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Non-relativistic quantum field theory of Verlinde's emergent entropic gravity

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ABSTRACT

Non relativistic quantum field theory propagators are indispensable tools in nuclear physics and condensed matter theory. Here we apply them to gravitation, that is, discuss a non relativistic quantum field theory for fermions and bosons in a Verlinde-information theory-scenario. In it, gravity becomes an emergent entropic force. In a first previous paper (Plastino and Rocca (2018)), we have shown, via statistical mechanics, that emergent gravity can indeed be thought of as an entropic force at a *classical* level. In two subsequent works (Plastino and Rocca etal (2019); Plastino and Rocca (2019)) we did the same at a *quantum level*, for bosons and fermions. Further, the two additional papers (Plastino and Rocca (2019)) discussed the first quantization of EEG for bosons and fermions, by solving the corresponding Schrödinger equations. *In the present effort* we deal with the pertinent Non-Relativistic (NR) Quantum Field Theory (QFT) corresponding to fermions and bosons' EEG. With this purpose, we use the results previously obtained in the above mentioned papers together with the formulation of the NR QFT described in the classical text-book by Fetter and Walecka. Note that we are speaking of an NR QFT because we are using Verlinde's gravitational potential (in relativistic QFT people do not use potentials).

1. Introduction

Non relativistic quantum field theory propagators are indispensable tools in nuclear physics and condensed matter theory [1]. Here we apply them to gravitation, that is, discuss a non relativistic

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quantum field theory for fermions and bosons in a Verlinde-information theory-scenario. To specify the locus of our work we can look at a cube whose sides are labeled by G, c, and $\hbar$ [see the first graph of the book [2] (Figure 1, page 13: The cube of physics)]. The locus is at the corner with $c^{-1} = 0$ and the other two constants $\neq 0$.

We emphasize that since we appeal to potentials entering Schrödinger Equation (SE), our treatment should be non-relativistic, because such is the character of SE.

1.1. Emergent entropy

Verlinde conceived the notion of linking gravity to an emerging entropic force back in (2011) [3]. His conjecture was later on proved valid (in a classical context) in [4].

In [3], gravity is conjectured to emerge as a result of information concerning the position of material bodies, conjoining a thermal treatment of gravitation with 't Hooft's celebrated holographic principle, so that gravitation ought to be viewed as an emergent phenomenon. Verlinde's notions were paid much attention to. For instance, see [5,6]. For an excellent review of the statistical mechanics of gravity consult Padmanabhan [7].

Verlinde's ideas motivated efforts in cosmology, the dark energy hypothesis, cosmological acceleration, cosmological inflation, and loop quantum gravity. The associated literature is large indeed [6]. We emphasize Guseo's work [8]. He showed that the local entropy function, linked to a logistic distribution, is a catenary and vice versa, an invariance that could be linked to Verlinde's conjecture. He also puts forward a novel interpretation of the local entropy in a system [8].

The Verlinde-concept of emerging entropic gravity (EEG) is a beautiful idea to be exploited in order to discuss gravity. In a first paper [4], we showed that Verlinde's emerging gravity is indeed an entropic force at a classical level, In two subsequent works [9,10], we did the same, at a quantum level, for bosons and fermions. Moreover, in two additional efforts [11,12], we tackled the first quantization of EEG for bosons and fermions by solving the corresponding Schrödinger equations. Here, we face the Non-Relativistic (NR) Quantum Field Theory (QFT) associated to the EEG for both types of particles. For this, we use the results previously obtained in the papers above mentioned plus the formulation of the NR QFT described in the classical text book of Fetter and Walecka's. We have taken into account the fact that the NR QFT of the EEG can become non-renormalizable both for bosons and for fermions (for the latter above the Fermi level). This inconvenience can be overcome by recourse 1) to techniques described in either [13–16], in which a complete treatment on the quantization of non-renormalizable QFT using ultrahyperfunctions is made, or 2) to the approach of [17], in which we generalize the Dimensional Regularization (DR) of Bollini and Giambiagi (BG) [18], showing that this generalization is apt to quantize non-renormalizable QFTs.

1.2. Organizing our material

In Section 2 we review relevant details of [9,10], more explicitly, Verlinde's emergent entropic forces for boson–boson and fermion–fermion gravitational interactions. We present in Section 3 the Verlinde's potentials associated to these forces, i.e., the mathematical forms for the concomitant quantum potentials, entering the Schrödinger equations employed in [11,12]. Approximate, but quite good forms for these potentials are given in Section 4. Section 5 is devoted to an explicit display of results belonging to [19], concerning non relativistic quantum field theory (NR QFT). In Section 6 we apply the results of Sections 3 and 4 so as to obtain the NR QFT of emerging entropic forces (EEG). We discuss, as examples, the calculation of the self-energy for fermions and the dressed propagator for both, bosons and fermions, to first order in perturbation theory.

2. Quantum entropic force

2.1. Fermionic entropic force

Verlinde pictures gravity as an emergent phenomenon. It derives from the quantum entanglement between small bits of space–time information [20]. Gravitation, regarded à la Verlinde as an emergent force, differs at short distances from Newton's form. The ensuing emergent gravitation-potential, when introduced into the Schrödinger equation (SE), yields quantized states. The associated energies may be viewed as novel energy-sources, not taken into account till now, with the exception of our previous treatments of Refs. [11,12].

In the present effort, we shall proceed, as we did in [10], using a statistical treatment of fermion gases. In [10] we encountered a fermion–fermion gravitational force therefrom, specifically, baryon–baryon. It turned out to be proportional to $1/r^2$ for r larger than one micron. For smaller r's, more involved contributions emerged. Thus, the pertinent potential $V_F(r)$ deviated from Newton's at short distances. The associated entropic force F_{eF} reads [10]

$$F_{eF} = \frac{4\pi\lambda k_B\,(\pi emE)^{\frac{3}{2}}}{(3N)^{\frac{3}{2}}\,h^3} r \left\{\ln\left[32\pi r^3\,(\pi emE)^{\frac{3}{2}} - (3N)^{\frac{5}{2}}h^3\right] - \ln\left[32\pi r^3\,(\pi emE)^{\frac{3}{2}}\right]\right\}. \tag{2.1}$$

This is the emergent Verlinde's gravity force between a couple of fermions, deduced in reference [10], that we use below.

2.2. Boson entropic force F_{eB}

The same procedure described above for fermions was also undertaken for bosons [9], finding a boson–boson gravitational force, that was again proportional to $1/r^2$ for distances larger than one micron. Once again, for smaller distances, novel and more involved contributions arose [9]. Accordingly, the ensuing potential $V_B(r)$ differed from the Newtonian one at short distances. We write down below the boson–boson entropic force of [9]

$$F_{eB} = \frac{4\pi\lambda k_B\,(\pi emE)^{\frac{3}{2}}}{(3N)^{\frac{3}{2}}\,h^3} r \left\{\ln\left[32\pi r^3\,(\pi emE)^{\frac{3}{2}} + (3N)^{\frac{5}{2}}h^3\right] - \ln\left[32\pi r^3\,(\pi emE)^{\frac{3}{2}}\right]\right\}. \tag{2.2}$$

This is the emergent Verlinde's gravity force between a couple of bosons, deduced in reference [9], that we use below.

3. Quantum gravitational potential $V(r)$

3.1. The gravitational potential function for N fermions of mass m

The following constants were introduced in [10]:

1. a and b of the form
2. $a = (3N)^{\frac{5}{2}}h^3$ and
3. $b = 32\pi(\pi emK)^{\frac{3}{2}}$, with
4. K the total energy of the N fermions (examples will be given below),
5. $r_2 = (a/b)^{1/3}$.
6. $A = Gm^2/r_2$.

It was shown in [10] that (G is the gravitational constant and k_B Boltzmann's constant) $\frac{\lambda 3Nk_B}{8\pi} = \frac{2}{3}Gm^2$, so that the ensuing potential energy $V_F(r)$ adopts the appearance

$$V_F(r) = -Gm^2\frac{2b}{3a}\left\{\frac{r^2}{2}\ln\left(1-\frac{a}{br^3}\right)\Theta\left[r-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right] - \frac{a^{\frac{2}{3}}}{2b^{\frac{2}{3}}}\left\{\frac{1}{2}\ln\left[\frac{\left[r-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right]^2}{r^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}r+\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right] + \sqrt{3}\left[\arctan\left[\frac{2r+\left(\frac{a}{b}\right)^{\frac{1}{3}}}{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right]-\frac{\pi}{2}\right]\right\}\right\}, \tag{3.1}$$

a central result for our present considerations ($\Theta(x)$ is the Heaviside step function).

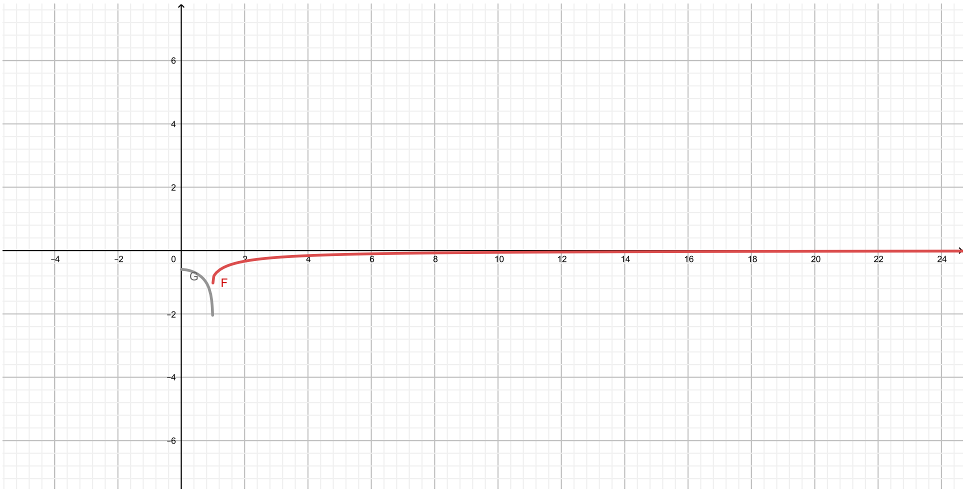

Fig. 1. Fermions. Plot of $V_F(x)/A$ (vertical axis) versus $x = r/r_2$, with $V_F \equiv V_F(r)$ as provided by (3.1). We have $F(x) = V_F(x)/A$ for $x > 1$ and $G(x) = V_F(x)/A$ for $x < 1$. Note that this discontinuity is only apparent (see estimates above). The value of A is determined in Section 3.3.

3.2. Plots

We will now plot, remembering the above definitions for r_2 and A, the potential form (dimension-less) $V_F(x)/A$ for fermions (for bosons $V_B(x)/A$) versus $x = r/r_2$ (also dimension-less). We give estimates for these two quantities in a subsection below. In it we estimate them, on the basis of the baryon-number N for the Universe and its total energy K. With these estimates one evaluates first r_2 and, afterwards, A. The ensuing potential form is plotted for fermions in Fig. 1 and for bosons in Fig. 3. These graphs are compared with their Newton's counterparts in Figs. 2 and 4.

3.3. Estimates for r_2 and A

In this subsection we will give numerical estimates for the values of r_2 and A, to be used in drawing our graphs below, both for fermions and bosons.

3.3.1. Fermions

As a typical example we give them for baryons in the universe. For them one estimates that, in the Universe one has [12] $N = 6.25 \times 10^{79}$, $K = mc^2 10^{53} Kg$, $m = 1.63 \times 10^{-27} Kg$, h is the Planck's constant, and G the gravitation constant. So they turn out: $r_2 = 5.8 \times 10^{35} m$ and $A = 8.2 \times 10^{-124} Nm$ This shows that the discontinuity of the potential corresponding to two baryons is absolutely negligible.

3.3.2. Bosons

As an example we focus attention on axions . For them one estimates that, in the Universe one has [11]: $N = 6 \times 10^{79}$, $K = mc^2 5.47 \times 10^{53} Kg$, and $m = 2.67 \times 10^{-39} Kg$ Thus, we obtain: $r_2 = 7.4 \times 10^{24} m$ and $A = 2.3 \times 10^{-91} Nm$

3.4. The gravitational potential function for bosons

The boson–boson gravitation potential $V_B(r)$, with masses $m_1 = m$ and $m_2 = M$, for an $N-$boson gas, was obtained in [9] using the micro-canonical treatment of Lemons [21]. In that paper, one dealt with identical bosons so that $m = M$. In [9], the entropy S for N bosons of total energy K

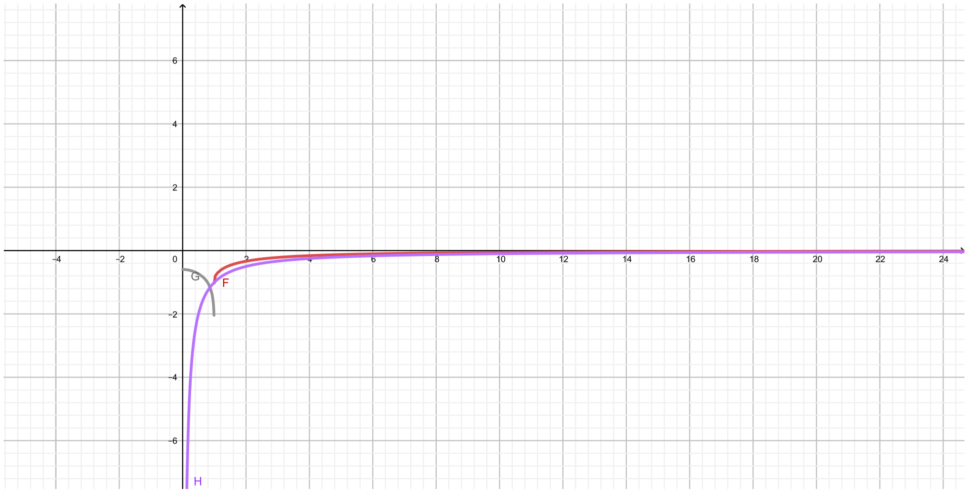

Fig. 2. Fermions. Graph of $V_F(x)/A$ vs. $x = r/r_2$, compared with the plot corresponding to Newton's gravity (in violet) $H(x) = -1/Ax$. The curve labeled G is the non-Newtonian part of $V_F(x)/A$. A is given in Section 3.3.

was derived. From S we can deduce an entropic force F_e. It, à la Verlinde, is associated to emerging gravity. The pertinent boson–boson potential $V(r)$ [9] will be discussed in this section.

In deriving $V(r)$ in [9], one defines two constants, a and b, for N bosons of total energy K, in the fashion (with k_B Boltzmann's constant, e Euler's number, and h Planck's constant)

$$a = (3N)^{\frac{5}{2}} h^3; \qquad b = 32\pi(\pi emK)^{\frac{3}{2}}. \tag{3.2}$$

The relation that defines the proportionality constant λ between F_e and the entropic gradient [9] (G is gravitation's constant) becomes

$$\lambda = 8\pi G m^2/3Nk_B. \tag{3.3}$$

It is then shown in [9] that $V(r)$ acquires the form.

$$V_B(r) = Gm^2 \frac{b}{a}\left\{\frac{r^2}{2}\ln\left(1+\frac{a}{br^3}\right) - \frac{a^{\frac{2}{3}}}{2b^{\frac{2}{3}}}\left\{\frac{1}{2}\ln\left[\frac{\left[r+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right]^2}{r^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}r+\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right] + \right.\right.$$
$$\left.\left.\sqrt{3}\left[\frac{\pi}{2} - \arctan\left[\frac{2r-\left(\frac{a}{b}\right)^{\frac{1}{3}}}{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right]\right]\right\}\right\}. \tag{3.4}$$

The ensuing potential form is plotted in Fig. 3 and compared with its Newton's counterpart in Fig. 4.

4. Taylor expansion (TA) for $V(r)$

4.1. Fermions

One cannot analytically deal with the SE for $V_F(r)$. As a consequence, we use the rigorous approximation for $V_F(r)$ given in Ref. [12]. We subdivide the r axis into four distinct regions: $0 < r < r_0$, $r_0 < r < r_1$, $r_1 < r < r_2$, and $r > r_2$. We take r_0 as 10^{-10} meters (a typical Hydrogen-atom's length), r_1 is 25 µm, and $r_2 = (a/b)^{\frac{1}{3}}$. Note that there is experimental evidence for choosing

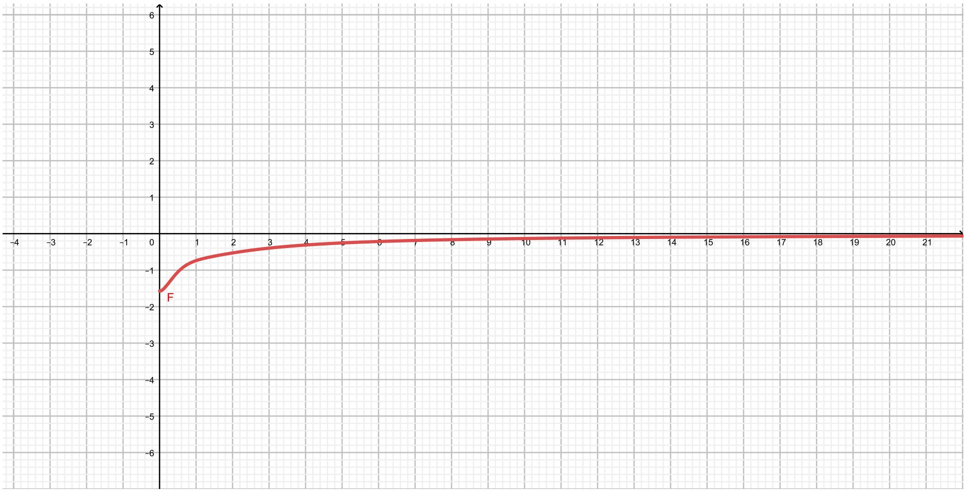

Fig. 3. Bosons. Plot of $V_B(x)/A$ vs. $x = r/r_2$. $A = Gm^2/r_2$ and $V_B \equiv V_B(r)$ is given by (3.4). A is determined in Section 3.3.

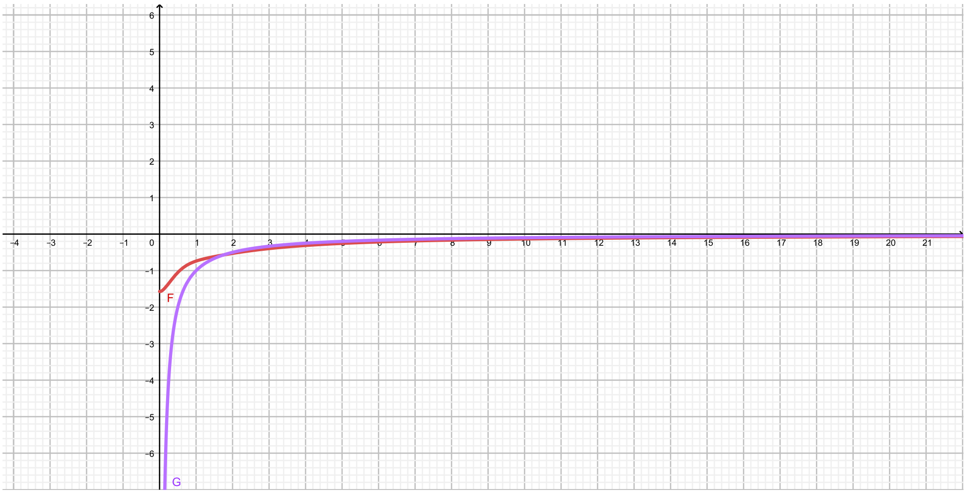

Fig. 4. Bosons. Graph of $V_B(x)/A$ vs. x, labeled F, compared with the graph corresponding to Newtonian gravity $G(x) = -1/Ax$ in violet. A is given in Section 3.3.

$r_1 = 25$ micrometers [22]. Accordingly,

$$V_F(r) \approx V_{F1}(r) + V_{F2}(r) + V_{F3}(r). + V_{F4}(r). \tag{4.1}$$

It has been proved in [12] that this approximation is exceedingly good.

It is convenient to define

$$V_{F0} = -Gm^2 \left(\frac{b}{a}\right)^{\frac{1}{3}} \frac{7\pi}{6\sqrt{3}} \tag{4.2}$$

and call V_{F1} the Taylor polynomial (TA), at zeroth order, for very small r.

$$V_{F1}(r) = -Gm^2 \left(\frac{b}{a}\right)^{\frac{1}{3}} \frac{7\pi}{6\sqrt{3}} \Theta(r_0 - r) = V_0 \Theta(r_0 - r). \tag{4.3}$$

For large r we have

$$V_{F3}(r) = -\frac{Gm^2}{r}[\Theta(r - r_1) - \Theta(r - r_2)]. \tag{4.4}$$

For intermediate $r-$ values, $r_0 < r < r_1$, the form $W(r) = 0$ was chosen in [10] for the interpolation between the two fixed points $r_1 - r_0$. Accordingly, as discussed in [10],

$$V_{F2}(r) = 0. \tag{4.5}$$

Finally, for $V_4(r)$, in [10]

$$V_{F4(r)} = -\frac{2Gm^2}{3r} \Theta(r - r_2), \tag{4.6}$$

was used.

4.2. Bosons

Schrödinger's equation with the above involved boson–boson potential is obviously not amenable for analytic dealing. For performing the exploratory study of [9], an approximate Taylor approximation was used that produced a suitable and rigorous approximation to $V_B(r)$. Thus, one had in [11]

$$V_B(r) \approx V_{B1}(r) + V_{B2}(r) + V_{B3}(r), \tag{4.7}$$

It has been proved in [11] that this approximation is exceedingly good

Here V_{B1} is the first order Taylor approach for r small enough. One does this for $0 < r < r_0$, with $r_0 = 10^{-10}$m (a typical Hydrogen-atom's length).

$$V_{B1}(r) = -\frac{\pi G m^2}{\sqrt{3}} \left(\frac{b}{a}\right)^{\frac{1}{3}} \Theta(r_0 - r) = V_{B0} \Theta(r_0 - r), \tag{4.8}$$

with $r_1 = 25.0$ micron, an empirical number [22], that minimum distance at which Newton's force that has been verified to work. The pertinent approximation for large distances of [11] is

$$V_{B3}(r) = -\frac{Gm^2}{r} \Theta(r - r_1). \tag{4.9}$$

For intermediate distances $r_0 < r < r_1$ one calls $W(r \propto) r^2$ the harmonic interpolating-form between r_1 and r_0. One had then (see details in [11])

$$V_{B2}(r) = W(r) = 0 \tag{4.10}$$

5. What we need from Fetter and Walecka's book

5.1. Preliminaries

In particular, we compute now self-energies. In quantum field theory, the energy that a particle acquires as a result of changes that it itself originates in its environment is denoted as the self-energy Σ. It represents the contribution to the particle's energy, or effective mass, due to interactions between this particle and its surrounding medium (SM). In a condensed matter environment corresponding to electrons moving in a material, Σ represents the potential felt by the electron due to the SM's interactions with it. Since electrons repel each other the moving electron polarizes the electrons in its neighborhood and then modifies the potential of the moving electron fields. These effects involve self-energy.

5.2. Dressed propagators for fermions

We will calculate here dressed propagators. For an accessible discussion of the concept see, for instance, [1]. In the book of Fetter and Walecka the authors comprehensively derived a fermion's NR QFT. For free fermions, they defined the following (current) propagator:

$$iG^0_{\alpha\beta}(\boldsymbol{x},t;\boldsymbol{x}',t') = \langle 0|T[\psi_\alpha(\boldsymbol{x},t)\psi^+_\beta(\boldsymbol{x}',t')]|0\rangle. \tag{5.1}$$

It reads

$$iG^0_{\alpha\beta}(\boldsymbol{x},t;\boldsymbol{x}',t') =$$
$$\frac{\delta_{\alpha\beta}}{(2\pi)^3}\int e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}e^{-\omega_k(t-t')}\left[\Theta(t-t')\Theta(k-k_F)-\Theta(t'-t)\Theta(k_F-k)\right]d^3k, \tag{5.2}$$

where Θ is the Heaviside's step function. We use now the well known formula

$$\Theta(t-t') = -\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{e^{-i\omega(t-t')}}{\omega+i0}d\omega, \tag{5.3}$$

and obtain

$$iG^0_{\alpha\beta}(\boldsymbol{x},t;\boldsymbol{x}',t') =$$
$$\frac{\delta_{\alpha\beta}}{(2\pi)^3}\int\int_{-\infty}^{\infty} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}e^{-\omega_k(t-t')}\left[\frac{\Theta(k-k_F)}{\omega-\omega_k+i0}-\frac{\Theta(k_F-k)}{\omega-\omega_k-i0}\right]d^3k\,d\omega. \tag{5.4}$$

Therefore, the associated expression in momentum space is

$$\hat{G}^0_{F\alpha\beta}(\boldsymbol{k},\omega) = \delta_{\alpha\beta}\left[\frac{\Theta(k-k_F)}{\omega-\omega_k+i0}+\frac{\Theta(k_F-k)}{\omega-\omega_k-i0}\right], \tag{5.5}$$

where

$$\frac{1}{\omega-\omega_k\pm i0} = PV\frac{1}{\omega-\omega_k}\mp i\pi\delta(\omega-\omega_k), \tag{5.6}$$

with $k = |\boldsymbol{k}|$ and $\omega_k = \sqrt{k^2/2m}$ (PV means "principal value of the given function"). Also, the system's interaction's Hamiltonian is defined by a two bodies V_F potential such that

$$V_F(\boldsymbol{x}_1-\boldsymbol{x}_2) = V_F(|\boldsymbol{x}_1-\boldsymbol{x}_2|)\mathbf{1}(1)\mathbf{1}(2), \tag{5.7}$$

with $\mathbf{1}$ the unity matrix. The dressed propagator of the theory satisfies

$$\hat{G}_{F\alpha\beta} = \delta_{\alpha\beta}\hat{G}_F, \tag{5.8}$$

that is, the dressed propagator is diagonal. In such a case you get ($\hat{G}^0_F(\boldsymbol{k},\omega)\equiv\hat{G}^0_F(k)$)

$$\hat{G}_F(k) = \hat{G}^0_F(k)+\hat{G}^0_F(k)\Sigma_F(k)\hat{G}^0_F(k), \tag{5.9}$$

where $\Sigma_F(k)$ is called the self-energy of the system, described above (in Preliminaries). One can then obtain its perturbative expansion. At first order we have

$$\Sigma^{(1)}_F(k)\equiv\Sigma^{(1)}(\boldsymbol{k}) = \frac{n}{\hbar}\hat{V}(0)-\frac{1}{(2\pi)^3\hbar}\int\hat{V}_F(\boldsymbol{k}-\boldsymbol{k}')\Theta(k_F-k')d^3k', \tag{5.10}$$

where $n = N/V$ and

$$\hat{V}_F(\boldsymbol{k}) = \int V_F(\boldsymbol{x})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}d^3x. \tag{5.11}$$

Thus, we have, up to first order,

$$\hat{G}^{(1)}_F(k) = \hat{G}^0_F(k)+\hat{G}^0_F(k)\Sigma^{(1)}_F(k)\hat{G}^0_F(k). \tag{5.12}$$

5.3. Dressed propagators for bosons

We appeal again to the book by Fetter and Walecka. For free bosons they define the following (usual) propagator in momentum space:

$$iG^0(\boldsymbol{x}, t; \boldsymbol{x}', t') = \langle 0|T[\phi(\boldsymbol{x}, t)\phi^+(\boldsymbol{x}', t')]|0\rangle. \tag{5.13}$$

It has the expression

$$\hat{G}^0_B(k) = \frac{1}{k_0 - \omega_k + i0}, \tag{5.14}$$

where $\omega_k = \sqrt{k^2/2m}$. For the dressed propagator we have then

$$\hat{G}_B(k) = -(2\pi)^4 n_0 i\delta(k) + \hat{G}'_B(k), \tag{5.15}$$

were the primed part refers to the noncondensate ($n_0 = N_0/V$)

$$\hat{G}'^{(1)}_B(k) = \frac{n_0}{h}\hat{G}^0_B(k)[\hat{V}_B(0) + \hat{V}_B(\boldsymbol{k})]\hat{G}^0_B(k), \tag{5.16}$$

and

$$\hat{V}_B(\boldsymbol{k}) = \int V_B(\boldsymbol{x})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}d^3x. \tag{5.17}$$

6. The non-relativistic QFT of emergent gravity

We remark that we have employed in this paper quantum potentials enter in the Schrödinger's equations, which precludes appeal to relativistic considerations.

6.1. Fermions

Our purpose in obtaining the NR QFT of the EEG is to calculate $\Sigma^{(1)}$ for the potential given in (3.1), with **1** the unity matrix,

$$V_F(r) = \left\{V_{F1}(r)\Theta(r_0 - r) - \frac{Gm^2}{r}\left[\Theta(r - r_1) - \Theta(r - r_2)\right] - \frac{2Gm^2}{3r}\Theta(r - r_2)\right\}\mathbf{1}. \tag{6.1}$$

For this purpose, we must evaluate the Fourier transform of that potential. For V_{F1} we have

$$\hat{V}_{F1}(0) = V_{F0}\int d^3x = 4\pi V_{F0}\int_0^\infty r^2 dr = 0. \tag{6.2}$$

To obtain this integral we have used the results of [23] concerning the regularization of integrals dependent on a power of x.

We now calculate the integral I_1 defined as

$$I_1 = \int_0^{2\pi}\int_0^{\pi}\int_0^{r_0} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}r^2\sin(\theta)d\phi d\theta dr =$$
$$\int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{r_2} e^{-ikr\cos(\theta)}r^2\sin(\theta)d\phi d\theta dr =$$
$$4\pi\left[\sin(kr_0)PV\frac{1}{k^3} - \cos(kr_0)PV\frac{1}{k}\right], \tag{6.3}$$

where PV means principal value. It satisfies

$$PV\frac{1}{k^n}|_{k=0} = 0. \tag{6.4}$$

Integral I_1 is the Fourier transform of the first term of (6.1). We now evaluate the integral I_2 defined as

$$I_2 = \int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{r_2} r^{-1}e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}r^2\sin(\theta)d\phi d\theta dr =$$
$$\int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{r_2} e^{-ikr\cos(\theta)}r\sin(\theta)d\phi d\theta dr =$$
$$4\pi[\cos(kr_1)-\cos(kr_2)]PV\frac{1}{k^2}. \tag{6.5}$$

Integral I_2 is the Fourier transform of the second term of (6.1).

$$I_3 = \int_0^{2\pi}\int_0^{\pi}\int_{r_2}^{\infty} r^{-1}e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}r^2\sin(\theta)d\phi d\theta dr =$$
$$\int_0^{2\pi}\int_0^{\pi}\int_{r_2}^{\infty} e^{-ikr\cos(\theta)}r\sin(\theta)d\phi d\theta dr =$$
$$4\pi\cos(kr_2)PV\frac{1}{k^2}. \tag{6.6}$$

Integral I_3 is the Fourier transform of the third term of (6.1). For the potential V we then have

$$\hat{V}_F(\boldsymbol{k}) = 4\pi\left\{V_{F0}\left[\sin(kr_0)PV\frac{1}{k^3}-\cos(kr_0)PV\frac{1}{k}\right]-\right.$$
$$\left.Gm^2[\cos(kr_1)-\frac{1}{3}\cos(kr_2)]PV\frac{1}{k^2}\right\}, \tag{6.7}$$

and as a consequence

$$\hat{V}_F(0) = 0. \tag{6.8}$$

Note that

$$\frac{1}{(2\pi)^3\hbar}\int\hat{V}_F(\boldsymbol{k}-\boldsymbol{k}')\Theta(k_F-k')d^3k' \simeq V_F(0) = -\frac{Gm^2}{r_2}\frac{7\pi}{4\sqrt{3}}, \tag{6.9}$$

where we have taken $k_F \to \infty$ to simplify the evaluation of the integral. We then obtain for the self-energy, up to first order

$$\Sigma_F^{(1)}(k) \simeq \frac{Gm^2}{r_2}\frac{7\pi}{4\sqrt{3}}\mathbf{1}. \tag{6.10}$$

We have included the unity matrix with the purpose of emphasizing the matrix character. Also, we obtain up to first order, for the dressed propagator, the expression

$$\hat{G}_F^{(1)}(k) \simeq \hat{G}_F^0(k)+\frac{Gm^2}{r_2}\frac{7\pi}{4\sqrt{3}}[\hat{G}_F^0(k)]^2. \tag{6.11}$$

When $k_F \to \infty$ we have:

$$\hat{G}^0(k) = \frac{1}{\omega-\omega_k-i0}. \tag{6.12}$$

In reference [13] it has been proved that:

$$PV\frac{1}{x^n}\delta^{(m)}(x) = \frac{(-1)^n}{2}\frac{m!}{(m+n)!}\delta^{(m+n)}(x). \tag{6.13}$$

Using then the result

$$PV\frac{1}{x^n}PV\frac{1}{x^m} = PV\frac{1}{x^{(n+m)}}, \tag{6.14}$$

we obtain

$$\frac{1}{\omega-\omega_k-i0}\frac{1}{\omega-\omega_k-i0}=\frac{1}{(\omega-\omega_k-i0)^2}, \tag{6.15}$$

so that

$$[\hat{G}^0(k)]^2=\frac{1}{(\omega-\omega_k-i0)^2}. \tag{6.16}$$

When $V\to\infty$, n finite, we obtain

$$\Sigma_F^{(1)}(k)\simeq 0, \tag{6.17}$$

and then

$$\hat{G}_F^{(1)}(k)\simeq \hat{G}_F^0(k). \tag{6.18}$$

6.2. Bosons

Our purpose in obtaining the NR QFT of the EEG is to calculate the dressed propagator for the potential given in (3.4)

$$V_B(r)=V_{B1}(r)\Theta(r_0-r)-\frac{Gm^2}{r}\Theta(r-r_1). \tag{6.19}$$

For this, we must calculate the Fourier transform of that potential. For V_{B1} we have

$$\hat{V}_{B1}(0)=V_{B0}\int d^3x=4\pi V_{B0}\int_0^\infty r^2dr=0. \tag{6.20}$$

So as to obtain this integral, we have used the result of [23] concerning the regularization of integrals dependent on a power of x.

We now calculate the integral I_1 defined as

$$I_1=\int_0^{2\pi}\int_0^{\pi}\int_0^{r_0} e^{-i\mathbf{k}\cdot\mathbf{x}}r^2\sin(\theta)d\phi d\theta dr=$$

$$\int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{r_2} e^{-ikr\cos(\theta)}r^2\sin(\theta)d\phi d\theta dr=$$

$$4\pi\left[\sin(kr_0)PV\frac{1}{k^3}-\cos(kr_0)PV\frac{1}{k}\right], \tag{6.21}$$

where PV means ”principal value of the given function”. It satisfies

$$PV\frac{1}{k^n}|_{k=0}=0. \tag{6.22}$$

Integral I_1 is the Fourier transform of the first term of (6.1). We now evaluate the integral I_2 defined as

$$I_2=\int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{\infty} r^{-1}e^{-i\mathbf{k}\cdot\mathbf{x}}r^2\sin(\theta)d\phi d\theta dr=$$

$$\int_0^{2\pi}\int_0^{\pi}\int_{r_1}^{\infty} e^{-ikr\cos(\theta)}r\sin(\theta)d\phi d\theta dr=$$

$$4\pi\cos(kr_1)PV\frac{1}{k^2}. \tag{6.23}$$

Integral I_2 is the Fourier transform of the second term of (6.1). For the potential V we then have

$$\hat{V}_B(\boldsymbol{k}) = 4\pi \left\{ V_{B0}\left[\sin(kr_0)PV\frac{1}{k^3} - \cos(kr_0)PV\frac{1}{k}\right] - Gm^2\cos(kr_1)PV\frac{1}{k^2}\right\}, \tag{6.24}$$

and, as a consequence, we obtain again

$$\hat{V}_B(0) = 0. \tag{6.25}$$

For the dressed propagator we obtain, up to first order

$$\hat{G}_B^{\prime(1)}(k) = 4\pi\frac{n_0}{h}\left\{ V_{B0}\left[\sin(kr_0)PV\frac{1}{k^3} - \cos(kr_0)PV\frac{1}{k}\right] - Gm^2\cos(kr_1)PV\frac{1}{k^2}\right\}[\hat{G}^0(k)]^2 \tag{6.26}$$

Proceeding as we did for the fermion propagator, we now have

$$[\hat{G}_B^0(k)]^2 = \frac{1}{(k_0 - \omega_k + i0)^2}. \tag{6.27}$$

7. Discussion

In this work we have constructed the non-relativistic quantum field theory (NR QFT) of emergent entropic gravitation (EEG), for pairs of fermions or bosons that interact between themselves via EEG. Our construction was based on

- the results of the book [19],
- the Verlinde's gravitational potentials obtained in [9,10].

These potentials coincide from all large distance down to atomic distances, with Newton's one. They do NOT diverge at the origin.

Our treatment generalizes the first quantization procedure of Refs. [11,12]. As examples, we have calculated the dressed propagator of the system, for both types of particles, up to first order in perturbations theory, and the self-energy for fermions, likewise.

The examples indicate that we have at our disposal, both for fermions and bosons, a viable non-relativistic quantum field theory of gravitation. Of course, we here speak only of a gravitation à la Verlinde, an emergent gravitational force, not an elementary one.

Now, if we were to regard our Verlinde-like potentials as phenomenological ones (not as derived from an underlying theory), these potentials could be viewed as quantum-generalized versions of Newton's classical one, that coincide with classical gravitation at macroscopic distances. Our graphs could be interpreted in this fashion.

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