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Modeling the lane changing behavior of major stream traffic due to U-turns

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ABSTRACT

The presence of U-turns at median openings affects the movement of approaching through vehicles in terms of either slowing down or lane changing. The approaching through vehicles change their lanes towards curb side to avoid possible conflict with the U-turns. The present study applies Markov's process to estimate lane changing patterns of approaching through vehicles due to the presence of U-turns. These vehicles generally start changing their lanes as they enter the slowdown section which is divided into equal parts of 10 m each. First, the probability matrices are obtained for the first 10 m of slowdown section and subsequently the transition matrices are defined for each lane. Finally, Markov procedure is applied to estimate the lane changing probabilities of the vehicles. These estimated probabilities by Markov's process are compared with the lane changing probabilities obtained from field data. The probability of lane changing is found to be more at low U-turning volumes whereas this probability decreases with an increase of U-turns.

1. Introduction

Research on traffic flow has become a hotspot in the development of the city for its potential economic value [12]. In a developing country, the urban infrastructure is not spared from the ill effects of urbanization because of burgeoning economic advancement. Urbanization has taken place so fast that the supply cannot meet the demand and the society is caught in the whirlpool of change in urban development [16]. To meet the demand of vehicular traffic, most of the urban roads are now constructed as multilane roads or existing two lane roads are being widened to multilane roads. The multilane roads are generally constructed with raised median in order to segregate the opposing traffic movements [4]. In the raised median opening, vehicles make an 180° turn to merge with the approaching through traffic stream and during this process the possibility of conflict develops. Upon arriving at the median opening, U-turning vehicles generally wait for an appropriate gap in the approaching through traffic stream to initiate the merging maneuver. However, the number of gaps accepted by U-turning vehicles during a unit of time usually decreases as the traffic volume of approaching through vehicles increases. This causes the U-turning drivers to wait for longer durations. Meanwhile, if the waiting duration goes beyond a specific threshold, U-turning vehicles initiate their merging process even in shorter gaps, compelling some of the vehicles in the approaching through traffic to compromise their speed and experience delays [15]. Because of such discourteous actions of the U-turning drivers, the rule of priority is habitually violated. In these circumstances, the priority of approaching through vehicles becomes shared, which is typically known as a limited priority situation [13]. In this situation, a typical action is taken either by the U-turning vehicle (by stopping at the median opening) or approaching through vehicles (either by braking, lane changing or a combination of both) [16]. The decision of approaching through vehicles (braking or lane changing or a combination of both) depends mainly on the U-turning traffic volume. Moreover the lane changing maneuvers are also maintained in such a way that driver safety is also maintained [22].

In the recent past, many researchers have discussed the causes and characteristic of lane changing behavior of different types of roadway locations. Gipps [6] proposed a model where a driver can choose the correct decision pertaining to lane change by considering the urban driving conditions of the road. Toledo et al. [21] developed an integrated lane-changing model and estimated the parameters of the model using detailed vehicle trajectory data. Lv et al. [12] described a microscopic lane-changing process (LCP) model and presented it by controlling fictitious cars in a car-following framework. In a different work the same authors [11] have investigated the generic lane-changing behavior considering the merging effect. Combining discretionary lane-changing and compulsive merging, they developed an integrative model which calculates the lane-changing probability. Shaban [19] studied the driver's lane changing behavior occurring on arterial streets under high traffic volumes. Wan et al. [23] modeled lane change behavior for autonomous vehicles based on surroundings recognition. Van Winsum et al. [22] examined the relation between perceptual information and the motor response during lane-change maneuvers in a fixed-based driving simula-

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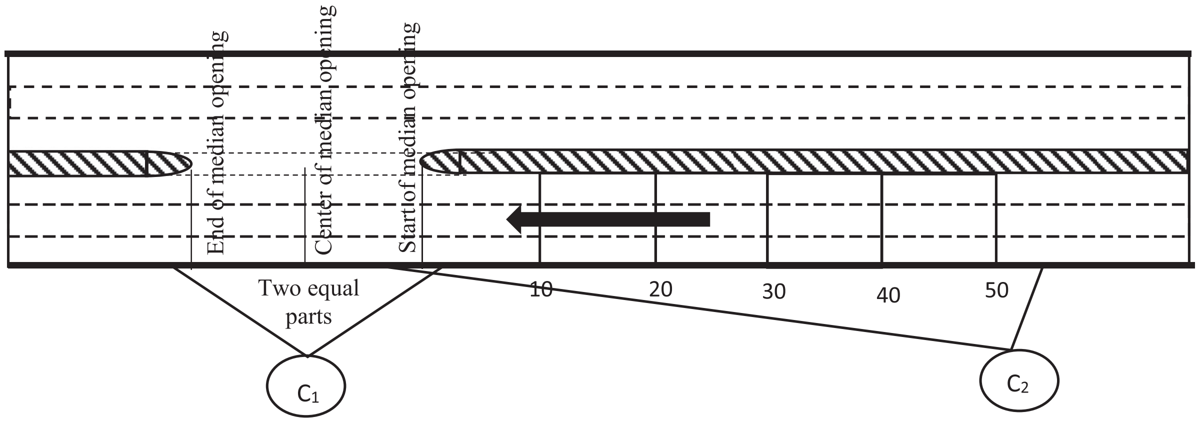

Fig. 1. Camera setup for field survey.

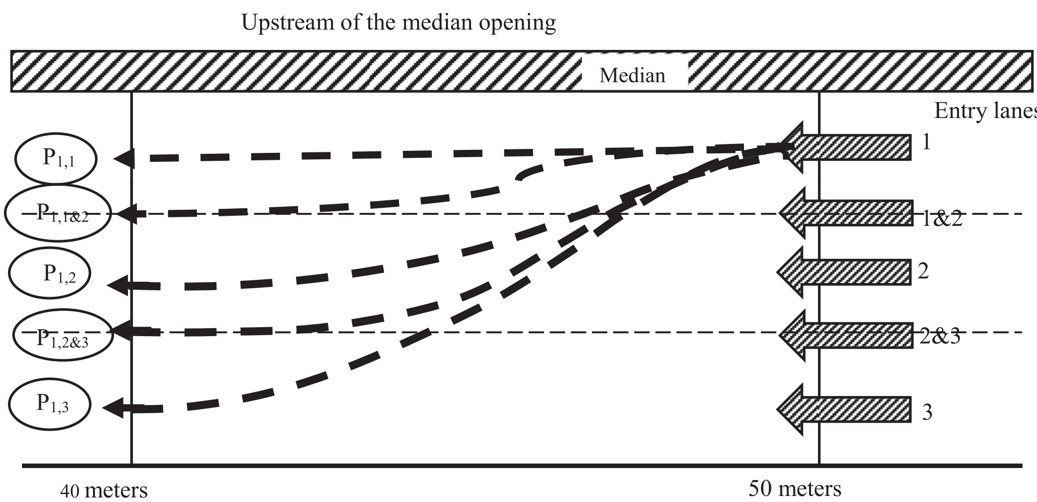

Fig. 2. Lane changing behavior of vehicles entering through lane 1.

Table 1
Geometric details of different test sections.

Section no.	1	2	3	4	5	6	7
Width of median opening, m	20	15.7	19.8	14.8	20	20.3	20.1
Width of raised median, m	1.3	1.0	1.2	1.2	1.3	1.2	1.3
Width of carriageway, m	9.6	9.6	9.8	9.5	9.4	9.5	9.4

tor. Laval and Daganzo [9] assessed lane changing in traffic streams and postulated that vehicles involved in lane-changing vehicles create voids in traffic streams and these voids reduce flow. Salvucci and Liu [18] explored the time course of a lane change in terms of the driver's control and eye movement behavior. In the recent past, many researchers [1, 10, 20, 24] have also implemented game theory to model lane changing patterns. According to Talebpour et al. [20], the game theory approach has two types of games: one where drivers are certain about other drivers' decisions; and the other one where drivers are uncertain about other drivers' decisions. According to them, this technology improves drivers' awareness about their surrounding traffic condition. Ali et al. [1] used CARRS-Q advanced driving simulator to collect high-quality vehicle trajectory data for applying game-theory approach. Liu et al. [10], recommended that this approach could effectively capture vehicle interactions at freeway merging sections with high accuracy of predicting vehicles' actions. Hill et al. [8] employed 46 research participants to drive an instrumented vehicle and performed a combined total of 726 freeway lane changes which was then used for analysis. Guo et al. [7] studied the effect of ramps, fast lanes and various vehicle types on lane changing behavior. The review of earlier studies reveals that most of the studies discuss about the lane changing behavior of vehicles at different highway sections mainly at merging sections. However, how the approaching through vehicles change their lanes as they approach towards the median opening have not been studied. Therefore, the present study attempts to study the lane changing behavior of approaching through vehicles due to U-turns by using Markov's process.

2. Field study

In order to study the lane changing behavior of approaching through vehicles, data were collected from seven different median openings on six-lane divided urban roads. The road sections were identified so that the traffic movement at these sites are not affected by horizontal curvature, presence of upstream or downstream intersection, bus stop, parked vehicles, pedestrian movements or any kind of side friction. All the road sections had road width in the range of 9.4 m to 9.8 m in each direction of travel with raised curb on either side of the road. At each test section, data were collected by video recording technique on typical weekdays. The geometrical details of all the seven median openings have been shown in Table 1.

Table 2
Estimation of lane-wise position of vehicles at 300–600 vph.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Entry through lane 1	$P_{1,1}$	0.745	0.556	0.413	0.308	0.230	0.171
	$P_{1,1\&2}$	0.255	0.395	0.459	0.474	0.459	0.427
	$P_{1,2}$	0.000	0.049	0.109	0.159	0.194	0.213
	$P_{1,2\&3}$	0.000	0.000	0.018	0.051	0.092	0.133
	$P_{1,3}$	0.000	0.000	0.001	0.008	0.025	0.056
Entry through lane 1&2	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.803	0.645	0.518	0.416	0.334	0.268
	$P_{1\&2,2}$	0.197	0.283	0.305	0.295	0.268	0.235
	$P_{1\&2,2\&3}$	0.000	0.070	0.149	0.211	0.251	0.270
	$P_{1\&2,3}$	0.000	0.002	0.028	0.078	0.147	0.227
Entry through lane 2	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.631	0.398	0.251	0.159	0.100	0.063
	$P_{2,2\&3}$	0.354	0.470	0.468	0.415	0.345	0.275
	$P_{2,3}$	0.015	0.132	0.281	0.426	0.555	0.662
Entry through lane 2&3	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.696	0.484	0.337	0.235	0.163	0.114
	$P_{2\&3,3}$	0.304	0.516	0.663	0.765	0.837	0.886
Entry through lane 3	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

Table 3
Estimation of lane-wise position of vehicles at 600–900 vph.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Entry through lane 1	$P_{1,1}$	0.786	0.618	0.486	0.382	0.300	0.236
	$P_{1,1\&2}$	0.214	0.337	0.398	0.418	0.411	0.389
	$P_{1,2}$	0.000	0.045	0.100	0.150	0.186	0.208
	$P_{1,2\&3}$	0.000	0.000	0.016	0.045	0.080	0.115
	$P_{1,3}$	0.000	0.000	0.001	0.005	0.023	0.052
Entry through lane 1&2	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.789	0.623	0.491	0.388	0.306	0.241
	$P_{1\&2,2}$	0.211	0.304	0.330	0.318	0.289	0.253
	$P_{1\&2,2\&3}$	0.000	0.073	0.152	0.210	0.242	0.252
	$P_{1\&2,3}$	0.000	0.000	0.027	0.084	0.163	0.254
Entry through lane 2	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.652	0.425	0.277	0.181	0.118	0.077
	$P_{2,2\&3}$	0.348	0.444	0.426	0.362	0.289	0.222
	$P_{2,3}$	0.015	0.131	0.297	0.457	0.593	0.701
Entry through lane 2&3	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.625	0.391	0.244	0.153	0.095	0.060
	$P_{2\&3,3}$	0.375	0.609	0.756	0.847	0.905	0.940
Entry through lane 3	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

Due to the limited priority situation, the approaching through vehicles generally start decelerating in the upstream of the median opening. Therefore, a possible slowdown section exists for the vehicles approaching towards the median opening. Ma et al. [13] reported 40 m before the starting of intersection to be the possible slowdown section. However, the situation is totally different at uncontrolled median openings and that too under mixed traffic condition in India, where lane discipline and rules of priority are hardly followed [15]. In a separate study, the authors reported that the possible slowdown section starts at a distance of 40–50 m in the upstream of median opening and end near the center of the median opening [15]. From the start of slowdown section to the start of median opening, the road segment is divided into five equal parts of 10 m each. The median opening portion is divided into two equal halves. Two cameras (C_1 and C_2) were used in the field to record the traffic data as shown in Fig. 1.

The recorded films were played on a large screen TV monitor and the following information were noted using the KINOVEA video editing software and a traffic data extractor.

i U-turning traffic volume.
ii The lane wise position of each and every approaching through vehicle when it crosses all the segments (50 m, 40 m, 30 m, 20 m, 10 m, 0 m, and center of median opening).

Table 4
Estimation of lane-wise position of vehicles at 900–1200 vph.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Entry through lane 1	$\mathbf{P_{1,1}}$	0.754	0.568	0.429	0.323	0.244	0.184
	$\mathbf{P_{1,1\&2}}$	0.246	0.376	0.431	0.439	0.419	0.384
	$\mathbf{P_{1,2}}$	0.000	0.056	0.126	0.191	0.240	0.272
	$\mathbf{P_{1,2\&3}}$	0.000	0.000	0.014	0.044	0.083	0.127
	$\mathbf{P_{1,3}}$	0.000	0.000	0.000	0.003	0.014	0.033
Entry through lane 1&2	$\mathbf{P_{1\&2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{1\&2,1\&2}}$	0.774	0.599	0.464	0.359	0.278	0.215
	$\mathbf{P_{1\&2,2}}$	0.226	0.342	0.389	0.392	0.371	0.338
	$\mathbf{P_{1\&2,2\&3}}$	0.000	0.059	0.134	0.204	0.259	0.296
	$\mathbf{P_{1\&2,3}}$	0.000	0.000	0.013	0.045	0.092	0.151
Entry through lane 2	$\mathbf{P_{2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,2}}$	0.740	0.548	0.405	0.300	0.222	0.164
	$\mathbf{P_{2,2\&3}}$	0.260	0.392	0.444	0.447	0.422	0.382
	$\mathbf{P_{2,3}}$	0.000	0.060	0.151	0.253	0.356	0.454
Entry through lane 2&3	$\mathbf{P_{2\&3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2\&3}}$	0.769	0.591	0.455	0.350	0.269	0.207
	$\mathbf{P_{2\&3,3}}$	0.231	0.409	0.545	0.650	0.731	0.793
Entry through lane 3	$\mathbf{P_{3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2\&3}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,3}}$	1.000	1.000	1.000	1.000	1.000	1.000

Table 5
Estimation of lane-wise position of vehicles at 1200–1500 vph.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Entry through lane 1	$\mathbf{P_{1,1}}$	0.913	0.834	0.761	0.695	0.634	0.579
	$\mathbf{P_{1,1\&2}}$	0.072	0.131	0.180	0.219	0.250	0.274
	$\mathbf{P_{1,2}}$	0.015	0.034	0.056	0.079	0.104	0.129
	$\mathbf{P_{1,2\&3}}$	0.000	0.001	0.003	0.006	0.010	0.016
	$\mathbf{P_{1,3}}$	0.000	0.000	0.000	0.001	0.002	0.002
Entry through lane 1&2	$\mathbf{P_{1\&2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{1\&2,1\&2}}$	0.913	0.834	0.761	0.695	0.634	0.579
	$\mathbf{P_{1\&2,2}}$	0.087	0.161	0.223	0.275	0.317	0.352
	$\mathbf{P_{1\&2,2\&3}}$	0.000	0.005	0.015	0.027	0.042	0.056
	$\mathbf{P_{1\&2,3}}$	0.000	0.000	0.001	0.003	0.007	0.013
Entry through lane 2	$\mathbf{P_{2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,2}}$	0.935	0.874	0.817	0.764	0.715	0.668
	$\mathbf{P_{2,2\&3}}$	0.065	0.117	0.157	0.187	0.210	0.227
	$\mathbf{P_{2,3}}$	0.000	0.009	0.026	0.049	0.075	0.105
Entry through lane 2&3	$\mathbf{P_{2\&3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2\&3}}$	0.857	0.734	0.629	0.539	0.462	0.114
	$\mathbf{P_{2\&3,3}}$	0.143	0.266	0.371	0.461	0.538	0.886
Entry through lane 3	$\mathbf{P_{3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2}}$	0.000	0.000	0.000	0.000	0.000	0.396
	$\mathbf{P_{3,2\&3}}$	0.000	0.000	0.000	0.000	0.000	0.604
	$\mathbf{P_{3,3}}$	1.000	1.000	1.000	1.000	1.000	1.000

3. Markov process

The theory of Markov process, or a Markov chain is concerned with a sequence of experiments in which the outcome of a trial depends on the outcome of the directly preceding trial (and only on it) [3]. Instead of thinking of a Markov chain as a sequence of events obtained from successive trials, it is convenient to regard it as a sequence of states through which a system passes at successive points in time, just as in queuing [5]. Many authors [2,5,17] have applied the Markov chain to predict the probabilities of outcomes beforehand.

The probability of being in a given state x_i at a given point in time t is $p_i(t)$. The probability of being in a state x_j at time $(t+1)$ is equal to the probability of being in state x_i at time t, multiplied by the transition probability summed over all possible states of x_i [5]. Mathematically,

$$p_j(t+1) = \sum_i^m p_i(t) p_{ij}, j = 1, 2 \ldots, \quad (1)$$

Where, $x_1, x_2, \ldots, x_n$ = states of the system,

$t_1, t_2, \ldots, t_n$ = points in time,

$p_i(t)$ = probability of being in state x_i at time t,

p_{ij} = probability of transition from x_i to x_j.

To make the algebra clear, the state probability at time $t+1$ (Eq. (1)) can be written as follows:

$$p_1(t+1) = p_1(t)p_{11} + p_2(t)p_{21} + \ldots + p_n(t)p_{n1}$$

$$p_2(t+1) = p_1(t)p_{12} + p_2(t)p_{21} + \ldots + p_n(t)p_{n2}$$

Table 6
Estimation of lane-wise position of vehicles at more than 1500 vph.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Entry through lane 1	$\mathbf{P_{1,1}}$	0.909	0.826	0.751	0.683	0.621	0.564
	$\mathbf{P_{1,1\&2}}$	0.091	0.171	0.240	0.301	0.353	0.398
	$\mathbf{P_{1,2}}$	0.000	0.003	0.009	0.016	0.024	0.034
	$\mathbf{P_{1,2\&3}}$	0.000	0.000	0.000	0.000	0.002	0.004
	$\mathbf{P_{1,3}}$	0.000	0.000	0.000	0.000	0.000	0.000
Entry through lane 1&2	$\mathbf{P_{1\&2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{1\&2,1\&2}}$	0.967	0.935	0.904	0.875	0.846	0.818
	$\mathbf{P_{1\&2,2}}$	0.033	0.062	0.088	0.110	0.130	0.147
	$\mathbf{P_{1\&2,2\&3}}$	0.000	0.003	0.007	0.013	0.020	0.028
	$\mathbf{P_{1\&2,3}}$	0.000	0.002	0.001	0.002	0.004	0.007
Entry through lane 2	$\mathbf{P_{2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,2}}$	0.916	0.839	0.769	0.704	0.645	0.591
	$\mathbf{P_{2,2\&3}}$	0.076	0.139	0.190	0.232	0.265	0.290
	$\mathbf{P_{2,3}}$	0.008	0.022	0.041	0.064	0.090	0.119
Entry through lane 2&3	$\mathbf{P_{2\&3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2\&3}}$	0.911	0.830	0.756	0.689	0.627	0.572
	$\mathbf{P_{2\&3,3}}$	0.089	0.170	0.244	0.311	0.373	0.428
Entry through lane 3	$\mathbf{P_{3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2\&3}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,3}}$	1.000	1.000	1.000	1.000	1.000	1.000

Table 7
Field observation of vehicular distribution at different lanes (300–600 vph).

		50 – 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Field data (300–600 vph)	$P_{1,1}$	0.745	0.618	0.436	0.273	0.200	0.194
	$P_{1,1\&2}$	0.255	0.382	0.491	0.546	0.473	0.483
	$P_{1,2}$	0.000	0.000	0.073	0.145	0.254	0.200
	$P_{1,2\&3}$	0.000	0.000	0.000	0.036	0.073	0.112
	$P_{1,3}$	0.000	0.000	0.000	0.000	0.000	0.011
	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.803	0.743	0.606	0.424	0.333	0.242
	$P_{1\&2,2}$	0.197	0.227	0.303	0.379	0.334	0.254
	$P_{1\&2,2\&3}$	0.000	0.030	0.091	0.167	0.227	0.262
	$P_{1\&2,3}$	0.000	0.000	0.000	0.030	0.106	0.242
	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.631	0.492	0.331	0.146	0.100	0.085
	$P_{2,2\&3}$	0.354	0.423	0.485	0.485	0.308	0.293
	$P_{2,3}$	0.015	0.085	0.184	0.369	0.592	0.622
	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.696	0.565	0.348	0.261	0.174	0.108
	$P_{2\&3,3}$	0.304	0.435	0.652	0.739	0.826	0.892
	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

$$p_n(t+1) = p_1(t)p_{1n} + p_2(t)p_{2n} + \ldots + p_n(t)p_{nn} \tag{2}$$

This group of equations (Eq. (2)) can be represented in matrix form as:

$$P(t+1) = T\,P(t) \tag{3}$$

Where, P represents the vector of state probabilities at time $(t+1)$ and t; and T is the matrix of transition probabilities. It can be shown by the process of induction that

$$P(t+1) = T^{(t+1)}P(0) \tag{4}$$

Where, $P(t+1)$ = probability of being in x_j at $(t+1)$,
T = probability of being in state x_i at time t i.e. initial state,
$P(0)$ = transition matrices when the system is in a stationary state.

3.1. Methodology

In the present study, this Markov process is used to assess the probabilities of lane changing of approaching through vehicles due to the presence of U-turns. Preliminary field observation revealed that the approaching through vehicles generally start to shift laterally as they enter the possible slowdown section. Moreover, this lateral shifting gradually increases as they enter the possible slowdown section. The amount of lateral shifting of each and every approaching through vehicle in the first 10 m (50–40 m) of the possible slowdown section has been noted from the recorded film. These data are used to obtain the probability matrices which are to be used for estimating the lane changing behavior at different segments in the downstream direction.

Table 8
Field observation of vehicular distribution at different lanes (600–900 vph).

		50 – 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Field data (600–900 vph)	$P_{1,1}$	0.786	0.657	0.529	0.471	0.372	0.256
	$P_{1,1\&2}$	0.214	0.314	0.371	0.343	0.371	0.412
	$P_{1,2}$	0.000	0.029	0.100	0.157	0.143	0.189
	$P_{1,2\&3}$	0.000	0.000	0.000	0.029	0.100	0.113
	$P_{1,3}$	0.000	0.000	0.000	0.000	0.014	0.030
	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.789	0.684	0.500	0.421	0.317	0.256
	$P_{1\&2,2}$	0.211	0.263	0.395	0.237	0.234	0.283
	$P_{1\&2,2\&3}$	0.000	0.053	0.105	0.289	0.264	0.233
	$P_{1\&2,3}$	0.000	0.000	0.000	0.053	0.185	0.228
	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.652	0.494	0.291	0.203	0.139	0.101
	$P_{2,2\&3}$	0.348	0.398	0.456	0.373	0.272	0.204
	$P_{2,3}$	0.000	0.108	0.253	0.424	0.589	0.695
	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.625	0.458	0.292	0.217	0.130	0.053
	$P_{2\&3,3}$	0.375	0.542	0.708	0.783	0.870	0.947
	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

Table 9
Field data for lane-wise position of vehicles at 900–1200.

		50 - 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Field data (900–1200 vph)	$\mathbf{P_{1,1}}$	0.754	0.622	0.475	0.361	0.262	0.225
	$\mathbf{P_{1,1\&2}}$	0.246	0.378	0.410	0.426	0.410	0.373
	$\mathbf{P_{1,2}}$	0.000	0.000	0.115	0.180	0.246	0.251
	$\mathbf{P_{1,2\&3}}$	0.000	0.000	0.000	0.033	0.082	0.148
	$\mathbf{P_{1,3}}$	0.000	0.000	0.000	0.003	0.000	0.003
	$\mathbf{P_{1\&2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{1\&2,1\&2}}$	0.774	0.679	0.509	0.339	0.283	0.231
	$\mathbf{P_{1\&2,2}}$	0.226	0.264	0.359	0.434	0.340	0.351
	$\mathbf{P_{1\&2,2\&3}}$	0.000	0.057	0.132	0.170	0.264	0.283
	$\mathbf{P_{1\&2,3}}$	0.000	0.000	0.000	0.057	0.113	0.135
	$\mathbf{P_{2,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2,2}}$	0.740	0.541	0.444	0.299	0.194	0.148
	$\mathbf{P_{2,2\&3}}$	0.260	0.431	0.458	0.451	0.389	0.426
	$\mathbf{P_{2,3}}$	0.000	0.028	0.098	0.250	0.417	0.426
	$\mathbf{P_{2\&3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{2\&3,2\&3}}$	0.769	0.615	0.500	0.385	0.269	0.235
	$\mathbf{P_{2\&3,3}}$	0.231	0.385	0.500	0.615	0.731	0.765
	$\mathbf{P_{3,1}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,1\&2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,2\&3}}$	0.000	0.000	0.000	0.000	0.000	0.000
	$\mathbf{P_{3,3}}$	1.000	1.000	1.000	1.000	1.000	1.000

Typical heterogeneous traffic is characterized by the presence of multiple vehicle types and non-lane-based movement [14]. In the absence of lane discipline and wide variation in sizes of different types of vehicles, they are found to move side by side on the road and it is quite common that 6–8 vehicles can be seen moving abreast in one direction of a 6-lane divided urban road [16]. Moreover, the vehicles are not uniformly distributed across the lanes. Most of the drivers have the tendency to follow the central portion of the carriageway. Therefore, according to the use of lanes by approaching through vehicles, the 3 lanes in one direction of travel have been divided into 5 parts as: median lane (henceforth lane 1), sharing median lane and central lane (henceforth lane 1&2), central lane (henceforth lane 2), sharing central lane and curb lane (henceforth lane 2&3), and curb lane (henceforth lane 3). The approaching through vehicles entering the slowdown section via any lane (lane 1 or lane 1&2 or lane 2 or lane 2&3 or lane 3) are found to shift laterally towards the curb as they move towards the median opening. Fig. 2 depicts the lane-changing behavior of approaching through vehicles which enters the slowdown section through lane 1.

In Fig. 2, $P_{1,1}$ represents the probability of a vehicle entering the slowdown segment (50 m) at lane 1 and leaving the immediate next segment (40 m) at lane 1. Likewise, $P_{1,1\&2}$ represents the probability of the approaching through vehicle entering the slowdown section via lane 1 and leaving the next segment at lane 1&2 and so on. Therefore, the sum of all the probabilities, i.e. $P_{1,1} + P_{1,1\&2} + P_{1,2} + P_{1,2\&3} + P_{1,3} = 1$. Similar exercise is carried out for approaching through vehicles entering the slowdown section through the other lanes (lane 1&2, lane 2, lane 2&3 and lane 3). This procedure is done at various U-turning traffic volumes. Thus the probability matrix at a definite U-turning traffic volume (T_V)

Table 10
Field observation of vehicular distribution at different lanes (1200–1500 vph).

		50 – 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Field data (1200–1500 vph)	$P_{1,1}$	0.913	0.841	0.754	0.667	0.594	0.545
	$P_{1,1\&2}$	0.072	0.130	0.203	0.275	0.290	0.295
	$P_{1,2}$	0.015	0.029	0.043	0.058	0.116	0.137
	$P_{1,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.023
	$P_{1,3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.913	0.848	0.761	0.717	0.653	0.623
	$P_{1\&2,2}$	0.087	0.152	0.239	0.283	0.304	0.324
	$P_{1\&2,2\&3}$	0.000	0.000	0.000	0.000	0.043	0.053
	$P_{1\&2,3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.935	0.852	0.834	0.805	0.746	0.715
	$P_{2,2\&3}$	0.065	0.148	0.154	0.172	0.207	0.208
	$P_{2,3}$	0.000	0.000	0.012	0.023	0.047	0.077
	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.857	0.786	0.643	0.571	0.425	0.425
	$P_{2\&3,3}$	0.143	0.214	0.357	0.429	0.575	0.575
	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

Table 11
Field observation of vehicular distribution at different lanes (> 1500 vph).

		50 – 40	40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Field data (> 1500 vph)	$P_{1,1}$	0.909	0.800	0.745	0.691	0.600	0.523
	$P_{1,1\&2}$	0.091	0.200	0.255	0.309	0.382	0.431
	$P_{1,2}$	0.000	0.000	0.000	0.000	0.018	0.046
	$P_{1,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1,3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{1\&2,1\&2}$	0.967	0.934	0.885	0.869	0.836	0.825
	$P_{1\&2,2}$	0.033	0.066	0.115	0.131	0.148	0.137
	$P_{1\&2,2\&3}$	0.000	0.000	0.000	0.000	0.016	0.034
	$P_{1\&2,3}$	0.000	0.000	0.000	0.000	0.000	0.004
	$P_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2,2}$	0.916	0.857	0.723	0.697	0.639	0.621
	$P_{2,2\&3}$	0.076	0.118	0.227	0.244	0.285	0.273
	$P_{2,3}$	0.008	0.025	0.050	0.059	0.076	0.106
	$P_{2\&3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{2\&3,2\&3}$	0.911	0.844	0.733	0.644	0.600	0.556
	$P_{2\&3,3}$	0.089	0.156	0.267	0.356	0.400	0.444
	$P_{3,1}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,1\&2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,2\&3}$	0.000	0.000	0.000	0.000	0.000	0.000
	$P_{3,3}$	1.000	1.000	1.000	1.000	1.000	1.000

is as follows.

$$T_V = \begin{bmatrix} P_{1,1} & P_{1,1\&2} & P_{1,2} & P_{1,2\&3} & P_{1,3} \\ P_{1\&2,1} & P_{1\&2,1\&2} & P_{1\&2,2} & P_{1\&2,2\&3} & P_{1\&2,3} \\ P_{2,1} & P_{2,1\&2} & P_{2,2} & P_{2,2\&3} & P_{2,3} \\ P_{2\&3,1} & P_{2\&3,1\&2} & P_{2\&3,2} & P_{2\&3,2\&3} & P_{2\&3,3} \\ P_{3,1} & P_{3,1\&2} & P_{3,2} & P_{3,2\&3} & P_{3,3} \end{bmatrix}$$

Where, $P_{i,j}$= probability of an approaching through vehicle entering the slowdown section through lane *i* and leaving the segment via lane *j*.

This probability matrix (T_V) is obtained for the 10 m section (50–40 m) where the approaching through vehicles are generally found to start the lateral shifting. If the proportion of vehicles that would shift laterally in the next 10 m (40 – 30 m) is to be calculated, then the probability matrix (T_V) is raised to the power 2. Subsequently, this matrix (T_V^2) is multiplied with transition matrix to obtain the probability for a vehicle arriving on a given lane as shown in Eq. (4). The transition matrix depends on the lane through which the approaching through vehicles enter the slowdown section. For a vehicle entering the slowdown section through lane 1, the transition matrix (P_1) is as given below.

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Similarly, if an approaching through vehicle enters the slowdown section through lane 1&2 or 2 or 2&3 or 3, then the respective transition matrices for the arrival are as follows.

$$P_{1\&2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Table 12
Comparison between estimated probabilities and field observations (300–600 vph).

300–600 vph		40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Difference in percentage	$P_{1,1}$	6.2	2.3	3.5	3.0	2.3
	$P_{1,1\&2}$	1.3	3.2	7.2	1.4	5.6
	$P_{1,2}$	4.9	3.6	1.4	6.0	1.3
	$P_{1,2\&3}$	0.0	1.8	1.5	1.9	2.1
	$P_{1,3}$	0.0	0.1	0.8	2.5	4.5
	$P_{1\&2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{1\&2,1\&2}$	9.8	8.8	0.8	0.1	2.6
	$P_{1\&2,2}$	5.6	0.2	8.4	6.6	1.9
	$P_{1\&2,2\&3}$	4.0	5.8	4.4	2.4	0.8
	$P_{1\&2,3}$	0.2	2.8	4.8	4.1	1.5
	$P_{2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,2}$	9.4	8.0	1.3	0.0	2.2
	$P_{2,2\&3}$	4.7	1.7	7.0	3.7	1.8
	$P_{2,3}$	4.7	9.7	5.7	3.7	0.4
	$P_{2\&3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2\&3}$	8.1	1.1	2.6	1.1	0.6
	$P_{2\&3,3}$	8.1	1.1	2.6	1.1	0.6
	$P_{3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2\&3}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,3}$	0.0	0.0	0.0	0.0	0.0

Table 13
Comparison between estimated probabilities and field observations (600–900 vph).

600–900 vph		40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Difference in percentage	$P_{1,1}$	3.9	4.3	8.9	7.2	2.0
	$P_{1,1\&2}$	2.3	2.7	7.5	4.0	2.3
	$P_{1,2}$	1.6	0.0	0.7	4.3	1.9
	$P_{1,2\&3}$	0.0	1.6	1.6	2.0	0.2
	$P_{1,3}$	0.0	0.0	0.5	0.9	2.2
	$P_{1\&2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{1\&2,1\&2}$	6.1	0.9	3.3	1.1	1.5
	$P_{1\&2,2}$	4.1	6.5	8.1	5.5	3.0
	$P_{1\&2,2\&3}$	2.0	4.7	7.9	2.2	1.9
	$P_{1\&2,3}$	0.0	2.7	3.1	2.2	2.6
	$P_{2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,2}$	6.9	1.4	2.2	2.1	2.4
	$P_{2,2\&3}$	4.6	3.0	1.1	1.7	1.8
	$P_{2,3}$	2.3	4.4	3.3	0.4	0.6
	$P_{2\&3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2\&3}$	6.7	4.8	6.4	3.5	0.7
	$P_{2\&3,3}$	6.7	4.8	6.4	3.5	0.7
	$P_{3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2\&3}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,3}$	0.0	0.0	0.0	0.0	0.0

$$P_{2\&3} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This methodology has been used in the present study to assess the lane changing behavior of approaching through vehicles as they move towards the median opening.

4. Results and analysis

The microscopic analysis of lane changing behavior of approaching through vehicles has been studied with a large amount of field data. The amount of lane changing by approaching through vehicles varies with the U-turning volume and this has been studied in detail. Therefore, the amount of lateral shifting in the first 10 m (50 to 40 m) of slowdown section have been studied at various U-turning volume levels (300–600 vph, 600–900 vph, 900–1200 vph, 1200–1500 vph, and more than 1500 vph). Around 2000 vehicles have been analyzed for the present study. The obtained probability matrices at different U-turning volume levels are given below.

$$T_{300-600} = \begin{bmatrix} 0.745 & 0.255 & 0 & 0 & 0 \\ 0 & 0.803 & 0.197 & 0 & 0 \\ 0 & 0 & 0.631 & 0.354 & 0.015 \\ 0 & 0 & 0 & 0.696 & 0.304 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 14
Comparison between estimated probabilities and field observations (900–1200 vph).

		40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Difference in percentage	$\mathbf{P_{1,1}}$	5.4	4.6	3.8	1.8	4.1
	$\mathbf{P_{1,1\&2}}$	0.2	2.1	1.3	0.9	1.1
	$\mathbf{P_{1,2}}$	5.6	1.1	1.1	0.6	2.1
	$\mathbf{P_{1,2\&3}}$	0.0	1.4	1.1	0.1	2.1
	$\mathbf{P_{1,3}}$	0.0	0.0	0.3	1.4	3.0
	$\mathbf{P_{1\&2,1}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{1\&2,1\&2}}$	8.0	4.5	2.0	0.5	1.6
	$\mathbf{P_{1\&2,2}}$	7.8	3.0	4.2	3.1	1.3
	$\mathbf{P_{1\&2,2\&3}}$	0.2	0.2	3.4	0.5	1.3
	$\mathbf{P_{1\&2,3}}$	0.0	1.3	1.2	2.1	1.6
	$\mathbf{P_{2,1}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{2,1\&2}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{2,2}}$	0.7	3.9	0.1	2.8	1.6
	$\mathbf{P_{2,2\&3}}$	3.9	1.4	0.4	3.3	4.4
	$\mathbf{P_{2,3}}$	3.2	5.3	0.3	6.1	2.8
	$\mathbf{P_{2\&3,1}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{2\&3,1\&2}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{2\&3,2}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{2\&3,2\&3}}$	2.4	4.5	3.5	0.0	2.8
	$\mathbf{P_{2\&3,3}}$	2.4	4.5	3.5	0.0	2.8
	$\mathbf{P_{3,1}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{3,1\&2}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{3,2}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{3,2\&3}}$	0.0	0.0	0.0	0.0	0.0
	$\mathbf{P_{3,3}}$	0.0	0.0	0.0	0.0	0.0

Table 15
Comparison between estimated probabilities and field observations (1200–1500 vph).

1200–1500 vph		40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Difference in percentage	$P_{1,1}$	0.7	0.7	2.8	4.0	3.4
	$P_{1,1\&2}$	0.1	2.3	5.6	4.0	2.1
	$P_{1,2}$	0.5	1.3	2.1	1.2	0.8
	$P_{1,2\&3}$	0.1	0.3	0.6	1.0	0.7
	$P_{1,3}$	0.0	0.0	0.1	0.2	0.2
	$P_{1\&2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{1\&2,1\&2}$	1.4	0.0	2.2	1.9	4.4
	$P_{1\&2,2}$	0.9	1.6	0.8	1.3	2.8
	$P_{1\&2,2\&3}$	0.5	1.5	2.7	0.1	0.3
	$P_{1\&2,3}$	0.0	0.1	0.3	0.7	1.3
	$P_{2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,2}$	2.2	1.7	4.1	3.1	4.7
	$P_{2,2\&3}$	3.1	0.3	1.5	0.3	1.9
	$P_{2,3}$	0.9	1.4	2.6	2.8	2.8
	$P_{2\&3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2\&3}$	5.2	1.4	3.2	3.7	2.9
	$P_{2\&3,3}$	5.2	1.4	3.2	3.7	2.9
	$P_{3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2\&3}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,3}$	0.0	0.0	0.0	0.0	0.0

$$T_{600-900} = \begin{bmatrix} 0.786 & 0.214 & 0 & 0 & 0 \\ 0 & 0.789 & 0.211 & 0 & 0 \\ 0 & 0 & 0.652 & 0.348 & 0 \\ 0 & 0 & 0 & 0.625 & 0.375 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{1200-1500} = \begin{bmatrix} 0.913 & 0.072 & 0.015 & 0 & 0 \\ 0 & 0.913 & 0.087 & 0 & 0 \\ 0 & 0 & 0.935 & 0.065 & 0 \\ 0 & 0 & 0 & 0.857 & 0.143 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{900-1200} = \begin{bmatrix} 0.754 & 0.246 & 0 & 0 & 0 \\ 0 & 0.774 & 0.226 & 0 & 0 \\ 0 & 0 & 0.74 & 0.26 & 0 \\ 0 & 0 & 0 & 0.769 & 0.231 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{>1500} = \begin{bmatrix} 0.909 & 0.091 & 0 & 0 & 0 \\ 0 & 0.967 & 0.033 & 0 & 0 \\ 0 & 0 & 0.916 & 0.076 & 0.008 \\ 0 & 0 & 0 & 0.911 & 0.089 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 16
Comparison between estimated probabilities and field observations (>1500 vph).

More than 1500 vph		40 - 30	30 - 20	20 - 10	10 - 0	0 - center of median opening
Difference in percentage	$P_{1,1}$	2.6	0.6	0.8	2.1	4.1
	$P_{1,1\&2}$	2.9	1.5	0.8	2.9	3.3
	$P_{1,2}$	0.3	0.9	1.6	0.6	1.2
	$P_{1,2\&3}$	0.0	0.0	0.0	0.2	0.4
	$P_{1,3}$	0.0	0.0	0.0	0.0	0.0
	$P_{1\&2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{1\&2,1\&2}$	0.1	1.9	0.6	1.0	0.7
	$P_{1\&2,2}$	0.4	2.7	2.1	1.8	1.0
	$P_{1\&2,2\&3}$	0.3	0.7	1.3	0.4	0.6
	$P_{1\&2,3}$	0.0	0.1	0.2	0.4	0.3
	$P_{2,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2,2}$	1.8	4.6	0.7	0.6	3.0
	$P_{2,2\&3}$	2.1	3.7	1.2	2.0	1.7
	$P_{2,3}$	0.3	0.9	0.5	1.4	1.3
	$P_{2\&3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{2\&3,2\&3}$	1.4	2.3	4.5	2.7	1.6
	$P_{2\&3,3}$	1.4	2.3	4.5	2.7	1.6
	$P_{3,1}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,1\&2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,2\&3}$	0.0	0.0	0.0	0.0	0.0
	$P_{3,3}$	0.0	0.0	0.0	0.0	0.0

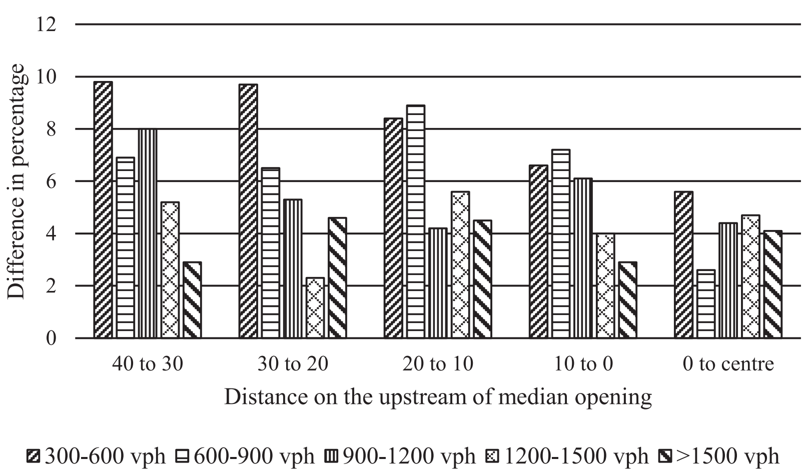

Fig. 3. Validation of Markov's process.

The above matrices have been used to estimate the proportion of approaching through vehicles involved in lane changing (lateral shifting towards curb) as they move towards the median opening.

The probability matrices at all U-turning traffic volumes are raised to the power 2 to estimate how the approaching through vehicles are distributed across the different lanes at a distance of 30 m upstream of the median opening. Likewise, the probability is raised to the power 3 to estimate the lane-wise position of the approaching through vehicles at a distance of 20 m upstream of median opening. The similar process is carried out to study the lane changing at all the segments. These probability matrices with raised powers (2 to 6 in the present study) are multiplied with the transition matrices to calculate the probability of lateral shifting of approaching through vehicles at various segments. Tables 2–6 shows this probability at all segments of road at different U-turning traffic volumes.

The above tables clearly show that the amount of lane changing is comparatively more when the U-turning volume is less. At low volume of U-turns, the traffic density in the median opening area is less. Therefore, the approaching through vehicles while crossing the median opening try to avoid the U-turns by shifting laterally towards the curb side. This results in more lane changing of approaching through vehicles. Whereas, at higher U-turning volume, a near congestion situation develops which considerably disrupt the overall traffic movements. Due to increase in traffic density, the approaching through vehicles find it very difficult to shift towards curb to avoid the U-turns at the median opening area. Therefore, the lane changing probabilities has been observed to decrease substantially when more number of U-turns are present in the median opening area.

5. Validation of Markov process

To validate the proposed Markov process used in the present study, lane changing behavior estimated by the use of Markov principle at all the U-turning volumes were compared to those obtained from the field data. The actual lane changing behavior at all the segments have been assessed from the field data and has been shown in Tables 7–11. The detailed comparison between the predicted and field values is provided in Tables 12–16. All these tables (Tables 12–16) show that the predicted values are in close agreement with the field values which indicates about the effectiveness of Markov's process to assess the lane changing behavior of vehicles.

In order to simplify the understanding of the validation process, a graphical plot has been presented in Fig. 3 below to show the maximum percentage of errors obtained while comparing the probabilities of Markov's process and field data. Fig. 3 represents that the maximum difference in percentage across all the traffic volume levels is less than 10%.

The distribution of approaching through vehicles at any segment of the slowdown section can be easily assessed if the distribution of the approaching through vehicles at the start of the slowdown section is known. Table 17 shows a generalized distribution of approaching through vehicles at the center of median opening at various U-turning volume levels. In the table, it is assumed that the number of vehicles entering the slowdown section through lane 1, lane 1&2, lane 2, lane 2&3, and lane 3 are a, b, c, d, and e respectively.

6. Conclusion

The priority of the approaching through vehicles at uncontrolled median openings is partly shared with the U-turns. As the vehicles approach towards the median opening they are found to either reduce their speed, or change lanes or to perform a combination of both to avoid conflict with the U-turning vehicles.

In the present study a new technique has been applied to predict the lane changing behavior of the approaching through vehicles due to the presence of U-turns. The proposed methodology is a direct application of the Markov process. The lane changing behavior of the approaching through vehicles are studied from the start of slowdown section to the middle of median opening at a regular interval of 10 m. In this exercise, the lane changing behaviors are assessed in the first 10 m (50–40 m) of slow down section and subsequently the probability matrix is obtained. Thereafter, five transition matrices were defined for the vehicles entering the slowdown section through any of the lanes (lane 1, lane 1&2, lane 2, lane 2&3, lane 3). Finally, Markov process has been applied to estimate the distribution of approaching through vehicles at different lanes after their lane changing as they move towards the median opening. This lane changing has been estimated at all the segments of 10 m interval from the start of slowdown section to the center of the median opening across various U-turning volume levels. The analysis of the study reveals that the probability of lane changing of approaching through vehicles decreases with an increase in U-turning volume. As the number of U-turns increase there is an overall increase in traffic density in the median opening area which reduces the flexibility to shift laterally for approaching through vehicles, and hence, rather than performing a risky lane change, they slow down and wait for some time to dissipate.

The applicability of the Markov process to study the lane changing behavior is tested by comparing the lane changing behavior calculated from Markov's process to those obtained from the field data. The comparison showed a remarkable level of accuracy. This technique is highly useful to assess the lane changing pattern of approaching through vehicles. This could help the practitioner engineers to adopt proper technical strategies for better traffic management at median openings. It can be used in ITS to augment the safety of vehicles on road and advise them regarding the safety aspect while changing lanes.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Table 17
Distribution of vehicles at the center of median opening.

	Number of vehicles Start of slow down section	Center of median opening				
		300–600 vph	600–900 vph	900–1200 vph	1200–1500 vph	> 1500 vph
Lane 1	a	0.17a	0.24a	0.18a	0.58a	0.56a
Lane 1&2	b	0.43a + 0.27b	0.39a + 0.24b	.38a + 0.22b	0.27a + 0.58b	0.4a + 0.82b
Lane 2	c	0.21a + 0.23b + 0.06c	0.21a + 0.25b + 0.08c	0.27a + 0.34b + 0.16c	0.13a + 0.35b + 0.67c	0.04a + 0.15b + 0.59c
Lane 2&3	d	0.13a + 0.27b + 0.28c + 0.11d	0.12a + 0.25b + 0.22c + 0.06d	0.13a + 0.3b + 0.38c + 0.21d	0.02a + 0.06b + 0.23c + 0.4d	0.03b + 0.29c + 0.57d
Lane 3	e	0.06a + 0.23b + 0.66c + 0.89d + e	0.05a + 0.25b + 0.7c + 0.94d + e	0.03a + 0.15b + 0.45c + 0.79d + e	0.01b + 0.1c + 0.6d + e	0.12c + 0.43d + e

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