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Propagation phenomena in a visco-thermo-micropolar elastic medium under the effect of micro-temperature

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ABSTRACT

The analysis is made on reflection of waves in thermoelastic micropolar medium. The medium is having an additional property of viscosity, while studying waves the effect of micro-temperature is also been considered. It is found that after reflection three longitudinal and three transverse waves propagate through the medium. Reflected coefficients are calculated for each wave to examine deviation of reflected waves. Results obtained theoretically are shown graphically against angle of incidence. It is analyzed that effect of viscosity and micro-temperature reaches to its maximum level during intermediate values of angle of incident.

Introduction

The linear theory of linear viscoelasticity is considered as very important branch of Elastodynamics. It was observed by Freudenthal [1] that, most of the solids when subjected to dynamic loading exhibit a viscous effect. Because of this viscous effect internal friction produces attenuation and dispersion. Initially, Biot [2,3] and Bland [4] linked the solution of linear viscoelastic problems with corresponding linear elastic solutions. A notable works in this field were the work of Gurtin and Sternberg [6], and Ilioushin [7] offered an approximation method for the linear thermal viscoelastic problems. Problems related with micropolar viscoelastic waves was initiated by McCarthy and Eringen [5]. They discussed the propagation conditions and growth equations which govern the wave propagation of waves in micropolar viscoelasticity. Some sources are considered on study of viscoelastic materials are, Othman and Fekry [8], they studied the effect of initial stress on generalized thermo-viscoelastic medium with voids and temperature-dependent properties under Green-Naghdi theory. Kumar and Choudhary [9] analyzed different wave problems in micropolar visco-elastic thermo elastic solid. Effect of rotation on generalized thermo-viscoelastic Rayleigh Lamb waves was discussed by Sharma and Othman [10].

The theory of micro temperatures deals with the propagation of the temperature wave in a rigid heat conductor which allows the variation of thermal properties at a microstructure level. The theory of thermodynamics for elastic material with inner structures was developed by Grot [11] according to which the molecules possess micro-temperatures along with macro-deformation of the body the micro temperatures depend homogeneously on the micro-coordinates of the microelement. The experimental data for the silicone rubber containing spherical aluminum particles and for human blood presented by Riha [12] conform closely to the predicted theoretical model of thermoelasticity for micro-temperatures. Some authors recently invested some results related with wave propagation [13–15].

Green and Naghdi developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labeled as model I, II and III [16–18]. The nature of these theories is such that when the respective theories are linearized, model I [16] reduces to the classical heat conduction based on Fourier's law. The linearized versions of model II and III permit propagation of thermal waves at finite speed. Model II, in particular, exhibits a feature that is not present in the other established thermoelastic

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models as it does not sustain dissipation of thermal energy [18]. In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green and Naghdi's third model [17] admits the dissipation of energy. In this model, the constitutive equations are derived by starting with the reduced energy equation, where the thermal displacement gradient and temperature gradient are among the constitutive variables. The uniqueness of the solution of governing equations for the GN type II model was established in [19]. Chandra Sekharaiah [20] studied the one dimensional thermal wave propagation in a half-plane based on the GN model. Some works on reflection waves in a half-space is discusser (see, Refs. [21–25]). Researchers as Othman and Song [26], Gupta and Rani [27] and Bayones and Abd-Alla [28] studied different type of waves propagating under different external influences. Kumar et al. [29] explained plane waves propagation in microstretch thermoelastic medium with micrtemperature. New features on waves reflection with an external parameters as magnetic field, initial stress and rotation has been investigated in (Refs. [30–36]).

In this article the authors are interested in the study of seismic waves and their reflection from a surface of thermoelastic medium. It is of great practical importance in geophysical investigations. Seismic signals carry a lot of information about the internal structure of the earth and this information is of great help in exploration of variable materials. We basically study the reflection of plane waves at the free surface of the micropolar generalized thermoelastic half space solid. The medium is naturally viscoelastic and the effect of micro temperature is also been considered while analyzing the amplitude of reflected waves. Green Naghdi theory of type III is considered to represent the heat waves conducting through the medium. Reflected coefficients for both transverse and longitudinal waves are obtained theoretically, analyzed and finally represented graphically against the incident angle.

Formulation of the problem

Cartesian Coordinates (x, y, z) are being selected to represent the system of problem. Origin is on surface $y = 0$ and z-axis directed along depth of solid. Basic governing equations for the problem are,

$$\rho \ddot{u}_i = (\lambda_I + \mu_I) u_{j,ji} + (\mu_I + k_I) u_{i,jj} + k_I \varepsilon_{ijk} \varphi_{k,j} - \beta T_{,i} \tag{1}$$

$$(\alpha_I + \beta_I' + \gamma_I)\, \vec{\nabla}(\vec{\nabla}.\vec{\varphi}) - \gamma_I \vec{\nabla} \times (\vec{\nabla} \times \vec{\varphi}) + k_I (\vec{\nabla} \times \vec{u}) - 2 k_I \vec{\varphi} - \mu_1 (\vec{\nabla} \times w) = j \rho \frac{\partial^2 \vec{\varphi}}{\partial t^2}, \tag{2}$$

$$k_6 \nabla^2 w + (k_4 + k_5) \nabla(\nabla.w) + \mu_1 (\vec{\nabla} \times \vec{\varphi}) - b \frac{\partial w}{\partial t} - k_2 w - k_3 \nabla T = 0, \tag{3}$$

$$K^* \nabla^2 T + K \nabla^2 \dot{T} + k_1 (\vec{\nabla}.\vec{w}) = \rho C_E \dot{T} + \beta T_0 \ddot{u}_{i,i}, \tag{4}$$

GN-II can be obtained by adjusting $K = 0$ in Eq. (4), the constitutive equations are

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2 \mu e_{ij} + k (u_{j,i} - \varepsilon_{ijk} \phi_k) - \beta T \delta_{ij} \tag{5}$$

$$m_{ij} = \alpha \phi_{l,l} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i}, \quad q_{ij} = -k_4 w_{l,l} \delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i} \quad j, i, l, k = 1, 2, 3. \tag{6}$$

Where $\alpha_I, \beta_I', \gamma_I, \mu_I, \lambda_I, k_i (i = 1, 2, \ldots, 6)$ are constitutive coefficients u_i, σ_{ij}, e_{ij}, m_{ij} are the components of displacement vector, of stress tensor, strain tensor and couple stress tensor respectively, j is the micro inertia moment, the mass density is ρ, the specific heat at constant strain is C_E, K^*, K are the thermal conductivity and the material characteristic respectively of the theory. T_0 is the reference temperature, $\beta = (3\lambda_I + 2\mu_I)\alpha_t$ where α_t are the coefficients of linear thermal expansion for the material.

Assuming the viscoelastic nature of the material [10],

$$\lambda_I = \lambda + \frac{\partial}{\partial t} \lambda_v, \quad \mu_I = \mu \left(1 + \frac{\partial}{\partial t} \tau_v\right), \quad k_I = k(1 + \frac{\partial}{\partial t} \tau_v),$$
$$\alpha_I = \alpha \left(1 + \frac{\partial}{\partial t} \tau_v\right), \beta_I = \beta(1 + \frac{\partial}{\partial t} \tau_v), \quad \gamma_I = \gamma \left(1 + \frac{\partial}{\partial t} \tau_v\right),$$

where, τ_v is the sensitive part representing the viscosity.

Displacement and microrotation components are taken as,

$$\vec{u} = (u_1(x_1, x_3), 0, u_3(x_1, x_3)) \quad \text{and} \quad \vec{w} = (w_1(x_1, x_3), 0, w_3(x_1, x_3)) \tag{7}$$

Following are the non-dimensional parameters introduced for the problem,

$$(x_i', u_i') = \frac{w^*}{c_1}(x_i, u_i), \quad t' = w^* t, \quad \phi_2' = \frac{w^{*2} j}{c_1^2} \phi_2, w^{*\prime} = \frac{K_I}{\rho j},$$
$$m_{ij}' = \frac{w^* \lambda_i^*}{c_1 \lambda_I}, \quad w_i' = \frac{c_1}{w^*} w_i \tag{8}$$

The component of displacement functions $(u_1, 0, u_3)$ and micro temperature $(w_1, 0, w_3)$ are connected with potential functions R, ψ and G, H respectively, by the relation [24],

$$u_1 = \frac{\partial R}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial R}{\partial z} + \frac{\partial \psi}{\partial x} \quad \text{and} \quad w_1 = \frac{\partial G}{\partial x} - \frac{\partial H}{\partial z},$$
$$w_3 = \frac{\partial G}{\partial z} + \frac{\partial H}{\partial x} \tag{9}$$

Making use of Eqs. (7)–(9) in (1)–(4) we obtained the following set of equations,

$$\left((\delta_1 + \delta_2)\nabla^2 - \frac{\partial^2}{\partial t^2}\right) R - \delta_4 T = 0, \tag{10}$$

$$\left(\delta_2 \nabla^2 - \frac{\partial^2}{\partial t^2}\right) \psi + \delta_3 \phi_2 = 0, \tag{11}$$

$$\left(\delta_5 \nabla^2 - 2\delta_3 - \frac{\partial^2}{\partial t^2}\right) \phi_2 + \delta_6 \nabla^2 \psi + \delta_7 \nabla^2 H = 0, \tag{12}$$

$$\left(K_6 \nabla^2 + \delta_8 \frac{\partial}{\partial t} + \delta_{10}\right) H - \delta_9 \phi_2 = 0, \tag{13}$$

$$\left((K_4 + K_5 + K_6)\nabla^2 - \delta_8 \frac{\partial}{\partial t} - \delta_{10}\right) G - \delta_{11} T = 0, \tag{14}$$

$$\left(\varepsilon_2 + \varepsilon_3 \frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2}\right) T + \delta_{12} \nabla^2 G - \varepsilon_1 \nabla^2 \ddot{R} = 0. \tag{15}$$

where,

$$\delta_1 = \frac{\lambda + \mu}{\rho c_1^2}, \quad \delta_2 = \frac{\mu + k}{\rho c_1^2}, \quad \delta_3 = \frac{k}{\rho \omega^{*2} j}, \quad \delta_4 = \frac{\beta T_0}{\rho c_1^2},$$
$$\delta_5 = \frac{\gamma}{j \rho c_1^2}, \quad \delta_6 = \frac{k}{\rho c_1^2}, \delta_7 = \frac{\mu_1 \omega^{*2}}{\rho c_1^4}, \quad \delta_8 = \frac{b c_1^2}{\omega^*}, \quad \delta_9 = \frac{\mu_1 c_1^4}{\omega^{*4} j},$$
$$\delta_{10} = \frac{K_2 c_1^2}{\omega^{*2}}, \quad \delta_{11} = \frac{K_3 c_1^2 T_0}{\omega^{*2}}, \quad \delta_{12} = \frac{K_1}{\rho C_E c_1^2 T_0}, \quad \varepsilon_1 = \frac{\beta}{\rho C_E},$$
$$\varepsilon_2 = \frac{K^*}{\rho C_E c_1^2} \quad \text{and} \quad \varepsilon_3 = \frac{K \omega^*}{\rho C_E c_1^2}.$$

Reflection coefficients

Let us consider a plane harmonic wave solution incident at the free surface $x_2 = 0$, making angleθ_0 with normal of the surface. After reflection six waves are generated as represented in figure below (See Fig. 1),

In order to solve the system of partial differential equations we have assumed the solution

$$(R, \psi, T, \phi_2, G, H) = (R', \psi', T', \phi_2', G', H')e^{i(l(x_1 \sin\theta + x_3 \cos\theta) - \omega t)}, \quad (16)$$

where $(R', \psi', T', \phi_2', G', H')$ are the amplitudes of the reflected waves, ω is the angular frequency and l is the wave number.

Substitute from Eq. (16) into Eqs. (10)–(15), we obtain

$$(\omega^2 - (\delta_1 + \delta_2)l^2)R - \delta_4 T = 0, \quad (17)$$

$$(\omega^2 - \delta_2 l^2)\psi + \delta_3 \phi_2 = 0, \quad (18)$$

$$(\delta_5 l^2 + 2\delta_3 + \omega^2)\phi_2 + \delta_6 l^2 \psi + \delta_7 l^2 H = 0, \quad (19)$$

$$(K_6 l^2 + i\delta_8 \omega - \delta_{10})H + \delta_9 \phi_2 = 0, \quad (20)$$

$$((K_4 + K_5 + K_6)l^2 - i\omega\delta_8 + \delta_{10})G + \delta_{11} T = 0, \quad (21)$$

$$((\varepsilon_2 - i\omega\varepsilon_3)l^2 + \omega^2)T + \delta_{12} l^2 G + \varepsilon_1 l^2 \omega^2 R = 0. \quad (22)$$

The above set of equations is basically depending on two sets. One set is related with longitudinal while the other with transverse waves propagating through the medium. For non trivial solution of the set depending on Eqs. (17), (21) and (22) related with the variables R, T and G is as,

$$\begin{vmatrix} \omega^2 - (\delta_1 + \delta_2)l^2 & -\delta_4 & 0 \\ 0 & \delta_{11} & (K_4 + K_5 + K_6)l^2 - i\omega\delta_8 + \delta_{10} \\ \varepsilon_1 l^2 \omega^2 & (\varepsilon_2 - i\omega\varepsilon_3)l^2 + \omega^2 & \delta_{12} l^2 \end{vmatrix} = 0, \quad (23)$$

It can also be represented as

$$l^6(\Gamma_4) + l^4(\Gamma_3) + l^2(\Gamma_2) + \Gamma_1 = 0 \quad (24)$$

where,

$$\Gamma_1 = i\omega^5\delta_8 - \omega^4\delta_{10},$$

$$\begin{aligned}\Gamma_2 = &-\omega^4 K_4 - \omega^4 K_5 - \omega^4 K_6 + i\omega^3\varepsilon_2\delta_8 + \omega^4\varepsilon_3\delta_8 - i\omega^3\delta_1\delta_8 \\ &- i\omega^3\delta_2\delta_8 + i\omega^3\varepsilon_1\delta_4\delta_8 - \omega^2\varepsilon_2\delta_{10} + i\omega^3\varepsilon_3\delta_{10} + \omega^2\delta_1\delta_{10} \\ &- \omega^2\delta_2\delta_{10} - \omega^2\varepsilon_1\delta_4\delta_{10} + \omega^2\delta_{11}\delta_{12},\end{aligned}$$

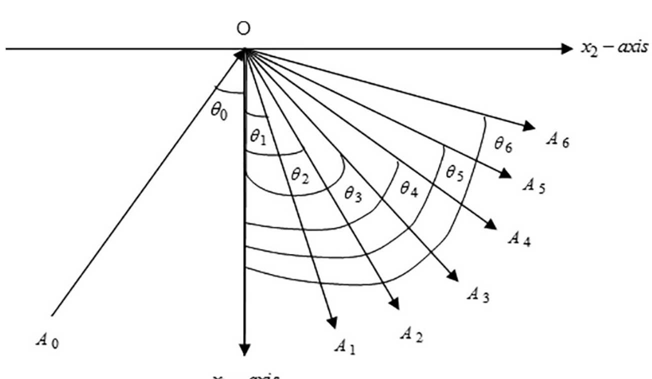

Fig. 1. Geometry of the problem.

$$\begin{aligned}\Gamma_3 = &-\omega^2 K_4\varepsilon_2 + i\omega^3\varepsilon_3 K_4 - \omega^2\varepsilon_2 K_5 + i\omega^3\varepsilon_3 K_5 - \omega^2\varepsilon_2 K_6 \\ &+ i\omega^3\varepsilon_3 K_6 + \omega^2 K_4\delta_1 + \omega^2 K_5\delta_1 + \omega^2 K_6\delta_1 + \omega^2 K_4\delta_2 + \omega^2 K_5\delta_1 \\ &+ \omega^2 K_6\delta_1 + \omega^2 K_4\delta_2 + \omega^2 K_5\delta_2 + \omega^2 K_6\delta_2 - \omega^2\varepsilon_1 K_4\delta_4 \\ &- \omega^2\varepsilon_1 K_5\delta_4 - \omega^2\varepsilon_1 K_6\delta_4 - i\omega\varepsilon_2\delta_1\delta_8 - \omega^2\varepsilon_3\delta_1\delta_8 - i\omega\varepsilon_2\delta_2\delta_8 \\ &- \omega^2\varepsilon_3\delta_2\delta_8 + \varepsilon_2\delta_1\delta_{10} - i\omega\varepsilon_3\delta_1\delta_{10} + \varepsilon_2\delta_2\delta_{10} - i\omega\varepsilon_3\delta_2\delta_{10} \\ &- \delta_1\delta_{11}\delta_{12} - \delta_2\delta_{11}\delta_{12},\end{aligned}$$

$$\begin{aligned}\Gamma_4 = &\varepsilon_2 K_4\delta_1 - i\omega\varepsilon_3 K_4\delta_1 + \varepsilon_2 K_5\delta_1 - i\omega\varepsilon_3 K_5\delta_1 + \varepsilon_2 K_6\delta_1 - i\omega\varepsilon_3 K_6\delta_1 \\ &+ \varepsilon_2 K_4\delta_2 - i\omega\varepsilon_3 K_4\delta_2 + \varepsilon_2 K_5\delta_2 - i\omega\varepsilon_3 K_5\delta_2 + \varepsilon_2 K_6\delta_2 \\ &- i\omega\varepsilon_3 K_6\delta_2.\end{aligned}$$

Similarly Eqs. (18)–(20) for the variables ψ, ϕ_2 and H could be represented as

$$\begin{vmatrix} \omega^2 - \delta_2 l^2 & \delta_3 & 0 \\ \delta_6 l^2 & \delta_5 l^2 + 2\delta_3 + \omega^2 & \delta_7 l^2 \\ 0 & \delta_9 & K_6 l^2 + i\delta_8\omega - \delta_{10} \end{vmatrix} = 0. \quad (25)$$

Can be represented as

$$l^6(\Gamma_8) + l^4(\Gamma_7) + l^2(\Gamma_6) + \Gamma_5 = 0 \quad (26)$$

where,

$$\Gamma_5 = i\omega^5\delta_8 + 2i\omega^3\delta_3\delta_8 - \omega^4\delta_{10} - 2\omega^2\delta_3\delta_{10},$$

$$\begin{aligned}\Gamma_6 = &\omega^4 K_6 + 2\omega^2 K_6\delta_3 - i\omega^3\delta_2\delta_8 - 2i\omega\delta_2\delta_3\delta_8 + i\omega^3\delta_5\delta_8 \\ &- i\omega\delta_3\delta_6\delta_8 + \omega^2\delta_7\delta_9 + \omega^2\delta_2\delta_{10} + 2\delta_2\delta_3\delta_{10} - \omega^2\delta_5\delta_{10} \\ &+ \delta_3\delta_6\delta_{10},\end{aligned}$$

$$\begin{aligned}\Gamma_7 = &-\omega^2 K_6\delta_2 - 2K_6\delta_2\delta_3 + \omega^2 K_6\delta_5 - K_6\delta_3\delta_6 - i\omega\delta_2\delta_5\delta_8 - \delta_2\delta_7\delta_9 \\ &+ \delta_2\delta_5\delta_{10},\end{aligned}$$

$$\Gamma_8 = -K_6\delta_2\delta_5,$$

Eqs. (24) and (26) are cubic in nature and having roots l_1, l_2, l_3 and l_4, l_5, l_6 respectively. Solution of the respective equations can be represented as follow:

$$\begin{aligned}R = &A_0 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &+ A_1 \exp[il_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] \\ &+ A_2 \exp[il_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] \\ &+ A_3 \exp[il_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t]\end{aligned} \quad (27)$$

$$\begin{aligned}T = &A_0\eta_1 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &+ A_1\eta_1 \exp[il_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] \\ &+ A_2\eta_2 \exp[il_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] \\ &+ A_3\eta_3 \exp[il_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t]\end{aligned} \quad (28)$$

$$\begin{aligned}G = &A_0\varpi_1 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &+ A_1\varpi_1 \exp[il_1(x\sin\theta_1 - z\cos\theta_1) - i\omega t] \\ &+ A_2\varpi_2 \exp[il_2(x\sin\theta_2 - z\cos\theta_2) - i\omega t] \\ &+ A_3\varpi_3 \exp[il_3(x\sin\theta_3 - z\cos\theta_3) - i\omega t]\end{aligned} \quad (29)$$

$$\begin{aligned}\psi = &B_0 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] \\ &+ B_4 \exp[il_4(x\sin\theta_4 - z\cos\theta_4) - i\omega t] \\ &+ B_5 \exp[il_5(x\sin\theta_5 - z\cos\theta_5) - i\omega t] \\ &+ B_6 \exp[il_6(x\sin\theta_6 - z\cos\theta_6) - i\omega t]\end{aligned} \quad (30)$$

$$\phi_2 = B_0\eta_4 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] + B_4\eta_4 \exp[il_4(x\sin\theta_4 - z\cos\theta_4) - i\omega t] + B_5\eta_5 \exp[il_5(x\sin\theta_5 - z\cos\theta_5) - i\omega t] + B_6\eta_6 \exp[il_6(x\sin\theta_6 - z\cos\theta_6) - i\omega t] \quad (31)$$

$$H = B_0\varpi_4 \exp[il_0(x\sin\theta_0 + z\cos\theta_0) - i\omega t] + B_4\varpi_4 \exp[il_4(x\sin\theta_4 - z\cos\theta_4) - i\omega t] + B_5\varpi_5 \exp[il_5(x\sin\theta_5 - z\cos\theta_5) - i\omega t] + B_6\varpi_6 \exp[il_6(x\sin\theta_6 - z\cos\theta_6) - i\omega t] \quad (32)$$

where

$$\eta_i = \begin{cases} \frac{(\omega^2 - l_i^2(\delta_1+\delta_2))}{\delta_4} & for\ i = 1,2,3 \\ \frac{-(\omega^2 - l_i^2\delta_2)}{\delta_3} & for\ i = 4,5,6. \end{cases},$$

$$\varpi_i = \begin{cases} \frac{-\delta_{11}(\omega^2 - l_i^2(\delta_1+\delta_2))}{\delta_4((K_4+K_5+K_6)l_i^2 - i\delta_8\omega + \delta_{10})} & for\ i = 1,2,3 \\ \frac{\delta_9(\omega^2 - \delta_2 l_i^2)}{\delta_3(l_i^2 K_6 + i\omega\delta_8 - \delta_{10})} & for\ i = 4,5,6. \end{cases}$$

Amplitude ratios of reflected and incident waves, $\frac{A_i}{A_0}, \frac{B_j}{B_0}$ for $i = 1,2,3$ and $j = 4,5,6$, gives the corresponding reflection coefficient ratio. Also, it may be noted that the angles θ, θ_j $(j = 1,2,\ldots,6\ )$, and the corresponding wave numbers, l_j, $j = 1,2,\ldots,6$ are to be connected by Snell's law as

$$l_0\sin\theta_0 = l_1\sin\theta_1 = l_2\sin\theta_2 = l_3\sin\theta_3 = l_4\sin\theta_4 = l_5\sin\theta_5 = l_6\sin\theta_6.$$

Boundary conditions

The boundary conditions are

(1) The mechanical boundary condition

$$\sigma_{zz}(x,0,t) = 0 = \sigma_{xx}(x,0,t) \quad (33)$$

$$\sigma_{zx}(x,0,t) = 0, \quad m_{zy} = 0 \quad (34)$$

(2) The thermal boundary condition

$$\frac{\partial T}{\partial z}(x,0,t) = 0, \quad (35)$$

(3) Heat flux moment

$$q_{zz}(x,0,t) = 0, \quad (36)$$

using Eqs. (4)–(7) and (21)–(26), in non dimensional boundary condition, algebraic equation for the incident waves becomes as follows:

$$\sum_{j=1}^{6}\Gamma_{ij}Z_j = b_i, \quad i,j = 1,2,\ldots 6,\ A_0 = B_0. \quad (37)$$

where

$$\Gamma_{1j} = \begin{cases} \pi_1 l_j^2 + \pi_3\eta_j + \pi_2 l_j^2\cos^2\theta_j & j = 1,2,3 \\ \pi_2 l_j^2\sin\theta_j\cos\theta_j & j = 4,5,6 \end{cases}$$

$$\Gamma_{2j} = \begin{cases} -l_j^2 - l_j^2\pi_4 - \pi_3\eta_j & j = 1,2,3 \\ \pi_4 l_j^2\sin\theta_j\cos\theta_j & j = 4,5,6 \end{cases}$$

$$\Gamma_{3j} = \begin{cases} (\pi_5 + \pi_6)l_j^2\sin\theta_j\cos\theta_j & j = 1,2,3 \\ (\pi_5 - \pi_6)l_j^2(\sin^2\theta_j - \cos^2\theta_j) + \pi_6\eta_j & j = 4,5,6 \end{cases}$$

$$\Gamma_{4j} = \begin{cases} i\eta_j l_j\cos\theta_j & j = 1,2,3 \\ 0 & j = 4,5,6 \end{cases}$$

$$\Gamma_{5j} = \begin{cases} 0 & j = 1,2,3 \\ i\eta_j l_j\cos\theta_j & j = 4,5,6 \end{cases}$$

$$\Gamma_{6j} = \begin{cases} l_j^2 K_4\omega_j - (K_5+K_6)\omega_j l_j^2\sin\theta_j\cos\theta_j & j = 1,2,3 \\ -(K_5+K_6)\omega_j l_j^2\sin\theta_j\cos\theta_j & j = 4,5,6 \end{cases}$$

$$b_1 = -\pi_1 l_0^2 - \pi_2 l_0^2\cos\theta_0\sin\theta_0 - l_0^2\pi_2\cos^2\theta_0 - \pi_3\eta_1,$$

$$b_2 = l_0^2 + \pi_4 l_0^2\sin^2\theta_0 - \pi_4 l_0^2\sin\theta_0\cos\theta_0 + \pi_3\eta_1,$$

$$b_3 = l_0^2(\pi_5+\pi_6)\sin\theta_0\cos\theta_0 + (\pi_6 - \pi_5)l_0^2(\sin^2\theta_0 - \cos^2\theta_0) - \pi_6\eta_4,$$

$$b_4 = il_0\eta_1\cos\theta_0,$$

$$b_5 = il_0\eta_4\cos\theta_0,$$

$$b_6 = -K_4 l_0^2\omega_1 - (K_5+K_6)l_0^2\omega_1\sin\theta_0\cos\theta_0 - (K_5+K_6)\omega_4 l_0^2\sin\theta_0 \times\cos\theta_0$$

$$\pi_1 = \frac{\lambda+2\mu}{\lambda},\ \pi_2 = \frac{k}{\lambda},\ \pi_3 = \frac{\beta T_0}{\lambda},\ \pi_4 = \frac{k+2\mu}{\lambda},\ \pi_5 = \frac{k+\mu}{\lambda},\ \pi_6 = \frac{\mu}{\lambda},\ \pi_7 = \frac{-kc_1^2}{\omega^{*2}}$$

The relevant parameters following Kumar and Sing [37] are represented as, $\aleph_1 = \aleph(1 - iQ_k^{-1})$ where $\aleph = \lambda,\mu,\beta,k,\alpha$ and γ.

Numerical results and discussion

In this section, let us consider the numerical example. For this purpose, Crust is taken as the thermoelastic material for which we take the following values of the different physical constants:

$$\lambda = \mu = 3\times10^{10}\ \text{N.m}^{-2},\ (K,K^*) = (3,0.1)\ \text{w.m}^{-1}.k^{-1},$$
$$T_0 = 300\ \text{K},\ \gamma = 1.6\times10^{11}\ \text{k}^{-1},\ \rho = 2900\ \text{kg.m}^{-3},$$
$$\omega^* = 0.02s^{-1},\ j = 0.1\ \text{m}^2,\ C_E = 1100\ \text{J.kg}^{-1}.\text{k}^{-1}.$$

The values of micro temperature constants are as

$$K_1 = 0.35\ \text{Ns}^{-1},\quad K_2 = 0.45Ns^{-1},\quad K_3 = 0.55\ \text{Ns}^{-1}\ \text{k}^{-1}$$
$$K_4 = 0.65\ \text{Ns}^{-1}\ \text{m}^2,\quad K_5 = 0.75\ \text{Ns}^{-1}\ \text{m}^2,\quad K_6 = 0.96Ns^{-1}\ \text{m}^2$$

In the theoretical presentation it was observed that there exist six reflected waves propagating through the medium three for longitudinal and three for transverse waves. For graphical representation of solution a particular medium is selected and Matlab-16 software has been used. Graphically we are focused on amplitude ratios of reflected waves against the incident angle for different conditions of angular velocity ω and viscosity factor τ_v. The figures are depending on two sets named as Fig. 2 and 3, each set is explained in the following paragraphs.

Fig. 2, represents the amplitude ratios of six waves propagating in the medium against incident angle for different values of angular velocity. The range of incident angle is taken as $0^\circ \leqslant \theta \leqslant 90^\circ$, while in this set of figures, viscosity of the medium is constant $\tau_v = 0.02$. First three figures are for the longitudinal waves while the last three are representing the transverse waves. It can be seen that the amplitude of longitudinal waves reaches the maximum

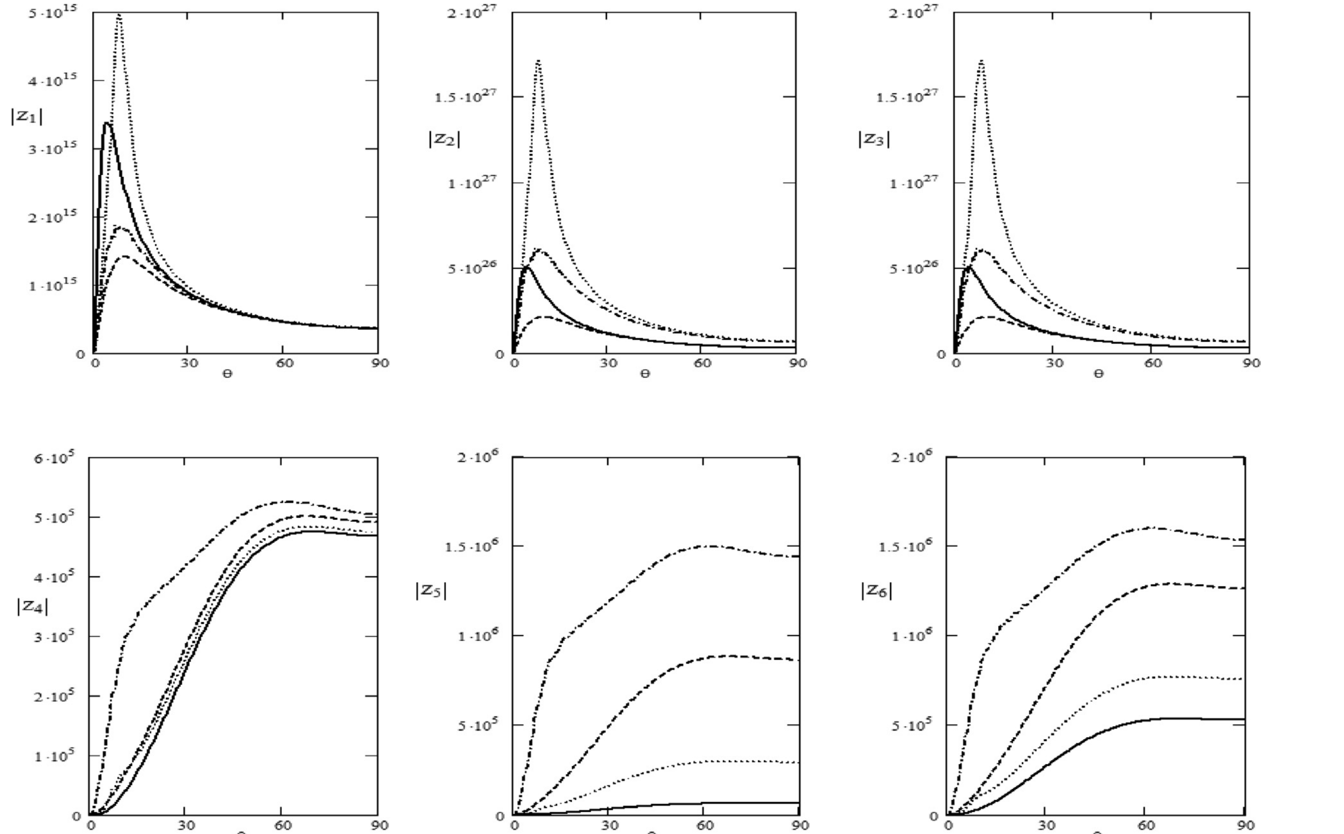

Fig. 2. Variation of the magnitude of amplitude ratios $Z_i (i = 1, 2, \ldots, 6)$ with respect to the angle of incidence θ for variation of $\omega = 0.1, 0.2, 0.3, 0.4$ when $\tau_v = 0.02$.

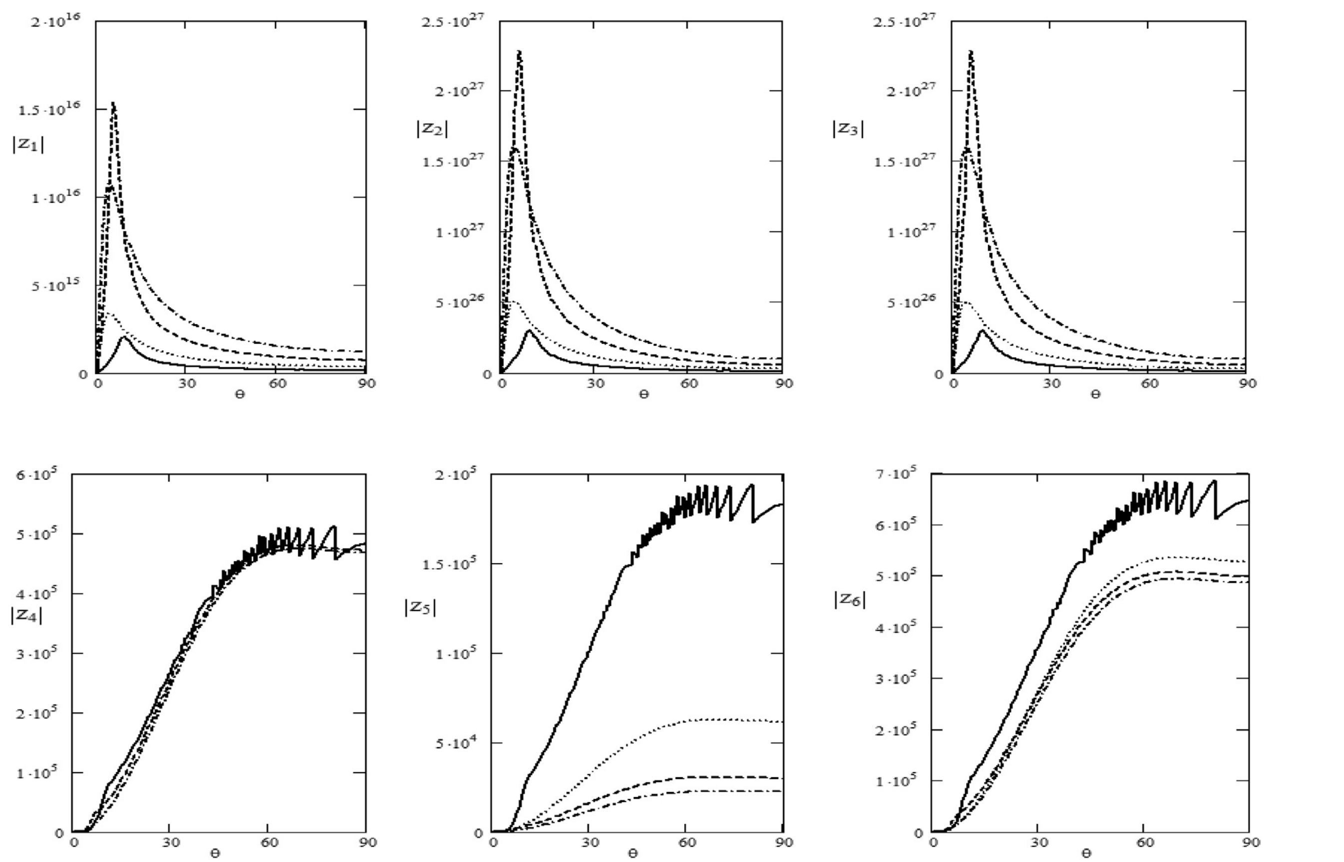

Fig. 3. Variation of the magnitude of amplitude ratios $Z_i (i = 1, 2, \ldots, 6)$ with respect to the angle of incidence θ for variation of $\tau_v = 0.01, 0.02, 0.03, 0.35$ when $\omega = 0.1$.

value for $\omega = 0.4$ which shows that greater the value of angular frequency of the medium greater will be the amplitude of longitudinal waves, but for the case of transverse waves the maximum amplitude is obtained for $\omega = 0.2$. Angle of incidence is also playing very important role in the propagation of these waves. Graphically it is observed that greater the angle of incidence the longitudinal waves decreases but the transverse waves have higher amplitude for large values of incident angle.

Fig. 3, depicts the six waves for different value of viscosity factor constant $\tau_v = 0.01, 0.02, 0.03, 0.04$ while the angular velocity remains fix $\omega = 0.1$. It is observed that viscosity factor is having a strong influence on propagation of reflected wave through the medium. Maximum amplitude for longitudinal wave is obtained for $\tau_v = 0.02$ and all the three curves have same kind of response to viscosity factor. All curves converge toward small values of reflected coefficients by increasing the angle from principal normal. Transverse waves are having complex oscillating response for the case of small viscosity factor and large angle of incidence.

Conclusions

By flowing through Theoretical and graphical representation we came across the following major points.

1. It is observed from Eqs. (17)–(22) that the total six reflected waves propagating through the medium. Three of them are longitudinal and three are transverse in nature.
2. Amplitude of longitudinal waves reduces by increasing the angle of incidence, while the amplitude of transverse waves increases by increasing the angle of incidence. Amplitude of reflected longitudinal wave is having highest curve for angle approximately equals $\theta = 15^\circ$ while the transverse waves have the maximum response at $\theta = 60^\circ$. Greater the value of angular frequency of the medium greater will be the amplitude of longitudinal waves but this behavior is not found in the case of transverse waves.
3. The viscosity factor is having a strong influence on the waves. Transverse waves have complex behavior for small value of viscosity and large value of angle of incidence. This indicates that viscosity makes the curves move in a smooth harmonic behavior.

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