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Multi-copy nested entanglement purification for quantum repeaters

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ABSTRACT

Photon decoherence is one of the main obstacles in global-scale quantum communication. Entanglement purification is a powerful approach to distill high quality entanglement ensembles from low quality entanglement ensembles. In this paper, we propose a multi-copy nested entanglement purification protocol (EPP). After performing the entanglement purification, we can distill a high fidelity entangled photon pair from multiple copies of low fidelity photon pairs. This protocol has several advantages. First, the double-pair noise components generated from the spontaneous parametric down conversion (SPDC) source can be automatically eliminated. Second, with the help of entanglement swapping, the distance of purified entangled state can be extended. Third, this EPP is based on current available technology. Fourth, we can obtain high fidelity by performing this EPP only once. This EPP may have potential application in future long-distance quantum communication.

1. Introduction

Entanglement plays an important role in many quantum communication protocols, such as quantum teleportation [1], quantum secret sharing (QSS) [2–4], quantum key distribution (QKD) [5],

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quantum secure direct communication (QSDC) [6–17] and some other interesting protocols [18–23]. In order to realize the global-scale quantum communication, quantum repeaters which combine entanglement generation, entanglement distribution, entanglement purification, entanglement swapping and entanglement storage are required to overcome the obstacle of environmental noise [24–29].

The environmental noise may degrade the entanglement of the quantum system to be a less-entangled pure state or a mixed state, which will decrease the fidelity of quantum teleportation or make quantum communication insecure. Entanglement concentration is a process for recovering the less-entangled pure state into the maximally entangled state [30–33]. Entanglement purification, which will be detailed here, is a process for distilling high quality entanglement from low quality entanglement ensembles [34]. In 1996, Bennett firstly proposed the concept of quantum entanglement purification [34]. Current entanglement purification protocols (EPPs) in an optical system usually can be divided into two groups. The EPPs in first group are based on linear optics [35–45] and the EPPs in the second group are based on nonlinear optics [46–55]. The advantage of the EPPs in first group is that they can be realized under current experimental condition. However, these EPPs exploit the post-selection principle. The purified high quality entangled photon states are destroyed after the photon detection and cannot be retained for further application. On the other hand, the EPPs in second group usually exploit the quantum nondemolition detection (QND), which does not destroy photons. As a result, the purified high quality entangled photon states can be retained. Moreover, such EPPs can be repeated to obtain higher fidelity entanglement. However, such QND-based EPPs are hard to realize under current experimental condition. In addition, most previous EPPs require perfect entanglement source [35,36,38–41,46,47]. In current experimental condition, the ideal entanglement source is unavailable. The spontaneous parametric down conversion (SPDC) source is commonly used, for it is currently available and easy to operate. However, the SPDC source does not work in the deterministic way, but has a probabilistic nature. Meanwhile, the SPDC source may generate double-pair noise components, which may disturb the resultant entangled state. All the above reasons hinder the practical applications of existing EPPs in quantum repeaters.

In 2017, Chen et al. proposed an architecture of nested entanglement purification [56]. In their protocol, a high fidelity entangled photon pair can be successfully distilled from two copies of low fidelity photon pairs. During the protocol, the double-pair noise components generated from the SPDC source are eliminated automatically. The purified entangled photon pairs can be used for future scalable entanglement connections to extend the communication distance. Inspired by their work, in this paper, we propose a multi-copy nested entanglement purification protocol. In general, there are two kinds of errors occurring in the degraded quantum system, say, the bit-flipping errors and phase-flipping errors. EPPs can directly purify the bit-flipping errors but cannot directly purify the phase-flipping errors. However, the phase-flipping errors can be converted to the bit-flipping errors by the Hadamard (H) operation and then be purified by EPPs. In this way, in our protocol, we only consider the purification of bit-flipping errors and our EPP can also be extended to deal with the general errors. After operating our EPP, we not only can eliminate the double-pair noise components, but also can obtain a higher fidelity entangled photon pair. This paper is organized as follows. In Section 2, we explain the basic principle of the two-copy nested EPP. In Section 3, we propose the multi-copy nested EPP. In Section 4, we provide a discussion and conclusion.

2. The nested entanglement purification

In this section, we explain the basic model of the nested EPP in Ref. [56]. The purification protocol comprises two parts. The first part is the entanglement swapping based on the Bell-state measurement (BSM), and the second part is the conventional two-copy entanglement purification. In the following two subsections, we briefly introduce the two parts, respectively.

2.1. Entanglement swapping part

The nested EPP adopts the conventional SPDC source. In SPDC source, the pump passes through a beta barium borate (BBO) crystal to produce entangled photon pairs in spatial modes 1 and 2 of

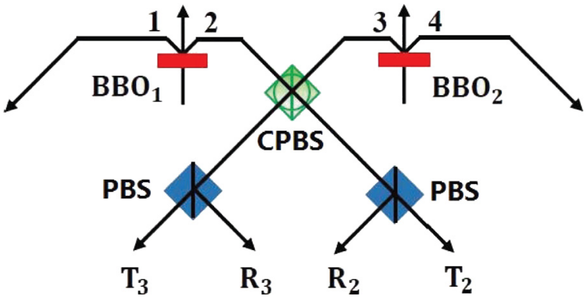

Fig. 1. The basic principle of the entanglement swapping part in the nested EPP. Two pumps pass through two beta barium borate (BBO) crystals, say, BBO_1 and BBO_2 to produce two entangled photon pairs in the spatial modes 1 and 2, 3 and 4, respectively. Photons in modes 2 and 3 are guided to a Bell-state measurement (BSM) to implement the entanglement swapping. Here, PBS represents the polarization beam splitter and CPBS represents the circular polarization beam splitter.

the form [56]

$$|\psi_{12}\rangle = |vac\rangle + \sqrt{p}(\frac{\hat{a}^{\dagger}_{H1}\hat{a}^{\dagger}_{H2} + \hat{a}^{\dagger}_{V1}\hat{a}^{\dagger}_{V2}}{\sqrt{2}})|vac\rangle + \frac{p}{2}(\frac{\hat{a}^{\dagger}_{H1}\hat{a}^{\dagger}_{H2} + \hat{a}^{\dagger}_{V1}\hat{a}^{\dagger}_{V2}}{\sqrt{2}})^2|vac\rangle. \tag{1}$$

Here, p is the single photon pair emission probability (The higher-order emissions bigger than p have been safely neglected since $p \ll 1$). $\hat{a}^{\dagger}_{H(V)}$ is the creation operator of a single photon in horizontal (vertical) polarization, and the subscripts 1 and 2 mean spatial modes. $|vac\rangle$ means that no photon is generated from the SPDC source.

Similarly, we can generate the entangled photon pair in spatial modes 3 and 4 of the same form

$$|\psi_{34}\rangle = |vac\rangle + \sqrt{p}(\frac{\hat{a}^{\dagger}_{H3}\hat{a}^{\dagger}_{H4} + \hat{a}^{\dagger}_{V3}\hat{a}^{\dagger}_{V4}}{\sqrt{2}})|vac\rangle + \frac{p}{2}(\frac{\hat{a}^{\dagger}_{H3}\hat{a}^{\dagger}_{H4} + \hat{a}^{\dagger}_{V3}\hat{a}^{\dagger}_{V4}}{\sqrt{2}})^2|vac\rangle. \tag{2}$$

The whole photon state $|\psi_{12}\rangle \otimes |\psi_{34}\rangle$ can be written as

$$\begin{aligned}|\psi_{12}\rangle \otimes |\psi_{34}\rangle =& |vac\rangle + \sqrt{p}(\frac{\hat{a}^{\dagger}_{H1}\hat{a}^{\dagger}_{H2} + \hat{a}^{\dagger}_{V1}\hat{a}^{\dagger}_{V2}}{\sqrt{2}})|vac\rangle + \sqrt{p}(\frac{\hat{a}^{\dagger}_{H3}\hat{a}^{\dagger}_{H4} + \hat{a}^{\dagger}_{V3}\hat{a}^{\dagger}_{V4}}{\sqrt{2}})|vac\rangle \\ &+ \frac{p}{2}(\frac{\hat{a}^{\dagger}_{H1}\hat{a}^{\dagger}_{H2} + \hat{a}^{\dagger}_{V1}\hat{a}^{\dagger}_{V2}}{\sqrt{2}})^2|vac\rangle + \frac{p}{2}(\frac{\hat{a}^{\dagger}_{H3}\hat{a}^{\dagger}_{H4} + \hat{a}^{\dagger}_{V3}\hat{a}^{\dagger}_{V4}}{\sqrt{2}})^2|vac\rangle \\ &+ p(\frac{\hat{a}^{\dagger}_{H1}\hat{a}^{\dagger}_{H2} + \hat{a}^{\dagger}_{V1}\hat{a}^{\dagger}_{V2}}{\sqrt{2}})(\frac{\hat{a}^{\dagger}_{H3}\hat{a}^{\dagger}_{H4} + \hat{a}^{\dagger}_{V3}\hat{a}^{\dagger}_{V4}}{\sqrt{2}})|vac\rangle + \cdots.\end{aligned} \tag{3}$$

Here, we omit the higher order items as they perform little contribution to the final results.

For simplicity, we suppose $|\Phi^{+}_{ij}\rangle = (\frac{\hat{a}^{\dagger}_{Hi}\hat{a}^{\dagger}_{Hj}+\hat{a}^{\dagger}_{Vi}\hat{a}^{\dagger}_{Vj}}{\sqrt{2}})|vac\rangle = \frac{1}{\sqrt{2}}(|HH\rangle+|VV\rangle)_{ij}$, where i and j represent spatial modes. From Eq. (3), the whole state can be described as follows. With the probability of p, the state is $|\Phi^{+}_{12}\rangle$ or $|\Phi^{+}_{34}\rangle$. With the probability of p^2, the state is $|\Phi^{+}_{12}\rangle|\Phi^{+}_{34}\rangle$. With the probability of $\frac{p^2}{4}$, the whole state is $|\Phi^{+}_{12}\rangle^{\otimes 2}$ or $|\Phi^{+}_{34}\rangle^{\otimes 2}$.

Then, we make the BSM for the photons in spatial modes 2 and 3. The basic principle of the BSM is shown in Fig. 1. We use two kinds of polarization beam splitters (PBSs), say, the normal PBS and the circular PBS (CPBS). The normal PBS fully transmits $|H\rangle$ polarized photon and reflects $|V\rangle$ polarized photon. The circular PBS fully transmits $|D\rangle$ polarized photon and reflects $|A\rangle$ polarized photon, where $|D/A\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. After the CPBS and PBSs, we select the cases where the output modes T_3T_2 or R_3R_2 each contain exactly one photon.

For example, from Eq. (3), with the probability of p^2, we can obtain the state $|\Phi^+_{12}\rangle|\Phi^+_{34}\rangle$ with the form of

$$\begin{aligned}|\Phi^+_{12}\rangle|\Phi^+_{34}\rangle &= \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes\frac{1}{\sqrt{2}}(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)\\ &= \frac{1}{2}(|H\rangle_1|H\rangle_4|H\rangle_2|H\rangle_3+|H\rangle_1|V\rangle_4|H\rangle_2|V\rangle_3\\ &+|V\rangle_1|H\rangle_4|V\rangle_2|H\rangle_3+|V\rangle_1|V\rangle_4|V\rangle_2|V\rangle_3).\end{aligned} \tag{4}$$

We pass the photons in modes 2 and 3 through the CPBS and PBSs, and select the cases where the spatial modes T_3T_2 or R_3R_2 each contain exactly one photon. In this way, $|\Phi^+_{12}\rangle|\Phi^+_{34}\rangle$ collapses to

$$|\varphi\rangle_1 = \frac{1}{2}(|H\rangle_1|H\rangle_4+|V\rangle_1|V\rangle_4)(|H\rangle_{T_2}|H\rangle_{T_3}+|V\rangle_{T_2}|V\rangle_{T_3}), \tag{5}$$

or

$$|\varphi\rangle_2 = \frac{1}{2}(|H\rangle_1|H\rangle_4+|V\rangle_1|V\rangle_4)(|H\rangle_{R_2}|H\rangle_{R_3}+|V\rangle_{R_2}|V\rangle_{R_3}). \tag{6}$$

With the probability of $\frac{p^2}{4}$, the whole state is $|\Phi^+_{12}\rangle^{\otimes 2}$ with the form of

$$\begin{aligned}|\Phi^+_{12}\rangle^{\otimes 2} &= \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes\frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\\ &= \frac{1}{2}(|H\rangle_1|H\rangle_2|H\rangle_1|H\rangle_2+2|H\rangle_1|H\rangle_2|V\rangle_1|V\rangle_2\\ &+|V\rangle_1|V\rangle_2|V\rangle_1|V\rangle_2).\end{aligned} \tag{7}$$

By passing the photons in mode 2 through the CPBS and PBSs and selecting the cases with the same principle, $|\Phi^+_{12}\rangle^{\otimes 2}$ collapses to

$$|\varphi\rangle_3 = \frac{1}{2}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1)(|H\rangle_{T_2}|H\rangle_{T_3}-|V\rangle_{T_2}|V\rangle_{T_3}), \tag{8}$$

or

$$|\varphi\rangle_4 = \frac{1}{2}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1)(|H\rangle_{R_2}|H\rangle_{R_3}-|V\rangle_{R_2}|V\rangle_{R_3}). \tag{9}$$

Similarly, with the probability of $\frac{p^2}{4}$, the whole state is $|\Phi^+_{34}\rangle^{\otimes 2}$. With the same operation and selection principle, $|\Phi^+\rangle^{\otimes 2}_{34}$ collapses to

$$|\varphi\rangle_5 = \frac{1}{2}(|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)(|H\rangle_{T_2}|H\rangle_{T_3}-|V\rangle_{T_2}|V\rangle_{T_3}), \tag{10}$$

or

$$|\varphi\rangle_6 = \frac{1}{2}(|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)(|H\rangle_{R_2}|H\rangle_{R_3}-|V\rangle_{R_2}|V\rangle_{R_3}). \tag{11}$$

On the other hand, it is obvious that $|\Phi^+_{12}\rangle$ or $|\Phi^+_{34}\rangle$ cannot make the spatial modes T_3T_2 or R_3R_2 each contain one photon. As a result, $|\Phi^+_{12}\rangle$ or $|\Phi^+_{34}\rangle$ can be eliminated automatically. Finally, by measuring the photons in the output modes T_3T_2 and R_3R_2, the whole state in Eq. (3) collapses to

$$|\Phi_{14}\rangle = \frac{p}{2}(|H\rangle_1|H\rangle_4+|V\rangle_1|V\rangle_4)+\frac{p}{4}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4). \tag{12}$$

From Eq. (12), the first item is the ideal Bell state $|\Phi^+_{14}\rangle$. It can be found that the SPDC sources also contribute some unwanted items, such as $|H\rangle_1|H\rangle_1$, $|V\rangle_1|V\rangle_1$, $|H\rangle_4|H\rangle_4$, and $|V\rangle_4|V\rangle_4$. We call these unwanted items as double-pair components, which are the noise items.

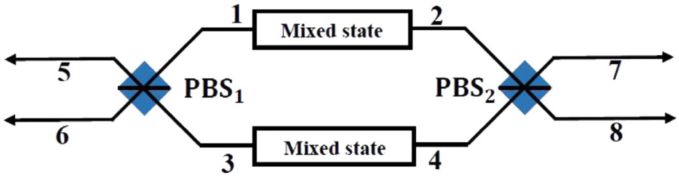

Fig. 2. Schematic principle of the entanglement purification part of the two-copy nested EPP. By selecting the "four-mode" cases, we can distill the high fidelity entangled state from two copies of mixed states with the form of Eq. (13).

2.2. Two-copy entanglement purification part

In this section, we perform a detailed explanation about the two-copy entanglement purification part based on the imperfect entangled state in Eq. (12). The basic principle of the entanglement purification part is shown in Fig. 2. Here, we consider a bit-flipping error occurring on the photon in spatial mode 4 with a probability of $1-F$, which makes $|\Phi_{14}\rangle$ in Eq. (12) become

$$|\Psi_{14}\rangle=\frac{p}{2}(|H\rangle_1|V\rangle_4+|V\rangle_1|H\rangle_4)+\frac{p}{4}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|V\rangle_4|V\rangle_4-|H\rangle_4|H\rangle_4). \tag{13}$$

As a result, $|\Phi_{14}\rangle$ degrades to a mixed state with the form of

$$\rho_{14}=F|\Phi_{14}\rangle\langle\Phi_{14}|+(1-F)|\Psi_{14}\rangle\langle\Psi_{14}|. \tag{14}$$

We suppose that two mixed states ρ_{12} and ρ_{34} are in the same form of Eq. (14). The whole state $\rho_{12}\otimes\rho_{34}$ in spatial modes 1, 2 and 3, 4 can be described as follows. With the probability of F^2, the state is $|\Phi_{12}\rangle|\Phi_{34}\rangle$. With the probability of $(1-F)^2$, the state is $|\Psi_{12}\rangle|\Psi_{34}\rangle$. With the equal probability of $F(1-F)$, the state is $|\Phi_{12}\rangle|\Psi_{34}\rangle$ or $|\Psi_{12}\rangle|\Phi_{34}\rangle$.

The state $|\Phi_{12}\rangle|\Phi_{34}\rangle$ can be written as

$$\begin{aligned}|\Phi_{12}\rangle|\Phi_{34}\rangle &= \frac{p^2}{4}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)\\ &+\frac{p^2}{8}(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)\\ &+\frac{p^2}{8}(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|H\rangle_2|H\rangle_2-|V\rangle_2|V\rangle_2)\\ &+\frac{p^2}{16}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|H\rangle_2|H\rangle_2-|V\rangle_2|V\rangle_2)(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3\\ &+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4).\end{aligned} \tag{15}$$

From Eq. (15), only the item $(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)$ represents the ideal sources, which can be purified as Ref. [35]. We make the photons in modes 1 and 3, 2 and 4 pass through PBS_1 and PBS_2, respectively. Then, $(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)$ evolves to

$$\begin{aligned}&(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)\\ =\ &|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4+|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_4+|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4+|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4\\ \to\ &|H\rangle_5|H\rangle_6|H\rangle_7|H\rangle_8+|H\rangle_7|V\rangle_7|H\rangle_8|V\rangle_8+|H\rangle_5|V\rangle_5|H\rangle_6|V\rangle_6+|V\rangle_5|V\rangle_6|V\rangle_7|V\rangle_8.\end{aligned} \tag{16}$$

Here, we select the cases where the spatial modes 5, 6, 7, and 8 each contain exactly one photon (four-mode cases) and the state in Eq. (16) collapses to

$$|\psi\rangle_1=\frac{1}{\sqrt{2}}(|H\rangle_5|H\rangle_6|H\rangle_7|H\rangle_8+|V\rangle_5|V\rangle_6|V\rangle_7|V\rangle_8). \tag{17}$$

On the other hand, after passing through two PBSs, the item $(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)$ evolves to

$$\begin{aligned}
&(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)\\
=\ & |H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_3-|H\rangle_1|H\rangle_2|V\rangle_3|V\rangle_3+|H\rangle_1|H\rangle_2|H\rangle_4|H\rangle_4-|H\rangle_1|H\rangle_2|V\rangle_4|V\rangle_4\\
+\ & |V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_3-|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_3+|V\rangle_1|V\rangle_2|H\rangle_4|H\rangle_4-|V\rangle_1|V\rangle_2|V\rangle_4|V\rangle_4\\
\to\ & |H\rangle_5|H\rangle_5|H\rangle_7|H\rangle_8-|H\rangle_7|V\rangle_7|V\rangle_7|H\rangle_8+|H\rangle_6|H\rangle_6|H\rangle_7|H\rangle_8-|H\rangle_7|V\rangle_8|V\rangle_8|H\rangle_8\\
+\ & |H\rangle_5|H\rangle_5|V\rangle_5|V\rangle_6-|V\rangle_5|V\rangle_6|V\rangle_7|V\rangle_7+|V\rangle_5|H\rangle_6|H\rangle_6|V\rangle_6-|V\rangle_5|V\rangle_6|V\rangle_8|V\rangle_8.
\end{aligned}\tag{18}$$

We can easily find that none items in Eq. (18) satisfies the four-mode cases. As a result, the item $(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)\otimes(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)$ can be eliminated automatically. Similarly, the items $(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)\otimes(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|H\rangle_2|H\rangle_2-|V\rangle_2|V\rangle_2)$ and $(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|H\rangle_2|H\rangle_2-|V\rangle_2|V\rangle_2)\otimes(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|H\rangle_4|H\rangle_4-|V\rangle_4|V\rangle_4)$ do not satisfy the four-mode cases, so that they also can be eliminated. Therefore, by selecting the four-mode cases, the state $|\Phi_{12}\rangle|\Phi_{34}\rangle$ collapses to $|\psi\rangle_1$.

With the probability of $(1-F)^2$, the whole state is $|\Psi_{12}\rangle|\Psi_{34}\rangle$ with the form of

$$\begin{aligned}
|\Psi_{12}\rangle|\Psi_{34}\rangle =\ & \frac{p^2}{4}(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2)(|H\rangle_3|V\rangle_4+|V\rangle_3|H\rangle_4)\\
&+\frac{p^2}{8}(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2)(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|V\rangle_4|V\rangle_4-|H\rangle_4|H\rangle_4)\\
&+\frac{p^2}{8}(|H\rangle_3|V\rangle_4+|V\rangle_3|H\rangle_4)(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|V\rangle_2|V\rangle_2-|H\rangle_2|H\rangle_2)\\
&+\frac{p^2}{16}(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|V\rangle_2|V\rangle_2-|H\rangle_2|H\rangle_2)\\
&\quad(|H\rangle_3|H\rangle_3-|V\rangle_3|V\rangle_3+|V\rangle_4|V\rangle_4-|H\rangle_4|H\rangle_4).
\end{aligned}\tag{19}$$

From Eq. (19), after passing through the PBSs, only $(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2)\otimes(|H\rangle_3|V\rangle_4+|V\rangle_3|H\rangle_4)$ can lead to the four-mode cases. As a result, by selecting the four-mode cases, the state in Eq. (19) collapses to

$$|\psi\rangle_2=\frac{1}{\sqrt{2}}(|H\rangle_5|V\rangle_6|H\rangle_7|V\rangle_8+|V\rangle_5|H\rangle_6|V\rangle_7|H\rangle_8).\tag{20}$$

With the equal probability of $F(1-F)$, the whole state is $|\Phi_{12}\rangle|\Psi_{34}\rangle$ or $|\Psi_{12}\rangle|\Phi_{34}\rangle$. With the same principle, we find that neither $|\Phi_{12}\rangle|\Psi_{34}\rangle$ nor $|\Psi_{12}\rangle|\Phi_{34}\rangle$ satisfies the four-mode cases, so that they can be eliminated automatically.

Based on above processes, by selecting the four-mode cases, we can obtain $|\psi\rangle_1$ and $|\psi\rangle_2$ with the probability of $\frac{p^2}{8}F^2$ and $\frac{p^2}{8}(1-F)^2$, respectively. Similarly with Ref. [35], by measuring the photons in the spatial modes 6 and 8 in the basis $|D\rangle/|A\rangle$, we ultimately obtain a new mixed state

$$\rho'_{56}=F_2|\Phi^+_{56}\rangle\langle\Phi^+_{56}|+(1-F_2)|\Psi^+_{56}\rangle\langle\Psi^+_{56}|,\tag{21}$$

with

$$F_2=\frac{F^2}{F^2+(1-F)^2}.\tag{22}$$

It is obvious that if $F>0.5$, the final fidelity F_2 is larger than original fidelity F, which means the purification is successful.

3. Multi-copy nested entanglement purification protocol

In the section, we propose our multi-copy nested EPP. This protocol also comprises of the entanglement swapping part and the multi-copy entanglement purification part. The entanglement swapping process is the same as that in Section 2, so that we do not explain it in this section. For

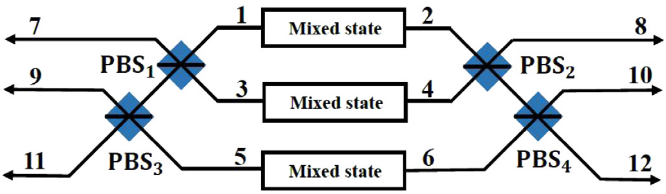

Fig. 3. Schematic principle of the entanglement purification part of the three-copy nested EPP.

clearly explaining the multi-copy entanglement purification part, we first discuss the three-copy entanglement purification whose basic principle is shown in Fig. 3.

We suppose that three copies of mixed states ρ_{12}, ρ_{34}, and ρ_{56} have the same form of the state in Eq. (14). The whole system $\rho_{12}\otimes\rho_{34}\otimes\rho_{56}$ can be described as follows. It is $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$ with the probability of F^3. It is $|\Psi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$, $|\Phi_{12}\rangle|\Psi_{34}\rangle|\Phi_{56}\rangle$, or $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Psi_{56}\rangle$ with the equal probability of $F^2(1-F)$. It is $|\Psi_{12}\rangle|\Psi_{34}\rangle|\Phi_{56}\rangle$, $|\Psi_{12}\rangle|\Phi_{34}\rangle|\Psi_{56}\rangle$, or $|\Phi_{12}\rangle|\Psi_{34}\rangle|\Psi_{56}\rangle$ with the equal probability of $F(1-F)^2$. It is $|\Psi_{12}\rangle|\Psi_{34}\rangle|\Psi_{56}\rangle$ with the probability of $(1-F)^3$.

As shown in Fig. 3, we make the photons in modes 1, 3, and 5 pass through PBS_1 and PBS_3 and the photons in modes 2, 4, and 6 pass through PBS_2 and PBS_4. After the PBSs, we select the cases where only one photon is in each of the six output modes 7–12 (six-mode cases). For example, we consider the whole state being $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$ with the probability of F^3. The state $|\Phi_{12}\rangle$ contains two parts, say, the state $|\Phi^+_{12}\rangle$ and the double-pair components $(|H\rangle_1|H\rangle_1-|V\rangle_1|V\rangle_1+|V\rangle_2|V\rangle_2-|H\rangle_2|H\rangle_2)$. The whole state $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$ can be written as eight parts in which only $|\Phi^+_{12}\rangle|\Phi^+_{34}\rangle|\Phi^+_{56}\rangle$ satisfies the six-mode cases. In detail, after passing through all PBSs, $|\Phi^+_{12}\rangle|\Phi^+_{34}\rangle|\Phi^+_{56}\rangle$ evolves to

$$\begin{aligned}
&(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)(|H\rangle_5|H\rangle_6+|V\rangle_5|V\rangle_6)\\
=\ &|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6+|H\rangle_1|H\rangle_2|H\rangle_3|H\rangle_4|V\rangle_5|V\rangle_6\\
+\ &|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6+|V\rangle_1|V\rangle_2|H\rangle_3|H\rangle_4|V\rangle_5|V\rangle_6\\
+\ &|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4|H\rangle_5|H\rangle_6+|V\rangle_1|V\rangle_2|V\rangle_3|V\rangle_4|V\rangle_5|V\rangle_6\\
\rightarrow\ &|H\rangle_7|H\rangle_8|H\rangle_9|H\rangle_{10}|H\rangle_{11}|H\rangle_{12}+|H\rangle_7|H\rangle_8|H\rangle_{11}|V\rangle_{11}|H\rangle_{12}|V\rangle_{12}\\
+\ &|H\rangle_7|V\rangle_7|H\rangle_8|V\rangle_8|H\rangle_9|H\rangle_{10}+|H\rangle_7|V\rangle_7|H\rangle_8|V\rangle_8|V\rangle_{11}|V\rangle_{12}\\
+\ &|V\rangle_7|V\rangle_8|H\rangle_9|V\rangle_9|H\rangle_{10}|V\rangle_{10}+|V\rangle_7|V\rangle_8|V\rangle_9|V\rangle_{10}|V\rangle_{11}|V\rangle_{12}.
\end{aligned}\tag{23}$$

By selecting six-mode cases, we can obtain

$$|\phi\rangle_1=\frac{1}{\sqrt{2}}(|H\rangle_7|H\rangle_8|H\rangle_9|H\rangle_{10}|H\rangle_{11}|H\rangle_{12}+|V\rangle_7|V\rangle_8|V\rangle_9|V\rangle_{10}|V\rangle_{11}|V\rangle_{12}).\tag{24}$$

The other items in $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$ do not satisfy the six-mode cases. For example, the state $(|H\rangle_1|H\rangle_2+|V\rangle_1|V\rangle_2)(|H\rangle_3|H\rangle_4+|V\rangle_3|V\rangle_4)(|H\rangle_5|H\rangle_5-|V\rangle_5|V\rangle_5+|H\rangle_6|H\rangle_6-|V\rangle_6|V\rangle_6)$ means the first two states are ideal Bell state and the third state are double-pair components. The double-pair components always make the two photons be in the same output mode, so that the whole state cannot satisfy the six-mode cases.

With the probability of $(1-F)^3$, the whole state is $|\Psi_{12}\rangle|\Psi_{34}\rangle|\Psi_{56}\rangle$. After passing the photons through the PBSs and selecting the six-mode cases, $|\Psi_{12}\rangle|\Psi_{34}\rangle|\Psi_{56}\rangle$ finally collapses to

$$|\phi\rangle_2=\frac{1}{\sqrt{2}}(|H\rangle_7|V\rangle_8|H\rangle_9|V\rangle_{10}|H\rangle_{11}|V\rangle_{12}+|V\rangle_7|H\rangle_8|V\rangle_9|H\rangle_{10}|V\rangle_{11}|H\rangle_{12}).\tag{25}$$

On the other hand, the $|\Psi_{12}\rangle|\Phi_{34}\rangle|\Phi_{56}\rangle$, $|\Phi_{12}\rangle|\Psi_{34}\rangle|\Phi_{56}\rangle$, $|\Phi_{12}\rangle|\Phi_{34}\rangle|\Psi_{56}\rangle$, $|\Psi_{12}\rangle|\Psi_{34}\rangle|\Phi_{56}\rangle$, $|\Psi_{12}\rangle|\Phi_{34}\rangle|\Psi_{56}\rangle$ and $|\Phi_{12}\rangle|\Psi_{34}\rangle|\Psi_{56}\rangle$ cannot satisfy the six-mode case, so that they can be eliminated automatically.

Finally, by measuring the photons in spatial modes 9, 10, 11 and 12 in the basis $|D\rangle/|A\rangle$, we obtain a new mixed state as

$$\rho'_{78}=F_3|\Phi^+_{78}\rangle\langle\Phi^+_{78}|+(1-F_3)|\Psi^+_{78}\rangle\langle\Psi^+_{78}|,\tag{26}$$

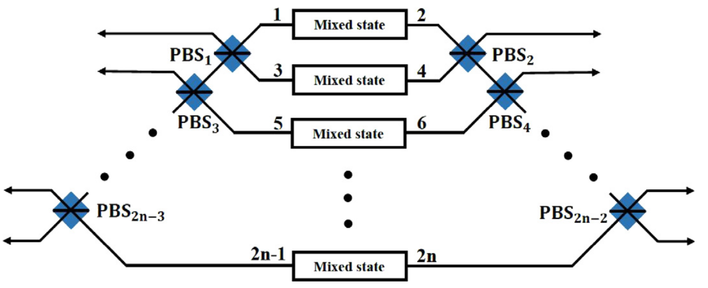

Fig. 4. Schematic principle of the entanglement purification part of the n-copy nested EPP.

with

$$F_3 = \frac{F^3}{F^3 + (1-F)^3}. \tag{27}$$

It can be found that $F_3 > F_2$ when $F > \frac{1}{2}$. As a result, the three-copy nested EPP has higher efficiency than the two-copy nested EPP.

Interestingly, this nested EPP can be further extended to the n-copy nested EPP. The principle of the entanglement purification part of the n-copy nested EPP is shown in Fig. 4. We use n copies of same mixed state in Eq. (14) to distill the high quality entanglement state. From Eq. (13), all the double-pair components can be written as $|HH\rangle_{ii}$ or $|VV\rangle_{ii}$. Here the subscript $i = 1, 2, \ldots, 2n$ is the spatial mode as shown in Fig. 4. After passing through the PBSs, the two photons in the double-pair components always appear in the same output mode. In this way, if the item contains the double-pair components, it cannot satisfy the $2n$-mode cases and can be eliminated automatically. Finally, by selecting $2n$-mode case, we can finally distill the new mixed state as

$$\rho_{out} = F_n|\Phi^+\rangle\langle\Phi^+| + (1-F_n)|\Psi^+\rangle\langle\Psi^+|, \tag{28}$$

with the fidelity F_n of

$$F_n = \frac{F^n}{F^n + (1-F)^n}. \tag{29}$$

When $F > \frac{1}{2}$, F_n increases with the growth of n.

4. Discussion and conclusion

So far, we have completely explained our multi-copy nested EPP. We first introduce the basic principle of two-copy nested EPP. In conventional EPPs, the entanglement sources generate the entangled photon pairs which are distributed to the distant parties directly. During the entanglement distribution, the entangled states may suffer from environmental noise and become mixed states. Two parties purify such mixed states with local operation and classical communication. In such EPPs, the imperfection of the entanglement sources disturb the purification directly. In nested EPPs, after the entanglement generation from the SPDC sources, they first perform the entanglement swapping. From Fig. 1 and Eq. (12), after performing the BSM, the imperfect items generated from SPDC sources become the double-pair noise components in Eq. (12). Interestingly, if a bit-flipping error occurs on the state in Eq. (12), the double-pair noise components do not change their polarization components. Moreover, after passing through the PBSs, the two photons in the double-pair noise components are always in the same output mode. Therefore, by selecting the four-mode case, the double-pair noise components can be eliminated automatically.

In Fig. 5, we calculate the fidelity F_n altered with the initial fidelity F. We let $n = 2$, 3 and 4, respectively, similar with Ref. [42]. The initial fidelity $F \in (0.5, 1)$. If the initial fidelity is $F = 0.6$, after purification, we can obtain the new fidelity 0.69 with $n = 2$, 0.77 with $n = 3$ and 0.83 with

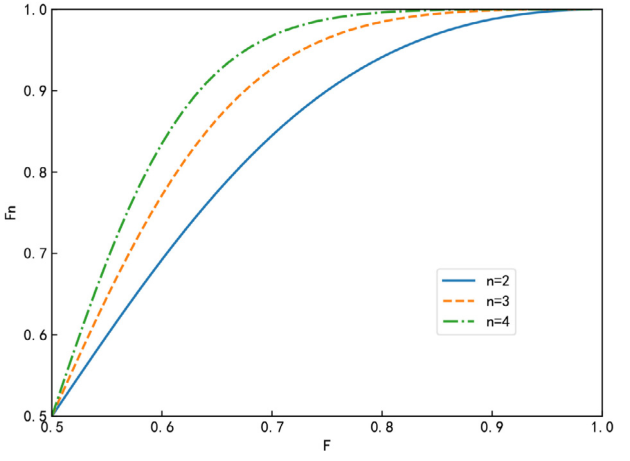

Fig. 5. The fidelity F_n altered with the initial fidelity F. n is the number of mixed states. Here we let $n = 2, 3$ and 4, respectively.

$n = 4$. In this way, increasing the copies of low fidelity mixed state can effectively increase the fidelity of the distilled target state.

How to obtain high fidelity entanglement rapidly is fundamental for many quantum communication protocols. In some traditional recyclable EPPs [34,46–49], they can improve the fidelity step by step and obtain a higher fidelity entangled state ultimately by repeating the protocols. However, these EPPs usually require the CNOT gates or QND measurements, which are hard to realize in current technology. The EPPs in linear optics can be realized in current technology. However, they can only be used once and their purification efficiency is relatively low [35–44]. The multi-copy entanglement purification provides us an efficient approach to obtain a high fidelity entangled state by only performing the EPP once.

In conclusion, we present the multi-copy nested entanglement purification protocol. We first explain the basic principle of two-copy nested EPP. Then, we extend this two-copy nested EPP to the arbitrary n-copy nested EPP. This protocol has several advantages. First, the double-pair noise components generated from the SPDC source can be eliminated. Second, with the help of entanglement swapping, the distance of purified entangled state can be extended. Third, this EPP is based on current available technology. Fourth, we can obtain a higher fidelity by performing this EPP only once by increasing the number of the initial low quality mixed states. This multi-copy nested EPP may be useful in future long-distance quantum communication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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