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How Probabilities Came to Be Objective and Subjective

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FOR DIRK STRUIK ON HIS 100TH BIRTHDAY

Between 1837 and 1842 at least six mathematicians and philosophers, writing in French, English, and German, and working independently of one another, introduced distinctions between two kinds of probability. Although the grounds, contents, and implications of these distinctions differed significantly from author to author, all revolved around a philosophical distinction between "objective" and "subjective" which had emerged ca. 1840. It was this new philosophical distinction which permitted the revisionist probabilists to conceive of the possibility of "objective probabilities," which would have been an oxymoron for classical probabilists such as Jakob Bernoulli and Pierre Simon Laplace. Without relinquishing the rigid determinism of the classical probabilists, the revisionists were nonetheless able to grant chance an objective status in the world by opposing it to the subjective variability of the mind. © 1994 Academic Press, Inc.

Zwischen 1837 und 1842 haben mindestens sechs Mathematiker und Philosophen, die auf Französisch, Englisch und Deutsch geschrieben haben und die unabhängig von einander gearbeitet haben, Unterscheidungen zwischen zweierlei Arten von Wahrscheinlichkeiten eingeführt. Obwohl die Begründungen, Inhalte und Implikationen dieser Unterscheidungen sehr verschiedend unter den Autoren waren, waren alle auf eine philosophische Unterscheidung zwischen "objektive" und "subjektiv," die rum 1840 erschien, zentriert. Diese philosophische Unterscheidung hat es ermöglicht, daß die revisionistische Probabilisten die Möglichkeit von "objektiven Wahrscheinlichkeiten" zulassen konnten—eine Möglichkeit die für die klassische Probabilisten wie Jakob Bernoulli und Pierre Simon Laplace ein Widerspruch an sich gewesen wäre. Ohne den strengen Determinismus der klassischen Probabilisten aufzugeben waren die revisionistischen Probabilisten bereit Zufall einen objektiven Status in der Welt zu verleihen, als Gegengewicht der subjektiven Variabilität des Geistes. © 1994 Academic Press, Inc.

Entre 1837 et 1842 au moins six mathématiciens et philosophes, écrivant en français, anglais, et allemand, et travaillant indépendamment l'un de l'autre, ont introduit distinctions entre deux espèces de probabilité. Quoique les raisons, qualités et implications de ces distinctions se diffèrent beaucoup entre les auteurs, elles tournaient toutes autour d'une distinction philosophique entre "objectif" et "subjectif" qui apparut ca. 1840. C'était cette nouvelle distinction philosophique qui permit les probabilistes revisionnistes de concevoir de la possibilité des "probabilités objectives," ce qui aurait été une alliance de mots pour les probabilistes classiques comme Jacques Bernoulli et Pierre Simon Laplace. Sans abandonner le déterminisme strict des probabilistes classiques, les revisionnistes pourraient

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donner au hasard une condition objective dans le monde, en l'opposant à la variabilité subjective de l'esprit.

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INTRODUCTION

In 1843 the French mathematician, philosopher, and economist A. A. Cournot wrote of "the double sense of probability, which at once refers to a certain measure of our knowledge, and also to a measure of the possibility of things [*possibilité des choses*] independently of the knowledge we have of them." He christened these two distinct senses "with the epithets of *subjective* and *objective,* which were necessary in order for me to distinguish radically [between] the two meanings of the term *probability*"[1] [9, 4–5]. Between 1837 and 1843 at least six authors—Siméon-Denis Poisson, Bernard Bolzano, Robert Leslie Ellis, Jakob Friedrich Fries, John Stuart Mill, and Cournot—approaching the topic as mathematicians and philosophers, writing in French, German, and English, and apparently working independently, made similar distinctions between the probabilities of things and the probabilities of our beliefs about things. Some, though not all, attached the terms "objective" and "subjective" to the two kinds of probability they were at pains to pull apart. All warned against the dangers of ignoring such distinctions in the application of mathematical probability, but they did not all agree on exactly where the dangers lay—the probability of judgments? the probability of causes? the Law of Large Numbers? Nor did they agree on exactly how to draw the distinction between the two kinds of probability, or even to what kind of events or entities probabilities could properly be said to apply.

In this essay I examine not the whys but the hows of this explosion of concern among probabilists ca. 1840.[2] That is, I will be concerned not with why ca. 1840 so many authors became simultaneously exercised about what mathematical probability could and could not mean,[3] but rather with how they went about making such distinctions. More specifically, I will be concerned with how such diverse arguments and examples could eventually, by the late 19th century, converge upon the same terminology of "objective" and "subjective" probabilities. What

[1] Unless otherwise noted, all translations are my own.

[2] Although Jakob Friedrich Fries's *Versuch einer Kritik der Principien der Wahrscheinlichkeitsrechnung* (1842) certainly belongs to this cluster of works, I do not discuss it in this paper. Fries's account of probability theory is embedded in an elaborate Kantian framework and must be set in the context of his other, extensive writings on the philosophy of the natural sciences and applied mathematics in order to appreciate its relation to his understanding of the meaning of objective and subjective. This is an undertaking which would require at least a paper in its own right. For a summary of those aspects relating to statistical regularities, see [37, 85–86].

[3] There already exists an historical literature on the "why" problem: [37, 77–86] on the influence of social statistics and criticisms of the probability of causes, [12, 210–224, 370–386] on the influence of the decline of associationist psychology and criticism of the probability of judgments, and [25, 95–98] on the influence of Joseph Fourier's *Recherches statistiques sur la ville de Paris* (1829) on Poisson in particular.

had these terms come to mean in the early decades of the 19th century, and why were they so eminently available to probabilists trained in such diverse mathematical and philosophical traditions, and with such contrasting visions of probability theory and its proper domain of applications? I want to argue three claims: first, that it was ca. 1830 that the words "objectivity" and "subjectivity" emerged in French, German, and English with new meanings that resonated with the new distinctions of the probabilists; second, that the probabilists were far from unanimous about the grounds for and the implications of their distinctions; and third, that the novelty of their distinctions lay not in any unified view about either the meanings or applications of mathematical probability, but rather in a new ontology of chance events that made it possible to understand "objective probability" as something other than a contradiction in terms.

THE NEW MEANINGS OF THE OBJECTIVE AND SUBJECTIVE

Cournot wrote that it was "the example of Jakob Bernoulli" that had stiffened his resolve to import the "metaphysical" language of objectivity and subjectivity into mathematics, and Bernoulli was indeed the first mathematical probabilist to use the terms "objective" and "subjective" in relation to probabilities. However, the sense in which he used them diverged significantly from Cournot's usage. In the opening passages of Part IV of the *Ars conjectandi* (1713) it is certainty (*certitudo*), not probability, which Bernoulli modifies by *objectivè*: "All things under the sun, past, present, and future, in themselves and objectively [*in se & objectivè*] always have the greatest certainty." In contrast, all probabilities, defined as "degrees of certainty that differ from [certainty] as part to whole" [3, 239], are incorrigibly subjective just because they fall short of total certainty. God's knowledge is certain and therefore objective; human knowledge is objective only insofar as it is certain. Galileo had daringly claimed that human knowledge about mathematics "equals the Divine in objective certainty [*certezza obiecttiva*]" because it is knowledge of necessity [21, 129]; Bernoulli similarly argued that causal knowledge of eclipses partook of the same necessity and therefore objectivity [3, 240]. Given this understanding of objectivity as certainty based on an understanding of necessary causes, "objective probabilities" would have been an oxymoron. We have recourse to probabilities when forced by our ignorance to traffic in contingencies rather than the objective necessity of things in themselves. For classical probabilists from Bernoulli through Pierre-Simon de Laplace, probabilities were officially subjective in Bernoulli's sense, figments of human ignorance for which an omniscient deity, or even a well-informed super-calculator, would have no need.

This does not imply that classical probabilists did not regularly make use of what after 1840 came to be known as objective probabilities. As Ian Hacking has shown, from the earliest stirrings of mathematical probability the subjective understanding of probabilities coexisted side by side with a conception of probabilities derived from observed frequencies (e.g., mortality statistics) or physical constitution (e.g., the symmetry of coins and dice) [24, 11–17]. Bernoulli himself, in the theorem which crowned the *Ars conjectandi*, aimed to connect the probabilities

of reasonable expectation with the actual frequencies of all manner of events, from wine harvests to storms. Eighteenth-century probabilists slid easily between sense of probabilities rooted in states of mind and in states of the world. Insofar as they remarked at all upon the gap between these senses, philosophers and mathematicians relied upon the associationist psychology of Locke, Hartley, Hume, and Condillac to correlate experience with expectation [12, 191–210]. Classical probabilists did not lack for applications of either subjective or objective probabilities in our sense, but they did lack any felt need to draw a sharp distinction between the two, much less a need to describe that distinction in the language of "objective" and "subjective," for them so redolent of late scholasticism.

For Galileo, Bernoulli, and other 17th-century writers the terminology of the objective and subjective still bore some of the marks of its 14th-century coinage in the works of Duns Scotus, William of Occam, and other nominalist Schoolmen on the status of universals and particulars. The objective in this context referred to the objects of thought, and the subjective to objects in themselves [35, A.2.a]. This (to modern ears) inverted sense survived well into the 18th century; witness, for example, the entry for "Objective/*objectivus*" in the 1728 edition of *Chamber's Dictionary:* "Hence a thing is said to *exist* OBJECTIVELY, *objectivè*, when it exists no otherwise than in being known; or in being an Object of the Mind" [6, 649]. The meanings of the terms had, however, already branched and crisscrossed in the 17th century in both Latin and in various vernaculars, although "objective" still generally modified thoughts rather than external objects. A famous example can be found in the *Meditationes* (1641) of René Descartes, in which he contrasted the "objective reality" of an idea—whether it represents its cause by perfection and/or content—with its "formal reality"—whether it corresponds to anything external to the mind [15, 40–42; 8, 136–137; 33] By the mid-18th century certain metaphysical texts drew an objective/subjective distinction along the lines of things in themselves versus thoughts, but even in such cases the mentalist associations were still strong: "One divides the truth into the objective or metaphysical [*objektivische oder metaphysische*], which is nothing other than the reality or possibility of the object itself ... [a]nd into the subjective or logical [*subjektive oder logikalische*], which is truth in a really existing mind ... All objective truth is thus in the divine mind a subjective truth" [11, 95].

But by the time these lines were published, the terminology of objectivity and subjectivity had an archaic and pedantic ring to it, and was confined to obscure, mostly German treatises on metaphysics and logic. In addition to its technical scholastic and theological ("God is our objective beatitude" [16]) senses, the most common eighteenth-century definition is the "objective" lens ("object glass") of a microscope or telescope. A quick survey of major dictionaries in French, English, and German reveals that the words surface again with something like their familiar modern meaning only in the 1830s—somewhat earlier in German. Already in 1820 a German dictionary defines *Objektivität* as "relation to an external object" and *Subjektiv* as "personal, inner, inhering in us, in opposition to objective" [27], and numerous editions of the Grimm brothers' etymological dictionary traced the newer philosophical senses of both *objektiv* and *subjektiv* directly to Kant [22;

23]. The early French entries were equally pointed in linking the new meaning of *objectif* as that "which is outside the thinking subject; all that is real and not at all ideal" with "new systems of philosophy" [4]. Littré's etymological dictionary of 1863 was still more explicit in crediting the "new sense" of *objectif* as "every idea which comes from objects exterior to the mind" as "due to the philosophy of Kant" [30].

English-language dictionaries were somewhat sluggish in picking up newfangled philosophical meanings, tending to assimilate them to 18th-century logical definitions.[4] But there is independent literary evidence that philosophical winds blowing from Kant's Königsberg also carried the new usage of "objective" and "subjective" to British readers. The poet Samuel Taylor Coleridge seems to have reintroduced the term back into general English usage in 1817, in the context of an exposition of Kantian and neo-Kantian philosophical systems he had learned about during a stay in Germany: "Now the sum of all that is merely OBJECTIVE we will henceforth call NATURE, confining the term to its passive and material sense, as comprising all the phenomena by which its existence is made known to us. On the other hand the sum of all that is SUBJECTIVE, we may comprehend in the name SELF or INTELLIGENCE. Both conceptions are in necessary antithesis" [7, 1:174]. By 1856 Thomas De Quincey could remark of the word "objective" that "[t]his word, so nearly unintelligible in 1821, so intensely scholastic, and ... yet, on the other hand, so indispensable to accurate thinking, and to wide thinking, has since 1821 become too common to need any apology" [14, 265].

To summarize: although a philosophical distinction between "objective" and "subjective" dated back to the 14th century, the terms languished in the late 17th and 18th centuries. Revived by Kant in his *Kritik der reinen Vernunft* (1781, 1787), which breathed wholly new meanings into them,[5] they became entrenched in general usage in German, French, and English in the period 1820–1840. But by 1840 this usage bore almost as little resemblance to the Kantian philosophy which had resurrected the terms as to the scholastic philosophy which had created them. For Kant, "object" (*Gegenstand*) and "objective validity" (*objektive Gültigkeit*) were quite distinct concepts, and it is the latter, understood as the synthetic *a priori* categories such as time, space, and causation that are preconditions for experience, which undergirds Kant's own distinction between the objective and subjective [28, 251–252, A201–202/B246–247; 1, 134–155]. Just how remote this usage is from our own is made clear by Kant's regular pairing of the "subjective"

[4] Samuel Johnson's *Dictionary of the English Language* defines one sense of "objective" from 1755 onwards as "[b]elonging to the object; contained in the object," a definition repeated verbatim (along with the illustrative quotation from Isaac Watts's *Logick* (1724)) by several other English-language dictionaries until late in the 18th century, e.g., [34]. Some mid-19th-century entries incorporate the new philosophical opposition of "objective" to "subjective," but preserve the 1724 quotation, e.g., [41].

[5] In the *Kritik der reinen Vernunft*, Kant discusses "probability [*Wahrscheinlichkeit*]" as well as the nature of the "objective" and "subjective," but the two discussions remain wholly unconnected. Moreover, Kant uses "probability" in a nonquantitative, almost archaic sense, reminiscent of classical treatises on rhetoric. Compare, for example, [28, 705–706, A775/B803] with [2, 2:1730, 1094b19–28].

with the "merely empirical." Yet the usage enshrined by 1840, despite its bows in the direction of the "new philosophy," drew the distinction between an "objective" external reality independent of all minds and "subjective" internal states dependent upon individual minds. This was the meaning of the distinction which crystallized in French, German, and English in the 1830s and upon which almost all of the distinctions between two kinds of probability made ca. 1840 relied, even if they did not invoke the newly fashionable terminology of the objective and subjective.

A PLURALITY OF DISTINCTIONS

An arresting temporal and conceptual coincidence links the emergence both of the new philosophical distinction between objective and subjective realms, and of various distinctions between two kinds of probability. But it would be rash to conclude to the existence of any straightforward connection between the philosophical and probabilistic distinctions: some of the probabilists—such as Cournot and Bolzano—explicitly couched their distinctions in the new philosophical terminology, but others did not. Moreover, even those who did speak the new language of objective and subjective probabilities did not agree with one another as to what these were. Upon closer inspection the relationship between the philosophical and probabilistic distinctions seems to be one of a shared ontology, one which seems to have become not only thinkable but self-evident only in the 1830s. This ontology not only carved up the world into what was inside and outside human minds—Descartes had already done as much—but also (*pace* Enlightenment rationalists) located reality and truth in the "outside" realm of objects, and (*pace* Enlightenment empiricists) further insisted on the mismatch not only between world and mind, but also among the minds of different individuals. That is, the new subjectivity was a threat to knowledge not only because it was insufficiently faithful to the reality of objects, but also because its contents and dictates varied from person to person. In order to understand how this ontology shaped the probabilistic distinctions, we must briefly examine the latter.

What is most striking upon a first composite reading of the works of Poisson, Cournot, Bolzano, Ellis, and Mill is the sharp divergence of motivations, formulations, and consequences of the distinctions they each draw between kinds of probability. Poisson, eager to rescue the mathematical theory of the probability of judgments from attacks recently launched against it by mathematicians, philosophers, and politicians [12, 342–369], and impressed by the criminal statistics recently gathered by the French Ministry of Justice [38, 186–194; 25, 87–104], emphasized the empirical foundations of probability theory. Bernoulli's theorem was but a special case of the Law of Large Numbers (Poisson's original coinage), which was a "general and incontestable fact, resulting from experience" of both physical and moral phenomena, and the "base of all applications of the calculus of probabilities" [36, 12]. Yet Poisson reserved the traditional term "probability" for its traditional definition as "the reason we have to believe that [an event] will or will not occur" [36, 30], letting "chance" designate "events in themselves and independent of the knowledge we have of them" [36, 31]. So, for example, the

"chances" of heads or tails for a given coin are unlikely to be equal because of the physical asymmetries of all such objects, but the "probability" is nonetheless equal because we know nothing about the coin's constitution. For Poisson, both the "chance" of underlying causes and the "probability" of our ignorance were equally legitimate interpretations of what it was that mathematical probabilities measured. He proposed to evict from mathematical probability theory neither the *probabilités* of belief, nor any of the much-maligned applications, such as the probabilities of causes and judgments, which depended on them. Clarification alone would suffice. All confusion and paradox would disappear, Poisson was confident, if probabilists would only learn to differentiate between *probabilité* and *chance*.

Cournot was considerably more severe about restricting both legitimate applications and interpretations of mathematical probabilities. Although he professed in his preface to be in total agreement with Poisson's distinction (at which Cournot had arrived independently) [9, 5–6], and although he defended the probability of judgments [9, 231], Cournot sharply criticized Condorcet and Laplace's Bayesian approach to the probability of causes, preferring an "experimental determination" based on Bernoulli's Theorem [9, 106]. The "subjective probabilities" based on equal ignorance of outcomes were fit only for the "frivolous use of regulating the conditions of a bet" [9, 111, 288], and were moreover the "cause of a crowd of equivocations [which] have falsified the idea one ought to have of the theory of chances and of mathematical probabilities" [9, 59]. What Cournot held against subjective probabilities was not so much that they were not empirical givens as that they varied "from one intelligence to another, according to their capacities and the data with which they are provided" [9, 106]. Far from equating objective probabilities with frequencies *tout court* as John Venn was later to do [40, 90], Cournot exhorted statisticians to purify the "immediate data of observation" from all features that depend "solely on the point of view in which the observer is situated," calling on theory and principles to help transcend mere "compilations of facts and figures" [9, 125].

None of the probabilists in this group, in fact, went so far as to identify probabilities baldly with frequencies, although John Stuart Mill came the closest in his attacks on the classical theory of probability as "ignorance . . . coined into science" by the mere manipulation of numbers[6] [31, 8:1142]. He roundly rejected both the probabilities of causes and judgments, condemning them for having made the calculus of probabilities "the opprobrium of mathematics." He further claimed that if the theory was to be profitably turned to any "scientific purpose," its foundations must rest on "the utmost attainable amount of positive knowledge" [31, 7:539]. Ideally, that knowledge would be of causes, for frequencies alone "can give rise to no other induction than that *per enumerationem simplicem;* and the precarious inferences derived from this" [31, 7:542].

[6] Mill's *Logic* went through seven editions during his lifetime: 1843, 1846, 1851, 1856, 1862, 1865, 1868. I here quote from the seventh edition, insofar as it agrees with the first edition. When the two diverge, I have indicated this by adding "1843" to the reference.

Still less reliable were estimates of equiprobability based on equal ignorance about possible outcomes. In the first 1843 edition of his *Logic,* Mill had dismissed all such applications of the Principle of Indifference as so much algebraic alchemy, transmuting base ignorance into the true metal of knowledge [31, 8:1141]. But in later editions, under the influence of John Herschel [39], Mill grudgingly conceded that "as a question of prudence" we might rationally assume that "one supposition is more probable *to us* than another supposition," and even bet on that assumption "if we have any interest at stake" and if we were in the desperate (and rare) situation of having no relevant experience whatsoever [31, 7:535–536].

Note that Mill's "scientific" probabilities did not precisely coincide with either Poisson's *chance* or Cournot's *probabilités objectives*. The latter referred to real states of the world, independent of all human knowledge, whereas Mill's probabilities, "scientific" or not, were always based upon experience, more or less complete, and therefore upon our knowledge: "We must remember that the probability of an event is not a quality of the event itself, but a mere name for the degree of ground which we, or some one else, have for expecting it" [31, 7: 535]. That knowledge coincided with things as they are only when it was full, causal knowledge. Despite his loud and repeated calls for probabilities founded on experience, Mill curiously remained the most traditional of the revisionists in his interpretation of all probabilities as epistemic.

Neither Mill nor Robert Leslie Ellis, the other British probabilist in my ca. 1840 sample, used the word "objective" in relation to the probabilities of experience. The omission perhaps reflects nothing more than the slight British lag-time in the reception of the new, pseudo-Kantian use of the word: Mill did use the word by 1863 in the context of the alleged "objective reality" of moral obligations [32, 43], and Ellis invoked the "merely subjective" character of secondary qualities in his 1857 introduction to the philosophical works of Francis Bacon [20, 75, 80–81]. Yet in Mill's case there may have been more principled reasons to deny merely epistemic probabilities, even those grounded in experience, an accolade reserved for things and events in the world. Mill believed these latter to be absolutely certain: "Every event is in itself certain, not probable" [31, 7:535].

The case of Ellis is considerably more convoluted. His essay occasionally glimmers with bits of neo-Kantian vocabulary, as when he insists on the importance of "*à priori* truths" and "the ideal elements of knowledge," as well as on the incompatibility of the classical theory of probabilities with "the views of the nature of knowledge, generally adopted at present"[7] [18, 1, 6]. Although in later articles on the Method of Least Squares Ellis seconded Mill's claim "that mere ignorance [here, of the specific law of error which applies for a given set of observations] is no ground for any inference whatever. *Ex nihilo nihil*" [19, 325], his 1842 essay

[7] The language of "ideal elements" is redolent of William Whewell's faintly Kantian *Philosophy of the Inductive Sciences* (1840), and it is quite possible that Whewell's views influenced Ellis directly through their association at Trinity College, Cambridge, where Whewell was appointed Master in 1841, and Ellis a fellow in 1840.

on the foundations of probability theory was a scathing critique of at least one kind of empiricism, the sensationalist philosophy of Condillac, because it "rejects all reference to *à priori* truths as such" [18, 1]. This would hardly have sat well with Mill's steadfast opposition to all *a priori* truths, even in mathematics and logic [31, 7:224–251]. Ellis, in contrast, argued that Bernoulli's theorem was not even the result of a mathematical deduction, much less a fact of experience in Poisson's sense. Rather, it was an *a priori* truth, established by "an appeal to consciousness" that revealed the impossibility of judging otherwise [18, 1–2]. Ellis insisted that judgments of probability were nonetheless "founded, not on the fortuitous and varying circumstances of each trial, but on those which are permanent—on what is called the nature of the case" [18, 3]. But what was permanent was the "fundamental axiom" that in the long run "the action of fortuitous causes disappears," an axiom supplied by neither mathematics nor experience but rather by "the mind itself, which is ever endeavouring to introduce order and regularity among the objects of its perceptions" [18, 3].

Ellis's talk of mind was not about mere psychology, and he resisted the suggestion that probabilities were "the measure of any mental state." Rather, they properly referred to "the number of ways in which a given event can occur, or the proportional number of times it will occur on the long run" [18, 3]. Note Ellis's cautious deployment of "can" and "will": probabilities cannot be simply read off of observed frequencies. The objections Ellis leveled against Laplace's "law of succession" and the probability of causes [12, 277–279]—namely, that the definition of what constituted the next event in a series of observations depended crucially on a judgment of similarity according to "a point of view" [18, 4–5]—are equally devastating for every attempt to gather statistics. Ellis did not so much condemn the dependence of probability judgments "on the mind which contemplates [an event]" as require the specification of viewpoint. What Cournot would have dismissed as the incorrigible subjectivity of the observer's particular perspective, Ellis saw as a necessary component of all valid probability judgments, which even "the most perfect acquaintance with the nature of the case" could not do without.

Bernard Bolzano's distinction between "real or objective [*wirkliche oder objektive*]" and "apparent or merely subjective [*vermeintliche oder bloß subjektive*]" probabilities strayed farthest of all from statistical frequencies. For Bolzano, probabilities measure the relationship between propositions, not events [5, 3:274, sect. 317]: the relation between the number of cases in which propositions A, B, C, D, ... are true, and the number of cases in which A, B, C, D, ... together with the additional proposition M are true can be represented by a fraction which equals one when M can be deduced from A, B, C, D, ... [5, 2:171–173, sect. 161]. If the propositions A, B, C, D, ... are all true, then the probability is "objective"; if some are false, then it is "subjective" [5, 3:266–267, sect. 317]. Both objective and subjective probabilities are judgments made by a "certain thinking being"; the objective/subjective distinction does not apply to probabilities as they relate to propositions *in se*, regardless of whether they are true or false,

or whether a thinking being can judge them to be so [5, 3:264, sect. 317]. That is, both objective and subjective probabilities are relative to a thinking mind, which can be objectively certain because the propositions it holds to be true are indeed so, or subjectively certain, because it mistakes false propositions for true. For God all probabilities are objective, because all that the divine mind holds to be certain is also true [5, 3:264, sect. 317]. For Bolzano, probability remained a measure of certainty, as it had been for the classical probabilists. But whereas they had considered probability immiscible with the objective judgments of God, Bolzano reconciles probability to even divine objectivity.

Bolzano was quite aware that his usage of "objective" with respect to a thinking subject could be attacked as Pickwickian, for by 1837 the objective was usually "understood as only that which can be thought [to be] without any relation to a subject" [5, 3:272, sect. 317]. But he defended his terminology by noting that rival candidates for the title "objective probabilities," such as the probabilities in games of chance, in fact rest upon states of knowledge that might in time be improved, so that eventually probabilities might become certainties, or at least change their value [5, 3:272, sect. 317]. Nor was Bolzano content with a distinction between quantitative and "philosophical" probabilities, of the sort Cournot elaborated in his later work [10, 1:71–101], for the boundary between the two seemed to him "often very subjective and variable" [5, 3:274, sect. 317].

THE OBJECTIVITY OF CHANCE IN A DETERMINISTIC WORLD

Even this briefest of surveys suffices to show how divergent the views of the ca. 1840 cluster of revisionist probabilists were on almost every significant point. All attacked some aspect of classical probability theory (usually making a target of some passage in Laplace's work), but not the same aspect. Poisson hoped to rescue the probability of judgments from certain pernicious assumptions; Cournot rejected the probability of causes; Mill lambasted both; Ellis challenged the mathematical demonstration of Bernoulli's theorem and Laplacean error theory; Bolzano objected to the application of probabilities to events rather than propositions. Their attitudes toward the relationship between probability and experience were similarly various. Mill insisted that "scientific" probabilities be anchored in the experience of causes or observed frequencies, but retained the contrast between the probabilities of the mind and the certainties of events; Cournot cautioned that probabilities derived from frequencies must be refined by the regulative principles of rationality; Ellis emphasized the role of those regulative principles in defining a reference class in securing the foundations of the mathematical theory such as Bernoulli's theorem; Poisson counted the Law of Large Numbers (of which Bernoulli's theorem was a special case) as an observational fact and the basis of all applications of mathematical probability, but nonetheless granted the *probabilités* of belief equal if separate status with the *chances* of events in themselves; Bolzano protested that probabilities were not about events at all, although they could be altered by advances in knowledge. Given this Babel of voices, it is tempting to conclude that these writers, far from all simultaneously arriving at

the same distinction between kinds of probabilities, were not in agreement about anything.

Yet there is a common theme which unites this cluster of ca. 1840 revisionists, and it is one that bears directly on the new meanings of "objective" and "subjective" that emerged at approximately the same time. Although all insisted on the principle of universal causation and the complete determinism of events in themselves, they nonetheless also carved out a place for chance in the world rather than in the mind. That is, in their writings, chance became objective without for one moment admitting the existence of genuine randomness. This was a position that would have been inconceivable for the classical probabilists, for whom *objectivè,* in the words of Jakob Bernoulli, was synonymous with certainty and necessity, and for whom chance, in the words of Abraham De Moivre, "can neither be defined nor understood: nor can any Proposition concerning it be either affirmed or denied, excepting this one, 'That it is a mere word' " [13, 253].

The symbiosis between determinism and classical probability theory was a deep and enduring one. What the revisionist probabilists did was not so much to abolish that alliance as to reinterpret it. With one crucial exception, all the elements of their reinterpretation were available in the writings of the classical probabilists, particularly in those of Laplace. These elements were a reaffirmation of a seamless causal order which determined all events necessarily, and a perturbational model of chance as the superimposition of weak, sporadic causes upon the action of strong, constant ones. At the same time—and this was the novelty—the revisionists opposed a stable objective order to a variable subjective one.

The revisionist probabilists wholeheartedly embraced the ironclad determinism famously set forth by Laplace in the *Essai philosophique sur les probabilités* (1814): "All events, even those which because of their smallness do not seem to hold to the great laws of nature, are a consequence of them as necessary as the revolutions of the sun" [29, vi]. But whereas Laplace and other classical probabilists since Bernoulli had concluded that mathematical probabilities must therefore measure degrees of belief, the revisionist probabilists did not draw this conclusion. Not that they were any less staunch in their allegiance to universal causation than Laplace had been: Mill elevated the "law of universal causation" to "the rigorous certainty and universality" of geometric truths [31, 7:325]; Cournot proclaimed that "no phenomenon or event is produced without [a] cause; this is the sovereign and regulative principle of human reason, in the investigation of real facts" [9, 53]. Rather, the revisionists grounded the reality of chance in another Laplacean dictum about ontology, this time about the nature of *hasard*: what we call "chance" is not randomness, but rather the interaction of uniform, constant causes with variable causes. In the long run regularity emerges as "the variable causes of this irregularity produce effects alternatively favorable and contrary to the regular course of events . . . mutually destroying one another in a large number of trials" [29, xlvii].

Reflections of this image of constant causes ultimately triumphing over the

fluctuations of variable ones can be found in the writings of almost all[8] of the revisionists, although they differed wildly as to whether it was a regulative principle of human reasoning, a mathematical theorem, or a fact derived from experience. Poisson's notion of *chance* approximated a constant underlying cause, while *hasard* encompassed "the collection of all causes which concur in the production of an event, without influencing the magnitude of its change" [36, 79–80]; Cournot described "fortuitous events" as the superimposition of independent causal chains [9, 55]; Mill reduced chance to the "composition of Causes ... [where] we have now one constant cause, producing an effect which is successively modified by a series of variable causes" [31, 7:530]. But whereas Laplace took his model to be a warrant (and even a moral imperative, in the case of the principles of sound government [29, xlviii–xlix]) for ignoring the effects of chance (in his sense of perturbations), the revisionists took it as a warrant for the real existence of chance.

What made it possible for the revisionists to reconcile the real existence of chance with watertight determinism? They rejected randomness with as much finality as Laplace—or for that matter, Jakob Bernoulli and De Moivre—had; moreover, they took over Laplace's ontological model of the fortuitous lock, stock, and barrel. But they imposed upon both determinist convictions and ontological model a new philosophical grid, Kantian in its inspirations if not in its substance, that divided the stable, external, "objective" world from the variable, internal, "subjective" world. Within the framework of classical probabilities causes and reasons had been systematically conflated, just as within 18th-century associationist psychology, experience and belief had been equally systematically linked [12, 191–210]. The distinction between objective and subjective which also emerged ca. 1840 destroyed the plausibility of any smooth meshing between the world of things and the world of the mind. This was not merely scepticism redux, for the champions of objectivity were buoyantly confident of human abilities to know the world of things. Nor was it simply a renewed warning against the Baconian idols of cave, tribe, marketplace, and theater: the main threat posed by subjectivity was not distortion but variability, and no one looked to the methods of "Tables of Essence and Presence" or "Prerogative Instances" as a suitable defense. When Cournot worried about "modifications" introduced by the differing viewpoints of observers [9, 125], or when Ellis thundered that probabilities cannot "be taken as the measure of any mental state" [18, 3], they were not simply affirming empiricism. On the contrary, both, as we have seen, argued vigorously for the centrality of regulative mental principles and *a priori* axioms of the mind to the precepts and practice of mathematical probability theory. Their target was not the mind *per se*—mental universals and regulative principles of thought were

[8] Bolzano is the one exception here, for his probabilities applied to the certainty of propositions and only indirectly and elliptically to the reality of events [5, 3:274, sect. 317]. But even Bolzano seems to have implicitly subscribed to such a model, claiming that if all circumstances for subsequent casts of a die were truly identical, then "not only 'no man' but God himself could not give grounds" why different faces turned up [5, 3:273, sect. 317].

still revered, as we have seen in the cases of Ellis and Cournot—but rather all that was idiosyncratic or variable in the mind.

This was the new core meaning of subjective: a capricious, arbitrary quality of the mind, responsible for not only inter- but also intra-individual differences. In contrast, the variability of irregular causes did not disturb the revisionist probabilists, for none of them believed that such fluctuations could, in the long run, occlude the constant causes that probabilities really measured. Much less did they believe that objective chance, defined in terms of causal variability, could topple the iron regime of determinism. Against this background it is perhaps not surprising that randomness first became thinkable when Gustav Theodor Fechner's psychophysical parallelism challenged not only this double standard of variability, but also the philosophical distinction between inner and outer experience [26]. It was this distinction that had made the objective probabilities of ca. 1840 suddenly conceivable, without abandonment of the determinism which had made subjective probabilities seemingly inevitable in the classical theory.

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