing to broader social ills like unemployment, crime, and substance abuse (Zweig, 2003). Research is needed to improve mathematics instruction in ways that will enable more students to succeed in learning mathematics and gain access to opportunities and information that can enhance their lives.

Low achievement and failure in mathematics also has serious consequences for public schools. Achievement test scores are the primary measure of school success under the No Child Left Behind Act (2002). Schools with consistently low test scores are likely to fall short of Adequate Yearly Progress (AYP) goals and become labeled as *failing schools*. Failure to improve in subsequent testing cycles can lead to additional penalties, including school improvement plans, reorganization, or even closure. In the event that a school is required to adopt an improvement plan, it is also required that the methods used to improve schools are supported by *scientifically-based research*. While this term is the subject of some controversy (Schoenfeld, 2006), the requirement itself nevertheless points to a need for educational researchers to aid teachers and schools in the effort to

identify instructional strategies and programs that are likely to improve mathematics learning among low achieving students. Such efforts stand to benefit teachers and schools by providing them with more choices concerning what methods are adopted to raise student achievement levels and avoid further sanctions. This may help diverse local school districts retain greater autonomy in developing improvement programs that accommodate the unique cultural values, curriculum objectives, instructional preferences, and diverse students within their communities.

The problem of low achievement in mathematics also affects mathematics teachers. The need for research to address this problem cannot overlook the important role teachers must play in any effort to improve mathematics instruction. These are the professional educators most directly responsible for teaching mathematics to young people. Mathematics teachers not only possess the most relevant experience and training to address this problem, but also have a professional responsibility to do so, as codified in The Professional Educators of Montana Code of Ethics. When students perform poorly on assignments or tests, teachers use their knowledge and skills to evaluate and diagnose the problems students are having, and then identify ways to help the students gain a better understanding of the material. This may mean adapting lessons to accommodate specific students' learning needs, or developing new lessons to engage student interests or explain mathematical concepts and skills differently. The process of evaluating the effectiveness of various lessons with respect to student learning is both complex and generally situated within the context of specific classrooms with specific students. In light of this, the role of research is not to displace teachers' professional judgment about what teaching methods are effective in a given situation, but rather to support teachers by providing

them with additional information and more choices about how to improve their lessons. Again research is needed to address the problem of low achievement in a way that supports teachers by expanding the scientific knowledge base that informs their instructional decisions.

Existing research on strategies to improve mathematics instruction is paradoxically both plentiful and yet very limited in what it can tell us. During the last several decades, there have been numerous studies to identify effective educational programs and practices. A wide variety of programs have been developed and evaluated in the effort to improve mathematics achievement among students. However, despite this large and growing body of research, few if any proven interventions are currently available to school districts seeking to improve mathematics achievement (What Works Clearinghouse, 2006). There are many reasons for this. Some studies and program evaluations failed to include sufficient documentation of their research methods or the instructional strategy under consideration to allow their findings to be reproduced (Slavin & Madden, 1989). Other studies provide information about general elements of effective instruction, but offer teachers little guidance on how to apply these as part of a coherent teaching strategy for specific classrooms (Carpenter & Fennema, 1991). Still other studies that purport to demonstrate the effectiveness of various curriculum packages or model programs are disputed due to ongoing disagreements concerning how best to measure student learning and hence program effectiveness (Schoenfeld, 2006). Consequently, more research is needed to clarify these problems and provide educators with reliable information that will help them address this problem effectively in the classrooms where math education takes place.

Student Discourse in Mathematics Instruction

The *Professional Teaching Standards for Teaching Mathematics*, published by the National Council of Teachers of Mathematics (NCTM) in 1991, recommends the use of teaching strategies that encourage student discourse and inquiry in mathematics. According to this document, student participation in classroom discourse is supposed to enhance development of conceptual understanding, mathematical vocabulary, communication skills, and problem solving skills by allowing students to share and explore their insights and questions among peers (NCTM, 1991). If accurate, this claim means that instructional discourse could become a valuable strategy for improving student learning and thereby raising achievement test scores. Conventional wisdom would suggest that students with better conceptual understanding, vocabulary, and problem solving skills should also perform better on achievement tests. At the present time, however, there is very little research to demonstrate any such connection between the instructional use of student discourse and mathematics achievement. While the *Professional Teaching Standards* have led to a recent surge of interest in mathematical discourse among teachers and educational researchers, most of the research on this topic is still in preliminary stages or exploratory in nature (Kysh, 1999; Moore 2000; Sfard, 2002). The question as to whether the instructional use of student discourse can improve student achievement in mathematics has not been answered.

The lack of research to establish student discourse as an effective strategy for improving student learning and achievement in mathematics constitutes an important gap in the current research on effective mathematics instruction. In the present situation, a teaching strategy that is being recommended by and for teachers as a standard component

of quality instruction, at the same time does not qualify under the NCLB law as a scientifically supported method of instructional improvement. Even if discursive pedagogy reflects a longstanding instructional tradition that can be traced back to Plato and the Socratic Method, neither this history nor the recommendation of professional educators is enough to satisfy the current law. Therefore, research is needed to reconcile this gap between what professional mathematics teachers have endorsed and what is supported by science. If discursive teaching can be shown to improve student achievement, this would bring professional practices and scientifically supported practices into agreement. Whether or not this is possible remains to be seen. On the other hand, if research indicates that discourse does not improve student achievement, this may nevertheless help clarify the role of discourse with respect to student achievement and other learning outcomes.

Specific Problem Addressed by this Research

This research will examine the use of student discourse as an instructional strategy to improve student learning and achievement among low achieving high school mathematics students. The study will employ a treatment-control group design in order to apply a common model of scientifically-based research. The findings will then be interpreted with reference to specific classroom environments and instructional activities to provide information that relates to situated instructional decision-making at the classroom level. This will provide information with which to address the question of whether instruction that provides opportunities for student discourse, as recommended by NCTM, can also provide teachers and schools with an effective strategy for improving student achievement in mathematics to meet the requirements of NCLB.

Research Questions

The central question to be addressed by this research is stated as follows:

Does the use of teaching methods that provide opportunities for student discourse improve student learning in mathematics among low achieving high school students?

This will be addressed through the following subsidiary questions:

1. Does the use of teaching methods that provide opportunities for student discourse improve mathematics achievement among low achieving high school students?
2. Does the use of teaching methods that provide opportunities for student discourse improve problem solving skills among low achieving high school students?
3. Does the use of teaching methods that provide opportunities for student discourse appear to influence student attitudes towards mathematics among low achieving high school students?

Importance of Study

Research is needed to identify effective strategies and programs to help more students succeed in learning mathematics. Students leaving high school without the confidence and skills needed to use mathematics effectively in their everyday lives will have less access to opportunities and information that can improve their lives. This includes opportunities for continued schooling and employment, and access to information that can help them make responsible and healthy choices. This research will

benefit students and society by helping more future citizens gain access to information and opportunities that can improve their lives.

This research will also help schools meet the requirements of No Child Left Behind. Under this law, Montana schools are required to demonstrate their competence by improving student achievement test scores in mathematics and other subjects on the annual MontCAS exam. Schools that fail to meet Annual Yearly Progress (AYP) goals may be labeled as *failing schools* and then required to adopt scientifically supported school improvement programs. Accordingly, research is needed to help schools identify effective instructional strategies that will enable them to meet AYP goals and avoid sanctions. This research will also benefit schools that fail to meet AYP by providing them with more information about instructional choices that are supported by scientific research. This may lead to new and better choices for schools and teachers seeking to improve student success in mathematics, and also contribute more generally to the mission of providing a high quality education for all students.

In addition, research on the effectiveness of instructional strategies benefits the math teachers who are responsible for instructing students, improving student learning outcomes, and making daily instructional decisions about how to do this. This research will contribute to the knowledge base that informs teachers' instructional decisions. This may provide teachers with new strategies for helping students learn math.

Finally, this will address a significant gap in the current research on mathematics education. Based on the *Professional Teaching Standards for Teaching Mathematics* (NCTM, 1991), teaching methods that provide opportunities for student discourse are supposed to improve student learning. However, little is known concerning whether this

type of teaching improves student achievement. Since schools and teachers facing sanctions under NCLB are required to adopt strategies that are supported by scientifically based research, research is needed to determine whether the teaching methods recommended by NCTM are effective methods for improving student achievement in mathematics. This research may also provide additional insights concerning the use of student discourse as an intervention for low achieving students. This may lead to new and better choices for schools and teachers seeking to improve student success in mathematics.

CHAPTER TWO

REVIEW OF LITERATURE

Low mathematics achievement among secondary school students is well documented by both national and state level data (Montana Office of Public Instruction, 2005; National Center for Education Statistics 2004, 2005). This establishes a need for educational interventions that will help more students succeed. But although this problem is widely acknowledged, there is much less agreement concerning the causes of low achievement and what solutions are needed. The effort to improve mathematics instruction is complicated by diverse perspectives and priorities among the many stakeholders of public education. For example, some critics interpret low achievement as a failing of teachers and schools, while others attribute this to societal inequities and cultural challenges faced by many students. At the same time, varying research paradigms and conceptual frameworks among researchers also contribute to the complexity of this issue. The resulting body of research on this issue spans a wide range of theories and variables thought to influence student success, both in general and with respect to specific content or populations.

This literature review begins with a consideration of general factors influencing student success in school. This addresses broad issues that affect student learning across the high school curriculum as well as important features of effective intervention programs. After considering these general factors on student success, the focus narrows to address research aimed specifically at improving mathematics learning and achievement among lower achieving students. This includes an accumulation of

evaluation reports from four decades of Title I programs and many other studies of programs and strategies ranging from after school tutoring to computer assisted learning. While some of these programs have sought to address factors identified in prior research, others were designed and implemented in response to social demands and only evaluated later. This haphazard development has contributed to a sprawling body of research with many threads that lack connection and consistency. Previous efforts to identify effective programs from this body of information have been impeded by variability and omissions with respect to key definitions and methodological features in these reports. The present review of this research seeks to identify the range of programs and teaching strategies that have been shown to measurably increase student success.

Finally, the present study also requires an overview of the available knowledge concerning the role of discourse in teaching and learning mathematics and how this relates to efforts to improve student learning. As noted previously, there has been a recent surge of interest in discourse among educators and researchers since publication of the *Professional Standards for Teaching Mathematics* (NCTM, 1991). This document emphasized the importance of providing opportunities for classroom discourse as a means to improve students' understanding of mathematical language and ideas, and help them develop communication and problem solving skills. However, most of this research is exploratory and qualitative in nature. Few studies have examined the use of classroom discourse with low achieving students and most of the other studies examine student interaction in specific classroom settings (Kysh, 1999; Li, 1998; Sfard, 2002). Accordingly, these findings have narrow applicability and make little comment on the issue of how discourse might affect quantitative measures of achievement. Nevertheless,

a review of these studies provides the context for further research on discourse by outlining the current knowledge base, methodologies, and theoretical frameworks applied in other studies of classroom discourse in mathematics. This is also needed to inform the research design and treatment in the present inquiry.

In sum, this chapter addresses three main topics: (a) general factors influencing student success in school, (b) effective programs and strategies for improving student success in school mathematics, and (c) research on the role and use of classroom discourse in mathematics education. These topics are considered in turn to provide a broad context in which to address the use of student discourse as a means to improve student learning in mathematics.

Factors Influencing Student Success in School

In order to help students succeed in mathematics, one must first understand why they are not succeeding already. Accordingly, this section provides an overview of factors thought to influence student success in school in general. This research does not address mathematics specifically, but looks beyond the details of mathematics instruction to identify broader factors that may underlie student success across the curriculum. These findings are therefore relevant to mathematics as a standard component of this curriculum. Awareness of these issues is important in the attempt to design effective ways to support student learning.

General Factors on Student Success

In recent decades, students who were deemed likely to fail classes or dropout of school were often described as *at-risk*, meaning simply that they were at-risk of failing, dropping out, or otherwise leaving high school without a diploma (Aron 2003; Raywid,

2001; Thomson, 1998). This designation has been used very broadly to identify students with special qualities or conditions found to be relatively common among other students who have already failed or dropped out. Indicators of at-risk status include some obvious individual factors, like having already failed many classes, as well as broader subgroup characteristics, like minority or low income status (Miller, 1999; National Center for Education Statistics, 2005a). However, the term also has been applied generally to any students who experience exceptional difficulties or simply *do not fit in* at their regular high schools (Raywid, 2001).

According to Aron (2003), students may be placed at-risk by a wide range of factors related to schools, individual students, communities, and families. Examples of school related factors include ineffective policies, cultural differences between school staff and students, lack of support for diverse learning styles, or lack of bilingual programs. Student related factors include gender, pregnancy, parenting, discipline problems, illness, low achievement or drug abuse. Factors in the community may include violent neighborhoods or poor relations between the school and community, while family related factors include situations such as poverty, abuse, high mobility, homelessness, language barriers, or lack of parental support (Aron, 2003). While this list is general and not exhaustive, it serves to show the wide variety of factors that may contribute to student failure in various situations.

Zweig (2003) characterizes such students as *disconnected youth*, or young people who are “disconnected from mainstream institutions and systems” (p. 1), such as supportive families, education, employment, marriage, military service, or other organizations that could prepare them for adulthood. In this account, becoming

disconnected is attributed to a number of factors including poverty, homelessness, teen pregnancy, ethnicity, language barriers, learning difficulties, incarcerated parents, foster care, or involvement in the juvenile justice system. According to Zweig, such disconnection leaves young people disadvantaged and vulnerable to other social ills such as crime, poverty and drug abuse. Since many communities lack resources and services to assist such young people, a high proportion end up unemployed, marginally employed, prone to substance abuse, on welfare, or in prison (Zweig, 2003). While schools may not be able to address all of these external causes of disconnection, an awareness of these is a first step towards identifying ways to retain students or re-connect them to schooling. For example, teen pregnancy can cause young mothers to drop out of high school for family related reasons. Some schools have recognized and removed this barrier by providing on-site day care facilities at high schools (Toch, 2003).

Given the wide range of factors that may disadvantage students in school, some researchers have adopted basic needs frameworks similar to that of Maslow (1968), who theorized that an individual's basic needs for safety, belongingness, love, respect, and power must be met before growth and learning can proceed. From this standpoint, the factors leading to disconnection are viewed as inhibitors of students' basic needs. This framework enables a shift in focus from grappling with a multiplicity of causes to providing for a smaller number of underlying needs.

According to Glasser (1988), students have basic needs for survival, love/belonging, power, freedom, and fun. He argues that schools seeking to retain students need to accommodate these basic needs with recognition that students have developed habits, expectations, and coping patterns based on previous experiences that

can continue to inhibit their progress, even in positive environments. Meier (2002) conveys a similar perspective in her discussion of the challenges involved in building trustful learning environments between students and adults. Similarly, Sagor and Cox (2004) identify basic emotional needs as five essential feelings: competence, belonging, usefulness, potency, and optimism. They also argue that school environments need to address these basic emotional needs in order to promote a sense of security and confidence that sets the stage for successful learning.

Similar themes are echoed in research addressing special challenges faced by minority students. As noted by Aron (2003), cultural and linguistic differences may contribute to student failure and dropout. Banks (2002) and Nieto (2004) provide comprehensive multicultural education frameworks to facilitate a clearer understanding of where and how such differences can influence student learning. Cultural differences can affect students' experiences of several components of the school environment, including curriculum, pedagogy, knowledge base, policies and racial attitudes. Students may find their culture excluded from the school curriculum, or find that culturally accepted behaviors and learning styles are not accepted at school. Culturally informed assumptions or meanings may be misinterpreted by others, or infuse a subtle bias into what gets represented as *knowledge*. Similar bias may influence school policies causing them to be inequitable, as in the case of an attendance policy that fails to accommodate cultural events. At the same time, low expectations among teachers may lead students to become indifferent about learning, while racist attitudes among peers can contribute to an atmosphere of hostility and violence that impedes student learning. Peer pressure may encourage such students to play down academic abilities in order to fit in with cultural

expectations or stereotypes (Banks, 2002; Nieto, 2004).

Research on immigrant children also indicates that students who experience school as cultural outsiders may feel un-welcome, confused, disempowered, ineffective or hopeless about succeeding academically (Igoa, 1995). For some students, not trying, quitting or dropping out may be perceived as a safer, more dignified, or more practical option than continued schooling (Suarez-Orasco & Suarez-Orasco, 2001). Accordingly, this body of research supports the view that a broad range of cultural, environmental, social and emotional factors should be considered when developing strategies to improve student success in school.

Finally, another issue to be aware of is students' sensitivity about being labeled as *at-risk* or enrolled in special *remedial* programs and how these terms may influence interpretations of low achievement or failure. Richardson, Casanova, Placier and Guilfoyle (1989), who studied the implications of being labeled at-risk among elementary school children, argued that the term *at-risk* inappropriately invokes an epidemiological model for understanding student failure as analogous to sickness. They argue that this analogy suggests that causes of failure are to be found in the student, and that remedies for failure involve treating the student. Accordingly, such terms may have negative consequences for students by influencing the perceptions of these children among teachers, parents, peers, community members, and even the children themselves. The implicit suggestion that there is something wrong with these students may affect their self-esteem and deflect criticism from other possible causes of their failure, such as qualities of the school climate, teaching styles, parents or society. Similarly, Rueda (1993) argues that the epidemiological model for understanding student failure is at least

partly responsible for a disproportionately high number of minority students placed in special education programs. That is, a narrow focus on student shortcomings may exclude due consideration of cultural and environmental influences on student success, leading some to misinterpret cultural differences as disabilities.

These issues are important to consider as part of the task of identifying effective interventions for students who perform poorly in mathematics. One of the most common assumptions about students who struggle or fail in school mathematics is that they are not smart enough to learn mathematics. This perspective may send a defeatist message to children who are perfectly capable of learning under the right circumstances. One must be wary of such assumptions, whether they stem from cultural bias or a tendency to venerate familiar customs and conventional practices as if these were beyond question. Either way, such assumptions do little to help students learn. As Hixson (as cited by North Central Regional Education Laboratory, 2004) points out,

Students are not 'at risk,' but are placed at risk by adults. ...when they experience a significant mismatch between their circumstances and needs, and the capacity or willingness of the school to accept, accommodate, and respond to them in a manner that supports and enables their maximum social, emotional, and intellectual growth and development. (Definition of At-Risk, ¶ 5)

Effective Intervention Programs

Another important source of information on factors affecting student success in school is research on programs designed to help them succeed. During the last several decades, a wide variety of intervention programs have been developed and implemented

to address many of the factors discussed above. This includes a wide range of vocational, remedial and detention programs (Aron, 2003; Raywid, 1994), as well as more recent models of alternative high schools, charter schools, theme schools, schools within schools, and smaller learning communities (Dessoiff, 2004; Martinez & Klopott, 2002; Meier, 2002; Toch, 2003). While these programs vary with respect to goals, structure, location, target populations, administration, and management (Aron, 2003), they also exhibit many common features, such as small class sizes, smaller learning communities, opportunities for a personal connection with teachers or other adults, and an emphasis on engaging students in meaningful learning (Meier, 2002; Raywid, 2001; Toch, 2003). Accordingly, these program features also appear to influence student success.

Efforts to identify effective intervention programs reference a variety of criteria, including achievement, graduation rates, student retention, grades, attendance, discipline referrals, and subsequent course placements (Aron, 2003; Griswold, Cotton & Hansen, 1986; Slavin, 1989). In a broad survey of research on alternative schooling, Aron (2003) found six independent efforts to characterize successful programs, each of which identified the following as key features: high academic standards, small schools, small class sizes, high quality student-centered programs, administrative autonomy, and voluntary participation by students and staff. Accordingly, the influence of these features on student success is supported by a high degree of consensus among researchers. While some of these factors address organizational or structural aspects of program success, others speak to student needs and interests.

In another analysis of effective alternative schooling practices, Thomson (1998) reports a major shift in effective practices away from remedial, or deficit-based models,

and towards more school-wide programs of a preventative nature. While early programs and research focused primarily on remedial programs within a traditional school framework, the newer emphasis is on redesigning schools to accommodate more diverse learners. Reflecting this shift, Thomson (1998) identified more features of affective programs that relate to student needs:

- *Whole student* focus (academic, behavioral, social)
- Staff who exhibit warmth and care
- Staff who also act as advisors and mentors
- A strong sense of community
- High student expectations
- Small class size and student/teacher ratio
- Experiential learning and work based learning components
- A safe environment
- Shorter blocks of time for credit accrual. (p. 16)

Here, several features appear to complement the basic needs framework discussed earlier, by addressing issues like safety, community, warmth and care as key elements of school design. Conversely, the basic needs framework may help explain why some program designs work better than others. For example, small class sizes may help promote a sense of belonging.

This shift in emphasis is also evident in different editions of the U.S. Department of Education's *Effective Compensatory Education Sourcebook* (Griswold, Cotton & Hansen, 1986), which identified components and examples of effective Title I programs. The first version of this *Sourcebook* set forth six instructional components of effective

programs, including appropriate instruction, academic learning time, frequent monitoring, high expectations, feedback to students, and recognition of student accomplishments. In contrast, the 1992 edition (Reisner & Haslam, 1992) lists nine instructional components of effective programs:

- Opportunities for students to use their own experiences as a foundation for learning
- Teaching that explains assumptions, expectations, and ways of doing things in school
- Curriculum that includes instruction in comprehension skills
- Curriculum that integrates instruction on basic skills with challenging content
- Instruction that highlights meaning & understanding
- Recognition that students sometimes learn best by directing their own learning and working together
- Assessment that informs students and others of students' progress
- Recognition and rewards for academic excellence
- Classroom management keyed to learning tasks.

Here again, there appears to be a shift away from general instructional elements of the traditional classroom to more complex features that acknowledge student diversity.

Building on student experiences, explaining assumptions about school learning, teaching comprehension skills, emphasizing meaning and understanding, incorporating different instructional styles are all strategies that acknowledge student differences influenced by different experience, social and cultural backgrounds. The emergence of these features as key elements of effective programs provides additional corroboration for the view that

effective programs to improve student learning are those that recognize and accommodate individual differences and student needs.

Summary: General Factors on Student Success

Students who do not succeed in school have been described as at-risk learners and disconnected youth. Research on broad social trends indicates that many such students lack meaningful connections to institutions such as school, employment, social organizations or military service (Zweig, 2003). Student success in school has also been linked to a wide range of factors related to individual students, their families, cultures, schools, and communities (Aron, 2003). Other research focused on individual learners suggests that many students do not succeed because their basic needs are not being met within the learning environment (Glasser, 1988; Sagor & Cox, 2004). Multicultural education research also supports the view that many learning environments do not accommodate student needs associated with diverse cultural backgrounds. Accordingly, research on student failure points to the importance of designing learning interventions that accommodate basic emotional needs and recognize the influence of broader social and cultural factors on student success. This view is also supported by analyses of effective programs, which indicate that student learning is supported by programs that recognize individual differences and student needs.

Research Addressing Low Achievement in Mathematics

Having considered at-risk learners and academic interventions in general, the next focus of inquiry is interventions specific to mathematics. This entails a large body of literature reflecting several decades of Title I programs and related research, plus many other studies. Unlike the interventions discussed above, efforts to improve learning and

achievement in mathematics are generally evaluated on the basis of students' mathematics achievement, course grades and course placement with respect to students' age or grade. Achievement is also the primary measure of student learning. While some studies have also examined longitudinal monitoring of students' subsequent course completion and success, mathematics achievement data is by far the most common measure of student learning and program success in this research. Accordingly, this section begins with a brief overview of achievement testing in order to provide a clearer understanding of what is meant by *program effectiveness*. The section then proceeds to consider the effectiveness of specific programs and elements of effective programs.

Achievement Testing as a Measure of Learning and Effectiveness

Research on low achievement in mathematics can be traced back to the development of intelligence testing and aptitude testing in the earlier part of the last century. Almost as soon as these tests were developed, theories began to surface concerning why some individuals performed better than others (Fancher, 1985; Freeman, 1939). It did not take long for standardized testing to begin to play a part in the evaluation of curricula and teaching methods. An early survey of curriculum research by Davis and Wilbur (1933) reported on trends among 381 studies during the preceding decade (1921-1931) and argued that "the most important factor in the study of the curriculum is the development of appropriate standardized measurements of the achievement of pupils" (p. 297).

Broader public concern over low achievement in mathematics did not emerge until the mid-twentieth century when two disparate reform movements brought attention to the issue. One thread can be traced to the Cold War era when success of the Soviet

satellite Sputnik in 1957 cast doubts on America's technological leadership in the race to space (Schoenfeld, 2006). Improvement in mathematics and science education was needed to prepare the workforce and lead advancements in an increasingly technological society and economy. Later, this concern shifted to broader issues of competition in the global marketplace and intellectual excellence as *A Nation at Risk* report renewed allegations that low standards in our nation's public schools had again weakened America's leadership in the world (National Commission on Excellence in Education, 1983). The second important thread is the emergence of the Civil Rights movement, which brought attention to inadequate schooling among minorities and the poor. Equal opportunity required access to a decent education to gain the literacy and mathematical skills needed for effective citizenship, economic advancement and political equality (National Council on Education and the Disciplines, 2001).

The growth of public concern over the quality of education led to a new era of federally funded research and support for educational improvement programs. Major milestones in this development included the National Defense Education Act (NDEA) of 1958, the establishment of Head Start under the Economic Opportunity Act (EOA) of 1964, and passage of the Elementary and Secondary Education Act (ESEA) of 1965. These laws brought new federal funding to many schools, as well as new requirements for accountability. Recipients of federal funds had to provide evidence of program effectiveness in order to continue receiving support. This greatly increased the importance of achievement testing as student success on achievement tests became established as the primary measure by which the quality of schooling and school improvement programs could be ascertained. At the same time, low achievement, as an

indicator of ineffective schools and programs, became an important problem for schools and researchers to address. This began a new era of government-sponsored research to identify effective teaching strategies and programs to improve the nation's schools.

Since 1965, Title I of the Elementary and Secondary Education Act (ESEA) has prompted thousands of school programs designed to improve reading and mathematics achievement among low-income public school children. Documentation of these programs is an important source of information concerning effective interventions for economically disadvantaged learners (Slavin and Madden, 1989). However, the quality and usefulness of this research varies considerably. For example, in a review of over 400 elementary reading and mathematics programs identified as successful, Slavin and Madden (1989) found only sixteen supported by *convincing evidence of effectiveness*, and these relied on evaluations in regular classrooms rather than actual Title I programs. Most program evaluation studies were determined to be of limited evidentiary value due to short duration, lack of random assignment or matched control groups, unreliable measures, or inadequate elucidation of program features. Other studies on improving achievement face similar quality issues, which has led to newly mandated effectiveness studies under No Child Left Behind (Institute of Education Sciences, 2006). After four decades of research focused on improving low achievement, the U.S. Department of Education's *What Works Clearinghouse* provides information on only 24 middle school mathematics programs, with only six of these identified as meeting evidence standards (What Works Clearinghouse, 2006, 2007).

Interpreting the available research on effective interventions is further complicated by incongruence among the assumptions, variables, definitions, scale,

populations and measures of different studies (Bauersfeld, 1988; Borman & D'Agostino, 2001). Several hypotheses exist concerning what variables are thought to influence achievement outcomes. Different studies have addressed a wide range of factors related to curriculum, instruction, teachers, school environments, and individual students (Crawford, 1989a). Definitions of key variables often do not agree. For example, in studies examining effects of *class size* on achievement, some researchers included data on individual tutoring, regarding this as a class size of one, while others excluded this as a categorically different intervention (Archambault, 1989). Since such distinctions influence findings, a high level of scrutiny is needed even when studies purport to examine the same variables. Moreover, specific delimitations concerning course content, grade level, geographical setting, or population subgroups generally restrict the applicability of findings.

The question of how achievement is measured is also a matter of controversy. Different studies have used different tests, ranging from curriculum and content-based assessments to a variety of broad-based standardized tests. Generally speaking, each approach has advantages and disadvantages. On the one hand, curriculum or content-based assessments offer a higher degree of content validity, due to the closer match between what is taught and what is assessed. On the other hand, standardized tests facilitate broader comparison of interventions across diverse instructional settings. Accordingly, standardized tests have been favored for comparing effectiveness of different interventions along a similar scale (Slavin & Madden, 1989; What Works Clearinghouse, 2006). However, content validity remains an important issue due to differences between the curricula or instructional programs of students measured by the

same test. (Cooley, 1993; Slavin & Madden, 1989). That is, test scores are likely to be higher among students in instructional programs with specific content similar to the test questions.

Additional disagreements relate to how tests are constructed to measure mathematical proficiency. Generally speaking, an achievement test score is intended to reflect the student's level of mathematics proficiency, or mastery, as indicated by successful answers to items addressing specific criteria. The selection of criteria concerning what constitutes mathematics proficiency has a major influence on an instrument's construct validity. According to Schoenfeld (2006), research in recent decades has improved constructs of mathematics proficiency by including higher order problem solving skills, whereas earlier constructs emphasized only factual knowledge and computational skills. This newer framework for proficiency is articulated in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), which recognizes five content standards as well as five process standards with respect to proficiency, which is measured using specific grade level benchmarks. While a new generation of tests now reflects these standards, most interventions are still evaluated using broad-based standardized tests that are based on earlier models of proficiency (Schoenfeld, 2006). As a result, most interventions identified as effective are those that were found to increase students' factual knowledge and computational skills, but may or may not improve complex problem solving. At the same time, programs that do increase complex problem solving and reasoning skills may not qualify as effective based on these measures. Schoenfeld (2006) argues that studies of program effectiveness should use the new generation assessments, such as the

Balanced Assessment (Mathematics Assessment Resource Service, 2000), in order to avoid misleading or invalid findings. He takes issue with the Department of Education's What Works Clearinghouse for favoring evidence from norm-referenced standardized tests without clarifying this issue. This may bias findings of effectiveness towards *Back to Basics* programs that emphasize basic facts and skills (Schoenfeld, 2006).

Finally, an additional layer of controversy relates to whether specific findings are based on norm-referenced or criterion-referenced tests. Since norm-referenced standardized tests base scores on their position in the overall distribution of scores, they do not reflect a clear level of proficiency with respect to established criteria. Instead, scores for identical answers on a test may be higher or lower at different times depending on how everyone else does on the test. Therefore the use of such tests for evaluating student learning and/or program effectiveness may be misleading to the extent that scores reflect overall performance as well as individual performance. Low achieving students may make substantial gains in skills and knowledge, but still receive low scores if higher achieving students improved more. Likewise, students may learn little but appear to perform better if others in the pool do poorly. In contrast, criterion referenced tests (CRT) determine individual scores based on objective standards, or benchmarks, of mastery (Airasian, 2005). While CRTs would seem to offer a more reliable indication of student competency in mathematics, these have also been criticized for setting standards too high (Popham, 2005). Despite these criticisms, norm-referenced tests are still widely used in program evaluation. With regard to program effectiveness, the expectation is that effective programs will improve student performance relative to the norm group mean. Accordingly, if two programs are compared via norm-referenced scores only, it should be

remembered that this indicates only relative success as compared to a norm group.

In summary, achievement testing is an important indicator of program success due to its wide usage, but needs to be interpreted with caution. Achievement tests do not always provide an accurate picture of student learning and program success. Many other factors can influence student test scores. In addition, norm-referenced tests provide only relative measures of student success that may not reflect actual gains in learning. Accordingly, multiple measures of student learning are needed to validate achievement outcomes and overcome these limitations.

Programs & Interventions to Improve Success in Mathematics

Recognizing the limitations and controversies associated with the use of student achievement data as a primary measure of student learning and program effectiveness was necessary in order to understand the findings of research on effective instruction and programs. As noted previously, this includes a large body of literature reflecting several decades of Title I programs and related research, plus many other studies. These intervention strategies can be classified into several generally descriptive categories including supplemental instruction, parent and community involvement programs, instructional innovations, and technological innovations.

Supplemental Instruction

Supplemental instruction includes a variety of programs designed to supplement students' regular mathematics classes, such as extra tutoring, after school programs, resource room assistance, summer programs, or other supplements to regular instruction. Recent studies by Carroll (2004), McLaughlin (2000), and Brown (1999) explore several models of supplemental instruction programs designed to improve student learning

among low achieving students. In addition, most Title I programs also fall under this heading due to statutory requirements that these interventions supplement, or add instructional time, rather than replace existing instruction (Slavin, 1989). This includes a variety of pullout programs, before or after school programs, and use of aides to increase instructional time for low-income students.

While there have been some studies that demonstrated the effectiveness of Title I programs as an intervention for low achieving mathematics students, these are few in number. For example, Carter (1984) showed in the *Sustaining Effects Study* that first through third grade students who participated in Title I programs made greater gains in achievement than similarly disadvantaged students who did not participate in Title I. However, this general finding did not address the characteristics of effective programs beyond a general description of Title I delivery models. Crawford (1989b) took a different approach in the Oklahoma City Study, which used a process-product research model to identify characteristics of effective Title I programs in both elementary and middle grades. This study found that higher mathematics achievement in the middle grades (5-8) was associated with judicious use of praise, higher numbers of student-initiated contacts, and higher numbers of private contacts between teacher and students, as well as longer periods. At the same time, mathematics achievement was found to correlate negatively with the proportion of questions students answered correctly and the frequency that teachers responded to incorrect answers by asking a different student. These findings suggested that effective mathematics interventions for older students were those that provided more opportunities for student questions and individual tutoring (private contacts), and included questions or problems that were challenging for students

with sustained teacher-student interactions following incorrect responses.

Most other Title I research has been less conclusive. In a meta-analysis to examine the general effectiveness of Title I programs since 1979, Borman and D'Agostino (2001) reviewed 150 studies and found only two that included control group comparisons and fifteen that provided norm-referenced evidence of program success. While these seventeen studies indicated that Title I has been modestly effective in improving student learning overall, the small number of program studies with adequate information for analysis led the authors to conclude that no summative evaluation of the program was likely to be accurate. Most evaluations of Title I simply did not provide sufficient documentation to allow any accurate estimates of program effectiveness.

Carroll (2004) examined a curriculum enrichment program for at-risk 9th graders, entering high school below grade level in mathematics. The program consisted of an after school tutoring and a summer program. This research compared average student achievement between three distinct subsets of program participants and a control group, as indicated by mathematics section scores on the Texas Assessment of Academic Skills (TAAS) and course grades in Algebra I. The three participant groups included (a) students who participated in after school tutoring only, (b) students who participated in the summer program only, and (c) those who did both. An analysis of covariance (ANCOVA) showed that students in the summer program performed somewhat better than the other groups on the achievement tests but not on course grades, with no other substantial differences among the groups. While the summer program therefore seemed promising as a means to increase achievement, it was not clear what program elements led to this outcome. Results were attributed to a complex of factors that distinguished the

summer program from the others, including mandatory attendance, study skills components, and course credit. It was also not clear why students in both programs did not show the same gains as students who participated in the summer program only.

McLaughlin (2000) looked at supplemental mathematics tutoring offered during the school day. In this case, a group of mostly Hispanic, at-risk tenth graders was enrolled in an extra period of *mathematics lab* as an elective to supplement their regular mathematics class. Key features of the mathematics lab course included maximum class size of 15, three adults scheduled for each period, access to manipulatives, games, peer tutoring, student-centered activities, informal discussion, focus on standardized test mastery, and attendance based credit. The program was evaluated using achievement data from TAAS mathematics scores and graduation rates. This analysis consisted of a post intervention t-test, with an analysis of variance (ANOVA) comparing gender and race subgroups. Results indicated 5% higher test scores among female Hispanic students who participated in the mathematics lab, with 90% confidence interval. While the mathematics lab appears to be a promising strategy for one group of minority students, it was not clear which features of the mathematics lab contributed to this result. More research is needed to identify what features of the intervention contributed to success for this group and to verify the consistency of this finding.

Brown (1999) took a somewhat different approach, designing an after school and summer enrichment program for at-risk elementary and middle school students and their parents. The program, “Peer and Group Enrichment” (PAGE ONE), was designed to improve student achievement and attitudes towards mathematics, while encouraging parent involvement. Program evaluation was based on quantitative student achievement

data and attitude data using post-treatment measures. Standardized achievement data was collected using the Metropolitan Achievement Test (MAT) and South Carolina's statewide Basic Skills Assessment Program (BSAP), while an attitude survey was developed by the researcher during the course of the study. Analysis of achievement data was conducted using an analysis of covariance (ANCOVA) to compare mean test scores, while a t-test was used to compare attitude ratings based on means of ordinal data. The study found no experimentally consistent improvement among either mathematics achievement or attitudes of participants.

Parental Involvement Programs

While Brown's study included some attention to parental involvement, the primary intervention was supplemental instruction. Other research has focused more exclusively on engaging parents as allies in the effort to improve achievement and promote positive attitudes about mathematics and schooling in general.

Mendoza (2003) developed an intervention to involve parents in mathematical learning through a series of evening workshops for Hispanic ninth graders and their parents. The weekly workshops promoted informal discourse about mathematics by engaging parents and students through puzzles, games, presentations, and sharing experiences. This was a means to overcome myths and misperceptions about mathematics, and address issues like anxiety or perceived cultural barriers. The study used a mixed methods, action research design that employed periodic surveys of participants to evaluate shifts in attitudes and teacher observation to assess improvements in student success. Survey results showed improvement in attitudes towards mathematics among both students and parents. The teacher/researcher also reported that students

participating in the program became more successful learners due to greater interest and positive attitudes resulting from the program.

Mendoza's study suggests that the use of informal discourse, engaging activities, and a culturally sensitive approach to learning had a strong impact on both students and parents. However, the connection between the shift in attitudes and improved student achievement is not clearly established. A better research design is needed to isolate possible teacher/researcher bias and provide a clearer measure of possible achievement gains attributed to the program. With regard to discourse, this intervention is interesting because it builds connections between the different languages and cultures of home and school. This may help culturally diverse students feel more accepted for who they are at school. At the same time, the broader, informal use of mathematical language at home may contribute to students' mathematical fluency and understanding. In contrast to efforts to build smaller learning communities in schools, this approach influences students' existing communities to take up mathematical ideas.

Instructional Innovations

Other research has focused on instructional innovations, where the focus is on using alternative curriculum materials or instructional styles in students' regular mathematics courses rather than adding extra programs. Lang (2001), for example, studied a two-week algebra unit designed to help at-risk students develop *self-instructional* strategies for solving word problems. In this case, the students' regular mathematics class became the site of an intervention targeting special needs students, English as second language (ESL) students, and other students at-risk for failure. The research design employed a treatment and control group to compare potential gains in

students' problem solving strategies and success, as well as attitudes. Measurements consisted of a ten-question word problem quiz given before and after the treatment and again later to assess retention, plus a Likert survey was used to assess student attitudes before and after the intervention. While this intervention did uncover experimentally consistent results¹, the outcome raises the question of whether such a short duration intervention is likely to generate substantial gains in learning.

Research on smaller learning communities also falls under this heading, since changing the classroom or school environment modifies the form of instruction in students' regular classes. Hall (2004) explored the impact smaller learning communities on student performance in mathematics. In this study, smaller learning communities were formed among *at-risk* students entering high school by block scheduling three core curriculum classes to promote a sense of community and more personalized learning environment. Participants were not selected randomly, but assigned to the program on the basis of academic status as *at-risk*, meaning "below grade level in one or more core subjects" (p. 12). Program evaluation used qualitative data and quantitative comparisons between program participants and a non-equivalent control group consisting of similarly *at-risk* students at a different school in the same district. Quantitative measures included students' mathematics test scores on the norm-referenced Stanford 9 (SAT9) achievement test and the criterion-referenced Arizona's Instrument to Measure Standards (AIMS) test, course grades in beginning algebra, course placements after three semesters, numbers of days absent, and numbers of discipline referrals. Qualitative data concerning student perceptions of their learning experiences was collected through three focus group

¹ Lang (2001) reports ANOVA results for both types of data, indicating a flawed analysis in which ordinal survey data was averaged.

interviews of students who participated in the intervention. Teacher interviews were also conducted and additional observations were made concerning program structure.

Results obtained from six analyses of covariance (ANCOVA) indicated that the smaller learning communities had mixed effects on student achievement and positive effects on other learning outcomes. Students who participated in the intervention ($n = 87$) had lower mean test scores on the SAT9 mathematics test by 10%, but higher mean test scores on the AIMS mathematics test by 4% as compared to their control group peers ($n = 89$). The intervention had positive impacts on average course grades (+ 4%), math placements (+ 1 semester), attendance (- 4 days), and discipline referrals (-0.6). In addition, the qualitative analysis of student interview data found that students expressed a strong sense of community, engagement, and personalized learning experiences. This was indicated through student statements that teachers understood their needs and were able to address these effectively by adapting instruction appropriately. Students also indicated that teachers cared about them as individuals as well as academically. However, the selection of focus group participants and influences of interaction between group members were not specified. Overall, this study helped validate several alleged benefits of smaller learning communities, but the specific findings concerning mathematics achievement were mixed. Although these results were not conclusive, they suggest that school designs that meet students' emotional needs like caring, belonging, and potency may also contribute to improved student performance in mathematics.

Technological Innovations

Finally, another important category of intervention research examines the use of instructional technology to supplement or modify mathematics instruction. This includes

use of adaptive learning technologies, tutoring software, and other computer based or technological innovations. An example of this type of research includes Shuck (2003), who examined the affects of Internet use in algebra classes among eighth grade Latino students. This study found that Internet use had little impact on learning basic skills but appeared to have positive affects as a research tool for more complex applications.

Another important area of research on technological innovations has been the development of Computer Assisted Instruction (CAI). For example, White (2005) explored the use of handheld computers in collaborative problem solving context among middle grade math students. This qualitative study provided an in depth picture of how students interacted with computers and one another during a five week summer program. The study found that the technology led to different gains in achievement depending on how well students collaborated in their groups.

Summary: Research Addressing Low Achievement in Mathematics

Research on strategies to improve mathematics achievement among at-risk learners can be grouped into four basic types: (a) supplemental instruction, (b) parental/community involvement, (c) instructional innovations, and (d) technology. In comparing the research on mathematics interventions to that addressing the more general intervention strategies discussed previously, there are some areas of overlap in the qualities that appear to help students overcome deficiencies. These include efforts to bridge cultural and linguistic differences through parent involvement, efforts to make teachers more accessible through small class sizes, and efforts to build smaller learning communities.

Of the interventions considered thus far, those that appear most effective are

smaller learning communities (Hall, 2004) and parent involvement workshops (Mendoza, 2003). It is notable that both of these interventions involved discourse rich treatments; in one case, a responsive sense of community developed between students and teachers, and in the other, a forum for sharing stories and experiences among parents, students, and teachers. Both of these interventions also involve elements from the list of recommended design features for intervention programs, in particular, a strong sense of community and teachers who exhibit warmth and care.

Two other interventions that appear promising are the mathematics lab intervention (McLaughlin, 2000), which improved test scores in one subgroup, female Hispanic students, and the summer enrichment program examined by Carroll (2004), which was also found to improve student test scores. Again, the mathematics lab environment described by McLaughlin also featured informal discourse and a low student to teacher ratio that facilitated a sense of community and personalized learning environment. In the case of the summer enrichment program, factors influencing the positive outcome were less clear and may have been influenced by other moderating factors. Overall, it appears that there is some evidence to suggest discourse is an important feature of successful interventions for at-risk learners in mathematics.

Discourse in Mathematics

While discourse plays a part in several mathematics interventions, none of these examined a direct relationship between discourse and success in mathematics. In order to understand this relationship more fully it is necessary to look beyond the research on interventions for at-risk students, to the more general category of research on mathematics education for the general student population. Another growing body of

literature is concerned with the role of discourse in mathematical understanding and achievement.

Instructional Standards

In 1991, the National Council of Teachers of Mathematics (NCTM) published *Professional Standards for Teaching Mathematics* that identified discourse as a key feature in three of the six standard areas. This included *Standard 2: The Teachers' Role in Discourse, Standard 3: Students' Role in Discourse, and Standard 4: Tools for Enhancing Discourse.* This brought new interest to research on discourse-based teaching as the research community took notice that these standards, or recommendations, while seemingly reasonable, lacked grounding in scientific evidence. Accordingly, the number of studies examining discourse has increased dramatically since the early 1990s.

The quality of discourse called for in the NCTM teaching standards is substantially different than traditional teacher led questioning. According to NCTM (1991) guidelines,

Students should engage in making conjectures, proposing approaches and solutions to problems, and arguing about the validity of particular claims. They should learn to verify, revise, and discard claims on the basis of mathematical evidence and use a variety of mathematical tools. Whether working in small or large groups, they should be the audience for one another's comments - that is, they should speak to one another, aiming to convince or to question their peers. Above all, the discourse should be focused on making sense of mathematical ideas, on using mathematical ideas sensibly in setting up and solving problems. (p. 45)

Thus, student discourse in mathematics should expand and reinforce students' understandings of important concepts and problem solving strategies.

Here, discourse is conceived broadly as a vehicle through which students can gain insight into how a procedure works, pose questions to peers, and compare their own perspectives to those of others. In other words, discourse is a vehicle for constructing knowledge; by using concepts interactively, the concepts themselves become clearer and more defined through the practice of relevant language. A classroom atmosphere that supports collaborative inquiry among students can also help students to bridge the difficulty of applying concepts to new problems and situations. As students become more willing to conjecture and try out potential strategies without fear of being wrong, they begin to participate in a process of mathematical discovery. This experience in turn enables students to understand mathematics as a field of exploration and discovery, rather than rote memorization and right answers.

In *Talking Mathematics: Supporting Children's Voices*, Corwin, Storeygard, and Price (1996) examined the role of conversation in teaching mathematics in the primary grades. They explained the benefits of discourse and provided examples of classroom discourse to clarify the difference between leading questions that encourage students to supply the *right answer* and the type of conversation that encourages students to explore mathematical ideas and solve problems creatively. According to these authors,

Participating in mathematical conversations is central to developing strong mathematical ideas. Talking allows students to compare their methods and discuss their ideas and theories with their classmates. Classmates'

questions or counter assertions often force a student to examine her own mathematical concepts or ideas. When students begin to comment on each other's methods and ask each other questions, confusion is clarified.

Expressing their assumptions in the context of a conversation helps students articulate and refine their ideas. (p. 2)

Proponents of multicultural education also have endorsed the use of informal discussion to engage students. Dalton (1998) reports on a separate set of five standards for effective pedagogy developed by the Center for Research on Education, Diversity and Excellence (CREDE) to reflect research on effective models for teaching minority students and other at-risk groups. Here, too, student discourse figures prominently as a means to support learning with opportunities for joint inquiry, meaningful learning and exploratory conversation. Hilberg, Doherty, Dalton, Youpa, & Tharp (2000), considered the CREDE standards in the context of mathematics, and agreed that mathematics teachers in culturally diverse settings should try to engage students in joint inquiry, promote the use of language, and make connections to students' lives. Again, discussion is regarded as a key classroom element that furthers all of these objectives. Informal discussion can help students develop vocabulary and facilitate peer tutoring to bridge cultural differences. Other proponents of multicultural education, such as Banks (2002) and Nieto (2004) also recognize discourse as an important tool for recognizing diverse perspectives and interpretations of knowledge.

At the same time, there are also challenges associated with implementing discourse. Students who are not accustomed to discursive learning may be confused by the different expectations associated with this form of instruction. Cazden (1988) points

out that both classroom norms and peer status relations may affect the way students interact with one another. On the one hand, established classroom practices and norms shape students' expectations. If students are used to listening to lectures and completing worksheets, they may be at a loss as to how to engage in group activities and discourse. At the same time, students who have more status among their peers may expect, or be expected by their peers, to take leadership roles in discourse. This may lead to balance of power issues within groups where the most powerful voices are not necessarily the most mathematically competent. While these types of factors do not make student discourse impossible, it may take time for classrooms and students to negotiate new expectations and norms for discursive learning.

Research on Implementing Discursive Teaching

While discourse promises many benefits, it also poses a substantial challenge for traditional teachers who may not have experience with discourse based pedagogy in mathematics. This obstacle to implementation raises other important research questions. Manouchehri (1997) argues that teacher education programs must be reformed to reflect the new 1991 NCTM teaching standards; "A substantially large body of research indicates that if teachers are to choose to teach according to the visions of mathematics reform, they must be convinced of their value and have exposure to similar learning environments firsthand as learners" (pp. 204-205). That is, if teachers have experienced mathematics as a collaborative inquiry, they have a working model of what it means to support this in their own classrooms. In contrast, teachers without this experience may be at a loss, perceiving neither the methods nor the potential benefits of a discursive teaching style. This has led to research examining the benefits of incorporating discourse

and collaboration into teacher education programs (Manouchehri, 2002).

Li (1998) addresses the difficulty faced by teachers who are struggling to incorporate discourse into their teaching styles by developing a framework to help them evaluate and structure the discourse in their classrooms. Li's main concern is with "epistemic" discourse, or *knowledge producing* discourse, which she addresses from the standpoint of a constructivist theoretical framework. She also delimits her study to address *whole class* discussions. Drawing on the field of communication, she takes a general model of "epistemic rhetorical discourse" (Li, 1998, p. 33) and adapts this for specific use in mathematics classrooms. The initial model developed by Cherwitz and Hikins (as cited in Li, 1998) identified five components of epistemic discourse in general: differentiative, associative, preservative, evaluative, and perspectival. These components are identified in the speech of discourse participants, and are thought to be necessary for individual and collective construction of new knowledge.

After using the Cherwitz and Hikins model to analyze discourse in three middle school classrooms, Li (1998) developed a revised model that speaks more directly to the concerns of mathematics teachers. Li proposes three guidelines for mathematics teachers working to generate productive whole class discourse:

- A. Make explicit distinctions and connections among mathematical concepts.
- B. Keep mathematical ideas alive.
- C. Explicitly negotiate mathematical meaning. (p. 86)

Here, the five original components are still present, but combined and restated as practical objectives. This practical framework is proposed for use by teachers in evaluating and planning lessons that use whole class discourse to guide students in

constructing mathematical knowledge.

Other research has taken a phenomenological approach, observing discursive teachers to identify effective strategies for engaging students in discourse. Based on observations of a fifth grade mathematics class, O’Conner (2002) describes several key strategies used by the teacher to facilitate a whole class discussion about whether every fraction can be expressed as a decimal. These included posing questions to solicit conjectures, challenging robust conjectures, encouraging exploration of examples, modeling confusion, and reviewing key points or examples. This work provides teachers with additional examples of successful strategies and ideas to draw on during class discussions. Identifying different types of teacher contributions and questions, and how these move the discussion in different directions, may help teachers accomplish the three general objectives outlined by Li (2002). Similarly, Forman and Ansell (2002) observed a third grade classroom to identify techniques of *discussion orchestration* and improve understanding of cultural factors on classroom discourse.

This type of research has also identified potential problems that occur during classroom discourse. For example, teachers may inadvertently favor students whose opinions or strategies resemble their own. In a study of a sixth grade mathematics classroom, Forman, McCormick, and Donato (1998) illustrated how a well-intentioned teacher promoted one student’s problem solving strategy over another’s that was equally valid, but different from her own. In this instance, the teacher failed to recognize the potential validity of a student’s proposed strategy, and consequently failed to acknowledge or encourage further development of that student’s thinking (Forman, McCormick, & Donato, 1998). This type of interaction can leave students confused,

unsupported in their efforts, and with the mistaken impression that the goal is to produce an answer the teacher wants to hear rather than develop their own mathematical thinking. In contrast, a more skilled teacher-facilitator could have responded by asking additional questions to develop and explore the student's ideas, possibly enriching the classroom discourse with a comparison of alternative problem solving strategies.

Role of Discourse in Learning

While some studies offer support to teachers who are inexperienced with discursive instruction, these often do not address the question of whether this is an *effective* teaching strategy in terms of student achievement. While the NCTM's *Professional Standards for Teaching Mathematics* (1991) clearly recommended discursive instruction, this is no substitute for a solid evidence base grounded in scientific research. In order to become recognized as a *best practice* or *effective practice*, research is needed to connect discursive instruction and mathematical achievement. Under the No Child Left Behind Act (2002), raising student test scores is an imperative for many districts and educators facing the stigma of being labeled *failing schools*. Accordingly, discourse based teaching is unlikely to be embraced widely unless it can be shown to correlate with improvements in student achievement. Accordingly, this raises a challenge to researchers to bridge this gap between professional standards and effective practices. Researchers like Moore (2000), Larriva (1998), and Kysh (1999) have begun the work of understanding how discourse relates to mathematics achievement.

The approach taken by Moore (2000) is based on in-depth interviews of successful, high achieving African American high school students. Moore asked five recent high school graduates about their experiences using discourse in mathematics and

whether they perceived this as an important element contributing to their success. Four of the five students interviewed indicated they did regard classroom discourse as an important factor in their mathematics achievement. Since a high proportion of African-American students are identified as at-risk students, this finding supports further investigation of discourse as an intervention strategy among similar students.

Another qualitative case study by Larriva (1998) utilized a micro-phenomenological framework, focusing on small group interactions of two Hispanic female high school students. This study described and contrasted the diverse communication styles of these students in relation to their academic success. The analysis was framed in terms of several discursive modes or features thought to affect the success of student discourse: politeness, mitigation, aggravation, intonation, turn-taking, classroom identity, and classroom goals. While this study was quite narrow in scope, it did provide some useful insights concerning the influence of student identity and social goals on students' discursive strategies displayed in group activities. In particular, students with more assertive behaviors tended to control discussions, sometimes overwhelming sound mathematical contributions from others. Students' perceptions of themselves and one another appeared to influence who got listened to and who took charge of the situation. Larriva concludes that, "teachers are in a position to influence participation by establishing classroom norms and expectations that will guide students to interact in more favorable ways. Situations for interaction can be created that open up opportunities for students... to contribute productively and to perhaps be recognized more readily as competent" (p. 136). This research suggests that students who are perceived as being at-risk or less competent by their peers may be disadvantaged as

participants in student discourse.

In contrast, Kysh (1999) conducted a phenomenological study to provide an in-depth look at student discourse during the course of a year long Algebra I class structured for small group work among students. At various times during the school year, tape recordings were made to document the discourse within teacher-student and student-student groups as they worked to solve mathematical problems. Dozens of such tapes were then transcribed and coded to compare the mathematical content and characteristics of student talk under different circumstances. Factors examined in the analysis included students' use of mathematical language, types of mathematical language, purposes of questions, variation related to problem type, individual learning styles, group interaction styles, teacher interaction styles, student written work, and class rank. Kysh also used descriptive statistics to analyze frequency of participation and types of language usage in different groupings, and explored possible correlations between class rank and use of discourse.

Kysh (1999) found that small groups greatly increased opportunities for participation among students. Students also used more mathematical language and gave more explanations in student-student group interactions than when they queried or responded to the teacher. She also found that teacher-student interactions tended to fall into a teacher-centered pattern of question-answer-verification (Q-A-V), also known as inquiry-response-evaluation (I-R-E) (Mehan, 1979), where the teacher directs classroom discourse. When students spoke among themselves, a wider variety of interaction patterns emerged, all of which had different strengths and weaknesses. Interestingly, higher ranked students did not use mathematical language any more or less than lower

ranked students, although the purpose of statements often varied. One element observed in many groupings was a high frequency of repetition, as students restated problem solving strategies for themselves. It also appeared that fragmented and incorrect usages were sometimes related to early stages of concept development, when students were beginning to grasp a new concept but had not yet mastered the language needed to describe it correctly.

Kysh (1999) illustrates many of the complex elements of classroom discourse and develops a multi-faceted model for analyzing discourse in the classroom setting. This study also provides some support for the NCTM claims concerning the instructional value of student discourse. However, since the study was limited to one classroom in one Sacramento high school, these results are not generalizable to other populations. Accordingly, more research is needed to verify whether similar results will occur among a wider variety of classroom settings, students, and teachers. In particular, this study did not address discourse among low achieving students. It is possible that less advanced students would exhibit different or less productive interaction patterns.

New Theoretical Frameworks

Other current literature on this topic is focused on broader, theoretical questions related to the use of a discursive framework in educational theory and research design (Lerman, 2002; Van Oers, 2002). This has led to a fresh look at discourse as the medium through which mathematical understanding and knowledge is constructed or developed in the individual through social and cultural interactions. In contrast to the classic Piagetian constructivist model, where individuals construct knowledge through experience, this work draws on a social constructivist perspective explored in the work of Vygotsky

(1934/1962, 1978), Wittgenstein (1922/1994, 1953/1994), and others (Lerman, 2002; Sfar, 2002; Van Oers, 2002). In this model, knowledge is viewed as inextricably social in the sense that it is bound up with language and communication and develops in the individual in response to social interaction and purposes. This shifts cultural and linguistic context to the forefront of knowledge and meaning construction processes.

Vygotsky argued that language is the primary medium of learning and knowledge. In his work, *Thought and Language* (1934/1962), he described the learning process as fundamentally social and rooted in language. That is, words and meanings are usually not made up out of thin air, but reflect the existing usages and meanings conveyed to a child by the people around her. In his research he observed that children learning new information would often mutter to themselves, or think aloud as they figured out a problem. Vygotsky conceptualized this as an intermediate step between gaining words from social interaction and fully internalizing their meanings as thought. Even in the case of nonsense speech, this was taken to reflect a semi-internalized thinking process with only some words being uttered audibly. This led to a larger conceptualization of learning as consisting of stages of internalized language meanings.

Vygotsky developed the concept of *zone of proximal development* to explain the relationship between learning and concept development (Vygotsky, 1978). The initial learning of a concept or word meaning was the beginning of concept development, which entailed a complex social process of connecting the new meaning to other meanings that have already been internalized and developed. The zone of proximal development speaks to the difference between fully internalized concepts and what concepts may potentially become internalized with the help of teachers or peers. Already internalized meanings

form the basis for understanding an array of more complex and dependent meanings, like a plateau from which one can reach the next level. The next level of conceptual complexity is thus within the zone of proximal development made accessible by the student's prior learning. Scaffolding, on the other hand, is the process of helping learners recognize and understand the more complex meanings within their zone of proximal development. Scaffolding may consist of formal lessons or informal social interactions that convey new meanings.

In contrast, Wittgenstein provides a more philosophical analysis of language and learning. In his early work, often described as logical positivism, he argued that the logic of language delimits what knowledge is possible. His *Tractatus Logico-Philosophicus* (Wittgenstein 1922/1994) consisted of a set of propositions defining all statements as assertions about objects, or concepts. These statements can be true or false. In this view, all knowledge can be reduced to true statements of facts that describe some object, and anything else is not knowledge. In other words, the structure of what it is possible to say delimits the range of what it is possible to know.

Later, however, Wittgenstein recognized language as far more complex than factual, *what it is* statements. Rather than focusing only on the picture metaphor, he also explored the use of language as a signifier, or cue, to act in a certain way. For example, in his *Philosophical Investigations* (Wittgenstein 1953/1994), he discusses how the names of different shapes could be used in the practice of building to mean *bring me a block*, or *bring me a cylinder*. Accordingly, the actual meaning of a word is not just a matter of describing facts, but also depends on usage within particular contexts. Wittgenstein described these contexts as *language games*, reflecting a particular set of usage rules and

meanings corresponding to various purposes. In this view, understanding new concepts and meanings may be complicated by overlapping usages reflecting different language games or unfamiliar language games.

The views of Vygotsky and Wittgenstein both have informed recent research on the role of student discourse in mathematics. Sfard (2002) discusses the need for a communicational research framework in mathematics education to provide knowledge that reflects a social discursive theory of learning. Whereas current research often reflects a theory of learning as concept acquisition, whether by construction or imprinting, this approach is limited because it treats the concepts themselves as fixed, or invariant. In contrast, a discursive theory of learning recognizes concepts as socially constructed shared meanings that admit of variability, depending on diverse social contexts. This may help explain how students arrive at misunderstandings of concepts as the result of insufficient or confused communication. Here, language takes on a primary role as the vehicle of social interaction which both defines and conveys shared meanings, as well as imperfectly shared meanings. If concepts are fixed, then the context of learning should not affect what is learned. But if knowledge is dependent on socially shared meanings, access to the social interaction in which these meanings are shared is critically important to what is learned. Sfard argues that a discursive conceptualization of learning will provide new insights about the role of discourse in student learning.

According to Van Oers (2002), the mathematics classroom is a *culture of practice* in which the many cultures that shape each individual come together with a culture of formal mathematics conveyed by the teacher. Here culture is understood very broadly to identify any set of shared meanings common to some group of people. This could be a

traditional culture delimited by geographical areas or ethnicity, or some other small or large association of individuals with shared symbolic meanings and practices. In this context, the mathematics classroom has its own developed culture of usages, practices, routines and norms that help define what takes place there and what counts as mathematical activity. The larger body of knowledge in the field of mathematics also reflects a culture of meanings and practices developed over many centuries and shared by mathematicians, students and teachers. At the same time, each individual student and teacher is affected by many overlapping cultures that shape their existing knowledge and understanding of symbolic meanings. The process of learning in the classroom therefore involves building connections between existing cultural understandings and the new information provided through instructional activities. This means that difficulties in learning may reflect overlapping usages or meanings that have not been clarified. Van Oers argues that classroom discourse is not just the dialogue among students, but rather a *polylogue* of all the historical voices that have contributed to the mathematical ideas or practices under consideration. Accordingly, a broad recognition of these diverse cultural and historical influences is needed to clarify mathematical meanings and support developing cultures of shared practice at the classroom level.

Lerman (2002) also emphasizes the influences of culture and prior experiences on such factors as classroom norms and expectations, interpretations of meaning, and instructional styles. Lerman proposes a *cultural discursive psychological* framework for mathematics education research that examines student learning as a culturally situated progression in time. This treats Vygotsky's notion of the zone of proximal development (ZPD) as culturally and temporally dependent. More generally, both Lerman (2002) and

Van Oers (2002) argue that a complex of diverse experiences and cultural influences affect each student and these need to be taken into account to obtain a true picture of how learning, or meaning construction, takes place in mathematics classrooms. Similarly, teachers are also influenced by prior experience and cultural factors, as illustrated by Forman and Ansell (2002) who describe how one teacher's past experiences help explain her instructional choices, even many years later.

Attention to the role of language and cultural influences on mathematics learning has led to new critiques of traditional educational research. Bauersfeld (1988) describes this as a scientific paradigm shift (Kuhn, 1970, as cited by Bauersfeld) in educational research. Whereas traditional educational research has focused on discrete elements of “‘didactical triad’ of subject matter-student-teacher” (Bauersfeld, p. 31), the newer socio-cultural paradigm recognizes the classroom setting as a dynamic intersection of these elements affected by individual differences and a constant potential for multiple perspectives to influence interpretations of classroom language and instructional activities. Accordingly, scientific studies need to take specific classroom settings as micro-cultures of meaning construction in order to understand or evaluate student learning outcomes.

Socio-Discursive Research Tools

In the effort to develop new research designs that address mathematical learning as a social process of meaning-construction, Sfard (2002) and Kieran (2002) have explored two new forms of qualitative analysis. One method, called *focal analysis*, compares students' statements, or stated focus, to what they are actually doing, or attended focus, as they solve problems. Sfard (2002) also addresses “‘intended focus”

(p. 34) which she derives from the theory that all communication implies intent. This type of analysis enables students' word usage and conceptual development to be compared to their implementation of problem solving strategies and skills. The second method, *pre-occupational analysis*, uses *interactivity flow charts* to map out the response patterns of student statements. This technique maps students interaction using directional arrows to indicate who is speaking and to whom, and whether the statement is addressed to a previous comment or initiating a new direction. The use of solid and dotted arrows adds an additional layer of information by identifying utterances as object-level or non-object level. Object level utterances are defined as those "related to the declared goal of a given activity" (p. 38). This method of analysis provides a graphic overview of student interaction patterns and material contributions to the discourse. Both of these methods were explored with reference to dyadic discourse between pairs of eighth grade mathematics students.

While Sfard (2002) introduces these techniques in the context of developing improved analytical tools for the study of classroom mathematical discourse, she also illustrates difficulties that inhere in some student pairs. In her example, two students with disparate mathematical abilities displayed a lopsided interaction in which one student led the discussion as the other tried to follow along, but appeared to be increasingly lost and confused. Kieran (2002) exhibits similar pairings, among others that seem more balanced. The ineffectiveness of some such pairings raises important questions about the relationship between discourse and achievement. This is especially true in Kieran's study, where some individuals performed poorly on individual work subsequent to ineffective pairings. While these studies are narrow in scope and limited to dyadic

discourse, any such indication of lower achievement after discourse is cause for concern. Additional research is needed to understand how these problems can be avoided or corrected when they occur. In general, the interactivity analysis is helpful in identifying whether student pairs are well matched or characterized by strong-weak interaction patterns. Additional research is needed to discover if more effective grouping practices can improve learning outcomes in these situations.

Summary: Discourse in Mathematics

Overall, it appears that many researchers are beginning to look seriously at the role of discourse in mathematics education, largely in response to the instructional standards propagated by NCTM. While some authors have addressed the rationale for using discourse, others have focused on challenges related to implementation. Still others are developing methods for analyzing classroom discourse to understand how it works and how it compares to other instructional methods. Despite this recent surge in activity, most of this work is in preliminary stages, with a great deal of work remaining to refine analysis tools and verify results across diverse classroom settings. Moreover, very little attention has been given to the use of discourse with low achieving or at-risk populations. With the exception of Moore (2000), most of the studies described above were set in classrooms where students were at or above grade level and already accustomed to the expectation of participating in classroom discourse. Accordingly, more research is needed to consider the use of discourse with at-risk populations and the special challenges or opportunities this may offer.

Chapter Summary

While a considerable body of research has addressed the needs of at-risk students, relatively little has focused on interventions for low achieving students in mathematics. We have some knowledge about the characteristics of at-risk students and what types of strategies appear to help them reconnect and succeed, but applying this within the field of mathematics education remains challenging. Available research on interventions for at-risk learners in mathematics suggests that discourse is an element in some effective interventions. However, these studies have not focused on discourse explicitly and other factors may explain the successful outcomes. Meanwhile, another growing body of research is focused on discourse in mathematics, but has not addressed its use with at-risk students. Accordingly, additional research is needed to bring these threads together to examine whether discursive instruction is effective as an intervention strategy for at-risk students. The present study is an effort to fill this knowledge gap.

CHAPTER THREE

METHODOLOGY

This study examined whether the use of teaching methods that provide opportunities for student discourse improve student learning in mathematics among low achieving high school students. A mixed methods case study design was used to compare outcomes between a treatment group of students in classrooms where student discourse was a key component of instruction and a control group of students in classrooms where student discourse was not a key component of instruction. This was facilitated by using an intervention strategy that employed role play scripts as a means to promote productive mathematical discourse among students in the treatment group classrooms. Quantitative and qualitative methods were used to compare learning outcomes, including achievement tests scores and problem solving success, as indicated by student performance on constructed response items. In addition, student attitudes were considered as a potential intervening variable affecting other learning outcomes. This research was undertaken to evaluate whether the use of student discourse is a promising strategy for improving learning among low achieving students in mathematics.

Research Design

This study combined qualitative and quantitative components in a mixed-method case study design. The central question concerning whether classroom discourse can lead to improved success in mathematics among low achieving learners was addressed through a comparison of two sets of classrooms, a treatment group that used activities to promote student discourse and a control group that did not. The primary analysis of these

classrooms used qualitative methods to describe the unique instructional features and characteristics of student discourse in each setting. This established the context for a situated understanding of the student learning outcomes in each setting. The specific treatment was then evaluated through multiple secondary analyses addressing quantitative measures of student achievement and problem solving skill, student attitudes towards mathematics, and student perceptions of the discourse activities used in their classrooms.

Quantitative methods were used to measure and compare student achievement and problem solving skills between the two sets of classrooms to address the effectiveness of student discourse as an intervention to improve student learning. A survey instrument was also used to identify whether the use of student discourse appeared to influence student attitudes towards mathematics and to identify any attitude differences between groups that may have influenced the observed or measured learning outcomes. Student interviews were also conducted after the intervention to provide an additional layer of direct feedback from students, to address their perceptions of student discourse activities and to provide an additional source of validation for the other analyses. In general, qualitative components of the study provided a source of internal validation and meaning clarification for the quantitative findings concerning achievement, problem solving, and attitudes.

Concurrent Triangulation Strategy

This study used a concurrent triangulation strategy in which collection of both qualitative and quantitative data was used to overcome limitations involved in using either method exclusively, while also providing a source of internal validation for

findings (Creswell, 2003). In this case, neither quantitative nor qualitative methods alone could have provided a complete picture of the relationship between discourse and achievement. While discourse has been studied using primarily qualitative methods, achievement has been evaluated using quantitative measures of student success. Accordingly, both types of methods are needed to relate discourse and achievement. As noted by Creswell, this poses challenges for interpreting and comparing the data drawn from distinct frameworks. In order to bridge this methodological difference, one must take care in the interpretation phase of research to identify any conflicting assumptions, and conceptual gaps or overlap between different frameworks. In this case, a cultural perspective was adopted to acknowledge the internal consistency of different research frameworks without insisting that one perspective is correct or supersedes another. That is, one may take the view that specific questions determine which view is necessary or sufficient in a given situation. Multiple perspectives are needed to understand complex issues like teaching and learning.

Visual Model

The flow chart, Figure 1, represents the basic design of this research. Quantitative components include pre-treatment and post-treatment phases of data collection for both student groups. Qualitative observations and descriptions of classroom interaction took place during the treatment phase of the study, with follow-up interviews conducted during the post-treatment phase of data collection.

Qualitative Components of the Study

As noted, qualitative components of this research were used to characterize the student discourse in each classroom setting. This included a description of the

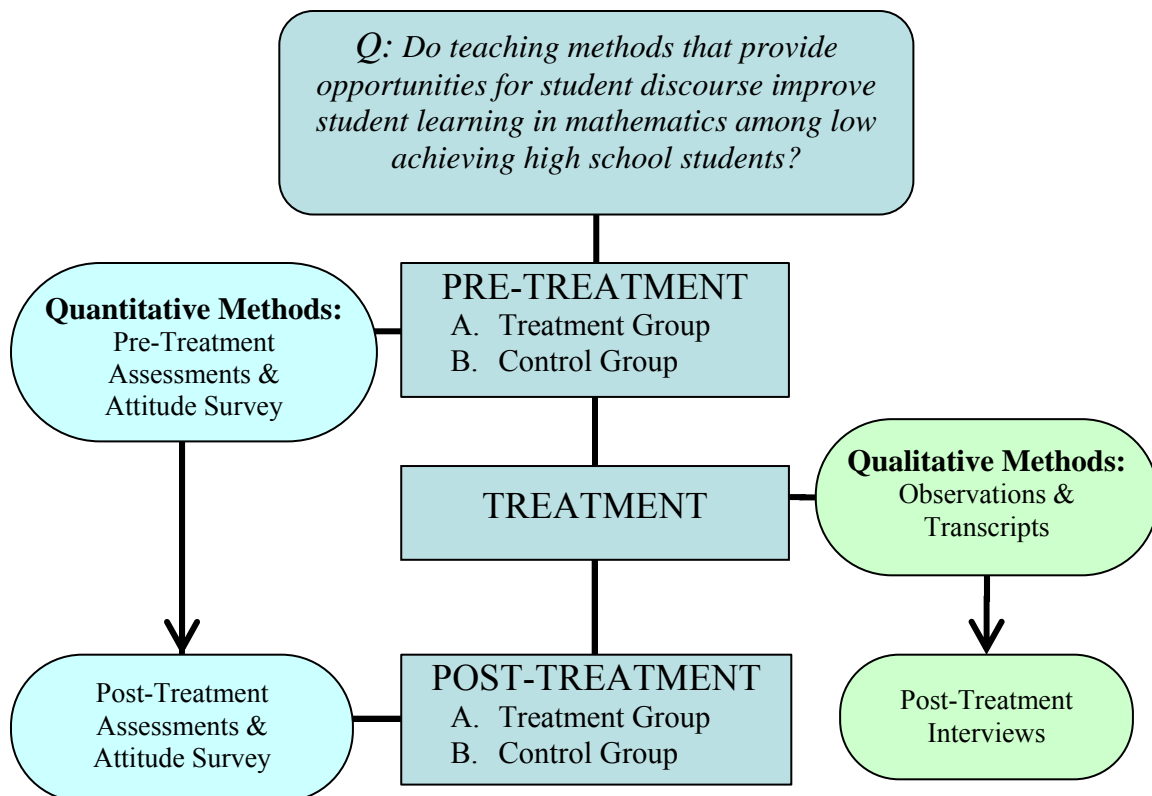


Figure 1. Visual model of research design.

intervention strategy used in the treatment classroom, observations of the implementation of the intervention, observations of both treatment and control group classrooms during the treatment phase of the research, and post-treatment interviews of selected students concerning their perceptions of the student discourse activities in their classrooms.

Audio recordings were used to facilitate analysis of classroom and interview transcripts.

Classroom transcripts were coded for three distinct purposes: (a) to characterize and compare the instructional activities and discourse that occurred in the different classroom settings, (b) to describe the discourse intervention and identify any variance in its implementation in different classroom settings, and (c) to identify potential discursive evidence of student learning and attitudes to corroborate quantitative findings. Interview

transcripts were used to provide feedback from students concerning their perceptions of the treatment activities and student discourse in general. This data and analysis were also submitted to an experienced teaching colleague to provide an independent perspective for peer review and validation of findings. These findings provided the basis for understanding and interpreting the quantitative findings by providing a window into the different classrooms to assist the identification of potential moderating factors and characteristics of student discourse that may have contributed to measurable outcomes.

Intervention: Mathematics Dialogue Activities

Because familiarity with discourse and classroom expectations relating to student discourse both vary greatly among classrooms and teachers, a course supplement was designed to establish a common basis for student discourse in treatment group classrooms. This consisted of a series of short scripted plays, called mathematics dialogues, which were used to introduce and model constructive discourse in the context of a specific mathematical unit. These dialogues portrayed different situations where students in a mathematics class are working together to solve problems. Characters in the play encounter difficulties that reflect common misunderstandings or communication problems that real students may encounter. Reading through or enacting the dialogues raises these issues as discussion topics, while providing students with examples of small group discourse.

The mathematics dialogue activities designed to support discourse in this research were based on the use of scripts, or vignettes, in previous research by DeJesus-Rueff (2006) and Walen & Hirstein (1995). In those studies, scripts were used to stimulate student discourse and enhance informal assessment opportunities. In this study, the main

purpose is simply to provide opportunities for student discourse. However, stimulating student discourse is a large part of this, especially among students who lack experience with this form of instruction. Mathematics dialogues provide a way to model constructive discourse while also promoting it through questions. The use of fictional characters may also provide students with a face-saving device, allowing them to discuss or critique what the characters say or do without implicating themselves or their peers.

In the present study, mathematics dialogues were developed to reflect the content of specific prealgebra lessons on one-step and two-step problem solving, as part of a larger instructional unit on algebraic problem solving. As a standard element of the prealgebra curriculum, this unit was selected to provide materials and activities that could be implemented in a wide variety of prealgebra classes. In general, this unit introduces the basic principles of algebraic reasoning used to solve for unknown quantities; the *addition property of equality* and the *multiplication property of equality*. It was selected both for its conceptual importance to the subsequent study of algebra and its commonality as a standard element of prealgebra courses. Accordingly, this unit is relevant to most prealgebra teachers.

The main treatment consisted of three mathematics dialogues that portrayed students working on problems corresponding to the three primary lessons of the problem solving unit; one-step problem solving using addition, one-step problem solving using multiplication, and two-step problem solving. Each of these was accompanied by a brief lesson plan to guide teachers in structuring the activity and provide initial discussion questions.

In general, the mathematics dialogue supplements were intended to follow the teacher's corresponding lessons. Students would be asked to form small groups of three or four and read through the script, adopting various character traits if they so wished. They could then be asked to perform these for the class or proceed to discussion questions, as directed by the teacher. This step was made optional to provide flexibility to accommodate time constraints and diverse needs of students. After reading or performing the dialogue, students were provided with several discussion questions to discuss and answer in their small groups. These generally addressed the action of the script and included 2-3 mathematics problems. After the groups had completed these questions, the teacher would facilitate a full class discussion by asking groups to report on their answers and group discussions. These activities were expected to last about 20-30 minutes.

An example of the mathematics dialogue scripts used in these activities can be seen in Figure 2. This script, designed to accompany a lesson on two-step problem solving, portrays four students working out a two-step problem. The discussion is intended to reflect a common error, or point of confusion, among beginning algebra students; namely, which step to do first. Accordingly, the students in the play model confusion and propose different ideas as they gradually narrow in on the correct solution. The following discussion questions accompanied this script:

1. What problem are these students having in the beginning of the play?
2. Would you want to be part of this group? Why or why not?
3. Write out the correct steps the group used to solve this problem. Then complete the other two problems from the play.

Dialogue #3: Mixing it up

Characters: Terry, Pat, Alex, and Jesse are four students in the same math class.

Scene: The teacher just finished the lesson and assigned students to groups to figure out some practice problems. The students are working on the first of these problems:

1. $6x - 8 = 22$
2. $3x + 7 = 31$
3. $16 = 42 - 3w$

Terry: I think we're supposed to subtract 8 here because the problem has subtraction in it.

Pat: Wait, uh, didn't she say to divide?

Alex: Subtraction doesn't work because that would make it negative sixteen.

Jesse: Yeah, we want to get rid of the 8, not make it bigger

Pat: So if we divide by eight, will it go away? So, $6x$ equals 22?

Terry: No, because 8 divided by 8 still leaves one.

Pat: Oh, so then we can subtract the one? So it's $5x$ equals 22?

Jesse: Hang on a minute. You can't subtract those -- they're different -- x terms and numbers don't mix.

Alex: I think we need to add 8 so it goes to zero.

Pat: I'm so confused.

Terry: Okay, Alex is saying we should subtract 8 to get $6x$ equals 22?

Jesse: Wait, we have to add 8 here too. So this should be 30.

Alex: Right. We add the same thing to both sides.

Terry: So $6x$ equals 30. Now what?

Pat: Is this where we divide?

Alex: Aha! Divide and conquer!

Terry: Divide by what?

Pat: Well, it's either the 6 or the 30.

Jesse: I'm pretty sure it's the 6 because then we get $1x$ all by itself.

Figure 2. Excerpt from Mathematics Dialogue

The complete set of mathematics dialogues is included in Appendix A, Mathematics Dialogue Activities.

Subsequent to the mathematics dialogue activities, students were given additional *unscripted* opportunities for student discourse during the remainder of the instructional unit. During a lesson on multi-step problem solving, students were asked to work in small groups to solve different multi-step problems and then try to identify differences and commonalities in their solutions. At other times, students simply worked together on their regular practice problems, which were treated as individual seatwork in the control group classrooms.

Data Collection

Qualitative data were collected through observations and field notes made during site visits, recordings of classroom discourse during the treatment stage of the unit, informal consultations with participating teachers, and post-treatment interviews of selected participants. The researcher visited and observed each participating classroom two to six times during the instructional unit. In treatment group classrooms, one visit occurred at the beginning of the intervention treatment to record initial implementation, with two to three follow-up visits to monitor the later development of the intervention. At least three lessons were recorded in each setting to provide for accurate characterizations of instructional activities and classroom discourse. Multiple recordings of small group discourse were also made during the mathematics dialogue activities, which were observed on four separate occasions in each setting.

Additional site visits were made to conduct follow-up interviews with students after completion of the mathematics dialogue unit. These were short interviews that

addressed student perceptions of the mathematics dialogue activities through three questions. Seven subjects were interviewed, including three students at the traditional school and four students at the alternative school. All seven students were volunteers who had participated in the mathematics dialogue activities and were the only students who returned completed consent forms. Interviews were recorded to provide documentation for transcription and analysis. A copy of the interview protocol is included in Appendix B.

Data Analysis

Coding and analysis of classroom transcripts drew on models from current research on discourse in mathematics instruction. Potential models included sequential analysis (Kysh, 1999; Mehan, 1979), focal analysis, and pre-occupational analysis (Kieran, 2002; Sfard, 2002).

Kysh (1999) applied two forms of sequential analysis to analyze classroom discourse and small group discussions in a ninth grade prealgebra class. One form of sequential analysis was adapted from earlier studies of classroom discourse in which each statement is identified with respect to its discursive purpose as an inquiry, response, or evaluation (Mehan, 1979). Mehan used this method to identify the sequence, inquiry-response-evaluation, or I-R-E, as a key characteristic of teacher-directed instruction. Kysh used a modified coding scheme in which the I-R-E pattern was reformulated as question-answer-verification, or Q-A-V, and provided additional codes to recognize other discursive purposes. Kysh also used a second classification system, adapted from Brenner (1995, as cited in Kysh, 1999), to identify how students expressed mathematical ideas and procedures, as instances of communicating *about mathematics*, *in mathematics*, *with*

mathematics, or *beyond mathematics*. These categories help identify the extent to which students use mathematical language, and whether they talk about specific problems or more general mathematical ideas during their classroom discourse.

Sfard (2002) and Kieran (2002) explored two more forms of analysis, focal analysis, which compares what students are *saying* to what they are *doing* as they solve problems, and pre-occupational analysis, which uses interactivity flow charts to examine the extent to which students respond to one another effectively. Interactivity flow charts display information about whether how students interact, such as whether they are responding to or directing specific threads of discussion, or just talking aloud to themselves. These elements are displayed graphically using arrows in columns to show the direction and content of statements by different students.

The final selection of the specific coding strategies to be used in this research was guided by three specific purposes: (a) to characterize classroom instruction and discourse in each setting, (b) to provide for the meaningful comparison of classroom discourse in each setting, and (c) to identify potential discursive evidence of student attitudes and student learning to corroborate quantitative findings.

Validation of Findings

Validation of qualitative findings was provided through the use of peer reviewers who were informed of the criteria of analysis, but naïve with respect to how this would influence the results of the research. Two reviewers, including an experienced teaching colleague and a fellow student with a background in sociology, provided the researcher with independent interpretations of the classroom discourse and student interview data. This enhanced the objectivity of findings by providing multiple perspectives from which

to identify points of agreement and disagreement. Agreement between multiple independent reviewers indicates a degree of consensus or objectivity, while disagreement between interpretations identifies points for further reflection and re-evaluation. Accordingly, these additional perspectives provided a check on the researcher's analysis and interpretation of the qualitative data.

In addition, an internal triangulation strategy was used to enhance validation of findings and increase objectivity. This included a comparison between the interpretation of classroom observations and student responses to interview questions, as well as a comparison between qualitative findings and quantitative findings to identify potential corroborating evidence. In the latter case, evidence of student learning identified from transcripts and interview responses was compared to evidence of achievement or problem solving gains provided by measurable learning outcomes. Again, agreement between independent analyses was taken as corroboration, indicating a degree of objectivity.

Quantitative Components of the Study

The quantitative components of the study compared achievement data, problem solving skills, and student attitudes in treatment and control groups. While achievement data was used to address the effectiveness of discourse as an intervention strategy for low achieving students, problem solving data provided a check on the validity of these results and contributed to a broader picture of student learning. A quasi-experimental design with pre-treatment and post-treatment measures was used to identify any differences in the average achievement gains and average problem solving gains between the two groups. Attitudinal data provided additional information to identify whether the use of student discourse appeared to influence student attitudes towards mathematics and

whether student attitudes appeared to influence other learning outcomes. Attitude data was compiled as classroom attitude profiles displaying frequency of student survey responses before and after the research.

The primary measurements are represented symbolically as follows:

<i>Group 1: Discourse Group</i>	<i>O</i>	<i>X₁</i>	<i>O</i>
	<i>O</i>	<i>Y₁</i>	<i>O</i>
<i>Group 2: Control Group</i>	<i>O</i>	<i>X₂</i>	<i>O</i>
	<i>O</i>	<i>Y₂</i>	<i>O</i>

Here the variables X and Y represent the mean difference between pre-treatment and post-treatment measurements of the achievement test scores (X), and the rubric-based problem solving scores (Y) for each group. The mark *O* indicates the measurements.

Research Questions

The central question addressed by this research was stated as follows:

Does the use of teaching methods that provide opportunities for student discourse improve student learning in mathematics among low achieving high school students?

This was addressed through the following subsidiary questions:

1. Does the use of teaching methods that provide opportunities for student discourse improve mathematics achievement among low achieving high school students?
2. Does the use of teaching methods that provide opportunities for student discourse improve problem solving skills among low achieving high school students?

3. Does the use of teaching methods that provide opportunities for student discourse appear to influence student attitudes towards mathematics among low achieving high school students?
4. Does the use of teaching methods that provide opportunities for student discourse appear to help teachers in the effort to improve student learning?

Does the use of teaching methods that provide opportunities for student discourse improve mathematics achievement among low achieving high school students?

This question was addressed through a quantitative analysis of achievement data collected from the treatment and control group. Pre-treatment and post-treatment tests were given to assess the gains in students' achievement test scores that occur during the intervention. Mean gains for each group were then compared to determine whether the discourse intervention led to any experimentally important and consistent differences in achievement test scores.

Does the use of teaching methods that provide opportunities for student discourse improve problem solving skills among low achieving high school students?

This question was addressed through quantitative analysis of rubric-based scores on constructed response items. Again, pre-treatment and post-treatment tests were given to assess any gains in students' problem solving skill that occurred during the intervention. Mean gains for each group were then compared to identify any experimentally important and consistent differences between the two groups. Results of this analysis also provided a source of corroboration for findings concerning achievement. Student interviews and classroom transcripts also were reviewed to identify any additional evidence of gains in student understanding.

Does the use of teaching methods that provide opportunities for student discourse appear to influence student attitudes towards mathematics?

This question was examined using descriptive statistics to characterize students' responses to a mathematics attitude survey given both before and after the intervention. The survey was based on Sandman's (1973) Mathematics Attitude Inventory (MAI), which generated scale scores indicating positive or negative attitudes towards mathematics among students. Since scale scores provided ordinal level data, a comparison of means was inappropriate. Instead, frequencies of scale scores were calculated to construct attitude profiles of each group and classroom before and after the intervention. This information was then used to assess any overall differences between the classrooms and to assess any changes in attitude that occurred during the intervention. Student interviews and classroom transcripts also were reviewed to identify additional evidence of student attitudes and potential attitude changes.

Does the use of teaching methods that provide opportunities for student discourse appear to help teachers in the effort to improve student learning?

This question was examined by comparing the characteristics of the classrooms using discourse to those of the classrooms not using discourse. Any characteristics found to be unique to classrooms using the student discourse interventions were then analyzed to identify their potential instructional value to teachers.

Definition of Terms

The following definitions of key terms were utilized in this research:

Classroom Discourse

The term *classroom discourse* was used broadly to refer to all forms of verbal or nonverbal communication taking place in a given classroom setting. Accordingly, the terms *discourse* and *communication* were used interchangeably in this research. In addition, the terms *discussion* and *conversation* were used to indicate sustained verbal exchanges between two or more individuals.

Dialogue

In the context of this research, the term *dialogue* was used to refer to the scripts and mathematics dialogue activities that were developed as lesson supplements and applied as an intervention in the treatment group classrooms.

Low Achieving Mathematics Students

For purposes of this research, *low achieving mathematics students* included any students enrolled in a high school mathematics course that is considered to be a remedial class with respect to standard high school course content. This included students enrolled in basic mathematics, mathematics topics, or prealgebra classes.

Mathematics Achievement

For the purpose of this research, *mathematics achievement* was taken to mean the level of success indicated by students' test scores on a multiple-choice format assessment reflecting the specific content of the mathematics unit taught during this study. A pre-test and post-test of student achievement were developed by the researcher in consultation with participating teachers to assure that the test questions reflect the instructional content of the unit. These assessments comprised the first 20 items on the Pre-Test and Post-Test included in Appendices C and D.

Mathematics Problem Solving Skill

For the purpose of this research, *mathematics problem solving skill* referred to students' demonstrated abilities to use mathematical operations, concepts and procedures, and respond effectively to constructed response questions, as indicated by students' rubric-based scores on these questions. A pre-test and post-test of problem solving skill, each of which consisted of two constructed response items, were developed by the researcher in consultation with participating teachers to assure that the test questions reflected the instructional content of the unit. These assessments comprised the last two items on the Pre-Test and Post-Test included in Appendices C and D. In addition, the scoring rubric used to evaluate student responses on these assessments is attached as Appendix E.

Student Attitudes Towards Mathematics

For purposes of this research, student attitudes towards mathematics referred to a positive or negative disposition towards mathematics classes or mathematical activities, as indicated by student responses on an anonymous attitude survey or other direct statements by students. The attitude survey used in this research was based on the Mathematics Attitude Inventory (Sandman, 1973, 1980) and is included in Appendix F.

Student Learning in Mathematics

In this study, *student learning in mathematics* referred broadly to any evidence of gains in students' mathematical knowledge, skills or understanding, as indicated by students' test scores, problem solving scores, written work, or discourse during classroom activities.

Teaching Methods that Provide Opportunities for Student Discourse

For the purpose of this research, *teaching methods that provide opportunities for student discourse* was taken to mean instructional activities that included discourse between students as an element intended to support student learning. This included small group discussions and activities, as well as other occasions when students can collaborate, help one another, or respond to one another during whole class instruction.

Population and Sample

The population for this study consisted of approximately 160 high school prealgebra students attending one of four different public high schools within the Missoula Valley, including three traditional high schools and one alternative high school program. Because prealgebra is generally expected to be learned during the middle school grades, this course is considered to be a remedial mathematics course when taught at the high school level. Accordingly, these students were low achieving mathematics students.

A preliminary screening of prealgebra teachers at these schools was undertaken to identify a pool of teachers who (a) were willing to participate in this research, (b) used similar curriculum materials, and (c) were using instructional methods that do not emphasize student discourse. These three criteria were developed to identify a pool of classes with similar content and lesson materials and teachers whose instructional style matched the conditions needed for the control group. After identifying a pool of eligible classes, cluster sampling was used to randomly assign intact prealgebra classes to a treatment group and a control group. Two classes were assigned to each group in the

effort to provide group sizes of 30-50 students, sufficient to meet the statistical requirements of the assumption of normality.

Measures of Student Learning

Variables and Levels of Data

The independent variable for quantitative analysis was student participation in the mathematics dialogue activities. This was a dichotomous categorical determination based on student enrollment in a class receiving the treatment or a class assigned to the control group. Accordingly the independent variable provides nominal level data.

There were two dependent variables, achievement and problem solving skill. The first dependent variable, *achievement*, was measured by student test scores on achievement tests given before and after the treatment. These scores were expressed as percentages, providing ratio level data. The second dependent variable, *problem solving skill*, was measured by student scores on rubric-based problem solving assessments given before and after the treatment. These scores were also expressed as percentages, providing ratio level data.

A third variable, *student attitude towards mathematics* was also included as a potential moderating variable. This variable was evaluated using a closed format survey instrument based on the Mathematics Attitude Inventory (Sandman, 1973, 1980). This instrument generated scale scores, which were categorized as high, medium, or low. These scale scores provided ordinal level data, which were compiled as frequency distributions of scale scores for each classroom.

Null Hypotheses

Given that this research measures two dependent variables, there were two corresponding null hypotheses.

The first research question addressed the variable, *achievement*, asking whether the use of teaching methods that provide opportunities for student discourse would improve mathematics achievement among low achieving high school students. The null hypothesis for this inquiry was stated as follows:

H₁: There will be no experimentally important or consistent difference ($X_2 - X_1$) between the mean gains in achievement test scores (X_1) of students in classrooms using instructional activities that provide opportunities for student discourse and the mean gains in achievement test scores (X_2) of students in classrooms that do not use instructional activities that provide opportunities for student discourse.

X_1 = Mean achievement gains in discourse group

X_2 = Mean achievement gains in control group

The second research question addressed the variable, *problem solving skill*, asking whether the use of teaching methods that provide opportunities for student discourse would improve problem solving scores among low achieving high school students. The null hypothesis for this inquiry was stated as follows:

H₂: There will be no experimentally important or consistent difference ($Y_2 - Y_1$) between the mean gains in problem solving skill (Y_1) of students in classrooms using instructional activities that provide opportunities for student discourse and the mean gains in problem solving skill (Y_2) of

students in classrooms that do not use instructional activities that provide opportunities for student discourse.

Y_1 = Mean problem solving gains in discourse group

Y_2 = Mean problem solving gains in control group

Definitions

With regard to measures of achievement and problem solving skill, experimental importance was defined as a mean difference of 10% or more between the two groups. This value was chosen to reflect the standard interval between letter grades used in evaluating student work. With regard to student attitudes, experimental importance was defined as a frequency difference of 10% between scale scores of different groups or applications of the survey. This level was chosen somewhat arbitrarily to allow a consistent definition of experimental importance for all three quantitative analyses. For comparisons with small sample size, experimental consistency was inferred from the similarity of findings across different classroom settings.

Statistical Procedures

For the purposes of this research, an analysis of mean gains was used to determine any experimentally important and consistent mean differences between the achievement or problem solving gains of students in classrooms using the mathematics dialogue activities and students in control group classrooms. Students' individual gains were calculated by subtracting pre-test scores from post-test scores for each individual. Mean gains were then calculated as the average of individual gains to compare the mean gains in achievement and problem solving for each group, and to provide classroom level comparisons for each teacher.

Attitude data was analyzed by using a point-based scoring procedure to generate a set of attitude scale scores for each completed survey. Frequency analysis was then used to compile the distribution of attitude scale scores within each group and classroom, before and after the treatment.

Delimitations and Limitations

Delimitations

This study is delimited by population selection and its specific interpretation of discursive instruction. The population is delimited to low achieving students enrolled in high school prealgebra classes at one of the four public high schools within the geographical area of Missoula Valley. This includes a geographical restriction on the population as well as an acknowledgement that the operational definition of low achieving students used in this research is a generalization that does not preclude the possibility that some more successful mathematics students may have been enrolled in these classes for various reasons. In addition, this study's interpretation of *teaching methods that provide opportunities for student discourse* is delimited to a specific set of instructional activities developed as lesson supplements for the treatment group. This is just one of many possible forms of providing opportunities for student discourse and the study makes no claim of comprehensiveness in covering this field. Accordingly, results are not generalizable beyond these parameters. Moreover, the selection of a case study framework provides for an examination of situated learning outcomes in specific classrooms, and is not intended to support generalization of findings beyond these settings.

Limitations

Other limits of the study relate to sampling issues inherent in a population of diverse individuals that spans multiple teachers, schools and classrooms. The effort to identify and randomly select a sufficient number of classrooms for treatment and control groups depended on several factors including teacher interest and experience, similarity of instructional methods, teaching schedules and sequencing of instructional content. Variations between distinct schools, classrooms, students, teaching styles, and other site related factors also influenced implementation and results.

Role of Researcher

The researcher acted as the primary research coordinator, classroom observer and data collector throughout the study. As coordinator, this included working with school district staff and participating teachers to obtain formal permission, access an appropriate sample, conduct preliminary screening, and develop suitable testing instruments. The researcher's role in classroom observations varied during the course of the study reflecting the expectations and needs of diverse students. In general, the researcher approached the study as an observer, questions from students or teachers and other interactions sometimes blurred the distinction between observer and participant observer. The researcher also conducted the data analysis and is solely responsible for the interpretation of results and recommendations for further research.

Potential Ethical Issues

There were no special ethical issues that affected this research. The study addressed alternative forms of instruction within a regular educational setting and therefore posed minimal risk to the subjects. A confidentiality plan was used to protect

the identity of individual subjects. This included the use of numeric coding to link pre-tests and post-tests for individual subjects, anonymous attitude surveys, and the use of fictional names in all reporting of classroom discourse episodes and student interview responses. In addition, formal consent was obtained for all student interviews, since this constituted an exception to regular school activities. These subjects were informed of the purpose of the study and of their right to discontinue involvement at any time. A formal proposal was also submitted to the Missoula County Public School District to obtain permission and assure compliance with all relevant school district policies and custodial responsibilities.

The potential for the researcher's interest in discourse as a teaching strategy to become a source of potential bias influencing results was acknowledged and addressed through the use of independent reviewers to provide critical feedback during data collection, analysis and interpretation. In addition, an internal triangulation strategy enabled further validation by comparing the quantitative and qualitative findings. While quantitative measures provided objective data to validate qualitative findings, the qualitative data provided contextual information to facilitate a situated and therefore more accurate interpretation of the quantitative findings. At the same time, multiple measures of student learning provided a check on individual measures by referencing the degree of agreement between them.

CHAPTER FOUR

QUALITATIVE CONTEXT

This study examined the effectiveness of student discourse as a teaching strategy for low achieving high school students in the context of two distinct schools and classroom settings and with two different teachers. In order to make a meaningful comparison of student learning outcomes from different classroom settings with different instructional styles, it is necessary to take a vantage point that will enable one to account for the possible influences of classroom and teacher differences. This requires clear descriptions of each setting and how the study was implemented in each setting.

In order to interpret student learning outcomes as accurately as possible, qualitative data was collected from classroom recordings and observations made during site visits to each participating classroom. This information formed the basis for a general description of each instructional setting and how the study was actually implemented in these settings. An analysis of classroom transcripts provided additional details concerning specific characteristics of student discourse in each setting. Accordingly, this chapter reports on the instructional setting and implementation of the study and provides a qualitative context to inform our interpretation of the resulting student learning outcomes.

The chapter begins with an overview of the development and implementation of the student discourse intervention. This involved a collaborative process between the researcher, participating teachers, and school district personnel. Second, findings from classroom observations and transcript analysis are presented to characterize the different

instructional settings and compare features of implementation and student discourse across multiple classroom settings. And third, findings concerning discursive evidence of student learning and student attitudes are also reported.

Development and Planning

Collaboration

During the planning and approval stages of the study, six prealgebra teachers were identified and contacted to provide them with information about the study. Only two of these volunteered to participate, with the rest declining for various reasons. The two volunteering teachers included one at a traditional high school and one at an alternative high school, each of whom taught two prealgebra classes. Due to the substantial differences between these two settings, including small class sizes at the alternative school, it was decided to assign one class in each setting to the student discourse treatment group and to assign one class in each setting to the control group. This optimized the degree of matching between the control group and treatment group by minimizing the influences of diverse instructional settings and teaching styles on student learning outcomes. It also addressed a concern raised by the school district that the study would provide teachers with information to enhance their instructional choices, and not be construed as comparing teacher effectiveness. Within these constraints, one class in each setting was randomly assigned to the treatment group, which also determined the control group.

Prior to the actual study, the researcher and teachers had two planning meetings. First an initial meeting with both teachers was arranged in conjunction with the school district curriculum office to address the general research plan, and then individual

meetings were arranged to address scheduling and logistics specific to each classroom setting. The purpose of the initial meeting was to provide teachers with an overview of the study, clarify what would be expected of them as participating teachers, and address any questions or concerns they may have about the study. This was also an opportunity for the school district to convey its support for the project and provide for their general oversight responsibilities with respect to any project taking place in the schools. One condition of school district approval was that teachers would not diminish the quality of their teaching to meet the conditions of the control group. Accordingly, teachers were encouraged to use their usual style of teaching for the main lessons in both classes, but to supplement this in the treatment group by using the student discourse activities provided by the researcher. Exactly how to do this in each setting was worked out individually with each teacher.

An initial presentation in each classroom was also planned to introduce the researcher and explain the study to students. An informational letter to parents with contact information for the researcher and the university was also distributed at this time. The Instructional Review Board determined that formal participant consent was not required for an in-school comparison of alternative curriculum materials and instructional methods.

Subsequent meetings with individual teachers addressed scheduling and how best to merge the treatment with their existing lesson plans. Both teachers' lessons followed a similar progression through the unit content, but their materials and teaching styles were quite different. Building schedules were also different. One school had 45 minute periods every day, while the other met for 90 minute periods on alternating days. This

situation recommended a schedule with one treatment activity for every 90 minutes of instruction to provide for similar duration of instruction and discourse activities in each setting. The resulting unit lasted three weeks, with approximately 450 total instructional minutes and five planned opportunities for student discourse in each treatment classroom.

Teachers were also asked to review the proposed treatment and assessment materials to identify any potential mismatch between these and their existing lesson plans for the unit. While the treatment materials were generally acceptable to both teachers, the draft assessments were edited slightly to reflect the given content more accurately, increase readability, and allow more space for student work. The final assessments are included in Appendices C and D.

Instructional Setting

The setting for the study included two distinct instructional settings reflecting different schools and different teachers. One pair of classes was located at a traditional high school with all the usual student activities and sports programs one would expect in a mid-sized high school. The other pair was at a much smaller alternative high school designed to support students who were not succeeding at the various traditional schools throughout the district. Each of these settings had its own distinct character that influenced student expectations and teachers' instructional strategies. Accordingly, these differences need to be considered in order to understand how the implementation of the mathematics dialogue activities varied between these settings. This information is also needed to inform the interpretation of student learning outcomes in each setting. Based on observations and recordings of classroom activities in both these settings, a general description of each setting and its characteristic form of instruction is reported below.

Traditional High School Setting

The traditional high school in the study was a mid-size school located near the edge of town. Partly due to this location, the school drew students from rural parts of the valley as well as the surrounding neighborhoods and across town. This contributed to student diversity, which was also enriched by Native American, Hmong and Russian students living in this area. Another special feature of this school was the use of a block schedule consisting of eight 90-minute periods that meet every two days. Accordingly, the mathematics classes in this setting met for 90 minute periods every other day. These long class periods were reflected in the teacher's instructional strategies and daily routines, which were structured to keep students involved and attentive for this length of time.

The mathematics classroom itself was large and spacious, sporting long whiteboards and blackboards along two walls, ideal for student board work. The other two walls were lined with computer stations, with bulletin boards and student posters above these on one side and windows into the hallway along the back. Six rows of desks faced the front board where an overhead projector marked the focal point of daily lessons. The teacher's desk sat in the back corner surrounded by photographs and certificates. Around the room, student papers, green plants and a poster of Einstein with his tongue sticking out added color and personality.

Most of the students in both the prealgebra classes were freshmen, although a few were older. Although the roster for the classes listed 20 to 22 students, only 15 to 17 were observed to attend on most days. While some of these absences were probably related to the winter flu season, the teacher indicated that several students had consistent

attendance issues and that students regularly transferred into or out of these classes throughout the school year. During the classes, most students appeared to be very attentive and engaged most of the time. The classroom atmosphere was generally friendly and good-humored, with only occasional instances in which students exhibited resistant attitudes.

Both classes observed in this setting had a well-established instructional routine. On each day, class began with students taking out their homework from the previous lesson. As this work was checked by the teacher, any students who were not prepared lost a specified number of points and moved to the back of the classroom where they continued to work on the assignment, facing away from both boards. Students who did complete the assignment were sent to the boards where each one put up one or more solutions from their paper. After these students returned to their seats, they exchanged papers. The teacher then checked each problem at the board, indicating corrections as needed, while students scored each others' papers. Students in the back kept working throughout, but could return to their seats and join in the corrections if they finished. At the end of corrections, completed papers were collected and all students returned to their seats. Anyone not finished had to complete this work later on their own time.

After corrections, which lasted about fifteen minutes, the new lesson began with review problems. Generally, the teacher would present two to four review problems, putting them on the overhead projector one at a time and then circulating around the classroom to check students' solutions as they attempted each problem. After allowing two or three minutes for students to complete the problem, the teacher returned to the overhead and called on students to guide him through the steps of the solution. This

review usually lasted about fifteen minutes. Then the new lesson for the day was presented in similar fashion. Again problems were introduced on the overhead one at a time and the teacher circulated as students attempted their solutions. Again students were called on to guide the teacher through the steps of the solution. This portion of the lesson also lasted about fifteen minutes.

At this point, about halfway through the ninety minute period, the teacher would direct all students to go to the long whiteboard and blackboard simultaneously. Once everyone had lined out and found themselves something to write with, the teacher gave instructions for writing a generic problem similar to those explored in the new lesson. Students had some leeway here to choose their own variable, like an initial, or pick a number from a range of possible values, resulting in an array of similar problems, each personalized by its student author. For example, on one day during the study, the teacher instructed students in the control group classroom as follows:

I want you to write... an even number. I don't care what it is; just an even number. Then go, plus $2x$. Then I want you to go, equals. And then I want you to choose any favorite even number. (Transcript 1, 35:55)

Students then proceeded to solve their problems, looking to neighbors for guidance and helping one another informally as needed. When everyone was ready, the teacher would check the problems from one end of the boards to the other, pointing out any needed corrections and praising each student for something they had done correctly. After this, everyone would erase the boards and repeat the process with a slightly more difficult problem. Usually, students solved two or three different problems at the board, taking anywhere from fifteen to twenty-five minutes. Finally, with fifteen to thirty minutes

remaining, students returned to their seats and received a practice worksheet to begin in class and complete for their homework.

The general strategy used to include the mathematics dialogue activities in this classroom routine was to shorten some parts of the routine to allow time for a small group activity after student board work. Usually this meant reducing the number of problems included during corrections, review, or board work. Students formed into small groups after board work, completed the dialogue activities, and then remained in small groups to begin their daily practice worksheets. This kept the teacher's form of instruction relatively constant between the treatment group and the control group, with the exception of the mathematics dialogue activities and associated opportunities for student discourse.

Alternative High School Setting

The alternative high school in the study was a small school of approximately 120 students, located near the center of town. This school was established about eight years earlier to provide a different type of educational program for students who were not succeeding at the district's traditional high schools. Featuring a relatively small staff, and small class sizes of 12 or less, this program offered students a more personalized education than larger high schools. Other unique features included a shorter school day, elective credits offered for documented employment, and shorter grading terms to facilitate early academic intervention. The program was also characterized by an emphasis on students' choices and the use of contractual agreements to address attendance, academic, and behavioral responsibilities.

The school had one small mathematics classroom with a sectioned whiteboard lining one wall and an assortment of tables and desks. On one side of the room, an old

blackboard posted the weekly schedule of assignments behind the teacher's desk and the room's one computer station. On the other side of the room the age of the building was evident from the old fashioned coat racks and locker shelves built into the wall. These shelves held the mathematics textbooks students use during class and check out as needed for homework. Opposite the whiteboard, traffic noise drifted in through a wall of windows overlooking the street. Several maps and posters decorated the walls, as well as a collection of intricate geometric patterns and project posters done by students.

As noted above, class sizes at the school had a maximum of twelve students. Due to the small size of the school, many mathematics classes were split between two or more courses taught during the same period. However, since both prealgebra classes were nearly full, with ten or eleven students in each at the start of the semester, no other mathematics courses had been scheduled for these periods. In addition, these prealgebra classes were shorter semester-long courses that had just started a few weeks before the study and met for 45 minute periods each day. Students in these classes were older, ranging from freshmen to seniors. Some of these students appeared to know a lot of the mathematics content from previous mathematics classes that they had failed to complete for various reasons. Resistant attitudes were fairly common among these students, many of whom frequently questioned why they had to show work or do more problems, occasionally complaining loudly. These behaviors were usually checked by simple reminders from the teacher, but occasionally led to private interventions or student behavior contracts.

The general instructional routine in these classrooms consisted of a short teacher-led lesson at the beginning of the period, followed by an assignment during which the

teacher tutored individual students as needed. The lessons presented at the board, sometimes with the help of a computer projector, followed a traditional pattern of teacher-led questioning with students supplying specific steps or calculations needed to complete the sample problems. Students sometimes volunteered responses but the teacher usually called on specific students to make sure everyone was included. After two to three sample problems lasting about ten minutes, students were given a problem set to work on individually. The teacher then circulated to check student work and respond to student questions. Some students asked for help frequently, while others completed problems swiftly with no need for additional assistance.

This format was flexible, allowing the teacher to provide tutoring where it was needed, or return to the board for additional direct instruction if it appeared most students were experiencing difficulty. Towards the end of the period, or when enough students had completed a portion of the problems, the teacher would refocus the group to check work. This was done by calling on students to report the steps of their solutions and answers for each problem. Generally, everyone took at least one turn and the solutions were corrected through guided questioning as needed. At the end of the period, students turned in their papers. It was rare for homework to be given in this setting. However, students could check out books if they needed to, or come in before or after school, or at other times, to complete their assignments.

The general strategy for incorporating the mathematics dialogues into this instructional setting was to substitute them for individual work on alternating days of the unit. This meant that both the treatment and control group classrooms spent two days on each lesson of the unit. The first day was similar in both settings, while the second day

varied to provide opportunities for student discourse in one classroom only.

Classroom Observations and Discourse Analysis

After developing a schedule and implementation plan with each teacher, the study commenced with introductory visits from the researcher the following week. The instructional unit then lasted three weeks in each setting, including pre-tests and post-tests. During this period multiple site visits were made to each classroom to collect information about how the activities were actually carried out and what types of discourse occurred in each setting. Based on field notes and audio-recordings from these visits, this section describes the actual implementation and resulting classroom discourse in each setting.

Methods

Classroom observations and recordings were conducted during six visits to the traditional school site and eight visits to the alternative school site. Classrooms using the discourse activities were observed on four days at the traditional school and six days at the alternative school, while comparison classrooms were observed on two days at the traditional school and four days at the alternative school. Here, the higher number of visits made to classrooms at the alternative school reflects the shorter length of those classes. After an initial screening for quality, recordings from ten different visits were reviewed to identify episodes of student discourse and whole class discourse which were then transcribed for further analysis.

The primary purposes of analyzing classroom transcripts were (a) to identify variation in how the mathematics dialogues were implemented in different settings, (b) to characterize and compare classroom discourse in each setting, and (c) to identify

potential discursive indicators of student learning and attitudes to corroborate quantitative findings.

The discourse analysis methods used in this study consisted primarily of sequential analysis to identify turn-taking patterns and utterance categories (Kysh, 1999; Mehan, 1979). Some use of interactivity flow charts (Sfard, 2002) was also included to illustrate small group interaction patterns and examine whether students sustained a shared mathematical focus. Focal analysis (Sfard, 2002) was also considered, but the lack of sufficiently detailed information on what students were writing and doing during specific moments of their discussions precluded this option.

Peer Review

For purposes of validation, one peer reviewer listened to several classroom recordings with the researcher and provided an independent perspective concerning the meaning or educational relevance of different classroom discourse episodes. During this process, the reviewer offered several interpretations of discourse episodes that were substantially different than the researcher's initial interpretations. This was especially true of episodes where students had expressed resistant attitudes or made negative comments about the activities or assignments they had been given. Initially, the researcher reacted to these with some dismay, perceiving them as having a negative influence on classroom discourse or detracting from the intended focus of the activities taking place. In contrast, the reviewer interpreted these as rare moments when students had embraced the opportunity to express themselves and gave their opinions about how they wanted to do or learn mathematics. This helped the researcher re-evaluate these episodes with less personal attachment to the particular activities and pre-conceived ideas

of how students should respond to these. Accordingly, the peer review process enabled the researcher to develop a more objective interpretation of the classroom discourse episodes that occurred during this research.

A second peer reviewer was provided with printed transcripts of the classroom discourse episodes, as based on the recordings made during ten site visits, together with a draft version of the description and analyses of classroom discourse presented in this chapter. This reviewer was asked to read through the transcripts and evaluate the accuracy of the proposed descriptions and characterizations of classroom discourse in each setting. In a subsequent meeting with the researcher, this reviewer confirmed most of the characterizations of student discourse, but raised important questions about some aspects of the classroom settings, student characteristics and grouping patterns that were not evident from the transcripts. These observations contributed to a more detailed description of the different instructional settings and greater clarification of factors affecting implementation in each setting.

Factors Affecting Implementation

The actual implementation of the mathematics dialogue activities varied from the planned implementation in both settings. This reflected a variety of factors including the different groups of students, diverse classroom norms, unrelated school activities that required teacher attention or affected the schedule, and frequent absences during flu season. The researcher's presence in the classrooms also affected the implementation.

Traditional School Implementation

Implementation of the mathematics dialogue activities in the traditional school setting got off to a slow start on the first day when the activity had to be postponed for

lack of time. This was due to a shortened school schedule and unplanned for classroom business at the beginning of the period. Therefore, instead of introducing the activity at the end of the lesson as planned, it was shifted to the beginning of the next period. This was a reasonable accommodation, but disrupted the established classroom routine of correction-review-lesson-practice and shifted the research schedule back a day. The activity itself went as planned, with students working in four groups for about twenty minutes, followed by a five minute whole class discussion of group results.

The second dialogue activity in this setting was also affected by unexpected circumstances. A situation during an assembly on the previous day required the teacher's attention and delayed the start of the class. Although the activity was not postponed on this occasion, it was reduced to fifteen minutes with no time for a follow up discussion. The teacher was also very busy and could not monitor the groups as actively as he usually did. Accordingly, it was not immediately noticed that students in two of the five groups had returned to their seats to work on the questions individually as soon as they finished reading the script. Some students also asked the researcher what to do, which introduced a new dimension concerning the role of the researcher in the study. The researcher responded by restating the directions for the group activity. However, students were accustomed to a well-established routine of working on problems individually and several continued in this pattern. After several minutes, the teacher intervened and redirected students to work together, which they did for the remainder of the activity. The other three groups did complete the activity, some with very detailed answers, but these were never shared with the class.

The third dialogue activity went more smoothly, with twenty minutes of group

work followed by seven minutes of whole class discussion. The teacher also circulated and kept students on track. During this activity, a worksheet assignment was handed out as students completed the discussion questions to allow them to continue working with one another on more problems. This had mixed results, as some students became preoccupied with getting their worksheets done and kept working right through the whole class discussion.

On the next day, students were given the group activity for solving multi-step equations. This lasted for about twenty minutes at the end of the class. The activity started off slowly as a number of students didn't read the instructions, but then went quite well after these were clarified by the teacher. As the class ran out of time, all of the groups appeared to be working together to identify similarities and differences between the equations they had been given, but once again, there was no time for a full class discussion or wrap-up.

In addition to these details of how the activities were implemented, frequent absences during flu season also affected the study. Several students missed one or more of the activities, or missed either the pre-test or the post-test, reducing the overall sample size in this setting. The teacher also had an unavoidable absence on the day of the control group classroom post-test, which was administered by a substitute teacher the morning after a long weekend.

Alternative School Implementation

The alternative school classroom presented a much different situation. This was a much smaller class that included some students who were older and students who had a variety of trust issues with adults and adult authority. Some students were very outspoken

and objected loudly when asked to do something outside their normal routine. Some students had learned problem solving previously. The classroom was also very small which allowed for exchanges between groups, especially when they overheard a funny comment or objection. Absences were also an issue. The class was originally supposed to have eleven students, but only eight attended regularly.

The implementation of the dialogues in this classroom stayed on schedule, but the whole class discussion component was more spontaneous and often emerged from informal discourse between students and the teacher. This usually happened during the discussion activities when an outspoken student made apparent negative comments about the questions or script that became a focal point for the rest of the class. Other students would look up and laugh or agree, sometimes expanding on what had been said with their own elaborations. The teacher intervened on these occasions by redirecting the whole class with exploratory questions to address why someone might have that opinion and whether there were other possible perspectives. These teachable moments often revealed some very thoughtful comments from students and ended with them resuming the activities. These exchanges displaced the need for further follow-up discussions of the scripts, although the teacher did sometimes redirect the whole class to address specific mathematics problems or procedures.

Trust issues among students also affected the way the activities were implemented. During the third dialogue activity, there was a disruption at the beginning of the class when one student mistook the teacher's instructions to re-read the script as a personal comment on his reading ability. The student walked out and an intervention episode ensued. This also disrupted the rest of the class as students speculated about

what had happened and whether someone was in trouble. This eventually got cleared up and the activity resumed, but the student in question was noticeably less communicative for the rest of the unit.

Trust issues also affected the way the researcher interacted with these students. During the second day of dialogue activities, students became obviously quieter when the recorders were set closer to the groups. Some students also asked about the recorders and whether the researcher was a psychologist. Consultation with the teacher about this revealed that one student had stated in a previous class that she thought the research was a secretive effort to shut down their school. In order to address student concerns and alleviate their suspicions, the researcher made it a point to visit with the different groups and explain more about why documentation is important and what the research is about. In order to build a more trustful relationship, the researcher also asked students how they were doing and occasionally shared perspectives with them about the different activities or questions they brought up. This allowed the students to evaluate the researcher for themselves, which did seem to make some of them more comfortable. Accordingly, the researcher had a stronger influence on students in this classroom.

Characteristics of Classroom Discourse in Each Setting

Transcript analysis was used to identify the common communication patterns in each classroom. This included identifying the sequence of speakers contributing to a given exchange, and identifying the general types of utterances in the exchange. For this initial analysis, utterances were classified according to their discursive function as questions (Q), answers (A), verifications (V), explanations (Ex) or redirections (R), with an additional category for replies that did not attempt to answer a question (N). This basic

coding scheme is adapted from that used by Kysh (1999) to characterize discourse in ninth grade algebra classrooms.

Transcript analysis indicated that three different types of discourse were common to all four settings in varying degrees. This included teacher-directed discourse during whole class instruction, tutoring exchanges when the teacher helped students individually, and informal student discourse that occurred at different times in each setting.

Teacher-Directed Whole Class Instruction

Transcripts from all four participating classrooms indicated that classroom discourse during whole class instruction was characterized by teacher-led questioning, with students contributing short answers. An example of this type of discourse can be seen in Transcript Excerpt 1, recorded during a regular lesson in the control group classroom at the traditional high school setting.

Transcript Excerpt 1. Whole Class Instruction, Traditional School

<u>Codes</u>	<u>Speaker</u>	<u>Utterance</u>
Q	Teacher:	Okay. Here we go. Eyes right here. Let me see here. Kelly, what did you do to both sides? [$d + 123 = 369$]
A	Kelly:	Uh, I did negative 123.
V	Teacher:	Minus 123.
A	Kelly:	Then negative 123 on the other side.
V, Q	Teacher:	That's 123. Then you drew a line all the way across. Steve, what did you do next?

- A Steve: Um. I got uh, (inaudible)
- V, Q Teacher: Nice. Six, four, two, is that right? (246 from right to left)
- A Steve: Uh huh.
- V, R, Q Teacher: OK. I think we're starting to grasp this. Okay? This is review. This is something we've seen. Now, draw a line on your paper. We're going to separate what you just did. ...Now guess what kind of problems we're going to be doing?
- A Students: Multiplication and division.
- V Teacher: Multiplication division. Everyone in this room knows that multiplication and division are different than addition and subtraction. Okay?
- [$3x = 12$]
- R, Ex Alright. Let's take a look. Number one. Three x equals twelve. Now I can tell you the answer, but I want you to show me the work. This is what you missed when you weren't here. Now the answer we know is four. Because three times four is twelve. See how that works?
- Q Now how do you actually have to show your math? Is this a times problem right here, or a division problem?
- A Students: Times.
- V, Q Teacher: Times. Is there a symbol in between that three and that x ? Do you see one in there?

- A Students: No.
- V, Q Teacher: I don't see one in there. There's nothing there. If you want, you can put a dot and dot means times, if that makes you happy. What's the opposite of multiplication?
- A Chris: Division.
- V, Q, Ex, Q Teacher: Division. Did you know every fraction is a division bar? Isn't that weird? Divide by the thing in front of the x ; three. I'm going to divide by the same thing on both sides. So wait a second. What?
- Q Nick: You mean you have to.. (inaudible)
- R, Q Teacher: So what did I mean by that? Let's take a look here. Three over three. Everything that's the same over something that's the same is going to be what?
- A Ashley: One.
- V, Q Teacher: One. Three over three is one. What's one times x ?
- A Ashley: x
- V, Ex, Q Teacher: It's just x . So that's our way of getting rid of those threes. OK. So I did some fancy math, and now I have only an x . That's what we're trying to do, get x by itself. So now we have x here. Twelve divided by three is?
- A Ashley: Four.
- V Teacher: Four. And you're done. That's what I need to see from you.

This is an example of teacher-led discourse, punctuated by short student responses. In this sequence of 25 turns, the teacher's contribution accounts for 13 turns. Eleven of the twelve remaining turns were brief responses from students and one turn was a student asking a clarifying question. The basic turn-taking pattern is teacher turns alternating with student turns, where every student turn is directed towards the teacher. Four distinct students each made 1-3 short responses to teacher questions, while multiple students responded to three teacher questions.

Coding for the discursive function of the utterances revealed the following pattern: QAVAVQAVQAVRQAVRExQAVQAVQAVQExQQRQAVQAVExQAV. Here there are ten separate occurrence of the sequence Q-A-V, question-answer-verify, which is the characteristic pattern of teacher-directed discourse (Kysh, 1999). This is the same basic pattern identified by Mehan (1979) as the basic sequence of teacher-led direct instruction; also known as I-R-E, or inquiry-response-evaluation. The analysis also shows three instances of explanations provided by the teacher, with no explanations contributed by students.

Transcripts from the alternative high school setting exhibited a similar discursive pattern during most lessons. Again, the whole class discussion was characterized by teacher-led questioning and brief student responses. An example of discourse from a typical lesson in the control group classroom is provided in Transcript Excerpt 2.

Transcript Excerpt 2. Whole Class Instruction, Alternative School

R, Q	Teacher:	Let's look at number one. The equation says $7x = 56$. So what do we need to get rid of to get x by itself?
A	Justin:	Divide, you need to get rid of the 7.

- Q Teacher: So what are you going to divide? $7x$ means...?
- A Justin: Seven times
- V, Ex, Q Teacher: Seven times. The opposite of multiplication is division. OK? So we're fine with number one, number two, number three. Where did you guys start to have a problem?
- A Justin: After we started with the fractions.
- Q Teacher: At the fraction ones? Ben?
- A Ben: One and one
- V, Q Teacher: OK. You went way far. That was the first time you started having a problem? You're good to go buddy. Did you have a place earlier on where you ran into some problems? No? Then you're doing good. Matt?
- A Matt: Uh, I don't know how to do the fraction ones.
- V, R Teacher: The fraction ones. So starting with number ten.
- A Matt: Yeah.
- $$\left[2d = \frac{7}{8} \right]$$
- Q Teacher: Okay. Now if I just said, I get some equation with something over here, and you're trying to get d by itself. What are you going to do to both sides?
- A Matt: You'd...uh...sub-, wait what?
- Q Teacher: To get d by itself. What do I need to do?

- A Ben: Divide.
- A Matt: Divide.
- Q Teacher: Why would you divide?
- A Matt: Because it's times.
- V, Ex, Q Teacher: Because that's multiplication. So to get rid of the 2 you would divide by 2. OK so our problem looks like this.
Make sense?
- A Sierra: Yeah.
- V, Q Teacher: So to get rid of the 2 we need to divide both sides by 2; $\frac{7}{8} \div 2$. But when we have a fraction, if we divide by 2, what else could we do that would be the exact same thing?
- A Matt: Multiply by a half.
- V Teacher: Multiply by a half. ...Multiply straight across, and there's your answer.

In this sequence of 24 turns, the teacher was the speaker for 12 turns. Short responses by four different students accounted for the remaining 12 turns. Again, the discourse was teacher-directed, with the teacher speaking in between student turns on all but one occasion when staggered responses from two students supplied the same answer. All student contributions to the discourse were directed to the teacher, and all mathematical explanations were provided by the teacher.

In this case, coding for discursive function of utterances revealed the following pattern: RQAQAVEQAQAVQAVRAQAQAAQAVEQAVQAV. This sequence

shows six occurrences of the Q-A-V pattern and four occurrences of an abbreviated Q-A pattern. Again, the repetition of this basic sequence characterizes the pattern as another example of teacher-directed discourse.

These examples indicate that a similar type of instructional discourse occurred in both settings. A similar discursive pattern was also observed during the whole class instruction in both classrooms using the mathematics dialogues. This reflected the fact that both classes in each setting received similar whole class instruction taught by the same teacher. Tapes and transcripts showed that both teachers generally went over the same sample problems for their central lesson in both classes. In the alternative school classrooms, the selection of specific sample problems sometimes varied to reflect different student questions in each class, but these were still presented in a similar fashion. Based on the observations and recordings made in each setting, teacher-directed discourse characterized the whole class instruction in all four settings.

However, it should also be noted that there were some variations in this pattern. Students occasionally made comments about another student's response or volunteered examples of something the teacher was explaining. Sometimes students answered the teacher differently which led to brief arguments between students. However, these occurrences were intermittent and occasional, rather than the norm. For example, during one lesson in the traditional high school classroom, three students disagreed about whether three negatives in a sentence would make it positive or negative. After voicing their disagreement for eight turns, including three attempted explanations, the teacher resumed questioning and guided them to the correct answer.

- V, Q Teacher: You've got to divide. What do we divide by?
- A Jacob: One over 3.5.
- V, R Teacher: Divide by 3.5. Just get out a calculator and divide 4.2 by
3.5. That's all there is to it.

This tutoring exchange began with the teacher responding to a student, Jacob, who had called the teacher over. The student's exact question was not audible on the tape, but the teacher's response indicated that it had to do with problem number thirteen in the assignment. The student appears to have been uncertain about which number to divide by or whether to multiply or divide by the reciprocal. Another student, Sierra, commented by offering an incorrect answer. The teacher responded by asking clarifying questions to help the students identify the right strategy for the problem.

The whole exchange was only 11 turns long, with both the teacher and initiating student taking five turns each. With respect to turn-taking, it is more balanced than the examples of whole class instruction discourse examined above. In this case, the student initiated the exchange rather than the teacher, making it somewhat more student-directed as well. Discursive function coding for this exchange renders the following sequence: QAQVQA RQAVAVQAVR. Here the Q-A-V pattern appears twice in the latter part of the exchange and another variation, Q-A-R, occurs once when the student's answer led the teacher to redirect his line of questioning. Accordingly, the exchange appears to shift back into the familiar pattern of teacher-directed discourse after the student's initiation of the sequence.

While tutoring exchanges like this were observed in both settings, and with greater frequency in the alternative school setting, most of these were not picked up by

the classroom recordings due to the low voices commonly used in these one-on-one teacher-student interactions.

Informal Student Discourse

A third type of discourse observed in these settings was informal conversations between students, often of a social nature, but occasionally concerning clarification or help on mathematical problems. In the traditional school setting, informal student discourse occurred in both classes at times when all of the students were at the board together and at times during lessons when some students completed a task before others and conversed as they waited for the lesson to resume. Some students also asked peers for feedback on specific problems as they began homework corrections or started new assignments at the end of the class. In the alternative school setting, informal student discourse occurred in both classes between students seated together at the same table, or adjacent tables. Again, students sometimes asked one another for feedback on specific problems or engaged in social conversations.

Most instances of informal student discourse about mathematics problems were very brief exchanges with one student asking about a specific problem and another student answering. Transcript Excerpt 4 shows some examples of informal student exchanges.

Transcript Excerpt 4: Informal Student Discourse

(16:30)

Q Kyle: What'd you guys get for number two?

A Ben: Negative eight.

(20:00)

- Q Kim: What did you get for five?
- A Laura: 12.8.
- V Kim: 12.8, so you like... (points to her paper)
- V Laura: Yeah.

In both of these instances, the initiating student appears to have been asking about a specific problem in order to verify the answer or to verify a problem solving strategy. The basic patterns, Q-A and Q-A-V, are similar to the pattern seen in teacher-directed discourse, but here the teacher role was assumed by a student and the exchange is student-directed.

Other informal student exchanges were somewhat longer and involved more peer-tutoring or explanations than those exhibited above. This type of exchange can be seen in Transcript Excerpt 5.

Transcript Excerpt 5: Informal Peer Tutoring

- Q Mike: What'd you get for eleven?
- [$-104 = 8x$]
- A Sarah: Think about it. All you do is divide negative 104 by 8.
- V Mike: Really?
- Ex Sarah: You do the opposite of what they make you do here. This is multiplied, so you divide.
- V Mike: Oh.

Here, Sarah replied by explaining the problem to Mike, rather than simply supplying him with an answer. While this exchange is not much longer than the one between Kim and Laura, it is quite different with respect to the mathematical content being expressed.

Whereas the previous exchanges simply compared answers, this exchange includes a student describing the step needed to solve a specific problem, and explaining this by articulating a general problem solving strategy. Again, the basic discursive pattern, QAVExV, resembles those in teacher-directed discourse, but in this case the exchange is entirely student-directed. This type of peer-tutoring exchange was observed to occur much less frequently than the simple verification exchanges described previously. Moreover, the majority of these peer-tutoring exchanges took place during the group activities in the discourse classrooms.

Nevertheless, informal student discourse did occur to some extent in all four classes. Like the tutoring exchanges, these exchanges were often hard to pick up on the recordings, either because students spoke to one another quietly in lowered voices or because the sound of multiple conversations during board work episodes drowned out the details. However, the direct observations made by the researcher during the study indicated that on most days in the traditional high school setting, several students helped one another during the board work tasks. And, on most days in both settings, students engaged in some amount of informal student discourse during their individual work on assignments. The latter occurrences were also more frequent in the alternative school setting where students spent a larger portion of their instructional time completing assignments and most students sat together at shared tables.

Small Group Discourse

In addition to the three types of discourse found common to all four classrooms, the two classrooms that used mathematics dialogue activities also exhibited a fourth type of discourse that occurred between students during small group activities. In these

classrooms, students were instructed to work in groups to answer questions and mathematics problems after reading each script. Transcripts from both dialogue classrooms indicated that the resulting small group discourse varied considerably, with different groups exhibiting a range of features from minimal exchanges with extensive periods of silence to longer episodes of peer tutoring or shared inquiry among students. Accordingly, small group discourse was not a homogenous category, but included several different forms, or discursive patterns, which are outlined below.

Silence-verification. One common form of small group discourse was for students to work on group activity questions individually and then briefly checked their answers with other group members. These episodes were characterized by long periods of silence in between brief exchanges to compare answers or clarify instructions. Transcript Excerpt 6 illustrates this type of interaction with a segment of student discourse from the second mathematics dialogue activity at the alternative school setting.

Transcript Excerpt 6: Silence-Verification Pattern

R	Jess:	OK. Identify something positive that in each character contributed to the group.
A	Bill:	They all talked together.
V	Jess:	Uh-huh.
Q	Katie:	What was number one?
A	Bill:	Because they got the question right.
Q	Jess:	Do they want us to solve number two and number three?
A	Bill:	Yeah.
V	Jess:	OK.

- (quiet as students solve problems)
- A Jess: Six.
- A Bill: Six.
- V, Ex Jess: $n = 6$. Okay, $16 = -8w$. Divide by -8 , divide by -8 .
Sixteen divided by -8 is negative two... $w = -2$. Bam.
Okay, there you go.

In this excerpt, all three students contributed to the discourse, but did little more than share answers. Jess appears to have led the discussion, while Katie asked one question and Bill supplied most of the answers. On the first math problem, both Bill and Jess solved it independently and offered the same solution. Jess then solved the second problem aloud, providing the solution and answer for her group. Out of a total of eleven turns, Jess took six and Bill took four. The discursive sequence, RAVQAQAVAAVEx, again resembles teacher-directed discourse, but in this case the role of the teacher was assumed by a student, Jess.

Sharing answers. At other times when students appeared to be discussing group questions more actively, the transcripts revealed that students were really just reading the questions and then accepting the first answer someone offered. The discourse in these episodes was characterized by students supplying one another with answers to get a task done quickly. Transcript Excerpt 7 displays this in a discourse segment from the first mathematics dialogue activity in the traditional school setting.

Transcript Excerpt 7: Answer Sharing in Small Groups

- Q Aaron: Did they figure out the problem correctly?
- A Nick: For real.

- Q Aaron: Was there a problem in this?
- A Carl: I don't know.
- A Nick: Well we didn't know the problem, so we couldn't solve it.
Just put yes.
- V Aaron: Yes.
- V Nick: Yes, indeedy.
- R Aaron: We are smart.
- R, Q, A Carl: Explain. How did we? Because it is right.
- R Aaron: Complete the other two problems with the students.
- Q Carl: What?
- R, A Aaron: $7 + x = 26$. Okay, so just minus seven. $26 - 7$, that's 19.
- Q Carl: What?
- Q Nick: Oh yeah?
- A Aaron: You have to know how to do this, dude.
- A Carl: Yeah.
 $[m - 12 = 30]$
- Ex, A Aaron: Plus 12, plus 12, $m = 42$. There.
- R Nick: Here, let me see that.
- R Aaron: No, no. It's easy. You've got to understand it.
- A Carl: Deal.
- Q Teacher: Are you guys working on the questions?
- A Carl: We're done.
- A Aaron: We finished too.

Here, three students engaged in discourse and complete the tasks of answering the questions, but in doing so made little effort to understand the questions or provide serious answers. Instead of looking to the script for clarification on the question of whether the characters solved their problem correctly, they simply dismissed the question and wrote “yes.” Similarly, instead of explaining their reasoning, they inserted, “because it is right.” One student, Aaron, did complete two algebra problems, but the other students only asked about his answers without attempting the problems themselves.

Again, the group discourse in this segment was characterized by answer sharing. Aaron appears to have led the conversation by redirecting its focus to specific questions and solving both math problems. The discursive sequence, QAQAAVVRRQARQRAQ QAAExARRAQ AA, has several repeated codes, few verifications and only one explanation. This suggests a conversation where speakers echo one another’s statements and are not concerned with verifying or explaining their answers.

Small group tutorials. Another common occurrence during the small group discourse activities was for students to call the teacher over to ask for help instead of directing their questions to the other students in their group. The teachers usually responded by listening to the question but then redirecting it to the group. At times the teacher stayed nearby to monitor or facilitate student interaction, which sometimes led to group tutorials with the teacher facilitating the discourse between two or more group members. One example of this type of teacher-group interaction is shown in Transcript Excerpt 8.

Transcript Excerpt 8: Small Group Tutorial

R Teacher: Mandi, explain number seven.

- A, Ex Mandi: Alright. So to do the opposite, you plus nine to the negative nine. Cross that out. Then you do it to this side, minus 13 plus nine, so that equals negative four.
- Q Teacher: Do you see what she saying? How do you get rid of minus nine?
- A Luke: You add it.
- R, Q Teacher: So underneath it why don't you write plus nine? What do you do to the other side then?
- A Luke: I don't really understand.
- Ex Teacher: Well, whatever you do to one side, you've got to do the exact same thing.
- V Luke: Oh, okay, okay.
- Q Teacher: So what are you going to write down?
- A Luke: Plus nine.
- V, Q, A, R Teacher: Plus nine. You see that? Negative four. OK, you've got some of these done. Why don't you keep working on this column? Talk about it.
- Ex Mandi: So, that's a positive three, so it's a negative three.
- Q Luke: Take that away from eight?
- V, A Mandi: Yeah, so that'd be five.
- V Luke: Five.
- R, Q Mandi: Plus fifteen to both sides. So it'd be negative 24?
- Ex Plus seven is twenty. (Quietly) Plus six, plus p equals...

A Mandi: ...Negative ten, eleven.

R Teacher: (Quietly) Keep talking, keep talking, keep talking.

This episode began with the teacher asking Mandi to explain the problem to the other students in her group, which included Luke and another student, James, who remained silent. Mandi explained, but then the teacher intervened to help Luke for another nine turns before returning the discourse to Mandi and Luke. Mandi and Luke continued to work together, with Mandi helping Luke and talking aloud as she attempted to figure out the next problems. The teacher was still nearby listening and quietly encourages the students to keep talking as the discourse began to trail off.

This example of group discourse has several important features. While part of the segment was teacher-directed, the students were also given responsibility to explain and help one another. Like the peer tutoring exchange in Transcript Excerpt 5, this exchange also involved a student explaining the mathematical content instead of relying on the teacher to do this. In this case, Mandi explained the problem and later continued to guide Luke as she moved on to additional problems. The teacher helped make this happen by directing Mandi to take this role, which also legitimated her as a source of mathematical knowledge. He then removed himself from the conversation, making room for the students to take over and direct the discourse on their own. Unlike the previous small group episode, Luke and Mandi both made a serious effort to figure out mathematical problems.

Collaboration. An additional form of discourse that was observed in several small groups was collaborative problem solving. This type of exchange was characterized by a more balanced interaction between students, where more than one

student offered explanations or tentative steps in the effort to answer questions or solve problems. An example of this type of discourse is included in Transcript Excerpt 9.

Transcript Excerpt 9: Collaborative Problem Solving 1

$$[15 + b = 23]$$

R, [1] Tara: OK. Number one, the opposite; you're trying to get rid
Ex of... Number one, you're trying to get rid of the fifteen
over there. So in order to do that, you have to subtract.

$$[26 = 8 + v]$$

Q [2] Jake: What plus eight equals 26?

V [3] Rob: Twenty-six minus eight. Uh, wait; uh wait.

Ex [4] Jake: That's what I look at.

V, Q [5] Rob: Twenty-six minus eight. What's twenty-six minus eight?

A [6] Jake: Eighteen right?

V [7] Rob: Eighteen, yeah, so

A [8] Rob, Jake: (in unison) v equals eighteen.

$$[3 + p = 8]$$

Q [9] Jake: Three plus what equals eight?

A [10] Rob: Four?

Q, A [11] Jake: Eight? No, five.

$$[15 + b = 23]$$

Q [12] Rob: Twenty-three minus fifteen equals what?

[13] Jake: Oosh, you guys want to run with all those big
numbers...

- A [14] Rob: Oh yeah, yeah.
- [15] Tara: hmm
- R [16] Rob: b equals...
- A [17] Jake: Eight, right?
- [18] Tara: (inaudible)
- V [19] Jake: Mm hmm. Thirty.

In this episode three students were working together to solve a series of problems after completing the dialogue activity. While Tara initiated the discourse, she then faded into the background as the other two students worked aloud to figure out a series of addition and subtraction steps they needed to complete a set of one-step algebra problems. Here, both students asked questions, and offered suggestions and answers to assist one another. This example of collaboration between students is distinct from the peer tutoring episodes in that none of the students took the leading role of tutor to the exclusion of the others. Instead, students shared this role and took turns leading the discourse at different times.

This interaction pattern can be seen more clearly using an interactivity flow chart, as developed by Sfard (2002) and Kieran (2002). An interactivity flow chart of the conversation between Tara, Jake and Rob is presented in Figure 3. Here the bracketed numbers represent turn numbers corresponding to the discursive sequence presented in Transcript Excerpt 9.

This flow chart illustrates the high degree of interactivity between Jake and Rob. Arrows pointing downwards indicate proactive statements that guide the conversation, while arrows pointing upwards indicate reactive comments that respond to something said previously. In the beginning of the exchange, Jake took the lead with more proactive

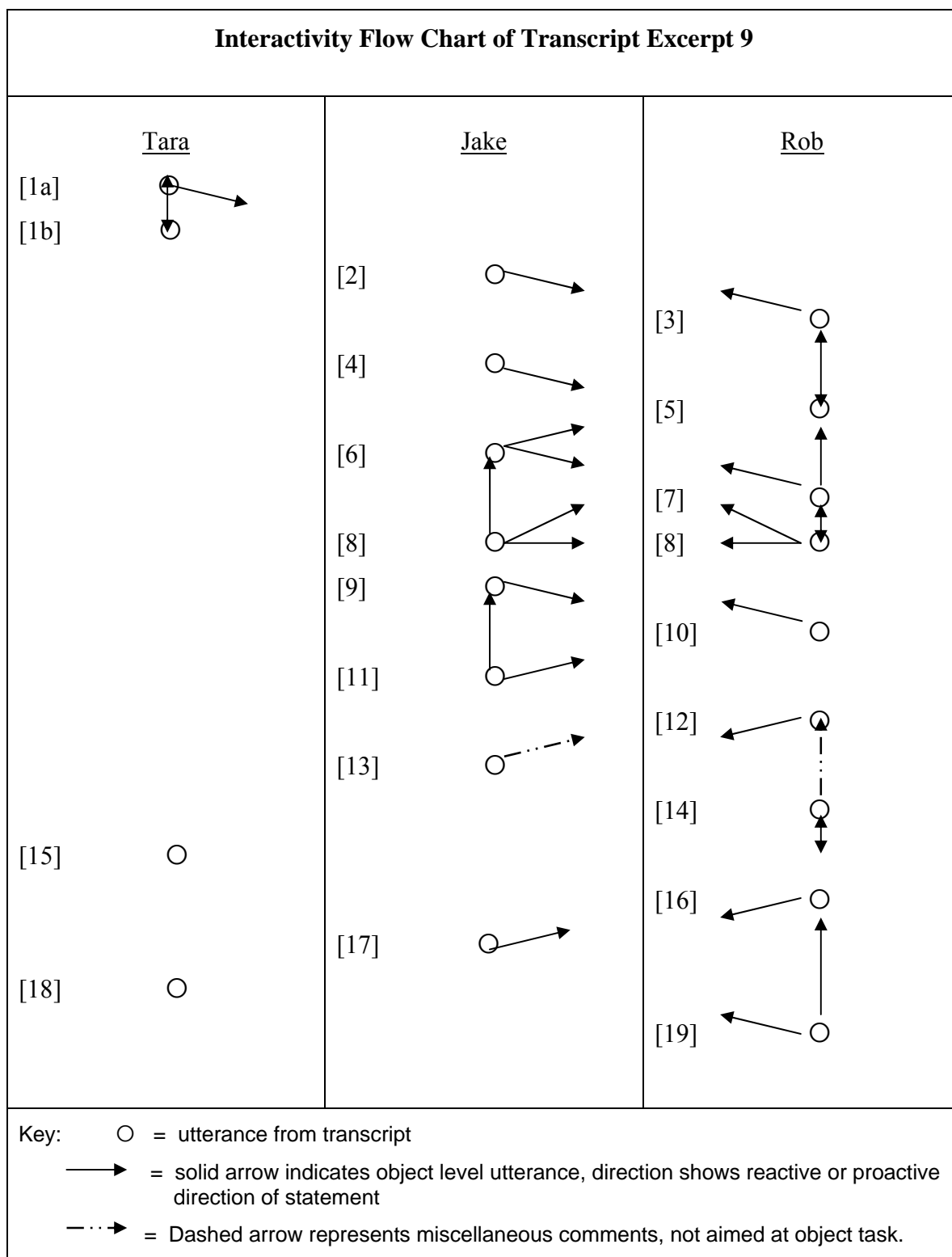


Figure 3. The interactivity flow chart illustrates the interaction pattern between Tara, Jake, and Rob from Transcript Excerpt 9.

comments, while Rob responded. Later in the exchange Rob took the lead and Jake responded. The solid arrows illustrate further that these students stayed on task, discussing the mathematics problems they were solving rather than some other topic. The pattern between the two boys therefore indicates a balanced exchange characteristic of small group collaboration. Tara, on the other hand, contributed little to the exchange after her initial statement that set the group to work. Although she made some indistinguishable sounds or comments near the end of the segment, she did not really take part in the collaboration even though she was part of the same group.

Compared to the other forms of small group discourse, occurrences of collaboration between students were rare. However, some instances of collaboration were evident in the transcripts from both settings. Most of these were also fairly short, like the exchange between Tara, Jake and Rob, which lasted for only 19 turns.

Overview: Small Group Discourse. Each of these four distinct patterns of small group discourse -- silence-verification, answer sharing, small group tutorials, and collaboration -- was observed repeatedly in both discourse classrooms. Of these, the most common patterns were silence-verification, answer sharing, and small group tutorials. There were also instances of peer tutoring between pairs of students within small groups and some instances of individual tutoring by the teacher.

Most of the recordings from both discourse classrooms included extended periods of silence or near-silence in between the transcribed episodes. These were periods when students worked independently in spite of the instructions to work together. This included occasions when students returned to their seats as soon as they had finished reading the scripts, not realizing that they were supposed to complete the questions as a group, and

occasions when students completed the activity questions quickly and cursorily and then resumed working individually. In general, it appeared that these students were unfamiliar with group activities and simply followed the routines they were accustomed to. Students expected to work individually on mathematics problems and expected the teacher, rather than other students, to answer their questions. However, every student group in both classes still completed the activities and participated in some form of small group discourse.

The teachers in both discourse settings also contributed by circulating around the groups and redirecting students who were working independently or off task. Teachers sometimes answered questions or raised questions about the group's progress while encouraging the students to work together and talk about the problems they were doing. These redirections often prompted students to resume peer-tutoring or answer sharing exchanges. This also gave students an opportunity to ask questions, which sometimes initiated group tutorials or individual tutoring exchanges. Tutoring exchanges during discourse activities were more common in the alternative school setting where this reflected the established instructional routine.

Summary: Characteristics of Classroom Discourse in Each Setting

The classroom discourse in both control group classrooms was characterized by a combination of teacher-directed whole class instruction, tutoring exchanges and informal student discourse, including answer-verification exchanges and peer-tutoring exchanges. Teacher-directed whole class instruction was more prominent in the traditional school setting, where it comprised 30-40 minutes of each class, as compared to about 10 minutes per class in the alternative high school setting. Tutoring exchanges were more prominent

in the alternative school setting where this was the primary form of instruction after a short introductory lesson. The tutoring exchanges accounted for 20-25 minutes of class in both settings. Informal student discourse also occurred intermittently in both settings. In the traditional school this usually took place during board work and sometimes during individual work, while students at the alternative school occasionally helped one another on practice problems.

The classroom discourse in the two treatment group classrooms was characterized by similar elements as were found in the control group classrooms from corresponding sites, but also included an additional component of small group discourse. In the traditional school setting, small group discourse lasted 15-20 minutes each day. In the alternative school setting, the activities lasted about 15 minutes each day, but students remained seated in their groups and continued to engage in small group discourse intermittently throughout the period. Small group discourse in both these settings included several identifiable forms described as silence-verification, answer sharing, group tutorials, and collaboration. Of these, tutoring exchanges were somewhat more common in the alternative school classroom. Student objections and negative comments about assignments were also a unique feature of this alternative school classroom.

Discursive Indicators of Student Learning and Attitudes

The third objective of transcript analysis was to identify potential indications of student learning and attitudes. Potential discursive indicators of student learning included student questions and responses to questions, student explanations of mathematical problems or concepts, and direct expressions of insight. Indicators of student attitudes included direct statements by students or observed behaviors. A summary of the

indicators observed in each classroom is presented below.

Traditional School Control Group Classroom

Student learning. In this classroom student learning was expressed most often through student responses to teacher-led questions. This enabled the teacher to identify whether students understood the new problem solving strategies. Student questions were also very informative, and sometimes showed that students were trying to connect new procedures to their prior learning. Student explanations of concepts were not frequent but occurred at various times during whole class instruction, when students offered an analogy or explained their steps on a problem, and during board-work, when students had an opportunity to help one another informally. Direct expressions of insight like, “oh, now I get it,” commonly ended tutoring exchanges and sometimes occurred during whole class instruction. Tutoring exchanges also provided additional opportunities for student questions and responses.

Student attitudes. There was little direct discursive evidence of student attitudes in this classroom. Most students were very attentive throughout the class and quick to comply with instructions. They seemed to like their teacher and laughed at his jokes. They were also very animated and talkative during board work.

Traditional School Discourse Classroom

Student learning. This classroom had similar indicators of student learning as the control classroom in this setting, but also provided more opportunities for student explanations, questions, and expressions of insight by virtue of the additional small group discourse activities. Peer tutoring episodes during these activities generally began with a student question, allowed another student to explain, and ended with an expression of

insight. Since student explanations displayed their understanding of concepts and procedures, this classroom had more discursive indicators of student understanding. Informal verification exchanges and answer sharing during group activities also provided additional indications of how students understood the questions and procedures they were working on. During follow-up discussions students expressed differing views concerning the learning value of the scripts; some indicated that the scripts were helpful while others found them to be confusing.

Student attitudes. Discourse in this classroom also provided limited evidence of student attitudes. There was one observation of a resistant attitude during direct instruction when a student did not respond to teacher-led questioning. There were also direct statements related to attitudes during the dialogue activities and follow-up discussions. One student expressed that he was bored and one stated that he didn't like activities that involved reading. Again, most students were very attentive during lessons and appeared to have positive attitudes, much like the other class in this setting.

Alternative School Control Group Classroom

Student learning. In this classroom, tutoring exchanges and problem corrections provided were the most prominent indicators of student learning. Tutoring exchanges were often initiated by student questions and provided opportunities for additional student responses, explanations, and expressions of insight that revealed their understanding of concepts and procedures. Problem corrections at the end of the period allowed students to report their problem solving strategies and solutions for different problems. Teacher-directed lessons provided additional opportunities for student responses and questions, while informal peer tutoring during practice work included

questions, explanations and expressions of insight.

Student attitudes. There was little discursive evidence of student attitudes in this group. In general, most of the students in this class appeared to exhibit positive attitudes. Most students were attentive and volunteered answers during lessons, and appeared to stay focused during their assignments. A few students who sat in the back were less frequent contributors and seemed to get sidetracked more easily, which led to more redirections from the teacher. However, there were no major expressions of resistance observed in this group. One student complained that the teacher always asked her to explain the difficult problems, but this was more in the spirit of playful banter than a serious objection. Some students were also occasionally side-tracked by social discussions, but seemed to respond well when redirected by the teacher.

Alternative School Discourse Classroom

Student learning. Indicators of student learning in this classroom were similar to the other class in this setting, but included more opportunities for student explanations during small group activities. Unlike the regular tutoring exchanges between teacher and student, peer tutoring and group tutorials generally included more instances where students did the explaining. Informal verification exchanges and answer sharing during group activities also provided additional indications of how students understood the questions and procedures they were working on. In this classroom, too, a number of students expressed that they thought the activities were too easy or found the scripts more confusing than just doing problems on their own. These comments also indicated that these students were fairly confident in their understanding of the basic problem solving steps.

Student attitudes. This classroom had the most discursive evidence of student attitudes, which consisted mostly of negative comments about the scripts or discussion questions and objections to group work and mathematics in general. Students also expressed the view that group work would be more helpful on harder questions. Other observations indicated that the most outspoken critic of the activities was also the most frequent peer-tutor in the class, and often increased her tutoring activity after having made her objections clear. Other students in this group exhibited trust issues, resistance to showing work on problems, and one pair of students was repeatedly side-tracked with social conversations. At the same time, several other students worked quietly and stayed on task most of the time. Students were also generally attentive and responded well during teacher-led lessons. In general, this was the most complex group and exhibited a mixture of positive and negative attitudes towards mathematics.

Summary: Indicators of Student Learning and Attitudes

In summary, the observations and transcripts in each classroom exhibited a variety of discursive indicators of student learning and attitudes. This included student questions and responses, explanations of mathematical problems or concepts, direct expressions of insight, direct expressions of attitudes and observed behavior patterns. While the variety and frequency of indicators varied according to the instructional routines in each setting, the classrooms that used the mathematics dialogue activities had more occurrences of student explanations and other forms of small group discourse that displayed student understanding. Whole class discussions associated with the activities also provided insight into student perceptions of learning. With respect to attitudes, most of the classrooms exhibited predominantly positive attitudes and attentive behaviors. The

one exception was the discourse classroom in the alternative school setting, where students expressed more resistant attitudes towards both the activities and other aspects of the class. This group was the most challenging to work with, but also provided the richest information about their perceptions of mathematics.

Chapter Summary

This chapter has provided an overview of the implementation of this study in four distinct classrooms, from teacher contact to the completion of the discourse intervention. This description provides detailed information about differences between the instructional settings, how the study was actually implemented in these settings, and characteristics of student discourse in each setting. All of these factors are important to consider when interpreting whether measurable student learning outcomes in these classrooms can be reasonably attributed to the mathematics dialogue intervention as applied in this study.

CHAPTER FIVE

RESULTS

The study took place in two pairs of prealgebra classrooms at two different school sites. One teacher at each school taught both participating classes, one in their usual style and one using the mathematics dialogue activities provided by the researcher. These activities were employed as lesson supplements, allowing both classes in each setting to receive similar forms of instruction. This chapter reports on the resulting student learning outcomes in each setting in terms of achievement, problem solving, and attitudes and presents the results of student interviews conducted several weeks after the intervention.

Achievement and Problem Solving

Pre-tests on achievement and problem solving were given on the class day immediately preceding commencement of the problem solving unit. These assessments were combined as a single 22 item test with 20 multiple choice items to measure achievement followed by two constructed response items to measure problem solving skill. Post-tests followed a similar format and were given at the close of the problem solving unit as a unit test.

For the purposes of this study, the achievement problems were scored based on students' numbers of correct answers, without reference to work shown. The constructed response problems were scored using a four scale rubric based on the *Mathematics Problem Solving Scoring Guide* distributed by the Northwest Regional Educational Laboratory (2000). Each constructed response solution was awarded from zero to three points in each of four categories: conceptual understanding, strategies and reasoning,

computation and execution, and communication. Each constructed response problem was therefore worth a maximum of 12 points, while the problem solving scores reported represent each student's percentage of points earned out of the twenty-four points possible for both problems (*See Appendix E*).

Students' test scores were analyzed to compare the mean gains in achievement and problem solving for each group, and to provide classroom level comparisons for each teacher. Mean gains were calculated as the average of individual gains, which were measured as the difference between pre-test scores and post-test scores for each student. Accordingly, the reported findings reflected only those students who completed both tests. Due to small class sizes and a high number of absences during the study, sample size was smaller than anticipated, precluding the use of more powerful Analysis of Covariance (ANCOVA) tests to calculate experimental importance and consistency.

Achievement

The total number of students who completed both pre-tests and post-tests for achievement was 22 in the group using the mathematics dialogue activities and 26 in the control group. Therefore, the analysis of student gains was conducted by assessing the mean individual gains in each group. Individual gains were calculated by subtracting pre-test scores from post-test scores for each individual. The average achievement test scores and mean gains for each group are reported in Table 1.

Students in classrooms using the mathematics dialogue activities had mean pre-test scores of 57% and mean post-test scores of 78%, while students in the control group classrooms had mean pre-test scores of 64% and mean post-test scores of 69%. The analysis of individual achievement gains indicated that students in classrooms using the

Setting	<u>Control Group</u>				<u>Discourse Group</u>			
	Pre	Post	Gain	n	Pre	Post	Gain	n
Combined	64%	69%	5%	26	57%	78%	21%	22
Traditional	62%	60%	-2%	16	54%	76%	22%	14
Alternative	67%	85%	18%	10	63%	81%	18%	8

mathematics dialogue treatment had mean gains in achievement of 21% and students in the control group classrooms had mean gains in achievement of 5%. Based on this comparison, the average gain in achievement among students in classrooms using the mathematics dialogue activities was 16% greater than that of students in the control group classrooms.

Subgroups: Achievement Comparisons by Setting

Traditional High School. In the traditional high school setting, the total number of students who completed both pre-tests and post-tests was 14 in the classroom using mathematics dialogue activities and 16 in the control group classroom. Students in the classroom using the mathematics dialogue activities had mean pre-test scores of 54% and mean post-test scores of 76%, while students in the control group classroom had mean pre-test scores of 62% and mean post-test scores of 60%.

An analysis of individual gains showed that students in the classroom using the mathematics dialogue activities had mean gains in achievement of 22%, while students in the control group classrooms had mean gains in achievement of – 2%. In other words, the average achievement tests score of students in the control group classroom dropped by 2% after they completed the instructional unit. Based on this comparison, the average

gain in achievement among students in classrooms using the mathematics dialogue activities was 24% greater than that of students in the control group classrooms. However, the reported decrease in achievement among students in the control group classroom is unusual and raises questions about the validity and reliability of these post-test scores. As noted in the previous chapter, the post-test in this classroom was administered to students by a substitute teacher on the morning after a long weekend. Accordingly, these scores appear to reflect an inconsistent testing condition and cannot be assumed to provide for a valid comparison of achievement between the two classrooms.

Alternative High School. In the alternative high school setting, the number of students who completed both pre-tests and post-tests was eight in the classroom using mathematics dialogue activities and ten in the control group classroom. Students in the classroom using mathematics dialogue activities had mean pre-test scores of 63% and mean post-test scores of 81%, while students in the control group classroom had mean pre-test scores of 67% and mean post-test scores of 85%. An analysis of individual gains indicated that students in both of these classrooms had mean gains of 18% during the instructional unit.

Problem Solving

The total number of students who completed both pre-tests and post-tests for problem solving was 22 in the group using the mathematics dialogue activities and 26 in the control group. Therefore, the analysis of student gains was again conducted by assessing the mean individual gains in each group. Individual gains were calculated by subtracting pre-test scores from post-test scores for each individual. The average problem

solving scores for each group are reported in Table 2.

Students in classrooms using the mathematics dialogue activities had mean pre-test scores of 32% and mean post-test scores of 56%, while students in the control group classrooms had mean pre-test scores of 34% and mean post-test scores of 47%. The analysis of individual achievement gains indicated that students in classrooms using the mathematics dialogue activities had mean gains in problem solving of 24% and students in the control group classrooms had mean gains in problem solving of 13%. Based on this comparison, the average gain in problem solving scores among students in classrooms using the mathematics dialogue activities was 11% greater than that of students in the control group classrooms.

Subgroups: Problem Solving Comparisons by Setting

Traditional High School. In the traditional high school setting, the total number of students who completed problem solving pre-tests and post-tests was 14 in the classroom using the mathematics dialogue activities and 16 in the control group classroom. Students in the discourse classroom had mean pre-test scores of 35% and mean post-test scores of 55%, while students in the control classroom had mean pre-test scores of 36% and mean post-test scores of 42%.

Analysis of individual gains showed that students in the discourse classroom had mean problem solving gains of 20%, while students in the control classroom had mean problem solving gains of 6%. Based on these results, the average problem solving gain among students in the classroom using the mathematics dialogue activities was 14% greater than that of students in the control group classroom. However, since the problem solving and achievement tests were given at the same time as a two-part exam, concerns

Setting	<u>Control Group</u>				<u>Discourse Group</u>			
	Pre	Post	Gain	<i>n</i>	Pre	Post	Gain	<i>n</i>
Combined	34%	47%	13%	26	32%	56%	24%	22
Traditional	36%	42%	6%	16	35%	55%	20%	14
Alternative	31%	57%	26%	10	27%	59%	32%	8

about the reliability of post-test achievement scores in the control group classroom must be extended to the problem solving post-test as well. Again, the control group tests in this setting may not provide for a valid comparison of problem solving skill between the two classrooms.

Alternative High School. In the alternative high school setting, the number of students who completed problem solving pre-tests and post-tests was eight in the classroom using student discourse activities and ten in the control group classroom. Students in the classroom using mathematics dialogue activities had mean pre-test scores of 27% and mean post-test scores of 59%, while students in the control classroom had mean pre-test scores of 31% and mean post-test scores of 57%. An analysis of individual gains showed that students in the classroom using the student discourse activities had mean problem solving gains of 32%, while students in the control group classrooms had mean problem solving gains of 26%. Accordingly, the average gains in problem solving among students in the classroom using mathematics dialogue activities was 6% greater than average gains of problem solving among students in the control classroom.

Follow-up Assessments

In order to address reliability issues concerning post-tests given in the traditional school setting and provide for a better comparison of achievement and problem solving

skills, an additional follow-up test was developed to assess students' retention of knowledge and skills. This test consisted of ten closed format achievement problems followed by one open-ended constructed response problem. A copy of this test is included in Appendix G.

The follow-up tests were administered approximately five weeks after the unit post-tests in each classroom. These were scored the same way as the pre-tests and post-tests, but in this case problem solving scores were based on 12 possible points rather than 24 points. An additional analysis of student gains in achievement and problem solving was based on the comparison of individual gains from pre-test scores to follow-up test scores for each group and school setting. The number of students who completed both follow-up tests and pre-tests was 20 students in the treatment classrooms and 24 students in the control group classrooms. The average achievement and problem solving scores from the follow-up tests and resulting mean gains for each group are reported in Table 3.

Follow-up Test on Achievement

Overall, students in classrooms using the mathematics dialogue activities had a mean achievement score of 85% on the follow-up test, while students in the control group classrooms had a mean achievement score of 75% on the follow-up test. An analysis of individual achievement gains comparing pre-test scores and follow-up test scores showed that students in classrooms using the mathematics dialogue treatment had mean gains in achievement of 29% and students in the control group classrooms had mean gains in achievement of 11%. Based on this comparison, the average gain in achievement among students in classrooms using the mathematics dialogue activities was 18% greater than that of students in the control group classrooms.

Table 3: Average Scores on Follow-up Tests of Achievement and Problem Solving								
ACHIEVEMENT								
Setting	<u>Control Group</u>				<u>Discourse Group</u>			
	Pre-Test*	Follow-up	Gain	n	Pre-Test*	Follow-up	Gain	n
Combined	64%	75%	11%	24	56%	85%	29%	20
Traditional	63%	79%	16%	14	54%	84%	30%	12
Alternative	67%	70%	3%	10	60%	85%	25%	8
PROBLEM SOLVING								
Setting	<u>Control Group</u>				<u>Discourse Group</u>			
	Pre-Test*	Follow-up	Gain	n	Pre-Test*	Follow-up	Gain	n
Combined	33%	54%	21%	24	33%	59%	26%	20
Traditional	35%	66%	31%	14	35%	75%	40%	12
Alternative	31%	38%	7%	10	30%	35%	5%	8
* Note: Mean pre-test scores reported here reflect the selection of pre-tests that correlated with follow-up tests and therefore vary from those reported in the post-test comparison.								

In the traditional school setting, students in the mathematics dialogue classrooms (n=12) had a mean score of 84% on the follow-up test and students in the control group classroom (n=14) had a mean score of 79%. A comparison of mean gains in achievement from pre-test to follow-up test for this subgroup showed that students in the mathematics dialogue classroom had mean gains of 30% as compared to mean gains of 16% in the control group classroom. Accordingly, this comparison indicates that the average gain in achievement among students using the mathematics dialogue activities was 14% greater than that of students in the control group classrooms.

In the alternative school setting, students in the mathematics dialogue classrooms (n=8) had a mean score of 85% on the follow-up test and students in the control group classroom (n=10) had a mean score of 70%. A comparison of mean gains in achievement from pre-test to follow-up test for this subgroup showed that students in the mathematics

dialogue classroom had mean gains of 25% as compared to only 3% in the control group classroom. This comparison indicates that the average gain in achievement among students using the mathematics dialogue activities was 22% greater than that of students in the control group classrooms.

Follow-up Test on Problem Solving

Overall, students in classrooms using the mathematics dialogue activities had a mean problem solving score of 59% on the follow-up test, while students in the control group classrooms had a mean problem solving score of 54% on the follow-up test. An analysis of individual problem solving gains comparing pre-test scores and follow-up test scores showed that students in the mathematics dialogue classrooms had mean gains of 26% as compared to mean gains of 21% in the control group classrooms. Accordingly, the average student in the mathematics dialogue classroom scored 5% higher than the average student in the control group classrooms.

In the traditional school setting, students in the mathematics dialogue classrooms (n=12) had a mean score of 75% on the follow-up test and students in the control group classroom (n=14) had a mean score of 66%. A comparison of mean gains in problem solving from pre-test to follow-up test for this subgroup showed that students in the mathematics dialogue classroom had mean gains of 40% as compared to mean gains of 31% in the control group classroom. Accordingly, this comparison indicates that the average gain in problem solving among students using the mathematics dialogue activities was 9% greater than that of students in the control group classrooms.

In the alternative school setting, students in the mathematics dialogue classrooms (n=8) had a mean score of 35% on the follow-up test and students in the control group

classroom (n=10) had a mean score of 38%. A comparison of mean gains in problem solving from pre-test to follow-up test for this subgroup showed that students in the mathematics dialogue classroom had mean gains of 5% as compared to 7% in the control group classroom. This comparison indicates that the average gain in problem solving among students using the mathematics dialogue activities was 2% less than that of students in the control group classrooms.

Summary: Achievement and Problem Solving

A summary of the results from the several analyses of student achievement tests scores and problem solving scores is presented in Table 4. Overall results indicate that students in classrooms that used the mathematics dialogue activities outperformed students in the control group classrooms with respect to achievement and problem solving. This pattern also held in the traditional classroom setting, but results in the

<i>Table 4: Summary of Mean Gains in Achievement and Problem Solving</i>				
	<u>ACHIEVEMENT</u>		<u>PROBLEM SOLVING</u>	
	<u>Pre-test to Post-test</u>	<u>Pre-test to Follow-up Test</u>	<u>Pre-test to Post-test</u>	<u>Pre-test to Follow-up Test</u>
<u>Combined Groups:</u>				
Math Dialogues	20%	29%	24%	26%
Control Group	6%	11%	13%	21%
<u>Traditional High School</u>				
Math Dialogues	22%	30%	20%	40%
Control Group	-2%	16%	6%	31%
<u>Alternative High School</u>				
Math Dialogues	18%	25%	32%	5%
Control Group	18%	3%	26%	7%

alternative school setting were mixed, with students in both groups performing equally as well in one comparison, and the control group outperforming the math dialogue classroom on the follow-up problem solving assessment.

Student Attitudes towards Mathematics

Student attitudes towards mathematics were assessed before and after the intervention using a 24-item ranked response survey instrument. This survey was based on Sandman's Mathematics Attitude Inventory (1973), which generates scale scores on six attitude subscales, including (a) perception of teacher, (b) anxiety towards mathematics, (c) value to society, (d) self-concept in mathematics, (e) enjoyment of mathematics, and (f) motivation in mathematics (Sandman, 1973).

The survey used in the present study included 23 items drawn from Sandman's survey and one item developed by the researcher to address student attitudes about working in groups in mathematics class. Several items were edited slightly to reflect the common language usage of contemporary high school students; for example, the statement, *mathematics is something I enjoy very much*, was changed to *math is something I enjoy*. While Sandman's survey included a total of 48-items, with eight items correlating to each subscale, the modified survey instrument includes four to five items on each subscale. The last two subscales, enjoyment of mathematics and motivation in mathematics, were also scored as a single combined measure because the distinction between these was not clear and did not seem relevant to the present study. The attitude survey used in this research is included in Appendix F.

Subscale scores for each respondent were calculated using Sandman's scoring method, but adjusted proportionally to reflect the smaller number of items used to

generate each score. These scores reflect a total number of points indicated by responses on the set of items associated with each subscale. Points on each item ranged from one to four, ascending from strongly agree to strongly disagree. Several items that were identified as reverse-scored items (Sandman, 1973) were adjusted to reflect a descending order of points by adding five and subtracting the assigned item point value. The possible scores on each subscale ranged from four to sixteen or five to twenty, depending whether the given subscale had four or five items. These were stratified as low, medium or high scores.

After calculating five subscale scores for each respondent, the frequency of low, medium and high scores on each subscale was determined for each application of the survey in each classroom. Low scores were those indicating a maximum of two points on each subscale item and high scores were those indicating a minimum of three points on each subscale item. Medium scores were those in between high and low, indicating a mix of high and low points on different scale items. The resulting frequency distributions provide an attitude profile for each classroom both before and after the treatment unit, as well as an overall distribution for the combined treatment group and control group. A separate analysis of student response frequencies was conducted for the one item addressing student attitudes about participating in group activities in mathematics class.

Attitude Survey Results

Overall Findings. Combined data from the control group classrooms at both schools as well as classrooms using the mathematics dialogue intervention at both schools resulted in the following distribution of student attitude scale scores as shown in Table 5. For each subscale, the reported percentages are based on the number of students

who responded to all of the items associated with that subscale. Some surveys were incomplete or included ambiguous responses that were omitted. This led to omission of one or more subscales scores depending on which items were missing.

The pre-test attitude ratings in each group indicate that groups were initially similar with respect to perception of teacher and self-concept in mathematics, with initial differences with respect to anxiety, value to society, and enjoyment/motivation. Students in the treatment group had more frequent low ratings for anxiety by 15%, more frequent high ratings for value to society by 11%, and more frequent high ratings for enjoyment/motivation by 30%.

A comparison of pre-test and post-test attitude distributions for each group indicates that the subscale distributions for perception of teacher and value to society

<u>CONTROL GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	40%	60%	20	0%	41%	59%	22
Anxiety	25%	70%	5%	20	43%	52%	4%	23
Value to Society	10%	38%	52%	21	5%	45%	50%	22
Self-Concept	19%	57%	24%	21	15%	50%	35%	20
Enjoyment/Motivation	30%	60%	10%	20	18%	59%	23%	22
<u>TREATMENT GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
<u>Scale</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	38%	63%	24	0%	29%	71%	21
Anxiety	40%	52%	8%	25	57%	38%	5%	21
Value to Society	0%	38%	63%	24	0%	43%	57%	21
Self-Concept	20%	56%	24%	25	5%	42%	53%	19
Enjoyment/Motivation	20%	40%	40%	25	14%	52%	33%	21
* Combined data from both control group classrooms and both treatment group classrooms.								

remained relatively constant during the intervention in both groups, while shifts of 10% or more occurred in the subscale scores for anxiety, self-concept and enjoyment/motivation. In the mathematics dialogue group, the number of students with high self-concept scores increased from 24% to 53%, a gain of 29%. At the same time, the number of students with low anxiety scores increased from 40% to 57%, a gain of 17%. In the control group, the number of students with high self-concept scores increased from 24% to 35%, a gain of 11%, while the number of students with high enjoyment/motivation scores increased from 10% to 23%, a gain of 13%. The number of students with low anxiety scores also increased from 25% to 43% in the control group, a gain of 18%.

Comparing the changes observed in each group shows that both groups exhibited similar decreases in math anxiety. Self-concept also improved in both groups, but the shift was much more pronounced among students in the discourse group, which exceeded the shift in the control group by 18%. In addition, the control group exhibited an increase of 13% in high scores for enjoyment/motivation that was not paralleled in the discourse group.

Classroom Attitude Profiles

Separate attitude profiles for each classroom allow a closer analysis of attitude changes specific to each setting and instructor.

Traditional School Attitude Profiles. Attitude data from the traditional high school classrooms is presented in Table 6, which shows the frequency of distribution of attitude subscale scores for each classroom. Again, several scores were omitted due to incomplete surveys. In particular, four out of fifteen students in the control group did not complete

<u>CONTROL GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
<u>Scale</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n*</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	30%	70%	10	0%	31%	69%	16
Anxiety	30%	70%	0%	10	50%	50%	0%	16
Value to Society	9%	27%	64%	11	0%	59%	41%	17
Self-Concept	18%	55%	27%	11	6%	65%	29%	17
Enjoyment/Motivation	9%	73%	18%	11	7%	67%	27%	15
<u>DISCOURSE GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
<u>Scale</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	25%	75%	16	0%	13%	87%	15
Anxiety	56%	38%	6%	16	73%	20%	7%	15
Value to Society	0%	25%	75%	16	0%	20%	80%	15
Self-Concept	6%	56%	38%	16	0%	36%	64%	14
Enjoyment/Motivation	6%	31%	63%	16	7%	47%	47%	15
* Several students in control group did not complete second page of survey on pre-test.								

the second page of the survey during the pre-test, rendering the subscales for these survey forms incomplete.

Based on this data, it appears that the discourse classroom and control group classroom in this setting exhibited initial differences with respect to anxiety, self-concept and enjoyment/motivation. In particular, a larger proportion of students in the discourse classroom had low anxiety scores, high self-concept scores, and high enjoyment/motivation scores. Here the most substantial difference was in the number of students rating high for enjoyment/motivation, which was 45% more frequent in the discourse classroom.

A comparison of the pre-test and post-test distributions in these classrooms indicates that the number of students with low scores for anxiety increased in both

classrooms. In the discourse classroom, the number of students with high scores for positive perception of teacher increased by 12%, the number of low anxiety scores increased by 17%, The number of high self-concept scores increased by 26%, and the number of high enjoyment/motivation scores decreased by 16%. These changes appear positive, except for the decrease in enjoyment/motivation. In the control classroom, the number of students with low anxiety scores increased by 20%, the number of high value to society scores decreased by 23%, and the number of low self-concept by decreased by 12%. These changes are positive, with the exception of lower ratings for value to society. Based on this data, the most substantial change in attitudes observed during the study was the 26% increase in high self-concept ratings among students in the discourse classroom.

Alternative School Attitude Profiles. Attitude data from the alternative high school classrooms is presented in Table 7, which shows the frequency of distribution of attitude subscale scores for each classroom. Again, several scores were omitted due to incomplete surveys.

Based on this data, it appears that the discourse classroom and control group classroom in this setting exhibited initial similarity with respect to positive perception of teacher and anxiety, and initial differences with respect to self-concept and enjoyment/motivation. A larger proportion of students in the discourse group rated low for self-concept by 24%, while a larger proportion of students in the control group classroom rated high for self-concept by 20%. In the control group classroom, more students also rated low for value to society by 10%, and more students rated low for enjoyment/motivation by 12%.

<u>CONTROL GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
<u>Scale</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	50%	50%	10	0%	50%	50%	10
Anxiety	20%	70%	10%	10	30%	60%	10%	10
Value to Society	10%	50%	40%	10	11%	44%	44%	9
Self-Concept	20%	60%	20%	10	29%	43%	29%	7
Enjoyment/Motivation	56%	44%	0%	9	30%	60%	10%	10
<u>DISCOURSE GROUP</u>	<u>PRE-TEST</u>				<u>POST-TEST</u>			
<u>Scale</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>	<u>Low</u>	<u>Med</u>	<u>High</u>	<u>n</u>
Positive Perception of Teacher	0%	63%	38%	8	0%	67%	33%	6
Anxiety	11%	78%	11%	9	17%	83%	0%	6
Value to Society	0%	63%	38%	8	0%	100%	0%	6
Self-Concept	44%	56%	0%	9	20%	60%	20%	5
Enjoyment/Motivation	44%	56%	0%	9	33%	67%	0%	6

A comparison of the pre-test and post-test distributions in these classrooms indicates that the only subscale distribution that remained relatively constant in both groups was perception of teacher. In the discourse classroom, the number of students with high anxiety scores decreased by 11%, the number of high scores for value to society decreased by 38%, the number of high self-concept scores increased by 20% as the number of low self-concept scores decreased by 24%, and the number of low enjoyment/motivation scores decreased by 11%. These changes are all positive, except for the substantial decrease in value to society. In the control classroom, the number of low anxiety scores increased by 10%, while low enjoyment/motivation scores decreased by 26% and high enjoyment/motivation scores increased by 10%. These changes all appear to be positive.

Based on this data, the most substantial change in attitudes associated with the mathematics dialogue activities in this setting was a substantial decrease in value to society and a substantial increase in self-concept among students in the discourse classroom.

Student Attitudes towards Group Activities

The final survey item was not drawn from the Mathematics Attitude Inventory (Sandman, 1973) but addressed student attitudes towards group work as versus individual work. Responses to this unique item are presented in Table 7.

These response frequencies indicate that students in three of the four classrooms exhibited more agreement with this statement on the post-test. In other words, more students expressed a preference for individual work at the end of the intervention unit. This included both of the classrooms that used the discourse activities. In fact, the largest

24. I would rather do math problems on my own than in a group activity.					
<u>Traditional HS</u>		<u>Strongly Agree</u>	<u>Agree</u>	<u>Disagree</u>	<u>Strongly Disagree</u>
CONTROL:	PRE-TEST	18%	27%	36%	18%
	POST-TEST	24%	29%	24%	24%
DISCOURSE:	PRE-TEST	6%	25%	56%	13%
	POST-TEST	13%	27%	40%	20%
<u>Alternative HS</u>					
CONTROL:	PRE-TEST	30%	20%	30%	20%
	POST-TEST	10%	30%	50%	10%
DISCOURSE:	PRE-TEST	0%	25%	38%	38%
	POST-TEST	17%	33%	33%	17%

shift in favor of individual work was in the discourse classroom at the alternative high school. The only exception to this trend was the alternative high school control classroom, which showed a 10% shift in favor of group work.

Student Interviews

After the completion of the instructional unit and post-tests, follow-up interviews were conducted with a few students from each classroom that participated in the mathematics dialogue activities to assess their perceptions of these activities as a means of learning mathematics. The interviewed students were all volunteers who returned formal consent forms. This included three students from the traditional school classroom and four students from the alternative school classroom.

The interview consisted of three questions:

1. Did you like using the dialogues and discussions groups in this unit?

Why or why not?

2. Do you think these activities helped you learn math? Explain why or why not.

3. What would you change about this unit, if anything?

A copy of the interview protocol is included in Appendix B. Information collected from these interviews is presented for each classroom.

Traditional School Interviews

The students interviewed in the traditional school setting included two girls and one boy, referred to here as Bonnie, Megan, and Thomas. The interviews took place in school, at the end of a math period approximately five weeks after the completion of the intervention unit. Each interview was conducted individually and took place in an

available teacher work room so students would not be influenced by their classmates.

Responses to each of the three questions are summarized below.

Did you like using the dialogues and discussions groups in this unit? Why or why not?

Two of the students, Megan and Thomas, stated that they liked the dialogue and discussion activities because they were fun and helped explain how to do the problems. Megan explained that doing mathematics in a group was more fun than sitting at your desk doing a worksheet. Thomas also elaborated that he liked seeing how other students tried to solve problems because it helped him correct his mistakes. Bonnie, on the other hand, said she thought the dialogues were kind of fun because she got to try something new, but they seemed too easy. According to Bonnie, there was not really any heart to it.

Do you think these activities helped you learn math? Explain why or why not.

On this question, Bonnie and Thomas both responded that the activities did help them learn mathematics. Bonnie explained that she did not understand how to do the problems until she went through the steps in the dialogues. This made the problems easy and then she could go back and check the steps again if she needed to. Thomas explained that it helped him to work with others and put ideas together on how to get answers. Megan offered a different perspective, responding that she already knew how to solve problems so the activities did not help her much. However, she also thought going through the problem steps was helpful and that this would probably be more helpful to someone who did not already know how to solve problems.

What would you change about this unit, if anything?

On the last question, each student had different ideas. Bonnie said she would make the dialogues more challenging, with questions that made you think harder to figure

stuff out, instead of just laying out the steps in the script. She also added that she would do the activities again if she could. Megan responded she would do more group work and have the groups go to the board to work out different problems. Thomas, on the other hand, thought it would help more students to perform the dialogue activities in front of the class. He explained that this would help students see different ways to do the problems.

Alternative School Interviews

The students interviewed in the alternative school setting included two boys and two girls, referred to here as Rob, James, Alicia, and Kelly. Each interview took place in school, at the end of a mathematics class, approximately four weeks after the completion of the intervention unit. The interviews were conducted individually and took place in the school conference room where students would not be influenced by their classmates. Student responses to each of the three questions are summarized below.

Did you like using the dialogues and discussions groups in this unit? Why or why not?

On this question, two students, Rob and James, said they did not really like the activities. Rob said they were okay, but he would rather work on his own. James said the activities were too complicated because most students will ask each other for help and work together if they need to without all the scripts. The other two students, Alicia and Kelly, both said they liked the activities to some extent, but preferred to work on their own most of the time. Alicia explained that group activities are okay in some classes, but in math she would rather just listen to the teacher and then go for it on her own; reading the scripts seemed pointless to her. Kelly said the activities were fun and she liked comparing answers, but not as much as working on her own.

Do you think these activities helped you learn math? Explain why or why not.

Student responses to this question again varied. Three students, James, Alicia, and Kelly, said the activities did help them learn mathematics to some extent. James said it helped him learn some of the problem steps to see them from a different point of view, but that was about it. Alicia explained more generally that working in groups was helpful because there were people to help you out if you did not understand something. Kelly, on the other hand, said that seeing how other students solved the problems helped her correct her work and learn a better way to do some of the problem solving steps. The fourth student, Rob, said he did not learn anything from the activities because he already knew how to solve the problems. Rob explained further that working with other students was helpful, but having too many students in the group made it confusing for him.

What would you change about this unit, if anything?

When asked how they would change the unit, each student had different ideas. Rob said he would probably not use scripts and keep the groups smaller. James said he thinks his teacher does a good job and likes working in groups because three brains are better than one and gives you a better chance of getting a right answer. On the other hand, he thought the scripts were too complicated and group work should be optional. Alicia said that using scripts would be better if the characters sounded more like real kids when they talked out the problems. She also said the problems were ridiculously easy and needed to be made more challenging to make you use your brain more to figure things out. Kelly, meanwhile, said the activities were fun, for math, but she would include more individual work with the group work, so there would be a better balance of both.

Summary: Student Interviews

In summary, students in the traditional setting seemed to like the activities and scripts, although one student thought they could be more challenging. Two of the three students from this class also thought that the activities helped them learn mathematics, while the third student said she already knew the material so it did not help her much. Students in this group suggested improving the activities by making them more challenging, or including more active elements like board work or performances.

In contrast, students in the alternative setting gave the activities mixed reviews, with two students who did not like the scripts and two who said they were okay. Three of the four students interviewed in this group also said that they preferred to work by themselves most of the time. Three of the four students also said that the activities helped them learn math, while the fourth said that large groups were too confusing. As for suggested improvements, two of these students said they would not use scripts and one student said the scripts should be more challenging and realistic. All of the students said they would keep some group work, with two students suggesting a better mix of group work and individual work, and one student recommending smaller groups.

Chapter Summary

This chapter reported student learning outcomes in terms of achievement, problem solving, student attitudes towards mathematics, and student perceptions of the mathematics dialogue activities. These results provide a multi-dimensional view of student learning and perceptions in each classroom setting. The next chapter will interpret these findings in relation to the qualitative features outlined in chapter four.

CHAPTER SIX

CONCLUSIONS

This study has examined the use of student discourse as an instructional strategy to improve student learning and achievement among low achieving high school mathematics students. The purpose of this inquiry was to address the problem of low achievement in mathematics by exploring whether instruction that provides opportunities for student discourse can be regarded as an effective strategy for teachers seeking to improve student learning and achievement in mathematics. Based on the preceding results from observations, discourse analysis, quantitative analysis of achievement and problem solving, classroom attitude profiles and student interview responses, we are now in a position to revisit the central questions of the study to interpret the meaning of these findings.

Research Questions

The central question addressed in this research was, *does the use of teaching methods that provide opportunities for student discourse improve student learning in mathematics among low achieving high school students?* Here, student learning was understood broadly to be reflected by a full range of identifiable gains in students' mathematical knowledge, skills or understanding, as evident in student test scores, problem solving scores, written work, or student discourse during classroom activities. In addition, student attitudes about mathematics were identified as a potential moderating factor on learning. Accordingly, four subsidiary questions were articulated to address evidence of student learning in terms of achievement, problem solving, attitudes towards

mathematics, and characteristics of student discourse. Each of these questions will now be considered in turn to evaluate whether the opportunities for student discourse provided by the mathematics dialogue activities improved student learning in any measurable or observable way.

Achievement

Does the use of teaching methods that provide opportunities for student discourse improve mathematics achievement among low achieving high school students? This first question concerns student achievement scores in mathematics, which are generally accepted as the primary measure of student learning and program effectiveness. Therefore, the results for student achievement speak to the larger issue of whether providing opportunities for student discourse can be regarded as an effective intervention for low achieving students. This question can now be reconsidered with reference to the analysis of student achievement gains and characteristics of each setting.

Based on the achievement test scores obtained from pre-tests, post-tests, and follow-up tests administered in each classroom, student gains in achievement were calculated to provide a comparison between achievement of students in classrooms using the mathematics dialogue activities and students in the control group classrooms. The combined data set from both school settings indicated that the average achievement gains among students in classrooms using the mathematics dialogues exceeded the average gains of students in control group classrooms by 14% on the pre-test to post-test comparison and by 18% on the pre-test to follow-up comparison, both of which represent experimentally important margins.

Classroom level comparisons in the traditional school setting also showed that the average achievement gain among students in the mathematics dialogue classroom exceeded that of students the control groups classroom by 24% on the pre-test to post-tests comparison and again by 14% on the follow-up tests. Again, both analyses indicated an experimentally important mean difference. However, it was noted that the 24% difference between these classes reflected unusually low post-test scores in the control group classroom. As expected, the follow-up tests provided a narrower margin between these two classes, but still indicated substantially higher scores in the mathematics dialogue classrooms.

At the alternative school, students in both classrooms had equivalent average gains of 18% on the pre-test to post-test comparison. However, the follow-up comparison showed that the average gain in the mathematics dialogue classroom exceeded that in the control group classroom by 22%. While this again indicates an experimentally important difference between the two groups, the discrepancy between these two findings raises questions about what would explain this difference. While it is not impossible that students in these groups have very different retention of mathematical knowledge, it seems more likely that some moderating factors may have affected student attitudes or motivation in one or both of these groups. Unfortunately, the qualitative data and attitude profiles offer little insight into this question since follow-up tests were given several weeks after classroom observations and attitude surveys were completed.

Nevertheless, the results from these assessments show that the average gains in achievement among students who participated in the mathematics dialogue activities exceeded those of students in the control group classrooms by an experimentally

important margin in five out of six comparisons. This demonstrates a high degree of consistency across multiple comparisons and suggests that student learning was enhanced by these activities. Accordingly, these findings do provide some support for the proposition that the use of mathematics dialogues to provide opportunities for student discourse is an effective strategy for improving achievement among low achieving students. At the same time, unanswered questions signal a note of caution concerning the reliability of these outcomes. Since small sample size prevented a more powerful statistical test of experimental importance and consistency, we cannot reject the null hypothesis under these circumstances.

Therefore, although the analysis of mean gains in achievement does support the use of mathematics dialogues as an intervention for low achieving mathematics students, this support remains tentative and more research is needed to address the consistency of these findings. In the meantime, the other measures and indicators of student learning considered in this study may help strengthen or clarify this tentative finding by providing additional confirming or disconfirming evidence.

Problem Solving

Does the use of teaching methods that provide opportunities for student discourse improve problem solving skills among low achieving high school students? This question addressed problem solving skills to provide a second measure of student learning to corroborate the findings on student achievement. Measurements of problem solving skill assess students' understanding of concepts and procedures by examining their problem solving strategies and explanations, as well as the correctness of their calculations. This was measured by using a four trait scoring rubric to evaluate student work on constructed

response items. Student interviews and classroom transcripts were also considered for additional indications of conceptual or procedural knowledge.

Problem solving scores from pre-tests and post-tests indicated that students who participated in the mathematics dialogue activities outperformed their control group peers in both settings. The reported difference in average gains between these groups was 11% overall, with 14% in the traditional school setting and 6% in the alternative school setting. This indicates experimentally important differences in two of the three comparisons. These results correspond to the post-test achievement gains, which also indicated experimentally important differences in the overall and traditional school comparisons, but not in the alternative school comparison.

The follow-up tests given five weeks later showed mixed results, with students who participated in mathematics dialogue activities still outperforming their control group peers overall, but in only one of the two settings. The overall results for both groups showed that the average gain in problem solving from pre-test to follow-up test was 5% greater among students in the mathematics dialogue classrooms. Results from the traditional school also indicated greater average gain of 9% among students in the dialogue classroom. However, at the alternative school, it was the control group that performed better by an average gain difference of 2%. Accordingly, there were no experimentally important differences in problem solving gains observed in results from the follow-up test. These findings also differ from those of the follow-up test on achievement where all three comparisons indicated greater gains among students in the discourse classrooms, all of which were also experimentally important.

To summarize these results, it appears that students in the dialogue classes did perform somewhat better in problem solving overall, but this finding was not consistent across classroom comparisons. Even though experimentally important mean differences occurred in the post-test results, these may have reflected the unfortunate timing of the test in the control group classroom. The larger number of students in the traditional school classes also skews the overall results to reflect this setting more than the alternative school, which means that the overall comparison reflects this same discrepancy. Therefore these findings may be misleading and we must look to the follow-up test as the better comparison between these groups. The follow-up comparison of gains between traditional school classrooms still comes close to experimental importance, which was set at 10% to reflect the common difference between letter grade intervals, but does not meet this threshold. Moreover, because the sample sizes were too small to allow for a more powerful statistical analysis, the degree of experimental consistency is uncertain. Therefore the null hypothesis cannot be refuted.

On the other hand, the findings are still of interest and suggest that the mathematics dialogue activities may have helped some students gain a better understanding of problem solving steps or perhaps improved their ability to explain these to others. Based on observations and the analysis of student discourse in each classroom, the classrooms that used mathematics dialogue activities had more occurrences of student explanations during small group discourse and peer tutoring episodes. These may have contributed to conceptual and procedural understanding both by providing students with informal opportunities to get help from peers or check their understanding, and by providing other students with opportunities to articulate and consolidate their

understanding. In Vygotsky's conception of the zone of proximal development, it is these timely social opportunities for concept-oriented discourse that enable learners to develop incipient concepts into more generalized understandings that can be applied effectively in a variety of situations (Vygotsky, 1978). While this includes the teacher's lessons and interactions with students, peer discourse provides additional channels for conceptual development and verification. Accordingly, these opportunities may help explain the modest advantage in problem solving among students who participated in the small group activities.

It is difficult to say why the differences between the classrooms at the alternative school shifted after five weeks, but this may reflect a combination of factors including the small degree of difference between the groups and the small number of students in the classrooms. The tests were also quite short which allowed minor errors to have a relatively strong influence on the scores. In addition, a number of students in both classrooms made no attempt to answer the constructed response problems, which suggests that student attitudes may have had something to do with these outcomes. Based on the attitude survey, it does appear that the control group classroom had higher ratings for motivation/enjoyment at the end of the instructional unit, which might account for higher scores on a later assessment. However, there is no evidence to support an assertion that this would still affect students five weeks later. The fact that several students in the group did not even attempt the questions would also contradict this theory. On the other hand, students in the dialogue classroom displayed an increase in self-concept on the post-unit survey that coincided with the timing of their post-tests. A temporary gain in self-concept may help account for their better performance at this time, but weaker

performance several weeks later when the influence of the activities had diminished. Here however, the supposition that self-concept would diminish five weeks later is also unfounded.

The interview responses from students who participated in the mathematics dialogue activities also address problem solving skills. Students in both groups expressed that the dialogue activities had helped them learn problem solving steps or helped them correct their mistakes. In the traditional school, all three students expressed something positive about the learning value of the activities. In contrast, two of the four students interviewed at the alternative school also said that they found the scripts or groups confusing. These conflicting views among students in the alternative school dialogue group corresponded to lower problem solving gains. Students becoming confused by the activities might also contribute to smaller problem solving gains in this setting.

Attitudes

Does the use of teaching methods that provide opportunities for student discourse appear to influence student attitudes towards mathematics? This question addressed student attitudes towards mathematics as a potential moderating factor on students' learning efforts and test performance. Student attitudes were assessed by developing attitude profiles for each classroom based on student survey responses before and after the intervention unit. The resulting profiles provided information about student's initial attitudes and any changes that occurred during the unit. This question is specifically concerned with identifying what if any attitude changes occurred in the dialogue classes that are noticeably different than the changes observed in the other classes. Some information from observations and classroom transcripts may also speak to this question.

Attitude profiles from the combined discourse and control groups indicated some shifts in attitude between the beginning and the end of the problem solving unit in both the dialogue classes and the control group classes. Both groups had lower anxiety scores and higher self-concept scores after the intervention unit, while the control group also exhibited higher enjoyment/motivation scores. Here, anxiety scores showed a similar shift in both groups, while the change in self-concept was 18% greater in the discourse group. Therefore, of the two attitude shifts that occurred among students in the discourse group, the only shift that was unique to this group was the large gain in self-concept. This is an experimentally important difference between the two groups and suggests that the mathematics dialogue activities had a positive affect on student self-concept in mathematics.

A comparison of classroom attitude profiles for each school also found that different attitude changes were associated with the mathematics dialogues in each setting. At the traditional high school, attitude changes unique to the dialogue classroom included a 12% increase in positive perception of the teacher and a 16% decrease in enjoyment/motivation. Self-concept scores increased in both classrooms, but again the change was more pronounced among students in the mathematics dialogue classrooms, where it exceeded the control group shift by an experimentally important difference of 14%. At the alternative high school, attitude shifts unique to the dialogue classroom included a 38% decrease in value to society and a 20% increase in self-concept. Comparing the findings from each setting again shows that the only consistent change in student attitudes across all settings is the increase in self-concept.

In answer to the question, then, it does appear that the mathematics dialogue activities used in this research led to an experimentally important and consistent increase in students' self-concept in mathematics. Therefore in this instance a teaching method that provided opportunities for student discourse does appear to have influenced student attitudes towards mathematics in a positive way. But to understand this assertion more fully, we need to consider the specific survey items that measured self-concept. These included the following items:

3. I am not very good at math.
7. Math is easy for me.
9. I usually understand what we are talking about in math class.
12. No matter how hard I try, I cannot understand math.
18. I am good at doing math problems.

Here, items 7, 9, and 18 were scored in reverse so higher point values corresponded to agreement with these statements. From these questions, it appears that self-concept is similar to self-efficacy or self-confidence and speaks to students' perceptions of their own mathematical understanding and abilities.

These findings indicate that the mathematics dialogues activities had a positive influence on students' perceptions of their ability to do and understand mathematics. This makes sense if we compare the different roles assumed by students during whole class instruction as versus small group activities. In whole class instruction, students provide short answers while the teacher provides the majority of the explanations and instructions. In small group activities, students are expected to take responsibility for managing the discourse and providing explanations or conjectures. While different

groups of students shared this responsibility in different ways, and perhaps imperfectly, the expectation that they could figure out problems among themselves was a sharp contrast to their usual reliance on the teacher. The implicit assumption of the small group activities is that the students are capable of doing mathematics and figuring out problems by working together. This new expectation may explain a change in the way these students perceived themselves.

Information drawn from the student interviews also connects the dialogue activities to increases in self-concept. For example, several of the interviewed students expressed that the mathematics dialogue activities helped them learn problem solving steps. This speaks directly to students perceptions of having gained understanding or become more competent as problems solvers. Other students expressed that the dialogues were too easy and should be more challenging. This also speaks to self-concept insofar as these students are exhibiting confidence about their ability to handle more challenging problems. Accordingly, these statements provide additional evidence of a connection between the mathematics dialogue activities and gains in self-concept.

Student comments about the activities being too easy also suggest another possible explanation for the increase in self-concept. That is, activities that were easy for students may have buoyed their self-confidence. However, only two of the seven students who were interviewed stated that they were too easy. Classroom observations also include many instances of students receiving help from peers or calling over the teacher for help during the activities. Therefore, the alleged easiness of the activities does not seem like an adequate explanation for the observed increase in self-concept.

The reasons for the other attitude shifts that were different in each setting are also unclear. This may say something about how the activities were received by different types of students or it may reflect other individual or classroom factors. One place to look for clues is the different classroom norms and routines inherent in each setting. Cazden (1988) discusses contextual factors on peer discourse, noting that students' expected roles and status among peers may influence how peers interact in small groups. Students in the traditional school setting had well established routines with clearly defined roles for both students and the teacher. The introduction of small group activities that diverged from the accepted classroom norms may have caused a degree of role confusion or discomfort that may help explain a decrease in enjoyment/motivation of mathematics. On the other hand, the increase in positive perceptions of the teacher could be a response to the greater responsibility entrusted to students to manage their own discussions. Students who experience greater self-concept may also credit the teacher for their newfound sense of competence. On the other hand, this could also be interpreted as indicating a preference for the teacher's regular form of instruction.

In the alternative school setting, the 38% decrease in value of mathematics to society is also hard to explain. However, this group of students was characterized by more negative opinions about mathematics and objections to group work and other tasks. When the teacher intervened to address these objections, there were times when he brought up the importance of being able to work together and communicate about mathematics in adult life. Some students responded to this by arguing that they would never have a job that involved math. These exchanges may have influenced some

students to continue expressing their resistance to the activities or to mathematics in general by rejecting the survey statements that asserted its value to society.

A final survey question addressed student attitudes about working in group activities versus working alone. Responses to this item showed that more students in both discourse groups favored individual work at the end of the activities. This was especially true in the alternative school setting where responses shifted by 25%. Possible explanations for this outcome include several of the observations made above. It may be that students in the discourse groups were less favorable about the activities because they were forced outside their usual comfort zone or accepted norms. A peer reviewer at the alternative school suggested that many of these students have had bad experiences in mathematics and would rather just do worksheet assignments because these are relatively safe and unthreatening; whereas trying something new makes them feel more vulnerable. In this case, rejection of something could be interpreted as a form of self protection. On the other hand, the dialogue activities may be poorly suited for some groups of students. The students who said the activities were too easy or too confusing might respond better to different activities that provide a better match for their knowledge base or learning styles. Individual teachers who are interested in using scripts may need to adapt these to specific groups of students.

Student Learning

Does the use of teaching methods that provide opportunities for student discourse appear to help teachers in the effort to improve student learning? This question returns us to a broad perspective on student learning. Whereas the previous questions have addressed specific learning outcomes in terms of achievement, problem solving and

attitudes, this question addresses the full range of information that teachers draw on to assess student learning and adjust lessons to meet individual needs. Achievement, problem solving and attitudes are all part of this picture, as are student assignments, but we also need to include the informal assessments opportunities afforded by various forms of classrooms discourse. Accordingly, we now ask the question, what do teachers have to gain from using these mathematics dialogue activities?

First, we did find substantial gains in achievement in both settings that used the dialogues. This evidence was not conclusive, but establishes the activities as a promising strategy for improving achievement. There were also modest gains in problem solving in both settings, but again, these findings were not conclusive. Information from transcripts and interviews indicates that some students perceived the activities as an aid to their learning, while a few found them confusing or pointless. Attitude data, meanwhile, showed that the dialogue activities were associated with improved self-concept in mathematics. These findings all suggest that the mathematics dialogue supplements may help teachers improve student learning.

The analysis of classroom discourse found that classes that used the mathematics dialogue activities included distinctive episodes of small group discourse that did not occur in the control classrooms. These episodes were generated by the activities and provided students with more opportunities to verify their work, check answers, get help from peers, explain problems or concepts, and collaborate with peers to solve problems. These exchanges provided additional indications of student learning in the form of questions and explanations to peers, and expressions of insight, as well as group answers to the activity questions. Accordingly one potential benefit to teachers is an additional

source of information for informal assessment of student progress and understanding. Teachers who have the option of listening in on student discussions, actively engaging with groups, or just evaluating and comparing group answers, can thus gain new insights into student learning.

Another potential benefit of the dialogue activities and small groups in general is the increased availability of peer tutoring as an alternative channel for students to get help as they are working on problems. In control group classes there were occasional instances of peer tutoring, but most students waited for the teacher to answer their questions. Since there was only one teacher in these classrooms, students sometimes waited for several minutes if the teacher was busy with someone else. In contrast, students in the discourse classrooms had readier access to peer tutoring. Instead of waiting for the teacher, students could compare their work easily and ask one another for explanations. While some students still preferred to wait for the teacher, and sometimes groups became stumped and needed additional help, the amount of waiting was still reduced by the increased availability of student tutors.

In addition, the increased number of opportunities for students to explain mathematical problems to their peers may also have important educational value as a means to develop student understanding. As noted earlier, Vygotsky (1978) argued that social opportunities for concept-development can help learners generalize their understanding of newly gained concepts. Accordingly, students who offer explanations to their peers can benefit from this practice. Even when student explanations are inaccurate, they still involve an effort to reformulate ideas or concepts, which in itself advances the speaker's concept development. Kysh (1999) also argued that inaccuracies

in student expressions may reflect an intermediate stage of concept development that students need to go through as they work to connect new concepts to prior learning. From this viewpoint, providing students with more opportunities to practice articulating mathematical concepts and applications is a way to promote student understanding of mathematical concepts and procedures. Whereas practice problems enable students to develop familiarity with procedural routines and algorithms, practicing explanations may also help students make connections between concepts and applications. While these claims are largely theoretical, the current research does provide some evidence of this from student interviews. Five of the seven students who were interviewed said that the dialogue activities helped them learn the steps involved in algebraic problem solving. This may also help explain the greater gains in student achievement among students who completed the dialogue activities.

On the other hand, some students indicated that they did not like the activities. Student attitudes in both dialogue classrooms showed an increased preference for working individually. Student attitude changes in one of these classrooms also indicated less enjoyment/motivation in mathematics, while the other classroom indicated less perceived importance of mathematics to society. While these attitudes may reflect the disruption of established classroom norms and role expectations, they may also become a factor on student learning. Student expressions of negative attitudes in one classroom posed a challenge to implementation. Accordingly, teachers may find that these types of activities work better with some groups of students than others.

Based on this research it does appear that the use of mathematics dialogue activities can help teachers in the effort to improve student learning. As compared to

their control group peers, students in classrooms that used dialogue activities made greater gains in achievement, equivalent gains in problem solving skill, and exhibited attitude gains in self-concept. The use of small group discourse in these classrooms also presents teachers with new opportunities for informal assessment, provides students with more channels to get help and feedback on their work, and may also provide students with more opportunities for concept development. While the occurrence of negative attitudes raises concerns about implementation and student effort in learning, we also saw that implementation could be adapted to accommodate different groups of students and still provide positive gains in achievement and increased self-concept in both settings. Accordingly, mathematics dialogue activities appear to offer teachers a promising strategy for supplementing their lessons to provide opportunities for student discourse that may improve student learning.

Implications

This research explored the question of whether the use of student discourse can improve student learning and achievement among low achieving students in high school mathematics. This question explores the relationship between the problem of low achievement and professionally recommended practices for teaching mathematics. On the one hand, there is little disagreement that widespread low achievement in mathematics is an important problem facing our public schools. The advent of No Child Left Behind (NCLB) has increased the importance of this issue by requiring schools to continually improve achievement or suffer consequences. This law also requires *failing schools* to utilize school improvement plans that are supported by scientifically based research. On the other hand, the *Professional Standards for Teaching Mathematics* (NCTM, 1991)

recommends teaching methods that provide opportunities for student discourse in mathematics. While these standards reflect decades of research and practice in mathematics education, they do not establish a clear scientific basis for the effectiveness of student discourse with respect to improving student achievement. Therefore, it is unclear whether teaching that reflects these standards would qualify as a scientifically supported strategy for improving instruction under NCLB.

This research examined this issue through a case study design that incorporated both qualitative and quantitative methods in order to provide a clear picture of the teaching strategy and instructional context as well as quantitative measures of effectiveness. Classroom observations and discourse analysis were used to delimit and characterize what type of opportunities for student discourse were being provided. Characteristics of each setting, including diverse teaching styles and students, were also included to provide a realistic picture of implementation and identify potential intervening factors on the quantitative outcomes.

The importance given to qualitative context in this study reflects an interest in providing information that teachers can relate to their instructional decision-making on the classroom scale. This research examined two very different classroom settings and shows how each teacher implemented the discursive activities differently to fit their unique instructional setting. This perspective also provides a realistic picture of the variety of contextual factors that can affect student learning outcomes in real classroom settings. Unanticipated school events, student attitudes, poor attendance, flu season, substitute teachers, and timing of tests all affected the implementation and outcomes of the present study. However, far from invalidating the study's findings, these factors serve

to illustrate the real challenges faced by teachers in the effort to improve student achievement. In order to evaluate student learning outcomes accurately we need to consider these factors.

In order to address the NCLB requirement that *failing schools* utilize improvement plans supported by scientifically based research, the present case study also incorporated a quasi-experimental design to compare achievement and problem solving gains between two matched pairs of classrooms. Each of two teachers taught two different classes; one using their regular form of instruction, and one where this was supplemented by mathematics dialogue activities designed to provide opportunities for student discourse. This allowed some degree of control over the influences of distinct teachers and school settings, while also providing each teacher with an opportunity to explore a new strategy for incorporating student discourse into their regular form of instruction. Accordingly, the quantitative measures in this study are intended to address the effectiveness of the mathematics dialogue activities in general, but also provide each participating teacher with information that will help them evaluate the advantages and disadvantages of using mathematics dialogue activities in future lessons.

In summary, this study demonstrates one method for providing opportunities for student discourse among high school prealgebra students. These activities were found to have positive affects on mathematics achievement in both settings, positive affects on problem solving skills in one setting, and positive affects on student attitudes concerning self-concept in both settings. Additional benefits to student learning were also identified. Accordingly, the preponderance of evidence provided by this research indicates that the use of mathematics dialogues is a promising strategy for improving student learning and

achievement. While the quantitative findings of this study are not conclusive due to small sample sizes, they do provide a rationale for further investigation of mathematics dialogue activities as a promising intervention for low achieving students. This study also provides a model for additional research to explore the reliability of these findings. Therefore, based on the evidence provided by this research, it does appear that the use of teaching methods that provide opportunities for student discourse may also provide teachers and schools with an effective strategy for improving student achievement to meet requirements of the No Child Left Behind Act.

Recommendations for Future Research

Additional research is needed to corroborate these findings in a study with larger sample sizes that will provide for a more powerful analysis of the experimental importance and consistency of the measured learning outcomes. While the present study provides a multi-dimensional picture of an intervention strategy and how it was implemented with varying success in two different classroom settings, this does not establish the effectiveness of the intervention beyond these isolated cases. Accordingly, work remains to be done to determine whether student discourse will prove to be effective as an intervention for other low achieving mathematics students.

Additional qualitative research would also be helpful to provide more feedback from teachers who have used the mathematics dialogue activities to provide opportunities for student discourse in their classrooms. Teacher interviews would complement the present study by providing teachers' perspectives on the experience of implementing these activities and their perceived educational value. This would offer additional support

for other teachers who are considering using these activities as supplements to their own lessons.

Finally, additional research is also needed to explore different variations of the mathematics dialogues activities that might increase their effectiveness among different groups of students. While this study illustrated how student attitudes could affect the implementation of dialogue activities, it is unknown whether different scripts or questions would have been received better by these students. The suggestions made by students during the interviews about how to improve the dialogue activities offer a starting point for continued exploration of scripts for different lessons, age groups, and settings. This could include scripts that portray students working to solve real life problems, or perhaps students can compose their own scripts that portray characters and situations they can relate to more easily. The use of performances and different types of group products may also enhance these activities. In general, this type of research would help refine and improve the mathematics dialogue activities for future implementations. This would provide teachers with a greater variety of scripts to apply in different instructional situations.

REFERENCES

- Airasian, P. W. (2005). *Classroom Assessment: Concepts and Applications* (5th ed.). Boston, MA: McGraw-Hill.
- Archambault, F. X. (1989). Instructional setting and other design features of compensatory education programs. In R. Slavin, N. Karweit & N. Madden (Eds.), *Effective Programs for Students At Risk* (pp. 220-263). Needham Heights, MA: Allyn and Bacon.
- Aron, L. Y. (2003). *Towards a typology of alternative education programs: A compilation of elements from the literature* (Electronic version). Washington, DC: Urban Institute.
- Banks, J. A. (2002). *An Introduction to Multicultural Education* (3rd ed.). Boston, MA: Allyn and Bacon.
- Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In D. A. Grouws, T. J. Cooney, & D. Jones (Eds.), *Perspectives on Research on Effective Mathematics Teaching* (pp. 27-46). Reston, VA: National Council of Teachers of Mathematics.
- Borman, G. D., & D'Agostino, J. V. (2001). Title I and student achievement: A quantitative synthesis. In G. D. Borman, S. C. Stringfield & R. E. Slavin (Eds.), *Title I: Compensatory Education at the Crossroads* (pp. 25-57). Mahwah, NJ: Lawrence Erlbaum Associates.
- Brown, D. C. (1999). The effects of Peer and Group Education (PAGE ONE), a comprehensive compensatory program for students at-risk for school failure, on mathematics achievement and student attitude (Doctoral dissertation, University

- of South Carolina, 1999). *Proquest Dissertations and Theses, 1999* (Publication No. AAT 9952705). Retrieved October 9, 2005 from ProQuest Digital Database.
- Carpenter, T. P., & Fennema, E. (1991). Research and Cognitively Guided Instruction. In E. Fennema, T. P. Carpenter, & S. Lamon (Eds.), *Integrating Research on Teaching and Learning Mathematics* (pp. 1-16). Albany, NY: State University of New York.
- Carroll, S. (2004). The effect of after school tutoring and summer enrichment programs on the Algebra I achievement of at-risk students (Doctoral dissertation, University of Houston, 2004). *Proquest Dissertations and Theses, 2004* (Publication No. AAT 3122343). Retrieved October 9, 2005 from ProQuest Digital Database.
- Carter, L. F. (1984). The Sustaining Effects Study of Compensatory and Elementary Education. *Educational Researcher, 13*(7), 4-13.
- Cazden, C. B. (1988). *Classroom Discourse: The Language of Teaching and Learning*. Portsmouth, NH: Heinemann Educational Books.
- Cooley, W. W. (1993). *The Difficulty of the Educational Task* (Pennsylvania Educational Policy Studies No.16). Pittsburgh, PA: Pennsylvania Learning Research and Development Center.
- Corwin, R. B., Storeygard, J., & Price, S. L. (1996). *Talking Mathematics: Supporting Children's Voices*. Portsmouth, NH: Heinemann.
- Crawford, J. (1989a). Instructional activities related to achievement gain in Chapter 1 classes. In R. Slavin, N. Karweit & N. Madden (Eds.), *Effective Programs for Students At Risk* (pp. 264-290). Needham Heights, MA: Allyn and Bacon.

- Crawford, J. (1989b). Teaching effectiveness in Chapter 1 classrooms. *The Elementary School Journal*, 90(1), 33-46.
- Creswell, J. W. (2003). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.
- Dalton, S. S. (1998). *Pedagogy Matters: Standards for Effective Teaching Practices* (Research Report No. 4). Santa Cruz, CA: Center for Research on Education, Diversity & Excellence.
- Davis, R. A., & Wilbur, M. J. (1933). Curriculum research. *Journal of Experimental Education*, 1(4), 294-297.
- DeJesus-Rueff, M. (2006). Teaching mathematical discourse through characters and scripts. In J. O. Masingila (Ed.), *Teachers Engaged in Research: Inquiry into Mathematics Classrooms, Grades 6-8* (pp. 247-280). Reston, VA: National Council of Teachers of Mathematics.
- Dessoff, A. (2004). Think small. *District Administration*, September 2004, 35-39.
- Fancher, R.E. (1985). *The Intelligence Men: Makers of the IQ Controversy*. New York, NY: W.W. Norton & Company.
- Forman, E., & Ansell, E. (2002). The multiple voices of a mathematics classroom community. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics Education* (pp. 115-142). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Forman, E. A., McCormick, D. E., & Donato, R. (1998). Learning What Counts as a Mathematical Explanation. *Linguistics and Education* 9(4), 313-339.

- Freeman, F. N. (1939). *Mental Tests: Their History, Principles and Applications*. Boston, MA: Houghton Mifflin Company.
- Glasser, W. (1988). *Choice Theory in the Classroom* (Rev. ed.). New York, NY: Harper Collins Publishers.
- Griswold, P. A., Cotton, K. J., & Hansen, J. B. (1986). *Effective Compensatory Education Sourcebook, Vol. I-II*. Washington, DC: U.S. Department of Education.
- Hall, T. I. (2004). The effects of a smaller learning community program on student academic performance in math and student engagement in high school (Doctoral dissertation, Arizona State University, 2004). *Proquest Dissertations and Theses, 2004* (Publication No. AAT 3123565). Retrieved October 9, 2005 from ProQuest Digital Database.
- Hilberg, R. S., Doherty, R. W., Dalton, S. S., Youpa, D., & Tharp, R. G. (2000). Standards for effective mathematics education for Native American students. In Secada, W.G. (Ed.) *Changing the faces of mathematics: Perspectives on multiculturalism and gender equity*. Reston, VA: National Council of Teachers of Mathematics.
- Igoa, C. (1995). *The Inner World of the Immigrant Child*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Institute of Education Sciences (2006). *National Assessment of Title I: Interim Report*. Washington, DC: U. S. Department of Education.
- Kieran, C. (2002). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics*

Education (pp. 188-228). Dordrecht, The Netherlands: Kluwer Academic Publishers.

- Kysh, J. M. (1999). Discourse in small groups in an Algebra 1 class (Doctoral dissertation, University of California, Davis, 1999). *Proquest Dissertations and Theses, 1999* (Publication No. AAT 9948708). Retrieved October 14, 2005 from ProQuest Digital Database.
- Lang, C. R. (2001). The effects of self-instructional strategies on problem solving in Algebra I for students with special needs (Doctoral dissertation, George Mason University, 2001). *Proquest Dissertations and Theses, 2001* (Publication No. AAT 3000279). Retrieved October 9, 2005 from ProQuest Digital Database.
- Larriva, C. M. (1998). A situated view of participation in a high school mathematics classroom (Doctoral dissertation, Stanford University, 1998). *Proquest Dissertations and Theses, 1998* (Dissertation # 9908801). Retrieved October 14, 2005 from ProQuest Digital Database.
- Lerman, S. (2002). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics Education* (pp. 188-228). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Li, B. B. (1998). Understanding the nature of epistemic discourse in middle-grade mathematics classrooms (Doctoral dissertation, Washington State University, 1998). *Proquest Dissertations and Theses, 1998* (Publication No. AAT 9917432). Retrieved October 14, 2005 from ProQuest Digital Database.

- Manouchehri, A. (1997). School mathematics reform: Implications for mathematics teacher preparation. *Journal for Teacher Education*, 48(3), 197-209.
- Manouchehri, A. (2002). Developing teaching knowledge through peer discourse. *Teaching and Teacher Education* 18(2002): 715-737.
- Martinez, M., & Klopott S. (2002). Beyond equal access. *Principal Leadership*, May 2002, 12-17.
- Maslow, A. (1968). *Toward a Psychology of Being* (2nd ed.). New York, NY: Litton Educational Publishing.
- Mathematics Assessment Resource Service. (2000). *Balanced Assessment in Mathematics: Form A, Grade 10* (Electronic version). Monterey, CA: CTB/McGraw-Hill.
- McLaughlin, N. L. (2000). Developmentally appropriate practices: Interventions to help at-risk high school students meet math graduation requirements (Doctoral dissertation, Baylor University, 2000). *Proquest Dissertations and Theses*, 2000 (Publication No. AAT 9978579). Retrieved October 9, 2005 from ProQuest Digital Database.
- Mehan, H. (1979) "What time is it, Denise?": Asking known information questions in classroom discourse. *Theory into Practice*, 18(4), 285-294.
- Meier, D. (2002). *In Schools We Trust*. Boston, MA: Beacon Press.
- Mendoza, Y. (2003). The development and implementation of a parental awareness program to increase parental involvement and enhance mathematics performance and attitude of at-risk high school students (Doctoral dissertation, Union Institute

- and University, 2003). *Proquest Dissertations and Theses, 2003* (Publication No. AAT 3136669). Retrieved October 9, 2005 from ProQuest Digital Database.
- Miller, P. (1999). *At-risk Students at Missoula County Public Schools* (Report prepared for Missoula County Public Schools). Missoula, MT: University of Montana.
- Montana Office of Public Instruction. (2005). *2004 CRT Results: Report by District*. Retrieved March 22, 2007 from <http://www.opi.mt.gov/Measurement>.
- Montana Office of Public Instruction. (2006). *2005 CRT Results: Reported by District*. Retrieved March 22, 2007 from <http://www.opi.mt.gov/Measurement>.
- Montana Office of Public Instruction. (2007). *Report on Assessment (CRT)*. Retrieved March 22, 2007 from Internet Retrieval Instructional Service (Iris Report No. 2007323144631705), <http://www.opi.state.mt.us/IRISReports>.
- Moore, J. B. (2000). The role of student discourse in the mathematics achievement of African American male high school students (Doctoral dissertation, University of Cincinnati, 2000). *Proquest Dissertations and Theses, 2000* (Publication No. AAT 9978979). Retrieved October 16, 2005, from ProQuest Digital Dissertations database.
- National Center for Educational Statistics. (2005a). *The Nation's Report Card: Mathematics 2005* (Publication No. 2006-453). Retrieved March 22, 2007, from <http://nces.ed.gov/nationsreportcard/mathematics>.
- National Center for Educational Statistics. (2005b). *The Nation's Report Card: Mathematics 2005 State Snapshot Reports for Grade 8* (Publication No. 2006-454MT8). Retrieved March 22, 2007, from <http://nces.ed.gov/nationsreportcard/pubs/stt2005>.

- National Commission on Excellence in Education. (1983). *A Nation at Risk: The Imperative for Educational Reform* (Electronic version). Washington, DC: U.S. Government Printing Office.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics* (Electronic version). Reston, VA: Author.
- National Council on Education and the Disciplines. (2001). *Mathematics and Democracy: The Case for Quantitative Literacy*. Woodrow Wilson National Fellowship Foundation.
- Nieto, S. (2004). *Affirming diversity* (4th ed.). Boston: Pearson Education, Inc.
- No Child Left Behind Act. (2002). Public Law 107-10, 107th Congress, 1st Session.
- North Central Regional Education Laboratory. (2004). *Critical Issue: Rethinking Learning for Students at Risk* (Online Report). Retrieved November 7, 2006 from <http://www.ncrel.org/sdrs/areas/issues/students/atrisk/at700.htm>. Naperville, IL: Author.
- Northwest Regional Education Laboratory. (2000). *Mathematics Problem Solving Scoring Guide* (Online Resource). Retrieved November 7, 2007 from <http://www.nwrel.org/msec>. Portland, OR: Author.
- O’Conner, M.C. (2002). “Can any fraction be turned into a decimal?”: A case study of a mathematical group discussion. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics Education* (pp. 143-185). Dordrecht, The Netherlands: Kluwer Academic Publishers.

- Popham, W. J. (2005). F for assessment. Retrieved April 10, 2005 from www.glef.org/assessment. *Edutopia* (April/May 2005), 38-41.
- Raywid, M. A. (1994). Alternative schools: The state of the art. *Educational Leadership* 52(1), 26-31.
- Raywid, M. A. (2001). What to do with students who are not succeeding (Electronic version). *Phi Delta Kappan* 82(8), 582-584.
- Reisner, E. R., & Haslam, M. B. (1992). *Effective Compensatory Education Sourcebook, Vol. I (Revised)*. Washington, DC: US Department of Education.
- Richardson, V., Casanova, U., Placier, P., & Guilfoyle, K. (1989). *School Children At-risk*. London, UK: Falmer Press.
- Rueda, R. (1993). An analysis of special education as a response to the diminished academic achievement of Chicano students. In R. R. Valencia (Ed.), *Chicano School Failure and Success: Research and Policy Agendas for the 1990s* (pp. 252-270). Bristol, PA: Falmer Press.
- Sagor, R., & Cox, J. (2004). *At-risk Students: Reaching Them and Teaching Them*. Larchmont, NY: Eye on Education.
- Sandman, R. S. (1973). *The Development, Validation, and Application of a Multi-Dimensional Mathematics Attitude Instrument* (Doctoral dissertation). Minneapolis, MN: University of Minnesota.
- Sandman, R. S. (1980). The mathematics attitude inventory: Instrument and user's manual. *Journal for Research in Mathematics Education*, 11, 148-149.

- Schoenfeld, A. H. (2006). What doesn't work: The challenge and failure of the What Works Clearinghouse to conduct meaningful reviews of studies of mathematics curricula. *Educational Researcher*, 35(2), 13-21.
- Schuck, R. W. (2003). *Responses to the use of the Internet with a California standards-based middle school algebra curriculum among at-risk, Latino students* (Doctoral dissertation, University of Southern California, 2003). *Proquest Dissertations and Theses, 2003* (Publication No. 3116784). Retrieved October 9, 2005 from ProQuest Digital Database.
- Sfard, A. (2002). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics Education* (pp. 59-83). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Slavin, R. E. (1989). Students at risk of school failure: The problem and its dimensions. In R. Slavin, N. Karweit & N. Madden (Eds.), *Effective Programs for Students At Risk* (pp. 3-19). Needham Heights, MA: Allyn and Bacon.
- Slavin, R. E., & Madden, N. A. (1989). Effective classroom programs for students at risk. In R. Slavin, N. Karweit & N. Madden (Eds.), *Effective Programs for Students At Risk* (pp. 23-51). Needham Heights, MA: Allyn and Bacon.
- Suarez-Orozco, C., & Suarez-Orozco, M. M. (2001). *Children of Immigration*. Cambridge, MA: Harvard University Press.

- Thomson, J. D. (1998). *What works with at risk students: Effective and promising practices for comprehensive program planning* (Report prepared for Missoula County Public Schools). Great Falls, MT: Thomson Consultation Services.
- Toch, T. (2003). *High Schools on a Human Scale*. Boston, MA: Beacon Press.
- Van Oers, B. (2002). Educational forms of initiation in mathematical culture. In C. Kieran, E. Forman, & A. Sfard (Eds.), *Learning Discourse: Discursive Approaches to Mathematics Education* (pp. 59-83). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Vygotsky, L. S. (1962). *Thought and Language* (E. Hanfmann & G. Vakar, Eds. and Trans.). Boston, MA: Massachusetts Institute of Technology Press. (Original work published 1934)
- Vygotsky, L. S. (1978). *Mind in Society: The Development of Higher Psychological Processes* (M. Cole, M. Lopez-Morillas, A. R. Luria, & J. Wertsch, Trans.). Cambridge, MA: Harvard University Press.
- Walén, S., & Hirstein, J. (1995). Classroom vignette: An alternative-assessment tool. *Teaching Children Mathematics*, 1(6), 362-365.
- What Works Clearinghouse. (2006). *Middle School Math Curricula*. Retrieved July 25, 2006 from <http://www.whatworks.ed.gov/>.
- What Works Clearinghouse. (2007). *Intervention Reports*. Retrieved April 9, 2007 from <http://www.whatworks.ed.gov/>.
- White, T. F. (2005). Encoding functions, decoding learning: Collaborative mathematics with wireless handheld computers (Doctoral dissertation, Stanford University,

2004). *Proquest Dissertations and Theses, 2005* (Publication No. 3153216).

Retrieved November 9, 2006, from ProQuest Digital Dissertations database.

Wittgenstein, L. (1994). *Tractatus Logico-Philosophicus*. In A. Kenny (Ed.), *The Wittgenstein Reader*. Oxford, UK: Basil Blackwell Ltd. (Original work published 1922).

Wittgenstein, L. (1994). *Philosophical Investigations* (G. E. M. Anscombe, Trans.). In A. Kenny (Ed.), *The Wittgenstein Reader*. Oxford, UK: Basil Blackwell Ltd. (Original work published 1953).

Zweig, J. M. (2003). *Vulnerable Youth: Identifying their Need for Alternative Educational Settings* (Electronic version). Washington, DC: Urban Institute.

Appendix A
Mathematics Dialogue Activities

Lesson Plan Supplement for Student Discussion Groups:**Dialogue #1: Getting Started**

Lesson: Solving one-step equations with the Addition Property of Equality

Materials: Copies of Script Handout (1 per student)

Time: 20-30 minutes

Procedure:

1. Arrange students in groups of four.
2. Pass out the script, instructing groups to read through it and assign character roles among themselves. Groups practice reading the script aloud in character. Teacher circulates and observes (5-10 min).
3. (Optional) Select one or more groups to act out the play (read aloud in character) for the class.
4. When they are done, ask students to discuss and answer the following questions within in their groups (10 min):
 - a) *Do you think these students are doing a good job of working together? Why or why not?*
 - b) *Did they figure out the problem correctly? Explain.*
 - c) *Complete the other two problems from the students' assignment in the play.*
5. Ask students in each group to share their answers in whole class discussion (5-10 min).

Dialogue #1: Getting Started

Characters: Jamie, Max, Cody and Natalie are four students in a math class.

Scene: The teacher has assigned these students to a problem solving group and handed out these problems:

1. $y - 4 = 18$
2. $7 + x = 26$
3. $m - 12 = 30$

Max: Okay, who wants the first one?

Natalie: Aren't we supposed to work together?

Max: It'll save time if we split it up, won't it?

Jamie: Uh, maybe, but I think Natalie's right.

Cody: Let's just start with the first one and get it done.
(Awkward silence)

Max: Okay, Okay. I get 22 for the first one.

Jamie: What? It says it equals 18.

Cody: Yeah, where did 22 come from?

Max: That's what goes in there; for the y .

Natalie: How'd you get that?

Max: In my head. If you take away 4 from 22, that's 18.

Jamie: Oh. So what do we write down?

Cody: Wait. Don't we have to add something to use this property thing?

Natalie: The rule says to add the same thing to both sides.

Jamie: Add 22?

Cody: I don't think so. It looks like you need to add one of the numbers from the problem. So either 4 or 18.

Natalie: What if we add 4?

Jamie: Let's try it. Uh... so, that's $y - 4 + 4$ on this side and then $18 + 4$ over here

Natalie: And $y + 0$ is just y , and that's 22, so $y = 22$

Max: Awesome.

Cody: Cool. Let's try the next one.

Lesson Plan Supplement for Student Discussion Groups:**Dialogue #2: Division of Labor**

Lesson: One-step problem solving; Multiplication/Division Property of Equality

Materials: Copies of Script Handout (1 per student)

Time: 20-30 minutes

Procedure:

1. Arrange students in groups of four.
2. Pass out the scripts, instructing groups to read through it and assign character roles among themselves. Groups then practice reading the script aloud, in character. Teacher circulates and observes (5-10 min.).
3. (Optional) Select one or more of the groups to act out the play (read aloud in character) in front of the class.
4. When they are ready, ask students to discuss and answer the following questions within in their groups (10 min.):
 - a) *Do you think these students are learning math? Why or why not?*
 - b) *Identify something positive that each character contributed to the group effort.*
 - c) *Try solving the other two problems from the play.*
5. Ask students to share answers in whole class discussion (5-10 min.).

Dialogue #2: Division of Labor

Characters: Paul, Jackie, Kelly, and Jordan are four students in the same math class.

Scene: The teacher has assigned these students to a problem solving group and handed out these practice problems:

1. $6x = 54$
2. $14n = 84$
3. $16 = -8w$

Jordan: What we're supposed to do with this?

Kelly: Uh, solve the problems maybe?

Jordan: But it already equals 54.

Paul: Yeah, but that's for $6x$. We need to figure out what x is by itself.

Jackie: I thought x could be anything if it's a variable?

Kelly: Maybe. But this is equal to 54, so both sides are supposed to match.

Paul: Right. We have to figure out what x -value will make this equal 54

Jordan: Okay. So how do we do that?

Jackie: I think we're supposed to do something with the Multiplication Property, but I don't get how it works.

Kelly: Isn't that where we multiply both sides by the same number and they come out equal.

Paul: Yeah. But it's also called the division property because you can divide both sides by the same thing too.

Jordan: Which is it?

Paul: Well, I think the answer should be nine; so how could we get that?

Jackie: Six times nine is 54. Yeah, that works.

Jordan: So just multiply by 9? On both sides?

Kelly: Wait. If we multiply 54 by 9 that's too big. It has to equal 54.

Paul: What if we divide by 9? Will that work?

Jackie: You mean $6x$ divided by 9?

Paul: Oops. That doesn't sound right. Maybe divide by 6.

Kelly: Okay, $6 \div 6 = 1$, so that leaves one x by itself.

Jordan: Hey, that's what you said we wanted.

Jackie: And then $54 \div 6 = 9$.

Jordan: So $x = 9$. Wow. That kind of makes sense now!

Lesson Plan Supplement for Student Discussion Groups:**Dialogue #3: Mixing it up**

Lesson: Two-step problem solving

Materials: Copies of Script Handout (1 per student)

Time: 20-30 minutes

Procedure:

1. Arrange students in groups of four.
2. Pass out the scripts, instructing groups to read through it and assign character roles among themselves. Groups then practice reading the script aloud, in character. Teacher circulates and observes (5-10 min.).
3. (Optional) Select one or more of the groups to act out the play (read aloud in character) in front of the class.
4. When they are ready, ask students to discuss and answer the following questions within in their groups (10 min.):
 - a) *What problem are these students having in the beginning of the play?*
 - b) *Would you want to be part of this group? Why or why not?*
 - c) *Write out the correct steps the group used to solve this problem. Then complete the other two problems from the play.*
5. Ask students to share answers in whole class discussion (5-10 min.).

Dialogue #3: Mixing it up

Characters: Terry, Pat, Alex, and Jesse are four students in the same math class.

Scene: The teacher just finished the lesson and assigned students to groups to figure out some practice problems. The students are working on the first of these problems:

1. $6x - 8 = 22$
2. $3x + 7 = 31$
3. $16 = 42 - 3w$

Terry: I think we're supposed to subtract 8 here because the problem has subtraction in it.

Pat: Wait, uh, didn't she say to divide?

Alex: Subtraction doesn't work because that would make it negative sixteen.

Jesse: Yeah, we want to get rid of the 8, not make it bigger

Pat: So if we divide by eight, will it go away? So, $6x$ equals 22?

Terry: No, because 8 divided by 8 still leaves one.

Pat: Oh, so then we can subtract the one? So it's $5x$ equals 22?

Jesse: Hang on a minute. You can't subtract those -- they're different -- x terms and numbers don't mix.

Alex: I think we need to add 8 so it goes to zero.

Pat: I'm so confused.

Terry: Okay, Alex is saying we should subtract 8 to get $6x$ equals 22?

Jesse: Wait, we have to add 8 here too. So this should be 30.

Alex: Right. We add the same thing to both sides.

Terry: So $6x$ equals 30. Now what?

Pat: Is this where we divide?

Alex: Aha! Divide and conquer!

Terry: Divide by what?

Pat: Well, it's either the 6 or the 30.

Jesse: I'm pretty sure it's the 6 because then we get $1x$ all by itself.

Terry: And then 30 divided by 6 is 5.

Pat: So that makes it $1x = 5$

Alex: Exactly. $x = 5$.

Appendix B
Interview Protocol

Interview Questions

Research Project: Student Perceptions of Math Dialogue Curriculum

Project Director: Susann M. Bradford, Doctoral student, The University of Montana,
Department of Curriculum and Instruction

Subject Code: _____

1. Did you like using the Dialogues and discussions groups in this unit? Why or why not?
2. Do you think these activities helped you learn math? Explain why or why not.
3. What would you change about this unit, if anything?

Appendix C

Pre-Test on Achievement and Problem Solving

Name: _____

Period: _____

Date: _____

Problem Solving Unit – Achievement Pre-Test

DIRECTIONS: Solve each problem and circle the letter of the correct answer. *Show work!*

1. $k + 6 = 9$	1. [A] -6 [B] 15 [C] 3 [D] -9
2. $3h = 27$	2. [A] 3 [B] 24 [C] $\frac{1}{3}$ [D] 9
3. $11 = 4 + w$	3. [A] 15 [B] -7 [C] -4 [D] 7
4. $5m = -35$	4. [A] 5 [B] -7 [C] -40 [D] -30
5. $40 = 4x + 6x$	5. [A] 4 [B] 10 [C] -10 [D] 30
6. $-54 = -9w$	6. [A] -6 [B] -45 [C] 6 [D] -63
7. $8 + h = 24$	7. [A] -8 [B] 16 [C] 3 [D] 14

Problem Solving Unit – Achievement Pre-Test

p. 2

8. $2p + 8 = 32$	8. [A] 20 [B] 3 [C] 12 [D] 4
9. $x - 3 = 14$	9. [A] 17 [B] 11 [C] -3 [D] 3
10. $30 = 4y - 6$	10. [A] 3 [B] -15 [C] 6 [D] 9
11. $-28 = 7k$	11. [A] 4 [B] -4 [C] -35 [D] $\frac{1}{4}$
12. $6w + 8 = 2w + 20$	12. [A] 3 [B] 2 [C] -3 [D] -2
13. $-4x - 6 = 2$	13. [A] -2 [B] 2 [C] 1 [D] -1
14. $2h + 7 = 8h - 11$	14. [A] -3 [B] 10 [C] 3 [D] 4

Problem Solving Unit – Achievement Pre-Test

p. 3

15. $42 = 10n - 4n$	15. [A] 7 [B] 6 [C] 3 [D] 36
16. $-3a - 9 = -24$	16. [A] -5 [B] 5 [C] -11 [D] 11
17. $14 - 7x = 42$	17. [A] -4 [B] 8 [C] -2 [D] -6
18. $23 = m - 17$	18. [A] 6 [B] -17 [C] -6 [D] 40
19. $12 - k = 3k$	19. [A] 3 [B] -4 [C] 6 [D] 4
20. $21 + 3w = 9w - 15$	20. [A] 4 [B] 6 [C] -4 [D] 2

Constructed Response Items – Pre-Test

Directions: Please write neatly and show all of your work in the box provided. Be sure to read carefully and respond to each part of the problem.

21. Solve the equation for x : $3x + 12 = -15$

- (a) Show all of your work.
- (b) Describe the steps taken in your solution.

Constructed Response Items – Pre-Test

Instructions: Please write neatly and show all of your work in the box provided. Be sure to read carefully and respond to each part of the problem.

22. Students at Central High School voted on whether to have pizza or hot dogs at their school carnival. A total of 280 students voted. If the number who voted for hot dogs was 40 less than the number who voted for pizza, how many students voted for each type of food?
- (a) Write an equation to represent the situation. Be sure to identify any variables.
 - (b) Solve the equation, showing your work.
 - (c) State your answer clearly.

Appendix D

Post-Test on Achievement and Problem Solving

Name: _____

Period: _____

Date: _____

Problem Solving Unit – Achievement Post-Test

DIRECTIONS: Solve each problem and circle the letter of the correct answer. *Show work!*

1. $k + 5 = 8$	1. [A] -3 [B] 13 [C] 3 [D] -13
2. $4h = 20$	2. [A] 16 [B] 5 [C] $\frac{1}{5}$ [D] 24
3. $13 = 7 + w$	3. [A] 6 [B] -7 [C] -6 [D] 20
4. $6m = -36$	4. [A] -30 [B] -6 [C] -42 [D] 42
5. $45 = 4x + 5x$	5. [A] 5 [B] 9 [C] 36 [D] 6
6. $-48 = -8w$	6. [A] 56 [B] -40 [C] 6 [D] -6
7. $9 + h = 27$	7. [A] -3 [B] 18 [C] -9 [D] 3

Problem Solving Unit – Achievement Post-Test

p. 2

8. $2p + 12 = 36$	8. [A] 24 [B] 3 [C] 12 [D] 8
9. $x - 4 = 12$	9. [A] 16 [B] -3 [C] 3 [D] 8
10. $28 = 4y - 8$	10. [A] 5 [B] -3 [C] 9 [D] 12
11. $-24 = 6k$	11. [A] 30 [B] -4 [C] -18 [D] $\frac{1}{4}$
12. $5m + 9 = 3m + 25$	12. [A] 8 [B] -7 [C] 2 [D] -17
13. $-3x - 5 = 4$	13. [A] $-\frac{1}{2}$ [B] 3 [C] -3 [D] $\frac{1}{3}$
14. $9h + 6 = 2h - 15$	14. [A] -1 [B] -3 [C] 2 [D] 7

Problem Solving Unit – Achievement Post-Test

p. 3

15. $36 = 12n - 3n$	15. [A] -8 [B] 3 [C] 4 [D] -9
16. $-4a - 6 = -18$	16. [A] -3 [B] 3 [C] -6 [D] 6
17. $15 - 6x = 45$	17. [A] -5 [B] 10 [C] -10 [D] 5
18. $28 = w - 14$	18. [A] 14 [B] -14 [C] -2 [D] 42
19. $15 - k = 2k$	19. [A] 7 [B] 15 [C] 5 [D] 3
20. $16 + 4w = 9w - 19$	20. [A] 2 [B] 4 [C] -2 [D] 7

Constructed Response Items – Post-Test

Directions: Please write neatly and show all of your work in the box provided. Be sure to read carefully and respond to each part of the problem.

21. Solve the equation for x : $4x + 16 = -12$

(a) Show all of your work.

(b) Describe the steps taken in your solution.

Constructed Response Items – Post-Test

Directions: Please write neatly and show all of your work in the box provided. Be sure to read carefully and respond to each part of the problem.

22. The population of Marbleton is 720 less than the population of Granville. If the combined population of both towns together is 2300, what is the population of each town?
- (a) Write an equation to represent the situation. Be sure to label any variables.
 - (b) Solve the equation, showing your work.
 - (c) State your answer clearly.

Appendix E

Problem Solving Scoring Rubric

Constructed Response Scoring Rubric

Instructions: Award points in each skill area according to the level of progress indicated by the student's work. Criteria for points are listed by column in the following table.

Points: Skill Areas:	3 Proficient	2 Intermediate	1 Beginning	0 Not Evident
Computation & Execution	<ul style="list-style-type: none"> Calculations are correct and complete. 	<ul style="list-style-type: none"> Some calculations omitted or minor errors Solution correct, or nearly correct 	<ul style="list-style-type: none"> Substantial errors in calculations, or key calculations omitted Solution incorrect 	<ul style="list-style-type: none"> No work
Conceptual Understanding & Application	<ul style="list-style-type: none"> Mathematical concepts and procedures applied effectively. Formula or principles used correctly. 	<ul style="list-style-type: none"> Application of concepts and procedures had minor errors Formula or principles sometimes applied incorrectly or inconsistently. 	<ul style="list-style-type: none"> Some use of applicable concepts and procedures. Incorrect/ incomplete application of formulae or principles. 	<ul style="list-style-type: none"> Procedure and concepts not clear. Not done
Strategies & Reasoning	<ul style="list-style-type: none"> Solution indicates logical reasoning in steps or written arguments. Strategy fits problem and leads to efficient solution. 	<ul style="list-style-type: none"> Some logical reasoning evident, but reasons or arguments not fully expressed or with minor errors Strategy less direct, but likely to lead to solution. 	<ul style="list-style-type: none"> Reasoning not clear, fragmented or hard to follow; substantial errors Strategy unlikely to lead to correct solution 	<ul style="list-style-type: none"> No evidence of strategy or logic Not done
Communication	<ul style="list-style-type: none"> Clarity of language, complete sentences and labels. Effective use of mathematical language and vocabulary 	<ul style="list-style-type: none"> Language less clear; sentence fragments, or labels omitted. Some use of math language and vocabulary, or minor errors 	<ul style="list-style-type: none"> Language & writing hard to interpret, few if any labels Incorrect use of terms, little or no use of math language 	<ul style="list-style-type: none"> Not legible Not done

Rubric Scoring Worksheet:

Subject# _____

Test: _____

Problem # _____

	Skill Area:	3 = Complete & Correct	2 = Partially Correct	1= Attempted	0 = Not Shown
Computations & Execution	Number Operations & Negative numbers				
Conceptual Understanding & Application	Use of Variables & Algebraic Steps to Solve				
Strategies and Reasoning	Strategy Fits Problem Steps Logical or Make Sense				
Communication	Answered Questions Clearly				
SUBTOTALS:					
TOTAL POINTS:					

Point Scored: _____

Points Possible: 12

Rubric Scoring Worksheet:

Subject# _____

Test: _____

Problem # _____

	Skill Area:	3 = Complete & Correct	2 = Partially Correct	1 = Attempted	0 = Not Shown
Computations & Execution	Number Operations & Negative numbers				
Conceptual Understanding & Application	Use of Variables & Algebraic Steps to Solve				
Strategies and Reasoning	Strategy Fits Problem Steps Logical or Make Sense				
Communication	Answered Questions Clearly				
SUBTOTALS:					
TOTAL POINTS:					

Point Scored: _____

Points Possible: 12

Combined Problem Solving Score	
$\frac{\quad}{24}$	%

Appendix F
Attitude Survey

School:

Per:

Math Attitude Survey

DIRECTIONS: Please read each statement carefully and decide whether it describes the way you feel about mathematics right now. For each item, select one of the four response choices to show how much you agree or disagree with the statement.

1. Math is useful in everyday life.

Strongly Agree Agree Disagree Strongly Disagree

2. Math is something I enjoy.

Strongly Agree Agree Disagree Strongly Disagree

3. I am not very good at math.

Strongly Agree Agree Disagree Strongly Disagree

4. My math teacher presents material in a clear way.

Strongly Agree Agree Disagree Strongly Disagree

5. There is little need for math in most jobs.

Strongly Agree Agree Disagree Strongly Disagree

6. Having to take a math class makes me unhappy.

Strongly Agree Agree Disagree Strongly Disagree

7. Math is easy for me.

Strongly Agree Agree Disagree Strongly Disagree

8. Math is helpful in understanding today's world.

Strongly Agree Agree Disagree Strongly Disagree

9. I usually understand what we are talking about in math class.

Strongly Agree Agree Disagree Strongly Disagree

10. My teacher makes math interesting.

Strongly Agree Agree Disagree Strongly Disagree

11. I don't like anything about math.

Strongly Agree Agree Disagree Strongly Disagree

12. No matter how hard I try, I cannot understand math.
- Strongly Agree Agree Disagree Strongly Disagree
13. My math teacher is willing to provide students with individual help.
- Strongly Agree Agree Disagree Strongly Disagree
14. I would really like to learn math.
- Strongly Agree Agree Disagree Strongly Disagree
15. It is important to know math in order to get a good job.
- Strongly Agree Agree Disagree Strongly Disagree
16. Sometimes I “freeze up” and forget what to do on math tests.
- Strongly Agree Agree Disagree Strongly Disagree
17. I would like a job that doesn’t use any math.
- Strongly Agree Agree Disagree Strongly Disagree
18. I am good at doing math problems.
- Strongly Agree Agree Disagree Strongly Disagree
19. You can get along perfectly well in life without learning math.
- Strongly Agree Agree Disagree Strongly Disagree
20. My math teacher likes students to ask questions.
- Strongly Agree Agree Disagree Strongly Disagree
21. It makes me nervous to even think about doing math.
- Strongly Agree Agree Disagree Strongly Disagree
22. I have a good feeling about math.
- Strongly Agree Agree Disagree Strongly Disagree
23. The only reason I’m taking math is because I have to.
- Strongly Agree Agree Disagree Strongly Disagree
24. I would rather do math problems on my own than in a group activity.
- Strongly Agree Agree Disagree Strongly Disagree

Appendix G

Follow-Up Test on Achievement and Problem Solving

Name: _____

Period: _____

Date: _____

Review Quiz: Problem Solving Check-up

DIRECTIONS: Solve for the variable in each problem and circle the correct answer. *Show work!*

1. $k + 9 = 16$	1. [A] -9 [B] 25 [C] 7 [D] 9
2. $48 = 6h$	2. [A] 42 [B] 8 [C] $\frac{1}{8}$ [D] 54
3. $60 = w - 15$	3. [A] 45 [B] 4 [C] -4 [D] 75
4. $9m = -54$	4. [A] -8 [B] -6 [C] 45 [D] -63
5. $6n + 18 = 54$	5. [A] 9 [B] 12 [C] -9 [D] 6
6. $-16 = 4 + 4g$	6. [A] -3 [B] -5 [C] -6 [D] -2
7. $20 + 8x = 60 + 3x$	7. [A] 8 [B] 10 [C] 16 [D] -5

Review Quiz: Problem Solving Check-up

p. 2

8. $32 - 4y = 8y - 4$	8. [A] 8 [B] -4 [C] 9 [D] 3
9. $16m + 25 = 20m - 15$	9. [A] 10 [B] -11 [C] 12 [D] -8
10. $-3a - 9 = -36$	10. [A] -9 [B] 15 [C] -15 [D] 9

Constructed Response Problem**DIRECTIONS: Show your complete solution and answer in the space below. Write neatly!**11. Use algebraic problem solving to find the value of x : $40x + 12 = 24x - 20$

(a) Show all of your work.

(b) Explain the steps in your solution.