Significant consequences of heat generation/absorption and homogeneous-heterogeneous reactions in second grade fluid due to rotating disk

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ABSTRACT

Flow of second grade fluid by a rotating disk with heat and mass transfer is discussed. Additional effects of heat generation/absorption are also analyzed. Flow is also subjected to homogeneous-heterogeneous reactions. The convergence of computed solution is assured through appropriate choices of initial guesses and auxiliary parameters. Investigation is made for the effects of involved parameters on velocities (radial, axial, tangential), temperature and concentration. Skin friction and Nusselt number are also analyzed. Graphical results depict that an increase in viscoelastic parameter enhances the axial, radial and tangential velocities. Opposite behavior of temperature is observed for larger values of viscoelastic and heat generation/absorption parameters. Concentration profile is increasing function of Schmidt number, viscoelastic parameter and heterogeneous reaction parameter. Magnitude of skin friction and Nusselt number are enhanced for larger viscoelastic parameter.

Introduction

Non-Newtonian fluids have extensive applications in technological and industrial sectors. Formulation of equations for these types of fluids is complex in comparison to the usual Naviers Stokes equation. It is due to extra rheological parameters in the constitutive relations of such materials. Applications related to geophysics, biological sciences and chemical processes involve non-Newtonian materials. Materials such as foams, pastes, ketchup, lubricant, certain oils, sugar solution, apple sauce, colloidal and suspension solutions, drilling muds, clay coating and soaps are the non-Newtonian fluids. Stagnation point flow of tangent hyperbolic nanofluid with nonlinear thermal radiation and double diffusive convection is examined by Hayat et al. [1]. Flow of micropolar fluid due to heated stretching sheet is analyzed by Turkyilmazoglu [2]. Rotating flow of Jeffrey fluid for Cattaneo-Christov heat flux is presented by Hayat et al. [3]. Rahman et al. [4] analyzed the effect of slip on flow of Jeffrey nanofluid through tapered artery with mild stenosis. Hayat et al. [5] studied the MHD flow of Jeffrey fluid accounting constructive and destructive chemical reactions. Das et al. [6] explored the heat/mass transfer of sec-ond grade fluid over a stretching sheet. Hayat et al. [7] analyzed the squeezing flow of second grade fluid with convective boundary conditions. Consequences of Soret/Dufour effects on flow of second grade fluid due to stretching cylinder are discussed by Majeed et al. [8]. Hayat et al. [9] examined the flow of second grade fluid in thermally stratified medium with non-Fourier heat flux. Ariel [10] studied the flow of second grade fluid by a rotating disk.

At present many researchers and scientists are considering flow problems due to rotating surfaces. It is due to its significance in engineering and industrial sectors involving electric power generating system, jet motors, turbine system and food processing. Karman [11] employed transformations in obtaining ordinary differential system for rotating disk flow by Navier Stokes equations. Cochran [12] used these transformations to analyze the rotating disk flow by numerical integration method. Stewartson [13] considered the flow between two rotating disks. Flow between stationary and rotating disks is examined by Chapple and Stokes [14]. Flow between rotating disks with thermal stratification is described by Hayat et al. [15]. Ellahi et al. [16] examined ferrofluid flow by stretchable rotating disk. Flow due to stretchable rotating disks with Cattaneo-Christov heat flux is addressed by Hayat et al. [17].

When there exists high temperature difference between the ambient fluid and surface then effects of heat source/sink are use-

ful. Heat generation works in exothermic or endothermic chemical reactions. Raju and Sandeep [18] considered heat source/sink effects in flow of Casson fluid. Effects of heat source/sink in flow of nanofluid over shrinking sheet is analyzed by Das et al. [19]. Radiative flow of nanofluid in presence of an inclined MHD and heat generation/absorption is explored by Hayat et al. [20]. MHD flow by an inclined permeable sheet is examined by Noor et al. [21]. Aziz [22] studied stagnation point flow in presence of heat source/sink.
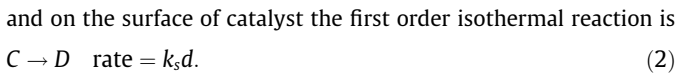
It should be noted that several chemical reacting systems consist of homogeneous-heterogeneous reactions. Except in the presence of catalyst some of the reactions does not work or may proceed very slow. Correlation between homogeneous-heterogeneous reactions is very complex. Ceramics, fog dispersion, food processing, crops damage via freezing, fog formation and polymer production, hydrometallurgical industry are applications of chemical reaction. Various scientists now are now engaged in the discussion of flows with homogeneous-heterogeneous different aspects [23–27].

Existing literature survey witnesses that flow of second grade fluid by stretchable rotating disks with homogeneous-heterogeneous reactions is not yet studied. Effects of heat generation absorption is also considered. The objective here is to carry out such analysis. Convergent series solutions are developed through HAM [28–36]. Plots explore the outcome of sundry variables for the velocity, temperature, concentration, skin friction coefficient and Nusselt number. Important points are concluded.

Modeling

Consider the steady axisymmetric flow of second grade fluid by a rotating disk. Disk at $z = 0$ rotate with angular velocity Ω_1. Stretching rate of disk is a_1. Disk and ambient temperatures are maintained at $\widehat{T}_w$ and $\widehat{T}_\infty$ respectively (see Fig. 1).

Effects of heat generation/absorption are considered. Homogeneous-heterogeneous reactions are also present. Homogeneous reaction for cubic autocatalysis is

$$C + 2D \rightarrow 3D \quad \text{rate} = k_c cd^2, \tag{1}$$

and on the surface of catalyst the first order isothermal reaction is

$$C \rightarrow D \quad \text{rate} = k_s d. \tag{2}$$

Here k_c and k_s are the rate constants and C and D are chemical species with concentrations c and d. We have used the cylindrical coordinates (r, θ, z) with velocity components $(\hat{u}, \hat{v}, \hat{w})$. The relevant equations are

$$\frac{\partial \hat{u}}{\partial r} + \frac{\hat{u}}{r} + \frac{\partial \hat{w}}{\partial z} = 0, \tag{3}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial r} + \hat{w}\frac{\partial \hat{u}}{\partial z} - \frac{\hat{v}^2}{r} = v\frac{\partial^2 \hat{u}}{\partial z^2} + \frac{\alpha_1}{\rho}\left(\hat{u}\frac{\partial^3 \hat{u}}{\partial r\partial z^2} + \hat{w}\frac{\partial^3 \hat{u}}{\partial z^3} + \frac{\partial \hat{v}}{\partial r}\frac{\partial^2 \hat{v}}{\partial z^2}\right.$$
$$\left. -\frac{1}{r}\left(\frac{\partial \hat{u}}{\partial z}\right)^2 - \frac{\partial^2 \hat{v}}{\partial z^2}\frac{\hat{v}}{r} + \frac{\partial \hat{v}}{\partial z}\frac{\partial^2 \hat{v}}{\partial r\partial z} + \frac{\partial \hat{w}}{\partial z}\frac{\partial^2 \hat{u}}{\partial z^2}\right.$$
$$\left. +3\frac{\partial \hat{u}}{\partial z}\frac{\partial^2 \hat{u}}{\partial r\partial z} + 2\frac{\partial \hat{u}}{\partial r}\frac{\partial^2 \hat{u}}{\partial z^2}\right) \tag{4}$$

$$\hat{u}\frac{\partial \hat{v}}{\partial r} + \hat{w}\frac{\partial \hat{v}}{\partial z} + \frac{\hat{u}\hat{v}}{r}$$
$$= v\frac{\partial^2 \hat{v}}{\partial z^2} + \frac{\alpha_1}{\rho}\left(\hat{u}\frac{\partial^3 \hat{v}}{\partial r\partial z^2} + \hat{w}\frac{\partial^3 \hat{v}}{\partial z^3} - 2\frac{\partial \hat{v}}{\partial z}\frac{\partial^2 \hat{u}}{\partial r\partial z} + \frac{\partial^2 \hat{v}}{\partial z^2}\frac{\hat{u}}{r} - \frac{1}{r}\frac{\partial \hat{u}}{\partial z}\frac{\partial \hat{v}}{\partial z}\right) \tag{5}$$
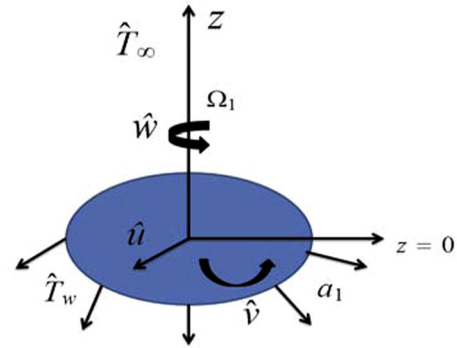

Fig. 1. Flow geometry.

$$(\rho c_p)\left(\hat{u}\frac{\partial \widehat{T}}{\partial r} + \hat{w}\frac{\partial \widehat{T}}{\partial z}\right) = k\frac{\partial^2 \widehat{T}}{\partial z^2} + Q^*(\widehat{T} - \widehat{T}_\infty), \tag{6}$$

$$\hat{u}\frac{\partial c}{\partial r} + \hat{w}\frac{\partial c}{\partial z} = D_C^*\left(\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2}\right) - k_c cd^2, \tag{7}$$

$$\hat{u}\frac{\partial d}{\partial r} + \hat{w}\frac{\partial d}{\partial z} = D_D^*\left(\frac{\partial^2 d}{\partial r^2} + \frac{1}{r}\frac{\partial d}{\partial r} + \frac{\partial^2 d}{\partial z^2}\right) + k_c cd^2, \tag{8}$$

with boundary conditions

$$\hat{u} = ra_1, \quad \hat{v} = r\Omega_1, \quad \hat{w} = 0, \quad \widehat{T} = \widehat{T}_w, \quad D_C^*\frac{\partial c}{\partial z} = k_s c,$$

$$D_D^*\frac{\partial d}{\partial z} = -k_s c \text{ at } z = 0,$$

$$\hat{u} = 0, \quad \hat{v} = 0, \quad \hat{w} = 0, \quad \widehat{T} = \widehat{T}_\infty, \quad c \rightarrow c_0, \quad d \rightarrow 0 \text{ at } z \rightarrow \infty, \tag{9}$$

where α_1 is material parameter of second grade fluid, ρ density, c_p specific heat, k thermal conductivity, v kinematic viscosity, Q^* heat generation absorption coefficient and D_C^* and D_D^* are diffusion species coefficients. Letting [15]:

$$\hat{u} = r\Omega_1\tilde{f}'(\xi), \quad \hat{v} = r\Omega_1\tilde{g}(\xi), \quad \hat{w} = -2h\Omega_1\tilde{f}(\xi), \quad \tilde{\theta} = \frac{\widehat{T} - \widehat{T}_\infty}{\widehat{T}_w - \widehat{T}_\infty},$$

$$\hat{p} = \rho_f\Omega_1 v_f\left(P(\xi) + \frac{1}{2}\frac{r^2}{h^2}\epsilon\right), \quad c = c_0\tilde{\phi}, \quad d = c_0\tilde{l}, \quad \xi = \frac{z}{h}, \tag{10}$$

the continuity equation is satisfied and Eqs. (4)–(9) are reduced to

$$\tilde{f}''' + We\text{Re}(2\tilde{f}''^2 + \tilde{g}'^2 - 2\tilde{f}\tilde{f}^{iv} + \tilde{f}'\tilde{f}''') - \text{Re}(\tilde{f}'^2 - 2\tilde{f}\tilde{f}'' - \tilde{g}^2) = 0, \tag{11}$$

$$\tilde{g}'' + We\text{Re}\left(2\tilde{f}'\tilde{g}'' - 2\tilde{f}\tilde{g}''' - 3\tilde{f}''\tilde{g}'\right) - \text{Re}(2\tilde{f}'\tilde{g} - 2\tilde{f}\tilde{g}') = 0, \tag{12}$$

$$\tilde{\theta}'' + \text{PrRe}(\tilde{f}'\tilde{\theta} - 2\tilde{f}\tilde{\theta}' + Q\tilde{\theta}) = 0, \tag{13}$$

$$\frac{1}{\text{Re}}\frac{1}{Sc}\tilde{\phi}'' + 2\tilde{f}\tilde{\phi}' - k_1\tilde{\phi}\tilde{l}^2 = 0, \tag{14}$$

$$\frac{\delta}{Sc}\frac{1}{\text{Re}}\tilde{l}'' + 2\tilde{f}\tilde{l}' + k_1\tilde{\phi}\tilde{l}^2 = 0, \tag{15}$$

with

$$\tilde{f}(0) = 0, \quad \tilde{f}(\infty) = 0, \quad \tilde{f}'(0) = A_1, \quad \tilde{f}'(\infty) = 0, \quad \tilde{g}(0) = 1,$$

$$\tilde{g}(\infty) = 0, \quad \tilde{\theta}(0) = 1, \quad \tilde{\theta}(\infty) = 0, \quad \tilde{\phi}'(0) = k_2\tilde{\phi}(0), \quad \tilde{\phi}(\infty) = 1,$$

$$\delta\tilde{l}'(0) = -k_2\tilde{\phi}(0), \quad \tilde{l}(\infty) = 0, \quad P(0) = 0, \tag{16}$$

$$\text{Re} = \frac{\Omega_1 h^2}{v}, A_1 = \frac{a_1}{\Omega_1}, We = \frac{\alpha_1}{\rho h^2}$$

$$\text{Pr} = \frac{\rho c_p v}{k}, Q = \frac{Q^*}{\rho c_p \Omega_1}, k_1 = \frac{k_c c_0^2}{\Omega_1}$$

$$k_2 = \frac{k_s h}{D_C^*}, \delta = \frac{D_D^*}{D_C^*}, Sc = \frac{v}{D_C^*}, \tag{17}$$

Here Re denotes Reynolds number, A_1 stretching parameter, *We* Weissenberg number, Pr Prandtl number, Q_0 heat generation/absorption parameter, k_1 and k_2 the homogeneous and heterogeneous reaction parameters respectively, δ ratio of diffusion coefficient and *Sc* Schmidt number. When diffusion coefficients D_C^* and D_D^* are equal for both chemical species i.e. $\delta = 1$ then

$$\tilde{\phi}(\xi) + \tilde{l}(\xi) = 1. \tag{18}$$

Now Eqs. (14) and (15) yield

$$\frac{1}{\text{Re}} \frac{1}{Sc} \tilde{\phi}'' + 2\tilde{f}\tilde{\phi}' - k_1 \tilde{\phi}(1 - \tilde{\phi})^2 = 0, \tag{19}$$

with boundary conditions

$$\tilde{\phi}'(0) = k_2 \tilde{\phi}(0), \quad \tilde{\phi}(1) = 1. \tag{20}$$

Skin friction coefficients in radial and tangential directions are C_{f_r} and C_{f_θ} respectively

$$C_{f_r} = \frac{\tau_{zr}}{\rho(r\Omega_1)^2}, \tag{21}$$

$$C_{f_\theta} = \frac{\tau_{z\theta}}{\rho(r\Omega_1)^2}, \tag{22}$$

Shear stresses τ_{zr} and $\tau_{z\theta}$ are defined by

$$\begin{aligned}
\tau_{zr} = &\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + \alpha_1\left(u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + 2\frac{\partial v}{\partial z}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) \\
&+ \frac{\partial u}{\partial r}\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial z} + 3\left(\frac{\partial u}{\partial r}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\frac{\partial w}{\partial z}\right) \\
&- \alpha_1\left(\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + \frac{\partial v}{\partial z}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)\right),
\end{aligned} \tag{23}$$

$$\begin{aligned}
\tau_{z\theta} = &\mu\frac{\partial v}{\partial z} \\
&+ \alpha_1\left(u\frac{\partial^2 v}{\partial r\partial z} + w\frac{\partial^2 v}{\partial z^2} + \frac{\partial u}{\partial z}\frac{\partial v}{\partial r} - \frac{v}{r}\frac{\partial u}{\partial z} + 3\frac{u}{r}\frac{\partial v}{\partial z} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial z}\right) \\
&- \alpha_1\left(\frac{\partial u}{\partial z}\frac{\partial v}{\partial r} - \frac{v}{r}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\frac{\partial v}{\partial r} - \frac{v}{r}\frac{\partial w}{\partial r} + 2\frac{u}{r}\frac{\partial v}{\partial z} + 2\frac{\partial v}{\partial z}\frac{\partial w}{\partial z}\right),
\end{aligned} \tag{24}$$

Skin friction coefficients C_{f_r} and C_{f_θ} in dimensionless forms are

$$\text{Re}_r C_{f_r} = \tilde{f}''(0) + \beta\text{Re}[3\tilde{f}'(0)\tilde{f}''(0) - 2\tilde{f}(0)\tilde{f}'''(0) + \tilde{g}(0)\tilde{g}'(0)], \tag{25}$$

$$\text{Re}_r C_{f_\theta} = \tilde{g}'(0) + \beta\text{Re}[4\tilde{f}'(0)\tilde{g}'(0) - 2\tilde{f}(0)\tilde{g}''(0)], \tag{26}$$

where local Reynolds number is $Re_r = \frac{r\Omega_1 h}{v}$.

Heat transfer rate is defined as

$$Nu_x = \frac{hq_w}{k(\widehat{T}_w - \widehat{T}_\infty)}\bigg|_{z=0}, \tag{27}$$

in which wall heat flux q_w is given by

$$q_w|_{z=0} = -k\frac{\partial \widehat{T}}{\partial z}\bigg|_{z=0}. \tag{28}$$

Nusselt number is

$$Nu_x = -\tilde{\theta}'(0). \tag{29}$$

Solutions methodology and convergence

Homotopy analysis method (HAM) leads to the solutions development. Thus initial guesses and auxiliary linear operators are

$$\tilde{f}_0(\xi) = A(1 - \exp(-\xi)), \tag{30}$$

$$\tilde{g}_0(\xi) = \exp(-\xi), \tag{31}$$

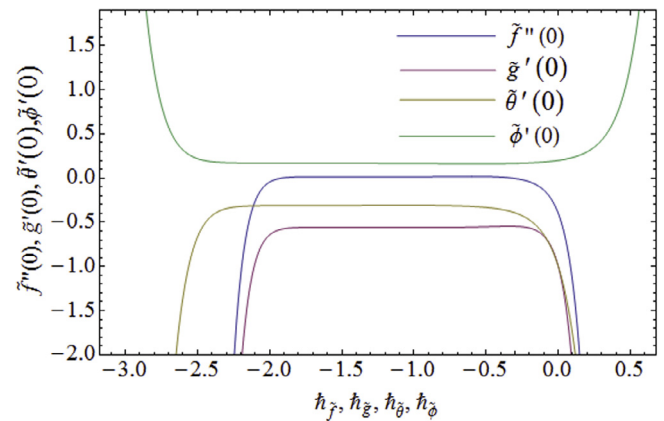$$\tilde{\theta}_0(\xi) = \exp(-\xi), \tag{32}$$

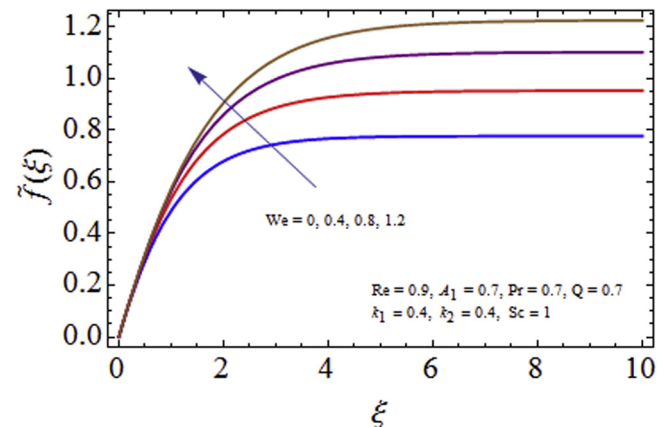**Fig. 2.** $\hbar$-curves for $\tilde{f}''(0), \tilde{g}'(0), \tilde{\theta}'(0)$ and $\tilde{\phi}'(0)$.

Fig. 3. Impact of *We* for $\tilde{f}(\xi)$.

Table 1
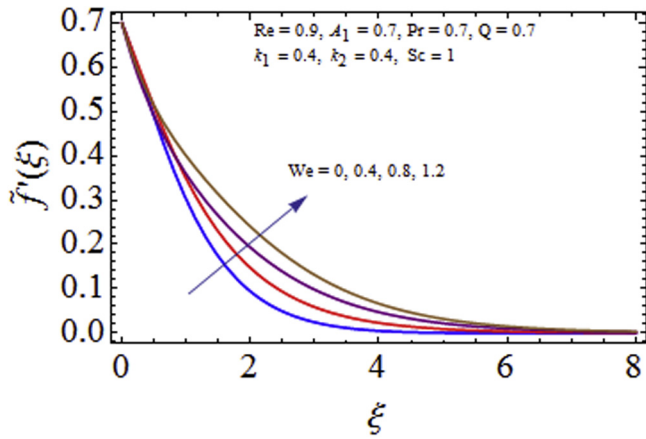Solutions convergence when $\text{Re} = 0.3, A_1 = 0.4, We = 0.01, \text{Pr} = 1.5, k_1 = k_2 = 0.4, Sc = 1$ and $Q = 0.2$.

Order of approximations	$\tilde{f}''(0)$	$-\tilde{g}'(0)$	$-\tilde{\theta}'(0)$	$\widetilde{\phi}'(0)$
1	0.0044	0.4300	0.6120	0.17741
5	0.0259	0.5479	0.3467	0.16148
10	0.0150	0.5599	0.3158	0.16483
22	0.0150	0.5573	0.3095	0.16958
23	0.0150	0.5572	0.3096	0.16959
24	0.0150	0.5572	0.3097	0.16961
26	0.0150	0.5572	0.3099	0.16961
30	0.0150	0.5572	0.3099	0.16961
40	0.0150	0.5572	0.3099	0.16961

Fig. 4. Impact of *We* for $\tilde{f}'(\xi)$.
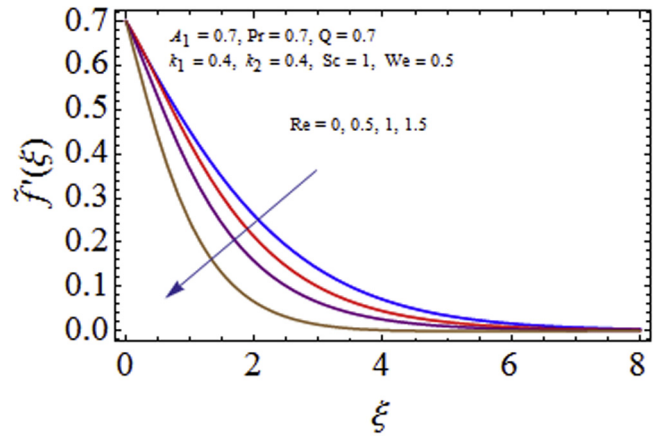

Fig. 7. Impact of *Re* for $\tilde{f}'(\xi)$.
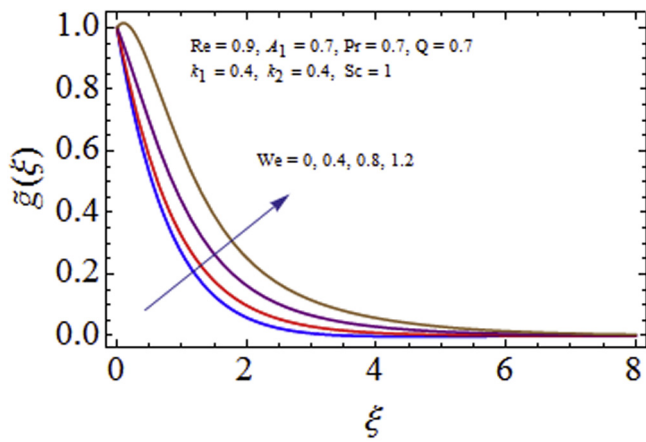

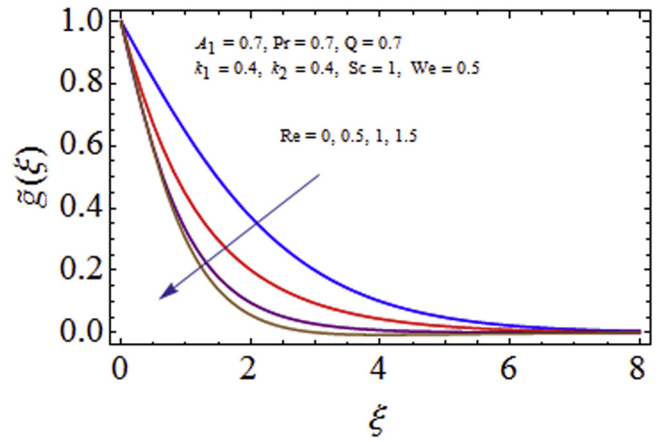**Fig. 5.** Impact of *We* for $\tilde{g}(\xi)$.

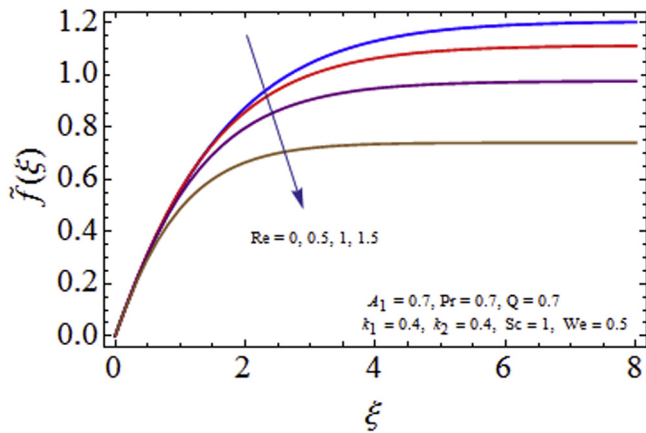**Fig. 8.** Impact of *Re* for $\tilde{g}(\xi)$.

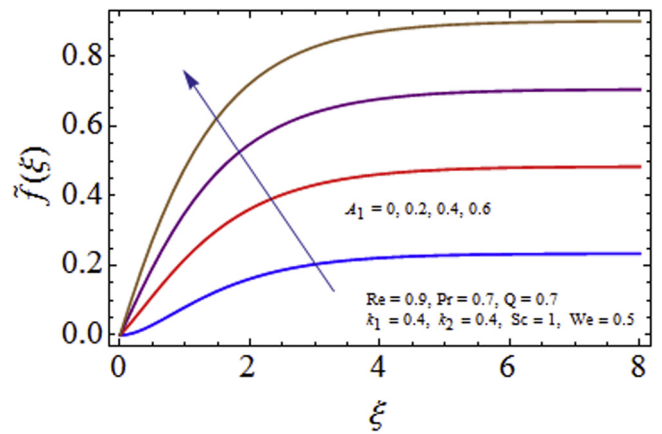**Fig. 6.** Impact of *Re* for $\tilde{f}(\xi)$.

Fig. 9. Impact of A_1 for $\tilde{f}(\xi)$.

$$\tilde{\phi}_0(\xi) = 1 - \frac{1}{2}\exp(-k_2\xi), \tag{33}$$

$$\mathcal{L}_{\tilde{f}} = \tilde{f}''' - \tilde{f}', \mathcal{L}_{\tilde{g}} = \tilde{g}'' - \tilde{g}, \mathcal{L}_{\tilde{\theta}} = \tilde{\theta}'' - \tilde{\theta}, \mathcal{L}_{\tilde{\phi}} = \tilde{\phi}'' - \tilde{\phi}, \tag{34}$$

with

$$\mathcal{L}_{\tilde{f}}\left[A_1 + A_2 e^{\xi} + A_3 e^{-\xi}\right] = 0, \tag{35}$$

$$\mathcal{L}_{\tilde{g}}\left[A_4 e^{\xi} + A_5 e^{-\xi}\right] = 0, \tag{36}$$

$$\mathcal{L}_{\tilde{\theta}}\left[A_6 e^{\xi} + A_7 e^{-\xi}\right] = 0, \tag{37}$$

$$\mathcal{L}_{\tilde{\phi}}\left[A_8 e^{\xi} + A_9 e^{-\xi}\right] = 0, \tag{38}$$

where A_i $(i = 1 - 9)$ are the constants. There is no doubt that series solutions involve auxiliary parameters $\hbar_{\tilde{f}}, \hbar_{\tilde{g}}, \hbar_{\tilde{\theta}}$ and $\hbar_{\tilde{\phi}}$. To acquire the admissible ranges we have drawn the $\hbar$−curves at 16th order of

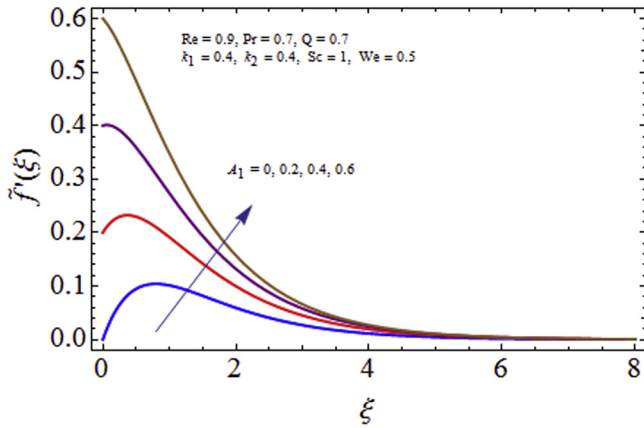**Fig. 10.** Impact of A_1 for $\tilde{f}'(\xi)$.

Fig. 11. Impact of A_1 for $\tilde{g}(\xi)$.
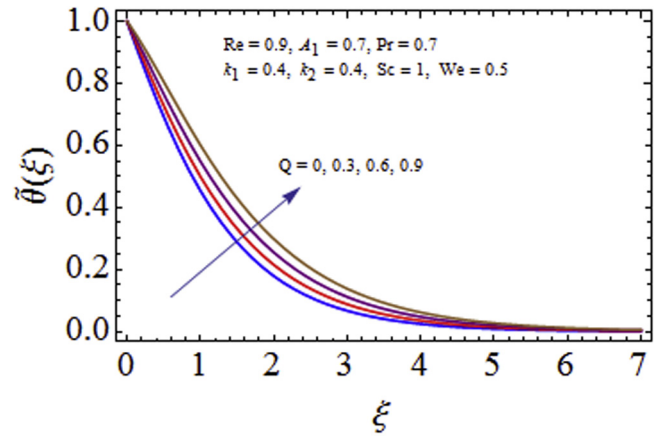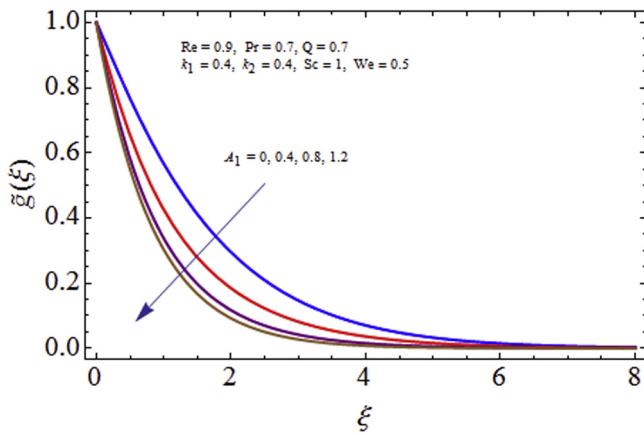

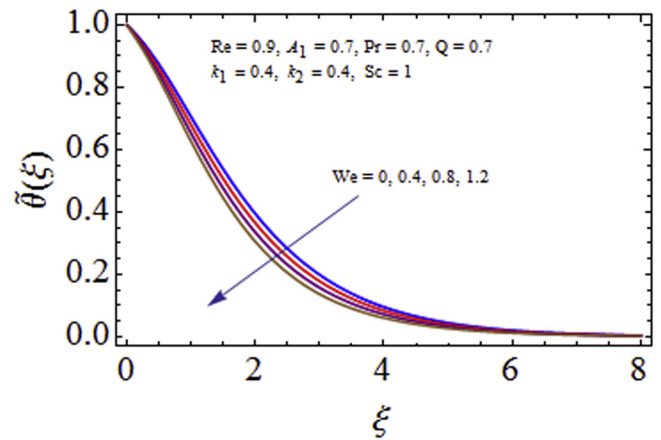**Fig. 12.** Impact of Pr for $\tilde{\theta}(\xi)$.

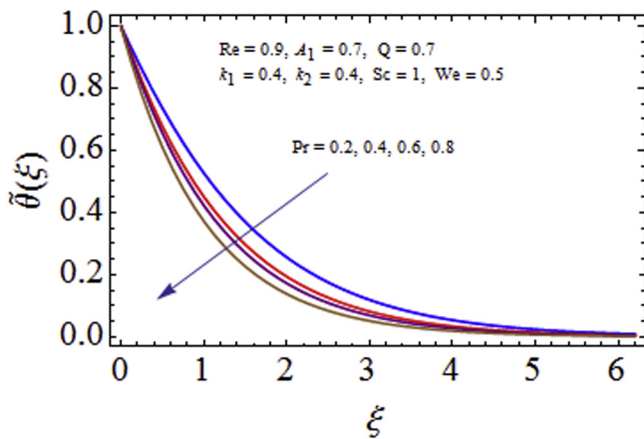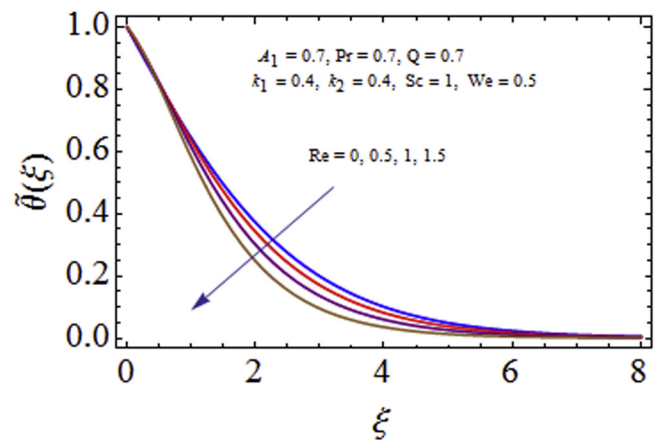**Fig. 13.** Impact of Q for $\tilde{\theta}(\xi)$.

Fig. 14. Impact of We for $\tilde{\theta}(\xi)$.

Fig. 15. Impact of Re for $\tilde{\theta}(\xi)$.

approximations. Convergence regions are $-1.9 \leqslant \hbar_{\tilde{f}} \leqslant -0.9$, $-1.8 \leqslant \hbar_{\tilde{g}} \leqslant -0.6$, $-1.5 \leqslant \hbar_{\tilde{\theta}} \leqslant -0.9$ and $-2.3 \leqslant \hbar_{\tilde{\phi}} \leqslant -1.3$ (see Fig. 2). Solution is convergent for entire region of $\xi(0 \leqslant \xi \leqslant \infty)$ when $\hbar_{\tilde{f}} = \hbar_{\tilde{g}} = -1.5 = \hbar_{\tilde{\phi}}$ and $\hbar_{\tilde{\theta}} = -1.3$. Table 1 is constructed to show the order of convergence. $\tilde{f}''(0), \tilde{g}'(0), \tilde{\theta}'(0)$ and $\tilde{\phi}'(0)$ converge at 10th, 23th, 26th and 24th order of approximations respectively.

Discussion

Present section is prepared just to investigate the effects of involved dimensionless variables on velocity (axial, radial, tangential), temperature, concentration, skin friction coefficient and Nusselt number.

0.1. Axial, radial and tangential velocity components

Impact of viscoelastic parameter *We* on axial, radial and tangential velocity profiles is shown in Figs. 3–5. For larger values of *We* fluid viscosity decreases because *We* is inversely proportional to the viscosity and hence fluid velocity (axial, radial and tangential) enhances. Influence of Re on axial, radial and tangential velocities is shown in Figs. 6–8. It is clear from these figures that for larger Reynolds number Re the axial, radial and tangential velocities are reducing. In fact Re is directly proportional to the density of fluid. When density enhances then velocities in radial, axial and tangential directions decay. Figs. 9–11 are portrayed to show the behavior of axial $\tilde{f}(\xi)$, radial $\tilde{f}'(\xi)$ and tangential $\tilde{g}(\xi)$ velocities for stretching parameter A_1. Magnitude of $\tilde{f}(\xi)$ and $\tilde{f}'(\xi)$ enhances for larger A_1 because stretching rate of disk is increasing through A_1 (see Figs. 9 and 10). Decreasing behavior is observed for tangential velocity $\tilde{g}(\xi)$ with an increase in stretching parameter A_1. With increase in A_1 the rotational velocity Ω_1 is reduced. It causes decrease in tangential velocity (see Fig. 11).

0.2. Temperature profile

Figs. 12–15 are made to examine the temperature profile. Temperature for Prandtl number Pr is shown is Fig. 12. Larger Pr gradually decreases the temperature of fluid because thermal diffusivity decays for larger Pr. Fig. 13 shows influence of heat source/sink parameter Q on $\tilde{\theta}(\xi)$. It shows that $\tilde{\theta}(\xi)$ is increasing
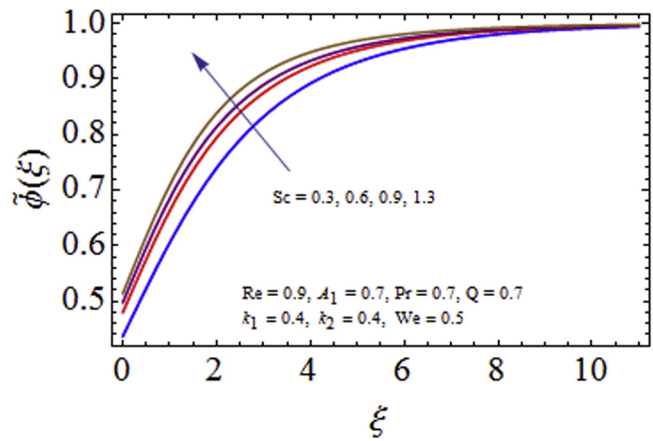

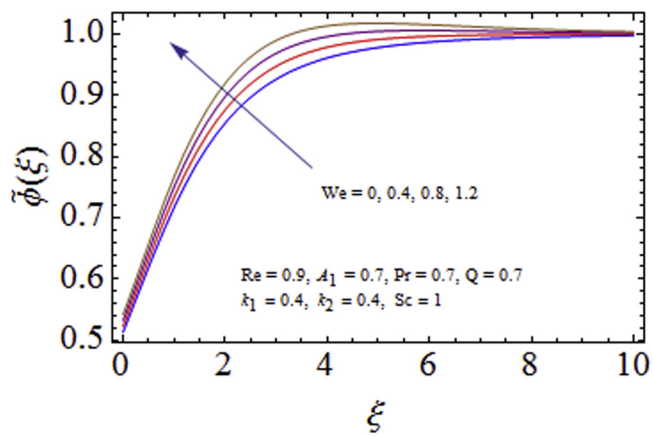**Fig. 16.** Impact of k_1 for $\bar{\phi}(\xi)$.

Fig. 17. Impact of k_2 for $\bar{\phi}(\xi)$.

Fig. 18. Impact of Sc for $\bar{\phi}(\xi)$.

Fig. 19. Impact of We for $\bar{\phi}(\xi)$.
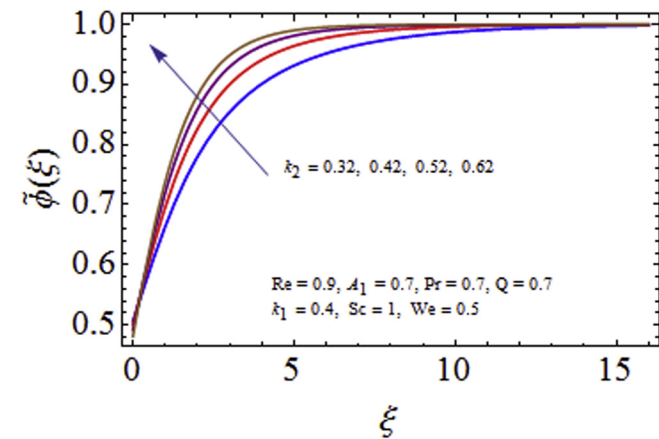

Fig. 20. Impact of Re for $\bar{\phi}(\xi)$.

function of Q. Impact of viscoelastic parameter *We* on temperature is depicted in Fig. 14. Here decreasing effects are captured for larger *We*. For larger Reynolds number Re there is decreasing impact of temperature (see Fig. 15).

0.3. Concentration profile

Figs. 16–20 are portrayed to show the influence of involved parameters on concentration profile. Impact of homogeneous reaction parameter k_1 is depicted in Fig. 16. As expected the concentra-
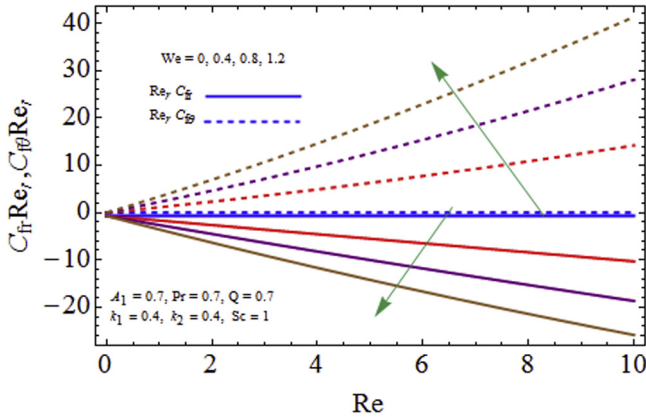
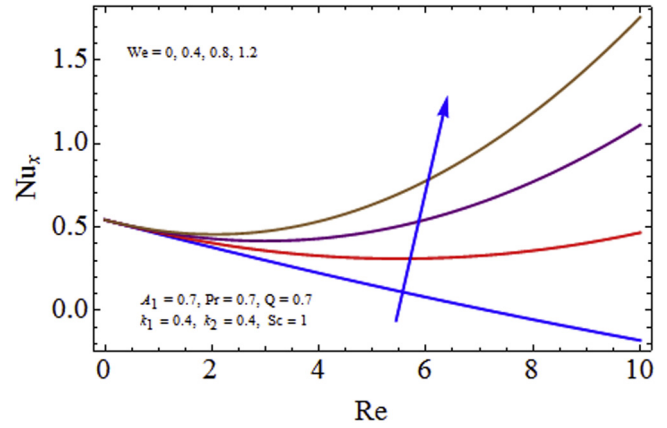**Fig. 21.** Impact of We for $C_{fr}Re_r$ and $C_{f\theta}Re_r$.
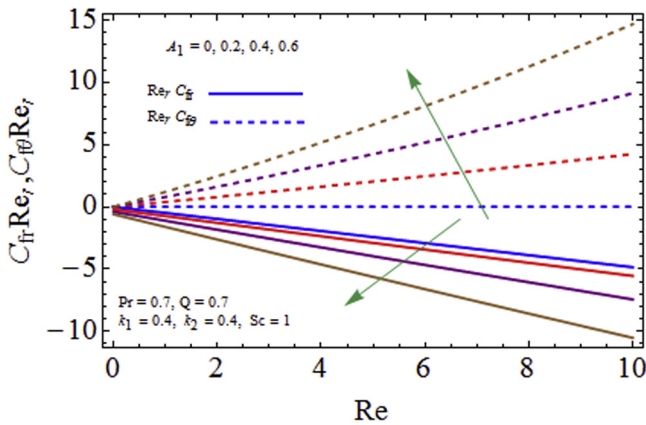

Fig. 22. Impact of A_1 for $C_{fr}Re_r$ and $C_{f\theta}Re_r$.
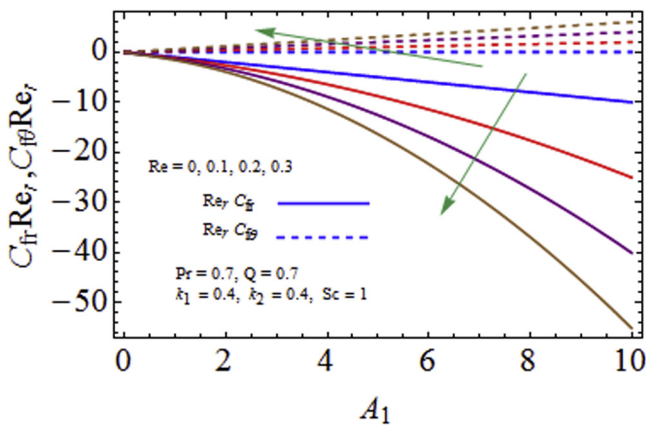
Fig. 23. Impact of Re for $C_{fr}Re_r$ and $C_{f\theta}Re_r$.

Fig. 24. Impact of We on Nu_x.

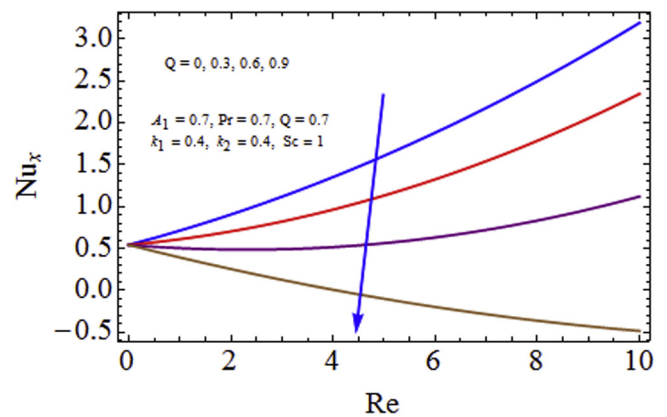**Fig. 25.** Impact of Pr on Nu_x.

Fig. 26. Impact of Q on Nu_x.

tion of fluid decays with an increase in k_1. It is through the fact that reactants are consumed during the homogeneous reaction. While opposite behavior of $\tilde{\phi}(\xi)$ is captured for larger heterogeneous reaction parameter k_2 (see Fig. 17). For increasing value of k_2 the diffusion coefficient reduces and less diffused particles enhance the concentration. Increment in concentration profile is noticed for larger Sc (see Fig. 18). As Sc is the ratio of momentum to mass diffusivity. Therefore momentum diffusivity enhances for larger Sc which in turn enhances the concentration profile. Fig. 19 shows that for larger We the concentration of fluid increases. Fig. 20 is

portrayed to show the influence of Re on concentration profile. It shows that $\tilde{\phi}(\xi)$ enhances with increase in Re.

0.4. Surface drag force

Impact of viscoelastic parameter We, stretching parameter A_1 and Reynolds number Re on skin friction coefficient are shown in Figs. 21–23. It is clear from the Figs. that magnitude of skin friction coefficient in radial and tangential directions enhances for larger We, A_1 and Re.

0.5. Nusselt number

Figs. 24–26 are sketched to analyze the effect of viscoelastic parameter *We*, Prandtl number Pr and heat generation/absorption parameter *Q* on Nusselt number. Fig. 24 shows that Nusselt number is increasing function of *We* while reduction in heat transfer rate is noticed for larger Prand *Q* (see Figs. 25 and 26).

Closing remarks

Here we consider the flow of second grade fluid accounting heat source/sink and homogeneous-heterogeneous reactions. The main findings are:

- Increasing behavior of axial, radial and tangential velocities is captured for larger viscoelastic parameter *We*.
- Temperature profile decreases for larger Prandtl number Pr while opposite behavior is noticed for increasing heat source/sink parameter *Q*.
- Temperature profile is decreasing function of viscoelastic parameter *We*.
- Concentration profile decays for larger k_1 while opposite behavior is examined for increasing k_2.
- Concentration profile shows similar behavior for larger Schmidt number *Sc* and viscoelastic parameter *We*.
- Viscoelastic parameter *We* give rise to surface drag force and Nusselt number.
- Heat transfer rate decays for larger Prandtl number Pr.

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