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Investment and financing for SMEs with bank-tax interaction and public-private partnerships☆

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ABSTRACT

In this paper, we consider a small- and micro-sized enterprise (SME) with assets-in-place invests a project. Especially, we assume that the fund needed by the investment are shared by the government's direct investment through public-private partnerships (PPPs) and the bank-tax interaction (BTI), which is a financial innovation instrument to solve the financing problem encountered by the SMEs. We address the pricing of corporate securities and optimal investment strategies. By providing a comparison with the pure private lending, we discover that the BTI makes the investment much cheaper and more attractive, and therefore mitigates the problem of under-investment in unstable markets. In addition, the BTI can increase the total firm value, reduce the leverage ratio and bankruptcy probability. The credit line of the BTI contract increases with the factors that influence the loan structure of the SME, such as the volatility, the tax rate, the investment cost sharing and the credit multiple. Most importantly, there exists an optimal investment cost sharing, such that the agency cost between SMEs and the government reaches zero due to the direct investment through PPPs.

1. Introduction

The financing problem encountered by the small- and micro-sized enterprises (SMEs, henceforth) is a very general and serious obstacle in present development in China. Compared to the mature international capital market, the Chinese market is still far from perfect, especially the guarantee and mortgage mechanism. Thus, a large number of SMEs are difficult or even impossible to obtain loans directly from banks due to their limited wealth, lack of collaterals, and other institutional frictions such as information asymmetry and agency costs. To solve this problem, China Banking Regulatory Commission and State Administration of Taxation jointly developed a new financial instrument in 2015, called the bank-tax-interaction (BTI, henceforth) in English.[1] The BTI is a three-party contract among a bank/lender, the tax department, and a SME/borrower. In the contract, the bank lends at a given interest rate to the SME mainly based on its historical tax payments provided by the tax department. Specially, the credit line is a certain multiple of the total tax payments of

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[1] The China Banking Regulatory Commission and the State Administration of Taxation jointly issued the notice on "Launching bank-tax-interaction to Boost SMEs Development Activities" on July 2015.

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the SME.[2] On the one hand, through the credit rating of the tax department, the asymmetric information between the SMEs and bank can be mitigated. Banks can make loans without collateral, which can be regarded as a solution to the financing problem of the SMEs. On the other hand, in reality, many enterprises evade taxes by various means for their own interests, and the BTI can encourage enterprises to pay tax in good faith and bring tangible benefits to those enterprises. In China, according to the statistics from the State Administration of Taxation up to the end of 2017, the national banking financial institutions have accumulatively issued a total of 632000 loans to trustworthy enterprises, including 504000 loans to SMEs. The amount of loans used by SMEs with the help of BTI has reached more than 710 billion RMB.[3]

For another, the government may address immediate needs considering economic growth, and thus provide direct subsidy for SMEs to stimulate investment, see Soumar (2016). In this paper, we consider a SME with asset-in-place invests a project. A part of the investment costs can be obtained from the government by the form of direct investment, in public-private partnerships (PPPs, henceforth). With direct investment, the government participates directly in the project by investing an amount of capital in return for taxes, thereby, sharing the project. The remaining costs needed by the project are financed by entering a bank-tax-interaction contract. If the credit line of the contract can not cover this part of costs, the SME utilizes the private lending to cover the funding gap. Due to the high financing cost of private lending, the SME raises the investment costs firstly by BTI. To the best of our knowledge, there are no papers to provide a quantitative research for BTI, let alone consider the impact of BTI on the investment and financing for SMEs with government investment subsidy. We want to know the effects of BTI on the investment decision, firm value, leverage and bankruptcy probability compared to the private lending? What factors may affect the loan structure(i.e. pure BTI or a mixture between BTI and private lending)? Is there an optimal investment cost sharing that eliminates the agency cost between the SME and government? In this paper, we aim to answer these questions.

Thus, in this paper, we develop a model to study the investment and financing for SMEs with BTI and government investment subsidy in a real options framework. We provide the closed-form solutions for the values of firm's securities, growth option and credit line. By providing a comparison with the private lending, we demonstrate that the BTI delays (accelerates) investment when the volatility of cash flow is low (high). This is due to the high financing cost of private lending. The BTI makes the investment much cheaper and more attractive, and therefore mitigates the under-investment in unstable markets. Furthermore, the BTI can increase the firm value and reduce the leverage ratio and bankruptcy probability, which could provide another explanation for why the BTI is so popular in China. The loan structure is affected by the factors such as the volatility, the tax rate, the investment cost sharing and the credit multiple. To be specific, the credit line of the BTI contract increases with these parameters. Most importantly, the agency cost due to the government's direct investment reaches at the minimum value zero if the cost sharing is designed optimally, which provides an insightful guide for the government when investing in a project through PPPs.

The structure of this paper is as follows. Section 2 provides a comprehensive overview of related literature. Section 3 develops a basic framework. Section 4 presents the pricing of corporate securities and the credit line of a BTI contract. Section 5 discusses two different investment strategies and gives a measure of agency costs between SMEs and the government. Section 6 provides numerical analysis and the economic implications. Section 7 concludes.

2. Related literature

Our research is related with the literature regarding the real option and capital structure. Brennan and Schwartz (1985) utilize option pricing theory to fix the value and optimal production policies of a natural resource investment. After that, real options theory which captures the value of flexibility under uncertainty has attracted growing attention following McDonald and Siegel (1986). Leland (1994) studies the corporate debt value and capital structure in a unified analytical framework. A large number of literature focus on the interaction between the financial structure and investment strategy. On the one hand, Myers (1977) argues that when a firm has outstanding risky debt, equityholders have an incentive to underinvest in future growth options. On the other hand, Jensen and Meckling (1976) argue that in some situations, equityholders have an incentive to overinvest in growth options. Thus, an important question is that how to measure and mitigate the conflicts over investment policies. Mauer and Sarkar (2005) examine the impact of a stockholder-bondholder conflict over the optimal investment policy for a project and provide a measure of this agency cost. Hackbarth and Mauer (2011) study the interaction between financing and investment decisions with multiple debt issues, and discover that jointly optimal capital and priority structures can virtually eliminate the agency cost of debt pointed out in Mauer and Sarkar (2005). Hirth and Uhrig-Homburg (2010) explore the impact of debt and liquidity on firm's investment, and then discover that there is an underinvestment for the firm with a high leverage and low liquidity. Lyandres and Zhdanov (2014) show that the convertible debt can alleviate and potentially totally eliminate the underinvestment problem because of the dilute effects of convertible debt. Sarkar and Zhang (2015) consider a investment model under performance-sensitive debt (PSD) financing, and then find that PSD can also mitigate and even completely eliminate the agency conflicts due to its risk-compensating nature. Sundaresan, Wang, and Yang (2015) investigate the investment problem for a firm with a growth option, which is financed with equity and straight debt. They get that dynamic investment and financing decisions lead to conservative leverage, thereby mitigating the underinvestment problem. Song and Yang (2016) incorporate contingent convertible bonds (CoCos) into the model of Mauer and Sarkar (2005), and examine how CoCos as a debt financing instrument affects a firm's investment policy, agency cost of debt, and capital structure. Luo and Yang (2017) extend the above model by taking into consideration the macroeconomic risk. They both find that there exists a unique CoCos' conversion ratio such that

[2] Data source: https://www.henan100.com/finance/2019/845477.shtml.

[3] Data source: https://baijiahao.baidu.com/s?id=1597502227581211831&wfr=spider&for=pc.

the agency cost between shareholders and bondholders arrives at the minimum value zero. Unlike the above literature, due to the fact that the government provides cost subsidy here in public-private partnerships in order to stimulate the firm investment for more tax, we focus on the agency conflicts between the entrepreneur and the government in this paper.

Also, our work is linked to the research on the investment and financing problem faced by the SMEs. To solve the financing constraints for SME, a variety of guarantee derivative instruments have appeared in China. For example, Yang and Zhang (2013), Wang, Yang, and Zhang (2015) and Luo, Wang, and Yang (2016) discuss the equity-for-guarantee swap; Yang and Zhang (2015) explore the option-for-guarantee swap; Gan, Luo, and Yang (2016) and Luo and Yang (2019) study the fee-for-guarantee swap. In the above literature, they develop structural models using the real-option approach to study the impacts of those financial instruments on the SMEs' investment and financing policies. However, here we consider a new financing tool for SMEs, called bank-tax-interaction. We try to conduct a quantitative study for this product to investigate the investment and financing problems of the SME who enters into a such contract. Lastly, this paper is also related to the research on the private-public partnership. Takashima, Yagi, and Takamori (2010) use a real option framework to study the interaction between a private firm and a government while in a PPP. Soumar (2016) compares two forms of government support: loan guarantee and direct investment through PPPs. They show that how the portion of shares given to the government can be a bargaining tool and mitigate information asymmetry when structuring PPPs.

3. Model setup

Following Goldstein, Ju, and Leland (2001), we assume a firm with assets in place generates the continuously instantaneous earnings X before interest and taxes (EBIT) driven by the following geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dZ_t, \quad X_0 = x > 0 \quad given. \tag{1}$$

where μ and $\sigma > 0$ are the constant drift and volatility of the cash flow, and especially $\mu < r$,[4] where r is the risk-free interest rate. Z is a standard Brownian motion defined on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ equipped with a filtration $\mathbb{F} = \{\mathscr{F}_t : 0 \le t < \infty\}$ satisfying the usual conditions. The filtration $\mathbb{F}$ describes the flow of information available to investors. The firm pays corporate income taxes at a constant tax rate, $\tau \in (0, 1)$.

Following Mauer and Ott (2000) and Hackbarth and Mauer (2011), we further assume that the firm has a growth option to expand by paying a fixed sunk cost I. Immediately upon exercise, the firm's cash flow increases immediately from X to QX, where $Q > 1$ is a constant, called growth ratio by us. Time is continuous, and indexed by $t \in [0, \infty)$.

Before investment, we assume the total initial assets in the firm is equity. In the spirit of Takashima et al. (2010) and Soumar (2016), the investment cost I is shared by the government and the private lending in the ratio of $1 - \varphi$ and φ respectively. Although the government is not motivated solely by the monetary benefits accruing from the project, the fees, which is a kind of tax, constitute a fair return for the government investment of $(1 - \varphi)I$. Especially, the fraction of the total cost φI can be financed through BTI or private lending. The proportion φ is an important parameter we will discuss in the following. If the part of investment cost φI is less than or equal to the credit line of the bank to the SME, the SME will only need obtain this part of cost through BTI. Otherwise, the SME utilizes the private lending to finance the funding gap with a high financing cost (the unit cost of private lending is h). Apparently, the investment decisions are influenced by the financing policies. For tractability reasons, we assume the bond has infinite maturity and requires a continuous coupon payment c per unit of time, until default.[5]

4. The pricing of corporate securities and credit line

Using the backward induction, we first price all the contingent claims after the growth option has been exercised and then turn to that prior investment. The superscripts a and b indicate the value of the claim after and before the exercising of the growth option, respectively.

After the option to invest has been exercised, we see that there are three kinds of claimants in all: shareholders, debtholders and the government. To price these corporate securities, we must specify the cash flows generated by them in advance.

To begin with, the value $E^a(x)$ of equityholders after investment, which yields a cash flow of $(1 - \tau)(Qx - c)$, is given by:

$$E^a(x) = \mathbb{E}\left[\int_t^{\tau_b} (1 - \tau)(Qx_s - c)ds | \mathscr{F}_t\right], \tag{2}$$

where $\mathbb{E}$ denotes the expectation operator and $\tau_c = inf\{t > 0 | x_t \le x_c\}$ is the optimal bankruptcy time and x_c is the bankruptcy threshold. Should the firm go default on its debt, the value of the firm's future cash flow is assigned to the debtholders and the equityholders' value reduces to zero, but a fraction, denoted by α, of the value of the future cash flow will lost due to bankruptcy costs, which is constant and called bankruptcy loss rate in the literature.

Noting that at the moment of bankruptcy, debtholders obtain all the liquidation value of the firm net of bankruptcy costs, which

[4] The assumption $r > \mu$ guarantees that the firm's value is finite, see Hackbarth and Mauer (2011) and Shibata and Nishihara (2015) for more details.

[5] Leland (1994) provides economic implications for the assumption. And this assumption is cited by a large number of documents, e.g. Goldstein et al. (2001), Mauer and Sarkar (2005) and Shibata and Nishihara (2015) among many others.

equals $(1-\alpha)(1-\tau)\frac{Qx_c}{r-\mu}$. If the firm is solvent, i.e. the current cash flow level x satisfies $x \geq x_c$, the value $D^a(x)$ of the issued debt after expansion is given by:

$$D^a(x) = \mathbb{E}\left[\int_t^{\tau_c} e^{-r(s-t)}cds + e^{-r(\tau_c-t)}(1-\alpha)(1-\tau)\frac{Qx_c}{r-\mu}\Big|\mathscr{F}_t\right], \tag{3}$$

In equation (3), the first term denotes the present value of the coupon payments and the second term represents the residual value at liquidation. The total firm value $V^a(x)$ after investment is the sum of equity and debt values, i.e. $V^a(x) = E^a(x) + D^a(x)$.

Lastly, the tax value $G^a(x)$ of the government after investment can be expressed as:

$$G^a(x) = \mathbb{E}\left[\int_t^{\tau_c} e^{-r(s-t)}\tau Qx_s ds + e^{-r(\tau_c-t)}\tau\frac{Qx_c}{r-\mu}\Big|\mathscr{F}_t\right], \tag{4}$$

Following the standard arguments (see Tan and Yang (2017)), the values of equity, debt and total firm after investment can be summarized by the following proposition.

Proposition 4.1. After investment, the equity value is given by:

$$E^a(x) = (1-\tau)\frac{Qx}{r-\mu} - (1-\tau)\frac{c}{r}\left[1-\left(\frac{x}{x_c}\right)^{\beta_-}\right] - (1-\tau)\frac{Qx_c}{r-\mu}\left(\frac{x}{x_c}\right)^{\beta_-}, \tag{5}$$

where $\beta_- = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$[6] is one of the solutions of the quadratic equation $\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0$. The debt value is:

$$D^a(x) = \frac{c}{r}\left[1-\left(\frac{x}{x_c}\right)^{\beta_-}\right] + (1-\alpha)(1-\tau)\frac{Qx_c}{r-\mu}\left(\frac{x}{x_c}\right)^{\beta_-}. \tag{6}$$

Thus, the total firm value is:

$$V^a(x) = (1-\tau)\frac{Qx}{r-\mu} + \frac{\tau c}{r}\left[1-\left(\frac{x}{x_c}\right)^{\beta_-}\right] - \alpha(1-\tau)\frac{Qx_c}{r-\mu}\left(\frac{x}{x_c}\right)^{\beta_-}, \tag{7}$$

and the tax value of government is

$$G^a(x) = \tau\frac{Qx}{r-\mu}. \tag{8}$$

Lastly, the optimal bankruptcy threshold x_c is[7]

$$x_c = \frac{\beta_-}{\beta_- - 1}\frac{r-\mu}{Q}\frac{c}{r}. \tag{9}$$

The implications for this proposition are as follows. Firstly, the first term on the right-hand side of Equation (5) represents the value of unlevered firm, and the last two terms are the costs due to the coupon payments and bankruptcy respectively. Secondly, Equation (6) shows that the debt value consists of the coupon payments before default and the residual firm value at default time. Thirdly, the first term on the right-hand side of Equation (7) is the unlevered firm value, the middle term is the expected present value of tax shields of debt and the last term is the expected present value of bankruptcy cost. Fourthly, Equation (8) shows that the tax value of government increases with the firm's cash flows. That is why the government is willing to provide investment subsidy for SMEs through PPPs in our model. Finally, from Equation (9), we can see that the default threshold x_c is independent of the current level of cash flow x due to the time-consistent structure of our model. Apparently, the default threshold decreases with growth scale, and increases with coupon payments.

The credit line. The credit line determined by the BTI contract is the key problem in the model. According to the BTI contracts in the

[6] Fan and Sundaresan (2000) provide an novel economic implication for the parameter β_-. They refer to it as the elasticity of the bankruptcy probability, $\left(\frac{x}{x_c}\right)^{\beta_-}$, with respect to the cash flow x, i.e. $\left(\partial\left(\frac{x}{x_c}\right)^{\beta_-}/\partial x\right)\left(x/\left(\frac{x}{x_c}\right)^{\beta_-}\right) = \beta_-$. The other solution is $\beta_+ = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1$, which will be used in the following. Similarly, the parameter β_+ can be interpreted as the elasticity of the investment probability, $\left(\frac{x}{x_i}\right)^{\beta_+}$, with respect to the cash flow x.

[7] Leland (1994) points out that the optimal default threshold is derived by smooth-pasting condition, i.e $\frac{\partial E^a(x)}{\partial x}|_{x=x_c} = 0$. In addition, the choice of default threshold can also be shown to differentiate equity value with respect to x_c, i.e. $\frac{\partial E^a(x)}{\partial x_c} = 0$.

market, the credit line, denoted by CL, is given by a linear continuous function of the total tax payments of the SME from the initial time to the investment time:

$$CL = \kappa H(x_i), \tag{10}$$

where κ can be viewed as a credit multiplier of the bank[8] and x_i is the optimal investment threshold that will be discussed in next section. $H(x_i)$, which is the total tax amount of the SME paid up until the point of investment, is given by:

$$\begin{aligned} H(x_i) &= \mathbb{E}\Big[\int_0^{\tau_i} e^{r(\tau_i - s)} \tau x_s ds \Big| \mathscr{F}_0\Big] \\ &= \tau \frac{x_0}{r-\mu}\left(\frac{x_i}{x_0}\right)^{\beta_+} - \tau \frac{x_i}{r-\mu}. \end{aligned} \tag{11}$$

Equation (10) implies that the larger the κ or the more tax paid by the SME, the more credit line the SME can obtain through the BTI.

As said before, SME obtains the part of investment cost φI by debt financing through BTI or private lending, so the value of issued debt should equal to this amount of cost, that is to say:

$$1_{\{CL \ge \varphi I\}} \varphi I + 1_{\{CL < \varphi I\}} [\varphi I + h(\varphi I - CL)] = D^a(x_i, c), \tag{12}$$

where h is the financing cost of the private lending. Both $1_{\{CL \ge \varphi I\}}$ and $1_{\{CL < \varphi I\}}$ are indicator functions that are equal to one when $CL \ge \varphi I$ and $CL < \varphi I$, respectively. Equation (12) states that the SME obtains the investment cost φI through the BTI if $CL \ge \varphi I$, on the contrary, the SME obtains the cost φI through both the BTI and private lending.

5. The investment policy and agency costs between the SME and government

We now turn to the optimal investment behavior of the firm. Especially, due to the fact that the government invests the amount of $(1-\varphi)I$ in the firm, we examine two different investment policies to maximize the equity value and the total social value, respectively. If the growth option is not exercised, to price the securities we must further specify the investment threshold, denoted by x_i, which is independent of time due to the time homogeneity in our model. For any given investment threshold x_i, prior to investment, the firm has only shareholders who receive the cash flow $(1-\tau)x$, and at the investment time, they get nothing more but a claim, of which the value equals $E^a(x_i) - (1_{\{CL \ge \varphi I\}} \varphi I + 1_{\{CL < \varphi I\}} [\varphi I + h(\varphi I - CL)] - D^a(x_i)) = E^a(x_i)$. For this reason, if the current cash flow level rate $x < x_i$, it implies that the value $E^b(x)$ of equity (which is also the firm value before investment) can be expressed by:

$$E^b(x) = \mathbb{E}\left[\int_t^{\tau_i} e^{-r(s-t)}(1-\tau)x_s ds + e^{-r(\tau_i - t)} E^a(x_i) | \mathscr{F}_t\right], \tag{13}$$

where the optimal stopping time $\tau_i = inf\{t > 0 | x_t \ge x_i\}$. Following the standard arguments, the equity value is given by:

$$E^b(x) = (1-\tau)\frac{x}{r-\mu} + \left[E^a(x_i) - (1-\tau)\frac{x_i}{r-\mu}\right]\left(\frac{x}{x_i}\right)^{\beta_+}, \tag{14}$$

where β_+ is given above in footnote 6 and $\left(\frac{x}{x_i}\right)^{\beta_+}$ can be interpreted as the present value of a dollar paid at investment time. Equation (14) implies that the equity value before investment equals the unlevered firm value plus the option value.

So, the investment threshold x_i is determined by solving the following optimization problem:

$$\max_{x_i} E^b(x; x_i), \tag{15}$$

For comparison, we consider the investment policy that maximizes the total social value which is given by the sum of the equity value and the government value before investment. Since the government invests $(1-\varphi)I$ in the firm as said before, the value $G^b(x)$ of the government before investment is expressed as:

$$G^b(x) = \mathbb{E}\left[\int_t^{\tau_i} e^{-r(s-t)} \tau x_s ds + e^{-r(\tau_i - t)}(G^a(x_i) - (1-\varphi)I) \Big| \mathscr{F}_t\right], \tag{16}$$

that is,

$$G^b(x) = \tau \frac{x}{r-\mu} + \left[\tau \frac{(Q-1)x_i}{r-\mu} - (1-\varphi)I\right]\left(\frac{x}{x_i}\right)^{\beta_+}, \tag{17}$$

[8] In the product "tax-easy-loans" issued by the China Construction Bank, the credit line is 3–10 times larger than the total tax payments of the SME.

where the first term denotes the tax payments by the unlevered firm before investment, and the second term is the present value of net profit of government at investment.

Using a similar procedure, we derive that the investment threshold x_i^f that maximizes the total social value $S(x) = E^b(x) + G^b(x)$ is determined by:

$$\max_{x_i^f} S(x). \tag{18}$$

Clearly, the investment threshold given by (15) for maximizing the equity value is different from that by (18) for maximizing the total social value. In the spirit of Mauer and Sarkar (2005) and Egami (2009), the agency cost between the SME and government can be defined as the difference in values between two investment policies: total social value versus equity value maximization, which is given by:

$$AC = S\left(x_i^f\right) - S(x_i). \tag{19}$$

Naturally, the agency cost due to the investment of government through PPP is always greater than or equal to zero. Intuitively, this cost depends on the investment cost sharing φ: A large (small) cost sharing may induce under-investment (over-investment) if other parameters are constant. Accordingly, we guess there is an optimal cost sharing φ such that the agency cost reaches the minimum value zero. We will prove this point in next section.

6. Model implications

Baseline parameters. In this section, we provide the quantitative analysis about the impacts of BTI on firm's financial policies and the agency cost between SME and the government. For the base case, we choose the following plausible parameter values to highlight the model's mechanism and main insights. Following Luo et al. (2016), we take the widely used value for the risk-free interest $r = 0.05$; the drift and volatility of the cash flow $\mu_0 = 0.01$ and $\sigma_0 = 0.3$ respectively; the bankruptcy cost $\alpha = 0.35$ and, the effective tax rate $\tau = 0.2$. As for the private loan cost, we set it at $h = 0.2$ since the average financing cost via private lending is about 20%~30% in China.[9] In the product tax-easy-loans issued by the bank, the credit line is about 3~10 times larger than the total tax payments of the SME[10], so we choose the credit multiple $\kappa = 3.5$. Finally, we set the ratio of the investment cost sharing at $\varphi = 0.7$, the growth scale at $Q = 2.5$, the investment cost at $I = 20$ and the current cash flow at $x_0 = 0.5$.

6.1. The effects of BTI

In this subsection, we focus on the effects of BTI on investment threshold, firm value, leverage ratio and bankruptcy probability respectively by comparison with pure private lending in order to highlight the advantage of BTI.

Compared to the pure private lending, the BTI delays (accelerates) investment when the volatility of the cash flow is low (high), as seen in Fig. 1(a). The intuition behind the results is as follows. Due to the high financing cost of private lending, in stable (or low-volatility) markets, the firm is willing to postpone investment in order to increase the credit line for BTI. However, when the risk of the firm is relatively high, BIT makes the investment much cheaper and more attractive, and therefore hastens investment. That is to say, BTI can mitigate the problem of under-investment due to costly private lending in more volatile markets.

Fig. 1(b) indicates that compared to the pure private lending, BTI significantly increases the firm value. On the one hand, BTI helps eliminate the financing constraints for SME and thereby reduces the financing cost. On the other hand, the default risk can be reduced by the BTI, which can be seen from Fig. 1(d). This result provides an explanation for why the BTI is so popular for solving financing constraints of SMEs in China.[11]

Following Leland (1994) and Goldstein et al. (2001), we define the leverage ratio as $D(x)/(D(x) + E(x))$. Fig. 1(c) shows that the leverage ratio of the SME financed through BTI is lower relative to pure private lending. Intuitively, Equation (12) shows that in order to gain the remaining cost φI needed by the investment, the SME with BTI financing issues less debt relative to that with pure private lending because the latter causes more financing cost. To be more specific, the debt value $D(x)$ for BTI is less than that for the private lending, which also implies that the firm with BTI financing pays less coupon to the bank. Thus, BTI decreases the default threshold and default probability (see Fig. 1(d)), and thereby increases the equity value. Eventually, the leverage ratio is reduced by the BTI. This conclusion provides another explanation for the popularity of BTI in China since the Chinese government recently implements a series of policies to reduce the firm's high leverage ratios.

6.2. Debt structure

In this subsection, we consider the debt structure of SME via comparing the credit line with the investment cost undertaken by the

[9] Data source: https://news.smm.cn/news/3827016.
[10] Data source: https://www.henan100.com/finance/2019/845477.shtml.
[11] Particularly, the State Administration of Taxation and the China Banking Regulatory Commission jointly issued the "Notice on further promoting the work of bank-tax-interaction" on June 2, 2017, which aims at improving the BTI to better help SMEs' development.

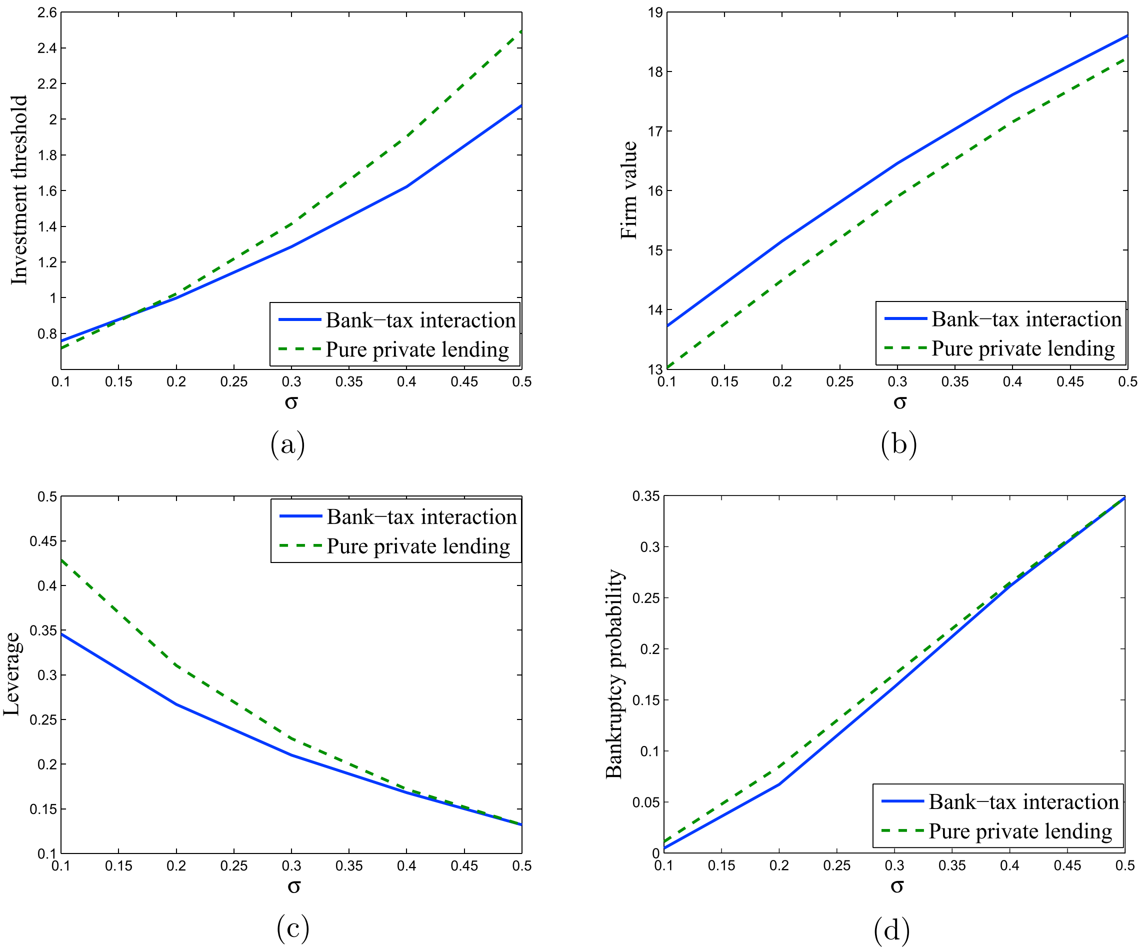

Fig. 1. The impact of volatility σ on (a) Investment threshold, (b) firm value, (c) leverage, and (d) bankruptcy probability.

SME. In our setting, the source of debt financing may consist of both the BTI and private lending when the credit line of BTI contract cannot fully cover the investment cost undertaken by the SME. In particular, we examine the effects of the volatility, tax rate, investment cost sharing and credit multiples on debt structures of the SME.

As mentioned before, after the direct investment we get from the government, the remaining cost required for the investment is $\varphi I = 14$ under the baseline parameter values, which can be seen in Fig. 2. Fig. 2(a) shows that the credit line increases with the volatility. The reasons are as follows. On the one hand, the investment threshold increases with the volatility of cash flows, as seen from Fig. 1(a). On the other hand, equation (10) shows that the credit line increases with total tax amount $H(x_i)$ of SME prior to investment. And meanwhile, it demonstrates that the total tax amount of SME prior to investment increases with investment threshold via differentiating $H(x_i)$ in equation (11) with respect to x_i.[12] Therefore, the larger the volatility of the cash flows, the higher the optimal investment threshold, the more the total amount of tax the firm paid, and thus the larger the credit line. Besides, the credit line is first lower, and then higher than φI, which means that the debt structure is a mixture of the BTI and private lending when the volatility is small, and pure BTI when the volatility is large. Fig. 2(b) shows that the credit line increases with the tax rate. Intuitively, according to the BTI contract, the larger the tax rate, the more the total tax payments of SMEs, and thus the more credit line from bank SMEs can get. Actually, the tax is always seen a cost for SMEs. This means that a higher tax rate reduces the profit of SMEs, and thus decreases the firm value. However, in our model, one benefit of the taxes is that the financing costs of SMEs can be significantly reduced through BTI compared to the pure private lending, as seen in Fig. 2(b). Thus at the same tax rate, Fig. 1(b) shows that the firm value with BTI financing is larger relative to the case with private lending.

A higher investment cost sharing implies that the SME should undertake more cost, and thus has more incentive to delay investment in order to increase the amount of tax payments and also the credit line of BTI contract, which can be seen from Fig. 2(c). Lastly, Fig. 2(d) demonstrates that the debt structure is a mixture of BTI and private lending for the lower level of credit multiples, and pure BTI for the high level of credit multiples. The credit multiples reflect the loan policy of the bank. Naturally, a high level of credit multiples means the

[12] Differentiating (11) with respect to x_i, we have $\frac{\partial H(x_i)}{\partial x_i} = \frac{\tau}{r-\mu}\left[\beta_+\left(\frac{x_i}{x_0}\right)^{\beta_+-1} - 1\right]$. Because of $\beta_+ > 1$ and $x_i > x_0$, hence, we derive that $\frac{\partial H(x_i)}{\partial x_i} > 0$.

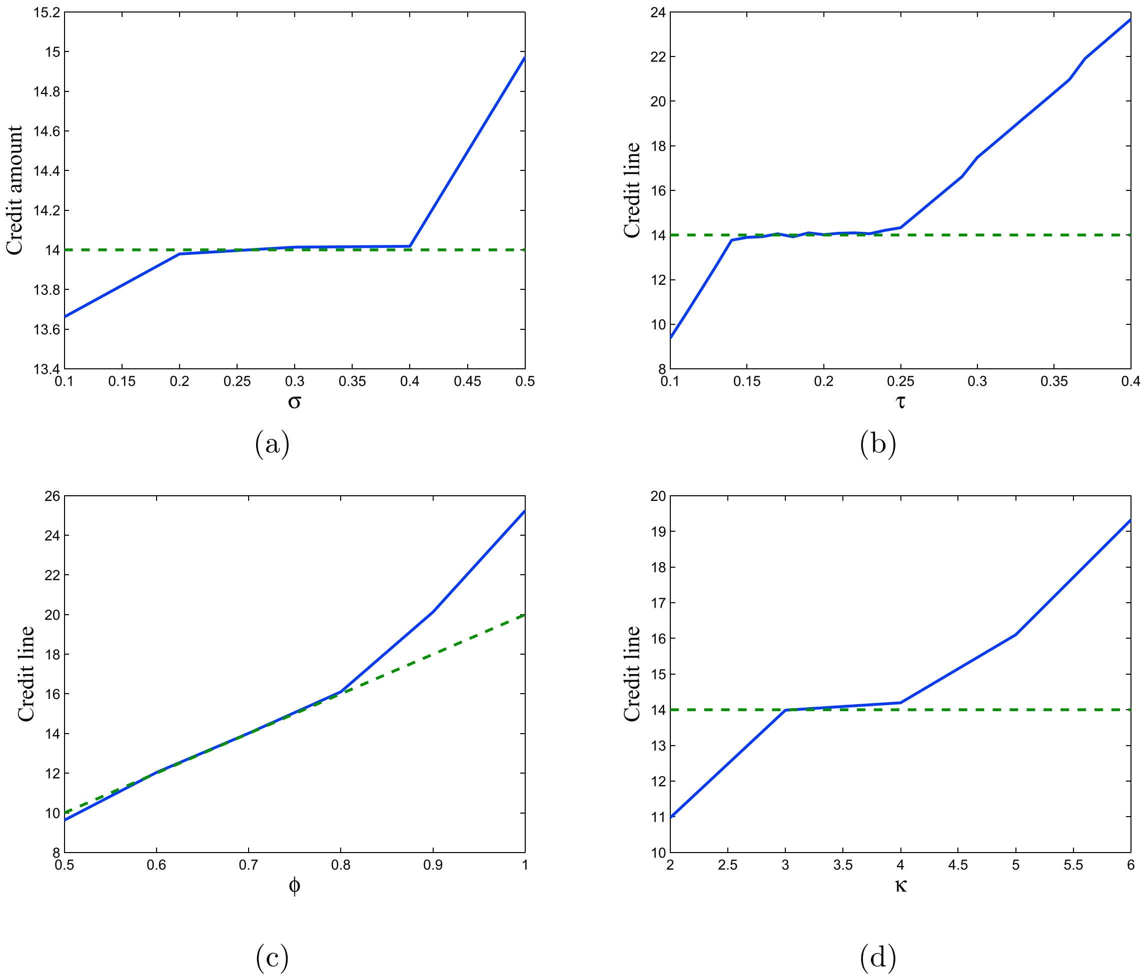

Fig. 2. The impact of (a) volatility σ, (b)tax rate τ, (c)investment cost sharing φ and (d) credit multiples κ on credit line. The solid lines plot the credit line, and the dashed lines plot the investment cost undertaken by SMEs.

easy monetary policy, so the SME can get a high credit line for financing. Otherwise, a low level of credit multiples implies the contractionary monetary policy, and thus the SME gets a low credit line.

6.3. Optimal cost sharing

In this subsection, we aim to explore the effects of the cost sharing by the PPP on the agency conflicts between the entrepreneur and the government.

As can be noted in Fig. 3(a), the two curves of investment thresholds under equity-value- and social-value-maximizing policies respectively intersect for some value of φ around 0.98. At the point of intersect, the under-or over-investment incentive is eliminated. This is the optimal investment cost sharing, since it ensures the entrepreneur will follow the social value maximizing investment policy, as a result the agency cost will be zero. Similarly, Fig. 3(b) indicates that under equity-value-maximizing policy, the total social value is a globally concave function of the cost sharing φ. On the contrary, under social-value-maximizing policy, it is a decreasing function of the cost sharing φ. Especially, with the optimal value of cost sharing, we get the same social value under both policies as seen in Fig. 3(b).

Furthermore, Fig. 4 depicts that the agency cost is a U-shape function of the investment cost sharing, which suggests that there is an interior optimal solution that minimizes the agency cost. If the government takes the optimal φ which is the crucial parameter, the agency costs due to the direct investment by the government reaches the minimum value zero as showing in Fig. 4. This result provides the government with an insightful guide when investing a project through PPP.

7. Conclusion

Recently, the BTI, as a new financial instrument, is getting more and more popular in China. However, there are no papers in the literature to quantitatively discuss the impacts of BTI on investment, debt structure and the agency costs between SMEs and the

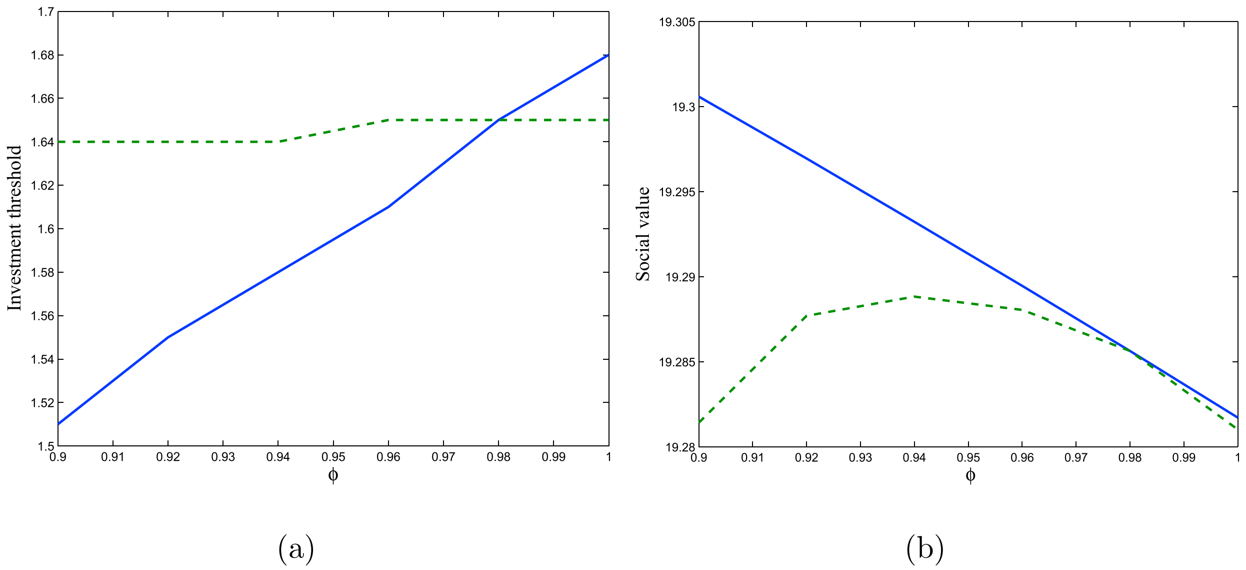

Fig. 3. The impact of investment cost sharing φ on (a) investment threshold and (b) social value. The solid line plots the total social value under social-value-maximizing policy, and the dashed line plots the total social value under equity-value-maximizing policy.

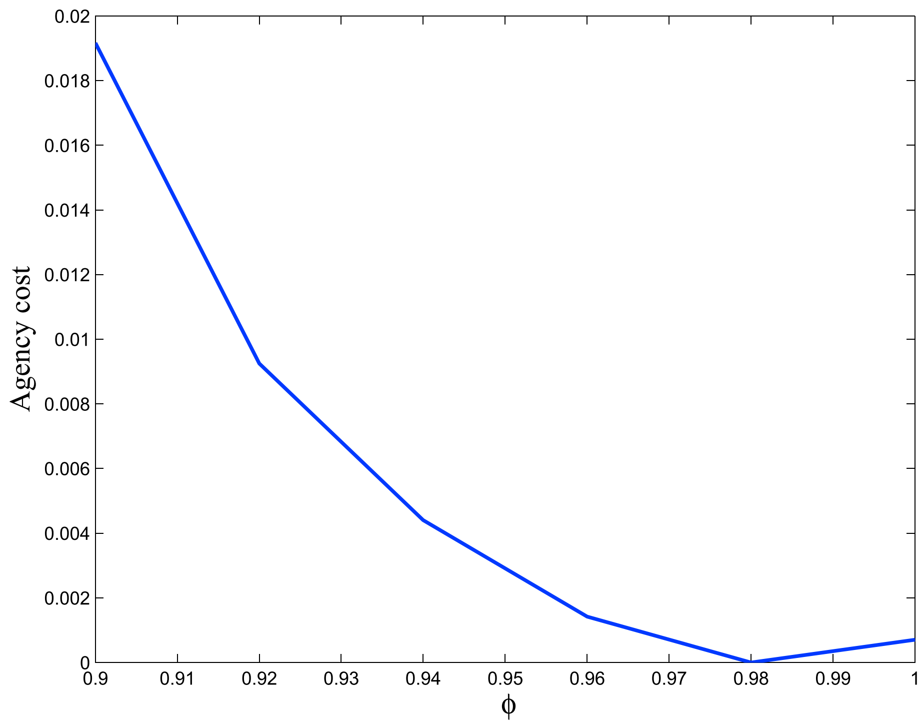

Fig. 4. The impact of investment cost sharing φ on agency costs.

government.

To make contributions along this research line, this paper builds the model to solve the investment and financing strategies for SMEs taking BTI and PPP into consideration. We find that BTI can mitigates the problem of underinvestment in unstable markets. In addition, compared to the pure private lending, BTI increases the firm value, reduces the leverage ratio and bankruptcy probability, which provides an explanation for why the BTI is popular in China. Then, we demonstrate that the loan structure of SMEs can be affected by the volatility, the tax rate, the investment cost sharing and the credit multiples. And most importantly, there exists a unique optimal investment cost sharing such that the agency cost arrives at the minimum value zero.

In practice, the cash flow of SMEs is always influenced by the jump shocks. There are two sources of jump shocks. First, the cash flow of SMEs is subject to the internal jump shock, and the current internal jump may increase the future jump risk. Second, the external jump shock affected by the SMEs may increase the internal jump risk. In other words, the jumps in the cash flow "self-excite" both in space (the external shocks) and in time (the internal future shocks). Fortunately, the Hawkes jump-diffusion can capture the above elements (Ait-Sahalia, Cacho-Diaz, and Laeven (2015)). Therefore, a promising direction for further research would to be introduce financial contagion and examine the impact of it on financial policies with BTI.

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