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KKL-model of MHD CuO-nanofluid flow over a stagnation point stretching sheet with nonlinear thermal radiation and suction/injection

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ABSTRACT

In this study, we analyzed numerically the effects of magnetic field and thermal radiation on forced-convection flow of CuO-water nanofluid past a stretching sheet with stagnation point in the presence of suction/injection. Considering the effects of Brownian motion, we applied the Koo–Kleinstreuer–Li (KKL) correlation to simulate the effective thermal conductivity and viscosity of the nanofluid. The equations governing the flow transformed to ordinary differential equations and solved numerically using the fourth-order Runge–Kutta integration scheme featuring a shooting technique. The influence of significant parameters such as the magnetic parameter, radiation parameter, suction/injection parameter and velocity ratio parameter on the velocity and temperature profiles are discussed and presented through graphs and analyzed for (CuO-water).

Introduction

Study of heat transfer through boundary layers over a stretching surface in the recent years has received significant attention due to its useful engineering applications such as thermal insulation, solar collectors, and designing building and cooling of electronic components. Crane [1] studied the two dimensional boundary layer flow of a viscous incompressible fluid over a stretching sheet. Dutta et al. [2] specified the temperature distribution of uniform heat flux in the flow over a stretching surface. Elbashbeshy et al. [3] studied the flow and heat transfer in a porous medium over a stretching surface with internal heat generation and suction/blowing when the surface is kept at constant temperature. In the presence of variable surface temperature, Chen et al. [4] investigated the heat transfer characteristics over a continuous stretching sheet. Chakrabarti et al. [5] studied the magnetohydrodynamic (MHD) flow of Newtonian fluids initially at rest, over a stretching sheet at different uniform temperatures. In view of these applications, Borkakoti et al. [6] studied the two-dimensional channel flows with heat transfer analysis of a hydromagnetic fluid where the lower plate was a stretching sheet. Carragher et al. [7] studied the heat transfer effects of a flow over a stretching surface with a temperature difference between the ambient fluid and the surface proportional to a power of distance from the fixed point. Grubka et al. [8] studied the effects of exerted uniform heat flux of heat transfer flow over a stretching surface. The effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet studied by Khan et al. [9], The rate type fluid with temperature dependent viscosity analyzed by Faraz et al. [10].

Nanofluid used to gain the maximum possible thermal properties of conventional basic fluids as water, oil or ethylene glycol by a suspension of solid nanoparticles of diameter (1–100 nm) in basic fluids at the minimum possible concentrations. Nanoparticles are made from various materials, such as Cu, Ag, Au metals, Al_2O_3, CuO, and TiO_2. It has been found that the presence of nanoparticles increases the thermal conductivity of the base fluid in the range of 15–40%. Nanofluids have several applications such as coolants for nuclear reactors, cancer therapy and safer surgery by cooling, vehicle computers and transformer cooling. Choi [11] advanced the concept of nanofluid, which is a kind of fluid with suspended nanoparticles. Elbashbeshy et al. [12] examined the nanofluids in the process of cooling during the heat treatment process of the metals. Bachok et al. [13] studied the flow two-dimensional stagnation point of a nanofluid over a stretching/shrinking sheet. Kameswaran et al. [14] studied the chemical reaction and viscous dissipation effects on MHD nanofluid flow over a stretching/shrinking sheet. Narayana et al. [15] studied the effect of laminar flow of a Nano liquid film over an unsteady stretching sheet. They investigated that the effect of the nanoparticle volume fraction is to reduce the axial velocity and free-stream velocity. Kang et al. [16] and Rudyak et al. [17] studied the thermo-physical properties of nanofluids such as thermal conductivity, diffusivity and viscosity. Due to the importance of MHD flow

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Nomenclature

Ai, (i = 1,…,5)	constants parameters
a	stretching sheet parameter
b	free stream velocity parameter
C_f	skin friction coefficient
f	dimensionless stream function
B_0	a uniform magnetic field
k	thermal conductivity
M	magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
q_r	the nonlinear radiation heat flux
Rd	radiation parameter
Re	local Reynolds number
U_w	the stretching velocity
U_∞	the free stream velocity
T	fluid temperature
T_∞	ambient temperature
(u, v)	velocity components in the (x, y) directions, respectively
(x, y)	Cartesian coordinates along x, y axes, respectively

Greek symbols

α	thermal diffusivity
η	similarity parameter
θ	similarity function for temperature
ρ	density
ϕ	nanoparticle volume fraction
μ	dynamic viscosity
ν	kinematic viscosity
ψ	stream function
λ	velocity ratio parameter
σ	electrical conductivity
σ^*	Stefan–Boltzmann constant
α^*	coefficient of mean absorption
θ_w	the temperature ratio parameter
f_w	the suction/injection parameter

Subscript

w	condition at the surface
∞	condition at infinity
nf	nanofluid
f	base fluids
s	nonsolid particles

of an electrically conducting fluid over stretching/shrinking sheet in modern metallurgy and metalworking processes. Elbashbeshy et al. [18] obtained an exact solution for the MHD boundary layer over a moving plate. Jafar et al. [19] studied the effects of external magnetic field, viscous dissipation and Joule heating on MHD flow and heat transfer over a stretching/shrinking sheet. Pavlov [20] studied the MHD flow of viscous incompressible fluid caused by deformation of a surface. Considering the effects of external magnetic field, Anderson [21] studied analytically the solution of two-dimensional Navier-Stokes' equation over a stretching sheet.

Thermal radiation plays an important role in space technology, high temperature processes and controlling heat transfer process in polymer processing industry. Hossain et al. [22] studied the effect of radiation on heat transfer problems. Zahmatkesh [23] found that a temperature distribution nearly uniform in the vertical sections within the enclosure makes the streamlines to be nearly parallel with the vertical walls due to the presence of thermal radiation makes. The radiation effects on combined with convection over a vertical flat plate embedded in a porous medium of variable porosity have investigated by Pal et al. [24]. The two-dimensional heat transfer flow with radiation in a channel with porous walls described by Hayat et al. [25].Chamkha et al. [26] investigated the thermal radiation effects on magnetohydrodynamic forced convection flow adjacent to a non-isothermal wedge. They showed that as the thermal radiation parameter increase the local Nusselt number predicted to decrease. Hayat et al. [27] studied the mixed convection boundary layer magnetohydrodynamic stagnation point flow through a porous medium bounded by a stretching vertical plate with thermal radiation. Das [28] studied the mixed convection heat transfer of stagnation point flow of Cu-water nanofluid towards a shrinking sheet. Khan et al. [29] studied a flow effects near the two-dimensional stagnation point on an infinite permeable wall with a homogeneous/heterogenous reactions. Nandkeolyar et al. [30] studied the nonlinear convection in a stagnation point nanofluid through a stretching sheet with homogeneous/heterogeneous reactions, viscous and Joule heating. Abdelwahed et al. [31] investigate the boundary layer behavior over a moving surface in a nanofluid under the influence of nonlinear thermal radiation and convective boundary conditions. Abdelwahed [32] extended his work [33] to study the variation of the surface thickness on the MHD boundary layer behavior. Abdelwahed et al. [34] studied analytically using KKL correlation the effect of hall current on MHD flow of a nanofluid due to a rotating disk in the presence of nonlinear thermal radiation

The aim of the present study is to investigate numerically the effects of magnetic field and thermal radiation on forced-convection flow of CuO-water Nanofluid past a stretching sheet in the presence of suction/injection using the fourth-order Runge–Kutta integration scheme featuring a shooting technique. The Koo–Kleinstreuer–Li (KKL) model used to simulate viscosity and thermal conductivity of the nanofluid.

Mathematical formulation

Assume steady, two-dimensional, incompressible flow of a nanofluid past a stretching sheet, at y = 0 the surface is subjected to magnetic field of strength B_0 normal to the surface The induced magnetic field is negligible due to small Reynolds number (see Fig. 1). Moreover,

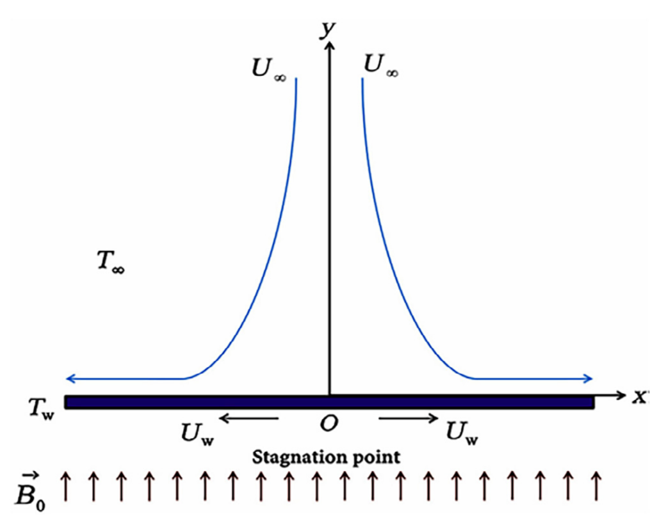

Fig. 1. Physical model of the problem.

Table 1
Thermo physical properties of water and CuO nanoparticles at room temperature [33].

	ρ (kg/m^3)	Cp (J/kg K)	k (W/m k)	dp (nm)
Pure water	997.1	4179	0.613	–
CuO	6500	540	18	29

Table 2
The coefficients of CuO-water nanofluids [34].

Coefficient values	CuO-water
a_1	−26.593310846
a_2	−0.403818333
a_3	−33.3516805
a_4	−1.915825591
a_5	$6.42185846658 \times 10^{-2}$
a_6	48.40336955
a_7	−9.787756683
a_8	190.245610009
a_9	10.9285386565
a_{10}	-0.72009983664

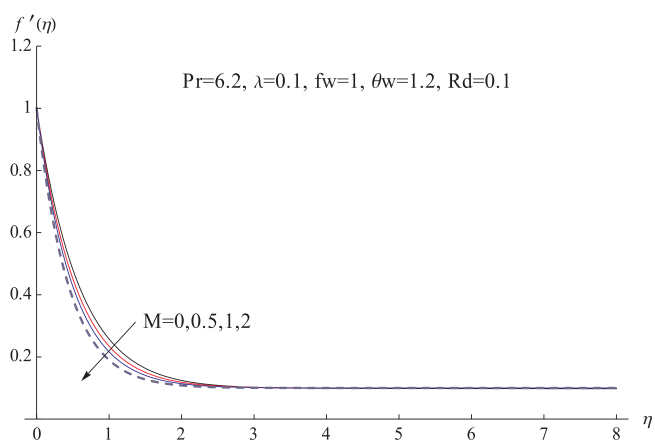

Fig. 2. Variation of velocity with increasing of magnetic parameter.

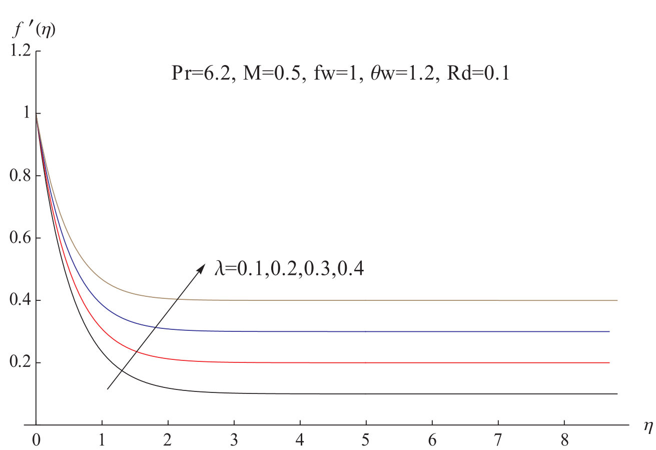

Fig. 3. Variation of velocity with increasing of velocity ratio.

the effect of nonlinear thermal radiation taken into account $U_w(x) = ax$ and $U_\infty(x) = bx$ are the stretching velocity, the free stream velocity respectively with x the distance from the stagnation point, where a and b are constants with $a > 0$ and $b \geqslant 0$. $T_w(x) = T_\infty + cx^n$ is the temperature of the sheet surface where T_∞ is the ambient fluid temperature, where c and n are constants with $c > 0$ (heated surface).

The governing equations of the problem are [29,30]:

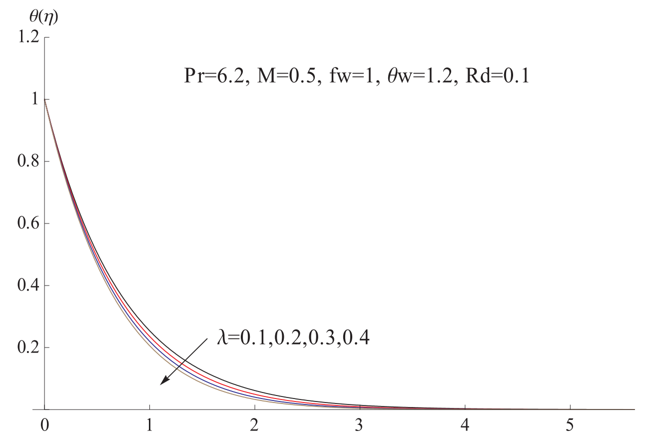

Fig. 4. Variation of temperature with increasing of velocity ratio.

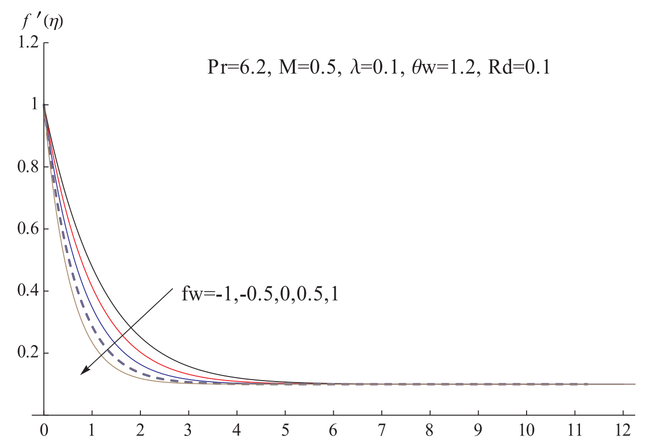

Fig. 5. Variation of velocity with increasing of suction/injection parameter.

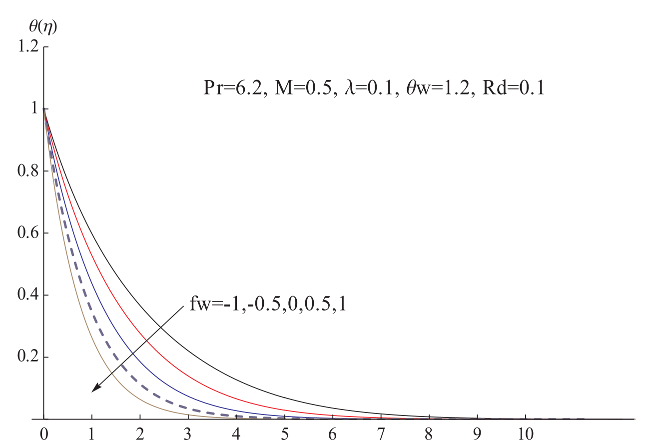

Fig. 6. Variation of temperature with increasing of suction/injection parameter.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{nf} B_0^2}{\rho_{nf}}(U_\infty - u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial y^2}\right) - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

With boundary conditions

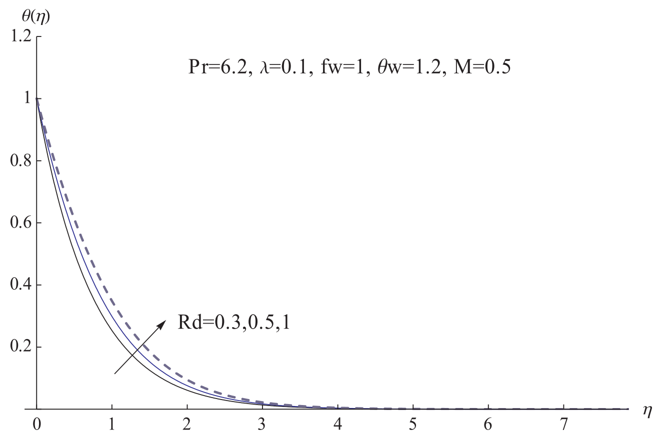

Fig. 7. Variation of temperature with increasing of radiation parameter.

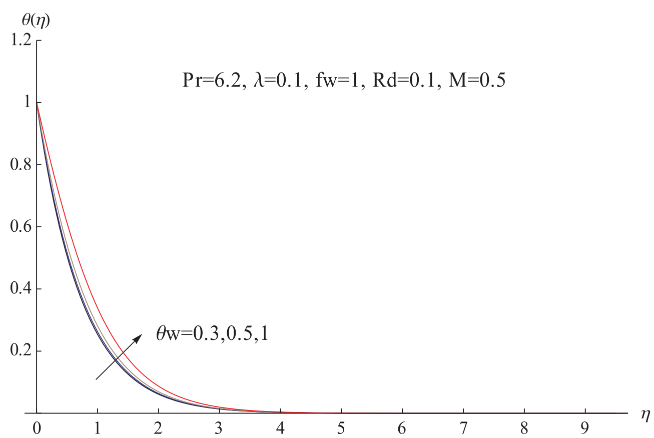

Fig. 8. Variation of temperature with increasing of temperature ratio.

Table 3
values of skin friction and Nusselt number for varies values of concentration.

ϕ	$f''(0)$	$\theta'(0)$	C_f	Nu
0.01	−1.69261	−1.66252	−1.75100	3.87627
0.05	−1.67473	−1.35241	−1.92068	3.50416
0.10	−1.65994	−1.14073	−2.17778	3.34709

Table 4
values of skin friction and Nusselt number for varies values of magnetic and suction parameters.

M	*fw*	$f''(0)$	$\theta'(0)$	C_f	Nu
0	−1.00	−0.60940	−0.59800	−0.79951	1.75463
	0.00	−0.96940	−0.94740	−1.27182	2.77983
	1.00	−1.52610	−1.49360	−2.00219	4.38246
1	−1.00	−0.91930	−0.52920	−1.20609	1.55276
	0.00	−1.29880	−0.88330	−1.70398	2.59175
	1.00	−1.82930	−1.45820	−2.39998	4.27859

Table 5
Values of skin friction and Nusselt number for varies values of temperature ratio and radiation parameters.

θw	$\theta'(0)$	Nu	Rd	$\theta'(0)$	Nu
1.00	−1.36420	3.10005	0.00	−1.36880	1.86630
1.20	−1.36050	3.99193	0.50	−1.36050	3.99193
1.50	−1.35230	5.99236	1.00	−1.35240	6.09238

$$y = 0: \ u = U_w(x), \quad v = v_w, \quad T = T_w(x) = T_\infty + Cx^n \tag{4}$$

$$y \to \infty: \ u \to U_\infty(x), \quad T \to T_\infty \tag{5}$$

where u and v are the velocity components along the x and y-axis, respectively and T is the fluid temperature.

By Rosseland's approximation the nonlinear radiation heat flux q_r is $q_r = -(4\sigma^*/3\alpha^*)(\partial T^4/\partial y)$ with $T^4 = T_\infty^4[\theta(\theta_w - 1) + 1]^4$ and $\theta_w = \frac{T_w}{T_\infty}$ the temperature ratio parameter.

Where σ^*, α^* the Stefan–Boltzmann constant and the coefficient of mean absorption, respectively

The physical properties of the nanofluids considered as the following forms [35]:

$$\begin{Bmatrix} \rho_{nf} = \rho_f(1-\phi) + \rho_s\phi \\ (\rho c_p)_{nf} = (\rho c_p)_f(1-\phi) + (\rho c_p)_s\phi \\ \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma_s/\sigma_f - 1)\phi}{(\sigma_s/\sigma_f + 21) - (\sigma_s\sigma_f - 1)\phi} \end{Bmatrix} \tag{6}$$

where σ_{nf} is the electrical conductivity of the nanofluid, the effective viscosity and effective thermal conductivity k_{eff} and composed of the particle's conventional static part and the Brownian motion part [36,37] are taken as:

$$\left.\begin{aligned} \mu_{eff} &= \mu_{static} + \mu_{Brownian} = \frac{\mu_f}{(1-\phi)^{2.5}} + \frac{k_{Brownian}}{k_f}\frac{\mu_f}{Pr_f} \\ k_{eff} &= k_{static} + k_{Brownian}, \ \frac{k_{static}}{k_f} = 1 + \frac{3\left(\frac{k_p}{k_f} - 1\right)\phi}{\left(\frac{k_p}{k_f} + 2\right) - \left(\frac{k_p}{k_f} - 1\right)\phi} \end{aligned}\right\} \tag{7}$$

Taking into account the effects of particle size, particle volume fraction, and temperature dependence as well as types of particle and base fluid combinations, where $k_{staticis}$ the static thermal conductivity based on the Maxwell classical correlation.

$$k_{Brownian} = 5 \times 10^4 \beta\phi\rho_f(C_p)_f\sqrt{\frac{K_bT}{\rho_p d_p}}f(T, \phi) \tag{8}$$

where β and f two empirical functions, for the thermo physical properties of water and CuO nanoparticles see Table 1.

Combined β and f functions in Eq. (8) to get a function g' depend on the types of fluid and nanoparticles takes the form [36]:

$$\begin{aligned} g'(T, \phi, d_p) = {} & (a_1 + a_2\ln(d_p) + a_3\ln(\phi) + a_4\ln(\phi)\ln(d_p) \\ & + a_5\ln(d_p)^2)\ln(T) + (a_6 + a_7\ln(d_p) + a_8\ln(\phi) \\ & + a_9\ln(\phi)\ln(d_p) + a_{10}\ln(d_p)^2) \end{aligned} \tag{9}$$

With a_i (i = 0–10) based on the type of nanoparticles [see Table 2].

Introducing the stream ψ function such that $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ and the transformation:

$$\eta = \sqrt{\frac{a}{v_f}}y, \quad \psi = x\sqrt{av_f}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

The momentum and energy Eqs. (2) and (3) can be transforming into the following ordinary differential equations:

$$f''' + ff'' - f'^2 + \lambda^2 + M\left(\frac{A_5}{A_1}\right)(\lambda - f') = 0 \tag{11}$$

$$\begin{aligned} & \left[1 + \frac{4Rd}{3A_3}(1 + (\theta_w - 1)\theta)^3\right]\theta'' + \frac{4Rd}{A_3}(\theta_w - 1)[1 + (\theta_w - 1)\theta]^2\theta'^2 \\ & + Pr\frac{A_4A_2}{A_3A_1}[f\theta' - nf'\theta] = 0 \end{aligned} \tag{12}$$

The boundary conditions (4) and (5) become:

$$f(0) = f_w, \quad f'(0) = 1, \quad \theta(0) = 1 \tag{13}$$

$$f'(\infty) \to \lambda, \ \theta(\infty) \to 0 \tag{14}$$

where $\lambda = b/a$ the velocity ratio parameter, $Pr = (\mu_f(\rho C_p)_f)/(\rho_f k_f)$ the Prandtl number, $M = \sigma_f B_0^2/(\rho_f a)$ the magnetic parameter, θ_w the temperature ratio parameter, f_w the suction/injection parameter, $R_e = \frac{a}{\nu_f}$ Reynolds number and $Rd = 4\sigma^* T_c^3/\alpha^* k_f$ the radiation parameter.

$$A_1 = \frac{\rho_{nf}}{\rho_f}, \quad A_2 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, \quad A_3 = \frac{k_{nf}}{k_f}, \quad A_4 = \frac{\mu_{nf}}{\mu_f}, \quad A_5 = \frac{\sigma_{nf}}{\sigma_f} \tag{15}$$

The coefficient of skin friction and the local Nusselt number can be expressed as:

$$C_f = A_4 \sqrt{R_e} f''(0) \tag{16}$$

$$Nu = -A_3 \sqrt{R_e} \left(1 + \frac{4Rd\theta_w^3}{3}\right) \theta'(0) \tag{17}$$

Results and discussion

The effects of magnetic field, and thermal radiation on the flow of a viscous, incompressible, and electrically conducting flow of CuO-water Nanofluid over a stretching surface is studied. The base fluid is water containing copper oxide (CuO) nanoparticle. To study the effect of stretching/shrinking parameter, thermal radiation, temperature ratio at the surface on the fluid velocity $f(\eta)$, fluid temperature $\theta(\eta)$. The profiles of these physical quantities are presented graphically in Figs. 2 to 8. We carry out computations for several non-dimensional parameters, namely magnetic parameter M, suction/injection parameter *fw*, radiation parameter Rd, temperature ratio parameter θw, and velocity ratio parameter λ respectively. Fig. 2 illustrates the effect of magnetic field parameter M. It is clear that increase in magnetic parameter M from hydrodynamic (M = 0) to hydromagnetic (M > 0) results in a strong reduction in dimensionless velocity $f'(\eta)$. The existence of magnetic field in an electrically conducting fluid is known to introduce a retarding body force acting transversely to the direction of the applied magnetic field. Further, it is also observed from Fig. 2 that the thickness of the momentum boundary layer is decreases by increasing the values of magnetic parameter M.

The ratio λ is the ratio between the stretching rates along the *y*–direction to the stretching rate along the *x*– direction. Figs. 3 and 4 show that, increasing the value of stretching rate ratio parameter reduces the velocity in *x*- direction but helps to enhance the velocity in the *y*-direction. Due to this reason, we have seen a decrease the temperature profiles of flow. To exhibit the effect of the suction/injection parameter *fw* on the dimensionless velocity $f'(\eta)$ we have plotted Fig. 5. From the graphs in Fig. 5, we see that the parameter *fw* has a profound influence on the boundary layer thickness. The flow is observed to be strongly decelerated with increasing *fw*. Suction/injection causes the boundary layer to adhere more closely to the wall and this destroys momentum leading to a fall in the velocity. Therefore, due to suction/injection, the momentum boundary layer thickness is decreased. The effects of suction/injection parameter *fw* on the dimensionless temperature are displayed in Fig. 6. The thermal boundary layer thickness is decreasing for increasing values of suction/injection parameter *fw*. Fig. 7 describes the effects of the thermal radiation parameter Rd on the temperature profile. Increase the values of Rd have the tendency to increase the conduction effect and to increase the thermal boundary layer. This part causes the temperature to increase at every point away from the surface. Fig. 8 shows the influence of temperature ratio parameter θw on the fluid velocity fluid temperature and species concentration. It is revealed from Fig. 8 that there are increments on fluid velocity, fluid temperature and species concentration with an increase in temperature ratio parameter θw. Therefore, one can conclude that the fluid velocity, fluid temperature and species concentration are being enhanced by temperature ratio parameter.

The numerical values of local skin friction factor and Nusselt number are exhibited in Tables 3, 4 and 5. In Table 3 the effect of nanoparticles concentration ϕ in the skin friction C_f and the Nusselt number Nu. It is evident from the table that increase in ϕ reduce the Nusselt number and decreases the coefficient of skin friction. The variation of $f''(0)$ and $\theta'(0)$ with respect to *fw* and *M* is given in Table 4. From Table 4, it can be seen that the local Nusselt number increase as magnetic and suction/injection parameters increase. Moreover, the results reveal that the skin friction coefficient decreases with an increase in *M* and *fw*. In Table 5 the effect of temperature ratio parameter θw and the radiation parameter Rd in the Nusselt number Nu. It is observed from the table that increase in θw and Rd enhance Nu.

Conclusions

The nonlinear thermal radiation effect on the steady two-dimensional forced convection stagnation point boundary layer flow of an incompressible, viscous, electrically conducting CuO-water nanofluid towards a linear stretching sheet are investigated. The effects of the magnetic parameter, radiation parameter, nanoparticle volume fraction, and velocity ratio parameter on the flow and heat transfer characteristics were determined and the results showed that.

- The fluid velocity decreases with increasing of suction/injection parameter and magnetic parameter but increasing with increases of velocity ratio.
- The fluid temperature decreases with increasing in the velocity ratio parameter and suction/injection parameter.
- The fluid temperature increases with increasing of radiation parameter and temperature ratio.

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