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Reliability evaluation of dielectric elastomer balloon subjected to harmonic voltage and random pressure

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ABSTRACT

Subjected to electric and/or mechanical stimuli, either deterministically or stochastically, dielectric elastomer structures may undergo remarkable oscillations which can deteriorate the operating performance or even induce complete failure. This work investigates the reliability evaluation of ideal dielectric elastomer balloon subjected to harmonic voltage and random pressure simultaneously, with the objective to provide some guidance on the design of dielectric elastomer structures/ components. The operating safety domain is determined by the material strength of dielectric elastomer material, while the reliability evaluation comes down to solving a first-passage failure problem in mathematics. The stochastic differential equations with respect to the first integral and phase difference are derived by executing a special transformation and stochastic averaging. The reliability function (*i.e.*, the probability of system states being in a specified safety domain in a given time interval) is then obtained by numerically solving the associated backward Kolmogorov equation. The influences of some crucial parameters (*e.g.*, the initial energy, the intensity of random pressure, the amplitude and frequency of harmonic voltage) on the reliability are discussed in detail.

1. Introduction

Due to its prominent advantages (*e.g.*, the capacity undergoing large stretch, capacity of rapid response to external stimuli, small mass density, and excellent compatibility with chemistry and biology), the dielectric elastomer (DE) material has attracted extensive attention in scientific and industrial communities [1,2]. DE film with compliant electrodes covered on its two surfaces constitutes the so-called *DE structures*, the in-plane dimensions of which can be easily changed by imposing an external voltage. By utilizing the electromechanical coupling property, DE structures have been extensively adopted as sensors for sensing external information and actuators for applying mechanical actions. The DE-based sensors and actuators possess plenty of advantages and are perfectly suitable for the soft robotics applications [3–15].

The quasi-static behaviors of various types of DE structures (*e.g.*, plane, tubular and balloon) were extensively investigated. In most practical applications, however, the soft devices composed of DE structures always operate in dynamic circumstances. Under deterministically time-varying voltage and/or mechanical load, the time-domain responses of DE structures were extensively investigated [16–24]. For a DE balloon, the prominent dependence of fundamental frequency on pressure difference, external voltage, and pre-stretch were discovered [23]. Some typical nonlinear behaviors such as multiple-frequency resonance, jump, and bifurcation were predicted theoretically [25,26].

Large stretchability and low stiffness are the notable advantages of DE structures. These properties however, induce its sensitivity to external disturbances, such as the pressure variation induced by temperature fluctuation and the voltage variation induced by electromagnetic field. Subjected to random voltage, random pressure, and combined excitations of harmonic voltage and random pressure, the stationary responses of DE structures were studied [27–29] and the asymmetric property of the stationary response of stretch ratio in the case with random voltage or random pressure is discovered as well as the phenomena of stochastic jump and bifurcation in the case with combined excitations. Also, the random responses of the DE generator and the mechanical system including DE components were investigated [30–31].

Dissimilar from a traditional metal material which has an almost perfect lattice micro-structure, DE material possesses inho-

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mogeneous long-chain structure. From the viewpoint of material strength, the amount of micro-defect and coupled interaction of strongly electric field and mechanical load render DE material easier to fail than the traditional material. Four typical modes of failure exist in DE structures, including electrical breakdown, electromechanical instability, loss of tension, and rupture induced by large stretch [32–36]. Early research focused on the pull-in failure [37,38]. Plante and Dubowsky established three large-scale failure criteria of DE actuators: pull-in, material strength, and dielectric strength [32]. Zhao and Suo studied the electro-mechanical instability of DE structures by the Hessian of free-energy function and found that the stability can be dramatically enhanced by pre-stresses [39]. More recently, buckling mode instabilities in DE structures were analyzed which could not be detected by the Hessian method [40–42]. For the DE structures with stretch ratio-dependent permittivity, Leng et al. analyzed the electro-mechanical stability and established the relation between nominal quantities and real quantities [43]. Chen et al. investigated the dynamic electro-mechanical instability of DE balloon subjected to parametric combinations of direct current (DC) voltage and alternate current (AC) voltage [44].

The existing literature regarding failure analysis of DE structures focuses on *the quasi-static case or the deterministically dynamic case*. The electric voltage and mechanical pressure imposed on DE structures, however, are inevitably perturbed by the circuit noise and temperature fluctuation. The random disturbance may induce large-amplitude responses, and it is just the large-amplitude random response which induces failure. This manuscript attempts to address the failure probability of DE structures under deterministic and random stimuli. In this work, the system reliability is described as a first-passage failure problem. The deterministic voltage is modeled by a harmonic function, while the pressure disturbance is modeled as Gaussian white noise. By combining the transformation technique, stochastic averaging and Markov diffusion process theory, the reliability function and various statistical quantities are obtained by numerically solving the backward Kolmogorov equation. Numerical results are given to demonstrate the implementation and accuracy of the proposed method and the influences of some crucial system parameters on system reliability.

2. Equation of motion of stretch ratio

As a typical configuration of DE structures, DE balloon achieves extensive applications, *e.g.*, as the main body of pump and loudspeaker. When used as a pump, the voltage with rectangular or triangular wave is imposed to periodically change its volume. When used as a soft loudspeaker, a constant voltage and a constant pressure are set to maintain its initial configuration, and then a deterministic harmonic voltage signal is imposed to excite the vibration around the initial configuration. The pressure difference however, is dramatically influenced by temperature change which is fluctuating constantly. Pressure disturbance may significantly deteriorate the operating performance of a soft loudspeaker as the frequency band of pressure disturbance covers its resonant frequencies. Stochastic jump and bifurcation occur in the DE balloon subjected to harmonic voltage and random pressure [28], which dramatically degrades the reliability.

Excited by the voltage and pressure, DE balloon will experience spherical symmetrical deformation as long as it does not lose its stability. In this case, the DE balloon behaves as a simple single-degree-of- freedom oscillator [28]. The average radii of the undeformed and deformed states are set as R and r, respectively. A DC voltage Φ_0 and a harmonic AC voltage are imposed on electrodes simultaneously. Random pressure difference p is imposed to inflate the balloon. Assume that the DE material is incompressible with ideal dielectric behavior, and the DE balloon is taken to deform under isothermal condition. Here, we adopt the well-known neo-Hookean model to describe the elastic energy of the ideal DE material. Then, the non-dimensional governing equation of the stretch ratio for DE balloon is derived as [28]

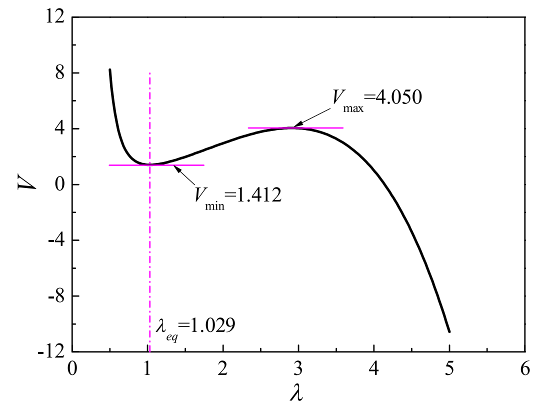


Fig. 1. Potential energy function of DE balloon. ($s_r = 0.1$, $s_f = 0.2$, $r_{ad} = 0.1$).

$$\ddot{\lambda} + c\dot{\lambda} + 2\lambda - 2\lambda^{-5} - \frac{pR}{\mu K}\lambda^2 - 2\frac{\varepsilon\Phi^2}{\mu K^2}\lambda^3 = 0 \tag{1}$$

in which, $\lambda = r/R$ is the stretch ratio; the symbol "dot" denotes the differentiation with respect to t; $t = \bar{t}R^{-1}\sqrt{\mu/\rho}$ is the non-dimensional time; $\bar{t}$ is the actual time; μ is the elastic modulus; ρ is the mass density; c is the non-dimensional viscous damping coefficient; ε is the permittivity; K denotes the thickness of balloon in free state. The total voltage $\Phi = \Phi_0[1 + r_{ad}\sin(\Omega t)]$, in which r_{ad} is the ratio of the amplitude of AC voltage to DC voltage; Ω is the non-dimensional frequency of AC voltage. The random pressure is $p = p_0[1 + \xi(t)]$, in which p_0 is the mean value of pressure; $\xi(t)$ is zero-mean Gaussian white noise with intensity $2D$. By substituting analytical expressions of voltage and pressure into the governing Eq. (1), we obtain,

$$\ddot{\lambda} + c\dot{\lambda} + g(\lambda) = s_r\lambda^2\xi(t) + s_f\lambda^3\left[2r_{ad}\sin(\Omega t) - r_{ad}^2/2\cos(2\Omega t)\right] \tag{2}$$

in which, the parameters $s_r = p_0R/(\mu K)$and $s_f = 2\varepsilon\Phi_0^2/(\mu K^2)$ measuring the mean pressure and mean voltage, respectively. The nonlinear restoring force is expressed as

$$g(\lambda) = 2\lambda - 2\lambda^{-5} - s_r\lambda^2 - s_f\left(1 + r_{ad}^2/2\right)\lambda^3 \tag{3}$$

Integrating the restoring force yields the potential energy of the system in Eq. (2), *i.e.*

$$V(\lambda) = \lambda^2 + \lambda^{-4}/2 - s_r\lambda^3/3 - s_f\left(1 + r_{ad}^2/2\right)\lambda^4/4 \tag{4}$$

The potential energy function only possesses one potential well and approaches a minimum value $V_{\min}$ at the sole stable equilibrium point λ_{eq}. The shape of the potential energy is asymmetric with respect to this equilibrium point as shown in Fig. 1. Once the dynamic response exceeds the critical value associated with the hump of the potential function, the response increases infinitely which corresponds to the strength failure and the loss of functionality. It is appropriate to set the local maximum value of potential energy $V_{\max}$ as *the threshold of failure*.

3. Transformation technique and stochastic averaging

The ultimate purpose of random analysis is to evaluate the reliability of systems/structures and further to provide some guidance to improve the reliability. There exist two major failure modes in random analysis, *i.e.*, first-passage failure and fatigue failure. First-passage failure postulates that once the dynamic response reaches a prespecified safety boundary for the first time, the system/structure fails. Fatigue failure occurs as the accumulated damage reaches a threshold and can also be described as a first-passage problem [45–47]. Herein, we describe the failure problem of DE balloon as a first-passage problem.

System in Eq. (2) possesses strongly nonlinear stiffness and is disturbed by the combined excitations of Gaussian white noise and two harmonic excitations with different frequencies. The modal damping is generally slight, while the pressure disturbance induced by temperature fluctuation is smaller compared to the static pressure, *i.e.*, the average pressure. Thus, it is suitable to investigate the reliability of the system in Eq. (2) through the stochastic averaging method [48].

Introduce a special transformation as follows [28]

$$sgn(\lambda-\lambda_{eq})U(\lambda)=\sqrt{H}\cos\varphi,\quad \dot{\lambda}=-\sqrt{2H}\sin\varphi \tag{5}$$

in which, φ is the phase, $H=\dot{\lambda}^2/2+U(\lambda)$ is the first integral of the system in Eq. (2) and the modified potential energy U is defined as,

$$U(\lambda)=V(\lambda)-V(\lambda_{eq}) \tag{6}$$

The Itô stochastic differential equations with respect to the first integral H and the phase φ are derived by starting from the Itô equations of stretch ratio and its ratio, and then using the Itô differential rule.

Due to the existence of harmonic excitations, the system in Eq. (2) may undergo resonant and non- resonant responses. Herein, only the resonant case is considered which corresponds to severe vibration and may induce failure in a short time interval. The phase difference Γ is introduced and expressed as,

$$\Gamma=2\Omega t-\frac{k}{q}\varphi \tag{7}$$

where, k and q are relatively prime positive integers and satisfy the relation,

$$\frac{2\Omega}{\omega(H)}=\frac{k}{q}+\sigma \tag{8}$$

in which, $\sigma = O(c)$ is the detuning parameter. The averaged frequency $\omega(H)$ can be calculated from $\omega(H)=2\pi/T_h(H)$, in which the first integral-dependent period $T_h(H)$ is expressed as,

$$T_h(H)=2\int_{\lambda_1}^{\lambda_2}\frac{1}{\sqrt{2H-2U(\lambda)}}d\lambda \tag{9}$$

in which, λ_1 and λ_2 correspond to the lower and upper bounds of motion with energy H, respectively, and can be determined by two adjacent roots of the equation $H-U(\lambda)=0$. It is obvious that $0<\lambda_1<\lambda_{eq}<\lambda_2$.

Here, we transform the original variable pair (H,φ) to a new variable pair (H,Γ). Note that the variances of the first integral H and the phase difference Γ are prominently slower than that of the phase φ. Performing the time averaging yields the following averaged Itô stochastic differential equations with respect to the new variable pair (H,Γ) [28], *i.e.*

$$dH=\overline{m}_1dt+\overline{\sigma}_1dB(t),\quad d\Gamma=\overline{m}_2dt+\overline{\sigma}_2dB(t) \tag{10}$$

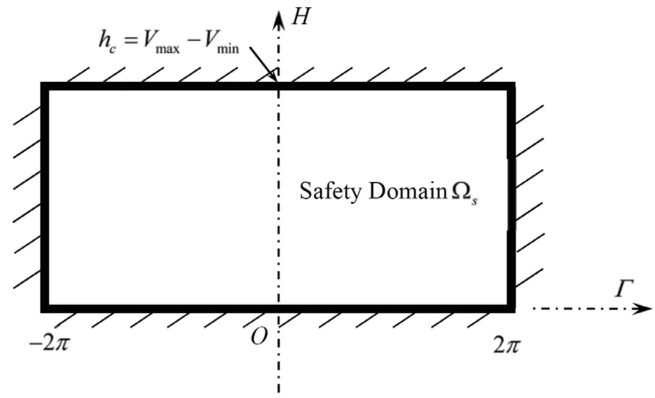


Fig. 2. Schematic of safety domain with respect to the energy and phase difference.

where the averaged drift and diffusion coefficients are calculated by

$$\overline{m}_1=\langle m_1\rangle_t,\ \overline{m}_2=\left\langle 2\Omega-\frac{k}{q}m_2\right\rangle_t,\ \overline{b}_{11}=\langle\sigma_1^2\rangle_t,\quad \overline{b}_{22}=\left\langle\frac{k^2}{q^2}\sigma_2^2\right\rangle_t,\ \overline{b}_{12}=\overline{b}_{21}=\left\langle\frac{k}{q}\sigma_1\sigma_2\right\rangle_t \tag{11}$$

in which,

$$\begin{aligned} m_i&=F_i+D\frac{\partial G_i}{\partial H}G_1+D\frac{\partial G_i}{\partial\varphi}G_2,\ \sigma_i^2=2DG_i^2\quad(i=1,2)\\ F_1&=-2cH\sin^2\varphi-\sqrt{2H}\sin\varphi s_f\lambda^3\\ &\quad\times\left[2r_{ad}\sin\left(\frac{\Gamma}{2}+\frac{k}{q}\frac{\varphi}{2}\right)-\frac{r_{ad}^2}{2}\cos\left(\Gamma+\frac{k}{q}\varphi\right)\right],\\ F_2&=-c\sin\varphi\cos\varphi+\frac{g(\lambda)}{\sqrt{2H}\cos\varphi}-\frac{\cos\varphi}{\sqrt{2H}}s_f\lambda^3\\ &\quad\times\left[2r_{ad}\sin\left(\frac{\Gamma}{2}+\frac{k}{q}\frac{\varphi}{2}\right)-\frac{r_{ad}^2}{2}\cos\left(\Gamma+\frac{k}{q}\varphi\right)\right],\\ G_1&=-\sqrt{2H}\sin\varphi s_r\lambda^2,\ G_2=-\frac{\cos\varphi}{\sqrt{2H}}s_r\lambda^2 \end{aligned} \tag{12}$$

The stretch ratio λ in Eq. (12) is expressed by the first integral H and the phase φ through the transformation in Eq. (5). Furthermore, replacing the time averaging by space averaging yields,

$$\langle\cdot\rangle_t=\frac{2}{T_h(H)}\int_{\lambda_1}^{\lambda_2}\frac{[\cdot]}{\sqrt{2H-2U(\lambda)}}d\lambda \tag{13}$$

4. Reliability evaluation

Define a conditional reliability function $R(t|h_0,\gamma_0)$ as $P\{(H(\tau),\Gamma(\tau))\in\Omega_s,\tau\in(0,t]|(h_0,\gamma_0)\in\Omega_s\}$, which measures the probability of system response keeping in a prespecified safety domain Ω_s during the time interval (0, t] under the condition of the initial state (h_0,γ_0) within the safety domain. Herein, the safety domain Ω_s is determined by the difference between the local maximum value $V_{\max}$ and the local minimum value $V_{\min}$ of the potential energy as shown in Fig. 2. The system fails if and only if the total energy exceeds the critical value $h_c=V_{\max}-V_{\min}$. The two-dimensional safety domain Ω_s adopted here is defined as $0\le H(\tau)\le h_c$, $-2\pi\le\Gamma(\tau)\le 2\pi$. It is worth to pointing out that both the mechanical and electric mechanisms are considered in the reliability analysis because the energy H adopted to measure system responses includes parameters s_r and s_f simultaneously.

The conditional reliability function $R(t|h_0,\gamma_0)$ is governed by the following backward Kolmogorov equation [49],

$$\frac{\partial R(t|h_0,\gamma_0)}{\partial t} = \bar{m}_1(h_0,\gamma_0)\frac{\partial R(t|h_0,\gamma_0)}{\partial h_0} + \bar{m}_2(h_0,\gamma_0)\frac{\partial R(t|h_0,\gamma_0)}{\partial \gamma_0} + \frac{1}{2}\bar{b}_{11}(h_0,\gamma_0)\times\frac{\partial^2 R(t|h_0,\gamma_0)}{\partial h_0^2} + \bar{b}_{12}(h_0,\gamma_0)\frac{\partial^2 R(t|h_0,\gamma_0)}{\partial h_0 \partial \gamma_0} + \frac{1}{2}\bar{b}_{22}(h_0,\gamma_0)\frac{\partial^2 R(t|h_0,\gamma_0)}{\partial \gamma_0^2} \tag{14}$$

The initial condition is as,

$$R(0|h_0,\gamma_0) = 1,\ (h_0,\gamma_0) \in \Omega_s \tag{15}$$

while the boundary conditions are,

$$R(t|0,\gamma_0) = \text{finite}, \quad R(t|h_c,\gamma_0) = 0, \quad R(t|h_0,\gamma_0 + 4n\pi) = R(t|h_0,\gamma_0) \tag{16}$$

The governing Eq. (14) with the initial and boundary conditions in Eqs. (15) and (16) constitutes a definitive problem with respect to the conditional reliability function. It is then numerically solved by finite difference method. For the lattice points within the safety domain Ω_s the explicit central difference scheme is adopted, while for the lattice points at the boundary $h_0 = 0$ the forward difference scheme is adopted. At the boundaries $\gamma_0 = \pm 2\pi$, the expressions $R(t|h_0,-2\pi - \Delta\gamma_0) = R(t|h_0,2\pi - \Delta\gamma_0)$ and $R(t|h_0,2\pi + \Delta\gamma_0) = R(t|h_0,-2\pi + \Delta\gamma_0)$ are adopted in the central difference scheme.

The conditional probability density of the first-passage time is,

$$p(T|h_0,\gamma_0) = -\partial R(t|h_0,\gamma_0)/\partial t|_{t=T} \tag{17}$$

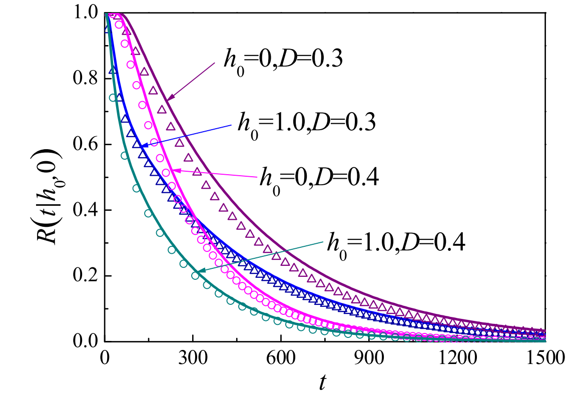
The first-order moment, *i.e.*, the mean first-passage time, possesses important practical significance and can be calculated by the following relation,

$$\mu_1(h_0,\gamma_0) = \int_0^\infty T p(T|h_0,\gamma_0)\mathrm{d}T \tag{18}$$

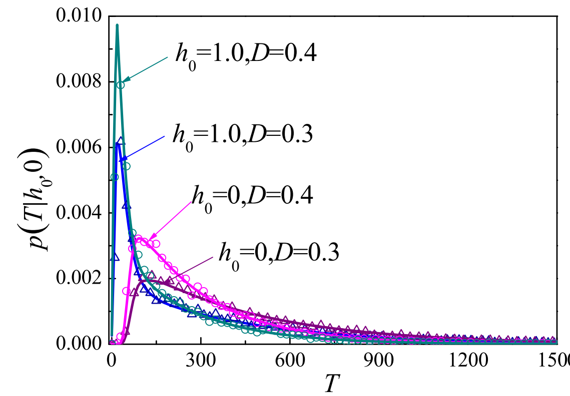
5. Numerical results and discussion

Numerical results are given in this section to demonstrate the implementation and accuracy of the established method for evaluating the first-passage failure of DE balloon by comparing with the results from Monte Carlo simulations (MCS) of the original system in Eq. (1). The conditional reliability function is first calculated, as well as the mean first-passage time. Then, the influences of the initial energy, the intensity of random pressure, the amplitude and frequency of harmonic voltage on the reliability are discussed in detail.

In the numerical example, the ratio of relatively prime positive integers is set as $k/q = 2$. As k and q are relatively prime positive integers, $k/q = 2$ actually indicates that $k = 2$ and $q = 1$. Referring to the parameter values in the existing literatures [22,25,28], the values of system and excitation parameters are set as: the non-dimensional damping coefficient $c = 0.01$, the amplitude ratio of the AC to DC components of the imposed voltage $r_{ad} = 0.1$, the non-dimensional frequency of harmonic voltage $\Omega = 2.95$, the non-dimensional quantities $s_r = 0.1$ and $s_f = 0.2$, the initial energy $h_0 = 0.0$, and the intensity of pressure disturbance $D = 0.3$, without otherwise mentioned. In this case, the stretch ratio associated with the stable equilibrium point is $\lambda_{eq} = 1.029$ while the minimum and maximum values of the potential energy are 1.412 and 4.050, respectively.



(a) Conditional reliability functions



(b) Conditional probability densities of the first-passage time

Fig. 3. Conditional reliability functions and conditional probability densities with several different combinations of initial energy and random pressure intensity.

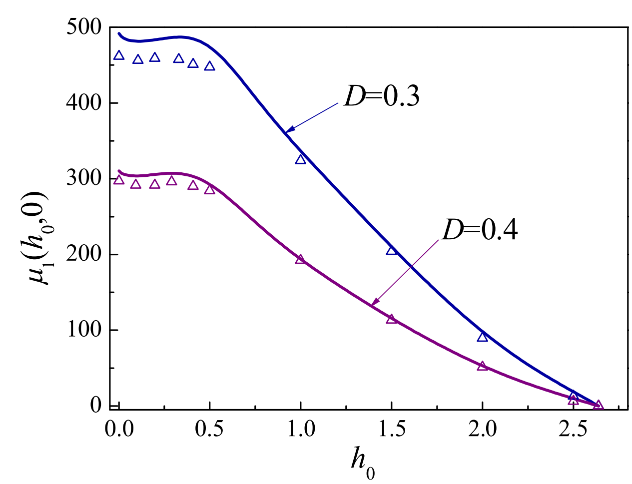


Fig. 4. Dependence of the mean first-passage time on the initial energy for two different values of random pressure intensity.

For the above values of system parameters, the DE balloon is in the resonant state. Fig. 3(a) depicts the conditional reliability as a function of time interval for several different combinations of initial energy values and excitation intensities and the conditional probability density of the first-passage time is shown in Fig. 3(b). Obviously, the present results (denoted by solid curves) coincide well with the MCS results (denoted by discrete symbols). The

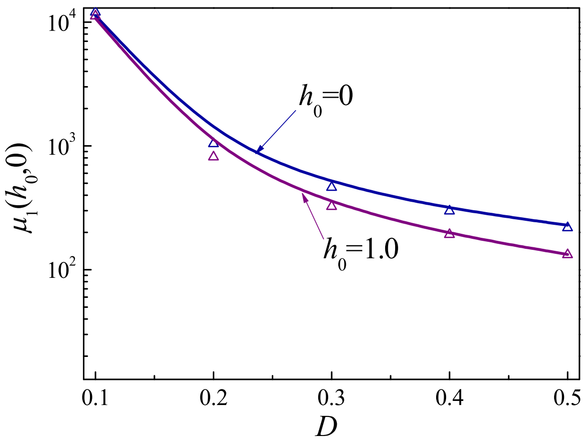


Fig. 5. Dependence of the mean first-passage time on the random pressure intensity for two different values of initial energy.

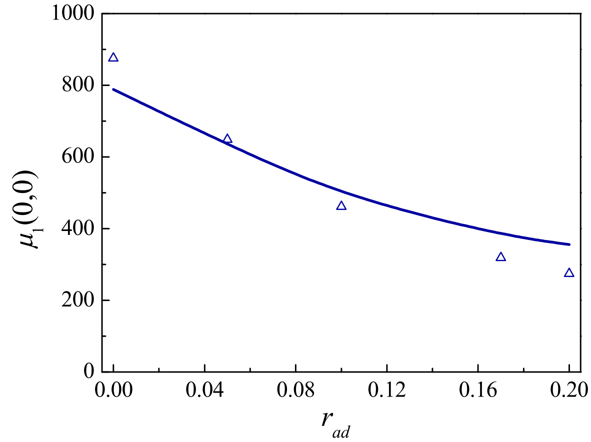
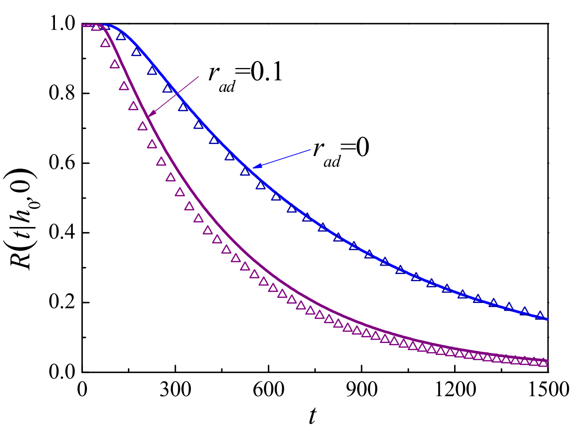
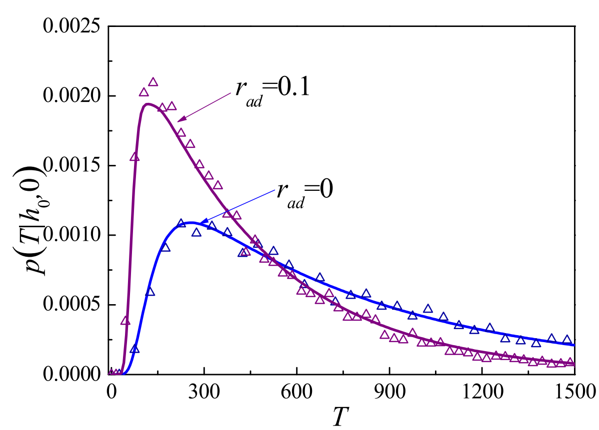


Fig. 7. Dependence of the mean first-passage time of DE balloon on the amplitude ratio of imposed voltage.

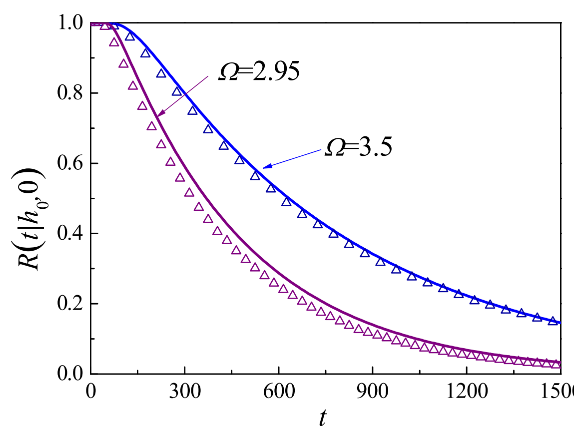


(a) Conditional reliability functions

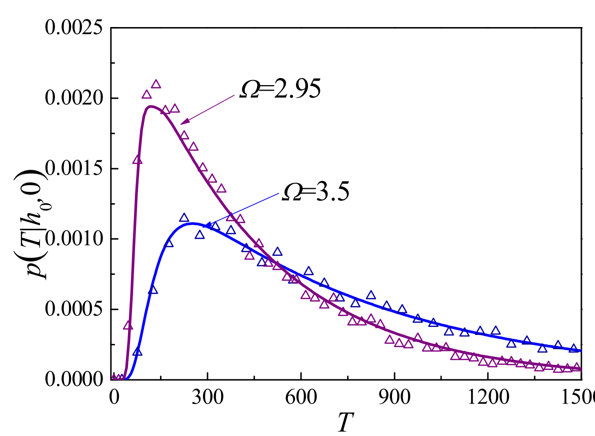


(b) Conditional probability densities of the first-passage time

Fig. 6. Conditional reliability functions and conditional probability densities with two different amplitude ratios of imposed voltage.



(a) Conditional reliability functions



(b) Conditional probability densities of the first-passage time

Fig. 8. Conditional reliability functions and conditional probability densities with two different frequencies of harmonic voltage.

reliability decreases with the increase of the initial energy value and/or the intensity of random pressure.

Figs. 4 and 5 depict the dependence of the mean first-passage time on the initial energy and the intensity of random pressure, respectively. As the initial energy h_0 is smaller than a critical value (*i.e.*, $h_0 < 0.5$ in this instance), the mean first-passage time of DE balloon slightly fluctuates with the change of initial energy. For large initial energy (*i.e.*, $h_0 > 0.5$ in this instance), the mean first-passage time sharply decreases with the initial energy value. From the view of randomness, the small initial energy can be regarded as an additional small disturbance and it has only negligible influence on the reliability. As shown in Fig. 5, the mean first-passage time of DE balloon monotonically decreases with the increase of the intensity of random pressure, and the descent rate

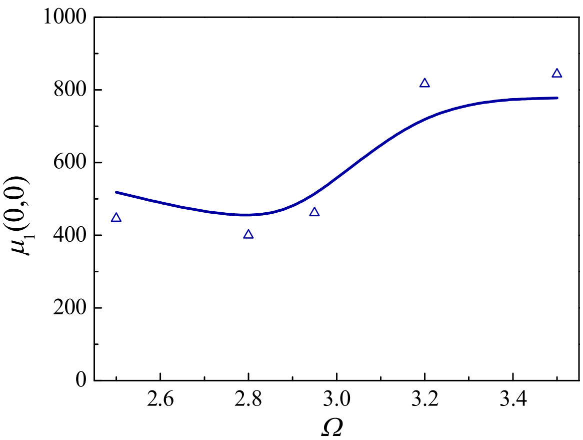


Fig. 9. Dependence of the mean first-passage time on the frequency of harmonic voltage.

tends to moderate with the increase of the intensity of random pressure. The large intensity of random pressure induces large random response and so small system reliability.

The influence of the amplitude ratio of AC component to DC component of imposed voltage on the reliability of DE balloon is shown in Fig. 6(a) for the conditional reliability function and Fig. 6(b) for the conditional probability density of first-passage time. As $r_{ad} = 0$, the DE balloon is subjected to a constant voltage and a time-varying pressure with static and random components. Although the DE balloon with $r_{ad} = 0$ belongs to the non-resonant case [28], the obtained results also possess high accuracy, as illustrated by compared with the MCS results. As shown in Fig. 6(a) and (b), DE balloon without harmonic voltage (*i.e.*, $r_{ad} = 0$) possesses the highest reliability compared with the cases with non-zero harmonic voltage (such as $r_{ad} = 0.1$). Given the initial energy $h_0 = 0$ and the initial phase difference $\gamma_0 = 0$, the relation of the mean first-passage time to the voltage ratio is depicted in Fig. 7. The mean first-passage time monotonically decreases with the increase of the amplitude ratio of imposed voltage.

The influences of the frequency of harmonic voltage on the reliability function and the probability density of first-passage time are depicted in Fig. 8(a) and (b), respectively. The relation of the mean first-passage time to the frequency of harmonic voltage is depicted in Fig. 9. It is worth to pointing out that for the frequency around the resonant frequency $\Omega \approx 2.95$, the system possesses the minimal reliability.

6. Conclusions

In this work, the reliability evaluation of a DE balloon subjected to a sinusoidal voltage and a random pressure disturbance has been investigated, which is described as a first-passage failure problem in mathematics. By adopting a special transformation and stochastic averaging, the backward Kolmogorov equation with respect to the conditional reliability function is derived. Numerically solving this equation yields the conditional reliability function, the conditional probability density of first-passage time, as well as various-order statistical moments of first-passage time. The present results coincide with the MCS results very well whether for the resonant case or for the non-resonant case. For small initial energy, the reliability of the DE balloon is insensitive to the variation of initial energy. However, for large initial energy, it monotonically decreases with initial energy. The reliability of the DE balloon monotonically decreases with the amplitude ratio of imposed voltage. The system possesses the minimal reliability for the frequency around the resonant frequency.

The objective of this work is to accurately predict the reliability of a DE balloon subjected to the combined excitation of harmonic voltage and random pressure disturbance, which is just the real excitation in the operating process of DE balloon actuators. It may provide some guidance on the design of DE-based components with the objective to maximize system reliability. By combining the present procedure and the stochastic dynamic programming principle or stochastic maximum principle, it is possible to develop a feedback control strategy to maximize system reliability. Furthermore, it is of great significance to generalize the present procedure to the cases with non-Gaussian or non-white noise. These are our further works.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

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References

[1] R. Pelrine, R. Kornbluh, Q. Pei, et al., High-speed electrically actuated elastomers with strain greater than 100%, Science 287 (2000) 836–839.
[2] Z.G. Suo, Theory of dielectric elastomers, Acta Mech. Solida Sin. 23 (2010) 549–578.
[3] G. Kovacs, L. During, S. Michel, et al., Stacked dielectric elastomer actuator for tensile force transmission, Sens. Actuators A – Phys. 155 (2009) 299–307.
[4] P. Brochu, Q. Pei, Advances in dielectric elastomers for actuators and artificial muscles, Macromol. Rapid Commun. 31 (2010) 10–36.
[5] Y. Ziser, G. Shmuel, Experimental slowing of flexural waves in dielectric elastomer films by voltage, Mech. Res. Commun. 85 (2017) 64–68.
[6] W. Kaal, S. Herold, Electroactive polymer actuators in dynamic applications, IEEE- ASME Trans Mech. 16 (2011) 24–32.
[7] U.K. Chakravarty, On the resonance frequencies of a membrane of a dielectric elastomer, Mech. Res. Commun. 55 (2014) 72–76.
[8] G.Y. Gu, J. Zhu, L.M. Zhu, et al., A survey on dielectric elastomer actuators for soft robots, Bioinspir. Biomim. 12 (2017) 011003.
[9] T. McKay, B. O'Brien, E. Calius, et al., An integrated, self-priming dielectric elastomer generator, Appl. Phys. Lett. 97 (2010) 062911.
[10] G. Moretti, G.P. Rosati Papini, L. Daniele, et al., Modelling and testing of a wave energy converter based on dielectric elastomer generators, P Roy Soc A-Math Phy 475 (2019) 20180566.
[11] G.P. Rosati Papini, G. Moretti, R. Vertechy, et al., Control of an oscillating water column wave energy converter based on dielectric elastomer generator, Nonlinear Dyn. 92 (2018) 181–202.
[12] E. Bortot, G. Shmuel, Tuning sound with soft dielectrics, Smart Mater. Struct. 26 (2017) 045028.
[13] G. Berselli, R. Vertechy, M. Babic, et al., Dynamic modeling and experimental evaluation of a constant-force dielectric elastomer actuator, J. Intell. Mater. Syst. Struct. 24 (2013) 779–791.
[14] Z. Lai, G. Thomson, D. Yurchenko, et al., On energy harvesting from a vibro-impact oscillator with dielectric membranes, Mech. Syst. Signal Process. 107 (2018) 105–121.
[15] G. Thomson, Z. Lai, D.V. Val, D. Yurchenko, Advantages of nonlinear energy harvesting with dielectric elastomers, J. Sound Vib. 442 (2019) 167–182.
[16] F. Liu, J.X. Zhou, Shooting and arc-length continuation method for periodic solution and bifurcation of nonlinear oscillation of viscoelastic dielectric elastomers, ASME J. Appl. Mech. 85 (2018) 011005.
[17] E. Bortot, Nonlinear dynamic response of soft thick-walled electro-active tubes, Smart Mater. Struct. 27 (2018) 105025.
[18] J.S. Zhang, H.L. Chen, B. Li, et al., Coupled nonlinear oscillation and stability evolution of viscoelastic dielectric elastomers, Soft Matter. 11 (2015) 7483–7493.
[19] H.D. Yong, X.Z. He, Y.H. Zhou, Dynamics of a thick-walled dielectric elastomer spherical shell, Int. J. Eng. Sci. 49 (2011) 792–800.
[20] J.W. Fox, N.C. Goulbourne, On the dynamic electromechanical loading of dielectric elastomer membranes, J. Mech. Phys. Solids 56 (2008) 2669–2686.
[21] S. Son, N.C. Goulbourne, Dynamic response of tubular dielectric elastomer transducers, Int. J. Solids Struct. 47 (2010) 2672–2679.
[22] J.J. Sheng, H.L. Chen, B. Li, et al., Nonlinear dynamic characteristics of a dielectric elastomer membrane undergoing in-plane deformation, Smart Mater. Struct. 23 (2014) 045010.
[23] T. Li, S. Qu, W. Yang, Electromechanical and dynamic analyses of tunable dielectric elastomer resonator, Int. J. Solids Struct. 49 (2012) 3754–3761.

[24] B.X. Xu, R. Mueller, A. Theis, et al., Dynamic analysis of dielectric elastomer actuators, Appl. Phys. Lett. 100 (2012) 112903.
[25] J. Zhu, S. Cai, Z. Suo, Nonlinear oscillation of a dielectric elastomer balloon, Polym. Int. 59 (2010) 378–383.
[26] J. Zhu, S. Cai, Z. Suo, Resonant behavior of a membrane of a dielectric elastomer, Int. J. Solids Struct. 47 (2010) 3254–3262.
[27] X.L. Jin, Z.L. Huang, Random response of dielectric elastomer balloon to electrical or mechanical perturbation, J. Intell. Mater. Syst. Struct. 28 (2017) 195–203.
[28] X.L. Jin, Y. Wang, M.Z.Q. Chen, et al., Response analysis of dielectric elastomer spherical membrane to harmonic voltage and random pressure, Smart Mater. Struct. 26 (2017) 035063.
[29] X.L. Jin, Y. Wang, Z.L. Huang, On the ratio of expectation crossing of random-excited dielectric elastomer balloon, Theor. Appl. Mech. Lett. 7 (2017) 100–104.
[30] H. Chen, Y. Wang, X.L. Jin, Z.L. Huang, Random vibration control for multi--degree-of-freedom mechanical systems with soft actuators, Int. J. Non-Linear Mech. 113 (2019) 44–54.
[31] G. Thomson, D. Yurchenko, D.V. Val, Z. Zhang, Predicting energy output of a stochastic nonlinear dielectric elastomer generator, Energy Convers. Manag. 196 (2019) 1445–1452.
[32] J.S. Plante, S. Dubowsky, Large-scale failure modes of dielectric elastomer actuators, Int. J. Solids Struct. 43 (2006) 7727–7751.
[33] M. Wissler, E. Mazza, Mechanical behavior of an acrylic elastomer used in dielectric elastomer actuators, Sens. Actuators A – Phys. 134 (2007) 494–504.
[34] M. Kollosche, G. Kofod, Electrical failure in blends of chemically identical, soft thermoplastic elastomers with different elastic stiffness, Appl. Phys. Lett. 96 (2010) 071904.
[35] X. Zhao, Q. Wang, Harnessing large deformation and instabilities of soft dielectrics: theory, experiment, and application, Appl. Phys. Rev. 1 (2014) 021304.
[36] L. Dorfmann, R.W. Ogden, Instabilities of soft dielectrics, Philos. Trans. R. Soc. A – Math. Phys. Eng. 377 (2019) 20180077.
[37] R. Pelrine, R. Kornbluh, J. Joseph, Electrostriction of polymer dielectrics with compliant electrodes as a means of actuations, Sens. Actuators A – Phys. 64 (1998) 77–85.
[38] C. Keplinger, M. Kaltenbrunner, N. Arnold, et al., Capacitive extensometry for transient strain analysis of dielectric elastomer actuators, Appl. Phys. Lett. 92 (2008) 192903.
[39] X.H. Zhao, Z.G. Suo, Method to analyze electromechanical stability of dielectric elastomers, Appl. Phys. Lett. 91 (2007) 061921.
[40] L. Dorfmann, R.W. Ogden, Instabilities of an electroelastic plate, Int. J. Eng. Sci. 77 (2014) 79–101.
[41] S.Y. Yang, X.H. Zhao, P. Sharma, Revisiting the instability and bifurcation behavior of soft dielectrics, ASME J. Appl. Mech. 84 (2017) 031008.
[42] E. Bortot, G. Shmuel, Prismatic bifurcations of soft dielectric tubes, Int. J. Eng. Sci. 124 (2018) 104–114.
[43] J.S. Leng, L.W. Liu, Y.J. Liu, et al., Electromechanical stability of dielectric elastomer, Appl. Phys. Lett. 94 (2009) 211901.
[44] F. Chen, J. Zhu, M.Y. Wang, Dynamic elecromechanical instability of a dielectric elastomer balloon, EPL-Europhys. Lett. 112 (2015) 47003.
[45] G.Q. Cai, W.Q. Zhu, Elements of Stochastic Dynamics, World Scientific, New Jersey, 2016.
[46] G.Q. Cai, Y.K. Lin, On statistics of 1st-passage failure, ASME J. Appl. Mech. 61 (1994) 93–99.
[47] J.B. Roberts, 1st-passage time for randomly excited nonlinear oscillators, J. Sound Vib. 109 (1986) 33–50.
[48] W.Q. Zhu, Nonlinear stochastic dynamics and control in hamiltonian formulation, Appl. Mech. Rev. 59 (2006) 230–248.
[49] Y.K. Lin, G.Q. Cai, Probabilistic Structural dynamics: Advanced Theory and Applications, McGraw-Hill, New York, 2004.