Results in Physics 10 (2018) 94–98

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Construction of lump soliton and mixed lump stripe solutions of (3+1)-dimensional soliton equation

Jiangen Liu, Yufeng Zhang*

School of Mathematics, China University of Mining and Technology, Xuzhou, Jiangsu 221116, People's Republic of China

ARTICLE INFO

Keywords:
Lump solutions
Mixed lump strip solutions
(3+1)-Dimensional soliton equation
Hirota bilinear form

ABSTRACT

In this letter, we apply two different ansatzs for constructing the lump soliton and mixed lump strip solutions of (3+1)-dimensional soliton equation, which is associating with the Hirota bilinear form. These lump soliton solutions rationally localized in all directions in the space. The solutions of interactions between a lump and a stripe are shown by graphic illustration of some special solutions which would give us a better understanding on the evolution of solutions of waves.

Introduction

In soliton theory [1–7], exact solutions, integrable systems, Painleve analysis and Hamiltonian structure are the hot topics. And the exact solutions of mathematical equations play a vital role in the proper understanding on qualitative features of the concerned phenomena and processes in nonlinear science, such as nonlinear optics, plasma physics and others. Deriving an exact solution of nonlinear partial differential equations (PDEs) is important, which could help us understand the complexity of the phenomena based on integer or fractional order derivatives. And the exact solution could also help us to analyze the stability of these systems and validate the results of numerical analysis in nonlinear PDEs. In recent years, localizing in all directions in the space, the lump solution attracted much attention [8–10], which is a kind rational function solution [11–13]. There are some recent studies on the topic have been obtained on lump solutions, lump-soliton solutions and its interactions, and others [14–19]. The Kadomtsev-Petviashvili equation [20] have been found to possess lump solutions. And lump solutions can be obtained through the Hirota bilinear form, which plays an important role in mathematical physics and engineering fields, or their generalized counterparts. Once a nonlinear equation is written in bilinear form by a dependent variable transformation, multi-soliton solutions, rational solutions, Wronskian and Pfaffan forms of N-soliton solution can be obtained [21–23].

In this paper, we consider the (3+1)-dimensional soliton equation [24] as follows,

$$3u_{xz}-(2u_t+u_{xxx}-2uu_x)_y+2(u_x\partial_x^{-1}u_y)_x=0, \tag{1}$$

under the transformation

$$u=-3(\ln f)_{xx}, \tag{2}$$

we can change the (3+1)-dimensional soliton equation into the bilinear form

$$(3D_xD_z-2D_yD_t-D_yD_x^3)f\bullet f=0, \tag{3}$$

Or, equivalently

$$3ff_{xz}-3f_xf_z-2ff_{yt}+2f_yf_t+f_{xxx}f_y+3f_{xxy}f_x-3f_{xx}f_{xy}-ff_{xxxy}=0. \tag{4}$$

Where $f=f(x,y,z,t)$ and the derivatives D_y,D_x,D_z are the Hirota operators [21] defined by

$$D_x^aD_t^b(f\bullet g)=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^a\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^b\times f(x,t)g(x,t)|_{x'=x,t'=t}.$$

The (3+1)-dimensional soliton Eq. (1) has been studied by many authors in recent years. for example, its algebraic-geometrical solutions have been explicitly given in the form of Riemann theta functions by using a nonlinearized method of Lax pair, the N-soliton solution and its Wronskian form of solution have been discussed and derived by using the Hirota method and Wronskian technique [23], the bilinear Backlund transformation and explicit solutions have also been obtained based on the Hirota bilinear method [25], and some periodic wave solutions have been found in Ref. [22].

The aim of this letter is to study the periodic wave solutions of (3+1)-dimensional soliton Eq. (1), which is associating with the Hirota bilinear form. And the graphic illustration of some special solutions would give us a better understanding on the evolution of solutions of waves.

* Corresponding author.
E-mail address: zhangyfcumt@163.com (Y. Zhang).

https://doi.org/10.1016/j.rinp.2018.05.022
Received 1 February 2018; Received in revised form 10 April 2018; Accepted 11 May 2018
Available online 17 May 2018

Lump solutions of the (3 + 1)-dimensional soliton equation

In this section, we consider the (3 + 1)-dimensional soliton equation, that is

$$3ff_{xz}-3f_xf_z-2ff_{yt}+2f_yf_t+f_{xxx}f_y+3f_{xxy}f_x-3f_{xx}f_{xy}-ff_{xxxy}=0. \tag{5}$$

Here, we will find the lump soliton solutions to Hirota bilinear Eq. (4) by making the following assumption:

$$f=g^2+h^2+a_{11}, \tag{6}$$

and

$$\begin{cases} g=a_1x+a_2y+a_3t+a_4z+a_5\\ h=a_6x+a_7y+a_8t+a_9z+a_{10}\end{cases} \tag{7}$$

where $a_i(1\leqslant i\leqslant 11)$ are all real parameters to be determined. To obtain the lump solutions, we notice that the conditions guaranteeing the good definiteness of f, positiveness of and localization of f in all directions in the space need to be satisfied.

To get the lump solutions, substituting (6) with (7) into Eq. (5), we get a polynomial of the variables x,y,z,t. Eliminating the coefficients of the polynomial yields a set of algebraic system in $a_i,(i=1,2,\dots,11)$. Solving this system of equations with the help of symbolic computation, we can obtain the following solutions of parameters:

Case 1.

$$\begin{aligned}&a_1=a_1,\ a_2=a_2,\ a_3=\frac{3a_1a_2a_4+3a_1a_7a_9-3a_2a_6a_9+3a_4a_6a_7}{2a_2^2+2a_7^2},\ a_4=a_4,\ a_5=a_5,\ a_6=a_6,\\ &a_8=\frac{3a_1a_2a_9-3a_1a_4a_7+3a_2a_4a_6+3a_6a_7a_9}{2a_2^2+2a_7^2},\ a_{11}=-\frac{(a_2^2+a_7^2)(a_1^2+a_6^2)(a_1a_2+a_6a_7)}{(a_1a_7-a_2a_6)(a_2a_9-a_4a_7)},\\ &a_7=a_7,\ a_9=a_9,\ a_{10}=a_{10},\end{aligned} \tag{8}$$

where $\frac{(a_1a_2+a_6a_7)}{(a_1a_7-a_2a_6)(a_2a_9-a_4a_7)}<0$ and $a_i(i=1,2,4,5,6,7,9,10)$ are arbitrary constants.

Case 2.

$$\begin{aligned}&a_1=a_1,\ a_2=a_2,\ a_3=\frac{3(a_1a_4+a_6a_9)(a_4-a_9)(a_4+a_9)}{2(a_4^2+a_9^2)a_2},\ a_4=a_4,\ a_5=a_5,\ a_7=\frac{2a_2a_9a_4}{a_4^2-a_9^2},\\ &a_6=a_6,\ a_8=-\frac{3(a_4-a_9)(a_4+a_9)(a_1a_9-a_4a_6)}{2(a_4^2+a_9^2)a_2},\ a_9=a_9,\ a_{10}=a_{10},\\ &a_{11}=\frac{(a_1^2+a_6^2)a_2(a_1a_4^2-a_1a_9^2+2a_4a_6a_9)(a_4^2+a_9^2)^2}{(a_4^2-a_9^2)a_9(2a_1a_4^3a_9+2a_1a_4a_9^3-a_4^4a_6+a_6a_9^4)},\end{aligned} \tag{9}$$

where $\frac{a_2(a_1a_4^2-a_1a_9^2+2a_4a_6a_9)}{(a_4^2-a_9^2)a_9(2a_1a_4^3a_9+2a_1a_4a_9^3-a_4^4a_6+a_6a_9^4)}>0$ and $a_i(i=1,2,4,5,6,9,10)$ are arbitrary constants.

Thus, these parameters in the **case 1** yields a class of positive quadratic solution to Eq. (5), have

$$\begin{aligned}f=&\left(\frac{t(3a_1a_2a_4+3a_1a_7a_9-3a_2a_6a_9+3a_4a_6a_7)}{2a_2^2+2a_7^2}+xa_1+ya_2+za_4+a_5\right)^2\\ &+\left(\frac{t(3a_1a_2a_9-3a_1a_4a_7+3a_2a_4a_6+3a_6a_7a_9)}{2a_2^2+2a_7^2}+xa_6+ya_7+za_9+a_{10}\right)^2\\ &-\frac{(a_2^2+a_7^2)(a_1^2+a_6^2)(a_1a_2+a_6a_7)}{(a_1a_7-a_2a_6)(a_2a_9-a_4a_7)},\end{aligned} \tag{10}$$

which, in turn, generates of lump solutions to Eq. (1) through transformation (2) as

$$u_1=\frac{12(a_1g+a_6h)^2-6(a_1^2+a_6^2)f}{f^2}, \tag{11}$$

where the function f is defined by (10), and the function g and h are given by

$$\begin{aligned}g=&\frac{t(3a_1a_2a_4+3a_1a_7a_9-3a_2a_6a_9+3a_4a_6a_7)}{2a_2^2+2a_7^2}+xa_1+ya_2+za_4+a_5,\\ h=&\frac{t(3a_1a_2a_9-3a_1a_4a_7+3a_2a_4a_6+3a_6a_7a_9)}{2a_2^2+2a_7^2}+xa_6+ya_7+za_9+a_{10}.\end{aligned} \tag{12}$$

By the same manipulation as illustrated above, we can obtain another lump solution for the **case 2**, that is

$$u_2=\frac{12(a_1g+a_6h)^2-6(a_1^2+a_6^2)f}{f^2}, \tag{13}$$

where the function f,g and h are given by

$$\begin{aligned}g=&\frac{3t(a_1a_4+a_6a_9)(a_4-a_9)(a_4+a_9)}{2(a_4^2+a_9^2)a_2}+xa_1+ya_2+za_4+a_5,\\ h=&-\frac{3t(a_4-a_9)(a_4+a_9)(a_1a_9-a_4a_6)}{2(a_4^2+a_9^2)a_2}+xa_6+2\frac{ya_2a_9a_4}{a_4^2-a_9^2}+za_9+a_{10},\\ f=&\left(\frac{3t(a_1a_4+a_6a_9)(a_4-a_9)(a_4+a_9)}{2(a_4^2+a_9^2)a_2}+xa_1+ya_2+za_4+a_5\right)^2\\ &+\left(-\frac{3t(a_4-a_9)(a_4+a_9)(a_1a_9-a_4a_6)}{2(a_4^2+a_9^2)a_2}+xa_6+\frac{2ya_2a_9a_4}{a_4^2-a_9^2}+za_9+a_{10}\right)^2\\ &+\frac{(a_1^2+a_6^2)a_2(a_1a_4^2-a_1a_9^2+2a_4a_6a_9)(a_4^2+a_9^2)^2}{(a_4^2-a_9^2)a_9(2a_1a_4^3a_9+2a_1a_4a_9^3-a_4^4a_6+a_6a_9^4)}\end{aligned} \tag{14}$$

Now we present graphic state of some special solutions which would help us be better to understand the lump solutions. At first, we give the choice of the parameters:

$a_1=1,\ a_2=1,\ a_4=1,\ a_5=1,\ a_6=2,\ a_7=3,\ a_9=1,\ a_{10}=1$, yields

$$u=600\,\frac{27t^2+120tx+180ty+100x^2+280xy+192y^2+120t+240x+352y-3372}{(45t^2+120tx+156ty+100x^2+280xy+200y^2+168t+240x+320y+3660)^2} \tag{15}$$

Their plots when $z=1$ and $t=0,10$ are depicted in 1 and 2, respectively.

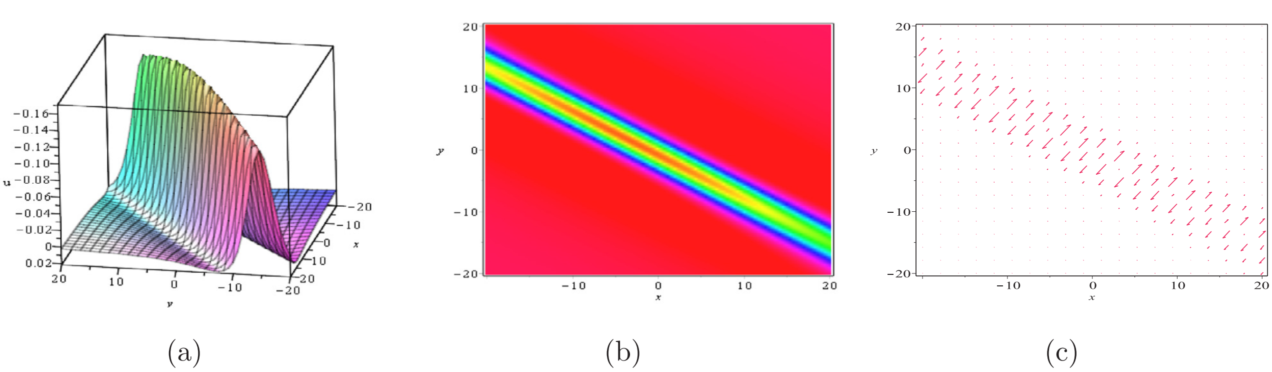


Fig. 1. This figure shows lump solution of Eq. (15) with $t=0$ (a) Perspective view of the wave. (b) 2D-Density plot. (c) 2D-gradient vector-field plot.

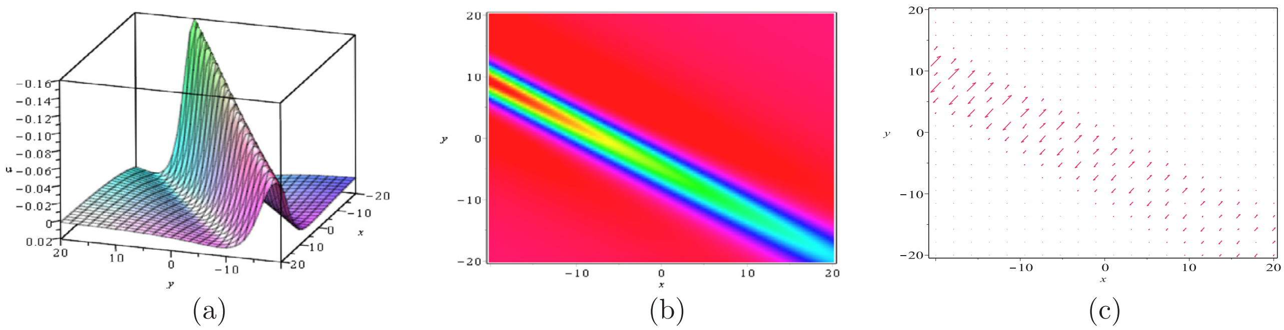


Fig. 2. This figure shows lump solution of Eq. (15) with $t = 10$ (a) Perspective view of the wave. (b) 2D-Density plot. (c) 2D-gradient vector field plot.

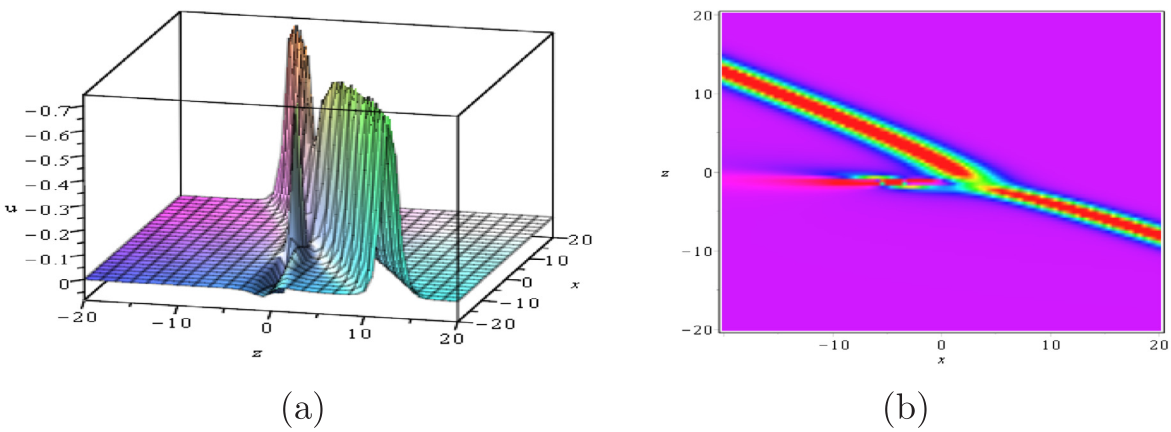


Fig. 3. This figure shows the mixed lump stripe solution of Eq. (25) with $y = 0$, $t = 0$: (a) Perspective view of the wave. (b) 2D-Density plot.

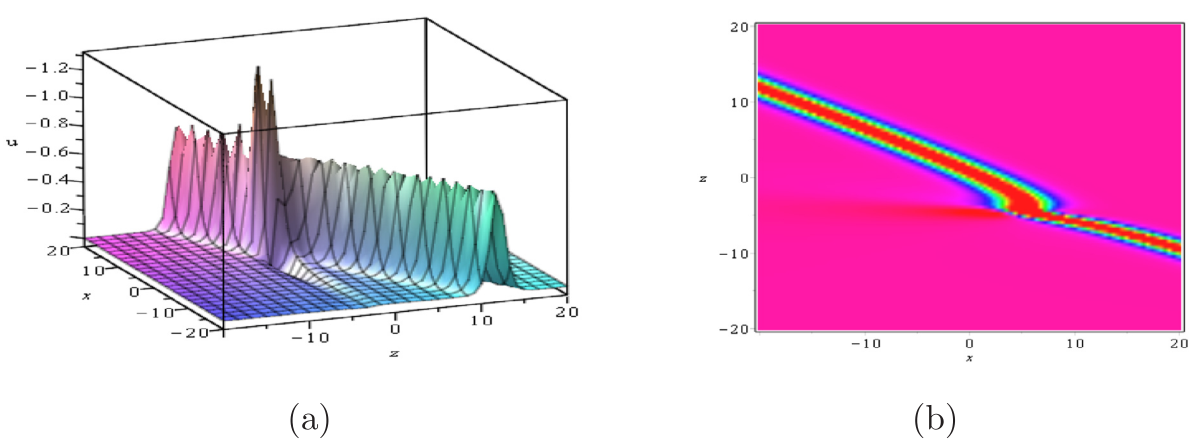


Fig. 4. This figure shows the mixed lump stripe solution of Eq. (25) with $y = 0$, $t = 8$: (a) Perspective view of the wave. (b) 2D-Density plot.

Mixed lump stripe solutions of the (3+1)-dimensional soliton equation

To get the mixed lump stripe solutions of Eq. (1), we consider

$$f = g^2 + h^2 + a_{11} + e^{tk_3+xk_1+yk_2+zk_4+k_5}, \tag{16}$$

and

$$\begin{cases} g = a_1x + a_2y + a_3t + a_4z + a_5 \\ h = a_6x + a_7y + a_8t + a_9z + a_{10} \end{cases} \tag{17}$$

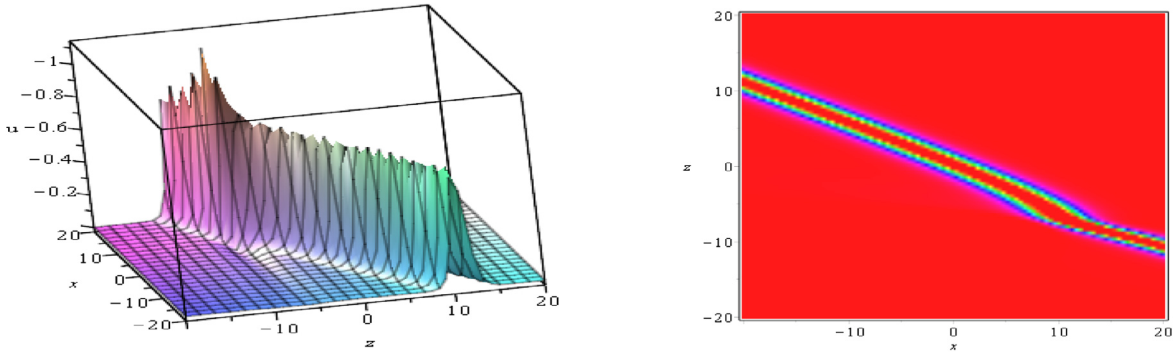
where $a_i (1 \leqslant i \leqslant 11)$ and $k_j (j = 1,2,3,4,5)$ are all real parameters to be determined.

To get the mixed lump stripe solutions, substituting (16) with (17) into Eq. (5), we get a polynomial of the variables x,y,z,t and $e^{tk_3+xk_1+yk_2+zk_4+k_5}$. Eliminating the coefficients of the polynomial yields a set of algebraic system in a_i, $(i = 1,2,...,11)$ and k_j $(j = 1,2,3,4,5)$. Solving this system of equations with the help of symbolic computation, we can obtain the following solutions of parameters:

Case 1.

$$\begin{aligned} &a_1 = \frac{a_2a_9^2k_1}{k_2(a_2^2k_1^4 + a_9^2)},\ a_2 = a_2,\ a_3 = -\frac{3a_2a_9^2k_1^3}{2k_2(a_2^2k_1^4 + a_9^2)},\ a_5 = a_5,\ a_6 = \frac{a_2{}^2a_9k_1^3}{k_2(a_2{}^2k_1^4 + a_9^2)}, \\ &a_4 = a_7 = 0,\ a_8 = \frac{3a_9{}^3k_1}{2k_2(a_2^2k_1^4 + a_9^2)},\ a_9 = a_9,\ a_{10} = a_{10},\ a_{11} = \frac{a_2{}^2a_9^2}{k_2^2(a_2{}^2k_1^4 + a_9^2)}, \\ &k_1 = k_1,\ k_2 = k_2,\ k_3 = -\frac{1}{2}k_1^3,\ k_4 = 0,\ k_5 = k_5, \end{aligned} \tag{18}$$

where k_j $(j = 1,2,5)$ and a_i $(i = 2,5,9,10)$ are arbitrary constants.



(a) (b)

Fig. 5. This figure shows the mixed lump stripe solution of Eq. (25) with $y = 0$, $t = 15$: (a) Perspective view of the wave. (b) 2D-Density plot.

Case 2.

$$a_1 = -\frac{a_2(a_2^2k_1^4-4a_9^2)k_1^3}{2k_4(a_2^2k_1^4+4a_9^2)},\ a_2 = a_2,\ a_3 = -\frac{3a_2a_9^2k_1^5}{k_4(a_2^2k_1^4+4a_9^2)},\ a_4 = 0,\ a_5 = a_5,$$
$$a_6 = \frac{2{a_2}^2a_9k_1^5}{k_4(a_2^2k_1^4+4a_9^2)},\ a_7 = 0,\ a_8 = -\frac{3a_9(a_2^2k_1^4-4a_9^2)k_1^3}{4k_4(a_2^2k_1^4+4a_9^2)},\ a_9 = a_9,\ a_{10} = a_{10},$$
$$a_{11} = -\frac{{a_2}^2(a_2^2k_1^4-4a_9^2)k_1^4}{16k_4^2a_9^2},\ k_1 = k_1,\ k_2 = 2\frac{k_4}{k_1^2},\ k_3 = \frac{1}{4}k_1^3,\ k_4 = k_4,\ k_5 = k_5, \tag{19}$$

where require $a_2^2k_1^4 < 4a_9^2, a_i$ $(i = 2,5,9,10)$, and k_j $(j = 1,4,5)$ are arbitrary constants.

Thus, these parameters in the **case 1** yields a class of positive quadratic solution to Eq. (16) as follows

$$f = \left(-\frac{3ta_2a_9^2k_1^3}{2k_2(a_2^2k_1^4+a_9^2)} + \frac{xa_2a_9^2k_1}{k_2(a_2^2k_1^4+a_9^2)} + ya_2 + a_5\right)^2$$
$$+ \left(\frac{3ta_9^3k_1}{2k_2(a_2^2k_1^4+a_9^2)} + \frac{x{a_2}^2a_9k_1^3}{k_2(a_2^2k_1^4+a_9^2)} + za_9 + a_{10}\right)^2$$
$$+ \frac{{a_2}^2a_9^2}{k_2^2(a_2^2k_1^4+a_9^2)} + e^{-1/2tk_1^3+xk_1+yk_2+k_5}, \tag{20}$$

which, in turn, generates of the mixed lump stripe solutions to Eq. (1) through transformation (2) as

$$u_2 = \frac{12(a_1g + a_6h)^2 - 6(a_1^2 + a_6^2)f}{f^2}, \tag{21}$$

where the function f is defined by (20), and the function g and h are given by

$$g = -\frac{3ta_2a_9^2k_1^3}{2k_2(a_2^2k_1^4+a_9^2)} + \frac{xa_2a_9^2k_1}{k_2(a_2^2k_1^4+a_9^2)} + ya_2 + a_5,$$
$$h = \frac{3ta_9^3k_1}{2k_2(a_2^2k_1^4+a_9^2)} + \frac{x{a_2}^2a_9k_1^3}{k_2(a_2^2k_1^4+a_9^2)} + za_9 + a_{10}. \tag{22}$$

By the same manipulation as illustrated above, we can obtain another lump solution for the **case 2**, that is

$$u_2 = \frac{12(a_1g + a_6h)^2 - 6(a_1^2 + a_6^2)f}{f^2}, \tag{23}$$

where the function f,g and h are given by

$$g = -\frac{3ta_2a_9^2k_1^5}{k_4(a_2^2k_1^4+4a_9^2)} - \frac{xa_2(a_2^2k_1^4-4a_9^2)k_1^3}{2k_4(a_2^2k_1^4+4a_9^2)} + ya_2 + a_5,$$
$$h = -\frac{3ta_9(a_2^2k_1^4-4a_9^2)k_1^3}{4k_4(a_2^2k_1^4+4a_9^2)} + \frac{2xa_2^2a_9k_1^5}{k_4(a_2^2k_1^4+4a_9^2)} + za_9 + a_{10},$$
$$f = \left(-\frac{3ta_2a_9^2k_1^5}{k_4(a_2^2k_1^4+4a_9^2)} - \frac{xa_2(a_2^2k_1^4-4a_9^2)k_1^3}{2k_4(a_2^2k_1^4+4a_9^2)} + ya_2 + a_5\right)^2$$
$$+ \left(-\frac{3ta_9(a_2^2k_1^4-4a_9^2)k_1^3}{4k_4(a_2^2k_1^4+4a_9^2)} + \frac{2xa_2^2a_9k_1^5}{k_4(a_2^2k_1^4+4a_9^2)} + za_9 + a_{10}\right)^2$$
$$- \frac{{a_2}^2(a_2^2k_1^4-4a_9^2)k_1^4}{16k_4^2a_9^2} + e^{1/4tk_1^3+xk_1+\frac{2yk_4}{k_1^2}+zk_4+k_5}. \tag{24}$$

Now, we present graphic state of some special solutions which would help better understand the lump solution (24). First, we give choice of the parameters:

$a_2 = 1$, $a_5 = 1$, $a_9 = 2$, $a_{10} = 3$, $k_1 = 1$, $k_4 = 2$, $k_5 = 1$, yields

$$u = \frac{-3\left(\frac{1}{8} + e^{\frac{1}{4}t+x+4y+2z+1}\right)}{\frac{9t^2}{16} - \frac{12ty}{17} + \frac{111t}{34} + \frac{1}{16}x^2 + \frac{15xy}{34} + \frac{39x}{34} + y^2 + 2y + \frac{2575}{256} + \frac{45tz}{17} + \frac{8xz}{17} + 4z^2 + 12z + e^{t/4+x+4y+2z+1}}$$
$$+ \frac{31\left(x/8 + \frac{15y}{34} + \frac{39}{34} + \frac{8z}{17} + e^{t/4+x+4y+2z+1}\right)^2}{\left(\frac{9t^2}{16} - \frac{12ty}{17} + \frac{111t}{34} + \frac{1}{16}x^2 + \frac{15xy}{34} + \frac{39x}{34} + y^2 + 2y + \frac{2575}{256} + \frac{45tz}{17} + \frac{8xz}{17} + 4z^2 + 12z + e^{t/4+x+4y+2z+1}\right)^2}, \tag{25}$$

First, the solution (25) evolution processes are depicted in Figs. 3–5, respectively.

Conclusion

In this paper, based on the Hirota bilinear form of the (3+1)-dimensional soliton equations, we search for lump solutions, mixed lump stripe solutions. Some obtained results are showed graphically in order to demonstrate that the method is quite effective for handling nonlinear evolution equations. Meanwhile, the performances of the mentioned methods above are substantially influential and absolutely reliable for finding new exact solutions of other NPDEs.

Acknowledgements

This work is supported by the Fundamental Research Funds for the Central Universities (No. 2017XKZD11).

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttp://dx.doi.org/10.1016/j.rinp.2018.05.022.

References

[1] Wang DS, Wei X. Integrability and exact solutions of a two-component Korteweg-de Vries system. Appl Math Lett 2016;51:60–7.
[2] Liu J, Zhang Y, Wang Y. Topological soliton solutions for three shallow water waves models. Waves Random Complex Media 2017;2:1–9.
[3] Dong H, Zhang Y, Zhang X. The new integrable symplectic map and the symmetry of integrable nonlinear lattice equation. Commun Nonlinear Sci Numer Simul 2016;36(2):354–65.
[4] Dong HH, Guo BY, Yin BS. Generalized fractional supertrace identity for Hamiltonian structure of NLS-MKdV hierarchy with self-consistent sources. Anal Math Phys 2016;6(2):199–209.
[5] Zhang Y, Fan E. Hamiltonian structure of the integrable coupling of the Jaulent-Miodek hierarchy. Phys Lett A 2006;348(3–6):180–6.
[6] Zhao Z, Zhang Y, Han Z. Symmetry analysis and conservation laws of the Drinfeld-Sokolov-Wilson system. Eur Phys J Plus 2014;129(7):1–7.
[7] Liu J, Zhang Y. Non-linear dynamics and exact solutions for the variable-coefficient modified Korteweg-de Vries equation. Z Naturforschung A 2018;73(2):143–9.
[8] Yang JY, Ma WX. Lump solutions to the BKP equation by symbolic computation. Int J Mod Phys B 2016;30(28):1640028.
[9] Ma WX, Zhou Y. Lump solutions to nonlinear partial differential equations via Hirota bilinear forms. J Differ Eq 2018;264(4):2633–59.
[10] Liu DY, Tian B, Xie XY. Lump solutions for the (3 + 1)-dimensional Kadomtsev-Petviashvili equation. Z Naturforschung A 2016.
[11] Yang JY, Ma WX. Abundant lump-type solutions of the Jimbo-Miwa equation in (3 + 1)-dimensions. Comput Math Appl 2016.
[12] Zhang Y, Ma WX. Rational solutions to a KdV-like equation. Appl Math Comput 2015;256:252–6.
[13] Zhang YF, Ma WX. A study on rational solutions to a KP-like eqation. Z Naturforschung A 2015;70:263–8.
[14] Zhang JB, Ma WX. Mixed lump-kink solutions to the BKP equation. Comput Math Appl 2017;74(3):591–6.
[15] Zhao H, Ma WX. Mixed lump-kink solutions to the KP equation. Comput Math Appl 2017;74(6):1399–405.
[16] Yang JY, Ma WX, Qin Z. Lump and lump-soliton solutions to the (2 + 1)-dimensional Ito equation. Anal Math Phys 2017:1–10.
[17] Ma WX, Yong X, Zhang HQ. Diversity of interaction solutions to the (2+ 1)-dimensional Ito equation. Comput Math Appl 2017.
[18] Yang HW, Chen X, Guo M, et al. A new ZK-BO equation for three-dimensional algebraic Rossby solitary waves and its solution as well as fission property. Nonlinear Dyn 2018;91(3):2019–32.
[19] Lu C, Fu C, Yang H. Time-fractional generalized Boussinesq equation for Rossby solitary waves with dissipation effect in stratified fluid and conservation laws as well as exact solutions. Appl Math Comput 2018;327:104–16.
[20] Ma WX. Lump solutions to the Kadomtsev-Petviashvili equation. Phys Lett A 2015;379(36):1975–8.
[21] Hirota R. Reduction of soliton equations in bilinear form. Physica D Nonlinear Phenomena 1986;18(s1–3):161–70.
[22] Liu Jiangen, Wu Pinxia, Zhang Yufeng, et al. New periodic wave solutions of (3 + 1)-dimensional soliton equation. Thermal Sci 2017;21.
[23] Geng X, Ma Y. N-soliton solution and its Wronskian form of a (3 + 1)-dimensional nonlinear evolution equation. Phys Lett A 2007;369(4):285–9.
[24] Jianping Wu, University Zhengzhou. Grammian determinant solution and pfaffianization for a (3 + 1)-dimensional soliton equation. Commun Theor Phys 2009;52(11):791–4.
[25] Jianping Wu. A Bilinear Backlund transformation and explicit solutions for a (3 + 1)-dimensional soliton equation. Chin Phys Lett 2008;25(12):4192–4.