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Modeling dependence structure between stock market volatility and sukuk yields: A nonlinear study in the case of Saudi Arabia

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Abstract

The aim of this paper is to investigate the dependence structure between sukuk (Islamic bonds) yields and stock market (returns and volatility) in the case of Saudi Arabia. We consider three Archimedean copula models with different tail dependence structures namely Gumbel, Clayton, and Frank. This study shows that the sukuk yields exhibit significant dependence only with stock market volatility. In addition, the dependence structure between sukuk yields and stock market volatility are symmetric and linked with the same intensity.

JEL classification: G02; G15

Keywords: sukuk; Conditional volatility; GARCH; Dependence; Copula

1. Introduction

The central characteristic of the Islamic finance is the prohibition of the payment and receipt of interest (or *riba*).[1] The best definition of "*riba*" is the prohibition of charging interest when lending money and of any addition to money that is unjustified (such as a penalty). Islamic principles permit instead for the replacement of interest by a return that is dependent upon the profitability of the underlying investment. Lending money by charging interest permits the lender to increase his capital without any effort because money by itself does not create "valued added". Islamic finance prohibits also investing in transactions involving gambling, alcohol, drugs, and transactions including uncertainty about the subject matter and contract terms. In addition, the transfer of debt and, therefore, the buying and selling of debt are prohibited under Islamic law. Various *Sharia* compliant financing and investment structures have been developed. The investment concept of sukuk was created as an alternative to interest-bearing instruments namely conventional bonds. The emergence of sukuk has been one of the most significant developments in Islamic capital markets in recent years.

The main motivation of this study arises from the perception that investigating dependence structure between sukuk and stock markets plays an important role in asset allocation as well as risk management. sukuk market may be influenced by local financial conditions and especially stock returns and volatility. Therefore, the purpose of this paper is to investigate the dependence structure for sukuk yields and stock market (returns and volatility) in the case of Saudi Arabia. The focus on Saudi Arabia is explained by the fact that sukuk market in this country is in development stage. In the GCC region and during the first three quarters of 2013, Saudi Arabia is the most market player with an issue of $8.69 billion sukuk followed UAE with an issue of $5.17 billion (Thomson Reuter's Zawya report, 2014). This paper differs from and add to the related literature on sukuk and stock market co-movements by investigating the nonlinear structure of dependence by using

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[1] The Islamic name for interest is Riba.

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Archimedean copula functions with different tail dependence structures namely Gumbel (Upper tail dependence), Clayton (Lower tail dependence) and Frank (symmetric dependence). Using daily data from 23 November 23, 2010 to October 6, 2014, our empirical results show negative and symmetric dependence structure between sukuk yields and stock returns volatility.

There are several advantages when we use copula functions in analyzing the dependence structure. First, copulas allow us to construct any multivariate distributions with given univariate margins. Second, are invariant to increasing and continuous transformations. For example, dependence structure with copula does not change with returns or logarithm of returns. This is not the case for the correlation, which is only invariant under linear transformations (Naifar, 2012). Third, the copula function can provide us the degree and the structure of dependence in the tail dependence. Tail dependence indicates the extreme co-movements and the potential of a simultaneous large loss in the equity markets. Furthermore, tail dependence is an appropriate measure for systematic risk in times of financial crisis and it allows investors and market participants to measure the probability of simultaneous extreme losses.

The remainder of the paper is organized as follows: Section 2 provides an overview of sukuk market. Section 3 presents literature review. Section 4 presents copula methodology. Section 5 presents data and preliminary statistics. Section 6 presents estimation and analysis of results. The article ends with a conclusion.

2. Sukuk market development

In May 2003, sukuk was defined officially by Auditing and Accounting Organization of Islamic Financial Institutions (AAOIFI) as "the certificates of equal value representing undivided shares in ownership of tangible assets, usufruct and services or in ownership of the asset of a particular project or special investment activity". sukuk securities have some similar features with conventional bond. It has fixed term maturity and is tradable in the normal yield price. However, there are major differences between sukuk and conventional bonds, including that conventional bond issuers pay interest to investors in regular intervals. sukuk avoids this type of interest and is based on the sharing of profit and loss between parties in a business transaction. In addition, sukuk are asset-based rather than asset-backed securities.

There are different types of sukuk, which can be arranged and ordered in the form of different financial transactions. The types of sukuk have to be reviewed and approved by "Sharia advisers" to ensure agreement with Sharia law. sukuk structures are based on Islamic mode of financing including "*Ijarah, Musharaka, Mudaraba and Murabaha*". The AAOIFI has laid down 14 types of sukuk. Ijara and musharaka sukuk have clearly emerged as the most popular sukuk structures for both investors and issuers. Ijarah sukuk are certificates associated with a leasing contract, which includes securities having equal value. Musharaka sukuk are certificates based on risk and profit sharing.

Malaysia was an early starter and has been the most dynamic in promoting and creating a exciting local-currency sukuk market through the provision of supportive banking and capital markets legislation. Outside the Gulf countries, Turkey is also beginning to actively develop its domestic sukuk market. The government is highly supportive of Islamic finance and sukuk instrument and has taken serious steps to support it through new legislation and large sovereign issuances. According to Islamic finance news guide (2014), the sukuk market will continue to develop at a good rate of growth. There may have been some ups and downs, but the overall market momentum over the past 10 years has been quite positive. We believe this positive momentum and growth will continue in the foreseeable future.

According to Thomson Reuters Zawya report (2014), the total number of sukuk issuance as at end Q3, 2013 was $79.70 billion (552 issues) compared to the higher $109 billion (532 issues) for the same period in 2012. In addition, the total sukuk issuances for Saudi Arabia from Jan 96-Sep13 is $ 39,296 million. In the first three quarter of 2013, Malaysia issued $54.33 billion sukuk, followed by Saudi Arabia ($8.69 billion), UAE ($5.17 billion) and Indonesia ($5.03 billion).

3. Literature review

Most of the research on sukuk instruments are theoretical studies and focuses mainly on explaining and developing sukuk structures with an emphasis on legal considerations (Abdel-Khaleq & Richardson, 2007; Tariq & Dar, 2007; Vishwanath & Azmi, 2009). It has been documented in literature that sukuk serve as a crucial tool for resource mobilization and a key instrument for the development of Islamic financial market (Jobst, Kunzel, Mills, & Sy, 2008; Wilson, 2008). In the recent years, some empirical studies are devoted either to research on structured sukuk instruments with case studies (Solé, 2008) or research on macroeconomic influences on sukuk issuance (Ahmad, Dauda, & Kefeli, 2012).

Other studies introduce the effects of stock market in explaining sukuk yield dynamics. Naifar and Mseddi (2013) analyse the links between sukuk yield yields, stock market conditions and macroeconomic variables in the case of United Arab Emirates (*thereafter* UAE) by using linear regression. They find that sukuk yield spread react positively to stock market implying that an increase in stock index return is accompanied with an increase in sukuk yield yields. Godlewski, Turk, and Weill (2014) analyze stock market reaction to types and characteristics of sukuk. Using event study framework and a sample of 131 sukuk of eight countries (Bermuda, Saudi Arabia, UAE, Malaysia, Qatar, Singapore, Caiman Islands and Indonesia) they find that Ijara sukuk excert a positive influence to stock prices of issuing firms. Alam, Hassan, and Haque (2013) examine the comparative wealth effect of sukuk and conventional bond announcements on stock returns in major Islamic financial market (Malaysia, Indonesia, Singapore, Pakistan, UAE, Bahrain and Qatar). They find that the stock market reaction is negative for the announcements of sukuk before and during 2007 global financial crisis. Aloui, Hammoudeh, and Ben Hamida (2015a) study the co-movement between the sharia-compliant stocks and sukuk in the Gulf Cooperation Council (GCC) countries using wavelet squared coherency approach. They find a strong dependence between sharia stock and sukuk indexes. Aloui, Hammoudeh, and Ben Hamida (2015b) investigate the volatility spillovers between sukuk and sharia-compliant stocks in GCC countries using DCC-GARCH model. They find a time-varying negative correlation between sukuk index and sharia-compliant stocks.

As the above literature recommend, sukuk yields can be linked to stock market conditions. Our study differs from previous research. First, we model the dependence structure between sukuk yields and their respective stock market in the case of Saudi Arabia. Second, we use a variety of copula functions that allow for asymmetric dependence and give the information about tail dependence and extreme events. Third, our recent sample include all liquid sukuk contract available on Thomson Reuters database covering the period from 23 November 2010 to 6 October 2014 that present regular daily data.

4. Archimedean copulas methodology

The theory of copula dates back to Sklar (1959). The copula function links the univariate margins with their full multivariate distribution. It presents a useful tool when modelling non Gaussian data since the Pearson's correlation coefficient is adapted for linear dependence and normal distribution. One appealing feature of a copula function is that the margins do not depend on the choice of the dependency structure and then, we can model and estimate the structure of dependency and the margins separately. Copula functions describe the whole multivariate distribution and present the property that it is invariant under strictly increasing transformations of the margins. Also, they present a basis for flexible techniques for simulating dependent random vectors.

For n uniform random variables $u_1, u_2, \ldots, u_n$, the joint distribution function C is defined as:

$$C(u_1, u_2, \ldots, u_n, \theta) = \Pr[U_1 \le u_1, U_2 \le u_2, \ldots, U_n \le u_n] \quad (1)$$

where θ the dependence parameter.

As we only need the concept of copulas for two dimensions, we present the following definition:

A copula function is the restriction to $[0,1]^2$ of a continuous bivariate distribution function whose margins are uniform on [0,1]. A (bivariate) copula is a function $C:[0,1]^2 \to [0,1]$ which satisfies the boundary conditions

$$C(t,0) = C(0,t) = 0 \text{ and } C(t,1) = C(1,t) = t \text{ for } t \in [0,1].$$

Similarly, copula satisfies the 2-increasing property: $C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0$ for all

u_1, u_2, v_1, v_2 in $[0,1]$ and $u_1 \le u_2$ and $v_1 \le v_2$.

A copula is symmetric if $C(u,v) = C(v,u)$ for all (u,v) in $[0,1]^2$ and is asymmetric otherwise.

Sklar (1959) shows the importance of copulas as a universal tool for studying multivariate distributions. Let F be a multivariate n-dimensional distribution function with marginals $F_1, \ldots, F_n$. then it exists a copula such that:

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)); \quad (x_1, \ldots, x_n \in \Re) \quad (2)$$

If the marginal distributions $F_1, \ldots, F_n$ are continuous, then C is unique.

By definition, applying the cumulative distribution function (CDF) to a random variable (r.v.) results in a r.v. that is uniform on the interval [0, 1]. Let X a random variable with continuous distribution function F_X, $F_X(X)$ is uniformly distributed on the interval [0,1]. This result is known as the probability integral transformation theorem and present many statistical procedures. With this result in hand, we may introduce the copula using basic statistical theory. In particular, the copula C for (X,Y) is just the joint distribution function for the random couple $F_X(X)$, $F_Y(Y)$ provided F_X and F_Y are continuous.

Numerous copulas can be found in the literature (see Nelson (1991)). The most commonly applied copula function (especially in finance modelling) is the normal copula.

The Archimedean copula has simplified the construction of bivariate distributions and it contains numerous families that present different dependency structures.

Let Φ denote a function $\Phi:[0,1] \to [0,\infty]$ which is continuous and satisfies:

- $\Phi(1) = 0$
- $\Phi(0) = \infty$
- For all $t \in (0,1)$, $\Phi'(t) < 0$, then Φ is decreasing.
- For all $t \in (0,1)$, $\Phi''(t) \ge 0$, then Φ is convex.

The function Φ has an inverse $\Phi^{-1}:[0,\infty] \to [0,1]$ which has the same properties except that $\Phi^{-1}(0) = 1$ and $\Phi^{-1}(\infty) = 0$. A copula is said to be an Archimedean copula if its distribution function can be written as follows:

$$C_{arch}(u,v) = \Phi^{-1}[\Phi(u) + \Phi(v)]$$

For all $0 \le u,v \le 1$ and Φ is called a generator function of copula that satisfies the following properties:

- C is symmetric; i.e., $C(u,v)) = C(v,u)$ for all $u, v \in [0,1]$.
- C is associative; i.e., $C(C(u,v),w) = C(u,C(v,w))$ for all $u, v, w \in [0,1]$.
- If $k > 0$ is any constant, then $k\Phi$ is also generator of Φ.

To take into account nonlinear relationships, we employ three Archimedean copula functions: Clayton, Frank, and Gumbel.

- ***Clayton copula***: This family is an example of an Archimedean copula and was proposed by Clayton (1978) as follows:

Let $\Phi(t) = \frac{(t^{-\theta}-1)}{\theta}$ with $\theta \in [-1, \infty)/\{0\}$, then:

$$C_{\theta}^{clayton}(u,v) = \max\left[\left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}}, 0\right] \tag{3}$$

The variable Φ is called a generator function of the copula and $\theta \in [-1, \infty)/\{0\}$ controls the degree of dependence between u and v. If $\theta > 0$, then $\phi(0) = \infty$, and we can simplify the above expression:

$$C_{\theta}^{clayton}(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{4}$$

The Clayton copula has lower tail dependence but not upper tail dependence. The contour or the level curves of a copula C are given by $\{(u,v) \in \mathrm{I}^2 / C(u,v) = t\}$ To illustrate the range of bivariate behaviour that can be represented by Archimedean copula, consider the following figures (see Fig. 1):

The Clayton copula has lower tail dependence but not upper tail dependence. The contour generated by the Clayton copula implies fat tailed distribution.

- ***Gumbel copula***: This copula, proposed by Gumbel (1960), is an extreme value copula. It is an asymmetric copula with higher probability concentrated in the right tail. It is suited not only to random variables that are positively correlated but also, particularly, to those in which high values of each are more strongly correlated than low values. It is given as follows:

Let $\Phi(t) = (-\ln t)^{\theta}$, with $\theta \geq 1$. Then,

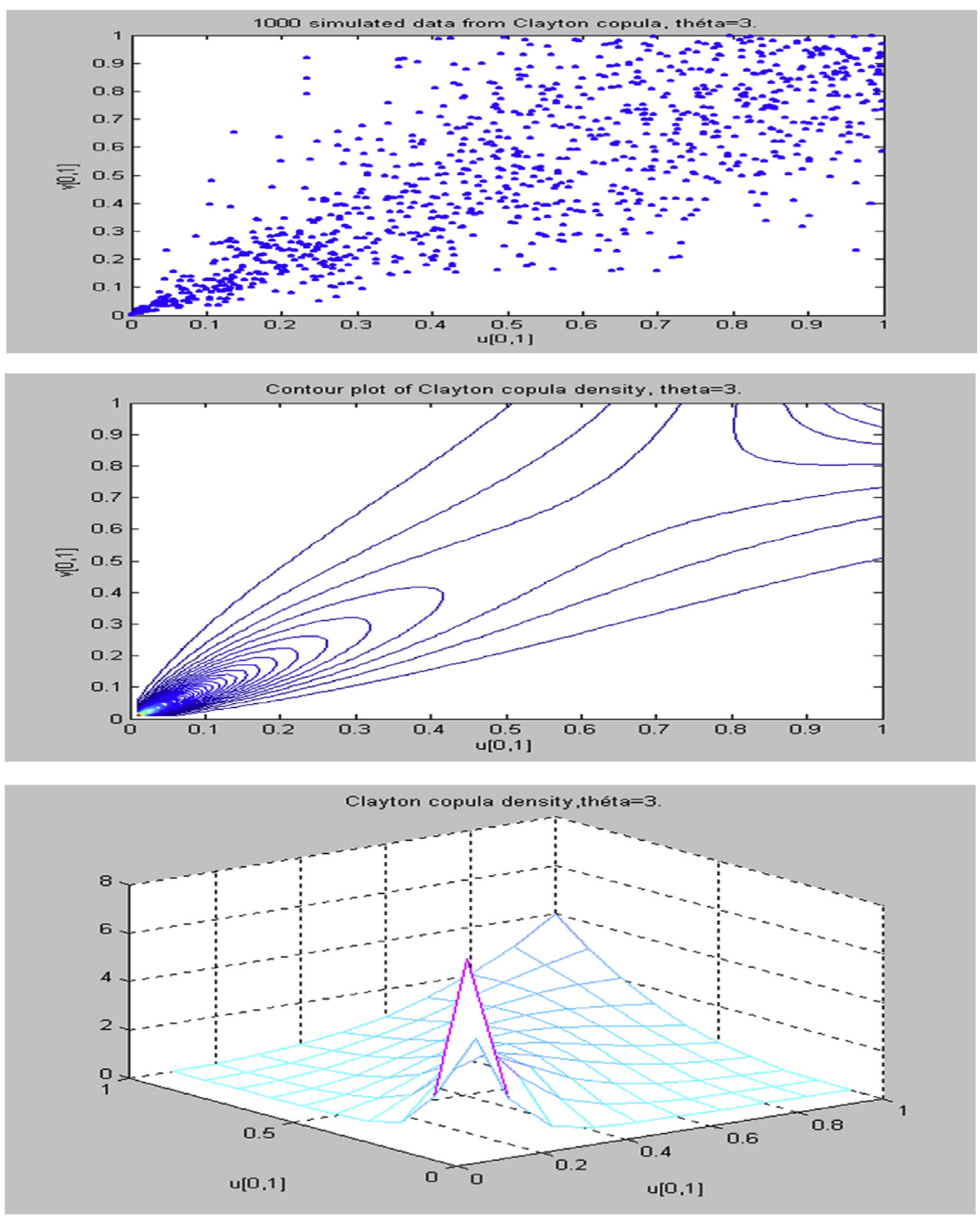

Fig. 1. Clayton copula.

$$C_{\theta}^{Gumbel}(u,v) = \exp\left(-\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{1}{\theta}}\right);\ 0 \le u, v \le 1, \tag{5}$$

where $\theta \in [1, \infty)$ controls the degree of dependence between u and v (see Fig. 2).

The Gumbel copula has Upper tail dependence but not lower tail dependence. Also, this copula does not allow for negative dependence and allow for positive right tail dependence, which means the probability that both variables are in their right tails is positive.

- ***Frank copula***: This family is proposed by Frank (1979) as follows:

Let $\Phi(t) = -\ln \frac{-e^{-\theta t}-1}{e^{-\theta}-1}$ with $\theta \in \Re/\{0\}$

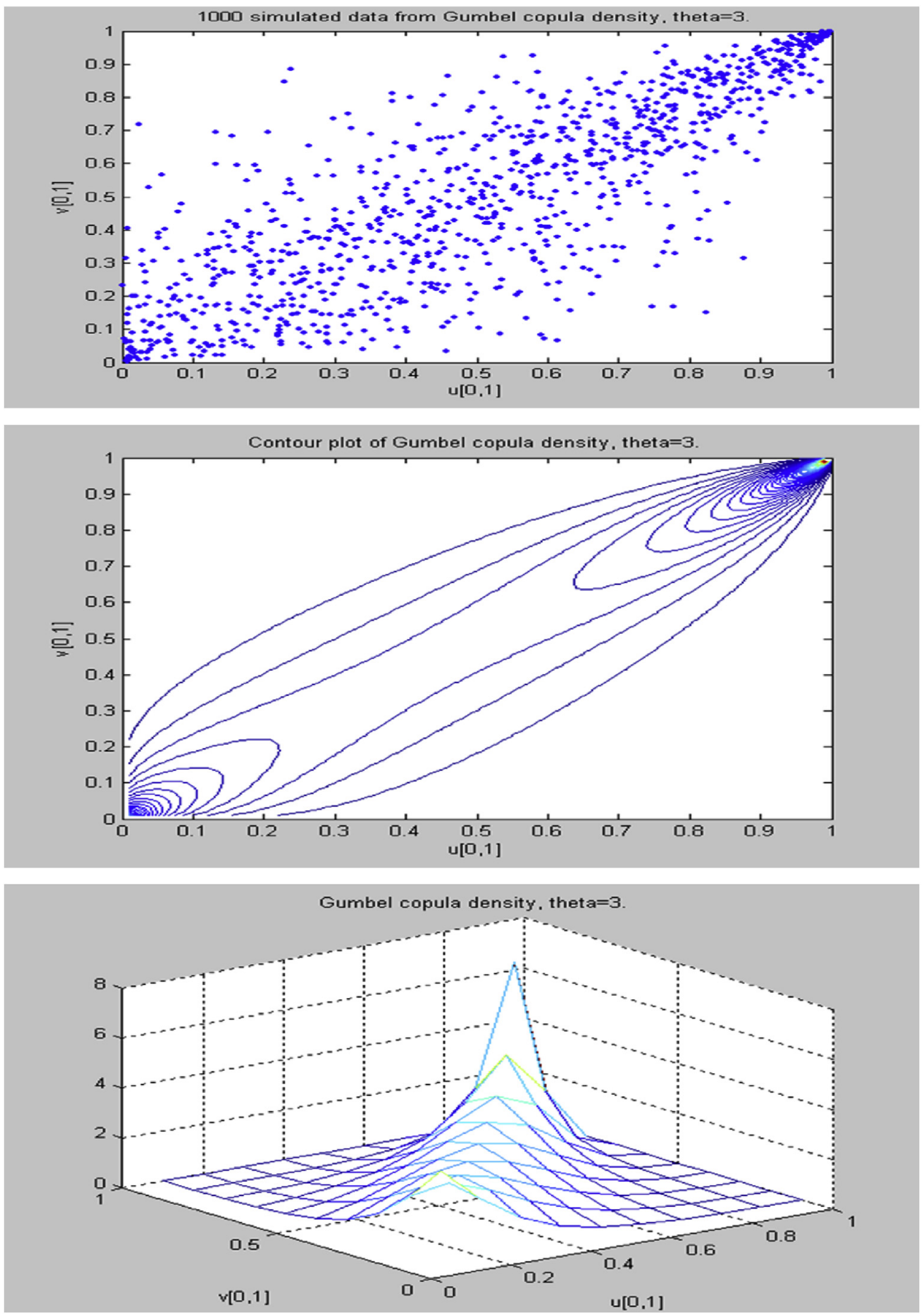

Fig. 2. Gumbel copula.

$$C_\theta^{Frank}(u,v) = -\frac{1}{\theta}\ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right). \quad (6)$$

The range of bivariate behaviour that can be represented by Frank copula is illustrated as follow (see Fig. 3):

The Frank copula is a symmetric Archimedean copula. It does not exhibit either upper or lower tail dependence. The Frank copulas do not exhibit either upper or lower tail dependence. They are the only radially symmetric Archimedean copulas.

5. Data description

The first data set consists of daily yield to maturity data of sukuk from Thomson Reuter's database from 23 November 2010 to 06 October 2014. We use only data for liquid sukuk that present regular daily data because they are completely negotiable and can be traded in the secondary markets. We construct an equally-weighted sukuk index for Saudi Arabia sukuk market. Table 1 lists the composition of the constructed sukuk index.

In Fig. 4, we plot the constructed sukuk index for Saudi Arabia.

Table 1
Composition of the constructed sukuk index.

Country	sukuk names
Kingdom of Saudi Arabia (KSA)	- Jadwa global sukuk - Itqan fund for murabaha & sukuk - AlAhli US Dollar sukuk and Murabaha Fund - Dar Al Arkan sukuk

Note: We use all liquid sukuk available on Thomson Reuter's database that present regular daily data covering the period from November 2010 to October 2014.

The second dataset consists of daily stock market index returns (Tadawul All-Share Index (TASI)), covering the period from 23 November 2010 to 06 October 2014. The daily stock index returns (R_t) are calculated as follows:

$$R_t = Ln\left(\frac{P_t}{P_{t-1}}\right) \times 100 \quad (7)$$

where P_t is the stock index at date t. the monthly stock index return is computed as the average of daily return.

The third dataset consists of conditional volatility of stock return. We estimate the conditional volatility of the Saudi

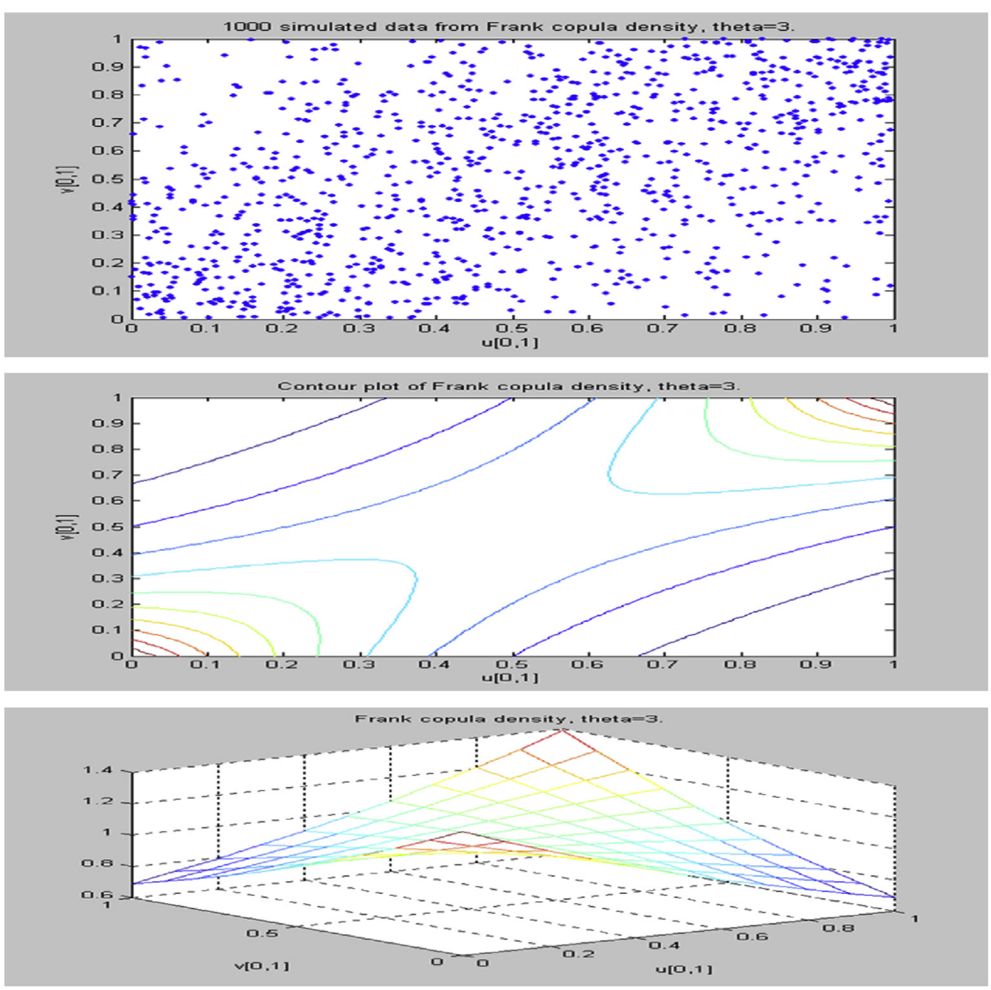

Fig. 3. Frank copula.

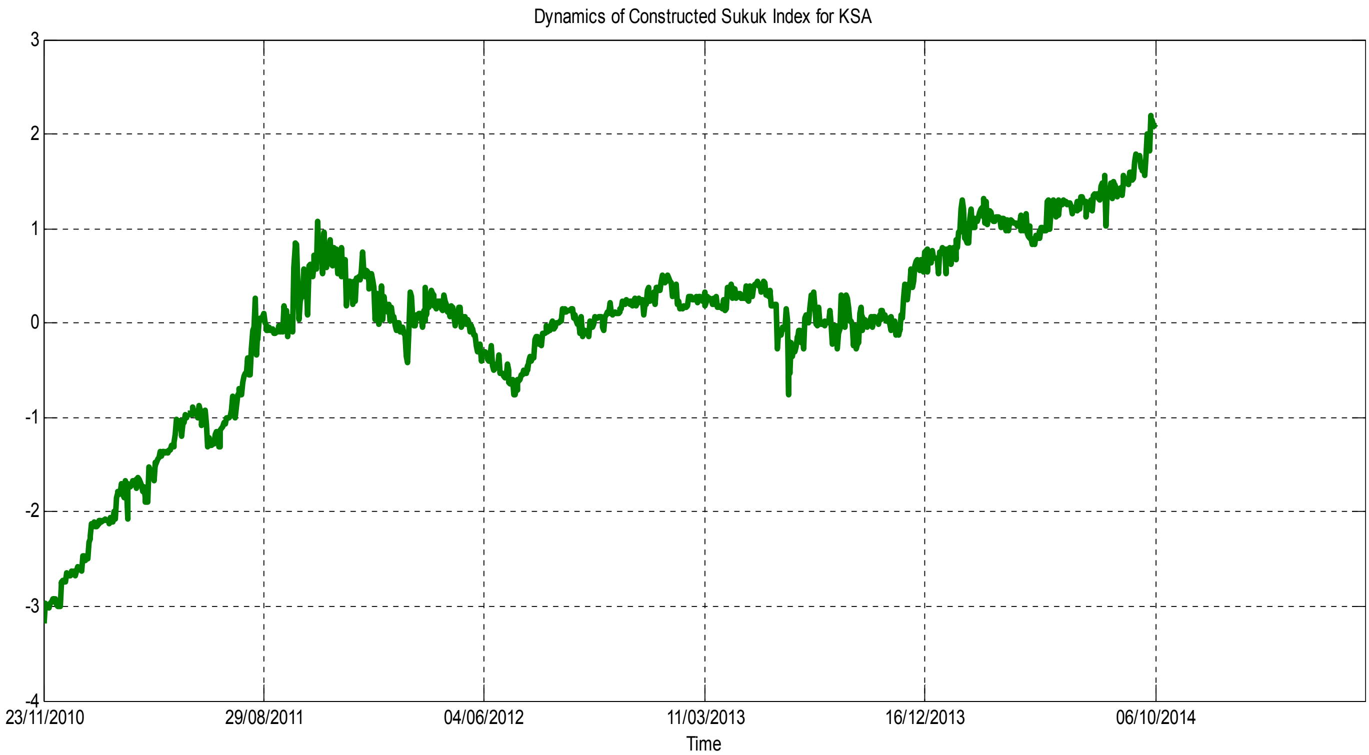

Fig. 4. Constructed sukuk index yields for KSA (an equally weighted sukuk index from Table 1).

Arabia stock market returns using GARCH models. We select the best specification according to the Bayesian information criterion (BIC). We include the following models (all with one lag of the innovation and one lag of volatility):

- GARCH [Bollerslev (1986)]:

$$h_t = w + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{8}$$

Exponential GARCH [EGARCH, Nelson (1991)]:

$$Ln(h)_t = w + \alpha\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \gamma\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta\ln(h_{t-1}) \tag{9}$$

- Threshold GARCH [TARCH, Zakoian (1994)]:

$$h_t^{\frac{1}{2}} = w + \alpha|\varepsilon_{t-1}| + \gamma I[\varepsilon_{t-1}\langle 0]|\varepsilon_{t-1}| + \beta h_{t-1}^{\frac{1}{2}}. \tag{10}$$

In these specifications, ω is the unconditional variance that satisfies the condition $\omega > 0$, ε_t is the innovation in the levels model and h_t is the conditional variance. The parameter γ is the asymmetry parameter that satisfies the condition $-1 < \gamma < 1$. The GARCH parameters $\beta < 1$ and $\alpha < 1$ measure the GARCH and ARCH effects, respectively. Table 2 presents the specifications of the selected GARCH processes (based on BIC criterion) and the estimated parameters.

From Table 2, we notice that the GARCH model selected for the Saudi Arabia stock returns excludes a significant asymmetric term. We report that GARCH (1,1) is the appropriate model and all the coefficients are significant, indicating that stock volatilities are characterized by a heteroscedastic process. In addition, the p-values corresponding to the Ljung–Box statistics are not significant, suggesting that we fail to reject the null of no serial correlation at the percent level. Then, the fitted GARCH (1,1) model is adequate in having captured all of the volatility dynamics. Table 3 summarizes the preliminary statistics from the data.

Table 3 illustrates that the skewness measure is negative which shows that excess-return time series are skewed to the left. The kurtosis statistic is more than 3 for all series, demonstrating that these series have fatter tails compared with the normal distribution. Finally, the Jarque–Bera test demonstrates that the null hypothesis of normality is rejected for all series.

6. Copula parameters estimation

A first explanatory tool to show the possible relations between variables is the scatter plot. A scatter plot can be a rich

Table 2
Univariate GARCH models estimations.

Index	Model selected	w	α	γ	β	LB (10)	LB (10)^2
TASI return	GARCH (1,1)	5.06E-06* (0.0000)	0.098095* (0.0000)	–	0.832770* (0.0000)	24.719 (0.006)	2.004 (0.996)

Note: We use the Bayesian information criterion (BIC) to select the appropriate GARCH specification. The models with the asymmetry parameter γ are EGARCH, TARCH. LB (10) and LB (10)^2 indicate the Ljung–Box statistics of the series for standardized errors and squared errors, respectively, up to the 10th lag. (.) indicates p-values and (*) indicates statistical significance at least at the 1% level.

Table 3
Preliminary statistics.

Name of the company	Mean	S.D	Max	Min	Skewness	Kurtosis	Jarque–Bera
Constructed SDY (KSA)	0.0001	0.0027	0.015	−0.018	−0.139	8.80	1423.3*
TASI returns	0.0005	0.008	0.070	−0.070	−1.27	21.41	14,545.67*

Note: (*) Indicate statistical significance at least at the 1% level.

source of information about association and dependence structure. We plot daily sukuk index yields and the stock market returns and volatility.

According to Fig. 5, we observe a priori the absence of a clear correlation. Significant deficiencies in the Pearson correlation coefficient encourage alternative dependence measures referred to as rank correlation. In this study, we focus on Kendall's tau and Spearman's rho. They are more useful in describing the dependence between random variables, because they are invariant to the choice of marginal distribution. Kendall's tau is just the number of observation pairs where both variables go in the same direction, minus the number of observations pairs where both variables go in the opposite direction, divided by the number of possible pairs. More both variables go in the same direction, the higher is tau. Kendall's tau and Spearman's rho can be computed based on the copula associated with the bivariate data. Schweizer and Wolff (1981) conclude that two standard nonparametric rank correlations can be expressed solely in terms of the copula function. Let $C(u_1,u_2)=F(F_X^{-1}(u_1),F_Y^{-1}(u_2))$ be the copula associated with the bivariate random vector X and Y. F_X and F_Y are the marginals of the distribution of the X and Y random variables, respectively, and $(u_1,u_2)\in[0,1]^2$. Spearman's rho (ρ) can be calculated as follows:

$$\rho=12\iint_{[0,1]^2} u_1u_2dC(u_1,u_2)-3 \tag{11}$$

Kendall's tau (τ) for two random variables X and Y is the probability of concordance minus the probability of discordance. Assume that (X,Y) and (X*,Y*) are two independent realizations of a joint distribution:

$$\tau=4\iint_{[0,1]^2} C(u_1,u_2)dC(u_1,u_2)-1 \tag{12}$$

In Table 4, we compute the empirical values of Kendall's tau and Spearman's rho rank correlations among the sukuk yields and stock market conditions.

In Table 4, we observe two insights related to dependence between sukuk yields and stock market conditions: (1) the absence of significant dependence (measured by Kendall's tau and Spearman's rho) between sukuk yields and stock market returns. (2) The dependence coefficients between sukuk yields and stock market volatility are significantly different from zero.

The next step in our analysis is to estimate the Archimedean copula parameters (Gumbel, Clayton and Frank). Following Genest and MacKay (1986), the Archimedean copula parameters can be expressed as a Kendall's tau statistic. Let's consider (X,Y) a pair of random variables whose distribution H is of the form: $[C_\Phi(x,y)=\Phi^{-1}\{\Phi(x)+\Phi(y)\}]$ for some Φ, then:

$$\tau=4\int_0^1\frac{\Phi(t)}{\Phi'(t)}dt+1. \tag{13}$$

Using Eq. (13), we can estimate the copula parameters, using a relationship between Kendall's tau and the Archimedean copula (Table 5).

Based on the values of Kendall's tau obtained from Table 4 and the relationship between Kendall tau and the generator function of Archimedean copula (Table 5), we can compute the dependence parameter θ and tail dependence coefficients of each retained Archimedean copula. We retain only the dependence structure between sukuk yields and stock market volatility since the dependence coefficients between sukuk yields and stock market returns are not significant (results from Table 4). The estimation results are given in Table 6.

According to Table 6, we notice the absence of upper and lower tail dependence between sukuk yields and all stock market return volatility. Frank copula gives a better fit of the dependence structure between KSA sukuk yields and stock market volatility. We conclude the existence symmetric upper and lower tail dependence, and this finding suggests that sukuk yields and stock market volatility are linked with the same intensity.

7. Conclusions

The correlation between sukuk and stock markets plays an important role in asset allocation as well as risk management. sukuk has emerged as one of the important components of global Islamic Financial System. The continual growth of sukuk has raised question whether sukuk can play the role of an alternative source of financing and investment which might replace the conventional bond. The findings of this study are pertinent to the recent issues related to Islamic financial instruments. We find symmetric dependence between sukuk and stocks market volatility and this finding suggests that sukuk yields and stock market volatility are linked with the same intensity and the linkage is immune to extreme events. This finding can be explained by the fact that, the Saudi Arabia stock market crash in February 2006 followed by the global financial crisis, have made investors be cautious. Then, any increase or decrease in the Saudi stock market has systematic repercussions manifested in their investment strategies in a symmetric way.

The knowledge of financial co-movement and dependence structure among sukuk and stock markets is important for

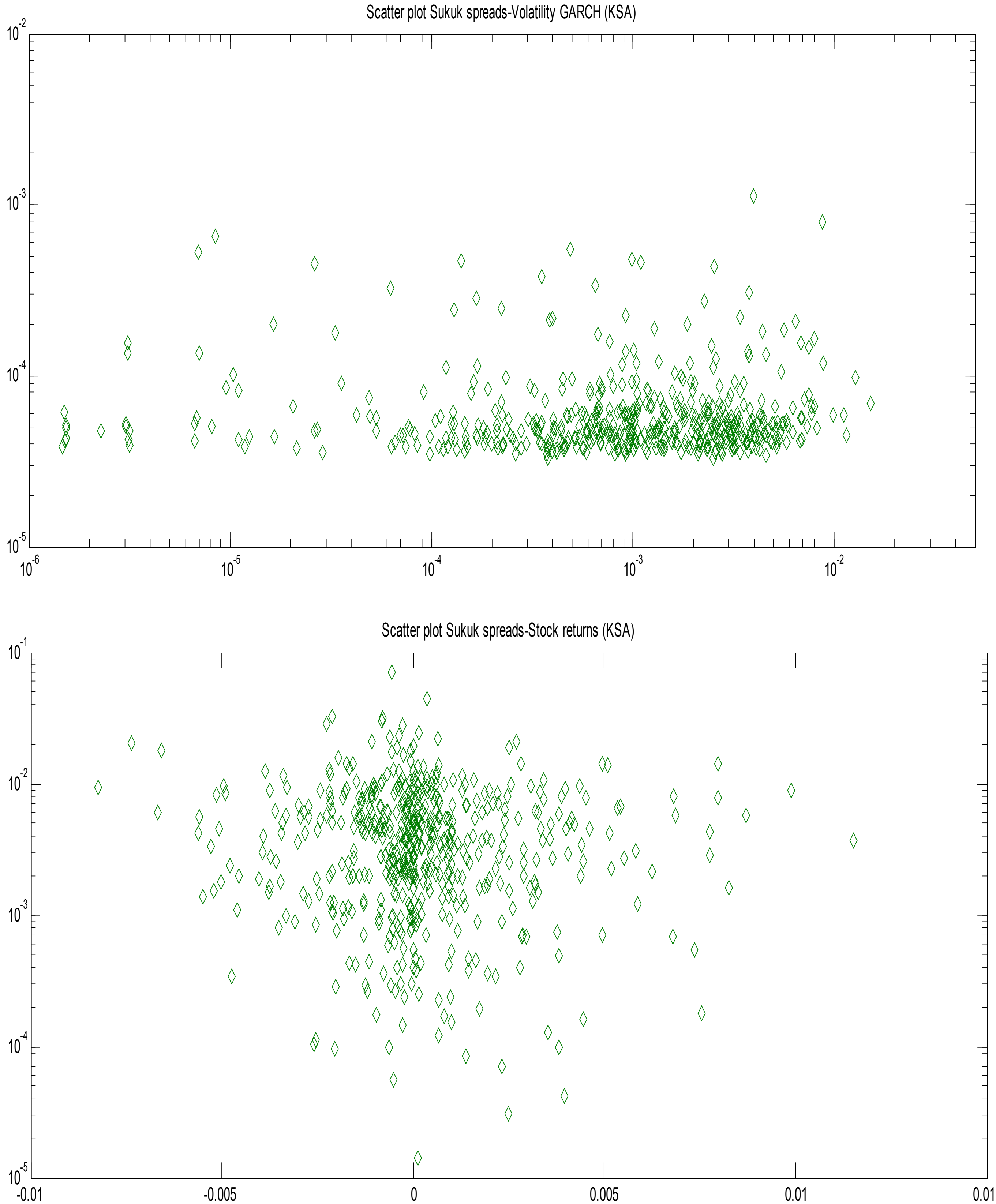

Fig. 5. Scatter plot of sukuk yields, stock returns and conditional volatility for KSA market.

Table 4
Rank correlation measures for different couples.

Pairs	Kendall's tau	Spearman's rho
sukuk yields (SY) − KSA(SMR)	0.0248 (0.2384)	0.0364 (0.2476)
sukuk yields (SY)-KSA (SMV)	−0.2610 (0.000)	−0.3844 (0.000)

Notes: SMR (Stock market return), SMV (Stock market volatility). The values in parentheses indicate the probability (p-value). P-values for Kendall's tau and Spearman's rho are using either the exact permutation distributions (for small sample sizes), or large-sample approximations.

Table 5
Relationship between Kendall tau and the generator function of Archimedean copula.

Family	Range of θ	$\Phi(u)$	τ
Gumbel	$\theta \in [1, \infty)$	$(-Ln(u))^{\theta}$	$\frac{\theta-1}{\theta}$
Clayton	$\theta \in [0, \infty)$	$u^{-\theta}-1$	$\frac{\theta}{\theta+2}$
Frank	$\theta \in (-\infty, +\infty)$	$-\ln \frac{e^{-\theta u}-1}{e^{-\theta}-1}$	$1-\frac{4}{\theta}[1-D_1(\theta)]$

Notes: $D_1(\theta)$ is the Debye function defined as $D_n(x) = \frac{n}{x^n}\int_0^x \frac{t^n}{e^t-1}dt$ for n positive integer.

Table 6
Nonparametric estimation of Archimedean copula parameters.

Country	Pairs	Gumbel	Frank	Clayton
KSA	(SY)-KSA(SMV)	ND	−2.4886	ND
	λ_{UR}	–	0	–
	λ_{LL}	–	0	–

Notes: ND means not defined in the corresponding value of Kendall tau. $\theta \in [1, \infty)$ for Gumbel copula, $\theta \in [0, \infty)$ for Clayton copula, $\theta \in (-\infty, +\infty)$ for Frank copula.

portfolio diversification and risk management. We observe a negative dependence between the sukuk yields and the volatility stock markets. Negative dependence across assets generally enhances diversification benefits, while asymmetric dependence limits diversification. Diversification benefits can be gained by investing in the Saudi sukuk combined with the local and regional stock markets. The symmetric volatility implies that volatility increases at the same intensity after a negative and a positive shock of the same magnitude. The portfolio managers can tap our empirical results by combining sukuk and stock as assets with symmetric dependence and stock and sukuk structure from other cooperation council countries with asymmetric dependence to hold optimal portfolio weights and hedge ratios in crisis periods and in different market episodes.

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