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Full Length Article

Testing and modeling of filter cake formation using new seepage-consolidation concept

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ABSTRACT

The behavior of filter cake formation has been investigated using new seepage-consolidation concept. This study is focused on the precious measurement of the fluid loss that affects several applications such as the interaction between the concrete and the pipe in the oil well application and bored piles, pipe stuck problems, and oil well production. Currently, American Petroleum Institute (API) model is being used to model the filter cake formation. The API-model has assumed several unrealistic assumptions such as infinity fluid loss at infinity time period, constant filter cake permeability during filter cake formation, constant relative solid content in the filter cake to the mud, and constant cake porosity during cake formation. Hence, the combination of seepage and consolidation phenomenon has been used to preciously model the filter cake formation. In addition, a new consolidation equation was derived based on the fact that the cake permeability is time dependent function. In the proposed solution, a coupling function of time and elevation was used to express the excess pore pressure function. The proposed solution was verified against Terzaghi consolidation solution and API model for long-term experimental results for both 2% and 8% bentonite drilling mud under a constant pressure of 690 kPa and different temperatures of 25°, 50°, 75°, and 100 °C. The verification included the variations of the fluid loss, permeability, coefficient of consolidation, and excess pore water pressure with the time. It was concluded that the new method has better prediction for the experimental results than both Terzaghi consolidation solution and API-model. The pore water pressure for 2% bentonite drilling mud at 25 °C decreased by 24% and 26% over 420 min using Terzaghi and new proposed method respectively.

1. Introduction

Drilling fluid consists of solids, liquids and chemicals where the liquid is the continuous phase. These components are important for the oil, gas and geothermal drilling industry because of their functions in transporting rock cuttings to the surface, lubricating the drilling bit, applying hydrostatic pressure in the wellbore to ensure the safety and decreasing the fluid loss by forming a filter cake on the inner face of the wellbore [1,2].

Filter cake forms over the face of the porous medium along the entire drilling operation and the filtrate is disappeared into the formation [3,4]. The skeleton of the filter cake contains larger particles of the slurry while the smallest particles migrate and deposit in the porous cake that is formed by the larger particles. As the drilling process is started, the liquid of the drilling fluid flows into the formation due to the variance of the temperature and formation permeability leading to the fluid loss. The rate of the fluid loss is mainly controlled by the formation and properties of the mud cake. In the formation with high permeability condition, the filtrate loss is dominated by the permeability of the mud cake. The analysis of cake filtration started with the classical work done by Ruth et al. (1933) [5] more than 80 years ago. Based on using a systematic investigation on the fluid loss and mud cake permeability, Williams and Cannon (1938) [6] determined that the filtration rate of the drilling fluid was primarily controlled by the amount and characteristics of solids in the drilling fluid. The fluid loss is increased with the addition of weighting materials. Other factors that affected the rate of filtration include the degree of solid dispersion, particle size distribution, degree of clay hydration, and presence of agglomerating dispersing agents [7,8]. Besides the accumulation of the drilling mud particles around the wellbore, the small particles in the mud occupy the formation initiating internal formation damage, starting from less than one inch to a maximum of one foot [9].

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List of Symbols

A and B	Arbitrary Constants
A_1	Constant at each time step (difference in the head over the distance)
A_o	The cross sectional area
C_v	Coefficient of consolidation
dx, dy, dz	Block dimensions in x, y, and z directions respectively
F(z)	Mathematical function expressing the excess pore water pressure in terms of depth
f_{sc}	Volume fraction of solids in the filter cake
f_{sm}	Volume fraction of solids in the mud
G(t)	Mathematical function expressing the excess pore water pressure in terms of time
h	The total head
H_1	Thickness of clay layer
h_e	Pressure head due to excess pore water pressure
k(t)	Time dependent value for the hydraulic conductivity of the filter cake
k_o	Initial value for the hydraulic conductivity of the filter cake
k_x, k_y, k_z	hydraulic conductivity in x, y, and z directions respectively
M	$(2m+1)\pi/2$
N	$N=\sqrt{2k\Delta p\left(\frac{f_{sc}}{f_{sm}}-1\right)}$
n	Mathematical constant
N_1	The number of data points
P	External applied pressure
R^2	Coefficient of Determination
RMSE	Root Mean Square Error
t	Time
T_v	Time factor
u	Excess pore water pressure
u_e	Excess pore water pressure
u_o	Initial excess pore water pressure
V_f	Fluid loss
V_o	Initial volume of fluid loss
v_x, v_y, v_z	The components of the discharge velocity in x, y, and z direction
x_i	The calculated value from the model
y_i	The actual value
Δp	Applied pressure (atm)
$\Delta Q_1(t)$	The amount of the fluid loss due to seepage
$\Delta Q_2(t)$	The amount of the fluid loss due to consolidation
$\Delta Q_T(t)$	The total amount of the fluid loss
Δu	Initial excess pore water pressure
m_v	Coefficient of volume compressibility
γ_w	Unit weight of water
$\underline{y}$	The mean of actual values
$\underline{x}$	The mean of calculated values
μ	Filtering fluid viscosity

Instantaneously, the cake may experience a consolidation progression due to high pressure and high temperature as the fluid flows within the cake [10]. The easiest and most usual approach to analyze the consolidation in practice is to rely on the one-dimensional (1-D) consolidation equation [11,12]. For the oil and gas drilling with a drilling fluid, mud cake is formed because the pressure in the wellbore is higher than the pore pressure in the rock [13,14]. The drilling string may get stuck in the cake because of the high net forces affecting the drilling string and push it against the wall. Sealing of different sections of the wall by packers, the interaction of these packers with the mud cake and the sealing efficiency are additional problems of the mud-cake in the wellbore [13].

The local permeability of the filter cake can be reduced because of the consolidation and cake clogging [10,15]. Cake consolidation occurs due to the compressive stress within the cake while the cake clogging is a result of the retention of fine particles [16]. Even though the amount of the fines involved in the cake is small, its effect on the permeability can be considerable. A simple approximation model has been obtained by Tiller (2002) [17] to illustrate the behavior of compactible cakes deposited under constant applied pressure. Zinati et al. (2009) [18] derived a simple model to predict the cake thickness and velocity profiles in the radial geometry for a suspension containing mono-sized particles.

Another equation was developed by Osisanya and Griffith (1997) [19] to evaluate the permeability of the filter cake using filtrate volume, shear stress, plastic viscosity, and yield point of the fluid. Khatib (1994) [20] investigated the impact of solids type, applied pressure, and oil presence on the permeability and porosity of thin cakes by using compression-permeability cell. The solids studied were iron hydroxide, iron sulfide, calcium carbonate and produced silt and clay. Based on the study, a correlation between permeability (K_c) and porosity (n) of silt/clay filter cake was obtained. Past studies on the growth of filter-cake were concentrated on the model tests in the laboratory. Cheng (2001) [21] performed laboratory tests for pure bentonite in medium coarse sand, and the results indicated that the density and viscosity had a big impact on the formation of the filter cake.

Generally, the bentonite content in the drilling muds varies from 2% to 5% (W/W). The testing time for the fluid loss varies from 20 to 600 min while current API fluid loss recommends a testing time of 30 min although drilling operations can vary in time from hours to days and weeks based on the project [22–24]. The standard API fluid loss model is built based on the assumption that both the permeability and solid fraction in the filter cake are constants and since then the fluid loss is directly proportional to the square-root of time with no limit for the maximum fluid loss [25–28]. In the real world situation, the fluid loss has a limit on the total volume depending on the type of the drilling mud and one of the time dependent relationships could be hyperbolic [29,30]. Hence, the filter cake formation is started initially with the water seepage in the absence of any cake formation. However, the filter cake then started to from with the time and the filter cake is going to be under the effects of both water seepage and cake consolidation.

The main objective of this study was to model the filter cake formation using new combined seepage-consolidation principle. The specific objectives of this study were as follows:

1. Investigate the filter cake formation for 2% and 8% bentonite drilling mud tested under normal applied pressure of 690 kPa at different temperatures ranged from 25 °C to 100 °C.
2. Examine the long-term formation of filter cake behavior using experimental tests extended up to 420 min.
3. Compare the new proposed combined seepage-consolidation model with the Terzaghi's one dimensional consolidation equation to study the filter cake formation.

2. Materials and methods

To perform the fluid loss experimental test, it is required to use the standard API filter press test that has a cross-sectional area of

45 cm^2 or the high-pressure and high temperature (HPHT) filter press with an area of 22.58 cm^2. In the HPHT test, 2% and 8% bentonite drilling mud were used and tested at a pressure of 690 kPa and temperature range between 25 °C and 100 °C. The tests were continued until zero fluid loss, about 420 min.

Selected amount of bentonite was mixed with water at room temperature for at least 5 min to prepare homogenous drilling mud. During the experimental tests, the variation of the fluid loss with the time is quantified. Then, the variations of coefficient of permeability, coefficient of consolidation, and pore water pressure are evaluated using Terzaghi and new proposed method. The unit weight and pH of the 2% drilling mud was 10.14 kN/m^3 and 9 respectively. The unit weight and pH of the 8% drilling mud was 10.61 kN/m^3 and 9.03 respectively. The initial measured electrical resistivity for 2% and 8% drilling mud were 6.29 Ohm·m and 2.87 Ohm·m respectively.

3. Comparison of model predictions

In order to determine the accuracy of any model predictions in the study, both the root mean square error (RMSE) and coefficient of determination (R^2) as defined in Eqs. (1) and (2) were quantified using:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - x_i)^2}{N_1}} \quad (1)$$

$$R^2 = \left(\frac{\sum_i \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\sum_i \left(x_i - \bar{x}\right)^2}\sqrt{\sum_i \left(y_i - \bar{y}\right)^2}}\right)^2 \quad (2)$$

where y_i is the actual value; x_i is the calculated value from the model; $\bar{y}$ is the mean of actual values; $\bar{x}$ is the mean of calculated values and N_1 is the number of data points.

4. Filter cake seepage-consolidation combination

A filter cake with an external applied pressure (P) is shown in Fig. 1. Due to the applied pressure on the surface of the cake, the cake is subjected to a combined effect of seepage and consolidation. Seepage-consolidation idealization during filter cake formation can be shown in Fig. 1. Initially, the drilling mud starts seeping through the rocks forming the filter cake. As soon as the filter cake starts to form and with continuous of pressure application, the process of seepage and filter cake consolidation start in parallel until the progression of drilling ends where very limited amount of water remains in the drilling mud.

To properly model the behavior of the filter cake, the boundary condition of the cake should be defined correctly for proper validation of the results with the experimental observations. The filter cake can be modeled using a 1-D simplification of both seepage and consolidation since the fluid loss can occur experimentally only through a small opening (diameter = 1 mm) in the bottom while the vertical sides are impervious. Hence, the total amount of the fluid loss can be identified as

$$\Delta Q_T(t) = \Delta Q_1(t) + \Delta Q_2(t) \quad (3)$$

where $\Delta Q_T(t)$ is the total amount of the fluid loss during the test, $\Delta Q_1(t)$ is the amount of the fluid loss due to seepage while $\Delta Q_2(t)$ is the amount of the fluid loss due to consolidation.

4.1. Seepage

The 3-D block has dimensions dx, dy, and dz. Let v_x, v_y, and v_z be the components of the discharge velocity in x, y, and z direction. The rate of flow of water into the elemental block in x, y, and z directions are v_x dy dz, v_y dx dz, and v_z dx dy respectively. Fig. 2 idealized the inflow and outflow in the 3-D scheme.

Assuming that the water is incompressible and no volume change in the solid mass occurs. Then, the total rate of inflow should equal the total rate of outflow.

With applying Darcy's law and assuming the soil to be isotropic with respect to hydraulic conductivity that ($k_x = k_y = k_z$), the continuity equation can be obtained as follows:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (4)$$

where h is the total head. The seepage model for 1-D condition can be represented as:

$$\Delta Q_1(t) = k(t) * \frac{\partial h}{\partial z} * A_o \quad (5)$$

where A_o is the cross sectional area. From Continuity (1-D): $\frac{\partial 2h}{\partial z^2} = 0 \Rightarrow \frac{\partial h}{\partial z} = A_1$ and:

$$\Delta Q_1(t) = k(t) * A_1 * A_o \quad (6)$$

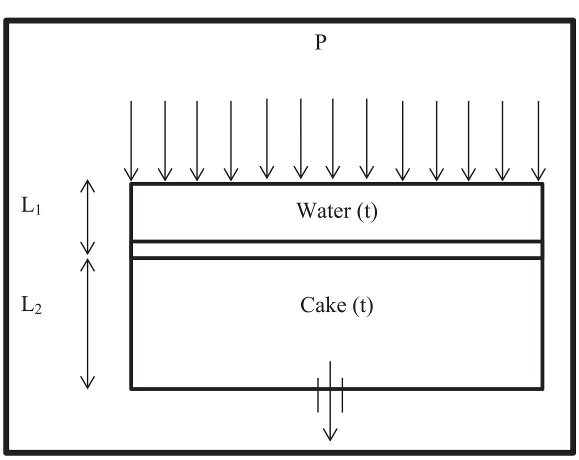

Fig. 1. Seepage-consolidation idealization during filter cake formation.

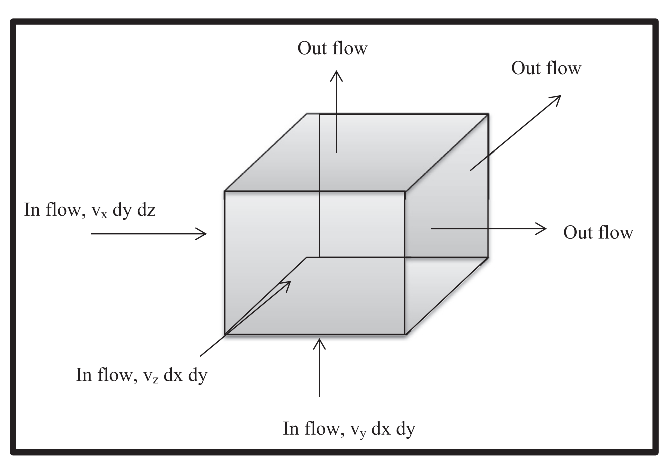

Fig. 2. Inflow and outflow idealization scheme in 3-D.

4.2. Consolidation

4.2.1. Terzaghi consolidation equation

The theory for the time rate of one-dimensional consolidation was first proposed by Terzaghi (1943) [11]. With the Terzaghi's assumptions, let us consider a clay layer of thickness H_1 as shown in Fig. 3.

The layer is located between two highly permeable sand layers. In this case of one-dimensional consolidation, the flow of water into and out of the soil element is in one direction only, i.e., in the z direction. Thus, the Terzaghi consolidation equation was derived to be as follows:

$$\frac{\partial u}{\partial t} = \frac{k_z}{\gamma_w m_v}\frac{\partial^2 u}{\partial z^2} = C_v\frac{\partial^2 u}{\partial z^2} \tag{7}$$

where C_v = coefficient of consolidation $= \frac{k_z}{\gamma_w m_v}$.

Eq. (7) is the basic differential equation of Terzaghi's consolidation theory and can be solved with proper boundary conditions. To solve the equation, assume u to be the product of two functions, i.e., the product of a function of z and a function of t, or

$$u = F(z)G(t) \tag{8}$$

The final solution for u can be represented as follows:

$$u = \sum_{m=0}^{m=\infty}\frac{2u_o}{M}\sin\frac{Mz}{H}\exp\left(-M^2T_v\right) \tag{9}$$

Then,

$$\Delta Q_2 = \frac{2k_oA_ou_o}{H\gamma_w}\sum_{m=0}^{m=\infty}\cos\frac{Mz}{H}\exp\left(-M^2T_v\right) \tag{10}$$

Substitute Eqs. (6) and (10) in Eq. (3), then

$$\Delta Q_T(t) = k(t) * A_o * A_1 + \left[\frac{2k_oA_ou_o}{H\gamma_w}\sum_{m=0}^{m=\infty}\cos\frac{Mz}{H}\exp\left(-M^2T_v\right)\right] \tag{11}$$

4.2.2. Method 1 (coupling solution)

In this method, a coupling function of time (t) and elevation (z) can be used to express the excess pore pressure function as:

$$u_e(z,t) = \frac{N * \gamma_{w*}z^{(n+1)}}{k_o * A_o * (n+1) * (A+Bt)} \tag{12}$$

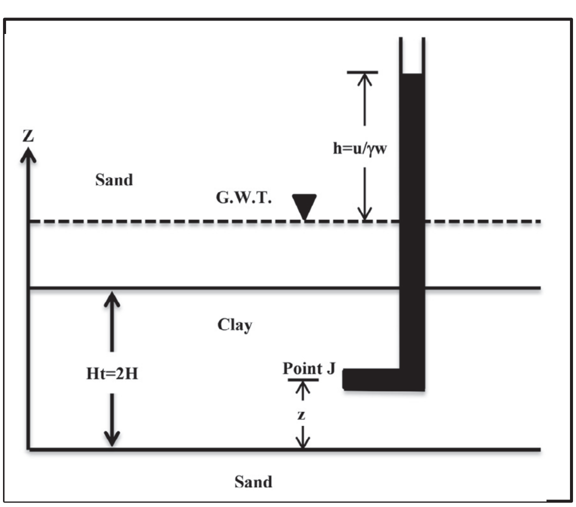

Fig. 3. Clay layer undergoing consolidation.

Eq. (12) can represent the best coupling function for excess pore water pressure including the effects of several characteristics such as the initial permeability of the studied medium, the cross sectional-area of the consolidated layer, applied pressure, solid content of the mud, and solid content for the time-dependent formed filter cake.

From Eq. (12), the first and second derivatives of excess pore water pressure with respect to time and depth can be represented respectively as:

$$\frac{\partial u}{\partial t} = \frac{-N * \gamma_{w*}z^{(n+1)} * B}{k_o * A_o * (n+1) * (A+Bt)^2} \tag{13}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{n * N * \gamma_{w*}z^{(n-1)}}{k_o * A_o * (A+Bt)} \tag{14}$$

Substitute Eqs. (13) and (14) in Eq. (7), then:

$$\frac{-N * \gamma_{w*}z^{(n+1)} * B}{k_o * A_o * (n+1) * (A+Bt)^2} = C_{v*}\frac{n * N * \gamma_{w*}z^{(n-1)}}{k_o * A_o * (A+Bt)} \tag{15}$$

To satisfy both sides of Eq. (15), then:

$$C_v = \frac{-B * z^{(n+1)}}{(n+1) * (A+Bt) * n * z^{(n-1)}} \tag{16}$$

The effect of n value on several parameters have been checked such as coefficient of consolidation, coefficient of permeability, excess pore water pressure, pressure head, fluid loss due to consolidation and consolidation discharge.

Thus, the n value has been optimized to be (−0.5), then

$$C_v = \frac{4 * B * z^2}{(A+Bt)} \tag{17}$$

$$u_e(z,t) = \frac{2 * N * \gamma_{w*}z^{0.5}}{k_o * A_o * (A+Bt)} \tag{18}$$

$$h_e = \frac{u_e(z,t)}{\gamma_w} = \frac{2 * N * z^{0.5}}{k_o * A_o * (A+Bt)} \tag{19}$$

$$\frac{\partial h_e}{\partial z} = \frac{N}{k_o * A_o * z^{0.5} * (A+Bt)} \tag{20}$$

$$k(t) = \frac{k_o * A * z^{0.5}}{(A+Bt)} \tag{21}$$

$$\Rightarrow \Delta Q_2(t) = \frac{dV_f}{dt} = \frac{k_o * A * z^{0.5}}{(A+Bt)} * \frac{N}{k_o * A_o * z^{0.5} * (A+Bt)} * A_o \tag{22}$$

$$V_f = \frac{N * t}{(A+Bt)} \tag{23}$$

Substitute Eqs. (6) and (23) in Eq. (3), then:

$$\Delta Q_T(t) = k(t) * A_o * \left(A_1 + \frac{N}{k_o * A_o * z^{0.5} * (A+Bt)}\right) \tag{24}$$

where

k(t): permeability (decreased with the time),
A and B: arbitrary constants,
A_1: constant at each time step (difference in the head over the distance),
A_o: cross-sectional area,
Δu: initial excess pore water pressure, and
C_v: coefficient of consolidation.

4.2.3. API model

This model was developed based on several conditions such as [31,26]:

1. The percentage of volume solid content in the forming cake (f_{sc}) is a constant.
2. The permeability of the forming cake (k) is constant.

API model is widely used to predict the amount of the fluid loss. The final form of the API model is as follows:

$$V_f - V_0 = \sqrt{2k_o\Delta p\left(\frac{f_{sc}}{f_{sm}} - 1\right)}\, A_o \frac{\sqrt{t}}{\sqrt{\mu}} \tag{25}$$

V_f = total volume of fluid loss (cm^3), V_o = initial volume of fluid loss (spurt) (cm^3), k_o = permeability of the filter cake (darcy), Δp = applied pressure (atm), f_{sc} = volume fraction of solid in cake, f_{sm} = volume fraction of solids in mud, A_o = filter area (cm^2), t = time (min), μ = filtering fluid viscosity (cP), $N = \sqrt{2k_o\Delta p\left(\frac{f_{sc}}{f_{sm}} - 1\right)}$.

5. Results and analysis

5.1. 2% bentonite drilling mud

The variations of fluid loss, coefficient of permeability, coefficient of consolidation, and pore water pressure with the time under a constant pressure of 690 kPa and various temperatures 25°, 50°, 75°, and 100 °C for 2% bentonite drilling mud shown in Figs. 4, 5, 6 and 7 respectively. In Fig. 4a, all three models predicted the fluid loss very well with higher accuracy for method 1 (R^2 = 0.99) compared to API model and Terzaghi method. In Fig. 4b, the final permeability value of the filter cake has been measured using falling head test. It is clearly shown that the coefficient of permeability was constant over the time in Terzaghi method while the coefficient of permeability decreased by 126,000 times over 420 min in method 1 approaching the final measured experimental value. Similarly, in Fig. 4c, the coefficient of consolidation was constant over the time in Terzaghi method while the coefficient of consolidation decreased by 126,000 times over 420 min in method 1. In Fig. 4d, the pore water pressure decreased by 24% and 26% over 420 min in Terzaghi and method 1 respectively.

In Fig. 5a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 where Terzaghi prediction was better than the API - model. In Fig. 5b, the final permeability value of the filter cake has been determined in falling head test. It is obviously shown that Terzaghi method predicts the coefficient of permeability to be constant over the time while the coefficient of permeability decreased by 109,847 times over 420 min in method 1 approaching the final measured experimental value. In addition, in Fig. 5c, Terzaghi method predicts the coefficient of consolidation to be constant over the time in while the coefficient of consolidation decreased by 109,847 times over 420 min in method 1. In Fig. 5d, the pore water pressure decreased by 24% and 25% over 420 min in Terzaghi and method 1 respectively.

In Fig. 6a, method 1 was the best model to predict the fluid loss with a coefficient of correlation (R^2) of 0.99 while Terzaghi model predication was better than the API - model. In Fig. 6b, the final permeability value of the filter cake has been computed in falling head test. It is visible that the coefficient of permeability decreased reasonably by 100,801 times over 420 min in method 1 to simulate the filter cake formation approaching from the final measured value while Terzaghi method fails to predict the changes in the coefficient of permeability of the filter cake. Similarly, in Fig. 6c, the coefficient

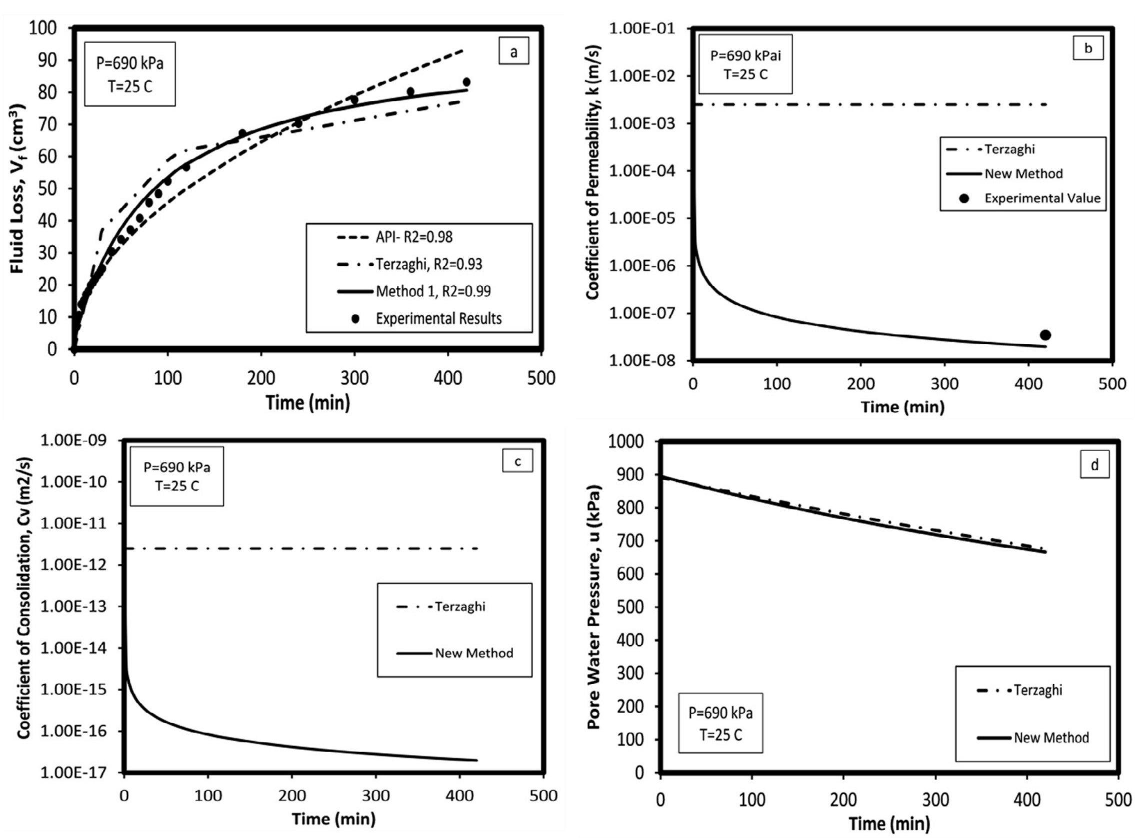

Fig. 4. Long-term model predictions of current study on 2% bentonite drilling mud at 25 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

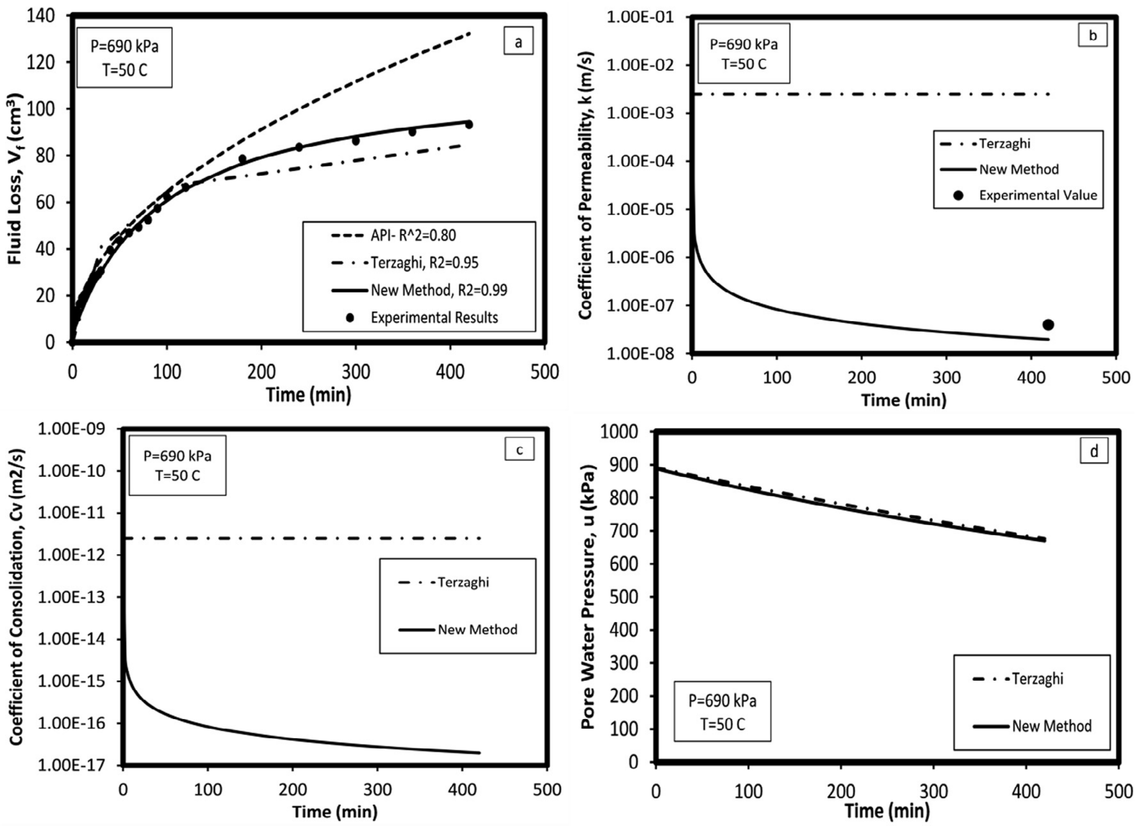

Fig. 5. Long-term model predictions of current study on 2% bentonite drilling mud at 50 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

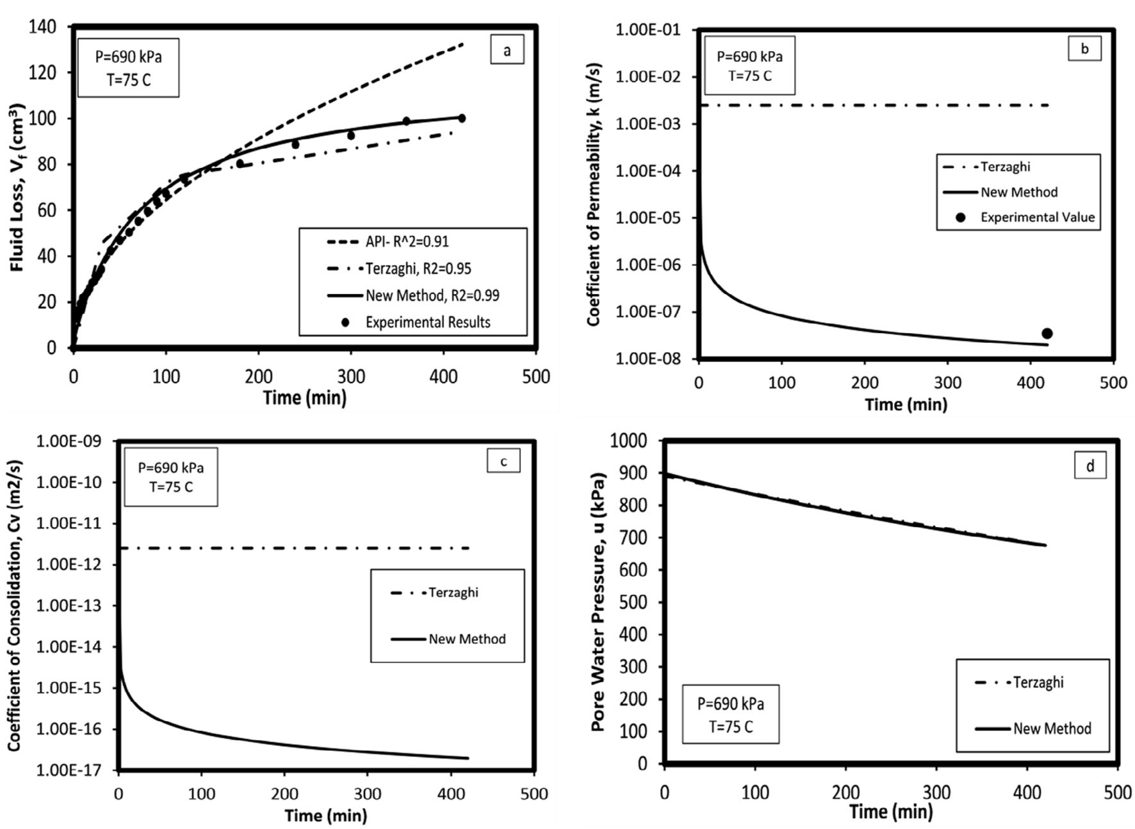

Fig. 6. Long-term model predictions of current study on 2% bentonite drilling mud at 75 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

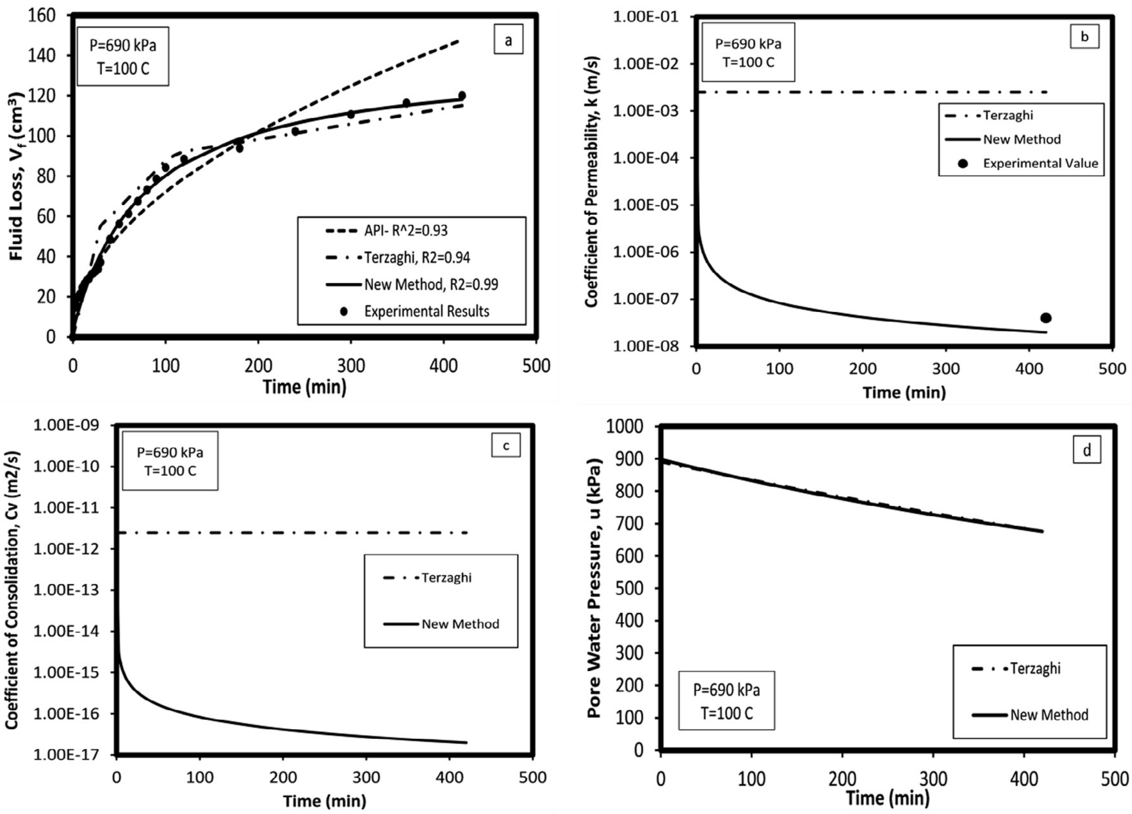

Fig. 7. Long-term model predictions of current study on 2% bentonite drilling mud at 100 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

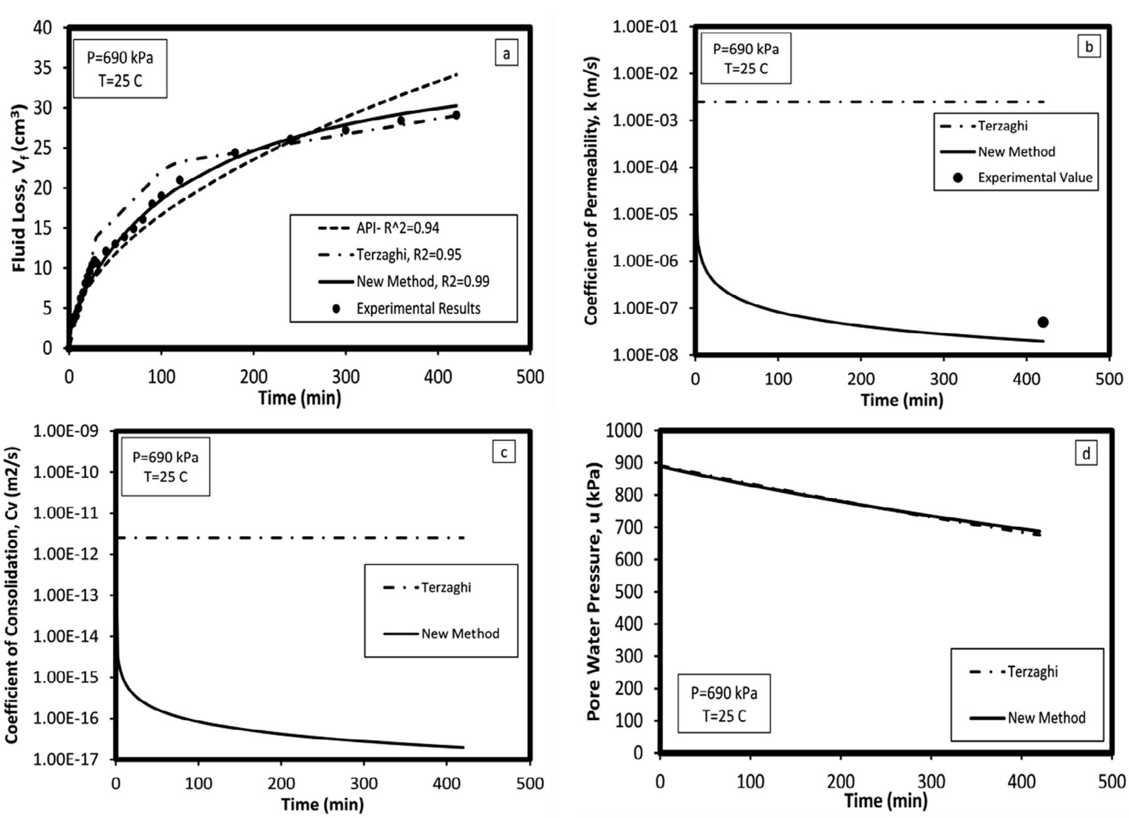

Fig. 8. Long-term model predictions of current study on 8% bentonite drilling mud at 25 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

of consolidation decreased by 100,801 times over 420 min in method 1 and no change in the coefficient of consolidation was predicted with the time in Terzaghi method. In Fig. 6d, the pore water pressure decreased by 24% and 25% over 420 min in Terzaghi and method 1 respectively. In Fig. 7a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 while API prediction was the worst with R^2 of 0.93. In Fig. 7b, the final permeability value of the filter cake has been measured using falling head test. It is clearly shown that the coefficient of permeability was constant over the time in Terzaghi method while the coefficient of permeability decreased by 100,801 times over 420 min in method 1 to approximately matches the final measured experimental value. Similarly, in Fig. 7c, the coefficient of consolidation was constant over the time in Terzaghi method while the coefficient of consolidation decreased by 100,801 times over 420 min in method 1. In Fig. 8d, the pore water pressure decreased by 24% and 25% over 420 min in Terzaghi and method 1 respectively. All model predictions for 2% bentonite drilling mud at different tested temperatures 25 °C, 50 °C, 75 °C, and 100 °C are summarized in Table 1. It is clearly shown that the method 1 is the most accurate model compared to API and Tezaghi models where method 1 had the highest R^2 and lowest RMSE compared to others.

5.2. 8% bentonite drilling mud

The variations of fluid loss, coefficient of permeability, coefficient of consolidation, and pore water pressure with the time under a constant pressure of 680 kPa and various temperatures 25°, 50°, 75°, and 100 °C for 8% bentonite drilling mud shown in Figs. 8, 9, 10 and 11 respectively. In Fig. 8a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 while API prediction was the worst with R^2 of 0.94. In Fig. 8b, the final permeability value of the filter cake has been determined using falling head test. It is obviously shown that the coefficient of permeability decreased by 365,396 times over 420 min in method 1 to reach almost the final experimental measured value while Terzaghi method predicted the coefficient of permeability to be constant. Similarly, in Fig. 8c, the coefficient of consolidation was constant over the time in Terzaghi method while the coefficient of consolidation decreased by 365,396 times over 420 min in both method 1. In Fig. 8d, the pore water pressure decreased by 24% and 23% over 420 min in Terzaghi and method 1 respectively.

In Fig. 9a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 while Terzaghi and API

Table 1
Fluid loss prediction for 2% bentonite drilling mud using three different models.

Drilling Mud (%)	Temp. C	API Model		Terzaghi Model		Method 1	
		R^2	RMSE (cm^3)	R^2	RMSE (cm^3)	R^2	RMSE (cm^3)
2	25	0.98	3.636	0.93	6.397	0.99	2.216
2	50	0.80	12.003	0.95	5.819	0.99	2.777
2	75	0.91	9.163	0.95	6.035	0.99	3.155
2	100	0.93	8.991	0.94	8.091	0.99	3.764

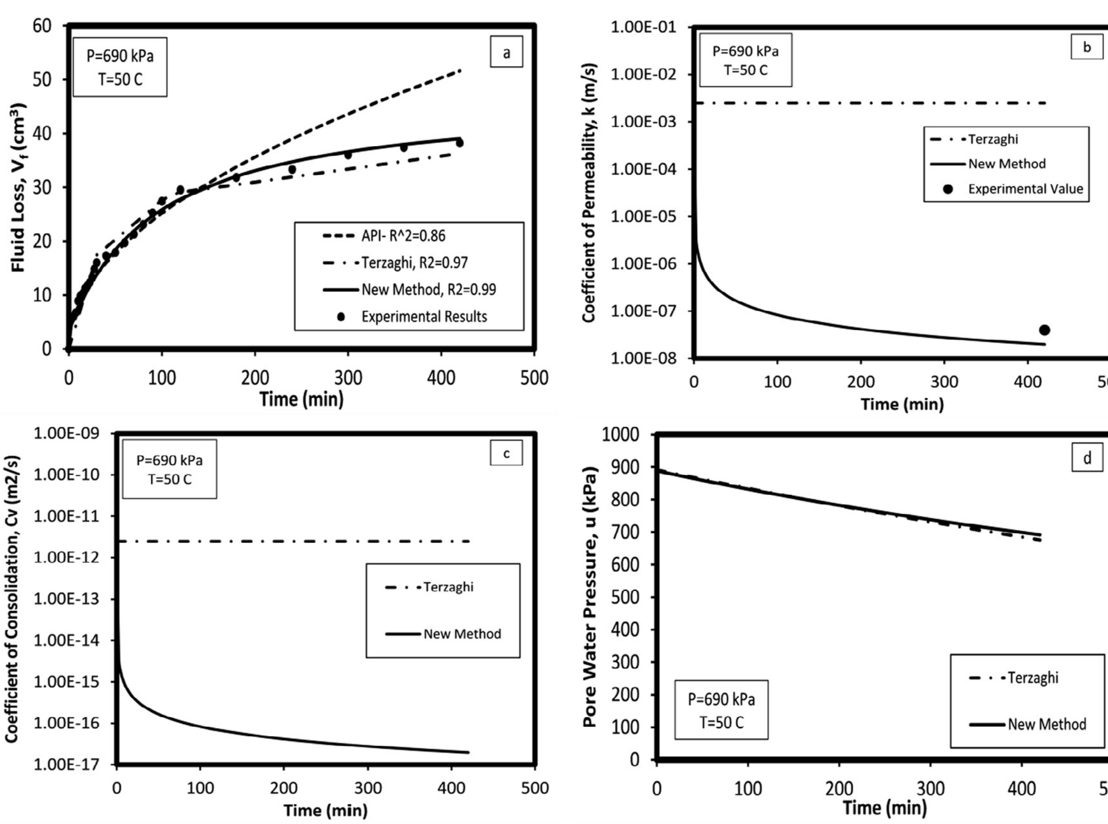

Fig. 9. Long-term model predictions of current study on 8% bentonite drilling mud at 50 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

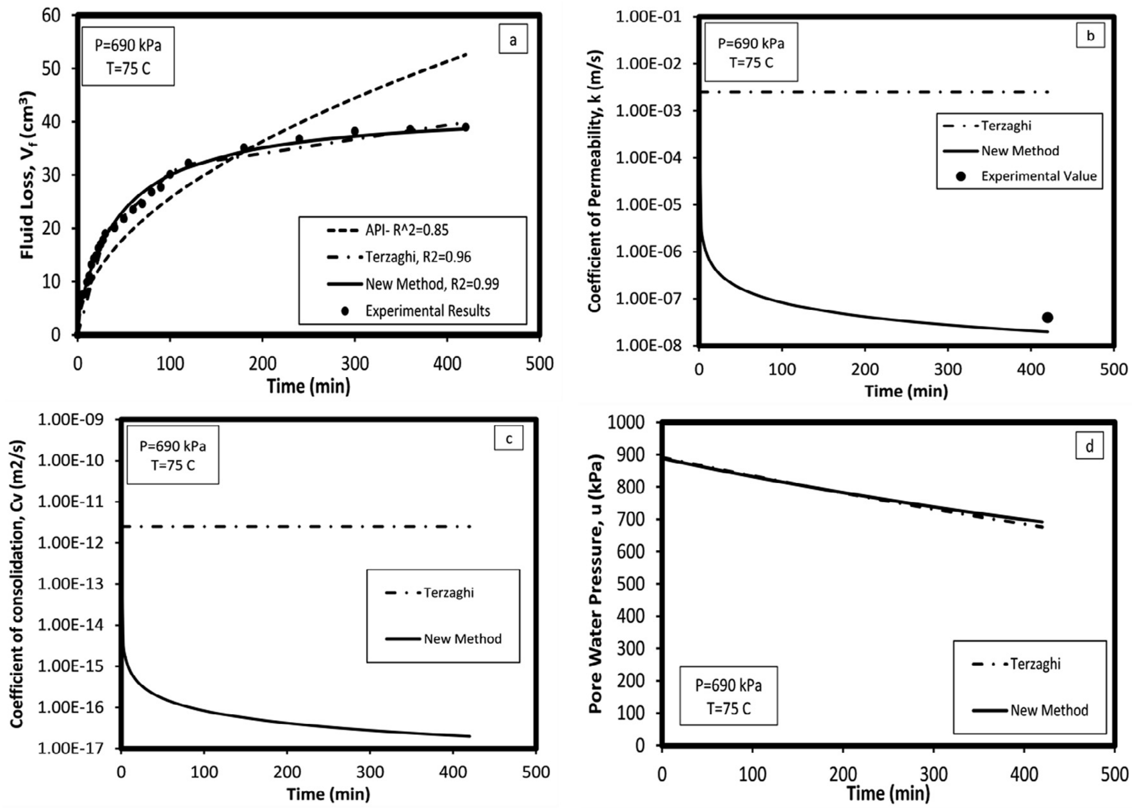

Fig. 10. Long-term model predictions of current study on 8% bentonite drilling mud at 75 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

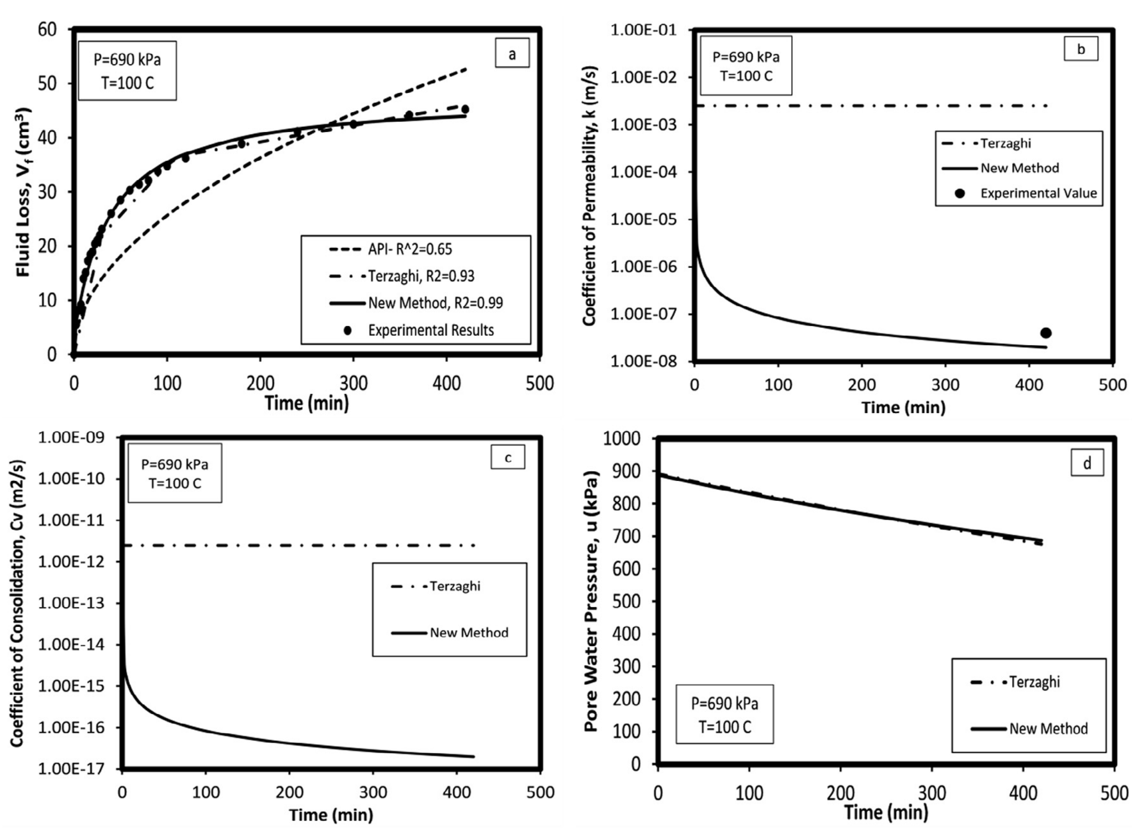

Fig. 11. Long-term model predictions of current study on 8% bentonite drilling mud at 100 °C (a) fluid loss versus time, (b) variation of permeability with time, (c) variation of coefficient of consolidation with time, and (d) variation of pore water pressure with time.

Table 2
Fluid loss prediction for 8% bentonite drilling mud using three different models.

Drilling Mud (%)	Temp. C	API Model		Terzaghi Model		Method 1	
		R^2	RMSE (cm^3)	R^2	RMSE (cm^3)	R^2	RMSE (cm^3)
8	25	0.94	1.820	0.95	2.005	0.99	0.828
8	50	0.86	3.958	0.97	1.935	0.99	1.096
8	75	0.85	4.717	0.96	2.139	0.99	0.932
8	100	0.65	7.266	0.93	3.201	0.99	0.803

coefficients of correlation were 0.97 and 0.86 respectively. In Fig. 9b, the final permeability value of the filter cake has been evaluated using falling head test. It is clearly indicated that the coefficient of permeability decreased by 255,523 times over 420 min in method 1 approximately reaching the final experimental value while the coefficient of permeability in Terzaghi method was constant. Similarly, in Fig. 9c, the coefficient of consolidation was constant over the time in Terzaghi method while the coefficient of consolidation decreased by 255,523 times over 420 min in method 1. In Fig. 9d, the pore water pressure decreased by 24% and 22% over 420 min in Terzaghi and method 1 respectively.

In Fig. 10a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 where Terzaghi and API coefficients of correlation were 0.96 and 0.85 receptively. In Fig. 10b, the final permeability value of the filter cake has been quantified using falling head test. It is clearly shown that the coefficient of permeability was constant over the time in Terzaghi method while the coefficient of permeability decreased by 235,740 times over 420 min in method 1 approaching the final measured experimental value. Similarly, in Fig. 10c, the coefficient of consolidation decreased by 235,740 times over 420 min in method 1 while the coefficient of consolidation was constant over the time in Terzaghi method. In Fig. 10d, the pore water pressure decreased by 24% and 22% over 420 min in Terzaghi and method 1 respectively.

In Fig. 11a, method 1 predicted the fluid loss very well with a coefficient of correlation (R^2) of 0.99 while Terzaghi and API coefficients of correlation were 0.93 and 0.65 respectively. In Fig. 11b, the final permeability value of the filter cake has been computed using falling head test. It is visible that the coefficient of permeability decreased by 202,999 times over 420 min in method 1 approximately matches the final experimental value while the coefficient of permeability was constant over the time in Terzaghi method. Similarly, in Fig. 11c, the coefficient of consolidation was constant over the time in Terzaghi method while the coefficient of consolidation decreased by 202,999 times over 420 min in method 1. In Fig. 11d, the pore water pressure decreased by 24% and 23% over 420 min in Terzaghi and method 1 respectively. All model predictions for 8% bentonite drilling mud at different tested temperatures 25 °C, 50 °C, 75 °C, and 100 °C are summarized in Table 2. It is clearly shown that the method 1 is the most accurate model compared to API and Tezaghi models where method 1 had the highest R^2 and lowest RMSE compared to others.

6. Conclusions

Based on the results of this study, the following conclusions can be advanced:

1. A new method for pore water pressure, coupling solution, was provided to solve consolidation equation. This method assumed the permeability and coefficient of consolidation of the filter cake to be a time dependent function.
2. The new method of consolidation solution was compared with Terzaghi solution and API- method. The new method and Terzaghi solution were used to predict the variations of the fluid loss, cake permeability, coefficient of consolidation of the cake, and the pore water pressure with the time. The new method incorporated better prediction for the experimental results than Terzaghi consolidation solution and API-model.
3. The coefficient of permeability for 2% bentonite drilling mud at 25 °C was constant over the time using Terzaghi method while the coefficient of permeability decreased by 126,000 times over 420 min in the new method 1.
4. The pore water pressure for 2% bentonite drilling mud at 25 °C decreased by 24% and 26% over 420 min using Terzaghi and method 1 respectively.
5. The coefficient of permeability for 8% bentonite drilling mud at 25 °C was constant over the time using Terzaghi method while the coefficient of permeability decreased by 365,396 times over 420 min in the new method 1.
6. The pore water pressure for 8% bentonite drilling mud at 25 °C decreased by 24% and 23% over 420 min using Terzaghi and method 1 respectively.

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