



2017-10-01

Considering Manufacturing in the Design of Thick-Panel Origami Mechanisms

Erica Brunson Crampton
Brigham Young University

Follow this and additional works at: <https://scholarsarchive.byu.edu/etd>

 Part of the [Mechanical Engineering Commons](#)

BYU ScholarsArchive Citation

Crampton, Erica Brunson, "Considering Manufacturing in the Design of Thick-Panel Origami Mechanisms" (2017). *All Theses and Dissertations*. 6567.

<https://scholarsarchive.byu.edu/etd/6567>

This Thesis is brought to you for free and open access by BYU ScholarsArchive. It has been accepted for inclusion in All Theses and Dissertations by an authorized administrator of BYU ScholarsArchive. For more information, please contact scholarsarchive@byu.edu, ellen_amatangelo@byu.edu.

Considering Manufacturing in the Design of Thick-Panel Origami Mechanisms

Erica Brunson Crampton

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of
Master of Science

Spencer P. Magleby, Chair
Larry L. Howell
Andrew George

Department of Mechanical Engineering
Brigham Young University

Copyright © 2017 Erica Brunson Crampton
All Rights Reserved

ABSTRACT

Considering Manufacturing in the Design of Thick-Panel Origami Mechanisms

Erica Brunson Crampton
Department of Mechanical Engineering, BYU
Master of Science

Origami has been investigated and demonstrated for engineering applications in recent years. Many techniques for accommodating the thickness of most engineering materials have been developed. In this work, tables comparing performance and manufacturing characteristics are presented. These tables can serve as useful design tools for engineers when selecting an appropriate thickness-accommodation technique for their application. The use of bent sheet metal for panels in thick-origami mechanisms shows promise as a panel design approach that mitigates several trade-offs between performance and manufacturing characteristics. A process is described and demonstrated that can be employed to use sheet metal in designs of origami-adapted mechanisms that utilize specific thickness-accommodation techniques. Data structures based on origami can be useful in the automation of thick-origami mechanism design. The use of such data structures is explained and shown in the context of a program that will automatically create the 3D CAD models and assembly of a thick-origami mechanism using the tapered panels technique based on the input origami crease pattern. Manufacturability in the design of origami-adapted mechanisms is discussed through presenting and examining three examples of origami-adapted mechanisms. As the manufacturability of origami-adapted products is addressed and improved, their robustness will also improve, thereby enabling greater use of origami-adapted design.

Keywords: origami, thick origami, origami-adapted design, sheet metal origami, origami design automation, manufacturing

ACKNOWLEDGMENTS

I would like to thank my committee—Dr. Spencer Magleby, Dr. Larry Howell, and Dr. Andy George—without whom this would not be possible. I express special thanks to Dr. Magleby for his guidance and mentoring in this research.

I would also like to acknowledge and thank my peers that have contributed to the work presented in this thesis. I thank Ariana Sellers for all of her work that went into our paper that is Chapter 4 of this thesis and Dr. John Salmon for his role in the paper, as well. Thank you to Kyler Tolman for his efforts and contributions to the review paper and other projects we worked on together. I also acknowledge and thank Sam Smith for his part in prototyping the sheet metal origami mechanisms that are presented here. Thank you to all those who contributed to the origami tooling and ballistic barrier projects which are described in part in Sections 5.2.1 and 5.2.2. And, of course, a thank you to the community of students and faculty that are the Compliant Mechanisms Research Group.

I also thank my family for their continued love and support. I express my greatest appreciation to my husband for his unending love, support, and sacrifice throughout my graduate school experience and without whom I would not be the person I am today.

The work presented in this thesis is based on research supported by funding from the National Science Foundation and the Air Force Office of Scientific Research under Grant No. EFRI-ODISSEI-1240417.

TABLE OF CONTENTS

LIST OF TABLES	vi
LIST OF FIGURES	vii
Chapter 1 Introduction	1
Chapter 2 A Comparison and Discussion of Thickness-Accommodation Techniques for Origami	5
2.1 Comparison and Discussion of Techniques	5
2.2 Manufacturing Considerations	8
Chapter 3 Realizing Origami Mechanisms From Metal Sheets	12
3.1 Introduction	12
3.2 Background	14
3.3 Application of Selected Techniques in Sheet Metal	16
3.3.1 Tapered Panels Technique	17
3.3.2 Hinge Shift Technique	18
3.3.3 Offset Panel Technique	20
3.4 Implementation	21
3.4.1 Process Illustration	21
3.4.2 Hardware	24
3.5 Analysis and Comparison	24
3.6 Conclusion	28
Chapter 4 Automating the Design of Thick-Origami Mechanisms	30
4.1 Introduction	30
4.2 Background	32
4.3 Crease Pattern Analysis	34
4.3.1 Lines	37
4.3.2 Vertices	37
4.3.3 Panels	38
4.4 Implementation	39
4.4.1 Program Workflow	39
4.4.2 Results/Testing	40
4.5 Discussion	44
4.5.1 Advantages	44
4.5.2 Limitations	45
4.6 Conclusion	46
Chapter 5 Considering Manufacturability in the Design of Deployable Origami-Adapted Mechanisms	47
5.1 Introduction	47

5.2	Examples	50
5.2.1	Manufacturing Process for Fabric-based Origami-adapted Products	50
5.2.2	Ballistic Barrier	53
5.2.3	Sheet Metal Panels	56
5.3	Manufacturability Lessons	60
5.4	Conclusion	61
Chapter 6	Conclusion	62
6.1	Conclusions	62
6.2	Future Work	63
REFERENCES	64

LIST OF TABLES

2.1	Comparison of thickness-accommodation techniques.	6
2.2	A comparison of manufacturing approaches for origami-inspired mechanisms. In the second column, dashed lines indicate layer or segment divisions within the panels and small circles represent stock hinges.	9
3.1	A comparison of panel approaches for origami-adapted mechanisms. In the second column, dashed lines indicate layer or segment divisions within the panels and small circles represent stock hinges.	27
4.1	Overview of steps the code takes to extract and analyze the required data. The location of each step in the code is also noted as either the main code (not in a separate class), the Vertex class, or the Panel class.	40
5.1	A comparison of panel approaches for two thickness-accommodation techniques for origami-adapted mechanisms. In the second column, dashed lines indicate layer divisions within the panels and small circles represent stock hinges. <i>Part count</i> indicates the relative part count of the mechanism where “baseline” is when the product has as many parts as the number of facets and creases in the paper origami model. If an approach is <i>conducive to sheet stock</i> , then materials in sheet form, such as sheet metals, can be easily used for the panels. The <i>second panel process</i> column lists any process in addition to a simple 2-D stock cutting process that is necessary to fabricate the panels. <i>Minimum number of processes</i> indicates the fewest number of distinct processes, including assembly, required to manufacture a mechanism using the given approach. (This table is a subset of what is shown in Table 3.1.)	58

LIST OF FIGURES

1.1	From left to right: tapered panels, hinge shift, offset panel, SORCE, and strained joint thickness-accommodation techniques.	2
3.1	From left to right: tapered panels, hinge shift, and offset panel thickness-accommodation techniques.	15
3.2	Cross-section of a bent panel for a simple pleat tapered panels mechanism relative to its zero thickness plane (shown in red) and “thick” counterpart.	17
3.3	Thick panel from a Miura-ori pattern and how the bent panel compares.	18
3.4	Tapered panels mechanism of a Miura-ori unit cell shown with all thick panels (left) and one of the thick panels substituted for a bent panel (right).	18
3.5	Miura-ori tapered panels mechanism implemented in sheet metal shown moving from flat to folded.	19
3.6	Square-twist hinge shift mechanism implemented in sheet metal shown moving from flat to folded.	19
3.7	Sheet metal square-twist mechanism designed using the offset panel technique shown in flat, folded, and partially folded states.	20
3.8	Thick-origami square-twist mechanism designed using the hinge shift technique shown moving from flat to folded.	22
3.9	Application of sheet metal to an edge panel of a square-twist mechanism. (a) A thick panel in context of the mechanism; (b) thick panel; (c) bent panel. Red lines signify hinge axes.	22
3.10	Application of sheet metal to the central square panel of a square-twist mechanism. (a) A thick panel in context of the mechanism; (b) thick panel; (c) bent panel. Red lines signify hinge axes.	23
3.11	Sheet metal square-twist mechanism designed using the hinge shift technique shown unfolded, partially folded, and fully folded.	25
3.12	Sheet metal Miura-ori mechanism designed using the tapered panels technique shown unfolded, partially folded, and fully folded.	26
4.1	Tapered panels technique.	33
4.2	Diagram detailing the length of a panel’s taper.	34
4.3	Example of a crease pattern sketch in Siemens NX.	35
4.4	An example crease pattern containing two vertices.	35
4.5	A degree-4 origami vertex shown flat (left) and partially folded (right).	36
4.6	The data structures used for analyzing crease patterns and the information contained in each.	36
4.7	Example of a thick-origami model of a square-twist crease pattern.	41
4.8	Example of a thick-origami model of a square-twist tessellation crease pattern.	42
4.9	A folded model of the thick-origami square-twist tessellation shown in Figure 4.8.	42
4.10	Example of a thick-origami model of a Miura-ori crease pattern.	42
4.11	Example of a thick-origami model of a hexagonal-twist crease pattern.	43
4.12	Example of a thick-origami model of a generic degree-4 crease pattern.	43
4.13	A folded model of the thick-origami mechanism shown in Figure 4.12.	44

5.1	Origami-adapted mechanism shown flat, partially folded, and folded.	49
5.2	Two identical mold halves used for creating a Miura-ori pattern using the double mold process shown open and together.	52
5.3	A sample of fabric that has been heat set using the double mold process with the mold shown in Fig. 5.2.	52
5.4	A reusable shopping bag that has been heat-set using the double mold process (including the second heat-set step) that can be easily rolled up after collapsing the origami pattern for simple storage.	53
5.5	Origami-inspired ballistic barrier in the deployed state.	54
5.6	Modified Yoshimura pattern used for design of an origami-inspired ballistic barrier. . .	55
5.7	Sheet metal square-twist mechanism designed using the hinge shift technique shown unfolded, partially folded, and fully folded.	57
5.8	Sheet metal Miura-ori mechanism designed using the tapered panel technique shown unfolded, partially folded, and fully folded.	59

CHAPTER 1. INTRODUCTION

The ancient art of origami can create extraordinary works of art from simple paper and folding. Origami possesses characteristics that are desirable for many engineering applications [1]. Many origami patterns are very compact in their folded state yet can be deployed to a much larger size. The characteristic that origami is folded from a single piece of paper is also desirable for simplifying shape and assembly. The impressively complex motion achieved by many origami patterns also provides a source of new motions not likely to be found through other avenues.

Adapting origami to be used with engineering materials such as plastics and metals, however, poses many challenges. These challenges are a result of the disparities between paper and most other engineering materials [2]. One disparity is that paper is “paper-thin” and allows for origami to be modeled as having approximately zero thickness. On the other hand, materials used in many engineering applications are significantly thicker and encounter issues of self-intersection in even a single-vertex origami pattern. Another key difference is that paper creases when folded to give hinge-like motion, whereas non-paper-like materials do not experience such a decrease in stiffness, thereby requiring another means to accomplish the folding hinge motion. These differences between paper and most engineering materials must be overcome to enable application of origami in engineering materials.

Much research has been done and several techniques developed to overcome these differences and provide ways to apply origami in non-paper-like materials [3]. These thickness-accommodation techniques vary in their approaches; some modify the panel geometry, and some modify the crease pattern as a step in accommodating material thickness. Figure 1.1 illustrates the general approach for applying several of these techniques to an origami pattern. The tapered panels technique [4] accommodates thickness in origami mechanisms by maintaining the original zero-thickness model within the thick panels and trimming away panel material around the hinges to avoid self-intersection. The offset panel technique [5] also preserves the original

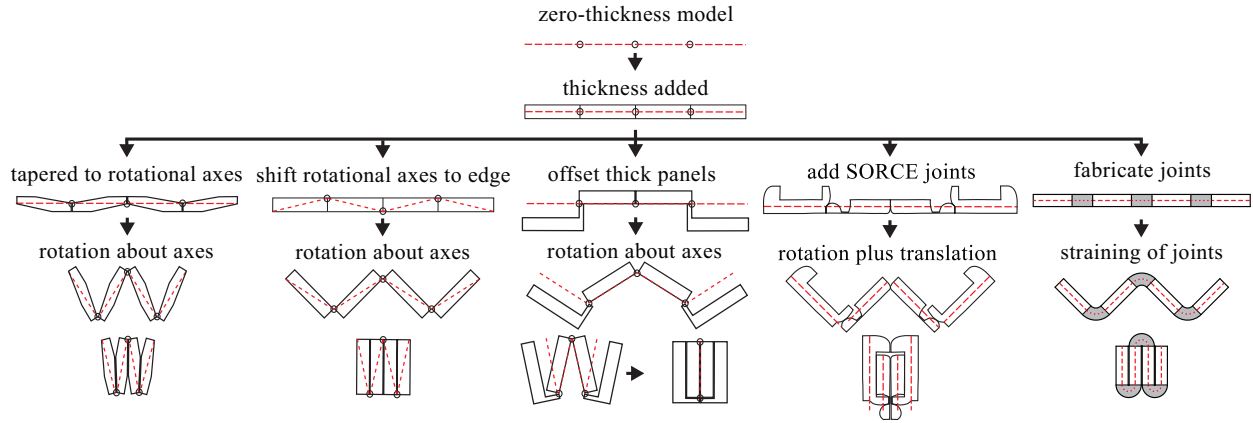


Figure 1.1: From left to right: tapered panels, hinge shift, offset panel, SORCE, and strained joint thickness-accommodation techniques.

zero-thickness origami model within the thick origami mechanism, but instead shifts the panels away from the zero-thickness plane using offsets in order to accommodate panel thickness. The hinge shift technique [6] shifts the rotational axes from the center planes of the thick panels to the outer edges to form a spatial mechanism at each vertex that allows for thick panels. To apply the Synchronized-Offset Rolling-Contact Elements (SORCE) technique [7], cam surfaces are designed for each crease so that the motion of the zero-thickness model is preserved through the combination of rotation and translation that occurs in the thick origami mechanism. The strained joint technique [8] allows for “folding” of thick material by using arrays of lamina-emergent torsional joints to introduce flexible joint areas in the thick material where “folding” can occur.

Even with the work that has been done in techniques to accommodate for the disparities between paper and other engineering materials, barriers still remain that hinder more widespread use of origami-adapted design—design that adapts origami to non-paper-like materials [9]. One barrier is that the designer must have an adequate knowledge of origami and understand the mathematics and kinematics that govern its motion. Knowledge of origami crease patterns and how to deal with the overconstrained nature of many origami mechanisms is also helpful in origami-adapted design, though not commonplace among engineers. Another common shortage among most engineers is an understanding of origami thickness-accommodation techniques as well as the amount of time and effort required to create 3D CAD models of even relatively simple thick-origami mechanisms.

There are also barriers to use of origami-adapted design that are related to manufacturing. Several design decisions must be made in the transition from a thick-origami model to a fully manufacturable design. Very little research has been done to bridge the gap between the thick-origami model and manufacturing. The designer faces challenges and design decisions such as how to address the manufacturing of such products in the design process, what are feasible ways to design the panels, and what fabrication approaches are best to achieve a given set of desired characteristics.

The objective of this research is to mitigate some of these barriers to more widespread use of origami-adapted design. The research presented in this thesis specifically works to allay barriers related to manufacturing through the use of sheet metal and ways to address manufacturing during design in addition to bridging the gap of origami and thickness-accommodation knowledge needed for origami-adapted design. As barriers are made to be less of a hindrance, origami-adapted design will become more accessible to a greater number of engineers and designers.

Chapter 2 gives a comparison of thickness-accommodation techniques for origami-adapted design. It is split into two primary sections; one comparing the performance characteristics of the techniques, and another comparing the fabrication approaches associated with the techniques. The tables presented in the chapter are useful design tools in selecting appropriate thickness-accommodation techniques for specific designs. The content for the chapter is drawn from a review paper that has been accepted for publication [3].

The use of sheet metal in conjunction with bending processes can mitigate many trade-offs that exist between manufacturing and performance characteristics of some origami thickness-accommodation techniques. Chapter 3 shows that the use of sheet metal in origami is possible and even useful. A process for applying sheet metal in the design of panels is described and demonstrated. The chapter has also been published as a conference paper [10].

One way to mitigate the lack of familiarity and knowledge of origami thickness-accommodation that hinders the use of origami-adapted design is to automate the process of accommodating for thickness in the design of a thick-origami mechanism. In Chapter 4, origami-based data structures are presented that can be used in the automation of thick-origami mechanism design. An example of how these data structures have been implemented is also explained.

Chapter 5 reviews three examples of origami-adapted design and focuses on the role of manufacturability in the designs. Several lessons on manufacturability of origami-adapted designs are identified for use and consideration in future designs. The chapter will be published in the proceedings of the symposium where it was presented [11].

CHAPTER 2. A COMPARISON AND DISCUSSION OF THICKNESS-ACCOMMODATION TECHNIQUES FOR ORIGAMI

This chapter compares and discusses the performance and manufacturing characteristics of many thickness-accommodation techniques. The techniques considered include the tapered panels [4], offset panel [5], hinge shift [6], doubled hinge [12], rolling contacts [7], membrane [13], and strained joint [8] techniques. These techniques are reviewed in detail in the preceding sections of the review paper that has been accepted for publication [3] from which this chapter is excerpted. The tables found in this chapter can serve as useful design tools for engineers when selecting an appropriate thickness-accommodation technique for their application.

2.1 Comparison and Discussion of Techniques

Each of the thickness-accommodation techniques that have been discussed here have their own advantages and disadvantages. For example, the strained joint technique has the advantages of maintaining a monolithic structure similar to paper origami, but loses its rigid single degree of freedom movement due to the flexibility of the joints. The offset panel technique, on the other hand, can maintain a rigid single degree of freedom movement but is unable to fold out to a flat planar state.

In this section, the strengths and weaknesses of each thickness-accommodation technique are summarized. We also discuss how the unique characteristics of each technique make them suitable for different applications. For this discussion and comparison, the specific implementations (where applicable) of techniques referred to are Chen et al.'s generalized implementation of the hinge shift technique, the offset crease implementation of the doubled hinge technique, and the SORCE (Synchronized-Offset Rolling-Contact Elements) implementation of the rolling contacts technique. In comparing each of the thickness-accommodation techniques that have been reviewed in Sections 3–9 of [3], we have chosen several characteristics that are of interest to designers and

Table 2.1: Comparison of thickness-accommodation techniques.

Technique	Equivalent kinematics	Preserves motion	Full ROM	Flat surface	Arbitrary patterns	Design complexity
Tapered Panels	Yes	Yes	No	No	Yes	Low
Offset Panel	Yes	Yes	Yes	No	Yes	Low
Hinge Shift	No	Yes	Yes	No*	No	Low/Med
Doubled Hinge	No	No	Yes	Yes	Yes	Med
Rolling Contacts	No	Yes	Yes	No*	Yes	High
Membrane	No	No	Yes	Yes	Yes	Low
Strained Joint	No	No	Yes	Yes	Yes	Med/High

* – except for special cases

that vary with each technique. These characteristics are listed in Table 2.1 and the definition of each of these metrics is as follows:

Equivalent kinematics: indicates if the technique is kinematically equivalent to the zero-thickness origami base model. This means that the mechanism must still contain the same spherical linkages that exist in the zero-thickness model. Therefore the same kinematic model used for the zero-thickness model can also be used to predict the motion, including the position and orientation, of the thick origami.

Preserves motion: indicates if the thick origami preserves the dihedral angles and degrees of freedom that are exhibited in the zero-thickness model without requiring additional constraints. When a thick origami mechanism exhibits these characteristics of the zero-thickness model, its motion is also the same. Some techniques preserve the dihedral angles by maintaining the original zero-thickness surface in the thick origami model, whereas other methods are able to preserve the motion through means such as offset functions and spatial linkages. It is also worth noting that techniques that do not always preserve the motion may be made to do so in specific configurations or through the addition of constraints that are designed to reduce the degrees of freedom to match the original zero-thickness model.

Full ROM: indicates if the technique preserves the range of motion (ROM) of the original zero-thickness origami model. This means that any thick origami based on a flat-foldable origami pattern would be able to start and end with the panel faces parallel to each other.

Flat surface: indicates if the thick origami mechanism has a flat surface in its unfolded state. Here we will use a working definition that a thick origami mechanism has a *flat surface* if

it can rest firmly on a flat face, such as a table top, in its unfolded state, with all panels in areal contact with the surface. This means that the large majority, if not all, of the lower panel faces must be coplanar (i.e., would rest on the “table top”) in the unfolded state, and so there cannot be any offsets protruding on one “face” of the mechanism or any significant variation in the distance of the panel face from the “table top”.

Arbitrary patterns: indicates whether the technique can be applied to any arbitrary origami crease pattern. Techniques that have this characteristic do not have limiting geometrical properties that make some patterns not possible for other techniques. Techniques lacking this property are only applicable to very specific fold patterns or particular geometries.

Design complexity: indicates the relative difficulty of synthesizing a thick origami mechanism based on a zero-thickness origami model using a specific thickness-accommodation technique. A technique with a “low” design complexity rating may require the use of a simple equation or two, if any at all, to develop a working mechanism model. A “high” design complexity rating, however, means that extensive computation and/or optimization is involved in the mechanism design process.

The importance of each of these characteristics depends on the application. For example, the ability to have a flat unfolded state may be important in the application of a folding table but may not be as important in the application of a folding tool box. A solar array would be insensitive to small offsets perpendicular to the panels, but a reflective antenna would be highly sensitive to the same. The single degree of freedom motion that comes with preserving the dihedral angles is desirable in most cases, however the ability to have multiple configurations, which often accompanies multiple degree of freedom mechanisms, may be desired in certain situations. So, for example, while the hinge shift technique is not applicable to arbitrary crease patterns, if the pattern to be used in the application maintains the conditions listed in [6], hinge shift can create a very rigid single degree of freedom mechanism.

One note of potential importance is that, even though most of the techniques are applicable to an arbitrary crease pattern, some techniques do not scale as well as others to varying panel thickness-to-size ratios. One example of this is the strained joint technique; if the pattern is scaled down to be smaller while using the same material for the mechanism, the joint areas must increase

in width because of their decreased length. Such scaling leads to progressively smaller panel areas until the areas from two joints meet to swallow-up the panel and there is no discernible pattern.

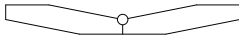
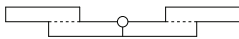
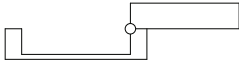
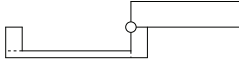
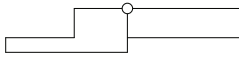
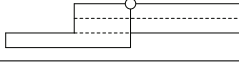
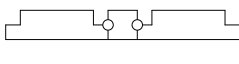
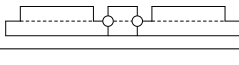



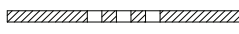
Another characteristic that may be of concern for some applications is whether or not the mechanism requires holes in the unfolded state. A mechanism designed using the offset crease technique, among others, will always have holes at interior vertices when in the unfolded configuration. The offset panel technique is also likely to have holes; however, these holes occur in the panels rather than at the vertices to allow all the hinges to penetrate to the zero-thickness surface. The strained joint technique also inherently leads to mechanisms with holes at interior vertices.

2.2 Manufacturing Considerations

For origami-inspired design to be widely used and practiced, how such products are to be manufactured must be considered. The thickness-accommodation technique used in the design of an origami-inspired product has a significant influence on the manufacturing approaches and costs of the product. Though all thickness-accommodation techniques make manufacturing more difficult than folding a piece of paper, some techniques make the transition from paper folding to manufacturing more challenging than others [12, 14]. The chosen approach determines not only the part count and what manufacturing processes are required, but also whether or not materials in sheet stock form can be easily used.

Several of the thickness-accommodation techniques previously discussed have been put into practice in more than one way. For example, Tachi has presented the tapered panels technique implemented in two ways: as panels composed of a single tapered piece, and as panels composed of two constant thickness pieces joined together [4], such as seen in the first rows of Table 2.2. Though these variations in how the mechanism is built do not affect the ideal performance and motion of the mechanism using the specific technique, they do have a significant impact on how the mechanism can be manufactured. In order to systematically consider the impact on manufacturing of each technique, the techniques have been divided (where applicable) into manufacturing approaches that have been suggested or employed before. A monolithic panel approach is likely to require processes to remove material to reach the final panel shape, whereas a layered or segmented approach would require additional assembly.

Table 2.2: A comparison of manufacturing approaches for origami-inspired mechanisms. In the second column, dashed lines indicate layer or segment divisions within the panels and small circles represent stock hinges.

Technique panel approach	Schematic representation	Part count	Conducive to sheet stock	Second panel process required	Minimum number of processes
Tapered Panels					
monolithic		Baseline	No	Yes	3
layered		High	Yes	Yes	2
Offset Panel					
monolithic		Baseline	No	Yes	3
segmented		High/Very High	Yes	Yes	2
Hinge Shift					
monolithic		Baseline	No	Yes	3
layered		High/Very High	Yes	Yes	2
Doubled Hinge					
monolithic		High	No	Yes	3
layered		Very High	Yes	Yes	2
Rolling Contacts					
integrated joints		Low	No	Yes	3
separate joints		Baseline	Yes	No*	3
Membrane					
		Low/Baseline	Yes	No	2
Strained Joint					
		Low	Yes	No	1

* – the SORCE joints themselves must be fabricated because stock hinges cannot be used

Table 2.2 lists thickness-accommodation techniques and several manufacturing characteristics relating to how these techniques are typically considered for manufacturing. The second column of Table 2.2 illustrates the manufacturing approaches for each technique. The characteristics relating to manufacturing shown as the other table column headings are described as follows:

Part count: indicates the relative part count of the origami-inspired mechanism. A part count is considered “baseline” when the product has as many parts as the number of facets and creases in the paper origami model. A part count may be considered “high” when it is roughly

twice the baseline count. The part count of a product is often an indicator of its complexity because more parts corresponds to a higher potential for problematic tolerance stack-ups that lead to poor product performance.

Conducive to sheet stock: indicates whether or not the manufacturing approach is conducive to the use of materials in sheet stock form for rigid panels without significant preprocessing. For the purposes of this discussion, a technique being conducive to sheet stock means that no three-dimensional process, such as milling, is necessary to fabricate any component of the mechanism. Therefore, using a thickness-accommodation technique that is conducive to sheet stock leads to simpler manufacturing processes and potentially lower production costs overall.

Second panel process required: indicates whether the fabrication of the panels using the specified approach would require a second process after initially cutting the nominal panel shape from the stock material. Possible second panel processes include milling and other material removal processes, joining, and assembly of each panel before assembly of the mechanism as a whole. This is based on an assumption that the initial process of cutting from stock material would use a machine with no more than two degrees of freedom, such as a saw or most lasers and waterjets. As these second panel processes usually require jigs and/or fixtures and significant setup/processing time, there may be a significant increase in production cost for using thickness-accommodation techniques requiring such processes.

Minimum number of processes: indicates the fewest number of processes required to fabricate a product using the specified manufacturing approach. Three possible processes were assumed: cutting panel shapes from stock material (1- or 2-dimensional process), material removal to achieve final 3-dimensional panel shape (3-dimensional process), and assembly of the mechanism (including any applicable panel assembly). Not all manufacturing approaches require all three processes. For example, the layered hinge shift approach requires two processes: one to cut the panel components to size from stock and one to assemble the panels and mechanism. The number of processes required to manufacture a product is important as it is an indicator as to how much labor and equipment may be needed.

When choosing a thickness-accommodation technique during product development, both performance and manufacturing should be considered. There are trade-offs involved with each technique and manufacturing approach. For example, the strained joint technique is generally

highly desirable from a manufacturing standpoint, but does not have the motion characteristics of preserving the dihedral angles and a single DOF. In comparison, a design that employs the SORCE rolling contacts technique would exhibit such motion characteristics, but does not have such favorable manufacturing characteristics because, in addition to requiring forming of the rolling contact surfaces, also requires tight tolerances for the surfaces, thereby furthering the increase in manufacturing cost.

One potential method of tailoring the design of an origami-inspired product to meet both the motion and manufacturing requirements is to utilize hybrid techniques. Techniques with particular motion characteristics can be implemented at some vertices in a pattern and techniques with favorable manufacturing characteristics implemented at others. By doing so, designers can pick and choose which techniques to use at various points in the pattern to reduce the overall manufacturing cost of the design while still retaining specific motion performance at critical vertices in the pattern.

CHAPTER 3. REALIZING ORIGAMI MECHANISMS FROM METAL SHEETS

Consideration of a product's manufacturability is a vital aspect of product design. When considering manufacturability of panels for origami-adapted products, there are trade-offs between panel design approaches as well as thickness-accommodation techniques. The use of bent sheet metal for panels shows promise as a panel design approach that mitigates several of these trade-offs. This chapter describes a process that can be employed to use sheet metal in designs of origami-adapted mechanisms that utilize specific thickness-accommodation techniques. The process is demonstrated for a square-twist mechanism designed using the hinge shift technique for accommodating thickness in origami patterns. A Miura-ori mechanism is also shown in sheet metal. The characteristics of these bent panel approaches are discussed and compared to other approaches for designing panels for manufacturing. The use of bent sheet metal panels allows for mitigation of several trade-offs and shows the applicability of origami-adapted design to sheet metal. This chapter has been presented at a conference and published in the proceedings of the conference [10].

3.1 Introduction

Origami is the art of paper folding that has recently seen an expansion into many fields, including engineering. When applying origami to create engineering solutions, the materials required, such as plastics and metals, usually behave differently than paper. Most applications necessitate engineering materials that are substantially thicker than paper and, therefore, cannot be approximated as having zero thickness. Another disparity between paper and engineering materials is seen in the result of folding. When paper is folded, it experiences a localized reduction in stiffness to form a crease that allows for repeated motion, whereas most engineering materials, including sheet metals, do not exhibit such a decrease in stiffness when folded [2]. These differences in material behavior present challenges that necessitate adaptation of origami patterns for

applications using engineering materials. Such designs and products that adapt origami to thick, non-creasing, or otherwise non-paper-like materials are called *origami-adapted* [1].

After an origami-adapted product design has accommodated for thickness, the next step to realization of the design is to consider manufacturing [15]. Several techniques for accommodating thickness in the design of origami-adapted products have been developed, and each has its own set of performance characteristics [3]. Once the thick origami mechanism has been designed, an evaluation of how it can be manufactured—and any subsequent design changes—must follow.

The approach used for designing the panels affects the manufacturing characteristics of the mechanism. Thus far, the approaches demonstrated in literature use either monolithic panels or panels composed of multiple layers or pieces of material [3]. As these panel approaches give rise to differing manufacturing characteristics, there are trade-offs between the disadvantages of each approach. A primary disadvantage of monolithic panel approaches is that fabrication of the panels requires extensive material removal. Layered approaches, though they do not usually require material removal, have a significantly higher part count than monolithic approaches. Such a high part count is a disadvantage as it can result in an increased chance of problematic tolerance stack-ups.

Trade-offs involving manufacturing and performance characteristics also arise between various thickness-accommodation techniques. Some techniques have very precise, predictable motion characteristics but require complexity in manufacturing, whereas other techniques that lead to more favorable manufacturing characteristics do not generally exhibit the same favorable performance characteristics. For example, mechanisms using the Synchronized-Offset Rolling-Contact Elements thickness-accommodation technique [7] exhibit the same single degree-of-freedom (DOF) motion as the original origami model but require complex, high precision rolling-contact surfaces that can lead to costly manufacturing. On the other hand, the Strained Joint Technique [8] has highly favorable manufacturing characteristics as it only requires cutting a single sheet to create flexible hinge-like sections. Nonetheless, the performance characteristics of the Strained Joint Technique are less favorable because the technique results in multi-DOF mechanisms that are prone to parasitic motion.

The use of sheet metal, however, provides an opportunity to achieve an improved set of manufacturing characteristics. An ideal panel approach leads to panels that would not require ma-

chining, be conducive to the use of sheet stock, and have a low part count. Although there are a few thickness-accommodation techniques whose panel approaches already meet this ideal, the techniques are not exempt from trade-offs between performance and manufacturing characteristics. Sheet metal is a readily available, relatively low cost material that is well suited to many applications. Through the use of bends, sheet metal allows for simplification of fabrication to meet this manufacturing ideal and mitigates the trade-offs associated with some thickness-accommodation techniques. Building on existing techniques for accommodating thickness in origami patterns, we explore and illustrate the use of metal sheet goods to realize selected thickness-accommodation techniques.

3.2 Background

To appropriately present and discuss panel approaches for origami thickness-accommodation techniques that involve bent sheet metal, some background knowledge is needed. It is necessary to understand techniques for accommodating thickness in origami and applicable design limitations. The current panel approaches associated with some thickness-accommodation techniques are briefly reviewed to enable comparison with the bent panel approaches that will be presented. Some previous work that has been done to use sheet metal in origami is also briefly summarized.

Most origami-adapted mechanisms are based on rigidly foldable origami patterns. A pattern is rigidly foldable if all deflection throughout the pattern motion occurs at the crease lines, thereby ensuring that the facets of the pattern can remain undeformed [16]. Research has been done in methods for determining rigid foldability of a given pattern [17] as well as the mathematical relationships necessary for rigid foldability [18, 19] and rigid foldability of specific patterns [20].

Many techniques have been developed to accommodate thickness in origami [3]. Some techniques make the transition to manufacturing more difficult than others [12, 14]. The three thickness-accommodation techniques addressed in this chapter for implementation in sheet metal are the tapered panels, hinge shift, and offset panel techniques. These techniques were chosen because not only could they be feasibly implemented in sheet metal, but doing so could also mitigate their trade-offs of manufacturing characteristics. Figure 3.1 illustrates the general approach for applying each of these three techniques to an origami pattern. The tapered panels

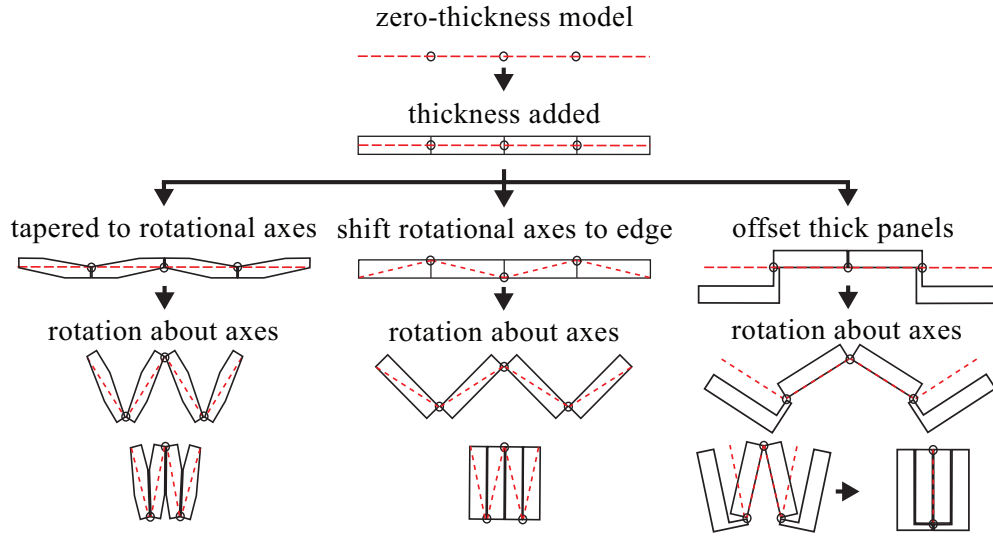


Figure 3.1: From left to right: tapered panels, hinge shift, and offset panel thickness-accommodation techniques.

technique [4] accommodates thickness in origami mechanisms by maintaining the original zero-thickness model within the thick panels and trimming away panel material around the hinges to avoid self-intersection. The offset panel technique [5] also preserves the original zero-thickness origami model within the thick origami mechanism, but instead shifts the panels away from the zero-thickness plane using offsets in order to accommodate panel thickness. The hinge shift technique [6] shifts the rotational axes from the center planes of the thick panels to the outer edges to form a spatial mechanism at each vertex that allows for thick panels.

Some thickness-accommodation techniques have been demonstrated with more than one approach to the panel design. In addition to monolithic panel approaches, layered or segmented panel approaches have been demonstrated for the three thickness-accommodation techniques discussed. For the layered approach using the tapered panels technique, each panel is composed of two layers of equal thickness that lie on either side of the origami model zero-thickness plane and any areas of the panel halves that would have been “tapered” are simply removed. The segmented approach for the offset panel technique is potentially more intuitive as it consists of separating the offsets from each panel. The layered approach for the hinge shift technique involves splitting each panel into layers, particularly at any planes where there is a change in the cross-sectional profile.

Some researchers have done work towards applying origami to sheet metal. Ferrell et al. presented designs for metal lamina emergent mechanisms that have the potential to be used as substitute creases in origami mechanisms [21]. Qattawi et al. explained a method for optimizing the flat pattern used for origami-based sheet metal folding [22–24]. Francis et al. characterized the origami crease-like abilities of some sheet metals [2]. Some have done prototype origami mechanisms using sheet metal panels with varying degrees of thickness-accommodation [25, 26], while others have investigated ways of imparting an origami pattern to sheet metal without the intention of having hinge-like motion [27, 28].

3.3 Application of Selected Techniques in Sheet Metal

The key motivation for applying origami thickness-accommodation techniques in sheet metal is to simplify fabrication. Therefore, the definition of sheet metal used here is: metal sheet stock for which bending is common and a more economical process than welding and machining. This is generally in following with a common industry convention that thicknesses of 7 gauge (3/16 inch or 5mm) and thinner are considered sheet metal while greater thicknesses are considered metal plate.

For designing an origami mechanism using a bent panel approach, steps of the general process are:

1. Design thick origami mechanism using chosen thickness-accommodation technique
2. Identify hinge axes in thick origami model and location of panel material relative to the hinges
3. Connect hinge axes with sheet metal for each panel

The guiding concept of applying sheet metal to a thick origami design is to connect the identified hinge axes with sheet metal while staying within the bounds of the thick-origami panels. To move forward from Step 1 in development of a sheet metal mechanism, the chosen thickness-accommodation technique must permit use of a bent panel approach (ex: the tapered panels, hinge shift, and offset panel thickness-accommodation techniques discussed here). Step 3 involves careful consideration and may require iteration with Step 2 to ensure that self-intersection between

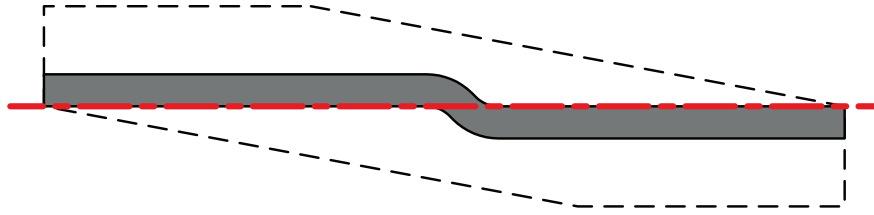


Figure 3.2: Cross-section of a bent panel for a simple pleat tapered panels mechanism relative to its zero thickness plane (shown in red) and “thick” counterpart.

panels does not occur. Step 3 uses the information from Step 2 to place bends where needed and keep the sheet metal panel within the bounds of the panel in the thick origami model. Some origami patterns and thickness-accommodation techniques may require offset bends, also known as jogs or z-bends, to ensure that the bent sheet metal panel stays within the bounds of the thick-origami panel.

The common characteristics pertaining to each bent panel approach are described in the following subsections.

3.3.1 Tapered Panels Technique

The use of the tapered panels technique in conjunction with a bent panel approach is more likely to require the use of offset bends than the bent approaches associated with the hinge shift and offset panel techniques. This is because the panel material must shift from one side of the zero-thickness plane to the other in order to avoid self-intersection. As there is no need to move the panel material any further from the zero-thickness plane, the offset required for these bends is always equal to the material thickness (assuming the hinge axes lie directly on panel edges). In a simple 2-dimensional pleat fold pattern, this is quite simple and straightforward, as seen in Figure 3.2. For 3-dimensional patterns, such as the Miura-ori, the complexity increases because the offset bend is now at an angle to split the mountain creases from the valley creases, as seen in Figure 3.3.

Another adjustment to the mechanism design that is required to implement the tapered panels technique in sheet metal is to trim away panel areas that come to a point rather than an edge. This trimming results from the need to keep the bent panel within the bounds of the thick

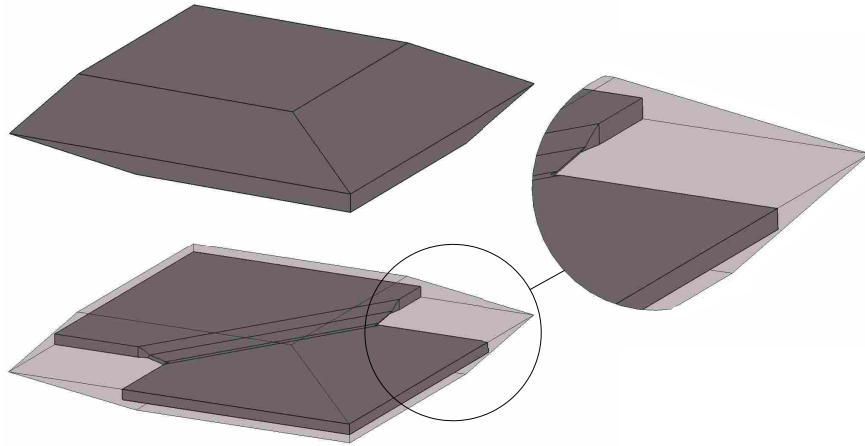


Figure 3.3: Thick panel from a Miura-ori pattern and how the bent panel compares.

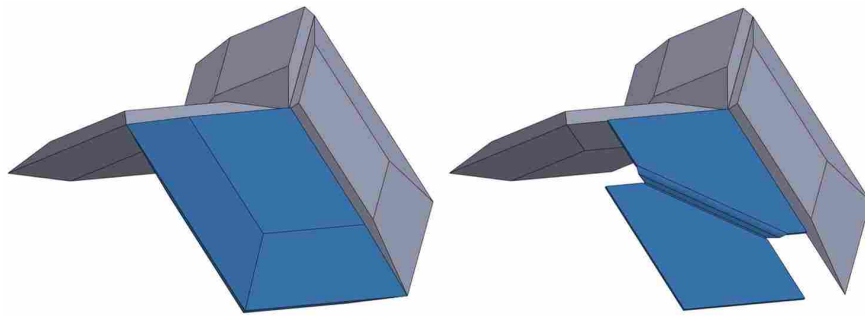


Figure 3.4: Tapered panels mechanism of a Miura-ori unit cell shown with all thick panels (left) and one of the thick panels substituted for a bent panel (right).

origami panel, thereby avoiding self-intersection of the panels. The detail view of Figure 3.3 illustrates how a bent panel used in the Miura-ori pattern could be trimmed.

Figure 3.4 shows how a bent panel compares to an equivalent thick panel for a single-cell Miura-ori mechanism that uses the tapered panels technique. Figure 3.5 then illustrates a tapered panels mechanism for a multi-cell Miura-ori tessellation implemented in sheet metal.

3.3.2 Hinge Shift Technique

Figure 3.6 shows the computer model of a hinge shift square-twist mechanism implemented in sheet metal. When implementing a hinge shift thick origami mechanism with bent panels, the placement of the sheet metal relative to the hinge planes may require more attention than other bent panel approaches for thick origami.

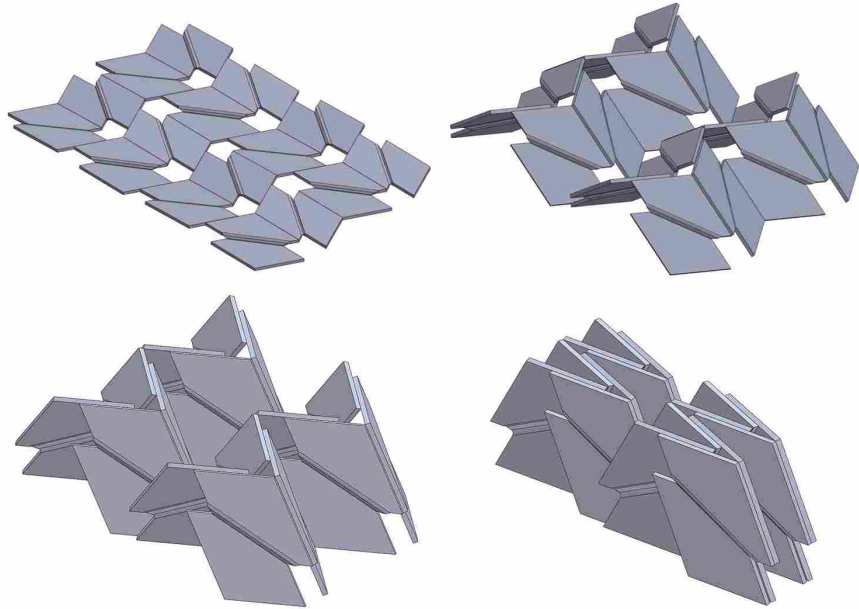


Figure 3.5: Miura-ori tapered panels mechanism implemented in sheet metal shown moving from flat to folded.

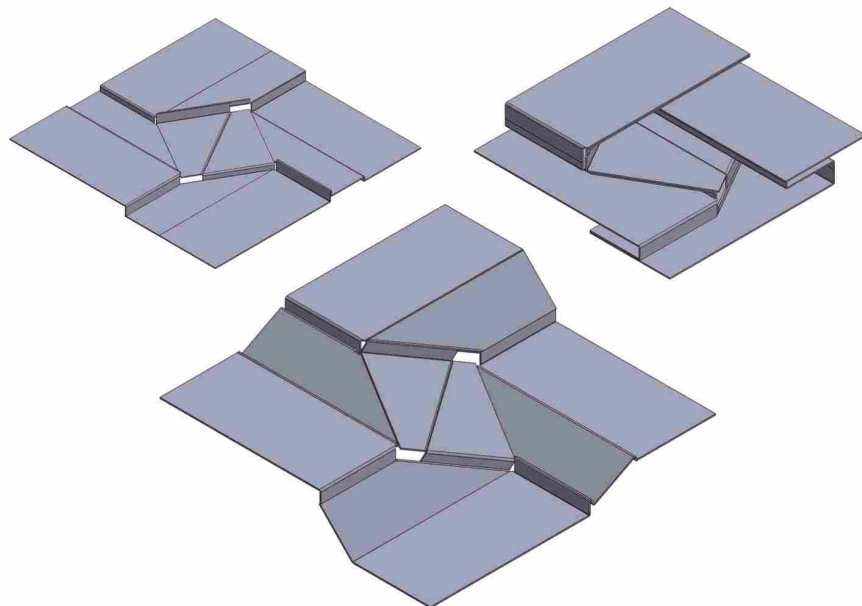


Figure 3.6: Square-twist hinge shift mechanism implemented in sheet metal shown moving from flat to folded.

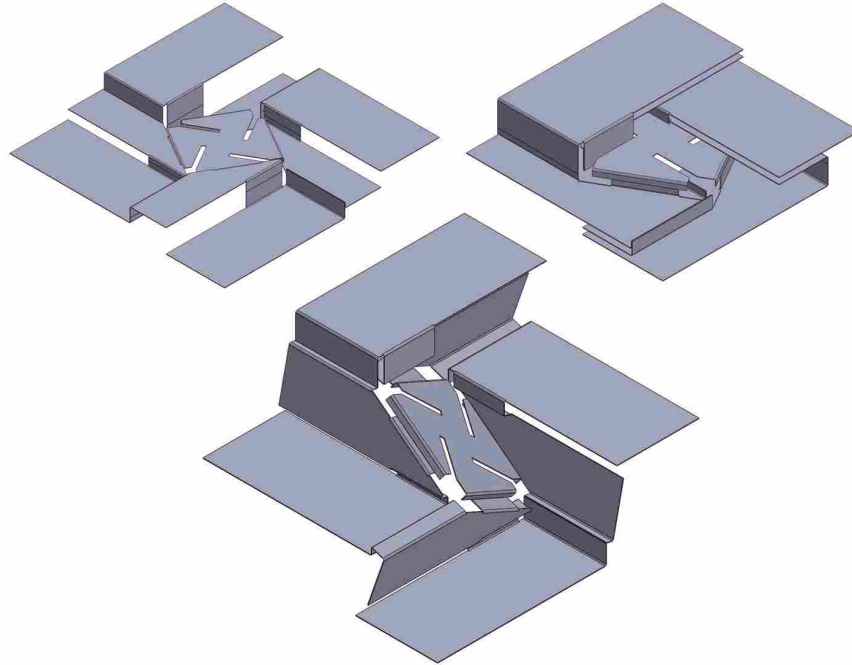


Figure 3.7: Sheet metal square-twist mechanism designed using the offset panel technique shown in flat, folded, and partially folded states.

To implement some patterns in sheet metal using the hinge shift technique, offset bends may be needed. Offset bends will be required when there are two creases of opposite parity (one mountain, one valley) across from each other that lie in the same hinge plane. One example of this is the central panel in the square-twist mechanism seen in Figure 3.6.

3.3.3 Offset Panel Technique

Implementation of the offset panel technique in sheet metal is relatively simple. The offsets that allow the hinges to reach the zero thickness plane (as shown in Fig. 3.1) simply become extensions of the panel rather than separate pieces to be attached. Figure 3.7 shows a square-twist mechanism designed using the offset panel technique implemented in sheet metal.

One item to consider in the design of sheet metal mechanisms using the offset panel technique is that it may be desirable to add bends to panels that would not require any offsets otherwise. In the absence of any bends, sheet metal is significantly more flexible. Bends may be added to panels that are coincident to the chosen joint plane to increase the rigidity of the panel. The central

panel of the square-twist mechanism shown in Figure 3.7 is an example where bends have been added to the panel edges, though not required for functionality of the mechanism.

3.4 Implementation

This section further details the process that was outlined in the previous section by completing the process for a specific mechanism.

3.4.1 Process Illustration

A square-twist origami pattern was chosen for this example of how the previously described process can be implemented.

Step 1.—Design thick origami mechanism using chosen thickness-accommodation technique

Figure 3.8 shows a mechanism that was designed using the hinge shift technique to accommodate for thickness in the square-twist origami pattern. The full design process to create this thick origami mechanism is not detailed here. See [6] for a detailed explanation of the hinge shift thickness-accommodation technique.

Step 2.—Identify hinge axes in thick origami model and location of panel material relative to the hinges

This step must be completed for each panel in the mechanism. Figures 3.9a and 3.9b illustrate this for an edge panel in the hinge shift square-twist mechanism and Figures 3.10a and 3.10b illustrate the step for the central square panel of the mechanism. Both the location of the hinge axes and where the panel material is relative to those axes must be considered in the design of bent panels. If both of these panel characteristics are not considered together, it becomes easy to put the sheet metal where it is not needed or in locations that would cause self-intersection of panels during motion.

It is also during this step that any need for offset bends can be identified. The central square panel of the mechanism provides an example of this (Figs. 3.10a–3.10b). The panel has two hinge axes in the same plane, but the panel material is on opposite sides of the plane at the

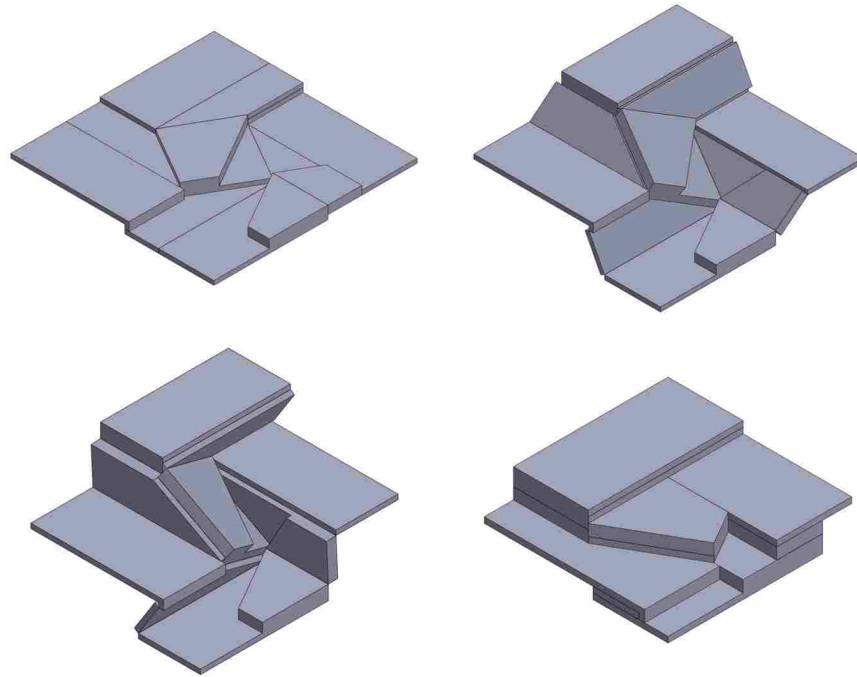


Figure 3.8: Thick-origami square-twist mechanism designed using the hinge shift technique shown moving from flat to folded.

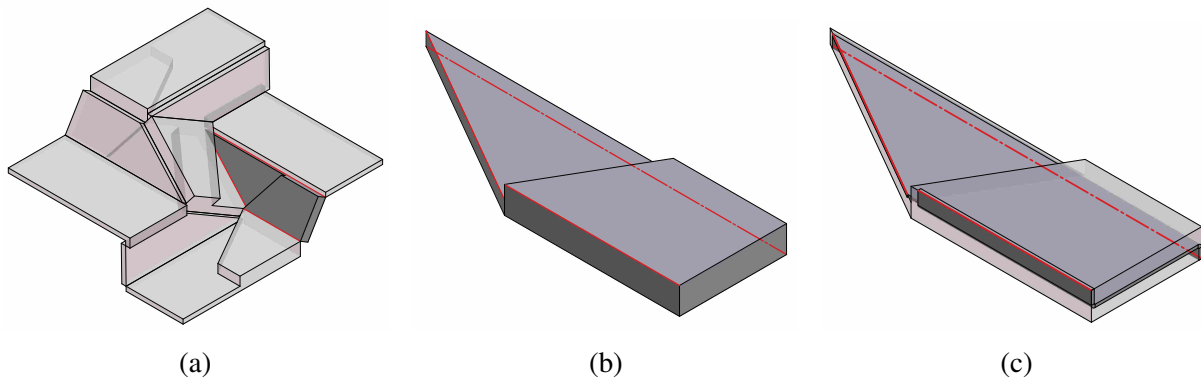


Figure 3.9: Application of sheet metal to an edge panel of a square-twist mechanism. (a) A thick panel in context of the mechanism; (b) thick panel; (c) bent panel. Red lines signify hinge axes.

hinge locations. Therefore, an offset bend is necessary to ensure the bent panel stays within the bounds of the thick origami panel.

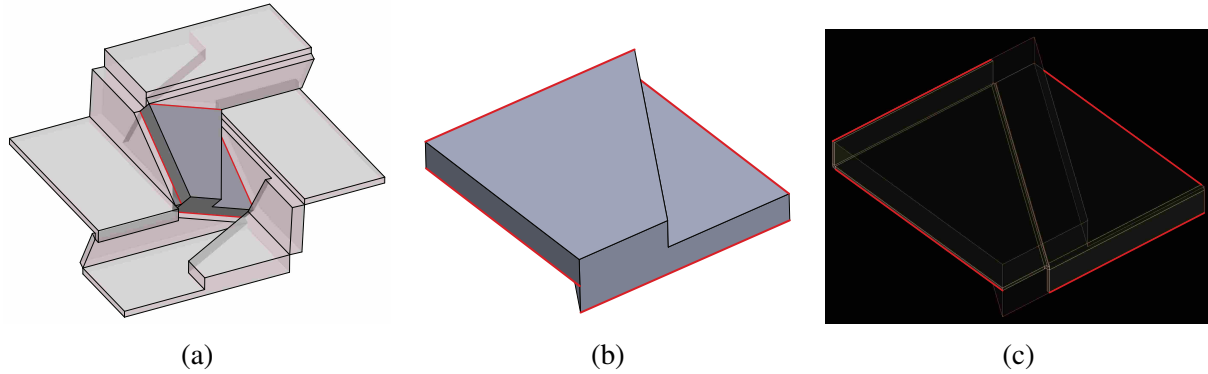


Figure 3.10: Application of sheet metal to the central square panel of a square-twist mechanism. (a) A thick panel in context of the mechanism; (b) thick panel; (c) bent panel. Red lines signify hinge axes.

Step 3.—Connect hinge axes with sheet metal for each panel

This step is illustrated in Figure 3.9c for an outside panel of the hinge shift square-twist mechanism and Figure 3.10c for the central square panel. This step of connecting the hinge points with sheet metal, and thereby forming the bent panel, relies on the information from Step 2. A complete computer model of the sheet metal mechanism is seen in Figure 3.6.

In many cases, this step may be simplified by first identifying where the primary panel face should be located then adding edge flanges for the sheet metal to meet all other hinge axes. In the example of the square-twist edge panel (Fig. 3.9), the location of the primary panel face is a result of the “middle” hinge axis that has one hinge above it and one below. From that face, only simple edge flanges are required to connect to the other hinge axes.

Another approach to help identify how and in what order to connect the hinges can be thought of as a “top to bottom, left to right” approach. This approach consists of choosing a starting hinge axis that is at a relative extreme location to the others for the panel then connecting that hinge axis to the others in a “top to bottom, left to right” fashion while staying within the bounds of the thick origami panel to finally end at the hinge axis that is furthest from the starting axis. This approach has the potential to be simpler for panels such as the central square panel shown in Figure 3.10.

Alternatively, this step of the process has the potential to be automated via software such as discussed in [29].

3.4.2 Hardware

Figure 3.11 shows the hinge shift square-twist mechanism physically implemented in sheet metal. A tapered panels Miura-ori tessellation physically implemented in sheet metal is shown in Figure 3.12. Both of these mechanisms were constructed from 20 gauge mild steel bent panels and unfinished steel piano hinges, then assembled via resistance spot welding and gas tungsten arc welding.

A few design changes were required to accommodate the use of piano hinges as the joints between panels. When using piano hinges to connect two panels, the center of rotation for the joint no longer lies at the panel edges. Therefore, because the axes of rotation must be at the same location as the model that does not use piano hinges, the panel designs must be altered slightly to ensure that the hinge axes are at the correct locations when assembled. Another modification required for the fabrication of the square-twist mechanism using piano hinges was to add edge flanges to some panels. These additional edge flanges facilitated assembly of the mechanism as the flange provided surface area to which the hinge could be attached.

3.5 Analysis and Comparison

Table 3.1 presents a comparison of these bent approaches for origami-adapted mechanisms to the other panel approaches that have previously been presented in literature. The table column headings are described as follows. *Part count* indicates the relative part count of the origami-adapted mechanism where a “baseline” part count is when the product has as many parts as the number of facets and creases in the paper origami model. If an approach is *conducive to sheet stock*, then materials in sheet form, such as sheet metals, can be easily used for the panels. If the approach requires another process in addition to a simple 2-D stock cutting process to fabricate the panels, that process is listed in the *second panel process* column. *Minimum number of processes* indicates the fewest number of distinct processes, including assembly, required to manufacture an origami-adapted mechanism using the given approach.

There are many similarities between the rows of Table 3.1 for the tapered panels, offset panel, and hinge shift techniques. These similarities reflect the common characteristics of the underlying panel approaches. However, the *Schematic representation* for each technique differs and

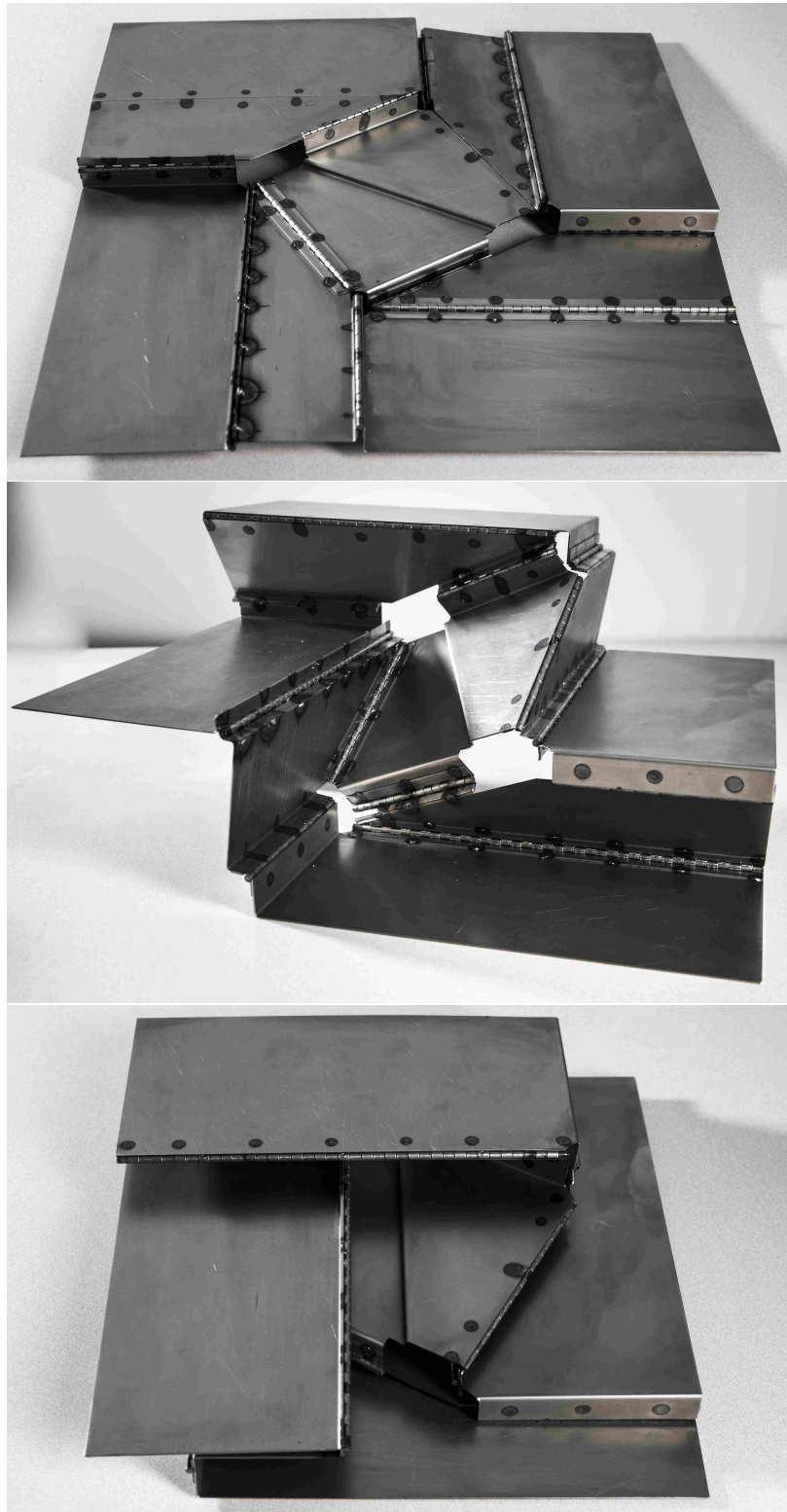


Figure 3.11: Sheet metal square-twist mechanism designed using the hinge shift technique shown unfolded, partially folded, and fully folded.

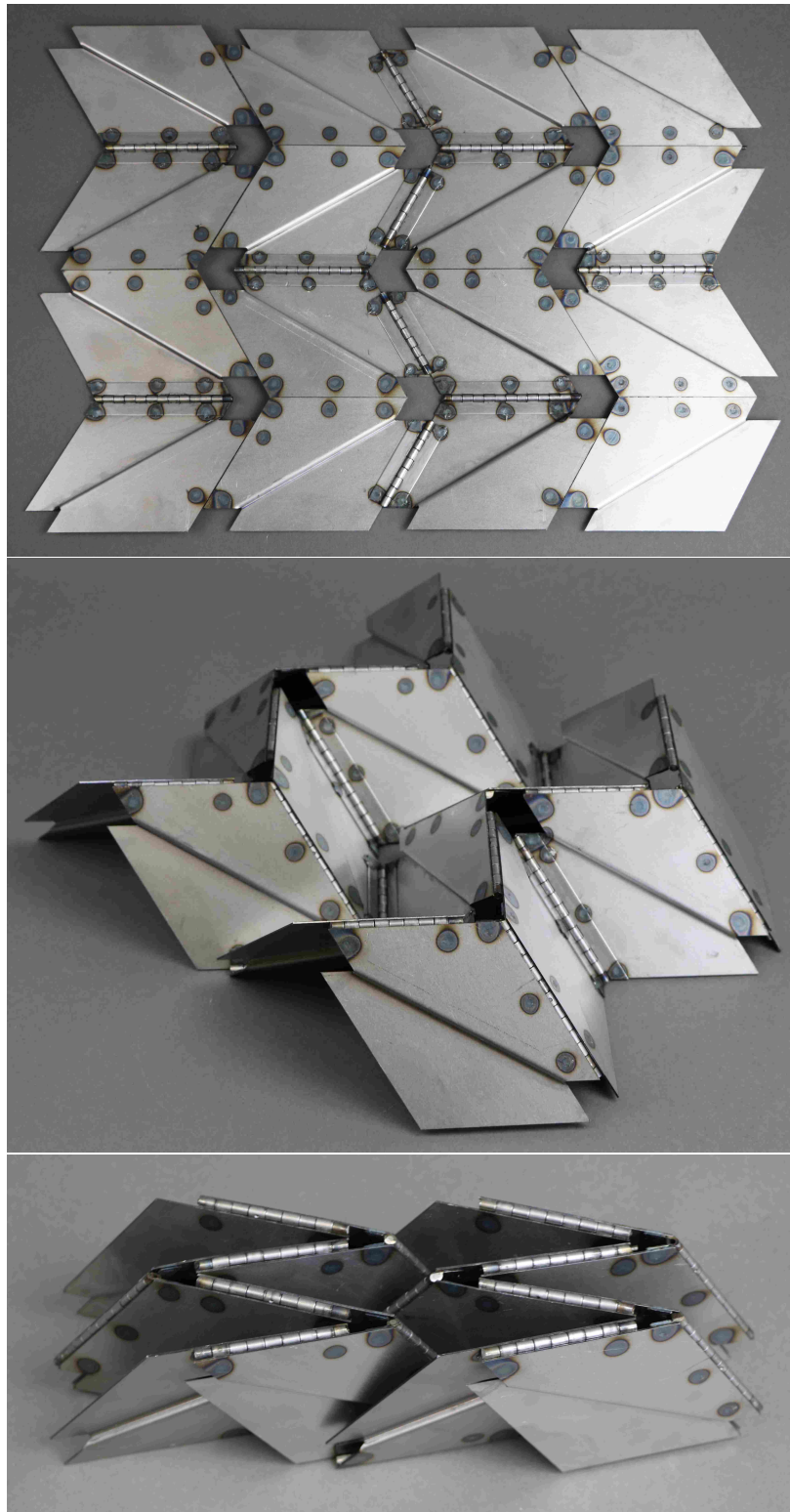
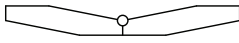
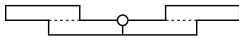
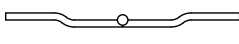
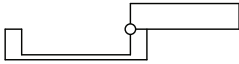
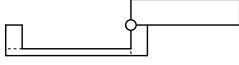
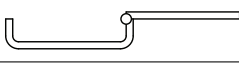
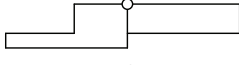
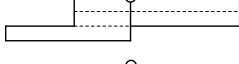

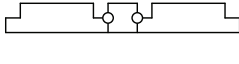
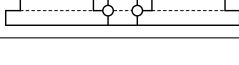


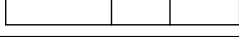
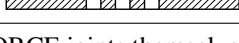


Figure 3.12: Sheet metal Miura-ori mechanism designed using the tapered panels technique shown unfolded, partially folded, and fully folded.

Table 3.1: A comparison of panel approaches for origami-adapted mechanisms. In the second column, dashed lines indicate layer or segment divisions within the panels and small circles represent stock hinges.

Technique panel approach	Schematic representation	Part count	Conducive to sheet stock	Second panel process	Minimum number of processes
Tapered Panels					
monolithic		Baseline	No	Material removal	3
layered		High	Yes	Assembly	2
bent		Baseline	Yes	Bending	3
Offset Panel					
monolithic		Baseline	No	Material removal	3
segmented		High/Very High	Yes	Assembly	2
bent		Baseline	Yes	Bending	3
Hinge Shift					
monolithic		Baseline	No	Material removal	3
layered		High/Very High	Yes	Assembly	2
bent		Baseline	Yes	Bending	3
Doubled Hinge					
monolithic		High	No	Material removal	3
layered		Very High	Yes	Assembly	2
Rolling Contacts					
integrated joints		Low	No	Material removal	3
separate joints		Baseline	Yes	None*	3
Membrane					
Membrane		Low/Baseline	Yes	None	2
Strained Joint					
Strained Joint		Low	Yes	None	1

* – the SORCE joints themselves must be fabricated because stock hinges cannot be used

gives visual representation of how the specific combination of thickness-accommodation technique and panel approach affects the overall architecture of the mechanism.

The bent approaches that have been presented here have advantages over several other approaches. They allow for the same “baseline” part count as the monolithic approaches, but without the disadvantage of requiring a 3D material removal process such as milling. Using sheet metal also allows for the use of sheet stock, but without the disadvantage of a high part count that exists with the use of layered or segmented approaches. By using a bent approach, the trade-off between part count and use of sheet stock that exists with the monolithic and layered/segmented approaches is eliminated.

Nevertheless, these bent approaches are not without their own design considerations. Mechanisms using the hinge shift or offset panel techniques are likely to have significant open spaces in the folded position when implemented in sheet metal. Using sheet metal with the tapered panels technique has the potential to increase the maximum fold angle of the mechanism, thereby increasing the density of the mechanism in its folded state. This still does not allow the mechanism to be fully compact, but potentially more compact than if other panel materials were used. It should be noted, however, that use of the bent (or layered) approach associated with the tapered panels technique also introduces holes at the vertices of the mechanism.

As with other panel approaches for origami-adapted products, the scale of the mechanism must be considered in the design of origami-adapted mechanisms using sheet metal. Design parameters including bend radius, sheet thickness (and resulting weight), and panel rigidity can all affect the feasible scale of the mechanism. The complexity of issues involving these parameters is likely to increase on both the small and large ends of the scale spectrum.

3.6 Conclusion

The viability of sheet metal use in the realization of three thickness-accommodation techniques has been illustrated and demonstrated. A general process for using sheet metal in origami-adapted mechanisms was described and demonstrated for a square-twist mechanism designed using the hinge shift technique to accommodate for thickness. The computer models and physical hardware shown validate the feasibility of these approaches for designing the panels of a thick origami mechanism. These bent panel approaches fill a gap in the design space to achieve favorable performance and manufacturing characteristics, thereby mitigating several trade-offs. The bent panel approaches lead to mechanisms that have lower part counts than those using layered or segmented

panel approaches while allowing for the use of sheet goods as well as avoiding potentially costly material removal processes.

Further work can yet be done to address the manufacturability of origami-adapted designs. Aside from manufacturability of the panels, the assembly of the mechanism as a whole also needs investigation because the highly constrained designs of most origami-adapted mechanisms require relatively tight tolerances in location of the hinge axes relative to one another. One approach to this problem may be to investigate feasible hinge solutions aside from traditional piano hinges such as those used for the hardware shown here.

Sheet metal is a viable option for panel material that can lead to advantageous manufacturing characteristics. By considering manufacturing as part of origami-adapted design, progress can be made to mitigate manufacturing disadvantages and enable more widespread use of origami-adapted design.

CHAPTER 4. AUTOMATING THE DESIGN OF THICK-ORIGAMI MECHANISMS

Applying an origami pattern to thick, non-paper-like materials is a challenging task. Though many techniques have been developed to accommodate thickness in origami, creating 3D models of such thick-origami mechanisms is complex. The time and knowledge required to manually model an origami mechanism can impede the exploration of the design space and creation of robust designs. This work presents data structures based on origami that can be used in the automation of thick-origami mechanism design. These structures are described and an example computer program that implements them is investigated. The program automatically generates all the necessary 3D CAD part models and an assembly model for a user-specified origami crease pattern. Models resulting from the program for several crease patterns are demonstrated with a discussion of the advantages and limitations of the system. With further development of the data structures and program, this framework has the potential to help mitigate some of the barriers to more widespread use of origami-based design.

4.1 Introduction

The ancient art of paper-folding, called origami, has experienced a resurgence of interest in the communities of science, technology, mathematics, and engineering in the past few decades. Applications of origami have been broad, including heart stents [30], DNA [31], solar arrays [13], and structures [32]. To employ origami in many engineering applications, it is necessary to use materials that cannot be approximated as having zero thickness like the assumptions applied when working with paper. *Origami-adapted* design transforms base origami designs to adjust for the non-paper-like characteristics of many materials such as thickness and the inability to have hinge-like creases [1].

Even with the development of several techniques for accommodating thickness in origami [3], barriers remain to widespread use of origami-adapted design. One of these barriers is the

time and effort required to create a 3D model of a thick-origami mechanism. Creating a 3D CAD model of such a mechanism requires a model for each panel and the full integrated assembly of the mechanism. The significant time investment required to create these models, especially those with large numbers of panels, such as tessellations, hinders exploration of the design space and can be costly.

The skill and familiarity with thick-origami mechanism design that is necessary to model these mechanisms is also an inhibitor to more widespread use of origami-adapted design. Aside from basic CAD skills, a designer of such mechanisms needs an understanding of the mathematics and theory behind origami that define the kinematics and folding motion of the patterns, as well as a knowledge of origami thickness-accommodation techniques and how to apply them—even with the overconstrained nature of many origami patterns.

Automation of thick-origami mechanism design would help to mitigate, if not eliminate, many of these barriers to implementation of origami-adapted design by reducing the knowledge and skill necessary to generate a design. Classes, a type of computer data structure, can be useful in such design automation. Classes are the fundamental building block of object-oriented programming (OOP). OOP allows designers to more easily create large, complex programs by requiring designers to carefully plan each data structure. Classes and other data structures can be created for storing and analyzing origami crease pattern information to make it useful for design automation. Origami-adapted design is one application that can benefit from the use of these structures.

The objective of this work is to explore and demonstrate how data structures based on origami can be used in the automation of thick-origami mechanism design. Automating the generation of models for these mechanisms will significantly reduce the time and skill required, enabling faster and better exploration of the design space for origami-adapted mechanisms. This will aid engineers and designers who are not familiar with origami-adapted design, as well as those who are, in generating better designs and meeting the needs of their applications.

The remainder of this chapter will provide some background on origami and related computer programs, present the data structures used by the authors to analyze crease patterns, and explain and discuss the example program that implements those data structures. The example program described in this work analyzes crease patterns that are rigid-foldable, flat-foldable networks

of degree-4 vertices for the purpose of generating thick-origami 3D CAD models of the crease patterns.

4.2 Background

The automatic generation of thick-origami models requires computer programs. A brief summary of several computer tools are presented below that support the design and visualization of origami. Furthermore, an explanation of the tapered panels technique, implemented in this study, for accommodating thickness in origami is also provided.

Several algorithms and computer programs have been written for the purpose of aiding in the design of origami patterns. *Origamizer* is a computer program that implements algorithms for generating a crease pattern for any given polyhedron [33, 34]. Another program, *Treemaker*, uses tree theory to generate a crease pattern based on the user's input tree diagram [35, 36]. Other programs allow for digitally folding origami for purposes of visualization, reasoning, and even creating fold diagrams of origami [37, 38]. A program called *FoldStar* [39, 40] generates a tessellation and allows for animation of its folding motion based on the user input "row-edge" and "column cross-section" lines that define the folded edges of the tessellation.

Some preliminary work has been done to address material thickness in computer modeling of origami. The program *iFold* [41] allows for visualization of zig-zag origami and includes minor accommodation for relatively small paper thickness by showing the folded layers as offset from one another. However, the length of material in the offset distances that define how parallel folded layers are connected together is not accounted for in the overall length of the material (or size of the "paper"). This is assumed because the primary purpose of the program is to aid visualization of folds in a more three-dimensional manner.

Ku and Demaine, as part of their work on a technique for accommodating material thickness in origami, created a web applet that allows the user to manipulate the modified crease pattern of the bird-base origami pattern to visualize the effect of material thickness when using the offset-crease thickness-accommodation technique [12, 42]. Ku has also created a program that can be used for generating the 2-dimensional sketches for cutting the parts necessary to assemble a single thick degree-4 vertex using a specific fabrication approach for the tapered panels technique [43, 44].

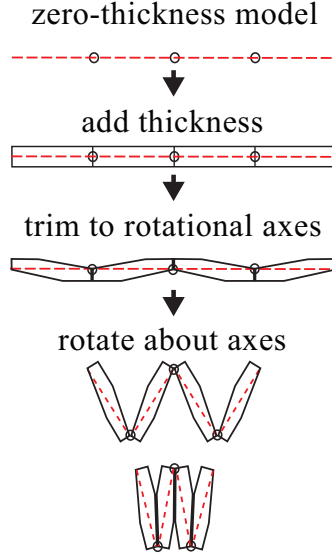


Figure 4.1: Tapered panels technique.

There are not yet any programs that can be used for the automatic generation of a 3-dimensional thick-origami CAD model that contains volumes.

However, many techniques for accommodating thickness in the design of origami mechanisms have been developed [3]. Some techniques alter the crease pattern as a step in the process of accommodating for thickness, while others do not. The tapered panels technique [4], shown in Figure 4.1, is one technique that does not alter the crease pattern. The principle behind this technique is that thickness is added with the crease pattern as the mid-plane, then the mechanism is partially folded and any self-intersecting material is trimmed away from the creases and vertices. Mechanisms using the tapered panels technique cannot be fully folded as doing so would result in panels with zero thickness.

Figure 4.2 describes the trimming of panels using the tapered panels technique. The length, L , of the taper for a given crease with dihedral angle, δ , is

$$L = t \cot(\delta/2), \quad (4.1)$$

where t is the panel half-thickness [4]. If using the fold angle of a crease γ (where $\gamma = 0$ when unfolded and $\gamma = \pm\pi$ when flat folded) rather than the dihedral angle, then the taper length can be

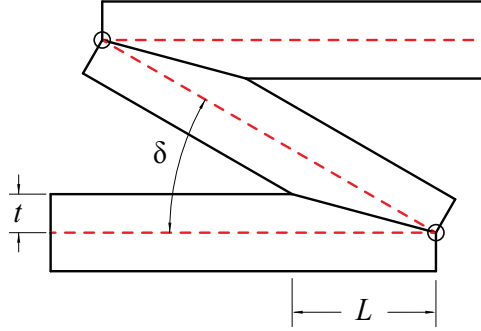


Figure 4.2: Diagram detailing the length of a panel's taper.

directly calculated via

$$L = t \cot\left(\frac{\pi - \gamma}{2}\right). \quad (4.2)$$

This knowledge of the tapered panels technique is part of the foundation used for the data structures that will be presented for use in origami crease pattern analysis.

4.3 Crease Pattern Analysis

In creating the method for analyzing origami crease patterns presented here, it was assumed that the crease pattern is provided in the form of a CAD sketch in Siemens NX, such as the sketch shown in Figure 4.3. It was also assumed that the crease pattern meets the criteria of rigid-foldable, flat-foldable, and a network of only degree-4 vertices (vertices where four crease lines meet). Figures 4.4 and 4.5 show an example crease pattern and vertex that have been labeled according to the terminology used throughout this chapter. This specific analysis was tailored to the tapered panels technique, although similar processes and data structures could be used for other thickness-accommodation techniques.

This data structure is based on the equations needed for the tapered panels technique presented above. These equations require the fold or dihedral angles at each vertex to calculate the taper lengths. Because the fold angles are dependent on the sector angles of the connected vertex, the data structure was organized around the vertices, creating a Vertex class, to store data and perform vertex related functions. The information stored in this class, including the sector angles and the order of the lines, can be generalized and used for other thickness-accommodation techniques.

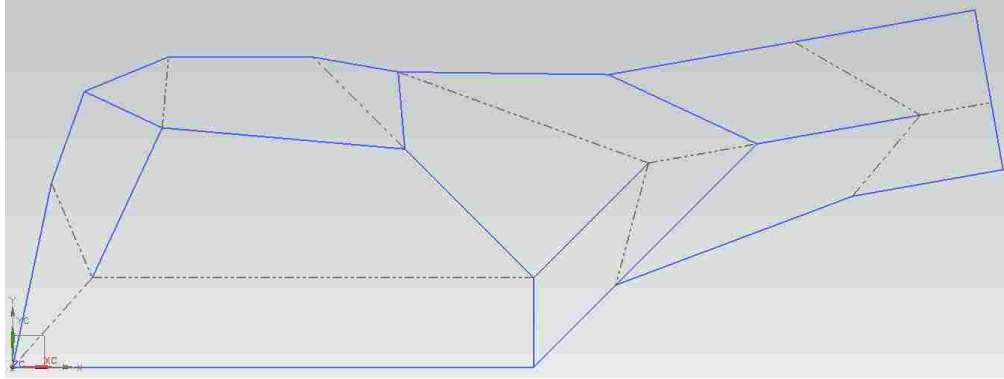


Figure 4.3: Example of a crease pattern sketch in Siemens NX.

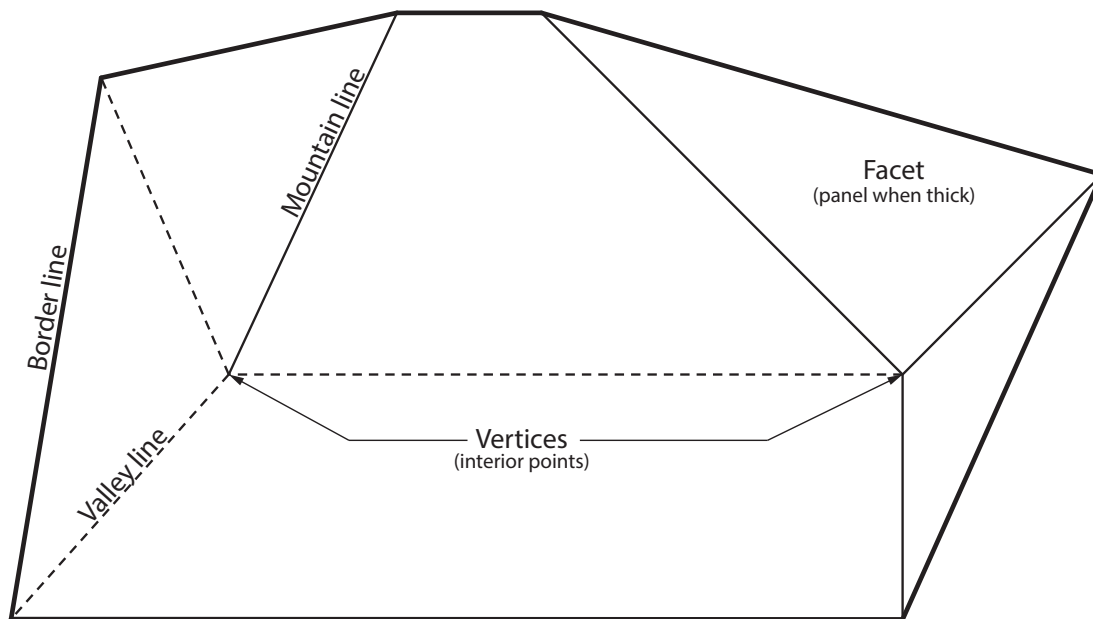


Figure 4.4: An example crease pattern containing two vertices.

The information in the Vertex class allows the performance of all necessary calculations for the tapered panels technique. Further information was gathered to facilitate physically modeling the panels. This information, such as what set of lines forms a panel, was stored using the Panel class. The Panel class also includes the modeling features and dimensions of each panel.

The Vertex and Panel classes organized the crease pattern lines for calculations and modeling. Two other classes were also created to store information for each individual line. These two custom line classes, smartLine and genLine, are shown in Figure 4.6.

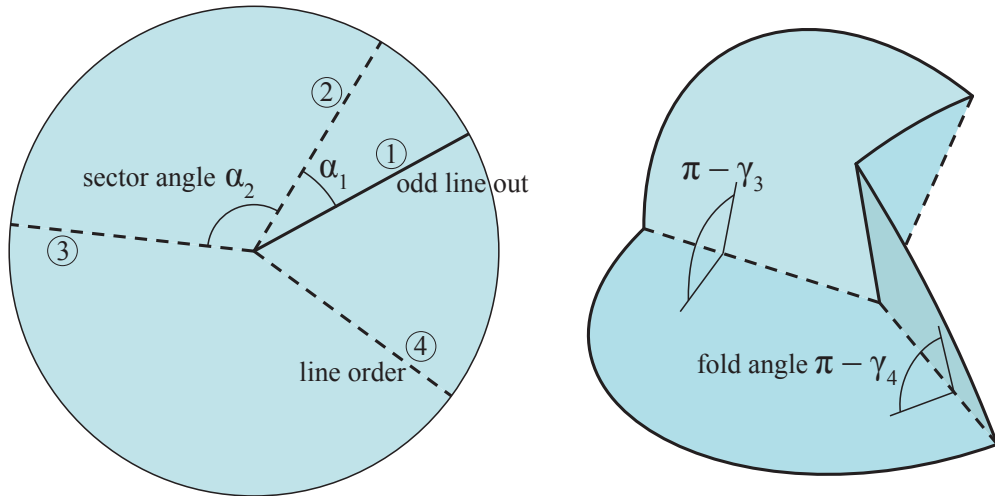


Figure 4.5: A degree-4 origami vertex shown flat (left) and partially folded (right).

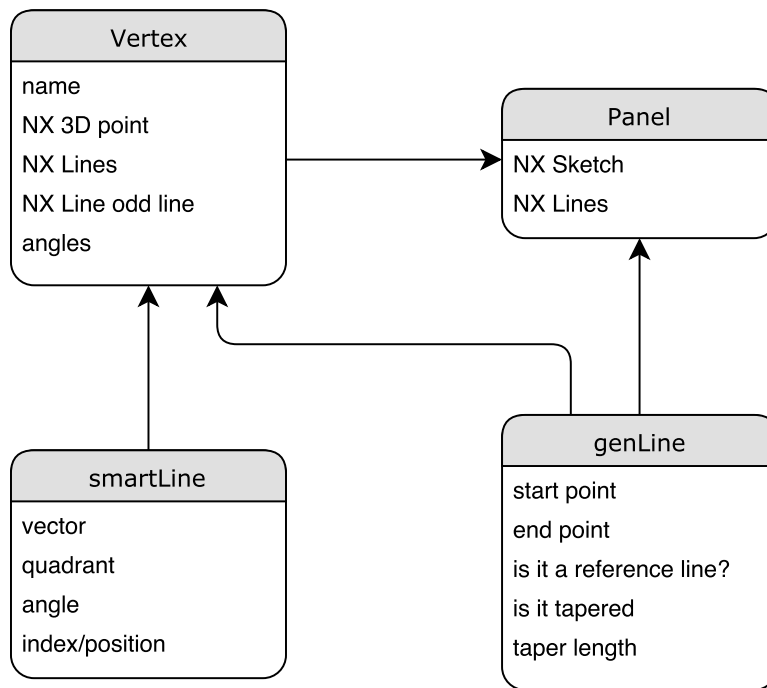


Figure 4.6: The data structures used for analyzing crease patterns and the information contained in each.

These four main data structures, shown in Figure 4.6, were used to extract information from the sketches and are explained in greater detail in the following subsections.

4.3.1 Lines

Two custom line classes were created, called `smartLine` and `genLine`. The `smartLine` class was used exclusively within the `Vertex` class and contained information required for line ordering as shown in Figure 4.6. The second line class, `genLine`, was used to store information about every line in the crease pattern, including reference line status (e.g. a dashed line indicating a valley crease line), if it is tapered (i.e. border lines are not tapered), and the taper length, if applicable. Both the `Vertex` and `Panel` classes contained lists of associated `genLines`.

4.3.2 Vertices

The `Vertex` class contains information about each internal point (any point not on the border), or origami vertex, of the crease pattern. This information about the vertices is needed for calculating taper lengths later on in the algorithm. The `Vertex` class also performs two vital functions: line ordering and sector angle calculation. Each `Vertex` object contains a name, the XYZ coordinates of the point it refers to, and all of the lines connected to the vertex in order, as shown in Figure 4.6. Each `Vertex` object also contains the “odd line out” and whether or not that line is a valley crease (indicated by a reference, construction, or otherwise dashed line). Of the four lines that meet at the vertex, the “odd line out” is the line of opposite crease parity—either mountain (normal object line) and the other three lines are valley, or vice versa where the odd line out is a valley crease line and the other three lines are mountain creases. Additionally, the `Vertex` class contains the `smartLine` class for storing the information needed to order the lines.

Line Ordering

An essential operation a `Vertex` object performs is ordering the lines around the vertex point, since all of the subsequent calculations rely upon this order. The order defined for this program is counterclockwise around the vertex point, starting with the odd line out. Each line is assigned a vector and quadrant in the `smartLine` class. This information is used to find the correct order of the lines, which is then stored for each `smartLine`.

Sector Angle Calculation

After the correct line order is determined, the first two sector angles—the angles between line 1 (the odd line out) and line 2, and between lines 2 and 3—are calculated using the dot product. Corrections are made based on the quadrant of the angle to account for the limitations of the tangent function. These sector angles are later used to calculate the taper lengths for each panel.

4.3.3 Panels

The Panel class contains the lines for each panel, the Vertex objects connected to those lines, and a sketch. A Panel object also performs three functions: finding lines to complete the panel, sketching the panel, and extruding the panel.

Completing the Loops

The most complex functionality of the Panel class is “connecting the dots”, or completing the loop of lines that define the panel. Each Panel object is initialized with all of the internal (i.e. non border) lines for that panel. If one or more border lines are required to complete a panel, they will be found by the object. This is performed by finding the open endpoints of the lines currently contained in the panel, and then finding the intersections between those lines and the border lines. Once the missing lines are found, they are added to the Panel object’s list of lines.

Modeling

The modeling performed by the Panel class includes sketching and extruding. The sketching function simply automates the sketching of each of the Panel object’s lines in CAD as part of the geometric modeling of the mechanism. The sketch is then stored in the object. Additional functions extrude the Panel object’s sketch by the user specified panel thickness. The solid feature produced is then tapered, or chamfered, on the model.

4.4 Implementation

The preceding data structure is tested by implementing a Siemens NX program using the NX Block UI Styler [45]. This program specifically uses a crease pattern with a network of degree-4 vertices that is both rigid-foldable and flat-foldable. The specific thickness-accommodation technique (i.e. tapered panels) was chosen due to the suitability of modeling the tapers programmatically in CAD programs.

4.4.1 Program Workflow

The following explains the general workflow of the program. The user is asked to input the following information into NX:

1. the desired file location for the finished assembly
2. a sketch containing just the border of the crease pattern
3. a sketch containing the rest of the crease pattern (can include the border)
4. a maximum fold angle
5. a panel thickness

The sketches must use construction, or reference, lines to refer to valley folds. Non-reference lines that are not border lines indicate mountain folds. Given the appropriate information in the correct format, the program then produces an assembly of the finished mechanism, with each panel in its appropriate location with the specified thickness and correct tapers.

Table 4.1 shows a broad overview of the steps the program takes to extract and analyze the data from the provided sketches and then generate the 3D model.

Step 8 (from Table 4.1) involves a branching algorithm to determine the correct tapers for each edge. When a crease pattern is symmetric (such as the Miura-ori or square-twist), there are only two unique taper lengths in the entire mechanism. However, if a pattern lacks symmetry, there will be more unique taper lengths. When there are more than two possible taper lengths, it becomes necessary to calculate the taper length using fold angles from previously tapered edges.

Table 4.1: Overview of steps the code takes to extract and analyze the required data. The location of each step in the code is also noted as either the main code (not in a separate class), the Vertex class, or the Panel class.

Step	Activity	Associated Class
1	Grab the two sketches	Main
2	Identify all the points and lines in each sketch	Main
3	Separate the border points and lines from the internal geometry	Main
4	Identify the internal points and create a Vertex object for each	Main
5	Order the lines around each vertex counterclockwise, starting with the odd line out	Vertex
6	Calculate the angles between the first three lines	Vertex
7	Determine which lines refer to tapered edges	Main
8	Calculate the tapers for each edge	Main
9	Create Panel objects for each panel, storing information needed	Panel
10	Model each panel in a new part	Panel
11	Chamfer each edge that needs to be tapered	Main
12	Assemble the individual parts	Main

This back-calculation process requires a branching algorithm to ensure that all taper lengths are dependent on each other.

Step 9 (of Table 4.1) involves a complex search algorithm to identify every panel and all of the lines connected to that panel. The algorithm identifies all of the internal (i.e. non border) lines defining a panel regardless of the number. It then branches off from the current panel to find others until all of the panels have been defined.

4.4.2 Results/Testing

The resulting program was able to successfully create assemblies for five different crease patterns, as well as variations on each pattern, to demonstrate it is capable of functioning for a variety of input crease patterns that fit the program criteria. The five test cases are discussed and explained below. Each of these example crease patterns meet the criteria of being rigid- and flat-foldable networks of degree-4 vertices.

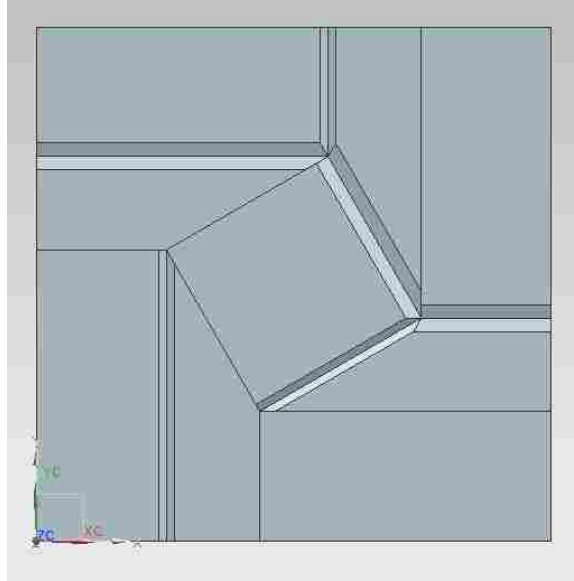


Figure 4.7: Example of a thick-origami model of a square-twist crease pattern.

Example Crease Patterns

The square-twist pattern, shown in Figure 4.7, provided a simple pattern for testing with only four vertices and two unique fold angles due to rotational symmetry. Variations of this square-twist pattern, that use a different angle of the center square, have also been tested successfully.

The program successfully handled a more complex pattern, still with only two unique fold angles, using a square-twist tessellation shown in Figure 4.8 and seen partially folded in Figure 4.9. The Miura-ori pattern, shown in Figure 4.10, demonstrates that the program can handle crease patterns with non-rectangular borders. The hexagonal-twist crease pattern, shown in Figure 4.11, was used to demonstrate that the program can successfully model thick-origami mechanisms with non-quadrilateral panels. Specifically, this pattern contains one hexagonal panel and several triangular panels.

An arbitrary degree-4 crease pattern, shown in Figure 4.12, was used to fully test the program. Figure 4.13 shows the model in a folded state. This pattern includes more than two unique taper lengths and fold angles, unlike each of the previous patterns. The additional taper lengths required further logic and were tested successfully. Each taper length relied on the fold angles and taper lengths of the previous vertex, necessitating the branching algorithm discussed in Sec-

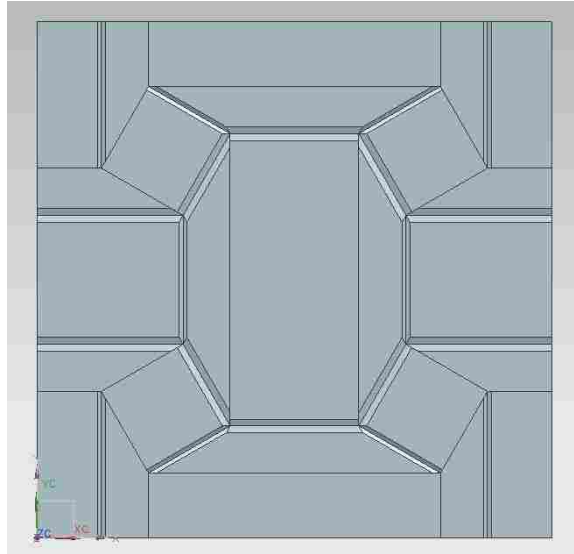


Figure 4.8: Example of a thick-origami model of a square-twist tessellation crease pattern.

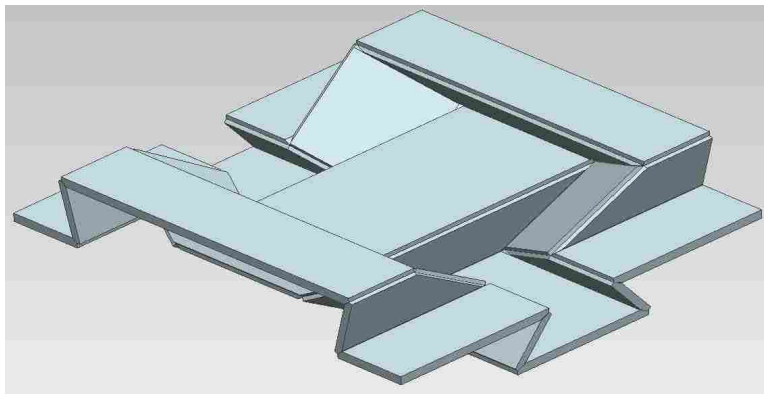


Figure 4.9: A folded model of the thick-origami square-twist tessellation shown in Figure 4.8.

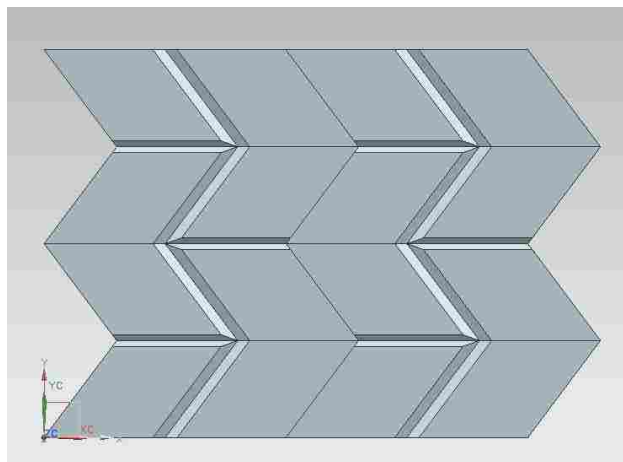


Figure 4.10: Example of a thick-origami model of a Miura-ori crease pattern.

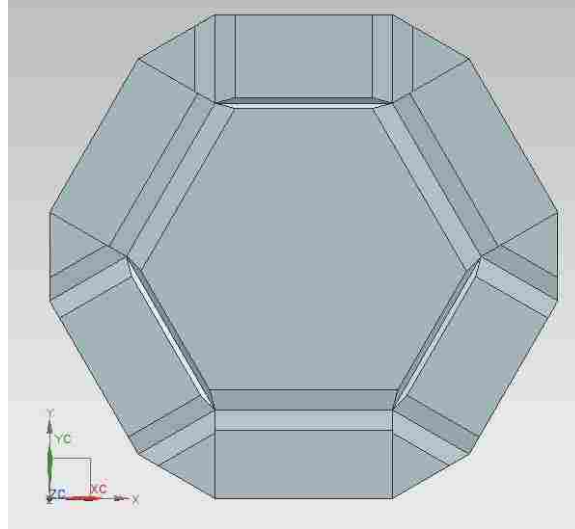


Figure 4.11: Example of a thick-origami model of a hexagonal-twist crease pattern.

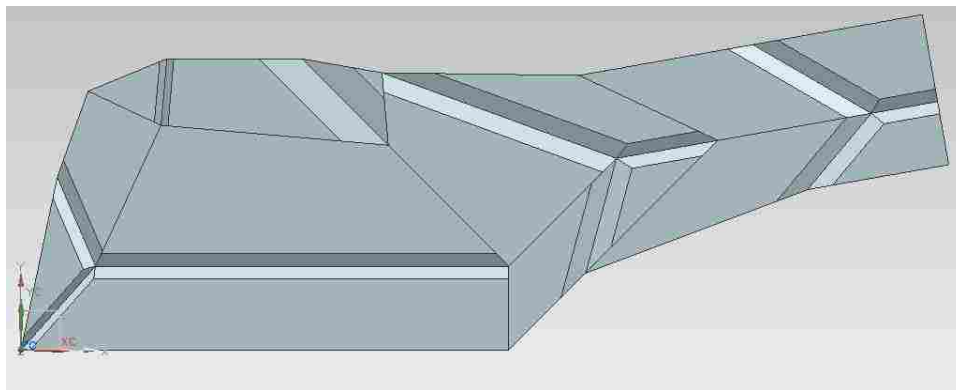


Figure 4.12: Example of a thick-origami model of a generic degree-4 crease pattern.

tion 4.4.1. Additionally, this pattern includes a branch of vertices to test the algorithm for finding each vertex and panel.

Time/Cost Savings

The program completes each of the discussed example crease pattern models in under 20 seconds. Comparatively, this is less time than most CAD modelers could complete assemblies of over eight components. Including the time required for calculating the various taper lengths for every single panel and modifying each component accordingly increases the manual modeling

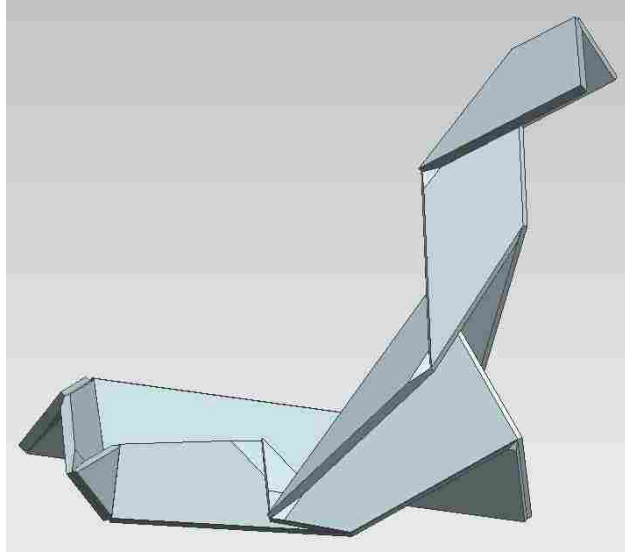


Figure 4.13: A folded model of the thick-origami mechanism shown in Figure 4.12.

time significantly. The program decreases both the modeling and calculation time, improving the time required for the overall design process.

Additionally, the program requires less expertise in the field of thickness-accommodated origami mechanisms to produce good results. Therefore, a wider variety of people will be able to create accurate 3D models of thick-origami mechanisms, enabling the growth of thick-origami design in industry.

4.5 Discussion

There are several advantages to this data structure, as well as some limitations, discussed below.

4.5.1 Advantages

The most noteworthy advantage of the presented program was its ability to drastically reduce the time needed to create a CAD model of a thick-origami mechanism. The time savings may enable researchers and designers to more fully explore the design space of various thick-origami mechanisms. Since the modeling time will be shorter, more designs can be explored in the same time that would previously be required to model one mechanism. This can help bridge

the gap between design and manufacturing in a variety of ways. If designs can be created more quickly, more thick-origami products may progress to the manufacturing state and full product realization. Similarly, this can help close the gap between research and product development in industry. A faster way to design thick-origami mechanisms makes it much more feasible for these mechanisms to be considered as viable designs. Furthermore, the framework demonstrated for automating thick-origami design will enable others to create automation systems for different thickness-accommodations techniques, eventually creating a robust set of methods for designing thick-origami mechanisms.

4.5.2 Limitations

The program presented in this work, though it exhibits many advantages, also has some limitations. Currently, the implementation is largely specific to the tapered panels technique. Also, it only works for crease patterns that are networks of degree-4 vertices, rigid-foldable, and flat-foldable. Crease patterns with an internal line co-linear with a border line are also improperly calculated as the co-linear edge will not have a taper, even though it should. However, this limitation is assumed to be rare in practice and can be easily avoided by a simple modification to the crease pattern.

While the program has limitations, it can handle a wide range of crease patterns, as demonstrated by the five test cases. Crease patterns outside of the specified restrictions (rigid-foldable, flat-foldable, network of degree-4 vertices) often have too many degrees of freedom to be handled well by CAD software. Moreover, the program is structured for the addition of future enhancements. Altering the code to handle patterns that are not flat-foldable, as well as patterns that contain degree-5 and degree-6 vertices could be implemented with reduced effort. Several other thickness-accommodation techniques would be straightforward to implement following the framework and data structures established.

4.6 Conclusion

This chapter has proposed and demonstrated a framework for automating the design of thick-origami mechanisms. This framework includes data structures for organizing the necessary information to automate the process. A program was implemented using the data structures to handle crease patterns fitting the following criteria: network of degree-4 vertices, flat-foldable, and rigid-foldable. The program correctly created 3D assemblies of five test crease patterns within the given criteria using the tapered panels thickness-accommodation technique. The five tests showed that the program and data structures can handle a variety of crease patterns with different border shapes, panel shapes, levels of complexity, and varying taper lengths.

The results of this implementation suggest that the framework can enable further implementations that automate the design of thick-origami mechanisms. The potential time savings of using an automated thick-origami generator have been demonstrated and show that using such a program can dramatically reduce the required time investment to design such mechanisms. Future work could continue to develop the potential of using data structures to automate the design of thick-origami mechanisms and further generalize the data structures and code to allow for use with other thickness-accommodation techniques, patterns that are not flat-foldable, and/or patterns containing higher degree vertices. Additionally, future work could implement intelligent selection of the starting vertex.

As this program and the data structures implemented in it are developed further and used in practice, some of the barriers preventing more widespread use of origami-adapted design will be mitigated.

CHAPTER 5. CONSIDERING MANUFACTURABILITY IN THE DESIGN OF DE- PLOYABLE ORIGAMI-ADAPTED MECHANISMS

Primary barriers to greater implementation of deployable origami-adapted mechanisms are their manufacturability and robustness. This chapter discusses manufacturability in the design of such mechanisms through presenting and examining three examples. Manufacturability lessons gathered from these examples include the importance of joint-panel interfaces and how techniques and approaches for origami-adapted design can be customized to meet the needs of a specific product. As the manufacturability of deployable origami-adapted products is addressed and improved, their robustness will also improve, thereby enabling greater use of origami-adapted design. This chapter was presented at a symposium and will be published in the proceedings [11].

5.1 Introduction

Origami has been increasingly applied in the realms of engineering, technology and mathematics. This ancient art of paper folding has found applications ranging from heart stents [30] to solar arrays [13] to kayaks [46]. Origami patterns that allow for movement are often well suited to application in engineering designs that require motion or the ability to collapse for compact storage. Such designs include many aerospace mechanisms where deployability is advantageous, if not necessary. Designs and products that do not directly apply an origami pattern but rather modify or adapt the pattern while still maintaining functionality are classified as origami-adapted [1]. One example of a deployable origami-adapted mechanism is shown in Figure 5.1.

This work aims to improve the robustness of deployable origami-adapted mechanisms by examining the manufacturing concerns associated with these mechanisms. As these concerns are addressed, a gateway is created to better origami-adapted deployable mechanisms that are more robust and easier to manufacture. Improving their robustness will lead to better deployable mechanisms that are suitable for use in more aerospace applications. This will enable the aerospace in-

dustry, including NASA, to accomplish things that were previously unattainable through the ability to use volume on spacecrafts more efficiently and have mechanisms with new useful functions that are robust and reliable.

Though several approaches have been developed to apply origami in materials that are not paper-like, issues still remain that hinder widespread use of origami-adapted design. Methods and techniques for accommodating thickness have been developed in response to the need for means to apply origami patterns using materials that cannot be approximated as having zero thickness, like paper [3]. Most of these thickness-accommodation techniques complicate manufacturing [12, 14] through high part counts, making many origami-adapted designs less practical and robust than designs that do not utilize origami. Key to the approaches is that the motion at creases in paper origami must be translated to “hinges” in thick origami. The challenge lies in finding a feasible fabrication approach that allows for efficient assembly of the product, yet still meets all the necessary motion and other design objectives. By addressing the fabrication concerns associated with deployable origami-adapted products, the robustness of these products can be improved.

The choices involved in deciding how to translate the motion at creases in paper origami to thick origami pose a significant barrier to successful implementation of origami-adapted designs. Ideally, an origami-adapted product would be manufactured from a single sheet of material. If the panel material is not conducive to compliant hinges, the entire origami pattern cannot be made of a single sheet of material. One such class of materials is metals; though metals can be in sheet form, they are not easily creased to create “hinges” that allow repeated motion [2]. Also, not all thickness-accommodation techniques allow for fabrication approaches that use a single flat sheet of material, such as stamping and laser cutting. For example, the offset-panel technique [5], which modifies the panel geometry through the addition of offsets to avoid panel interference, does not allow for fabrication using a single process because of the offsets.

High design complexity stemming from translating creases to hinges leads to low robustness in origami-adapted products. High part count in a design causes high complexity. Depending on the hinge choice, it is possible to have as many parts as there are faces and creases in the origami pattern. This high part count results in a large number of interfaces between parts. It is not uncommon for several panels in a pattern to have three or more interfaces. Each interface becomes a place where alignment error can be introduced, thereby causing a significant tolerance stack-up



Figure 5.1: Origami-adapted mechanism shown flat, partially folded, and folded.

over the whole product. Consequently, the robustness of the design suffers as the likelihood of joints binding or becoming too sloppy for proper functionality and deployment increases.

Because of all of the problems associated with hinges, many origami-adapted designs have not made it past the prototyping stage. The one-off prototypes made of foam board and tape make the issue of robustness apparent in all but the highest construction quality prototypes. These tape hinges do not usually translate well to final designs because hinges made of tape do not have sufficient performance for most applications.

There is a need for a clear way to address the manufacturing requirements of an origami-adapted product during the design phase. Improving the manufacturability of the design will lead to a more robust product. This chapter reviews and summarizes ways that manufacturability can be approached in the design of origami-adapted products.

5.2 Examples

This section reviews three examples of how manufacturability can be considered and the role it plays in the design of origami-adapted products. The first example is a manufacturing process that was developed to improve the manufacturability of fabric-based origami-adapted products. The second and third examples illustrate how manufacturing was considered for design of an origami-inspired ballistic barrier and two sheet metal origami mechanisms, respectively.

5.2.1 Manufacturing Process for Fabric-based Origami-adapted Products

Sandwich molding is a traditional process that is used for creating highly complex pleats in fabric. The current sandwich molding process involves creating complex molds by hand based on unfolded origami patterns. The molds are made from a stiff laminate and the pattern is made twice, creating two halves of a single mold. A piece of fabric is then placed in the mold and then the assembly is refolded. This assembly is then placed in an oven. Once cooled, the molds are removed from the fabric, leaving the pattern imprinted in the fabric. Due to the complexity of this process, each item is assembled, folded and removed by hand. The patterns that can be made are complex, but the requirement that each piece be handmade limits the scale of production and leaves many opportunities for errors. [47]

Pleat rolling is an industrial scale process used to manufacture patterned materials. The fabric or paper is put into a roller that crimps the material as it passes through, such as seen in the patent from Painter et al. [48]. Pleat rolling can produce large amounts of a simple pattern, but lacks the ability to create complex patterns yet.

For this example of origami and manufacturing, a manufacturing process is explained that was designed to have the same technical capabilities as sandwich molding in addition to being scalable to meet the demands of large scale industrial orders. By creating a two part mold that describes a partially folded pattern, it is possible to effectively stamp the material with the full pattern and set the default relaxed state of the material.

The steps of this manufacturing process, which is here called the *double mold* process, are as follows:

1. Mold: Design and machine the mold, consisting of two halves that mesh together, out of high density urethane foam (ex.: see Fig. 5.2).
2. Sandwich/Clamp: Sandwich the fabric between the two mold halves and clamp this assembly of the mold and fabric together to apply pressure. Preheating the molds before assembly and clamping aids the heat distribution during the heat-setting step.
3. Heat-set: Place the assembly in an oven for 30 minutes at 300°F to set the crease pattern in the fabric.
4. Cool: After removing from the oven, cool the assembly. Cooling time necessary will vary depending on the size of the molds and environment used for cooling; a large mold that has base dimensions of 12” by 12” requires approximately 2 hours in a refrigerator.
5. Release: Release the clamps and remove the finished patterned fabric.
6. Second Heat-set (optional): If a further folded set position is desired, fold the fabric to the collapsed configuration using the pattern already set in the fabric, then clamp the folded fabric between two pieces of a flat rigid material such as cardboard or metal (that can withstand the temperatures required for heat-setting the fabric). Then repeat the “Heat-set”, “Cool”, and “Release” steps.

A sample of fabric that has been heat-set using the double mold process (not including the optional second heat-set step) can be seen in Figure 5.3. This process has given the best results when using 100% polyester fabrics. Figure 5.4 shows a potential application for the double mold process: a reusable shopping bag that was heat-set once to imprint the pattern, fully folded using the pattern, then heat-set a second time to make it easier to fully collapse and roll up the bag for neat and compact storage.

There are a few key characteristics of the double mold process. One important characteristic of the process is that it allows for setting the default state of the material to an intermediate fold state, rather than completely folded or completely unfolded. There is, however, still the option of doing a secondary heat-set to set the default state at the completely folded state. This ability to have an intermediate fold state as the default may be very useful, if not necessary, for some



Figure 5.2: Two identical mold halves used for creating a Miura-ori pattern using the double mold process shown open and together.

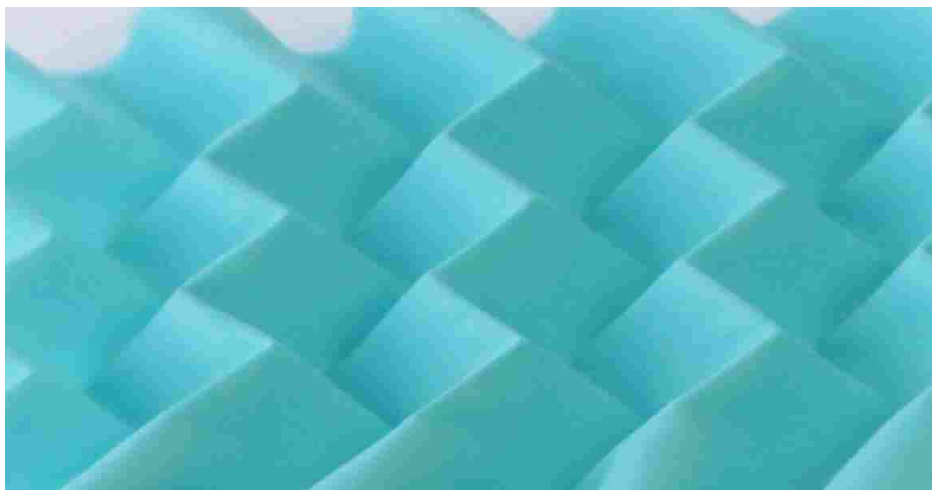


Figure 5.3: A sample of fabric that has been heat set using the double mold process with the mold shown in Fig. 5.2.

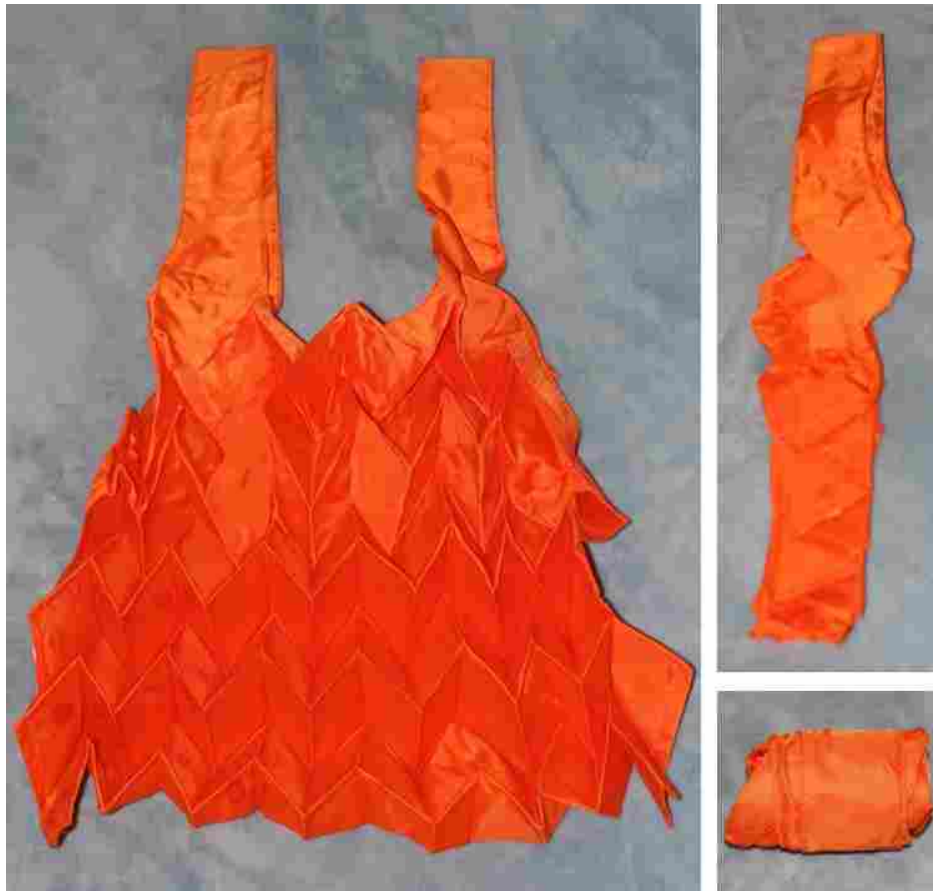


Figure 5.4: A reusable shopping bag that has been heat-set using the double mold process (including the second heat-set step) that can be easily rolled up after collapsing the origami pattern for simple storage.

applications. Another characteristic is that the process has the capability of working for complex origami patterns, not just simple pleats, while being a more scalable process than the traditional sandwich molding process that requires a great deal of time and skill.

5.2.2 Ballistic Barrier

In addition to providing ballistic protection, the origami-inspired barrier shown in Figure 5.5 was designed to be portable, deployable, and self-standing. For anything to be deployable, it must have hinges, joints, or some other way to allow for the deploying motion. As a good ballistic barrier ought not to have any significant weak areas, use of joints such as traditional hinges posed serious weaknesses that could be difficult to mitigate while maintaining portability. Therefore,



Figure 5.5: Origami-inspired ballistic barrier in the deployed state.

using ballistic fabrics, such as the aramid fabric Kevlar, as the material to provide both ballistic protection and motion at the joints made the most sense because such fabrics can also be flexible to allow for the necessary motion of deploying the barrier.

Manufacturability was a characteristic considered during selection of the origami pattern for use in the ballistic barrier design. The origami pattern selected can affect such characteristics as part count of the mechanism, what methods may be feasible for assembly and the order in which manufacturing operations can be done. The modified Yoshimura pattern that was chosen for the

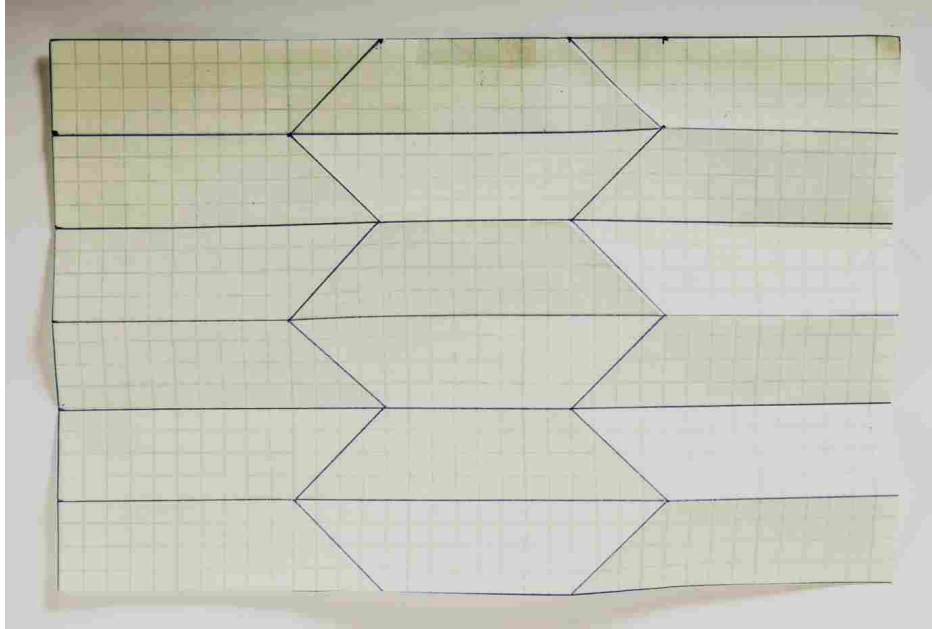


Figure 5.6: A modified Yoshimura pattern used for the design of an origami-inspired ballistic barrier.

barrier (seen in Figure 5.6) has a moderate number of panels while still meeting the stability and deployability design requirements.

A key challenge resulting from the use of fabric as the primary material for the barrier that involved considering manufacturability was how to make areas of the fabric rigid to be the panels of the origami pattern while still allowing for the flexibility necessary at the creases. Because the barrier design required twelve layers of aramid fabric, the layers must be bound together for function as a single mechanism. Many different methods of assembling fabric layers and achieving rigidity with local flexibility were explored and prototyped to determine a suitable method. A light rigid material was cut into panel shapes to give rigidity to the panel areas of the origami pattern.

The membrane thickness-accommodation technique [13] was used as a basis for determining how to accommodate for thickness in the barrier. However, the membrane technique alone was not sufficient for accommodating the thickness of the barrier because the membrane technique assumes that the “membrane” connecting the panels has negligible thickness as compared to the panels. The twelve layers of aramid fabric together were approximately twice the thickness of the center panel material being used, thereby not allowing for use of the membrane technique as it has been previously described in [13].

Another issue that arises when folding several layers of such fabric is that, if all layers have the same length of “folding section”, then the fixed length of the outermost layer causes bunching of the other layers as the radius around which they fold becomes smaller towards the inside of the fold. To address this, concepts from the strained joint technique [8] were used in conjunction with the membrane technique. By drawing upon both thickness-accommodation techniques, a different approach was developed that consisted of sandwiching the rigid panels between the layers of aramid fabric, six fabric layers on each side, and leaving sufficient space between the rigid panels for the fabric layers to fold. The gap between the panels provided space for the fabric to fill as it bunched during folding, thereby reducing the effect of bunching on the motion of the mechanism.

To bind all the layers of the mechanism together, a spray adhesive was used to bond the fabric layers together and to the rigid panels. The adhesive used was 3M™ Super 77™ Multipurpose Spray Adhesive. One advantage found when using a spray adhesive that never fully hardens is that some push and pull between the fabric layers can occur during initial folding of the mechanism. Therefore, the spacing between the rigid panels during fabrication does not require overly strict tolerances to still result in a functioning mechanism.

5.2.3 Sheet Metal Panels

Sheet metal manufacturing processes provide different capabilities on which manufacturing approaches can be based. A common and economical sheet metal process is bending. The “bent” panel approaches shown in Table 5.1 have been developed to take advantage of this process for origami-adapted mechanisms using sheet metal panels [10].

Figure 5.7 shows a mechanism that is based on the square twist origami pattern. The mechanism uses the hinge shift technique to accommodate for thickness and the associated bent panel approach to allow for use of sheet metal as the panel material. The design of the panels using the bent approach is not trivial as it requires careful consideration of where the panel material must be located to allow for the necessary interfaces with other panels while also avoiding intersection with other panels during the mechanism’s motion. The information needed to properly design the panels can be obtained from the thick origami model just created by accommodating for thickness using the chosen thickness-accommodation technique (not specifically considering sheet metal).

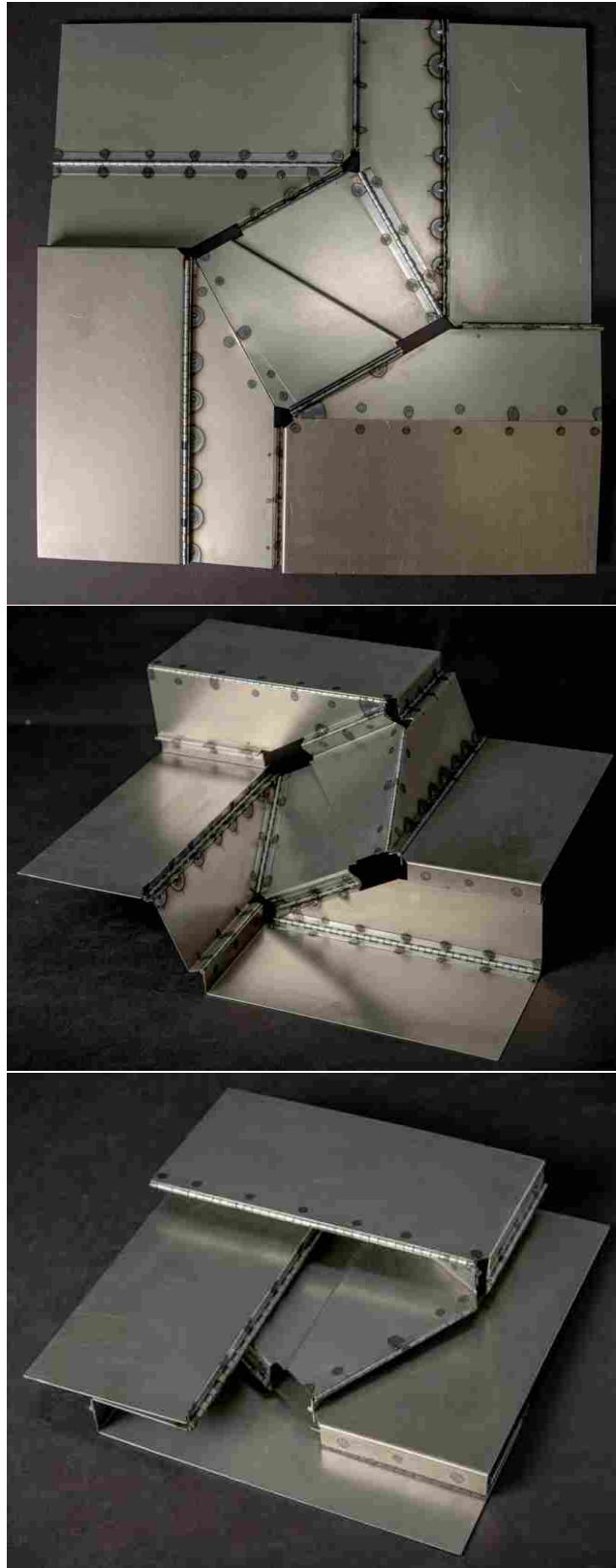
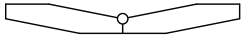
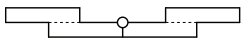
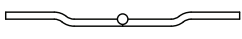
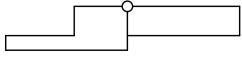
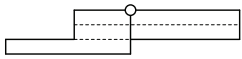
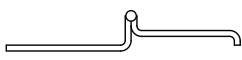


Figure 5.7: Sheet metal square-twist mechanism designed using the hinge shift technique shown unfolded, partially folded, and fully folded (this mechanism is also seen in Fig. 3.11).

Table 5.1: A comparison of panel approaches for two thickness-accommodation techniques for origami-adapted mechanisms. In the second column, dashed lines indicate layer divisions within the panels and small circles represent stock hinges. *Part count* indicates the relative part count of the mechanism where “baseline” is when the product has as many parts as the number of facets and creases in the paper origami model. If an approach is *conducive to sheet stock*, then materials in sheet form, such as sheet metals, can be easily used for the panels. The *second panel process* column lists any process in addition to a simple 2-D stock cutting process that is necessary to fabricate the panels. *Minimum number of processes* indicates the fewest number of distinct processes, including assembly, required to manufacture a mechanism using the given approach. (This table is a subset of what is shown in Table 3.1.)

Technique panel approach	Schematic representation	Part count	Conducive to sheet stock	Second panel process	Minimum number of processes
Tapered Panels					
monolithic		Baseline	No	Material removal	3
layered		High	Yes	Assembly	2
bent		Baseline	Yes	Bending	3
Hinge Shift					
monolithic		Baseline	No	Material removal	3
layered		High/Very High	Yes	Assembly	2
bent		Baseline	Yes	Bending	3

Shown in Figure 5.8 is another sheet metal mechanism. This mechanism, however, uses the Miura-ori tessellation as the base origami pattern and the tapered panels technique for accommodating thickness. This mechanism is somewhat simpler as each panel requires only an offset bend for the panel material to match the interfaces. However, another consideration of using a bent panel approach with the tapered panels technique is that panel material at the vertices must be cut away. If the material surrounding the vertex is not removed, then the mechanism will fail to function because of interference occurring between the panels.

In both of these sheet metal mechanisms, tolerances during assembly were a significant concern. Both the Miura-ori tessellation and square twist patterns are overconstrained by geometry. If it were not for the symmetry that exists in the patterns, the mechanisms would have zero (or negative) degrees of freedom, therefore not allowing for any motion. To avoid binding of any

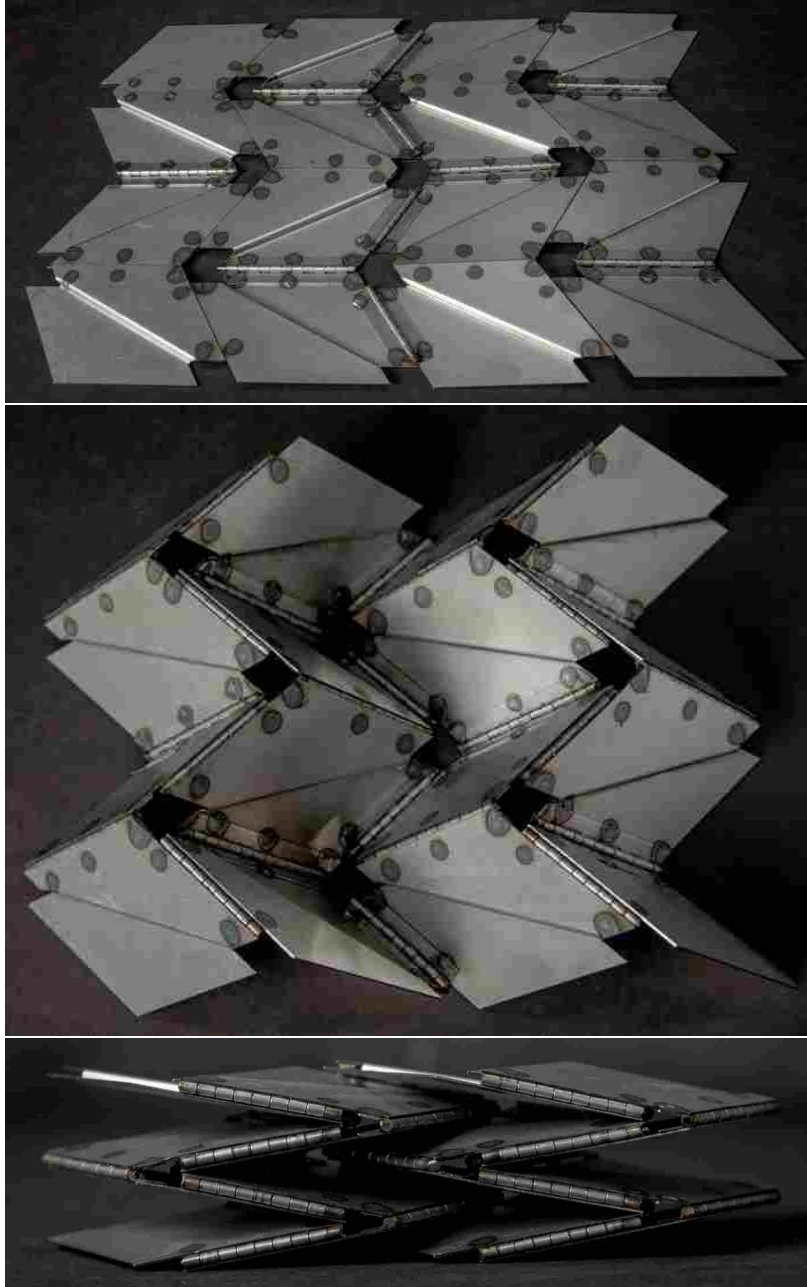


Figure 5.8: Sheet metal Miura-ori mechanism designed using the tapered panel technique shown unfolded, partially folded, and fully folded (this mechanism is also seen in Fig. 3.12).

of the hinges due to misalignment, the assembly of the hinges and panels was completed very carefully to ensure adequate tolerancing for the assembly to function.

5.3 Manufacturability Lessons

From the examples reviewed in the previous section, several lessons concerning manufacturability in origami-adapted mechanisms can be gathered. Some lessons are seen from the specific examples, whereas others can be gathered by examining more than one of the examples.

One lesson from the example of a manufacturing process for fabric-based origami-adapted products (Sec. 5.2.1), is that another viable way for modifying and processing sheet goods is by heating. This can be particularly useful when trying to impart patterns to fabrics and such materials that can be heat-set. Heat-setting, using a process such as the double-mold process, can also be used when an intermediate fold state, rather than the fully folded or unfolded state, is desired as the default state of a product. A key lesson from the ballistic barrier example (Sec. 5.2.2) is that thickness-accommodation techniques can be customized and hybridized for specific products. The approach of using a thick membrane that was employed in the design of the ballistic barrier can also be applied for other systems where use of flexible layers for the joints between panels is desired. From the example of using sheet metal panels in origami-adapted mechanisms (Sec. 5.2.3), there are lessons in how to address shapes and thicknesses of panels, but not hinges.

In both the ballistic barrier and sheet metal panels examples, there are such lessons as that the hinges/joints of an origami-adapted mechanism matter and can have a significant effect on the mechanism as a whole as well as the lesson that integration of the panels and hinges is a vital part of the design that must be considered. In the example of the ballistic barrier, the requirement that the joints also have adequate ballistic protection governed the design to result in a different technique to accommodate thickness. In the sheet metal panels example, the hinges posed challenges because of the tight assembly tolerances required to avoid binding of the joints in both of the over-constrained mechanisms. To mitigate those tolerancing issues, the lesson from the ballistic barrier example of customizing and hybridizing the techniques and fabrication approaches to suit the design-specific needs could be applied.

Another key lesson that can be seen from more than one of the examples is that a more broad system approach is useful in examining how each aspect of a design affects its manufacturability. Things that may not initially be seen as impacting manufacturability, such as the origami pattern used for the ballistic barrier, may indeed have significant affects. Continually considering how design decisions will affect the product's manufacturability throughout the design process and

making adjustments accordingly will help to improve the product's manufacturability and robustness.

5.4 Conclusion

Improving the manufacturability of deployable origami-adapted products is key to improving their robustness. This work has examined some ways in which manufacturability can be considered in the design of deployable origami-adapted products by reviewing examples and highlighting lessons related to manufacturability from these examples. These lessons included customizing the thickness-accommodation technique and fabrication approach for the specific product, using a holistic approach to improve manufacturability by considering how each design decision impacts manufacturability, and the interfaces between joints and panels are essential to proper functionality of a deployable origami-adapted mechanism.

Future work that remains to be done includes developing guidelines for considering and improving manufacturability of deployable origami-adapted mechanisms as well as examining options for panel-joint interfaces. As work to address manufacturability of these mechanisms progresses and their robustness improves, better deployable origami-adapted mechanisms will be developed that will enable new accomplishments in many fields, including aerospace.

CHAPTER 6. CONCLUSION

6.1 Conclusions

There are several thickness-accommodation techniques that have been developed, each with its own set of performance and manufacturing characteristics. A comparison and discussion of the techniques and their characteristics has been presented. There are trade-offs of performance and manufacturing between thickness-accommodation techniques and the differences in characteristics make some thickness-accommodation techniques more desirable for some applications over others. The tables of performance and manufacturing characteristics presented can be used as design tools for determining which thickness-accommodation techniques are appropriate for a given application.

The bent approaches for designing panels of origami-adapted mechanisms that were presented here can mitigate many of the performance and manufacturing trade-offs that exist with some thickness-accommodation techniques. The bending process used with sheet metal is what enables these bent panel approaches to have the favorable characteristics of both monolithic and layered panel approaches. Mechanisms using bent approaches can have as low a part count as monolithic panel approaches yet still be able to use sheet stock and have a simpler second panel process like the layered panel approaches. These bent panel approaches fill a gap in the design space and pave the way for origami-adapted products that use sheet metal panels.

Origami-based data structures can be used in the automation of thick-origami mechanism design. These data structures provide a way to gather and organize the information contained in an origami crease pattern. A program using these data structures that automates the 3D CAD model creation of tapered panel mechanisms has been demonstrated. By automating the process of modeling a thick-origami mechanism, origami-adapted design becomes more accessible to engineers as less time and thickness-accommodation expertise is required to create a model of a thick-origami mechanism.

By observing past origami-adapted designs, we have been able to glean several lessons that can be used in the improvement of origami-adapted design manufacturability. These lessons included customizing and/or combining thickness-accommodation techniques to meet the specific needs of the application, consideration of the interface between hinges and panels, and taking a system approach to how each design decision affects manufacturability of the design. As the manufacturability and, by extension, robustness of origami-adapted products is improved, origami-inspired design will be a more viable design approach.

6.2 Future Work

It is proposed that the following areas be considered for continued research to further increase the accessibility and feasibility of origami-adapted design in industry:

- Investigating options for hinges in origami-adapted mechanisms and how they can be integrated into the mechanisms: This would include investigating use of hinges aside from piano hinges and other traditional hinges as well as ways of attaching hinges aside from welding. Also, further analysis could be done in considering where some hinges in a pattern could be removed altogether because of overconstraints.
- Applying origami-based data structures to automating the design of thick-origami mechanisms using other thickness-accommodation techniques: This may include automating the design for crease patterns that have higher degree vertices and incorporating the offset panel technique as well as ways to approach design automation for thickness-accommodation techniques that involve modification of the crease pattern, such as the offset crease technique.
- Developing design for manufacture and assembly (DFMA) guidelines specific to origami-adapted mechanisms: Potential DFMA guidelines include: Use hybrid/composite thickness-accommodation techniques where advantageous; Choose appropriate panel design approach; Eliminate overconstraints in the mechanism; Selectively use surrogate folds; Consider assembly sequence; and Use jigs and fixtures as appropriate.

REFERENCES

- [1] Morgan, J., Magleby, S. P., and Howell, L. L., 2016. “An approach to designing origami-adapted aerospace mechanisms.” *Journal of Mechanical Design*, **138**(5), p. 052301. 1, 13, 30, 47
- [2] Francis, K. C., Blanch, J. E., Magleby, S. P., and Howell, L. L., 2013. “Origami-like creases in sheet materials for compliant mechanism design.” *Mechanical Sciences*, **4**(2), pp. 371–380. 1, 12, 16, 48
- [3] Lang, R. J., Tolman, K. A., Crampton, E. B., Magleby, S. P., and Howell, L. L., 2016. “A review of thickness-accommodation techniques in origami-inspired engineering.” *Applied Mechanics Reviews*, **TBD**(Accepted for Publication). 1, 3, 5, 13, 14, 30, 33, 48
- [4] Tachi, T., 2011. “Rigid-foldable thick origami.” In *Origami*⁵. CRC Press, pp. 253–264. 1, 5, 8, 15, 33
- [5] Edmondson, B. J., Lang, R. J., Morgan, M. R., Magleby, S. P., and Howell, L. L., 2016. “Thick rigidly foldable structures realized by an offset panel technique.” In *Origami*⁶, K. Miura, T. Kawasaki, T. Tachi, R. Uehara, R. J. Lang, and P. Wang-Iverson, eds. American Mathematical Society, pp. 149–161. 1, 5, 15, 48
- [6] Chen, Y., Peng, R., and You, Z., 2015. “Origami of thick panels.” *Science*, **349**(6246), pp. 396–400. 2, 5, 7, 15, 21
- [7] Lang, R. J., Nelson, T. G., Magleby, S. P., and Howell, L. L., 2016. “Thick rigidly foldable origami mechanisms based on synchronized offset rolling contact elements.” In *Proceedings of the ASME 2016 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference Paper #IDETC2016-59747*. 2, 5, 13
- [8] Pehrson, N. A., Magleby, S. P., Lang, R. J., and Howell, L. L., 2016. “Introduction of monolithic origami with thick-sheet materials.” In *Proceedings of the IASS Annual Symposium 2016 “Spatial Structures in the 21st Century”*, K. Kawaguchi, M. Ohsaki, and T. Takeuchi, eds. 2, 5, 13, 56
- [9] Morgan, J., Lang, R. J., Magleby, S. P., and Howell, L. L., 2015. “A preliminary process for origami-adapted design.” In *Proceedings of the ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference Paper #DETC2015-47559*. 2
- [10] Crampton, E. B., Magleby, S. P., and Howell, L. L., 2017. “Realizing origami mechanisms from metal sheets.” In *Proceedings of the ASME 2017 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference Paper #DETC2017-68025*. 3, 12, 56

- [11] Crampton, E. B., and Magleby, S. P., 2017. “Considering manufacturability in the design of deployable origami-adapted mechanisms.” In *Proceedings of the Utah NASA Space Grant Consortium 23rd Annual Fellowship Student Symposium*. 4, 47
- [12] Ku, J. S., and Demaine, E. D., 2016. “Folding flat crease patterns with thick materials.” *Journal of Mechanisms and Robotics*, **8**(3), p. 031003. 5, 8, 14, 32, 48
- [13] Zirbel, S. A., Lang, R. J., Thomson, M. W., Sigel, D. A., Walkemeyer, P. E., Trease, B. P., Magleby, S. P., and Howell, L. L., 2013. “Accommodating thickness in origami-based deployable arrays.” *Journal of Mechanical Design*, **135**(11), p. 111005. 5, 30, 47, 55
- [14] Trautz, M., and Buffart, H., 2013. “Construction approach for deployable folded plate structures without transversal joint displacement.” In *Proceedings of the First Conference Transformables*. 8, 14, 48
- [15] Francis, K. C., Rupert, L. T., Lang, R. J., Morgan, D. C., Magleby, S. P., and Howell, L. L., 2014. “From crease pattern to product: Considerations to engineering origami-adapted designs.” In *ASME 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers Paper #DETC2014-34031. 13
- [16] Evans, T. A., Lang, R. J., Magleby, S. P., and Howell, L. L., 2015. “Rigidly foldable origami gadgets and tessellations.” *Royal Society Open Science*, **2**(9), p. 150067. 14
- [17] Watanabe, N., and Kawaguchi, K.-i., 2009. “The method for judging rigid foldability.” In *Origami⁴*, R. J. Lang, ed., A K Peters, pp. 165–174. 14
- [18] Tachi, T., 2009. “Generalization of rigid foldable quadrilateral mesh origami.” In *Symposium of the International Association for Shell and Spatial Structures (50th. 2009. Valencia). Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures: Proceedings*, Editorial Universitat Politècnica de València. 14
- [19] Tachi, T., 2010. “Geometric considerations for the design of rigid origami structures.” In *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium*, Vol. 12, pp. 458–460. 14
- [20] Evans, T. A., Lang, R. J., Magleby, S. P., and Howell, L. L., 2016. “Rigidly foldable origami twists.” In *Origami⁶*. American Mathematical Society, pp. 119–130. 14
- [21] Ferrell, D. B., Isaac, Y. F., Magleby, S. P., and Howell, L. L., 2011. “Development of criteria for lamina emergent mechanism flexures with specific application to metals.” *Journal of Mechanical Design*, **133**(3), p. 031009. 16
- [22] Qattawi, A., Mayyas, A., Thiruvengadam, H., Kumar, V., Dongri, S., and Omar, M., 2014. “Design considerations of flat patterns analysis techniques when applied for folding 3-d sheet metal geometries.” *Journal of Intelligent Manufacturing*, **25**(1), pp. 109–128. 16
- [23] Qattawi, A., Abdelhamid, M., Mayyas, A., and Omar, M., 2014. “Design analysis for origami-based folded sheet metal parts.” *SAE Int. J. Mater. Manf.* 16

- [24] Qattawi, A., 2016. “Optimizing origami-based sheet metal parts using traversal algorithms and manufacturing based indices.” In *International Manufacturing Science and Engineering Conference*, American Society of Mechanical Engineers Paper #MSEC2016-8754. 16
- [25] Wu, W., and You, Z., 2011. “A solution for folding rigid tall shopping bags.” *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, **467**(2133), pp. 2561–2574. 16
- [26] Cash, T. N., Warren, H. S., and Gattas, J. M., 2015. “Analysis of miura-type folded and morphing sandwich beams.” In *Proceedings of the ASME 2015 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, American Society of Mechanical Engineers Paper #DETC2015-46380. 16
- [27] Schenk, M., Allwood, J., and Guest, S., 2011. “Cold gas-pressure folding of miura-ori sheets.” In *Proceedings of International Conference on Technology of Plasticity (ICTP 2011)*. 16
- [28] Ma, J., 2011. “Thin-walled tubes with pre-folded origami patterns as energy absorption devices.” *Doctoral dissertation, University of Oxford, Oxford, UK*. 16
- [29] Patel, J., and Campbell, M. I., 2008. “An approach to automate concept generation of sheet metal parts based on manufacturing operations.” In *Proceedings of the ASME 2008 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference* Paper #DETC2008-49648. 23
- [30] Kuribayashi, K., Tsuchiya, K., You, Z., Tomus, D., Umemoto, M., Ito, T., and Sasaki, M., 2006. “Self-deployable origami stent grafts as a biomedical application of ni-rich tni shape memory alloy foil.” *Materials Science and Engineering: A*, **419**(1), pp. 131–137. 30, 47
- [31] Rothmund, P. W., 2006. “Folding DNA to create nanoscale shapes and patterns.” *Nature*, **440**(7082), pp. 297–302. 30
- [32] Filipov, E. T., Tachi, T., and Paulino, G. H., 2015. “Origami tubes assembled into stiff, yet reconfigurable structures and metamaterials.” *Proceedings of the National Academy of Sciences*, **112**(40), pp. 12321–12326. 30
- [33] Tachi, T. Software: Freeform origami, origamizer, rigid origami simulator <http://www.tsg.ne.jp/TT/software/>. 32
- [34] Tachi, T., 2009. “3d origami design based on tucking molecules.” In *Origami⁴*, R. J. Lang, ed. A K Peters, pp. 259–272. 32
- [35] Lang, R. J., 2003. Treemaker <http://www.langorigami.com/treemaker.htm>. 32
- [36] Lang, R. J., 1996. “A computational algorithm for origami design.” In *12th ACM Symposium on Computational Geometry*, pp. 98–105. 32
- [37] Ida, T., Takahashi, H., Marin, M., Kasem, A., and Ghourabi, F., 2009. “Computational origami system eos.” In *Origami⁴*, R. J. Lang, ed. A K Peters, pp. 285–293. 32

- [38] Fastag, J., 2009. “egami: Virtual paperfolding and diagramming software.” In *Origami*⁴, R. J. Lang, ed. A K Peters, pp. 273–283. 32
- [39] Kling, D. H., 2005. Patterning technology for folded sheet structures, Aug. 30 US Patent 6,935,997. 32
- [40] Folded Structures Company LLC FoldStar <http://www.foldstar.com>, retrieved 2017-07-17. 32
- [41] Hasinoff, S., 2000. Exploring computational origami with ifold <http://people.csail.mit.edu/hasinoff/pubs/origami>, retrieved 2017-07-28. 32
- [42] Ku, J. S., 2016. Thick folding <http://jasonku.mit.edu/thick/>, retrieved 2017-07-28. 32
- [43] Ku, J. S., 2017. “Folding thick materials using axially varying volume trimming.” In *Proceedings of the ASME 2017 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference* Paper #DETC2017-67577. 32
- [44] Ku, J. S., 2017. Four crease hinge layout <http://jasonku.mit.edu/four-crease-hinge-layout/>, retrieved 2017-09-01. 32
- [45] Siemens NX Block UI styler https://docs.plm.automation.siemens.com/tdoc/nx/10/nx_api#uid:index_blockstyler:intro_int_ov_v1, retrieved 2017-08-10. 39
- [46] Willis, A. M., 2012. Collapsible kayak, Nov. 27 US Patent 8,316,788 B2. 47
- [47] Gardiner, M., 2013. *Designer Origami*. Hinkler Books Pty Ltd. 50
- [48] Painter, E. V., Grange, L., Mesek, F. K., Grove, D., and Shepherd, R. C., 1968. Method of pleating sheet materials US Patent 3,390,218. 50