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Behavior of surface plasmon polaritons at the interface of metal and a non-integer dimensional (NID) dielectric medium

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ABSTRACT

The behavior of surface plasmon polaritons (SPPs) at a planar non-integer dimensional dielectric-metal interface has been studied for the first time. The dimension of the interface is shown to have a strong effect on the behavior of SPPs. Dimensions of both the half spaces are different from each other which can be used to distinguish between them besides their different wavenumbers. Lossless dielectric medium has non-integer dimensions while metal has integer dimension. Surface plasmon polaritons (SPPs) only exist for p-polarization and no surface modes exist for s-polarization because the partnering non-integer dimensional dielectric material is homogeneous (Polo and Lakhtakia, 2011). Two methods are used in order to verify the proposed idea. First method is named as so-called indirect method, by using this method, it has been observed that SPP waves are excited only at one incident angle [$\theta_i = 54.87°$] for p-polarized incidence. It has been investigated that the behavior of SPPs changes with change in order of non-integer dimension. The absorption of SPPs increases by decreasing dimensionality for p-polarization at the excitation angle. Second method is named as so-called direct method, according to this method, by solving the presented idea directly using SPP waves fields expressions in NID space, it is conceived that SPPs with same polarization state, but with different phase speeds, attenuation rates, along with propagation lengths can be directed by such interface which incorporates metal and a non-integer dimensional dielectric medium. The results achieved by both methods (so-called indirect and direct methods) complement each other tremendously, validating the efficiency of our proposed concept regarding propagation of SPP waves at a planar non-integer dimensional dielectric-metal interface. Furthermore, the effect of relative permittivity of non-integer dimensional dielectric medium on SPPs is also discussed. The conventional results for integer dimensional space can be recovered as a special case.

Introduction

Highly complicated structures, for example, roughness of ocean floor, dust particles and snow etc., cannot be explained by using Euclidean geometry but idea of NID space can be taken as a healthy tool in modeling such geometries. Mandelbrot [1] introduced the term, “Fractal” for the first time in order to provide explanation of complex structures. The special property of self-similarity and repetition of fractals at different scales is a key in giving a solution regarding complex geometries because less number of parameters are enough to model them as compared to Euclidean geometry. Fractals are modeled by a non-integer dimension. Complex structures can be made distinguished at both microscopic and macroscopic levels [2,3]. The non-integer dimensional analysis has been studied by many researchers for the past two decades due to its importance and versatility [4–9].

For the purpose of getting large benefits of these non-integer dimensional (NID) modes, the electromagnetic theory must be analyzed in NID space. To remark this, solutions to Poisson’s and Laplace equations in NID space have been investigated in [10,11]. A lot of studies have been done in very recent times which incorporate non-integer dimensional (NID) space to examine different EM phenomenons [12–19].Electromagnetic wave propagation and solutions for planes, cylindrical and spherical waves in NID space have been studied by Zubair et al. [20]. In the last few years, it has also been a subject of interest to analyze the electromagnetic radiation from non-integer dimensional structures [21–24]. Recently, quasi-static analysis of scattering from a radially uniaxial dielectric sphere in fractional space has been described by Nisar et al. [25]. Nisar et al. further analyzed the

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cloaking and magnifying in non-integer dimensional space by using radial anisotropy [26]. Perfect electric conductor waveguide and non-integer dimensional tunnel has been analyzed in [27]. Circuit elements have been explained in non-integer dimensional space at optical frequency in [28]. EM behavior of a planar interface of NID spaces has been investigated by Naqvi in [29]. Asad et al. and Omar et al., explored the behavior of electromagnetic waves at dielectric-fractal [30] and dielectric fractal-fractal interface [31], respectively. Very recently, a study regarding analysis of reflection and transmission from a NID-interface/NID-dielectric interface in the presence of losses has been reported [32]. This investigation has been a source of inspiration for us to study the behavior of surface plasmon polariton waves at the interface of metal and the NID dielectric medium. The analysis of SPP waves in non-integer dimensional (NID) space has been presented for the first time in order to understand the effect of dimensionality variation on them.

An electromagnetic SPP wave is a combination of two materials. The SPP wave bestrides the planar interface which has two half spaces, either side of it, is tied up by a different material; eliminate the interface by making the two collaborating materials indistinguishable, and the SPP wave disappears. An interface of a metal and a dielectric medium is used in driving an SPP wave along that interface [33]. The metal and the dielectric material are taken as isotropic. The real part of permittivity of metal is taken as negative, meanwhile the real part of permittivity of dielectric is taken as positive and each collaborating material is assumed as isotropic, linear, non-magnetic, homogeneous and achiral [34]. The propagation of SPPs can occur along any direction which is parallel to the interface but after a specific distance by the interface, its amplitude decreases on both sides of the interface. This property of boundedness is an auspicious phenomena that is used in some highly sensitive biochemical sensors [35]. Recently, SPP waves are used in some newly emerging technologies, like, near-field spectroscopy [36]. Theory of EM and surface waves is given in [37,38], respectively. The SPPs are excited by impinging the linearly p-polarized incident wave only at the metal/dielectric interface because partnering dielectric material is homogeneous [39]. Quasi-static analysis of scattering from a layered plasmonic sphere in fractional space has been investigated in [40] while power tunneling and rejection has been discussed in [41] by using fractal chiral-chiral interface. Dispersion analysis of deep-subwavelength-decorated metallic surface using field-network joint solution is presented in [42].

In this paper, the absorption and different characteristics of SPP waves are seen and analyzed by using so-called indirect and direct methods, respectively, for p-polarization at the interface of metal and a non-integer dimensional dielectric medium. Both methods exhibit the similar results by showing that the non-integer dimension have a strong affect on the propagation of SPP waves, giving us a strong validation of the proposed concept. The conventional results can be achieved from non-integer dimensional results when dimensionality is an integer.

We selected to examine the proposed concept regarding surface wave behavior by the interface which consists of a metal and the non-integer dimensional dielectric medium in the canonical boundary-value problem in order to eradicate the presence of waveguide modes that's why half-space has been assigned to each involved material. A brief explanation regarding formalism of the problem has been shown in Section "Problem formulation" while numerical results have been presented and elaborated in Section "Numerical results and discussion". Conclusions have been demonstrated in Section "Conclusions". An $\exp(i\omega t)$ time-dependence has been assumed with ω indicating the angular frequency, t represents the time, and $i = \sqrt{-1}$. Furthermore, the free-space wavenumber has been denoted by $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$, where ε_0 and μ_0 are permittivity and permeability of free space, with $\lambda_0 = 2\pi/k_0$. Vectors are in boldface and Cartesian unit vectors are identified as $\hat{\mathbf{a}}_\mathbf{x}$, $\hat{\mathbf{a}}_\mathbf{y}$, and $\hat{\mathbf{a}}_\mathbf{z}$.

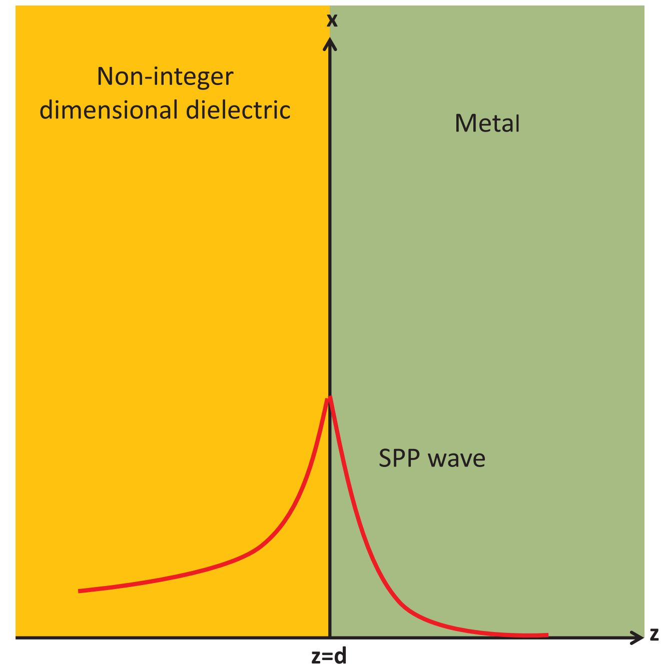

Fig. 1. Typical variation of the amplitude of the electric field phasor of an SPP wave as a function of distance from the interface of metal and a non-integer dimensional dielectric medium at $z = d$, in the presence of homogeneous, isotropic, non-magnetic, and achiral partnering materials, same as [34], only dimensions of dielectric medium are different.

Problem formulation

A planar interface of two materials is considered as shown in Fig. 1. The interface is assumed to be located at $z = d$, with metal filling the half space $z > d$ whereas half space $z < d$ is of non-integer dimensional dielectric medium. Constitutive parameters and dimensions D for material filling half space $z < d$ are (ε_1, μ_1, $1 < D \leqslant 2$) and for half space $z > d$ are (ε_2, μ_2, $D = 2$). Where $\varepsilon_1 = \varepsilon_0\varepsilon$, $\mu_1 = \mu_0\mu$, and ε/μ is relative permittivity/permeability of the non-integer dimensional dielectric medium. For metallic medium, which is taken to be bulk aluminium, $\varepsilon_2 = \varepsilon_0\varepsilon_m$, $\mu_2 = \mu_0\mu_m$ having: $\varepsilon_m = -56 - i21$ and $\mu_m = 1$.

P-polarized case

Fig. 2 illustrates the incident wave for p-polarization and along with it's reflection and transmission from the presented interface of metal and non-integer dimensional dielectric medium which is centered at $z = d$. Expression of fields for the incident wave with it's reflected and transmitted parts can be written as given below:

$$\mathbf{E}_i^{fd} = E_0(\hat{\mathbf{a}}_\mathbf{x}\cos\theta_i - \hat{\mathbf{a}}_\mathbf{z}\sin\theta_i)\exp(-ik_1\sin\theta_i x)(k_1\cos\theta_i z)^{n_1} H_{n_1}^{(2)}(k_1\cos\theta_i z), \tag{1}$$

$$\mathbf{H}_i^{fd} = \hat{\mathbf{a}}_\mathbf{y}\frac{E_0}{\eta_1}\exp(-ik_1\sin\theta_i x)(k_1\cos\theta_i z)^{n_{h1}} H_{n_{h1}}^{(2)}(k_1\cos\theta_i z), \tag{2}$$

and

$$\mathbf{E}_r^{fd} = (\hat{\mathbf{a}}_\mathbf{x}\cos\theta_r + \hat{\mathbf{a}}_\mathbf{z}\sin\theta_r)E_0\Gamma\exp(-ik_1\sin\theta_r x)(k_1\cos\theta_r z)^{n_1} H_{n_1}^{(1)}(k_1\cos\theta_r z), \tag{3}$$

$$\mathbf{H}_r^{fd} = -\hat{\mathbf{a}}_\mathbf{y}\frac{E_0\Gamma}{\eta_1}\exp(-ik_1\sin\theta_r x)(k_1\cos\theta_r z)^{n_{h1}} H_{n_{h1}}^{(1)}(k_1\cos\theta_r z) \tag{4}$$

where $k_1 = \omega\sqrt{\mu_1\varepsilon_1}$ is wavenumber of NID dielectric medium, $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ is the intrinsic impedance, Γ is the reflection coefficient. The exponential function has been used to explain the propagation of wave in x direction and Hankel function of order n has been utilized to

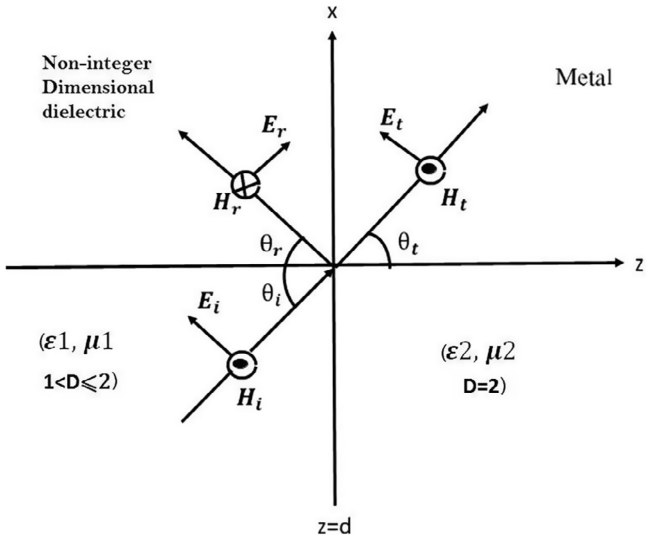

Fig. 2. P-polarized incident wave at the interface of metal and non-integer dimensional (NID) dielectric medium.

indicate propagation of wave in z direction. Hankel function of first kind of order n and second kind of order n have been chosen to illustrate waves propagating away and towards the interface, respectively [30,31,43]. Order of the Hankel functions are selected as $n_1 = |3-D|/2$ and $n_{h1} = |D-1|/2$, where $(1 < D \leqslant 2)$ is the dimension of NID space. It may be noted that superscript fd has been used for fields in non-integer dimensional dielectric medium and superscript m has been used for fields in metal.

Similarly, the electric and magnetic fields inside the metal can be written as,

$$\mathbf{E}_t^m = (\hat{\mathbf{a}}_\mathbf{x}\cos\theta_t - \hat{\mathbf{a}}_\mathbf{z}\sin\theta_t)E_0 T\exp(-ik_2\sin\theta_t x)\exp(-ik_2\cos\theta_t z) \quad (5)$$

$$\mathbf{H}_t^m = \hat{\mathbf{a}}_\mathbf{y}\frac{E_0 T}{\eta_2}\exp(-ik_2\sin\theta_t x)\exp(-ik_2\cos\theta_t z) \quad (6)$$

where $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$ is wave number of metallic media, $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$ is the intrinsic impedance, T is the transmission coefficient. The unknown reflection and transmission coefficients can be determined by using boundary conditions on the continuous tangential components of electric and magnetic fields at the interface $(z = d)$, as follows,

$$E_i^{fd}(z = d) + E_r^{fd}(z = d) = E_t^m(z = d), \quad (7)$$

$$H_i^{fd}(z = d) + H_r^{fd}(z = d) = H_t^m(z = d). \quad (8)$$

Above mentioned boundary conditions have been used in order to find unknown coefficients Γ and T after the substitution of electric and magnetic fields into them.

The law of conservation of energy must be satisfied that is given by,

$$1 = \Gamma + T, \quad (9)$$

and absorption for p-polarized incidence can be calculated by using following relation in order to observe the existence of SPP waves,

$$A_p = 1-(\Gamma + T), \quad (10)$$

This method can be termed as so-called “INDIRECT METHOD” to analyze the effect of non-integer dimension on the behavior of SPP waves at the interface of metal and non-integer dimensional dielectric medium.

Numerical results and discussion

Let us investigate the behavior of SPP waves at interface of metal and non-integer dimensional dielectric medium for linearly p-polarized incidence. To determine the absorption spectra, the numerical data for

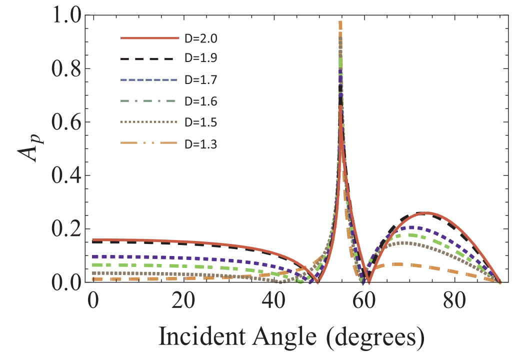

Fig. 3. Normalized absorption spectrum of SPP waves in case of p-polarized incidence for different non-integer values and varying incident angles.

lossless non-integer dimensional dielectric medium is taken as $\varepsilon = 90$ and $\mu = 1$. The operating wavelength is chosen as 633 nm, the metal is taken as bulk aluminum having: $\varepsilon_m = -56-i21$, $\mu_m = 1$, while incident angles and dimensions are kept variable in order to observe the behavior of SPP waves at the interface of metal and the non-integer dimensional dielectric medium.

SPP waves are excited only at one incident angle (54.87°) at interface of metal and non-integer dimensional dielectric medium which are shown in Fig. 3 for p-polarized incidence. In Fig. 3, it can be observed that SPPs are strongly effected by changing the order of non integer dimension. When the order of non-integer dimension increases, the amplitude of absorption demonstrates decreasing behavior, showing that the confinement of SPP waves at the interface is maximum for the lowest value of non-integer dimension, i.e., $(D = 1.3)$. Moreover, it can be analyzed that strong SPP waves are excited for p-polarized incidence in case of integer dimensional space as well.

In order to verify the above mentioned absorption spectrum of SPP waves for p-polarized incidence, which is obtained by using an indirect method, we will now apply so-called “DIRECT METHOD” to investigate the effect of non-integer dimension on different characteristics of SPP waves [controllable phase speeds, propagation length, and attenuation rates] using the expressions of fields of SPP waves in NID space and matching the results in both cases will strengthen our idea of SPP waves propagation at the interface of metal and a non-integer dimensional dielectric medium. These expressions can be derived from Eqs. (1) and (2) for p-polarized incident wave by applying suitable boundary conditions. Suitable values of the above mentioned parameters are chosen for this method, different than which have been used in indirect method in order to maintain the coherence between the behavior of the results obtained by both the methods.

It is obvious by looking at the results of indirect method that for each value of non-integer dimension, there exists one separate SPP wave. Little different approach have been used in direct method. In direct method, we plotted relative wavenumber, propagation length and relative phase speed versus dimensionality (D) keeping all the other parameters constant for convenience, because it is easy and direct way to give the clearest picture that how the order of non-integer dimensions affects the behavior of SPP waves. It is also convenient way to compare it with the results of indirect method. But if someone wants to examine each SPP wave at each and every corresponding value of non-integer dimension, then he must plot relative wavenumber, propagation length and relative phase speed versus any suitable parameter mentioned above in indirect method, and vary the dimensionality as a third parameter just like the plot of absorption spectrum obtained by indirect method. In that case, each separate SPP wave for each and every separate value of dimensionality can be drawn. For example, in indirect method, six values of non-integer dimension give six SPP waves. Similarly, in direct method, by taking dimensionality as a third parameter, six exponentially decaying or increasing curves of SPPs can be obtained for six values of dimensionality separately, instead of one exponentially decaying or increasing curve of SPPs for all values of dimensionality as we are going to use in our paper, because it is convenient in order to obtain the desired results. If we plot relative wavenumber, propagation length and relative phase speed versus dimensionality (D) keeping all the other parameters same and if we plot relative wavenumber, propagation length and relative phase speed versus any appropriate parameter mentioned above in indirect method by varying the dimensionality as a third parameter just like the plot of absorption spectrum obtained by indirect method, in both ways, the amplitude of SPP waves will be and must be different corresponding to each value of non-integer dimension but the behavior of SPP waves will be same.

In this article, relative wavenumber, propagation length and relative phase speed have been plotted versus dimensionality (D) keeping all the other parameters same so one exponentially decaying or increasing curve of SPP waves has been plotted for all values of dimensionality, because it is convenient and gives clear insight regarding behavior of SPPs at the interface of metal and a non-integer dimensional dielectric medium.

First of all, in order to observe how the behavior of relative wavenumber of SPP waves is affected due to non-integer dimension, the relative wavenumber as a function of dimensionality has been analyzed.

Fig. 4 illustrates that the relative wavenumber of p-polarized SPP waves is effected by changing the order of non-integer dimension. From Fig. 4, it is clear that SPP waves propagate very efficiently in non-integer dimensional space, whereas attenuation rate of SPP waves decreases by increasing order of non-integer dimension which demonstrates that the SPP waves become loosely bound to the interface in case of integer dimensional space (D = 2) when compared to non-integer dimensional space (D = 1.1 to D = 1.9), satisfying the absorption spectrum given in Fig. 3. So, non-integer dimensions is an important parameter in order to enhance the absorption of SPP waves from the interface of metal and a non-integer dimensional dielectric medium.

The expression of propagation length,

$$\Delta_{xp} = 1/Im(wavenumber)$$

along the direction of propagation has been shown in Fig. 5 with respect to dimensionality. Where wavenumber indicates the wavenumber of p-polarized SPP waves. The explanation of propagation length Δ_{xp} can be given in such a way that it is the distance traveled by SPP waves along the direction of propagation $\hat{\mathbf{x}}$ after that the amplitude of the electric and magnetic field decays to $exp(-1)$ of its value at $x = 0$. Δ_{xp} stands for propagation length of p-polarized SPP waves.

The propagation length of SPP waves increases by increasing dimensionality for p-polarized SPP waves. It shows that SPP waves become more and more tightly confine to the interface as we go down into the non-integer dimensional space, i.e., (D = 2 to D = 1.3) and it's confinement is highest at (D = 1.3), which verifies the absorption spectrum given in Fig. 3, obtained by so-called indirect method.

The relative phase speed

$$v_p/c_0 = k_0/Re(wavenumber),$$

where c_0 is depicted as speed of light in free space, is plotted in Fig. 5, having same parameters as Fig. 4. The figure illustrates that the increase in the order of non-integer dimension increases the relative phase speed of the SPP waves, matching exactly with the results of absorption spectrum, drawn in Fig. 3.

In order to observe that, can the relative permittivity of the non-integer dimensional dielectric medium effect the absorption of SPP waves at the interface of metal and the non-integer dimensional dielectric medium? To answer this question, the absorption have been plotted as function of relative permittivity and dimensionality for p-polarized incident waves.

From Fig. 6, it is seen that absorption decreases by increasing order of non-integer dimension and it increases by increasing relative permittivity of non-integer dimensional dielectric medium for p-polarized incidence at the excitation angle, which is also in accordance with the absorption spectrum shown in Fig. 3. It means that the greater the value of relative permittivity of non-integer dimensional dielectric medium, the higher is the localization of SPP waves to the interface.Therefore, it has been observed that relative permittivity of non-integer dimensional dielectric medium has a strong effect on the behavior of SPP waves.

In view of this, we can see that the physical insight achieved by absorption spectrum of SPP waves given in Fig. 3, [obtained by so-called indirect method], and different characteristics of SPP waves [phase speeds, propagation length, and attenuation rates] obtained from SPP field expressions in non-integer dimensional space at the interface of metal and a non-integer dimensional dielectric medium [obtained by so-called direct method], is fully matched with each other, proving the validity of the proposed idea.

Conclusions

In this article, two methods named as so-called indirect and direct methods, are used to analyze the behavior of SPP waves propagation at the interface of metal and a non-integer dimensional dielectric medium. Firstly, it has been observed that by using so-called indirect method, the SPP waves are excited at the interface of metal and the non-integer dimensional dielectric medium for p-polarized incident waves at one incident angle. It is analyzed that due to the interface of metal and non-integer dimensional dielectric medium, the absorption of SPP waves changes by changing the order of non-integer dimension at the excitation angle ($\theta_i = 54.87°$), it increases by decreasing the order of non-integer dimension, i.e., (D = 2 to D = 1.1). Secondly, the so-called direct method which uses SPPs field expressions in NID space revealed that SPP waves having same frequency, linear polarization state and direction of propagation but with different phase speeds, attenuation rates, and propagation lengths can be controlled by the interface of metal and a non-integer dimensional dielectric medium. Both methods complement each other results very efficiently, leaving us with a strong authenticity of the proposed concept.The results show that non-integer dimensional dielectric-metal interface leads to the excitation of

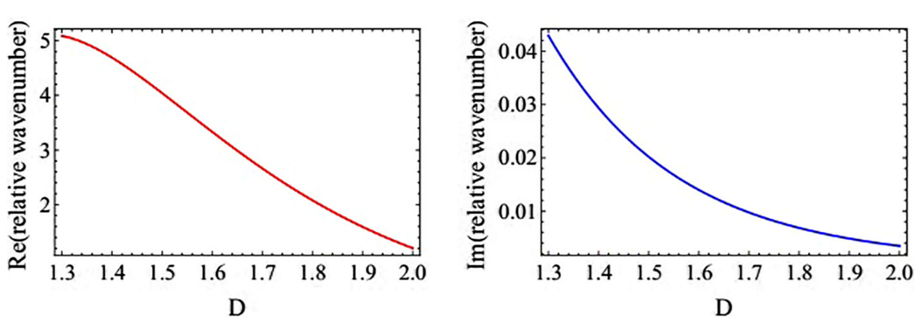

Fig. 4. Real and Imaginary parts of relative wavenumber of p-polarized SPP waves as a function of varying dimensionality at excitation angle = 54.87°.

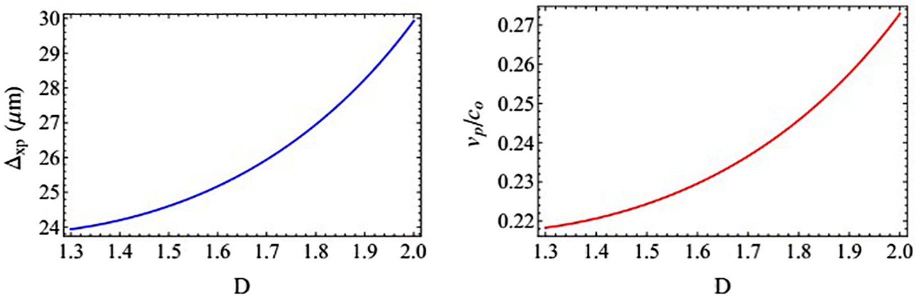

Fig. 5. Propagation length Δ_{xp} and relative phase speed v_p/c_0 of p-polarized SPP waves for different non-integer values having same parameters as Fig. 4, at excitation angle = 54.87°.

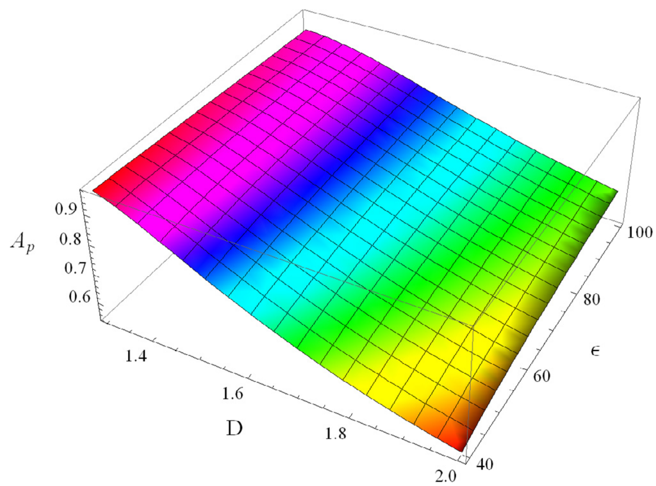

Fig. 6. Same as Fig. 3 except varying the order of non-integer dimension and relative permittivity of lossless non-integer dimensional dielectric medium at θ_i = 54.87°.

different SPP waves patterns at single excitation angle (θ_i = 54.87°), in case of p-polarized incident wave. It enlightens that many options of partnering non-integer dimensional dielectric mediums are accessible for the exploitation of p-polarized SPP waves. The effect of relative permittivity of non-integer dimensional dielectric medium on the behavior of SPP waves has also been presented. So, it can be concluded that the non-integer dimensions of the interface, in addition to relative permittivity of non-integer dimensional dielectric medium, can be used for controlling the power of SPP waves from an interface.The conventional results other than non-integer dimensional, can be obtained by putting the integer values of dimension in place of non-integer dimension.

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