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Full Length Article

# Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation

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ABSTRACT

Numerous practical applications like robot balancing, segway and hover board riding and operation of a rocket propeller are inherently based on Inverted Pendulum (IP). The control of an IP is a sophisticated problem due to various real world phenomena that make it unstable, non-linear and under-actuated system. This paper presents a comparative analysis of linear and non-linear feedback control techniques based on investigation of time, control energy and tracking error to obtain best control performance for the IP system. The implemented control techniques are Linear Quadratic controller (LQR), Sliding Mode Control (SMC) through feedback linearization, Integral Sliding Mode Control (ISMC) and Terminal Sliding Mode Control (TSMC). Considering cart position and pendulum angle, the designed control laws have been subjected to various test signals so as to characterize their tracking performance. Comparative results indicate that ISMC gives a rise time of 0.6 s with 0% overshoot and over-performs compared to other control techniques in terms of reduced chattering, less settling time and small steady state error.



## 1. Introduction

Recent advancements in the domain of control systems are completely reshaping mechatronics [1] and robotic systems [2]. Inverted Pendulum (IP) is a challenging problem studied in the field of control systems theory. It is a highly non-linear, unstable and under-actuated Multiple Input and Multiple Outputs (MIMO) mechanical system [3]. Consequently, such a system, requiring a sophisticated control law [4], is considered as a benchmark to develop the ideas relevant to the robust systems to characterize and compare the performance of the classical and modern control strategies on a large scale. It has wide range of industrial applications e.g. two wheeled self-balancing vehicles (seg-way), rockets, guided missiles, intelligent robots, and other systems exhibiting crane model. Various control strategies have been implemented on IP. Trivial control algorithms like Proportional Integral Derivatives (PID) [5] are not well suited and incapable of handling for such a complex and non-linear system because they cannot handle inherent uncertainties and disturbances [6]. The robustness of the system decreases with parametric and structural uncertainties consequently making tuning of gains in PID control law a very itchy task [7]. Thus, robust control laws [8] are needed to achieve a high level of precision and accuracy resulting in a more reliable and flexible system to converge the state trajectories into the system in finite time. These challenges highlight the role of more advanced and sophisticated control strategies like Linear Quadratic Regulator (LQR) and Sliding Mode Control (SMC) or its variants, which can provide a systematic way to accurately track the desired trajectories [9].

LQR is a purely linear control technique used for the linear systems while SMC is a robust control technique which deals with the complex systems where uncertainties and disturbances are present. SMC, Integral SMC (ISMC) [10] and Terminal SMC (TSMC) are robust control techniques so, there is always an inconsistency between mathematical and actual model for designing a controller [11]. Unknown external disturbances like matched and unmatched uncertainties are the source of discrepancies between the actual and mathematical model of the system [12,13].

When a pendulum moves in the upright position, stability requirements necessitate the use of robust control technique which should be capable of dealing with the fast dynamics of the

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system. The chattering phenomena present in first order SMC, law can be handled by higher order sliding mode control technique which also improves the system performance [14].

In this paper a comparative analysis of control strategies i.e. LQR, SMC, ISMC, TSMC have been presented. All these algorithms have been implemented on IP. The comparison is carried out based on various parameters like time, control energy and chattering phenomena. The results depict efficacy of ISMC over LQR, SMC and TSMC. The remaining paper is organized as follows: Section 2 derives linear and non-linear model of IP. Section 3 details the control techniques under study while, Section 4 presents the results and discussion of the implemented control strategies. Finally, Section 5 presents the conclusion of this paper.

## 2. Methodology – Mathematical modelling

Mathematical model is required to design the control law. Therefore, Newton law based model of the IP has been derived. The IP consists of a moveable cart rail system and a swing-able pole connected to the cart as shown in Fig. 1. Cart position is controlled with DC motor.

The non-linear mathematical model of the IP is derived using the Newton law approach. Vertical force does not affect the cart position and the horizontal movement is controlled by the forces applied through DC motors [15,16]. The obtained non-linear mathematical model of system is given by (1) and (2). The nomenclature is explained in Table 1, and system specifications are provided in Table 2.

$$(M+m)\ddot{x} + mL\ddot{\theta}\cos\theta - -ml\dot{\theta}^2\sin\theta + B\dot{x} = F \tag{1}$$

$$(I+ml^2)\ddot{\theta} + mglsin\theta = -ml\ddot{x}cos\theta \tag{2}$$

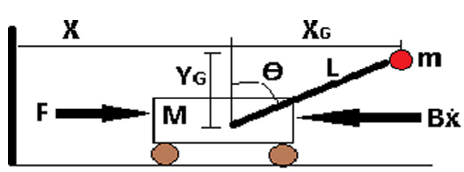


**Fig. 1.** Physical model of IP.

**Table 1**
System states and parameters.

| Symbol | Description |
|---|---|
| $B\dot{x}$ | Cart friction |
| $F$ | External force for horizontal movement |
| $x, \dot{x}, \ddot{x}$ | Cart position, velocity, acceleration respectively |
| $M$ | External pole moment |
| $m$ | Pole friction moment |
| $\theta, \ddot{\theta}$ | Angular velocity and angular acceleration |
| $l$ | Length of the pendulum |

**Table 2**
Assigned values to system.

| Symbol | Quantity/Meaning | Value | Unit |
|---|---|---|---|
| $m$ | Pendulum mass | 2.4 | kg |
| $M$ | Cart mass | 0.23 | kg |
| $I$ | Inertia | 0.38 | $kg/m^2$ |
| $g$ | Gravity | 9.8 | $m/s^2$ |
| $B\dot{x}$ | Friction of cart | 0.005 | N |
| $l$ | Length of pendulum | 45 | cm |
| $L$ | Cart length | 91 | cm |

To implement LQR [17] on this set of equations, we need to linearize the non-linear terms $\dot{\theta}^2$ When pendulum is stable at $\theta \cong 0, \dot{\theta}^2 \cong 0$ and $\cos(0) = 1$, the mathematical model is reduced to (3) and (4).

$$(M+m)\ddot{x} + B\dot{x} - ml\ddot{\theta} = F \tag{3}$$

$$(I+ml^2)\ddot{\theta} - mgl\theta = ml\ddot{x} \tag{4}$$

The transfer functions of the cart position and angle of pendulum is given as,

$$\frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - mgl}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \tag{5}$$

$$\frac{\theta(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \tag{6}$$

where, q is defined as follow:

$$q = [(M+m)(I+Ml^2) - (ml^2)] \tag{7}$$

Converting (5) and (6) to the equivalent state space form given as,

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0194 & 0.2188 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0128 & 6.5848 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.3887 \\ 0 \\ 0.2552 \end{bmatrix} u(t) \tag{8}$$

The output matrix can be written as,

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(t) \tag{9}$$

The system has four poles with two in the right half plane which makes the system unstable. Therefore, a linear controller needs to be designed to force the poles in the left half plane. The calculated open loop poles are located at: S = 0–5.6041 5.5651–0.1428.

## 3. Methodology – Control techniques

In this section, LQR based control technique, feedback linearization based on SMC, ISMC and TSMC are described in details. On the basis of system performance, parameters like setting time, rise time, steady state error and overshoot are calculated.

### *3.1. Linear Quadratic Regulator (LQR)*

LQR is a linearized and optimal control technique which provides optimum gains for the systems. It is more suitable for the linear systems having no uncertainties or disturbances. The major benefit of this technique is that it gives the gains to minimize the cost function [18,19] represented by (10). For an nth order system the general cost function of LQR is given as,

$$J = \int_0^\infty [x^T(t)Q(t)x(t) + U^T(t)R(t)\mathrm{U}(t)]\mathrm{dt} \tag{10}$$

where, $Q \in \mathcal{R}^{n\times n}$ is positive definite or positive semi definite Hermitian matrix (or real symmetric matrix), $R \in \mathcal{R}^{r\times r}$ is a positive definite Hermitian matrix (or real constant number), $S \in \mathcal{R}^{n\times n}$ is a positive definite Hermitian matrix (or real symmetric matrix).

The LQR gain is computed as,

$$K = R^{-1}B^TP \tag{11}$$

The Riccati equation is given in (12) whereas the generic representation of the linear system is given in (13).

$$A^TP + PA - PBR^{-1}B^TR + Q = 0 \tag{12}$$

$$\dot{x}(t) = Ax(t) + BU(t) \tag{13}$$

The control law for the linear system is given as,

$$U(t) = -Kx(t) \tag{14}$$

For computing the gains open loop response of the system can be written as,

$$\dot{x}(t) = Ax(t) - BKx(t) \tag{15}$$

$$\dot{x}(t) = (A - BK)x(t) \tag{16}$$

### 3.2. Feedback linearization

Feedback linearization is a common approach to control the behavior of the non-linear systems. A system is said to be feedback linearizable if, non-linear dynamics of the system can be transformed into an equivalent fully or partially controllable linear system dynamics cancelling the nonlinearities [20]. In our system relative degree is two and order of the system is four which is greater than the relative degree of the system so, we use input output linearization control technique to make relative degree equal to the order of the system. Transform (3) and (4) to get the canonical form given as,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = o_1 + i_1F$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = o_2 + i_2F$$

where x is the state space vector, F is the force need to move cart and $o_1$, $o_2$, $i_1$ and $i_2$ are nominal non-linear functions is defined as follows:

$$o_1 = \frac{mlx_4^2 sinx_3 - mgsinx_3 - cosx_3\dot{r}_3 = r_4}{M + msin^2x_3} \tag{17}$$

$$o_2 = \frac{(M+m)gsinx_3 - mlx_4^2cosx_3}{l(M + msin^2x_3)} \tag{18}$$

$$i_1 = \frac{1}{M + msin^2x_3} \tag{19}$$

$$i_2 = \frac{-cosx_3}{l(M + msin^2x_3)} \tag{20}$$

Let,

$$F = M + msin^2x_3u - (mlx_4^2sinx_3 - mgsinx_3cosx_3) \tag{21}$$

and

$$\dot{X} = f(x) + g(x)u \tag{22}$$

After mapping new coordinates for stabilization of the non-linear system, are given below as,

$$\dot{r}_1 = r_2$$

$$\dot{r}_2 = r_3$$

$$\dot{r}_3 = r_4$$

$$\dot{r}_4 = L_f^4(T^{-1}(r)) + L_gL_f^3h(T^{-1}(r))u$$

$L_fh(x)$ is the Lie derivative of $h(x)$ along vector $f(x)$. The output of the system is defined as,

$$y = h(x) = x_1 + l\ln\left(\frac{1 + sinx_3}{cosx_3}\right) \tag{23}$$

The transformed states thus obtained through feedback linearization are given in the following equations

$$r_1 = h(x) = x_1 + l\ln\left(\frac{1 + sinx_3}{cosx_3}\right) \tag{24}$$

$$r_2 = L_xh(x) = x_2 + \frac{lx_4}{cosx_3} \tag{25}$$

$$r_3 = L_{f^2}h(x) = tanx_3\left(g + \frac{lx_4^2}{cosx_3}\right) \tag{26}$$

$$r_4 = L_{f^3}h(x) = \left(\frac{2}{cos^2x_3} - \frac{1}{cosx_3}\right)lx_3^4 + \left(\frac{3g}{cos^2x_3} - 2g\right)x_4 \tag{27}$$

where,

$$f(x) = \left(\frac{6sinx_3}{cos^4x_3} - \frac{sinx_3}{cos^2x_3}\right)l^4x_4 + \left(\frac{6gsinx_3}{cos^3x_3}x_4^2\right) + \left(\frac{2gsinx_3}{cos^3x_3} - \frac{gsinx_3}{cosx_3}\right)3x_4^2 + \left(\frac{3g}{cos^2x_3} - 2g\right)\frac{gsinx_3}{l} \tag{28}$$

$$g(x) = \frac{-6\psi_4^2}{cos^2\psi_3} - \frac{3g}{lcos\psi_3} + 3\psi_4^2 + \frac{2gcos\psi_3}{l} \tag{29}$$

Feedback linearizable non-linear equation in the 'r' system is shown as,

$$\dot{r}_1 = r_2$$

$$\dot{r}_2 = r_3$$

$$\dot{r}_3 = r_4$$

$$\dot{r}_4 = f(r) + g(r)u$$

$$y = r_1$$

### 3.3. Sliding mode control (SMC)

SMC is a robust control technique used for the higher order non-linear dynamic systems having uncertainties and disturbances. It has fast dynamic response and is insensitive to variety of external disturbances [21]. Due to inconsistency between an actual plant and its mathematical model, matched and unmatched uncertainties, parametric uncertainties and external disturbances need to be taken into account while designing the control law [22]. SMC can better handle such discrepancies due to its robust nature and ability to handle fast dynamic response while ensuring global stability. Tracking the reference can be achieved with the first order SMC which rejects the external disturbances and improves system stability [23]. SMC has two phases; (i) Reaching phase (ii) Sliding phase. In the reaching phase, system reaches the desired sliding surface, defined as,

$$\dot{x} = f(x) + g(x)u + \Delta(x,t) \tag{30}$$

$f(x)$ and $g(x)$ are vector fields representing matched uncertainties. The sliding surface is chosen as,

$$S = \left(\frac{d}{dt} + \lambda\right)^3 e \tag{31}$$

where $\lambda$ is a positive constant and $e$ is the tracking error signal and is the difference between actual and desired output i.e. $e = e_1 - e_d$

$$s = \dddot{e} + \lambda^3 e + 3\lambda^2 \dot{e} + 3\lambda\ddot{e} \tag{32}$$

Sliding surface defined in (32) is based on the tracking function, when sliding surface converges to zero the tracking error automatically converges to zero. SMC based control law consist of two parts; Equivalent control ($u_{eq}$) and discontinuous control ($u_{dis}$). The control input $u_{eq}$ is obtained by putting the sliding surface at $\dot{S} = 0$. Afterwards, derivative of the sliding surface is taken by putting the value of different states and forcing the time derivative to zero, the equivalent controller for the system is obtained as,

$$u_{eq} = \frac{f(x) + (3\lambda e_4) + (3\lambda^2 e_3) + (\lambda^3 e_2)}{g(x)} + \frac{\dddot{e}_d + (3\lambda^2 \ddot{e}_d) + (3\lambda\dddot{e}_d) + (\lambda^3 \dot{e}_d)}{g(x)} \tag{33}$$

The second part of the controller $u_{dis}$ ensures that that the system stays on the sliding surface and usually designed by the Sign function given as,

$$u_{dis} = -kSign(s) \tag{34}$$

Finally, we get

$$u = u_{eq} + u_{dis}$$

First order SMC suffers from the chattering phenomena, which is an oscillatory motion occurring around the sliding manifold. To ensure the stability of the system, the Lyapunov function is defined as,

$$V = \frac{1}{2}S^2 \tag{35}$$

$$\dot{V} = S\dot{S} \tag{36}$$

Taking the time derivative of (32) and putting them in above expression, (37) is obtained

$$\dot{V} = S(-ksign(s)) \tag{37}$$

$$\dot{V} \leqslant -k|s| \tag{38}$$

### 3.4. Integral sliding mode control (ISMC)

High frequency turbulences in SMC damage the system and reduce the life of actuators while the settling time also gets enhanced. ISMC minimize the cost function and chattering in which system dynamic are weak against the uncertainties [24]. System trajectories start from the sliding surface while, the reaching phase is eliminated in the integral mode sliding control. The ISMC improves the results by generalizing the higher order system derivatives and by involving system trajectories into the sliding surface. The performance of the error is also improved in ISMC. By using the reaching phase, it handles the matched uncertainties and stabilizes system asymptotically. States are directed towards sliding manifolds and stabilization depends on sliding constraints. ISMC are most suitable for complex and MIMO system. In ISMC, sliding surface is chosen as (39)

$$S = \left(\frac{d}{dt} + \lambda\right)^3 e + z \tag{39}$$

where,

$$e = e_1 - e_d \tag{40}$$

$$s = \dddot{e} + \lambda^3 e + 3\lambda^2 \dot{e} + 3\lambda\ddot{e} + z \tag{41}$$

$$s = \dddot{e} + \lambda^3 (x_1 - x_d) + 3\lambda^2 (x_2 - \dot{x}_d) + 3\lambda(x_3 - \ddot{x}_d) + z \tag{42}$$

After sliding mode is established, it must satisfy the condition at $\dot{s} = 0$. The sliding mode involves $n - 1$ states, thus reducing the system uncertainties and achieving robustness. Taking the derivative of sliding surface and force the surface at $\dot{s} = 0$, equivalent control part of the system is thus obtained as

$$\dot{s} = \ddot{e} + \lambda^3 (x_2 - \dot{x}_d) + 3\lambda^2 (x_3 - \ddot{x}_d) + 3\lambda(x_4 - \ddot{e}_d) + \dot{z} \tag{43}$$

$$u_{eq} = \frac{f(x) + \lambda^3 (x_2 - \dot{x}_d) + 3\lambda^2 (x_3 - \ddot{x}_d) + 3\lambda(x_4 - \ddot{e}_d) + \dot{z}}{g(x)} \tag{44}$$

$$u_{dis} = -ksign(s) - \zeta s \tag{45}$$

Lyapunov stability based analysis ensures the stability for the system. The Lyapunov energy function is defined as

$$V = \frac{1}{2}S^2 \tag{46}$$

$$\dot{V} = S\dot{S} \tag{47}$$

$$\dot{V} = S(f(x) + g(x)(u_{eq} + u_{dis}) + \lambda^3 (x_2 - \dot{x}_d) + 3\lambda^2 (x_3 - \ddot{x}_d) + 3\lambda(x_4 - \ddot{e}_d) + \dot{z}) \tag{48}$$

$$\dot{V} = S(-ksign(s) - \zeta s) \tag{49}$$

$$\dot{V} \leqslant -k|s| - \zeta s^2 \tag{50}$$

where $kand\zeta$ are positive constants and their value is greater than zero. As $\dot{V}$ is negative definite, hence the system dynamics will converge to the sliding surface in finite time.

### 3.5. Terminal sliding mode control (TSMC)

The convergence of state in SMC is infinite time while in TSMC the state convergence is in finite time. The precise sliding surface is designed in terminal sliding mode control and the purpose of control law is to retain the system on the defined surface. When the illustrative point of the system move on the sliding surface, TSMC is established and infinite convergence is guaranteed [25]. Sliding surface for terminal sliding mode control is chosen as

$$S = \left(\frac{d}{dt} + \lambda\right)^3 e + \beta e_1^{\frac{q}{p}} \tag{51}$$

$$S = \dddot{e} + \lambda^3 (x_1 - x_d) + 3\lambda^2 (x_2 - \dot{x}_d) + 3\lambda(x_3 - \ddot{x}_d) + \beta\frac{q}{p} e^{\frac{q}{p}-1} e_1 \tag{52}$$

The time derivative of sliding surface is given in (53).

$$\dot{S} = \ddot{e} + \lambda^3 (x_2 - \dot{x}_d) + 3\lambda^2 (x_3 - \ddot{x}_d) + 3\lambda(x_4 - \ddot{e}_d) + \beta\frac{q}{p} e^{\frac{q}{p}-1} \dot{e}_1 \tag{53}$$

Putting the sliding surface at $\dot{s} = 0$, the equivalent control part for TSMC is given in (54).

$$u_{eq} = \frac{f(x) + \lambda^3 (x_2 - \dot{x}_d) + 3\lambda^2 (x_3 - \ddot{x}_d) + 3\lambda(x_4 - \ddot{e}_d) + \beta\frac{q}{p} e^{\frac{q}{p}-1} e_2}{g(x)} \tag{54}$$

$$u_{dis} = -k\,sign(s) - \zeta s \tag{55}$$

where, $u = u_{eq} + u_{dis}$. For Lyapunov stability using energy function (56).

$$V = \frac{1}{2}S^2 \tag{56}$$

$$\dot{V} = S\dot{S} \tag{57}$$

Putting (53) in the above expression, (58) is obtained.

$$\dot{V} = S(-k\,sign(s) - \zeta s) \tag{58}$$

$$\dot{V} \leqslant -k|s| - \zeta s^2 \tag{59}$$

where $k$and$\zeta$ is positive constant and greater than zero. The derivative of Lyapunov function $\dot{V}$ becomes negative definite and system dynamics converges to origin in finite time.

## 4. Results and discussions

The simulations involved in the proposed work were performed on MATLAB/Simulink, with simulation time of 50 s. The performance of LQ, SMC, ISMC and TSMC is compared to stabilize cart position of an IP along with pendulum angle for upright position.

The parameters for the aforementioned control techniques are given in Table 3. To validate the control performance, two types of signals are used i.e. a unit step position and a sinusoidal position signals for cart position to stabilize the pendulum.

Figs. 2 and 3 show the comparison of different control techniques for a step input. The stability of the cart at desired reference position and the pendulum angle at its upright position is achieved after 2 s. for ISMC, which is far superior than the linear technique LQR. The stabilization time for SMC and TSMC is 3 s. and 2.5 s. respectively. Fig. 3 shows that the IP attains its upright position after initial dynamics, for all the applied control techniques.

Fig. 4 shows the simulation results of the input torque applied to the pendulum cart to achieve the desired control task. It is evident from the figure that SMC exhibits undesirable chattering phenomenon occurred in control input which is effectively reduced by control laws based on TSMC and ISMC.

Fig. 5 gives the time response for the sliding surfaces defined for SMC, ISMC and TSMC. Here s denotes the sliding surface.

**Table 3**
Tuning parameters of control strategies.

| Parameters | LQR | SMC | ISMC | TSMC |
|---|---|---|---|---|
| $r$ | 1 | – | – | – |
| $k$ | −70, 37, | | | |
| -105,20 | 5 | 3 | – | |
| Λ | – | 20 | 1 | – |
| P | – | – | – | 10 |
| $q$ | – | – | – | 20 |
| $\alpha$ | – | – | – | 30 |
| $\beta$ | – | – | – | 40 |

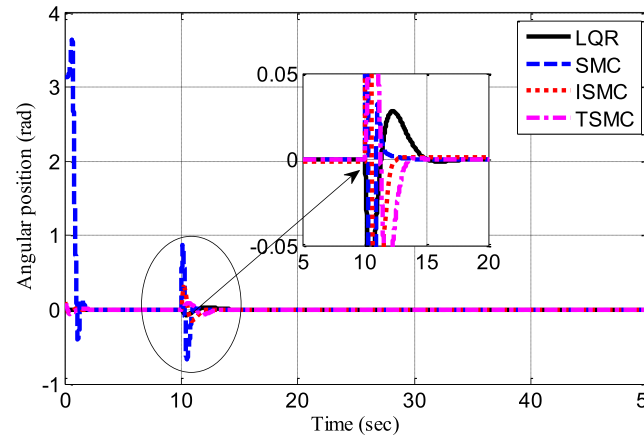


**Fig. 3.** Step response of pendulum angle based on LQR, SMC, ISMC and TSMC laws.

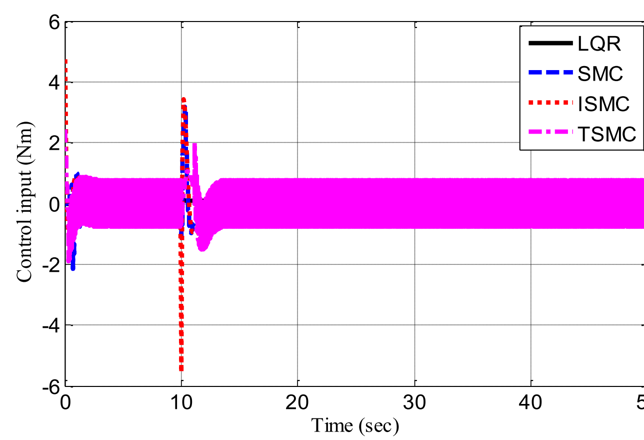


**Fig. 4.** Control input using LQR, SMC, ISMC and TSMC laws.

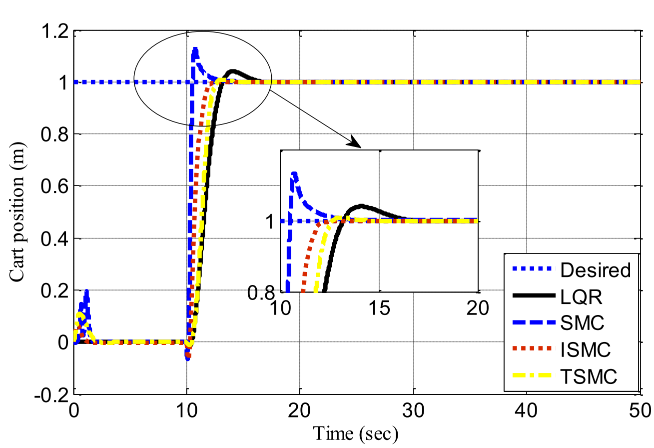


**Fig. 2.** Step response of cart position based on LQR, SMC, ISMC and TSMC laws.

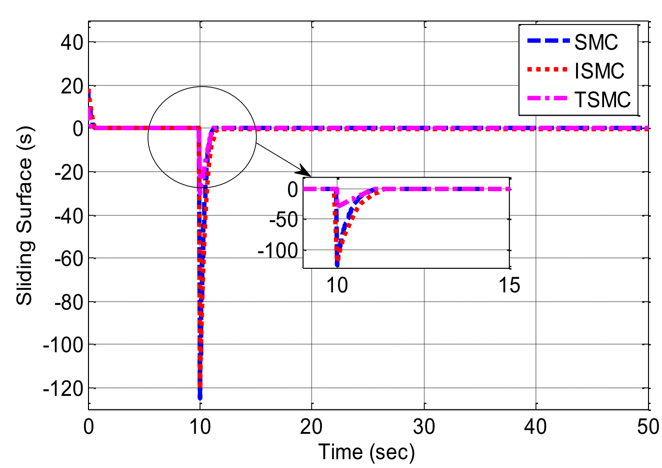


**Fig. 5.** Sliding surface based on SMC, ISMC and TSMC laws.

Second validation test was performed using a sinusoidal reference input signal of the form given in (60). Where, a = 0, b = 1, f = 0.157 and $\theta = 0$.

$$y_d = a + b\sin(f(t) + \theta) \tag{60}$$

Figs. 6 and 7 respectively show the simulation results of SMC, ISMC and TSMC against the reference sinusoidal signal for cart position and pendulum angle. As is evident from Fig. 6 that the cart reaches the desired sinusoidal trajectory for all the control techniques, keeping the inverted pendulum stable to its upright position. The tracking performance of ISMC is better as compared to the other techniques. All the controllers exhibit good steady state performance with ISMC having relatively least settling time, rise time and steady state errors.

Figs. 8 and 9 show the corresponding control input and the sliding surface for sinusoidal reference trajectory respectively. Results obtained using LQR technique was not included, as linearization is no more valid for a time varying input signal.

Table 4 summarizes the performance based comparative analysis of linear control technique (LQR) and non-linear control techniques (SMC, ISMC and TSMC) for cart position while keeping the pendulum stable to its upright position. The performance parameters show that ISMC provides the best steady state and transient behavior while LQR technique, being the linear technique, fails to provide satisfactory results especially in the case of sinusoidal reference input signal.

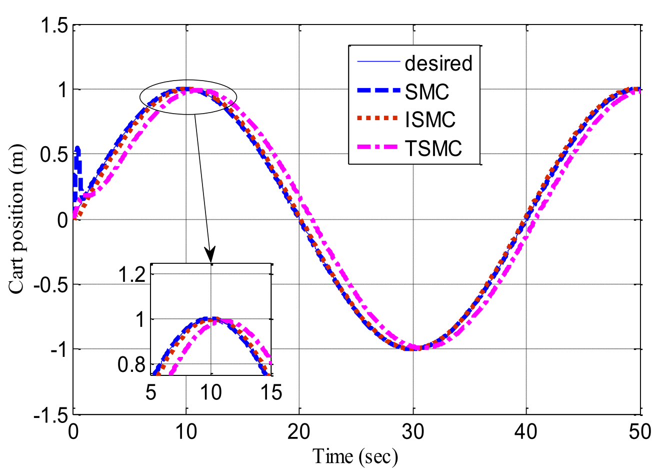


**Fig. 6.** Sinusoidal response of cart position based on SMC, ISMC and TSMC laws.

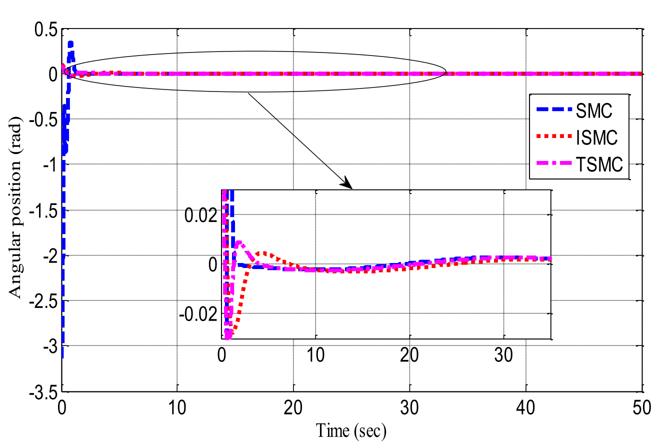


**Fig. 7.** Sinusoidal response of pendulum angle based on SMC, ISMC and TSMC laws.

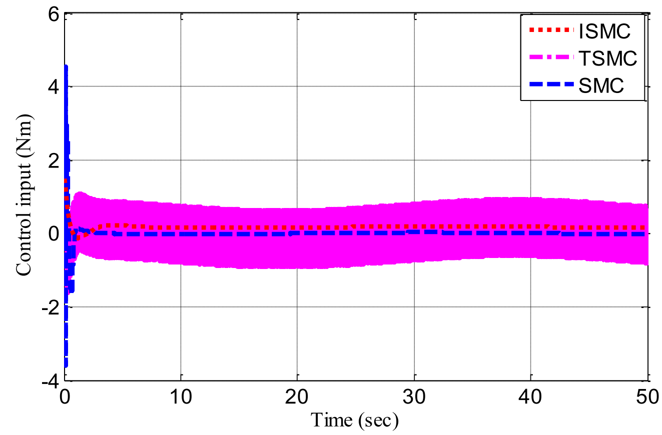


**Fig. 8.** Control input by using SMC, ISMC and TSMC laws.

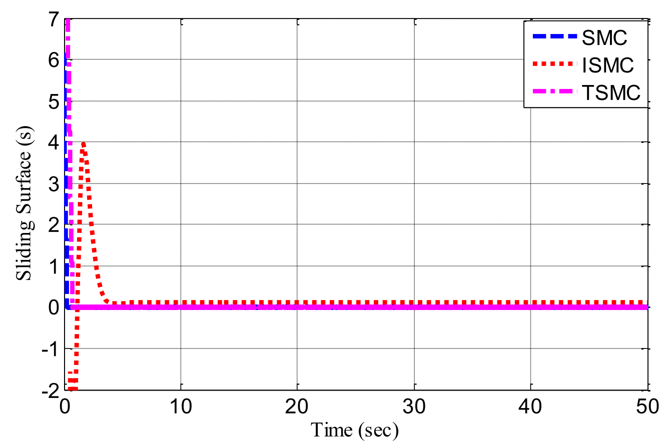


**Fig. 9.** Sliding surface based on SMC, ISMC and TSMC laws.

**Table 4**
Performance analysis of control strategies.

| Response Specification | LQR | SMC | ISMC | TSMC |
|---|---|---|---|---|
| Settling Time (sec) | 6.2 | 3 | 2 | 2.5 |
| Rise Time (sec) | 3.2 | 0.9 | 0.6 | 0.65 |
| Steady state error ($e_{ss}$) | 0.02 | 0.0012 | 0.0001 | 0.001 |
| (%) Overshoot | 0.01 | 0.001 | 0 | 0 |

The performance of the implemented control techniques i.e. LQR, SMC, TSMC and ISMC are also tested in the presence of disturbances, the amplitude of the disturbance is set to 0.1 rad/sec$^2$ which is then, introduced in the plant to unbalance the upright position of the pendulum. Figs. 10 and 11 show the simulation results for cart position and pendulum angle. The step signal of amplitude 0.1 rad/sec$^2$ is introduced after 25 s. Simulations results show that all the three techniques based on sliding mode control show robust performance, however, LQR technique fails to stabilize the cart at 1 m.

All the control techniques presented in this paper are model based assuming that all the states are observable. Possible directions in future include observer design, consideration of faults in sensors/actuators, adaptation of SMC-based law and hybridizing SMC with other robust techniques like H∞. Observer design finds its potential in numerous real-world applications since practically it may not be possible to measure all states of a system at all times due to cost and viability issues [26]. Another pertinent issue of

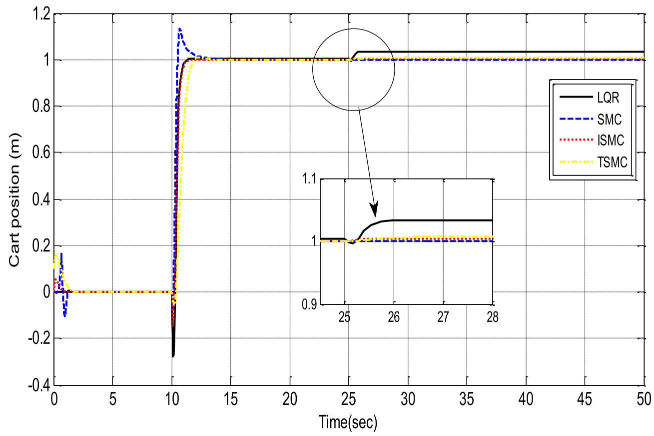


**Fig. 10.** Cart position with disturbance based on SMC, ISMC and TSMC laws.

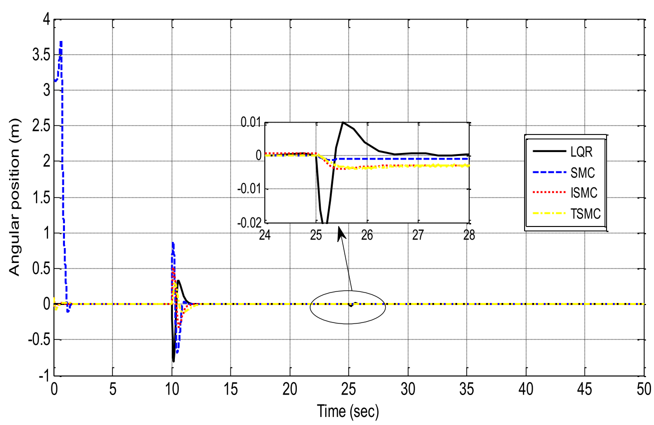


**Fig. 11.** Pendulum angle with disturbance based on SMC, ISMC and TSMC laws.

utmost importance is a mechanism to deal with consequences of components failures. Reliable control ensures that the desired specifications of a system are met even in case of breakdown in one or more components [27]. Reliability function may be based on Riccati equation, linear matrix identity method, coprime factorization and Lyapunov function. An adaptive SMC based law finds its potential in cases where complete information about external disturbance, limits on actuator faults and nonlinearity bounds is not available [28]. To simultaneously handle several of the aforementioned issues, research works in [29–31] highlights the benefits of robust $H_\infty$ SMC.

## 5. Conclusion

In this research paper linear and non-linear control strategies has been successfully designed and simulated. Robust control techniques have been discussed for IP to ensure the stability to achieve better response of system. Robust control techniques for this study include SMC, ISMC and TSMC. Based on analysis and design structure, all the laws are capable of controlling the IP. Result of each controller has been plotted and compared. Overall performance comparison concludes that ISMC gives the best performance over the other control techniques.

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