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Combining Newton's second law and de Broglie's particle-wave duality

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ABSTRACT

All matter can exhibit wave-like behaviour, and Louis de Broglie first predicted light to display the dual characteristics as both a collection of particles, called photons, or in some respects as a wave. The particle velocity is the group velocity of the wave, and if the particle velocity u_g is subluminal then the associated wave or phase velocity u_p through the de Broglie relation $u_g u_p = c^2$ is necessarily superluminal. This is believed not to contradict the fact that information cannot be carried faster than the velocity of light c because the wave phase is supposed to carry no energy. However, the superluminal phase velocity may well be physically significant, and here we propose that the sub particle world and the super wave world might be equally important, and that each might exert an influence on the other, such that any mechanical equations must not only be Lorentz invariant but they must also be invariant under the transformation connecting the sub and super worlds. Following this approach, Einstein's equation $\mathcal{E} = mc^2$ becomes simply $\mathcal{E} = (m + m')c^2$, where m and m' are masses given by Einstein expressions arising from the perceived sub and superluminal velocities u_g and u_p respectively. This modification, although superficially simple, results from non-conventional physics and gives rise to an extension of Newton's second law, that might well account for the extra energy and mass that is known to exist in the universe, and referred to as dark energy and dark matter. An explicit solution for photons and light predicts a non-zero photon rest-mass $m_0 = h\nu/2c^2$, where h is Planck's constant and ν is the light frequency. Interestingly, the associated energy of this mass is the zero-point energy, believed to be the lowest energy that a quantum mechanical system may possess.

1. Introduction

While Einstein's formula $m(u) = m_0[1 - (u/c)^2]^{-1/2}$, for the variation of mass m with its velocity u, where m_0 denotes the rest mass, has been overwhelmingly verified in our own local environment, it is clear that on a cosmological scale our understanding of mass and matter are not so successful, and issues such as dark energy and dark matter remain improperly understood. In our local environment the rest mass m_0 is deemed to be the sole critical parameter, and yet the mysteries associated with dark energy and dark matter indicate that matter itself may adopt other forms or possess other defining characteristics or that our present accounting for energy is flawed. (see for example Saari [1]).

de Broglie's particle-wave duality was originally formulated within the context of quantum mechanics and connects the subluminal particle and superluminal wave formulations through the relation $u_g u_p = c^2$, where u_g is the particle velocity or group velocity of the wave and u_p is the associated wave or phase velocity u_p. In an abbreviated notation, the relation $uu' = c^2$ arises from the underlying space–time transformation $x' = ct$ and $t' = x/c$, such that the critical speed of light c acts as a hinge about which the sub and super worlds turn, and the low velocity Newtonian world connects with the high velocity wave world. We may imagine a symmetrically folded sheet with the fold corresponding to the speed of light c, such that the sub and super worlds lie on either side of the fold, and any prescribed data at one edge of the sheet (say at $u = 0$) must be reconciled with data at the other edge of the sheet (say $u' = \infty$).

In the one dimensional extension of special relativity beyond the speed of light, as proposed by Hill and Cox [2,3], Vieira [4] and others, the velocity addition formula still applies and is non-singular at the speed of light. In the limit for infinite relative frame velocity v, the de Broglie's formula remarkably also emerges from Einstein's velocity addition formula. On the one hand, de Broglie's formula is formulated from quantum mechanics while on the other hand, the velocity addition formula is basically kinematical. Further, if with respect to some fixed frame, all subluminal frames are grouped together S_{sub} and all superluminal frames are grouped together S_{sup}, then there is no objective way to decide in which set of frames we belong.

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In this paper, we propose that the sub and superluminal worlds might be equally important, and that each might exert an influence on the other, such that any mechanical equations must not only be Lorentz invariant but they must also be invariant under the above transformation connecting the sub and super worlds. This idea gives rise to an extension of Newton's second law involving not just the rest mass m_0 but a second parameter termed here p_∞, and corresponding to the constant limiting momentum at infinite velocity. With this definition of mass and force, involving the two constants m_0 and p_∞, it is not difficult to envisage that the additional degree of freedom embodied in p_∞ might well be exploited to account for the extra energy and mass that is known to exist in the universe, and referred to as dark energy and dark matter.

We emphasise that the confluence of the de Broglie's formula with that arising from one dimensional extended special relativity for superluminal motion only forms part of the motivation for the theory proposed here, and that a full three dimensional extension of the theory proposed here hinges only on the existence of a three dimensional version of the de Broglie formula, and not on the validity of three dimensional extended relativity for superluminal motions. According to Guemez et al. [5] the special relativistic three dimensional version of de Broglie's formula becomes simply the scalar product $\mathbf{u_g}.\mathbf{u_p} = c^2$, which is the critical formula necessary to develop the three dimensional version of the theory presented here.

The underlying philosophy in this paper, is to recognise the importance of the Einstein theory of special relativity, and to seek to develop a theory in a manner that embraces the essential features of the existing theory. Now given the veracity of the special theory, it may not be too unreasonable to expect that somewhere embodied within the theory are clues as to the notions which have been termed dark matter and dark energy. We also need to remember that general relativity arose out of special relativity, which means that if we can appropriately modify the special theory then corresponding extensions to the general theory might be made. However, since the special theory deals only with non-accelerating frames, we certainly would not expect any such extension to tell the complete story, but we might expect some definite pointers as to how a more complete picture may be subsequently developed. The superluminal phase velocity may well be physically significant, and in this paper we propose that the sub particle world and the super wave world might be equally important, and that each might exert an influence on the other, such that any mechanical equations must not only be Lorentz invariant but they must also be invariant under the transformation connecting the sub and super worlds.

We propose that both the particle and wave worlds contribute equally to the mechanics and energy accounting. In consequence, and as a word of caution to the reader, this implies that our present physical intuition must be moderated to grasp the fact that now there are two contributions in play; and more importantly, when say the particle world is contributing the least, the wave world is contributing the greatest. This means that on face value, we might obtain some apparently surprising results, and we need to bear in mind that the relative contribution might be insignificant locally and only significant on a global scale.

As an illustrative example of the approach adopted here, we examine particles which are assumed to have a non-zero rest mass m_0 but capable of travelling at the speed of light. Conventional special relativity, for the case of photons travelling at the speed of light, assigns the rest-mass of a photon to be zero, so that Einstein's formula $m(u) = m_0[1-(u/c)^2]^{-1/2}$ is sensible in the limit $u \to c$ and the limiting values $p = h\nu/c$ and $\mathcal{E} = h\nu$ are assigned, where h is the usual Planck constant and ν is the light frequency. The theory presented here allows an alternative formulation with the same limiting values, but with a non-zero photon rest-mass $m_0 = h\nu/2c^2$. As fully detailed below, adopting zero as the datum energy at $u = 0$, namely $\mathcal{E}^*(0) = 0$ in (4.8) gives the final summary formulae (4.9), and the equation $m_0 = h\nu/2c^2$, which gives a numerical value for m_0 of about 2.22×10^{-36} kg for typical light frequencies in the range $4-8 \times 10^{14}$ Hz. Interestingly, the quantity $h\nu/2$ is well known as the zero-point energy or the ground state energy, and is believed to be the lowest possible energy that a quantum mechanical system may have. The existence of this explicit mathematical solution is a non-trivial consequence of the theory that is based upon the key hypothesis that Newton's second law can be replaced by a formulation that remains invariant under $x' = ct$ and $t' = x/c$.

The well known formula for the mass m; namely $m(u) = m_0[1-(u/c)^2]^{-1/2}$ is fundamental to special relativity, and can be verified as follows. From Newton's second law, the applied force f balances the rate of change of momentum $f = d(mu)/dt$, where $u = dx/dt$ is the velocity. Together with the energy or work equation that expresses the fact that any increase in energy arises from the work done $d\mathcal{E} = fdx$, which is often expressed as the rate-of-work equation $d\mathcal{E}/dt = fu$. Assuming $\mathcal{E} = mc^2$ there now follows from the above equations the identity:

$$m\frac{d\mathcal{E}}{dt} - (mu)\frac{d(mu)}{dt} = c^2\frac{d\left[m^2\left(1-(u/c)^2\right)\right]}{dt} = 0, \tag{1.1}$$

and from this identity follows $m(u) = m_0[1-(u/c)^2]^{-1/2}$. We comment that this formula is one of many expressions showing a particular variation of mass with its velocity, and has a long history involving many eminent scientists such as Abraham, Bücherer, Lorentz, Ehrenfest, Kaufmann and of course Einstein, some of whom first grappled with the notion that the 'transverse and longitudinal' masses may be distinct. The story describing the development of the Einstein expression is fully detailed by Weinstein [6].

In the following section we present a brief summary of the basic equations of one dimensional special relativity and of the extended theory for superluminal velocities. In the subsequent section we comment how the de Broglie relation also emerges from the Einstein velocity addition law in the limit of infinite frame velocity v, and we discuss the underlying space – time transformation connecting the particle and wave speeds. We then propose an extension of Newton's second law purposely designed not only to remain invariant under the Lorentz transformations but also under the space-time transformation connecting the sub and super worlds. From this modification of Newton's second law emerges the expression $\mathcal{E} = (m+m')c^2$, where m and m' are the perceived sub and superluminal masses respectively. We also briefly mention the three dimensional extension of the proposed theory based on the special relativistic extension of de Broglie's formula Guemez et al. [5]; as well as an extended version of the theory necessary for particles capable of travelling at the speed of light. In the section thereafter, we apply the new formulae to the special case of photons moving at the speed of light, and their connection with formulae arising from the special relativistic Doppler effect is discussed in the subsequent section. In the final section of the paper we make some brief concluding remarks.

2. Special relativity and extended theory

We consider a rectangular Cartesian frame (X, Y, Z) and another frame (x, y, z) moving with constant velocity v relative to the first frame and the motion is assumed to be in the aligned X and x directions as indicated in Fig. 1. We note that the coordinate notation adopted here is slightly different to that normally used in special

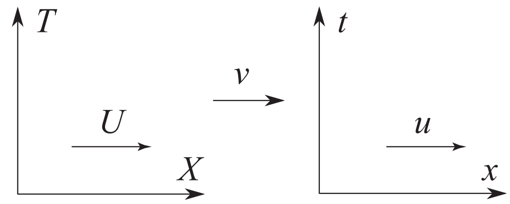

Fig. 1. Two inertial frames moving along x-axis with relative velocity v.

relativity involving primed and unprimed variables. We do this purposely because in [2,3] it is convenient to view the relative velocity v as a parameter measuring the departure of the current frame (x, y, z) from the rest frame (X, Y, Z) which is the notation employed in nonlinear continuum mechanics. Time is measured from the (X, Y, Z) frame with the variable T and from the (x, y, z) frame with the variable t. Following normal practice, we assume that $y = Y$ and $z = Z$, so that (X, T) and (x, t) are the variables of principal interest.

For $0 \leqslant v < c$, the standard Lorentz transformations are

$$X = \frac{x + vt}{[1 - (v/c)^2]^{1/2}}, \quad T = \frac{t + vx/c^2}{[1 - (v/c)^2]^{1/2}},$$

with the inverse transformation characterised by $-v$, thus

$$x = \frac{X - vT}{[1 - (v/c)^2]^{1/2}}, \quad t = \frac{T - vX/c^2}{[1 - (v/c)^2]^{1/2}}, \tag{2.1}$$

and various derivations of these equations can be found in many standard textbooks such as Feynmann et al. [7] and Landau & Lifshitz [8], and other novel derivations are given by Lee & Kalotas [9] and Levy-Leblond [10]. The above equations reflect, of course, that the two coordinate frames coincide when the relative velocity v is zero, namely $x = X, t = T$ when $v = 0$. We comment that for $c < v < \infty$, basically the same equations apply except that the denominator becomes appropriately modified to be well-defined and gives (3.2) in the limit $v \to \infty$, thus (see Hill and Cox [2] for a formal derivation)

$$x = \frac{-X + vT}{[(v/c)^2 - 1]^{1/2}}, \quad t = \frac{-T + vX/c^2}{[(v/c)^2 - 1]^{1/2}}, \tag{2.2}$$

and similar transformations with (X, T) replaced by $(-X, -T)$ can be found in [2]. With velocities $U = dX/dT$ and $u = dx/dt$, both (2.1) and (2.2) yield the same addition of velocity law, namely

$$u = \frac{U - v}{(1 - Uv/c^2)}, \tag{2.3}$$

which is well known and due to Einstein.

We note that as an aside, an immediate consequence of (2.3) is the identity

$$[1 - (u/c)^2](1 - Uv/c^2)^2 = [1 - (v/c)^2][1 - (U/c)^2], \tag{2.4}$$

which is not so well-known, but is nevertheless fundamental to the development of special relativity. Another not so well-known formula arising from (2.3) is

$$\left(\frac{1 - U/c}{1 + U/c}\right) = \left(\frac{1 - u/c}{1 + u/c}\right)\left(\frac{1 - v/c}{1 + v/c}\right), \tag{2.5}$$

and both (2.4) and (2.5) apply for both sub and super luminal motion. The latter formula reveals that at least one of the velocities u, v or U must not exceed the speed of light, and both formulae need re-arrangement depending upon the particular values of the three velocities. For subluminal relative frame velocities v, either both u and U are subluminal or both are superluminal, while for superluminal frame velocities one of u and U is subluminal and one is superluminal.

We further comment that independently and approximately at the same time, but by entirely different formulations, both Hill and Cox [2] and Vieira [4] generated precisely the same extended Lorentz transformations applicable to superluminal motions. The approach adopted in [2] is mathematically motivated, while that in [4] is physically based following the numerous standard derivations of the Lorentz transformations of special relativity such as those cited above ([7–10]). The almost simultaneous appearance of the same extended Lorentz transformations for superluminal motion is entirely positive, in that there is some commonality of agreement in the basic equations underlying superluminal motion. Nevertheless, other authors have raised certain difficulties such as Jin and Lazar [11] who raise questions relating to the isotropy of space and to the sign of the space–time Jacobian. Vieira [4] also argues that the proposed extended Lorentz transformations exhibit certain difficulties when attempting to generalise to four dimensions (three space and one time) and quite recently Andreka et al. [12] have confirmed this. In response to such criticism, we may only remark that the physics underpinning superluminal motion has yet to be established, and in the absence of experimental evidence, it remains unclear as to which of the existing principles might apply, and which might only be applicable in a subluminal world.

3. de Broglie's particle and wave duality

All matter exhibits wave-like behaviour, so that for example light possesses a dual character, in the sense that in some respects it behaves like a collection of particles, called photons, and in other respects like a wave. The same is true for electrons and other elementary particles, and indeed quantum mechanics associates waves with every kind of elementary particle, such that the energy and momentum E and p respectively are linked with the wavelength λ and frequency ν through the relations $\lambda = h/p$ and $\nu = E/h$ where h is the Planck constant. The wave or phase velocity $u_p = \lambda\nu$, and from these elementary relations we may readily deduce the de Broglie formula $u_g u_p = c^2$, connecting the particle or group velocity of the wave u_g with the wave or phase velocity u_p. Further, Guemez et al. [5] have recently provided the special relativistic four vector extension of the de Broglie relation valid for three spatial dimensions as simply the scalar product $\mathbf{u_g} . \mathbf{u_p} = c^2$.

In two papers Hill and Cox [2,3] have proposed an extension of special relativity to incorporate superluminal velocities for which the Einstein law for the addition of velocities (2.3) still applies, and in particular for infinite relative frame velocity $v \to \infty$ there follows from (2.3) the relation $uU = c^2$, which is formally equivalent to that proposed by de Broglie $uu' = c^2$, arising from quantum mechanics and connecting the particle and wave speeds u and u' respectively.

Although the de Broglie relation originally arose from quantum mechanical considerations, it formally arises from the underlying space–time transformation $x' = ct$ and $t' = x/c$, for which $u' = dx'/dt' = c^2 dt/dx = c^2/u$. This space–time transformation is formally equivalent to the limiting extended Lorentz transformation in the limit $v \to \infty$ (see Hill and Cox [3]) and has been widely used to connect the Galilean and Carroll transformations as significant limits of Lorentz invariant theories, for example in electromagnetism. The Carrollian transformations were originally introduced by Jean-Marc Levy-Leblond and their origin and development is fully detailed by Rousseaux [13] and Houlrik and Rousseaux [14] including extensive referencing.

In [2,3] it is proposed that if with respect to some fixed frame, all subluminal frames are grouped together S_{sub} and all superluminal frames are grouped together S_{sup}, then in both special relativity and the proposed extension of special relativity for speeds beyond the speed of light, there is no objective test that might distinguish between S_{sub} and S_{sup}. If this were true then it would imply that all the basic mechanical laws should not only be Lorentz invariant, but in addition should be invariant with respect to the space–time transformation $x' = ct$ and $t' = x/c$. Indeed, in the context of special relativity, the inference is that both the $v = 0$ and $v = \infty$ data might play equally important roles; namely

$$x = X, \quad t = T, \quad u = U, \quad v = 0, \tag{3.1}$$

and

$$x = cT, \quad t = X/c, \quad uU = c^2, \quad v = \infty. \tag{3.2}$$

The symmetry and equity in this approach then leads us to speculate that moving particles are in fact subject to two forces; namely a force f that we recognise as arising from the particle (Newtonian) $v = 0$ perspective, and a force f' as arising from the wave or superluminal $v = \infty$ perspective, so that

$$f = \frac{d(mu)}{dt}, \quad f' = \frac{d(m'u')}{dt'}, \tag{3.3}$$

where the primed variables are such that $x' = ct, t' = x/c$, $u' = dx'/dt' = c^2/u$, and $m(u)$ and $m'(u')$ are certain perceived sub and superluminal masses. In order to propose a mass $m'(u')$ associated with the superluminal wave phase, the guiding principle is that both phases might contribute symmetrically as viewed from either perspective. Accordingly, if in the particle phase we assume the Einstein formula $m(u) = m_0[1-(u/c)^2]^{-1/2}$, then bearing in mind the formal calculation leading to (1.1), we are led to propose the following Einstein expressions

$$m(u) = \frac{m_0}{[1-(u/c)^2]^{1/2}}, \quad m'(u') = \frac{p_\infty/c}{[(u'/c)^2-1]^{1/2}}, \tag{3.4}$$

where m_0 and p_∞ denote respectively the constant rest mass and the constant limiting momentum at infinite velocity, and assuming that u is subluminal so that u' is superluminal through the relation $uu' = c^2$. Of course, it is not usual to identify a mass associated with the wave phase, but it is accepted to assign an associated momentum and energy, so formally we might first propose a momentum $p'(u') = p_\infty/cu'/[(u'/c)^2-1]^{1/2}$ that is associated with the wave phase, and then define the associated mass as given by p'/u'; and such an approach is formally identical to that described above. Noting also that the superluminal relation implies that the mass $m'(u')$ is necessarily zero as $u' \to \infty$ while the limiting momentum remains finite. This means physically that for small u, the effective mass as defined below, is virtually the Newtonian mass since in these circumstances $m'(u')$ is negligible.

We now propose that the actual physical quantities such as mass m^*, momentum p^* and force f^* arise as the sum of the particle (Newtonian) and wave contributions, namely

$$m^* = m(u) + m'(u'),$$

$$p^* = p(u) + p'(u'),$$

$$f^* = f(u) + f'(u') = \frac{d(mu)}{dt} + \frac{d(m'u')}{dt'}, \tag{3.5}$$

where we use $*$ to designate the observed physical quantity. We further assume that any increase in energy or work done arises in consequence of both the sub and superluminal worlds; thus

$$d\mathcal{E}^* = fdx + f'dx', \tag{3.6}$$

which on using the force expressions (3.3) can be re-written as the rate-of-work equation

$$\frac{d\mathcal{E}^*}{dt} = u\frac{d(mu)}{dt} + u'\frac{d(m'u')}{dt}. \tag{3.7}$$

If we now assume that $\mathcal{E}^* = (m+m')c^2$ where m and m' are given by (3.4) and $uu' = c^2$, then by performing all the various time differentiations in (3.7) it is a straightforward matter to show that the equation is properly satisfied, and for $0 \leqslant u < c$ we may readily deduce the expression

$$\mathcal{E}^* = \frac{(m_0c^2 + p_\infty u)}{[1-(u/c)^2]^{1/2}} + \mathcal{E}_0^*, \tag{3.8}$$

where $\mathcal{E}_0^*$ denotes an arbitrary additive constant. Alternatively, for superluminal $c < u' < \infty$ we may deduce the expression

$$\mathcal{E}^* = \frac{(p_\infty + m_0u')c}{[(u'/c)^2-1]^{1/2}} + \mathcal{E}_0^*, \tag{3.9}$$

and both of (3.8) and (3.9) might be modified to be symmetric for negative velocities by replacement of u and u' with $|u|$ and $|u'|$ respectively. Strictly speaking this means that the new term is a generalised function, and the resulting energy function although continuous at zero velocity, nevertheless admits a jump in the gradient of the energy at zero velocity. Although, the physics behind (3.5) for the net force is non-conventional, surprisingly the mathematics for a relativistic mechanics problem is not greatly changed. For example, for $0 \leqslant u < c$ we have from (3.5) and the formal relation $m'(u') = (p_\infty/m_0c^2)m(u)u$ that the net force (or equivalently the net rate-of-change of momentum) becomes

$$\begin{aligned} f^* &= \frac{d(mu)}{dt} + \frac{d(m'u')}{dt'} = \frac{d(mu)}{dt} + \frac{p_\infty c}{m_0 u}\frac{dm}{dt} \\ &= \frac{(m_0 + p_\infty/c)}{[1-(u/c)^2]^{3/2}}\frac{du}{dt}, \end{aligned} \tag{3.10}$$

and (3.10) differs from the conventional formula only by the constant factor $(m_0 + p_\infty/c)$. This means that many of the established relativistic mechanics calculations might be readily extended.

Formally, we may give an extension of the above theory to three spatial dimensions (x, y, z) through the three dimensional extension of the de Broglie's formula, namely $\mathbf{u_g}.\mathbf{u_p} = c^2$ due to Guemez et al. [5]. Using the obvious abbreviated formalism $\mathbf{u}.\mathbf{u}' = c^2$, one possible coordinate decomposition of this formula might be $\mathbf{r}' = ct\mathbf{r}/r$ and $t' = r/c$, where $\mathbf{r}$ and $\mathbf{r}'$ are the obvious position vectors and $r = (x^2+y^2+z^2)^{1/2}$. Noting especially the identical inverse transformations $\mathbf{r} = ct'\mathbf{r}'/r'$ and $t = r'/c$ where $r' = (x'^2+y'^2+z'^2)^{1/2}$; and that this coordinate decomposition, although apparently a natural extension of the one dimensional coordinate transformation $x' = ct$ and $t' = x/c$ for $uu' = c^2$ may not be unique. Indeed, the one dimensional transformation itself may not provide a unique decomposition of the equation $uu' = c^2$. The coordinate transformation $\mathbf{r}' = ct\mathbf{r}/r$ might well apply in a predominantly Friedman-LeMaitre universe in which there is a preferred reference frame in which the cosmic microwave background is isotropic, since the transformation is spatially spherically symmetric and polar angles remain unchanged and $r' = ct$ and $t' = r/c$. Accordingly, the transformation $\mathbf{r}' = ct\mathbf{r}/r$ is essentially one dimensional, and the above Eqs. (3.4)–(3.10) can be readily duplicated as the basis for a fully three dimensional version of the theory proposed here.

Of course it is very hard to formally reconcile a space–time reciprocity between a particle and its tachyonic partner as in (3.2) when there are three space dimensions and only one time dimension, and which does not avoid the fact that we are embedded in more than one space dimension. An alternative approach might be to simply propose, in the spirit of full reciprocity, that the partner tachyonic world has three time dimensions and only one space dimension.

In the following section we deal with the special case of photons travelling at the speed of light for which we need a slightly more general extension of the above formulae which may be readily verified, namely

$$m(u) = \frac{n_0}{[1-(u/c)^2]^{1/2}} + \alpha, \quad m'(u') = \frac{q_\infty/c}{[(u'/c)^2-1]^{1/2}} + \beta,$$

$$\mathcal{E}^* = \frac{n_0c^2}{[1-(u/c)^2]^{1/2}} + \frac{q_\infty c}{[(u'/c)^2-1]^{1/2}} + \frac{\alpha u^2}{2} + \frac{\beta u'^2}{2} + \mathcal{E}_0^*,$$

where α and β denote arbitrary constants, and the constants m_0 and p_∞ have been purposely changed to n_0 and q_∞ respectively to reflect the fact that their prior meanings might be altered with the presence of the new constants α and β. We note that the new terms are reminiscent of the classical kinetic energy terms, and that these slightly more general expressions appear not to change the above significantly, noting especially that if we restrict attention to $0 \leqslant u \leqslant c$ then it is immediately apparent that both $q_\infty = -n_0c$, and the new constant β is zero, for the energy expression to remain finite at both end-points of the interval.

4. Light and photons

In this section, we apply the theory of the previous section to the case of photons capable of travelling at the speed of light. In special relativity, the rest-mass of a photon is assigned to be zero so that Einstein's formula $m(u) = m_0[1-(u/c)^2]^{-1/2}$ is sensible in the limit of the velocity u tending to the velocity of light c. Further, conventional special relativity for light and photons uses the formula $\mathcal{E}^2 = (pc)^2 + (m_0c^2)^2$ and $m_0 = 0$, to assign the limiting values $p = h\nu/c$ and $\mathcal{E} = h\nu$ where ν denotes the frequency and h is the usual Planck constant. Here we attempt to duplicate these conventionally accepted characteristics in the context of the above theory, to produce alternative formulae with a non-zero photon rest-mass.

For $0 \leqslant u \leqslant c$, we have from (3.4) that a combined mass $m^*(u)$ might be given by

$$m^*(u) = m(u) + m'(u') = \frac{(n_0 + (q_\infty/c^2)u)}{[1-(u/c)^2]^{1/2}} + \alpha, \tag{4.1}$$

and for this expression to remain finite at $u = c$ we require that $q_\infty = -n_0c$, in which case (4.1) becomes

$$m^*(u) = m(u) + m'(u') = n_0\left(\frac{1-u/c}{1+u/c}\right)^{1/2} + \alpha, \tag{4.2}$$

and with a corresponding combined momentum

$$p^*(u) = m(u)u + m'(u')u' = -n_0c\left(\frac{1-u/c}{1+u/c}\right)^{1/2} + \alpha u. \tag{4.3}$$

Further, with $q_\infty = -n_0c$, we have from the equation corresponding to (3.8) that the energy expression becomes

$$\mathcal{E}^* = n_0c^2\left(\frac{1-u/c}{1+u/c}\right)^{1/2} + \frac{\alpha u^2}{2} + \mathcal{E}_0^*. \tag{4.4}$$

The above two Eqs. (4.2) and (4.3) admit the following expansions for small u:

$$p^*(u) = -n_0c^2\left[1 - \frac{u}{c} + \frac{1}{2}\left(\frac{u}{c}\right)^2 + \ldots\ldots\ldots\right] + \alpha u, \tag{4.5}$$

$$\mathcal{E}^* = n_0c^2\left[1 - \frac{u}{c} + \frac{1}{2}\left(\frac{u}{c}\right)^2 + \ldots\ldots\ldots\right] + \frac{\alpha u^2}{2} + \mathcal{E}_0^*, \tag{4.6}$$

from which we observe that the appropriate linear momentum m_0u and the correct classical kinetic energy expression $m_0u^2/2$ emerge since from (4.2) the rest-mass $m_0 = n_0 + \alpha$.

Using the formula $\mathcal{E}^2 = (pc)^2 + (m_0c^2)^2$, conventional special relativity for light and photons sets the value $m_0 = 0$, and assigns the limiting values $p = h\nu/c$ and $\mathcal{E} = h\nu$ where h is the usual Planck constant. Here we adopt the same values $p(c) = h\nu/c$ and $\mathcal{E}^*(c) = h\nu$, which yield from (4.3) and (4.4) the values

$$\alpha = h\nu/c^2 \qquad \mathcal{E}_0^* = h\nu/2, \tag{4.7}$$

which altogether from (4.4) provides a rest-mass energy

$$\mathcal{E}^*(0) = n_0c^2 + h\nu/2 = m_0c^2 - h\nu/2, \tag{4.8}$$

on using $m_0 = n_0 + \alpha$. For conventional special relativity, $\mathcal{E} = mc^2$ for light gives the rest-mass energy to be zero, since for light m_0 is assumed to be zero. In order to be consistent with special relativity for light, we propose here assigning the value zero for the datum energy of light. We note however, that the constants of the above theory may be alternatively prescribed, and that the choice of a datum energy is arbitrary. In the above we have followed a strategy that attempts to mimic presently accepted features of light and photons. The choice of zero datum energy for light predicts an associated photon rest-mass $m_0 = h\nu/2c^2$, giving a numerical value for m_0 of about 2.22×10^{-36} kg for typical light frequencies in the range $4-8 \times 10^{14}$ Hz.

In summary, we have the following formulae for $0 \leqslant u \leqslant c$:

$$m^*(u) = \frac{h\nu}{c^2}\left[1 - \frac{1}{2}\left(\frac{1-u/c}{1+u/c}\right)^{1/2}\right],$$

$$p^*(u) = \frac{h\nu}{c}\left[\left(\frac{u}{c}\right) + \frac{1}{2}\left(\frac{1-u/c}{1+u/c}\right)^{1/2}\right], \tag{4.9}$$

$$\mathcal{E}^*(u) = \frac{h\nu}{2}\left[1 + \left(\frac{u}{c}\right)^2 - \left(\frac{1-u/c}{1+u/c}\right)^{1/2}\right],$$

so that $p^* + m^*c = 2m_0(u+c), \mathcal{E}^* + p^*c = m_0(u+c)^2$, noting especially the plus sign in the left-hand sides of these latter two relations, and that the theory predicts non-zero momentum $p^*(0) = m_0c$ at zero velocity. Further, as previously noted for the above momentum and energy solutions (4.5) and (4.6) the correspondence principle applies and the conventional rest-mass momentum m_0u and rest-mass kinetic energy $m_0u^2/2$ are obtained for small u/c. However, the first term in the expansion (4.5) reflects the non-zero momentum at zero velocity, and the second term of (4.6) is non-standard. All these are purposeful features of a new theory which is the combination of two conventional theories, indicating unusual physics and the existence of a contribution from the wave phase even in circumstances in which this would not be expected.

Finally in this section, we comment that clearly the square-root terms in the above equations are reminiscent of the Doppler effect (see for example French [15], page 134), so this topic is discussed in the following section.

5. Relation with Doppler effect

For a light source of frequency N_0 situated at the origin of the (X, T) frame the frequency ν observed from the moving frame (x, t) is well known (French [15], page 134) to be given by the equation

$$\nu = N_0\left(\frac{1-v/c}{1+v/c}\right)^{1/2}, \tag{5.1}$$

while if a light source of frequency ν_0 is located at the origin of the moving frame (x, t), then the observed frequency N from the stationary frame is given by

$$N = \nu_0 \left(\frac{1 + v/c}{1 - v/c} \right)^{1/2}, \tag{5.2}$$

and evidently (5.2) arises from (5.1) simply by reversing the sign of the velocity v. These two results may be formally deduced from the two invariants

$$\frac{x - ut}{\left(1 - (u/c)^2\right)^{1/2}} = \frac{X - UT}{\left(1 - (U/c)^2\right)^{1/2}},$$

$$\frac{t - ux/c^2}{\left(1 - (u/c)^2\right)^{1/2}} = \frac{T - UX/c^2}{\left(1 - (U/c)^2\right)^{1/2}}, \tag{5.3}$$

which may be verified by direct substitution of (2.1) into the left-hand sides, and noting that the first arises from the second

$$\frac{t' - u'x'/c^2}{\left(1 - (u'/c)^2\right)^{1/2}} = \frac{T' - U'X'/c^2}{\left(1 - (U'/c)^2\right)^{1/2}},$$

and vice versa on using the relations $x' = ct, t' = x/c, u' = c^2/u$. From the condition given by Moller [16] (page 57) that the phase in either the unprimed or primed coordinate systems must be an invariant, thus

$$\nu\left(t - \frac{x}{u'}\right) = N\left(T - \frac{X}{U'}\right), \quad \nu'\left(t' - \frac{x'}{u}\right) = N'\left(T' - \frac{X'}{U}\right),$$

and using the elementary relations $x' = ct, t' = x/c, u' = c^2/u$ and corresponding results for (X, T) these relations can be shown to become

$$\nu\left(t - \frac{xu}{c^2}\right) = N\left(T - \frac{XU}{c^2}\right), \quad \nu'u'\left(t - \frac{x}{u'}\right) = N'U'\left(T - \frac{X}{U'}\right).$$

On using (5.3), (2.3) and (2.4) these two relations become simply

$$\frac{\nu}{N} = \left(\frac{1 - (U/c)^2}{1 - (u/c)^2} \right)^{1/2} = \frac{(1 - Uv/c^2)}{(1 - (v/c)^2)^{1/2}}, \tag{5.4}$$

$$\frac{\nu'}{N'} = \frac{\nu u}{NU} = \frac{(1 - Uv/c^2)}{(1 - (v/c)^2)^{1/2}} \frac{(U - v)}{U(1 - Uv/c^2)},$$

and the latter equation readily simplifies to yield the anticipated fully symmetric relation

$$\frac{\nu'}{N'} = \frac{(1 - v/U)}{(1 - (v/c)^2)^{1/2}} = \frac{(1 - U'v/c^2)}{(1 - (v/c)^2)^{1/2}}.$$

If now a source of light of frequency N_0 is situated at the origin of the (X, T) frame then for an observer moving away from the source, the frequency ν as observed from the moving (x, t) frame (5.1) emerges immediately from (5.4) noting that in this situation $u = U = c$. Alternatively, if the light source of frequency ν_0 is located at the origin of the moving frame (x, t) then for an observer moving towards the source, (5.2) also emerges from (5.4) noting again that $u = U = c$.

It is not altogether clear why the term arising in Eqs. (4.2), (4.3) and (4.4) also arises in the Doppler Eq. (5.1), noting especially that the latter equation is kinematic in nature while Eqs. (4.2), (4.3) and (4.4) arise from a mechanical proposal. It is however, very interesting that the Einstein formula $m(u) = m_0[1 - (u/c)^2]^{-1/2}$ and the critical term in these equations formally emerge from appropriate limits of the inclined Doppler equation (see for example Resnick [17], page 89); thus

$$\nu = N_0 \frac{(1 - (v/c)\cos\theta)}{(1 - (v/c)^2)^{1/2}},$$

from the respective limits $\theta = \pi/2$ and $\theta = 0$.

6. Conclusions

In special relativity, the two formulae $\mathcal{E} = mc^2$ and $m(u) = m_0[1 - (u/c)^2]^{-1/2}$ describe energy and mass of a moving particle as a function of its velocity u in terms of a single arbitrary constant m_0, termed the rest mass. While these formulae have been thoroughly successful locally, the dark matter and dark energy issues indicate that on a cosmological scale, perhaps an essential ingredient is lacking. The critical speed of light c acts as a hinge about which the sub and super worlds turn, such that the subluminal low velocity Newtonian or particle world relates to the high velocity superluminal wave world, and we may visualise a symmetrically folded sheet with the fold corresponding to the speed of light c and the sub and super worlds on either side of the fold, and any prescribed data at one edge of the sheet is inherited at the other edge of the sheet.

Motivated by the parity and symmetry operating in the sub and superluminal worlds S_{sub} and S_{sup} as first proposed by Hill and Cox [3], we have suggested that both sets of data (3.1) and (3.2) play equally vital roles, and that in any modification of the Einstein theory, the superluminal world must be involved in an equitable manner. Accordingly, we have proposed that associated with the de Broglie's particle-wave duality, we might assign a superluminal mass m' and momentum $m'(u')u'$ that are coupled with the superluminal wave speed u'. In addition, proceeding formally and based on the rate-of-work Eq. (3.6), we have verified that the natural and obvious modification of Einstein's famous formula, namely $\mathcal{E} = (m + m')c^2$ applies. We have purposely adopted an entirely formal approach, since firstly we recognise the success of the Einstein theory locally, and secondly we require that any modification be invariant not just with respect to the Lorentz group, but also with respect to the underlying space–time transformation $x' = ct$ and $t' = x/c$, that connects the sub and superluminal worlds. Together with the four vector formulation of the de Broglie relation given by Guemez et al. [5], the approach presented here presents a particular generalisation to three spatial dimensions.

If in (3.8) the ratio p_∞/m_0c is small, then for the experimental data that is used to confirm the validity of $\mathcal{E} = mc^2$, the new term will not materially alter the outcome. However, on a cosmological scale the new term involving p_∞ might easily generate the magnitude of dark energies that are observed. In order to get some idea of the additional energy that is predicted by these expressions, we might for example, evaluate the ratio of a measure of the total energies; thus

$$\frac{\int_0^c (m + m')c^2\, du}{\int_0^c mc^2\, du} = 1 + \frac{2p_\infty}{\pi m_0 c}, \tag{6.1}$$

which is evidently indicative only, but nevertheless from (6.1) it is apparent that the ratio p_∞/m_0c might well be chosen to give the order of magnitude of additional energy that is actually observed in the universe.

At first sight, the approach might well appear naive and an over simplification of a complex physical situation, and the notion of assigning mass and momentum characteristics to a wave might well fly in the face of conventional physical wisdom. However, as we have already stated, we know that for moderate relative velocities the Einstein theory gives excellent agreement with experiment, and therefore any new theory must incorporate this fact. In addition, we believe that the theory must reflect equally the particle and wave formulations, so that the physical world looks the same from either perspective.

We emphasise that the simple additive formula $\mathcal{E} = (m + m')c^2$ arises from entirely different physics to calculating in the classical manner the energy corresponding to the mass $m + m'$. This latter calculation appears as an illustrative example in [2] and the derived energy expression given there is quite different to that proposed here. The formula $\mathcal{E} = (m + m')c^2$ results from non-conventional physics using non-conventional equations, and the elegance of this result in itself is suggestive of the veracity of the approach adopted here.

The existence of the simple explicit mathematical solution summarized by (4.9) is a non-trivial consequence of the theory that predicts a non-zero photon rest-mass $m_0 = h\nu/2c^2$, where h is Planck's constant and ν is the light frequency. This gives a numerical value for m_0 of about 2.22×10^{-36} kg for typical light frequencies in the range $4 - 8 \times 10^{14}$ Hz. As previously noted, the quantity $h\nu/2$ is known as the zero-point energy or the ground state energy, and is thought to be the lowest possible energy state of a quantum mechanical system. We have noted the connection of the formula and the Einstein expression $m(u) = m_0[1 - (u/c)^2]^{-1/2}$ with the relativistic Doppler equation, and that the critical expressions both emerge from appropriate limits of the inclined Doppler equation.

In terms of future outstanding issues, we have not answered the question as to whether the new constant p_∞ might involve an additional invariant arising from the group-theoretic basis of the emergence of mass and spin as a full set of fundamental Casimir invariants within irreducible unitary representations of the inhomogeneous Lorentz group, as first described in the 1940s by Wigner ([18]), and possibly arising from a larger group, that includes the Lorentz group, space–time translations, plus space–time reciprocity and perhaps more. Further, if de Broglie's classical theory is to hold its ground within a space–time reciprocal tachyonic theory, then there is possibly a connection with Bohm's pilot-wave extension of de Broglie's classical matter wave, which has been gaining attention recently (see for example [19]) but we have not attempted to make that connection here.

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