



All Theses and Dissertations

2012-04-19

Reducing Residual Drift in Buckling-Restrained Braced Frames by Using Gravity Columns as Part of a Dual System

Megan Boston

Brigham Young University - Provo

Follow this and additional works at: <https://scholarsarchive.byu.edu/etd>



Part of the [Civil and Environmental Engineering Commons](#)

BYU ScholarsArchive Citation

Boston, Megan, "Reducing Residual Drift in Buckling-Restrained Braced Frames by Using Gravity Columns as Part of a Dual System" (2012). *All Theses and Dissertations*. 3204.

<https://scholarsarchive.byu.edu/etd/3204>

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in All Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact scholarsarchive@byu.edu, ellen_amatangelo@byu.edu.

Reducing Residual Drift in Buckling-Restrained Braced Frames by Using
Gravity Columns as Part of a Dual System

Megan Boston

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of
Master of Science

Paul W. Richards, Chair
Fernando S. Fonseca
Richard J. Balling

Department of Civil and Environmental Engineering
Brigham Young University
June 2012

Copyright © 2012 Megan Boston
All Rights Reserved

ABSTRACT

Reducing Residual Drift in Buckling-Restrained Braced Frames by Using Gravity Columns as Part of a Dual System

Megan Boston

Department of Civil and Environmental Engineering, BYU
Master of Science

Severe earthquakes cause damage to buildings. One measure of damage is the residual drift. Large residual drifts suggest expensive repairs and could lead to complete loss of the building. As such, research has been conducted on how to reduce the residual drift. Recent research has focused on self-centering frames and dual systems, both of which increase the post-yield stiffness of the building during and after an earthquake. Self-centering systems have yet to be adopted into standard practice but dual systems are used regularly. Dual systems in steel buildings typically combine two types of traditional lateral force resisting systems such as buckling restrained braced frames (BRBFs) and moment resisting frames (MRFs). However, the cost of making the moment connections for the MRFs can make dual systems costly.

An alternative to MRFs is to use gravity columns as the secondary system in a dual system. The gravity columns can be used to help resist the lateral loads and limit the residual drifts if the lateral stiffness of the gravity columns can be activated. By restraining the displacement of the gravity columns, the stiffness of the columns adds to the stiffness of the brace frame, thus engaging the lateral stiffness of the gravity columns. Three methods of engaging the stiffness of the gravity columns are investigated in this thesis; one, fixed ground connections, two, a heavy elastic brace in the top story, and three, a heavy elastic brace in the middle bay.

Single and multiple degree of freedom models were analyzed to determine if gravity columns can be effective in reducing residual drift. In the single degree of freedom system (SDOF) models, the brace size was varied to get a range of periods. The column size was varied based on a predetermined range of post-yield stiffness to determine if the residual drift decreased with higher post-yield stiffness. Three and five story models were analyzed with a variety of brace and column sizes and with three different configurations to activate the gravity columns.

Using gravity columns as part of a dual system decreases the residual drift in buildings. The results from the SDOF system show that the residual drift decreased with increased post-yield stiffness. The three and five story models showed similar results with less residual drift when larger columns were used. Further, the models with a heavy gravity column in the top story had the best results.

Keywords: residual drift, cantilever columns, buckling restrained braces, self-centering frames, post-yield stiffness

ACKNOWLEDGMENTS

I would like to thank Dr. Paul W. Richards for all the help he has given me in my research. Without his help, this thesis would not have been possible.

I would like to thank my husband Taylor for everything he has done to help me with this thesis. He has helped me with the figures, formatting and learning new software. He has also been patient with me as I have worked through my thesis. He has helped make the final documentation of this thesis possible.

I would also like to thank my family for their love and support while I have been in graduate school.

TABLE OF CONTENTS

LIST OF TABLES	ix
LIST OF FIGURES	xi
NOMENCLATURE	xiii
Chapter 1 Introduction	1
1.1 Motivation	1
1.2 Background	1
1.3 Outline	2
Chapter 2 Literature Review	5
2.1 Overview	5
2.2 Residual Drift	5
2.2.1 Residual Drift and Post Yield Stiffness	6
2.2.2 Residual Displacement as a Performance Parameters	6
2.3 Residual Drifts for Traditional Systems	7
2.3.1 Moment Frames	7
2.3.2 Buckling Restrained Braced Frames	8
2.4 Self-Centering Frames	9
2.4.1 Parameters for Self-Centering Moment Frames	10
2.4.2 Self-Centering Moment Frames	10
2.4.3 Self-Centering Braced Frames	12
2.4.4 Rocking Frames	13
2.5 Dual Systems	13
2.5.1 Designing a Dual System	14
2.5.2 BRBFs as the Primary System	14
2.5.3 MRFs as the Primary System	15
2.5.4 BRBFs with Secondary Gravity Columns	16
Chapter 3 Single Degree of Freedom Study	17
3.1 Method	17
3.2 Model	17
3.2.1 Parameter Variation	19
3.3 OpenSees	19
3.4 Earthquake Ground Motions	20
3.5 Output	20
3.6 Results and Discussion	20
3.6.1 Maximum Displacement	22
3.6.2 Residual Displacement	24
3.6.3 Implications for Buildings	26
3.7 SDOF Conclusions	26

Chapter 4	Three Degree of Freedom Study	27
4.1	Overview	27
4.2	Model	27
4.3	Modeling and Analysis Techniques	28
4.4	Parameter Variation	28
4.4.1	Brace Parameter	28
4.4.2	Column Variation	29
4.4.3	Model Configuration	29
4.4.4	Earthquake Ground Motions	29
4.5	Output	29
4.6	Results and Discussion	30
4.6.1	Maximum Drift Results And Discussion	30
4.6.2	Residual Drift Results and Discussion	33
4.6.3	Moment Ratios	36
4.7	Three Story Frame Conclusions	40
Chapter 5	Five Story Case Study	41
5.1	Design	41
5.2	Baseline Model and Analysis	42
5.3	Variants of Baseline Model	43
5.4	Results	44
5.4.1	Baseline Model	44
5.4.2	Fixed Base Gravity Columns	45
5.4.3	Heavy Top Brace	46
5.4.4	Heavy Middle	47
5.4.5	Comparison Between the Baseline and Three Variations	49
5.5	Conclusions	50
Chapter 6	Summary and Conclusions	51
6.1	Summary	51
6.2	Conclusions	52
6.2.1	SDOF Study	52
6.2.2	Three Story Study	52
6.2.3	Five Story Case Study	53
REFERENCES		55
Appendix A	SDOF Parameter Study Results for Individual Earthquakes	59
Appendix B	Three Story Frame Calculations	65
Appendix C	Design of Five Story Frame	67
Appendix D	OpenSees Code	71

D.1	SDOF Model	71
D.2	Three Story Models	75
D.2.1	Fixed Gravity Columns	75
D.2.2	Heavy Top Brace	80
D.2.3	Heavy Middle Brace	85
D.3	Five Story Model	90
D.4	Misc. Files	98

LIST OF TABLES

3.1	Earthquake Records	21
5.1	Five Story Design	42
5.2	Column Size Variations	44
B.1	Column Size Variations	65

LIST OF FIGURES

2.1	Self-Centering Force Deformation Diagrams	9
2.2	Weld Moment Frame and Self-Centering Moment Frame Hysteretic	10
2.3	Self-Centering Post-Tensioned Moment Frame Beam-Column Connection	11
3.1	SDOF Model	18
3.2	System Force Deformation Diagrams	18
3.3	Earthquake Response Spectra	21
3.4	Maximum Displacement from a Single Earthquake	22
3.5	Maximum Displacement from SDOF Models to a Suite of Earthquakes	23
3.6	Maximum Displacement for Systems with Low Periods	23
3.7	Residual Displacement for Periods Less than 4 Seconds	25
3.8	Residual Displacement from SDOF models to a Suite of Earthquakes	25
4.1	Frame Configurations	28
4.2	Maximum Drift in the First Story of Three Story Frames	31
4.3	Maximum Drift in the Second Story of Three Story Frames	32
4.4	Residual Drift in the First Story of Three Story Frames	34
4.5	Residual Drift in the First Story of Three Story Frames	35
4.6	Moment Ratios for the First Floor	37
4.7	Moment Ratios for the Second Floor	38
4.8	Moment Ration for Thrid Floor	39
5.1	Elevation and Plan View of Building Used for the Case Study	41
5.2	Five Story Case Study Model	43
5.3	Five Story Baseline Drifts	45
5.4	Five Story Drifts with Fixed Gravity Columns	45
5.5	Five Story Drifts with a Heavy Top Brace	47
5.6	Five Story Drifts with a Heavy Middle Brace	48
5.7	Five Story Drifts Comparison	49
A.1	SDOF Results for the Canoga Park Earthquake	59
A.2	SDOF Results for the Beverly Hills Earthquake	60
A.3	SDOF Results for the Hollywood Earthquake	60
A.4	SDOF Results for the Sun Valley Earthquake	61
A.5	SDOF Results for the Hollister Array Earthquake	61
A.6	SDOF Results for the Hollister City Hall Earthquake	62
A.7	SDOF Results for the Gilroy No. 3 Earthquake	62
A.8	SDOF Results for the Gilroy No. 4 Earthquake	63
A.9	SDOF Results for the Parachute Test Site 1 Earthquake	63
A.10	SDOF Results for the Parachute Test Site 2 Earthquake	64
B.1	Three Story Configurations	66

NOMENCLATURE

A_{brace}	Cross-sectional Area of the Brace
BRBF	Buckling Restrained Brace Frame
γ	Post-yield Stiffness, Percent of Initial Stiffness
k_i	Initial Stiffness
k_{col}	Stiffness of the Column
k_{brace}	Stiffness of the Brace
I_{col}	Moment of Inertia of the Column
MRF	Moment Resisting Frame
SMRF	Special Moment Resisting Frame

CHAPTER 1. INTRODUCTION

1.1 Motivation

Severe earthquakes cause considerable damage to buildings. One manifestation of damage from earthquakes is residual drift, the permanent deformation of a building after it has stopped shaking. Large residual drifts cause buildings to be unsafe for human occupancy and suggest the need for expensive repairs. As such, recent research has focused on ways to improve building designs to decrease or even eliminate the residual drift resulting from a severe earthquake.

The residual drift that occurs in a building is a function of post-yield stiffness. Buildings with higher post-yield stiffness have less residual drift. Methods for increasing the post-yield stiffness of buildings are currently being researched.

1.2 Background

Different types of lateral force resisting systems have been used to mitigate earthquake damage to buildings, however, most traditional systems may have large residual drifts even when they perform as designed. Buckling restrained braced frames (BRBF) and moment resisting frames (MRF) are among the most popular lateral force resisting systems in high seismic areas. Both of these systems are expected to have expensive repair cost or total loss of the building after severe earthquakes due the large residual drifts.

New methods are being researched to improve the performance of buildings with either non-traditional lateral force resisting systems or dual systems.

Non-traditional lateral force resisting systems include the self-centering moment frames, the self-centering braced frames, and rocking frames. These systems are designed to completely mitigate the residual drift in a building by allowing the building elements to move or rotate with the earthquake but then have other components that pull the structural elements back into place

after the earthquake. While these non-traditional systems appear to perform well, they have not been widely used in construction because of the difficulty in implementing them.

Other research has focused on creating dual systems that combine traditional lateral force resisting systems. These system may have a BRBF as the primary system and a MRF as a secondary systems. It is assumed that the secondary system will remain essentially elastic during the earthquake. Since the secondary system remains elastic, it contributes to the post-yield stiffness of the building and reduces the residual drift. While there has been some success in this research, the moment frame connections can be expensive, so alternative dual systems should be investigated to see if the residual drift in BRBFs can be mitigated more economically.

This thesis explores a dual system that combines BRBFs with “gravity columns” in a building that have been activated for lateral resistance. Three different methods of activating the gravity columns are analyzed to determine which performed the best. The first method utilizes gravity columns with fixed ground connections (cantilevered columns). The second and third method have pinned connections between the gravity columns and the ground, but the columns are restrained against story drifts at one or more levels using heavy braces that are designed to remain elastic. Restraining the columns activates the lateral stiffness in the columns. The heavy braces are placed in either the top story or the middle story. Engaging the gravity columns should increase the post-yield stiffness of the system and help limit the residual drift. Increasing the size of the gravity columns should further increases the post-yield stiffness and decreases the residual drift.

1.3 Outline

This thesis is divided into several chapters to best present the research done. Chapter 2 is a literature review discussing the relevant research on residual drift, post-yield stiffness, traditional lateral load resisting frames, non-traditional frames and dual systems.

Chapter 3 and Chapter 4 present the results of parameter studies investigating the reduction of residual drift with varying levels of post-yield stiffness. Chapter 3 is a single degree of freedom study and Chapter 4 is a three degree of freedom study. The three story study in Chapter 4 compares frames with fixed gravity columns, a heavy top brace, or a heavy middle brace.

Chapter 5 is a five story case study. A traditional BRBF is analyzed and compared to systems where the gravity columns have been activated. Finally, Chapter 6 is a summary of key results and conclusions.

CHAPTER 2. LITERATURE REVIEW

2.1 Overview

Buildings that are designed according to current codes have permanent displacement at the end of severe earthquakes. Code compliant buildings experience large displacements during severe earthquakes causing some of the structural components to yield or break; the yielding in the members results in permanent displacement. This permanent displacement is the residual displacement. The residual drift is the residual displacement normalized of the story height or total height. Even structures that perform as expected during design level earthquakes sustain residual drift [1–3]. Residual drift in a structure could be theoretically prevented by designing all the structural components to remain elastic during an earthquake. However, it is not economically feasible to do this with traditional systems.

Several approaches have been investigated or developed to limit the amount of residual drift in buildings including self-centering frames and dual systems. Self-centering frames combine different structural elements to create a flag shaped hysteretic resulting in no residual drift. Dual systems combine traditional systems, such as MRFs and BRBFs, to resist the lateral loads and limit residual drift.

The following four sections review the literature related to residual drift, traditional systems, self-centering frames, and dual systems. Finally, the use of gravity columns as part of a dual system is introduced.

2.2 Residual Drift

Residual drift in buildings cause economic liabilities and safety hazards; therefore, they should be understood and reduced. High residual drift can lead to expensive repair costs or total loss of the building [3]. McCormick et al. [4] considered permissible levels of residual deformation

in buildings based on human comfort, safety and the cost of repair. They found that drifts larger than 0.5% caused building occupants to experience nausea and headaches. The same level of drifts made it economically unviable to repair the building and made the building more prone to damage from aftershocks or future earthquakes. To help limit the amount of residual drift to 0.5% or less in future buildings from earthquake loading, it is important to understand what factors impact residual drift, and how residual drift can be mitigated.

2.2.1 Residual Drift and Post Yield Stiffness

Past research has indicated that the amount of residual drift a system experiences is dependent on post-yield stiffness. The post yield stiffness is the amount of stiffness remaining in the building after key structural components have yielded. MacRae et al. [5] analyzed bilinear oscillators under earthquake records and reported the amount of residual drift. From their analysis, they found that the amount of residual drift depended more on the amount of post yield stiffness than on the ground motion. Systems that have a positive post yielding stiffness have lower residual drift than those with negative post-yield stiffness. Further research done by Borzi [6] confirmed that the residual drift is dependent on the ductility requirements and the post-yield stiffness of the system.

In addition to post-yield stiffness, residual drifts are also related to maximum drifts. In an analytical study by Borzi [6], it was found that the residual drifts represented a high percentage of the maximum drifts reached by the system. Hatzigeorgiou et al. [7] use the relationship between maximum and residual drifts to estimate the amount of maximum drift after an earthquake from the measured residual drift and post-yield stiffness of the structure. Yazgan and Dazio [8] also use this relationship to estimate the amount of damage to a structure by similarly finding the maximum drift from the residual drift. Erochko et al. [3] also found that the residual and maximum drift were related and give limits for the maximum drift in order to keep the residual drift to an acceptable range.

2.2.2 Residual Displacement as a Performance Parameters

In the companion papers by Christopoulos et al. and Pampanin et al. [2,9], examining SDOF and MDOF systems respectively, a Residual Deformation Damage Index (RDDI) is suggested for

assessing a building after an earthquake. This index considers the maximum and residual drift of the structure and the residual drift from structural and non-structural elements. This index was created as a way to assess the potential damage to a building by considering only the maximum and residual responses of the building to an earthquake.

Ruiz-Garcia and Miranda [10] suggested using a residual displacement ratio, the residual displacement divided by the peak elastic displacement, to further assess the performance of a structure. They found that the residual displacement ratio is dependent on the period of vibration, the lateral strength ratio, and the type of hysteretic behavior. The lateral strength ratio is defined as the mass of the system multiplied by the spectral acceleration divided by the lateral yield strength. They found that for periods less than 0.5 seconds, the residual demand was greater than the peak elastic demands. For periods greater than 1.0 second the displacement ratio is not as dependent on the period of vibration as it is on the lateral strength ratio and the hysteretic behavior. Systems with positive post yield stiffness ratios had smaller residual displacements and demands compared to systems with elastic plastic behavior. These results are similar to the conclusions made in the previous sub-section.

2.3 Residual Drifts for Traditional Systems

Traditional systems such as moment frames and braced frames experience large displacements during severe earthquakes, and have minimal post-yield stiffness, leading to residual drifts.

2.3.1 Moment Frames

A common type of lateral load resisting frame is the moment resisting frame (MRF). MRFs are built with moment-resisting connections between the beams and columns. Special moment resisting frames (SMRF) are detailed to provide ductile response during an earthquake. SMRFs tend to have high strength-to-stiffness ratios which makes them susceptible to large drifts during ground motions. To reduce the amount of drift in a SMRFs, the frames are designed and sized according to FEMA 350 [11] which accounts for both the required stiffness of the frame, and drift limitations. To meet these requirements, the SMRFs members are much heavier than the gravity columns and beams in the system. SMRFs are expensive to use because of the heavier members

and the cost of the moment connections. Residual drifts in SMRFs are limited, in theory, by the maximum drifts.

In a study by Erochko [3], buildings with SMRFs were designed according to ASCE 7-05 [12]. The designs were subjected to earthquake ground motions to determine the amount of residual drift; they found the residual drift for the SMRFs to range between 0.5 to 1.55%. Since the residual drift is higher than 0.5%, it is large enough to be noticeable to building occupants and too expensive to fix. They suggested that in order to keep the residual drifts less than 0.5%, the maximum drifts had to be less than 1.5% for the SMRF.

2.3.2 Buckling Restrained Braced Frames

Another common type of lateral force resisting systems is braced frames. In braced frames, the stiffness is based on the cross-sectional area of the brace. In a typical braced frame, the brace has strength in both tension and compression. However, when the brace is in compression, the strength is limited by the buckling strength of the brace. Performance problems arise when the compression and tensile capacities are different [13]. These problems can be solved through the use of a buckling restrained brace (BRB).

A BRB is restrained along the length of the brace. This allows the brace material to yield in compression but not to buckle. Constraining the brace against buckling gives the brace similar tensile and compression strengths. To restrain the brace, the brace core is encased in a concrete filled steel tube with a lubricant to prevent bonding between the core and the tube. The core is allowed to yield longitudinally but is restrained against buckling.

The previously mentioned study by Erochko [3] also looked at BRBFs designed according to ASCE 7-05 [12]. The BRBFs were found to have residual drifts ranging between 0.8 to 1.8%. These values are also higher than acceptable for human comfort and to be fixed economically. To reduce the amount of residual drift in BRBFs to below 0.5%, Erochko suggested that the maximum displacement cannot be higher than 1.0%. Further, Tremblay et al. [14] performed a nonlinear analysis on buckling restrained braces and found that for BRBFs varying between 2 and 16 stories, the residual drift varies between 0.84 and 1.38%.

BRB have a low post-yield stiffness which can cause the frame to have large maximum and residual displacements [13]. These large displacements can make it so that the building is

very expensive to repair. Choi et al. [15] compared BRBF and MRF performance using push over analysis. BRBFs had the largest residual drifts with the drift increasing with building height. MRFs performed similar to the BRBF but with slightly lower values of both maximum and residual displacements for buildings less than eight stories. MRFs with more than ten stories had smaller displacements to BRBFs.

2.4 Self-Centering Frames

Recent research has focused on non-traditional frames with much lower residual drift than the traditional frames discussed above. Self-centering frames follow a flag-shaped hysteretic pattern. There are a variety of systems that have this hysteretic behavior: self-centering moment frames, self-centering brace frames, and rocking frames. The damage to structural members in self-centering frames is limited due to the behavior of the frame which reduces the amount of residual drift.

The flag shape hysteretic as shown in Figure 2.1 is formed by combining the hysteretic of the different component of the self-centering frame. These systems generally have posttensioned elements (bi-linear hysteretic) and energy dissipating devices (elasto-plastic hysteretic) which make the flag-shaped hysteretic. The inelastic behavior of the frame is prevented under dynamic loading so the hysteretic behavior is dependent on the posttensioning elements and the energy dampeners.

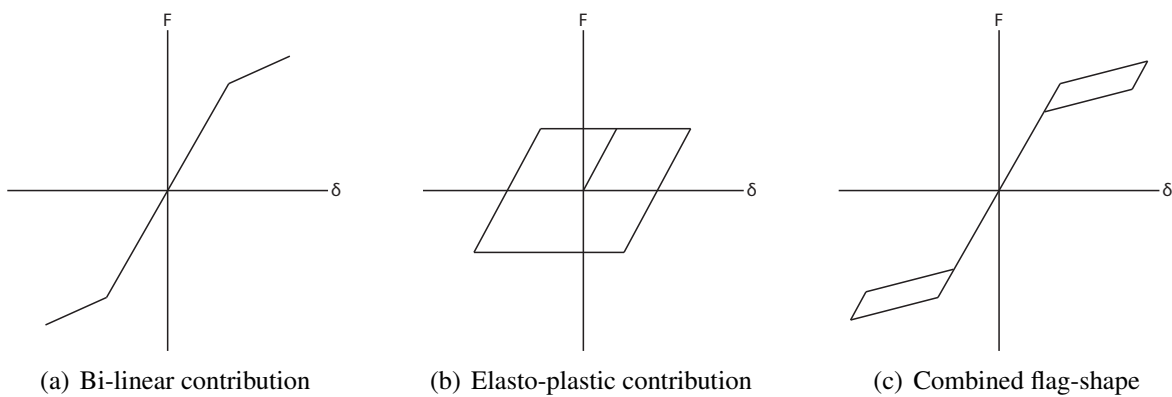


Figure 2.1: The flag-shaped hysteretic for self-centering frames comes from a combination of a bi-linear system and an elasto-plastic system.

2.4.1 Parameters for Self-Centering Moment Frames

Christopoulos et al. [16] looked at single degree of freedom systems with self-centering capacity. They compared the response of systems with flag-shaped hysteretic, representing self-centering systems, with bilinear elasto-plastic systems representing moment resisting frames. Examples of the two hysteretic are shown in Figure 2.2. It was found that a flag-shaped hysteretic system with the same period and strength as the bilinear system, had lower displacement ductility, which is the maximum displacement divided by the yield displacement. The seismic response of both systems were similar, but the flag-shaped hysteretic could perform better than the bilinear elasto-plastic by adjusting α , the post-yield stiffness coefficient, and β , the energy dissipation capacity of the system. Further, the bilinear elasto-plastic hysteretic system exhibited residual drift, particularly when the system had short periods and low stiffness. The flag-shaped hysteretic system however showed no residual drift.

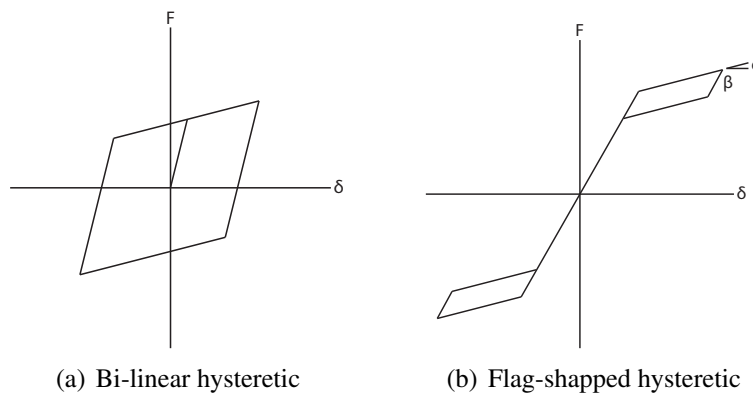


Figure 2.2: The two frames hysteretic systems compared by Christopoulos [16]. The Bi-linear system represents a welded moment frame and the flag-shaped system is a self-centering moment frame.

2.4.2 Self-Centering Moment Frames

Self-centering moment frames are similar to traditional moment frames, however, the connections and method of energy dissipation allow the system to follow a flag-shaped hysteretic instead of a bilinear hysteretic. In a self-centering moment frame, beam-column connections are made with posttensioned elements instead of welded or bolted connections [Figure 2.3(a)]. The

posttensioned elements hold the beams and columns together, but still allow some rotation or separation between the beam and column. During dynamic loading from an earthquake, gaps forms between the beams and the columns that have been post-tensioned [Figure 2.3(b)]. The post-tensioning in the beam-column connection pulls the beam and the column back together when the earthquake stops. This action pulls the system back to center eliminating residual drift in the structure. Energy in this frame is dissipated through friction dampeners or other elements and not through damage to the structural elements.

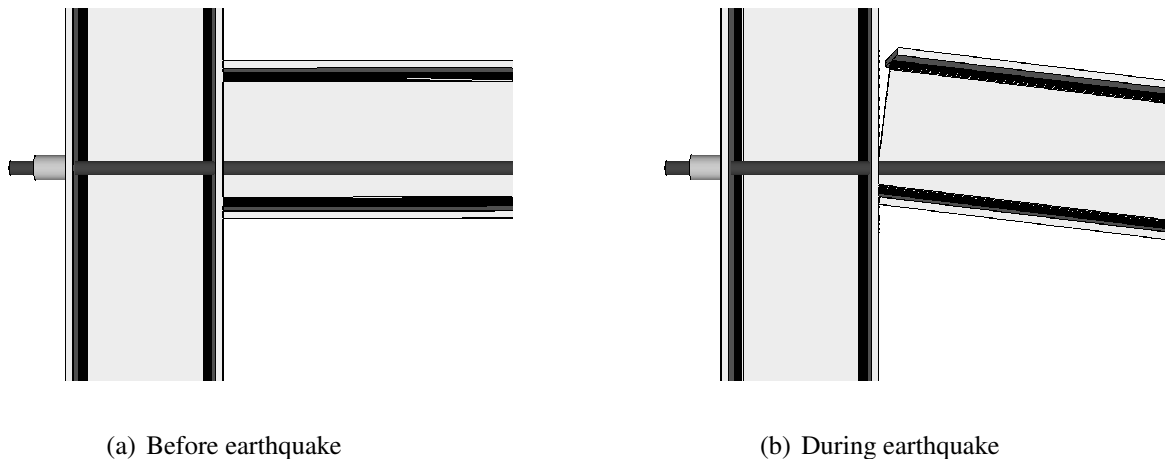


Figure 2.3: Beam column connection in a self-centering moment frame before and during an earthquake.

Garlock et al. [17] examined the behavior of post-tensioned beam-to-column connections. In these connections, the posttensioned high strength strands run parallel to the beams, compressing the beam flanges to the column flanges to resist moments. The posttensioned strands also act to self-center the structure. This study looked at six different posttension connections under cyclic inelastic loading. Frames with these connections were found to self-center when the beams did not experience local buckling and the posttension strands did not yield. When either of the above occurred, the frame did not fully self-center. It is suggested that the number of posttensioned strands can be increased to prevent yielding in the strands and the beam should be designed to avoid local buckling. A design procedure for the posttensioned steel systems is presented by Garlock et al. [18].

Chou et al. [19] further investigated the work done by Garlock [17] looking at the post-tensioned beam-to-column connections. In this study, the authors used reduced flange plates as the energy dissipating device. This system was able to dissipate energy and experienced a flag-shaped hysteretic. However, similar to Garlock [17], the system showed a loss of strength, stiffness, and self-centering capacity when the post-tensioned strands yielded or the beam experienced local buckling.

Kim and Christopoulos [20] present an alternative design procedure to eliminate the problems given above. This procedure aims to keep the post-tensioning strands elastic and to prevent local buckling of the beam to help achieve self-centering behavior. Further, this procedure also aims to achieve similar response indices to welded moment frames, but with less residual drift. Their results showed that both the moment frame and the self-centering frame had similar initial stiffness as well as maximum interstory drifts and accelerations. Further, the self-centering frame remained ductile under ultimate loads, and experienced almost no residual drift above the first floor while the moment frame had significant residual drifts.

A self-centering moment frame with a self-centering friction connections was proposed by Kim and Christopoulos [21]. The self-centering friction connection has frictional energy dissipating devices on the top and bottom of the beams. The frictional energy dissipating devices are activated when gaps form between the beams and the columns and absorb energy in the frame. The authors conducted tests comparing the response of the frame with only the friction device, then with only the post-tensioning strands, and then with both the friction device and the post-tensioning strands. The friction dissipating device was shown to have good capacity. Under self-centering limits, the frame with both the friction dissipating device and the post-tensioned strands had no residual drift. The frame also had no structural damage.

2.4.3 Self-Centering Braced Frames

Another type of self-centering frame is the self-centering energy dissipating brace. This system is introduced by Christopoulos et al. [16]. In one form of this system, there are two bracing members that are connected to the dissipating mechanism. The two braces slide against each other, but the post-tensioned element of the brace pulls the element brace back and re-centers the frame. In tests done by Choi et al. [15] comparing the seismic response of self-centering energy

dissipating braces, buckling restrained braces, and moment resisting frames through a pushover analysis, the self-centering energy dissipating brace had lower residual drift than the other two systems. Further, in a nonlinear analysis done by Tremblay et al. [14] comparing buckling restrained braces to self-centering energy dissipating braces, the self-centering braces did not have residual drift, but the BRBF had high residual drifts.

Another type of self-centering brace frame was described by Zhu and Zhang [22]. This brace is a self-centering friction damping brace that follows a flag-shaped hysteretic. The brace uses shape memory alloys to obtain self-centering capacity and friction to dissipate energy. Zhu and Zhang compared their self-centering friction damping brace with buckling restrained braces. They found that under the design base earthquake suite, the self-centering friction damping brace had greater maximum drift than a buckling restrained brace, but while the BRBF had residual drift, the self-centering frame had no residual drift.

2.4.4 Rocking Frames

Another type of self-centering frame that is being developed to reduce residual drift is a rocking frame. A rocking frame allows uplift in the column through yielding of the column base plate; this allows the frame to rock during the ground motion and uses gravity as the restoring force. Midorikawa et al. [23] conducted shake table experiments on steel braced frames that allowed for the columns to have uplift to determine if the rocking vibration and uplift reduces the seismic damage to a building. The results of the shake table analysis were compared to the result of a fixed base structure. They found that allowing uplift in the columns reduced the base shear. Further, the maximum roof displacements for the uplift system were similar to displacements for an elastic fixed base system.

2.5 Dual Systems

The self-centering systems described in the previous section have not been implemented because they are unconventional. Another alternative to the self-centering frames is to combine two traditional systems together. This type of system is called a dual system.

Since BRBFs and MRFs both have residual displacements after severe earthquakes, several researchers have suggested combining lateral load resisting systems to try to mitigate residual drift in a structure. A dual system has a primary lateral load resisting system that resists most of the load and a secondary system that is designed to resist the remaining lateral load and provide post-yield stiffness.

2.5.1 Designing a Dual System

Pettinga et al. [24] suggested a method for designing a secondary system based on the primary system and the desired amount of post-yield stiffness. Particularly they focused on designing a secondary moment frame out of existing elements from the gravity load resisting system. The secondary system is designed to resist a certain portion of the lateral load as determined by the desired post-yield stiffness. The secondary system is designed to remain elastic during the earthquake excitation. Yielding in the building should only occur in the primary system. Since the secondary system does not yield during the earthquake, the secondary system should act as a self-righting mechanism to re-center the structure after the earthquake.

To limit the amount of residual drift in a system, the secondary system should be designed to resist a certain percentage of the lateral force to help preserve a certain percentage of the pre-yield stiffness. The amount of post-yield stiffness to limit the amount of residual displacement has been suggested by Pettinga et al. [24] to be somewhere between 5 and 10% of the initial stiffness. Increasing the post-yield stiffness further does not significantly decrease the residual drift.

2.5.2 BRBFs as the Primary System

Researchers have suggested reducing residual drift in BRBFs by turning some of the gravity beams and columns into moment resisting frames. Kiggins and Uang [25] tested a building with BRBFs as the main lateral force resisting system with added moment frames as a back-up system. The MRFs were designed to resist 25% of the seismic base shear, and the BRBFs sections were reduced to match the sizes required for 75% of the base shear. Their study concluded that the maximum drifts could be reduced by 10-12% with the addition of a secondary system. The residual drifts were decreased by about 50%, dropping from 0.0039 to 0.0021 for a three story frame and

from 0.0029 to 0.0013 for a six-story frame. Xie [26] followed a similar procedure to that of Kiggins and Uang and concluded that the addition of a secondary system would reduce drifts due to the added stiffness of the secondary frame.

Maley et al. [27] used a displacement based design method to design a dual lateral force resisting system that combines MRFs with BRBFs. Two designs were made of a six story system with the strength divided between the two systems. The first design divided the base shear equally with both systems being designed to resist half of the base shear. In the second design, the MRF was designed for 40% of the base shear and the BRBF was designed for 60% of the base shear. They did find a reduction of residual drift when the MRF was designed to resist more of the base shear. They concluded that this happened because the post yield stiffness of the system increases as the moment frame resists more of the load.

2.5.3 MRFs as the Primary System

Apostolakis and Dargush [28] looked at retrofitting moment resisting frames with yielding metallic buckling restrained braces or friction dampeners. In their study, they utilized a genetic algorithm to optimize the frame by varying the position and type of the energy dampener. They also varied the yield or slip load and the stiffness of the brace. The original moment resisting frames were compared to the optimized retrofitted design with the dampers or braces. Each frame was tested under a suite of non-linear time histories where parameters such as residual drift, maximum drift, and maximum acceleration were compared. The positions of the braces or dampeners were allowed to vary across the length of the building. They found that contrary to common design where all of the braces are located in either interior or exterior bays, the best frames had a topological distribution that crossed the building and alternated the braces between the internal and external bays. In each comparison between the original and the retrofitted design, the optimized frames performed better than the original. The optimized frames had smaller maximum displacements and small to negligible residual drift while the original frames had noticeable amounts of residual drifts. By following this procedure, they were able to decrease the amount of residual drift in a system by adding braces or frictions dampeners as the secondary system.

2.5.4 BRBFs with Secondary Gravity Columns

Several studies have been completed on how gravity columns add to the strength and stiffness of a building. Sabelli et al. [13] accounted for the effects of the gravity columns in his models by adding an additional column to the model with the moment of inertia and moment capacity equal to the sum of the gravity columns in the building. The column was assumed to provide little resistance to the lateral loads, but still provided some resistance. The column also worked to redistribute loads across the story. Further, MacRae et al. [29] examined the effects that the column stiffness has on the braced frame. They found that as the combined stiffness of the column increases that the story drift decreases. Thus gravity columns do add to the structure's lateral stiffness, and increasing the column size decreases the systems drift.

As suggested earlier in the research, dual systems should be made out of structural elements already available in the gravity system. Gravity columns have lateral stiffness that could be combined with a BRBF to increase the system stiffness and the post-yield stiffness. In previous work, this has been accomplished by turning the gravity columns and beams into moment frames. This thesis will explore two other ways to activate the gravity columns. The gravity columns need to be constrained so that they resist some of the lateral loads. This can be done by fixing the column connection to the ground, or, by designing a brace to remain elastic and limit the displacement in that story. These methods are looked at in the following chapters of this thesis.

CHAPTER 3. SINGLE DEGREE OF FREEDOM STUDY

To determine if heavier gravity columns might be effective in reducing the amount of residual drift in a system, single and multiple degree of freedom models were studied under several earthquake ground motions. The single degree of freedom (SDOF) study is presented in this chapter.

3.1 Method

A parametric study was conducted using SDOF models subjected to earthquake loading. The parameters considered were braces size, column size, and earthquake loading. Brace and column sizes were varied to get a range of periods and post-yield stiffness for each earthquake. Thus, for each earthquake considered, various response spectra were generated as described in the following sections.

3.2 Model

The SDOF model shown in Figure 3.1 was used for the study. The two dimensional model consisted of a cantilever column and a brace that meet at a node. The height of the column, h , is 3.96m (13ft). The brace length, L_{br} , is 9.96m (32.7ft). A mass, m , of 184×10^3 kg (1.048 kip-sec²/in) was assigned to the top node. Young's Modulus, E , was taken as 200Gpa (29000ksi). The angle, θ , is the angle between the ground and the brace, and constrained by L and h . The cross-sectional area of the brace, A_{brace} , and the moment of inertia of the column, I_{col} , were varied in the parameter study. The procedure to vary A_{brace} and I_{col} is given later in this chapter.

The SDOF model has an elasto-plastic brace and an elastic column. Considering the brace only, the force-displacement diagram for the model is shown in Figure 3.2(a). The stiffness of the elasto-plastic brace before yielding is k_{brace} , and the stiffness after yielding is zero. Next, considering the column only, the force-displacement diagram for the model is shown in Figure 3.2(b).

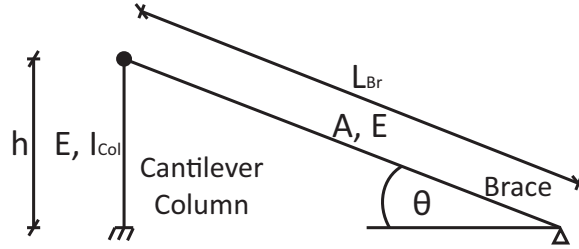


Figure 3.1: The SDOF model geometry used in the study.

Since the column is an elastic material, it has constant stiffness, k_{col} . The stiffness of the entire system [Figure 3.2(c)], k_i , is the sum of the stiffness of the two elements,

$$k_i = k_{col} + k_{brace} \quad (3.1)$$

The post-yield stiffness of the system is also the sum of the brace and column after yielding. Since the column does not yield, but the brace does, the post yield stiffness of the system is equal to k_{col} , as indicated in Figure 3.2(c). It is convenient to introduce a factor γ that relates the initial and post-yield stiffness of the model.

$$\gamma = \frac{k_{col}}{k_{col} + k_{brace}} \quad (3.2)$$

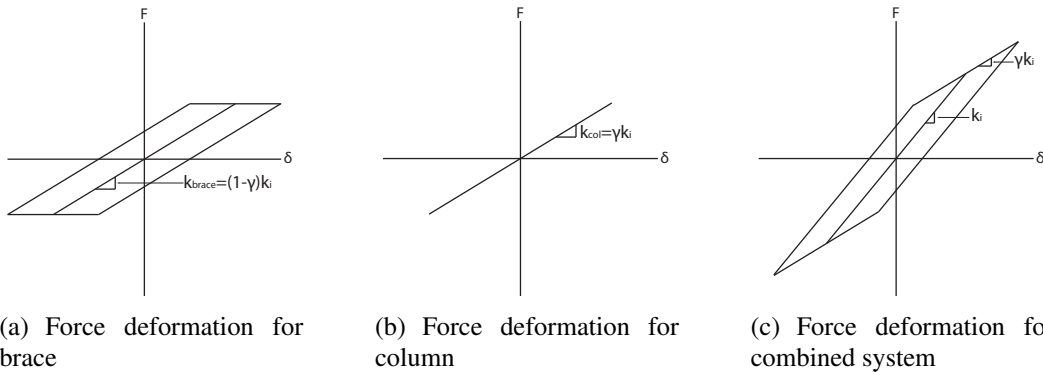


Figure 3.2: Force deformation diagrams for elasto-perfectly plastic system, elastic system, and a combined system.

It is helpful for later discussion to write the expressions for k_{col} and k_{brace} in terms of k_i and γ .

$$k_{col} = \gamma k_i \quad (3.3)$$

$$k_{brace} = (1 - \gamma)k_i \quad (3.4)$$

The equations above make it possible to determine values of k_{col} and k_{brace} based on desired values for k_i and γ .

3.2.1 Parameter Variation

For the parameter study, SDOF models with a range of k_i values [$k_i = 17.5$ kN/m (0.1 kip/in) to 2900000 kN/m (16552 kip/in)] and a few distinct values of γ ($\gamma = 0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.5, 0.99$) were produced. Various families were generated for specific value of γ by the following procedure.

First, a range of brace sizes was selected [$A_{brace} = 0.025$ to 38 cm² (0.01 to 15 in²)] to achieve the desired initial stiffness. Second, for each brace size and desired γ , the corresponding stiffness was determined as follows,

$$k_{brace} = \frac{EA_{brace}}{L_{br}} \cos^2 \theta \quad (3.5)$$

$$k_i = \frac{k_{brace}}{1 - \gamma} \quad (3.6)$$

and, third, for each combination of brace size and γ , the desired I_{col} was determined using,

$$I_{col} = \frac{k_{col} h^3}{3E} \quad (3.7)$$

$$k_{col} = \frac{3EI_{col}}{h^3} \quad (3.8)$$

3.3 OpenSees

The models were analyzed using the program OpenSees (Open System for Earthquake Engineering Simulation). The OpenSees software simulates the behavior of structures to earth-

quakes [30]. To analyze the models in OpenSees, the column was modeled with an elastic beam column (elasticBeamColumn) element. The element was assigned the I_{col} that was calculated previously. The cross-sectional area of the column was approximated to be the square root of I_{col} . It was not important for the area to be exact since the column does not have axial loads (see Figure 3.1). The brace was modeled as a corotational truss (CorotTruss) element. Corotational truss elements were used because they account for large deformation effects. The brace material was modeled as an elastic-perfectly plastic material. The yield stress of the brace, F_{ybrace} , was assumed to be 317MPa (46ksi). The brace was assumed to have the same stiffness in both tension and compression. The yield strain was determined by $\epsilon = F_{ybrace}/E$. The model had fixed connections at the base (see Figure 3.1). A mass of $m = 183705$ kg (1.05 kip-sec²/in) was assigned at the node where the column and brace met.

3.4 Earthquake Ground Motions

All of the varieties of the SDOF model were analyzed under ten different earthquakes. The earthquake records are the same as were used in Richards (2009) and all from California events. The ground acceleration records were each scaled up to match a particular design spectra. The factors are shown in Table 3.1. The individual and mean response spectra for the earthquake records are shown in Figure 3.3. Each earthquake time history was padded with zero acceleration at the end to allow the system to come to rest, for accurate measurement of the residual displacement in the system.

3.5 Output

Each variety of the model was analyzed under each earthquake. The maximum and residual drifts were calculated. The natural period was also calculated for each frame.

3.6 Results and Discussion

After the dynamic analysis was performed on the systems with varying periods and post-yield stiffness, the results were plotted in the form of response spectra; outputs were plotted versus the natural period of each system and points with common gamma were connected. The results

Table 3.1: Earthquake records used in analysis

Record	PGA (g)	Scale Factor
1994 Northridge		
Canoga Park, NORTHR/CNP196	0.42	2.06
90013 Beverly Hills, NORTHR/MUL279	0.52	1.07
90018 Hollywood, NORTHR/WIL108	0.25	1.85
90006 Sun Valley, NORTHR/RO3090	0.44	1.27
1989 Loma Prieta		
1656 Hollister Array, LOMAP/HDA255	0.28	2.2
Hollister City Hall, LOMAP/HCH180	0.22	1.54
Gilroy No. 3, LOMAP/GO3090	0.37	2.6
Gilroy No. 4, LOMAP/GO4090	0.21	2.53
1987 Superstition Hills		
Parachute Test Site, SUPERST/B-PTS225	0.28	2.2
Parachute Test Site, SUPERST/B-PTS315	0.22	1.54

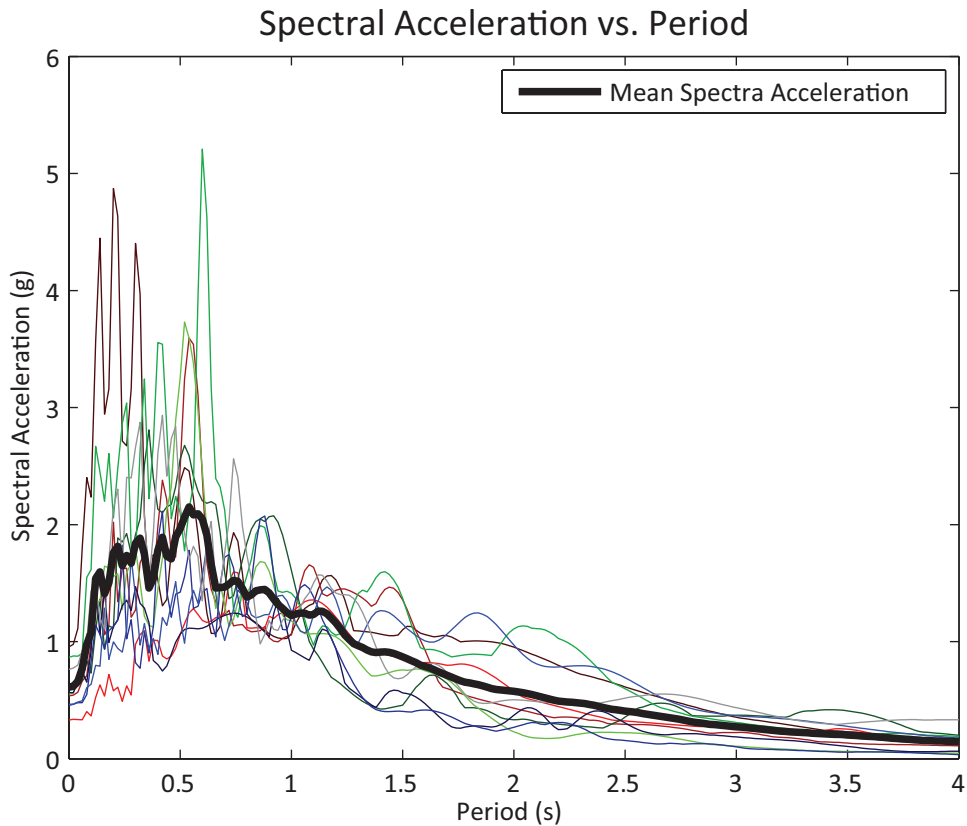


Figure 3.3: Acceleration response spectra for the suite of earthquakes.

were plotted for each of the ten earthquakes. For example, Figure 3.4 shows the plots below shows the displacement spectra for the Canoga Park record. Average displacement spectra for the suite of earthquakes will be presented in the rest of this chapter. Results from the individual earthquakes are included in Appendix A.

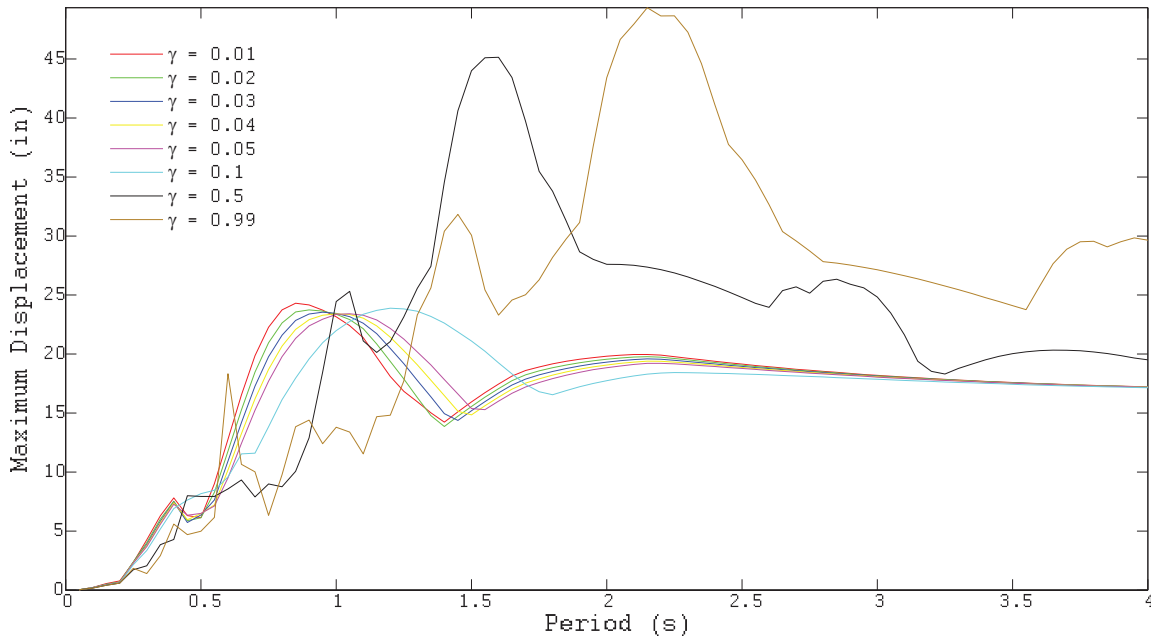


Figure 3.4: Maximum displacement response spectra for the 1994 Northridge recorded at Canoga Park.

3.6.1 Maximum Displacement

The maximum displacements for the SDOF models are more dependent on the period of the systems than the post yield stiffness, γ . The averages over the suite of earthquakes of the maximum displacements versus period are shown in Figure 3.5. This plot shows the ranges of maximum displacement for periods up to 20 seconds to give an idea of what is happening in the system. Another plot, Figure 3.6 shows the spectra for periods less than 4 seconds to give a close up of the region of interest.

The displacement spectra for the models with $\gamma = 0.99$ is approximately equivalent to the elastic response spectra for the suite of earthquakes. The elastic response spectra has low maximum

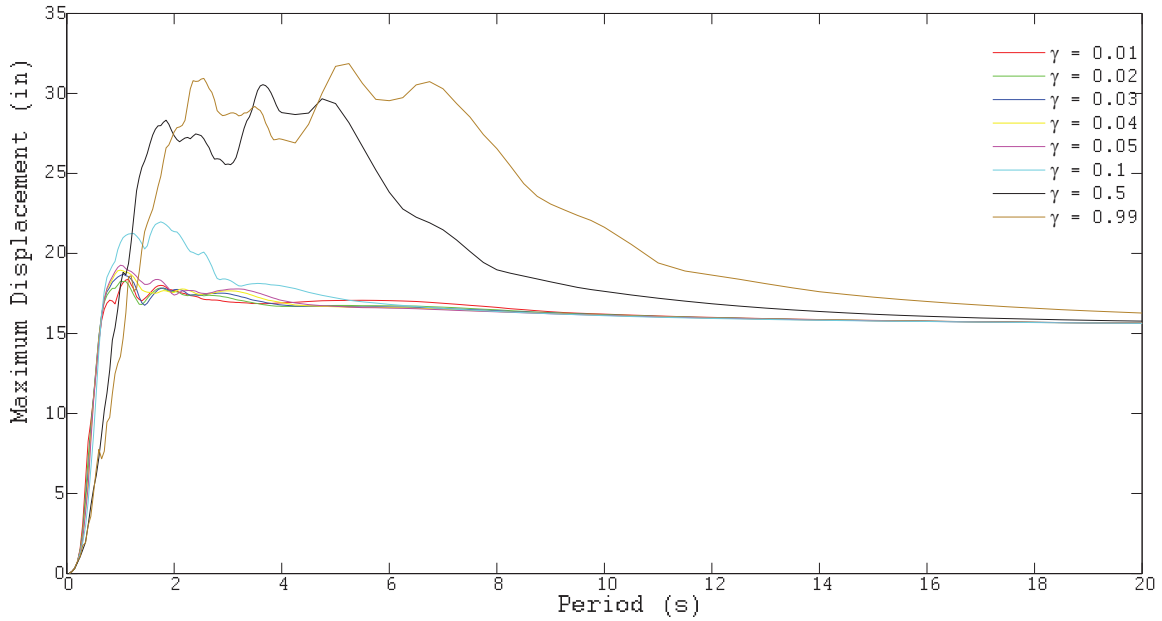


Figure 3.5: The mean maximum displacement response spectra.

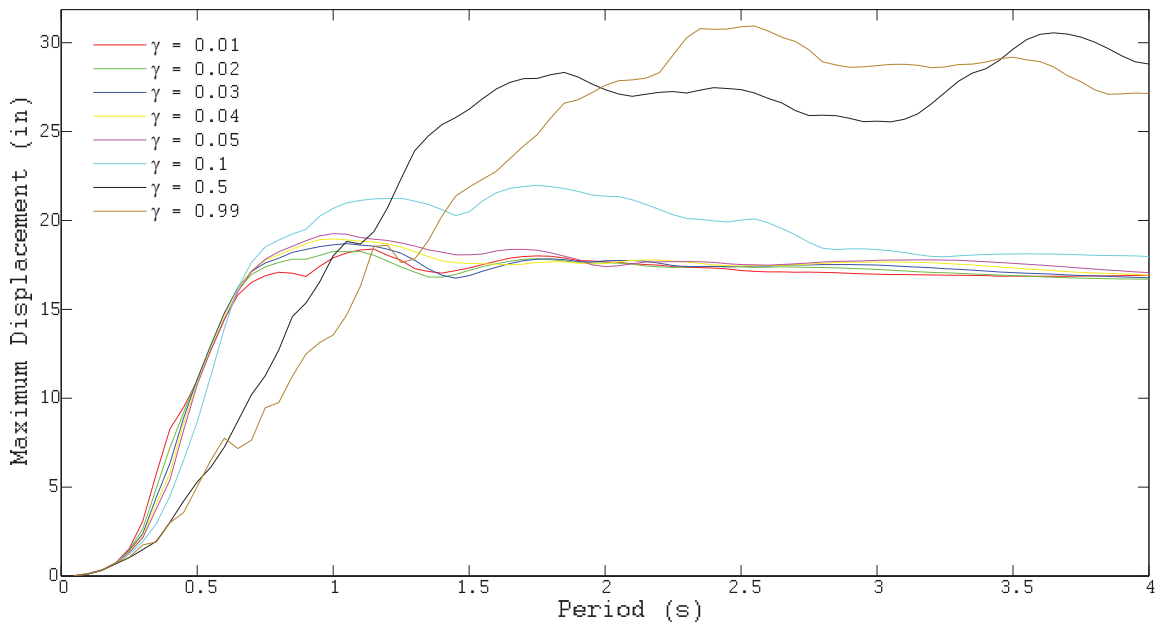


Figure 3.6: The mean maximum displacement response spectra for periods less than 4 seconds.

displacements for periods less than 0.2 seconds. For periods ranging from 0.2 seconds to about 2 seconds the maximum displacement of the system increases until it reaches a maximum value. The response decreases as periods become long. For elastic structures with long periods, the top

of the structure essentially does not move and the maximum relative displacement is the same as the ground displacement.

The displacement spectra for the models with $\gamma = 0.01$ is approximately the elastic perfectly plastic response spectra for the suite of earthquakes. Yielding in the braces helps to limit the maximum displacement of the system. Figure 3.5 and Figure 3.6 show that the elasto-plastic systems have low maximum displacements for $T < 0.2$ seconds because the systems do not yield. As the natural period increases, there is a sharp increase in displacement ($0.2 < T < 0.6$ seconds). At approximately $T = 0.6$ seconds, the elasto-plastic spectra reaches a maximum displacement and levels off. The maximum displacements for the elasto-plastic systems ($\gamma = 0.01$) is smaller than the elastic systems ($\gamma = 0.99$) because yielding prevents resonance.

3.6.2 Residual Displacement

Residual displacements are the focus of the present study. Figure 3.7 shows the residual displacements for the systems analyzed. The residual displacements for the region of interest for this study, $T < 4$ seconds, are shown in Figure 3.8. High values of γ represent more elastic systems, with $\gamma = 0.99$ representing an essentially elastic system.

The residual displacement of the models was found to be dependent on the period. All of the models had no residual displacement for very low periods, $0 < T < 0.2$ seconds since the maximum displacement was also low and the models do not yield. As such, the systems performed elastically and did not have permanent deformation. As the period increases, the residual drift increases because the systems are less strong (flexible) and have yielding in the braces. Yielding in the braces leads to permanent deformation of the brace and residual displacement in the frame.

The residual displacement was also dependent on γ . Systems with higher values of γ have little to no residual displacement while systems with lower values of γ , $0.01 < \gamma < 0.05$, have much higher residual displacement. For example, for a system with a period of 0.5 seconds and $\gamma = 0.1$ seconds the residual displacement is 0.4 inches. However, the same system with $\gamma = 0.01$ the residual displacements is 3 inches. The residual displacement for the system with $\gamma = 0.01$ is 7.5 times greater than the residual displacement of the system with $\gamma = 0.1$. The largest decrease in residual drift occurs when γ is increased from 0.01 to 0.1. Systems with a post-yield stiffness greater than $\gamma = 0.1$ have decreased residual drift, but the return for increased stiffness is less.

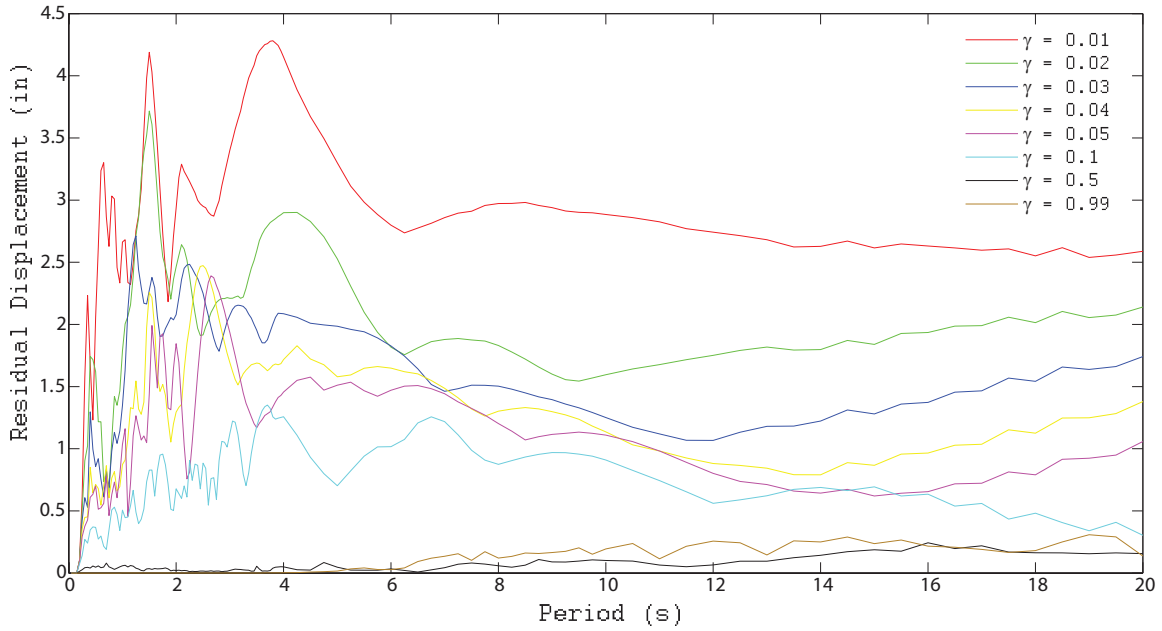


Figure 3.7: The mean residual displacement response spectra for the suite of earthquakes for periods less than 4 seconds.

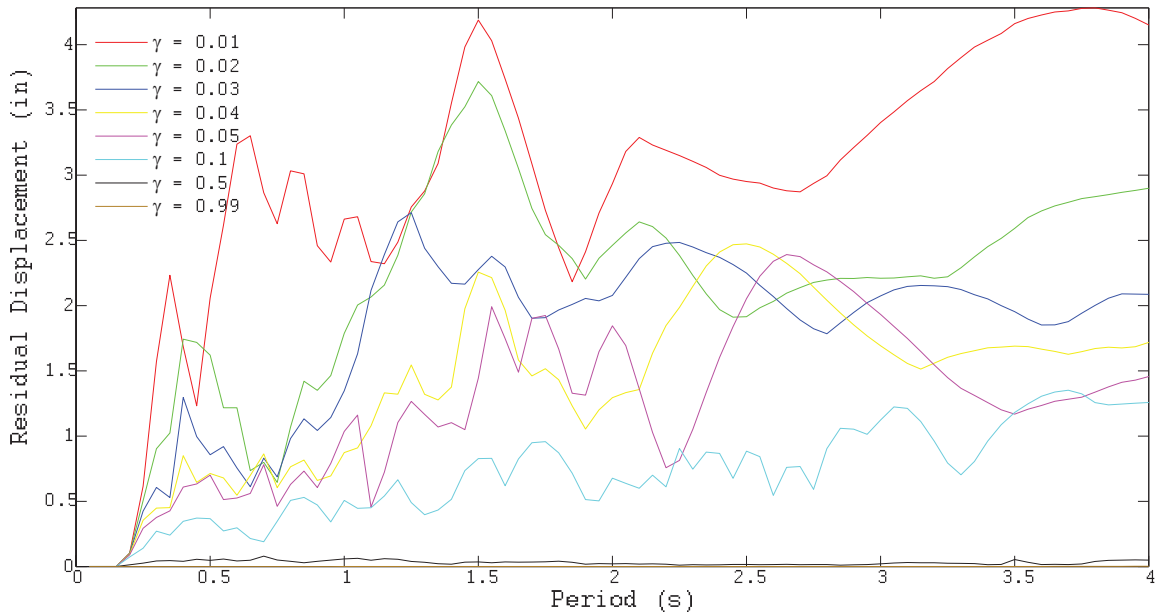


Figure 3.8: The mean residual displacement response spectra.

These results are similar to results from Pettinga [24], Kiggins and Uang [25] and Xie [26].

3.6.3 Implications for Buildings

The levels of γ can represent different building structures. Systems with low post-yield stiffness represent structures with very little gravity column stiffness; the majority of the stiffness comes from the brace. These systems can have high residual displacement [3, 13, 15]. Increasing the post-yield stiffness would be equivalent to having a large column and very small brace. A larger column adds more stiffness to the system after the brace yields. This stiffness works as a self-centering system to bring the structure back to center after the earthquake. Larger columns will result in less residual displacement in the system.

Post-yield stiffness reduces the amount of residual displacement. Systems studied with high values of post-yield stiffness, $\gamma > 0.10$, have much smaller residual displacements than systems with very low, $\gamma < 0.05$, post-yield stiffness. As suggested by Pettinga et al. [24], the return for increased post-yield stiffness does begin to taper off when the post-yield stiffness is between 5 – 10%, $\gamma = 0.05$ to 0.1.

3.7 SDOF Conclusions

From the SDOF study, it was found that the residual drift is dependent on the period of the system and the post-yield stiffness. Higher post-yield stiffness results in smaller residual displacement. The greatest return for decreasing residual drift come when the residual drift is increased by 5-10%, $\gamma = 0.05$ or $\gamma = 0.1$. Systems with periods between 0.3 and 3 seconds had the largest drop in residual drift when γ was increased to 0.1.

CHAPTER 4. THREE DEGREE OF FREEDOM STUDY

4.1 Overview

Three story structures were investigated to determine how involving gravity columns and increasing the post-yield stiffness would decrease the amount of residual drift for multiple degree of freedom systems. Three story models were selected because hand calculations for the stiffness of the brace system and the cantilever column were still easily performed.

A parametric study was conducted on three story models, similar to that done for SDOF systems. The parameters considered were brace size, column moment of inertia, and earthquake loading. An additional parameter, frame configuration was also considered. The frame configurations considered represent different ways in which the gravity columns might be activated in practice. The first configuration activated the stiffness of the gravity columns by having fixed connections between the columns and the ground. The second and third configurations activated the gravity columns by having an extremely stiff story (either in the second or third story), acting in some respect as an outrigger story.

4.2 Model

The basic designs for the three story frames are shown on the next page, Figure 4.1. The frame consists of gravity columns, braces, and beams. The height of each story, h , was taken as 3.96m (13ft). The brace length, L_{br} , was 9.96m (32.7ft). Young's modulus, E , was taken as 200Gpa (29000ksi) for all members. The mass assigned to each story is 184×10^3 kg (1.048 kip-sec²/in). The mass was assigned to the top node of each brace.

The right column represents all the gravity columns. The column stiffness (I_{col}), was varied for the different models, but were always the same for all stories in a particular model. The moment of inertia was found by taking the moment of inertia of a single column and multiplying it by the

number of gravity columns it was suppose to represent, in this study, the column represented eight gravity columns (see Appendix B for a description of a prototype system represented by the model).

The braced frame was designed with the same columns as used for the gravity analysis. The brace size (A_{brace}) was varied for the different models.

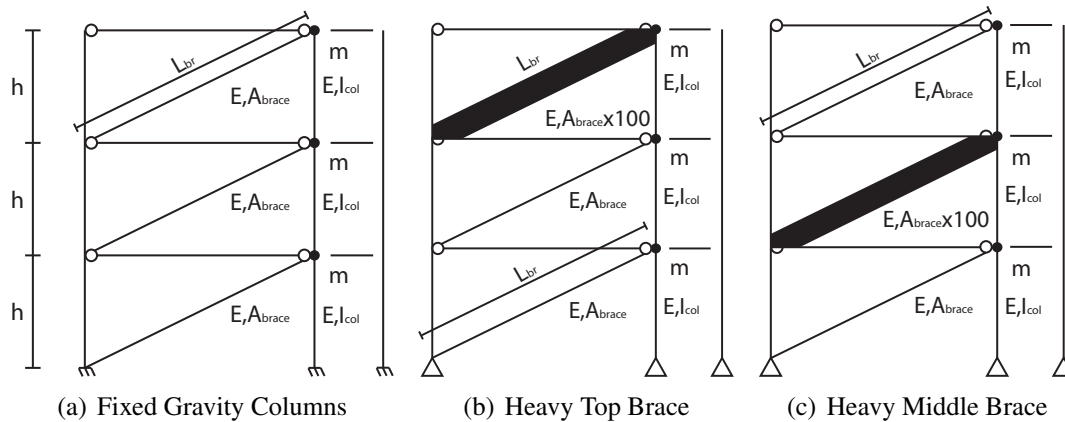


Figure 4.1: The three frame configurations used for the three story models.

4.3 Modeling and Analysis Techniques

The three story models were built and analyzed with OpenSees [30]. The right columns were modeled to resist earthquake loads were assumed to be elastic columns and were modeled with the elasticBeamColumn elements in OpenSees. The column area was assumed to be the square root of the moment of inertia. The other columns and beams were both assumed to be elastic materials as well, but without flexural stiffness. The braces were modeled as elasto-perfectly plastic materials with the same properties as the SDOF systems.

4.4 Parameter Variation

4.4.1 Brace Parameter

As for the SDOF systems, the brace size was a varied parameter. Varying the brace size allowed frames with different periods to be investigated. The brace sizes ranged from 0.065 cm^2 to 645 cm^2 (0.01 in^2 to 100 in^2). The same size of brace was used for all stories in a particular

model. The exception to this is for the second and third configurations where the brace on either the second or third story was 100 times larger than the other braces.

4.4.2 Column Variation

To create different spectra for the three story frames, different column sizes were used for the gravity columns. The first value for the columns, 19771 cm^4 ($I = 3800 \text{ in}^4$), came from a gravity design of a basic three story building with bays spaced 9.144 m (30 ft) apart in both directions and a seismic weight of 4.79 kPa (100 psf). This value represents the moment of inertia of eight W12x58 columns (see Appendix B). The additional values for the columns represent cases where the initial column stiffness has been increased.

4.4.3 Model Configuration

Models were built with each combination of brace size, column moment of inertia, and frame configuration. Three configurations were investigated to find the best way to activate the gravity columns. Fixing the gravity columns at the bottom can be complicated and expensive, but putting a heavy brace in one story is easy. Therefore if the lateral stiffness of the gravity columns can be activated with the heavier brace, it would be easy to implement in practice.

4.4.4 Earthquake Ground Motions

The same suite of earthquakes was used for the three story models as was used for the single degree of freedom systems. The ground motions were padded with zeros so the systems would have time to come to rest for a better measurement of the residual drift.

4.5 Output

The maximum and residual drift for each level were output for each model and ground motion. The natural period of the structure was also output. Drift is the amount of horizontal displacement divided by the height of the story.

It was also desired to know if the column would truly remain elastic since the columns were modeled as elastic elements. To determine if they theoretically should have yielded, the maximum moments and axial forces in the columns were also recorded.

The elastic section modulus, S_x was calculated as $S_x = \frac{2I_{col}}{d}$, where d is the depth of the column. The yield stress from the axial loads was found by $f_{axial} = P/A_{col}$, where P is the axial load on the column and A_{col} is the cross-sectional area of the column. The remaining yield stress in the columns, f'_y is found by taking $f_y - F_{axial}$. M_y was then found.

$$M_y = S_x f'_y = \frac{2I_{col}}{d} f'_y \quad (4.1)$$

The column would have yielded if the ratio of the maximum moment M_{max} to M_y exceeded 1.0.

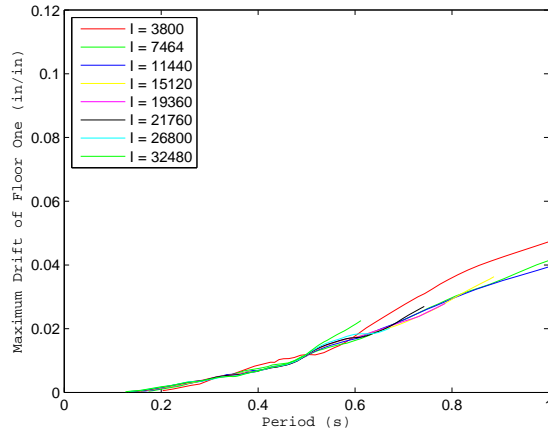
4.6 Results and Discussion

The maximum drift for each structure was output for each floor and earthquake. The maximum drifts were averaged over the suite of earthquakes, and the average was then plotted versus the period for each of the floors. The residual drift were also average over the suite of earthquakes and plotted versus period of each of the floors. Additionally, the moment ratios were also average and plotted for each of the floors. The results for the three different systems are discussed below.

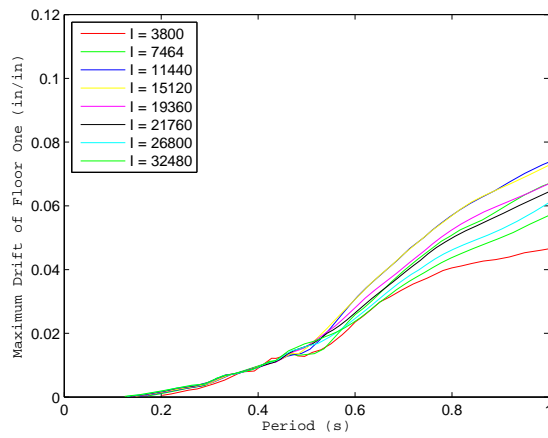
4.6.1 Maximum Drift Results And Discussion

The maximum drifts for the three systems depend on the system type, story height, natural period of the structure and the column size. The maximum drifts for the first floor are shown in Figure 4.2, and the maximum drifts for the second story are in shown Figure 4.3. The drifts for the third story are not shown because they are very small.

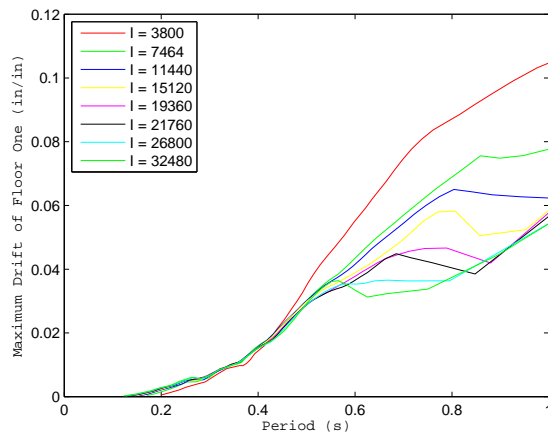
The moment of inertia of the gravity columns affected the maximum drift of the different configurations. At the first floor, the three different configurations show a similar trend for maximum drift. The drift is small for stiff structures with low periods. As the period increases, the maximum drift also increases.



(a) Fixed Gravity Columns

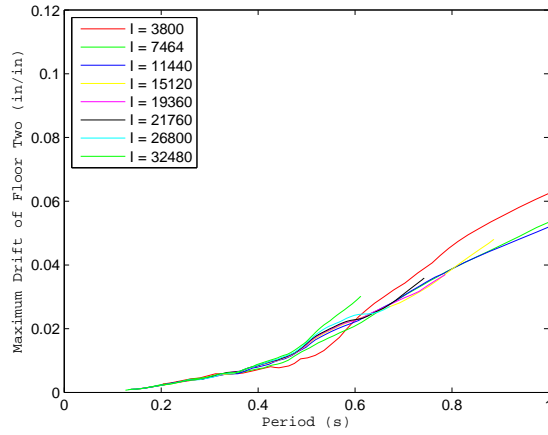


(b) Heavy Top Brace

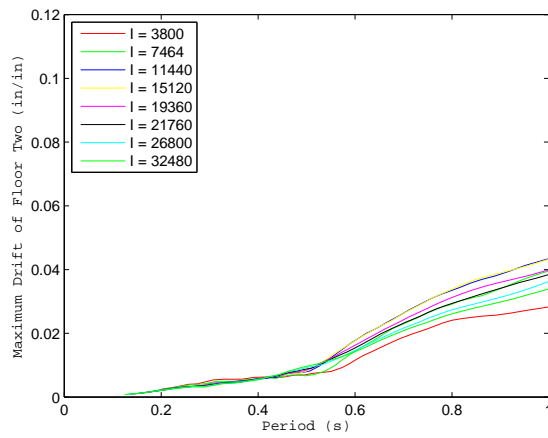


(c) Heavy Middle Brace

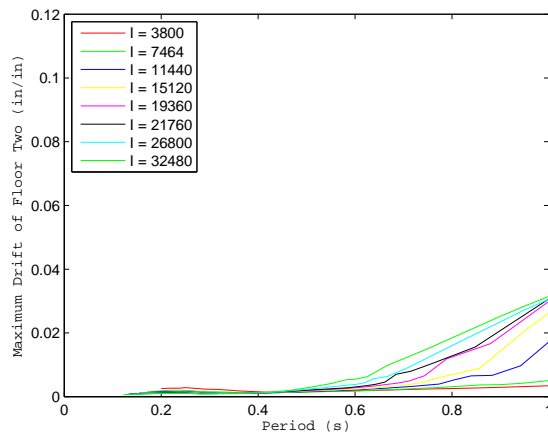
Figure 4.2: Maximum drift in the first story of the three different types of frames tested.



(a) Fixed Gravity Columns



(b) Heavy Top Brace



(c) Heavy Middle Brace

Figure 4.3: Maximum drift in the second story of the three different types of frames tested.

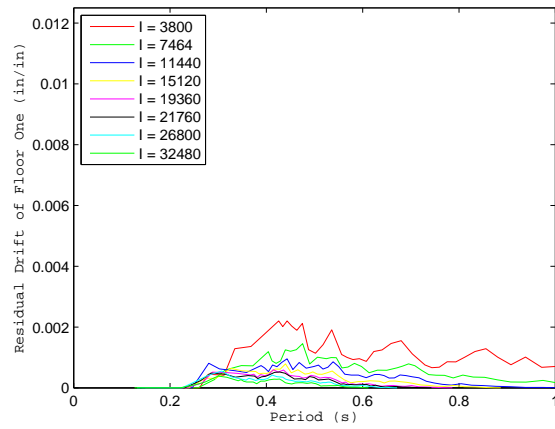
For periods greater than 0.5 seconds, the moment of inertia of the column affects how much maximum drift the frame experiences. The amount of drift is also dependent on the frame configuration. The maximum drifts for the first systems are relatively uniform, and all of the spectra have relatively the same drifts, but there begins to be slight variations between the drifts as the period increases. This variation is more apparent in the second and third configurations with the third configuration having the widest variations.

Adding a heavy brace to the second story reduces the maximum drift of the frame. At the second floor (Figure 4.3), the maximum drift is the highest for the first configuration with the fixed gravity columns. The maximum drift from the second configuration are similar, but slightly lower. The lowest drifts are from the third configuration where a heavy brace was placed in the second story. The maximum drifts for this story are almost zero for periods less than 0.6 seconds. The brace does not yield for these lower periods because it is very stiff. Periods greater than 0.6 seconds have some drift, however the drift is still lower than the other configurations.

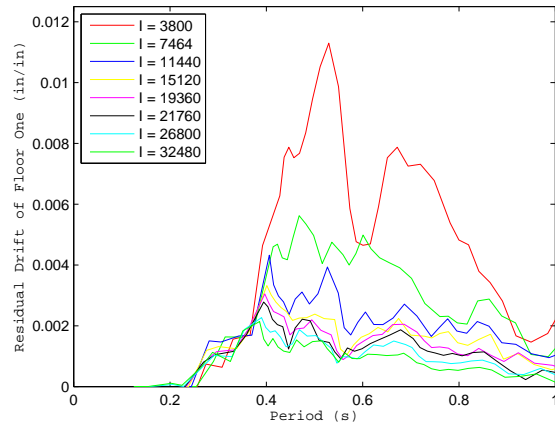
Drift for all three configurations were very small, almost non-existent for the third floor (not plotted). This is due to the design of the building. Since the same size of brace and column was used for all stories, the drifts were concentrated in the bottom two stories. A more conventional design, where the brace size decreased with building height would have more uniform drifts. Further, the second configuration would have no drifts on the top story due to the heavy brace at that story.

4.6.2 Residual Drift Results and Discussion

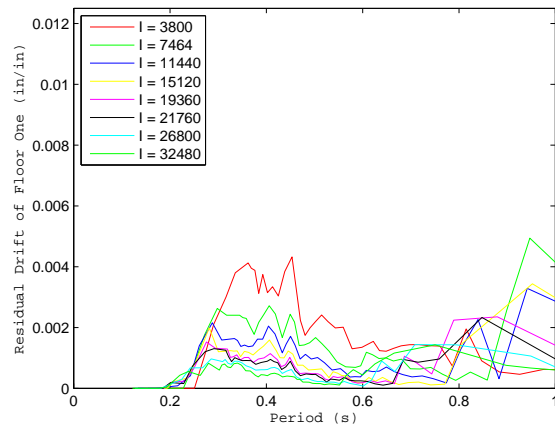
The residual drifts for the first and second floors are shown in Figures 4.4 and 4.5. The residual drifts for the third story are not shown since they are very small for all of the systems. The drifts are small at the third floor since a uniform brace size was used at all of the levels and the drifts were concentrated at the first and second floors.



(a) Fixed Gravity Columns

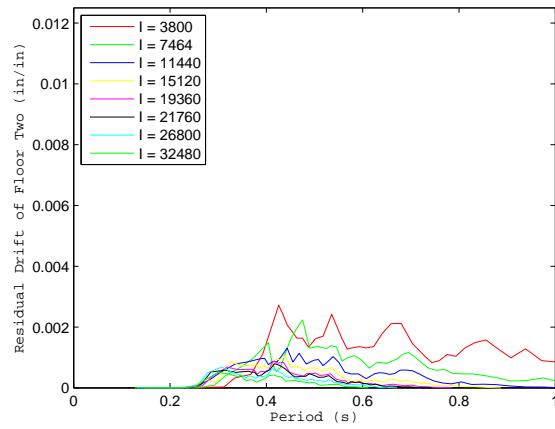


(b) Heavy Top Brace

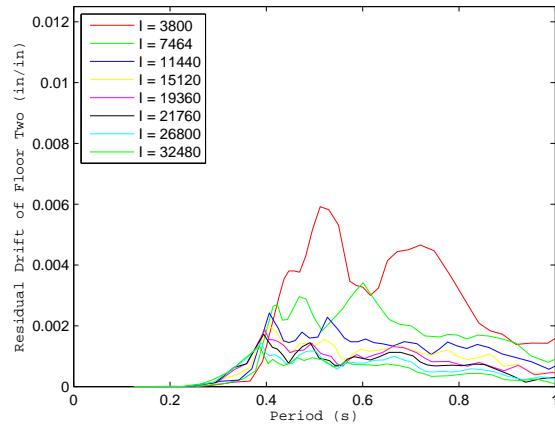


(c) Heavy Middle Brace

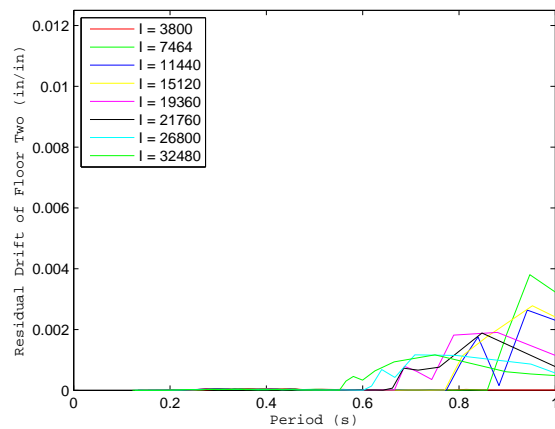
Figure 4.4: Residual drift in the first story of the three different types of frames tested.



(a) Fixed Gravity Columns



(b) Heavy Top Brace



(c) Heavy Middle Brace

Figure 4.5: Residual drift in the second story of the three different types of frames tested.

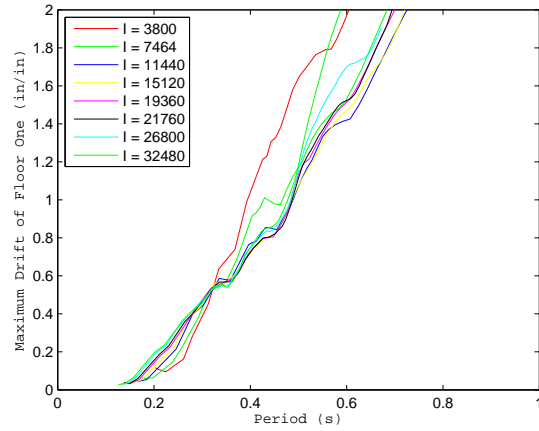
For all three of the systems, frames with larger column stiffness had lower residual drift. This is true for all floors. This is the same pattern as was seen with the SDOF systems. Increasing the stiffness of the columns does help to reduce the amount of residual drift. For example, in the system with the heavy top brace at the first floor with a period of 0.5 seconds, the residual drift is about 0.01 for the smallest column size, $I = 3800 \text{ in}^4$, but decreases to about 0.005 when the column increase to $I = 7464 \text{ in}^4$.

Overall, the fixed gravity columns system had the lowest residual drifts in the first and second story. The system with the heavy top brace had the highest residual drifts. The residual drifts at the third story are interesting because the heavy brace restricted displacements in the second floor. At the first floor, the residual drifts are slightly higher than the drift with fixed gravity columns, but at the second level, the residual drifts are negligible until a period of about 0.6 seconds. At these higher periods, the residual drift indicates that either the heavy brace did yield or that the system had not completely come to rest.

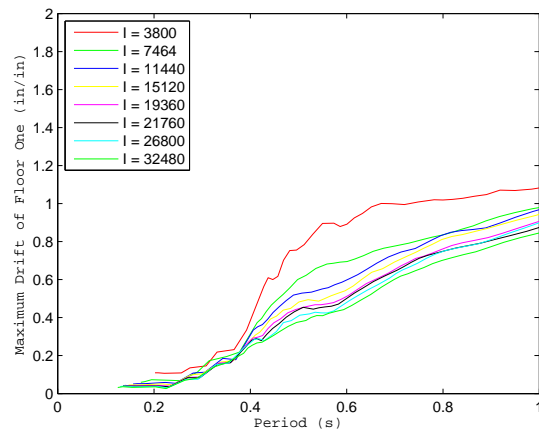
4.6.3 Moment Ratios

The moment ratios give an idea if yielding would have occurred in the columns and if the residual drift shown above would be the actual residual drift of the system. When the moment ratios are greater than one, the column would have yielded introducing more residual drift into the system. The moment ratios for the three systems are shown in Figure 4.6 for the first floor, Figure 4.7 for the second floor, and Figure 4.8 for the third floor.

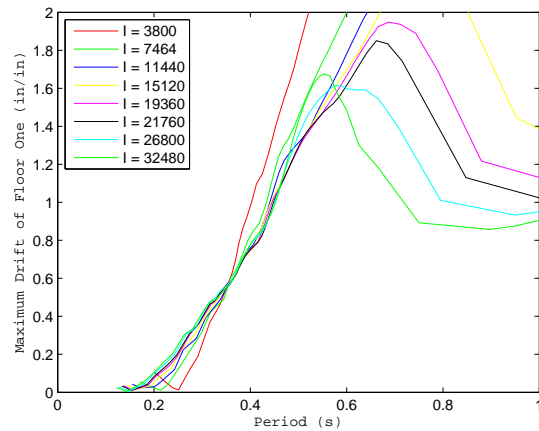
When the systems were run with elastic columns, the fixed base frame had the lowest residual drift. However, the moment ratios for many of the brace and column combinations were greater than one. The amount of residual drift that would have occurred if column yielding had been modeled is unknown. The system with the fixed gravity columns would have had yielding in the columns at all three of the stories.



(a) Fixed Gravity Columns

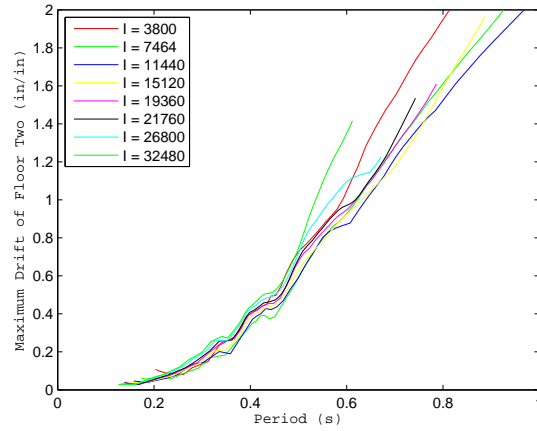


(b) Heavy Top Brace

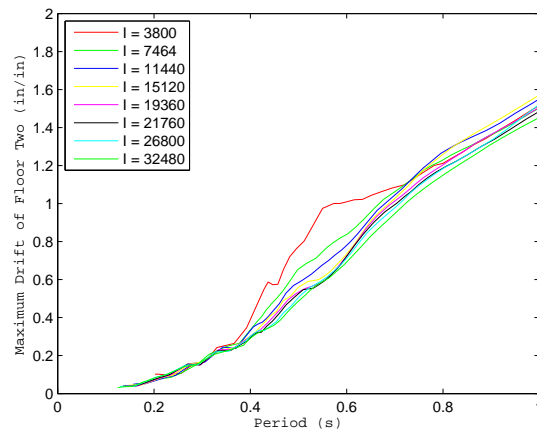


(c) Heavy Middle Brace

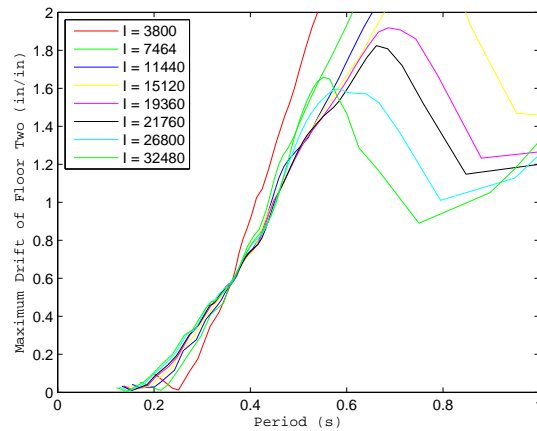
Figure 4.6: Moment ratios at the first floor. Ratios greater than one mean that the column would have yielded.



(a) Fixed Gravity Columns

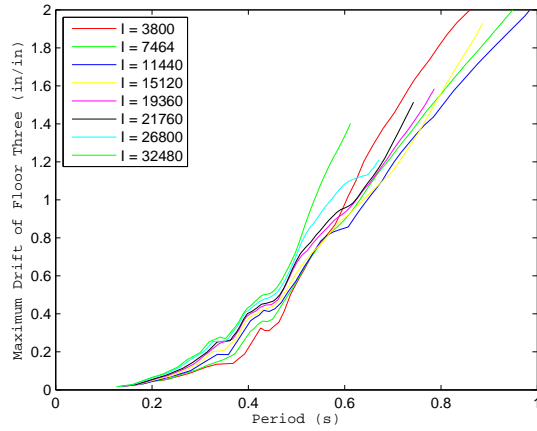


(b) Heavy Top Brace

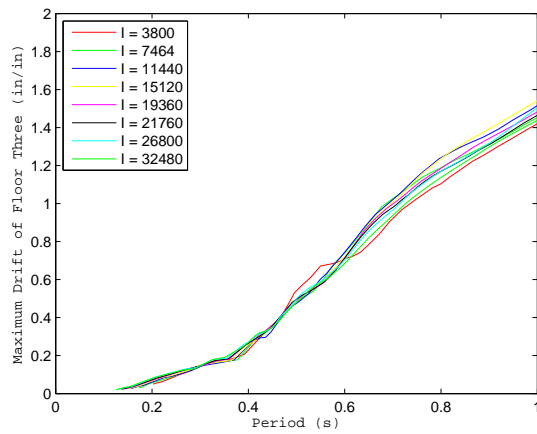


(c) Heavy Middle Brace

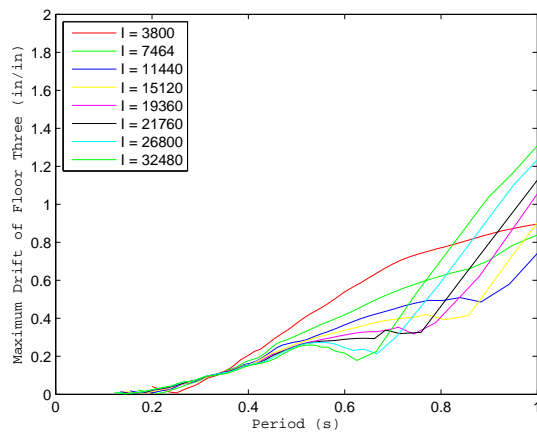
Figure 4.7: Moment ratios at the second floor. Ratios greater than one mean that the column would have yielded.



(a) Fixed Gravity Columns



(b) Heavy Top Brace



(c) Heavy Middle Brace

Figure 4.8: Moment ratios at the third floor. Ratios greater than one mean that the column would have yielded.

When the systems were run with elastic columns, the fixed base frame had the lowest residual drift. However, the moment ratios for many of the brace and column combinations were greater than one. The amount of residual drift that would have occurred if column yielding had been modeled is unknown. The system with the fixed gravity columns would have had yielding in the columns at all three of the stories.

The system with the heavy top brace appears to have performed the best, no yielding would have occurred in the first story, with the exception of the model with the smallest column, and yielding in the middle and top story only would have occurred when the period is greater than 0.7 seconds or later depending on the column size. The residual drifts for this system are the highest, but for most of the spectra that is shown, the residual drift is what would be expected if the column yielding had been modeled.

The system with the heavy middle column would have experienced yielding in the columns when the period is greater than 0.4 seconds in the first and second floor. This system had the highest moment ratios, indicating that the residual drift may be much higher than what was recorded if columns had been modeled to yield. Also, since the moment ratios are so high in the first and second floors, the drifts could be concentrating even more on the first floor.

4.7 Three Story Frame Conclusions

The three story frame had similar results to the single degree of freedom systems, the residual drift decreased as the post-yield stiffness (column stiffness) increased. As with the single degree of freedom system, the largest increase in performance is in the first increase in column stiffness. Larger increases in column stiffness continue to reduce the residual drift, but with less improvement for each increase.

The residual drift was the smallest for the fixed gravity columns case, but columns would have yielded so the data is hard to compare. The heavy middle brace had the next smallest residual drift from the brace, but had large moment ratios suggesting yielding would have occurred, increasing the residual drift. The heavy top brace, while having higher residual drifts, had very little yielding in columns. Further, as the column size was increased for each of these cases, the moment ratio decreased.

CHAPTER 5. FIVE STORY CASE STUDY

A five story BRBF was designed and analyzed to demonstrate the effectiveness of using a BRBF with gravity columns for a dual system as compared to a traditional BRBF or dual system. A five story building was selected because it is more feasible to keep the gravity columns in the elastic range for taller buildings.

5.1 Design

The building used for the case study was designed following the Equivalent Lateral Force procedure (ELF) from ASCE 7-10 [31]. The five story building considered was assumed to be located in downtown Los Angeles. The design spectra accelerations used are the same as used in the study by Richards [32]. The short period acceleration, S_{DS} , used is 1.11g and the one second acceleration is 0.61g. The building site was assumed to have site classification D soil conditions. The building is 3500 m² (37674 ft²) per level with a seismic weight of 4.0 kPa (83.5 psf), [Figures 5.1(a) and 5.1(b)]. The bays are 10 m (32.8 ft) long with 7 bays in one direction and 5 bays in the other. The height of each story is 5 m (16.4 ft).

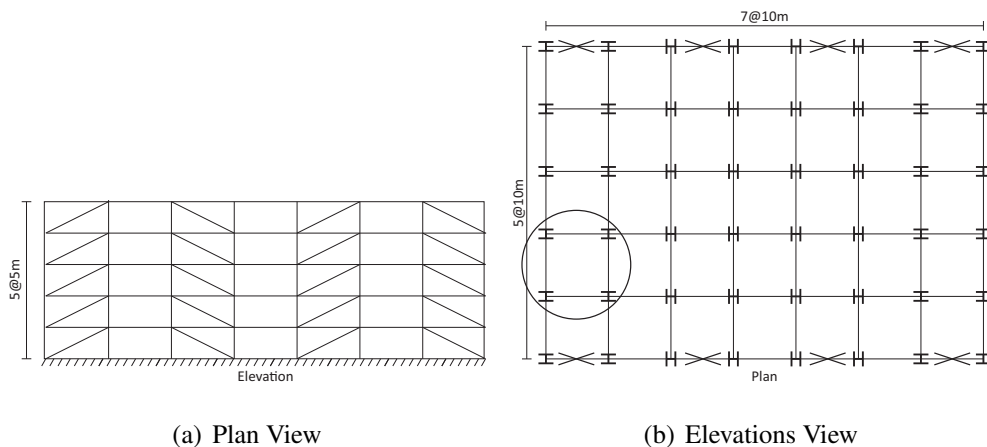


Figure 5.1: Elevation and Plan View of building used for the case study.

5.2 Baseline Model and Analysis

A baseline building was designed using BRBFs as the lateral load resisting system. There were four braces along each side of the building resulting in eight braced bays to resist the loads in each direction.

The BRBFs were designed according to ASCE 7-10 [31]. The braces were sized by distributing the base shear along the height of the building. Each brace was designed to resist one eighth of the lateral load at each story. The calculations used in the design are shown in Appendix C. The columns were sized based on both the forces from the braces and the gravity loads. For this building, the gravity loads governed the column design. The columns were sized based on the demands at the first and third story and changed size accordingly. The design of the columns is also shown in Appendix C. The baseline building properties are given in Table 5.1, and shown in Figure 5.2.

Table 5.1: Design value for the five story case study

Story	Column Size	Brace Size (in ²)
1	W12x79	4.5
2	W12x79	4.5
3	W12x53	4.0
4	W12x53	3.0
5	W12x53	2.0

The baseline building was analyzed in OpenSees under the same suite of earthquake ground motions as used for the one and three story studies (see Table 3.1). The Rayleigh damping for the model was defined as 2% in the first and third modes. The model was tested with both elastic and inelastic columns. The inelastic columns were modeled using rotational springs at the top and bottom of the elastic column elements. The moment of inertia and area values for the column were multiplied by 4 to represent the gravity columns that would be contributing to the stiffness of the

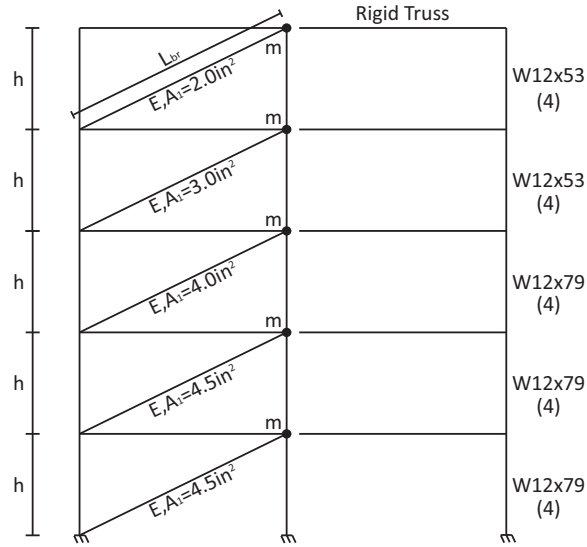


Figure 5.2: Model of the five story frame used in the case study.

braced frame. The braces were modeled with corotational truss elements with elastic perfectly plastic behavior so they would not contribute any stiffness to the structure after yielding.

5.3 Variants of Baseline Model

Three variations of the baseline model were analyzed and compared with the baseline model to see if residual drifts could be mitigated by activating the stiffness of the gravity column. For the first variation, the gravity columns were all fixed to the ground. The second variation, the gravity columns were pin connected to the ground, but a heavy brace [$A_{brace} = 645 \text{ cm}^2$ (100 in^2)] was used in the top bay. The third variation also had pin connections to the ground, but the heavy brace was placed in the third story to limit drifts at the third floor. Both elastic and inelastic models were analyzed.

All three of the variations were tested with four different column stiffness. The baseline moment of inertia of the top and bottom column were multiplied by 2, 3, and 4 and then a column in the W12 series was selected with moment of inertia value close to the calculated value. The column sizes were varied to determine how the added stiffness would reduce the residual drift of the building. The variation of column sizes and stiffness is shown in Table 5.2.

Table 5.2: Variations of the column sizes to increase the post-yield stiffness

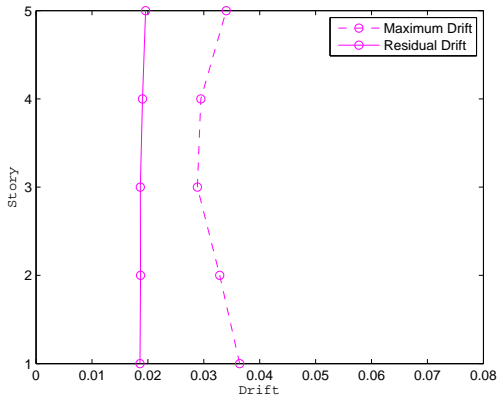
	Shape	Actual Increase in I
Story 1 and 2		
I = 1x	W12x79	1.0
I = 2x	W12x152	2.16
I = 3x	W12x190	3.04
I = 4x	W12x252	4.11
Story 3,4 and 5		
I = 1x	W12x53	1.0
I = 2x	W12x106	2.2
I = 3x	W12x136	2.92
I = 4x	W12x190	4.45

5.4 Results

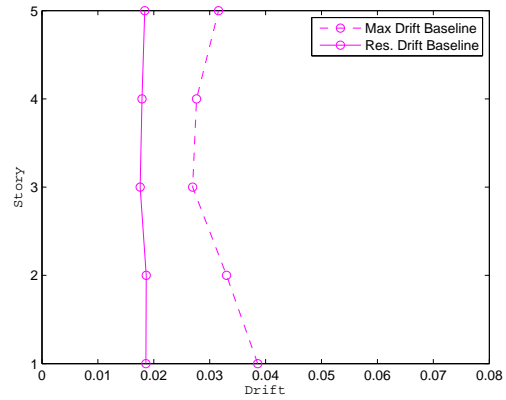
The maximum and residual drift for the baseline model and the three variants were output and plotted. The maximum and residual drifts for each earthquake were averaged together. The results are presented in the following subsections.

5.4.1 Baseline Model

The maximum and residual drifts for the baseline model are shown in Figure ?? for the elastic model and in Figure 5.3(b). The baseline building performed as expected with residual drifts around 2%. The maximum drifts range between 3-3.5%. Both the elastic and inelastic models had similar results indicating that there was not much yielding in the columns.



(a) Elastic

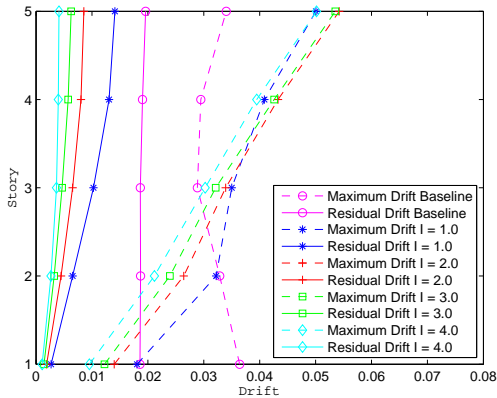


(b) Inelastic

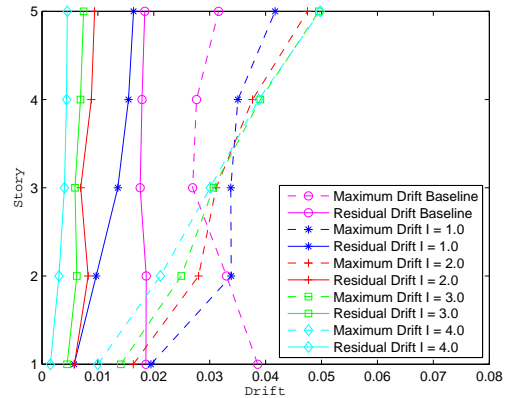
Figure 5.3: Maximum and residual drifts for the five story baseline building modeled with elastic and inelastic columns.

5.4.2 Fixed Base Gravity Columns

The first variation of the baseline building is the building with fixed gravity columns. The results from this variation with four different column stiffness are compared to the baseline design in Figure 5.4(a) for the elastic model and in Figure 5.4(b) for the inelastic model.



(a) Elastic



(b) Inelastic

Figure 5.4: Maximum and residual drifts for the fixed gravity column variation compared to the baseline building.

For the fixed base building, the maximum drift varies over the height of the building. In the baseline system, the maximum displacement is fairly uniform over the height of the building. In contrast, when the gravity columns are fixed at the base, the maximum drift at the lower stories decreases and the maximum drift at the top stories increases. For example, for the elastic model, when the gravity columns are fixed and twice as stiff as normal, the maximum drift is 0.014 at the first story and 0.054 at the fifth story as compared to 0.036 and 0.034 respectively for the baseline model. This configuration does decrease the residual drift in the system, but it increases the maximum drifts.

Fixing the gravity column greatly reduces the residual drift in the building. By simply fixing the column at the base, the residual drift is decreased from 0.019 at the first floor to 0.0026. For the elastic model the drifts at the top floor are decreased from 0.020 to 0.014. For the inelastic columns, the residual drift at the top is not reduced significantly, however, by doubling the moment of inertia of the column, the residual drift at the top floor does decrease. As the stiffness of the gravity column continues to increase, there is a decrease in the amount of residual drift in the building. The largest decrease in residual drift occurs when the column stiffness is doubled. The residual drift decreases further when the column stiffness is tripled or quadrupled, however, the amount that the residual drift decreases becomes less significant.

The difference in drifts between the elastic and inelastic models show that there is some yielding that occurs in the columns. The yielding occurs mostly in the bottom stories of the structure. The stiffer the columns are, the less they will yield and add to the residual drift.

5.4.3 Heavy Top Brace

The second variation of the base line building has a heavy brace at the top story and pinned connections between the gravity columns and the ground. Again, the column size was varied between one and four times the initial stiffness. The results of this variation compared to the baseline design are shown in Figure 5.5(a) and in Figure 5.5(b) for the elastic and inelastic cases respectively.

Placing a heavy brace in the top story greatly reduces the amount of maximum drift in the system. By placing a heavy brace in the top frame, the maximum drifts in the lower stories remain about the same as the baseline building. However, in upper stories, the maximum drift begins to

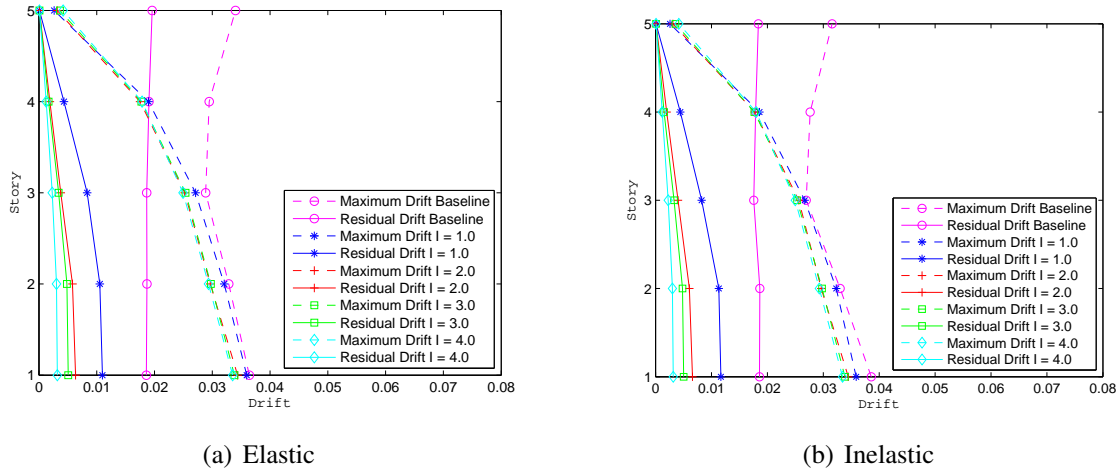


Figure 5.5: Maximum and residual drifts for the building with a heavy top brace and varying column sizes compared to the baseline building.

be a smaller portion of the baseline maximum drift. At the fifth story, the maximum drift is very close to zero. This drift is close to zero because the brace is very stiff and does not yield during the earthquake.

The residual drifts for this building are much lower than they were for the baseline building. The residual drift in the first story decreases from 0.018 to 0.011 and in the fifth story, it decrease from 0.020 to 0. The residual drift is zero at the top story because of the heavy brace. Further reduction to the residual drift is achieved by stiffening the gravity columns. Again, for this case, the residual drift decreases the most when the column stiffness is doubled. Increasing the columns stiffness further does decrease the residual drift, but the amount the residual drift decreases becomes smaller.

The results for the elastic and inelastic models for this configuration are almost identical indicating there the is little to no yielding of the columns.

5.4.4 Heavy Middle

The last variant of the baseline that was analyzed had a heavy brace at the third story and pinned ground connections. The results for this case with varying column stiffness are shown in Figures 5.6(a) and 5.6(b) and compared to the baseline.

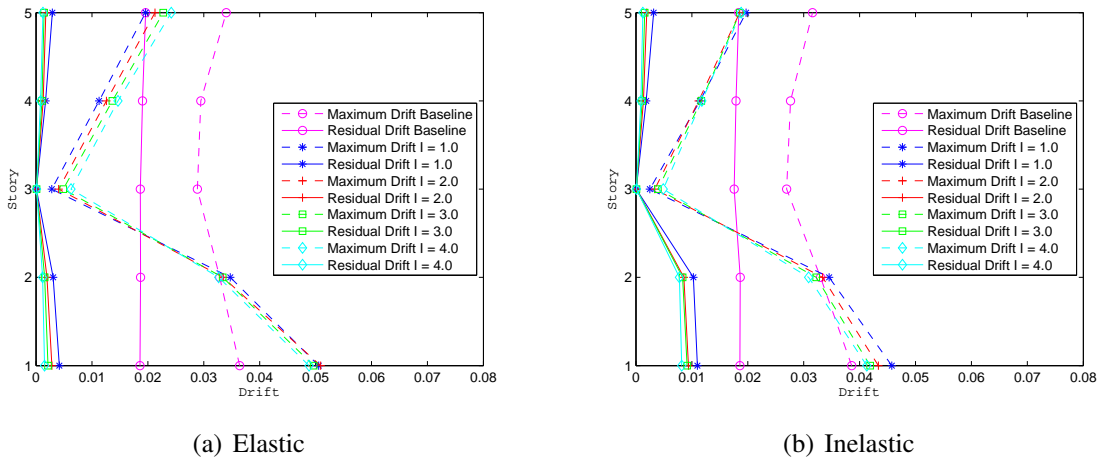


Figure 5.6: Maximum and residual drifts for the building with a heavy middle brace and varying column sizes to the baseline building.

The maximum drifts from this case remain about the same regardless of the column stiffness, but are very different from the maximum drifts of the baseline. In this variation, the drifts tend to be concentrated in the lower stories of the building. For the elastic model, the maximum drifts for the first story are increased to about 5%. The drifts at the third story are close to zero because of the heavy brace in that story. The maximum drifts increase again for the top two stories and are lower than the baseline maximum drift. The maximum drifts are smaller for the inelastic model due to yielding of the columns. The drift in the first two stories better matches the drift of the baseline model. For the third through fifth story, the inelastic and elastic model match, indicating that the columns are not yielding in the upper stories.

Adding a stiff brace to the middle story decreases the residual drifts for all of the stories. This variant of the baseline model had the lowest residual drifts. Since the heavy brace in the middle constrains the residual drift at the third story to be zero, the residual drift in the other stories is also limited and remains small. The columns do contribute some residual drift in the first two stories as seen in Figure 5.6(b). There is again some variation between the drifts and the column stiffness, but for all of the variations in stiffness, the difference in residual drift is small.

5.4.5 Comparison Between the Baseline and Three Variations

In order to compare the baseline and the three variants better, they are plotted together in Figures 5.7(a) and 5.7(b). The models where the column stiffness was doubled provided the best increase in performance without increasing cost of material by too much. As such, the results for the three variations are plotted for the case when the column stiffness is doubled.

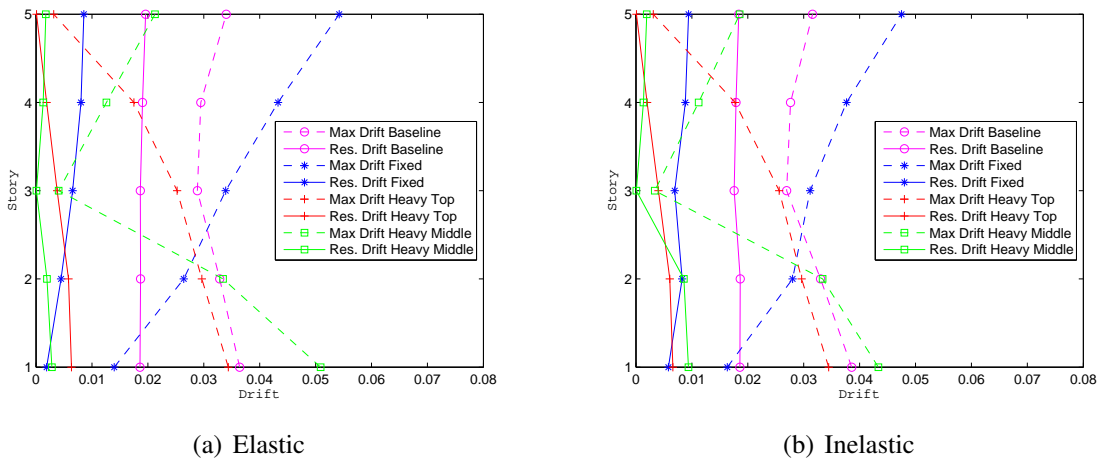


Figure 5.7: A comparison of the maximum and residual drifts for the five story baseline frame and the three variants with a column stiffness twice the size of the original stiffness.

Activating the lateral stiffness of the gravity columns changes the maximum drift of the models. The variant with fixed gravity columns had the lowest maximum drift at the first story, but the highest drift at the top. In contrast, the system with a heavy brace at the third story had large drifts at the the first story, very small drifts in the middle, and larger drifts again at the top. The last variant with the heavy top brace was the only system that constantly had maximum drift less than the baseline example and had the smallest drift at the top story.

The residual drift varied based on how the lateral stiffness of the gravity columns was activated. For the lower stories, all three of the configurations had similar drifts. The upper stories had more variation in residual drift. The system with the fixed gravity column had the highest drifts of the three systems. This configuration does have the most uniform residual drifts. The second and third configurations both had very low residual drift on the upper stories. Yielding in the columns occurs for two of the variations, the fixed gravity columns and the heavy middle brace.

This yielding increases the residual drift of the buildings and must be considered in addition to the residual displacement of the braces.

5.5 Conclusions

As expected, all three of the variant cases had lower residual drifts than the baseline building. Adding the column stiffness to the frame reduces the amount of residual drift in the system. Increasing the column stiffness further helped to decrease the residual drift with the largest increase coming when the column stiffness was doubled.

The largest maximum drifts occurred in the first and fifth story for the variant with the heavy middle brace and the fixed gravity columns respectively, make these systems less practical as the drifts are large enough to cause yielding in the columns leading to larger residual drifts.

The second variant with the heavy brace in the top story appears to be the best approach to mitigating residual drift in buildings while allowing gravity columns to remain elastic. This variant had lower maximum drifts than the baseline building for all of the stories, and had low residual drifts with no drift at the top story. This system also had little to no yielding in the columns.

CHAPTER 6. SUMMARY AND CONCLUSIONS

6.1 Summary

This thesis examines the possibility of using gravity columns as part of a dual system with BRBFs. The goal was to reduce the residual drift in buildings by increasing the post-yield stiffness through better use of the gravity columns. By utilizing the gravity columns as an elastic secondary system to BRBFs, the residual drift of the system can be reduced without adding moment connections.

Chapter 3 of this thesis presented a parametric study of SDOF braced frame systems. The brace area and the post-yield stiffness (γ) were varied to create response spectra for different systems. The columns were assumed to be elastic and the braces were elasto-plastic. Each version of the model was analyzed under a suite of ground motions and the maximum and residual drifts were output.

Chapter 4 presented the parametric study for a three story building. The three story frame was analyzed with three different configurations; fixed gravity columns, heavy top brace, and heavy middle brace. The brace area and the moment of inertia of the column were varied. To create the spectra, each configuration was analyzed under a suite of ground motions for a range of column and brace sizes. The maximum and residual drifts for each story were output for comparison between the systems.

Chapter 5 examined a five story BRBF. A traditional frame was designed and analyzed and compared to three variations of the frame with activated gravity columns. These variations are: having fixed gravity columns, a heavy top brace, or a heavy middle brace. Each of the variations was tested with the original columns and columns that were approximately 2, 3, and 4 times stiffer than the original.

6.2 Conclusions

From the analysis of the SDOF, three story frame, and five story case study, it appears that gravity columns can be effective as a dual system, and increasing the column size decreases the residual drift.

Other conclusions are summarized in the following subsections.

6.2.1 SDOF Study

- The residual drifts depends on the period and the post-yield stiffness.
- Systems with small periods have low maximum drifts because they are exceptionally stiff (and strong) and do not yield.
- Systems with longer periods had higher maximum drifts because there is some resonance.
- High post-yield stiffness decreased the residual drift with the greatest return occurring when γ is 0.05 to 0.1.

6.2.2 Three Story Study

- The residual drift decreased as the column size increased. The highest decrease came when the column size was doubled.
- The systems with fixed gravity columns had the lowest residual drifts, but did have large moment ratios so the columns would have yielded increasing the residual drift. In practice, it would be expensive to “fix” all of the gravity columns.
- The systems with a heavy top brace had the highest residual drifts, however, the moment ratios were mostly below one so the columns are not yielding.
- For the heavy middle brace systems, there is some residual drift at the first floor, but almost zero at the second floor. The moment ratios for these two stories however are very large and column would have yielded. Even though the residual drift from the braces is low, additional residual drift from the columns would be expected.

6.2.3 Five Story Case Study

- All of the variations of the model performed better than the baseline model. Residual drifts were decreased for all of the cases.
- The fixed gravity column variation had lower residual drifts than the baseline building with the residual drifts increasing with the story height. The maximum drifts however were higher than the base line for higher stories. There is yielding in some of the columns for this systems which add to the residual drift.
- The variation with the heavy top brace performed better than the baseline design for all of the floor levels for both maximum and residual drifts. The residual drift for this variation at the top story was zero.
- The variation with the heavy middle column had the lowest overall residual drift with no residual drift at the third story. However, the maximum drifts for this case were very large for the first and second stories. There was yielding that occurred in the bottom columns, which increased the residual drift at these floor, however, this drift is still lower than the drift for the baseline building.

REFERENCES

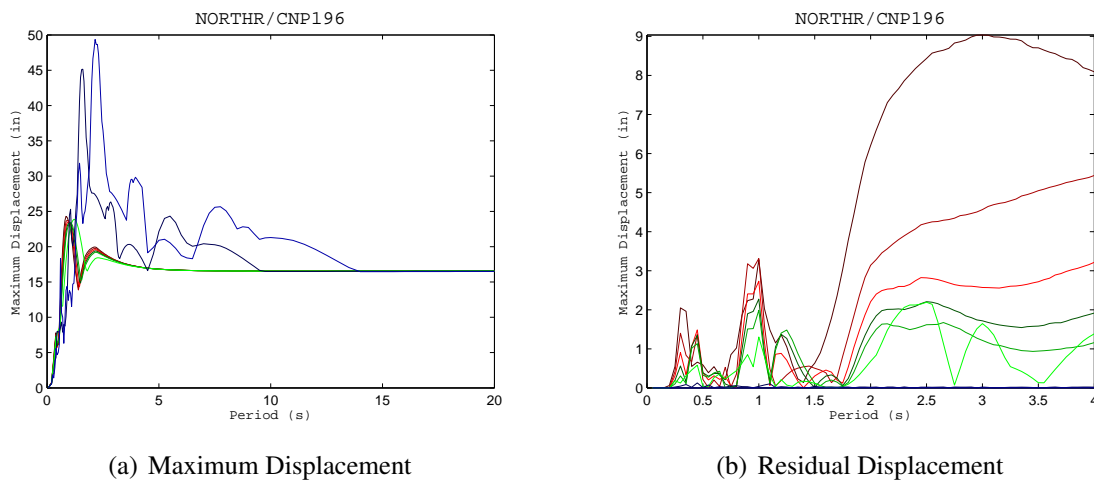
- [1] C. Christopoulos and S. Pampanin, "Towards performance-based seismic design of mdof structures with explicit consideration of residual deformation," *ISET Journal of Earthquake Technology*, vol. 41, no. 1, pp. 53–73, 2004, paper No. 440. 5
- [2] C. Christopoulos, S. Pampanin, and M. J. Nigel Priestley, "Performance-based seismic response of frame structures including residual deformations: Part i: Single-degree of freedom systems," *Journal of Earthquake Engineering*, vol. 7, no. 1, p. 97, 2003. 5, 6
- [3] J. Erochko, C. Christopoulos, R. Tremblay, and H. Choi, "Residual drift response of smrfs and brb frames in steel buildings designed according to asce 7-05," *Journal of Structural Engineering*, vol. 137, no. 5, pp. 589–599, 2011. 5, 6, 8, 26
- [4] J. McCormick, H. Aburano, M. Ikenaga, and M. Nakashima, "Permissible residual deformation levels for building structures considering both safety and human elements," in *Proc. 14th World Conference of Earthquake Engineering*. Seismological Press of China, 2008. 5
- [5] G. A. Macrae and K. Kawashima, "Post-earthquake residual displacements of bilinear oscillators," *Earthquake Engineering Structural Dynamics*, vol. 26, no. 7, pp. 701–716, 1997. 6
- [6] B. Borzi, G. M. Calvi, A. S. Elnashai, E. Faccioli, and J. J. Bommer, "Inelastic spectra for displacement-based seismic design," *Soil Dynamics and Earthquake Engineering*, vol. 21, no. 1, pp. 47–61, 2001. 6
- [7] G. D. Hatzigeorgiou, G. A. Papagiannopoulos, and D. E. Beskos, "Evaluation of maximum seismic displacement of sdof system from thier residual deformation," *Engineering Structures*, vol. 33, pp. 3422–3431, 2011. 6
- [8] U. Yazgan and A. Dazio, "Post-earthquake damage assessment using residual displacements," *Earthquake Engineering Structural Dynamics*, pp. n/a–n/a, 2011. [Online]. Available: <http://dx.doi.org/10.1002/eqe.1184> 6
- [9] S. Pampanin, C. Christopoulos, and M. J. Nigel Priestley, "Performance-based seismic response of frame structures including residual deformations: Part ii: Multi-degree of freedom systems," *Journal of Earthquake Engineering*, vol. 7, no. 1, p. 119, 2003. 6
- [10] J. Ruiz-Garca and E. Miranda, "Residual displacement ratios for assessment of existing structures," *Earthquake Engineering Structural Dynamics*, vol. 35, no. 3, pp. 315–336, 2006. [Online]. Available: <http://dx.doi.org/10.1002/eqe.523> 7
- [11] FEMA, "Recomended seismic design provisions for new moment frame buildings report fema 350," 2000. 7

- [12] ASCE, *Minimum Design Loads for Buildings and Other Structures*. Reston, VA: ASCE/SEI 7-05, 2005, vol. 1. 8
- [13] R. Sabelli, S. Mahin, and C. Chang, “Seismic demands on steel braced frame buildings with buckling-restrained braces,” *Engineering Structures*, vol. 25, no. 5, pp. 655–666, 2003. 8, 16, 26
- [14] R. Tremblay, M. Lacerte, and C. Christopoulos, “Seismic response of multistory buildings with self-centering energy dissipative steel braces,” *Journal of Structural Engineering*, vol. 134, no. 1, pp. 108–120, 2008. 8, 13
- [15] H. Choi, J. Erochko, C. Christopoulos, and R. Tremblay, “Comparison of the seismic response of steel building incorporating self-centering energy dissipative braces, buckling restrained braces and moment resisting frames,” in *Proc. of the 14th World Conference on Earthquake Engineering*, 2008. 9, 12, 26
- [16] C. Christopoulos, R. Tremblay, H. J. Kim, and M. Lacerte, “Self-centering energy dissipative bracing system for the seismic resistance of structures: Development and validation,” *Journal of Structural Engineering*, vol. 134, no. 1, pp. 96–107, 2008. 10, 12
- [17] M. M. Garlock, J. M. Ricles, and R. Sause, “Experimental studies of full-scale posttensioned steel connections,” *Journal of Structural Engineering*, vol. 131, no. 3, pp. 438–448, 2005. [Online]. Available: <http://link.aip.org/link/?QST/131/438/1> 11, 12
- [18] M. M. Garlock, R. Sause, and J. M. Ricles, “Behavior and design of posttensioned steel frame systems,” *Journal of Structural Engineering*, vol. 133, no. 3, pp. 389–399, 2007. [Online]. Available: <http://link.aip.org/link/?QST/133/389/1> 11
- [19] C.-C. Chou, J.-H. Chen, Y.-C. Chen, and K.-C. Tsai, “Evaluating performance of post-tensioned steel connections with strands and reduced flange plates,” *Earthquake Engineering Structural Dynamics*, vol. 35, no. 9, pp. 1167–1185, 2006. [Online]. Available: <http://dx.doi.org/10.1002/eqe.579> 12
- [20] H.-J. Kim and C. Christopoulos, “Seismic design procedure and seismic response of post-tensioned self-centering steel frames,” *Earthquake Engineering Structural Dynamics*, vol. 38, no. 3, pp. 355–376, 2009. 12
- [21] ———, “Friction damped posttensioned self-centering steel moment-resisting frames,” *Journal of Structural Engineering*, vol. 134, no. 11, pp. 1768–1779, 2008. 12
- [22] S. Zhu and Y. Zhang, “Seismic behaviour of self-centring braced frame buildings with reusable hysteretic damping brace,” *Earthquake Engineering Structural Dynamics*, vol. 36, no. 10, pp. 1329–1346, 2007. 13
- [23] M. Midorikawa, T. Azuhata, T. Ishihara, and A. Wada, “Shaking table tests on seismic response of steel braced frames with column uplift,” *Earthquake Engineering Structural Dynamics*, vol. 35, p. 1767, 2006. 13

- [24] D. Pettinga, C. Christopoulos, S. Pampanin, and N. Priestley, “Effectiveness of simple approaches in mitigating residual deformations in buildings,” *Earthquake Engineering Structural Dynamics*, vol. 36, no. 12, pp. 1763–1783, 2007. 14, 25, 26
- [25] S. Kiggins and C.-M. Uang, “Reducing residual drift of buckling-restrained braced frames as a dual system,” *Engineering Structures*, vol. 28, no. 11, pp. 1525–1532, 2006. 14, 25
- [26] X. Qiang, “Dual system design of steel frames incorporating buckling-restrained braces,” in *The 14th World Conference on Earthquake Engineering*, 2008. 15, 25
- [27] T. J. Maley, T. J. Sullivan, and G. D. Corte, “Development of a displacement-based design method for steel dual systems with buckling-restrained braces and moment-resisting frames,” *Journal of Earthquake Engineering*, vol. 14, no. S1, pp. 106–140, 2010. 15
- [28] G. Apostolakis and G. F. Dargush, “Optimal seismic design of moment-resisting steel frames with hysteretic passive devices,” *Earthquake Engineering Structural Dynamics*, vol. 39, no. 4, pp. 355–376, 2010. 15
- [29] G. A. MacRae, Y. Kimura, and C. Roeder, “Effect of column stiffness on braced frame seismic behavior,” *Journal of Structural Engineering*, vol. 130, no. 3, pp. 381–391, 2004. 16
- [30] B. C. Pacific Earthquake Engineering Research Center, University of California, “Open system for earthquake engineering simulation (opensees),” Pacific Earthquake Engineering Research Center, University of California, Berkeley, CA, 2000. 20, 28
- [31] ASCE, *Minimum Design Loads for Buildings and Other Structures*. Reston, VA: ASCE/SEI 7-10, 2010, vol. 1. 41, 42
- [32] P. W. Richards, “Seismic column demands in ductile braced frames,” *Journal of Structural Engineering*, vol. 135, no. 1, pp. 33–41, 2009. 41

APPENDIX A. SDOF PARAMETER STUDY RESULTS FOR INDIVIDUAL EARTHQUAKES

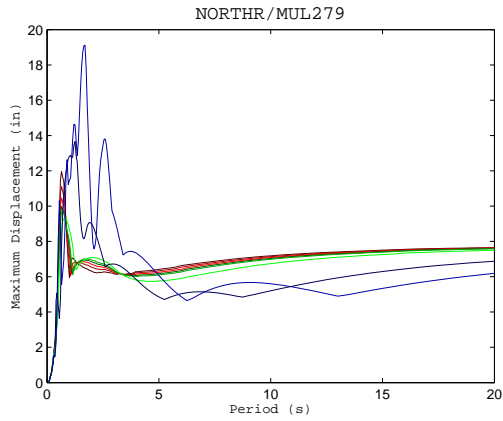
This Appendix shows the maximum and residual displacement for the individual earthquakes for the SDOF parameter study.



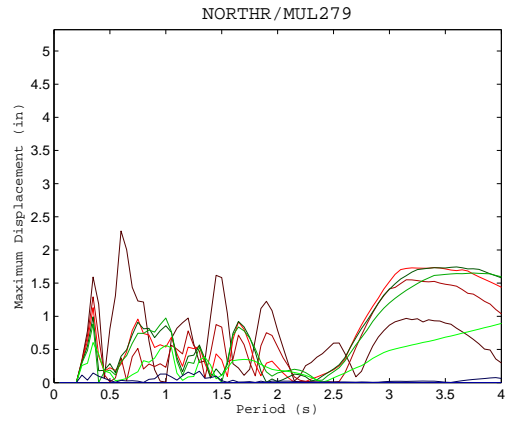
(a) Maximum Displacement

(b) Residual Displacement

Figure A.1: The maximum (a) and residual (b) displacements for the Canoga Park earthquake for the SDOF parameter study.

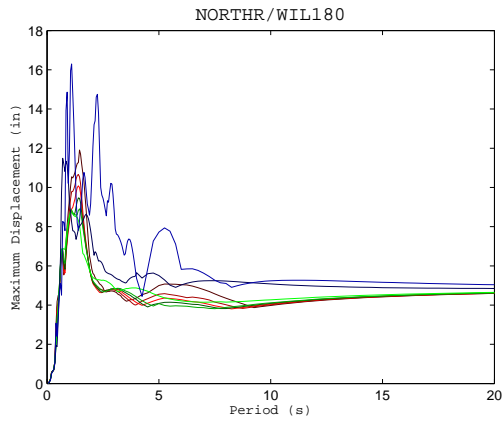


(a) Maximum Displacement

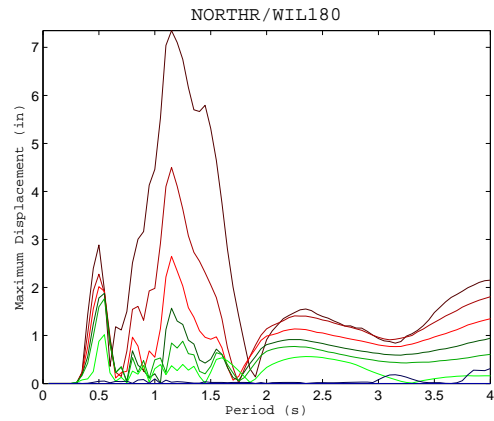


(b) Residual Displacement

Figure A.2: The maximum (a) and residual (b) displacements for the Beverly Hills earthquake for the SDOF parameter study.

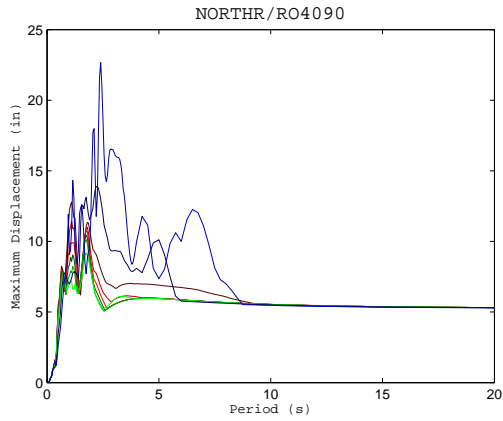


(a) Maximum Displacement

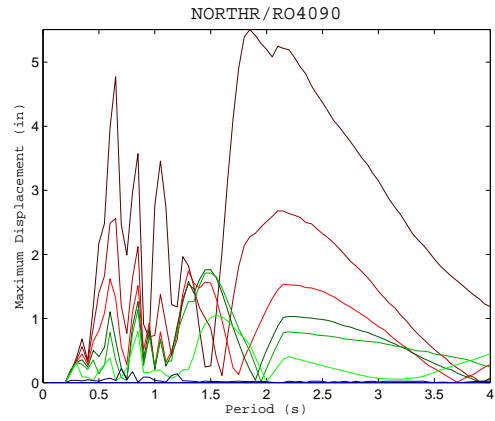


(b) Residual Displacement

Figure A.3: The maximum (a) and residual (b) displacements for the Hollywood earthquake for the SDOF parameter study.

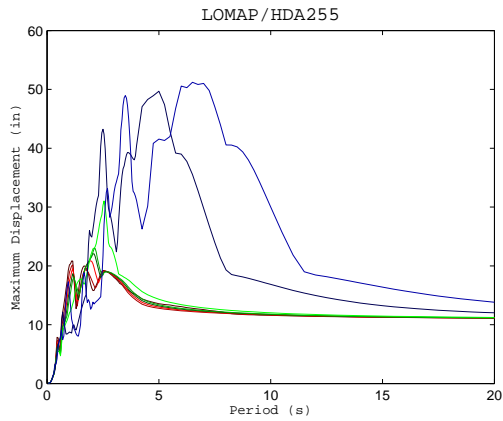


(a) Maximum Displacement

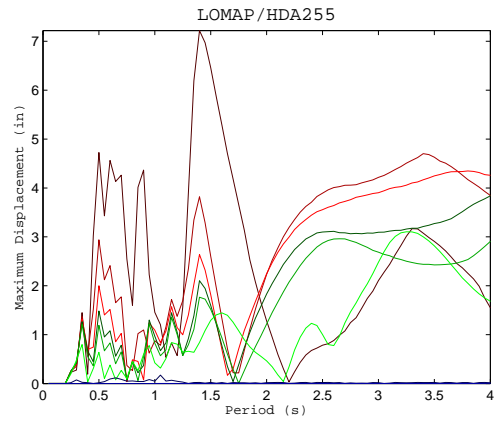


(b) Residual Displacement

Figure A.4: The maximum (a) and residual (b) displacements for the Sun Valley earthquake for the SDOF parameter study.

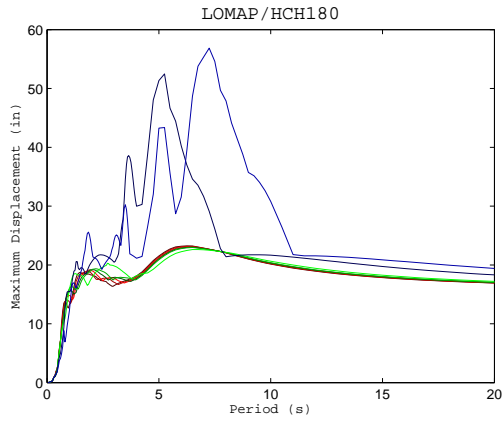


(a) Maximum Displacement

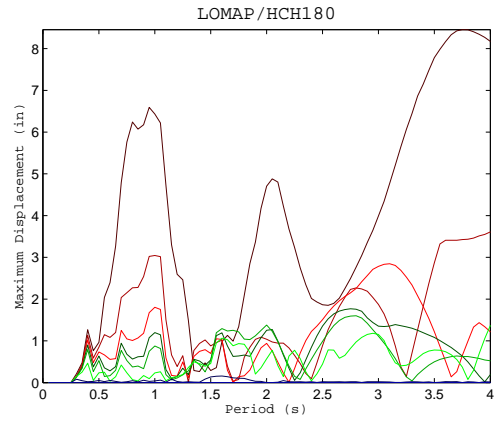


(b) Residual Displacement

Figure A.5: The maximum (a) and residual (b) displacements for the Hollister Array earthquake for the SDOF parameter study.

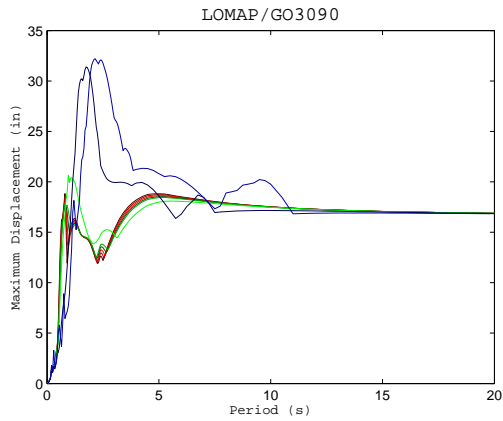


(a) Maximum Displacement

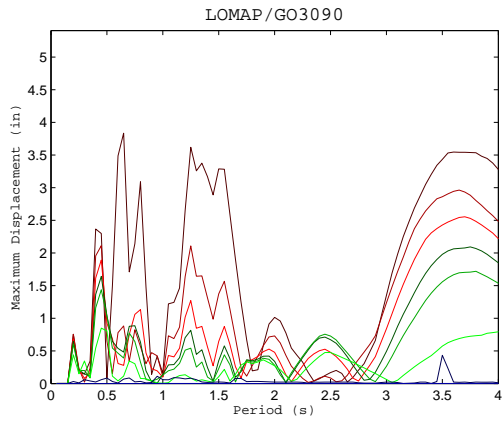


(b) Residual Displacement

Figure A.6: The maximum (a) and residual (b) displacements for the Hollister City Hall earthquake for the SDOF parameter study.

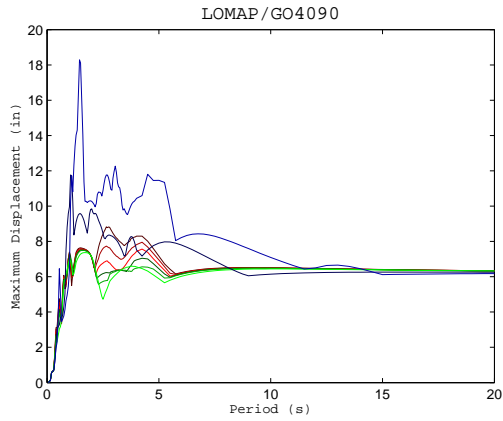


(a) Maximum Displacement

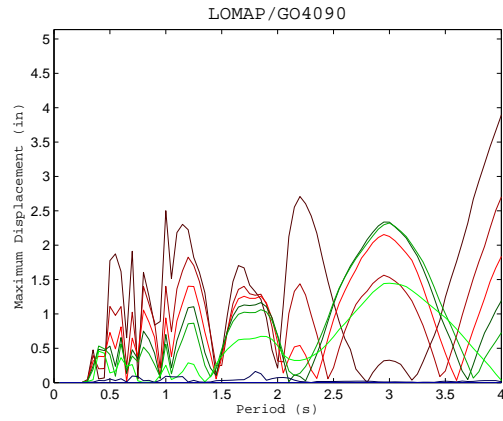


(b) Residual Displacement

Figure A.7: The maximum (a) and residual (b) displacements for the Gilroy No. 3, earthquake for the SDOF parameter study.

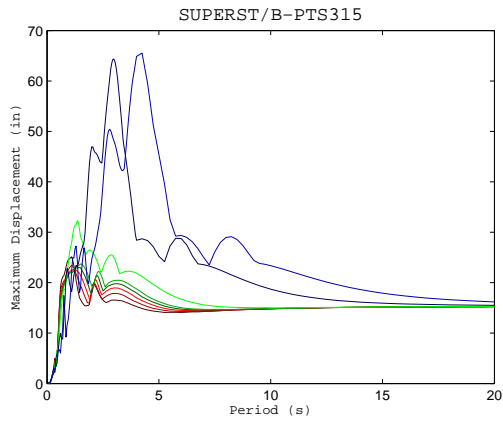


(a) Maximum Displacement

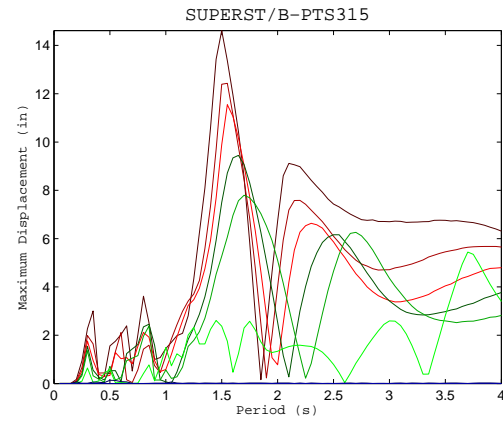


(b) Residual Displacement

Figure A.8: The maximum (a) and residual (b) displacements for the Gilroy No. 4 earthquake for the SDOF parameter study.

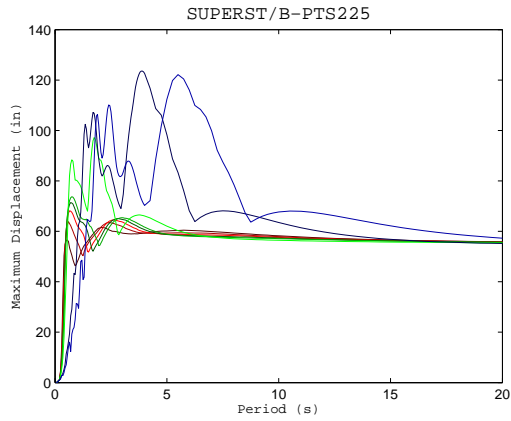


(a) Maximum Displacement

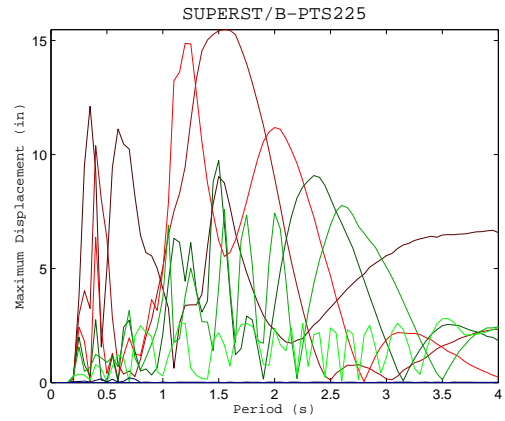


(b) Residual Displacement

Figure A.9: The maximum (a) and residual (b) displacements for the Parachute Test Site 1 earthquake for the SDOF parameter study.



(a) Maximum Displacement



(b) Residual Displacement

Figure A.10: The maximum (a) and residual (b) displacements for the Parachute Test Site 2 earthquake for the SDOF parameter study.

APPENDIX B. THREE STORY FRAME CALCULATIONS

The three story frame was designed as a portion of larger building. The building was a three by three bay buildings, with columns spaced 9.144 m (30 ft) apart. The height of each story was 3.96 m (13 ft). The dead load of the building was assumed to be 4.79 kPa (100 psf) per floor. The building had one brace bay at each story. The column analyzed in OpenSees represents the sum of eight gravity columns. The brace size was varied over a wide range of values to give a range of periods.

The size of the columns was determined according to a gravity analysis of the building. The original size of the columns needed for a gravity analysis is a W12x58. This column has a moment of inertia of 19771 cm^2 (475 in^4). For the analysis, this system was multiplied by 8 to get 158168 cm^4 (3800 in^4). The other column sizes can from multiplying this number by 2, 3, 4,...8 and then finding a column in the W12 series that was close to this value. The variation of column size is shown in Table B.1. And the basic set up for all three configurations is in shown in Figure B.1.

Table B.1: Variations of the column sizes to increase the post-yield stiffness

	new I	Closest Shape	Actual I
I = 1x	475	W12x53	475
I = 2x	950	W12x106	933
I = 3x	1425	W12x152	1430
I = 4x	1900	W12x190	1890
I = 5x	2375	W12x230	2420
I = 6x	2850	W12x252	2720
I = 7x	3325	W12x305	3550
I = 8x	3800	W12x336	4060

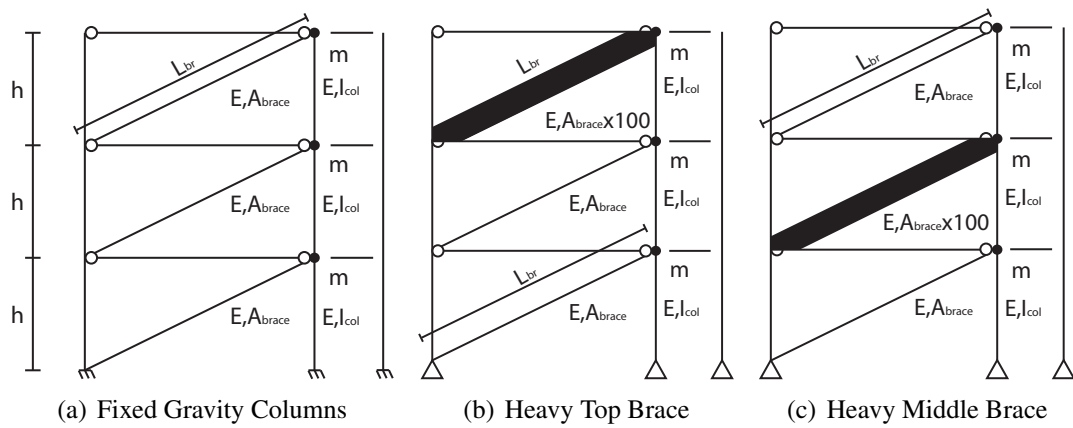


Figure B.1: The three configurations used for the three story models

APPENDIX C. DESIGN OF FIVE STORY FRAME

Five Story

Building Dimensions

Number of bays E-W = 7

Number of bays N-S = 5

Length of bays E-W = 32.808 ft

length of bays N-S = 32.808 ft

Number of Stories = 5

Height of story, $h = 16.404$ ft

Height of building $H = 5 * 16.404 = 82.02$ ft

$L_{br} = (16.404^2 + 32.808^2)^{1/2} = 36.6804591$ ft

$Perimeter = (5 * 32.808 + 7 * 32.808) * 2 = 787.4$ ft

$FloorArea = (5 * 32.808 + 7 * 32.808) * 2 = 37672$ ft²

Loads

$DeadLoadFloor = 83.5$ psf

$DeadLoadRoof = 83.5$ psf

$LiveLoadFloor = 50$ psf

$LiveLoadRoof = 50$ psf

$PartitionWallWeight = 10$ psf

$ExteriorWallWeight = 25$ psf

$Seismicweight\ per\ floor = FloordeadLoad * FloorArea = (83.5\ psi) * (37672\ ft^2) / 1000$
 $= 3147.325$ kip

$FloorWeight = 4 * 3147.325\ kip = 12589.3$ kip

$RoofWeight = 3147.325$ kip

$TotalSeismicWeight = 12589.3\ kip + 3147.325\ kip = 15736.625$ kip

Estimate Period and Coefficient

$$S_{ds} = 1.39g$$

$$S_{d1} = 0.77g$$

$$C_t = 0.03$$

$$x = 0.75$$

$$R = 8$$

$$I = 1(\text{importance factor})$$

$$T_a = C_t * H^x = 0.03 * 82.02^{0.75} = 0.817638021\text{sec}$$

$$C_u = 1.4$$

$$T = T_a * C_u = 1.144693229$$

$$C_{s1} = S_{ds}/(R/I) = 1.39g/(8/1) = 0.17375$$

$$C_{s2} = S_{d1}/(T * R/I) = 0.77/(1.144 * 8/1) = 0.084083663$$

$$C_s = \min(C_{s1}, C_{s2}) = 0.084083663$$

$$\text{BaseShear, } V = \text{seismicweight} * C_s = 1323.193077\text{kip}$$

Distribute Base Shear

$$k = 1.322346615$$

$$\text{Floor Weight (w) Height (h) } w * h^k \quad c_{vx} = w * h^k / (\text{sum}(wh^k)) \quad F_x(\text{kip})$$

$$\text{Roof } 3147.325 \quad 82.02 \quad 1068580 \quad 0.3745 \quad 495.6$$

$$4 \quad 3147.325 \quad 65.616 \quad 795533 \quad 0.279 \quad 368.9$$

$$3 \quad 3147.325 \quad 49.212 \quad 543808 \quad 0.191 \quad 252.2$$

$$2 \quad 3147.325 \quad 32.808 \quad 318120 \quad 0.111 \quad 147.5$$

$$1 \quad 3147.325 \quad 16.404 \quad 127211 \quad 0.045 \quad 59.0$$

Find Forces in Braces Forces per side:

$$\text{roof} = 495.6/2 = 247.8\text{kip}$$

$$4 = 368.9/2 = 184.45\text{kip}$$

$$3 = 252.2/2 = 126.10\text{kip}$$

$$2 = 147.5/2 = 73.76\text{kip}$$

$$1 = 59/2 = 29.50\text{kip}$$

Forces per frame:

$$\text{roof} = 247.8\text{kip}/4\text{bracesperside} = 61.95\text{kip}$$

$$4 = 61.95 + 184.45/4 = 108.1kip$$

$$3 = 108.1 + 126.1/4 = 139.6kip$$

$$2 = 139.6 + 73.76/4 = 158kip$$

$$1 = 158 + 29.5/4 = 165.4kip$$

Find Brace Forces

Story story shear (kip) F_{br}

$$F_{br5} = 61.94 * L_{br}/h = 69.256kip$$

$$F_{br4} = 108.06 * L_{br}/h = 120.815kip$$

$$F_{br3} = 139.58 * L_{br}/h = 156.059kip$$

$$F_{br2} = 158.02 * L_{br}/h = 176.677kip$$

$$F_{br1} = 165.40 * L_{br}/h = 184.922kip$$

$$F_{ybrace} = 46ksi$$

Size Braces

Story Size Required,

$$5 = F_{br5}/(0.9 * F_{ybrace}) = 1.67, A_5 = 2in^2$$

$$4 = F_{br4}/(0.9 * F_{ybrace}) = 2.92, A_4 = 3in^2$$

$$3 = F_{br3}/(0.9 * F_{ybrace}) = 3.77, A_3 = 4in^2$$

$$2 = F_{br2}/(0.9 * F_{ybrace}) = 4.27, A_2 = 4.5in^2$$

$$1 = F_{br1}/(0.9 * F_{ybrace}) = 4.47, A_1 = 4.5in^2$$

Size Columns

Ultimate brace capacity

$$\beta * \omega * R_y = 1.8$$

$$F_{ultroof} = \beta * \omega * R_y * F_{ybrace} * A_5 = 124.2kip$$

$$F_{ult4} = \beta * \omega * R_y * F_{ybrace} * A_4 = 207kip$$

$$F_{ult3} = \beta * \omega * R_y * F_{ybrace} * A_3 = 248.4kip$$

$$F_{ult2} = \beta * \omega * R_y * F_{ybrace} * A_2 = 289.8kip$$

$$F_{ult1}) = \beta * \omega * R_y * F_{ybrace} * A_1 = 331.2kip$$

Column Loads

$$roof = F_{ultroof} * h/L_{br} = 55.54392856kip$$

$$4 = F_{ult4} * h/L_{br} = 148.1171428kip$$

$$3 = F_{ult3} * h / L_{br} = 259.205kip$$

$$2 = F_{ult2} * h / L_{br} = 388.8074999kip$$

$$1 = F_{ult1} * h / L_{br} = 536.9246428kip$$

$$KLx = Kly = 16.404ft$$

$$P_u = 536.9246428kip$$

Column ϕP_n (kip) Wt (lb/ft)

W12x72 684 72 bottom

W12x53 427 53 3rd floor

APPENDIX D. OPENSEES CODE

D.1 SDOF Model

```
# -----
# by Megan Boston, 2011

set dataDir spectra
file mkdir $dataDir
source LibUnits.tcl; # define units and constants
source LibGMfact.tcl
source LibGMfiles.tcl
source DisplayPlane.tcl; # procedure for displaying a plane in model
source DisplayModel2D.tcl; # procedure for displaying 2D perspective of model

# an array of periods to test the structure under
set iTs "0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8
0.85 0.9 0.95 1 1.05 1.1 1.15 1.2 1.25 1.3 1.35 1.4 1.45 1.5 1.55 1.6 1.65 1.7
1.75 1.8 1.85 1.9 1.95 2 2.05 2.1 2.15 2.2 2.25 2.3 2.35 2.4 2.45 2.5 2.55 2.6
2.65 2.7 2.75 2.8 2.85 2.9 2.95 3 3.05 3.1 3.15 3.2 3.25 3.3 3.35 3.4 3.45 3.5
3.55 3.6 3.65 3.7 3.75 3.8 3.85 3.9 3.95 4 4.25 4.5 4.75 5 5.25 5.5 5.75 6 6.25
6.5 6.75 7 7.25 7.5 7.75 8 8.25 8.5 8.75 9 9.25 9.5 9.75 10 10.5 11 11.5 12 12.5
13 13.5 14 14.5 15 15.5 16 16.5 17 17.5 18 18.5 19 19.5 20"
set iGs "0.01 0.02 0.03 0.04 0.05 0.1 0.5 1.0"
#set iGs "0.25";
# in this case looking at different levels of post yields stiffness,
# this value is going to be different values of gamma,
# where gamma is the post yield stiffness multiplied by some ratio
# when gamma is 1, then the post-yield stiffness system is equal to the initial stiffness
# otherwise it is some other value
set counter 0;

foreach GroundFile $iGroundFile {

puts $GroundFile
set GMfact [lindex $iGMfact $counter]
#set GMfact 1.0; # Ground-motion scaling factor
puts $GMfact
#set GroundFile "NORTHR.CNP196"
source ReadSMDFile.tcl
set inFile $GroundFile.at2;
puts $inFile
set outFile $GroundFile.g3; # define output files
ReadSMDFile $inFile $outFile dt; # call procedure to convert the ground-motion file
set GMfatt [expr $g*$GMfact];
puts $dt
puts $GMfact

foreach T $iTs {
set omega [expr 2*PI/$T] ;
```

```

#puts $omega
puts $T

foreach G $iGs {
puts $G
wipe; # get rid of previous models
set DtAnalysis [expr 0.005*$sec]; # time-step Dt for lateral analysis
set TmaxGround [expr 100*$sec]; # maximum duration of ground-motion analysis
set Nsteps [expr int($TmaxGround/$DtAnalysis)];
set g 386.4;
set Weighth 405; #weight of half of my structure in kips
set MASS [expr $Weighth/$g];

# Stiffness of the system
set K [expr $MASS*pow($omega,2)]; #calculate what k is based on the period and the mass
#puts $K

# find the stiffness of the column and brace
set kbrace [expr $K*(1-$G)];
set kcol [expr $K-$kbrace];
#puts $kcol
#puts $kbrace

set Lbay 360.0;
set LCol 156.0;
set Lbr [expr pow(pow($Lbay,2)+pow($LCol,2),0.5)];

set Es 29000.0;
set Fybr 46.0;
set numCol 8;

set cost [expr $Lbay/$Lbr]
set Abrace [expr ($kbrace*$Lbr)/($Es*pow($cost,2))];
set IzCol [expr $kcol*pow($LCol,3)/(3*$Es)];
set AgCol [expr pow($IzCol,0.5)]
#puts $Abrace
#puts $IzCol
#puts $AgCol
set deltax [expr ($Fybr*$Lbr)/($Es*pow($cost,3))]
#puts $deltax

# Setup Model
wipe; # clears existing models
model BasicBuilder -ndm 2 -ndf 3; # Sets dimensions and degrees of freedom

# Define Geometry
node 1 0.0 0.0;
node 2 0.0 $LCol;
node 3 $Lbay 0.0;

# Define boundary conditions
fix 1 1 1 1; # fixed support at base of column
fix 3 1 1 1; # pinned support at base of brace, fix all three since brace is modeled with a truss member

# Define geomtric Transformation
set ColTransfTag 1;
set IDBraceTransf 103; # all brace members
geomTransf Linear $ColTransfTag; # options are linear, P-Delta, and Corotational
geomTransf Corotational $IDBraceTransf;

# Define Materials

```

```

set BrMat 5;
set BraceIE 15;
# set BcMat 10;
uniaxialMaterial Elastic $BrMat $kcol;
# inelastic material for column, future beams
# uniaxialMaterial Steel02 $BcMat $Fy $Es $Bs $RO_BC $cR1_BC $cR2_BC;
# inelastic material for brace
# uniaxialMaterial Steel02 $BraceIE $Fy $Es $BsBrace $RO_Brace $cR1_Brace $cR2_Brace
# set deltax [expr ($Fybr*$Lbr)/($Es*(%cost)^3)]

if {$kbrace != 0} {
uniaxialMaterial ElasticPP $BraceIE $Es 0.001596
}
if {$kbrace == 0} {
uniaxialMaterial ElasticPP $BraceIE $Es $deltax
}

# Define Element connectivity
element elasticBeamColumn 1 1 2 $AgCol $Es $IzCol $ColTransfTag;
#element CorotTruss 20 2 3 $Abrace $BrMat;

# Define Sections Assuming all columns are the same shape
# Wsection 12136 $BcMat $d $bf $tf $tw $nfdw $nftw $nbf $nftf

# Define element connectivity with inelastic materials
#element nonlinearBeamColumn 1 1 2 $np 12136 $ColTransfTag;
element CorotTruss 100 2 3 $Abrace $BraceIE;

# Define the mass
mass 2 $MASS 1e-9 0.0; # place mass on top node

# Dynamic Analysis of frame

# Define recorders
recorder Node -file $dataDir/residualdisp$GroundFile$G$T.out -node 2 -dof 1 disp;
recorder EnvelopeNode -file $dataDir/maxdisp$GroundFile$G$T.out -node 2 -dof 1 disp;
recorder Drift -file $dataDir/Drift$GroundFile$G$T.out -iNode 1 -jNode 2 -dof 1 -perpDirn 2;

timeSeries Path 10 -dt $dt -filePath $outFile -factor $GMfatt

# DYNAMIC EQ ANALYSIS -----
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1; # ground-motion direction

# set up ground-motion-analysis parameters
set DtAnalysis [expr 0.02]; # time-step Dt for lateral analysis
set TmaxAnalysis [expr 300.]; # maximum duration of ground-motion analysis -- should be 50*$sec

# display deformed shape:
#set ViewScale 5; # amplify display of deformed shape
#DisplayModel2D DeformedShape $ViewScale ; # display deformed shape, the scaling factor needs to be adjusted for each model

# DYNAMIC ANALYSIS PARAMETERS
constraints Transformation ;

# DOF NUMBERER (number the degrees of freedom in the domain): (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/366.htm)
numberer Plain

# SYSTEM (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/371.htm)
system SparseGeneral -piv

```



```

# TEST: # convergence test to
set Tol 1.e-8;                # Convergence Test: tolerance
set maxNumIter 10;           # Convergence Test: maximum number of iterations that will be performed before "failure to converge" is returned
set printFlag 0;             # Convergence Test: flag used to print information on convergence (optional)
# 1: print information on each step;
set TestType EnergyIncr; # Convergence-test type
test $TestType $Tol $maxNumIter $printFlag;

# Solution ALGORITHM: -- Iterate from the last time step to the current (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/682.htm)
set algorithmType ModifiedNewton
algorithm $algorithmType;

# Transient INTEGRATOR: -- determine the next time step for an analysis including inertial effects
set NewmarkGamma 0.5; # Newmark-integrator gamma parameter (also HHT)
set NewmarkBeta 0.25; # Newmark-integrator beta parameter
integrator Newmark $NewmarkGamma $NewmarkBeta

# ANALYSIS -- defines what type of analysis is to be performed (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/324.htm)
analysis Transient

# define DAMPING-----
# apply Rayleigh DAMPING from $xDamp
#  $D = \alpha M + \beta K_{curr} + \beta K_{comm} + \beta K_{init}$ 
set xDamp 0.02; # 2% damping ratio
set alphaM 0.; # M-prop. damping; D = alphaM*M
set betaKcurr 0.; # K-proportional damping; +betaKcurr*KCurrent
set betaKcomm [expr 2.*$xDamp/($omega)]; # K-prop. damping parameter; +betaKcomm*KlastCommitt
set betaKinit 0.; # initial-stiffness proportional damping +betaKinit*Kini
# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm; # RAYLEIGH damping

# ----- perform Dynamic Ground-Motion Analysis
# Uniform EXCITATION: acceleration input
set IDloadTag 400; # load tag
#set dt 0.02; # time step for input ground motion
set GMfatt [expr $G*$GMfact]; # data in input file is in g Units -- ACCELERATION TH
#set AccelSeries "Series -dt $dt -filePath $GroundFile -factor $GMfatt" ; # time series information
#pattern UniformExcitation $IDloadTag $GMDirection -accel $AccelSeries ; # create Uniform excitation

pattern UniformExcitation 100 1 -accel 10

set Nsteps [expr int($TmaxAnalysis/$DtAnalysis)];
set ok [analyze $Nsteps $DtAnalysis]; # actually perform analysis; returns ok=0 if analysis was successful

if {$ok != 0} { # if analysis was not successful.
# change some analysis parameters to achieve convergence
# performance is slower inside this loop
# Time-controlled analysis
set ok 0;
set controlTime [getTime];
while {$controlTime < $TmaxAnalysis && $ok == 0} {
set ok [analyze 1 $DtAnalysis]
set controlTime [getTime]
set ok [analyze 1 $DtAnalysis]
if {$ok != 0} {
puts "Trying Newton with Initial Tangent .."
test NormDispIncr $Tol 1000 0
algorithm Newton -initial
set ok [analyze 1 $DtAnalysis]
test $TestType $Tol $maxNumIter 0
algorithm $algorithmType
}
}
}

```

```

}
if {$ok != 0} {
puts "Trying Broyden .."
algorithm Broyden 8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying NewtonWithLineSearch .."
algorithm NewtonLineSearch .8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
}
}; # end if ok !0

remove recorders

} ;# end of gamma value
} ; # end of period
wipe;

set counter [expr $counter + 1]
} ; #end earthquake ground motion

```

D.2 Three Story Models

D.2.1 Fixed Gravity Columns

```

# -----
# by Megan Boston, 2011

set dataDir normal3storyInertia
file mkdir $dataDir
source LibUnits.tcl; # define units and constants
source LibGMfact.tcl
source LibGMfiles.tcl
set GMfact 1.0; # Ground-motion scaling factor

#set IcolTest 10000
set iGs "0.01 0.02 0.03 0.04 0.05 0.1 0.5 0.99"
set iGs "0.25";
# in this case looking at different levels of post yields stiffness,
# this value is going to be different values of gamma,
# where gamma is the post yield stiffness multiplied by some ratio
# when gamma is 1, then the post-yield stiffness system is equal to the initial stiffness
# otherwise it is some other value

set Es 29000.0;
set Fybr 46.0;
set LCol 156.0
set Lbay 360.0
set Lbr [expr pow(pow($Lbay,2)+pow($LCol,2),0.5)];
set cost [expr $Lbay/$Lbr]
set g 386.4;
set Weigth 405; #weight of half of my structure in kips
set MASS [expr $Weigth/$g];
set brstrain [expr $Fybr/($Es*$cost)]
set counter 0; #counts the earthquakes
set GammaCounter 0; #for rotation through the gammas

```

```

set AreaCounter 1; # for the period number ranges between 1 and 80

#set iGroundFile {"NORTHR.CNP196" }

foreach GroundFile $iGroundFile {

set GMfact [lindex $iGMfact $counter]
#set GMfact 1.0; # Ground-motion scaling factor
puts $GMfact
#set GroundFile "NORTHR.CNP196"
source ReadSMDFile.tcl
set inFile $GroundFile.at2;
puts $inFile
set outFile $GroundFile.g3; # define output files
ReadSMDFile $inFile $outFile dt; # call procedure to convert the ground-motion file
set GMfatt [expr $g*$GMfact];
puts $dt
puts $GMfact

set IcolTest "3800 7464 11440 15120 19360 21760 26800 32480"

foreach IcolTest $IcolTest {
#foreach G $iGs
#foreach IcolTest $IcolTest

#puts "G $G"
set iAbraces "0.0001 0.00025 0.0005 0.00075 0.001 0.0025 0.005 0.0075 0.01 0.025 0.05 0.075
0.1 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5 5.5 6 6.5
7 7.5 8 8.5 9 9.5 10 11 12 13 14 15 16 17 18 19 20 25 30 40 50 75 100 250 500 1000"

set GammaCounter [expr $GammaCounter + 1]

foreach Abrace $iAbraces {

set kbrace [expr 3*$Es*$Abrace*pow($cost,2)/(7*$Lbr)]

set G [expr (18*$IcolTest*$Es)/(18*$IcolTest*$Es+113*pow($LCol,3)*$kbrace)]
puts $GroundFile
puts "G $G"
puts "Abrace $Abrace"
#post stiffness strength in the column only
set kinitial [expr $kbrace/(1-$G)];
set kposty [expr $G*$kinitial];
set IzCol [expr 113*$kposty*pow($LCol,3)/(18*$Es)]
#set kcol [expr $kbrace*$G/(1-$G)];
#set IzCol [expr $kcol*pow($LCol,3)/(12*$Es)];
set AgCol [expr pow($IzCol,0.5)];

#puts $Abrace
puts $IzCol
#puts $AgCol

# Setup Model
wipe; # clears existing models
model BasicBuilder -ndm 2 -ndf 3; # Sets dimensions and degrees of freedom

# Define Geometry
node 1 0.0 0.0;
node 2 0.0 $LCol;
node 3 $Lbay 0.0;
node 4 $Lbay $LCol;

```

```

node 5 0.0 [expr $LCol*2]
node 6 $Lbay [expr $LCol*2]
node 7 0.0 [expr $LCol*3]
node 8 $Lbay [expr $LCol*3]

# Define boundary conditions
fix 1 1 1 1; # fixed support at base of column
fix 3 1 1 1; # pinned support at base of brace, fix all three since brace is modeled with a truss member

# Define geomtric Transformation
set ColTransfTag 1;
set IDBraceTransf 103; # all brace members
geomTransf Linear $ColTransfTag; # options are linear, P-Delta, and Corotational
geomTransf Corotational $IDBraceTransf;

# Define Materials
set BrMat 5;
set BraceIE 15;
uniaxialMaterial ElasticPP $BraceIE $Es $brstrain

# Define Element Connectivity
#Columns, load bearing
element elasticBeamColumn 1 1 2 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 2 2 5 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 3 5 7 $AgCol $Es $IzCol $ColTransfTag;

#Braces
element CorotTruss 101 2 3 $Abrace $BraceIE;
element CorotTruss 102 5 4 $Abrace $BraceIE;
element CorotTruss 103 7 6 $Abrace $BraceIE;

#Non-flexural Columns
element elasticBeamColumn 21 3 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 22 4 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 23 6 8 $AgCol $Es 0.001 $ColTransfTag;

#Beam no flexural strength
element elasticBeamColumn 31 2 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 32 5 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 33 7 6 $AgCol $Es 0.001 $ColTransfTag;

# Define the mass
mass 2 $MASS 1e-9 0.0; # place mass on top node
mass 5 $MASS 1e-9 0.0;
mass 7 $MASS 1e-9 0.0;

# Dynamic Analysis of frame

# Define recorders
#recorder Node -file $dataDir/residualdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;

#Drift Recorders
recorder Drift -file $dataDir/Drift1$GroundFile$GammaCounter$AreaCounter.out -time -iNode 1 -jNode 2 -dof 1 -perpDirn 2;

```

```

recorder Drift -file $dataDir/Drift2$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 5 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift3$GroundFile$GammaCounter$AreaCounter.out -time -iNode 5 -jNode 7 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/RoofDrift$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 7 -dof 1 -perpDirn 2;

#Element Recorders to get the moment in the columns
recorder EnvelopeElement -file $dataDir/StoryForces1$GroundFile$GammaCounter$AreaCounter.out -time -ele 1 localForce;
recorder EnvelopeElement -file $dataDir/StoryForces2$GroundFile$GammaCounter$AreaCounter.out -time -ele 2 localForce;
recorder EnvelopeElement -file $dataDir/StoryForces3$GroundFile$GammaCounter$AreaCounter.out -time -ele 3 localForce;

timeSeries Path 10 -dt $dt -filePath $outFile -factor $GMfatt

# DYNAMIC EQ ANALYSIS -----
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1; # ground-motion direction

# set up ground-motion-analysis parameters
set DtAnalysis [expr 0.02]; # time-step Dt for lateral analysis
set TmaxAnalysis [expr 200.]; # maximum duration of ground-motion analysis -- should be 50*$sec

# DYNAMIC ANALYSIS PARAMETERS
constraints Transformation ;

# DOF NUMBERER (number the degrees of freedom in the domain): (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/366.htm)
numberer Plain

# SYSTEM (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/371.htm)
system SparseGeneral -piv

# TEST: # convergence test to
set Tol 1.e-8; # Convergence Test: tolerance
set maxNumIter 10; # Convergence Test: maximum number of iterations that will be performed before "failure to converge" is returned
set printFlag 0; # Convergence Test: flag used to print information on convergence (optional)
# 1: print information on each step;
set TestType EnergyIncr; # Convergence-test type
test $TestType $Tol $maxNumIter $printFlag;

# Solution ALGORITHM: -- Iterate from the last time step to the current (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/682.htm)
set algorithmType ModifiedNewton
algorithm $algorithmType;

# Transient INTEGRATOR: -- determine the next time step for an analysis including inertial effects
set NewmarkGamma 0.5; # Newmark-integrator gamma parameter (also HHT)
set NewmarkBeta 0.25; # Newmark-integrator beta parameter
integrator Newmark $NewmarkGamma $NewmarkBeta

# ANALYSIS -- defines what type of analysis is to be performed (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/324.htm)
analysis Transient

# define DAMPING-----
# apply Rayleigh DAMPING from $xDamp
# D=$alphaM*M + $betaKcurr*Kcurrent + $betaKcomm*KlastCommit + $beatKinit*$Kinitial

set xDamp 0.02; # damping ratio
set MpropSwitch 1.0;
set KcurrSwitch 0.0;
set KcommSwitch 1.0;
set KinitSwitch 0.0;
set nEigenI 1; # mode 1
set nEigenJ 3; # mode 3
set lambdaN [eigen [expr $nEigenJ]]; # eigenvalue analysis for nEigenJ modes
set lambdaI [lindex $lambdaN [expr $nEigenI-1]]; # eigenvalue mode i

```

```

set lambdaJ [lindex $lambdaN [expr $nEigenJ-1]]; # eigenvalue mode j
set omegaI [expr pow($lambdaI,0.5)];
set omegaJ [expr pow($lambdaJ,0.5)];
set alphaM [expr $MpropSwitch*$xDamp*(2*$omegaI+$omegaJ)/($omegaI+$omegaJ)]; # M-prop. damping; D = alphaM*M
set betaKcurr [expr $KcurrSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # current-K; +beatKcurr*KCurrent
set betaKcomm [expr $KcommSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # last-committed K; +betaKcomm*KlastCommitt
set betaKinit [expr $KinitSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # initial-K; +beatKinit*Kini
set Period [expr 2*$PI/$omegaI];
puts "period $Period"

# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm; # RAYLEIGH damping

# ----- perform Dynamic Ground-Motion Analysis
# Uniform EXCITATION: acceleration input
#set IDloadTag 400; # load tag
#set dt 0.02; # time step for input ground motion
#set GMfatt [expr $g*$GMfact]; # data in input file is in g Unifts -- ACCELERATION TH
#set AccelSeries "Series -dt $dt -filePath $outfile -factor $GMfatt"; # time series information
#pattern UniformExcitation $IDloadTag $GMDirection -accel $AccelSeries ; # create Uniform excitation

pattern UniformExcitation 100 1 -accel 10

set Nsteps [expr int($TmaxAnalysis/$DtAnalysis)];
set ok [analyze $Nsteps $DtAnalysis]; # actually perform analysis; returns ok=0 if analysis was successful

if {$ok != 0} { # if analysis was not successful.
# change some analysis parameters to achieve convergence
# performance is slower inside this loop
# Time-controlled analysis
set ok 0;
set controlTime [getTime];
while {$controlTime < $TmaxAnalysis && $ok == 0} {
set ok [analyze 1 $DtAnalysis]
set controlTime [getTime]
set ok [analyze 1 $DtAnalysis]
if {$ok != 0} {
puts "Trying Newton with Initial Tangent .."
test NormDispIncr $Tol 1000 0
algorithm Newton -initial
set ok [analyze 1 $DtAnalysis]
test $TestType $Tol $maxNumIter 0
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying Broyden .."
algorithm Broyden 8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying NewtonWithLineSearch .."
algorithm NewtonLineSearch .8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
}
}; # end if ok != 0

#set Period [expr 2*$PI/$omega]

```

```

#Create file that will show the Earthquake, Gamma, Area of brace, and Period
set outfile $dataDir/output$GroundFile$GammaCounter$AreaCounter.out
set outFileID [open $outfile w]
puts $outFileID $GroundFile
puts $outFileID $G
puts $outFileID $Abrace
puts $outFileID $IzCol
puts $outFileID $Period
close $outFileID

remove recorders

set AreaCounter [expr $AreaCounter + 1]
} ;# end of period values

set AreaCounter 1

} ; # end of gamma

set GammaCounter 0; #for rotation through the gammas
set counter [expr $counter + 1]

} ; #end earthquake ground motion

```

D.2.2 Heavy Top Brace

```

# -----
# Modified by Megan Boston, 2011

set dataDir StiffTop3storyInertia
file mkdir $dataDir
source LibUnits.tcl; # define units and constants
source LibGMfact.tcl
source LibGMfiles.tcl
set GMfact 1.0; # Ground-motion scaling factor

#set IcolTest 10000
set iGs "0.01 0.02 0.03 0.04 0.05 0.1 0.5 0.99"
set iGs "0.25";
# in this case looking at different levels of post yields stiffness,
# this value is going to be different values of gamma,
# where gamma is the post yield stiffness multiplied by some ratio
# when gamma is 1, then the post-yield stiffness system is equal to the initial stiffness
# otherwise it is some other value

set Es 29000.0;
set Fybr 46.0;
set LCol 156.0
set Lbay 360.0
set Lbr [expr pow(pow($Lbay,2)+pow($LCol,2),0.5)];
set cost [expr $Lbay/$Lbr]
set g 386.4;
set Weigth 405; #weight of half of my structure in kips
set MASS [expr $Weigth/$g];
set brstrain [expr $Fybr/($Es*$cost)]
set counter 0; #counts the earthquakes
set GammaCounter 0; #for rotation through the gammas
set AreaCounter 1; # for the period number ranges between 1 and 80

#set iGroundFile {"NORTHR.CNP196" }

```

```

foreach GroundFile $iGroundFile {

set GMfact [lindex $iGMfact $counter]
#set GMfact 1.0; # Ground-motion scaling factor
puts $GMfact
#set GroundFile "NORTHR.CNP196"
source ReadSMDFile.tcl
set inFile $GroundFile.at2;
puts $inFile
set outFile $GroundFile.g3; # define output files
ReadSMDFile $inFile $outFile dt; # call procedure to convert the ground-motion file
set GMfatt [expr $g*$GMfact];
puts $dt
puts $GMfact

set IcolTest "3800 7464 11440 15120 19360 21760 26800 32480"

foreach IcolTest $IcolTest {
#foreach G $iGs
#foreach IcolTest $IcolTest

#puts "G $G"
set iAbraces "0.0001 0.00025 0.0005 0.00075 0.001 0.0025 0.005 0.0075 0.01 0.025 0.05 0.075
0.1 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5 5.5 6 6.5
7 7.5 8 8.5 9 9.5 10 11 12 13 14 15 16 17 18 19 20 25 30 40 50 75 100 250 500 1000"

set GammaCounter [expr $GammaCounter + 1]

foreach Abrace $iAbraces {

set kbrace [expr 6*$Es*$Abrace*pow($cost,2)/(11*$Lbr)]

set G [expr (3*$IcolTest*$Es)/(3*$IcolTest*$Es+24*pow($LCol,3)*$kbrace)]
puts $GroundFile
puts "G $G"
puts "Abrace $Abrace"
#post stiffness strength in the column only
set kinitial [expr $kbrace/(1-$G)];
set kposty [expr $G*$kinitial];
set IzCol [expr 24*$kposty*pow($LCol,3)/(3*$Es)]
#set kcol [expr $kbrace*$G/(1-$G)];
#set IzCol [expr $kcol*pow($LCol,3)/(12*$Es)];
set AgCol [expr pow($IzCol,0.5)];

#puts $Abrace
#puts $IzCol
#puts $AgCol

# Setup Model
wipe; # clears existing models
model BasicBuilder -ndm 2 -ndf 3; # Sets dimensions and degrees of freedom

# Define Geometry
node 1 0.0 0.0;
node 2 0.0 $LCol;
node 3 $Lbay 0.0;
node 4 $Lbay $LCol;
node 5 0.0 [expr $LCol*2]
node 6 $Lbay [expr $LCol*2]
node 7 0.0 [expr $LCol*3]
node 8 $Lbay [expr $LCol*3]

```



```

# Define boundary conditions
fix 1 1 1 0; # fixed support at base of column
fix 3 1 1 0; # pinned support at base of brace, fix all three since brace is modeled with a truss member

# Define geomtric Transformation
set ColTransfTag 1;
set IDBraceTransf 103; # all brace members
geomTransf Linear $ColTransfTag; # options are linear, P-Delta, and Corotational
geomTransf Corotational $IDBraceTransf;

# Define Materials
set BrMat 5;
set BraceIE 15;
uniaxialMaterial ElasticPP $BraceIE $Es $brstrain

# Define Element Connectivity
#Columns, load bearing
element elasticBeamColumn 1 1 2 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 2 2 5 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 3 5 7 $AgCol $Es $IzCol $ColTransfTag;

#Braces
element CorotTruss 101 2 3 $Abrace $BraceIE;
element CorotTruss 102 5 4 $Abrace $BraceIE;
element CorotTruss 103 7 6 [expr $Abrace*100] $BraceIE;

#Non-flexural Columns
element elasticBeamColumn 21 3 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 22 4 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 23 6 8 $AgCol $Es 0.001 $ColTransfTag;

#Beam no flexural strength
element elasticBeamColumn 31 2 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 32 5 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 33 7 6 $AgCol $Es 0.001 $ColTransfTag;

# Define the mass
mass 2 $MASS 1e-9 0.0; # place mass on top node
mass 5 $MASS 1e-9 0.0;
mass 7 $MASS 1e-9 0.0;

# Dynamic Analysis of frame

# Define recorders
#recorder Node -file $dataDir/residualdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;

#Drift Recorders
recorder Drift -file $dataDir/Drift1$GroundFile$GammaCounter$AreaCounter.out -time -iNode 1 -jNode 2 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift2$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 5 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift3$GroundFile$GammaCounter$AreaCounter.out -time -iNode 5 -jNode 7 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/RoofDrift$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 7 -dof 1 -perpDirn 2;

```

```

#Element Recorders to get the moment in the columns
recorder EnvelopeElement -file $dataDir/StoryForces1$GroundFile$GammaCounter$AreaCounter.out -time -ele 1 localForce;
recorder EnvelopeElement -file $dataDir/StoryForces2$GroundFile$GammaCounter$AreaCounter.out -time -ele 2 localForce;
recorder EnvelopeElement -file $dataDir/StoryForces3$GroundFile$GammaCounter$AreaCounter.out -time -ele 3 localForce;

timeSeries Path 10 -dt $dt -filePath $outFile -factor $GMfatt

# DYNAMIC EQ ANALYSIS -----
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1; # ground-motion direction

# set up ground-motion-analysis parameters
set DtAnalysis [expr 0.02]; # time-step Dt for lateral analysis
set TmaxAnalysis [expr 200.]; # maximum duration of ground-motion analysis -- should be 50*$sec

# DYNAMIC ANALYSIS PARAMETERS
constraints Transformation ;

# DOF NUMBERER (number the degrees of freedom in the domain): (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/366.htm)
numberer Plain

# SYSTEM (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/371.htm)
system SparseGeneral -piv

# TEST: # convergence test to
set Tol 1.e-8; # Convergence Test: tolerance
set maxNumIter 10; # Convergence Test: maximum number of iterations that will be performed before "failure to converge" is returned
set printFlag 0; # Convergence Test: flag used to print information on convergence (optional)
# 1: print information on each step;
set TestType EnergyIncr; # Convergence-test type
test $TestType $Tol $maxNumIter $printFlag;

# Solution ALGORITHM: -- Iterate from the last time step to the current (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/682.htm)
set algorithmType ModifiedNewton
algorithm $algorithmType;

# Transient INTEGRATOR: -- determine the next time step for an analysis including inertial effects
set NewmarkGamma 0.5; # Newmark-integrator gamma parameter (also HHT)
set NewmarkBeta 0.25; # Newmark-integrator beta parameter
integrator Newmark $NewmarkGamma $NewmarkBeta

# ANALYSIS -- defines what type of analysis is to be performed (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/324.htm)
analysis Transient

# define DAMPING-----
# apply Rayleigh DAMPING from $xDamp
#  $D = \alpha M + \beta K_{curr} + \beta K_{comm} + \beta K_{kinit}$ 

set xDamp 0.02; # damping ratio
set MpropSwitch 1.0;
set KcurrSwitch 0.0;
set KcommSwitch 1.0;
set KinitSwitch 0.0;
set nEigenI 1; # mode 1
set nEigenJ 3; # mode 3
set lambdaN [eigen [expr $nEigenJ]]; # eigenvalue analysis for nEigenJ modes
set lambdaI [lindex $lambdaN [expr $nEigenI-1]]; # eigenvalue mode i
set lambdaJ [lindex $lambdaN [expr $nEigenJ-1]]; # eigenvalue mode j
set omegaI [expr pow($lambdaI,0.5)];
set omegaJ [expr pow($lambdaJ,0.5)];
set alphaM [expr $MpropSwitch*$xDamp*(2*$omegaI*$omegaJ)/($omegaI+$omegaJ)]; # M-prop. damping; D = alpha*M

```

```

set betaKcurr [expr $KcurrSwitch*2.*$xDamp/($omegaI+$omegaJ)];      # current-K;      +beatKcurr*KCurrent
set betaKcomm [expr $KcommSwitch*2.*$xDamp/($omegaI+$omegaJ)];    # last-committed K; +betaKcomm*KlastCommitt
set betaKinit [expr $KinitSwitch*2.*$xDamp/($omegaI+$omegaJ)];    # initial-K;      +beatKinit*Kini
set Period [expr 2*$PI/$omegaI];
puts "period $Period"

# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm; # RAYLEIGH damping

# ----- perform Dynamic Ground-Motion Analysis
# Uniform EXCITATION: acceleration input
#set IDloadTag 400; # load tag
#set dt 0.02; # time step for input ground motion
#set GMfatt [expr $g*$GMfact]; # data in input file is in g Unifits -- ACCELERATION TH
#set AccelSeries "Series -dt $dt -filePath $outfile -factor $GMfatt" ; # time series information
#pattern UniformExcitation $IDloadTag $GMdirection -accel $AccelSeries ; # create Uniform excitation

pattern UniformExcitation 100 1 -accel 10

set Nsteps [expr int($TmaxAnalysis/$DtAnalysis)];
set ok [analyze $Nsteps $DtAnalysis]; # actually perform analysis; returns ok=0 if analysis was successful

if {$ok != 0} { ; # if analysis was not successful.
# change some analysis parameters to achieve convergence
# performance is slower inside this loop
# Time-controlled analysis
set ok 0;
set controlTime [getTime];
while {$controlTime < $TmaxAnalysis && $ok == 0} {
set ok [analyze 1 $DtAnalysis]
set controlTime [getTime]
set ok [analyze 1 $DtAnalysis]
if {$ok != 0} {
puts "Trying Newton with Initial Tangent .."
test NormDispIncr $Tol 1000 0
algorithm Newton -initial
set ok [analyze 1 $DtAnalysis]
test $TestType $Tol $maxNumIter 0
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying Broyden .."
algorithm Broyden 8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying NewtonWithLineSearch .."
algorithm NewtonLineSearch .8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
}
}; # end if ok !0

#set Period [expr 2*$PI/$omega]

#Create file that will show the Earthquake, Gamma, Area of brace, and Period
set outfile $dataDir/output$GroundFile$GammaCounter$AreaCounter.out
set outFileID [open $outfile w]
puts $outFileID $GroundFile

```

```

puts $outFileID $G
puts $outFileID $Abrace
puts $outFileID $IzCol
puts $outFileID $Period
close $outFileID

remove recorders

set AreaCounter [expr $AreaCounter + 1]
} ;# end of period values

set AreaCounter 1

} ; # end of gamma

set GammaCounter 0; #for rotation through the gammas
set counter [expr $counter + 1]

} ; #end earthquake ground motion

```

D.2.3 Heavy Middle Brace

```

# -----
# by Megan Boston, 2011

set dataDir StiffMiddle3storyInertia
file mkdir $dataDir
source LibUnits.tcl; # define units and constants
source LibGMfact.tcl
source LibGMfiles.tcl
set GMfact 1.0; # Ground-motion scaling factor

#set IcolTest 10000
#set iGs "0.01 0.02 0.03 0.04 0.05 0.1 0.5 0.99"
#set iGs "0.25";
# in this case looking at different levels of post yields stiffness,
# this value is going to be different values of gamma,
# where gamma is the post yield stiffness multiplied by some ratio
# when gamma is 1, then the post-yield stiffness system is equal to the initial stiffness
# otherwise it is some other value

set Es 29000.0;
set Fybr 46.0;
set LCol 156.0
set Lbay 360.0
set Lbr [expr pow(pow($Lbay,2)+pow($LCol,2),0.5)];
set cost [expr $Lbay/$Lbr]
set g 386.4;
set Weigth 405; #weight of half of my structure in kips
set MASS [expr $Weigth/$g];
set brstrain [expr $Fybr/($Es*$cost)]
set counter 0; #counts the earthquakes
set GammaCounter 0; #for rotation through the gammas
set AreaCounter 1; # for the period number ranges between 1 and 80

#set iGroundFile {"NORTHR.CNP196" }

foreach GroundFile $iGroundFile {

set GMfact [lindex $iGMfact $counter]

```

```

#set GMfact 1.0; # Ground-motion scaling factor
puts $GMfact
#set GroundFile "NORTH.R.CNP196"
source ReadSMDFile.tcl
set inFile $GroundFile.at2;
puts $inFile
set outFile $GroundFile.g3; # define output files
ReadSMDFile $inFile $outFile dt; # call procedure to convert the ground-motion file
set GMfact [expr $g*$GMfact];
puts $dt
puts $GMfact

set IcolTest "3800 7464 11440 15120 19360 21760 26800 32480"

foreach IcolTest $IcolTest {
#foreach G $IGs
#foreach IcolTest $IcolTest
puts "I $IcolTest"
#puts "G $G"
set iAbraces "0.0001 0.00025 0.0005 0.00075 0.001 0.0025 0.005 0.0075 0.01 0.025 0.05 0.075
0.1 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5 5.5 6
6.5 7 7.5 8 8.5 9 9.5 10 11 12 13 14 15 16 17 18 19 20 25 30 40 50 75 100 250 500 1000"

set GammaCounter [expr $GammaCounter + 1]

foreach Abrace $iAbraces {

set kbrace [expr 2*$Es*$Abrace*pow($LCol,2)/(3*$Lbr)]

set G [expr ($IcolTest*$Es)/($IcolTest*$Es*pow($LCol,3)*$kbrace)]
puts $GroundFile
puts "G $G"
puts "Abrace $Abrace"
#post stiffness strength in the column only
set kinitial [expr $kbrace/(1-$G)];
set kposty [expr $G*$kinitial];
set IzCol [expr $kposty*pow($LCol,3)/($Es)]
#set kcol [expr $kbrace*$G/(1-$G)];
#set IzCol [expr $kcol*pow($LCol,3)/(12*$Es)];
set AgCol [expr pow($IzCol,0.5)];

#puts $Abrace
puts $IzCol
#puts $AgCol

# Setup Model
wipe; # clears existing models
model BasicBuilder -ndm 2 -ndf 3; # Sets dimensions and degrees of freedom

# Define Geometry
node 1 0.0 0.0;
node 2 0.0 $LCol;
node 3 $Lbay 0.0;
node 4 $Lbay $LCol;
node 5 0.0 [expr $LCol*2]
node 6 $Lbay [expr $LCol*2]
node 7 0.0 [expr $LCol*3]
node 8 $Lbay [expr $LCol*3]

# Define boundary conditions
fix 1 1 1 0; # fixed support at base of column

```

```

fix 3 1 1 0; # pinned support at base of brace, fix all three since brace is modeled with a truss member

# Define geomtric Transformation
set ColTransfTag 1;
set IDBraceTransf 103; # all brace members
geomTransf Linear $ColTransfTag; # options are linear, P-Delta, and Corotational
geomTransf Corotational $IDBraceTransf;

# Define Materials
set BrMat 5;
set BraceIE 15;
uniaxialMaterial ElasticPP $BraceIE $Es $brstrain

# Define Element Connectivity
#Columns, load bearing
element elasticBeamColumn 1 1 2 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 2 2 5 $AgCol $Es $IzCol $ColTransfTag;
element elasticBeamColumn 3 5 7 $AgCol $Es $IzCol $ColTransfTag;

#Braces
element CorotTruss 101 2 3 $Abrace $BraceIE;
element CorotTruss 102 5 4 [expr $Abrace*100] $BraceIE;
element CorotTruss 103 7 6 $Abrace $BraceIE;

#Non-flexural Columns
element elasticBeamColumn 21 3 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 22 4 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 23 6 8 $AgCol $Es 0.001 $ColTransfTag;

#Beam no flexural strength
element elasticBeamColumn 31 2 4 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 32 5 6 $AgCol $Es 0.001 $ColTransfTag;
element elasticBeamColumn 33 7 6 $AgCol $Es 0.001 $ColTransfTag;

# Define the mass
mass 2 $MASS 1e-9 0.0; # place mass on top node
mass 5 $MASS 1e-9 0.0;
mass 7 $MASS 1e-9 0.0;

# Dynamic Analysis of frame

# Define recorders
#recorder Node -file $dataDir/residualdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp1$GroundFile$G$Abrace.out -time -node 2 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp2$GroundFile$G$Abrace.out -time -node 5 -dof 1 disp;

#recorder Node -file $dataDir/residualdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;
#recorder EnvelopeNode -file $dataDir/maxdisp3$GroundFile$G$Abrace.out -time -node 7 -dof 1 disp;

#Drift Recorders
recorder Drift -file $dataDir/Drift1$GroundFile$GammaCounter$AreaCounter.out -time -iNode 1 -jNode 2 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift2$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 5 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift3$GroundFile$GammaCounter$AreaCounter.out -time -iNode 5 -jNode 7 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/RoofDrift$GroundFile$GammaCounter$AreaCounter.out -time -iNode 2 -jNode 7 -dof 1 -perpDirn 2;

#Element Recorders to get the moment in the columns
recorder EnvelopeElement -file $dataDir/StoryForces1$GroundFile$GammaCounter$AreaCounter.out -time -ele 1 localForce;
recorder EnvelopeElement -file $dataDir/StoryForces2$GroundFile$GammaCounter$AreaCounter.out -time -ele 2 localForce;

```

```

recorder EnvelopeElement -file $dataDir/StoryForces3$GroundFile$GammaCounter$AreaCounter.out -time -ele 3 localForce;

timeSeries Path 10 -dt $dt -filePath $outFile -factor $GMfatt

# DYNAMIC EQ ANALYSIS -----
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1; # ground-motion direction

# set up ground-motion-analysis parameters
set DtAnalysis [expr 0.02]; # time-step Dt for lateral analysis
set TmaxAnalysis [expr 200.]; # maximum duration of ground-motion analysis -- should be 50*$sec

# DYNAMIC ANALYSIS PARAMETERS
constraints Transformation ;

# DOF NUMBERER (number the degrees of freedom in the domain): (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/366.htm)
numberer Plain

# SYSTEM (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/371.htm)
system SparseGeneral -piv

# TEST: # convergence test to
set Tol 1.e-8; # Convergence Test: tolerance
set maxNumIter 10; # Convergence Test: maximum number of iterations that will be performed before "failure to converge" is returned
set printFlag 0; # Convergence Test: flag used to print information on convergence (optional)
# 1: print information on each step;
set TestType EnergyIncr; # Convergence-test type
test $TestType $Tol $maxNumIter $printFlag;

# Solution ALGORITHM: -- Iterate from the last time step to the current (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/682.htm)
set algorithmType ModifiedNewton
algorithm $algorithmType;

# Transient INTEGRATOR: -- determine the next time step for an analysis including inertial effects
set NewmarkGamma 0.5; # Newmark-integrator gamma parameter (also HHT)
set NewmarkBeta 0.25; # Newmark-integrator beta parameter
integrator Newmark $NewmarkGamma $NewmarkBeta

# ANALYSIS -- defines what type of analysis is to be performed (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/324.htm)
analysis Transient

# define DAMPING-----
# apply Rayleigh DAMPING from $xDamp
# D=$alphaM*M + $betaKcurr*Kcurrent + $betaKcomm*KlastCommit + $beatKinit*Kinitial

set xDamp 0.02; # damping ratio
set MpropSwitch 1.0;
set KcurrSwitch 0.0;
set KcommSwitch 1.0;
set KinitSwitch 0.0;
set nEigenI 1; # mode 1
set nEigenJ 3; # mode 3
set lambdaN [eigen [expr $nEigenJ]]; # eigenvalue analysis for nEigenJ modes
set lambdaI [lindex $lambdaN [expr $nEigenI-1]]; # eigenvalue mode i
set lambdaJ [lindex $lambdaN [expr $nEigenJ-1]]; # eigenvalue mode j
set omegaI [expr pow($lambdaI,0.5)];
set omegaJ [expr pow($lambdaJ,0.5)];
set alphaM [expr $MpropSwitch*$xDamp*(2*$omegaI*$omegaJ)/($omegaI+$omegaJ)]; # M-prop. damping; D = alphaM*M
set betaKcurr [expr $KcurrSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # current-K; +beatKcurr*KCurrent
set betaKcomm [expr $KcommSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # last-committed K; +betaKcomm*KlastCommitt
set betaKinit [expr $KinitSwitch*2.*$xDamp/($omegaI+$omegaJ)]; # initial-K; +beatKinit*Kini

```

```

set Period [expr 2*$PI/$omega];
puts "period $Period"

# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm; # RAYLEIGH damping

# ----- perform Dynamic Ground-Motion Analysis
# Uniform EXCITATION: acceleration input
#set IDloadTag 400; # load tag
#set dt 0.02; # time step for input ground motion
#set GMfatt [expr $g*$GMfact]; # data in input file is in g Units -- ACCELERATION TH
#set AccelSeries "Series -dt $dt -filePath $outfile -factor $GMfatt" ; # time series information
#pattern UniformExcitation $IDloadTag $GMDirection -accel $AccelSeries ; # create Uniform excitation

pattern UniformExcitation 100 1 -accel 10

set Nsteps [expr int($TmaxAnalysis/$DtAnalysis)];
set ok [analyze $Nsteps $DtAnalysis]; # actually perform analysis; returns ok=0 if analysis was successful

if {$ok != 0} {
    ; # if analysis was not successful.
    # change some analysis parameters to achieve convergence
    # performance is slower inside this loop
    # Time-controlled analysis
    set ok 0;
    set controlTime [getTime];
    while {$controlTime < $TmaxAnalysis && $ok == 0} {
        set ok [analyze 1 $DtAnalysis]
        set controlTime [getTime]
        set ok [analyze 1 $DtAnalysis]
        if {$ok != 0} {
            puts "Trying Newton with Initial Tangent .."
            test NormDispIncr $Tol 1000 0
            algorithm Newton -initial
            set ok [analyze 1 $DtAnalysis]
            test $TestType $Tol $maxNumIter 0
            algorithm $algorithmType
        }
        if {$ok != 0} {
            puts "Trying Broyden .."
            algorithm Broyden 8
            set ok [analyze 1 $DtAnalysis]
            algorithm $algorithmType
        }
        if {$ok != 0} {
            puts "Trying NewtonWithLineSearch .."
            algorithm NewtonLineSearch .8
            set ok [analyze 1 $DtAnalysis]
            algorithm $algorithmType
        }
    }
}; # end if ok != 0

#set Period [expr 2*$PI/$omega]

#Create file that will show the Earthquake, Gamma, Area of brace, and Period
set outfile $dataDir/output$GroundFile$GammaCounter$AreaCounter.out
set outFileID [open $outfile w]
puts $outFileID $GroundFile
puts $outFileID $G
puts $outFileID $Abrace
puts $outFileID $IzCol

```



```

puts $outFileID $Period
close $outFileID

remove recorders

set AreaCounter [expr $AreaCounter + 1]
} ;# end of period values

set AreaCounter 1

} ; # end of gamma

set GammaCounter 0; #for rotation through the gammas
set counter [expr $counter + 1]

} ; #end earthquake ground motion

```

D.3 Five Story Model

```

# Five Story Case Study,
# by Megan Boston

#delete all previously constructed objects
wipe;

# create data directory
set dataDir modes
file mkdir $dataDir;

source LibUnits.tcl
source DisplayPlane.tcl; # procedure for displaying a plane in model
source DisplayModel2D.tcl; # procedure for displaying 2D perspective of model
source LibGMfact.tcl
source LibGMfiles.tcl

set counter 0

foreach GroundFile $iGroundFile {

#puts $iGroundFile
set GMfact [lindex $iGMfact $counter]
#set GMfact 1.0; # Ground-motion scaling factor
puts $GMfact
#set GroundFile "NORTHR.CNP196"
source ReadSMDFile.tcl
set inFile $GroundFile.at2;
puts $inFile
set outFile $GroundFile.g3; # define output files
ReadSMDFile $inFile $outFile dt; # call procedure to convert the ground-motion file
set GMfatt [expr $g*$GMfact]; # data in input file is in g Units -- ACCELERATION TH
#puts $dt

#set input variables
#-----

#mass
set Weight 393.0;
set g 386.0;

set m [expr $Weight/$g/2]

```

```

#number of modes
set numModes 5

#material
#normal W12x79
set A 23.2
set I 662.0
#normal W12x53
set At 15.6;
set It 425;

set I 4;
if {$I == 1} {
#Strong gravity columns
#W12x79 I = 1
set Ag1 23.2;
set Ig1 662;
#W12x53
set Ag2 15.6
set Ig2 425
}
if {$I == 1.5} {
#I = 1.5
#W12x106
set Ag1 31.2;
set Ig1 993;
#W12x79
set Ag2 23.2;
set Ig2 662;
}
if {$I == 2} {
#I = 2.0
#W12x152
set Ag1 44.7;
set Ig1 1430;
#W12x106
set Ag2 31.2;
set Ig2 933;
}
if {$I == 3} {
#I = 3.0
#W12x190
set Ag1 55.8;
set Ig1 1890;
#W12x136
set Ag2 39.9;
set Ig2 1240;
}
if {$I == 4} {
#I = 4.0
#W12x252
set Ag1 44.7;
set Ig1 2720;
#W12x190
set Ag2 55.8;
set Ig2 1890;
}
puts "Ig1 $Ig1"
puts "Ig2 $Ig2"

set numCol 6;

```

```

set Ag1 [expr $Ag1*$numCol];
set Ig1 [expr $Ig1*$numCol];
set Ag2 [expr $Ag2*$numCol];
set Ig2 [expr $Ig2*$numCol];

set E 29000.0
set A1 4.5;
set A2 4.5;
#set A3 4.0;
set A3 100.0;
set A4 3.0;
set A5 2.0;
#set A5 100;

#geometry
set L 394.
set h 197.

# define the model
#-----
#model builder
model BasicBuilder -ndm 2 -ndf 3

# nodal coordinates:
node 1 0. 0. ;
node 2 $L 0. ;
node 13 [expr 2*$L] 0 ;

node 3 0. $h ;
node 4 $L $h ;
node 33 0. $h ;
node 43 $L $h ;
node 14 [expr 2*$L] $h ;

node 5 0. [expr 2*$h] ;
node 6 $L [expr 2*$h] ;
node 53 0. [expr 2*$h] ;
node 63 $L [expr 2*$h] ;
node 15 [expr 2*$L] [expr 2*$h] ;

node 7 0. [expr 3*$h] ;
node 8 $L [expr 3*$h] ;
node 73 0. [expr 3*$h] ;
node 83 $L [expr 3*$h] ;
node 16 [expr 2*$L] [expr 3*$h] ;

node 9 0. [expr 4*$h] ;
node 10 $L [expr 4*$h] ;
node 93 0. [expr 4*$h] ;
node 103 $L [expr 4*$h] ;
node 17 [expr 2*$L] [expr 4*$h] ;

node 11 0. [expr 5*$h] ;
node 12 $L [expr 5*$h] ;
node 113 0. [expr 5*$h] ;
node 123 $L [expr 5*$h] ;
node 18 [expr 2*$L] [expr 5*$h] ;

#rigid diaphragm
#equalDOF 3 4 1 2

```

```

#pinned connections
equalDOF 3 33 1 2
equalDOF 4 43 1 2
equalDOF 5 53 1 2
equalDOF 6 63 1 2
equalDOF 7 73 1 2
equalDOF 8 83 1 2
equalDOF 9 93 1 2
equalDOF 10 103 1 2
equalDOF 11 113 1 2
equalDOF 12 123 1 2

# Single point constraints -- Boundary Conditions
fix 1 1 1 0;
fix 2 1 1 0;
fix 13 1 1 0;

# assign mass
mass 3 $m 0. 0. ;
mass 4 $m 0. 0. ;
mass 5 $m 0. 0. ;
mass 6 $m 0. 0. ;
mass 7 $m 0. 0. ;
mass 8 $m 0. 0. ;
mass 9 $m 0. 0. ;
mass 10 $m 0. 0. ;
mass 11 $m 0. 0. ;
mass 12 $m 0. 0. ;

# define geometric transformation:
set TransfTag 1;
geomTransf Linear $TransfTag ;

# define elements:
# columns
element elasticBeamColumn 11 1 3 $A $E $I $TransfTag;
element elasticBeamColumn 12 2 4 $A $E $I $TransfTag;
element elasticBeamColumn 13 3 5 $A $E $I $TransfTag;
element elasticBeamColumn 14 4 6 $A $E $I $TransfTag;
element elasticBeamColumn 15 5 7 $At $E $It $TransfTag;
element elasticBeamColumn 16 6 8 $At $E $It $TransfTag;
element elasticBeamColumn 17 7 9 $At $E $It $TransfTag;
element elasticBeamColumn 18 8 10 $At $E $It $TransfTag;
element elasticBeamColumn 19 9 11 $At $E $It $TransfTag;
element elasticBeamColumn 20 10 12 $At $E $It $TransfTag;

# beams
element elasticBeamColumn 201 33 43 $A $E $I $TransfTag;
element elasticBeamColumn 202 53 63 $A $E $I $TransfTag;
element elasticBeamColumn 203 73 83 $At $E $It $TransfTag;
element elasticBeamColumn 204 93 103 $At $E $It $TransfTag;
element elasticBeamColumn 205 113 123 $At $E $It $TransfTag;

#gravity Columns
element elasticBeamColumn 301 13 14 $Ag1 $E $Ig1 $TransfTag;
element elasticBeamColumn 302 14 15 $Ag1 $E $Ig1 $TransfTag;
element elasticBeamColumn 303 15 16 $Ag2 $E $Ig2 $TransfTag;
element elasticBeamColumn 304 16 17 $Ag2 $E $Ig2 $TransfTag;
element elasticBeamColumn 305 17 18 $Ag2 $E $Ig2 $TransfTag;

```

```

# define rigid links, truss between frame and gravity column
set TrussMatID 600; # define a material ID
set Arigid 1000.0; # define area of truss section (make much larger than A of frame elements)
set Irigid 100000.0; # moment of inertia for p-delta columns (make much larger than I of frame elements)
uniaxialMaterial Elastic $TrussMatID $E; # define truss material

# rigid links
element truss 401 4 14 $Arigid $TrussMatID; # Floor 2
element truss 402 6 15 $Arigid $TrussMatID; # Floor 3
element truss 403 8 16 $Arigid $TrussMatID; # Floor 4
element truss 404 10 17 $Arigid $TrussMatID; # Floor 5
element truss 405 12 18 $Arigid $TrussMatID; # Floor 6

#braces
# Braces
set BraceIE 15; #material for Brace in Brace frame
#uniaxialMaterial Elastic $BraceIE $E
set FyBrace 46.
set cost 0.906
uniaxialMaterial ElasticPP $BraceIE $E [expr $FyBrace/$E*$cost]
element corotTruss 101 1 4 $A1 $BraceIE
element corotTruss 102 3 6 $A2 $BraceIE
element corotTruss 103 5 8 $A3 $BraceIE
element corotTruss 104 7 10 $A4 $BraceIE
element corotTruss 105 9 12 $A5 $BraceIE

# record eigenvectors
#-----
for { set k 1 } { $k <= $numModes } { incr k } {
    recorder Node -file [format "modes/mode%i.out" $k] -nodeRange 1 6 -dof 1 2 3 "eigen $k"
}

# perform eigen analysis
#-----
set lambda [eigen $numModes];

# calculate frequencies and periods of the structure
#-----
set omega {}
set f {}
set T {}
set pi 3.141593

foreach lam $lambda {
    lappend omega [expr sqrt($lam)]
    lappend f [expr sqrt($lam)/(2*$pi)]
    lappend T [expr (2*$pi)/sqrt($lam)]
}

puts "periods are $T"

# write the output file consisting of periods
#-----
set period "modes/Periods.txt"
set Periods [open $period "w"]
foreach t $T {
    puts $Periods " $t"
}
close $Periods

# Define GRAVITY
pattern Plain 1 Linear {

```

```

load 14 0 -$Weight 0
load 15 0 -$Weight 0
load 16 0 -$Weight 0
load 17 0 -$Weight 0
load 18 0 -$Weight 0
}

# ----- apply gravity load
set Tol 1.0e-8; # convergence tolerance for test
constraints Plain; # how it handles boundary conditions
numberer Plain; # renumber dof's to minimize band-width (optimization), if you want to
system BandGeneral; # how to store and solve the system of equations in the analysis
test NormDispIncr $Tol 6 ; # determine if convergence has been achieved at the end of an iteration step
algorithm Newton; # use Newton's solution algorithm: updates tangent stiffness at every iteration
set NstepGravity 10; # apply gravity in 10 steps
set DGravity [expr 1./$NstepGravity]; # first load increment;
integrator LoadControl $DGravity; # determine the next time step for an analysis
analysis Static; # define type of analysis static or transient
analyze $NstepGravity; # apply gravity
# ----- maintain constant gravity loads and reset time to zero
loadConst -time 0.0

#Dynamic Analysis

# Dynamic Analysis of frame
#recorder Node -file $dataDir/DispFree$GroundFile.out -time -node 4 6 8 10 12 -dof 1 disp;
#recorder Drift -file $dataDir/Drift$GroundFile.out -time -iNode 2 4 6 8 10 -jNode 4 6 8 10 12 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift1$GroundFile.out -iNode 2 -jNode 4 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift2$GroundFile.out -iNode 4 -jNode 6 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift3$GroundFile.out -iNode 6 -jNode 8 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift4$GroundFile.out -iNode 8 -jNode 10 -dof 1 -perpDirn 2;
recorder Drift -file $dataDir/Drift5$GroundFile.out -iNode 10 -jNode 12 -dof 1 -perpDirn 2;

recorder Node -file $dataDir/VBase$GroundFile.out -node 1001 22 132 -dof 1 reaction;

timeSeries Path 10 -dt $dt -filePath $outFile -factor $GMfatt

# DYNAMIC EQ ANALYSIS -----
# Uniform Earthquake ground motion (uniform acceleration input at all support nodes)
set GMdirection 1; # ground-motion direction

# set up ground-motion-analysis parameters
set DtAnalysis [expr 0.02]; # time-step Dt for lateral analysis
set TmaxAnalysis [expr 100.]; # maximum duration of
ground-motion analysis -- should be 50*$sec

# display deformed shape:
set ViewScale 5; # amplify display of deformed shape
DisplayModel2D DeformedShape $ViewScale ; # display deformed shape,
the scaling factor needs to be adjusted for each model

# DYNAMIC ANALYSIS PARAMETERS
constraints Transformation ;

# DOF NUMBERER (number the degrees of freedom in the domain): (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/366.htm)
numberer Plain

# SYSTEM (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/371.htm)
system SparseGeneral -piv

# TEST: # convergence test to

```

```

set Tol 1.e-8;                # Convergence Test: tolerance
set maxNumIter 10;           # Convergence Test: maximum number of
iterations that will be performed before "failure to converge" is returned
set printFlag 0;             # Convergence Test: flag used
to print information on convergence (optional)
# 1: print information on each step;
set TestType EnergyIncr; # Convergence-test type
test $TestType $Tol $maxNumIter $printFlag;

# Solution ALGORITHM: -- Iterate from the last time step to the current (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/682.htm)
set algorithmType ModifiedNewton
algorithm $algorithmType;

# Transient INTEGRATOR: -- determine the next time step
for an analysis including inertial effects
set NewmarkGamma 0.5; # Newmark-integrator gamma parameter (also HHT)
set NewmarkBeta 0.25; # Newmark-integrator beta parameter
integrator Newmark $NewmarkGamma $NewmarkBeta

# ANALYSIS -- defines what type of analysis is to be performed (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/324.htm)
analysis Transient

# define DAMPING-----
# apply Rayleigh DAMPING from $xDamp
#  $D = \alpha M + \beta_{curr} K_{curr} + \beta_{kcomm} K_{klastComm} + \beta_{kinit} K_{kinit}$ 
#set xDamp 0.02; # 2% damping ratio
#set lambda [eigen 1]; # eigenvalue mode 1
#set omega [expr pow($lambda,0.5)];
#puts "omega"
#puts $omega
#set period [expr 2*$PI/$omega];
#puts $period
#set stiffness [expr pow($omega,2)*$MASS];
#puts $stiffness
#set alphaM 0.; # M-prop. damping;  $D = \alpha M$ 
#set betaKcurr 0.;
# K-proportional damping;  $\beta_{curr} K_{curr}$ 
#set betaKcomm [expr 2.*$xDamp/($omega)];
# K-prop. damping parameter;  $\beta_{kcomm} K_{klastComm}$ 
#set betaKinit 0.;
# initial-stiffness proportional damping  $\beta_{kinit} K_{kinit}$ 
# define damping
#rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm; # RAYLEIGH damping
# ----- define & apply damping
# RAYLEIGH damping parameters, Where to put M/K-prop damping, switches (http://opensees.berkeley.edu/OpenSees/manuals/usermanual/1099.htm)
#  $D = \alpha M + \beta_{curr} K_{curr} + \beta_{kcomm} K_{klastComm} + \beta_{kinit} K_{kinit}$ 
#set xDamp 0.02; # damping ratio
set MpropSwitch 1.0;
set KcurrSwitch 0.0;
set KcommSwitch 1.0;
set KinitSwitch 0.0;
set nEigenI 1; # mode 1
set nEigenJ 3; # mode 3
set lambdaN [eigen [expr $nEigenJ]]; # eigenvalue analysis for nEigenJ modes
set lambdaI [lindex $lambdaN [expr $nEigenI-1]]; # eigenvalue mode i
set lambdaJ [lindex $lambdaN [expr $nEigenJ-1]]; # eigenvalue mode j
set omegaI [expr pow($lambdaI,0.5)];
set omegaJ [expr pow($lambdaJ,0.5)];
set alphaM [expr $MpropSwitch*$xDamp*(2*$omegaI*$omegaJ)/($omegaI+$omegaJ)];

```

```

# M-prop. damping; D = alpha*M*M
set betaKcurr [expr $KcurrSwitch*2.*$xDamp/($omegaI+$omegaJ)];
# current-K;      +beatKcurr*KCurrent
set betaKcomm [expr $KcommSwitch*2.*$xDamp/($omegaI+$omegaJ)];
# last-committed K;  +betaKcomm*KlastCommitt
set betaKinit [expr $KinitSwitch*2.*$xDamp/($omegaI+$omegaJ)];
# initial-K;      +beatKinit*Kini
set period [expr 2*$PI/$omegaI];
puts $period

# define damping
rayleigh $alphaM $betaKcurr $betaKinit $betaKcomm;
# RAYLEIGH damping

# perform Dynamic Ground-Motion Analysis
# Uniform EXCITATION: acceleration input
#set IDloadTag 400; # load tag

#set AccelSeries "Series -dt $dt -filePath $outFile -factor
$GMfatt"; # time series information
#pattern UniformExcitation $IDloadTag $GMDirection -accel
$AccelSeries ; # create Uniform excitation

pattern UniformExcitation 100 1 -accel 10

set Nsteps [expr int($TmaxAnalysis/$DtAnalysis)];
set ok [analyze $Nsteps $DtAnalysis]; # actually perform analysis;
returns ok=0 if analysis was successful

if {$ok != 0} {      ; # if analysis was not successful.
# change some analysis parameters to achieve convergence
# performance is slower inside this loop
# Time-controlled analysis
set ok 0;
set controlTime [getTime];
while {$controlTime < $TmaxAnalysis && $ok == 0} {
set ok [analyze 1 $DtAnalysis]
set controlTime [getTime]
set ok [analyze 1 $DtAnalysis]
if {$ok != 0} {
puts "Trying Newton with Initial Tangent .."
test NormDispIncr $Tol 1000 0
algorithm Newton -initial
set ok [analyze 1 $DtAnalysis]
test $TestType $Tol $maxNumIter 0
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying Broyden .."
algorithm Broyden 8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
if {$ok != 0} {
puts "Trying NewtonWithLineSearch ..."
algorithm NewtonLineSearch .8
set ok [analyze 1 $DtAnalysis]
algorithm $algorithmType
}
}
}; # end if ok != 0

```



```

wipe;

set counter [expr $counter + 1]

}

```

D.4 Misc. Files

```

# -----
# LibUnits.tcl -- define system of units
# Silvia Mazzoni & Frank McKenna, 2006
#

# define UNITS -----
set in 1.; # define basic units -- output units
set kip 1.; # define basic units -- output units
set sec 1.; # define basic units -- output units
set LunitTXT "inch"; # define basic-unit text for output
set FunitTXT "kip"; # define basic-unit text for output
set TunitTXT "sec"; # define basic-unit text for output
set ft [expr 12.*$in]; # define engineering units
set ksi [expr $kip/pow($in,2)];
set psi [expr $ksi/1000.];
set lbf [expr $psi*$in*$in]; # pounds force
set pcf [expr $lbf/pow($ft,3)]; # pounds per cubic foot
set psf [expr $lbf/pow($ft,2)]; # pounds per square foot
set in2 [expr $in*$in]; # inch^2
set in4 [expr $in*$in*$in*$in]; # inch^4
set cm [expr $in/2.54]; # centimeter, needed for displacement input in
MultipleSupport excitation
set PI [expr 2*asin(1.0)]; # define constants
set g [expr 32.2*$ft/pow($sec,2)]; # gravitational acceleration
set Ubig 1.e10; # a really large number
set Usmall [expr 1/$Ubig]; # a really small number

#GMfact.tcl

#Scaling factors for ground motions

set iGMfact "2.06 1.07 1.85 1.27 2.20 1.54 2.20 1.54 2.60 2.53 1.02 2.02"
#GroundFile {"NORTHR.CNP196" "NORTHR.MUL279" "NORTHR.WIL180"
"NORTHR.R03090" "LOMAP.HDA255" "LOMAP.HCH180" "LOMAP.G03090" "LOMAP.G04090"
"UPERST.B-PTS225" "UPERST.B-PTS315"}

#GMfiles.tcl

#Set up Ground Motions
#Labes for Ground-MotionFiles

set iGroundFile {"NORTHR.CNP196" "NORTHR.MUL279" "NORTHR.WIL180"
"NORTHR.R03090" "LOMAP.HDA255" "LOMAP.HCH180" "LOMAP.G03090" "LOMAP.G04090"
"UPERST.B-PTS225" "UPERST.B-PTS315"}

#####
# ReadSMDFile \${inFilename} \${outFilename} \${dt}
#####
# read gm input format
#
# Written: MHS
# Date: July 2000
#

```

```

# A procedure which parses a ground motion record from the PEER
# strong motion database by finding dt in the record header, then
# echoing data values to the output file.
#
# Formal arguments
# inFilename -- file which contains PEER strong motion record
# outFilename -- file to be written in format G3 can read
# dt -- time step determined from file header
#
# Assumptions
# The header in the PEER record is, e.g., formatted as follows:
# PACIFIC ENGINEERING AND ANALYSIS STRONG-MOTION DATA
# IMPERIAL VALLEY 10/15/79 2319, EL CENTRO ARRAY 6, 230
# ACCELERATION TIME HISTORY IN UNITS OF G
# NPTS= 3930, DT= .00500 SEC

proc ReadSMDFile {inFilename outFilename dt} {
# read gm input format

# Pass dt by reference
upvar $dt DT

# Open the input file and catch the error if it can't be read
if [catch {open $inFilename r} inFileID] {
puts stderr "Cannot open $inFilename for reading"
} else {
# Open output file for writing
set outFileID [open $outFilename w]

# Flag indicating dt is found and that ground motion
# values should be read -- ASSUMES dt is on last line
# of header!!!
set flag 0

# Look at each line in the file
foreach line [split [read $inFileID] \n] {

if {[length $line] == 0} {
# Blank line --> do nothing
continue
} elseif {$flag == 1} {
# Echo ground motion values to output file
puts $outFileID $line
} else {
# Search header lines for dt
foreach word [split $line] {
# Read in the time step
if {$flag == 1} {
set DT $word
puts $DT
break
}
# Find the desired token and set the flag
if {[string match $word "DT="] == 1} {set flag 1}
}
}

# Close the output file
close $outFileID

```

```

    # Close the input file
    close $inFileID
}
};

#####

proc DisplayPlane {ShapeType dAmp viewPlane {nEigen 0} {quadrant 0}} {
#####
## DisplayPlane $ShapeType $dAmp $viewPlane $nEigen $quadrant
#####
## setup display parameters for specified viewPlane and display
## Silvia Mazzoni & Frank McKenna, 2006
##
## ShapeType : type of shape to display. # options: ModeShape ,
NodeNumbers , DeformedShape
## dAmp : relative amplification factor for deformations
## viewPlane : set local xy axes in global coordinates (XY,YX,XZ,ZX,YZ,ZY)
## nEigen : if nEigen not=0, show mode shape for nEigen eigenvalue
## quadrant: quadrant where to show this figure (0=full figure)
##
#####

set Xmin [lindex [nodeBounds] 0];
# view bounds in global coords - will add padding on the sides
set Ymin [lindex [nodeBounds] 1];
set Zmin [lindex [nodeBounds] 2];
set Xmax [lindex [nodeBounds] 3];
set Ymax [lindex [nodeBounds] 4];
set Zmax [lindex [nodeBounds] 5];

set Xo 0; # center of local viewing system
set Yo 0;
set Zo 0;

set uLocal [string index $viewPlane 0];
# viewPlane local-x axis in
global coordinates
set vLocal [string index $viewPlane 1];
# viewPlane local-y axis in
global coordinates

if {\ $viewPlane == "3D" } {
set uMin $Zmin+$Xmin
set uMax $Zmax+$Xmax
set vMin $Ymin
set vMax $Ymax
set wMin -10000
set wMax 10000
vup 0 1 0; # dirn defining up direction of view plane
} else {
set keyAxisMin "X $Xmin Y $Ymin Z $Zmin"
set keyAxisMax "X $Xmax Y $Ymax Z $Zmax"
set axisU [string index $viewPlane 0];
set axisV [string index $viewPlane 1];
set uMin [string map $keyAxisMin $axisU]
set uMax [string map $keyAxisMax $axisU]
set vMin [string map $keyAxisMin $axisV]
set vMax [string map $keyAxisMax $axisV]
if {\ $viewPlane == "YZ" || $viewPlane == "ZY" } {

```

```

set wMin $Xmin
set wMax $Xmax
} elseif {$viewPlane == "XY" || $viewPlane == "YX" } {
set wMin $Zmin
set wMax $Zmax
} elseif {$viewPlane == "XZ" || $viewPlane == "ZX" } {
set wMin $Ymin
set wMax $Ymax
} else {
return -1
}
}

set epsilon 1e-3;
# make windows width or height not zero when the Max
and Min values of a coordinate are the same

set uWide [expr $uMax - $uMin+$epsilon];
set vWide [expr $vMax - $vMin+$epsilon];
set uSide [expr 0.25*$uWide];
set vSide [expr 0.25*$vWide];
set uMin [expr $uMin - $uSide];
set uMax [expr $uMax + $uSide];
set vMin [expr $vMin - $vSide];
set vMax [expr $vMax + 2*$vSide];
# pad a little more on top, because of
window title
set uWide [expr $uMax - $uMin+$epsilon];
set vWide [expr $vMax - $vMin+$epsilon];
set uMid [expr ($uMin+$uMax)/2];
set vMid [expr ($vMin+$vMax)/2];

# keep the following general, as change the X and Y and Z
# for each view planenext three commands define viewing
# system, all values in global coords
vrp $Xo $Yo $Zo;
# point on the view plane in global coord, center of local
viewing system
if {$vLocal == "X"} {
vup 1 0 0; # dirn defining up direction of view plane
} elseif {$vLocal == "Y"} {
vup 0 1 0; # dirn defining up direction of view plane
} elseif {$vLocal == "Z"} {
vup 0 0 1; # dirn defining up direction of view plane
}
if {$viewPlane == "YZ" } {
vpn 1 0 0; # direction of outward normal to view plane
prp 10000. $uMid $vMid ;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "ZY" } {
vpn -1 0 0; # direction of outward normal to view plane
prp -10000. $vMid $uMid ;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "XY" } {
vpn 0 0 1; # direction of outward normal to view plane
prp $uMid $vMid 10000;
# eye location in local coord sys defined by viewing system

```

```

plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "YX" } {
vpn 0 0 -1; # direction of outward normal to view plane
prp $uMid $vMid -10000;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "XZ" } {
vpn 0 -1 0; # direction of outward normal to view plane
prp $uMid -10000 $vMid ;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "ZX" } {
vpn 0 1 0; # direction of outward normal to view plane
prp $uMid 10000 $vMid ;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} elseif {$viewPlane == "3D" } {
vpn 1 0.25 1.25;
# direction of outward normal to view plane
prp -100 $vMid 10000;
# eye location in local coord sys defined by viewing system
plane 10000 -10000;
# distance to front and back clipping planes from eye
} else {
return -1
}
}
# next three commands define view, all values in local coord system
if {$viewPlane == "3D" } {
viewWindow [expr $uMin-$uWide/4] [expr $uMax/2] [expr $vMin-0.25*$vWide]
[expr $vMax]
} else {
viewWindow $uMin $uMax $vMin $vMax
}
projection 1; # projection mode, 0:perspective, 1: parallel
fill 1; # fill mode; needed only for solid elements

if {$quadrant == 0} {
port -1 1 -1 1 # area of window that will be drawn into
(uMin,uMax,vMin,vMax);
} elseif {$quadrant == 1} {
port 0 1 0 1 # area of window that will be drawn into
(uMin,uMax,vMin,vMax);
} elseif {$quadrant == 2} {
port -1 0 0 1 # area of window that will be drawn into
(uMin,uMax,vMin,vMax);
} elseif {$quadrant == 3} {
port -1 0 -1 0 # area of window that will be drawn into
(uMin,uMax,vMin,vMax);
} elseif {$quadrant == 4} {
port 0 1 -1 0 # area of window that will be drawn into
(uMin,uMax,vMin,vMax);
}
}

if {$ShapeType == "ModeShape" } {
display -$nEigen 0 [expr 5.*$dAmp]; # display mode shape for mode
$nEigen
} elseif {$ShapeType == "NodeNumbers" } {

```

```

display 1 -1 0 ; # display node numbers
} elseif {$ShapeType == "DeformedShape" } {
display 1 2 $dAmp; # display deformed shape the 2 makes the nodes
small
}
};
#####

proc DisplayModel2D { {ShapeType nill} {dAmp 5} {xLoc 10} {yLoc 10}
{xPixels 512} {yPixels 384} {nEigen 1} } {
#####
## DisplayModel2D $ShapeType $dAmp $xLoc $yLoc $xPixels $yPixels $nEigen
#####
## display Node Numbers, Deformed or Mode Shape in 2D problem
## Silvia Mazzoni & Frank McKenna, 2006
##
## ShapeType : type of shape to display. # options: ModeShape ,
NodeNumbers , DeformedShape
## dAmp : relative amplification factor for deformations
## xLoc,yLoc : horizontal & vertical location in pixels of graphical
window (0,0=upper left-most corner)
## xPixels,yPixels : width & height of graphical window in pixels
## nEigen : if nEigen not=0, show mode shape for nEigen eigenvalue
##
#####
global TunitTXT; # load time-unit text
global ScreenResolutionX ScreenResolutionY; # read global values for
screen resolution

if { [info exists TunitTXT] != 1} {set TunitTXT ""};
# set blank if it has not been defined previously.

if { [info exists ScreenResolutionX] != 1} {set ScreenResolutionX 1024};
# set default if it has not been defined previously. 1024
if { [info exists ScreenResolutionY] != 1} {set ScreenResolutionY 768}
; # set default if it has not been defined previously. 768

if {$xPixels == 0} {
set xPixels [expr int($ScreenResolutionX/2)];
set yPixels [expr int($ScreenResolutionY/2)]
set xLoc 10
set yLoc 10
}
if {$ShapeType == "nill"} {
puts ""; puts ""; puts "-----"
puts "View the Model? (N)odes, (D)eformedShape, anyMode(1),(2),(#).
Press enter for NO."
gets stdin answer
if {[length $answer]>0} {
if {$answer != "N" & $answer != "n"} {
puts "Modify View Scaling Factor=$dAmp? Type factor, or press
enter for NO."
gets stdin answerdAmp
if {[length $answerdAmp]>0} {
set dAmp $answerdAmp
}
}
if {[string index $answer 0] == "N" || [string index $answer 0]
== "n"} {
set ShapeType NodeNumbers
} elseif {[string index $answer 0] == "D" || [string index $answer 0]

```

```

== "d" } {
set ShapeType DeformedShape
} else {
set ShapeType ModeShape
set nEigen $answer
}
} else {
return
}
}

if {$ShapeType == "ModeShape" } {
set lambdaN [eigen $nEigen]; # perform eigenvalue analysis for
ModeShape
set lambda [lindex $lambdaN [expr $nEigen-1]];
set omega [expr pow($lambda,0.5)]
set PI [expr 2*asin(1.0)]; # define constant
set Tperiod [expr 2*$PI/$omega]; # period (sec.)
set fmt1 "Mode Shape, Mode=%.1i Period=%.3f %s "
set windowTitle [format $fmt1 $nEigen $Tperiod $TunitTXT]
} elseif {$ShapeType == "NodeNumbers" } {
set windowTitle "Node Numbers"
} elseif {$ShapeType == "DeformedShape" } {
set windowTitle "Deformed Shape"
}

set viewPlane XY
recorder display $windowTitle $xLoc $yLoc $xPixels $yPixels -wipe ;
# display recorder
DisplayPlane $ShapeType $dAmp $viewPlane $nEigen 0
}

```