

Anomalous transverse transformation of astigmatic four-petal Gaussian beams in isotropic nonlocal nonlinear media

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ABSTRACT

In this paper, the evolution dynamics of astigmatic four-petal Gaussian beams in isotropic nonlocal nonlinear media is investigated. A series of analytical expressions are derived to describe the beam evolution, the beam width, and the critical power. The anomalous transverse transformation is presented, i.e., an astigmatic four-petal Gaussian beam is compressed in one transverse direction and it is broadened in the other transverse direction. These evolution characteristics of astigmatic four-petal Gaussian beams are illustrated through some numerical simulations.

Nonlinearity exists in nature widely, and optical solitons are typical nonlinear phenomena in optics [1–5]. Recently, nonlocal nonlinear media [6,7] have attracted much attention, because of the discovery of many new novel nonlocal solitons [8–13]. The evolution of optical beams in nonlocal nonlinear media also indicates many novel properties [14–16]. In this paper, the evolution dynamics of astigmatic four-petal Gaussian beams (AFPGBs) in strongly isotropic nonlocal nonlinear (SINN) media is investigated. The anomalous transverse transformation of AFPGBs is illustrated.

The evolution dynamics of AFPGBs in nonlinear media is governed by the nonlinear Schrödinger equation (NLSE). For the beam evolution in SINN media, the NLSE is reduced into a simplified form [7,8,13]

$$2ik \frac{\partial \Psi}{\partial z} + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi - k^2 \gamma^2 P_0 (x^2 + y^2) \Psi = 0, \quad (1)$$

where k represents the wave number, γ is a material constant, z indicates the direction of beam propagation, and $P_0 = \iint |\Psi|^2 dx dy$ denotes the input power of AFPGBs.

The electric field of an AFPGBs in rectangular coordinates at the input plane ($z = 0$) is expressed as [17,18]

$$\Psi(x, y, 0) = C_0 \left(\frac{x}{w_{0x}} \frac{y}{w_{0y}} \right)^{2n} \exp \left(-\frac{x^2}{w_{0x}^2} - \frac{y^2}{w_{0y}^2} \right), \quad (2)$$

where $C_0 = 4^n \sqrt{(2P_0)/(w_{0x} w_{0y})} / \Gamma(2n + 1/2)$, $\Gamma(\cdot)$ denotes the Euler gamma function, $n = 0, 1, 2, 3 \dots$ denotes the order of the beam, w_{0x} and w_{0y} denote the size of the beam spot in x and y directions, respectively. In this paper, we take $\eta_0 = w_{0y}/w_{0x}$ to describe the asymmetric degree of AFPGBs at the input plane.

Resorting to the similar method in our previous work [13,18], one can obtain the evolution expression of AFPGBs,

$$\Psi(x, y, z) = \frac{iC_0 k}{2\pi \mathcal{L}_p \sin \xi} \psi(x) \psi(y), \quad (3)$$

where

$$\psi(j) = 2w_{0j} \left(\frac{2\mathcal{L}_p}{ikw_{0j}^2 + 2\mathcal{L}_p} \right)^n \exp \left[-\frac{ik \cos^2 \xi j^2}{2\mathcal{L}_p \sin \xi} \right] \times \left[\frac{{}_1F_1 \left(n + \frac{1}{2}, \frac{1}{2}, \beta \right) \Gamma \left(n + \frac{1}{2} \right) \sqrt{\mathcal{L}_p \sin \xi}}{\sqrt{4\mathcal{L}_p \sin \xi + 2ikw_{0j}^2 \cos \xi}} \right. \\ \left. - \frac{{}_1F_1 \left(n + 1, \frac{3}{2}, \beta \right) \Gamma(n + 1)j}{w_{0j} \cos \xi - 2i\mathcal{L}_p \sin \xi} \right], \quad (4)$$

$$\beta = -\frac{k^2 w_{0j}^2 j^2}{2ikw_{0j}^2 \mathcal{L}_p \cos \xi \sin \xi + 4\mathcal{L}_p^2 \sin^2 \xi}, \quad (5)$$

$j = x$ or y , $\mathcal{L}_p = 1/\sqrt{\gamma^2 P_0}$, $\xi = z/\mathcal{L}_{p,1}$, $F_1(a; b; z)$ denotes the Kummer confluent hypergeometric function. In order to show the good gross information on AFPGBs during propagation, we employ the second-order moment to describe the beam width of AFPGBs. Thus, the beam width in x or y direction is defined as $w_j(z) = [4 \iint j^2 |\Psi(x, y, z)|^2 dx dy / \iint |\Psi(x, y, z)|^2 dx dy]^{1/2}$. Then one can obtain

$$w_x(z) = w_{0x} \left[(1 + 4n) \cos^2 \xi + \frac{(1-8n) \mathcal{L}_p^2 \sin^2 \xi}{(1-4n)z_r^2} \right]^{\frac{1}{2}}, \quad (6)$$

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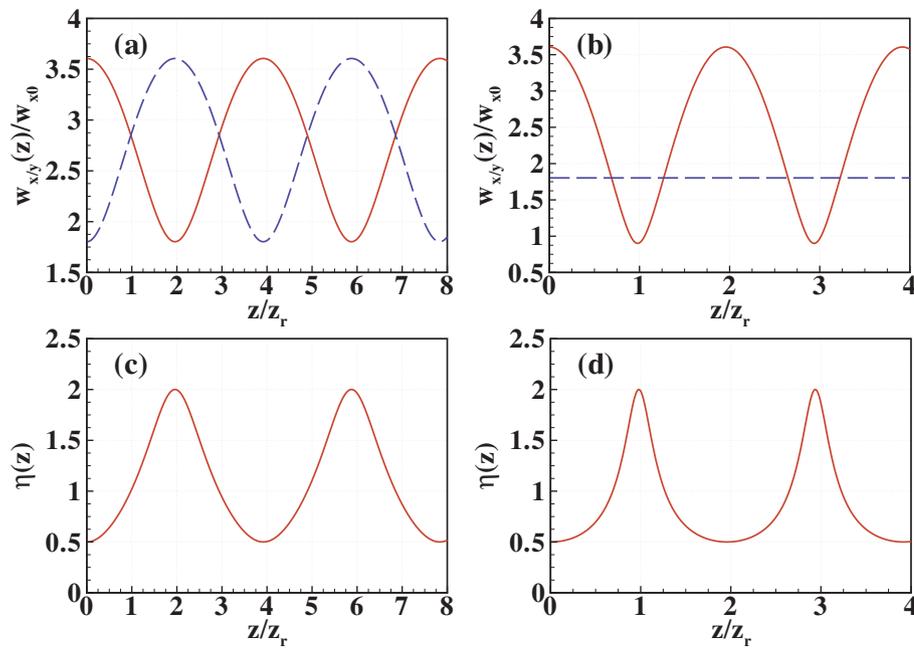


Fig. 1. (a) and (b): Evolutions of beam widths of AFPGBs; (c) and (d): Evolutions of asymmetric degree of AFPGBs corresponding to (a) and (b), respectively. The solid and dashed lines in (a) and (b) represent $w_x(z)$ and $w_y(z)$, respectively. The parameters are $P_0 = P_e^{(n=3)}$ for (a) and (c), $P_0 = P_{cy}^{(n=3)}$ for (b) and (d), and $n = 3$, $w_{0x} = 1$, $w_{0y} = 1/2$ for all cases.

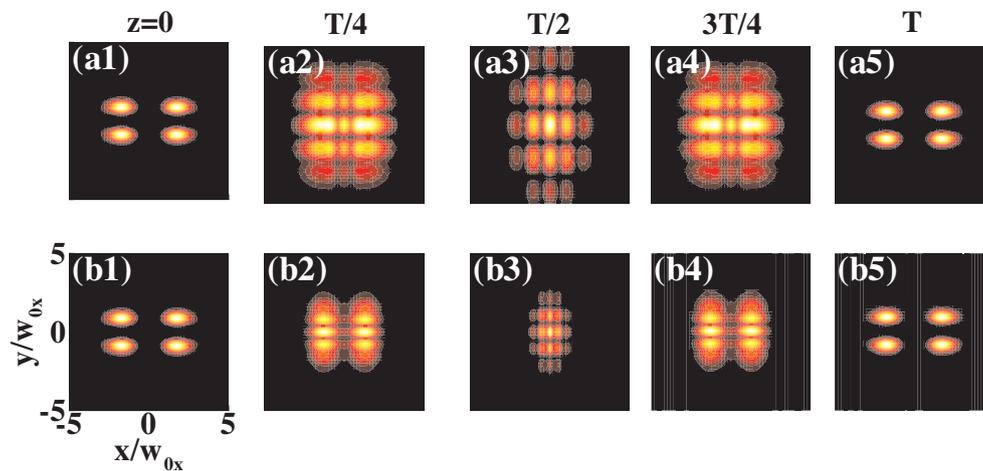


Fig. 2. Intensity patterns of AFPGBs at different output planes shown on the top in one evolution period. The parameters in the first row (a1)–(a5) and the second row (b1)–(b5) are the same as those in Fig. 2 and Fig. 1(b), respectively.

$$w_y(z) = \eta_0^2 w_{0x} \left[(1 + 4n)\cos^2\xi + \frac{(1-8n)\mathcal{L}_p^2 \sin^2\xi}{(1-4n)\eta_0^4 z_r^2} \right]^{\frac{1}{2}}, \tag{7}$$

where $z_r = kw_{0x}^2/2$. Therefore, the asymmetric degree of AFPGBs during evolution is $\eta(z) = w_y(z)/w_x(z)$. Based on Eq. (6), one can easily find that if the input power of an AFPGB takes a critical power $P_{cx}^{(n)} = (1-8n)P_{cg}/(1-16n^2)$, the beam width in x-direction keeps invariant during evolution, where $P_{cg} = 1/(\gamma^2 z_r^2)$ is the power of a Gaussian nonlocal soliton [8]. Similarly, for y-direction, the critical power is . The evolution of AFPGBs in SINN media is periodical and the period is $T = \pi \mathcal{L}_p$.

If the input power is between $P_{cx}^{(n)}$ and $P_{cy}^{(n)}$ for $\eta_0 \neq 1$, there exists the anomalous transverse transformation, namely, an AFPGB is compressed in one transverse direction and it is broadened in the other transverse direction. Especially, if $P_0 = P_e^{(n)} = (1-8n)P_{cg}/[\eta_0^2(1-16n^2)]$, the variational region of the beam widths in two transverse directions are equal [see (a) in Fig. 1]. For another special case, if $P_0 = P_{cy}^{(n)}$, the

beam width of AFPGBs in one transverse direction keeps invariant during evolution, and it varies periodically in the other transverse direction [see (b) in Fig. 1]. Although the evolutions of AFPGBs in two transverse directions have several different transformation forms, the asymmetric degree of AFPGBs at the middle of each evolution period is always $1/\eta_0$, i.e. $\eta_{max}\eta_{min} \equiv 1$ [see (c) and (d) in Fig. 1]. Fig. 2 gives the intensity distributions of AFPGBs at different evolution planes. The evolution period of AFPGBs is associated with the input power, hence, in Fig. 2 the actual distances are $T \approx 3.92z_r$ for (a) and $T \approx 1.96z_r$ for (b), respectively. These evolution properties are much different from the standard symmetrical four-petal Gaussian beams, i.e. the case of $\eta_0 = 1$ [18].

In conclusion, the evolution of AFPGBs in SINN media is investigated. The analytical expressions of the beam evolution and the beam width are derived. The anomalous transverse transformation are presented and investigated in detail.

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