2018-12-01

Origami-Based Design of Fold States and Stability

Alex Avila
Brigham Young University

Origami-Based Design of Fold States and Stability

Alex Avila

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Science

Spencer P. Magleby, Chair
Larry L. Howell
John L. Salmon

Department of Mechanical Engineering

Brigham Young University

ABSTRACT

Origami-Based Design of Fold States and Stability

Alex Avila
Department of Mechanical Engineering, BYU
Master of Science

Origami is a potentially elegant and powerful source of inspiration for many engineering designs. The viable shapes (fold states) of a single device allow it to perform multiple, seemingly contradictory, functions. The fold state is a large factor in the device's performance, but there are challenges in selecting and maintaining those fold states. In this thesis we analyze existing concepts for overcoming these challenges. Those concepts are compared with those that occur in origami-based devices. From this analysis fundamental gaps were identified, specifically, shortcoming in the terminology used to refer to (1) non-flat origami states and (2) sets of facets and creases. Likewise we found a need for a comprehensive categorization method of fold states. Fold states are divided into seven types based on the set of fold angles they contain: U, P, F, UP, UF, PF, and UPF. The origami-based devices are analyzed based on their functional fold states, showing an emphasis on P and PF fold states. The fold states and their functions are tabulated. We demonstrate the table as a tool in an origami-based design method.

Selecting fold states for an application is just the first step for effective use of origami. Once selected, the origami fold state must be maintained during use to perform its functions. This thesis also outlines the Origami Stability Integration Method (OSIM) for integrating a wealth of stability techniques. These techniques are categorized and analyzed to assist designers in selecting a technique for a device's application.

Both methods, the fold-state selection method and the OSIM, are demonstrated in designing an origami-based ballistic barrier. The barrier is designed to stow in a compact fold state and deploy to a partially folded state to provide protection during armed conflicts. Quick deployment and a stable structure make the barrier a valuable example of origami-based design, demonstrating these two methods in addressing some of origami's design challenges.

Keywords: origami, fold states, origami linkage, stability, design, spherical mechanism

ACKNOWLEDGMENTS

I would first and foremost like to thank my Heavenly Father for leading me through this process. My Advisor Dr. Spencer Magleby for his infinite patience and encouragement. My committee Dr. Larry Howell and Dr. John Salmon for their guidance. My family for supporting me in many ways. Dr. Robert Lang for his collaboration on the fold states paper. A special thanks is due to Jacob Greenwood for his invaluable help on the stability chapter. I would like to thank Kyler Tolman and Erica Crampton for the excellent ground work they provided in thick origami and their mentorship. Kenny Seymour for collaborating on the ballistic barrier and subsequent paper. Jared Butler, Todd Nielson, and Nathan Pehrson for their sagely wisdom. Hannah Lutz for artistic consultation. All the other CMR students for contributing through their positive attitudes and encouragement.

This work is based upon research supported by funding from the National Science Foundation and the Air Force Office of Scientific Research under Grant No.1240417. Any opinions, findings, and conclusions or recommendations expressed in this thesis are those of the authors and do not necessarily reflect the views of the National Science Foundation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1. INTRODUCTION

Origami is a powerful source of inspiration in engineering. Origami-based devices can be manufactured from sheet materials. It scales to apply to many fields, micro to macroscopic. Origami devices often have low degrees of freedom for simple deployment. Origami's changing properties and behaviors make it so a single origami-based device can perform functions that are normally contradictory. For example, the origami solar array large enough to power a space station folds small enough to fit in the cargo bay of a spaceship. The three-dimensional robot developed by MIT has spacial motion but was made from flat sheets of material [1, 2], and origami antennas can deploy to the range of reception frequencies [3].

Developments in origami are making it a valuable design tool. Researchers have developed programs for designing origami patterns based on mathematical models such as Treemaker by Lang [4] and Origamizer by Tachi et al. [5]. Other developments have focused on materials and manufacturing techniques. Most engineering materials do not have negligible thickness, so methods that accommodate for thickness have been heavily researched [6–9].

Thickness accommodation has led to other advances in manufacturing techniques, such as sheet metal origami and a volume-trimming fabrication technique [10, 11]. Advances in hinge materials have pushed the research of stimulus-driven origami devices that self actuate [1, 12]. Origami is finding its way into robotics, space exploration, medical technology, and many consumer goods [13–16].

Despite the many advances, origami-based design still has challenges and opportunities for exploration. A large portion of origami research has focused on origami patterns that can fold flat. However, the large range of three-dimensional origami shapes deserve focus. These 3D shapes can be structures, stools, receptacles, and boats, to name a few [17–20]. In contrast to the variety of non-flat origami application, scant research exists on how to create and maintain these states.

Chapter 2 in this thesis lays crucial groundwork for further study of these states, by developing a comprehensive method for classifying all rigidly foldable origami. In chapter 2 we categorize the shapes of origami, or their *fold states*, providing designers with the terminology to discuss non-flat origami and to evaluate existing origami beyond just its crease pattern. To further assist designers, we analyzed and tabulated the fold state and functions of 69 origami-based devices. These tables are a resource for designers to identify fold states with similar functions to those in their application. We demonstrate the use of these tables as they apply to the origami ballistic barrier [21].

Another difficulty in origami-based design is maintaining origami in a particular fold state. In many applications this is vital to the device's performance. For example, the origami star shade needs to maintain its shape at a high level of integrity to avoid creating artificial bright spots [22]. Many stability techniques for maintaining a fold state have been developed [8, 12, 23–33]. However, there is no method for choosing a stability technique or for integrating multiple techniques during the design process. Chapter 3 provides a method for visualizing and combining stability techniques. In Chapter 3 we also categorize and evaluate a number of techniques to facilitate their selection.

Chapters 2 is taken from a paper written in conjunction with Spencer P. Magleby, Robert J. Lang, and Larry L. Howell. It is planned for future publication in the journal of *Mechanical Science* [34]. Chapter 3 is material taken from papers written in conjunction with Jacob Greenwood, Spencer P. Magleby, and Larry L. Howell. Its location of publication is yet to be determined.

CHAPTER 2. ORIGAMI FOLD STATES

2.1 Introduction

Origami has become an effective source of inspiration for engineering solutions. The scalable nature of origami, its inherent motion, and potential for reconfigurability make its influence versatile and applicable in many fields. Origami has inspired a range of innovations including an origami-based helmet for emergencies [35], an unfolding telescope for space exploration [36], and re-configurable origami furniture for homes [37].

One reason origami is an elegant and natural source for solutions to many engineering challenges is that an origami-based device has different properties and behaviors in each fold state—an arrangement of the facets and creases along with its fold angles. The different properties of each fold state, such as shape, dimensions, and projected surface area, allow a device based on a single origami crease pattern to perform multiple functions. For example, an origami-based ballistic barrier was designed for storage and transportation in one fold state and then partially unfolded to provide ballistic protection in another fold state (see Fig. 2.1) [21].

Specific origami fold states, such as the unfolded and flat-folded states, have garnered interest in the artistic and mathematic fields because of their unique properties and behaviors. The properties and behaviors of these fold states make them especially useful for certain engineering applications. For example, engineers use unfolded states for planar manufacturing and flat-folded states for storage or transportation of devices.

While researchers have studied the unfolded and flat-folded states extensively, and engineers have used them in various devices [38–40], there are many other fold states that occur in origami art and origami-based devices. For example, the origami starshade is not completely flat-folded when it is stowed [41], and the origami-based ballistic barrier is not deployed completely flat. There are no widely accepted terms that refer to these fold states.

Figure 2.1: An origami-based ballistic shield in its closed state (top) and deployed state (bottom).

Lacking a comprehensive list of fold states, designers limit their ability to discuss many common fold states and risk neglecting fold states when designing origami-based devices.

In this chapter, we form a list of seven types of origami fold states. Examples of origami-based devices for each type of fold state are identified and discussed, along with their properties and behaviors. We demonstrate how fold-state types can be used in origami-based design. The terms, examples, and tables presented in this chapter offer a way to discuss origami fold states more thoroughly, identify uses for each fold state, and design origami-based devices.

2.2 Definitions

Origami nomenclature is an eclectic assortment of artistic, mathematical, and engineering terms [40, 42, 43]. Most terms are clearly defined, however,some are used inconsistently. For example, origami in the fully folded state can refer to origami that (1) can no longer fold in a fold direction [44], (2) is folded flat (all the fold angles are 180° or -180°) [45], or (3) is in its final fold state irrespective of its fold angles [46].

To create a comprehensive and consistent list of fold types, we must first define fold-angle sets and other ancillary terms. The first section contains those foundational definitions, and the second section defines the specific fold types and fold-state types.

The parentheses and braces used in the definitions and in the rest of the chapter follow list and set notation.

2.2.1 Foundational Definitions for Fold Types

A *crease* is a linear feature along which a fold takes place. A crease can be *unfolded*, *partially folded*, or *fully (flat) folded* [40]. While the term crease typically means a revolute hinge joint formed from material deformation, we use the term crease to refer to any revolute-like joint in an origami-based device, e.g. hinge joints, compliant joints, and rolling contact joints.

A *fold* is a crease with an associated fold angle or assignment (mountain or valley).

A *fold angle* is the signed angle between the normal vectors of two facets that meet at a fold [40].

An *origami fold state* or a *fold state* is a 2D or 3D arrangement of facets that are connected to each other by folds and vertices, plus any layer-order information for facets that are pairwise coplanar. Terms, such as origami configuration, origami figure, and folded form, have similar meanings with origami fold state, but in this chapter we will use the term fold state exclusively.

A *fold-angle set* is any subset of the fold angles of a fold state, together with the mapping between fold angle and fold. The fold number and corresponding fold angle are listed as a pair. For example, in Fig. 2.2, one fold-angle set of the fold state is $((1,-90^\circ),(3,-90^\circ),(4,135^\circ))$.

A *complete fold-angle set* of a fold state is a fold-angle set that includes all folds of the fold state. The mapping of the fold angle with a fold can be implicit. For example, rather than writing the complete fold-angle set of the fold state in Fig. 2.2 as $((1,-90^\circ),(2,-135^\circ),(3,-90^\circ),(4,135^\circ))$ we can write it as $(-90^\circ,135^\circ,-90^\circ,135^\circ)$.

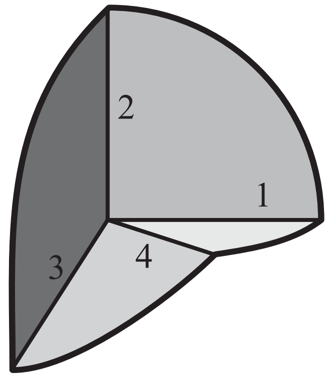


Figure 2.2: A degree four vertex shown in a partially folded state [40]. It has a complete fold-angle set of $((1,-90°),(2,-135°),(3,-90°),(4,135°))$.

2.2.2 Fold Types

A *fold type* is a property that conveys information about the value(s) of fold angles, fold-angle sets, or fold states. We specify whether a fold type refers to a fold, fold-angle set, or fold state by including it in the term, e.g. fold-angle type, fold-angle set type, fold-state type.

An angle or a fold-angle set is *unfolded* (U) if the angle or all angles in the set are in the set $U \equiv \{0°\}$.

An angle or a fold-angle set is *partially folded* (P) if all angles in the set are in $P \equiv (-180°,180°)\backslash\{0°\}$ (the open interval from $-180°$ to $180°$, with the value $0°$ removed).

An angle or a fold-angle set is *fully folded* (F) if all angles in the set are in $\{F \equiv -180°,180°\}$.

An angle or a fold-angle set is *flat-folded* if all angles in the set are in the set $\{-180°,0°,180°\}$. We do not discuss the flat-folded state further because we categorize the fold-angle sets it describes using two more specific fold types. See Fig. 2.4.

A fold-angle set is *mixed-folded* if its fold angles are not a single type. For example, the fold state shown in Fig. 2.3 is a mixed fold state because it has a complete fold-angle set ($90°$, $90°$, $90°$, $-180°$, $180°$), which come from two sets, the P and F.

We define the labels for the fold-angle sets based on the minimal sets (U,P,F) that include their fold angles. For example, we label a fold-angle set as U whose angles are in the U set, and UP labels a fold-angle set whose angles are in the $U \cup P$ set. There are three single fold-angle

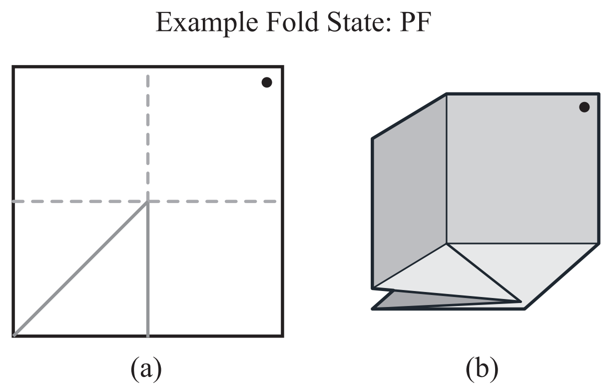


Figure 2.3: A crease pattern (a) with fold-angle assignment that will result in a mixed fold state (b). The fold state is a PF fold state with a complete fold-angle set of $(90°, 90°, 90°, -180°, 180°)$.

set types: U, P, and F. There are four mixed-fold-angle set types: UP, UF, PF, and UPF. Fig. 2.4 shows the relationship between fold-angle sets (capital Roman letters) and lists the minimal sets they include (italics and set notation).

A fold state is one of these types if its complete fold-angle set is that type. For example, a fold state with a complete fold-angle set of $(-180°, 180°, 180°, 180°)$ is fully folded (F). We label fold states with the same label as their complete fold-angle set.

2.3 Functions and Fold States of Origami-Based Devices

The value of fold states in origami-based design stems from the fact that fold states of a type share properties and behaviors characteristic of that type. Designers can then generalize those properties as they evaluate or design patterns for an application. The properties and behaviors of the fold state of a device determine, in part, how well the device performs the various functions of the application. This results in correlations between fold states of a device and functions it performes. We analyzed these correlations exhibited in a large number of origami-based devices, ranging from research concepts to commercially available products [1, 16–21, 30, 35, 36, 46–85].

The list of devices is not comprehensive, but we attempted to find a large variety by looking for devices in multiple fields (e.g. medical, outdoors, furniture, cooking, and space exploration) and devices that are unique in the functions they perform or the fold states they use. We believe the selection is a healthy, representative sample of the current technology. The devices were all

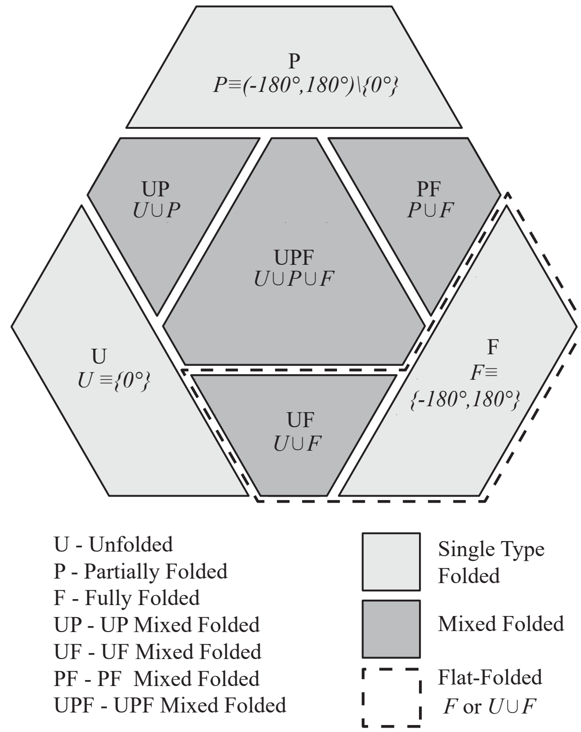


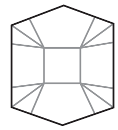
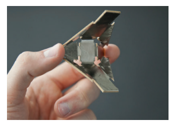
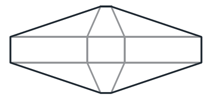
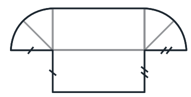
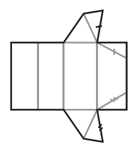
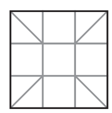
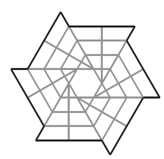
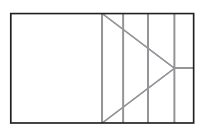
Figure 2.4: A graphical representation of how the fold-angle space is divided into sets (set notation and italic letters) and their corresponding fold-angle sets (capital Roman letters). The light gray regions are single type folded, the dark gray regions are mixed folded, and the two regions (F and UF) grouped using the dotted line are flat-folded.

found using a google and google scholar search and had to meet the criteria given in the following subsection.

2.3.1 Device Criteria

Devices in this study have at least one interior vertex (to exclude simple fan-folding devices), revolute-like joints and at least two different fold states (to exclude devices that look like origami but do not fold like origami, such as a cast ceramic figure). Table 2.1 gives an example of an origami-based device for each fold-state type. The table also shows the crease patterns for each device. Table 2.2 gives a complete list of the 69 origami-based devices in this study.

Table 2.1: An example of origami-based device for each fold state. Bowl (Fozzils™) [55], Forceps [21], Colander (B&R Plastics Inc™) [62], Glasses Case (Warby Parker™) [69], Camping Pot (Bear Minimum™) [56], Solar Array [86], and Tablet Case (Pipetto™) [81]. The hashes on the crease pattern indicate where facets are connected.

Fold State Type	**Origami-Based Device**	**Crease Pattern**
Unfolded Folded (U)	 Bowl	
Partially Folded (P)	 Forceps	
Fully Folded (F)	 Colander	
Mixed Folded (UP)	 Glasses Case	
Mixed Folded (UF)	 Camping Pot	
Mixed Folded (PF)	 Solar Array	
Mixed Folded (UPF)	 Tablet Case	

2.3.2 Function Criteria

Functions are divided into two groups: (1) those performed by the device (Table 2.4) and (2) those performed on the device (Table 2.3). For example, the origami blanket insulates a user; the function is performed *by* the blanket [16]. A worker assembles the fairing, the function is performed *on* the truck fairing [67].

Functions are labeled in Tables 2.3 and 2.4 using the functional basis developed by Hirtz et al. [87].

2.3.3 Fold State Criteria

Fold state(s) for each function were determined by using images or by folding the crease pattern. Only folds connected to an interior vertex were used to determine the fold state of the device. This is to avoid including simple flaps in the fold state. We list the device number in rows of the functions performed on or by the device. The column(s) correspond with the fold state(s) of the device.

2.3.4 Correlations Between Device Fold States and Their Functions

Given the emphasis on flat-foldable origami, Tables 2.3 and 2.4 illustrate some unexpected results. P and PF are the most common fold states and F and UF are the least common when a device performs a function (see Fig. 2.5). The frequent use of the P and PF states suggests that additional emphasis should be placed on researching ways to create and maintain P and PF states.

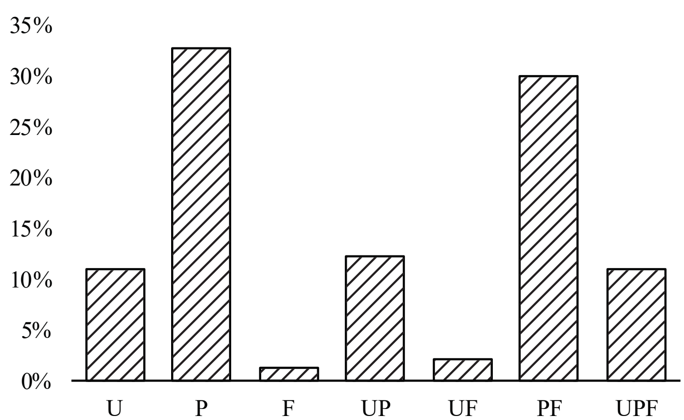


Figure 2.5: The frequency (as a percentage) of the device fold states when performing a function.

U, F and UF fold states are common states for storage and transportation (see Fig. 2.6). This is not surprising because these states often have high spatial densities.

Table 2.2: A list of the origami-based devices used in Tables 2.3 and 2.4, their device number and reference number.

Device Number	Product Description	Ref.
1	Antenna	[47]
2	Backpack	[21]
3	Baffling	[48]
4	Bag	[49]
5	Ballistic Barrier	[21]
6	Bath Tub	[50]
7	Bath Tub	[50]
8	Battery	[51]
9	Bellows	[52]
10	Blanket	[16]
11	Boat	[53]
12	Boat	[54]
13	Boat	[20]
14	Boat	[20]
15	Boots	[16]
16	Bowl	[55]
17	Camping Pot	[56]
18	Canoe	[57]
19	Chair	[58]
20	Chair	[59]
21	Chair	[60]
22	Chair	[61]
23	Colander	[63]

Device Number	Product Description	Ref.
24	Colander	[62]
25	Cup	[20]
26	Curtain	[64]
27	Cutting Board	[65]
28	Cutting Board	[66]
29	Fairing	[67]
30	Forcpdf	[68]
31	Glasses Case	[69]
32	Green House	[20]
33	Helmet	[35]
34	Ice Bucket	[16]
35	Kayak	[16]
36	Kiosk	[16]
37	Lamp	[20]
38	Lampshade	[70]
39	Phone	[71]
40	Planter	[16]
41	Ring Box	[72]
42	Robot	[1]
43	Robot	[30]
44	Shelter	[17]
45	Shelter	[20]
46	Shelter	[73]

Device Number	Product Description	Ref.
47	Shroud	[21]
48	Sofa	[74]
49	Sofa	[75]
50	Solar Array	[88]
51	Solar Array	[86]
52	Speaker	[20]
53	Spoon	[20]
54	StarShade	[76]
55	SunShade	[77]
56	Stent	[46]
57	Stool	[20]
58	Stool	[18]
59	Table	[20]
60	Table	[78]
61	Tablet Case	[79]
62	Tablet Case	[80]
63	Tablet Case	[81]
64	Telescope Lens	[36]
65	Utensils	[82]
66	Waste Bin	[19]
67	Wheel	[83]
68	Wheel	[84]
69	Wine Tote	[85]

We only listed fold states that were clearly used for manufacturing in Table 2.3. For example, devices manufactured from a planar sheet, such as the camping pot [56], require at least one manufacturing step in the unfolded state. Devices assembled from multiple individual parts, such as the fairing, do not have a clear manufacturing fold state. Of the fold states recorded for manufacturing, 80% were the unfolded state (see Fig. 2.7). This percentage is probably inappropriately amplified because manufacturing in the unfolded state is easy to positively identify. Nevertheless, the majority of the origami-based devices in this study are manufactured using at least one continuous sheet.

Table 2.3: We list each device's number in rows corresponding with the functions performed on the device and in columns of its corresponding fold states. See Table 2.2 for device numbers and references.

Functions *on* the Device	**U**	**P**	**F**	**UP**	**UF**	**PF**	**UPF**
Manufactured by Dividing Material	1, 8, 9, 11, 12, 13, 16, 17, 18, 21, 22, 25, 32, 35, 38, 39, 41, 47, 51, 64, 65, 66, 67	-	-	-	-	-	-
Manufactured by Joining Material	1, 2, 5, 8, 10, 17, 39, 42, 48, 52, 55, 56, 59, 60, 61, 65	1, 2, 9, 36, 66, 67	-	14, 49	12	33	-
Manufactured by Removing Material	43	-	-	-	-	-	-
Stored	4, 6, 11, 16, 19, 20, 23, 27, 28, 39, 43, 48, 63	-	1, 5, 24, 29, 31, 36, 41, 44, 45, 46, 47, 57, 58, 67	18	7, 11, 13, 14, 17, 33, 34, 35	25, 40	62
Transported	4, 11, 16, 20, 43	-	5, 44, 45, 46, 50	2, 4, 18, 30	7, 11, 13, 14, 17, 35, 55, 56	52, 54	-

2.4 Fold State Properties and Behaviors

In this section we discuss some common properties and behaviors of each fold-state type, as well as the functions performed by devices in these fold states. We will assume that the origami patterns are rigidly foldable and have zero thickness [43].

For each fold state, there is a figure containing a crease pattern on the left (a) and a corresponding fold state on the right (b). In these figures the dashed, solid, and dotted lines respectively represent unfolded, partially folded, and fully folded folds, as seen in Fig. 2.8.

Table 2.4: We list each device's number in rows corresponding with the functions performed by the device and columns of its corresponding fold states. See Table 2.2 for device numbers and references.

Functions *by* the Device	**U**	**P**	**F**	**UP**	**UF**	**PF**	**UPF**
Change Mechanical Force	-	30	-	-	-	-	-
Channel Liquid	-	-	-	56	-	-	-
Collect Electromagnetic Energy	50, 51	-	-	-	-	-	-
Contain Material	-	2, 7, 30, 40, 42, 43	-	2, 4, 40	-	6, 16, 17, 25, 27, 28, 40, 53, 66	34, 65
Convert Magneticomotive Force	-	42	-	-	-	-	-
Convert Rotational Angular Velocity	-	43	-	-	-	-	-
Decrement Electromagnetic Intensity	26, 54, 55	-	-	-	-	38	26
Decrement Pneumatic Pressure	-	-	-	29	-	-	-
Distribute Electromagnetic Intensity	26	-	-	-	-	38	26
Export Material	6, 16, 17, 23, 30, 65	7	-	24	-	-	-
Extract Liquid	-	-	-	24	-	23, 27, 28	-
Import Electromagnetic Intensity	-	-	-	-	-	55	-
Import Material	-	45	36	-	-	-	-
Increment Acoustic Pressure	-	-	-	52	-	-	-
Increment Electromagnetic Intensity	64	1	-	-	-	-	-
Inhibit Liquid	-	11	-	14	-	11, 12	13, 18, 35
Inhibit Material	27, 28	2, 5, 9, 15, 36, 45, 46, 47	-	2, 31, 44	-	-	-
Inhibit Mechanical Force	61	5, 7, 69	-	4, 31	62, 63	6, 33	-
Inhibit Thermal Heat Flow	10, 48	7, 32, 69	-	-	-	6	34
Measure Material	-	-	-	-	-	-	65
Position Material	43, 64	1, 37, 39, 41, 43, 68	-	52, 63	-	61	62, 63
Regulate Acoustic Pressure	3	-	-	-	-	3	-
Regulate Electromagnetic Intensity	-	1	-	-	-	-	-
Regulate Mechanical Torque	-	67	-	-	-	-	-
Rotate	-	9	-	-	-	9	-
Secure Material	-	15	-	-	-	33	-
Stabilize Material	-	5, 20	-	49	-	19, 21, 22, 48	-
Store Electrical Energy	-	-	8	-	-	-	-
Support Material	59	60, 67, 68	-	56, 63	-	61, 65	62, 63
Support Mechanical Force	-	11, 20, 45, 46, 58	-	14, 44, 49	-	11, 12, 19, 21, 22, 48, 57	13, 18, 35
Translate Material	-	42, 43	-	-	-	-	-
Transmit Electromagnetic Intensity	-	32	-	-	-	-	-
Transmit Mechanical Force	-	30, 67, 68	-	-	-	-	-
Transport Material	-	11, 69	-	14	-	11, 12, 53	13, 18, 35

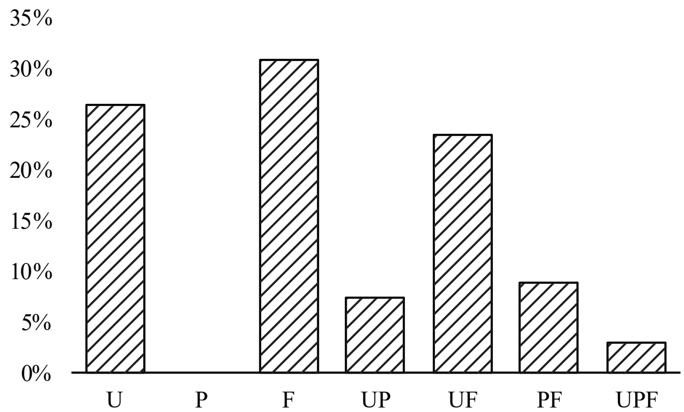


Figure 2.6: The frequency (as a percentage) of the device fold states when the device is transported or stored.

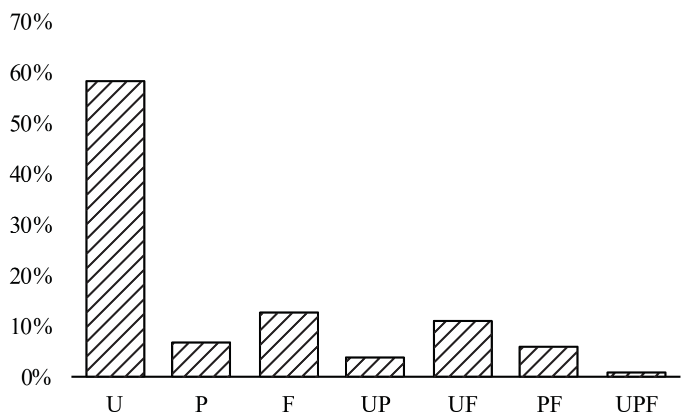


Figure 2.7: The frequency (as a percentage) of the device fold states when the device is manufactured. These values only represent devices with clear manufacturing fold states.

2.4.1 Unfolded State (U)

Unfolded states are essentially planar surfaces with designated crease lines. However, the unfolded state is important for several reasons: (1) the majority of origami-based devices are manufactured in the unfolded state from sheetlike materials, (2) unfolded states are bifurcation points in the origami's path of motion, (3) most crease patterns are a 2D embedding of the unfolded state, as seen in Fig. 2.9.

Figure 2.8: Legend giving the fold angles assignment of the example fold states.

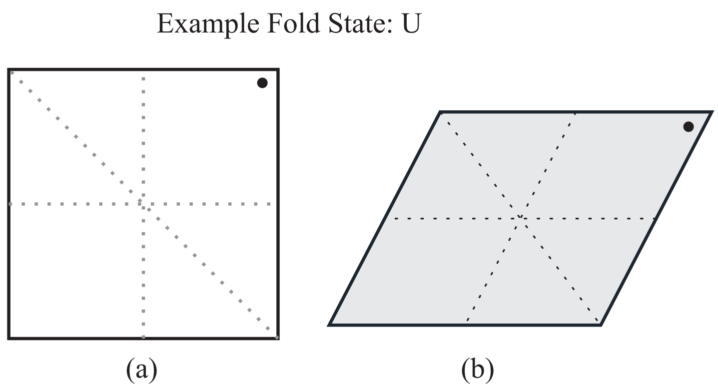


Figure 2.9: A crease pattern (a) with fold-angle assignment that will result in an unfolded fold state (b). The legend for fold-angle assignment is given in Fig. 2.8. Although the crease pattern appears the same as the fold state, the crease pattern is a planar embedding of the fold state.

All the creases in the unfolded state are coplanar, which creates the bifurcation point in the fold motion, as seen in Fig. 2.10 [26, 89]. In a degree four vertex in the unfolded state, the minor and major folds form simultaneously; this is not necessarily true for the UP, UF, or UPF states. When origami bifurcates, the fold order changes, forming new fold states with different properties [26].

Unfolded states are often used for storage and transportation because they have low thickness and high spatial density. However, they also typically have large dimensions. The unfolded state is especially common for devices that are transported right after being manufactured in the unfolded state.

Unfolded states have the largest projected area for a given origami pattern, which is one reason why it has been used for the space telescope, flasher solar array, and starshade [36, 41, 86].

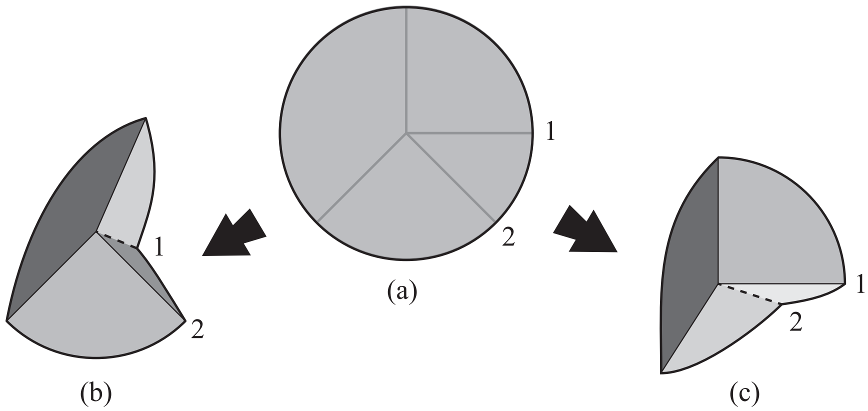


Figure 2.10: Crease patterns in the unfolded state (a) can bifurcate so that either fold 1 in fold state (b) or 2 in fold state (c) (represented by the dotted line) has a fold angle with an opposite sign to the other fold angles.

2.4.2 Partially Folded State (P)

For a given crease pattern there are a finite number of fully folded and unfolded states, which means that those states are especially useful for communicating information about a fold state. On the other hand, there is a certain amount of ambiguity communicated with the term partially folded state. This is because, unlike the unfolded and fully folded states, the partially folded is not based on a finite set of fold angles.

A partially folded state is guaranteed to have a non-planar shape because the facets of the origami are not coplanar. Likewise, the partially folded set *P* is the basis for three mixed fold states, which are potentially finite for a given origami pattern.

The partially folded state is the most common fold state for devices performing a function (see Fig. 2.5). This is because devices often perform functions that interact with our 3D world, not just in a plane. One example is loading conditions. The ballistic barrier is deployed to the partially folded state with an out-of-plane base to prevent tipping [21]. The partially folded state also provides higher stiffness in bending than the unfolded state.

2.4.3 Fully Folded State (F)

The fully folded state is distinct from other fold states because each pair of adjacent facets is coplanar and they intersect.

Example Fold State: P

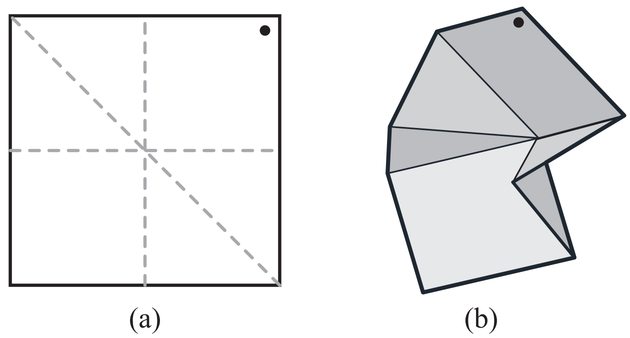


Figure 2.11: A crease pattern (a) with fold-angle assignment that will result in a partially folded state (b). The legend for fold-angle assignment is given in Fig. 2.8. The partially folded state is one of the most common fold states for performing a function. The fold state (b) shown in Fig. 2.9 has the same crease pattern, demonstrating how a single crease pattern can have multiple fold states.

In a zero-thickness model, the facets of the fully folded state intersect, creating a hard stop in one fold direction. By constraining the crease in the opposite fold direction, the crease is completely constrained, as seen in Fig. 2.12. When the loading on a fully folded crease results in a moment in the constrained direction, additional constraints might not be necessary. This loading situation is an elegant way to create a stable fold state without adding hardware.

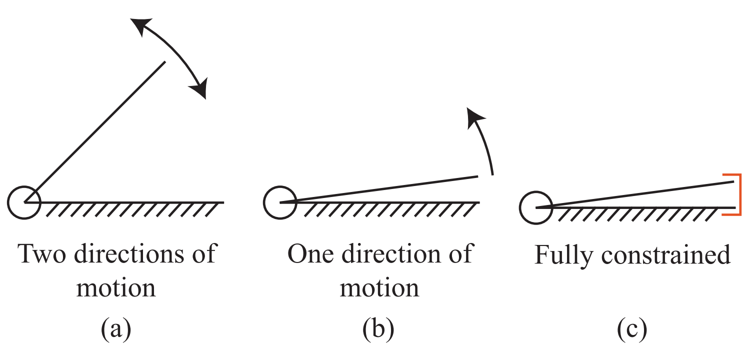


Figure 2.12: A crease can fold in two directions (a) until it reaches the fully folded state (b). In the fully folded state the motion of one facet is limited by intersection with the other facet. An external constraint in the opposite direction fully constrains the crease (c).

Designers often choose the fully folded state for storing or transporting a device because it typically has high spatial density and small dimensions relative to other states of the same origami pattern.

Only origami patterns that are flat-foldable have fully folded states.

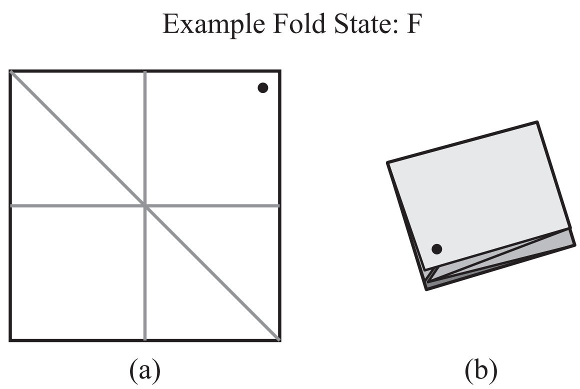


Figure 2.13: A crease pattern (a) with fold-angle assignment that will result in a fully folded state (b). The legend for fold-angle assignment is given in Fig. 2.8. The fully folded state is often used for transportation or storage, as seen in Fig. 2.6.

2.4.4 Mixed Fold States

Mixed fold states (UF, UP, PF, or UPF) are the groups of fold states that have fold angles from more than one set of fold angle values (U, P, F). Mixed fold states combine the properties and behaviors that come from having U, P, and F fold angles. For example, the UP, PF, and UPF fold states all have P fold angles and are non-planar fold states.

We discuss the mixed fold states that have unfolded fold angles (UP, UF, and UPF) as a group, because they share some similar properties and behaviors. Each mixed fold state is then discussed individually.

Unfolded Creases in Mixed Fold States

The UP, UF, and UPF fold states all contain at least one unfolded fold angle. These states are interesting because the unfolded state on first inspection appears to add no contribution to the overall shape of the fold state. The fold state would be the same shape if the two facets adjacent to the unfolded crease were combined into one facet, by removing the unfolded crease. For example, the fold state in Fig. 2.11 is the same shape as in Fig. 2.14. However, there are some reasons why an unfolded fold angle is included in a mixed fold state, as follows:

1. To make an origami pattern rigidly foldable. For example, the metal bag designed by Wu et al. [90].

2. To allow origami with the same overall shape an additional form of motion. For example, the fold state in Fig. 2.21 and 2.14 have the same shape, but only the fold state in Fig. 2.21 is flat-foldable.

3. As a by-product of the fabrication process. For example, pre-creasing in traditional paper folding.

4. Because the origami is reconfigurable and has inactive creases, such as the superimposed origami patterns by Liu et al. [91].

UP Mixed Fold State (UP)

The UP fold state occurs frequently in devices based on origami tessellations with many facets and degrees of freedom, such as the curtain, backpack, sofa, and bag [21, 49, 64, 75]. These devices are made of soft materials and are designed to conform to user input.

Devices with the UP fold state often are designed to have large cavities, such as the stent, shelter, planter, and tablet case. These devices maintain their shape because of global interference rather than the local interference of a fully folded crease.

Another common occurrence of the UP fold state is when the device forms a cavity and is non-developable, such as the colander, speaker, fairing, or glasses case [20, 62, 67, 69]. The unfolded crease allows the device to fold flat.

Sometimes the unfolded creases are prevented from folding by the partially folded creases and can only fold in the fully folded or unfolded state. This is illustrated in Fig. 2.15 and is used by Liu et. al [91].

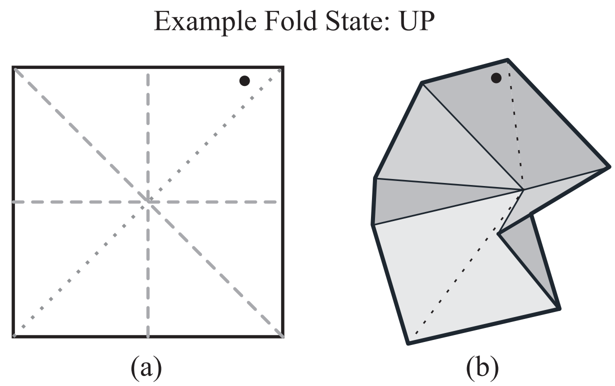


Figure 2.14: A crease pattern (a) with fold-angle assignment that will result in a UP fold state (b). The legend for fold-angle assignment is given in Fig. 2.8. The UP fold state is often used in devices that conform to the material they contain or support. It also occurs in devices with large cavities, especially those that are non-developable.

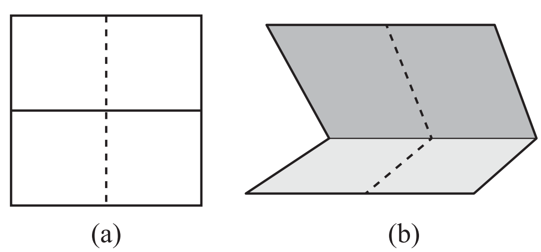


Figure 2.15: The UP state (b) based on the crease pattern (a), cannot fold along the creases represented by the dotted line because the creases are not collinear.

UF Mixed Fold State (UF)

The UF fold state is typically used for storage (Fig. 2.6) in two scenarios: (1) When a device is folded along one pattern to perform its main function and folded along another superimposed pattern for storage, such as the kayak and boats [16, 20]. (2) When a device is non-developable, such as the bath tub, ice bucket, and helmet [16, 35, 50].

All origami that folds sequentially passes through either the UF or UPF mixed fold states. This is because these states acquire an additional path of motion when two unfolded creases become collinear (assuming that the rest of the pattern allows folding). For examples, see Figs. 2.16 and 2.17.

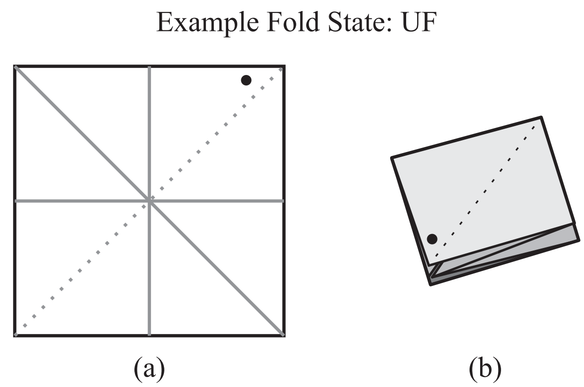


Figure 2.16: A crease pattern (a) with fold-angle assignment that will result in a UF fold state (b). The legend for fold-angle assignment is given in Fig. 2.8. The UF state can gain added degrees of freedom if the unfolded creases are collinear. For example, the fold state could fold along the dotted line.

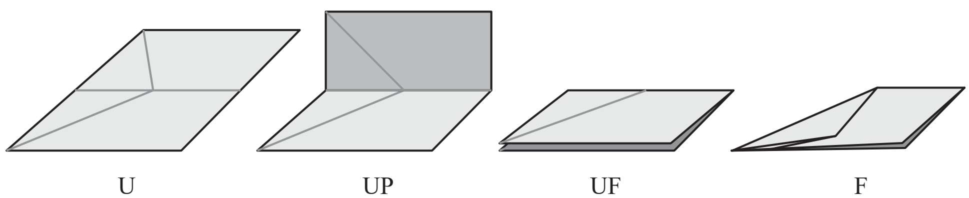


Figure 2.17: A degree four vertex that is sequentially folded. In the UF state the two diagonal creases are collinear which allows them to fold.

PF Mixed Fold State (PF)

The PF state is often used to create three-dimensional structures because the PF states share similarities with both fully folded states and partially folded states. The partially folded fold angles ensure a three-dimensional configuration and the fully folded fold angles decrease the degrees of freedom in a fold direction. See Fig. 2.18.

A single degree of freedom origami pattern has a PF fold state if it is not flat-foldable.

The fully folded fold angles results in small dimensions and high spatial density locally, but the partially folded fold angles ensure that the overall shape is non-planar. The combination of these fold angles makes this state a strong candidate for storage or transportation if the device will be stowed in a three-dimensional shape, as is the flasher solar array [86] or the origami planter [16].

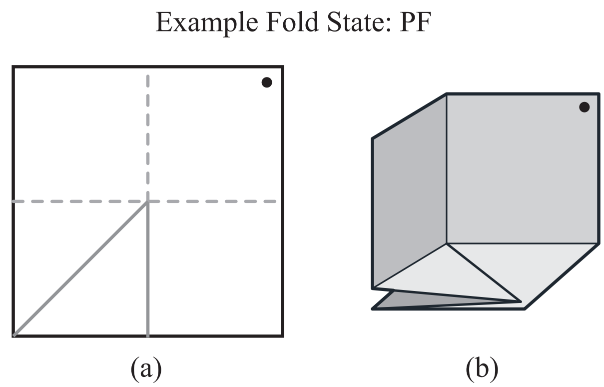


Figure 2.18: A crease pattern (a) with fold-angle assignment that will result in a PF fold state (b). The legend for fold-angle assignment is given in Fig. 2.8. The PF fold state is useful for creating 3D structures with reduced degrees of freedom.

Including fully folded fold angles in a fold state is an effective method for creating a specific shape. A fold state with two adjacent, fully folded angles forms a flap that is coplanar with an adjacent facet. This essentially removes the flap from the overall shape of the origami (as long as the flap is bounded by the facet). The seam formed at the base of the flap remains an unbroken surface, unlike if the flap were actually cut out.

The camping pot in Table 2.1 is an example of a device in which this can be useful. The corner facets are fully folded, making them coplanar with facets that form the side walls of the pot. This “removes” the corner pieces from the geometry to create the box shape while providing watertight seams, as seen in Fig. 2.19.

UPF Mixed Fold State (UPF)

The UPF state occurs frequently when considering the complete fold-angle set of the entire device; however, it seldom occurs in a single vertex. Of the eight devices that have a UPF fold state, only two have a vertex with unfolded, partially folded, and fully folded fold angles.

The tablet case in Table 2.1 is an excellent example of how three types of fold angles are combined to result in a fold state with specific properties and behaviors. The tablet case has unfolded fold angles because it is reconfigurable, partially folded fold angles because it is 3D, and fully folded fold angles which limits the fold motion to support the device.

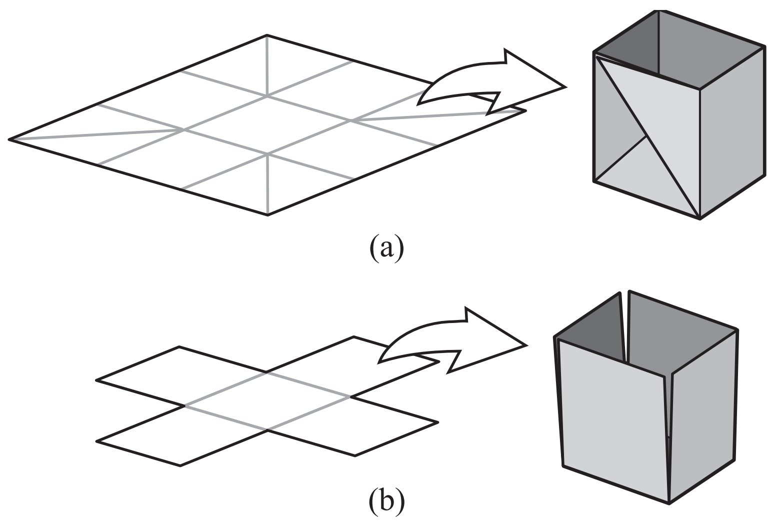


Figure 2.19: Both fold states shown in (a) and (b) have a similar overall shape when folded, but the fold state in (a) has a watertight seam because the surface is unbroken.

2.5 Designing Devices Using Fold States

One of the primary steps for designing an origami-based device is selecting an origami pattern. Fold states do not determine the origami pattern; however, they can be useful in directing a designer towards certain patterns. This is because some patterns are more conducive for achieving certain fold states. For example, the UP fold state typically occurs in non-developable origami patterns. Fully-folded states are only achieved by flat-foldable patterns. Mixed fold states with unfolded fold angles typically occur in patterns that are superimposed to be reconfigurable.

Two potential methods of selecting fold states are (1) selecting combination fold-angle types that will result in a fold state with desired properties, (2) using Tables 2.3 and 2.4 to find fold states that commonly perform the desired set of functions.

2.5.1 Composing Fold States from Fold Angles

We typically consider that a fold state defines the facets, creases, and complete fold-angle set. However, for design it is potentially useful to look at view the other way around, that the facets, creases, and complete fold-angle set defines the fold state. This allows a designer to look at each component individually to consider what properties or behavior it contributes to the whole.

As discussed in the previous section, each fold-angle type contributes distinct properties. When designing an origami-based device, an engineer can combine specific fold-angle types to result in a fold state with the those properties. Figure 2.20 shows an example of how individual fold angles are combined in the fold state of various vertices.

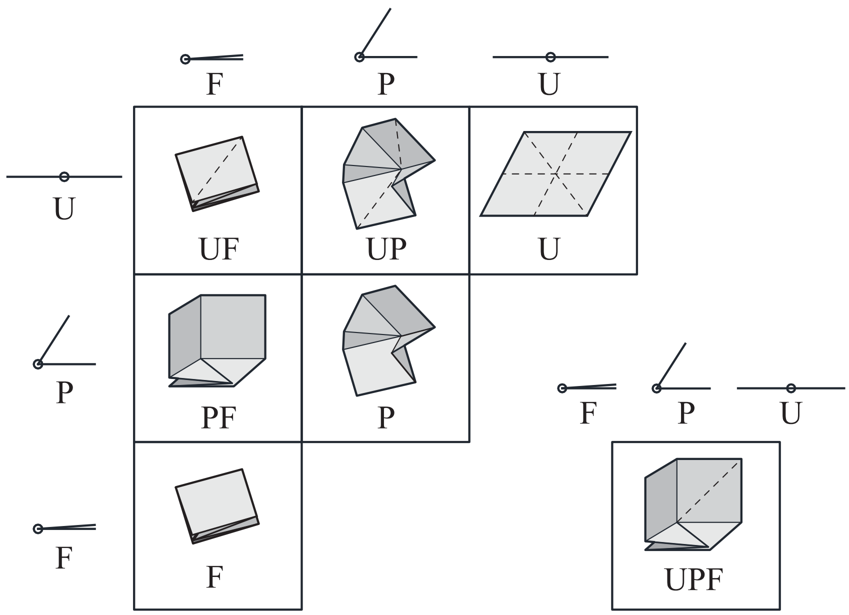


Figure 2.20: An example of how fold angles from each set (U, P, and F) can be combined to form different fold states. Each vertex is an example of one of the seven fold states.

For example, if we were designing an origami-based chair, we might select a partially folded fold angles so the chair has a 3D shape and fully folded fold angles to limit the chair's fold motion. If the U and P fold angles were the only types of fold angles in the complete fold-angle set, the fold state would be a PF fold state. This indicates that the PF fold state is a potential fold state for the device.

2.5.2 Using the Tables to Select Fold States

By referring to Tables 2.3 and 2.4 designers can limit the fold states to consider by looking at which ones are commonly used for a similar function. These fold states are likely to have desirable properties. By identifying fold states commonly used by the functions performed simultaneously, a designer can further limit the number to consider. An example of this process:

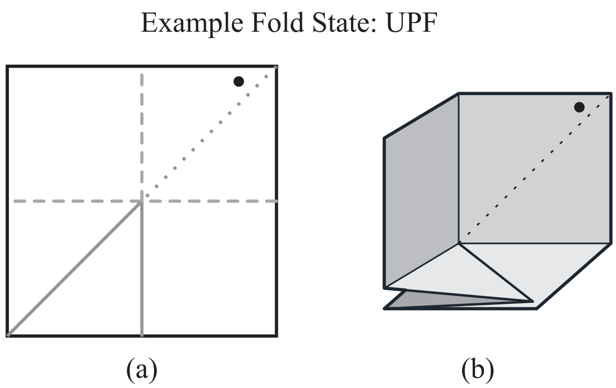


Figure 2.21: A crease pattern (a) with fold-angle assignment that will result in a UPF fold state (b). The legend for fold-angle assignment is given in Fig. 2.8. The UPF state has at least two pairs of coplanar facets, one pair separated by a fully folded crease and another by an unfolded crease.

1. List device functions.

   Function 1
   Function 2
   Function 3
   Function 4

2. Group functions that the device performs simultaneously.

Group 1	Group 2
Function 1	Function 2
Function 4	Function 3

3. Using Tables 2.3 and 2.4, list fold states commonly used for each function.

Group 1	Group 2
Function 1: U, P, F	Function 2: P, UP, PF
Function 4: P, F, PF	Function 3: U, F, UF

4. Identify fold states that are shared between grouped functions.

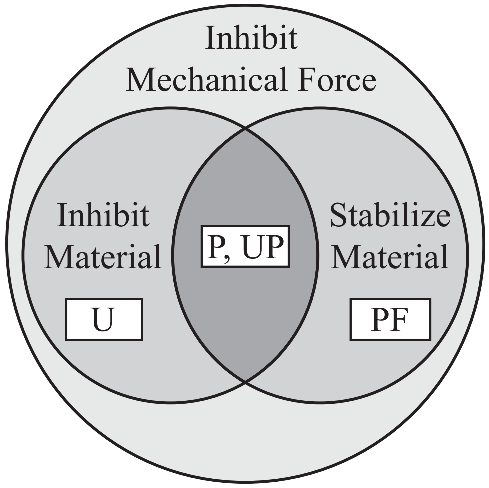


Figure 2.22: Devices in the P and UP fold states commonly perform the functions: inhibiting material, inhibiting mechanical force, and stabilizing material. The Venn diagram shows the overlap of the fold states between these three functions.

Group 1	Group 2
Functions 1 & 4: P, F	Functions 2 & 3: None

Fold states P and F are candidate fold states for performing functions 1 and 4. The designer could then consider the benefits of each fold state or simply begin investigating origami patterns that can achieve either fold state.

Functions 2 and 3 do not share any fold states in common, so the designer will have to evaluate which function is the most critical for the device, or decide which function will be less affected by the fold state.

This process could be used for designing the ballistic barrier. See Fig. 2.1. The barrier needs to: (1) inhibit material (stop bullets), (2) inhibit mechanical force (stop bullets), (3) stabilize material (stand on its own), and (4) be stored. From Table 2.4 the designer can see that fold states U, P, and UP have been used for inhibiting material; U, P, UP, and PF for inhibiting mechanical force; and P, UP, and PF for stabilizing material; U, F, UP, UF, PF, and UPF for storage.

Since the device will need to perform the first three functions simultaneously, the designer could select only fold states that are shared among all of the functions—P and UP (see Fig. 2.22). The designer selects the P fold state because the UP fold state can have additional forms of motion, which would be undesirable.

Tables 2.3 and 2.4 can also initiate out-of-the-box thinking by indicating less common fold states for a function. For example, the barrier designer could consider potential advantages that come from using the UF or F fold states. In these states the barrier would have multiple layers, meaning it would provide changeable levels of ballistic protection. However, it is also important to consider why these fold states are not currently used to perform a function. For example, a barrier in the UF or F fold states could provide less coverage than the same pattern in another fold state.

2.6 Conclusion

Origami fold states communicate valuable information about the properties and behaviors of origami. The list of fold-state types based on fold-angle sets is comprehensive of all fold states. Although the fold angles of a fold state do not determine all of the properties and behaviors the fold states of each type are similar in a way that makes them useful for performing similar functions. Future work could be done on integrating other properties of the fold states, such as facet dimensions, and sector angles in future categorizations.

From the 69 origami-based devices analyzed, we found strong correlations between the devices' fold states and their functions. For example, non-planar fold states are most common for performing functions, while planar fold states are typically used for transportation, storage, and manufacturing. We anticipate these particular trends will continue in future origami-based devices, making them useful for identifying research directions.

Fold states for origami-based design can be selected, using the properties of individual fold-angle types or by using the correlations established in this chapter. We illustrate both methods, selecting a fold state for an origami-based chair and ballistic barrier.

Fold states provide a way to think about and discuss the fold states used in origami-based devices. The properties and behaviors associated with fold states make them a valuable tool in origami-based design.

CHAPTER 3. ORIGAMI STABILITY

3.1 INTRODUCTION

Many people think of origami simply as the art of folding paper. However, in recent years, origami has inspired designs that surpass this traditional definition—designs extending well beyond art pieces using materials other than paper. In a broader sense, origami is the manipulation of planar materials using folds. Origami has been a source of inspiration for a wide variety of devices, from tiny, minimally invasive surgical tools [68] to massive folding solar arrays [88], and from ballistic barriers [21] to baby baths [50].

Many advantages of origami capitalize on the shape-changing ability of origami—how origami moves as it folds. Devices based on origami can be simple to manufacture, starting in a planar state then folding into shape [10,30,33]. Origami can provide complex motions with simple actuation and low degrees of freedom (DOF) [8,9,92]. This allows a device to quickly transform from a compact shape to a deployed shape. The simple actuation of origami allows it to be scaled for applications—massive or miniature.

While the foldability of origami is key to many of its benefits, foldability is also a challenge when trying to maintain a device in a desired state. For example, how does one make an origami-inspired stool that can support a person's weight, but also folds up and stores flat? Without careful planning, the folds that allow an origami-based product to actuate will decrease the stability of the product and cause it to fail its function. Many techniques for achieving stability have been implemented in existing products and explored in literature [8, 12, 24–33, 93]. A wealth of techniques and a number of reviews [12, 94] have been presented, and the field would benefit from a comprehensive method for designing stable equilibrium at specific fold states in origami. Additionally, the techniques that are used could be categorized for design purposes.

This chapter will provide a new way for designers to integrate stability techniques in origami-based designs by: (1) reviewing origami vocabulary and presenting new terms, (2) in-

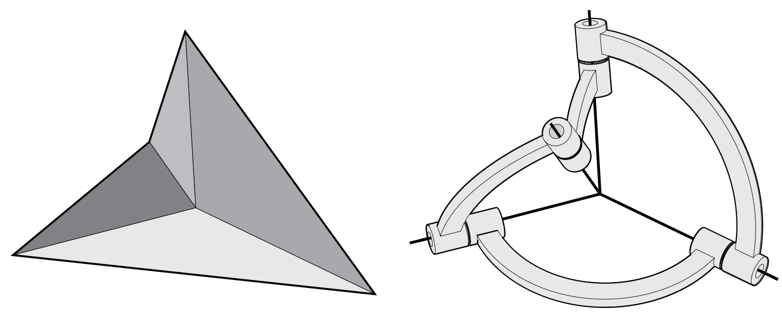


Figure 3.1: The motion of an origami vertex can be modeled as a spherical mechanism.

troducing the *origami stability integration method (OSIM)*, a method for planning stable states in origami, and (3) evaluating and classifying stability techniques, specifically how they can be used in conjunction with the OSIM. The OSIM and various stability techniques are also illustrated in the origami ballistic barrier [95].

3.2 Background

Researchers have shown that origami can be modeled as a kinematic linkage [96–98]; the facets (or panels) are modeled as links, the creases as hinge joints, and origami vertices as spherical mechanisms [27], as shown in Fig. 3.1.

This relationship between origami and kinematic linkages provides a useful framework for evaluating origami stability in two main ways:

1. Applying stability techniques from traditional linkages to origami (e.g. using the Grübler criterion to analyze the degrees of freedom of origami [99]).

2. Identifing the kinematic properties of stability techniques found in existing origami-based products (e.g. modeling the folds of non-rigidly foldable origami as an over-constrained linkages with compliant members [42]).

Many of the techniques for creating stability focus on strain energy stored in the joints during folding [26, 45, 100–104]. Other techniques introduce strain energy at the joints using other stimuli, such as heat, magnetic fields, or surface tension [12, 30–33]. One benefit of stimuli-driven techniques is that fold angles are often controlled by adjusting the amount of exposure to the stimulus [30]. Some of these techniques even have reversible folding capabilities [31, 105].

Figure 3.2: The origami kayak (Oru Kayaks™) uses several stability techniques such as supporting bulkheads, straps, and retaining channels to keep its folded state. Image downloaded from www.orukayak.com December 3, 2018.

In origami-based products, several alternate techniques for creating stability are used, such as: clips, magnets, actuators, and other constraints [16, 20]. For example, origami kayaks use supporting bulkheads, straps, and retaining channels to keep their folded state. (Installation of the retaining channels is shown in Fig. 3.2 .)

While there are a variety of stability techniques and even a review of many of them [12], there is no method that integrates the techniques comprehensively for planning stable states in origami-based products.

3.2.1 Visualizing Stability

A *stable equilibrium* is a state of equilibrium of a body (such as a pendulum hanging directly downward from its point of support) such that when the body is slightly displaced, it tends to return to its original position [106]. The method presented here can be modeled using the common "ball on a hill" method, where all energy in the system is modeled as potential energy [107]. The stable equilibria are the local minima, as seen in Fig. 3.3.

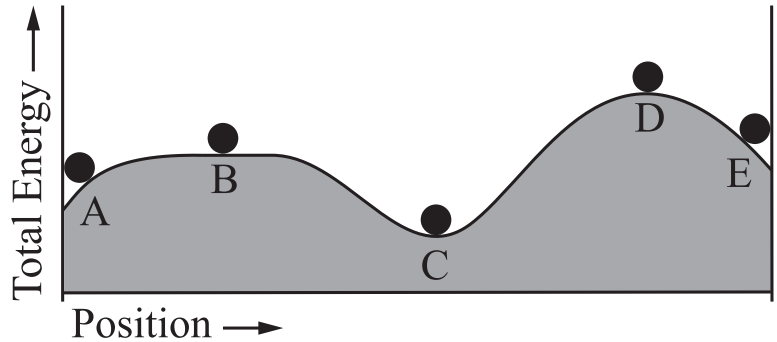


Figure 3.3: A "ball on a hill" diagram for visualizing stability through potential energy. Positions A, C, and E are stable equilibrium positions. Position D is an unstable equilibrium position. Position B is neutrally stable.

3.3 Origami Linkage

Discussing origami as a kinematic linkage, using the common, existing terms for origami can be difficult because the terms either do not refer to a generalized set of facets and creases, or they imply static fold angles. For example, an *origami vertex* refers to facets and creases independent of fold angle, but is limited to only those sets whose creases all meet at a single point. Other terms, such as *origami figure*, *configuration*, and *fold state*, refer to general sets of facets and creases. However, these terms, by definition, have static fold angles.

The term *crease pattern* is often used to refer to a set of facets and creases. However, a crease pattern is a mapping of creases but not the facets and creases themselves [40].

Referring to a set of facets and creases as a fold state is akin to referring to H_20 by one of its phases: steam, ice, or liquid water. Referring to a set of facets and creases as a crease pattern is akin to referring to a 2D diagram of the molecule as the molecule itself. In cases where the phase is unknown or unimportant, or when referring to the molecule itself, a general term is needed, H_20. The term *origami linkage* is proposed as a general term to refer to any set of facets and creases.

3.3.1 Definition

An *origami linkage* is a set of planar facets and their creases. This term builds on the traditional use of the term linkage in engineering to emphasize an undefined position and to strengthen the concept of origami as an engineering tool.

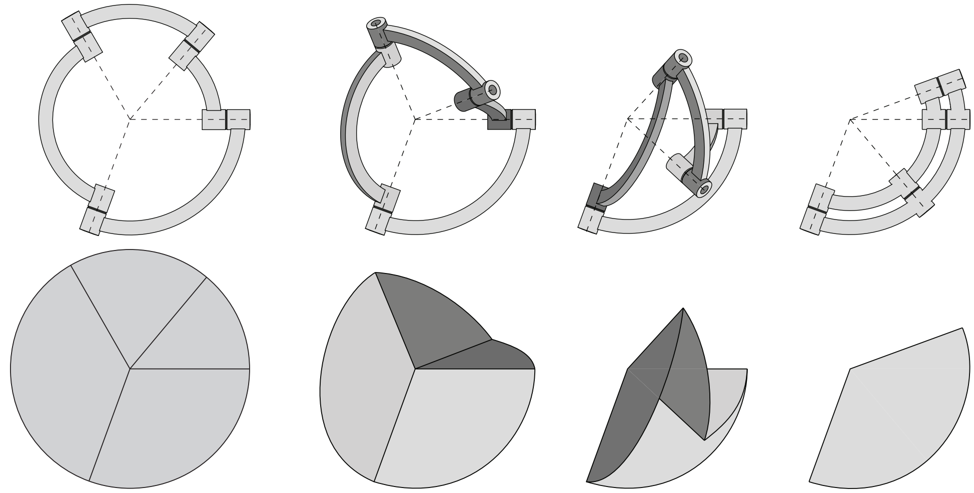


Figure 3.4: An origami linkage (bottom) with a kinematically equivalent spherical linkage (top). This origami linkage is also an origami vertex because all creases intersect at a single point.

The term origami linkage works well with existing terms. Origami vertices are a subset of origami linkages whose creases all meet at a single point (see Fig. 3.4). The Miura Ori, Yoshimura, and Square Twist are other subsets of origami linkages that satisfy specific vertex and sector angle requirements. A fold state is an origami linkage in a defined position (see Fig. 3.5).

3.3.2 Selecting an Origami Linkage for Origami-Based Design

Often an origami linkage, in rough or final form, is conceived during the concept development stage of the product design process. An origami linkage can also be selected based on its properties or the properties of its fold states. For example, if the product needs to fold completely flat, then an origami linkage that is flat foldable should be chosen. Resources for origami linkage design are given by Morgan et al., Hernandez et al., and Lang [34, 108–110].

3.4 Fold-State Continuum of Origami Linkages

As an origami linkage folds, it progresses through an infinite number of fold states; any continuous set of fold states forms a continuum. Often, specific ranges of fold states are useful for a designer. Some common examples of these ranges include the states near the fully folded or unfolded states. While stability may be desired in the fully folded state, a certain range of fold

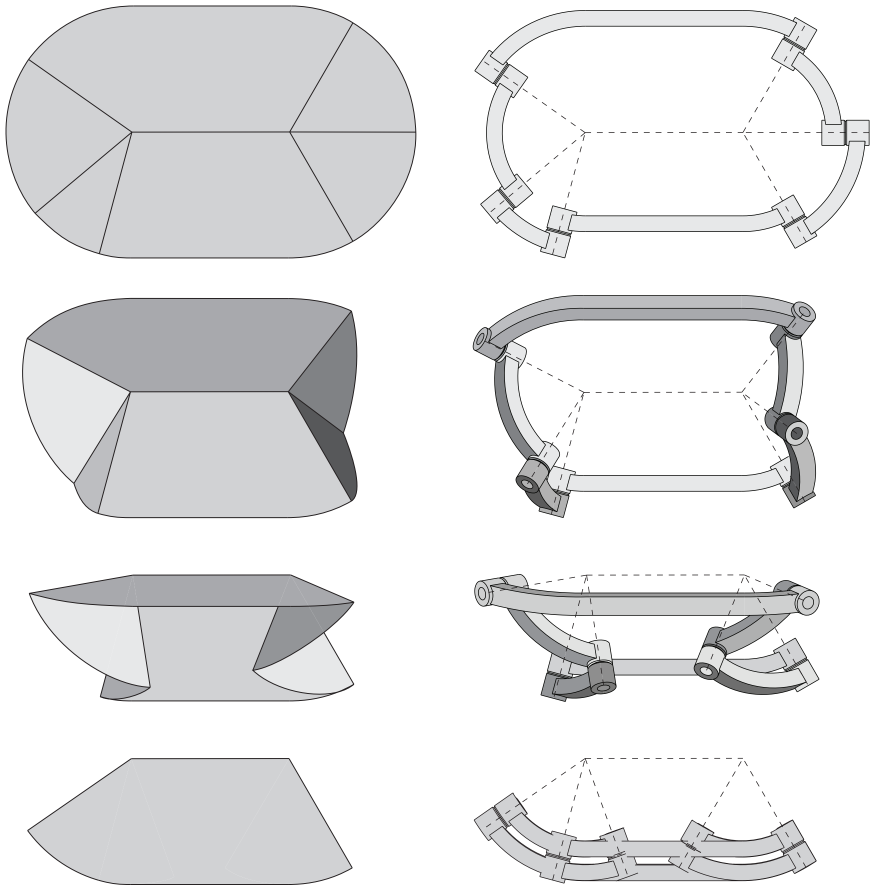


Figure 3.5: An origami linkage represented as a paper model (left) and a linkage (right). The top images show the unfolded state. The bottom images show the fully folded state. This origami linkage is flat-foldable.

states could be considered fully folded for the application. These ranges are represented by pairs of vertical, dashed lines. Figure 3.6 shows an example fold-state continuum of an origami linkage.

A fold-state continuum can be a useful tool in analyzing and understanding the range of motion of an origami linkage and the properties of the linkage throughout this range. The fold-state continuum for 1 DOF origami linkages can often be plotted along a single line, as seen in Fig. 3.6. However, care should be taken to ensure that origami linkages deploy correctly from any change-point position (such as the fully unfolded state). This 1D continuum representation can be used to predict and plan the stability of certain ranges of fold states.

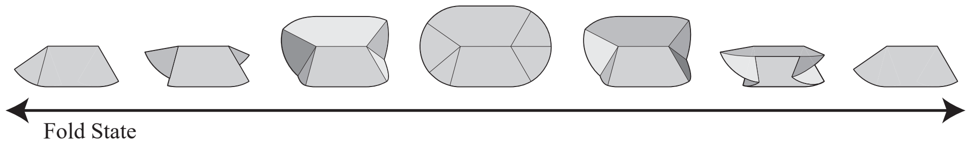


Figure 3.6: A 1D fold-state continuum of the origami linkage in Fig. 3.5.

3.5 Origami Stability Integration Method (OSIM)

The *origami stability integration method (OSIM)* is a design tool for visualizing and planning stable equilibria in an origami linkage at desired ranges along a 1D fold-state continuum. The x-axis is the 1D fold-state continuum, and the y-axis is the amount of potential energy stored in the system. Because the OSIM is intended as a conceptual tool, the mapping of the fold states to a continuum for the method does not need to be exact. For example, a designer may elongate sections of the continuum to focus on an important range of fold states.

(See section 3.2.1 and Figure 3.3.) The steps are as follows:

1. Select an origami linkage. (See section 3.3.2.)
2. Assemble a 1D fold-state continuum for the linkage. (See section 3.4.)
3. Designate the energy conditions for fold-state ranges along the continuum as stable, blocked, or undefined [111].
    (a) Stable: The origami linkage should be stable within this range.
    (b) Blocked: The origami linkage should not be able to reach this range.
    (c) Undefined: The stability of the origami is undefined in this range. Meaning that the range is not blocked and typically contains an unstable or neutrally stable equilibrium.
4. Determine the energy components inherent in the system, such as strain gravity, strain in the hinges, and other loading conditions. Graph the energy stored by each component along the fold-state continuum.
5. Sum the energy components from step 4.
6. Draw a basic energy curve that satisfies the desired energy conditions from step 3.

7. Select stability techniques to obtain this energy curve. An overview of pros and cons for each type of technique is given in Table 3.3. Additional details and examples are given in section 3.6.

8. Sum each energy component from steps 4 and 7.

Different combinations of techniques and linkages can produce favorable results. As such, the steps of the OSIM are meant to be an iterative process. Throughout the process, the origami linkage should be analyzed to determine if it should be modified or replaced. A well-designed origami linkage and loading conditions can reduce the number of techniques.

3.5.1 Example of the OSIM

Suppose a designer is making a device that will have three stable fold states, as a container, channel, and compact shape. Descriptions of the steps for designing the device are given below, and corresponding illustrations are given in Fig. 3.7 and Fig. 3.8.

Step 1: A degree-5 origami vertex is chosen as the origami linkage because it can create a convex hull shape, and it can fold flat along select creases for storage. Figure 3.7 step 1 shows two fold states of the origami linkages.

Step 2: The fold states are placed along a continuum. In practice, the continuum would only begin with the left-most fold state depicted since fold states to the left are blocked. However, for the example, the continuum is extended to illustrate multiple inherent energy components.

Step 3: The continuum is divided into six ranges of fold states. Three of the ranges are designated as stable states, one corresponding with each function of the device. The left-most stable range corresponds when the linkage is a container, the center when the linkage is a channel, and the right-most when the device is stored. The range left of the left-most stable range should be blocked to prevent the contents being crushed. The remaining ranges are undefined.

Step 4: There are two inherent energy components: (1) facet interference and (2) energy storage in the creases as they fold from the unfolded state.

Step 5: The sum of the inherent energy components would result in one stable equilibrium at the unfolded state. The desired stable ranges are not accomplished by the inherent energy conditions.

Step 6: A desired energy path is superimposed on the graph. This allows the designer to predict which techniques will produce the desired energy path.

Step 7: The designer selects two intrinsic differentiable techniques. (See section 3.6.) The first is a spring that reaches max extension in the unfolded state. The second technique is a magnet that increases the energy state pseudo-uniformly for all states except near the last, stable state.

Step 8: The sum of the techniques and inherent energy components satisfy the desired conditions. At this point, a physical prototype would be built to verify that the energy conditions are achieved.

An additional example of this process is given in section 3.8 as a case study.

3.6 Technique Classification Criteria

All stability techniques can be classified into four groups. The groups are based on two criteria: (1) whether the technique is intrinsic or extrinsic to the origami linkage, and (2) whether the techniques result in differentiable or non-differentiable energy storage.

The purpose of this section is to explain the criteria used for categorizing stability techniques and help designers decide which group of techniques might best address their needs. This is done by defining the criteria and comparing their benifits and drawbacks. (Details about each group and its techniques are given in section 3.7.)

3.6.1 Intrinsic vs. Extrinsic Techniques

An *intrinsic stability technique* is a technique that assists in realizing stable equilibria, using only the creases and facets of an origami linkage. These are techniques such as hinge interference, compliant joints, and non-rigidly foldable linkages [8].

In this study, actuated techniques are also considered intrinsic despite the need for outside influence; the outside influence is considered part of the loading condition rather than the technique.

A *extrinsic stability technique* is any technique that does not use just the facets and creases of an origami linkage to help realize stable equilibria. There are a wide variety of extrinsic techniques, such as clips, magnets, straps, and actuators.

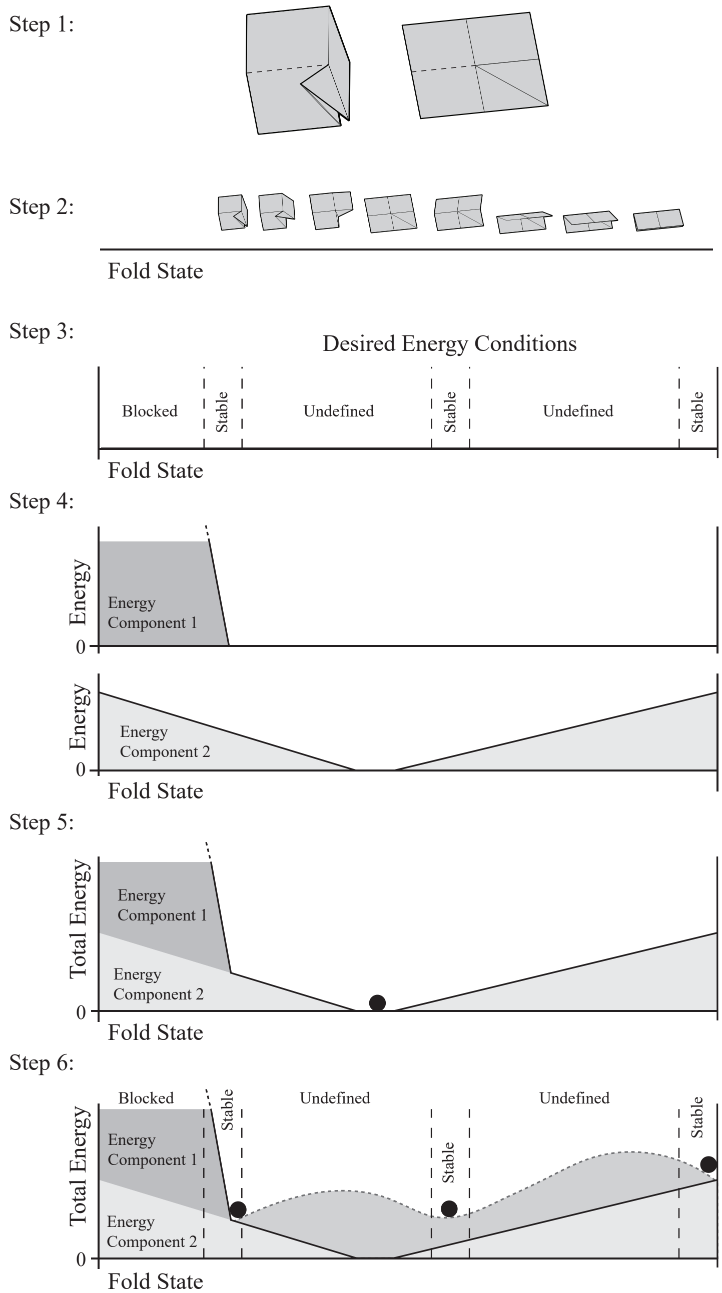


Figure 3.7: Steps 1 through 6 of the origami stability integration method (OSIM) demonstrated with a fictitious example in subsection 3.5.1. Steps are continued in Fig. 3.8.

Intrinsic Technique Design Considerations

The primary advantage of intrinsic techniques is that they only involve the members of the origami linkage. For applications that are concerned with appearance these techniques are beneficial because they highlight the simple, elegant nature of origami.

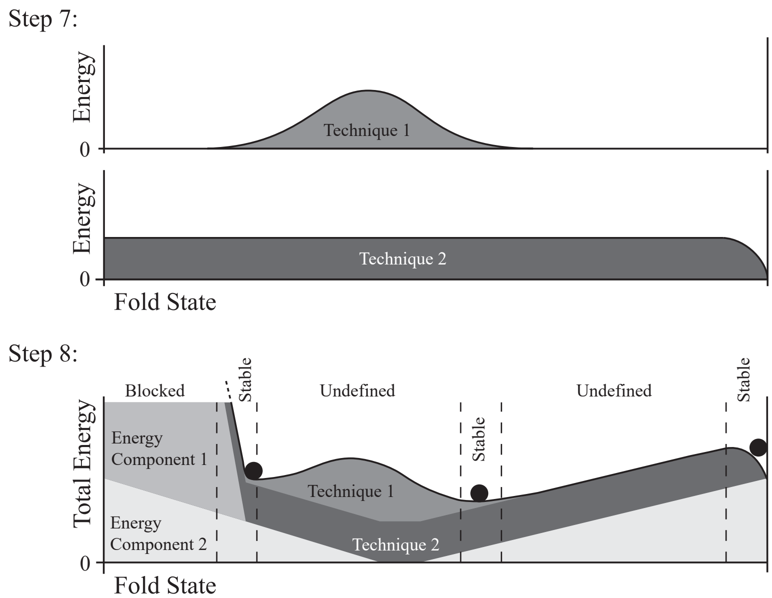


Figure 3.8: Steps 7 and 8 of the origami stability integration method (OSIM) demonstrated with a fictitious example in subsection 3.5.1. Steps are continued from Fig. 3.7.

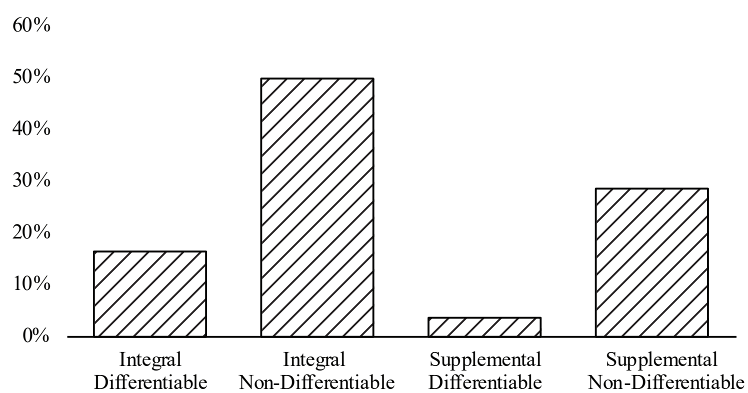


Figure 3.9: The number of implementations of each type of stability techniques in a sample of 69 origami-based devices [34]. Devices have multiple implementations; the implementations are counted once for each stable equilibrium to which they contribute.

One drawback of these techniques is that they are constrained by the material of the origami linkage. For example, the origami ballistic barrier is made from sheets of aramid fabric, which only allows for small deformation. However, advances in materials and methods for increasing their compliance are making intrinsic techniques more accessible.

If an intrinsic stability technique is selected for an application, how the technique will be implemented should considered when the origami linkage is selected because the two affect each other. For example, if the facet interference technique is selected to help maintain a non-planar fold state, the origami linkage must either be non-flat foldable or have more than one degree of freedom [38, 111].

Extrinsic Technique Design Considerations

For application where the origami linkage should not be adjusted, extrinsic techniques are useful because they do not alter the kinematics. The origami linkage provides the motion and the separate techniques provide the stability.

Extrinsic techniques are not limited to the materials used for the origami linkage. For example, compliant materials could be integrated into a device to act as springs.

Despite their versatility, extrinsic techniques only occurred in 37 of the 69 devices in Chapter 2.

3.6.2 Differentiable vs. Non-differentiable Techniques

This criteria is meant to differentiate between techniques that cause a sudden change in energy (non-differentiable) from techniques that have a gradual or constant effect (differentiable).

A *differentiable stability technique* creates a change in the energy of the linkage that is differentiable at all fold states in the continuum. Examples include torsional and linear springs, gravity, magnets, and non-rigidly foldable linkages.

Differentiable techniques technically influence the energy state of all the fold states in the continuum. However, their influence may be negligible over a range of fold states (such as a spring that goes slack). Technique two in Fig. 3.8 illustrates this.

Non-differentiable stability techniques have a non-differentiable change in energy in the origami linkage. Examples include hard stops, clips, and panel interference.

Differentiable Technique Design Considerations

If an origami-based device is made from a rigid material or if the loads are large, deformation in the facets or creases may be undesirable or insufficient for creating a stable state. For example, the origami telescope's frame is made from aluminum and could plasticly deform [36].

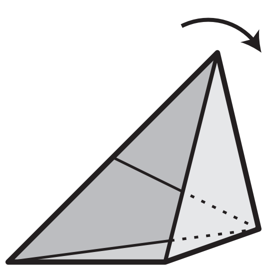
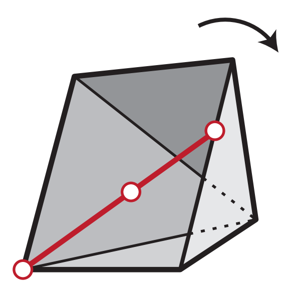
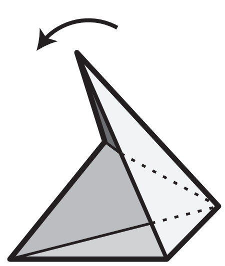
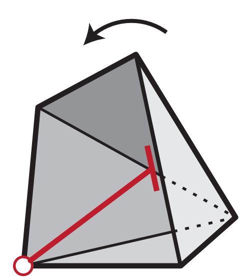
Differentiable techniques are useful for assisting actuation because their influence typically extended over a range of folds states. For example, the torque-adaptive wheel stores energy in the creases and facets, resulting in a bias towards the unfolded state [83].

Differentiable techniques have received notable attention in academia [26,29]. In products, however, differentiable techniques are used much less than non-differentiable techniques, as seen in Fig. 3.9.

Non-Differentiable Technique Design Considerations

The majority of non-differentiable techniques are hard stops: offsets, strings, clips, retaining channels, detents, and facet interference. A *hard stop* is a mechanism that inhibits a device's motion due to interference [93]. Hard stops come in two flavors: compression hard stops and tension hard stops. Table 3.1 illustrates examples of both types as intrinsic and extrinsic techniques in an origami linkage.

Table 3.1: Origami linkages with intrinsic and extrinsic hard stops in tension and compression.

	Intrinsic	Extrinsic
Tension		
Compression		

When an application needs a static equilibrium position to fall on a specific fold state, or in a narrow range of fold states, non-differentiable techniques are valuable because they can create distinct, stable equlibiria.

Non-differentiable techniques usually have high rates of energy storage. This is useful for creating blocked conditions.

Non-differentiable stability techniques are the most common type of technique used in the origami-based products sampled in this chapter, as seen in Fig. 3.9.

Differentiable behaviors can be a result of a user toggling the technique on and off. This is valuable for affecting only energy storage when the linkage in in a specific fold state.

3.7 Technique Reference Guide

This section is a technique selection reference guide. Intrinsic techniques are discussed in detail, because they are limited in number and specific to origami. Discussion of each intrinsic technique consists of (1) a brief description, (2) design considerations, (3) resources, and (4) examples.

The behaviors and design considerations of extrinsic techniques are discussed generally because the techniques are numerous and not specific to origami linkages.

Common techniques for each type are also given in Table 3.2. A summary of benefits and drawbacks (including those from section 3.6) is given in Table 3.3.

3.7.1 Intrinsic Differentiable Techniques

Origami linkages have several differentiable ways of storing energy in their facets and creases: non-rigid foldability, compliant joints, and stimuli-actuated joints.

Non-Rigid Origami

Description Non-rigidly foldable origami linkages require deformation of the facets or creases in order to fold. In addition to allowing motion, the deformed members also store energy.

Table 3.2: Examples of each type of stability technique.

	Differentiable	Non-Differentiable
Intrinsic	Non-Rigid Origami Compliant Joints Stimuli Actuated Joints	Hinge Interference Limited Facet Extension Global Facet Interference Adjacent Facet Interference
Extrinsic	Magnets Torsional Springs Linear Springs Gravity	Offsets Snaps, Clips, Buckles Telescoping Poles Velcro Retaining Channels

A non-rigidly foldable origami linkage behaves similar to a traditional over-constrained, compliant mechanism [42, 112].

Design Considerations This technique is generally useful for creating monostable energy behaviors. However, some linkages, such as the flasher, triangulated tube, and square twist, can be bistable [92, 113].

One variation on this technique is to make the origami non-rigidly foldable by offsetting hinges in thick origami. This was demonstrated by creating bistability in an origami antenna [101].

Resources A useful method for calculating energy storage in non-rigidly foldable origami is given by Saito et al. [28].

See the following resources for creating bistablity in traditional compliant mechanisms [112, 114–116].

Examples The foam origami tub in Fig. 3.10 is based on a non-rigidly foldable origami linkage. The largest facet bends as it folds, storing energy which biases the tub towards the unfolded state, shown in parts (a) and (c).

Table 3.3: Typical benefits (+) and drawbacks (-) of the four types of stability techniques. Explanations are given in their respective subsections.

	Differentiable	Non-Differentiable
Intrinsic	+ No added parts + Helps actuation - Hinders actuation - Limited by material - Complex to design - Low energy threshold	+ No added parts + Easy to design + Distinct stable points + High energy threshold + Versatile energy behavior
Extrinsic	+ Typically passive + Helps actuation - Hinders actuation - Added parts - Low energy threshold	+ High energy threshold + Easiest to implement + Diode effect - Added parts - User input usually required

Compliant Joints

Description The crease themselves are compliant joints that store strain energy like a torsional spring.

Design Considerations Material is a major consideration in this technique. The materials must be compliant enough to handle maximum deflections, while remaining stiff enough to provide the desired energy storage.

Like the non-rigidly foldable technique, one of the lowest energy states occurs at the manufactured fold state, unless the creases have been modified through plastic deformation or stimuli. (See section 3.7.1.)

Resources Research has shown that the stiffness and low-energy state of a crease in a degree-4 vertex can be tailored to create up to six stable states [26].

See the follow resource for an evaluation of various creased materials [117].

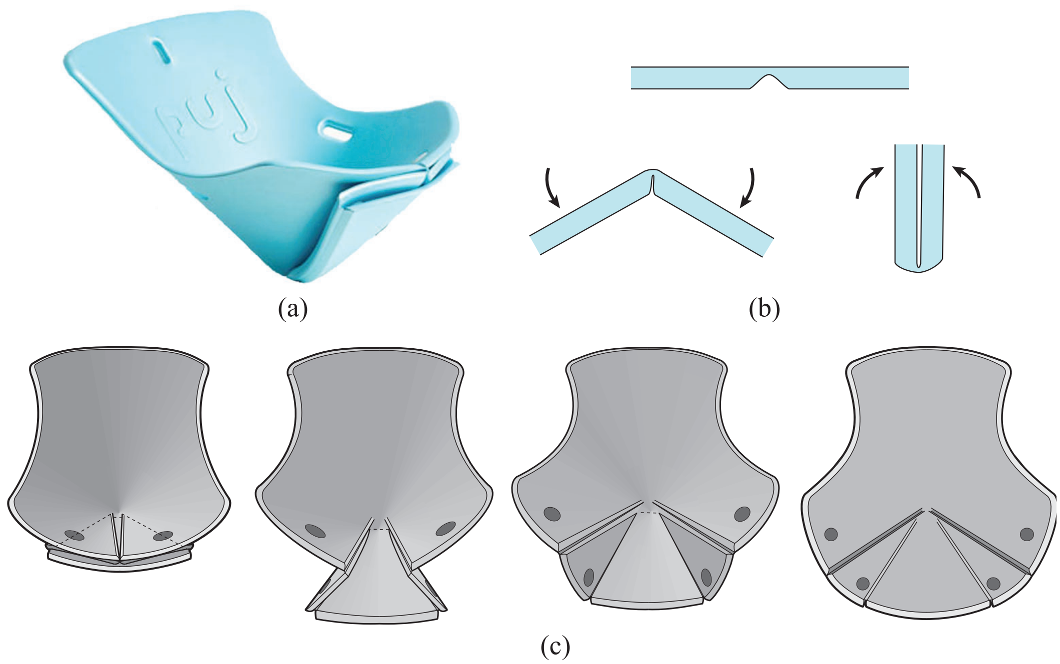


Figure 3.10: The origami-based tub (a) uses several stability techniques discussed in section 7. The linkage is blocked from folded the wrong way because of hinge interference (b). The creases and facets store strain energy, returning the origami to the unfolded state shown in part (c). The fully folded state (shown in (a) and (c)) is maintained using magnets and adjacent facet interference.

The geometry of the material can be altered to facilitate folding, for example, by using a LET array [25].

Examples Stable behavior outside of the unfolded state can be seen in even paper origami. Such as the waterbomb base and kaleidocycle [27, 102].

A number of the origami-based products (colander, bathtub, Kayak, bowl, and glasses case) use living hinges to fold and store energy [16, 50, 55, 63, 69].

Because of its limited use and the high potential, this is a technique that would benefit from further research into potential applications and design methods.

Stimuli Actuated Joints

Description Stimuli actuated joints are different from other intrinsic differentiable techniques because often the energy storage is not due to elastic deformation. The energy storage is more akin to plastic deformation, where the lowest energy state of the hinges is redefined completely.

Design Considerations On the microscopic level, these techniques have a distinct advantage because direct manipulation by a user is minimized [118].

Resources

See section 3.2 for more resources.

Examples One of the few macroscopic examples, the origami robot, uses heat to actuate the joints [30].

3.7.2 Intrinsic Non-Differentiable Techniques

Intrinsic non-differentiable techniques are primarily caused by self-interference—between adjacent facets, from sequential folding, between creases with thickness, between non-adjacent facets, and from facets in tension.

Adjacent Facet Interference

Description The adjacent facet interference technique is very intuitive—when adjacent facets interfere with each other they will stop folding.

Design Considerations For applications that need a large exposed surface area this technique is not ideal because facets double back on one another. For example, the ballistic barrier's coverage would be reduced if it had adjacent facets in the deployed state.

Adjacent facet interference only blocks one fold direction. Thus, in applications that require fully constrained states, additional techniques will be needed. One benefit is that it is relatively easy to constrain adjacent facets using bolts, magnets, and hook and loop fasteners.

This technique is often used in load bearing applications, such as the origami-based chairs and tablet cases [58, 79–81].

This is the most prevalent technique in the sample of origami-based devices. 51 of the 79 intrinsic non-differentiable techniques are adjacent facet interference. 29 of those are when the device is either stored or transported.

Resources A resource for designing origami linkages to interfere in non-planar fold states is given by Foschi et al. [111].

Examples

Technically the bathtub is fully folded when it functions as a tub, but because it is not rigidly foldable it has a non-planar shape (Fig. 3.10 part (a)). The adjacent facets are a good technique for this application because the facets are sandwiched between the baby and the sink. The adjacent facets are also constrained by magnets.

Sequential Flap Interference

Description This technique is where two creases are made collinear to form a flap in a folds state that can bifurcate. The flap is then folded, blocking other bifurcation modes.

Design Considerations This technique can be difficult to implement if an application requires thick materials because creases that form the flap must be nearly collinear. This means that, without modifications, hinge interference will occur in either fold direction. (See section 3.7.2.)

Resources See the following resources for methods allow sequential folding in thick materials [7, 119].

Examples This is an uncommon technique; in the sample of devices, it only occurs in the battery, ice bucket, and sofa [16, 74]. Flaps are formed in a number of other devices such as the boat, bowl, and camping pot, but they are a slight variation on the technique where the flaps does not create the need for sequential folding [20, 55, 56].

This is one of the few intrinsic techniques that can be toggled on and off by a user. Only the ice bucket uses the flap in this way.

Hinge Interference

Description Hinge interference occurs when facets with non-zero thickness obstruct folding of the crease.

Design Considerations This technique is a low-profile intrinsic technique that is beneficial because it only affects the kinematics of the linkage once it is blocked. Facet chamfer angles determine when the facet interferes.

Applications that use thin material are not conducive to this technique because the technique only occurs in linkages with out-of-plane thickness.

Resources See the following resources for fold angle equations by Huffman and Lang et al. [92, 120].

Tachi shows an implementation of this technique in the tapered panel technique (a thickness accommodation technique) [8].

Ku gives a novel variation for creating creases interference [11].

Directional hinge techniques that bridge the gap between extrinsic and intrinsic techniques are provided by Shemenski et al. [93].

Examples The linkage shown in Fig. 3.11 is blocked from reaching the unfolded state by hinge interference.

Figure 3.10 (b) shows the crease interference technique used to block the tub's folding in the wrong direction.

Global Facet Interference

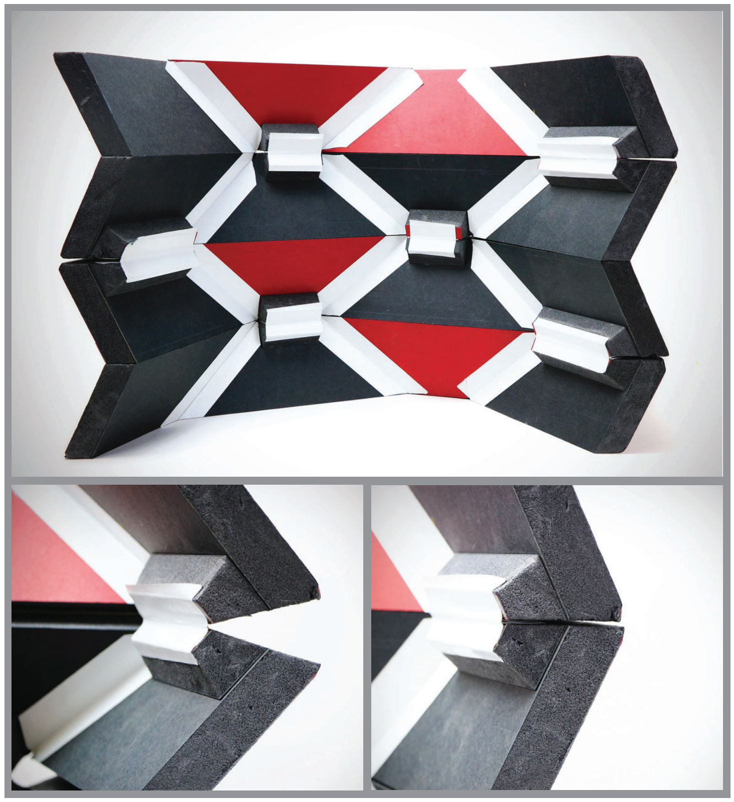
Description Global facet interference is when two or more non-adjacent facets interfere.

Design Considerations Calculating global interference is more involved than calculating local interference because the entire fold state must be defined.

Resources One method for simplifying implementation of this technique is the use of symmetry. An example of how symmetry is used to achieve global interference is shown in Fig. 3.12.

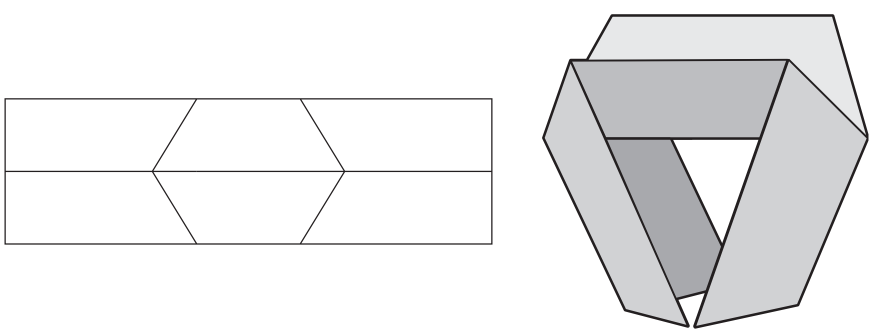
Examples The majority of the devices that use this technique use symmetry—he kayak (Fig. 3.2), canoe, forceps, and several of the chairs [57, 59–61, 68].

(a)



(b) (c)

Figure 3.11: The facets of the thick origami linkage (a) are negatively chamfered to create a stable partially folded state [21]. Figures (b) and (c) show details of the hinge interference.



(a) (b)

Figure 3.12: A symmetric crease pattern (a) and a fold state with global facet interference (b).

Limited Facet Extension

Description Limited facet extension occurs when the crease in between the actuated facet and a grounded facet are unfolded, as seen in Table 3.1. This point is known as a change point, toggle point, or singularity.

Design Considerations The limited facet extension technique is useful for creating containers or devices that are kept in tension. The technique usually only works if the origami linkage is non-developable.

Examples The origami colander in Fig. 3.13 has a stable state that uses intrinsic hard stops in tension. Since the origami linkage is non-developable the facets that make up the sides reach full extension when the other facets are non-coplanar.

The colander is a good example of a design that uses the loading condition to bias towards a stable equilibrium; the feet placement of the colander (shown in (b) of Fig. 3.13) results in forces that keep the side facets in tension. If the feet had been attached in a different manner, shown in (c), the resulting forces would put the side facets in compression, pushing the linkage out of equilibrium.

3.7.3 Extrinsic Differentiable Techniques

For the origami ballistic barrier the designers wanted to use a differentiable technique that would assist actuation. However, the barrier was too heavy for strain energy stored in the hinges to overcome the gravitational potential energy. So the designers added gas springs with a low-energy state in the deployed state.

There are a plethora of these techniques: bolts, hook and loop fasteners, snaps, clips, safety pins, ties, springs, pistons, linear actuators, elastics, cables, restraining channels, latches, and telescoping poles, to name a few.

Another example of a extrinsic differentiable technique is the magnets that secure the puj tub in the folded state.

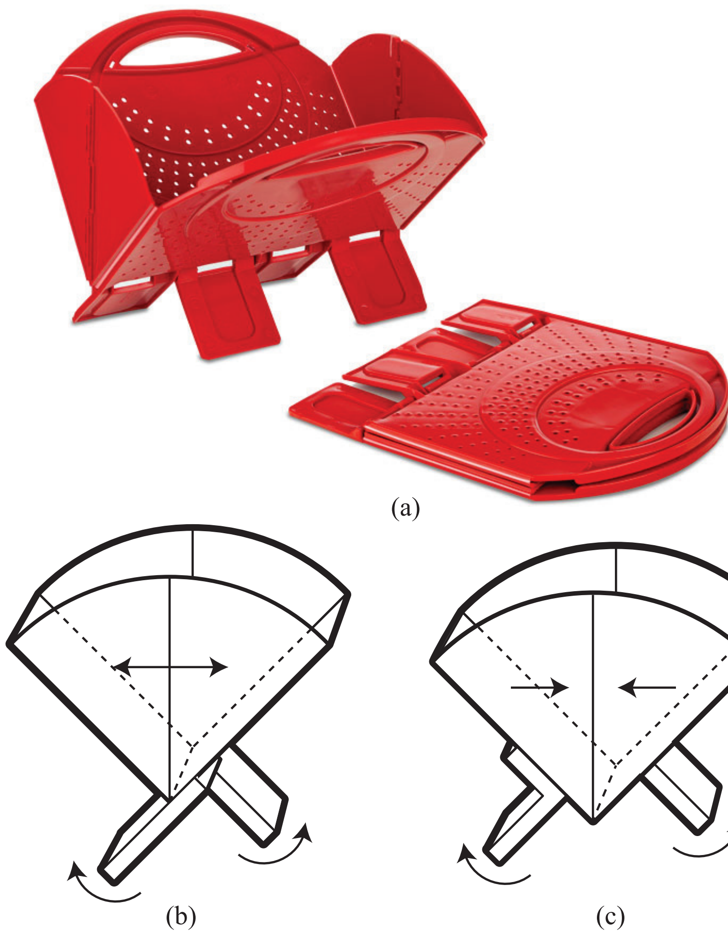


Figure 3.13: An origami-based colander in the unfolded and UP fold states (a). The feet in the actual embodiment put the unfolded facets further in tension (b). A less ideal placement of the feet would have resulted in the unfolded facets being placed in compression (c).

3.7.4 Extrinsic Non-differentiable Techniques

Extrinsic non-differentiable techniques come in a wide variety and are common in origami-based devices. Snaps, clips, fasteners, and restraints are easy to add to a device at any fold state.

Extrinsic non-differentiable techniques are generally used in applications that permit user assembly, such as setting the clip and retaining channel in the origami kayak (Fig. 3.2). However, this gives the techniques versatility; they are implemented only when a user wants them. They can have a one-way or diode effect on energy storage, where the energy threshold for entering a fold state is low, but exiting is high. This can be accomplished using a detent or clip. In the OSIM this one-way effect is represented using a diode symbol, as seen in Fig. 3.14.

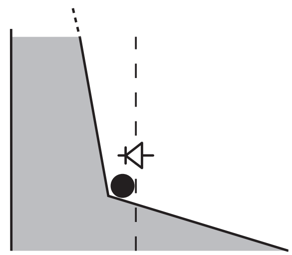


Figure 3.14: Some extrinsic non-differentiable techniques have a one-way effect that have low barriers of entry to a fold state but block returning to the previous fold state. In the OSIM these techniques are represented using a diode symbol.

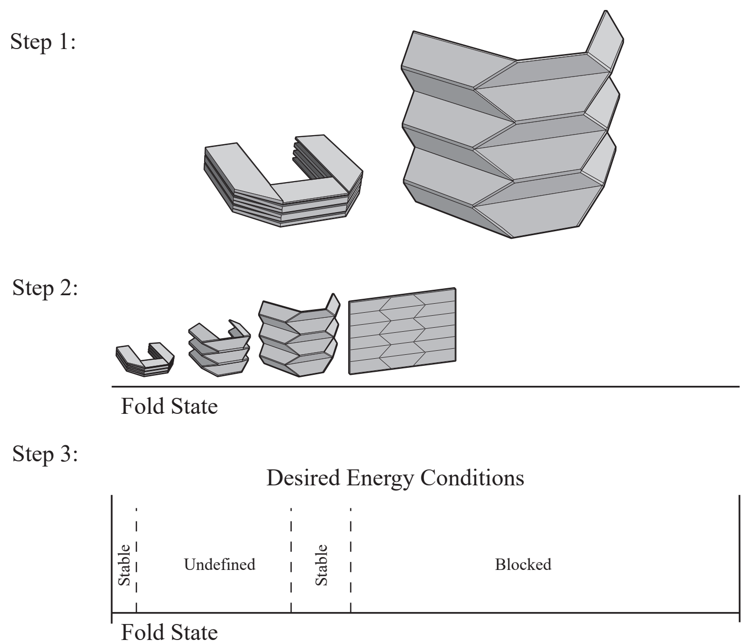


Figure 3.15: The first three steps of the OSIM for the origami ballistic barrier.

3.8 Case study

The OSIM is used to design stability in the origami ballistic barrier [21]. The origami ballistic barrier is a deployable, bullet-resistant shield that stores fully folded and opens to be a self-standing protection.

OSIM steps 1-3 are in Fig. 3.15. Steps 4-8 are repeated for two separate loading conditions, shown in separate figures: Fig. 3.16 and Fig. 3.17. Steps 7 and 8 are repeated and are labelled 7a and 8a, 7b and 8b.

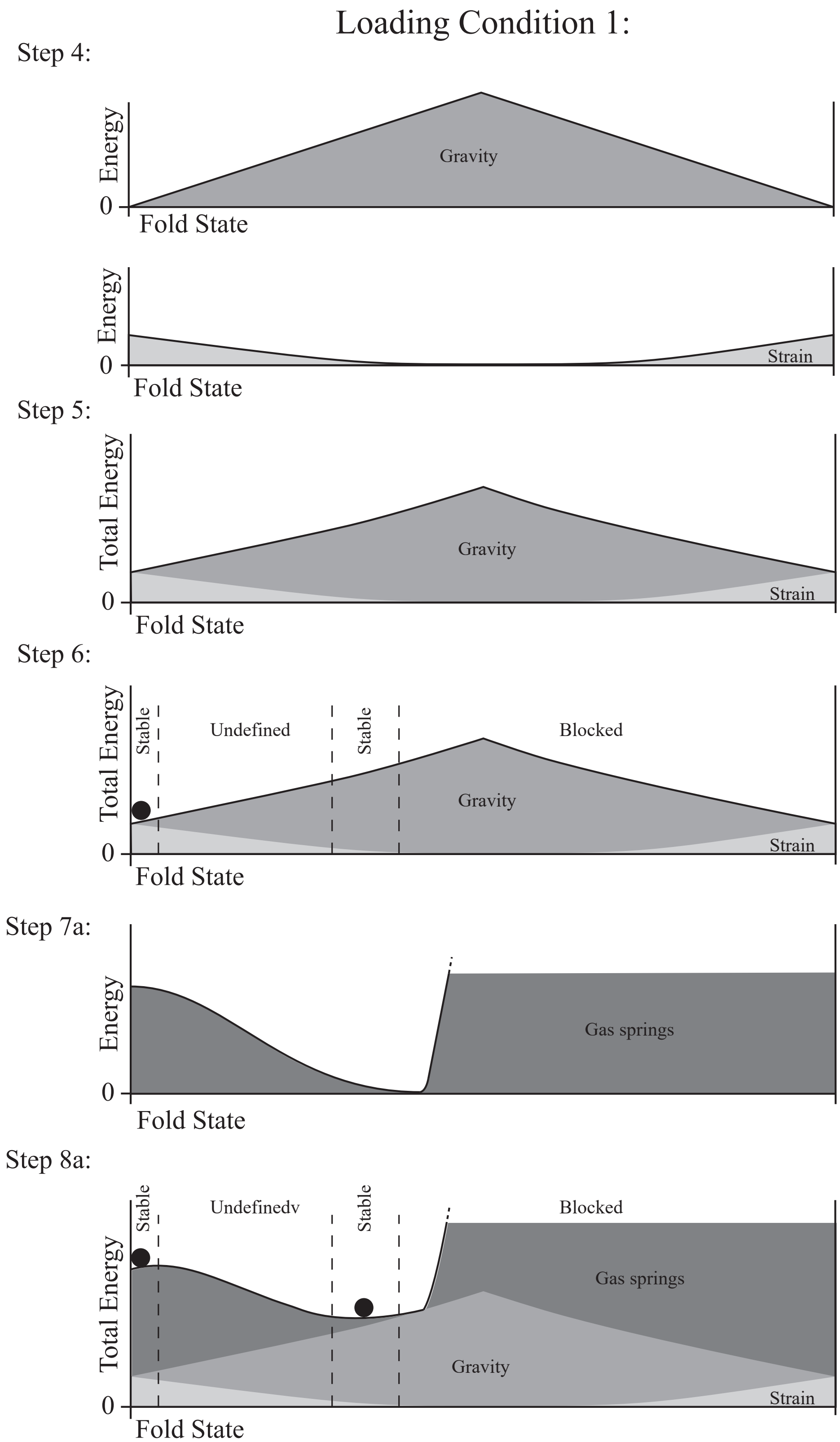


Figure 3.16: The barrier has two loading conditions that result in different energy components. This is steps 4-8 of the OSIM for loading condition one where the barrier is resting on the ground. Step 7a and 8a correspond to steps 7a and 8a to the in Fig. 3.17.

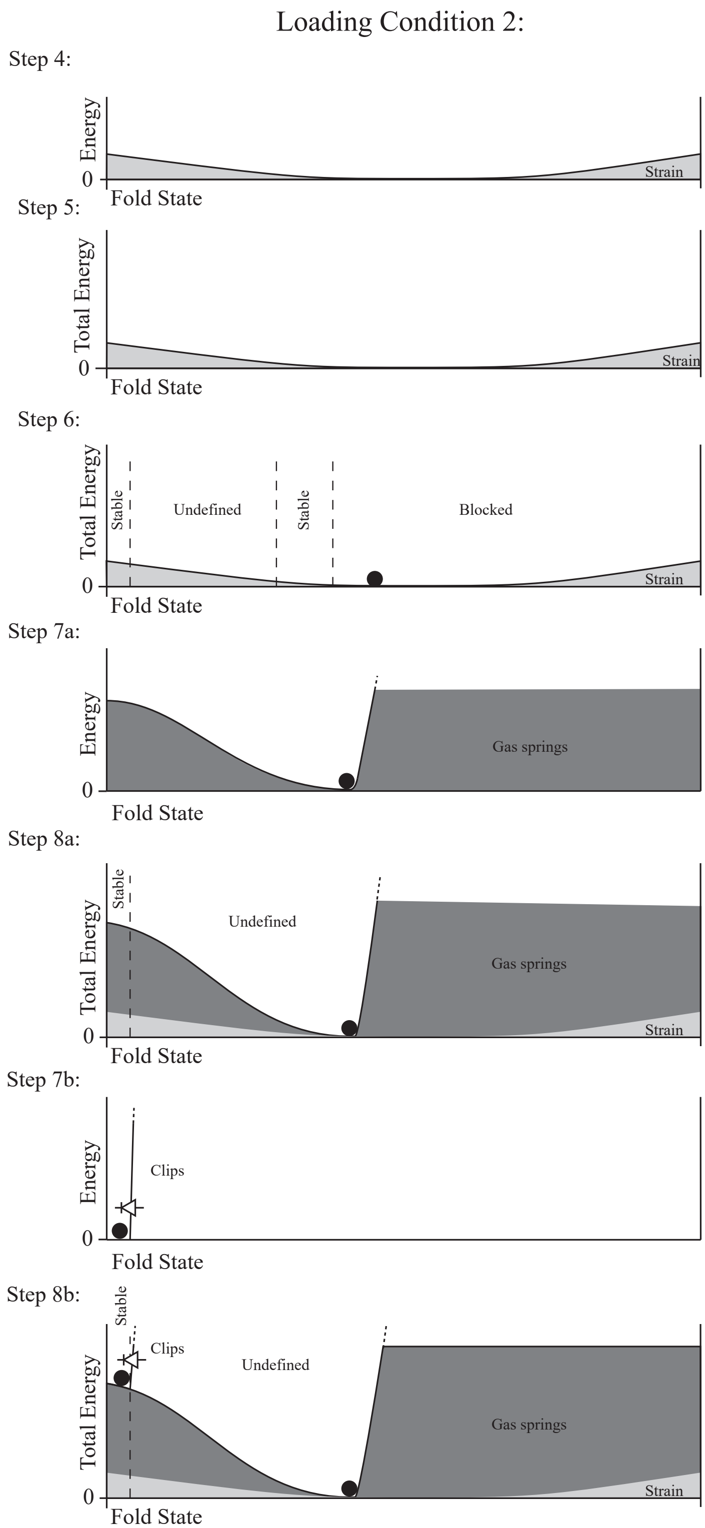


Figure 3.17: The barrier has two loading conditions that result in different energy components. This is steps 4-8 of the OSIM for loading condition two where the barrier is on resting on an edge. Step 7a and 8a correspond to steps 7a and 8a to the in Fig. 3.16. For details on the diode symbol used in 7b and 8b see subsection 3.7.4.

Step 1: The origami linkage is selected based on its crescent shape in the deployed state and its compact stowed state.

Step 2: Assemble the fold-state continuum. The origami linkage has a single degree of freedom and is flat foldable which means that the range of the fold-state continuum is from one fully folded state to the other.

Step 3: The barrier needs to be stable in the fully folded state and in a partially folded state. It is helpful if the energy in the undefined range biases the barrier towards the unfolded state to help with actuation. The barrier should not reach the unfolded state because in that state the origami linkage can bifurcate into undesired modes.

Step 4: The barrier has two different loading conditions: when it is (1) deployed, and (2) stored or transported standing on one end. In each loading condition, the creases are biased towards the unfolded state. The difference is in the role that gravitational potential energy plays. In loading condition 1, the barrier is resting on the ground so that the gravitational potential energy biases the barrier towards the fully folded state. In loading condition 2, gravity does not result in significant energy storage as the barrier folds. Each loading condition must be summed and evaluated separately.

Step 5: Sum the inherent energy conditions.

Step 6: Superimposing the desired energy conditions shows that in loading condition 1 the barrier will be stable in the fully folded state, but in loading condition 2 is biased towards the unfolded state. The origami linkage and loading conditions are viable because no desired stable ranges are blocked.

Step 7a: Selecting stability techniques.

When choosing techniques types there are number of considerations to keep in mind: (1) The barrier should be kept as light as possible, which indicates that an intrinsic technique could be valuable because no parts are added. (2) The barrier needs to deploy quickly and require little-to-no assembly. This indicates a differentiable technique could be useful. (3) Because the device has a desired blocked condition, a non-differentiable technique could be needed. (4) The barrier is heavy, so a differentiable technique could be useful for assisting actuation.

Because the partially folded state is the more difficult state to maintain, its stability techniques are considered first. Considerations 1, 2, and 4 above indicate that an intrinsic differentiable techniques should be investigated first.

Evaluating the intrinsic differentiable techniques:

Making the barrier non-rigidly foldable towards the unfolded state is a simple change. The facets of the barrier are made from sheets of aramid fabric and a rigid panel. The choice of material for the rigid panels is flexible, meaning that a material that can store a large amount of energy could be selected. This technique is viable. However, one drawback of the technique is that the panels would have significant deformation in the fully folded state, making the barrier less compact during storage. Likewise energy storage increases as the barrier folds, making it more difficult to keep the barrier fully folded.

The aramid fabric does not make a good torsional spring, so compliant joints are not viable.

The stimuli-actuated joint would most likely be too slow, so it is also not considered for use.

While the non-rigidly foldable technique is viable, its drawbacks encourage investigation of other techniques. Next, extrinsic differentiable techniques are considered.

Evaluating the extrinsic differentiable techniques:

Gravity cannot be used to keep the barrier open, and since there are no adjacent facets, magnets are difficult to implement effectively.

Springs are a viable candidate, either torsional springs added at the creases or linear springs attached to facets. Both methods seem viable and have few drawbacks; both are prototyped. Torsional springs in the form of spring-steel sheets are added to the creases, and linear springs in the form of gas springs are also added.

Gas springs are selected because (1) As the barrier folds the mechanical advantages of the springs decreases. (See the energy behavior of the gas spring in Fig. 3.16 step 7a.) This creates a stable equilibrium in the fully folded state. (See step 8a.) (2) The gas springs are attached to non-adjacent facets to counteract parasitic motion with less hardware. (3) The gas springs double as handles. (4) The gas springs are also a hard stop, blocking the barrier from entering the unfolded state.

Step 8a: The combined energies in loading condition 1 results in bistability, with a stable equilibrium in both the fully folded and partially folded state. However, in loading condition 2 the barrier is only stable in the partially folded state. Thus, the design is returned to step 7.

Step 7b: Rather than redesign the gas-spring technique, another technique is added. Because speed is not a concern during transportation and storage, a technique that toggles, based on user input, is acceptable. A bag with clips is used to store and transport the barrier.

Step 8b: The new energy conditions are satisfactory.

As a note, once it was determined that clips would be needed to keep the barrier in the fully folded state, the bistable behavior from the gas springs was not needed in loading condition 1. Because of this, implementing a new technique that strongly biased towards the unfolded state might have served better. This could have meant further exploration of the torsional springs or the attachment points of the gas springs, points that did not reduce the mechanical advantage as much.

3.9 Conclusion

The method, vocabulary, techniques, and examples presented in this chapter are effective tools for planning stable states in origami. As shown in the case study, the OSIM reveals information about how energy components interact in origami based-design. The categorization of stability techniques facilitates their comparison and selection.

Future work in this area could include investigating the relationship between the path used to generate the fold state continuum and the energy states; for origami linkages with bifurcation or multiple degrees of freedom may have different stable states along one fold path than another. Likewise the dynamic of the linkage entering a stable state could also be considered.

CHAPTER 4. CONCLUSION

4.1 Conclusion

Precise terminology is critical for effective discussion of origami in an engineering design setting, such as fold state and origami linkage. Terms build on one another and an nucleation points for concepts; for example the Origami Stability Integration Method builds on fold state continuums which in turn builds on fold states.

There is a logical, comprehensive way to categorize non-planar folds states. This categorization method, based on a simple property (fold angles), can predict trends in properties and behaviors of a fold state, such as, its exposed surface area or compactness. Because of this, the fold-state type of an origami-based device correlates strongly with the types of functions it performs.

Based on the fold state analysis of origami-based devices, we learn that the previously named fold states, unfolded and flat folded, are not necessarily the most common states for performing functions. We expect these trends to continue in future origami-based devices, and encourage further investigation of the properties and behaviors of non-flat origami. Origami fold states are a valuable design tool for selecting relevant linkages and applications.

While foldability is key to many of origami's advantages, however, so is stability. Visualizing the energy behavior of an origami linkage gives designers insight into what types of stability techniques to implement.

We anticipate the contents of this thesis will inform choices for designers of origami-based devices, allowing them to step decisively through the creative process.

4.2 Future Work

Future work should expand the fold-state types to account for states that are non-rigidly foldable and based on kirigami. Additional classifications could be made using other fold state properties such as facet dimensions and sector angles.

The concept of a fold-state continuum is an innovative and flexible concept that warrants further exploration as a 1D representation of reconfigurable origami or origami with multiple degrees of freedom. The effect of the fold path used to compose the fold state continuum on th energy states could also be explored.

Further research in origami-stability techniques, such as methods for decoupling the facet interference behaviors from the behavior of the rest of the origami linkage, could increase the use of the technique for creating non-planar fold states.

REFERENCES

[1] Miyashita, S., Guitron, S., Ludersdorfer, M., Sung, C. R., and Rus, D., 2015. "An untethered miniature origami robot that self-folds, walks, swims, and degrades." In *Robotics and Automation (ICRA), 2015 IEEE International Conference on*, IEEE, pp. 1490–1496. 1, 7, 11

[2] Sreetharan, P. S., Whitney, J. P., Strauss, M. D., and Wood, R. J., 2012. "Monolithic fabrication of millimeter-scale machines." *Journal of Micromechanics and Microengineering,* **22**(5), p. 055027. 1

[3] Yao, S., Liu, X., Georgakopoulos, S. V., and Tentzeris, M. M., 2014. "A novel reconfigurable origami spring antenna." In *Antennas and Propagation Society International Symposium (APSURSI), 2014 IEEE*, IEEE, pp. 374–375. 1

[4] Lang, R. J., 1996. "A computational algorithm for origami design." In *Proceedings of the twelfth annual symposium on Computational geometry*, ACM, pp. 98–105. 1

[5] Demaine, E. D., and Tachi, T., 2017. "Origamizer: A practical algorithm for folding any polyhedron." In *LIPIcs-Leibniz International Proceedings in Informatics*, Vol. 77, Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik. 1

[6] McAdams, D. A., 2018. "Discussion of a review of thickness-accommodation techniques in origami-inspired engineering(lang, rj, tolman, ka, crampton, eb, magleby, sp, and howell, ll, 2018, asme appl. mech. rev., 70 (1), p. 010805." *Applied Mechanics Reviews,* **70**(1), p. 015504. 1

[7] Tolman, K. A., Lang, R. J., Magleby, S. P., and Howell, L. L., 2017. "Split-vertex technique for thickness-accommodation in origami-based mechanisms." In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT08A054–V05BT08A054. 1, 46

[8] Tachi, T., et al., 2011. "Rigid-foldable thick origami." *Origami,* **5**, pp. 253–264. 1, 2, 28, 36, 47

[9] Chen, Y., Peng, R., and You, Z., 2015. "Origami of thick panels." *Science,* **349**(6246), pp. 396–400. 1, 28

[10] Crampton, E. B., Magleby, S. P., and Howell, L. L., 2017. "Realizing origami mechanisms from metal sheets." In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT08A055–V05BT08A055. 1, 28

[11] Ku, J. S., 2017. "Folding thick materials using axially varying volume trimming." In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT08A044–V05BT08A044. 1, 47

[12] Peraza-Hernandez, E. A., Hartl, D. J., Malak Jr, R. J., and Lagoudas, D. C., 2014. "Origami-inspired active structures: a synthesis and review." *Smart Materials and Structures,* **23**(9), p. 094001. 1, 2, 28, 29, 30

[13] Rus, D., and Tolley, M. T., 2018. "Design, fabrication and control of origami robots." *Nature Reviews Materials*, p. 1. 1

[14] Cowan, B., and Von Lockette, P. R., 2017. "Fabrication, characterization, and heuristic trade space exploration of magnetically actuated miura-ori origami structures." *Smart Materials and Structures,* **26**(4), p. 045015. 1

[15] Johnson, M., Chen, Y., Hovet, S., Xu, S., Wood, B., Ren, H., Tokuda, J., and Tse, Z. T. H., 2017. "Fabricating biomedical origami: a state-of-the-art review." *International journal of computer assisted radiology and surgery,* **12**(11), pp. 2023–2032. 1

[16] Morris, E., McAdams, D. A., and Malak, R., 2016. "The state of the art of origami-inspired products: A review." In *ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT07A014–V05BT07A014. 1, 7, 9, 11, 20, 21, 30, 44, 46

[17] Designboom, 2012. Cardboard Shelter kernel description `https://www.designboom.com/design/tricycle-house-and-garden-by-peoples-architecture-office/` Accessed: 2018-05-16. 1, 7, 11

[18] Yong, J., 2013. A furniture set that can be folded like origami paper `http://designtaxi.com/news/358580/A-Furniture-Set-That-Can-Be-Folded-Like-Origami-Paper/` Accessed: 2018-05-16. 1, 7, 11

[19] bltd, 2010. Waste Bin kernel description `http://www.betterlivingthroughdesign.com/accessories/polywrap-wastepaper-bin/` Accessed: 2018-05-16. 1, 7, 11

[20] Cafarelli, M., Motta, M., and Storto, M., 2013. Origami: Evoluzione e Ispirazione Thesis available at `https://issuu.com/mauriziosturt/docs/tesi_origami_-_evoluzione_e_ispiraz`. 1, 7, 11, 19, 20, 30, 46

[21] Morgan, D. C., Halverson, D. M., Magleby, S. P., Bateman, T. C., and Howell, L. L., 2017. *Y Origami?: Explorations in Folding*., Vol. 104 American Mathematical Soc. vi, ix, 2, 3, 7, 9, 11, 16, 19, 28, 48, 51

[22] Seager, S., Turnbull, M., Sparks, W., Thomson, M., Shaklan, S. B., Roberge, A., Kuchner, M., Kasdin, N. J., Domagal-Goldman, S., Cash, W., et al., 2015. "The exo-s probe class starshade mission." In *Techniques and Instrumentation for Detection of Exoplanets VII*, Vol. 9605, International Society for Optics and Photonics, p. 96050W. 2

[23] Shemenski, P. D., and Trease, B. P., 2018. "Compact directional and frictional hinges for flat folding applications.". 2

[24] Yan, Z., Zhang, F., Wang, J., Liu, F., Guo, X., Nan, K., Lin, Q., Gao, M., Xiao, D., Shi, Y., et al., 2016. "Controlled mechanical buckling for origami-inspired construction of 3d microstructures in advanced materials." *Advanced functional materials,* **26**(16), pp. 2629–2639. 2, 28

[25] Jacobsen, J. O., Chen, G., Howell, L. L., and Magleby, S. P., 2009. "Lamina emergent torsional (let) joint." *Mechanism and Machine Theory,* **44**(11), pp. 2098–2109. 2, 28, 44

[26] Waitukaitis, S., Menaut, R., Chen, B. G.-g., and van Hecke, M., 2015. "Origami multistability: From single vertices to metasheets." *Physical review letters,* **114**(5), p. 055503. 2, 15, 28, 29, 40, 43

[27] Zhang, H., Zhu, B., and Zhang, X., 2018. "Origami kaleidocycle-inspired symmetric multistable compliant mechanisms." *Journal of Mechanisms and Robotics*. 2, 28, 29, 44

[28] Saito, K., Tsukahara, A., and Okabe, Y., 2015. "New deployable structures based on an elastic origami model." *Journal of mechanical design,* **137**(2), p. 021402. 2, 28, 42

[29] Silverberg, J. L., Evans, A. A., McLeod, L., Hayward, R. C., Hull, T., Santangelo, C. D., and Cohen, I., 2014. "Using origami design principles to fold reprogrammable mechanical metamaterials." *science,* **345**(6197), pp. 647–650. 2, 28, 40

[30] Felton, S., Tolley, M., Demaine, E., Rus, D., and Wood, R., 2014. "A method for building self-folding machines." *Science,* **345**(6197), pp. 644–646. 2, 7, 11, 28, 29, 45

[31] Na, J.-H., Evans, A. A., Bae, J., Chiappelli, M. C., Santangelo, C. D., Lang, R. J., Hull, T. C., and Hayward, R. C., 2015. "Programming reversibly self-folding origami with micropatterned photo-crosslinkable polymer trilayers." *Advanced Materials,* **27**(1), pp. 79–85. 2, 28, 29

[32] Hayes, G. J., Liu, Y., Genzer, J., Lazzi, G., and Dickey, M. D., 2014. "Self-folding origami microstrip antennas." *IEEE Transactions on Antennas and Propagation,* **62**(10), pp. 5416–5419. 2, 28, 29

[33] Bircan, B., Miskin, M., Dorsey, K., McEuen, P., and Cohen, I., 2018. "Bidirectional folding with nanoscale sheets for autonomous micro-origami." *Bulletin of the American Physical Society*. 2, 28, 29

[34] Avila, A., Lang, R., Howell, L., and Magleby, S., 2019. "Origami fold states: Concept and design tool." *Journal of Mechanical Science* The paper is currently in review. viii, 2, 32, 38

[35] Yang, Y., Nara, C., Chen, X., and Hagiwara, I., 2017. "Investigation of helmet based on origami structures." In *ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT08A040–V05BT08A040. 3, 7, 11, 20

[36] Early, J., Hyde, R., and Baron, R., 2003. Twenty meter space telescope based on diffractive fresnel lens Tech. rep., Lawrence Livermore National Lab., CA (US). 3, 7, 11, 15, 39

[37] Lin, J., 2011. "A nomadic furniture design for college students." Available at http://hdl.handle.net/2142/18533. 3

[38] Bern, M., and Hayes, B., 1996. "The complexity of flat origami." In *SODA*, Vol. 96, pp. 175–183. 3, 39

[39] Rabinovich, M., Hoffmann, T., and Sorkine-Hornung, O., 2018. "Discrete geodesic nets for modeling developable surfaces." *ACM Transactions on Graphics (TOG),* **37**(2), p. 16. 3

[40] Lang, R. J., 2017. "Twists, tilings, and tessellations: Mathematical methods for geometric origami.". vii, 3, 4, 5, 6, 31

[41] Lo, A. S., 2009. "Starshade technology development." In *astro2010: The Astronomy and Astrophysics Decadal Survey*, Vol. 2010. 3, 15

[42] Greenberg, H., Gong, M. L., Magleby, S. P., and Howell, L. L., 2011. "Identifying links between origami and compliant mechanisms." *Mechanical Sciences,* **2**(2), pp. 217–225. 4, 29, 42

[43] Tachi, T., 2010. "Geometric considerations for the design of rigid origami structures." In *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium*, Vol. 12, pp. 458–460. 4, 12

[44] Wu, W., and You, Z., 2010. "Modelling rigid origami with quaternions and dual quaternions." In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 466, The Royal Society, pp. 2155–2174. 5

[45] Pagano, A., Yan, T., Chien, B., Wissa, A., and Tawfick, S., 2017. "A crawling robot driven by multi-stable origami." *Smart Materials and Structures,* **26**(9), p. 094007. 5, 29

[46] Kuribayashi, K., Tsuchiya, K., You, Z., Tomus, D., Umemoto, M., Ito, T., and Sasaki, M., 2006. "Self-deployable origami stent grafts as a biomedical application of ni-rich tini shape memory alloy foil." *Materials Science and Engineering: A,* **419**(1), pp. 131–137. 5, 7, 11

[47] Liu, X., Yao, S., Georgakopoulos, S. V., Cook, B. S., and Tentzeris, M. M., 2014. "Reconfigurable helical antenna based on an origami structure for wireless communication system." In *Microwave Symposium (IMS), 2014 IEEE MTT-S International*, IEEE, pp. 1–4. 7, 11

[48] Thün, G., Velikov, K., Ripley, C., Sauvé, L., and McGee, W., 2012. "Soundspheres: resonant chamber." *Leonardo,* **45**(4), pp. 348–357. 7, 11

[49] Wee, D., 2014. Bao bao issey miyake launches new distortion series Available at http://www.blouinartinfo.com/news/story/1038919/bao-bao-issey-miyake-launches-new-distortion-series, Accessed: 2018-05-16. 7, 11, 19

[50] puj, n.d.. Bath Tub kernel description https://puj.com/ Accessed: 2018-05-16. 7, 11, 20, 28, 44

[51] Cheng, Q., Song, Z., Ma, T., Smith, B. B., Tang, R., Yu, H., Jiang, H., and Chan, C. K., 2013. "Folding paper-based lithium-ion batteries for higher areal energy densities." *Nano letters,* **13**(10), pp. 4969–4974. 7, 11

[52] Butler, J., Morgan, J., Pehrson, N., Tolman, K., Bateman, T., Magleby, S. P., and Howell, L. L., 2016. "Highly compressible origami bellows for harsh environments." In *ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT07A001–V05BT07A001. 7, 11

[53] Simpson, K., and Elkins, P., n.d. CoroPlast Boat kernel description `https://www.christinedemerchant.com/boat-styles-coroplast.html` Accessed: 2018-05-16. 7, 11

[54] Griffiths, S., 2015. The origami boat a human can row: Vessel made from a 300ft-long roll of paper carries an adult across a lake `http://www.dailymail.co.uk/sciencetech/article-2911327.html` Accessed: 2018-05-16. 7, 11

[55] Fozzils, n.d.. Fozzils Ultralight Backpacking Bowl kernel description `https://www.fozzils.com/` Accessed: 2018-05-16. vi, 7, 9, 11, 44, 46

[56] Minimum, B., n.d.. Bear Bowl kernal description `https://www.bearminimum.org/` Accessed: 2018-05-16. vi, 7, 9, 11, 46

[57] Onak, n.d.. Origami Canoe kernel description `http://onakcanoes.com/` Accessed: 2018-05-16. 7, 11, 47

[58] Brownell, B. E., 2006. *Transmaterial: A Catalog of Materials, Products and Processes that are Redefining Our Physical Environment*. Princeton Architectural Press. 7, 11, 45

[59] Elliott, A., 2013. Origami Chair Sheet Metal kernel description `https://www.youtube.com/watch?v=lgMZsHXJr6w` Accessed: 2018-05-16. 7, 11, 47

[60] Bachrach, J., 2015. Computational design + fabrication: 2d design `https://inst.eecs.berkeley.edu/~cs194-28/fa15/lectures/2d-design.pdf` Accessed: 2018-05-16. 7, 11, 47

[61] Staff, D., n.d.. Origami style: Paper-thin, patio-ready white folding chairs `https://dornob.com/origami-style-paper-thin-patio-ready-white-folding-chairs/?ref=search` Accessed: 2018-05-16. 7, 11, 47

[62] B&R Plastics inc., n.d.. B&R Plastic inc. Origami Colander kernel description `https://www.brplastics.com/folding-colanders.html` Accessed: 2018-05-16. vi, 7, 9, 11, 19

[63] DesignSwan, 2011. Every origami: 15 origami inspired product designs `https://www.designswan.com/archives/every-origami-15-origami-inspired-product-designs.html` Accessed: 2018-05-16. 7, 11, 44

[64] Kraüti, F., n.d. Magnetic Curtain kernel description `http://www.kraeutli.com/index.php/2008/01/31/magnetic-curtain/` Accessed: 2018-05-16. 7, 11, 19

[65] Junkie, G., n.d.. Oriboard origami cutting board `http://www.gadgetify.com/oriboard-origami-cutting-board/` Accessed: 2018-05-16. 7, 11

[66] DesignSwan, 2013. 8 cool multifunctional cutting boards https://www.designswan.com/archives/8-cool-multifunctional-cutting-boards.html Accessed: 2018-05-16. 7, 11

[67] Smith, A. F., Horrell, C. M., Grossmann, J. J., Feldman, J. R., and Bruccoleri, A. R., 2017. Rear-mounted aerodynamic structure for truck cargo bodies, Jan. 17 US Patent 9,545,960. 7, 9, 11, 19

[68] Edmondson, B. J., Bowen, L. A., Grames, C. L., Magleby, S. P., Howell, L. L., and Bateman, T. C., 2013. "Oriceps: Origami-inspired forceps." In *ASME 2013 conference on smart materials, adaptive structures and intelligent systems*, American Society of Mechanical Engineers, pp. V001T01A027–V001T01A027. 7, 11, 28, 47

[69] Parker, W., n.d.. Warby Parker Glasses Case kernel description https://i.warbycdn.com/s/f/97582daa7f3b79eb0f42d698b1dafa893dea1517?width=1200&quality=80 Accessed: 2018-05-16. vi, 7, 9, 11, 19, 44

[70] Nellianna, n.d.. Origami Lampshade kernel description https://www.etsy.com/listing/82568715/chestnut-paper-origami-lampshade-canary Accessed: 2018-05-16. 7, 11

[71] Fubar, K., 2011. Origami and phones, together at last https://www.slashgear.com/origami-and-phones-together-at-last-15146447/ Accessed: 2018-05-16. 7, 11

[72] Fubar, K., 2011. With clifton flat engagement ring case, your bride-to-be would never see a proposal coming https://mikeshouts.com/clifton-flat-engagement-ring-case-bride-never-see-proposal-coming/ Accessed: 2018-05-16. 7, 11

[73] Thrall, A., and Quaglia, C., 2014. "Accordion shelters: A historical review of origami-like deployable shelters developed by the us military." *Engineering structures,* **59**, pp. 686–692. 7, 11

[74] Hussey, M., 2014. Origami sofa by Yumi Yoshida unfolds to become a floor mat Available at https://www.dezeen.com/2014/03/09Accessed: 2018-05-16. 7, 11, 46

[75] Rehn, A., 2011. Cay Sova kernel description https://www.youtube.com/watch?v=tLgRisKD41w Accessed: 2018-05-16. 7, 11, 19

[76] Suzanne Casement, Martin Flannery, T. G. A. L., 2012. "Starshade design driven by stray light from edge scatter." *Proc.SPIE,* **8442**, pp. 8442 – 8442 – 11. 7, 11

[77] Schielke, T., 2014. Light matters: Mashrabiyas - translating tradition into dynamic facades https://www.archdaily.com/510226 Accessed: 2018-05-16. 7, 11

[78] Morgan, M. R., Lang, R. J., Magleby, S. P., and Howell, L. L., 2016. "Towards developing product applications of thick origami using the offset panel technique." *Mechanical Sciences,* **7**(1), p. 69. 7, 11

[79] Ihnatko, A., 2013. Kindle fire hdx 7-inch review: Third time's the charm https://www.pcworld.com/article/2051202/kindle-fire-hdx-7-inch-review-third-times-the-charm.html Accessed: 2018-05-16. 7, 11, 45

[80] Swanner, N., 2015. Leather Tablet Case kernel description https://www.slashgear.com/moshi-versacover-review-origami-cool-for-the-ipad-22380289/ Accessed: 2018-05-16. 7, 11, 45

[81] Pipetto, n.d.. Pipetto Tablet Cases kernel description https://www.pipetto.co.uk/skin/frontend/default/pipetto/images/highresimages/iPad Accessed: 2018-05-16. vi, 7, 9, 11, 45

[82] Yang, L., 2017. These futuristic origami-style utensils should be in every kitchen check them out http://www.businessinsider.com/origami-inspired-kitchen-utensils-kickstarter-2017-4 Accessed: 2018-05-16. 7, 11

[83] Felton, S. M., Lee, D.-Y., Cho, K.-J., and Wood, R. J., 2014. "A passive, origami-inspired, continuously variable transmission." In *Robotics and Automation (ICRA), 2014 IEEE International Conference on*, IEEE, pp. 2913–2918. 7, 11, 40

[84] Lee, D.-Y., Kim, J.-S., Kim, S.-R., Koh, J.-S., and Cho, K.-J., 2013. "The deformable wheel robot using magic-ball origami structure." In *ASME 2013 international design engineering technical conferences and computers and information in engineering conference*, American Society of Mechanical Engineers, pp. V06BT07A040–V06BT07A040. 7, 11

[85] Hugue, M., 2017. Six ways to bring the beauty of origami to your home decor https://www.theglobeandmail.com/life/home-and-garden/decor/six-standout-pieces-of-origami-inspired-furniture-and-decor/article15156918/ Accessed: 2018-05-16. 7, 11

[86] Zirbel, S. A., Lang, R. J., Thomson, M. W., Sigel, D. A., Walkemeyer, P. E., Trease, B. P., Magleby, S. P., and Howell, L. L., 2013. "Accommodating thickness in origami-based deployable arrays." *Journal of Mechanical Design,* **135**(11), p. 111005. vi, 9, 11, 15, 21

[87] Hirtz, J., Stone, R. B., McAdams, D. A., Szykman, S., and Wood, K. L., 2002. "A functional basis for engineering design: reconciling and evolving previous efforts." *Research in engineering Design,* **13**(2), pp. 65–82. 10

[88] Miura, K., and Natori, M., 1985. "2-d array experiment on board a space flyer unit." *Space Solar Power Review,* **5**(4), pp. 345–356. 11, 28

[89] Tachi, T., and Hull, T. C., 2017. "Self-foldability of rigid origami." *Journal of Mechanisms and Robotics,* **9**(2), p. 021008. 15

[90] Wu, W., and You, Z., 2011. "A solution for folding rigid tall shopping bags." In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, Vol. 467, The Royal Society, pp. 2561–2574. 19

[91] Liu, X., Gattas, J. M., and Chen, Y., 2016. "One-dof superimposed rigid origami with multiple states." *Scientific Reports,* **6**. 19

[92] Lang, R. J., Magleby, S., and Howell, L., 2016. "Single degree-of-freedom rigidly foldable cut origami flashers." *Journal of Mechanisms and Robotics,* **8**(3), p. 031005. 28, 42, 47

[93] Shemenski, P. D., and Trease, B. P., 2018. "Compact directional and frictional hinges for flat folding applications." In *ASME 2018 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT07A064–V05BT07A064. 28, 40, 47

[94] Ning, X., Wang, X., Zhang, Y., Yu, X., Choi, D., Zheng, N., Kim, D. S., Huang, Y., Zhang, Y., and Rogers, J. A., 2018. "Assembly of advanced materials into 3d functional structures by methods inspired by origami and kirigami: A review." *Advanced Materials Interfaces*, p. 1800284. 28

[95] Seymour, K., Burrow, D., Avila, A., Bateman, T., Morgan, D. C., Magleby, S. P., and Howell, L. L., 2018. "Origami-based deployable ballistic barrier." In *Origami 7: The proceedings from the 7th international Meetin on Origami in Science, Mathmatics, and Education*, Vol. 3, Tarquin, pp. 763–777. 29

[96] Wei, G., and Dai, J. S., 2014. "Origami-inspired integrated planar-spherical overconstrained mechanisms." *Journal of Mechanical Design,* **136**(5), p. 051003. 29

[97] Bowen, L. A., Grames, C. L., Magleby, S. P., Howell, L. L., and Lang, R. J., 2013. "A classification of action origami as systems of spherical mechanisms." *Journal of Mechanical Design,* **135**(11), p. 111008. 29

[98] Wiener, M. R., 2016. "A foundation for analysis of spherical system linkages inspired by origami and kinematic paper art.". 29

[99] Yellowhorse, A., and Howell, L. L., 2016. "Creating rigid foldability to enable mobility of origami-inspired mechanisms." *Journal of Mechanisms and Robotics,* **8**(1), p. 011011. 29

[100] Li, S., and Wang, K., 2015. "Fluidic origami with embedded pressure dependent multi-stability: a plant inspired innovation." *Journal of The Royal Society Interface,* **12**(111), p. 20150639. 29

[101] Pehrson, N. A., Magleby, S. P., and Howell, L. L., 2018. "An origami-based thickness-accommodating bistable mechanism in monolithic thick-sheet materials." In *2018 International Conference on Reconfigurable Mechanisms and Robots (ReMAR)*, IEEE, pp. 1–7. 29, 42

[102] Hanna, B. H., Lund, J. M., Lang, R. J., Magleby, S. P., and Howell, L. L., 2014. "Waterbomb base: a symmetric single-vertex bistable origami mechanism." *Smart Materials and Structures,* **23**(9), p. 094009. 29, 44

[103] Yasuda, H., Chen, Z., and Yang, J., 2016. "Multitransformable leaf-out origami with bistable behavior." *Journal of Mechanisms and Robotics,* **8**(3), p. 031013. 29

[104] Yasuda, H., and Yang, J., 2015. "Reentrant origami-based metamaterials with negative poissons ratio and bistability." *Physical review letters,* **114**(18), p. 185502. 29

[105] Mu, J., Hou, C., Wang, H., Li, Y., Zhang, Q., and Zhu, M., 2015. "Origami-inspired active graphene-based paper for programmable instant self-folding walking devices." *Science advances,* **1**(10), p. e1500533. 29

[106] stable equilibrium `Merriam-Webster.com` Accessed 2018-10-25. 30

[107] Jensen, B., Howell, L., and Salmon, L., 1999. "Design of two-link, in-plane, bistable compliant micro-mechanisms." *Journal of Mechanical Design,* **121**(3), pp. 416–423. 30

[108] Morgan, J., Magleby, S. P., Lang, R. J., and Howell, L. L., 2015. "A preliminary process for origami-adapted design." In *ASME 2015 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers, pp. V05BT08A053–V05BT08A053. 32

[109] Hernandez, E. A. P., Hartl, D. J., and Lagoudas, D. C., 2019. "Introduction to active origami structures." In *Active Origami*. Springer, pp. 1–53. 32

[110] Lang, R. J., 2011. *Origami design secrets: mathematical methods for an ancient art*. AK Peters/CRC Press. 32

[111] Foschi, R., and Tachi, T., 2018. "Designing self-blocking systems with non-flat-foldable degree-4 vertecies." In *Origami 7: The proceedings from the 7th international Meetin on Origami in Science, Mathmatics, and Education*, Vol. 3, Tarquin, pp. 795–809. 34, 39, 46

[112] Howell, L. L., Magleby, S. P., and Olsen, B. M., 2013. *Handbook of compliant mechanisms*. John Wiley & Sons. 42

[113] Silverberg, J. L., Na, J.-H., Evans, A. A., Liu, B., Hull, T. C., Santangelo, C. D., Lang, R. J., Hayward, R. C., and Cohen, I., 2015. "Origami structures with a critical transition to bistability arising from hidden degrees of freedom." *Nature materials,* **14**(4), p. 389. 42

[114] Chen, G., Gou, Y., and Zhang, A., 2011. "Synthesis of compliant multistable mechanisms through use of a single bistable mechanism." *Journal of Mechanical Design,* **133**(8), p. 081007. 42

[115] Oh, Y. S., and Kota, S., 2009. "Synthesis of multistable equilibrium compliant mechanisms using combinations of bistable mechanisms." *Journal of Mechanical Design,* **131**(2), p. 021002. 42

[116] Alfattani, R., and Lusk, C., 2018. "A lamina-emergent frustum using a bistable collapsible compliant mechanism." *Journal of Mechanical Design,* **140**(12), p. 125001. 42

[117] Francis, K., Blanch, J., Magleby, S., and Howell, L., 2013. "Origami-like creases in sheet materials for compliant mechanism design." *Mechanical Sciences,* **4**(2), pp. 371–380. 43

[118] Rogers, J., Huang, Y., Schmidt, O. G., and Gracias, D. H., 2016. "Origami mems and nems." *Mrs Bulletin,* **41**(2), pp. 123–129. 45

[119] Wang, C., Li, J., and You, Z., 2018. "A kirigami-inspired foldable model for thick panels." In *Origami 7: The proceedings from the 7th international Meetin on Origami in Science, Mathmatics, and Education*, Vol. 3, Tarquin, pp. 715–730. 46

[120] Huffman, D. A., 1976. "Curvature and creases: A primer on paper." *IEEE Transactions on computers*(10), pp. 1010–1019. 47