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Note

The recognition of the class of indecomposable digraphs under low hemimorphy

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ABSTRACT

Given a digraph $G = (V, A)$, the subdigraph of G induced by a subset X of V is denoted by $G[X]$. With each digraph $G = (V, A)$ is associated its dual $G^\star = (V, A^\star)$ defined as follows: for any $x, y \in V$, $(x, y) \in A^\star$ if $(y, x) \in A$. Two digraphs G and H are hemimorphic if G is isomorphic to H or to $H^\star$. Given $k > 0$, the digraphs $G = (V, A)$ and $H = (V, B)$ are k-hemimorphic if for every $X \subseteq V$, with $|X| \leq k$, $G[X]$ and $H[X]$ are hemimorphic. A class $\mathcal{C}$ of digraphs is k-recognizable if every digraph k-hemimorphic to a digraph of $\mathcal{C}$ belongs to $\mathcal{C}$. In another vein, given a digraph $G = (V, A)$, a subset X of V is an interval of G provided that for $a, b \in X$ and $x \in V - X$, $(a, x) \in A$ if and only if $(b, x) \in A$, and similarly for (x, a) and (x, b). For example, $\emptyset$, $\{x\}$, where $x \in V$, and V are intervals called trivial. A digraph is indecomposable if all its intervals are trivial. We characterize the indecomposable digraphs which are 3-hemimorphic to a non-indecomposable digraph. It follows that the class of indecomposable digraphs is 4-recognizable.

1. Introduction

A *directed graph* or simply *digraph* G consists of a finite and nonempty set V of *vertices* together with a prescribed collection A of ordered pairs of distinct vertices, called the set of the *arcs* of G. Such a digraph is denoted by (V, A). For example, given a set V, $(V, \emptyset)$ is the *empty* digraph on V whereas $(V, (V \times V) - \{(x, x);\ x \in V\})$ is the *complete* digraph on V. Given a digraph $G = (V, A)$, with each nonempty subset X of V associate the *subdigraph* $(X, A \cap (X \times X))$ of G induced by X denoted by $G[X]$. In another respect, given digraphs $G = (V, A)$ and $G' = (V', A')$, a bijection f from V onto V' is an *isomorphism* from G onto G' provided that for any $x, y \in V$, $(x, y) \in A$ if and only if $(f(x), f(y)) \in A'$. Two digraphs are then *isomorphic* if there exists an isomorphism from one onto the other. Finally, a digraph H *embeds* into a digraph G if H is isomorphic to a subdigraph of G.

With each digraph $G = (V, A)$ associate its *dual* $G^\star = (V, A^\star)$ and its *complement* $\overline{G} = (V, \overline{A})$ defined as follows. Given $x \neq y \in V$, $(x, y) \in A^\star$ if $(y, x) \in A$, and $(x, y) \in \overline{A}$ if $(x, y) \notin A$. The digraph $\overrightarrow{G} = (V, \overrightarrow{A})$ is then defined by $\overrightarrow{A} = A - A^\star$. Given digraphs G and H, G and H are *hemimorphic* if G is isomorphic to H or $H^\star$. Given an integer $k > 0$, consider digraphs $G = (V, A)$ and $H = (V, B)$. The digraphs G and H are *k-hemimorphic* if for every subset X of V, with $|X| \leq k$, the subdigraphs $G[X]$ and $H[X]$ are hemimorphic. A digraph G is *k-forced* (up to duality) if G and $G^\star$ are the only digraphs k-hemimorphic to G.

We need some notations. Let $G = (V, A)$ be a digraph. For $x \neq y \in V$, $x \longrightarrow_G y$ or $y \longleftarrow_G x$ means $(x, y) \in A$ and $(y, x) \notin A$, $x \longleftrightarrow_G y$ means $(x, y), (y, x) \in A$ and $x \cdots_G y$ means $(x, y), (y, x) \notin A$. For $x \in V$ and $Y \subseteq V$, $x \longrightarrow_G Y$ signifies that for every

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$y \in Y$, $x \longrightarrow_G y$. For $X, Y \subseteq V$, $X \longrightarrow_G Y$ signifies that for every $x \in X$, $x \longrightarrow_G Y$. For $x \in V$ and for $X, Y \subseteq V$, $x \longleftarrow_G Y$, $x \longleftrightarrow_G Y$, $x \cdots_G Y$, $X \longleftrightarrow_G Y$ and $X \cdots_G Y$ are defined in the same way. Furthermore, an equivalence relation, denoted by $\equiv_G$, between the ordered pairs of distinct vertices of a digraph $G = (V, A)$ is defined in the following way. For $x \neq y \in V$ and for $u \neq v \in V$, $(x, y) \equiv_G (u, v)$ if the function, which attributes u to x and v to y, is an isomorphism from $G[\{x, y\}]$ onto $G[\{u, v\}]$. Equivalently, $(x, y) \equiv_G (u, v)$ if $x \longrightarrow_G y$ and $u \longrightarrow_G v$ or $x \longleftarrow_G y$ and $u \longleftarrow_G v$ or $x \longleftrightarrow_G y$ and $u \longleftrightarrow_G v$ or $x \cdots_G y$ and $u \cdots_G v$. The negation is denoted by $(x, y) \not\equiv_G (u, v)$. Given a subset X of V, an element x of $V - X$ is a *separator* of X if there exist $u, v \in X$ such that $(x, u) \not\equiv_G (x, v)$. The set of the separators of X is denoted by $S_G(X)$.

A digraph $G = (V, A)$ is a *poset* provided that for $x, y, z \in V$, if $x \longrightarrow_G y$ and $y \longrightarrow_G z$, then $x \longrightarrow_G z$. With each poset $Q = (V, A)$ associate its *comparability digraph* $C(Q) = (V, A \cup A^\star)$. Given a digraph $G = (V, A)$, distinct vertices x and y of G form a *directed* pair if either $x \longrightarrow_G y$ or $y \longrightarrow_G x$. A digraph is a *tournament* if all its pairs are directed. For example, $(\{0, 1, 2\}, \{(0, 1), (1, 2), (2, 0)\})$ is a tournament, called 3-*cycle* and denoted by C_3. A *total order* is both a poset and a tournament. Given a total order $O = (V, A)$, $x < y$ means $x \longrightarrow_O y$ for $x, y \in V$.

Given a digraph $G = (V, A)$, a subset X of V is an *interval* [3,7] (or an *autonomous* set [5,8,9] or a *clan* [4] or a *homogeneous* set [2,6] or a *module* [11]) of G if $S_G(X) = \emptyset$. For instance, $\emptyset$, V and $\{x\}$, where $x \in V$, are intervals of G called *trivial* intervals. A digraph is *indecomposable* [3,7,10] (or *prime* [2] or *primitive* [4]) if all its intervals are trivial; otherwise, it is *decomposable*. The indecomposability bears a certain rigidity. The next result illustrates this fact in the case of the posets.

Theorem 1 (*[5,9]*). *Let $Q = (V, A)$ be an indecomposable poset. For every poset $Q' = (V, A')$, if $C(Q') = C(Q)$, then $Q' = Q$ or $Q' = Q^\star$.*

Given a poset Q, any digraph G, 3-hemimorphic to Q, is a poset such that $C(G) = C(Q)$. Therefore, every indecomposable poset is 3-forced. To obtain an analogue of Theorem 1 for the tournaments, the comparability digraph is replaced by the C_3-structure. Given a tournament $T = (V, A)$, the family of the subsets X of V, such that $T[X]$ is isomorphic to C_3, is called the C_3-*structure* of T and denoted by $C_3(T)$.

Theorem 2 (*[1]*). *Let $T = (V, A)$ be an indecomposable tournament. For every tournament $T' = (V, A')$, if $C_3(T') = C_3(T)$, then $T' = T$ or $T' = T^\star$.*

In other words, every indecomposable tournament is 3-forced. To generalize the two theorems above, we have to disallow the embedding of the following digraph and its dual. The digraph $(\{0, 1, 2\}, \{(0, 2), (2, 0), (0, 1)\})$ is denoted by F. The digraphs F and $F^\star$ are called *flags*. A digraph G is then said to be without flags when F and $F^\star$ do not embed into G.

Theorem 3 (*[1]*). *An indecomposable digraph without flags is* 3*-forced.*

The flags are generalized in the following way. Given an integer $n \geq 4$, consider a permutation σ of $\{0, \ldots, n-2\}$. The digraph $F_n(\sigma)$ is defined on $\{0, \ldots, n-1\}$ in the following manner:

(1) $F_n(\sigma)[\{0, \ldots, n-2\}]$ is the total order $\sigma(0) < \cdots < \sigma(n-2)$;
(2) given $m \in \{0, \ldots, n-2\}$, either m is even and $(m, n-1)$, $(n-1, m)$ are arcs of $F_n(\sigma)$ or m is odd and $(m, n-1)$, $(n-1, m)$ are not.

Given $n \geq 4$, $F_n(\mathrm{Id}_{\{0,\ldots,n-2\}})$ is simply denoted by F_n (see Fig. 1). For $k \geq 2$, the digraphs F_{2k} and $\overline{F_{2k}}$ (resp. F_{2k+1} and $(F_{2k+1})^\star$) are called *generalized flags*. By definition, $F_3(\mathrm{Id}_{\{0,1\}}) = F$. We may verify that for a permutation σ of $\{0, \ldots, n-2\}$, where $n \geq 3$, $F_n(\sigma)$ is decomposable if and only if there is $i \in \{0, \ldots, n-3\}$ such that $\sigma(i)$ and $\sigma(i+1)$ share the same parity. Therefore, the generalized flags are indecomposable. Furthermore, given an indecomposable digraph G, if I is an interval of $\overrightarrow{G}$, then the digraph obtained from G, by reversing all the arcs included in I, is 3-hemimorphic to G. Sometimes, intervals are created in this way so that the obtained digraph equals neither G nor $G^\star$. For instance, given $n \geq 4$, consider the generalized flag F_n and an integer $i > 0$ such that $2i \leq n-2$. Clearly, $\{1, \ldots, 2i\}$ is an interval of $\overrightarrow{F_n}$. From F_n, we obtain by reversing the arcs contained in $\{1, \ldots, 2i\}$ the digraph $F_n(\sigma_i)$, where σ_i is the permutation of $\{0, \ldots, n-2\}$ which interchanges j and $2i - j + 1$ for $1 \leq j \leq 2i$. The pair $\{0, 2i\}$ forms an interval of $F_n(\sigma_i)$. Consequently, the generalized flags are not 3-forced since F_n and $F_n(\sigma_i)$ differ regarding the indecomposability. Incidently, the problem of the recognition of the class of indecomposable digraphs also occurs. Precisely, given $k > 0$, a class $\mathcal{C}$ of digraphs is *k-recognizable* if every digraph k-hemimorphic to a digraph of $\mathcal{C}$ belongs to $\mathcal{C}$ as well. As showing by F_n and $F_n(\sigma_i)$, the class of indecomposable digraphs is not 3-recognizable. We reconsider these counter-examples with the following observation: $\{0, \ldots, 2i\}$ is an interval of $\overrightarrow{F_n}$ and for every $x \in \{0, \ldots, n-1\} - \{0, 2i\}$, we have $(x, 0) \not\equiv_{F_n} (x, 2i)$ if and only if $0 < x < 2i$. Generally, consider an indecomposable digraph $G = (V, A)$. Given vertices α and β of G such that $\alpha \longrightarrow_G \beta$, the pair $\{\alpha, \beta\}$ is *weakly separated* if $\{\alpha, \beta\} \cup S_G(\{\alpha, \beta\})$ is an interval of $\overrightarrow{G}$ and if $\alpha \longrightarrow_G S_G(\{\alpha, \beta\}) \longrightarrow_G \beta$. The main result consists of the following characterization.

Theorem 4. *Let G be an indecomposable digraph. There exists a decomposable digraph* 3*-hemimorphic to G if and only if G admits a weakly separated pair.*

As an immediate consequence, we obtain:

Theorem 5. *The class of indecomposable digraphs is* 4*-recognizable.*

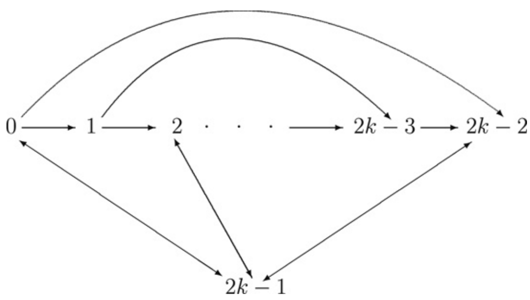

Fig. 1. The generalized flag F_{2k}.

2. The Gallai decomposition theorem

We begin with a well-known property of the intervals. Given a digraph $G = (V, A)$, if X and Y are disjoint intervals of G, then $(x, y) \equiv_G (x', y')$ for any $x, x' \in X$ and $y, y' \in Y$. This property leads to consider *interval partitions* of G, that is, partitions of V, all the elements of which are intervals of G. The elements of such a partition P become the vertices of the *quotient* $G/P = (P, A/P)$ of G by P defined as follows: given $X \neq Y \in P$, $(X, Y) \in A/P$ if $(x, y) \in A$ for $x \in X$ and $y \in Y$. To state the Gallai decomposition theorem below, we need the following strengthening of the notion of interval. Given a digraph $G = (V, A)$, a subset X of V is a *strong interval* [5,9] of G provided that X is an interval of G and for each interval Y of G, we have: if $X \cap Y \neq \emptyset$, then $X \subseteq Y$ or $Y \subseteq X$. The family of the maximal strong intervals under inclusion which are distinct from V is denoted by $P(G)$.

Theorem 6 *([5,9]). Given a digraph $G = (V, A)$, with $|V| \geq 2$, the family $P(G)$ constitutes an interval partition of G. Moreover, the corresponding quotient $G/P(G)$ is a complete digraph or an empty digraph or a total order or an indecomposable digraph.*

The next result follows from Theorem 3.

Corollary 7 *([1]). Given digraphs G and H without flags, if G and H are 3-hemimorphic, then $P(G) = P(H)$.*

3. Proof of Theorems 4 and 5

Lemma 8. *Consider 3-hemimorphic digraphs $G = (V, A)$ and $H = (V, B)$. Given an interval I of G such that $|I| \geq 2$, if $\overrightarrow{G}[I]/P(\overrightarrow{G}[I])$ is not a total order, then I is an interval of H.*

Proof. Given $x \in V - I$, since I is an interval of G, we have: $x \longleftrightarrow_G I$ or $x \cdots_G I$ or $x \longrightarrow_G I$ or $x \longleftarrow_G I$. In the first two instances, it follows from the 2-hemimorphy that $x \longleftrightarrow_H I$ or $x \cdots_H I$. In the last two ones, since $\overrightarrow{G}[I]/P(\overrightarrow{G}[I])$ is not a total order, $P(\overrightarrow{G}[I \cup \{x\}]) = \{I, \{x\}\}$. As $\overrightarrow{G}$ and $\overrightarrow{H}$ are 3-hemimorphic digraphs without flags, it follows from Corollary 7 that $P(\overrightarrow{H}[I \cup \{x\}]) = \{I, \{x\}\}$. Consequently, either $x \longrightarrow_H I$ or $x \longleftarrow_H I$. $\square$

Corollary 9. *Consider 3-hemimorphic digraphs G and H. If G is indecomposable, then for every interval I of H, $H[I]$ is a total order.*

Proof. Consider an interval I of H. By the previous lemma, $\overrightarrow{H}[I]/P(\overrightarrow{H}[I])$ is a total order. We denote the elements of $P(\overrightarrow{H}[I])$ by $X_1, \ldots, X_q$ in such a way that $\overrightarrow{H}[I]/P(\overrightarrow{H}[I])$ is the total order $X_1 < \cdots < X_q$. For a contradiction, suppose that there is $i \in \{1, \ldots, q\}$ such that $|X_i| \geq 2$. Since I is an interval of H, X_i is also. It follows from the preceding lemma that $\overrightarrow{H}[X_i]/P(\overrightarrow{H}[X_i])$ is a total order as well. By interchanging H and $H^\star$, we can assume that $i < q$. By denoting by Y the largest element of $\overrightarrow{H}[X_i]/P(\overrightarrow{H}[X_i])$, we obtain that $Y \cup X_{i+1}$ would be an interval of $\overrightarrow{H}[I]$, which contradicts the fact that X_i is a strong interval of $\overrightarrow{H}[I]$. Consequently, for each $i \in \{1, \ldots, q\}$, $|X_i| = 1$, that is, $H[I]$ is a total order. $\square$

Theorem 10. *Consider 3-hemimorphic digraphs $G = (V, A)$ and $H = (V, B)$. If G is indecomposable and if H is decomposable, then there exist $\alpha \neq \beta \in V$ such that $\{\alpha, \beta\}$ is an interval of H which is weakly separated in G.*

Proof. Given a non-trivial interval I of H, by the preceding corollary, $H[I]$ is a total order. Denote by α and β the first two elements of this total order, with $\alpha \longrightarrow_G \beta$. Clearly, $\{\alpha, \beta\}$ is an interval of H. Consider the smallest interval $\overrightarrow{J}$ of $\overrightarrow{G}$ containing α and β. We use Theorem 6. Firstly, suppose that $\overrightarrow{G}[\overrightarrow{J}]/P(\overrightarrow{G}[\overrightarrow{J}])$ is empty. Since $\{\alpha, \beta\}$ is directed, there is an element of $P(\overrightarrow{G}[\overrightarrow{J}])$ containing α and β, which contradicts the minimality of $\overrightarrow{J}$. Secondly, assume that $\overrightarrow{G}[\overrightarrow{J}]/P(\overrightarrow{G}[\overrightarrow{J}])$ is indecomposable. As $\overrightarrow{G}[\overrightarrow{J}]$ and $\overrightarrow{H}[\overrightarrow{J}]$ are 3-hemimorphic digraphs without flags, it follows from Corollary 7 that $P(\overrightarrow{G}[\overrightarrow{J}]) = P(\overrightarrow{H}[\overrightarrow{J}])$. Since $\{\alpha, \beta\}$ is an interval of H, $\{\alpha, \beta\}$ is an interval of $\overrightarrow{H}[\overrightarrow{J}]$. We obtain the same contradiction because $\overrightarrow{H}[\overrightarrow{J}]/P(\overrightarrow{H}[\overrightarrow{J}])$ is indecomposable by Theorem 3. Therefore, $\overrightarrow{G}[\overrightarrow{J}]/P(\overrightarrow{G}[\overrightarrow{J}])$ is a total order. We denote the elements of $P(\overrightarrow{G}[\overrightarrow{J}])$ by $X_1, \ldots, X_q$ in such a way that the corresponding quotient is $X_1 < \cdots < X_q$. By the minimality of $\overrightarrow{J}$, $\alpha \in X_1$ and $\beta \in X_q$. As previously noticed, $P(\overrightarrow{H}[\overrightarrow{J}]) = \{X_1, \ldots, X_q\}$ and hence $\overrightarrow{H}[\overrightarrow{J}]/P(\overrightarrow{H}[\overrightarrow{J}])$ is a total order as well. Since $\{\alpha, \beta\}$ is an interval of H, $\{\alpha, \beta\}$ is an interval of $\overrightarrow{H}[\overrightarrow{J}]$. As X_1 and X_q are strong intervals of $\overrightarrow{H}[\overrightarrow{J}]$, $\{\alpha, \beta\} = X_1 \cup X_q$ or, equivalently, $X_1 = \{\alpha\}$ and $X_q = \{\beta\}$. To conclude, we verify that $\{\alpha, \beta\}$ is weakly separated in G. It suffices to show that for every $x \in V - \{\alpha, \beta\}$, $(x, \alpha) \not\equiv_G (x, \beta)$ if and only if $x \in \overrightarrow{J} - \{\alpha, \beta\}$. Clearly, if $x \in \overrightarrow{J} - \{\alpha, \beta\}$, then $\alpha \longrightarrow_G x \longrightarrow_G \beta$ and hence $(x, \alpha) \not\equiv_G (x, \beta)$. Conversely, consider an element u of $V - \overrightarrow{J}$. If $\{u, \alpha\}$ is directed, then $(u, \alpha) \equiv_G (u, \beta)$ because $\overrightarrow{J}$ is an interval of $\overrightarrow{G}$. Otherwise, $(u, \alpha) \equiv_G (u, \beta)$ because $\{\alpha, \beta\}$ is an interval of H. □

The proof of the main result follows.

Proof of Theorem 4. Consider an indecomposable digraph $G = (V, A)$. If there is a decomposable digraph 3-hemimorphic to G, then, by Theorem 10, G possesses a weakly separated pair. Conversely, consider $\alpha \neq \beta \in V$ such that $\{\alpha, \beta\}$ is a weakly separated pair of G. Since $\{\alpha, \beta\} \cup S_G(\{\alpha, \beta\})$ is an interval of $\overrightarrow{G}$, $\{\beta\} \cup S_G(\{\alpha, \beta\})$ is also. Consequently, by reversing all the arcs contained in $\{\beta\} \cup S_G(\{\alpha, \beta\})$, we obtain a digraph H which is 3-hemimorphic to G. The pair $\{\alpha, \beta\}$ is then an interval of H and thus H is decomposable. □

The next result follows from Theorem 10.

Corollary 11. *Consider 3-hemimorphic digraphs $G = (V, A)$ and $H = (V, B)$ such that G is indecomposable and H is decomposable. There exists a subset X of V, with $|X| = 4$, such that $G[X]$ is indecomposable and $H[X]$ is decomposable. More precisely, $G[X]$ is isomorphic to F_4 (resp. $\overline{F_4}$) and $H[X]$ is isomorphic to $F_4(\sigma)$ (resp. $\overline{F_4(\sigma)}$), where σ is the permutation of $\{0, 1, 2\}$ which interchanges either 0 and 1 or 1 and 2.*

Proof. By Theorem 10, there are $\alpha \neq \beta \in V$ such that $\{\alpha, \beta\}$ is an interval of H which is weakly separated in G. If $\{\alpha, \beta\} \cup S_G(\{\alpha, \beta\}) = V$, then $\{\beta\} \cup S_G(\{\alpha, \beta\})$ would be an interval of G. Consequently, $\{\alpha, \beta\} \cup S_G(\{\alpha, \beta\}) \neq V$ and hence $\{\alpha, \beta\} \cup S_G(\{\alpha, \beta\})$ is an interval of $\overrightarrow{G}$ and not of G. Therefore, there exist $s \in S_G(\{\alpha, \beta\})$ and $u \notin \{\alpha, \beta\} \cup S_G(\{\alpha, \beta\})$, such that $\{\alpha, \beta, s\}$ is an interval of $\overrightarrow{G}[\{\alpha, \beta, s, u\}]$ and not of $G[\{\alpha, \beta, s, u\}]$. It follows that $\{\alpha, u\}$, $\{s, u\}$ and $\{\beta, u\}$ are not directed. For example, assume that $\alpha \longleftrightarrow_G u$. Since $u \notin S_G(\{\alpha, \beta\})$, $\beta \longleftrightarrow_G u$ and, necessarily, $s \cdots_G u$. Furthermore, $G[\{\alpha, \beta, s\}]$ is the total order $\alpha < s < \beta$ or $\beta < s < \alpha$ because $s \in S_G(\{\alpha, \beta\})$. In both cases, $G[\{\alpha, \beta, s, u\}]$ is isomorphic to F_4. As G and H are 3-hemimorphic, we have $\alpha \longleftrightarrow_H u$, $s \cdots_H u$, $\beta \longleftrightarrow_H u$ and $H[\{\alpha, \beta, s\}]$ is a total order. To end, it is sufficient to recall that $\{\alpha, \beta\}$ is an interval $H[\{\alpha, \beta, s\}]$. □

Theorem 5 is directly deduced. Finally, Corollary 11 leads to the following.

Remark 12. To obtain Theorem 5, it is not necessary to assume that the considered digraphs $G = (V, A)$ and $H = (V, B)$ to be 4-hemimorphic. It suffices to require that G and H are 2-hemimorphic and that for every subset X of V, with $|X| = 3$ or 4, the subdigraphs $G[X]$ and $H[X]$ are both indecomposable or not.

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