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Homotopy perturbation method for nonlinear oscillators with coordinate-dependent mass

Yue Wu[a], Ji-Huan He[b,*]

[a] College of Economics and Management, Shanghai University of Political Science and Law, 7989 Waiqingsong Road, Qingpu Area, Shanghai 201701, People's Republic of China

[b] National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, Suzhou, People's Republic of China

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ABSTRACT

Nonlinear oscillators with no linear term or negative linear term are difficult to be solved analytically. This paper shows that the homotopy perturbation method can be effectively applied to such kinds of nonlinear oscillator by the parameter expansion technology. A nonlinear oscillator with coordinate-dependent mass is used as an example to elucidate the effectiveness and convenience of the method. The condition for periodic solution is obtained.

Introduction

Recently Lev, Tymchyshyn & Zagorodny proposed a nonlinear oscillator with coordinate-dependent mass [1]

$$(1+\alpha x^2)\ddot{x}+\alpha x\dot{x}^2-x(1-x^2)=0 \tag{1}$$

with initial conditions

$$x(0)=A,\ \dot{x}(0)=0 \tag{2}$$

Eq. (1) can describe phase transitions in physics, and plays an important role in quark confinement, cosmos-logical model, and spinodal decomposition [1].

We re-write Eq. (1) in the following equivalent form

$$\ddot{x}-x+\alpha x^2\ddot{x}+\alpha x\dot{x}^2+x^3=0 \tag{3}$$

This nonlinear oscillator with negative coefficient of linear term is difficult to be solved by the perturbation method. This paper applies the homotopy perturbation method [2–9] to find the approximate period of Eq. (1).

Homotopy perturbation method

In order to apply the homotopy perturbation method, we construct the following homotopy equation [3,4].

$$\ddot{x}+(-1)x+p(\alpha x^2\ddot{x}+\alpha x\dot{x}^2+x^3)=0 \tag{4}$$

We expand the coefficient of the linear term (-1) and the solution into the following forms [3,4].

$$-1=\omega^2+a_1p+a_2p^2+\cdots \tag{5}$$

$$x=x_0+x_1p+x_2p^2+\cdots \tag{6}$$

The parameter-expanding (parameter-expansion) method is widely used in dealing with nonlinear problems [10,11].

Submitting Eqs. (5) and (6) into Eq. (4), and processing as that by the perturbation method, we obtain following linear equations:

$$\ddot{x}_0+\omega^2x_0=0,\ \ x_0(0)=A,\ \dot{x}_0(0)=0 \tag{7}$$

$$\ddot{x}_1+\omega^2x_1+a_1x_0+\alpha x_0^2\ddot{x}_0+\alpha x_0\dot{x}_0^2+x_0^3=0,\ \ x_1(0)=A,\ \dot{x}_1(0)=0 \tag{8}$$

It is easy to solve x_0 from Eq. (7), which reads

$$x_0=A\cos\omega t \tag{9}$$

Substituting Eq. (9) into Eq. (8), we obtain

$$\ddot{x}_1+\omega^2x_1+a_1A\cos\omega t-\alpha A^3\omega^2\cos^3\omega t+\alpha A^3\omega^2\cos\omega t\sin^2\omega t+A^3\cos^3\omega t=0 \tag{10}$$

By a simple calculation, we have

$$\ddot{x}_1+\omega^2x_1+\left(a_1A+\alpha A^3\omega^2+\frac{3}{4}A^3(-2\alpha\omega^2+1)\right)\cos\omega t+\frac{1}{4}A^3(-2\alpha\omega^2+1)\cos 3\omega t=0 \tag{11}$$

No secular term in x_1 requires that

$$a_1A+\alpha A^3\omega^2+\frac{3}{4}A^3(-2\alpha\omega^2+1)=0 \tag{12}$$

If the first order approximate is searched for, from Eq. (5) we have

* Corresponding author.
E-mail addresses: needlewu@sina.com (Y. Wu), Hejihuan@suda.edu.cn (J.-H. He).

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$$-1 = \omega^2 + a_1 \tag{13}$$

From Eqs. (12) and (13) the following frequency can be obtained

$$\omega = \sqrt{\frac{\frac{3}{4}A^2 - 1}{\left(1 + \frac{1}{2}A^2\alpha\right)}} \tag{14}$$

Eq. (14) is valid for the case when

$$A > \frac{2}{\sqrt{3}} \tag{15}$$

When $A < 2/\sqrt{3}$, there is no periodic solution to Eq. (1). The real number could be more or less, and its accuracy can be improved if a higher order approximate solution is solved, but Eq. (15) gives an idea of the small amplitude for non-periodic solutions.

Conclusions

This short paper is to give an alternative solution to Eq. (1), and elucidates that the homotopy perturbation method is valid for nonlinear oscillators with negative linear terms, and conditions for the periodic solutions can be easily obtained. The idea can be extended to other nonlinear problems, and the present paper can be used as paradigm for other applications.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.rinp.2018.06.015.

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