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A novel technique of reduce order modelling without static correction for transient flow of non-isothermal hydrogen-natural gas mixture

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ABSTRACT

In this effort towards zero carbon emission of energy generation, Hydrogen Natural Gas Mixture (HCNG) is considered as energy source. This paper focuses on transient flow of HCNG in a pipelines under non-isothermal condition, using a modern technique of Reduced Order Modelling (ROM) technique. The effect of mass ration, body force and pipeline environmental temperature profile is investigated. The pipeline surrounding temperature effect HCNG flow parameters as shown in the results. The increase in hydrogen lead to hold up, variation on pressure and celerity wave. Good agreement is observed with new method and the usual reduced order method from the solutions published. The results presented give good agreement with experimental results and the new develop model improves on the accuracy.

Introduction

As world energy requirement is continuously increases, also the search of alternation energy with aim of reducing air pollution due to energy generation. Hence, the use of hydrogen been a zero carbon emission gas is worldwide considered. These are achieved by natural gas enrichment with hydrogen gas, known as hydrogen natural gas mixture HCNG. Furthermore, there is a need of it effective transportation and the cheapest means is pipeline.

Hydrogen been the most abundant and cleanness gas in the world it consideration as a fuel alternative in our future energy demand is a clear welcome development. At standard temperature (273.15 K) and pressure (100 kPa), hydrogen is a colourless, tasteless, nontoxic and highly combustible. Hydrogen can play an important role in a sustainable energy supply, since the utilization of H_2 yields no carbon dioxide (CO_2). Hydrogen is the most common element in the universe, but it never occurs by itself on the earth. It is always combines with other elements such oxygen or carbon [23].

In many applications of gas dynamics temperature is a function of both time and space, the inclusion of energy equation in the gas flow analysis is therefore, required [22]. For effective transportation of gas in a long distance pipeline sufficient energy is required. In the process of transporting gas, energy and pressure are lost periodically and there is a need to monitor them for effective demanding capacity meet [17].

In an inviscid fluid flow analysis external work done or convective heat transfer is a dominant stage of the heat transfer mode hence one-dimensional energy conservation equation is required for effective analysis [21]. In two-phase fluid flow magnitude of change caused by transient is higher compared to single phase fluid flow [1].

Natural gas is normally operated in single phase mode initially, liquid phase are formed within the pipeline during operation due to temperature and pressure change [2]. For accurate analysis of transient flow behaviour of high pressured fluid temperature change is important. For every flow in pipelines, the pipe wall constitutes the direct neighbour of the fluid, with the exception of some cases. Generally, external heat transferred into the fluid is influenced by pipe wall [11].

Pipes are at many times in different condition due to pipeline networks design and sometimes the environmental conditions. This condition will affect the flow parameters, although at a glance it may appear to have no significant effect on flow. From the experiment result small change in fluid body force due to pipeline the inclination angle reduces pipe storage capacity and pressure drop loss, while the breakdown rate will be higher. This is as result of

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Nomenclature

e'	total energy change
ρ	density of fluid mixture
f	coefficient of pipe friction
P	pressure
D	pipe diameter
g	gravitational acceleration
θ	the pipe inclination
c	celerity wave
e	internal energy per unit mass
h	specific enthalpy
ϕ	mass ratio of HCNG
φ	hydrogen ratio
m_i	mass
n_i	polytropic index
L	pipe length

Subscript nomenclature

h	hydrogen gas
g	Natural gas
o	Standard condition
d	dynamic condition
s	steady condition

low gas accumulation leads to low fluid velocity [20], but more of on two phase flow analyses.

The studies of fluid properties are more accurately achieved if the reality condition of pipeline are considered [19]. Therefore there is a need for fully defining the pipeline situation in the study of HCNG transient flow. The transient behaviours of HCNG was presented on the previous published work using different numerical methods.

Governing equation

Non-isothermal conditions are governed by:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0 \tag{2.1}$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + (\mathbf{u}\cdot\nabla)\rho\mathbf{u} + \nabla p = \nabla\sigma + \rho\mathbf{F} \tag{2.2}$$

$$\rho\left(\frac{\partial e'\mathbf{u}}{\partial t} + \nabla(e'\mathbf{u})\right) = \dot{Q}_g - \nabla\cdot q + \nabla\cdot(\mathbf{u}\cdot\sigma) \tag{2.3}$$

where e' is total energy change.

Numerical procedure

One dimensional flow equation is sufficient in the description of compressible gas in pipe effectively [9]. Therefore, the set of partial differential equations describing the general gas flow dynamics through a pipeline is obtained from (2.1–2.3) which are the vector for of conservation of mass, momentum and energy equations.

Hence, the one dimensional form are

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \tag{3.1}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2 + \rho c^2)}{\partial x} + \frac{f\rho u|u|}{2D} + \rho g\sin\theta = 0 \tag{3.2}$$

$$\frac{\partial}{\partial t}\left[\left(e + \frac{u^2}{2}\right)\rho\right] + \frac{\partial}{\partial x}\left[\left(h + \frac{u^2}{2}\right)\rho u\right] = \rho q - \rho ug\sin\theta \tag{3.3}$$

where ρ is the density of fluid mixture, f is coefficient of pipe friction, P the pressure is equal to ρc^2, u is the velocity of fluid mixture, D is the pipeline diameter, g is the gravitational acceleration, θ is the pipe inclination and c is a celerity wave. The internal energy per unit mass is e and h the specific enthalpy.

If the kinetic energy and wave effect on temperature change are neglected Eq. (3.3) reduces into

$$\frac{\partial}{\partial t}[\rho e] + \frac{\partial}{\partial x}[\rho ue] = \rho q - \rho ug\sin\theta \tag{3.4}$$

Since the two mixed components are in gaseous form, them homogeneous mixture is assumed. For a polytropic flow the density of the resultant mixture will depend on mass ratio of each gas. To develop density of the mixture ρ of HCNG, hydrogen fluid mass ratio will be used as the determining factor of the density. Therefore, the mass ratio of the mixture is defined by

$$\phi = m_h/(m_h + m_g) \tag{3.5}$$

where ϕ, m_i are the mass ratio of HCNG, mass of hydrogen and natural gas for $i = h$ *or* g respectively.

From the definition of density of gas

$$\rho = \frac{m_m}{V_m}, \rho_h = \frac{m_h}{V_h} \text{ and } \rho_g = \frac{m_g}{V_g}$$

where V_m, V_h, V_g, ρ_h and ρ_g are volume of gas mixture.(HCNG), hydrogen and natural gas density respectively taking the reciprocal of mixture density we get

$$\frac{1}{\rho} = \frac{V_m}{M_m}$$

Furthermore, $V_m = V_h + V_g$ hence we get

$$\frac{1}{\rho} = \frac{V_h + V_g}{M_m} = \frac{m_h V_h}{M_m m_h} + \frac{m_g V_g}{M_m m_g} \tag{3.6}$$

From density definition on we get

$$\frac{1}{\rho} = \frac{1}{\rho_h}\left(\frac{m_h}{M_m}\right) + \frac{1}{\rho_g}\left(\frac{m_g}{M_m}\right)$$

Simplifying using density definition and Eq. (3.5)

$$\frac{1}{\rho} = \frac{\phi}{\rho_h} + \frac{(1-\phi)}{\rho_g}, \rho = \left[\frac{\phi}{\rho_h} + \frac{(1-\phi)}{\rho_g}\right]^{-1} \tag{3.7}$$

Assuming the flow inlet is constant under polytropic process then using polytropic index the densities of hydrogen and natural gas are expressed as:

$\frac{p}{p_0} = \left(\frac{\rho_h}{\rho_{h_0}}\right)^{n_h}$ and $\frac{p}{p_0} = \left(\frac{\rho_g}{\rho_{g_0}}\right)^{n_g}$ substituting into Eq. (3.7)

$$\rho = \left[\frac{\varphi}{\rho_h} + \frac{1-\varphi}{\rho_g}\right]^{-1} = \left[\frac{\varphi}{\rho_{h0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n'}} + \frac{(1-\varphi)}{\rho_{g0}}\left(\frac{p}{p_0}\right)^{\frac{1}{n''}}\right]^{-1} \tag{3.8}$$

From the definition of celerity pressure wave and taken the derivative of (3.8) with respect to p and simplifying we have

The Celerity of pressure wave is defined as [3]:

$$c = \left(\frac{d\rho}{dp}\right)^{-1/2}$$

Integrating and taken into account (3.8) to obtained the celerity wave

Initial and boundary conditions

The flow is initially assumed to be at steady state condition, therefore the initial condition will be given as:

$$\frac{\partial\rho(x,0)}{\partial x} = 0 \quad (3.10a)$$

$$\frac{\partial\rho u(x,0)}{\partial x} = -c^2\frac{\partial\rho}{\partial x} - f\frac{\rho u^2}{2D} \quad (3.10b)$$

$$\frac{\partial}{\partial x}[\rho ue] = \rho q - \rho ug\sin\theta \quad (3.10c)$$

Boundary condition depends on valves operational time and locations coupled with compressor supply and pressure regulator. This work involved valves, which are placed in upstream and downstream of pipe with different valves and operational times.

The Automatic Closure Valve (ACV) is placed at the upstream and Rapid Closure Valve (RCV) at the downstream, boundary condition are based on reaction and actuation time effect on the inlet and outlet fluid.

At $x = 0$

$$\rho(x,t) = \rho_0(t), \frac{\partial u}{\partial x} = u_0(t)\ T(x,t) = T_0 \quad (3.11a,b,c)$$

At x = L

$$\rho u(L,t) = \rho u_L, \frac{\partial u}{\partial x} = u_L(t), \rho eu = h_L(t) \quad (3.11d,e,f)$$

where ρ, $\rho u = m_{0,1}$ and $\rho eu = h_{0,1}$ are the density and the inlet and outlet gas mass flux respectively.

Solution procedure

To construct ROM, the set of partial differential equation are written in vector flux form and descritazed using forward and backward difference on time and space. The scheme is refer to as implicit Steger-Warming Splitting Scheme. The scheme is then linearized using steady state solution and a small perturbation over the solution. The resulting scheme is referred to as linear form of implicit Steger-Warming Splitting Scheme and then written in the of Eigen value problem form. Eigen mode analysis is perform and subsequent construction of ROM by setting the flow analysis with Vortex Lattices Method (VLM) [5].

Writing Eqs. (3.1), (3.2), (3.4) in matrix form we have

$$\frac{\partial Q}{\partial t} = -\frac{\partial E(Q)}{\partial x} + H(Q) \quad (4.1)$$

where

$$E = \begin{bmatrix} \rho u \\ \rho u^2 + c^2\rho \\ \rho ue + pu \end{bmatrix} \quad (4.2)$$

$$H = \begin{bmatrix} 0 \\ \frac{f\rho u|u|}{2d} - \rho g\sin(\theta) \\ q\rho - \rho gu\sin(\theta) \end{bmatrix} \quad (4.3)$$

Differentiating with respect to Q Eqs. (4.1) and (4.2) we get

$$\frac{E(Q)}{\partial Q} = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 & 2u & r-1 \\ -ue & e & u \end{bmatrix} \quad (4.4)$$

$$\frac{\partial H(Q)}{\partial Q} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{fu|u|}{2d} - g\sin(\theta) & \frac{f|u|}{2d} & 0 \\ q - gu\sin(\theta) & -g\sin(\theta) & 0 \end{bmatrix} \quad (4.5)$$

Let $A = \frac{\partial E(Q)}{\partial Q}$ and $B = \frac{\partial H(Q)}{\partial Q}$ where A and B are Jacobian matrix of *E (Q)* and *H(Q)* respectively.

To obtain implicit Steger-Warming flux vector splitting scheme the following procedure is applied. Since the flux vector is $E = AQ$ and E homogenous, hence there exit two sub vector associated with positive and negative eigenvalues of it jacobian matrix such that their sum is equal to E. Similarly A can be split in the same manner as did on flux vector.

Such that

$$A = A^+ + A^-$$

$$E = E^+ + E^-$$

where A^+, A^- is transpose of a matrix whose rows are right and left eigenvector of matrix of A and E^+ is the produce of A^+ with E. Similarly $E^- = A^-E$

$$A^+ = \frac{u}{2c}\begin{bmatrix} -u+c & \frac{u+c}{u} & \frac{(r-1)(c-u)}{cu} \\ -(u^2+c^2) & \frac{(u+c)^2}{u} & \frac{(r-1)(-u^2+2uc+c^2)}{cu} \\ \frac{-c^2(u+c)}{r-1} & \frac{c^2(u+c)}{u(r-1)} & \frac{c(u+c)}{u} \end{bmatrix} \quad (4.6)$$

$$A^- = \frac{u}{2c}\begin{bmatrix} u-c & \frac{-(u-c)}{u} & \frac{(r-1)(u-c)}{cu} \\ (u-c)^2 & \frac{-(u-c)^2}{u} & \frac{(r-1)(u-c)^2}{cu} \\ \frac{c^2(u-c)}{r-1} & \frac{-c^2(u-c)}{u(r-1)} & \frac{c(u-c)}{u} \end{bmatrix} \quad (4.7)$$

$$E^+ = \begin{bmatrix} \frac{\rho(u+c)}{2} \\ \frac{\rho(u+c)^2}{2} \\ \frac{\rho c^2(u+c)}{2(r-1)} \end{bmatrix}, E^- = \begin{bmatrix} \frac{\rho(u-c)}{2} \\ \frac{\rho(u-c)^2}{2} \\ \frac{\rho c^2(u-c)}{2(r-1)} \end{bmatrix} \quad (4.8a,b)$$

To obtain implicit Steger-Warming flux vector splitting scheme using Taylor series expansion

By Taylor series expansion the left hand side of (3.1) in order 2

$$Q^{n+1} = Q^n + \frac{1}{2}\left[\left(\frac{\partial Q}{\partial t}\right)^n + \left(\frac{\partial Q}{\partial t}\right)^{n+1}\right]\Delta t + O(\Delta t)^3 \quad (4.9)$$

Substituting (3.1) into (4.9)

$$Q^{n+1} - Q^n = -\frac{1}{2}\left[\left(\frac{\partial E}{\partial x} - H\right)^n + \left(\frac{\partial E}{\partial x} - H\right)^{n+1}\right]\Delta t + O(\Delta t)^3 \quad (4.10)$$

Linearize the nonlinear term (4.10) by Taylor series expansion, $\left(\frac{\partial E}{\partial x} - H\right)^{n+1}$ we have

$$E^{n+1} = E^n + \Delta t\left(\frac{\partial E}{\partial x}\right)^n + O(\Delta t)^2$$
$$E^{n+1} = E^n + A^n(Q^{n+1} - Q^n) \quad (4.11a)$$

Similarly

$$H^{n+1} = H^n + B^n(Q^{n+1} - Q^n) \quad (4.11b)$$

$$\left(\frac{\partial E}{\partial x}-H\right)^{n+1}=\left(\frac{\partial E}{\partial x}-H\right)^{n}+\left(\frac{\partial A}{\partial x}-B\right)^{n}(Q^{n+1}-Q^{n}) \quad (4.11c)$$

where A and B are Jacobian matrices of E(Q) and H(Q) respectively.

Substituting (4.11a), (4.11b) and (4.11c) into (4.10) and simplifying

$$Q^{n+1}-Q^{n}+\frac{1}{2}\Delta t\left(\frac{\partial A}{\partial x}-B\right)^{n}(Q^{n+1}-Q^{n})=-\Delta t\left(\frac{\partial E}{\partial x}-H\right)^{n} \quad (4.12)$$

Substituting the split matrices into (4.12)

$$Q^{n+1}-Q^{n}+\frac{1}{2}\Delta t\left(\frac{\partial(A^{+}+A^{-})}{\partial x}-B\right)^{n}*(Q^{n+1}-Q^{n}) = -\Delta t\left(\frac{\partial(E^{+}+E^{-})}{\partial x}-H\right)^{n} \quad (4.13)$$

Taking backward and forward difference space step on positive and negative part of split matrices (4.13) and simplifying [16]

$$\left[1+\tfrac{\Delta t}{\Delta x}(A_i^{n(+)}-A_i^{n(-)})-\Delta tB_i^{n}\right](Q_i^{n+1}-Q_i^{n})-\left(\tfrac{\Delta t}{\Delta x}A_{i-1}^{n(+)}\right)(Q_{i-1}^{n+1}-Q_{i-1}^{n}) +\left(\tfrac{\Delta t}{\Delta x}A_{i+1}^{n(-)}\right)(Q_{i+1}^{n+1}-Q_{i+1}^{n})=-\tfrac{\Delta t}{\Delta x}[E_i^{n(+)}-E_{i-1}^{n(+)}+E_{i+1}^{n(-)}-E_i^{n(-)}]+\Delta tH_i^{n}$$

$$\left[1+\frac{\Delta t}{\Delta x}\left(A_i^{n(+)}-A_i^{n(-)}\right)-\Delta tB_i^{n}\right]\Delta Q_i-\left(\frac{\Delta t}{\Delta x}A_{i-1}^{n(+)}\right)\Delta Q_{i-1} +\left(\frac{\Delta t}{\Delta x}A_{i+1}^{n(-)}\right)\Delta Q_{i+1}=-\frac{\Delta t}{\Delta x}\left[E_i^{n(+)}-E_{i-1}^{n(+)}+E_{i+1}^{n(-)}-E_i^{n(-)}\right]+\Delta tH_i^{n} \quad (4.14)$$

(4.14) is referred to as the implicit Steger-warming flux vector splitting method (FSM) using backward time difference scheme deformation [18].

The linearized form of FSM is obtain using steady state and perturbation, (4.14) becomes (4.15).

$$-\left(\frac{\Delta t}{\Delta x}A_{j+1}^{0}\right)\widehat{Q}_{j-1}^{n+1}+\left(1+\frac{\Delta t}{\Delta x}(A_j^{0+}-A_j^{0-})-\Delta tB_j^{0}\right)\widehat{Q}_j^{n+1} +\left(\frac{\Delta t}{\Delta x}A_{j+1}^{0-}\right)Q_{j+1}^{n+1} =\widehat{Q}_j^{n} \quad (4.15)$$

Written as

$$W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}+V^{n+1} \quad (4.16)$$

where V^{n+1} and W^{0} are imposed value from the boundary condition and the coefficient matrix of the unknown term $\hat{Q}_j^{n+1}$ respectively.

Construction of ROM using Vortex lattice method (VLM)

Vortex lattice method is use to construct ROM that requires static correction and ROM that does not require static correction.

ROM with static correct requirement

The homogeneous part of (4.16) $W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}$ is satisfied by setting

$$(Q_j=x_j\exp(-i\omega_jt)\alpha_j\exp(iz_jx) \quad (4.17)$$

Then (4.17) is the general eigenmode [14]

Considering the VLM for small perturbation from (4.15), then (4.17) reduces to

$$\widehat{Q}_j=x_j\exp(\lambda_jt), z_i=\exp(\lambda_i\Delta t) \quad (4.18)$$

where λ_i, x_i eigenvalue and the corresponding eigenvector.

From the eigenvalue problem the following generalization is made

$$z_jW^{0}x_j=Ix_j \quad (4.19a)$$

$$ZW^{n}X=IX \quad (4.19b)$$

(4.19b) is the general right eigenvalue.

where Z and X are the diagonal matrix of eigenvalue and a matrix whose columns are the corresponding eigenvector.

Similarly considering the left eigenvalue problem of the homogenous part of (4.16) $W^{o}\widehat{Q}^{n+1}=I\widehat{Q}^{n}$ we have

$$(W^{0})^{T}YZ=IY \quad (4.20)$$

For normalized eigenvalue the orthogonal conditions are satisfied

$$Y^{T}W^{0}X=I, Y^{T}IX=Z \quad (4.21a,b)$$

where Y is the matrix whose columns are the left eigenvectors of W^{0}

From eigenmode analysis base on time step, (4.18) described the fluid flow behaviour at individual nodes

Therefore

$$\widehat{Q}=X\,\widehat{c} \quad (4.23a)$$

where $\widehat{c}$ is the vector of the normal node coordinate [6]

From (4.23a)

$\hat{Q}=\begin{bmatrix}Q_1\\Q_2\\ \vdots\\Q_n\end{bmatrix}$, and $\hat{c}=\begin{bmatrix}\hat{c}_1\\\hat{c}_2\\ \vdots\\\hat{c}_n\end{bmatrix}$ normal node coordinates.

Since $\hat{Q}$ is known at the initial point then using orthogonal conditions (4.21a,b) and multiplying (4.23a) by $Y^{T}W^{0}$ and evaluation to obtain

$$\hat{c}=Y^{T}W^{0}\hat{Q} \quad (4.23b)$$

Going back to (4.17) and multiplying through by Y^{T} considering the orthogonal conditions we get.

$$\widehat{c}^{n+1}-Z\widehat{c}^{n}=Y^{T}V^{n+1} \quad (4.24)$$

The left-hand side of (4.24) is now the diagonal of each mode that marched forward in time independently and less computational cost. The results are then reassembled using (4.23a) to obtain the fluid flow bahaviours. The advantage of this modal is to satisfactory ROM can construct with few numbers of the original modes. In the present work m modes are retain with the largest eigenvalues z_i for $m<N$ where N is the total number of original modes. With this the order of X, Y are reduced to $N\times m$ matrices, and Z is reduced to $m\times m$ matrix. Unfortunately, satisfactory results cannot give a satisfactory result unless a large number of m is used. This is as a result of neglected modes which are not orthogonal to the downwash and however does not participate in the response [15].

For a satisfactory ROM to be constructed, (4.16) is decomposed into two system part known as quasisteady and systems dynamic part.

$$Q^{n}=Q_s^{n}+Q_d^{n} \quad (4.25)$$

where Q_s^{n}, and Q_d^{n} are quasisteady and dynamic parts.

Such that

$$Q_d^{n}=XC_d^{n} \quad (4.26)$$

Substituting (4.25) into (4.16) and multiply through by Y^{T} and simplifying we have

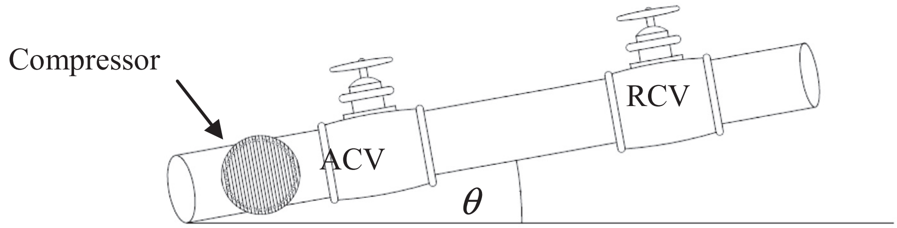

Fig. 1. Pipeline installation.

$$C_d^{n+1} = ZC_d^n + Y^T V^{n+1} - Y^T(W^0 Q_s^{n+1} + IQ_s^n) \tag{4.27}$$

Then (4.24) is replaced by (4.26) in the construction of ROM. Since static correction is carried out on (4.26) it will give satisfactory result where the system is dominated by few eigen mode [5].

ROM without static correction

The construction of ROM is unsatisfactory and requires static correction due to existence of zero eigenvalue in the eigensystem [13]. However, the zero eigenvalues can be removed by defining a new eigenvalue problem whose eigenvalues are non-zero of the previous eigenvalue problem [7]. In high pressure HCNG transient flow analysis we let Q_b and Q_s to be the vectors of fluid motion due to disturbance and steady state fluid respectively. Eq. (4.17) can now be written as

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix}^{n+1} + \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix}^{n} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}^{n+1}$$

where $W_{i,j}$, $I_{i,j}$ are sub square matrices from W^o and I with $1 \leqslant i,j \leqslant 3$

Expanding (4.27) we have

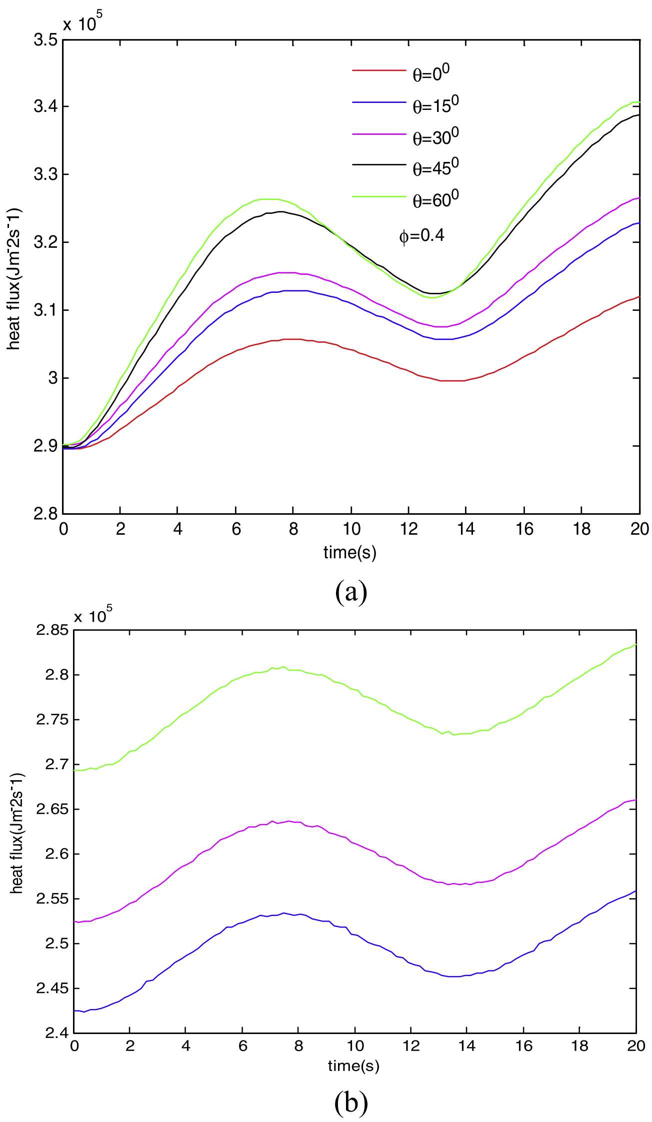

Fig. 2. Heat flux (a) different inclination angle; (b) different mass ratio.

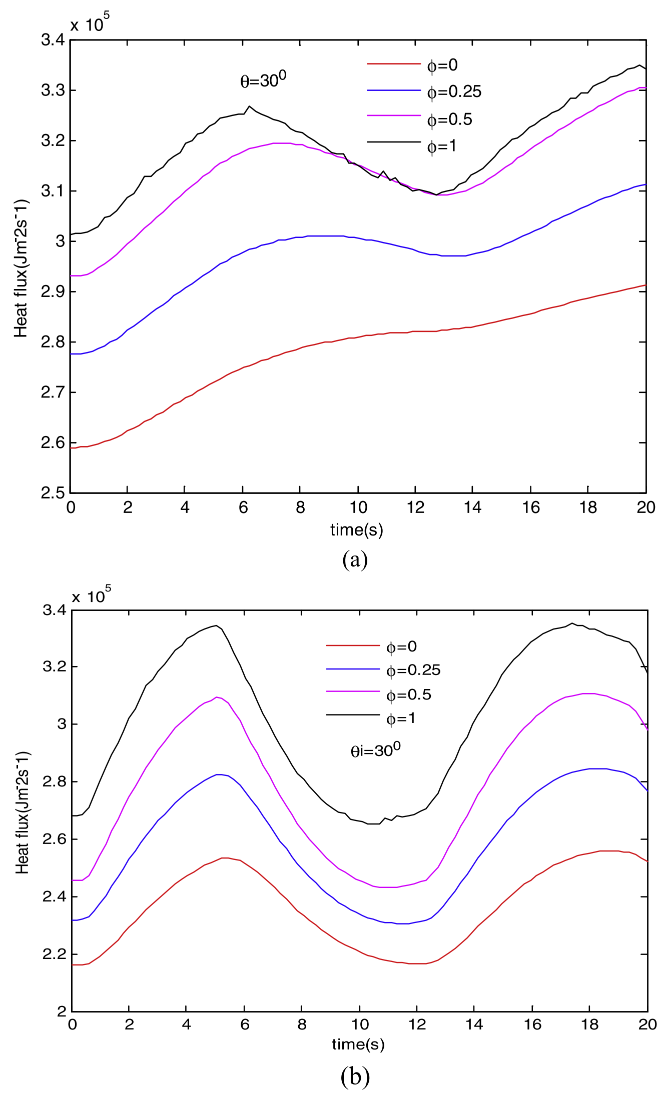

Fig. 3. Effect of valve on heat flux distribution (a) after RCV (b) before ACV.

$$W_{11}\Gamma_1^{n+1} + W_{12}\Gamma_2^{n+1} + W_{13}\Gamma_3^{n+1} + I_{11}\Gamma_1^n + I_{12}\Gamma_2^n + I_{13}\Gamma_3^n = V_1^{n+1} \quad (4.28a)$$

$$W_{21}\Gamma_1^{n+1} + W_{22}\Gamma_2^{n+1} + W_{23}\Gamma_3^{n+1} + I_{21}\Gamma_1^n + I_{22}\Gamma_2^n + I_{23}\Gamma_3^n = V_2^{n+1} \quad (4.28b)$$

$$W_{31}\Gamma_1^{n+1} + W_{32}\Gamma_2^{n+1} + W_{33}\Gamma_3^{n+1} + I_{31}\Gamma_1^n + I_{32}\Gamma_2^n + I_{33}\Gamma_3^n = V_3^{n+1} \quad (4.28c)$$

Since the matrices W^o and I are tridigonal and identity matrices respectively then following sub matrices are zero matrix $W_{1,3}$ and $W_{3,1}$ while $I_{i,j} = 0$ if $i \neq j$, therefore equations (4.28a) and (4.28c) reduces and hence are simplified to get

$$\Gamma_1^{n+1} = W_{11}^{-1}[V_1^{n+1} - (W_{12}\Gamma_2^{n+1} + I_{11}\Gamma_1^n)] \quad (4.29a)$$

$$\Gamma_3^{n+1} = W_{33}^{-1}[V_3^{n+1} - (W_{32}\Gamma_2^{n+1} + I_{33}\Gamma_3^n)] \quad (4.29b)$$

Substituting (4.29a) and (4.29b) into (4.28b) and evaluating we get

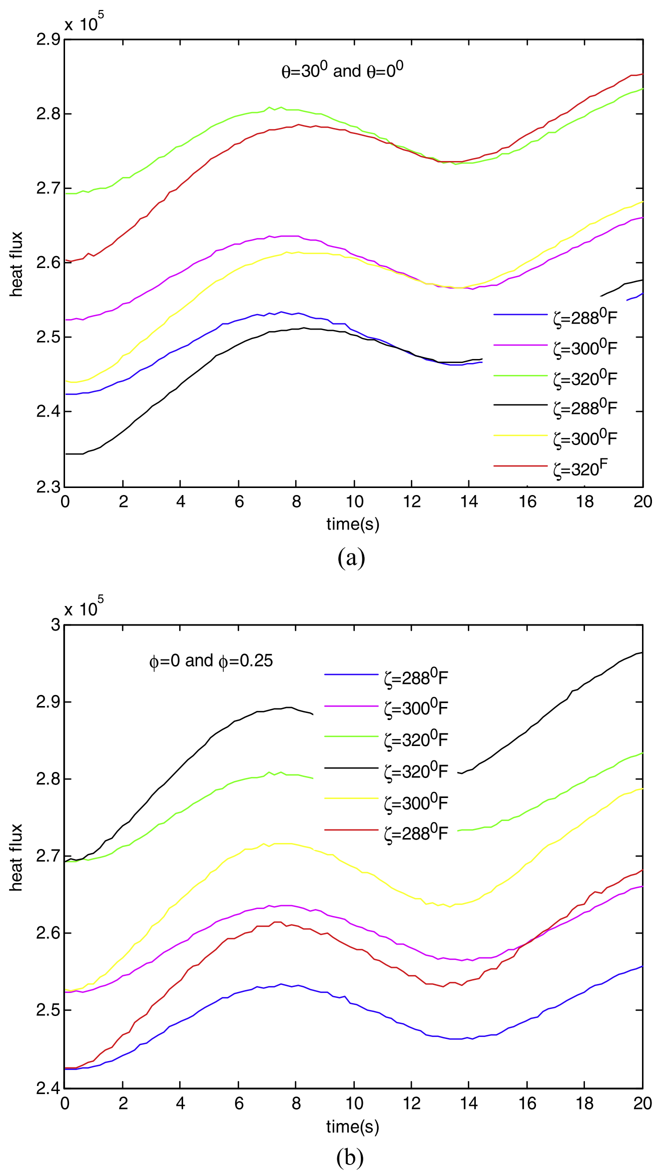

Fig. 4. Heat flux for different surrounding temperature (a) after ACV (b) before RCV.

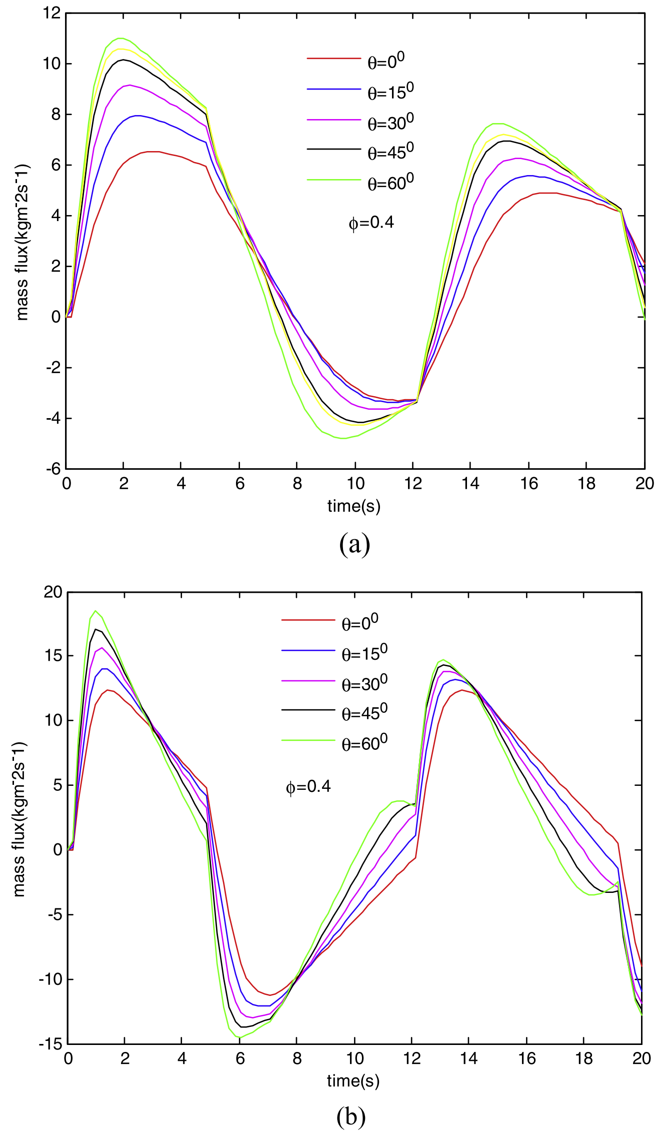

Fig. 5. Mass flux at the steady state condition (a) after ACV (b) before RCV.

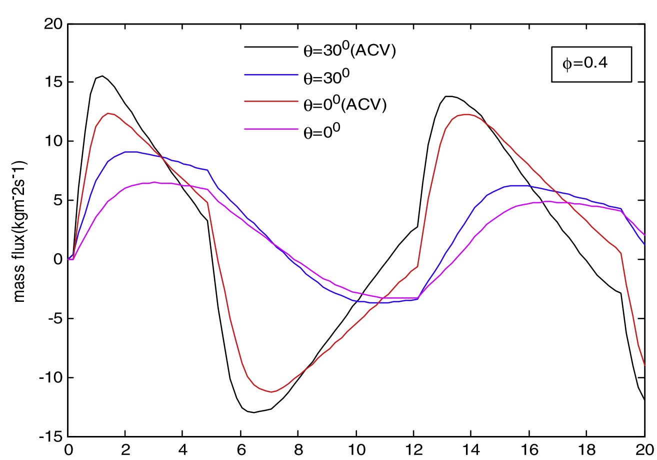

Fig. 6. Mass flux of transient due to operation valve.

$$[W_{22} - W_{21}W_{11}^{-1}W_{12} - W_{23}W_{33}^{-1}W_{32}]\Gamma_2^{n+1} - W_{21}W_{11}^{-1}\Gamma_1^n - W_{23}W_{33}^{-1}\Gamma_3^n = V_2^{n+1} - W_{31}W_{11}^{-1}V_1^{n+1} - W_{23}W_{33}^{-1}V_3^{n+1} \quad (4.30)$$

Assuming $\Gamma_1^n = \Gamma_3^n = -\Gamma_2^n$ then (4.30) become

$$W_{new}^o\Gamma_s^{n+1} + B_{new}\Gamma_s^n = V_{new}^{n+1} \quad (4.31)$$

where $W_{new}^o = [W_{22} - W_{21}W_{11}^{-1}W_{12} - W_{23}W_{33}^{-1}W_{32}]$, $B_{new} = (-W_{21}W_{11}^{-1} - W_{23}W_{33}^{-1})$ and $V_{new}^{n+1} = V_2^{n+1} - W_{31}W_{11}^{-1}V_1^{n+1} - W_{23}W_{33}^{-1}V_3^{n+1}$

However (4.31) is eigenvalue problem that fully define the previous problem and its eigenvalues are nonzero on which ROM can now be constructed without static correction. Then ROM can now be constructed without static correction following as above up to (4.24).

To analysed dynamic behaviour of HCNG, for change on surrounding temperature to be achieved (4.31) and (4.28) are used for when static correction is required and without static correction requirement.

Problem description

The initial flow rate is assumed $m_0 = 10\ kg/s$ at fluid temperature of 15 °C, with absolute pressure of $p = 20\ bar$, the specific heat of both gases given following [12] at permanent pressure of $p = 20\ bar$ with the surrounding of three different cases.

Body force due to inclination is varies for different angles. The mass ratio is varied to test its effect to other parameters, and become a single at $\phi = 0$ and $\phi = 1$ as natural gas and hydrogen gas respectively. The properties of hydrogen and natural gas used in the calculations are presented by [12]. For this problem to be realistic the pipeline surrounding temperature is considered and assumed to be constant at 340 °K.

Fig. 1 illustrates the pipeline installation where the ACV is placed just after the compressor while the Rapid Control Valve (RCV) is placed at the downstream of the pipe.

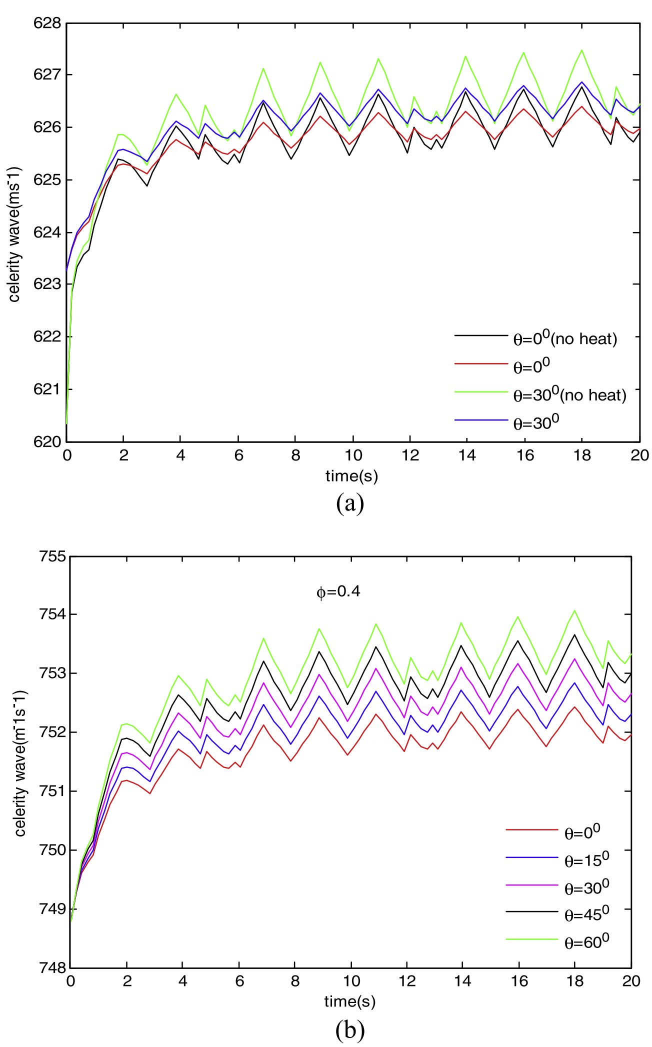

Fig. 7. Celerity wave due to body force (a) comparison with isotherm (b) different inclination.

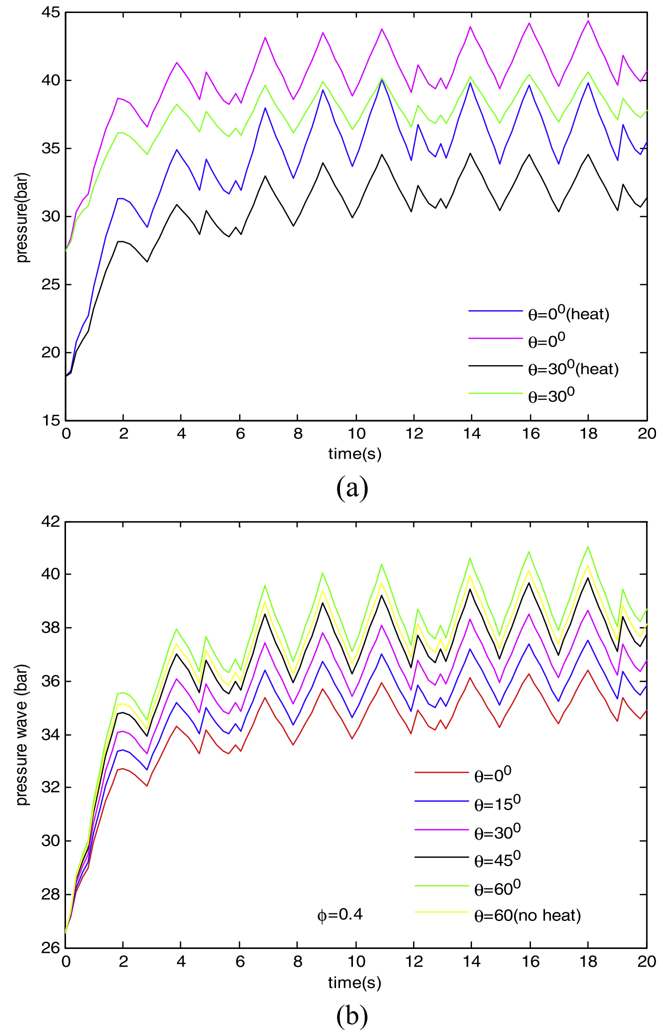

Fig. 8. Pressure wave oscillation due to temperature (a) comparison with isotherm (b) different inclination.

Results and discussion

From Fig. 2 the inclination angle as shows influence on heat flux and almost equal between times 8 and 14 s at the pipeline outlet. When the angle 45 and 60. At the initial condition obtain by solving the Eqs. (3.10a,b,c). The inlet the heat flux is equal for any value of the pipeline inclination. Fig. 2(b) shows different surrounding temperature for steady mass ratio and body force.

Valve operation effect heat flux distribution behaviour as show from Fig. 3(a) and (b) which also depend on fluid type. After ACV for mass ratio more than 0.5 the heat flux are the same within some simulation time. Before the fluid get to RCV heat flux distribution is steady for at mass ratio as show in Fig. 3(b).

From Fig. 4 surrounding temperature have shown the effect on heat flux subsequently will effect velocity and pressure drop. The mass flux distribution shows no effect at the start with same values for different mass ratio and begins with different values for different inclination angle.

From Fig. 5 mass flux is more at the downstream of the pipe before RCV. The behaviour is different as shown in the graphs with peak at the same simulation time of about 2sec for both Fig. 5(a,b). From the graph mass flux value are sometimes equal. This occurs 3 times from Fig. 5(a) after ACV and 6times from Fig. 5(b) before RCV. Increased in mass flux for increase in mass ratio is as a result of burning capacity of hydrogen gas (Fig. 6).

Transient flow due to valve operation influence mass flux throughout the simulation period. Decreased in mass flux value is observed at any pipeline position when compared to steady state. This is as a result of change in velocity due to valve closure and show the transient flow on hydrogen natural gas mixture at a given mass ratio and agree with the early published result by Subani et al. [25] (Fig. 7).

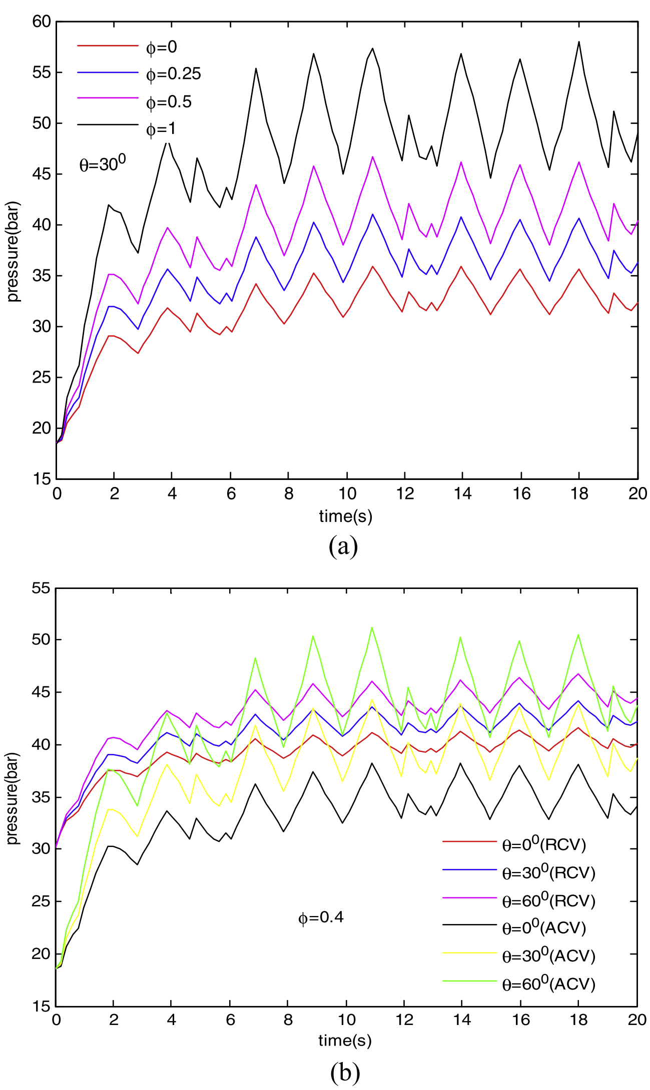

Fig. 9. Pressure wave for various in mass ratio (a) different flow position (b) different mass ratio.

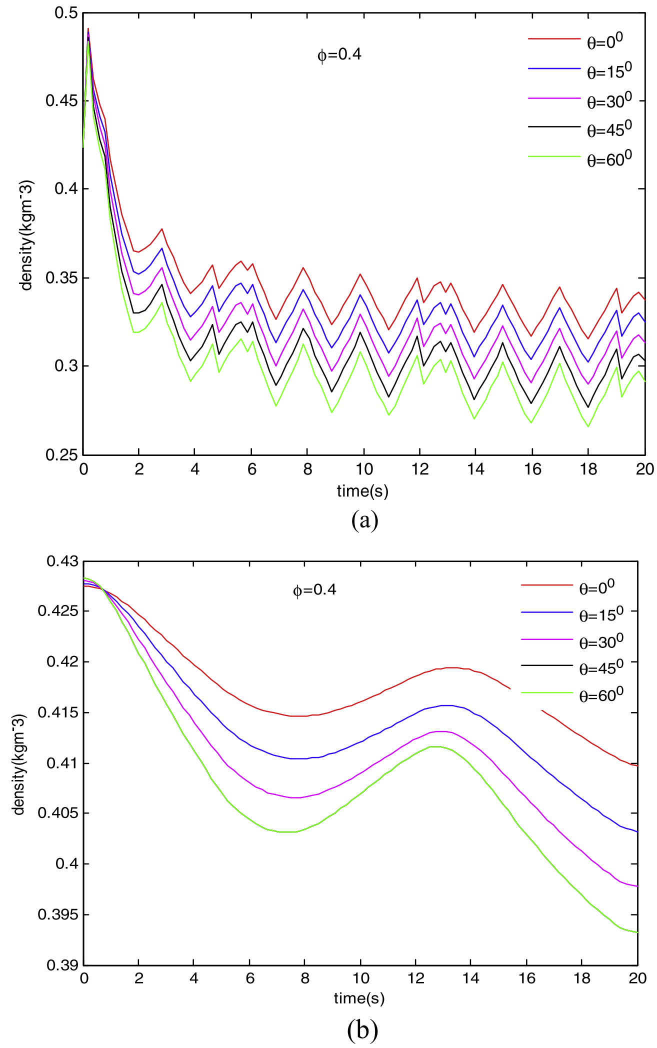

Fig. 10. Density at (a) ACV and (b) RCV for change body force effecct.

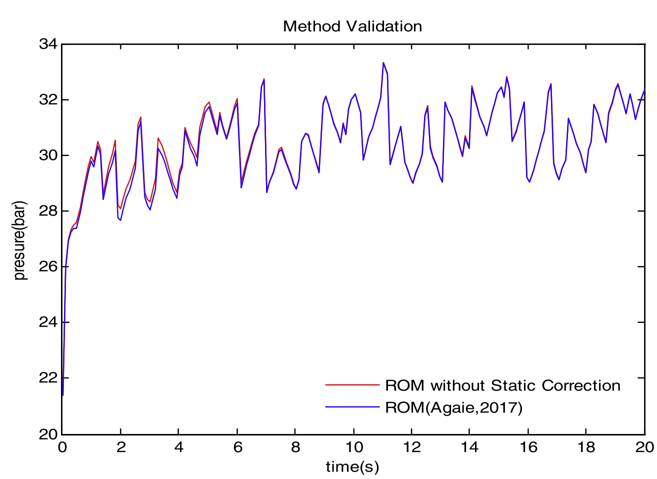

Fig. 11. Method Validation on pressure drop.

The effect of surrounding temperature is observed on the celerity wave from the initial simulation period, but sometime equal in some point and becomes the same after 18secs. The effect of body force is observed in celerity wave at HCNG transient with on effect during the first second and proposition to difference between the inclination angles after 2 s for any mass ratio. The different in celerity we is high for any change in mass ratio.

In Fig. 8(a) we presented the surrounding temperature effect on pressure wave for different body force. A decrease on pressure is observed Fig. 8(a). Fig. 8(b) shows effect of body force (inclination angle) on pressure wave.

In Fig. 9 effect of mass ratio is presented, high pressure wave is observe for $\theta = 1$ and reduces for decrease on θ throughout the pipeline since similar observation is made at any point. The results also agrees with the published works on hydrogen natural gas flow leakage detection and reduced order methods work when correction technique is required [3,4].

From Fig. 10 density continuously varies during simulation. The density wave and value different is observed on the presented results above. Density increases toward RCV is due to HCNG hold on in the pipeline. The results presented agree with on the experimental study on heat transfer effect on gas -solid flow projection angle (inclination) effect flow parameters [28]. Heat transfer from pipeline surrounding show effect on accuracy of predicted flow parameters [8].

In Fig. 11 ROM without static correction requirement was compared with the conventional ROM generate a satisfactory result. With an advantage of non-existence of zero eigenvalue, the computational time is reduced. Shows agreement with early published result, which indicates computational efficiency in the approach and accuracy.

The numerical technique ROM without static is said to be stable and will give a satisfactory result on the transient behaviour of gas mixture [5]. Since the eigenvalues involved in eigensystem are non-zero and the real part of the eigenvalues are negative. Therefore, there is no need to carry out static correction and that also reduce the computational time of simulation.

As presented in the results the presence of hydrogen leads to decrease on heat flux, this also agrees with early reported of [26]. Heat flux for any case is observed to be high if body force is considered in the computation and similar to what was obtain in micro channel as presented by [10]. The storage and transportation of HCNG will involve substantial challenges on heat and mass transfer [27]. It is observed that the mass of hydrogen pressure must always be taken into for any HCNG flow analysis, this is also in line with the experimental and development of model on active cooling system for hydrogen storage [24].

Conclusion

The temperature is quite important in the study of transient behaviour of HCNG all flow parameter presented. For good prediction of HCNG flow parameters, energy equation is required in the description of HCNG along the pipeline. From our results, surrounding temperature effect on celerity wave is higher than other parameters such as pressure. The heat and mass transfer are important in the analysis of hydrogen natural gas mixture.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.rinp.2018.01.052.

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